

Age discontinuity and nonemployment benefit policy evaluation through the lens of job search theory

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Abstract: A recent strand of papers use sharp regression discontinuity designs (RDD) based on age discontinuity to study the impacts of minimum income and unemployment insurance benefit extension policies. This design challenges job search theory, which predicts that such RDD estimates are biased. Owing to market frictions, people below the age threshold account for future eligibility to the policy. This progressively affects their search outcomes as they get closer to entitlement. Comparing them to eligible people leads to biased estimates because both groups of workers are actually treated. We provide a nonstationary job search model and illustrate the theoretical biases on the datasets used in the literature. Our results suggest that the employment impact of minimum income policies are (significantly) under-estimated, whereas the impacts of benefit extensions on nonemployment duration are (not significantly) over-estimated.

Keywords: RDD; Age discontinuity; Nonstationary job search theory

1 Introduction

Several papers use an age discontinuity to evaluate the impacts of nonemployment benefits on job search outcomes. Lemieux and Milligan (2008) on Canadian data and Bargain and Doorley (2011, 2017) on French data examine the minimum income impact on employment. Lalive (2008) on Austrian data, Caliendo et al. (2013) and Schmieder et al. (2012a,b, 2016) on German data study the effects of unemployment benefit extensions on unemployment duration and post-unemployment wages. In all these papers, the fate of people right after entitlement is compared to the fate of those right before it.

This design challenges job search theory. On the ground that this theory offers a comprehensive view of labor market phenomena, RDD estimates based on age discontinuity should be biased.

In this paper, we highlight the roles played by market frictions and workers' expectations. Owing to frictions, workers below the threshold age respond to future labor market conditions in a continuous way. Age-conditional policies affect their current behavior, implying that people non-eligible to the policy are actually treated. Neglecting such effects leads to (significantly) under-estimate the minimum income

impacts on Canadian and French data and (not significantly) over-estimate the benefit duration impacts in Austria and Germany (though this may still matter in the latter case).

We start by discussing the likely bias affecting RDD when using an age discontinuity to evaluate minimum income and benefit extension policies (Section 2). We first provide simplified arguments to understand the nature of the bias and guess the key factors behind its magnitude. We argue that search frictions imply that individuals are treated before entitlement and that the magnitude of pre-entitlement effect should be larger in case of minimum income policies than in case of benefit extension policies. Then we examine the Local Average Treatment Effect (LATE) and decompose the RDD estimate expectation into the true effect and the potential bias due to the combination of frictions and rational expectations. The bias should be negative in case of minimum income policies and positive in case of benefit extension policies.

We then provide a job search model tailored to study age-conditional minimum income and unemployment benefit potential duration (Section 3). This model features three key elements of job search theory: (i) frictions prevent workers from finding jobs instantaneously; (ii) search effort can increase the arrival rate of job offers; (iii) individuals rationally anticipate policy eligibility. The typical job search problem is nonstationary: as time passes, individuals get closer to entitlement. This continuously affects the current value of future streams of income and, therefore, the time pattern of search efforts.

We use our model to quantify the impact of age-conditional minimum income policies on employment (Section 4). We calibrate its structural parameters on the Canadian data used by Lemieux and Milligan (2008) and the French data used by Bargain and Doorley (2011, 2017). Well before entitlement, job-seekers are already impacted because they have a large chance to remain unemployed after the age threshold. Therefore they progressively reduce search effort. The comparison of individuals shortly before and after entitlement is misleading because they are similarly exposed to the policy. Our calibrations suggest that this effect quantitatively matters and theory predicts that the papers previously quoted underestimate the minimum income impact.

We also use our model to quantify the impact of age-conditional unemployment benefit extensions on employment (Section 5). We calibrate its structural parameters on the Austrian data used by Lalive (2008) and the German data used by Caliendo et al. (2013) and Schmieder et al. (2012a,b, 2016). Before the age threshold, job-seekers expect that they will have access to longer benefits in case they find a job and lose it afterwards. This provides early incentive to search. Therefore the comparison of individuals before and after the age threshold overestimates the benefit duration impact on unemployment duration. However, our calibrations reveal that this effect is quantitatively small, especially in the Austrian case. The pattern of transitions involved in the reasoning does not frequently occur, i.e., over a year, the probability of finding a job times the probability of losing it can be neglected.

Our paper applies search theory arguments to typical labor market issues where this theory is used. Therefore, it is not only food for thought for researchers specialized in public policy evaluation, but also a test of the theory with regard to its ability to reproduce actual labor market outcomes. Search theory provides a consistent framework to describe how fast people find jobs and how much they get paid. This framework is based on the notion of search frictions: people cannot instantaneously match and spend

time and resources to get matched. Alternatively, people may find jobs very rapidly and only those who cannot or do not want to find jobs stay nonemployed. Unobserved worker heterogeneity then gives the false impression that everyone needs time. In the Walrasian limit of our model, people wait the very last second to modify their behavior; this implies that the RDD is perfectly valid.

This paper owes a lot to the literature on nonstationary search. Several papers focus on the potential duration of benefits and show that the search effort increases and the reservation wage decreases as the exhaustion date gets closer (see, e.g., Mortensen, 1977; van den Berg, 1990; Cockx et al., 2018). Eligibility effects are important and their consideration improves our understanding of job-finding rates by unemployment duration. In this paper, we model the benefit duration as a Poisson process instead of a deterministic duration. We do so because there are additional eligibility criteria to UI benefits so that the potential UI duration varies more across unemployed workers than expected. Having a random potential UI duration is a shortcut to account for such effects. The model can easily be modified to account for a fixed potential duration.

Closer to us, there is a growing literature on life-cycle effects in job search models. Chéron et al. (2013); Hairault et al. (2010, 2012, 2015) focus on the retirement age and its impact on the pattern of training and search investments decisions. Menzio et al. (2016) analyze job-to-job transitions and the wage distribution during the life-cycle. The main lesson is that the retirement age affects workers well before retirement. This explains why older workers search less when unemployed and receive less training when employed. This also explains wage and employment patterns by age. We use insights from these models to quantify the theoretical bias of RDD based on an age discontinuity.

Schmieder et al. (2012a, 2016) provide nonstationary job search models to explain their results. Nonstationarity is not based on age eligibility to benefits. Rather, it involves the standard effects of the potential duration of benefits and the duration spent in unemployment. As time passes, the wage offer distribution worsens. Job-seekers reduce their search effort and receive lower wages. We do not examine wages – here again, the interested reader can modify our model to allow for duration-dependence in the wage offer distribution. Instead, we complete their analysis by emphasizing the role of the age threshold. This allows us to show that their RDD is theoretically valid despite workers are actually treated before entitlement.

There is already a literature on the evaluation of pre-treatment effects in the context of policy discontinuity with respect to time (be it calendar time, age or unemployment duration). These papers focus on training policies and their impacts on employment or hazard rates to employment (see, e.g., Crépon et al., 2009, 2018; van den Berg et al., 2018). This literature is dominated by the ‘No anticipation assumption’ whereby individuals do not modify their behavior in expectation of the treatment date (see Abbring and van den Berg, 2003). Though excluding the nonstationarity featured in this paper, this assumption is actually compatible with ex-ante policy effects, individuals acting in expectation of being treated at some future unknown date. The closest papers in spirit evaluate anticipated impacts through the individual disclosure of information on the treatment date (see, e.g., Black et al., 2003; Giorgi, 2005; Crépon et al., 2018; van den Berg et al., 2018, for participation to job search assistance or training programmes, and Lalive et al., 2005, for UI benefit sanctions). This consists in comparing hazard rates for those who are notified some months or even weeks in advance to those who are not. These papers find significant effects

of notification. Our paper complements this literature in three ways. First, we analyze a different economic issue, namely age-conditional nonemployment benefits instead of active labor market programmes. Second, we provide a nonstationary job search model that explicitly predicts the magnitude and direction of the RDD bias as a function of worker age. Third, we calibrate this model to illustrate the theoretical biases in several studies.

2 Main intuitions

We first provide a detailed intuition on the reason why matching frictions imply that workers below the age discontinuity are affected by age-conditional benefits. We then discuss the potential bias that affects RDD estimates.

We are interested in the two types of labor market policies discussed in Introduction: age-conditional minimum income and age-conditional UI extensions. Both policies are likely to affect job search behavior. Only the people above the threshold age A are eligible to such policies. However, people below A are also treated when

1. individuals can significantly impact the outcome variable;
2. individuals anticipate that they will be treated later;
3. tomorrow "matters" for today's decision.

The first condition justifies that there is a policy evaluation. The second condition requires that individuals are forward-looking and sufficiently well-informed on the policy. The last condition depends on the magnitude of search frictions, which we now explain.

2.1 Heuristic argument

We explain why search frictions and expectations imply that workers are treated by age-conditional benefits before being entitled to them. We also discuss the sign of this pre-treatment impact on workers' welfare as well as its likely magnitude according to worker age.

Age is continuous and indexed by $a \geq 0$. Unemployed worker find jobs at rate m and lose them at rate q . The wage is w per unit of time. We consider two different types of age-conditional benefits. In both cases, unemployment income is b_0 before age A . In the first case, all unemployed workers receive $b_1 \geq b_0$ after age A . In the second case, only those who lost their job after age A receive b_1 , whereas the others receive b_0 . This case mimics benefit extension policies where the benefit level does not change but the potential benefit duration increases after age A .

We start with the first case where all unemployed receive b_1 after age A . We denote by $S(a)$ the match surplus, i.e., the asset value differential between employment and unemployment. This quantity

is important because it indicates how much the job-seekers are willing to find jobs. For all $a \leq A$,

$$rS(a) = w - b_0 - (m + q)S(a) + S'(a), \quad (1)$$

$$rS(A) = w - b_1 - (m + q)S(A). \quad (2)$$

These arbitrage equations receive the following interpretation. The left-hand side is the financial return to match surplus. The right-hand side is its labor market return. It is composed of the income differential between employment and unemployment, $w - b_0$, plus the loss of surplus induced by the transition rate m (while unemployed) and q (while employed), plus appreciation due to age as the worker becomes closer to entitlement to b_1 . This third term is zero after age A .

The pre-entitlement match surplus solving this problem is

$$S(a) = S_0(1 - F(a)) + S_1F(a), \quad (3)$$

where $S_i = (w - b_i)/(r + m + q)$ is the stationary match surplus associated to benefit level b_i , $i = 0, 1$, and $F(a) = \exp[-(r + m + q)(A - a)]$ is the effective discount factor.

The match surplus at age a is a weighted mean of the two stationary match surpluses. The effective discount factor increases with age and decreases with the the sum of interest rate, r , and transitions rates, m and q . It is equal to one when $a = A$. Therefore the match surplus decreases with age from S_0 to S_1 .

Let $\varepsilon(a) = d \ln S(a) / d \ln b_1$ be the elasticity of match surplus with respect to age-conditional benefit b_1 . When $b_0 = b_1$, we have

$$\varepsilon(a) = \varepsilon(A)F(a) < 0. \quad (4)$$

The pre-entitlement match surplus negatively responds to an increase in post-entitlement benefits. The magnitude of the pre-entitlement elasticity, $\varepsilon(a)$, is proportional to the post-treatment one, $\varepsilon(A)$. The factor of proportion is lower than one and increases with age to reach one at age A where the two elasticities coincide. It also decreases with the job-finding rate, reaching 0 in the Walrasian case where the job-finding rate tends to infinity.

Next, we consider the second case where age-conditional benefit b_1 is restricted to those who lost their job after age A . For all $a \leq A$,

$$rS(a) = w - b_0 - (m + q)S(a) + S'(a), \quad (5)$$

$$rS(A) = w - b_0 - q\Sigma(A) - mS(A), \quad (6)$$

$$r\Sigma(A) = w - b_1 - (m + q)\Sigma(A). \quad (7)$$

Workers after age A are not automatically entitled to higher benefits. They need to experience an employment episode. This is why the computation of stationary match surplus for nonentitled workers, $S(A)$, involves the stationary match surplus for those entitled, $\Sigma(A)$. The transition towards entitlement occurs at rate m . Once entitled, workers will never be nonentitled again.

The pre-entitlement match surplus solving this problem is

$$S(a) = S_1 + \frac{b_1 - b_0}{r + m + q} \left[1 + \frac{q}{r + m} F(a) \right]. \quad (8)$$

and tends to $S(A) = S_1 + (b_1 - b_0)/(r + m) \geq S_1$, inequality being strict when $b_1 > b_0$. These workers are still nonentitled to b_1 after age A because they have not experienced a job episode lasting after this age.

Let $e(a) = d \ln S(a) / d \ln b_1$. For $b_0 = b_1$, we have

$$e(a) = -\varepsilon(a) \frac{q}{r + m} > 0. \quad (9)$$

Unlike the previous elasticity $\varepsilon(a)$, the elasticity $e(a)$ is positive. An increase in age-conditional benefits paid to job losers increases the value of matching for those nonentitled. This type of phenomenon is well-known in the search literature as the entitlement effect (see, e.g., Mortensen, 1973). The quantitative impact is proportional to the elasticity $\varepsilon(a)$. This means that $|e(a)|$ increases with age and decreases with the job-finding rate. Theoretically, the factor of proportion $q/(r + m)$ can be lower or greater than one. In practice, $q \ll m$ so that $|e(a)| < |\varepsilon(a)|$. Therefore, the pre-entitlement impact of benefits restricted to job losers after age A is likely much smaller than the impact of benefits unconditionally paid to all unemployed after this age.

