Proxy-SVAR as a Bridge for Identification with Higher Frequency Data*

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Abstract

High frequency identification around key events has recently solved many puzzles in empirical macroeconomics. This paper proposes a novel methodology, the *Bridge Proxy-SVAR*, to identify structural shocks in Vector Autoregressions (VARs) by exploiting high frequency information in a more general framework. Our methodology comprises three steps: (I) identify the structural shocks of interest in high frequency systems; (II) aggregate the series of high frequency shocks at a lower frequency employing the correct filter; (III) use the aggregated series of shocks as a proxy for the corresponding structural shock in lower frequency VARs. Both analytically and through simulations, we show that our methodology significantly improves the identification of VARs, recovering the true impact effect. In a first empirical application on US data, we show that financial shocks identified at daily frequency produce unambiguously macroeconomic effects consistent with a demand shock. In a second application, we identify U.S. monetary policy shocks that are highly correlated with the series of monetary policy surprises but, contrary to the latter ones, are invertible and so valid external instruments for low-frequency VARs.

JEL classification: C32; C36; E52; E58; E44

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1 Introduction

The identification of causal relationships in empirical macroeconomics is a challenging task due to the simultaneous co-movement among economic variables. Isolating an exogenous variation of interest at high frequency arguably relies on weaker assumptions than the typical restrictions applied at monthly or quarterly frequency. For instance, Gertler and Karadi (2015) employ the aggregated series of monetary policy surprises, defined as the change in the price of futures contracts in a 30 minute window around FOMC meetings (see Gurkaynak et al., 2005), for the identification of a monthly structural Vector Autoregression (VAR). This identification strategy is based on the assumption that within this short window only monetary policy news hit the economy, which is arguably more convincing than the recursive restrictions traditionally imposed in monthly or quarterly systems. High frequency (HF) identification is currently performed exclusively by exploiting the movements of some variables around specific events, which limits the implementability of this type of analysis.[1]

This paper fills such a gap in this line of research by developing a novel methodology, labeled "*Bridge Proxy-SVAR*" (Bridge-PSVAR). The Bridge-PSVAR exploits HF information to identify VARs through the Proxy-SVAR (Stock and Watson, 2012; Mertens and Ravn, 2014) in a general framework. The methodology consists of three steps. First, we identify the structural shock of interest in HF systems, hence in systems that are not subject to time aggregation and so characterized by less severe identification challenges. Second, we aggregate the series of shocks to the lower frequency, e.g. monthly or quarterly for macroeconomic variables, employing the correct filter. We test that the aggregated series of shocks are invertible (see Forni and Gambetti, 2014; Stock and Watson, 2018; and Miranda Agrippino and Ricco, 2018), which means that they cannot be predicted using past information and that the HF-VAR is not information

[1] Other papers that apply a similar strategy for identification are: Auerbach and Gorodnichenko (2016), Piffer and Podstawski (forth), and Kanzig (2018)

deficient. Third, we use the aggregated series of shocks as a proxy for the corresponding structural shock at the lower frequency (LF).[2] Namely, we draw identifying restrictions for the LF representation from HF information. HF identification employs the correct information set of agents when they take decisions, without mixing it with their endogenous responses.

The Bridge-PSVAR offers two important advantages over event-based analysis. First, HF identification in a time series model yields structural shocks, orthogonal to the past information set and to other shocks that contemporaneously affect the economy.[3] Second, our methodology expands the implementability of HF identification to a wider set of cases, where events selection is not straightforward, not appropriate, or the few events do not convey enough information for reliable statistical inference. The lack of availaility of lower frequency variables in the first stage, such as GDP and CPI at the daily frequency, does not pose a problem *per se*. This potential omitted variable bias, which is common to any small scale VAR, independently of the frequency of the data, can be tested via an information sufficiency test as in Forni and Gambetti (2014).

Consistently with this analysis, the contribution of this paper is twofold. First, we formally show that if the underlying HF process is a VAR, the HF shocks correctly aggregated recover the true causal impact effects. Importantly, we derive the correct filter to aggregate the HF shocks to lower frequencies for different cases. Second, we argue that HF identification is not limited by the cases of uncontroversial event-based analysis but can be generalized by employing standard time series tools as VARs at HF.

Besides the analytical analysis of external instruments within a temporal aggregation framework, we rely on Monte Carlo experiments to provide a general assessment of the

[2]While local projections, more robust to misspecification, can be alternatively employed to compute Impulse Response Function (IRFs), we prefer the Proxy-SVAR as the benchmark because it is more robust to measurement errors. See the Appendix for a discussion of the advantages and disadvantages of Proxy-SVAR and local projections.

[3]The structural shock is defined as a martingale difference with respect to the past information set, based on the Wold Representation Theorem. Regarding the multiple shocks captured by event-based analysis, see for instance Campbell et al. (2012),Campbell et al. (2016), Karadi & Jarokinsky (2018), Andrade & Ferroni (), ... on the information content in monetary policy surprises.

small-sample performances of the Bridge-PSVAR and to quantify the gains compared to the common practice of identification at lower frequency. We compare our procedure to a VAR on temporally aggregated data (LF-VAR) and to the best possible and counter-factual HF estimation (HF-VAR). Our results show that the Bridge-PSVAR is a suitable method for approximating the true underlying responses under different data generating processes. In fact, the Bridge-PSVAR greatly outperforms the LF-VAR, with a reduction in the temporal aggregation bias ranging between 19% to 85% depending on the specification; and yields similar but less precise estimates than the (counterfactual) HF-VAR.

The potential of the methodology is exemplified in two applications. First, we show that important differences emerge when comparing quarterly versus daily identification of financial shocks. The conclusion on the response of the price level and consequently on the nature of financial shocks relies on the recursive ordering chosen at quarterly frequency using VARs similar to Gilchrist et al. (2014). The same holds for the responses of the short and long-term interest rates. Conversely, identifying financial shocks in a daily VAR imposes lower constraints on the data and different recursive orderings imposed at this frequency yield consistent results: the price level falls after a financial shock and monetary policy reacts to it, in line with the findings of Caldara and Herbst (2018). Second, we recover monetary policy shocks in a daily VAR applying Independent Component Analysis. Our identified shocks are highly correlated with the series of monetary policy surprises computed by Gurkaynak et al. (2005) and traditionally employed in the literature. While our identified series of monetary policy shocks are both invertible and strong external instruments, the aggregated series of monetary policy surprises are contaminated by previous macroeconomic conditions. Our methodology, extending HF identification to HF-VARs, can thus tackle problems of predictability and information release arising in event-based HF identifications.

This paper is related to the Proxy-SVAR methodology (or VAR identified via external instruments), a very recent development in the identification of SVAR, developed by

Stock and Watson (2012) and Mertens and Ravn (2013).[4] This method employs an exogenous variation as an external instrument to identify SVARs. In other words, this external information is a proxy for the (unobserved) structural shock of interest. The proxy is assumed to be correlated with the latter but orthogonal to other structural shocks. In practice, the proxy constitutes an instrument for the reduced form residuals of the VAR and is used for (partial) identification of the covariance matrix of the structural shocks. The clear advantage of this technique is that, as long as the proxy is a relevant and valid instrument, the identification relies on a much weaker set of assumptions than other identification schemes. For example, no assumptions are made on the contemporaneous relationship among the variables in the system. Moreover, Carriero et al. (2015) have shown through Monte Carlo experiments that the Proxy-SVAR is robust to measurement errors. Proxies are usually extracted from a narrative description of policy decisions or exploiting HF identification around some key events. The BP-SVAR generalizes the Proxy-SVAR to those cases where there are no key events or when their selection is troublesome and arbitrary.[5] Due to its flexibility, our methodology can also be combined with identifications that nest event based approaches. For example, HF identification based on heteroskedasticity (Rigobon, 2003) or narrative sign restrictions (Antolin-Diaz and Rubio-Ramirez 2016) exploits all the daily observations for identification. Narrative evidence employed at HF can very valuable because it imposes only minimal constrains on the data.

The severity of temporal aggregation biases in VAR models is illustrated in Marcellino (1999) and Foroni and Marcellino (2016).[6] Impulse response functions (IRFs) and forecast error variance decomposition (FEVD) can be strongly biased due to

[4]The first seeds of the methodology can be found already in Beaudry and Saito (1998).

[5]Our approach remotely resembles the bridging equations which link data available at different frequencies through linear regression to produce nowcast and short-term forecast; e.g. Baffigi, Golinelli, and Parigi (2004) and Diron (2008).

[6]A complete mathematical characterization of temporal aggregation is developed in Hansen et al. (1991) (Ch.10 by Albert Marcet).

temporal aggregation (see Sims, 1971; Hendry, 1992; and Swanson and Granger, 1997).[7] This paper contributes theoretically to the relationship between external instruments and temporal aggregation. To our knowledge, we provide the first proof that shocks identified at HF (also in the special case of event-based analyses), correctly aggregated and employed as external instruments in SVAR identification can estimate the true causal effects of the shocks.

The remainder of this paper is organized as follows. Section 2 develops an analytical example to illustrate the econometric problem. Section 3 describes the BP-SVAR methodology and its analytical properties. Section 4 presents the Monte Carlo experiments employed for testing the methodology. Section 5 is devoted to study the dynamic effects of financial shocks employing the Bridge-PSVAR. Section 6 studies monetary policy in the US by applying the BP-SVAR. Finally, Section 7 concludes.

2 An Illustrative Case

In this Section, we describe a simple case to illustrate the econometric problem and the intuition behind our proposed solution, the Bridge-PSVAR. We postpone the full characterization of the temporal aggregation framework to Section 3. Consider a simple bivariate $VAR(1)$ process at the HF frequency $t = 1, 2, ..., T$ that is assumed to be covariance stationary and causal:

[7]To avoid this bias, the literature has proposed MF-VARs as the standard tool to handle data sampled at different frequencies. There are two main approaches to estimate VARs with mixed frequency data. The most popular one, developed by Zadrozny (1988), is based on a state space representation (a dynamic linear model). The system is driven by latent shocks whose economic interpretation is not straightforward. Moreover, the computational intensity of this technique increases exponentially with the number of states and the frequency mismatch. The second approach, proposed by Ghysels (2016), is more similar to standard VARs in being driven only by observable shocks. This particular VAR deals with series sampled at different frequencies through stacking: a HF variable is decomposed into several LF variables and directly employed in the VAR. The shortcoming consists of the curse of dimensionality, i.e. parameters proliferation. Moreover, recovering the HF structural shocks from those in the stacked LF-VAR is not necessarily straightforward.

$$
\begin{bmatrix} x_t \\ y_t \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} b_{11} & 0 \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^y \end{bmatrix} \tag{1}
$$

where x_t, y_t are the two endogenous variables, $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is the autoregressive matrix, $B = \begin{bmatrix} b_{11} & 0 \\ b_{21} & b_{22} \end{bmatrix}$ is the impact matrix that determines the instantaneous feedback from the structural innovations $\varepsilon_t = \left[\varepsilon_t^x \, \varepsilon_t^y\right]'$ to the endogenous variables. We assume B to have a Cholesky structure without loss of generality. Those innovations are structural as $\mathbb{E}\left[\varepsilon_t \varepsilon_t^{'}\right] = \mathcal{I}_2$ but only a linear combination of them is observable to the econometrician by estimating the reduced form system. These residuals $u_t = B\varepsilon_t$ are correlated: $\mathbb{E}\left[u_t u_t^{'}\right] = \Sigma_u = BB'$. In this case, given u_t, the knowledge of the B matrix is sufficient for the correct identification of the shocks.

Often applied researchers dispose of variables that are observable at different frequencies. For instance, CPI is available at the monthly frequency whereas GDP only at the quarterly frequency. We consider an even simpler case: let y be observable at semi-annual frequency and x only at the annual frequency. This frequency mismatch (equal to 2) generates an estimation problem as the SVAR in eq.(1) is unobservable. The problem is commonly solved by temporally aggregating y to the lower frequency at which x is observable. As we show below, this procedure is not cost-free.

