

Job Ladders and Labor Productivity Dynamics*

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Abstract

Recent microeconomic evidence suggests that the composition of match qualities among employed workers deteriorates in recessions. We interpret this as evidence of the destruction of valuable job ladders, a form of intangible capital. This paper builds an equilibrium search model with a stylized job ladder to study the relationship between the composition of match qualities and the dynamics of aggregate productivity and output. Our results show that shocks which destroy high quality matches and their associated job ladders can have significant and very persistent effects on labor productivity and output, even after aggregate employment has recovered.

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1 Introduction

The allocation of workers across different types of jobs can play a fundamental role in explaining income levels as well as the dynamics of business cycles fluctuations. Given technology and factor endowments, if every worker was always allocated to her more productive use, output would be maximized as the labor allocation would be optimal in any given moment. Across time, if workers could always move immediately to their new most productive use, the real effects of fluctuations associated to sectorial shocks and changes in relative prices would be smoothed out. However, the presence of search frictions can imply that, even with fully flexible wages, workers might not be allocated to their most productive use, and that the adjustment of the labor market to cyclical fluctuations will not be automatic.

One way to rationalize the dynamics of the allocation of workers to jobs are job ladders. The basic idea of the job ladder¹ is that workers are more productive in some jobs than others, and that finding better jobs can take time. Workers employed in jobs where their productivity is relatively poor - low quality matches - will search on-the-job for better options. Once they find a better match, they'll move up to the job ladder. The process will continue until the worker's productivity is such that the expected value of search is low enough, and the worker wishes to stay in her current job, or until the match breaks and the worker is pushed to unemployment. The latter can bring the worker back to the bottom of the ladder and, at least in expected terms, search frictions will imply that climbing back can take time.

The distribution of job ladders for individual workers, associated to the quality of their matches, has direct implications for aggregate productivity. The structure of existing job ladders in the economy, reflecting the job histories of individual workers and the time they have invested for jobs that are better matches, is a form of intangible capital. For the same labor endowment and production technologies, an economy with richer ladders - a better labor allocation - will have higher productivity and output. By a similar argument, the effects of an aggregate shock on productivity dynamics will depend on its impact on the distribution of match qualities. A recent paper by Mueller (2017) presents evidence that suggests that, in fact, the distribution of match qualities deteriorates in recessions, as the destruction of high wage workers increases proportionally more than the destruction of low wage workers. Conceptually, this can have first order effects on the evolution of productivity and output, as the intangible capital associated to high quality matches is destroyed. Aggregate shocks that increase unemployment but also destroy a large share of high quality matches can have severe and persistent effects on productivity, both due to the direct effect of destruction and to the fact that rebuilding the individual job ladders can take time. This is, the overall efficiency of labor allocations in the economy can not only deteriorate on impact, but can remain below its initial level for a relatively long period as finding those high quality matches is a slow process. Even after unemployment rate has returned to its initial level, aggregate productivity and output can remain below as long as workers remain mismatched.

This paper elaborates on this argument by building an equilibrium search model with a stylized job ladder to study the relationship between the allocation of workers across jobs and the dynamics of aggregate productivity and output. In our model, while workers are *ex ante* homogeneous, their productivity varies across different matches. The productivity of a match in a given

¹See Moscarini and Postel-Vinay (2018) for a review on this subject.

period is determined by: a measure of aggregate productivity, which proxies the economy’s cyclical stance; a permanent idiosyncratic component, which measures the match’s persistent overall quality; and a temporary idiosyncratic component. Workers can search on the job for matches of better quality, and will move up the job ladder if they find one. The model also allows for match separations that leave the worker unemployed, either exogenously - as in the standard separation shock - or endogenously in response to aggregate productivity and the temporary idiosyncratic shock.

We calibrate the model using a novel dataset of matched employer-employee tax records for the Chilean economy. This data, which is a census of all formal wage employment in the economy, allows us to get a rich characterization of the structure of job qualities, job transition flows and probabilities, and the distribution of wages. We then use the calibrated model to simulate the effect of two separate shocks, each of them temporarily increasing unemployment in 1%. The first shock is a temporary reduction in aggregate productivity, which has a direct impact on endogenous separations across different types of jobs. The second shock is an exogenous destruction shock, which temporarily increases job separations across all types of jobs.

The results of this simulation exercise illustrate the main theoretical insights of the model. The large value of high quality matches for workers and firms implies that they have a high tolerance to temporary productivity drawbacks. Thus, an aggregate productivity shock has a smaller effect on the endogenous separation rate of high quality jobs relative to low quality jobs, unlike the empirical evidence in Mueller (2017). In consequence, the productivity shock changes the composition of matches, increasing average quality and dampening the effect on aggregate labor productivity. Job ladders are too valuable and costly to replace, so destruction is rarely optimal. Results are dramatically different in the case of the separation shock, which destroys jobs across the quality spectrum. In that case, the economy loses part of its intangible asset, and recovery is very slow as search frictions delay the reconstruction of the high quality matches that were destroyed by the shock. In this context, one must interpret the separation shock as the reduced form of an underlying economic mechanism which is not explicitly incorporated in the model. As we discuss later, financial frictions, which can force firms and workers to destroy valuable matches when hit by temporary shocks, are a natural candidate.

In that regard, this paper contributes to previous literature by analyzing the macroeconomic implications of job destruction shocks that deteriorate the composition of match qualities, which is consistent with the microeconomic evidence, but which is typically not an equilibrium outcome in search and matching models. We show that such shocks can have persistent and quantitatively large effects in labor productivity and output.

2 Related Literature

The dynamic process of reallocation of factors throughout the business cycle is a long standing issue of research and debate. Ever since the insights of Schumpeter (1939) the hypothesis of crises-driven creative destruction has given a silver lining to recessions, where the most inefficient arrangements in the economy are wiped out, improving the average efficiency.² In some cases, re-

²Arrangements can be understood as a wide definition of economic relationships. Some authors study the cleansing of entire firms during crises as Lee and Mukoyama (2007) do, while Mortensen and Pissarides (1994) for

allocation of factors to more productive uses also increases because the opportunity cost to do so is low. Caballero and Hammour (1994, 1996) among others formalize this mechanism in a model and call it the *cleansing effect* of recessions. It is natural to think of this mechanism working in matching models of the labor market such as Mortensen and Pissarides (1994), where search frictions may prevent instant formation of highly efficient firm-worker matches.

However, the empirical evidence of the cleansing effect in the labor market is rather ambiguous. On one hand, there is evidence of cleansing in manufacturing from late 1940s to 1990s, and for the entire private sector from 1990s to early 2000s, according to Davis and Haltiwanger (1990, 1992, 1999); Davis et al. (2006, 2012) respectively.³ On the other hand, many studies do not find this cleansing effect at all: Mustre-del Río (2012) finds no relationship between match quality and the business cycle conditions when said match ends; Liu and Tybout (1996), Nishimura et al. (2005) and Hallward-Driemeier and Rijkers (2010) find no or little relationship between firm productivity and exit; Baily et al. (1992) and Griliches and Regev (1995) do not find evidence of an increased contribution of reallocation to productivity growth in recessions.

This may be due to other mechanisms at work operating in recessions. In fact, many authors have opposed the Schumpeterian view, positing that recessions have a scarring effect: severe negative and long lasting consequences for output and productivity. This is documented for financial and political crises in Cerra and Saxena (2008) and for all crises in Cerra and Saxena (2017), and in particular the labor share decreases and recovers only slowly and partially, especially for financial crises according to Diwan (2001). Models such as Barlevy (2002, 2003) and Ouyang (2009) posit different mechanisms consistent with these pieces of evidence. In particular, Barlevy (2002) uses a search and matching model of the labor market with on the job search and studies the sully effect of recessions: firms post less vacancies, which implies fewer job-to-job transitions which diminishes productivity-enhancing reallocation, and thus match efficiency and average labor productivity. He finds that it can be substantially larger than the cleansing effect, also incorporated in his model through endogenous separation of matches.

Evidence at the micro level supports the sully effect, as jobs created during recessions are usually less productive, less well-paid, and less likely to last: see Bills (1985), Shin (1994), Bowlus (1995) and Davis et al. (1996). However, standard models with endogenous separation such as Barlevy's cannot explain the evidence in Mueller (2017) discussed in the previous section. As mentioned, Mueller (2017) finds that in recessions the pool of unemployed shifts toward workers with high wages in their previous job. Moreover, this compositional shift is almost entirely driven by separation rates instead of job finding rates, due to the high cyclicality of the former for high wage workers. That is, in recessions we can expect the destruction of high wage workers to increase proportionally more than the destruction of low wage workers, which is opposite to what most models that study endogenous separations and the cleansing effect predict.⁴

example focus on worker-firm matches. This paper studies the latter arrangements.

³It must be noted that this effect seems to lessen for the great financial crisis, and there are theoretical reasons to think it lessens with financial crises in general: Foster et al. (2016), Barlevy (2003) and Eslava et al. (2010).

⁴Models such as Barlevy (2002), den Haan et al. (2000) and the benchmark model in Mueller (2017) assume that matches break when their surplus is below a certain threshold, which implies (under wages set by Nash-bargaining, which is standard) that workers with low wages (say, the lower half of the wage distribution) will suffer stronger separation rates during recessions, while workers with high wages will continue to suffer separations only an exogenous rate which is constant.

If wages are related to productivity, then the job destruction margin may also be a source of scarring in the labor market during recessions. We contribute to the literature by filling this gap, as we study job destruction in a standard search and matching model augmented with a job ladder and endogenous separation. We show that the sullyng effect as understood by Barlevy (2002) is quantitatively small, and job destruction shocks consistent with Mueller’s findings are needed to account for the scarring effect of recessions.

3 Model

3.1 Setup

The model is a discrete time version of the standard search framework by Mortensen and Pissarides (1994), with three main departures. First, there is wage rigidity as in Hall (2005).⁵ Second, the model allows for both endogenous as well as exogenous match separations, as in den Haan et al. (2000). Third, and most importantly for our main argument, there is a job ladder as workers may move from their current job towards higher productivity matches. Parameter specifications guide the intensity of these three departures, therefore our model nests the standard discrete time DMP search and matching model.

There are two types of ex ante homogeneous agents: workers and firms. There is one homogeneous good, produced by one-to-one firm-worker teams, who find each other through a matching function as in Mortensen and Pissarides (1994). All agents are risk neutral and discount the future by a factor of $\beta \in (0, 1)$.

There exists a continuum of workers of mass 1 which are either employed or unemployed. Unemployed workers, with aggregate mass u_t , receive utility b from leisure and search costlessly for a job. Employed workers receive a per period payment in the form of a wage. Employed workers can also search costlessly for a new job. All job searchers, whether employed and unemployed, encounter jobs at random at an endogenous probability p_t per unit of search ability. However, we allow search ability to differ depending on the worker’s status. We normalize the search ability of unemployed workers to 1, while the search ability of employed workers is s . Thus, the job finding rate for employed workers is $p_t s$. Search ability is exogenous in the model, and can reflect that employed workers have access to better networks or search technologies (which would imply $s > 1$) or that they implicitly can allocate less time search activities (which would suggest $s < 1$).