We summarize the main lessons taught by this simple model.

- (i) Age-conditional benefits affect the pre-entitlement match surplus. It follows that workers are actually treated before entitlement age.
- (ii) The magnitude of pre-entitlement impact increases with worker age and search frictions. In the Walrasian case where m is arbitrarily large, the impact is zero.
- (iii) Age-conditional benefits paid to all unemployed after age A reduce the pre-entitlement match surplus.
- (iv) Age-conditional benefits restricted to job losers after age A increase the pre-entitlement match surplus.
- (v) Given realistic values for m and q , benefits paid to all unemployed after age A have a larger impact on pre-entitlement match surplus than those restricted to job losers after this age.

2.2 RDD bias

We now turn to a description of the RDD bias. This description involves the fact that search efforts increase with expected match surplus. This is a very intuitive idea that we will formally establish later.

Age-conditional minimum income and UI extensions are binary treatments that impact job search outcomes like the employment probability, unemployment duration or job-finding rate. To fit with the

previous discussion, we discuss the case of the hazard rate from unemployment to employment. Individuals are indexed by i and their age is a_i . We neglect other forms of observable heterogeneity to simplify exposition.

Let D_i denote treatment exposure with $D_i = 1$ if treated and $D_i = 0$ otherwise. We have $D_i = 1$ when $a_i \geq A$. The potential outcomes are Y_i^0 when untreated and Y_i^1 when treated. Econometricians do not simultaneously observe Y_i^1 and Y_i^0 . Therefore they estimate a local average treatment effect (LATE) $\mathbb{E}[Y_i^1 - Y_i^0 | a_i = A]$ in a regression discontinuity design (RDD).

The key problem comes from those who are too young and supposedly non-treated but who already respond to the treatment in expectation of future entitlement. Thus we need three different potential outcome variables: Y_i^1 for those with age $a_i \geq A$; Y_i^0 the ideal outcome for those who do not expect treatment exposure; $\tilde{Y}_i^0$ for those not currently exposed, $a_i \leq A$, but expecting future entitlement.

The assignment variable is $D_i = \mathbf{1}(a_i \geq A)$. The observed outcome is

$$Y_i = D_i Y_i^1 + (1 - D_i) \tilde{Y}_i^0 = \begin{cases} Y_i^1 & \text{if } a_i \geq A \\ \tilde{Y}_i^0 & \text{if } a_i < A \end{cases}, \quad (10)$$

which departs from the ideal outcome $D_i Y_i^1 + (1 - D_i) Y_i^0$. Econometricians can observe Y_i^1 after entitlement. However, they cannot observe Y_i^0 before it. Instead, they observe $\tilde{Y}_i^0$.

The two necessary conditions to estimate a LATE are (i) the treatment is "as good as random", i.e., $Y_i^0, Y_i^1 \perp\!\!\!\perp D_i$, and (ii) conditional expectations $\mathbb{E}[Y_i^0 | a_i = a]$ and $\mathbb{E}[Y_i^1 | a_i = a]$ are continuous in a . Sticking to these assumptions, we can estimate the standard econometric model

$$Y_i = \beta_0 + \beta_1 \mathbf{1}(a_i \geq A) + \delta(a_i) + \epsilon_i, \quad (11)$$

where δ is a continuous function.¹

The RDD estimator, $\hat{\beta}_1$, is a consistent estimator of $\mathbb{E}[Y_i^1 - \tilde{Y}_i^0 | a_i = A]$. This expectation, that we call the RDD estimand, may differ from the LATE, which can be decomposed as follows:

$$\mathbb{E}[Y_i^1 - Y_i^0 | a_i = A] = \underbrace{\mathbb{E}[Y_i^1 - \tilde{Y}_i^0 | a_i = A]}_{\text{RDD estimand}} + \underbrace{\mathbb{E}[\tilde{Y}_i^0 - Y_i^0 | a_i = A]}_{\text{Bias}} \quad (12)$$

The RDD estimand coincides with the LATE when $\tilde{Y}_i^0$ is very close to Y_i^0 around A . Otherwise, the RDD estimand features a positive bias with respect to the LATE when the behavioral response of individuals below A is qualitatively similar to the response of those above A . The bias is negative when the two responses are negatively correlated.

Figures 1 and 2 illustrate the bias in the two different situations of interest. Figure 1 focuses on age-conditional minimum income. This corresponds to benefits paid to all unemployed after age A in the previous sub-section. Therefore the pre-entitlement expected match surplus decreases with age and converges to the post-entitlement expected match surplus. As individuals make search effort on the basis

¹We can of course add controls X_i to estimate the expectation conditional on X_i .

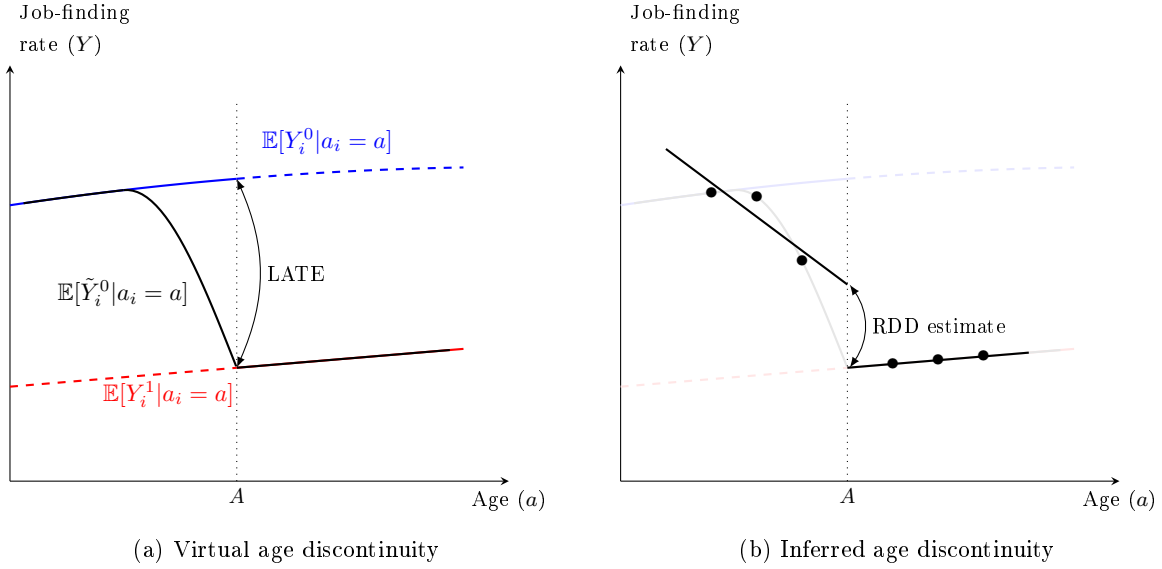


Figure 1: RDD bias in case of age-conditional minimum income

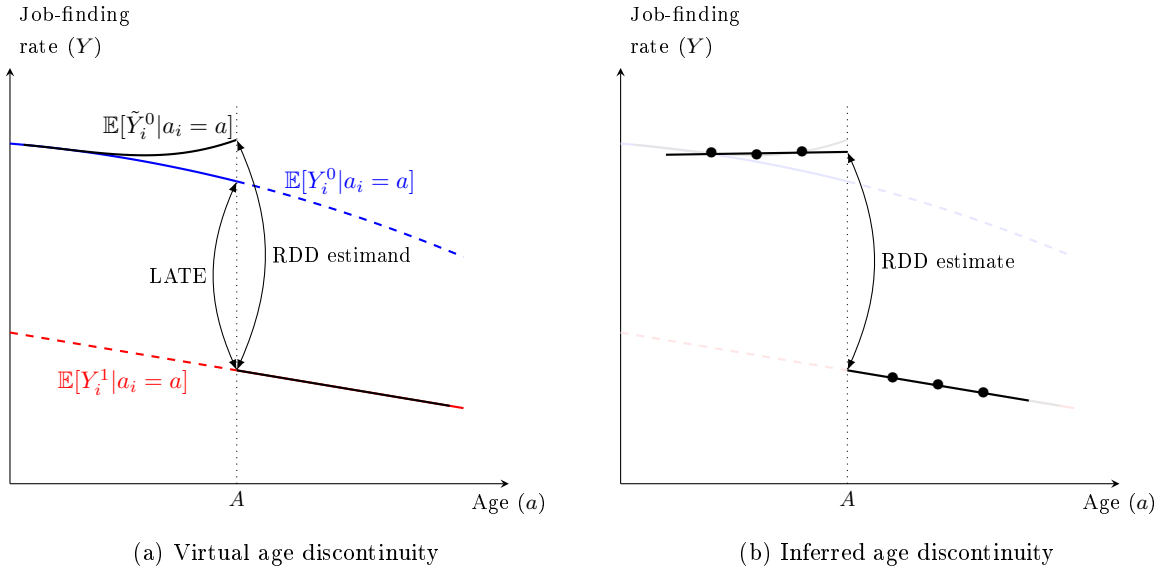


Figure 2: RDD bias in case of age-conditional UI benefit extension

Note. The left-hand side graphs depict the job-finding rate as a function of age in the three possible cases: treated, Y_i^1 (in red), non-treated, Y_i^0 (in blue), treated in expectation, $\tilde{Y}_i^0$ (in black). The right-hand side graphs illustrate the estimation with actual observations. The narrative of the RDD estimation says we *can* infer $\mathbb{E}[Y_i^0|a_i = a]$ for $a_i < a$ and $\mathbb{E}[Y_i^1|a_i = a]$ for $a_i > a$. What we *do* infer, however, is $\mathbb{E}[\tilde{Y}_i^0|a_i = a]$ for $a_i < a$.

of expected match surplus, search efforts gradually decrease with age well before entitlement. Slightly before the threshold age A , individuals are almost fully treated and behave as their older peers. Continuity implies that the RDD estimand is 0 in A , i.e. $\mathbb{E}[Y_i^1 - Y_i^0 | a_i = A] = 0$.

In practice, econometricians do not observe the continuous curve depicted by the left-hand side figure. Instead, individuals are observed at discrete time intervals as shown in the right-hand side figure. The linear regression provides the RDD estimate through extrapolation, choosing the degree of sophistication with the function δ . Here we show a linear spline model.² As evidenced by the right-hand side figure, the RDD underestimates the policy impact on the job-finding rate. Moreover the extrapolation procedure strengthens the bias.

Figure 2 focuses on age-conditional UI benefit extensions. This corresponds to benefits paid to those who lost their job after age A in the previous sub-section. To be eligible, nonemployed individuals need first to find a job and lose it after age A . As explained in the previous subsection, the pre-entitlement expected match surplus increases with age to reach a higher stationary level than for those who are entitled, i.e., those who remain unemployed after age A still need to find a job to become eligible to the policy. This explains why the pre-entitlement hazard rate increases with age and why the comparison of those eligible right after age A to those right before this age involves a discontinuity in the hazard rate.

Here again, the RDD estimate significantly differs from the LATE. The right-hand side figure shows that the RDD overestimates the policy impact.

The bias can be reduced by expanding the observation window around the discontinuity. If workers change their behavior one year before eligibility, we can simply drop all individuals who will be eligible in less than a year. This strategy of voluntarily losing observations in a window $[A - \varepsilon, A]$ before the threshold has two limits. First, at individual level, the age pattern right before $A - \varepsilon$ becomes more difficult to extract when ε increases. Second, population heterogeneity between individuals at age $A - \varepsilon$ and at age just above A also increases with ε . The design no longer responds to the criteria defining a randomized experiment, which is usually a strength of RDD (Lee and Lemieux, 2010).

Expected effects of future treatment are already known in the literature on policy evaluation. However, identification involves the 'No expectation' assumption whereby workers do not modify their behavior in expectation of the treatment date (see Abbring and van den Berg, 2003). This assumption forbids the type of patterns reported by Figures 1 and 2. Within this framework, Crépon et al. (2018) and van den Berg et al. (2018) exploit heterogeneous notification dates on future treatment to measure expected effects. This consists in comparing those notified at a given date with those who are not at the same date.

3 Job search with age entitlement to nonemployment benefits

We provide a continuous-time model with age-conditional benefits and search efforts. This model is as simple as possible: workers are homogenous and risk-neutral, and there is no on-the-job search.

²The linear spline specification consists in estimating $\delta(a)$ as a linear function of a with a different slope below and above A .