The temporally aggregated system includes the variables that are observable once every two HF periods t. It is thus denoted by the lower frequency index

$\tau = 2, 4, ..., T$:[8]

$$\begin{bmatrix} x_\tau \\ y_\tau \end{bmatrix} = \begin{bmatrix} a_{11}^2 + a_{12}a_{21} & a_{11}a_{12} + a_{12}a_{22} \\ a_{11}a_{21} + a_{21}a_{22} & a_{12}a_{21} + a_{22}^2 \end{bmatrix} \begin{bmatrix} x_{\tau-1} \\ y_{\tau-1} \end{bmatrix} + \begin{bmatrix} \xi_\tau^x \\ \xi_\tau^y \end{bmatrix} \tag{2}$$

where the reduced-form residuals ξ_τ are combination of the structural ones over periods $t-1$ and t:

$$\begin{bmatrix} \xi_\tau^x \\ \xi_\tau^y \end{bmatrix} = \begin{bmatrix} b_{11}\varepsilon_t^x + (a_{11}b_{11} + a_{12}b_{21})\,\varepsilon_{t-1}^x + a_{12}b_{22}\varepsilon_{t-1}^y \\ b_{21}\varepsilon_t^x + b_{22}\varepsilon_t^y + (a_{21}b_{11} + a_{22}b_{21})\,\varepsilon_{t-1}^x + a_{22}b_{22}\varepsilon_{t-1}^y \end{bmatrix} \tag{3}$$

Notice that this process is still a $VAR(1)$ process but the variance-covariance matrix of the residuals is affected by this transformation: $\mathbb{E}\left[\xi_\tau \xi_\tau^{'}\right] = \Sigma_\xi = (\mathcal{I} + A)\,BB'\,(\mathcal{I} + A)^{'}$. It is straightforward to verify that $\Sigma_\xi \neq \Sigma_u$. Thus, even the correct Cholesky structure implicit in B, imposed on ξ_τ, cannot (in general) recover the shocks ε. In other words, temporal aggregation makes impossible to recover the instantaneous causal impact of the innovations.

The Bridge-PSVAR follows a different approach that rests on the proper use the external information on ε_t^y in order to achieve a correct identification of the LF-VAR in eq.(2). We estimate a VAR at the "semester" frequency t, for which the critical property to take into account concerns the partial invertibility of the identified shocks.[9] This property is related to the informational sufficiency of this higher frequency VAR and depends on the other variables included in the system (denoted as the block Ω_t). While we develop this concept in Section 3.3.3, for now it suffices to say that this is a common issue with any quarterly or monthly small scale VAR typically employed in the

[8]We apply temporal aggregation via skip-sampling over 2 periods, meaning that the variables are observable only once every two periods (and consider the last) but are not transformed. This corresponds to applying the filters $D(L) = I + AL$ and $W(L) = I$ to to $[x_t\, y_t]^{'}$, where L is the lag operator. For the full discussion of temporal aggregation refer to Section 3.

[9]Alternatively, the external information may come from narrative sources or event-studies, as typically done in the Proxy-SVAR literature. To apply the correct filter to aggregate the shocks to the lower frequency, one still needs to assume a specific structure for the underlying process.

literature. The HF-VAR can be written as:

$$\begin{bmatrix} \Omega_t \\ y_t \end{bmatrix} = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & d_{22} \end{bmatrix} \begin{bmatrix} \Omega_{t-1} \\ y_{t-1} \end{bmatrix} + \begin{bmatrix} C_{11} & 0 \\ C_{21} & c_{22} \end{bmatrix} \begin{bmatrix} \boldsymbol{\varepsilon}_t^{\Omega} \\ \varepsilon_t^{y} \end{bmatrix} \tag{4}$$

where $D = \begin{bmatrix} D_{11} & D_{12} \\ D_{21} & d_{22} \end{bmatrix}$ is the autoregressive matrix and $C = \begin{bmatrix} C_{11} & 0 \\ C_{21} & c_{22} \end{bmatrix}$ the contemporaneous one. The informational sufficiency assumption states that $\boldsymbol{\varepsilon}_t^{\Omega} \supseteq \varepsilon_t^{x}$ and, under those conditions, we may recover ε_t^{y}, or at least a proxy for it, which we denote again z_t^{y} to avoid a confusing notation in the remainder of this section. Under this assumption, $z_t^{y} = \varepsilon_t^{y} + \eta_t \ \ , \eta_t \sim wn(0, \sigma_\eta^2) \ \ , \eta_t \perp \varepsilon_t^{y}$.

In this illustrative case, properly employing z_t^{y} means to use exclusively the observations corresponding to $h = 2, 4, ..., T$, i.e. the observation in the second semester of each year. We denote such a series as $z_{t,h}^{y}$. Once the correct filter is applied to z_t^{y}, such that we obtain $z_\tau^{y} = z_{t,h}^{y}$, we employ z_τ^{y} as an external instrument (or proxy) for the structural shock ε_t^{y}. This strategy is valid under the typical assumption in the Proxy-SVAR literature on the exogeneity of z_t^{y}, i.e. $\mathbb{E}\left[\left(z_{t,h}^{y}\right)' \varepsilon_t^{x}\right] = 0$, and its strength, i.e. $\mathbb{E}\left[\left(z_{t.h}^{y}\right)' \varepsilon_t^{y}\right] \neq 0$. In this way, we correctly identify the second column of the B matrix, up to the scale factor μ:

$$\hat{b}_{22} = \mathbb{E}\left[\left(z_\tau^{y}\right)' z_\tau^{y}\right]^{-1} \mathbb{E}\left[\left(z_\tau^{y}\right)' \xi_\tau^{y}\right] = \mu b_{22} \tag{5}$$

$$\hat{b}_{12} = \mathbb{E}\left[\left(\hat{b}_{22}\xi_\tau^{y}\right)' \hat{b}_{22}\xi_\tau^{y}\right]^{-1} \mathbb{E}\left[\left(\hat{b}_{22}\xi_\tau^{y}\right)' \xi_\tau^{x}\right] = 0 \tag{6}$$

In this way, the ratio $\frac{\hat{b}_{12}}{\hat{b}_{22}} = \frac{b_{12}}{b_{22}}$ is correct.

A natural question that may arise concerns the actual relevance of the correct aggregation of z_t^{y} for identification. To highlight the critical role of the filter transforming z_t^{y} into z_τ^{y} (from HF to LF), we repeat the same exercise as above for

$h^{'} = 1, 3, ..., T$ maintaining all the previous assumptions. In other words, we employ now the observations of z_t^y in the first semester of each year. To avoid a confusion with the previous results, we denote $\tilde{z}_\tau^y = z_{t,h^{'}}^y$. Employing $\tilde{z}_\tau^y$ as external instrument yields

$$\tilde{b}_{22} = \mathbb{E}\left[\left(\tilde{z}_\tau^y\right)^{'} \tilde{z}_\tau^y\right]^{-1} \mathbb{E}\left[\left(\tilde{z}_\tau^y\right)^{'} \xi_\tau^y\right] = \mu a_{22} b_{22} \tag{7}$$

$$\tilde{b}_{12} = \mathbb{E}\left[\left(\hat{b}_{22}\xi_\tau^y\right)^{'} \hat{b}_{22}\xi_\tau^y\right]^{-1} \mathbb{E}\left[\left(\hat{b}_{22}\xi_\tau^y\right)^{'} \xi_\tau^x\right] = \mu a_{12} b_{22} \tag{8}$$

In this case the ratio $\frac{\tilde{b}_{12}}{\tilde{b}_{22}} = \frac{a_{22}}{a_{12}} \neq 0$. Therefore, the correct use of HF external information is crucial to correctly estimate the impact causal effects. In Section 3.3.1 we fully develop the econometric framework and derive the correct aggregation filter for each case.

3 Methodology

This section generalizes the description of the Bridge Proxy-SVAR. Section 3.1 presents the econometric framework used for the analysis. Sections 3.2 to 3.4 describe in detail the different steps of the methodology.

The framework that we consider is the following: i) the vector of innovations that drive the economy can be partitioned as $\varepsilon = \left[\varepsilon^1 \, \varepsilon^{\bar{1}}\right]$; ii) the researcher wants to identify the effect the innovation ε^1 on a vector of endogenous variables y; iii) ε^1 can be recovered at the frequency $t = 1, 2, ..., T$ higher than the frequency τ at which y is observable ($\tau = m, 2m, ..., T$ where $m > 1$ is the frequency mismatch). The Bridge-PSVAR comprises three steps: 1) identify ε_t^1; 2) correctly aggregate ε_t^1 transforming it into ε_τ^1; 3) estimate a VAR on y_τ and use ε_τ^1 as external instrument for a to estimate the causal impact effect of ε^1 on y.

3.1 Econometric Framework

Consider a vector of n time series modeled as a causal and covariance stationary SVAR of lag-length p:

$$y_t = A_1 y_{t-1} + ... + A_p y_{t-p} + B\varepsilon_t \tag{9}$$

Such a process can be expressed via compact notation through the polynomial $A(L) = \mathcal{I} - A_1 L - A_2 L^2 - ... - A_p L^p$ (where L is the lag operator such that $L^i y_t = y_{t-i}$):

$$A(L)y_t = B\varepsilon_t \tag{10}$$

where ε_t is a vector of stochastic innovations and B is a $n \times n$ matrix whose coefficients determine how ε_t affect contemporaneously the variables y_t. The SVAR is not observable itself, but corresponds to a multiplicity of reduced-form VAR representations

$$A(L)y_t = u_t \tag{11}$$

In what follows, we focus exclusively on the problem of identification of the matrix B under temporal aggregation. Specifically, we study how HF shocks should be used as external instrument for the LF-VAR in order to recover the underlying impact effects.

Temporal aggregation can be expressed as a two-step filer (see Marcellino, 1999 for a more detailed discussion of the topic). First, the data are observable only once every m periods, which represent the frequency mismatch. As a matter of notation, $t = 1, 2, ..., T$ stands for the high frequency (HF, e.g. days), whereas $\tau = m, 2m, ..., T$ indexes the low frequency (LF, e.g. months). To convert the process in eq.(10) to the LF, we apply the filter $D(L) = \mathcal{I} + D_1 L + D_2 L^2 + ... + D_{pm-p} L^{pm-p}$. The specification of $D(L)$ has to be such that $D(L)A(L)$ only contains powers of L^m, meaning that only the observable datapoints enter the transformed process. The conditions for the existence of such a filter, as well as the values taken by the matrices D_i are derived in Marcellino (1999). The second filter depends on the temporal aggregation scheme chosen: skip-sampling (or point-in-time sampling) usually applied to stock variables (e.g. prices) or averaging applied to flow variables (e.g. volumes). In the former case, the second filter $W(L) = \mathcal{I}$ does not modify the data. For instance, consider the $VAR(1)$ process $y_t = A_1 y_{t-1} + B\varepsilon_t$ and $m = 2$. The filter $D(L)W(L) = \mathcal{I} + A_1 L$ transforms the original process into $y_t (\mathcal{I} + A_1 L) = A_1 y_{t-1} (\mathcal{I} + A_1 L) + B\varepsilon_t (\mathcal{I} + A_1 L)$ which can be simplified as $y_\tau = A_1^2 y_{\tau-1} + B\varepsilon_t + A_1 B\varepsilon_{t-1}$. In the latter case, $W(L) = \mathcal{I} + L + L^2 + ... + L^{m-1}$ as the variables are averaged (summed) . In the previous example $W(L) = \mathcal{I} + L$ and $D(L)W(L) = (\mathcal{I} + A_1 L)(\mathcal{I} + L)$. The temporally aggregated process would then become $\bar{y}_\tau = A_1^2 \bar{y}_{\tau-1} + B(\varepsilon_t + \varepsilon_{t-1}) + A_1 B (\varepsilon_{t-1} + \varepsilon_{t-2})$.