Firms own the match’s production flow, Y , but compensate the worker with a wage. Firms can post vacancies v_t at a cost of ψ per period. The probability of filling a vacancy in any given period is q_t .

⁵We incorporate this feature to obtain a reasonable response of unemployment to productivity shocks, allowing us to compare the size of the change in TFP to the change in match quality, which Barlevy (2002) cannot do properly

3.2 Output and Productivity in Individual Matches

The idiosyncratic productivity of any active match is given by a permanent component, y^i , and a transitory component, x . The permanent component is a random variable that is perfectly revealed when both agents meet for the first time, and can take two values, $y^h > y^l > 0$. The ex ante probability of the permanent component in a match being y^h is λ . The permanent component reflects the overall quality of the match, and can be thought as summarizing how a particular job suits a particular worker.

We refer to matches with permanent component y^h as high quality matches, while matches with the y^l component are low quality. The permanent component remains fixed throughout the existence of the match. In the model, job ladders can be defined as transitions between these two match qualities: workers that successfully move up the job ladder are those that are able to find a job that suits them better, reallocating towards a high quality match.

The transitory component changes every period and is drawn from a quality-specific distribution with cdf $F^i(x)$, $i \in \{h, l\}$. Throughout the paper, we assume that the transitory component is *i.i.d.*, so its expected value for the next period is independent of the current realization. The transitory component is drawn and known every period before production takes place.

Every period the output of a match with permanent productivity y^i and transitory productivity x is given by $Y(A_t, y^i, x)$, where Y is an increasing function in all its arguments and A_t is the aggregate productivity of the economy. A_t follows a log AR(1) process with steady state value of $\bar{A}$, persistence ρ_A , and volatility σ_A .

Let $\hat{Y}_t^i(A_t)$ be the expected value of output of a match of quality $i \in \{h, l\}$ ⁶ across x (further explained in definition 1). We assume for all t

$$E_t \left\{ \hat{Y}_{t+1}^h(A_{t+1}) \right\} > E_t \left\{ \hat{Y}_t^l(A_t) \right\}, \quad (1)$$

where E_t is the expectation operator. Then, the expected value of output in high-quality matches is always higher than the expected output for low-quality matches.

We argue below that, conditional on the quality of the match and the economy's aggregate productivity, there exists a lower threshold on the temporary idiosyncratic component, κ_t^i , $i \in \{h, l\}$, such that all matches with temporary draws below their corresponding threshold will optimally break up prior to production. This will be the source of endogenous job destruction in the model.

The exact timing of the model in any given period is as follows:

1. The starting set of matches is formed by previous matches that were active in production the last period, as well as new matches formed at the end of the previous period that have not yet had a chance to produce. The permanent component of all these matches is fully known.

⁶For the entire paper, i is used to refer to both types of match quality, $i \in \{h, l\}$

2. Nature reveals the realizations of aggregate productivity A_t and idiosyncratic temporary productivities x across all matches for this period..
3. A share δ_t of current matches across both types is destroyed exogenously. Given the optimal cutoffs for the transitory components, shares $F^i(\kappa_t^i)$ for each type $i \in \{h, l\}$ are destroyed endogenously. Job destruction, either exogenous or endogenous, also impacts newly formed matches coming from the end of the previous period, even if they never had the chance of engaging in production activities.
4. Unemployed workers, including the newly unemployed whose matches were destroyed at the previous stage, receive b . Surviving matches produce and bargain over wages.
5. After production takes place, workers (unemployed and employed) can search for a job. Firms place vacancies. New matches are formed at the end of the period.

3.3 Labor Market Dynamics

Given the nature of the idiosyncratic components, assumption (1), and the wage bargaining process detailed below (for now suffice to say that higher match output implies higher value for the worker), the existence of on the job search will be depend solely on the permanent component.

Workers in high quality matches have no incentive to look for a new job, as our assumptions imply that the expected value of any new high quality match is identical to their current match, while the expected value of any low quality match is strictly smaller. Thus, in this simplified setup, there are no direct transitions between jobs of the same permanent quality, or job to job movements downwards the job ladder, as expected gains are always non-positive. On the other hand, workers in low quality jobs will always search, as the expected value of search is strictly positive given they are always better off in expected terms if they can land a high quality match.

Therefore, in any given period t the number of workers searching for a job, adjusted for their search ability, is $u_t + sn_t^l$, where n_t^l is the mass of workers employed in a low quality match. Given vacancies, which are ex ante identical, labor market tightness is $\theta_t = v_t / (u_t + sn_t^l)$.

As in most of the literature, the matching technology is given by a Cobb-Douglas production function, where the number of matches is given by the function $m_t = \Lambda (u_t + sn_t^l)^\kappa (v_t)^{1-\kappa}$, with $\Lambda > 0$, $\kappa \in (0, 1)$. This function defines the equilibrium job finding probability as $p_t = \Lambda \theta_t^{1-\kappa}$ and the vacancy filling probability as $q_t = \Lambda \theta_t^\kappa$.

At the beginning of the next period there are $\tilde{n}_{t+1}^h$ workers in matches with permanent component y^h and $\tilde{n}_{t+1}^l$ workers in matches with permanent component y^l . The former are given by all high quality matches that were active the previous period, plus a share λp_t of those that looked for a job, both from unemployment and on the job in low quality matches. Therefore, $\tilde{n}_{t+1}^h$ is given by

$$\tilde{n}_{t+1}^h = n_t^h + \lambda p_t (u_t + sn_t^l). \quad (2)$$

Similarly, the mass of workers in low quality matches at the beginning of the period, $\tilde{n}_{t+1}^l$, is given by:

$$\tilde{n}_{t+1}^l = (1 - \lambda p_t s) n_t^l + (1 - \lambda) p_t u_t. \quad (3)$$

A share $\lambda p_t s$ of the workers in low quality matches in t moved up the job ladder to a better match, while $(1 - \lambda) p_t$ of the unemployed workers found a job in a low quality match. Including job destruction yields the dynamics of employment in matches with permanent productivity $i \in \{h, l\}$:

$$n_{t+1}^i = \rho_{t+1}^i \tilde{n}_{t+1}^l, \quad (4)$$

where

$$\rho_{t+1}^i = (1 - \delta_{t+1}) (1 - F^i(\kappa_{t+1}^i)) \quad (5)$$

is the probability of match survival, which takes into account both the exogenous (δ_t) and endogenous ($F^i(\kappa_{t+1}^i)$) probabilities of job destruction. δ_t follows a log AR(1) process with steady state value of $\bar{\delta}$, persistence ρ_δ , and volatility σ_δ .

3.4 Value Functions

Before stating the formal recursive equations of the model, the following definition is useful:

Definition 1 For $i = \{h, l\}$, let κ_t^i be the productivity cutoff such that whenever $x < \kappa_t^i$, the match is destroyed. Then for any variable $X_t(x)$,

$$\hat{X}_t^i := \int_{\kappa_t^i}^{\infty} \frac{X_t^i(x)}{1 - F^i(\kappa_t^i)} dF^i(x)$$

is its expected value across x .

The value function of being employed in an active match with permanent productivity h at time t and transitory productivity x , $W_t^h(x)$, is:

$$W_t^h(x) = w_t^h(x) + \beta E_t \left\{ \rho_{t+1}^h \widehat{W}_{t+1}^h + (1 - \rho_{t+1}^h) U_{t+1} \right\}, \quad (6)$$

The first term on the right is the current wage, $w_t^h(x)$, and the second is the continuation value, which admits two cases. First, with probability $1 - \delta_{t+1}$ the match survives the exogenous destruction shock. However, only $1 - F^h(\kappa_{t+1}^h)$ of them will actually draw a transitory component x good enough to keep the match alive, so the continuation value is given by the expected value of a high productivity match next period $\widehat{W}_{t+1}^h$. Secondly, in any other case, the match is broken and the worker goes to unemployment with value function U_{t+1} .

In the case of a worker in an active low productivity match at time t the value function is given by:

$$W_t^l(x) = w_t^l(x) + \beta E_t \left\{ \lambda p_t s \rho_{t+1}^h \widehat{W}_{t+1}^h + (1 - \lambda p_t s) \rho_{t+1}^l \widehat{W}_{t+1}^l + (1 - \lambda p_t s \rho_{t+1}^h - (1 - \lambda p_t s) \rho_{t+1}^l) U_{t+1} \right\}. \quad (7)$$

As before, the first term in the right is the current wage and the second the continuation value. These workers will search with ability s for a new job, such that $\lambda p_t s$ of them will move up the job ladder to a better match, and $(1 - \lambda p_t s)$ will not. However, only with probability ρ_{t+1}^i the match of quality $i \in \{h, l\}$ survives both kinds of job destruction, in the same fashion as described above. These matches will actually produce, holding a continuation value of $\widehat{W}_{t+1}^i$. The rest will go to unemployment.

The value for an unemployed worker is:

$$U_t = b + \beta E_t \left\{ \begin{array}{l} p_t \left(\lambda \rho_{t+1}^h \widehat{W}_{t+1}^h + (1 - \lambda) \rho_{t+1}^l \widehat{W}_{t+1}^l \right) \\ + (1 - p_t \lambda \rho_{t+1}^h - p_t (1 - \lambda) \rho_{t+1}^l) U_{t+1} \end{array} \right\}, \quad (8)$$

where b is the flow payoff from non-employment. With probability p_t the unemployed will contact a firm. A proportion λ of them will end up in a high quality match, while the rest will be matched to a low quality job. If there is no match, the worker will be unemployed again next period.

The value for the firm of an active job is determined by the output of the match net of the wage paid to the worker, plus its discounted continuation value. The latter is given by the chance that the match is alive next period times the expected value of having an active position. That is:

$$J_t^h(x) = Y(A_t, y^h, x) - w_t^h(x) + \beta E_t \left\{ \rho_{t+1}^h \widehat{J}_{t+1}^h \right\}, \quad (9)$$

for a high quality match, and

$$J_t^l(x) = Y(A_t, y^l, x) - w_t^l(x) + \beta E_t \left\{ (1 - \lambda p_t s) \rho_{t+1}^l \widehat{J}_{t+1}^l \right\}, \quad (10)$$

for a low quality one. On the other hand, the value of an open vacancy is given by

$$V_t = -\psi + q_t \beta E_t \left\{ \lambda \rho_{t+1}^h \widehat{J}_{t+1}^h + (1 - \lambda) \rho_{t+1}^l \widehat{J}_{t+1}^l \right\}, \quad (11)$$

where ψ is the cost of posting a vacancy. Free entry implies $V_t = 0$, which leads to the standard job creation condition.