The time unit is the month. Benefits respond to age in the two ways previously discussed by the literature. At age A , assistance benefits suddenly increase from $b_0^{\min}$ to $b_0^{\max}$. This corresponds to the Canadian case examined by Lemieux and Milligan (2008) and the French one analyzed by Bargain and Doorley (2011, 2017). Moreover, workers dismissed after age A can receive UI benefits for a longer period. We capture this as a smaller rate of losing benefit entitlement: the Poisson rate goes from $\lambda_{\max}$ before age A to $\lambda_{\min}$ after this age. This corresponds to the Austrian case studied by Lalive (2008) and the German case highlighted by Schmieder et al. (2012a,b, 2016). The reason why we treat the loss of benefit entitlement as a random event is because there are additional criteria to get entitled to UI. Therefore the max duration, in practice, varies across individuals of the same age and same date of job loss.

We model the job search behavior of a typical nonemployed individual. Before age A , the model is nonstationary because the worker's situation changes as the threshold age gets closer. Let $S(a, w, b)$ be the match surplus for a worker of age $a \leq A$, when the wage is w and the worker is entitled to benefit b . In practice the benefit can take three values: b_{UI} , the regular UI benefit, and $b_0^{\min}$ and $b_0^{\max}$, the assistance benefits when workers are, respectively, below and above age A . After age A , the model is stationary as the expected utility of a worker does not directly depend on age. However, unemployed workers who lose their job before age A experience the higher rate of losing benefits $\lambda_{\max}$, whereas the others are submitted to the lower risk $\lambda_{\min}$. We denote by $\Sigma(w, b, \lambda)$ the match surplus for a worker of age $a > A$ when the wage is w , the worker is entitled to nonemployment benefit b and the risk of losing UI benefit is λ .

Workers make search efforts. The search effort s comes at cost $c(s)$, with $c'(\cdot) > 0$, $c''(\cdot) > 0$ and $c(0) = c'(0) = 0$. In exchange for the cost, workers receive wage offers at rate ms . Each wage offer is a random draw from the cumulative distribution function Φ . The corresponding probability density function is φ . In the remaining, we suppose that the lower bound of the support of the distribution – the minimum wage – is higher than the reservation wage. Employed workers lose their job at rate q .

We have

$$rS(a, w, b) = \min_{s \geq 0} \{w - qS(a, w, b_{UI}) - b + c(s) - ms\bar{S}(a, b) + \lambda_{\max}(S(a, w, b_0^{\min}) - S(a, w, b)) + S_a(a, w, b)\}, \quad (13)$$

$$r\Sigma(w, b, \lambda) = \min_{s \geq 0} \{w - q\Sigma(w, b_{UI}, \lambda_{\min}) - b + c(s) - ms\bar{\Sigma}(b, \lambda) + \lambda(\Sigma(w, b_0^{\max}, \lambda) - \Sigma(w, b, \lambda))\}, \quad (14)$$

$$S(A, w, b) = \max\{\Sigma(w, b, \lambda_{\max}), \Sigma(w, b_0^{\max}, \lambda_{\max})\}, \quad (15)$$

where $\bar{S}(a, b) = \int_{w_{\min}}^{\infty} S(a, w, b)\varphi(w)dw$ and $\bar{\Sigma}(b, \lambda) = \int_{w_{\min}}^{\infty} \Sigma(w, b, \lambda)\varphi(w)dw$.

The first two equations describe non-arbitrage equations before and after age A . We explain the first one, the second one receiving a similar interpretation. The left-hand side is the financial return on match surplus, whereas the right-hand side is its market return. Job seekers choose their search investment s . This comes at cost $c(s)$ and gives access to the average match surplus $\bar{S}(a, b)$ with flow probability ms . The match surplus increases with the wage w and decreases with nonemployment income b . When hit by job destruction, workers join the pool of unemployed and receive UI benefits. This is why the right-hand side is decreased by $qS(a, w, b_{UI})$. Workers receiving UI benefits may also lose UI benefit entitlement. Therefore the right-hand side is also decreased by $\lambda_{\max}(S(a, w, b_0^{\min}) - S(a, w, b))$. Lastly,

the term $S_a(a, w, b)$ accounts for appreciation with age because assistance benefits increase after A and the rate of losing UI entitlement goes down.

The purpose of unemployed workers is to minimize the match surplus, i.e., make their welfare as high as possible while unemployed. The first-order conditions to the optimization problem give

$$c'(s(a, b)) = m\bar{S}(a, b), \quad (16)$$

$$c'(\sigma(b, \lambda)) = m\bar{\Sigma}(b, \lambda), \quad (17)$$

where the functions s and σ stand for the optimal search effort before and after age A . In both cases, the marginal cost of search is equal to its marginal gain, i.e., the contact rate by unit of search, m , times the average match surplus corresponding to their situation.

In the parameterized version of the model used for calibration, we suppose that the cost of effort is isoelastic with $c(s) = s^{1+\alpha}$, $\alpha > 0$. Let also $B_1 = \alpha(m/(1+\alpha))^{\frac{1+\alpha}{\alpha}}$. Then stationary average match surpluses solve

$$r\bar{\Sigma}(b, \lambda) = \bar{w} - b - q\bar{\Sigma}(b_{UI}, \lambda_{\min}) - B_1\bar{\Sigma}(b, \lambda)^{\frac{1+\alpha}{\alpha}} + \lambda(\bar{\Sigma}(b_0^{\max}, \lambda) - \bar{\Sigma}(b, \lambda)), \quad (18)$$

for $b = b_{UI}, b_0^{\max}$ and $\lambda = \lambda_{\min}, \lambda_{\max}$.

The worker starts with $S(0, b_0^{\min})$. Then, the dynamics of the nonstationary match surplus is, for all $a < A$ and $b = b_{UI}, b_0^{\min}$,

$$r\bar{S}(a, b) = \bar{w} - b - q\bar{S}(a, b_{UI}) - B_1\bar{S}(a, b)^{\frac{1+\alpha}{\alpha}} + \lambda_{\max}(\bar{S}(a, b_0^{\min}) - \bar{S}(a, b)) + \bar{S}_a(a, b), \quad (19)$$

with the terminal conditions $\bar{S}(A, b) = \max\{\bar{\Sigma}(b, \lambda_{\max}), \bar{\Sigma}(b_0^{\max}, \lambda_{\max})\}$.

The search efforts are $s(a, b) = B_2\bar{S}(a, b)^{1/\alpha}$ and $\sigma(b, \lambda) = B_2\bar{\Sigma}(b, \lambda)^{1/\alpha}$, where $B_2 = (m/(1+\alpha))^{1/\alpha}$.

The model is stationary after age A . Job seekers differ in two dimensions: whether they are entitled to UI benefits or not, and, for the entitled, their risk of losing entitlement. That $b_0^{\max} < b_{UI}$ implies $\bar{\Sigma}(b_0^{\max}, \lambda) > \bar{\Sigma}(b_{UI}, \lambda)$ for $\lambda = \lambda_{\min}, \lambda_{\max}$. Therefore $\sigma(b_0^{\max}, \lambda) > \sigma(b_{UI}, \lambda)$ and the nonentitled find jobs more rapidly than the entitled at given rate of losing UI benefits. That $\lambda_{\min} < \lambda_{\max}$ implies that $\bar{\Sigma}(b, \lambda_{\max}) > \bar{\Sigma}(b, \lambda_{\min})$ for $b = b_0^{\max}, b_{UI}$. Therefore $\sigma(b, \lambda_{\max}) > \sigma(b, \lambda_{\min})$ and those who lose their jobs after age A find jobs less rapidly than those who lose their jobs before age A .

The dynamics of the different populations of unemployed is characterized in Appendix A. This is necessary because the typical outcome used to evaluate age-conditional minimum income policies is the employment rate - and not the job-finding rate as discussed in the previous section.

We now turn to the two cases of interest, i.e., age entitlement to assistance benefits and age entitlement to extended UI benefits.

4 Age entitlement to assistance benefits

This section focuses on age entitlement to assistance benefits. Our purpose is to illustrate the theoretical bias of the RDD used by Lemieux and Milligan (2008) on Canadian data and Bargain and Doorley (2011, 2017) on French data. We first discuss how different groups of workers should react to an age-conditional increase in assistance benefits. Then we calibrate our model on Canadian and French data.

4.1 Model

We suppose $b_0^{\min} < b_0^{\max} < b_{UI}$ and $\lambda_{\min} = \lambda_{\max} = \lambda$. An increase in $b_0^{\max}$ affects all workers: not only those above the age threshold actually receiving $b_0^{\max}$ but also those below this age who modify their job search behavior in expectation of future entitlement.

Consider first workers above age A . We simply denote $\bar{\Sigma}(b) \equiv \bar{\Sigma}(b, \lambda)$. We solve

$$r\bar{\Sigma}(b_{UI}) = \bar{w} - b_{UI} - q\bar{\Sigma}(b_{UI}) - B_1\bar{\Sigma}(b_{UI})^{\frac{1+\alpha}{\alpha}} + \lambda(\bar{\Sigma}(b_0^{\max}) - \bar{\Sigma}(b_{UI})), \quad (20)$$

$$r\bar{\Sigma}(b_0^{\max}) = \bar{w} - b_0^{\max} - q\bar{\Sigma}(b_0^{\max}) - B_1\bar{\Sigma}(b_0^{\max})^{\frac{1+\alpha}{\alpha}}. \quad (21)$$

We first find $\bar{\Sigma}(b_0^{\max})$ and replace it in (20) to find $\bar{\Sigma}(b_{UI})$.

Before age A ,

$$r\bar{S}(a, b_{UI}) = \bar{w} - b_{UI} - q\bar{S}(a, b_{UI}) - B_1\bar{S}(a, b_{UI})^{\frac{1+\alpha}{\alpha}} + \lambda(\bar{S}(a, b_0^{\min}) - \bar{S}(a, b_{UI})) + \bar{S}_a(a, b_{UI}), \quad (22)$$

$$r\bar{S}(a, b_0^{\min}) = \bar{w} - b_0^{\min} - q\bar{S}(a, b_0^{\min}) - B_1\bar{S}(a, b_0^{\min})^{\frac{1+\alpha}{\alpha}} + \bar{S}_a(a, b_0^{\min}). \quad (23)$$

with the terminal conditions $\bar{S}(A, b_{UI}) = \bar{\Sigma}(b_{UI})$ and $\bar{S}(A, b_0^{\min}) = \bar{\Sigma}(b_0^{\max})$. Here also, we first integrate (23) to find the function $\bar{S}(a, b_0^{\min})$ and then replace it in (22) to find the function $\bar{S}(a, b_{UI})$.

Suppose there is an increase in $b_0^{\max}$. For workers above age A nonentitled to UI benefits, the match surplus $\bar{\Sigma}(b_0^{\max})$ decreases as they receive higher nonemployment income. Therefore these workers reduce search investment $\sigma(b_0^{\max}) \equiv \sigma(b_0^{\max}, \lambda)$ and the job-finding rate decreases. For workers entitled to UI benefits, the decline in match surplus is lower because they first need to lose entitlement before getting the new level of assistance benefits. The qualitative impact on the search effort $\sigma(b_{UI}) \equiv \sigma(b_{UI}, \lambda)$ is the same as for the nonentitled and the job-finding rate also decreases. However the quantitative impact is smaller.

For workers below age A , the terminal conditions $\bar{S}(A, b_{UI}) = \bar{\Sigma}(b_{UI})$ and $\bar{S}(A, b_0^{\min}) = \bar{\Sigma}(b_0^{\max})$ imply that the pre-entitlement match surpluses decrease at all ages, just like in Section 2. The value of nonemployment situations in the future rises and this is instantaneously reflected by lower match surpluses, i.e., both $\bar{S}(A, b_{UI})$ and $\bar{S}(A, b_0^{\max})$ decrease. Therefore these workers search less intensively in expectation of future entitlement. This effect grows as worker get closer to entitlement.

Our model predicts the behavioral response of workers before entitlement. The magnitude of this response governs the theoretical bias discussed in this paper. We now quantify it on French and Quebecois

data.

4.2 Quebecois data

From 1969 to 1989, social assistance benefits in Quebec were conditional on claimants' age with a discontinuity at 30. Lemieux and Milligan (2008) use the age discontinuity to quantify the impact of assistance benefits on the employment rate.

We parameterize our model to quantify the theoretical bias associated with this design. We use information available in Lemieux and Milligan (2008) to fix income parameters. We then set the elasticity of the marginal search cost α , the job-finding rate per unit of search m , the job separation rate q to match the age pattern of the employment rate around 30. The rate of losing UI benefits λ is deduced from the expected potential UI duration. We then reproduce the RDD estimations of Lemieux and Milligan (2008) on the simulated employment rate.

We follow Lemieux and Milligan (2008) and focus on drop-out men without dependent children. The annual interest rate is 5%. Lemieux and Milligan (2008) provide income figures for 1986. The mean yearly wage was \$13,924 and nonemployed workers received \$185 per month of assistance benefits below 30 and \$507 after 30. By normalizing the mean wage to one, we obtain $b_0^{min} = 0.159$ and $b_0^{max} = 0.437$. We get details on the UI scheme in 1986 from Lin (1998). The replacement rate was 60% and, therefore, we set $b_{UI} = 0.6$.