The effect of the innovations ε_t on y_t at the horizon k are captured by the impulse response functions. Denote the true IRF at HF ass Θ_k^t, which is unequivocally defined on impact as $\Theta_0^t = \mathbb{E}_{\mathcal{F}_{t-1}}[y_t/\varepsilon_t = 1]$ - $\mathbb{E}_{\mathcal{F}_{t-1}}[y_t/\varepsilon_t = 0]$ with $\mathcal{F}_{t-1} = \{y_{t-1}, ..., y_{t-p}\}$. Conversely, there are two possible definitions of temporally aggregated IRFs Θ_0^τ. This choice crucially depends on the information set chosen to define the IRFs at low frequency and, in turns, determines how to correctly use external HF information on ε_t for LF-VAR identification. A first possibility is to aggregate the IRFs directly via the temporal aggregation filter, implicitly using the information set $\mathcal{F}_{t-1}$. In this case, the LF-IRFs are defined directly as $\Theta_0^\tau = D(L)W(L)\Theta_0^t$. A second possibility employs the relevant information set for the LF representation: the information set is arguably $\mathcal{F}_{\tau-1} = \{y_{\tau-1}, y_{\tau-2}, ...\}$ and consequently $\Theta_0^\tau = \mathbb{E}_{\mathcal{F}_{\tau-1}}[y_\tau/\varepsilon_\tau = 1]$ - $\mathbb{E}_{\mathcal{F}_{\tau-1}}[y_t/\varepsilon_\tau = 0]$ where $\varepsilon_\tau = \{\varepsilon_{t,1}, \varepsilon_{t,2}, ..., \varepsilon_{t,m-1}\}$. This definition considers as shocks all the innovations occurring between $\tau - 1$ and τ. While both definitions are formally correct, we regard the second as the most interesting one from a macroeconomic perspective. Still, we differentiate the discussion across these two cases.

3.2 First Step: Identification at High Frequency

The first step concerns the identification of a shock or proxy ε_t^1 at the high frequency $t = 1, 2, ..., T$. ε_t^1 may come from narrative sources and event-studies, as in previous works, or from the estimation and identification in a HF-VAR. The HF-VAR has to be specified such that it is informationally sufficient. This means that the control variables Ω_t, included in the HF-VAR, allow the correct identification of ε_t^1 (without incorporating the other shocks $\varepsilon_t^{\bar{1}}$). Consider

$$\mathring{A}(L)\begin{bmatrix}\Omega_t \\ y_t^1\end{bmatrix} = \mathring{B}\begin{bmatrix}\boldsymbol{\varepsilon}_t^{\Omega} \\ \varepsilon_t^1\end{bmatrix} \tag{12}$$

The underlying assumption is that the set Ω is sufficient to distinguish ε_t^1 from $\varepsilon_t^{\bar{1}}$ and thus $\boldsymbol{\varepsilon}_t^{\Omega} \supseteq \varepsilon_t^{\bar{1}}$. Whereas there are obviously no direct diagnostic test over the identification assumption's employed, tests are available to test the informational deficiency of a VAR and will be discussed within the description of the next step.

3.3 Second Step: Correct Aggregation and Information Sufficiency Test

In the second step, we aggregate the shocks identified at high frequency and test for information sufficiency. For the first step, we consider two alternative definitions of the IRFs that have different implications for the correct aggregation of the HF shocks.

3.3.1 IRFs Aggregated via Temporal Filtering

The most natural definition of temporally aggregated IRFs is obtained by employing the temporal aggregation filter applied to the data to the HF-IRFs themselves. Thus, we differentiate this section across the two temporal aggregation schemes potentially applied to the data: skip-sampling and averaging.

Skip-Sampling

The skip-sampling case is simple because the second step of the temporal aggregation process consists of the trivial filter $W(L) = \mathcal{I}$ the leaves the variables unaffected. Skip-sampling is usually applied by taking the last value: for example the last daily observation within the month. We focus on this skip-sampling scheme without loss of generality.[10] Eq.(10) is modified by temporal aggregation as:

$$D(L)A(L)y_t = D(L)B\varepsilon_t \tag{13}$$

Under skip-sampling, the impact effect of ε on y is trivially given $\Theta_0^\tau = \Theta_0^t = B$. Suppose that the HF shocks $\varepsilon_{t,1}$, $\varepsilon_{t,2}$, ..., $\varepsilon_{t,m-1}$, which are indexed by the position within the LF periods, are identified under the assumptions described in Section 12. For instance, $\varepsilon_{t,m-1}$ is the last shock within the LF period and $\varepsilon_{t,1}$ is the first shock within the LF period.

Proposition I. *Given an underlying HF-VAR temporally aggregated via skip-sampling, the IRF $\Theta_0^\tau = B$ can be recovered by projecting the reduced form residuals estimated from the LF-VAR ξ_τ on the last HF shock within the LF period $\varepsilon_{t,m-1}$. Thus the correct filter $J(L)$ applied to ε_t is $J(L) = \mathcal{I}$ such that $\varepsilon_\tau = \varepsilon_{t,m-1}$.*

Proof: see appendix.

Averaging

The averaging case is more complex as the second filter is $W(L) = \mathcal{I} + L + L^2 + ... + L^{m-1}$. We use the summing filter, which is equivalent to averaging up to a constant. Consequently, the system becomes

$$A(L)D(L)W(L)y_t = D(L)W(L)B\varepsilon_t \tag{14}$$

The HF impact effect is again $\Theta_0^t = B$ but the IRFs must be consistently temporally

[10] Notice that the same results that we provide hold simply by using the shock corresponding to the skip-sampling scheme (e.g. take the first shock if skip-sampling is performed using the first HF value)

aggregated if we want to dispose of a reliable metric of comparison. Under summing, the impact effect of ε on y is given by $\Theta_0^\tau = \Theta_0^t + \Theta_1^t + \ldots + \Theta_{m-1}^t$.

Proposition II. *Given an underlying HF-VAR temporally aggregated via averaging, the IRF* Θ_0^τ *can be recovered by projecting the reduced form residuals estimated from the LF-VAR* ξ_τ *on the first HF shock within the LF period* $\varepsilon_{t,1}$*. Thus the correct filter* $J(L)$ *applied to* ε_t *is* $J(L) = L^{m-1}$ *such that* $\varepsilon_\tau = \varepsilon_{t,1}$.

Proof: see appendix.

3.3.2 IRFs Aggregated via Information Set

An alternative definition of IRFs, perhaps encompassing a more interesting concept for macroeconomists, considers the response of the LF-VAR to all the innovations occurring between two LF periods. In this perspective, the impulse is defined in terms of the LF information set $\mathcal{F}_\tau$. Let $\varepsilon_\tau = \{\varepsilon_{t,1}, \varepsilon_{t,2}, ..., \varepsilon_{t,m-1}\}$ such that $\varepsilon_\tau = 1$ implies $\{\varepsilon_{t,1} = 1, \varepsilon_{t,2} = 1, ..., \varepsilon_{t,m-1} = 1\}$. Then $\Theta_0^\tau = \mathbb{E}_{\mathcal{F}_{\tau-1}}[y_\tau/\varepsilon_\tau = 1]$ - $\mathbb{E}_{\mathcal{F}_{\tau-1}}[y_t/\varepsilon_\tau = 0]$ means that all the innovations occurring between $\tau - 1$ and τ are considered when defining the IRFs.. Define $\mathit{\Gamma}_0, \mathit{\Gamma}_1, ..., \mathit{\Gamma}_{m-1}$ the impact response to the shocks $\varepsilon_{t,1}, \varepsilon_{t,2}, ..., \varepsilon_{t,m-1}$ consistently aggregated as in the previous sections: via $W(L) = I$ in the point-in-time sampling case and $W(L) = I + L + ... + L^{m-1}$ in the averaging case. For instance, $\mathit{\Gamma}_{m-1} = \Theta_0^t = B$ and $\mathit{\Gamma}_{m-2} = \Theta_0^t + A_1\Theta_0^t = (\mathcal{I} + A_1)B$.

Then the aggregated response at LF is given by

$$\Theta_0^\tau = \sum_{i=0}^{m-1} \mathit{\Gamma}_j \tag{15}$$

In this case, there is no need of a specific exposition across skip-sampling and averaging temporal aggregation, as explained in Proposition III.

Proposition III. *Given an underlying HF-VAR temporally aggregated either via skip-sampling or averaging, the IRF* Θ_0^τ *can be recovered by projecting the reduced form* *residuals estimated from the LF-VAR ξ_τ on the average of the HF shock within the LF period $\bar{\varepsilon}_t$. Thus the correct filter $J(L)$ applied to ε_t is $J(L) = I + L + ... + L^{m-1}$.*

Proof: see appendix.

3.3.3 Partial Invertibility

Our HF identification is performed in a system that does not include the lower frequency endogenous variables. This feature may be considered problematic. For instance, in case of a frequency mismatch daily-monthly, this means that macroeconomic variables are not employed for identification but only in the third step of the Bridge-PSVAR as endogenous variables. The presence of endogenous macroeconomic variables is neither a necessary nor a sufficient condition to achieve a correct identification. A wide literature, starting from Sims (1998), has advocated the use of financial variables in VAR to exploit their information content. More importantly, any small scale VAR suffers the same problem of potential omitted variables. In fact, the literature has proposed a number of tests to detect whether omitted variables contaminate the identified shocks. The most popular is the Forni and Gambetti (2014) invertibility test. Such test is discussed in the framework of external instruments both in Stock and Watson (2018) and Miranda Agrippino and Ricco (2018). For this reason, the partial invertibility test of the aggregated shocks is a fundamental step in our methodology.

In practice, following Forni and Gambetti (2014), we regress ε^y_τ on the lags of factors or principal components Λ of large datasets. The following relationship should hold:

$$\mathbb{E}\left[\varepsilon^1_\tau \Lambda_{\tau-k}\right] = 0 \quad \forall k \in \mathbb{R}^+ \tag{16}$$

This test is particularly important because it assures that the shocks are not predictable and are proper innovations. Moreover, in our framework it constitutes a further diagnosis inherent to the omission of LF variables from the HF model, where identification is performed.

3.4 Third Step: External Instruments and Identification of the LF-VAR

The last step is twofold. First, the LF-VAR of order p in eq.(17) has to be estimated in a standard fashion. This yields the vector of reduced form residuals ξ_τ.

$$y_\tau = \tilde{A}_1 y_{\tau-1} + \tilde{A}_2 y_{\tau-2} + ... + \tilde{A}_p y_{\tau-2} + \xi_\tau \tag{17}$$

Second, the causal impact effect of ε^1 on y is identified by employing ε^1_τ as external instrument or proxy for the LF-VAR by projecting ξ_τ on ε^1_τ:

$$\Theta^\tau_0 \propto \mathbb{E}\left[\varepsilon^1_\tau \xi_\tau\right] \tag{18}$$

The inference is valid under three conditions as stated in Stock and Watson (2018) and Miranda Agrippino and Ricco (2018):

i) exogeneity: $\mathbb{E}\left[\tilde{\varepsilon}^1_\tau \varepsilon^{\bar{1}}_\tau\right] = 0$

ii) strength: $\mathbb{E}\left[\tilde{\varepsilon}^1_\tau \varepsilon^1_\tau\right] \neq 0$

iii) limited lag-lead exogeneity: $\mathbb{E}\left[\tilde{\varepsilon}^1_\tau \xi^{\bar{1}}_{\tau+k}\right] \neq 0$ for $\forall k \in \mathbb{R}$ and $j \neq 0$; testable via eq.(16).