Now, let $S_t^i(x)$ represents the surplus of a match with permanent productivity y^i and transitory productivity x . The surplus given by:

$$S_t^i(x) = J_t^i(x) + W_t^i(x) - U_t. \quad (12)$$

Finally, the job destruction conditions, derived in appendix A from the condition $S_t^i(\kappa_t^i) = 0$, are given by:⁷

⁷In this model, job destruction conditions consider only socially efficient separations, as they are determined considering match surplus instead of worker or firm value functions separately. This is irrelevant with nash bargaining and flexible wages, however with rigid wages it may imply - given a certain sequence of shocks - that continuation of the match will not be optimal for some workers and/or firms, which calls for occasionally binding

$$Y(A_t, y^h, \kappa_t^h) = b - \theta_t \psi - \beta E_t \left\{ \rho_{t+1}^h \widehat{S}_{t+1}^h - p_t \left[\lambda \rho_{t+1}^h \widehat{S}_{t+1}^h + (1 - \lambda) \rho_{t+1}^l \widehat{S}_{t+1}^l \right] \right\}, \quad (13)$$

$$Y(A_t, y^l, \kappa_t^l) = b - \theta_t \psi - \left\{ \begin{array}{l} (1 - \lambda p_t s) \rho_{t+1}^l \widehat{S}_{t+1}^l + \lambda p_t s \rho_{t+1}^h \left(\widehat{W}_{t+1}^h - U_{t+1} \right) \\ - p_t \left[\lambda \rho_{t+1}^h \widehat{S}_{t+1}^h + (1 - \lambda) \rho_{t+1}^l \widehat{S}_{t+1}^l \right] \end{array} \right\}. \quad (14)$$

Given the difference in their permanent component, for a common distribution of the transitory component across both types of matches the social value of high quality matches is always larger. This implies that the optimal policy function will be such that the productivity cutoff for low quality is always more strict. In consequence, high quality matches have a higher tolerance to temporary negative shocks. Thus, in equilibrium endogenous separations will be more frequent in low quality matches.

Moreover, as workers can additionally separate from low quality jobs to move up the job ladder, the average tenure in high quality jobs will always be larger, coherent with previous papers with heterogeneous matches such as Jovanovic (1979), Neal (1999) or Krolikowski (2017). This insight will be important for our calibration strategy.

3.5 Bargaining and Wages

Each period, every match Nash-bargains over the wage that the firm pays the worker. The outcome of such a negotiation will be a notional wage $\widetilde{w}_t^i(x)$ for $i = \{h, l\}$. However, actual wages will be given by an adaptive rule:

$$w_t^i(x) = \rho_w w_{t-1}^i(x) + (1 - \rho_w) \widetilde{w}_t^i(x), \quad (15)$$

where $\rho_w \in [0, 1)$ is a wage rigidity parameter in the spirit of Hall (2005).⁸ As is common in the literature we assume a linear surplus sharing rule, where $\widetilde{w}_t^i$ is such that the worker receives a fraction η of the match surplus and the firm the remaining fraction. This is:

$$\widetilde{W}_t^i(x) - U_t = \eta S_t^i(x), \quad (16)$$

$$\widetilde{J}_t^i(x) = (1 - \eta) S_t^i(x), \quad (17)$$

where $\widetilde{W}_t^i$ and $\widetilde{J}_t^i$ are the respective value functions considering notional (nash-bargained) wages.

Both employed and unemployed workers negotiate their wage using unemployment as their outside

constraints as in Tortorice (2013). Our solution algorithm does not consider such constraints, however we ran a simulation exercise calculating the wage adjustment needed to comply with this issue: $\widehat{w}_t^l$ would change 0.33% and $\widehat{w}_t^h$ 0.007% on average. Therefore correcting this issue would yield only negligible changes in the model's dynamics, and we disregard this issue to be of much importance, as Krause and Lubik (2007) do and Hall (2005) suggests. Simulation results upon request.

⁸Combining idiosyncratic productivity with rigid wages yields the question of whether the wage norm (the wage multiplied by ρ_w) should consider the same idiosyncratic productivity of the current match, x , or an average of said component, such as $\widehat{x}_t^i$. We follow Tortorice (2013) in choosing the former option.

option. We also assume that employed workers that receive an outside offer cannot use it to negotiate with their current employer. These assumptions are similar to the ones in Krolkowski (2017).

Wages are set period by period. Following the derivations in appendix A, notional flow wages are a weighted average of two components: the period production plus recruitment savings, and unemployment benefits.⁹

$$\tilde{w}_t^h(x) = \eta [Y(A_t, y^h, x) + \theta_t \psi] + (1 - \eta) b, \quad (18)$$

$$\tilde{w}_t^l(x) = \eta [Y(A_t, y^l, x) + \theta_t \psi] + (1 - \eta) b - \eta (1 - \eta) \beta E_t \left\{ \lambda p_t s \rho_{t+1}^h \hat{S}_{t+1}^h \right\}. \quad (19)$$

Finally, average wages are given by:

$$w_t = \frac{n_t^h \hat{w}_t^h + n_t^l \hat{w}_t^l}{n_t}. \quad (20)$$

Note that:

$$\frac{dw_t}{w_t} = \frac{\hat{w}_t^h - \hat{w}_t^l}{w_t} d\chi_t + \zeta_t^w \frac{d\hat{w}_t^h}{\hat{w}_t^h} + (1 - \zeta_t^w) \frac{d\hat{w}_t^l}{\hat{w}_t^l}, \quad (21)$$

where $\chi_t = \frac{n_t^h}{n_t}$ is the share of workers in a high quality match, and $\zeta_t^w = \frac{n_t^h}{n_t} \frac{\hat{w}_t^h}{w_t}$ and $1 - \zeta_t^w = \frac{n_t^l}{n_t} \frac{\hat{w}_t^l}{w_t}$ are the contributions of high and low matches to the change in average wages. The first term on the right reflects the job ladder.

3.6 Output and Productivity

Assume that the function Y is linear on x . Then, total output is given by

$$Y_t = n_t^h Y(A_t, y^h, \hat{x}_t^h) + n_t^l Y(A_t, y^l, \hat{x}_t^l). \quad (22)$$

Measured TFP for the economy, which in this context is just average labor productivity (ALP), is given by

$$ALP_t = \frac{Y_t}{n_t} = \chi_t ALP_t^h + (1 - \chi_t) ALP_t^l, \quad (23)$$

where $ALP_t^i = Y(A_t, y^i, \hat{x}_t^i)$.

Now, assume that the match production function is such that $Y(A_t, y^i, x) = A_t y^i + x$. Then, the growth rate of ALP (measured TFP) is given by

$$\frac{dALP_t}{ALP_t} = \underbrace{\zeta_t^A \frac{dA_t}{A_t}}_{\text{TFP effect}} + \underbrace{\frac{ALP_t^h - ALP_t^l}{ALP_t} d\chi_t}_{\text{Extensive margin}} + \underbrace{\zeta_t^{x^h} \frac{d\hat{x}_t^h}{\hat{x}_t^h} + \zeta_t^{x^l} \frac{d\hat{x}_t^l}{\hat{x}_t^l}}_{\text{Intensive margin}}, \quad (24)$$

⁹In the case of low quality matches, the last term indicates the gain in continuation value for low quality workers due to the opportunity of searching on the job, weighted by $(1 - \eta)$

where ζ_t are weights that add up to 1.¹⁰ ALP growth can be decomposed in three channels. First, the direct growth in the economy's aggregate TFP shock, A_t . The two additional channels come from changes in match quality through the composition of active matches. On one hand, an increase in the share of high quality matches in the economy, χ , has a direct impact in ALP as jobs with a higher permanent component are more productive on average. We call this the adjustment of the extensive margin of match quality. On the other hand, changes in the productivity cutoffs associated to the transitory components affect the average productivity levels of each type of match. For instance, eliminating a bigger chunk of the lower tail of the x distribution raises the average idiosyncratic productivity across both types of matches. We call this the adjustment of the intensive margin of match quality. Our closest benchmark model in the literature, Barlevy (2002), considers the extensive margin of adjustment, but not the intensive margin.

The cleansing effect of recessions¹¹ can be found most clearly in the intensive margin, as within each type of match, those with lower idiosyncratic productivity x are separated in recessions. But it can also be found in the extensive margin if in response to a shock a bigger proportion of low quality matches are destroyed relative to high quality matches, as we show in section 6. The sullyng effect of Barlevy (2002) can only be found in the extensive margin, as lower job-to-job transitions will result in a lower χ , not affecting the active distribution of x .

3.7 Equilibrium

An equilibrium in this economy consists of paths for unemployment u_t , employment in high and low quality matches n_t^h and n_t^l , vacancies v_t , actual wages w_t^i and notional wages $\tilde{w}_t^i$, production y_t , productivity cutoffs κ_t^h and κ_t^l and a market tightness, θ_t , such that, given shocks A_t and δ_t :

1. u , n^h and n^l are consistent with labor market dynamics given θ_t and κ_t^i ;
2. v_t is the outcome of the free entry condition given θ_t , κ_t^i and $\hat{S}_{t+1}^i$;
3. κ_t^h and κ_t^l are consistent with the job destruction conditions given θ_t , κ_t^i and $\hat{S}_{t+1}^i$;
4. U_t , $W_t^i(x)$, $J_t^i(x)$ and $S_t^i(x)$ are as defined in subsection 3.4 given θ_t and κ_t^i ;
5. $\tilde{w}_t^i(x)$ divide the match surplus by nash bargaining given θ_t , κ_t^h and $\hat{S}_{t+1}^h$; and $w_t^i(x)$ result as the outcome of wage rigidity given $\tilde{w}_t^i(x)$ and $w_{t-1}^i(x)$;
6. The labor market clears, $n_t^h + n_t^l = 1 - u_t$;
7. Production is given by the production function (22).

4 Data

The availability of new administrative datasets has become a major asset for economic research. Such datasets provide rich micro data that can uncover new stylized facts, challenging many

¹⁰In particular, $\zeta_t^A = \frac{A_t(\chi_t y^h + (1-\chi_t)y^l)}{ALP_t}$, $\zeta_t^{x^h} = \frac{\chi_t \hat{x}_t^h}{ALP_t}$ and $\zeta_t^{x^l} = \frac{(1-\chi_t)\hat{x}_t^l}{ALP_t}$

¹¹See Caballero and Hammour (1994) for early ideas and Barlevy (2002) for its incorporation in a model of search in the labor market

existing theories and prompting the creation of new ones. The data we use to calibrate our model corresponds to this kind: a 2005-2016 census of matched employer-employee records for the Chilean economy (see section 4 for a detailed description).

Along with using macroeconomic statistics from the Central Bank of Chile’s database, we calibrate the model using rich micro data from a 2005-2016 census of matched employer-employee records for the Chilean economy. Our data comes from Chile’s Internal Revenue Service (*Servicio de Impuestos Internos*, SII). The SII dataset has unique identifiers for both workers and companies, allowing us to track individuals and firms over time. SII uses identifiers that guarantee the anonymity of both firms and individuals. All formal firms in the country must report to the SII, so the data is a census of all firms and all employment relationships with a wage contract in Chile (58% of all active workers). Each firm must present an annual statement (DJ1887) reporting the sum of wages, overtime wages, labor earnings and any other similar income (excluding disability, pensions and retirement payments) for each individual worker with a labor contract. While the statement (and the income information) is annual, firms must also report the specific months in which each worker was employed in the firm. Thus, for any given month, we can identify the employment status of an individual worker, and a measure of her average monthly labor income in that year. This information can be complemented with additional tax forms (F22 and F29) to obtain additional information on the firm, such as sales, production costs, and capital. In the context of the model, the data set allows us to build up a detailed characterization of the distribution of monthly labor transitions, job flows and wages.