The potential duration of unemployment was limited to 50 weeks and had two components: a work component based on weeks worked and a regional component increasing in unemployment rate. We set λ such that the model-based expectation of the potential UI duration, $1/\lambda$, equalizes the expectation of the potential duration based on the 1986 rules linking the max benefit duration to the previous employment duration. We only consider the last employment spell. If this spell is t , then the individual can receive UI benefits for at most $f(t)$, where f is defined from Lin (1998). Parameter λ solves

$$\frac{1}{\lambda} = \int_0^\infty f(t) \cdot t e^{-qt} dt.$$

We simulate the pattern of unemployment by age to broadly match the empirical pattern. We start at age 25 and suppose that 68% of these workers are employed as reported by Lemieux and Milligan (2008). None of the unemployed is initially entitled to UI benefits. In practice we choose the parameter combination that minimizes the sum of squared deviations between each target and its simulated counterpart.

Intuitively, parameters α and m are identified by the employment rate and its change after entitlement. The job separation rate, q , is identified by the speed of employment rate adjustment. The rate of losing UI benefits, λ , is identified by the relationship linking $1/\lambda$ to the theoretical distribution of employment spells and its induced impact on the mean potential UI benefit duration.

Table 1 displays the calibrated parameters, whereas Table 2 shows the moments that this calibration

matches.

Table 1: Parameters in the Quebecois case

A	$\bar{w}$	$b_0^{\min}$	$b_0^{\max}$	b_{UI}	r	α	m	q	λ
30	1	0.159	0.437	0.6	0.0048	0.54	0.149	0.0504	0.11

Table 2: Moments to fit the values of α , m and q , Quebecois case

Moments	Target	Model
Employment rate between 27 and 28	66.4%	67.0%
Employment rate between 28 and 29	66.2%	66.7%
Employment rate between 29 and 30	66.0%	64.8%
Employment rate between 30 and 31	61.1%	60.7%
Employment rate between 31 and 32	59.8%	59.1%
Employment rate between 31 and 32	57.6%	58.8%

Notes. Targeted moments are borrowed from Lemieux and Milligan (2008).

Table 3: Regression discontinuity estimates, Quebecois case

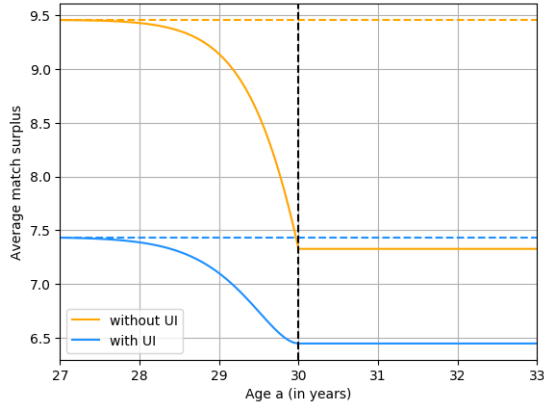
Polynomial specification for age	Estimated effect on the employment rate	
	actual data	simulated data
Linear	-0.041	-0.064
Quadratic	-0.051	-0.047
Cubic	-0.048	-0.047
Linear spline	-0.049	-0.048
Quadratic spline	-0.056	-0.037
LATE	X	-0.082

Note. The dependent variable is the employment rate. The empirical estimates are taken from Table 1, second column, in Lemieux and Milligan (2008). The simulated data are the dots depicted on Figure 3d. The LATE is the difference between the two counterfactual employment rates at the age threshold.

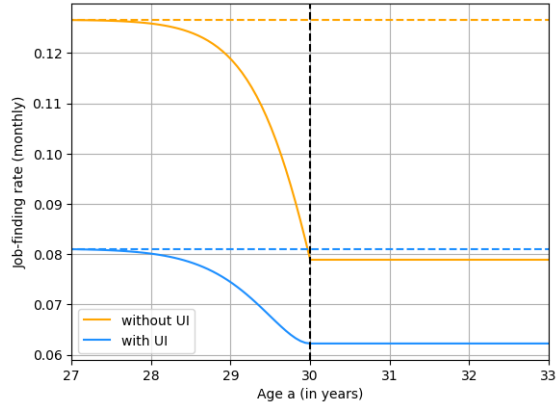
The elasticity α is 0.54, which implies that the elasticity of the search effort to the match surplus is about two. The job separation rate, q , is 0.05, which corresponds to a mean employment duration of 20 months. This is remarkably short but not so surprising for the population of interest. However, given this rate is relatively large, in the Appendix we calibrate the model with alternative values for q . Lastly, the rate of losing UI benefits, λ , is 0.11.

Figures 3a to 3c depict the outcomes of the calibrated model. Each figure displays four curves. The continuous lines correspond to nonemployed workers under UI (blue curve) and social assistance (orange curve) when assistance benefits are conditional on age. The dashed lines correspond to alternative scenarios where assistance benefits do not change with age.

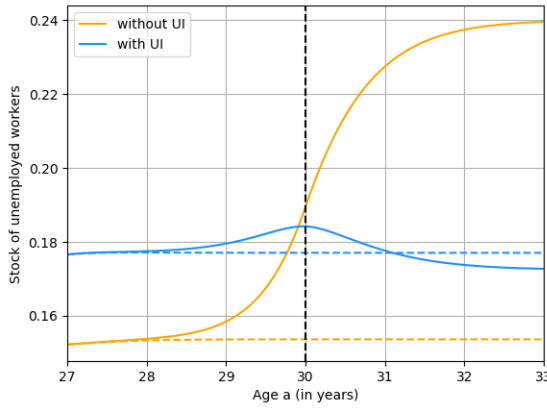
Figures 3a and 3b reveal that workers respond to the policy change well before 30. In Figure 3a, match surpluses start to decrease at 28. Then they drop by 15-20% depending on the group of nonemployed workers. Workers gradually decrease search effort as a result. In Figure 3b, the job-finding rate of workers under social assistance decreases by 35%, whereas it decreases by 25% for workers under UI.



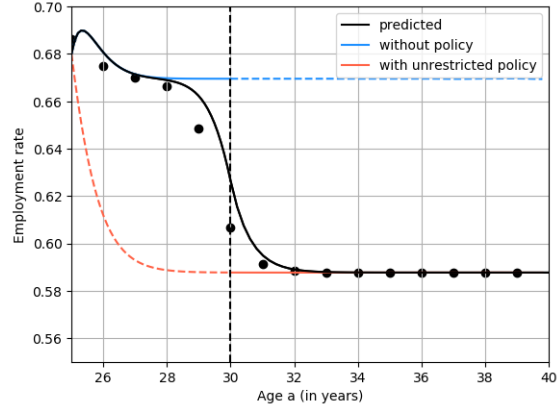
(a) Average match surplus



(b) Job-finding rates



(c) Population of nonemployed workers



(d) Employment rate

Figure 3: Simulation with Quebecois data

Notes for figures 3a, 3b and 3c. Continuous curves correspond to unemployed workers under UI benefits (blue curve) and social assistance (orange curve) confronted to the age-conditional policy. Dashed curves represent the counterfactual scenarios without policy change at the age threshold.

Notes for figure 3d. The black curve is the predicted employment rate when assistance benefits are conditional on age. The blue and red curves correspond to the counterfactual employment rates when assistance benefits do not depend on age. Nonemployment income is b_0^{min} when the curve is blue and b_0^{max} when red.

Figure 3c shows the implied changes in both stocks of unemployed workers. The percentage of workers under UI slightly decreases, whereas the percentage of workers under social assistance dramatically increases from 15% to 24%.

Figure 3d displays the age pattern of the employment rate. The black curve shows the simulated employment rate with age-conditional assistance benefits. Dots show the mean employment rate over a full year, i.e., the dot at age a is the mean employment rate between a and $a + 1$.³

The two other curves depict the counterfactual scenarios where benefits do not change with age. The employment rate starts decreasing at 28 and converges towards the red counterfactual employment rate. The convergence process is almost completed at 32. Changes in job-finding rates immediately impact the

³The discrete approximation of the running variable, here age, generates a bias in the RDD estimates (see Dong, 2015). Dots are below the continuous employment curve because the curve is decreasing. For instance at 29, the dot corresponds to the average employment rate of workers between 29 and 30 and is lower than the employment rate at 29.

flow of workers finding a job, but need time to affect the stock of employed workers.

Figure 3d visually depicts the quantitative impact of the policy as the spread between the two counterfactual scenarios. The effect of raising assistance benefits by 170% is a decline in employment rate by 8 percentage points. Using the black curve in the neighborhood of the age discontinuity leads to a different computation. Comparing the employment rate at 29 to the one at 30 for instance underestimates the policy impact by 4 percentage points.

Shortening the age span increases the bias, whereas opening the age window reduces it. Of course this latter statement does not account for the risk of increasing heterogeneity between the two groups of workers, a major preoccupation of the RDD.

Table 3 shows the RDD estimates of Lemieux and Milligan (2008) and our own estimates based on simulated data. We choose their specification with the employment rate at the Census age ("ERC"). Our simulated data are represented by the dots on Figure 3d. We provide several estimates, corresponding to two different kinds of polynomial specifications for age. We also report the LATE, as defined by the difference between the blue curve and the red one at age 30.

The estimated effects, whether on actual or simulated data, are always lower than the LATE. The spread amounts to between 2 to 4 percentage points. Though the two sets of estimates are actually close, there is a subtle difference between them. The impact decreases with the polynomial order when using simulated data, whereas it increases when using actual data. With simulated data, increasing the polynomial order captures the declining employment pattern due to pre-entitlement effect. The estimated impact at the age discontinuity is correspondingly reduced. Lemieux and Milligan (2008) do not obtain this property because the age polynomial captures a differential employment trend before and after 30 at work on real data. This differential trend is not predicted by our model and likely not related to minimum income.

One may worry that our measures of the bias heavily depend on the finding of a relatively large job separation rate. Our simulations exposed in Appendix XX reveal that the magnitude of the bias decreases with the job separation rate. From this perspective, the estimates that we report here can be seen as conservative.

4.3 French data

From 1988 to 2009, the French *revenu minimum d'insertion* (RMI) was an assistance benefit paid to individuals older than 25. Bargain and Doorley (2011) use this discontinuity to evaluate the impact of assistance benefits on the employment rate. We proceed similarly to the Quebecois case.

We apply the model to the population of single male high-school dropouts. The annual interest rate is 5%. Bargain and Doorley (2011) provide relevant income figures for 1999. Unemployed workers below 25 receive on average 206 euros per month of housing benefits. After 25, the sum of housing benefits and RMI is 539 euros. Moreover, the ratio of this income to the median wage income is about 66%. We suppose that the ratio of assistance benefits to mean wage is the same.

We use details about the French UI scheme in 1999 from Pommier and Cohen-Solal (2001). The replacement rate was 68%, which leads to $b_{UI} = 0.68$. We use the French policy rules to define λ as a function of q like in the Quebecois case.

We simulate the pattern of employment by age. We start at age 20 and suppose that 64% of the corresponding workers are employed, the employment rate reported by Bargain and Doorley (2011). None of the unemployed is initially entitled to UI benefits.

The employment rate before 25 strongly increases with age in the data. Our model cannot well explain these dynamics. We therefore reduce the window around the entitlement age (compared with the Quebecois case) and consider only three employment rates. The model perfectly matches these three moments as there are three free parameters. As in the Canadian case, parameters m and α are identified with the level of employment rate and its change, whereas parameter q is identified by the adjustment speed.

Table 4 displays the different parameters and key model outcomes. The elasticity α is smaller than in the Canadian case, 0.32 against 0.54. This indicates that the search effort is more responsive to changes in match surplus in the French case (the elasticity is 3) than in the Canadian one (the elasticity is 2). As in the Canadian case, the job separation rate is very large at 0.0737, implying that jobs last 14 months on average. This comes as a surprise given the French separation rate is usually low by international standard. However, the average rate hides strong disparities between young workers and the rest of the crowd. The French youth were (and still are) disproportionately exposed to short-term contracts and associated precarity. This phenomenon was particularly striking for the low skilled, here composing the population of interest. However, and like the Canadian case, we consider alternative values for q in the Appendix.