The first condition implies that the proxy has to be (contemporaneously) uncorrelated with the other structural shocks of the system. This condition, analogous to the exclusion restriction for the IV estimator, cannot be tested directly and rests on the goodness of the identification strategy. The second condition is related to the relevance of the instrument. Montiel Olea et al. (2018) derive the conditions for this condition and compute the critical value of the F-Stat, which is informative if possible weak instruments concern. The critical value of this test in the context of the Proxy-SVAR with 5% critical value is 3.84, lower than in the traditional IV literature. The third conditions is related to partial invertibility as in eq.(16). Stock and Watson (2018) explicitly relate invertibility with omitted variables.

3.5 Discussion

Several issues related to our econometric framework and methodology are worth discussing.

First, the Bridge-Proxy SVAR can recover the correct impact $IRFs$ but the estimated autoregressive matrix of the LF-VAR may be biased because a VAR is in general transformed by temporal aggregation into a VARMA. However, depending on the cases and sample size, a VAR may well approximate the VARMA structure. Due to this reason, VAR models are typically used in empirical application instead of VARMA that suffer from high parametrization and severe problems in defining an economic interpretable structure. For a complete discussion of this issue, see Kilian and Lütkepohl (2017, Ch.2).

Second, in principle, one could compute the LF-IRFs both via Proxy-SVAR and LP-IV. However, our framework rests on a VAR structure at HF and without this assumption, nothing guarantees that the aggregation of the shocks employed as IV can recover the causal effect in a type of LP-IV regression.

Third, the confidence bands for the IRFs can be computed using different methods. While *Wild Bootstrap* is the most popular one (see for instance Mertens and Ravn, 2013 and Gertler and Karadi, 2015), Jentsch and Lunsford (forth.) pose some doubts on its properties in the external instrument framework and propose a residual-based moving block bootstrap procedure. Montiel Olea et al. (2018) employ instead the Delta method to compute the confidence sets. Mertens and Ravn (2018) conclude that the differences between these methods become larger when the F-statistic for the instrument is lower.

4 Monte Carlo Experiments

4.1 Experimental Design

We rely on Monte Carlo experiments to test the performance of the BP-SVAR in identifying the correct impact matrix in a general setup and finite samples. As we regard it as the most interesting IRF concept for macroeconomic analysis, our baseline design follows the definition of IRFs in Section 3.3.2.[11] We compare the performances of the HF-VAR (high frequency data), LF-VAR (time aggregated data), and the Bridge-PSVAR in recovering the underlying DGP. In our experiments, the Bridge-PSVAR identifies the shocks in a HF system that is informationally sufficient and then uses the aggregated shock to instrument the reduced form residual of the LF-VAR, which consists of the temporally aggregated variables. The LF-VAR and the BP-SVAR temporally aggregate information in antithetical ways. In a LF-VAR, the aggregation occurs before identification while the Bridge-PSVAR identifies the shocks at HF and then correctly filters them at the LF. Since we apply the correct filter, our methodology correctly recovers the impact causal effects. Conversely, the LF-VAR identifies IRFs that diverge from those implied by the data generating process.

Our "experimental" design is similar to that in Foroni and Marcellino (2016). The DGP described in eq.(19) is a $VAR(1)$ process driven by Cholesky innovations such that the different methodologies applied the same (correct) identification scheme.

$$\begin{pmatrix} x_t \\ y_t \end{pmatrix} = \begin{pmatrix} \rho_l & \delta_l \\ \delta_h & \rho_h \end{pmatrix} \begin{pmatrix} x_{t-1} \\ y_{t-1} \end{pmatrix} + \begin{pmatrix} b_{11} & 0 \\ b_{21} & b_{22} \end{pmatrix} \begin{pmatrix} e_t^x \\ e_t^y \end{pmatrix} \tag{19}$$

where $\begin{pmatrix} e_t^x \\ e_t^y \end{pmatrix} \sim \mathcal{N}(0, \mathcal{I}_2)$. The simulations consider the cases of temporal aggregation via skip-sampling and averaging for both the frequency mismatch m equal to 3 (monthly-quarterly case) and to 30 (daily-monthly case). We compare two different

[11] The same exercise run using the other definition of IRFs in Section 3.3.1 yields very similar results, actually quantitatively more favorable to the Bridge-PSVAR.

LF sample sizes $\mathcal{T}$ to take into account both short and large samples. Specifically, we set $\mathcal{T} = 100$ and $\mathcal{T} = 1000$. We avoid an arbitrary choice of the DGP parameters and thus randomly generate the parameters of the autoregressive and contemporaneous matrices. The autoregressive coefficients are drawn from a uniform distribution as $\{\rho_l, \rho_h, \delta_l, \delta_h\} \sim \mathcal{U}(-1, 1)$. We select those parametrizations with real eigenvalues that belong to the set $(0.7, 0.95)$ to avoid non-stationarity and to impose some persistence in the IRFs.[12] The impact coefficients $\{b_{11}, b_{22}\} \sim \mathcal{U}(0, 1)$ and $b_{12} \sim \mathcal{U}(-1, 1)$. We retain those parametrization that satisfy $b_{21} < b_{11}$ and $b_{21} < b_{22}$ to maintain a mapping between shocks and variables which is important to limit the problem of weak instruments. For the same reason, we impose $b_{11} > 0.1$ and $b_{22} > 0.1$. In order to provide a representative measure of the parameter space, we repeat the simulations for 100 random parametrizations.

Each experiment is repeated over 1000 simulations and the performances of the different methods are evaluated via the *Mean Absolute Distance* (MAD) between the true IRFs and the estimated one (cumulated over 8 horizons). Notice that our synthetic measure takes into account the precision of the estimates as the MAD is computed for each replication and then averaged over the whole set of replications. The MAD is defined as:

$$mad_{i,j}^{id} = \sum_{h=1}^{8} \left| \Theta(h)_{i,j} - \hat{\Theta}^{id}(h)_{i,j} \right| \quad i, j = \{1, 2\}$$

for $id = \{$HF-VAR, LF-VAR, Bridge-PSVAR$\}$. This metric is then aggregated, for each parametrization k, over variables j and shocks i as $MAD_k^{id} = \sum_{i=1}^{2} \sum_{j=1}^{2} mad_{i,j}^{id}$. Finally, a unique metric across all parametrization is obtained as $MAD^{id} = \sum_{k=1}^{100} MAD_k^{id}$.

[12] This is important because IRFs aggregated at LF are an uninteresting case without persistence, yielding zero effect independently of the impact matrix.

4.2 Specific Parametrization

Before presenting the general performances of the methodologies, this section illustrates graphically the results from a specific parametrization of the DGP. We select

$$\begin{pmatrix} \rho_l & \delta_l \\ \delta_h & \rho_h \end{pmatrix} = \begin{pmatrix} 0.71 & -0.82 \\ 0 & 0.82 \end{pmatrix} \quad \begin{pmatrix} b_{11} & 0 \\ b_{12} & b_{21} \end{pmatrix} = \begin{pmatrix} 0.28 & 0 \\ 0.23 & 0.95 \end{pmatrix}$$

for the parameters of eq.(19). We focus on the temporal aggregation via skip-sampling for a monthly-quarterly case ($m = 3$) with $\mathcal{T} = 1000$. Figures 1 and 2 display the IRFs of the system to a shock e_t^x and to a shock e_t^y, respectively. There are significant differences in the estimated IRFs by the HF-VAR (blue), Bridge-PSVAR (red), and LF-VAR (green), together with the IRFs implied by the DGP (black). The HF-VAR recovers perfectly the IRFs, whereas the LF-VAR completely misses the shape and sign of the impact effects of e^x on y, which in turns implies a wrong estimated dynamic effect also on x itself. The LF-VAR constraints the impact of e^y on x to be 0, missing in this way the actual negative effect. The Bridge-PSVAR closely replicates the performances of the HF-VAR but it is less efficient being a two stage estimation (IV versus OLS).

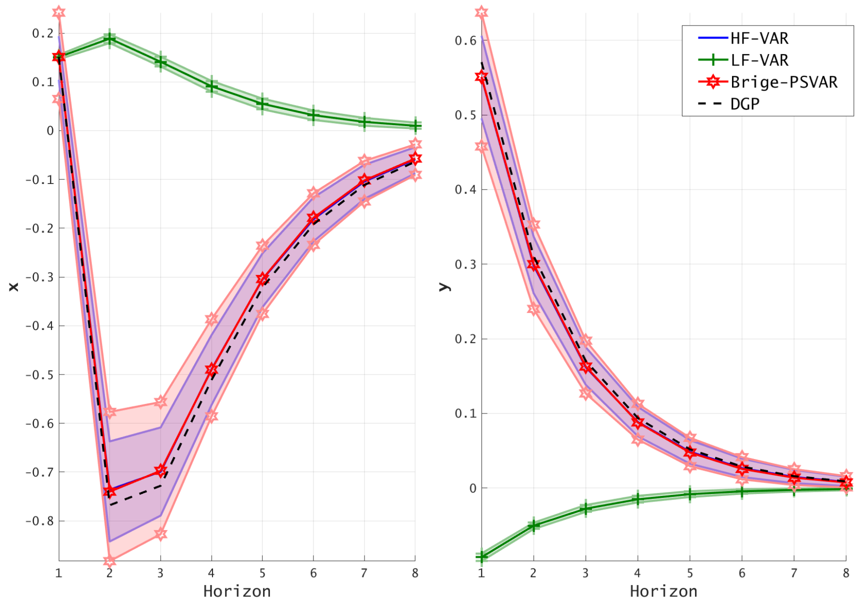

Figure 1: IRF TO ε^x - MONTE CARLO EXPERIMENT

IRFs to a shock in the first variable in the bivariate system (x). The true IRF is represented by the dotted black line. The autoregressive matrix is $\begin{bmatrix} 0.71 & -0.82 \\ 0 & 0.82 \end{bmatrix}$ and the impact matrix is $\begin{bmatrix} 0.28 & 0 \\ 0.23 & 0.95 \end{bmatrix}$. The shock is identified through the correct recursive structure in the HF system (blue), LF system (green) and Bridge-PSVAR (red). Shaded areas correspond to the 90% percentiles across 1000 replications. Time aggregation follows a skip-sampling scheme and the frequency mistmatch is 3 (monthly-quarterly case).

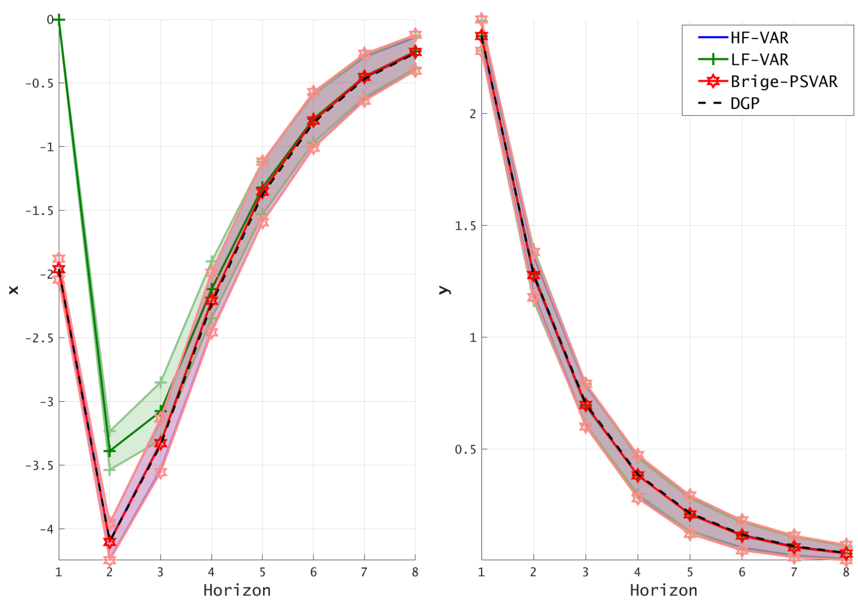

Figure 2: IRF TO ε^y - MONTE CARLO EXPERIMENT

IRFs to a shock in the second variable in the bivariate system (y). The true IRF is represented by the dotted black line. The autoregressive matrix is $\begin{bmatrix} 0.71 & -0.82 \\ 0 & 0.82 \end{bmatrix}$ and the impact matrix is $\begin{bmatrix} 0.28 & 0 \\ 0.23 & 0.95 \end{bmatrix}$. The shock is identified through the correct recursive structure in the HF system (blue), LF system (green) and Bridge-PSVAR (red). Shaded areas correspond to the 90% percentiles across 1000 replications. Time aggregation follows a skip-sampling scheme and the frequency mistmatch is 3 (monthly-quarterly case).