Some workers in DJ1887 may have more than one ongoing job relationship in any given month. Therefore, to construct transition data that matches the model, we must define a single main job for each worker-month. Thus, when more than one job relationship is active in any given month, we define the worker’s main job as the one with the largest overall tenure. If two or more jobs have the same overall tenure, we choose the job with the largest monthly wage.

Stylized Facts using microdata for the Chilean economy

A number of studies have already established several facts using this dataset. First, the existence of job ladders in the labor market, as described in section 1, is strongly suggested by the data. Figure 1 shows average wage gains or losses for job transitions during the sample period. Noticeably, transitions that are directly from one job to another are associated with significant wage gains (presumably, workers will switch jobs if they are moving up the ladder), while those workers that go through non-employment before finding another job frequently incur no gains and even losses. And this is not just a wage phenomenon as the conclusions stand when analyzing productivity: J-t-J transitions are associated with workers switching to more productive companies, which is not the case for J-U-J transitions, as seen in figure 2. The cyclical properties are also apparent, as the proportion of all transitions that correspond to job-to-job transitions falls during the 2009 crisis, and rises in the following boom.

But what happens if a worker falls off the job ladder? One of the cleanest kind of events to study this question are sudden firm closures. These will usually have no or little relation with worker characteristics, as they imply a job loss for all of the workers in the closing firm. Figure 3 shows that the wage losses after such an event are large and persistent, evidence which is in line with the findings of Krolikowski (2017). Such losses are also found to be more severe during a recession, as is the case for Chile for the years 2008-2009. It’s especially interesting to notice that

these losses are larger for workers with higher tenure, see figure 4. This will relate directly to the implications of our model, and it strongly suggests that workers higher up the job ladder suffer especially hard from falling off it. Implicitly, this assumes that workers with better jobs have longer tenure, which is an implication many job ladder models including ours, see Krolikowski (2017) for a discussion on this issue.

All of these data suggests that job ladders play a significant role at the micro level, explaining both strong wage gains and losses through a worker’s career. But do job ladders have macro implications? A quick look at job destruction series from other studies shows their significant countercyclicality, and large spikes during crises, as shown in figure 5. Another series we want to look at is labor productivity, as we expect changes in average match quality to be reflected in this indicator. From figure 6, we can observe strong losses in labor productivity growth during crises. Moreover, from stronger crises which have a financial component we can observe a very slow recovery of the level of labor productivity. This is specially the case of the Asian crises, which hit Chile hardest during 1999.

Summing up, the above evidence seems consistent with our model by strongly suggesting the presence of job ladders in the micro data. The macro series, on the other hand, are consistent with the idea that strong episodes of job destruction are not only relevant for individual workers, they may explain movements in average labor productivity as well, through a scarring of the labor market as many high quality matches may break during these events, and will be hard to recover in the future.

Reconciling these micro and macro facts drives the simulation exercises presented in the following sections.

5 Calibration

Parameters in the model are calibrated directly from the Chilean data, indirectly through the model’s steady state outcomes, or by following the standards from previous literature. See Table 2 in appendix C for a summary of the following section.

A central element in the calibration is how to define job ladders in the data. While the model clearly differentiates between high and low quality matches, this is an unobservable characteristic. What we can observe, however, are matches with high and low tenure. We can then make use of the model’s ability to generate a distribution of tenure in order to compute moments that match the data. See appendix B for the details regarding the model’s tenure distribution.

In the current calibration exercise, we define jobs with complete tenure of at least 60 months as high tenure jobs, and the rest as low tenure jobs. Once we have defined these two job categories, we can get moments on wages and transition probabilities to unemployment and across jobs. All moments are calculated using the data for 2011, which is at the middle of the sample period. The definition of high tenure jobs not only includes existing jobs in 2011 that already had a tenure of 60 months up that point, but also relatively newer jobs that attained that tenure in the future.

Table 1 summarizes the calibration strategy. The model is calibrated at a monthly frequency. The discount factor is set to $\beta = 0.9959$ implying a 5% real interest rate. As in most of the literature, the worker's bargaining power η is set at 0.5, and the matching function elasticity $\kappa = 0.5$ follows Hosios (1990). From previous work using Chilean data, the wage rigidity parameter is set to $\rho_W = 0.9398$.

We choose two functional forms: for the match production function, we choose $Y(A_t, y^i, x) = A_t y^i + x$. For the distribution of idiosyncratic productivities, we choose a normal distribution with mean μ_x and standard deviation σ_x^i . Following these choices, we make three normalizations: $\bar{A} = 1$, $y^l = 1$ and $\mu_x = 0$. Persistence and volatility for shocks A_t and δ_t are chosen for the simulation exercises in the next section.

Next, we impose the steady state values for the following endogenous variables of the model: from Chilean macro data $u = 0.073$, and from the literature $q = 0.3381$.¹² From appendix B which uses the DJ1887 micro data, we find $\chi = 0.5396$, $\rho^h = 0.9969$, $\rho^l = 0.9295$, and $p^{EE} = \lambda ps = 0.0031$ are consistent with observable moments. This delivers endogenously the parameter values $s = 0.9417$ and $\lambda = 0.0071$.

Finally, $\bar{\delta} = 0.002$, $\sigma_x^l = 1$ and $\sigma_x^h = 35.25$ are chosen to match relevant statistics from the micro data, such as the wage gains from job-to-job transitions and wage dispersion. The remaining parameters are obtained endogenously: $y^h = 1.8279$, $\Lambda = 0.3964$, $\psi = 0.3812$ and $b = 0.6583$. The last three are in line with plausible ranges in the literature, see for example Shimer (2005), Krause and Lubik (2006), Hagedorn and Manovskii (2008), Tortorice (2013), Krolikowski (2017).

Relevant statistics that are indirectly targeted include the wage gains from movements up the job ladder. The data shows a 36.28% average gain from low to high tenure jobs, which is matched by the corresponding model moment of 36.275%. The model also matches reasonably well wage volatility: standard deviation of log wages is 0.9153 against the empirical counterpart of 0.9333. This is a welcome surprise in the literature on job ladders, which typically faces a trade-off between correctly matching job transition gains and accounting for wage dispersion. See Mukoyama (2014) for a discussion on this issue.

6 Results

To study the effects of job destruction in this model, as well as the cleansing and sullyng effects discussed by the literature, we analyze the impact of two separate shocks that increase the unemployment rate. The first is a standard temporary reduction in aggregate productivity or TFP, A_t , while the second shock is a temporary increase in the exogenous job separation rate, δ_t . We show that, depending on the type of shock, the job ladder structure embedded in the model plays a distinct role, with relevant consequences in terms of the size and persistence of the effects on output.

Both shocks are normalized to generate a 1% increase in unemployment rate. The productivity shock is chosen to have a persistence matching Chilean data, $\rho_A = 0.9166$, while the destruction

¹²den Haan et al. (2000) use a vacancy filling probability of 71% for one quarter, which translates to 33.81% monthly.

shock is chosen to have a persistence of $\rho_\delta = 0.75$ in order to obtain unemployment dynamics similar to the case of the productivity shock.

Figure 7 tracks the evolution of average labor productivity, ALP (black line), using the decomposition in section 3.6 to separate the effects associated to the growth of aggregate TFP (blue), the intensive margin of match quality associated to the change in productivity cutoffs (yellow), and the extensive margin of match quality associated to movements up and down the job ladder (red). All units are measured in % deviations from steady state, while the x-axis indicates model periods (quarters). Figures 8 and 9 track the impulse response functions of relevant macro variables, where unemployment and χ_t are displayed as a %, while output is normalized to 1 in steady state.

The left hand panel of Figure 7 shows that, in response to a standard productivity shock, the reduction in average labor productivity is partially offset by both margins of match quality. Within each class of match quality, the intensive margin will work through more stringent productivity cutoffs, eliminating the least productive matches as they are no longer viable (their surplus is no longer positive). This is the cleansing effect of recessions found in most search and matching models of the labor market with endogenous separations. On the other hand, the increase in endogenous separations is stronger for low quality jobs, such that the composition of surviving job types shifts towards high quality matches: a cleansing effect through the extensive margin as well. The sully effect of Barlevy (2002) is present too: lower TFP will deter firms from posting vacancies, lowering labor market tightness and thus the probability of performing a job-to-job transitions, which will imply a sully of match quality, or reduction or worsening of the extensive margin in our terms. As in Barlevy's model, on impact a productivity shock produces a rise in match quality, meaning that the cleansing effect is stronger in the short run. Thus, the job ladder embedded in the model can buffer the effect of productivity shocks in the short run, as the optimal response of firms and workers in high quality matches in the face of an adverse temporary shock is to keep most of those matches due to their high option value, and fire low quality matches. Recall that this contrasts the evidence provided by Mueller (2017), as discussed in section 2.

When the productivity shock dies off, productivity cutoffs and vacancies go back to their steady state levels. As unemployed workers get rehired, the amount low quality matches, which are easier to get, move towards its initial level. However, the extensive margin of the labor composition exhibits a persistent, negative effect for an extended period of time. Following the shock, the number of high quality matches is reduced both due to an increase in their endogenous separation and a sully effect as described in Barlevy (2002). Even if the reduction is relatively small, because good quality matches are hard to replace, their share in the economy remains below its steady state level long after the productivity shock faded.

The compositional dynamics, however, drive a very small part of the movements following a productivity shock. Employment, output and the ALP are mainly driven by the direct effect of the productivity shock. For the most part, they all recover as the shock dies out. Barlevy (2002) posits that the sully effect will be enough to produce a significant impact on average labor productivity. In our model, standard productivity shocks are not enough to produce a significant and persistent scarring of the labor market, even considering the intensive margin, as firms will

optimally keep most of the high quality matches, which are valuable and difficult to replace¹³.

Now we look towards the second kind of shock in the model. The right hand panel of Figure 7 shows the impact of a job separation shock that exogenously destroys jobs equally across all match qualities. Figure 9 shows that the response of unemployment to this shock is quite similar, especially in the short run. But the similarities end at that: the extensive margin shows a large and very persistent fall in match quality, which drives the scarring of the labor market in ALP and output. The much larger proportion of high quality matches which was destroyed in this case drives the results, whereas the intensive margin only plays a small role at the beginning of the shock.¹⁴ And in this case, job ladders play a scarring role from the beginning, as the extensive margin contributes to a fall in ALP even in the short run. Unemployment recovers quickly to a level reasonably close to its steady state due to the recovery of low quality matches, while the share of high quality jobs in the economy recovers very slowly, which explains the persistent reduction in output and ALP. This is, the characteristics of the search process imply that building a stock of high quality matches takes time, as reallocation of workers towards more productive jobs - moving up the job ladder - is a slow process.