Table 4: Parameters in the French case

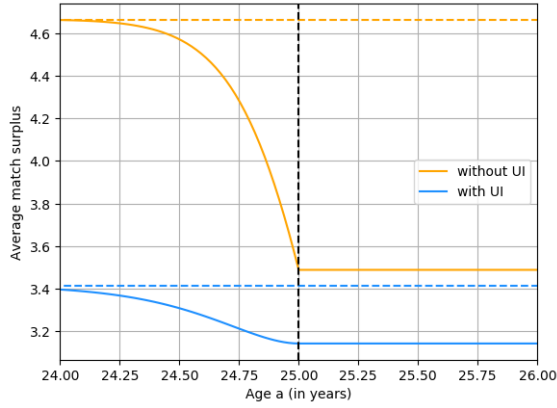
A	$\bar{w}$	$b_0^{\min}$	$b_0^{\max}$	b_{UI}	r	α	m	q	λ
25	1	0.23	0.602	0.68	0.0048	0.319	0.315	0.0737	0.068

Table 5: Moments to fit the values of α , m and q , French case

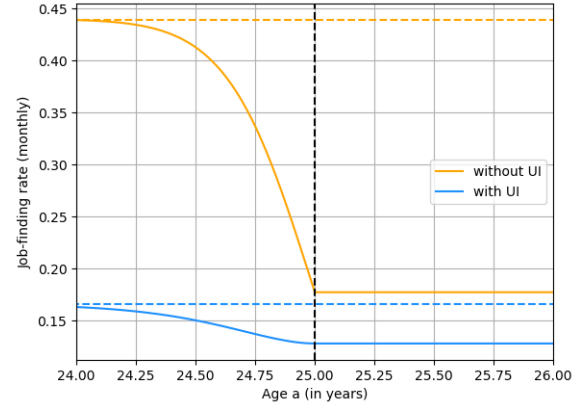
Moments	Target	Model
Employment rate between 24 and 25	72.1%	72.1%
Employment rate between 25 and 26	66.8%	66.8%
Employment rate between 26 and 27	65.8%	65.8%

Notes. Targeted moments are borrowed from Bargain and Doorley (2011).

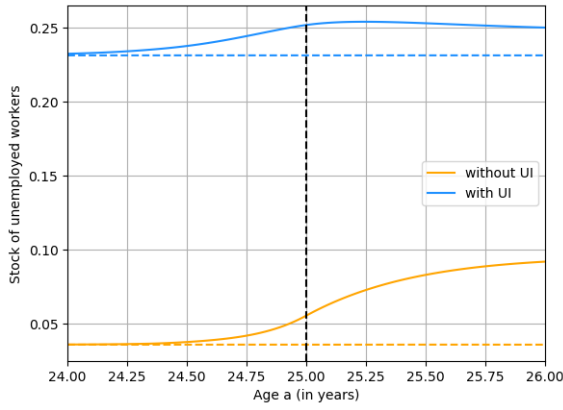
Figures 4a and 4b show the simulated patterns of match surpluses and job-finding rates by age. Each figure displays four curves. The solid curves correspond to nonemployed workers under UI (blue curve) and social assistance (orange curve) when assistance benefits are conditional on age. The dashed curves correspond to the counterfactual scenario where assistance benefits do not increase at 25. Figure 4c depicts the resulting patterns for the two stocks of nonemployed workers.



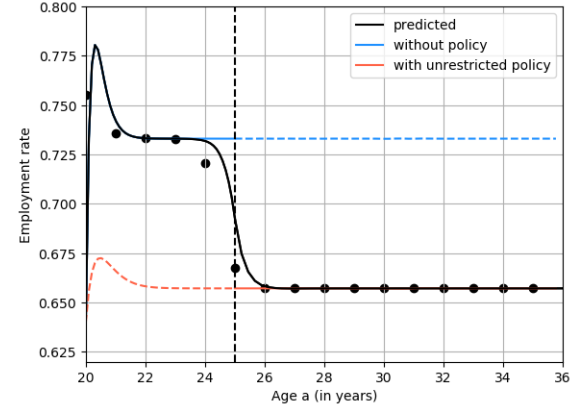
(a) Average match surplus



(b) Job-finding rates



(c) Population of nonemployed workers



(d) Employment rate

Figure 4: Simulation with French data

Notes for Figures 4a, 4b and 4c. Continuous curves correspond to unemployed workers under UI benefits (blue curve) and social assistance (orange curve) confronted to the age-conditional policy. Dashed curves represent the counterfactual scenarios without policy change at the age threshold.

Notes for Figure 4d. The black curve is the predicted employment rate when assistance benefits are conditional on age. The blue and red curves correspond to the counterfactual employment rates when assistance benefits do not depend on age. Nonemployment income is b_0^{min} when the curve is blue and b_0^{max} when red.

The match surplus decreases by 7 to 25% between 24 and 25. Correspondingly, job-finding rates decrease from 50% at 24 to 20% at 25 for workers under social assistance and from 19% to 15% for workers under UI. The stock of workers under social assistance starts increasing after 24 to reach 0.10 at 25 instead of 0.03 in the counterfactual scenario.

Note that the job-finding rates are quite large for low-skilled French workers, especially for those under social assistance. This result is induced by the high job loss rate chosen by our calibration procedure. To match a given employment rate, short employment durations must be balanced by short nonemployment durations.

Figure 4d displays the simulated employment rate and compares it to two counterfactual scenarios. When assistance benefits remain at b_0^{min} , the blue curve shows that the employment rate converges rapidly to 73%. When assistance benefits are (unexpectedly) set to b_0^{max} at age 20, the red curve shows

Table 6: Regression discontinuity estimates, French case

Polynomial specification for age:	Estimated effect on the employment rate			
	Empirical		Simulated	
	Age in: years	quarters	years	quarters
Linear	0.027	0.028	−0.069	−0.068
Quadratic	0.067	−0.069	−0.053	−0.052
Cubic	−0.065	−0.067	−0.055	−0.054
Linear spline	−0.049	−0.058	−0.053	−0.056
Quadratic spline	−0.067	−0.068	−0.059	−0.052
Linear spline, with age window widths:	Age in: years	quarters	years	quarters
±5 around 25	−0.052	−0.058	−0.051	−0.052
±4 around 25	−0.06	−0.069	−0.056	−0.054
±3 around 25	−0.065	−0.72	−0.052	−0.050
±2 around 25	−0.067	−0.079	−0.043	−0.040
LATE	X		−0.076	

Note. The dependent variable is the employment rate. The empirical estimates are taken from Table 1 in Bargain and Doorley (2011). The simulated data are the dots depicted by Figure 3d. The LATE is the difference between the two counterfactual employment rates at the age discontinuity on Figure 4d.

that employment rate converges to 66%. The simulated employment rate initially sticks to the blue curve, separates from it at age 24 and decreases to reach the red curve at age 26. By age 25, the employment rate is 69%. The speed of adjustment is twice faster than in Canada, a phenomenon mostly due to the high job separation rate.

The policy impact is the difference between the two counterfactual scenarios. This gives 7 percentage points at age 25. By comparing employment rates at 24 and 25, the estimated impact is lower by 2 percentage point. Here again, shortening the age window further reduces the estimated impact, whereas expanding the window increases the estimated impact.

In Table 6, we simulate a counterfactual dataset from the theoretical model and apply the same linear regressions as Bargain and Doorley (2011). They aggregate the employment rate by age group using years and quarters for workers between 20 and 35. They also play with the age window for robustness purpose. We reproduce their estimates in the two columns entitled "Simulated". We also report the LATE, here again defined as the difference between the two counterfactual scenarios at age 25.

Despite the first line of Table 6 displays a positive impact on actual data, Bargain and Doorley (2011) see their estimates in the range "between −0.069 and −0.049". We obtain simulated coefficients of the same magnitude. The LATE is above the upper bound of the estimates, implying a theoretical bias of about 1 to 2 percentage points.

In the Appendix, we assess the robustness of our findings by considering alternative job loss rates. Like the Canadian case, the magnitude of the bias decreases with the job loss rate.

5 UI benefit extension

We now focus on age-conditional extensions of UI benefits. Lalive (2008) on Austrian data and Schmieder et al. (2012a,b, 2016) on German data study experimental settings where the potential duration of benefits is conditional on age at dismissal. Though they associate their findings to search theory, they never discuss the non-stationarity induced by age.

5.1 Model

We suppose $\lambda_{\min} < \lambda_{\max}$ and $b_0^{\min} = b_0^{\max} = b_0$. Like the previous section, we argue that a decrease in $\lambda_{\max}$ affects all workers: not only those receiving extended benefits, but also those below the age threshold who take care of future employment gains in terms of benefit entitlement.

Consider first workers above age A . We have

$$\begin{aligned} r\bar{\Sigma}(b_{UI}, \lambda_{\min}) &= \bar{w} - b_{UI} - q\bar{\Sigma}(b_{UI}, \lambda_{\min}) - B_1\bar{\Sigma}(b_{UI}, \lambda_{\min})^{\frac{1+\alpha}{\alpha}} + \lambda_{\min}(\bar{\Sigma}(b_0, \lambda_{\min}) - \bar{\Sigma}(b_{UI}, \lambda_{\min})) \\ r\bar{\Sigma}(b_0, \lambda_{\min}) &= \bar{w} - b_0 - q\bar{\Sigma}(b_{UI}, \lambda_{\min}) - B_1\bar{\Sigma}(b_0, \lambda_{\min})^{\frac{1+\alpha}{\alpha}}, \\ r\bar{\Sigma}(b_{UI}, \lambda_{\max}) &= \bar{w} - b_{UI} - q\bar{\Sigma}(b_{UI}, \lambda_{\min}) - B_1\bar{\Sigma}(b_{UI}, \lambda_{\max})^{\frac{1+\alpha}{\alpha}} + \lambda_{\max}(\bar{\Sigma}(b_0, \lambda_{\max}) - \bar{\Sigma}(b_{UI}, \lambda_{\max})) \\ \bar{\Sigma}(b_0, \lambda_{\max}) &= \bar{\Sigma}(b_0, \lambda_{\min}). \end{aligned} \tag{24}$$

We can solve it by blocks. We solve the first two equations in $\bar{\Sigma}(b_{UI}, \lambda_{\min})$ and $\bar{\Sigma}(b_0, \lambda_{\min})$ and then replace these values in the last two equations.

Before age A ,

$$\begin{aligned} r\bar{S}(a, b_{UI}) &= \bar{w} - b_{UI} - q\bar{S}(a, b_{UI}) - B_1\bar{S}(a, b_{UI})^{\frac{1+\alpha}{\alpha}} + \lambda_{\max}(\bar{S}(a, b_{UI}) - \bar{S}(a, b_0)), \\ r\bar{S}(a, b_0) &= \bar{w} - b_0 - q\bar{S}(a, b_0) - B_1\bar{S}(a, b_0)^{\frac{1+\alpha}{\alpha}}, \end{aligned} \tag{28}$$

with the terminal conditions $\bar{S}(A, b_{UI}) = \bar{\Sigma}(b_{UI}, \lambda_{\max})$ and $\bar{S}(A, b_0) = \bar{\Sigma}(b_0, \lambda_{\max})$. We first integrate (29) to find the function $\bar{S}(a, b_0)$ and then replace it in (28) to find the function $\bar{S}(a, b_{UI})$.

The pre-entitlement match surpluses $\bar{S}(a, b_0)$ and $\bar{S}(a, b_{UI})$ increase with age as workers get closer to entitlement. Therefore search efforts and the resulting job-finding rates also increase with age prior to entitlement age.

The calibration procedure exposed hereafter involves the average nonemployment duration and the average benefit duration. We do so because they are outcomes used to evaluate the impact of benefit extension policies. Both durations can be computed from the trajectories of the various job-finding rates. Appendix A1 shows the details of the computation.

5.2 Austrian data

We now focus on the Austrian data used by Lalive (2008). From 1988 to 1993, the maximum UI duration after 50 was extended in some regions in Austria, from 39 weeks to 209 weeks. We follow Lalive and calibrate the model on males between 46 and 54 who have experienced continuous employment - women are affected by the early retirement policy in Austria. We then reproduce the RDD estimation on the simulated average nonemployment duration.

The annual interest rate is 5%. Here again, we normalize the average wage to one. Lalive (2008) mentions a gross replacement rate of UI benefits at 0.4 for the median wage earner without family allowance. We thus set the replacement rate of UI benefits at 0.6. Assistance benefits amount to 70% of UI benefits, the ratio of median assistance to median UI benefits in the data.

Lalive only selects workers who are eligible to the maximum UI duration. Therefore we set λ_{max} and λ_{min} so that the mean potential durations of benefits are, respectively, 39 and 209 weeks.

We simulate the pattern of unemployment by age from age 46 and focus on a particular outcome of interest, the average unemployment duration. As in the Quebecois and French cases, we are left with three parameters: the elasticity of the marginal search cost, α , the contact rate by unit of search, m , and the job loss rate, q . Parameters α and m are calibrated to match the the average unemployment durations before and after the reform. In practice, we choose the average unemployment duration for workers between 49.75 and 50 and for workers between 50 and 50.25.

As the average unemployment duration is non-continuous at the threshold, the job loss rate cannot be calibrated as in the Quebecois and French cases. Here we use external information on job destruction. Winter-Ebmer (2003) study the same reform and find that between 25 and 22% of employed workers lose their job within a year. We accordingly set q to 0.02. In the Appendix, we report simulation results with alternative values for q .