4.3 General Results

This section describes the general results from our Monte Carlo simulations. Our goal is to provide a synthetic measure of the differences in the accuracy of the three methodologies in estimating the IRFs. For this reason, for each of the 100 parametrizations of the DGP, we sum the (replication's mean) MAD errors for each IRF (two for each of the two shocks). This absolute measure is transformed into a relative summary statistics as the MAD percentage reduction compared to the LF-VAR that can be expressed as $M\tilde{A}D^{id} = \frac{MAD^{id} - MAD^{LF}}{MAD^{LF}}$ for $id = \{$HF-VAR, Bridge-PSVAR$\}$. Table 1 reports such a synthetic measure across the following crucial crucial features of the experiment: the definition of the IRFs (via the LF information set or the temporally aggregated), the temporal aggregation scheme (skip-sampling or averaging), the sample size at LF (small - 100 or large - 1000), and the frequency mismatch (3 or 30 representing the monthly-quarterly or daily-monthly cases).[13]

The MAD gains from the (counterfactual) HF-VAR represent the best possible estimation available both for the estimation of the VAR and for the identification, which is performed on data at the correct frequency. Our simulations confirm that the informational sufficient Bridge-PSVAR, by applying the appropriate temporal aggregation filter to the HF shocks, correctly recovers the contemporaneous matrix of the SVAR. The lower MAD gains reported for the BP-SVAR are due the loss of accuracy in a two stage approach, which resemble a standard loss of efficiency when using IV estimation. Nonetheless, the MAD gains are sizable and in many cases quite close to the performances of the HF-VAR. The gains of the Bridge-PSVAR increase significantly with a larger frequency mismatch for the averaging temporal aggregation case. Important

[13]Notice that whereas a skip-sampled VAR(1) remains a VAR(1), temporal aggregation by averaging transform the process into a VARMA. Consequently, the estimated autoregressive matrix of the LF-VAR is biased. This bias is decreasing in the sample size. On the one hand, the impact effect identified by the Bridge-PSVAR is not affected by the misspecification of the LF-VAR. On the other hand, the biased autoregressive matrix is common across the LF-VAR and the Bridge-PSVAR in computing the dynamic effects.

insights on the asymptotic properties of our methodology are revealed by the MAD gains increasing with the sample size in all cases. The increase in the precision of the estimation with the sample size is analogous to the asymptotic properties of the IV.

MAD gains over LF-VAR	IRFs - LF information set		IRFs - temporal aggregation	
	Skip-sampling	**Averaging**	**Skip-sampling**	**Averaging**
Small (LF) sample size T=100				
Frequency Mismatch: Monthly-Quarterly Case (3)				
HF-VAR	65%	83%	56%	61%
Bridge-PSVAR	59%	19%	54%	42%
Frequency Mismatch: Daily-Monthly Case (30)				
HF-VAR	80%	97%	72%	90%
Bridge-PSVAR	43%	42%	57%	74%
Large (LF) sample size T=1000				
Frequency Mismatch: Monthly-Quarterly Case (3)				
HF-VAR	87%	95%	57%	61%
Bridge-PSVAR	85%	21%	57%	42%
Frequency Mismatch: Daily-Monthly Case (30)				
HF-VAR	93%	90%	99%	90%
Bridge-PSVAR	79%	79%	93%	79%

Table 1

Performance comparison across the counter-factual HF-VAR, the LF-VAR, and the Bridge-PSVAR . *Performances are evaluated in terms of the Mean Absolute Distance (MAD) between the true IRFs and the estimated IRFs in 100 randomly parametrized DGPs. One summary statistic is computed as mean across all combinations of shocks-variables in the system. The gains are expressed as percentage MAD gains over the LF-VAR. As described in Section 3.3, the IRFs are defined in 2 ways: via the LF information set and the IRFs temporally aggregated. We analyze different cases for a VAR(1) DGP by varying: I) temporal aggregation scheme: skip-sampling or averaging; II) the frequency mismatch between HF and LF is either 3 (monthly-quarterly case) or 30 (monthly-daily case); III) sample size: small (100 LF observations) or large (1000 LF observations).*

5 Robust Macroeconomic Effects of Financial Shocks

During the Global Financial Crisis, corporate credit spreads have fluctuated considerably, reflecting both current and expected economic conditions and risks to the economic outlook. Previous works have examined the link between credit spreads and economic activity (e.g. Gilchrist and Zakrajsek, 2012). Specifically, Gilchrist et al. (2014) highlight the interaction between financial conditions and uncertainty and study their causal effects on the economy. In order to identify the effects of financial shocks, they estimate a quarterly VAR for the U.S. economy that includes the following variables: real business fixed investment, real personal consumption expenditures on durable goods, real consumption expenditures on non-durable goods and services, real GDP, GDP deflator, nominal effective federal funds rate, 10 year BBB-Treasury credit spread, and a proxy for idiosyncratic uncertainty at the aggregate level. To identify the effects of financial shocks, they consider the recursive ordering described above (i.e. financial shocks affect only uncertainty within the same quarter and can affect the remaining variables of the system only with a delay of one quarter). In order to assess the robustness of their results, they also consider an alternative ordering where the credit spread is ordered after the proxy for uncertainty. In this section, we estimate a quarterly VAR over the sample 1997:Q1-2017:Q3 that includes a similar set of variables to Gilchrist et al. (2014), replacing the U.S. BBB corporate spread with the excess bond premium (EBP, i.e the popular proxy for financial conditions proposed in Gilchrist and Zakrajsek, 2012).[14] Following Gilchrist et al. (2014), we apply two Cholesky identification schemes at this frequency. The first Cholesky decomposition places the EBP on top of the financial block (sixth), whereas the alternative decomposition places the EBP at the bottom of the financial block. When the EBP is placed on top (bottom) of the financial block, financial conditions are assumed not to respond to fluctuations in the VXO equity implied volatility index, fed fund rate, and 10-year spread for one quarter. Viceversa when the EBP is placed at the bottom, the other variables are allowed to respond to financial shocks within the same quarter; conversely, EBP is assumed not react to fluctuations in the other financial variables. These timing restrictions imposed on financial variables may not be appealing since HF variables react instantaneously to shocks to other variables. Thus, these restrictions tightly constrain the data, potentially affecting the estimated dynamic effects. This dependence is clearly depicted in Figure 3,

[14] The sample starts in 1997 due to the availability of high frequency data. In particular, the objective of this application is to use exactly the same sample for both identification strategies so that the potential difference in results does not come from the different information set. The conclusions we obtain are comparable to the ones of Gilchrist et al. (2014). Similar results hold using the BBB spread (see Appendix).

which displays the IRFs to a financial shock using both orderings. The responses of inflation and non-durable consumption (but also of VXO, fed fund rate, and the 10-year rate) depend on the Cholesky ordering chosen, which is in line with the lack consensus on the effects of financial shocks on prices in the existing literature (e.g., Gilchrist and Zakrajsek, 2012 and Abbate et al., 2016). Both in the theoretical and empirical literature, financial shocks may produce effects consistent with demand or supply shocks depending on the assumptions underlying the model (may it be theoretical or econometric). On the one hand, an exogenous reduction of the EBP may act as a demand shock since it increases asset prices and generates a positive wealth effect, increasing consumption. On the other hand, the reduction of the EBP reduces firms' borrowing rate, reducing their marginal costs and inducing disinflationary effects. Finally, the response of the Fed Funds rate also depends on the ordering of the variables and this is a crucial issue for monetary policy (Peek et al., 2016; Caldara and Herbst, 2018).

The lack of robustness in these results likely depends on the timing restrictions imposed at the quarterly frequency on financial variables like the VXO and the BBB spread. As asset prices update continuously, incorporating new information, their co-movement tends to increase as the data is aggregated at lower frequencies, irrespective of which was the original source of fluctuations. To avoid imposing tight identifying assumptions, we rely on a daily VAR estimated on US data over the sample 1997m1-2017m10. In the daily analysis we employ the BBB spread because EBP is not available at daily frequency.[15] The system includes the (option adjusted) U.S. BBB corporate spread (FRED: BAMLC0A4CBBB), SP&500, VXO, Brent spot oil price (OILBREN), the BLS spot commodity price index (CRBSPOT), EuroDollar spot exchange rate (USEURSP), Dollar-Pound spot exchange rate (USDOLLR), gold spot price index (GOLDHAR), economic policy uncertainty index, Libor 3M, the 1Y Treasury term premium (USTTP1Y), and 10Y Treasury term premium (USTTP10). The conditional correlation between the BBB corporate spread and the VXO falls from 0.69 to 0.24 when

[15] The two variables are highly correlated (0.95) at the monthly frequency.

we change the frequency of the system. We identify a shock to the U.S. BBB corporate spread using two identification schemes comparable with the quarterly analysis previously described. First, we order the BBB corporate spread first in the daily system (i.e. a financial shock can affect all the other variables of the system within the same day). Second, we order the BBB spread after VXO, Libor, and 10Y spread (i.e. assuming that a financial shocks cannot affect these variables within the same day). Then, we aggregate the two series of shocks identified test if they are invertible. Table 2 reports the results from a regression of the identified financial shocks aggregated at the quarterly frequency on the lagged factors from the McCracken and Ng (2016) database.