Finally, we look at the effects of both shocks with no persistence, that is $\rho_A = \rho_\delta = 0$. Shock sizes are also calibrated to generate a response of a 1% increase in the unemployment rate. The ALP decomposition is shown in figure 10. The main takeaway is that with job destruction shocks the ALP deterioration is persistent even if the underlying shock is not, as opposed to productivity shocks. In the latter case, ALP, output and employment all recover as the shock dies out, even if the shock lasted a single period. This further stresses the point that across the board job destruction has persistent effects due to the nature of the recovery instead of the nature of the shock. Note also that now the intensive margin plays no role whatsoever in the δ shock, as there is no anticipated rise in exogenous job destruction, therefore no reason to endogenously terminate more or less matches than in steady state.

Clearly, these exercises must be seen as a preliminary analysis, intended to provide an illustration of the qualitative features of the model and the data. The complete version of the paper must include a careful quantitative analysis, relying on alternative calibration strategies and a more precise assessment of the relevant shocks.

More fundamentally, while the intuition behind the differences between both shocks is very transparent, the economic forces that could underlie an exogenous separation shock need to be addressed more precisely. The productivity shocks suggests that high quality matches are hard to break endogenously, as their high continuation value prevents separation in the face of adverse temporary shocks. Thus, the model must incorporate additional elements to endogenously deliver separations across the board that are similar to the exogenous separation shock. Financial frictions and liquidity constraints, that can limit the ability of firms and workers to sustain short term losses, seems

¹³Notice that, even as the focus of Barlevy (2002) is the sullyng effect of productivity shocks on the labor force composition, he needs a long lasting productivity shock of 72% in order to generate a sullyng effect of only 1% 5 years after the shock is active.

¹⁴Productivity cutoffs also become more stringent in this case because as destruction rates are expected to stay high for some time due to shock persistence, the continuation value of matches falls and thus the entire surplus. This is not an issue which drives the results, as the scarring described here applies also to unexpected shocks, which do not have any effect in the intensive margin.

like a promising venue. They may permit sudden firm closures which may have effects similar as those documented in figure 4, where workers are fired regardless of any characteristic, such as tenure or productivity. Under that setup, financial crises that endogenously destroy a significant share of high quality jobs can have persistent effects on output and ALP through the destruction of job ladders. Notice that, under that setup, even if financial stress was a temporary event, it would have a persistent effect, as the technological restriction on building high quality jobs makes recovery slow. Even in the absence of further financial restrictions, rebuilding the job ladder is slow and costly. We plan to include this extension in the completed version of the paper.

7 Conclusions

In this paper we seek to understand the dynamics of labor productivity during the business cycle, focusing on the cleansing and scarring effects of recessions. Motivated by the evidence in Mueller (2017) we build a model to analyze and quantify the effects of job destruction during a recession, filling this gap in the literature. We argue that a standard search and matching model augmented with endogenous separation and job ladders is a suitable choice to study these effects. This model is able to replicate micro stylized facts reported in the introduction, such as significant wage gains associated to direct job transitions, and the large and persistent earnings losses for workers with high tenure who are displaced. But more importantly for our point: it can explain the difference between job destruction episodes which entailed only temporary losses in labor productivity, and those that presented very slow recoveries, such as the Asian crisis for the case of Chile.

We use this model to understand how the origin of the shock matters for productivity dynamics. Our results indicate, first of all, that standard productivity shocks cannot account for a sizable sullyng effect, as understood by Barlevy (2002) and contrary to his results. Second, we find that across the board job destruction shocks produce a lasting drop in average labor productivity through the extensive margin of match quality, even if the shock is temporary, because high quality matches lost to the shock are difficult to replace.

This suggests that standard search and matching models with a single value of match quality may miss a relevant state of the labor market: the distribution of jobs across the job ladder. In these models, higher unemployment leads to lower output because of a decrease in labor as a production factor, and this is independent of the shock which raised unemployment in the first place. We show that this is not the case in our model which considers job ladders, where shocks which destroy jobs across the board will produce a scarring in the labor market and output, through a lasting decrease in average match quality due to the difficulty in recovering high quality matches. In this case unemployment, even though it recovers quickly back to pre-crisis levels, is not a sufficient statistic to gauge the recovery of the labor market. Therefore studying a crisis through the lens of a standard DMP model may leave out information which could be valuable in assessing the consequences of the episode.

For future work, the economic forces that could underlie an exogenous separation shock need to be addressed more precisely. The productivity shocks suggests that high quality matches are hard to break endogenously, as their high continuation value prevents separation in the face of adverse temporary shocks. Thus, the model must incorporate additional elements to endogenously deliver separations across the board that are similar to the exogenous separation shock. Financial

frictions and liquidity constraints, that can limit the ability of firms and workers to sustain short term losses, seems like a promising venue. They may permit sudden firm closures which may have effects similar as those documented in figure 4, where workers are fired regardless of any characteristic, such as tenure or productivity. Under that setup, financial crises that endogenously destroy a significant share of high quality jobs can have persistent effects on output and ALP through the destruction of job ladders. Notice that, under that setup, even if financial stress was a temporary event, it would have a persistent effect, as the technological restriction on building high quality jobs makes recovery slow. Even in the absence of further financial restrictions, rebuilding the job ladder is slow and costly.

A Model Solution

The value of an open vacancy, combined with the free entry condition, leads to the job creation condition:

$$\frac{\psi}{q_t} = \beta E_t \left\{ \lambda \rho_{t+1}^h \widehat{J}_{t+1}^h + (1 - \lambda) \rho_{t+1}^l \widehat{J}_{t+1}^l \right\} \Leftrightarrow \theta_t \psi = p_t \beta E_t \left\{ \lambda \rho_{t+1}^h \widehat{J}_{t+1}^h + (1 - \lambda) \rho_{t+1}^l \widehat{J}_{t+1}^l \right\}. \quad (\text{A.1})$$

From equations (6), (8) and (9) we can construct the surplus for high quality matches, that is:

$$S_t^h(x) = J_t^h(x) + W_t^h(x) - U_t, \quad (\text{A.2})$$

$$S_t^h(x) = Y(A_t, y^h, x) - b + \beta E_t \left\{ \rho_{t+1}^h \widehat{S}_{t+1}^h - p_t \left[\lambda \rho_{t+1}^h \left(\widehat{W}_{t+1}^h - U_{t+1} \right) + (1 - \lambda) \rho_{t+1}^l \left(\widehat{W}_{t+1}^l - U_{t+1} \right) \right] \right\}. \quad (\text{A.3})$$

Now, replacing $\widehat{W}_{t+1}^h - U_{t+1}$ with $\widehat{S}_{t+1}^h - \widehat{J}_{t+1}^h$ and using (A.1), the surplus can be written recursively as:

$$S_t^h(x) = Y(A_t, y^h, x) - b + \theta_t \psi + \beta E_t \left\{ \rho_{t+1}^h \widehat{S}_{t+1}^h - p_t \left[\lambda \rho_{t+1}^h \widehat{S}_{t+1}^h + (1 - \lambda) \rho_{t+1}^l \widehat{S}_{t+1}^l \right] \right\}. \quad (\text{A.4})$$

Following analogous steps, the surplus for low quality matches can be written as:

$$S_t^l(x) = Y(A_t, y^l, x) - b + \theta_t \psi + \beta E_t \left\{ \begin{aligned} & (1 - \lambda p_t s) \rho_{t+1}^l \widehat{S}_{t+1}^l + \lambda p_t s \rho_{t+1}^h \left(\widehat{W}_{t+1}^h - U_{t+1} \right) \\ & - p_t \left[\lambda \rho_{t+1}^h \widehat{S}_{t+1}^h + (1 - \lambda) \rho_{t+1}^l \widehat{S}_{t+1}^l \right] \end{aligned} \right\}. \quad (\text{A.5})$$

Defining the thresholds for match destruction for $i = \{h, l\}$ as

$$\{\kappa_t^i\} : S_t^i(\kappa_t^i) = 0, \quad (\text{A.6})$$

we can use $x = \kappa_t^i$ in (A.4) and (A.5) to obtain the job destruction conditions:

$$Y(A_t, y^h, \kappa_t^h) = b - \theta_t \psi - \beta E_t \left\{ \rho_{t+1}^h \widehat{S}_{t+1}^h - p_t \left[\lambda \rho_{t+1}^h \widehat{S}_{t+1}^h + (1 - \lambda) \rho_{t+1}^l \widehat{S}_{t+1}^l \right] \right\}, \quad (\text{A.7})$$

$$Y(A_t, y^l, \kappa_t^l) = b - \theta_t \psi - \left\{ \begin{aligned} & (1 - \lambda p_t s) \rho_{t+1}^l \widehat{S}_{t+1}^l + \lambda p_t s \rho_{t+1}^h \left(\widehat{W}_{t+1}^h - U_{t+1} \right) \\ & - p_t \left[\lambda \rho_{t+1}^h \widehat{S}_{t+1}^h + (1 - \lambda) \rho_{t+1}^l \widehat{S}_{t+1}^l \right] \end{aligned} \right\}. \quad (\text{A.8})$$

We can also define match surpluses as:

$$S_t^i(x) = Y(A_t, y^i, x) - Y(A_t, y^i, \kappa_t^i). \quad (\text{A.9})$$

Notional wages can be obtained starting from (16) and (17), which yield:

$$(1 - \eta) \left(\widetilde{W}_t^i(x) - U_t \right) = \eta \widetilde{J}_t^i(x) \quad (\text{A.10})$$

Using equations (6), (8) and (9) with notional wages in (A.10) for $i = h$ yields

$$\begin{aligned} (1 - \eta) \left[\widetilde{w}_t^h(x) - b + \beta E_t \left\{ \rho_{t+1}^h \left(\widehat{W}_{t+1}^h - U_{t+1} \right) - p_t \left[\lambda \rho_{t+1}^h \left(\widehat{W}_{t+1}^h - U_{t+1} \right) + (1 - \lambda) \rho_{t+1}^l \left(\widehat{W}_{t+1}^l - U_{t+1} \right) \right] \right\} \right] \\ = \eta \left[Y(A_t, y^h, x) - \widetilde{w}_t^h(x) + \beta E_t \left\{ \rho_{t+1}^h \widehat{J}_{t+1}^h \right\} \right]. \end{aligned} \quad (\text{A.11})$$

Using (16) and (17) for the continuation values¹⁵ yields:

$$\begin{aligned} (1 - \eta) \left[\widetilde{w}_t^h(x) - b + \beta E_t \left\{ \rho_{t+1}^h \eta \widehat{S}_{t+1}^h - \left(\frac{\eta}{1 - \eta} \right) p_t \left[\lambda \rho_{t+1}^h \widehat{J}_{t+1}^h + (1 - \lambda) \rho_{t+1}^l \widehat{J}_{t+1}^l \right] \right\} \right] \\ = \eta \left[Y(A_t, y^h, x) - \widetilde{w}_t^h(x) + \beta E_t \left\{ \rho_{t+1}^h (1 - \eta) \widehat{S}_{t+1}^h \right\} \right]. \end{aligned} \quad (\text{A.12})$$

Using (A.1) and reordering yields the equilibrium notional wage for high quality matches:

$$\widetilde{w}_t^h(x) = \eta \left[Y(A_t, y^h, x) + \theta_t \psi \right] + (1 - \eta) b. \quad (\text{A.13})$$

Following analogous steps for low quality matches:

$$\widetilde{w}_t^l(x) = \eta \left[Y(A_t, y^l, x) + \theta_t \psi \right] + (1 - \eta) b - \eta (1 - \eta) \beta E_t \left\{ \lambda p_t s \rho_{t+1}^h \widehat{S}_{t+1}^h \right\}. \quad (\text{A.14})$$

B Model Calibration by Tenure Data

In any given period, workers may either be unemployed (u), employed in a low quality match (n^l), or employed in a high quality match (n^h). In steady state, the transition probabilities between u , n^l and n^h are given by:

Table 1: Transition probabilities

Origin \ Destination	u	n^l	n^h
u	$1 - p \left[\lambda \rho^h + (1 - \lambda) \rho^l \right]$	$p (1 - \lambda) \rho^l$	$p \lambda \rho^h$
n^l	$1 - \rho^l - \lambda p s (\rho^h - \rho^l)$	$(1 - \lambda p s) \rho^l$	$\lambda p s \rho^h$
n^h	$1 - \rho^h$	0	ρ^h

¹⁵This implies assuming that the nash bargaining process considers notional wages for the future as well as the present, which is an assumption that simplifies equilibrium results. The alternative assumption, that future wages during the bargaining are those resulting from the wage rigidity rule (as in Tortorice (2013)) yields similar results, therefore we choose the former alternative.