Table 7: Parameters in the Austrian case

A	$\bar{w}$	b_0	b_{UI}	r	α	m	q	λ_{max}	λ_{min}
50	1	0.28	0.6	0.0048	0.0195	0.0687	0.02	0.111	0.0207

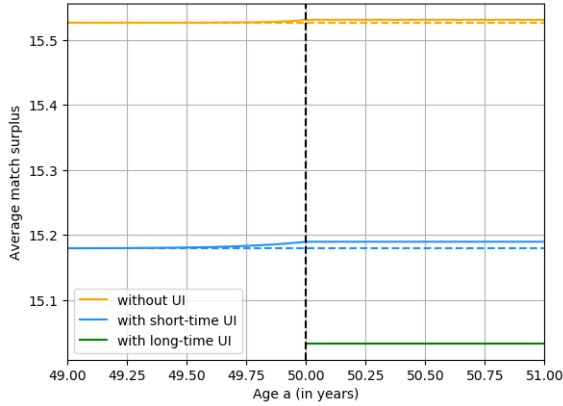
Table 8: Moments to fit the values of α and m , Austrian case

Moments	Target	Model
Average nonemployment duration between 49.75 and 50	15.2 weeks	15.2 weeks
Average nonemployment duration between 50 and 50.25	29.6 weeks	29.6 weeks

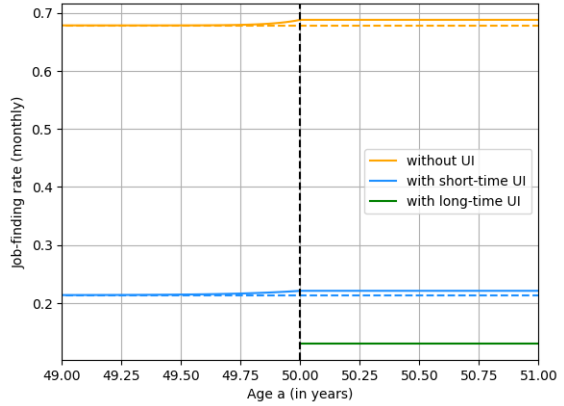
Note. Targeted moments are borrowed from Lalive (2008). The age is at start of unemployment spell.

We find a low elasticity of the marginal cost, α , compared with the Canadian and French cases of the previous section, 0.02 against 0.54 and 0.32. This indicates a very large behavioral response to changes in match surplus: the corresponding elasticity is about 50.

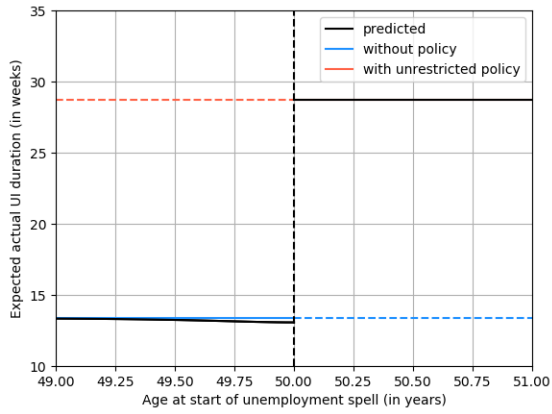
Figure 5a displays the age pattern of match surpluses, whereas Figure 5b shows the corresponding



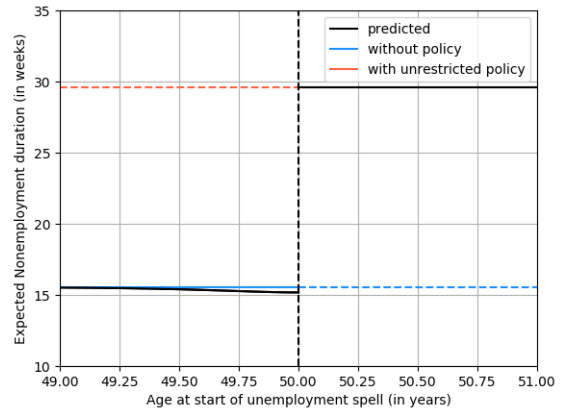
(a) Average match surplus



(b) Job-finding rates



(c) Expected UI duration



(d) Expected nonemployment duration

Figure 5: Simulation with Austrian data

Notes for figures 5a and 5b. The blue and orange continuous curve respectively correspond to the case of an unemployed worker receiving UI benefits for a short period and an unemployed worker receiving the social assistance. The green curve represents the case of a worker who is eligible to longer UI benefit. The dashed curves represent the counterfactual scenario without policy change at the age threshold.

Notes for figures 5c and 5d. The black curve is the predicted expected unemployment duration. The blue and red curves correspond to the counterfactual durations when eligibility to UI does not depend on age. People lose UI entitlement at rate λ_{max} in blue and λ_{min} in red.

job-finding rates. In each case there are five curves. The solid curves in blue and orange are associated to unemployed workers under regular UI (blue curve) and assistance benefits (orange curve) when job losers have access to extended UI benefits after 50. The solid green curve corresponds to workers eligible to extended benefits. In practice, this can only happen after 50, but we also show what would happen if workers could get extended benefits after 48. The dashed curves show the counterfactual scenario where the potential UI benefit duration does not vary with age.

The main lesson displayed by Figures 5a and 5b is that pre-entitlement effects are small. Unemployed workers increase their search effort to account for the possibility of extended UI after job loss. But this increase is quantitatively small and does not start before age 49.5, i.e., 6 months before the threshold age. This small behavioral response of non-eligible workers goes along with a large impact of the policy on eligible workers. Their job-finding rate is half the one of non-eligible ones under UI.

Table 9: Regression discontinuity estimates, Austrian case

Polynomial specification for age	Estimated effect on nonemployment duration (in weeks)	
	Empirical	Simulated
No trend	14.6	14.1
Linear spline	14.8	14.2
Cubic spline	11.15	14.5
LATE	X	14.1

Note. The dependent variable is the expected nonemployment duration (in weeks) at the age at start of the spell. The empirical estimates are taken from Table 2 in Lalive (2008). The LATE is the difference between the two counterfactual nonemployment duration at the age threshold in figure 5d.

Figures 5c and 5d depict, respectively, the age patterns of the expected UI duration and expected nonemployment duration. The UI duration differs from the nonemployment duration when the worker exhausts her benefits. In each case, the black line shows the simulated duration with UI extension after 50. The dashed lines display the two counterfactual scenarios when benefits are extended since 46 (red curve) or never extended (blue curve). The true policy impact is measured by the difference between the red curve and the blue one; the RDD exploits the change in the black curve around age 50.

Lalive focuses on the expected unemployment duration. Figure 5d shows that the red dashed line is very close to the black line before 50, whereas the blue dashed line is very close to the black line after 50. Workers increase search effort before turning 50 and the expected unemployment duration slightly declines. Therefore comparing the expected unemployment durations of workers dismissed before and after 50 overestimates the policy impact. But the bias can hardly be seen.

Table 9 replicates the RDD based on age discontinuity in Lalive (2008). He runs regressions at individual level. We run similar regressions at age cell level and monthly frequency. We assume the worker age is uniformly distributed and compute the average unemployment duration for each age group.

The results confirm that the bias is very small. The estimated impact is about 14 weeks of additional unemployment and we reproduce it by applying the same strategy on the simulated data. It is slightly larger than the LATE computed as the difference between the red dashed curve and the blue one in Figure 5d.

Considering alternative values for the job loss rate does not invalidate the results reported here. In the Appendix, we numerically show that the bias increases with the job loss rate. It amounts to between 1.1 (estimate on simulated data) and 1.8 weeks (exact LATE computed from the counterfactual trajectories of the average unemployment duration) when the job loss rate reaches 0.1, a very large value.

5.3 German data

From 1987 to 1999, German workers who started their unemployment spell before 42 were entitled to 12 months at best of UI benefits. If their unemployment spell started between 42 and 44, they were entitled to 18 months. Schmieder et al. (2012a) use this age discontinuity to quantify the impact of

benefit extensions on average nonemployment and benefit durations.

We apply the model to the population of households with children. The annual interest rate is 5%. Income in the different states is borrowed from Schmieder et al. (2012a). The replacement rate for UI benefit is 68%, which corresponds to the average for individuals with children. We set the replacement rate for social assistance at 35%, the average for married individuals. As in the Austrian case, we suppose that parameters m and q do not change with age.

We simulate the pattern of unemployment by age starting from age 48. Schmieder et al. (2012a) provide the average UI duration before and after the threshold. We use these moments to calibrate the rates at which workers lose UI entitlement, λ_{max} and λ_{min} . Like in the Austrian case, α and m are calibrated to match the average nonemployment durations. We do not have external information on the job loss rate, q . Instead, we calibrate q using information about cumulated nonemployment spells in the five years after the start of the first nonemployment spell. From Schmieder et al. (2012b), we set the average number of days of nonemployment over this period to 907 and convert it in months.⁴

With five empirical targets and five free parameters, we exactly match the moments.

Table 10: Parameters in the German case

A	$\bar{w}$	b_0	b_{UI}	r	α	m	q	λ_{max}	λ_{min}
42	1	0.35	0.68	0.0048	0.739	0.109	0.0259	0.0941	0.0663

Table 11: Moments used to fit the values of α , m , q , λ_{max} and λ_{min} , German case

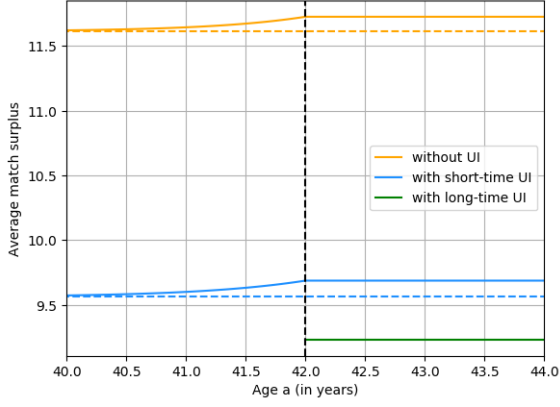
Moments	Target	Model
Average nonemployment duration before 42	15.6 months	15.6 months
Average nonemployment duration after 42	16.4 months	16.4 months
Average cumulated nonemployment duration after 42	29.8 months	29.8 months
Average nonemployment duration before 42	6.7 months	6.7 months
Average nonemployment duration after 42	8.5 months	8.5 months

Note. Targeted moments are borrowed from Schmieder et al. (2012a) and Schmieder et al. (2012b). We define ages just before and just after 42 at 41.99 and 42.01. The age is at start of unemployment spell.

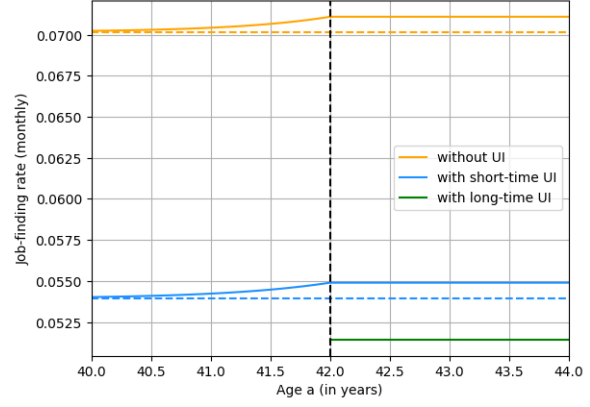
Parameters λ_{max} and λ_{min} are slightly above the values matching the potential UI durations before and after 42. This implies that, on average, workers are entitled to UI benefits for (slightly) shorter durations than indicated above. We interpret this finding as resulting from additional eligibility conditions that typically reduce the potential UI duration.

Figure 6a shows the average match surpluses in three cases. The green curve depicts $\bar{X}_1$. It is constant and expected by workers who get dismissed after age A , thereby receiving b_1 . The blue curve depicts $\bar{X}_0(a)$, which is of interest for workers dismissed before age A . It gradually increases until A is reached and then stays constant at $\bar{X}_0$. Lastly, the orange curve shows the counterfactual average match surplus when the benefit level is b_0 irrespective of age. It is initially very close to $\bar{X}_0(a)$ and the spread gradually

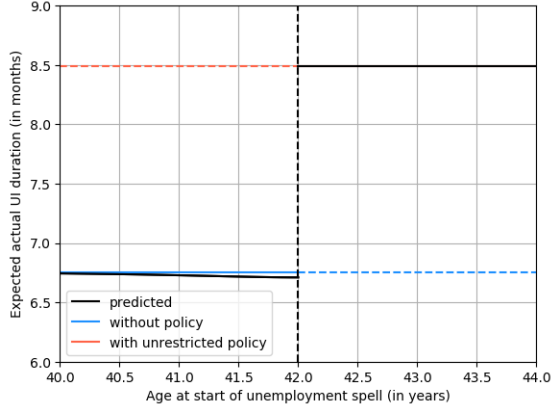
⁴The theoretical distribution of the number of nonemployment days can be recovered from the theoretical distribution of nonemployment and employment spells. In practice, we use a Monte-Carlo method to compute the expected number of nonemployment days.