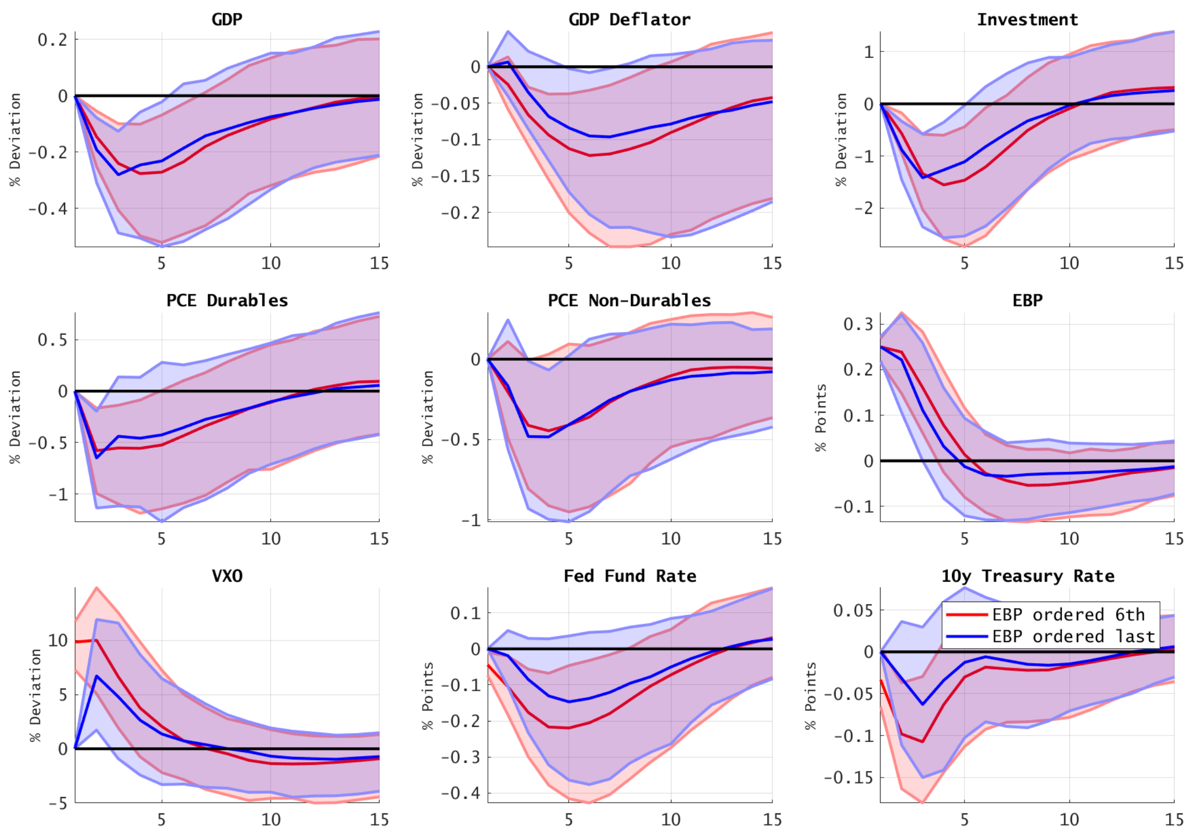

Figure 3: IRF to a Financial Shock - Gilchrist et al. (2014)

IRFs to a financial shock, shock to the EBP, identified using a Cholesky decomposition. The VAR includes [GDP, GDP Deflator, Investment, Consumption of Durable Goods, Consumption of Non Durable Goods, Execess Bond Premium, VXO; Fed Fund rate, 10Y Treasury rate] and it is estimated in log-levels with the optimal number of lags (2) and includes a deterministic constant. The read (blue) line denote the median impulse response when EBP is ordered in the fifth (last) place of the system. Shaded areas correspond to 95% bootstrapped confidence bands from 1000 replications.

	$\varepsilon_{1,\tau}$	$\varepsilon_{2,t}$
$f_{1,t-1}$	0.04	0.04
	(0.06)	(0.06)
$f_{2,t-1}$	0.13	0.15*
	(0.09)	(0.09)
$f_{3,t-1}$	-0.05	-0.05
	(0.05)	(0.05)
$f_{4,t-1}$	0.03	0.02
	(0.10)	(0.10)
$f_{5,t-1}$	0.07	0.05
	(0.11)	(0.11)
$f_{6,t-1}$	-0.25**	-0.22*
	(0.11)	(0.11)
$f_{7,t-1}$	0.04	0.04
	(0.11)	(0.11)
Observations	83	83
R^2	0.11	0.11
F-stat	1.10	1.11

Table 2: Invertibility Test - Financial Shocks
*Regressions include one lag of the dependent variable and a constant. $\varepsilon_{1,\tau}$ and $\varepsilon_{2,t}$ denote the aggregated financial shock identified at daily frequency using two different Cholesky decompositions. F-stat denotes the value of the F test statistic. Standard errors are reported in parenthesis, *** $p<0.01$, ** $p<0.05$, * $p<0.1$. The optimal number of factors for FRED- database is 7 for the sample 1997m1-2017m10.*

The lagged factors do not have explanatory power for the identified financial shocks since the F-statistics is not statistically different from zero (see Forni and Gambetti, 2014). Thus, the identified financial shocks are invertible and can be employed as a proxy for the EBP in the quarterly VAR (see Miranda Agrippino and Ricco, 2018). The quarterly system, which includes the same variables as Figure 3, is estimated using a sample which is consistent with the daily VAR.[16] Figure 4 displays the IRFs to a financial shock identified with the BP-SVAR, employing these two series of financial shocks identified at daily frequency.

[16]Results are robust to using the U.S. BBB corporate spread instead of the excess bond premium in the quarterly VAR. These results are presented in the online appendix XX.

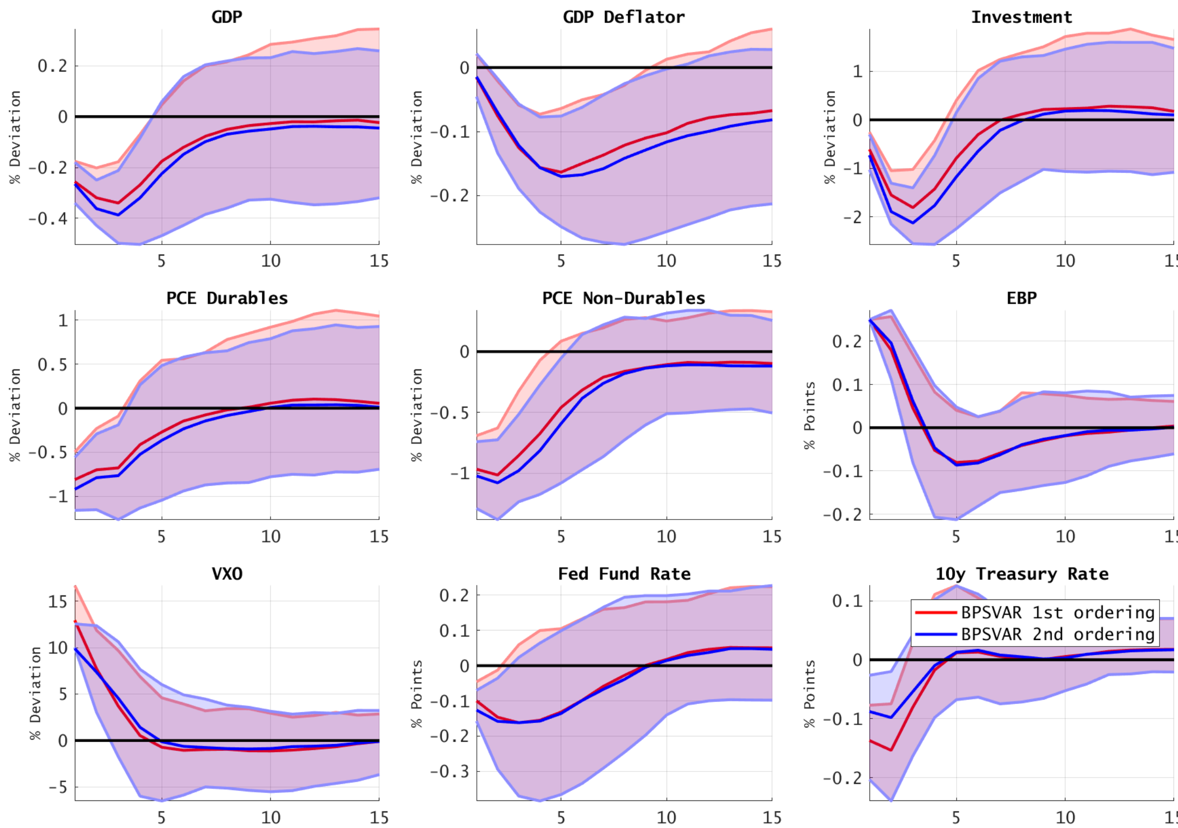

Figure 4: IRF to a Financial Shock

IRFs to a financial shock identified by instrumenting the Excess Bond Premium with the series of shocks in the U.S. BBB corporate spread. The shocks to the U.S. BBB corporate spread are identified using a daily VAR with two different recursive ordering, one where the BBB corporate spread is ordered before VXO (first order) and another where BBB corporate spread is ordered after the VXO, Libor, and 10Y spread (second order). The daily series of shocks to the BBB corporate spread are aggregated to the monthly frequency as an average. The first stage results are $F - stat = 35.7$ *and* $R^2 = 0.32$ *with the first daily ordering and* $F - stat = 29.62$ *and* $R^2 = 0.28$ *with the second daily ordering. The VAR includes [GDP, GDP Deflator, Investment, Consumption of Durable Goods, Consumption of Non Durable Goods, Execess Bond Premium, VXO; Fed Fund rate, 10Y Treasury rate] and it is estimated in log-levels with the optimal number of lags (2) and includes a deterministic constant. Shaded areas correspond to 95% bootstrapped confidence bands from 1000 replications.*

Unlike the identification based on quarterly data, the IRFs displayed in Figure 4 are robust and the economic implications do not hinge on the specific identifying assumptions: a financial shock induces a significant decline in economic activity and in prices. The BP-SVAR employs the timing assumptions that constrain the data only for one day versus one quarter of a standard VAR. Although the results are not dramatically different from those in Gilchrist et al. (2014), there are some important differences. First, the decline of the CPI is statistically significant, contributing to characterize this shock as a demand one (prices and quantities decrease). Second, the financial shock induces a comparable decline in durable and non-durable consumption, whose dynamics are important to shed light on the transmission channels to these shocks. Finally, the financial shock induces a significant and immediate decline of the interest rate, both short and 10Y rate. This mechanism, which is key for policy analysis, is identified only with the BP-SVAR but disappears with the quarterly one. In this section we have focused on the identification of financial shocks using the same restrictions as Gilchrist and Zakrajsek (2012) and Gilchrist et al. (2014) at higher different frequencies, which deliver more robust implications. We do not analyze the role of uncertainty because Alessandri et al. (2018) is totally devoted to assessing the macroeconomic effects of financial shocks through the BP-SVAR.

6 Macroeconomic Effects of Monetary Policy Shocks

The macroeconomic effects of monetary policy is a widely studied and debated topic. One of the key challenges is that central banks use the interest rate to stabilize the economy which makes it difficult to disentangle a causal relationship. The other crucial challenge is related to the anticipation in the movements of short-term interest rates. Previous works in the literature have identified monetary policy surprises as the variation in the Fed Funds future contracts in a 30 minute window around FOMC meetings.[17] Focusing on a tight window around these events helps to isolate monetary policy news from other types of shocks.

We follow a similar approach by employing Fed Funds future contracts in a daily VAR. The identification of monetary policy shocks in a more structural model offers a higher degree of robustness relative to the monetary policy surprises. Three features are particularly appealing in light of recent findings in the literature. First, Peek et al. (2016) and Caldara and Herbst (2018) have documented that the Federal Reserve reacts to financial conditions. Thus, the identification of monetary policy shocks at high

[17] Moreover, over this sample, we do not face an additional identification challenge due to the presence of another component of monetary policy, i.e. the asset purchases programs.

frequency, which controls timely for financial conditions, can be key to isolate an exogenous variation. Second, Neuhierl and Weber (2018a) provide evidence of predictability of the standard monetary policy surprises in stock market returns 25 days before the FOMC dates. Our daily VAR includes lagged asset prices, thus making our monetary policy shocks orthogonal to past equity excess returns. Third, Neuhierl and Weber (2018b) document that monetary policy affects stock market throughout the year and not only on FOMC dates. Specifically, the speeches by the chair and vice-chair of the Fed constitute monetary policy news captured by the stock market. The daily VAR that we employ in this section captures monetary policy news continuously and can account for monetary policy events outside FOMC dates.

We estimate a daily VAR on the sample 1991:m10-2008:m6 to avoid the period of the zero lower bound, which may be regarded as a different regime. Moreover, over this sample, we do not face an additional identification challenge due to the presence of another component of monetary policy, i.e. the asset purchases programs. The daily VAR consists of the following variables: the front Fed Funds future contract, S&P500 index, VXO index, Brent crude oil price, the BLS spot commodity price index (CRBSPOT), gold spot price index (GOLDHAR), U.S. BAA corporate spread, Cleveland FED financial stress index, Arouba-Diebold-Scotti business conditions index, 1-year treasury yield. The front Fed Fund Future (FF1) is our indicator of monetary policy. The system is estimated in (log)-levels and includes 35 lags for two reasons. First, every time there is an FOMC meeting, the VAR includes information from the previous FOMC meeting. Second, including financial variables further back in time tackles the concerns raised in Neuhierl and Weber (2018a) on the predictability of monetary policy surprises.