Where the row indicates the original employment status, and the column indicates the final status. With this matrix, and given λ , s , and steady state values for u , n^l , n^h , p , ρ^h and ρ^l we can compute the steady state mass of workers by status and tenure.

Imagine a generation of newly hired workers: those that were matched from unemployment the previous period, implying a tenure of one month in the model. In steady state, newly hired workers with low quality matches will have a mass of $up(1-\lambda)\rho^l = n_{ten=1}^l$ ¹⁶, and those with high quality matches will have a mass of $up\lambda\rho^h = n_{ten=1}^h$.

Next period, of the low quality workers with a month of tenure, $n_{ten=1}^l$, a proportion $(1-\lambda ps)\rho^l$ will remain in low quality matches, while $\lambda ps\rho^h$ will switch to a high quality match (the rest will return to unemployment). Out of previous high quality matches, $n_{ten=1}^h$, a fraction ρ^h will survive as such. Therefore, $n_{ten=2}^l = n_{ten=1}^l(1-\lambda ps)\rho^l$ and $n_{ten=2}^h = n_{ten=1}^h\rho^h + n_{ten=1}^l\lambda ps\rho^h$. Generalizing, for all tenures greater than 1 it will be true that:

$$\begin{aligned} n_{ten=t}^l &= n_{ten=t-1}^l(1-\lambda ps)\rho^l \\ n_{ten=t}^h &= up(1-\lambda)\rho^l[(1-\lambda ps)\rho^l]^{t-1}, \end{aligned} \quad (B.15)$$

and

$$\begin{aligned} n_{ten=t}^h &= n_{ten=t-1}^h\rho^h + n_{ten=t-1}^l\lambda ps\rho^h \\ n_{ten=t}^h &= up\lambda(\rho^h)^t + up(1-\lambda)\rho^l\lambda ps\rho^h \sum_{i=1}^{t-1} (\rho^h)^{t-1-i} ((1-\lambda ps)\rho^l)^{i-1}. \end{aligned} \quad (B.16)$$

Cross-sectional probabilities are then easily calculated. Conditional on observing a worker/match with high tenure (of at least T), call $P(i|ten \geq T)$ the probability that it is of quality $i = \{h, l\}$. Analogously call $P(i|ten < T)$ the probability that a worker with low tenure has a match of quality i . Then,

$$P(i|ten \geq T) = \frac{\sum_{t=T}^{\infty} n_{ten=t}^i}{\sum_{t=T}^{\infty} (n_{ten=t}^l + n_{ten=t}^h)}, \quad (B.17)$$

$$P(i|ten < T) = \frac{\sum_{t=1}^{T-1} n_{ten=t}^i}{\sum_{t=1}^{T-1} (n_{ten=t}^l + n_{ten=t}^h)}. \quad (B.18)$$

It is also trivial to calculate the proportion of workers with high or low tenure in each kind of match quality, which can also be interpreted as the probability of observing a match with certain tenure conditional on observing matches of a given quality:

$$P(ten \geq T|i) = \frac{\sum_{t=T}^{\infty} n_{ten=t}^i}{\sum_{t=1}^{\infty} n_{ten=t}^i}, \quad (B.19)$$

$$P(ten < T|i) = \frac{\sum_{t=1}^{T-1} n_{ten=t}^i}{\sum_{t=1}^{\infty} n_{ten=t}^i}. \quad (B.20)$$

Using these probabilities, we can construct the moments that we observe in the data. In particular, let χ^{ten} be the proportion of high-tenured workers ($ten \geq T$) over those employed. In the model, in steady state:

¹⁶ up is the number of matches formed in one period, $1-\lambda$ is the probability said matches will be of low quality, and ρ^l is the proportion of matches that will survive to be active next period.

$$\chi^{ten} = \chi P(ten \geq T|h) + (1 - \chi) P(ten \geq T|l) \quad (B.21)$$

Next, let $\rho^{ten \geq T}$ be the probability that a high tenure match does not enter unemployment next month, and $\rho^{ten < T}$ for low tenure matches. Probabilities of match survival by tenure will be a weighted average of probabilities of match survival by match quality, where the weights will indicate which match quality is more present in each tenure category:

$$\rho^{ten \geq T} = \rho^h P(h|ten \geq T) + \rho^l P(h|ten \geq T) \quad (B.22)$$

$$\rho^{ten < T} = \rho^h P(h|ten < T) + \rho^l P(h|ten < T) \quad (B.23)$$

Finally, we observe from the data the transition probability from low tenure matches to high tenure matches, which we denote by p_{ten}^{EE} . In the model, $p^{EE} \equiv \lambda ps$ is the employment-employment transition probability, which is exclusively from low quality to high quality matches. However, the observable can easily be computed as:¹⁷

$$p_{ten}^{EE} = \frac{n^l p^{EE} P(ten \geq T|h)}{n^h P(ten \geq T|h) + n^l P(ten \geq T|l)} \quad (B.24)$$

Note that the numerator corresponds to all job-to-job transitions in the model ($n^l p^{EE}$) which ended in a high tenure match ($P(ten \geq T|h)$), while the denominator includes all low tenure matches.

With the conditional probabilities and the observables χ^{ten} , $\rho^{ten \geq T}$, $\rho^{ten < T}$ and p_{ten}^{EE} it is possible to use equations (B.21) - (B.24) to infer the steady state values of χ , ρ^h , ρ^l and p^{EE} . However, the conditional probabilities themselves are a function of χ , ρ^h and ρ^l , which we do not observe. Therefore, numerical methods are needed to obtain these values. We use a simple iterative algorithm:

1. Provide guess values for χ , ρ^h , ρ^l and p^{EE} .
2. Obtain λ , p and s from equations (2) - (4) and the definition of p^{EE} . Note that we use u as an observable, allowing $n^h = (1 - u)\chi$ and $n^l = (1 - u)(1 - \chi)$.
3. Compute the probabilities given by (B.17) - (B.20).
4. Using the observed values of χ^{ten} , $\rho^{ten \geq T}$, $\rho^{ten < T}$ and p_{ten}^{EE} in equations (B.21) - (B.24), compute updated values for χ , ρ^h , ρ^l and p^{EE} .
5. If the updated values approximately coincide with the values used in step 2, keep those values as true. Otherwise, use the updated values and return to step 2.

¹⁷As of this version of the draft, using equation (B.24) to calibrate yields conflicting results. This is probably due to the model's strong assumption regarding job-to-job transitions: that they only occur from low to high quality matches, which is difficult to conciliate with the data. For now, instead of using (B.24) we simply assume $p_{ten}^{EE} = p^{EE}$.

C Tables and Figures

Figure 1: Wage gains (losses) of job transitions:



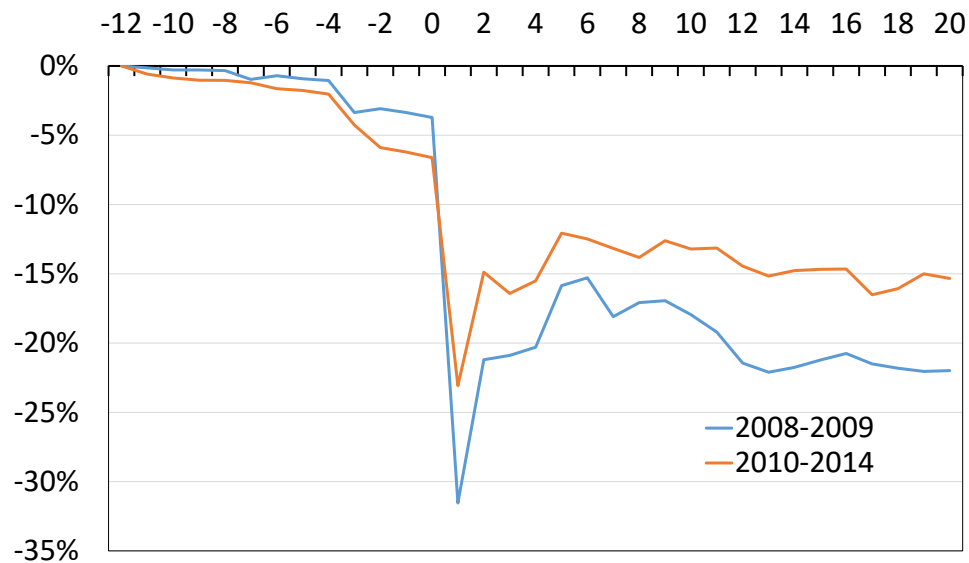
Source: Albagli et al. (2018) with data from the *Servicio de Impuestos Internos*. Wage gains are calculated from real wages, after controlling for year and age gains.