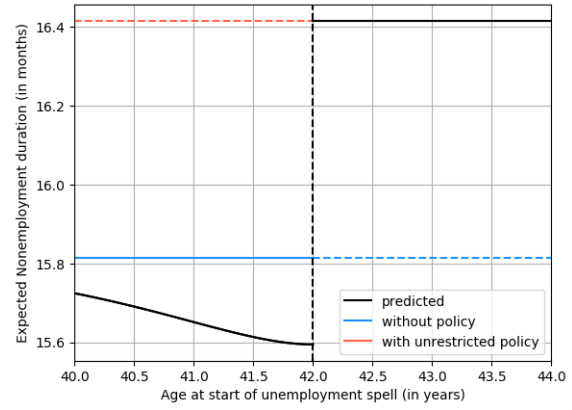
(a) Average match surplus



(b) Job-finding rates



(c) Expected UI duration



(d) Expected nonemployment duration

Figure 6: Simulation with German data

Notes for figures 6a and 6b. The continuous curves correspond to unemployed workers receiving unextended UI benefits (blue curve) and under social assistance (orange curve). The dashed curves represent the counterfactual scenario without policy change at the age threshold. The green curve correspond to unemployed workers receiving extended UI benefits. Notes for figures 6c and 6d. The black curve is the predicted expected unemployment duration. The blue and red curves correspond to the counterfactual durations when eligibility to UI does not depend on age. People lose UI entitlement at rate λ_{max} in blue and λ_{min} in red.

Table 12: Regression discontinuity estimates, German case

	Estimated effect on duration (in months)			
	Empirical		Simulated	
Bandwidth in years:	UI duration	Nonemployment duration	UI duration	Nonemployment
± 2 around 42	1.78	0.78	1.78	0.833
± 1	1.82	0.92	1.78	0.828
± 0.5	1.73	1.04	1.78	0.823
± 0.2	1.65	0.79	1.78	0.821
LATE	X	X	1.74	0.6

Note. The dependent variables are the UI benefit duration and nonemployment duration. The empirical estimates are taken from Table 2 in Schmieder et al. (2012a). The LATE is the difference between the red curve and the blue one, as displayed by Figures 6c and 6d.

increases, reflecting the possibility for workers to get access to b_1 in case they accept a job offer.

Figure 6b shows the resulting job-finding rates. The job-finding rate linearly increases with the match surplus. The difference between the actual case and the counterfactual one increases with age but remains small.

Figures 6c and 6d show the average UI duration and nonemployment duration as a function of age at start of unemployment spell. The curves slightly decrease before the threshold because workers search more. Jobs are more valuable to workers because they give entitlement to extended benefits in case of job loss. Given the job loss rate, workers who find a job at 41.5 expect to keep the job sufficiently long to pass the threshold and enter the more generous UI system. The curves are discontinuous at 42. Workers losing their job just before 42 and workers losing their job just after 42 do not similarly behave because they enter two different systems of UI benefits.

Like the Austrian case, the RDD overestimates the policy impact. The quantitative effect is more pronounced in the German case: the impact on the expected unemployment duration is overestimated by 0.2 month, about a fourth of the reported impact.

Table 12 reproduces the two regressions in Schmieder et al. (2012a). As in the Austrian case, we mimic their regressions performed on individual data by aggregating workers by age cell in our simulations. Age is in days. The first set of regressions uses the actual UI duration as the dependent variable and the second one uses the nonemployment duration. We report the results with a linear spline and different bandwidths.

The regressions on simulated data reproduce quite well the results on actual data. The estimated impact is around 0.8 months of nonemployment and overestimates the actual impact by 0.2 months. The estimated impact on the UI duration is both stronger and closer to the actual one.

5.4 Discussion

The bias is induced by a particular transition pattern between unemployment and employment. Namely, it increases with the likelihood of finding a job and losing it afterwards. This likelihood is small when compared with alternative transition patterns. Therefore the expectation of eligibility to extended benefits does not impact much the search behavior of workers prior to the eligibility age.

Figures 7a and 7b show the theoretical bias associated with the RDD. It consists of the job-finding rate differential between unemployed workers receiving UI benefits just before the entitlement age threshold and counterfactual workers in a world without policy change at age threshold. The bias is represented as a function of parameters m and q . For the baseline parameterizations, indicated by crosses, the bias is the gap between the dashed and the continuous blue curves in Figures 5b and 6b.

Parameter m has a non-monotonic impact. The differential is small when m is low – why would workers search if they had no chance of finding a job? – and when m is large – workers can simply wait the very last minute to take a job. Conditional on the job-finding rate, the differential is maximized for intermediate values of m .

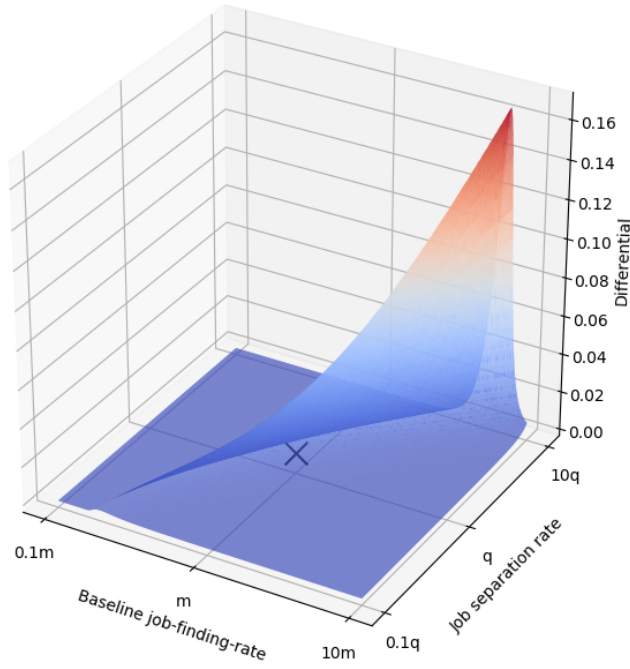
Parameter q increases the differential. When q is very small, UI benefits do not matter and the differential is low. As q increases, the pre-entitlement match surplus becomes more impacted by post-entitlement UI benefits. This affects the pre-entitlement behavioral response of the job-seekers, thereby increasing the RDD bias.

Figures 7a and 7b reveal that the magnitude of the bias can be very large, but only for parameterizations involving large values of m and q . In particular, the job loss rate must be way higher than empirically plausible values. This is especially true in the Austrian case where q needs to be more than 0.2 to see anything on the job-finding rate differential. This amounts to ten times the value used for the simulations.

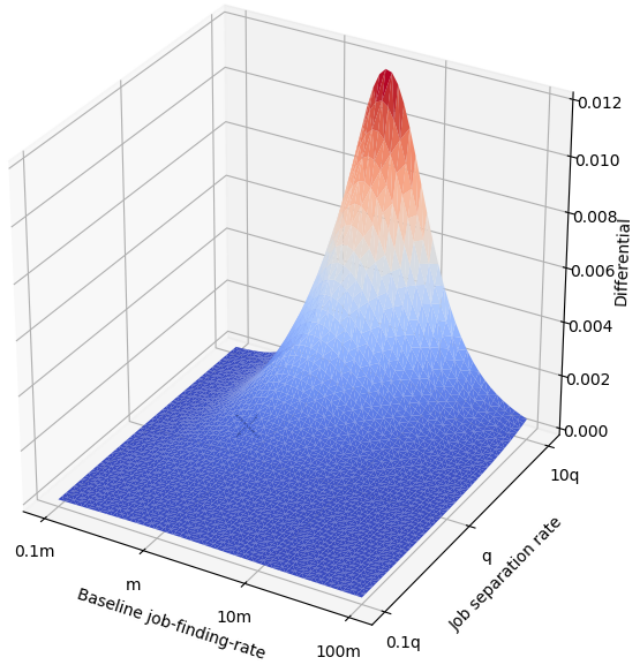
It follows that the differential must be small for reasonable values of m and q . Therefore it makes sense to compare unemployed workers before and after the threshold age. Though both workers are actually treated, the behavioral response of the supposedly nontreated can be neglected.

6 Conclusion

Job search theory is a consistent and powerful framework to describe labor market phenomena. We use it to discuss the potential bias of popular RDD to evaluate minimum income and UI benefit extension policies. The design involves an age discontinuity in treatment exposure, i.e., only those above a specific age have access to minimum income or extended benefits. Search frictions imply that this design is biased: individuals should progressively modify their search behavior before entitlement. We calibrate a nonstationary job search model on the various datasets used by the literature. According to our calibrations, the impact of age-conditional minimum income is under-estimated and this quantitatively matters, whereas the impact of age-conditional UI benefit extensions is over-estimated but the bias can



(a) Austrian case



(b) German case

Figure 7: Theoretical bias as a function of m and q

Notes. This figure depicts the difference in job-finding rates between workers under UI just before the age threshold and counterfactual workers who are not confronted to such a policy change. The job-finding rate differential is represented as a function of parameters m (left axis) and q (right axis) in base-10 log scale. In the Austrian case, $m = 0.041$ and $q = 0.02$. In the German case, $m = 0.00147$ and $q = 0.0203$.

be neglected.

These results do not prove that the designs are biased per se. The biases are conditional on the fact that job search theory is an accurate description of actual labor markets. The simple models that we use certainly overstate the magnitude of search frictions because they do not feature worker heterogeneity or participation choices. Therefore a first task consists in enriching this model to make more reliable counterfactual predictions. Nevertheless, labor market researchers should be careful while interpreting the results of an RDD based on age discontinuity. After all, the burden of proof falls on them. At least they should explain why the bias is small and why we should trust their estimates.

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A The different populations of unemployed by age

The size of the labor force is normalized to one. At age a , the mass of unemployed workers receiving social assistance is $u_0(a)$. Similarly, $u_{UI}^{short}(a)$ and $u_{UI}^{long}(a)$ are the masses of unemployed receiving UI benefits for, respectively, a regular period and an extended one.

When $a \leq A$,

$$\frac{du_0}{da}(a) = \lambda_{max} u_{UI}^{short}(a) - ms(a, b_0^{min}) u_0(a) \quad (30)$$

$$\frac{du_{UI}^{short}}{da}(a) = q[1 - u_0(a) - u_{UI}^{short}(a)] - [ms(a, b_{UI}) + \lambda_{max}] u_{UI}^{short}(a) \quad (31)$$

$$u_{UI}^{long}(a) = 0 \quad (32)$$

Workers receiving UI benefits lose entitlement and enter the social assistance regime at rate λ_{max} . Meanwhile, workers under social assistance find jobs at rate $ms(a, b_0^{min})$. In the second differential equation, the inflow of workers receiving benefits is composed of those who lose their job at rate q . The outflow is made of workers who find jobs at rate $ms(a, b_{UI})$ or lose benefit entitlement at rate λ_{max} .

When $a \geq A$,

$$\frac{du_0}{da}(a) = \lambda_{max} u_{UI}^{short}(a) + \lambda_{min} u_{UI}^{long}(a) - m\sigma(b_0^{max}, \lambda_{min}) u_0(a) \quad (33)$$

$$\frac{du_{UI}^{short}}{da}(a) = -[m\sigma(b_{UI}, \lambda_{max}) + \lambda_{max}] u_{UI}^{short}(a) \quad (34)$$

$$\frac{du_{UI}^{long}}{da}(a) = q[1 - u_0(a) - u_{UI}^{short}(a) - u_{UI}^{long}(a)] - [m\sigma(b_{UI}, \lambda_{min}) + \lambda_{min}] u_{UI}^{long}(a) \quad (35)$$

The inflow of workers receiving social assistance now has two components: those who lose UI entitlement at rate λ_{max} and those who lose it at rate λ_{min} . There is no inflow of workers in the short-period entitlement regime. Workers who lose their job at rate q get access to extended benefits.

B Computing the age-conditional average nonemployment and benefit durations

Let B be the UI benefit duration and D the non-employment duration. After age A , the Poisson rate of quitting benefit entitlement is $\lambda_{min} + m\sigma(b_{UI}, \lambda_{min})$, i.e., rate of losing entitlement plus job-finding rate. Before age A , the rate of quitting benefit entitlement is age-dependent.

The probability density function (pdf) of UI benefit duration B , conditional on age at the start of the

unemployment spell a , is

$$f_{UI}(B|a) = \begin{cases} \exp \left[- \int_0^B (\lambda_{max} + m\tilde{s}(a+t, b_{UI})) dt \right] \cdot (\lambda_{max} + m\tilde{s}(a+B, b_{UI})) & \text{if } a < A, \\ \exp [-(\lambda_{min} + m\sigma(b_{UI}, \lambda_{min}))B] \cdot (\lambda_{min} + m\sigma(b_{UI}, \lambda_{min})) & \text{if } a \geq A. \end{cases} \quad (36)$$

with $\tilde{s}(a+t, b_{UI}) = s(a+t, b_{UI})$ if $a+t < A$, or $\tilde{s}(a+t, b_{UI}) = \sigma(b_{UI}, \lambda_{max})$ if $a+t > A$.

The pdf is the product of two terms. The first term with the exponential is the survival probability, i.e., the probability of receiving UI benefits for *at least* duration B . The second term is the hazard rate of losing UI benefits at $a+B$.