As previous studies have shown that monetary policy surprises affect financial variables at daily frequency (e.g. Gurkaynak et al., 2005), timing assumptions (i.e Cholesky decomposition) are not appealing to identify monetary policy shocks (even at daily frequency). Thus, we identify shocks in the FF1 employing a more agnostic identification strategy: Independent Component Analysis (ICA).[18] ICA has been widely used to recover the original underlying signals from the observation of mixtures of them, which is exactly the identification problem in SVARs. Intuitively, ICA can be seen as a generalization of principal component analysis. While principal component analysis searches for uncorrelated latent components, ICA maximizes the statistical independence among such components. If the distribution is normal, the two concepts are equivalent because two gaussian random variables are independent if and only if they are uncorrelated. The covariance between the two random variables provides, in this case, all the useful information on independence. However, when departing from gaussianity, independence depends also on higher order moments, and ICA also exploits these higher order moments. In fact, in a VAR framework, the reduced form residuals can be decomposed in uncorrelated structural shocks in infinite ways because the covariance matrix is symmetric.[19] ICA uses moments different from covariances in the search for the (unique) combination of the most statistically independent components. Minimizing the deviations from independence is equivalent to minimize the distance between the joint distribution of the shocks and the product of their marginal distribution. Both visual inspection and the Kolmogorov-Smirnov test reject the normality of all reduced form residuals. Therefore, we exploit the deviations from normality for identification.

A useful check on our identification is the comparison with the Gurkaynak et al. (2005) surprises (GSS). Focusing only on FOMC meetings dates, our identified shocks correlate 0.84 with the current factor of monetary policy surprises (GSS1) and 0.73 with the surprises in the front Fed Funds future contract. Moreover, as we expect *a priori* for

[18] Lanne and Lutkepohl (2010) and Lanne et al. (2017) suggest that non-Gaussianity can be exploited for the identification of the structural shocks in a SVAR model. Recent applications of ICA to VARs are Hyvärinen et al (2010), Capasso and Moneta (2016), Herwartz and Plodt (2016), and Gouriéroux & Monfort, (2018). Gourieroux et al. (2017)derive the statistical properties of the ICA estimates. Bonhomme and Robin (2009) provide Monte Carlo evidence on the accuracy of the technique.

[19] We miss $\frac{N(N-1)}{2}$ moments from the data to achieve identification, where N is the number of variables in the system.

monetary policy shocks, the standard deviation of the monetary policy shocks is (2.5 times) larger on FOMC dates versus non-FOMC dates.[20]

We aggregate the shock to the monthly frequency using two different aggregation schemes. First, we pick only the monetary policy shocks that occurred on FOMC meeting days and compute a monthly average of this series. Second, we compute the monthly average using the daily series of monetary policy shocks (i.e. averaging all the daily shocks in the month). Considering that monetary policy has been increasingly relying on communication, the daily shocks outside FOMC dates capture monetary policy events that could be related to speeches and announcements between FOMC dates. We regress the surprises on the lags of the factors extracted from the macroeconomic data set developed by McCracken and Ng (2016). Table 3 displays the results of the invertibility test on our two series of shocks identified with the Bridge Proxy-SVAR. The first one considers daily shocks on FOMC dates only ($\varepsilon^{M}_{DVAR-ALL,\tau}$), while the second one is the monthly average of all the daily shocks ($\varepsilon^{M}_{DVAR-FOMC,t}$). As in Miranda-Agrippino and Ricco (2017) and Miranda Agrippino and Ricco (2018), we show that the raw monetary policy surprises, aggregated as a simple average as in Caldara and Herbst (2018) and Stock and Watson (2018), do not pass the invertibility test proposed by Forni and Gambetti (2014).[21] Thus, those surprises are not suitable as external instruments for VAR identification. Conversely, the series of monetary policy shocks identified using the Bridge Proxy-SVAR are orthogonal to the lagged factors, meaning that these shocks are invertible and are valid proxies for monetary policy shocks.

[20]The Online Appendix contains a detailed description of the main monetary policy events identified with this identification strategy and a comparison with other series of monetary policy shocks.

[21]The test reject even more neatly the invertibility if we use the surprises aggregated as monthly moving averages as in Gertler and Karadi (2015).

	$\varepsilon^M_{RAW,\tau}$	$\varepsilon^M_{DVAR-ALL,\tau}$	$\varepsilon^M_{DVAR-FOMC,t}$
$f_{1,t-1}$	0.03**	-0.14*	-0.07**
	(0.01)	(0.08)	(0.03)
$f_{2,t-1}$	0.01	-0.01	-0.01
	(0.01)	(0.08)	(0.03)
$f_{3,t-1}$	-0.04	0.11	-0.02
	(0.01)	(0.11)	(0.04)
$f_{4,t-1}$	0.01	-0.05	0.06
	(0.02)	(0.13)	(0.05)
$f_{5,t-1}$	0.03**	0.01	0.03
	(0.02)	(0.12)	(0.04)
$f_{6,t-1}$	-0.02	-0.11	-0.02
	(0.02)	(0.13)	(0.05)
$f_{7,t-1}$	-0.04**	0.04	-0.03
	(0.01)	(0.11)	(0.04)
Observations	199	199	199
R^2	0.12	0.06	0.05
F stat	3.37***	1.48	1.23

Table 3: Invertibility Test - Monetary Policy Shocks

Regressions include one lag of the dependent variable and a constant. $\varepsilon^M_{GK15,\tau}$*,*$\varepsilon^M_{ALL,\tau}$ *, and* $\varepsilon^M_{FOMC,t}$ *denote the aggregated series of monetary policy surprises of Gertler and Karadi (2015) and the monetary policy shocks identified with the Bridge Proxy-SVAR using all dates and FOMC meeting days only, respectively. Standard errors are reported in parenthesis, *** p<0.01, ** p<0.05, * p<0.1. The optimal number of factors for FRED- database is 7 for the sample 1991:m10-2008:m6.*

In order to identity the macroeconomic effects of monetary policy shocks, we use the aggregated series as a proxy for the structural shock of interest in a monthly VAR. The monthly VAR includes the following variables: Fed Funds rate, consumer price index, industrial production, and excess bond premium. We estimate the VARs in (log)-levels, including a constant with 2 lags, as suggested by the information criteria and we evaluate the strength of the instrument using the criterion developed by Stock and Yogo (2005). The F-statistics that we report for each of our first stage regressions are always above the critical values. Figure 5 displays the IRFs to a monetary policy shock.

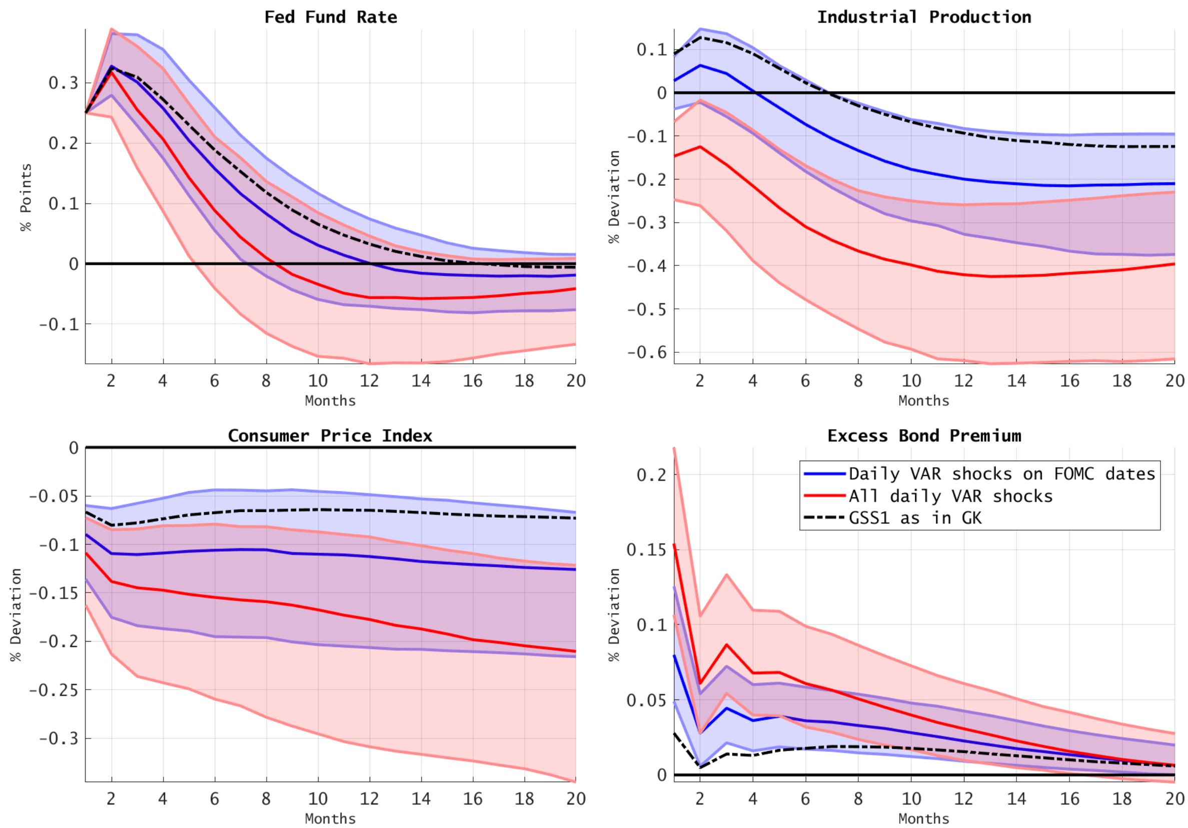

Figure 5

IRFs to a monetary policy shock identified by instrumenting the Fed Fund Rate with the series of daily shocks on FOMC dates (in blue) and on all the dates (in red), and using the series of surprises of Caldara and Herbst (2018). The first stage estimates are: $F - stat = 9.3$ and $R^2 = 0.04$ (Daily shocks on FOMC dates), $F - stat = 8.5$ and $R^2 = 0.04$ (all daily shocks), and $F - stat = 14$ and $R^2 = 0.06$ (Gertler and Karadi, 2015 shocks),. The VAR includes [FFR, CPI, Industrial Production] and it is estimated in log-levels with the optimal number of lags (2) and includes a deterministic constant. Shaded areas correspond to 95% bootstrapped confidence bands from 1000 replications.

The figure shows that the effect of monetary policy shocks is robust across the two aggregation schemes. Industrial production declines after some months, reaching its minimum at around 16 months after the shock. The consumer price index also declines significantly and reaches its minimum around 6 months after the shock. Finally, the monetary policy shock induces a significant and persistent increase in the excess bond premium. All these results are in line, both qualitatively and quantitatively, with previous studies (see for example Gertler and Karadi, 2015). Another interesting finding is that using only the shocks at FOMC meeting dates and using all the daily shocks as a proxy yield comparable impulse responses. This fact reinforces the relevance of FOMC meetings as the main monetary policy events.

7 Conclusions

High frequency identification is a powerful tool that has solved some puzzles in empirical macroeconomics. However, HF identification is currently performed exclusively by exploiting the movement of particular variables around specific events, which are limited to special cases and sometimes arbitrary.

To exploit HF identification more efficiently and in a larger set of cases, this paper proposes a new methodology, the *Bridge Proxy-SVAR* (Bridge-PSVAR) that attenuates temporal aggregation biases. Structural shocks are recovered in high frequency systems, correctly aggregated to the lower frequency, and used as a proxy for a structural shock of interest in lower frequency VARs. In this way, identification restrictions are derived from high frequency information. The methodology exploits high frequency data for identification by controlling for the correct information set of agents when making decisions. We derive analytically the correct aggregation filter for the Bridge-PSVAR across different cases. This paper provides the first proof that shocks identified at HF, correctly aggregated to the lower frequency, and used as external instruments for the identification of SVAR recover the true causal impact effect without temporal aggregation bias.