Figure 2: Productivity gains (losses) of job transitions:



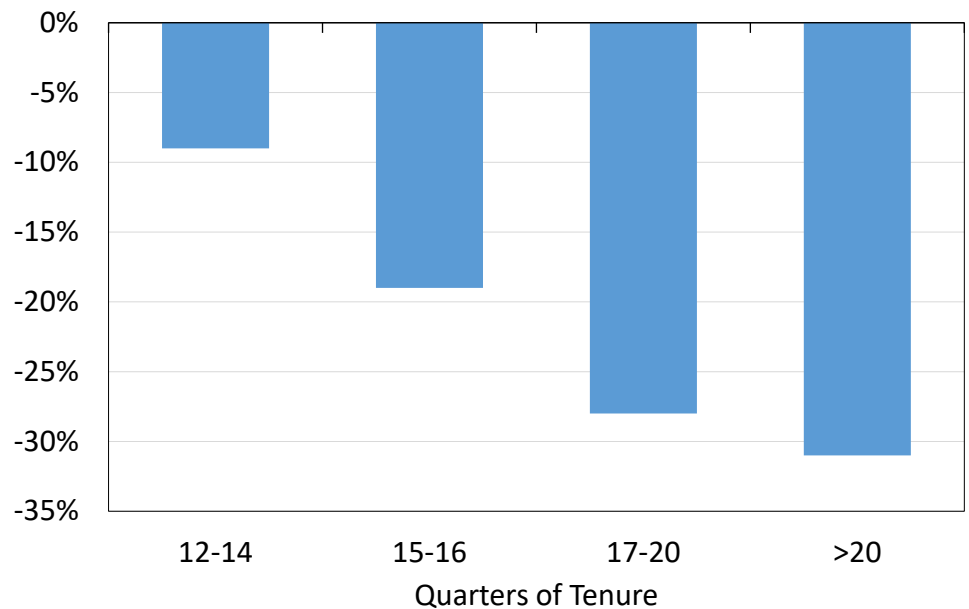
Source: Albagli et al. (2018) with data from the *Servicio de Impuestos Internos*. They use mean labor productivity, calculated as sales over number of employed workers. Productivity gains are calculated as differentials between the firms of origin and destination, controlling for year and sector gains.

Figure 3: Wage difference of employed workers to control group before and after a sudden firm closure, by quarter:



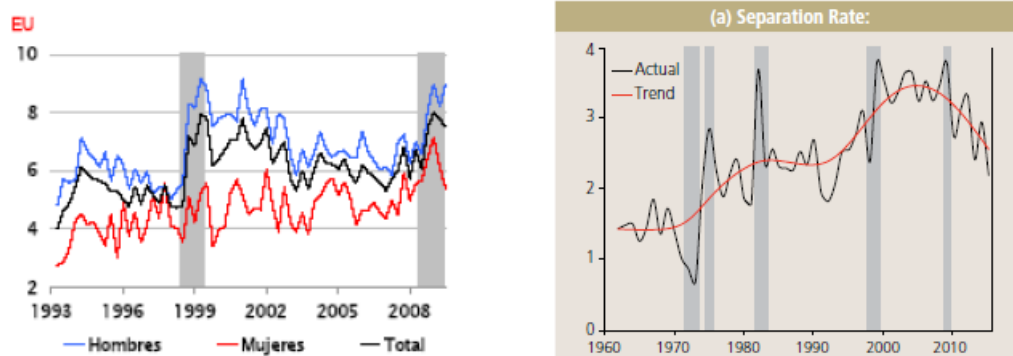
Source: Albagli et al. (2018) with data from the *Servicio de Impuestos Internos*. Wage difference is normalized to be 0% at 12 quarters prior to closure.

Figure 4: Wage difference of employed workers to control group 5 years after closure, by tenure:



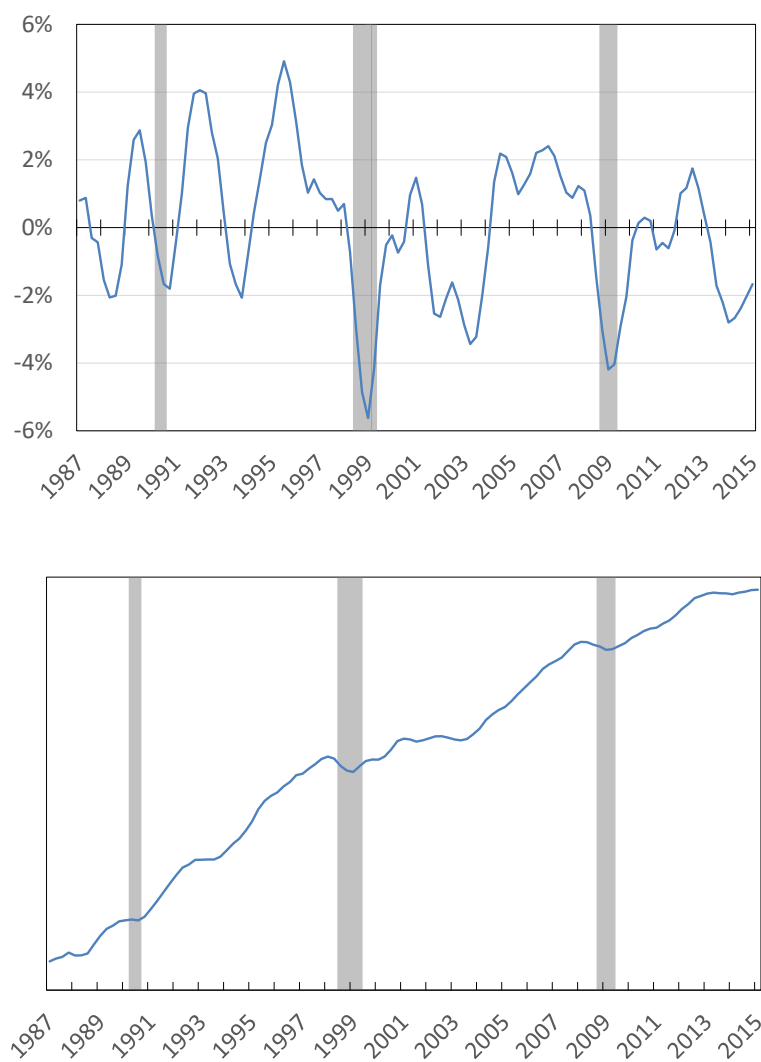
Source: Albagli et al. (2018) with data from the *Servicio de Impuestos Internos*.

Figure 5: Employment-unemployment flows and separation rate



Sources: García and Naudon (2012) using data from the *Encuesta Nacional de Empleo* and Naudon and Pérez (2018) using data from the *Encuesta de Ocupación y Desocupación*. Employment-Unemployment flows are gross flows as % of working age population. The separation rate is monthly and calculated as the arrival rate of a Poisson process.

Figure 6: Labor productivity growth and level (logs)



Source: Own elaboration with Central Bank of Chile data. Labor productivity measured as real GDP (not including natural resources) over number of employed workers. Growth is over same quarter of previous year, demeaned and smoothed with a moving average. Episodes of crises shaded in grey.

Table 2: Calibration

Parameter	Symbol	Value	Source
Discount Factor	β	0.9959	5% real interest rate
Worker Bargaining Power	η	0.5	Literature Standard
Matching Elasticity	κ	0.5	Hosios (1990)
Wage Rigidity	ρ_w	0.9398	Chilean Data
Steady state TFP	$\bar{A}$	1	Normalization
Low-Quality Persistent Productivity	y^l	1	Normalization
Idiosyncratic Productivity Mean	μ_x	0	Normalization
Employed Search Ability	s	0.9417	Matches labor flows data
Prob. of High-Quality Match	λ	0.0071	Matches labor flows data
High-Quality Persistent Productivity	y^h	1.828	Obtained Endogenously
Matching Efficiency	Λ	0.3964	Obtained Endogenously
Vacancy Posting Cost	ψ	0.3812	Obtained Endogenously
Unemployment Benefit	b	0.6583	Obtained Endogenously
SS Exogenous Destruction Rate	$\bar{\delta}$	0.002	Matches Relevant Statistics
High-Quality x Volatility	σ_x^h	35.25	Matches Relevant Statistics
Low-Quality x Volatility	σ_x^l	1	Matches Relevant Statistics