The expected benefit duration is $\mathbb{E}(B|a) = \int_0^\infty B f_{UI}(B|a) dB$. For $a \geq A$, this conditional expectation is

$$\mathbb{E}(B|a) = (\lambda_{min} + m\sigma(b_{UI}, \lambda_{min}))^{-1} \quad \text{if } a \geq A. \quad (37)$$

We now characterize the age-conditional pdf of non-employment duration D . If $a < A$,

$$f_{NE}(D|a) = f_{UI}(D|a) \cdot \frac{m\tilde{s}(a+D, b_{UI})}{\lambda_{max} + m\tilde{s}(a+D, b_{UI})} + \int_0^D f_{UI}(B|a) \cdot \frac{\lambda_{max}}{\lambda_{max} + m\tilde{s}(a+B, b_{UI})} \cdot \exp \left[- \int_B^D m\tilde{s}(a+t, b_0) dt \right] \cdot m\tilde{s}(a+D, b_0) dB \quad (38)$$

We distinguish two situations: either the worker finds a job before losing UI benefits, $D = B$, or after benefit exhaustion, $D > B$. The pdf f_{NE} is the sum of two terms. The first term corresponds to $D = B$. This is the pdf of the benefit duration f_{UI} weighted by the proportion of exits to employment among those who stop receiving benefits. The second term corresponds to $D > B$. This is an integral over all possible values of B between 0 and D . The quantity $f_{UI}(B|a) \frac{\lambda_{max}}{\lambda_{max} + m\tilde{s}(a+B, b_{UI})}$ is the pdf of receiving UI benefits for duration B multiplied by the proportion of exits to assistance benefits among those who stop receiving UI benefits. The exponential term is the probability of remaining unemployed between duration B and D . The term $m\tilde{s}(a+D, b_0)$ is the job-finding rate for nonentitled workers who entered unemployment at age a and are still unemployed at age D .

If $a \geq A$,

$$\begin{aligned} f_{NE}(D|a) &= e^{-(\lambda_{min} + m\sigma(b_{UI}, \lambda_{min}))D} \cdot m\sigma(b_{UI}, \lambda_{min}) \\ &+ \int_0^D e^{-(\lambda_{min} + m\sigma(b_{UI}, \lambda_{min}))B} \cdot \lambda_{min} \cdot e^{-m\sigma(b_0, \lambda_{min})(D-B)} \cdot m\sigma(b_0, \lambda_{min}) dB \\ &= \frac{(\lambda_{min} + m\sigma(b_{UI}, \lambda_{min}))(m\sigma(b_0, \lambda_{min}) - m\sigma(b_{UI}, \lambda_{min}))}{m\sigma(b_0, \lambda_{min}) - (\lambda_{min} + m\sigma(b_{UI}, \lambda_{min}))} e^{-(\lambda_{min} + m\sigma(b_{UI}, \lambda_{min}))D} \\ &- \frac{\lambda_{min} m\sigma(b_0, \lambda_{min})}{m\sigma(b_0, \lambda_{min}) - (\lambda_{min} + m\sigma(b_{UI}, \lambda_{min}))} e^{-m\sigma(b_0, \lambda_{min})D} \end{aligned} \quad (39)$$

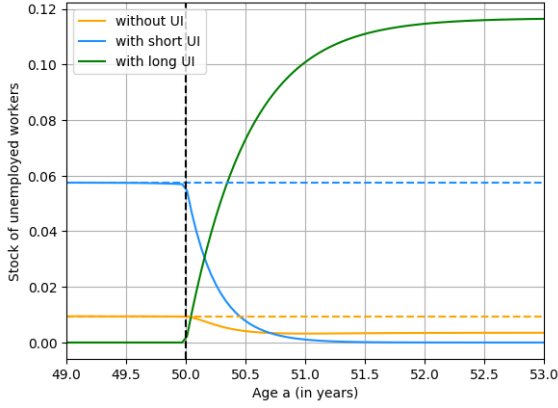
$$(40)$$

The expected nonemployment duration is $\mathbb{E}(D|a) = \int_0^\infty D f_{NE}(D|a) dD$. This expectation has a simple

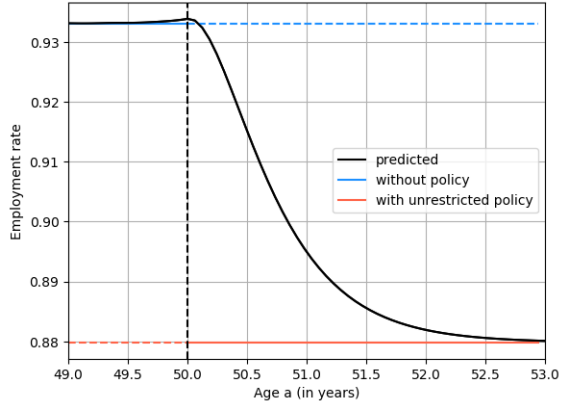
form when age is above the threshold. For $a \geq A$,

$$\mathbb{E}(D|a) = \frac{m\sigma(b_0, \lambda_{min}) - m\sigma(b_{UI}, \lambda_{min})}{(\lambda_{min} + m\sigma(b_{UI}, \lambda_{min}))(m\sigma(b_0, \lambda_{min}) - \lambda_{min} - m\sigma(b_{UI}, \lambda_{min}))} - \frac{\lambda_{min}}{m\sigma(b_0, \lambda_{min})(m\sigma(b_0, \lambda_{min}) - \lambda_{min} - m\sigma(b_{UI}, \lambda_{min}))} \quad (41)$$

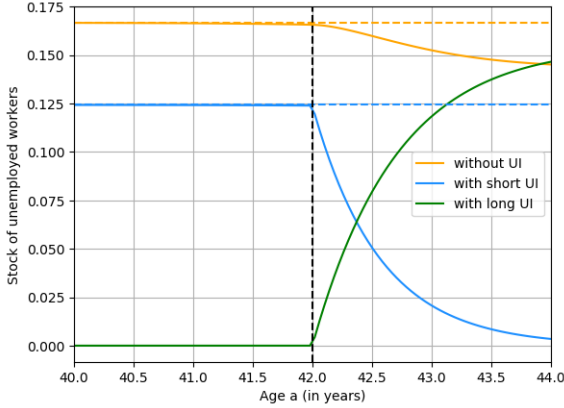
C Employment with Austrian and German data



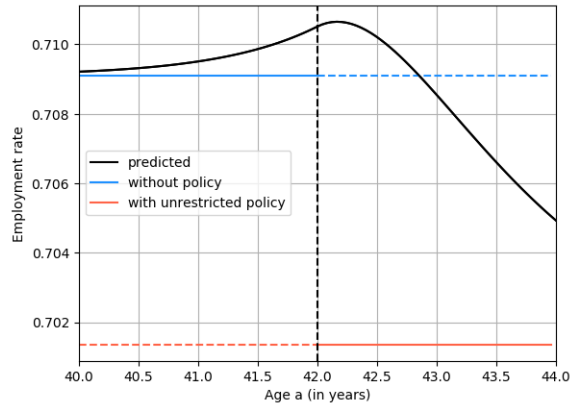
(a) Population of nonemployed workers, Austrian case



(b) Employment rate, Austrian case



(c) Population of nonemployed workers, German case



(d) Employment rate, German case

Figure 8: Employment dynamics with Austrian and German data

Notes for figures 8a and 8c. The continuous curves depict the stocks of unemployed under UI (blue curve) and social assistance (orange curve) when unemployed above 50 receive extended UI benefits. The dashed curves represent the counterfactual scenario without benefit extension.

Notes for figures 8b and 8d. The black curve is the predicted employment rate when unemployed workers get extended UI benefits after 50. The blue and red curves correspond to the counterfactual employment rates in absence of age discontinuity. People lose UI eligibility at rate λ_{max} when the curve is blue and λ_{min} when red.

D Robustness with respect to the job destruction rate

Each of the calibrations exposed in Sections 4 and 5 involve a particular assumption to set the job loss rate, q . Tables 13 to 16 report the results for alternative job loss rates. We find

- i) the absolute value of the LATE decreases with q ,
- ii) the estimate on simulated data tends to increase with q .

Table 13: Robustness, Quebecois case

Fixed parameters	q	0.005	0.01	0.025	0.05	0.1
Calibrated parameters	α	0.133	0.298	0.526	0.541	0.44
	m	0.0163	0.0293	0.0736	0.148	0.292
Employment rate (%)	27-28	67.5	67.5	67.4	67	66.6
	28-29	66.1	66.2	66.4	66.7	66.6
	29-30	64.2	64.2	64.4	64.8	65.4
	30-31	61.9	61.7	61.2	60.7	60.1
	31-32	59.8	59.6	59.3	59.1	59.2
	32-33	57.8	58	58.4	58.8	59.2
Rate of loosing benefits	λ	0.0888	0.0908	0.0975	0.11	0.139
Job-finding rate at 27	$ms(27, b_0^{min})$	0.00899	0.0205	0.0568	0.125	0.294
Job-finding rate after 30	$m\sigma(b_0^{max})$	0.00328	0.0111	0.0354	0.078	0.174
Estimate on actual data		-0.049	-0.049	-0.049	-0.049	-0.049
Estimate on simulated data		-0.028	-0.037	-0.044	-0.048	-0.053
LATE		-0.19	-0.14	-0.097	-0.082	-0.074

Note. The job-finding rates correspond to workers nonentitled to unemployment insurance. The estimates corresponds to the estimated effects from simulated data with a linear spline specification. The estimate on actual data comes from Lemieux and Milligan (2008) reported in Table 3.

Table 14: Robustness, French case

Fixed parameters	q	0.005	0.01	0.025	0.05	0.1
Calibrated parameters	α	0.106	0.262	0.355	0.330	0.332
	m	0.0217	0.0408	0.106	0.214	0.43
Employment rate (%)	24-25	71.4	71.5	71.8	72	72.1
	25-26	68.1	68	67.6	67.1	66.6
	26-27	65.1	65.1	65.3	65.5	65.9
Rate of losing benefits	λ	0.0351	0.0369	0.0429	0.0546	0.086
Job-finding rate at 22	$ms(22, b_0^{min})$	0.0224	0.0429	0.126	0.286	0.587
Job-finding rate after 25	$m\sigma(b_0^{max})$	0.00249	0.0139	0.0502	0.115	0.242
Estimate on actual data		-0.049	-0.049	-0.049	-0.049	-0.049
Estimate on simulated data		-0.076	-0.089	-0.073	-0.056	-0.054
LATE		-0.33	-0.2	-0.11	-0.085	-0.071

Note. The job-finding rates correspond to workers nonentitled to unemployment insurance. The estimates corresponds to the estimated effects from simulated data with a linear spline specification in years. The estimate on actual data comes from Bargain and Doorley (2011) reported in Table 6.

Table 15: Robustness, Austrian case

Fixed parameters	q	0.005	0.01	0.025	0.05	0.1
Calibrated parameters	α	0.00784	0.0118	0.0234	0.042	0.077
	m	0.0269	0.0408	0.0826	0.152	0.282
Job-finding rate at 49, with λ_{max}	$ms(49, b_{UI})$	0.218	0.217	0.212	0.205	0.195
Job-finding rate after 50, with λ_{max}	$m\sigma(b_{UI}, \lambda_{max})$	0.22	0.22	0.221	0.222	0.224
Job-finding rate after 50, with λ_{min}	$m\sigma(b_{UI}, \lambda_{min})$	0.13	0.13	0.13	0.13	0.13
Estimate on actual data		14.8	14.8	14.8	14.8	14.8
Estimate on simulated data		14.4	14.3	14.2	14	13.7
LATE		14.3	14.2	14	13.6	13

Note. The job-finding rates correspond to workers receiving unemployment insurance. The estimates corresponds to the estimated effects from simulated data with a linear spline specification in years. The estimate on actual data comes from Lalive (2008) reported in Table 9.

Table 16: Robustness, German case

Fixed parameters	q	0.005	0.01	0.025	0.05	0.1
Calibrated parameters	α	0.361	0.48	0.775	1.18	1.86
	m	0.0618	0.0787	0.108	0.129	0.137
	λ_{max}	0.0943	0.0942	0.094	0.0937	0.0932
	λ_{min}	0.066	0.0659	0.0657	0.0655	0.0652
Job-finding rate at 40, with λ_{max}	$ms(40, b_{UI})$	0.0548	0.0547	0.0545	0.0542	0.054
Job-finding rate after 42, with λ_{max}	$m\sigma(b_{UI}, \lambda_{max})$	0.055	0.0551	0.0553	0.0556	0.0561
Job-finding rate after 42, with λ_{min}	$m\sigma(b_{UI}, \lambda_{min})$	0.0517	0.0518	0.0519	0.0521	0.0524
<i>UI duration</i>						
Estimate on actual data		1.78	1.78	1.78	1.78	1.78
Estimate on simulated data		1.80	1.80	1.80	1.80	1.80
LATE		1.79	1.78	1.76	1.73	1.70
<i>Nonemployment duration</i>						
Estimate on actual data		0.78	0.78	0.78	0.78	0.78
Estimate on simulated data		0.80	0.81	0.81	0.82	0.82
LATE		0.75	0.70	0.59	0.48	0.36

Note. The job-finding rates correspond to workers receiving unemployment insurance. The estimates corresponds to the estimated effects from simulated data with a 2-year bandwidth. The estimate on actual data comes from Schmieder et al. (2012a) reported in Table 12.