Figure 7: ALP decomposition of shocks

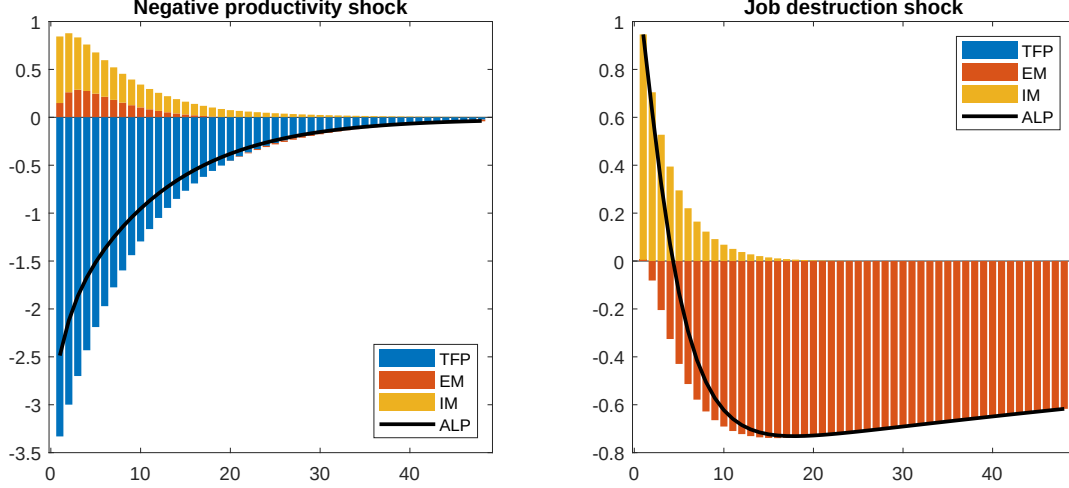


Figure 8: Response to a TFP shock

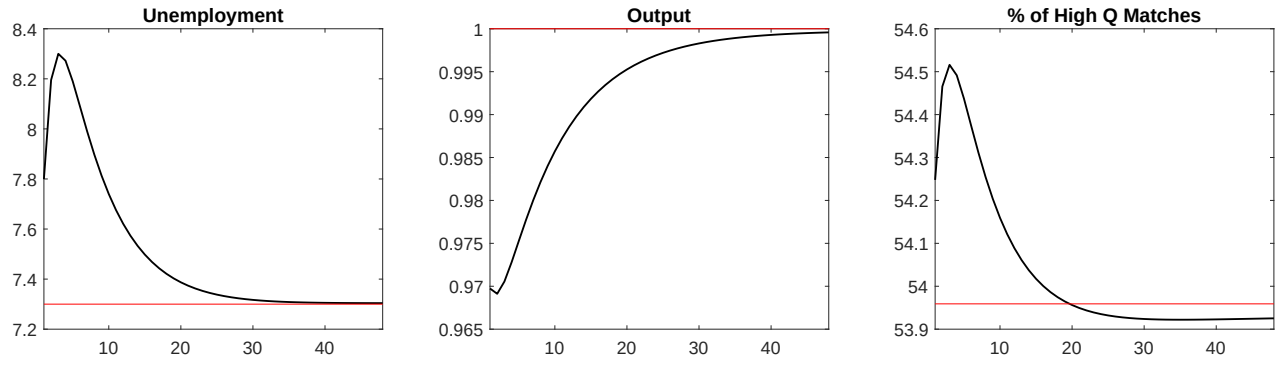


Figure 9: Response to a job destruction shock

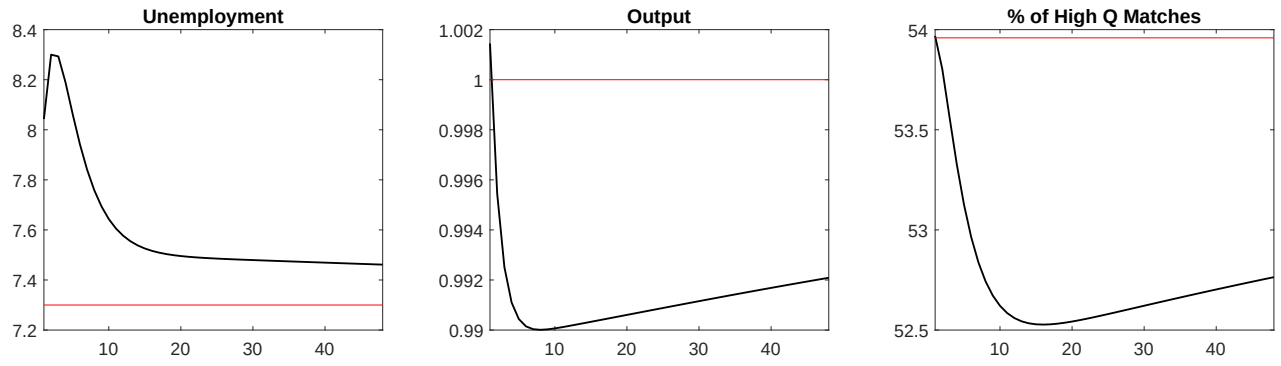
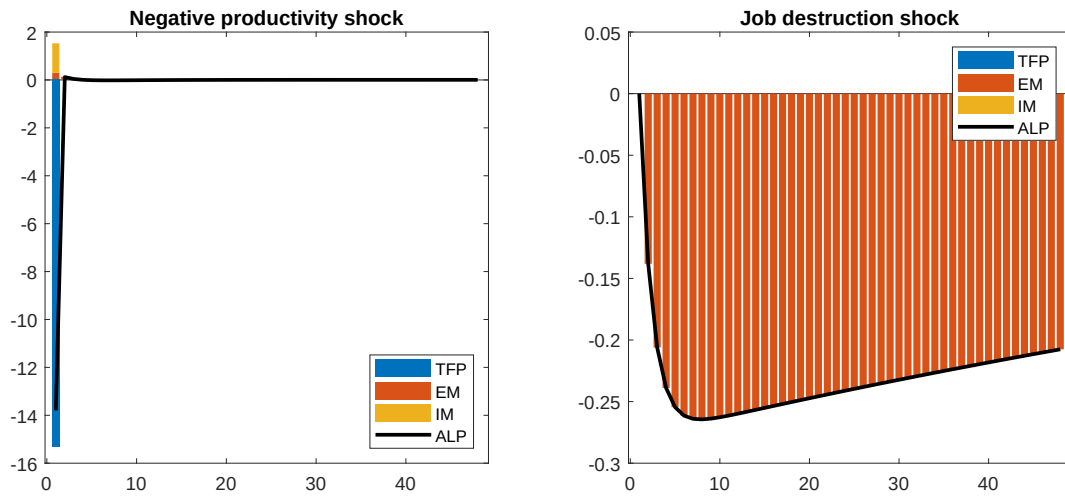


Figure 10: ALP decomposition of shocks with no persistence



References

- Albagli, E., M. Canales, C. Syverson, M. Tapia, and J. Wlasiuk (2018). Firm Productivity and Job Transitions: The Reallocation Puzzle. *Mimeo, Banco Central de Chile*.
- Albagli, E., M. Marcel, A. Martner, and M. Tapia (2018). Sudden Firm Closures and the Effects on Displaced Workers. *Mimeo, Central Bank of Chile*.
- Baily, M., C. Hulten, and D. Campbell (1992). The Distribution of Productivity in Manufacturing Plants. *Brookings Papers: Microeconomics* 4, 187–226.
- Barlevy, G. (2002, January). The Sullyng Effect of Recessions. *Review of Economic Studies* 69(1), 65–96.
- Barlevy, G. (2003, November). Credit Market Frictions and the Allocation of Resources Over the Business Cycle. *Journal of Monetary Economics* 50(8), 1795–1818.
- Bils, M. (1985, August). Real Wages over the Business Cycle: Evidence from Panel Data. *Journal of Political Economy* 93(4), 666–689.
- Bowlus, A. (1995, April). Matching Workers and Jobs: Cyclical Fluctuations in Match Quality. *Journal of Labor Economics* 13(2), 335–350.
- Caballero, R. and M. Hammour (1994, December). The Cleansing Effect of Recessions. *American Economic Review* 84(5), 1350–1368.
- Caballero, R. and M. Hammour (1996, August). On the Timing and Efficiency of Creative Destruction. *The Quarterly Journal of Economics* 11(3), 805–852.
- Cerra, V. and S. Saxena (2008, March). Growth Dynamics: The Myth of Economic Recovery. *American Economic Review* 98(1), 439–457.
- Cerra, V. and S. Saxena (2017). Booms, Crises, and Recoveries: A New Paradigm of the Business Cycle and Its Policy Implications. *IMF Working Paper No. 17/250*.
- Davis, S., J. Faberman, and J. Haltiwanger (2006). The Flow Approach to Labor Markets: New Data Sources and Micro-Macro Links. *Journal of Economic Perspectives* 20(3), 3–26.
- Davis, S., J. Faberman, and J. Haltiwanger (2012, January). Labor Market Flows in the Cross Section and Over Time. *Journal of Monetary Economics* 59(1), 1–18.
- Davis, S. and J. Haltiwanger (1990). Gross Job Creation and Destruction: Microeconomic Evidence and Macroeconomic Implications. *NBER Macroeconomics Annual* 5, 123–186.
- Davis, S. and J. Haltiwanger (1992, August). Gross Job Creation, Gross Job Destruction, and Employment Reallocation. *Quarterly Journal of Economics* 107(3), 819–863.
- Davis, S. and J. Haltiwanger (1999, December). On the Driving Forces behind Cyclical Movements in Employment and Job Reallocation. *American Economic Review* 89(5), 1234–1258.
- Davis, S., J. Haltiwanger, and S. Schuh (1996). Job Creation and Destruction. *Cambridge, MA: MIT Press*.

- den Haan, W. J., G. Ramey, and J. Watson (2000, June). Job Destruction and Propagation of Shocks. *American Economic Review* 90(3), 482–498.
- Diwan, I. (2001). Debt as Sweat: Labor, Financial Crises, and the Globalization of Capital. *Processed, World Bank*.
- Eslava, M., A. Galindo, M. Hofstetter, and A. Izquierdo (2010). Scarring Recessions and Credit Constraints: Evidence from Colombian Plant Dynamics. *CEDE Working Paper*.
- Foster, L., C. Grim, and J. Haltiwanger (2016, January). Reallocation in the Great Recession: Cleansing or Not? *Journal of Labor Economics* 34(1), S293–S331.
- García, M. and A. Naudon (2012). Dinámica Laboral en Chile. *Working Papers Central Bank of Chile* 659.
- Griliches, Z. and H. Regev (1995, January). Firm Productivity in Israeli Industry: 1979 - 1988. *Journal of Econometrics* 65(1), 175–203.
- Hagedorn, M. and I. Manovskii (2008, September). The Cyclical Behavior of Equilibrium Unemployment and Vacancies Revisited. *American Economic Review* 98(4), 1692–1706.
- Hall, R. E. (2005, March). Employment Fluctuations with Equilibrium Wage Stickiness. *American Economic Review* 95(1), 50–65.
- Hallward-Driemeier, M. and B. Rijkers (2010). Do Crises Catalyze Creative Destruction? Firm-Level Evidence from Indonesia. *Processed, World Bank*.
- Hosios, A. (1990, April). On the Efficiency of Matching and Related Models of Search and Unemployment. *The Review of Economic Studies* 57(2), 279–298.
- Jovanovic, B. (1979, October). Job Matching and the Theory of Turnover. *Journal of Political Economy* 87(5), 972–990.
- Krause, M. and T. Lubik (2006). The Cyclical Upgrading of Labor and On-the-Job Search. *Labour Economics* 13, 459–477.
- Krause, M. U. and T. A. Lubik (2007, April). The (ir)relevance of real wage rigidity in the New Keynesian model with search frictions. *Journal of Monetary Economics* 54(3), 706–727.
- Krolikowski, P. (2017, April). Job Ladders and Earnings of Displaced Workers. *American Economic Journal: Macroeconomics* 9(2), 1–31.
- Lee, Y. and T. Mukoyama (2007). Entry, Exit, and Plant-level Dynamics over the Business Cycle. *Federal Reserve Bank of Cleveland Working Paper* 07-18.
- Liu, L. and J. Tybout (1996). Productivity Growth in Chile and Colombia: The Role of Entry, Exit, and Learning. *M. Roberts, and J. Tybout, eds., Industrial Evolution in Developing Countries. Oxford: Oxford University Press*, 73–103.
- Mortensen, D. T. and C. A. Pissarides (1994, July). Job Creation and Job Destruction in the Theory of Unemployment. *Review of Economic Studies* 61(3), 397–415.

- Moscarini, G. and F. Postel-Vinay (2018, August). The Cyclical Job Ladder. *Annual Review of Economics* 10(1), 165–188.
- Mueller, A. (2017, July). Separations, Sorting, and Cyclical Unemployment. *American Economic Review* 107(7), 2081–2107.
- Mukoyama, T. (2014). The Cyclical Job-to-Job Transitions and its Implications for Aggregate Productivity. *Journal of Economic Dynamics and Control* 539, 1–17.
- Mustre-del Río, J. (2012). Job Duration and the Cleansing and Sullyng Effects of Recessions. *Research Working Paper RWP 12-08, Federal Reserve Bank of Kansas City*.
- Naudon, A. and A. Pérez (2018, April). Unemployment Dynamics in Chile: 1960-2015. *Economia Chilena* 21(1), 004–033.
- Neal, D. (1999, April). The Complexity of Job Mobility among Young Men. *Journal of Labor Economics* 17(2), 237–261.
- Nishimura, K., T. Nakajima, and K. Kiyota (2005, September). Does the Natural Selection Mechanism still Work in Severe Recessions? Examination of the Japanese Economy in the 1990. *Journal of Economic Behavior and Organization* 58(1), 53–78.
- Ouyang, M. (2009, March). The Scarring Effect of Recessions. *Journal of Monetary Economics* 56(2), 184–199.
- Schumpeter, J. (1939). Business Cycles: A Theoretical, Historical and Statistical Analysis of the Capitalist Process. *New York: McGraw-Hill*.
- Shimer, R. (2005, March). The Cyclical Behavior of Equilibrium Unemployment and Vacancies. *American Economic Review* 95(1), 25–49.
- Shin, D. (1994). Cyclical Job-to-Job Transitions and its Implications for Aggregate Productivity. *Economics Letters* 46, 137–42.
- Tortorice, D. L. (2013). Endogenous separation, wage rigidity and the dynamics of unemployment. *Journal of Macroeconomics* 38(PB), 179–191.