The quantitative performances of the methodology, also in small-samples, are tested through Monte Carlo simulations. Our methodology largely outperforms a LF-VAR using temporally aggregated data, which is the common naive practice in applied macroeconomics. The BP-SVAR can replicate the performances of a counter-factual HF-VAR, which constitutes the best possible estimation, with lower precision. Unlike existing mixed frequency techniques that are subject to computational challenges, especially under irregular frequencies, our methodology can exploit daily data in large dimensional systems to improve the identification of SVARs.

The potential of the methodology is exemplified in two applications. First, important differences emerge when comparing quarterly versus daily identification of financial shocks. The macroeconomic effects of financial shocks are highly dependent on the identifying assumptions if those are imposed at quarterly frequency. The conclusion on the response of the price level and so on the nature of financial shocks depends on the recursive ordering chosen in the VARs of Gilchrist et al. (2014). The same holds for the response of monetary policy and long-term rates. Conversely, identifying financial shocks in a daily VARs imposes lower constraints on the data and different recursive orderings imposed at this frequency yield consistent results: the price level falls after a financial shock and monetary policy reacts to it. Second, we recover monetary policy shocks in a daily VAR applying independent component analysis. Our identified shocks are highly correlated with the Gurkaynak et al. (2005) surprises traditionally employed in the literature but, while the former are invertible and are valid external instruments, the latter are contaminated by previous macroeconomic innovations. Our methodology can thus tackle problems of predictability and information release in monetary policy surprises.

Importantly for future research, the Bridge-PSVAR extends the implementability of the high frequency identification of SVARs to the vast number of analysis that cannot rely on a definite set of events to isolate shocks. High frequency identification employs the correct information set of agents, not mixing the real innovations with the endogenous response of the system to the shocks. Among the many possible applications, the Bridge-PSVAR is particularly promising to improve structural analyses on the macroeconomic effects of financial shocks, which are characterized by a wide frequency mismatch, irregular frequency, and large dimensionality.

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Appendix

A Proofs

A.1 Proposition I

The correct impact matrix can be recovered simply projecting u_τ on $\varepsilon_{t,m-1}$. The LF reduced form residuals are given by

$$\xi_\tau = D(L)u_t = D(L)B\varepsilon_t \tag{A.1}$$

Recall that $D(L) = \mathcal{I} + D_1 L + D_2 L^2 + ... + D_{pm-p}L^{pm-p}$ always contains the identity matrix as first term. Thus

$$D(L)B\varepsilon_t = B\varepsilon_t + D_1 B\varepsilon_{t-1} + D_2 B\varepsilon_{t-2} + ... + D_{pm-p}B\varepsilon_{t-pm+p} \tag{A.2}$$

and independently of the values of D_i (that depend on the HF-VAR lag length p and frequency mismatch m) the following relationship holds:

$$Proj\left(\xi_\tau/\varepsilon_t\right) = B = \Theta_0^\tau \tag{A.3}$$

A.2 Proposition II

It is convenient to express the IRFs at horizon k through the companion form of the VAR:

$$x_t = F\,x_{t-1} + \eta_t \tag{A.4}$$

where

$$F = \begin{bmatrix} A_1 & A_2 & A_3 & \dots & A_p \\ \mathcal{I} & 0 & 0 & \dots & 0 \\ 0 & \mathcal{I} & & \ddots & \vdots \\ \vdots & \ddots & \ddots & & \\ 0 & & & \mathcal{I} & 0 \end{bmatrix} \tag{A.5}$$

$$\eta_t = \begin{bmatrix} u_t = B\varepsilon_t \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{A.6}$$

$x_t = [y_t \, y_{t-1} \ldots y_{t-p}]'$ where p is the lag length of the VAR, $x_{t-1} = Lx_t$ and $\eta_t = [\varepsilon_t \, 0 \ldots 0]'$. Then given $\Theta_0^t = B$ the dynamic effects can be written as well in companion form as :

$$\tilde{\Theta}_k^t = F\tilde{\Theta}_{k-1}^t \, k > 0 \tag{A.7}$$

We are interested in the matrix that contains the impulse response at horizon k, positioned in $\tilde{\Theta}_k^t(1,1) = \Theta_k^t$.

A result that we use in what follows comes from Kalman (1982) and Kilic (2007) who have shown that this formulation of the IRFs corresponds to $S_p(A_1, ..., A_p)$, a generalized Fibonacci sequence of order p in the matrices $A_1, A_2, ..., A_p$ (also referred to as generalized order p Fibonacci polynomial). Given an initial condition B, $S_p(A_1, ..., A_p)$ generates a sequence whose elements are a linear combination of the previous terms in the sequence weighted by $A_1, ..., A_p$. A_1 is the weight associated to the previous element of the sequence whereas A_p multiplies the $p-th$ previous element. For instance $S_0 = B$, $S_1 = A_1S_0$, $S_2 = A_1S_1 + A_2S_0$, and so on and so forth.

Following Marcellino (1999), we define the following vector of matrices D^v and A^v with dimension $1 \times pm$, where m denotes the frequency mismatch, and the matrix of matrices G with dimension $(pm-p) \times pm$

$$\begin{aligned}
D^v &= (D_1, D_2, ..., D_{pm-p}) \\
A^v &= (A_1, A_2, ..., A_p, 0, ..., 0) \\
G &= \begin{bmatrix}
-\mathcal{I} & A_1 & A_2 & ... & A_{p-1} & A_p & 0 & 0 & \dots & & & 0 \\
0 & -\mathcal{I} & A_1 & A_2 & ... & A_{p-1} & A_p & 0 & 0 & \dots & & 0 \\
0 & 0 & -\mathcal{I} & A_1 & A_2 & ... & A_{p-1} & A_p & 0 & 0 & \dots & 0 \\
\vdots & 0 & 0 & -\mathcal{I} & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\
 & \vdots & 0 & \ddots & \ddots & & & & & & & \\
 & & \vdots & & & & & & & A_{p-1} & A_p & 0 \\
0 & \cdots & & 0 & \dots & 0 & -\mathcal{I} & A_1 & A_2 & ... & A_{p-1} & A_p
\end{bmatrix}
\end{aligned}$$

Marcellino (1999) shows that $D(L)$ exists if $|G_{-k}| \neq 0$ where G_{-k} corresponds to the G matrix whose columns multiple of m have been deleted $(m, 2m, ...)$. Then, the coefficients of $D(L)$ are given by $D^v = -A^v_{-k}\left(G_{-k}\right)^{-1}$. The intuition is that they have to be such that only the powers of L^m can have a coefficient different from 0 (since the variables are unobservable between L^m and L^{2m}).

Since our goal is the identification of the impact effect Θ_0^τ, we focus exclusively on the upper square block of G of dimension $m-1 \times m-1$, denoted by $\tilde{G}$. Consistently, we denote $\tilde{A}^v$ and $\tilde{D}^v$ the vector containing the first $m-1$ elements of the original A^v and D^v vectors. The matrix $\tilde{G}$ is a very special matrix, being a Toeplitz upper triangular matrix with main diagonal $-\mathcal{I}$. Sahin (2018) shows that, if invertibility is satisfied, the inverse of this class of matrices, denoted in our case by $\tilde{G}^{-1}$, contains the elements of the

Fibonacci sequence in the matrix $A_1, ..., A_p$. By considering that

$$\tilde{D}^v = -\tilde{A}^v \tilde{G}^{-1}$$

it follows that the elements of $\tilde{D}^v$ correspond to $S_p\left(A_1, ..., A_p\right)$, the Fibonacci sequence in $A_1, ..., A_p$. Disregarding the initial condition B, the IRFs $\Theta_0^t, \Theta_1^t, ..., \Theta_{m-1}^t$ and the elements of the temporal aggregation filter $\mathcal{I}, D_1, D_2, ..., D_{m-1}$ are generated by the same generalized Fibonacci sequence $S_p\left(A_1, ..., A_p\right)$. Thus, we conclude that they are the same. Finally, recall that the temporally aggregated residuals are given by $\xi_\tau = D(L)W(L)\varepsilon_t$. The filter $W(L) = \mathcal{I} + L + ... + L^{m-1}$ implies the first shock within the LF period, i.e. $\varepsilon_{t,1}$, enter u_τ through all the terms $\Theta_0^t, \Theta_1^t, ..., \Theta_{m-1}^t$ and thus recovers the correct $\Theta_0^\tau = \Theta_0^t + \Theta_1^t + ... + \Theta_{m-1}^t$. Thus, the projection of y_τ on ε_{t-m+1} yield the correct IRFs Θ_0^τ:

$$Proj\left(\xi_\tau / \varepsilon_{t-m+1}\right) = \Theta_0^\tau \tag{A.8}$$

A.2.1 Direct Computation

This result can be also checked by direct computation. Here we focus on the case $m = 4$ and $p > 2$.

The HF-IRFs are given by

$$\Theta_0^t = B$$

$$\Theta_1^t = A_1 \Theta_0^t = A_1 B$$

$$\Theta_2^t = A_1 \Theta_1^t + A_2 \Theta_0^t = B\left(A_1^2 + A_2\right)$$

$$\Theta_3^t = A_1 \Theta_2^t + A_2 \Theta_1^t + A_1 \Theta_0^t = B\left(A_1^3 + A_1 A_2 + A_2 A_1 + A_3\right)$$

$$\Theta_0^\tau = \Theta_0^t + \Theta_1^t + \Theta_2^t + \Theta_3^t = B\left(I + A_1 + A_1^2 + A_2 + A_1^3 + A_1A_2 + A_2A_1 + A_3\right)$$

The temporal aggregation matrix is

$$\tilde{G} = \begin{bmatrix} -\mathcal{I} & A_1 & A_2 \\ 0 & -\mathcal{I} & A_1 \\ 0 & 0 & -\mathcal{I} \end{bmatrix}$$

and

$$\tilde{G}^{-1} = \begin{bmatrix} -\mathcal{I} & -A_1 & -A_1^2 - A_2 \\ 0 & -\mathcal{I} & -A_1 \\ 0 & 0 & -\mathcal{I} \end{bmatrix}$$

Consistently,

$$\tilde{A}^v = (A_1 \, A_2 \, A_3)$$

$$\begin{aligned} \tilde{D}^v &= -\left(A_1 \, A_2 \, A_3\right) \begin{bmatrix} -\mathcal{I} & -A_1 & -A_1^2 - A_2 \\ 0 & -\mathcal{I} & -A_1 \\ 0 & 0 & -\mathcal{I} \end{bmatrix} \\ \tilde{D}^v &= \begin{bmatrix} A_1, & A_1^2 + A_2, & A_1^3 + A_2A_1 + A_2A_1 + A_3 \end{bmatrix} \end{aligned}$$

The vector $\tilde{D}^v$ corresponds to the IRFs by period until period $m-1$. The temporally aggregated residuals are given by:

$$\begin{aligned}\xi_\tau &= D(L)W(L) = W(L)B\left[\varepsilon_t + BA_1\varepsilon_{t-1} + \left(A_1^2 + A_2\right)\varepsilon_{t-2} + \left(A_1^3 + A_2A_1 + A_2A_1 + A_3\right.\right.\\ &= B\left(\varepsilon_t + \varepsilon_{t-1} + \varepsilon_{t-2} + \varepsilon_{t-3}\right) + BA_1\left(\varepsilon_{t-1} + \varepsilon_{t-2} + \varepsilon_{t-3} + \varepsilon_{t-4}\right)\\ &+ \left(A_1^2 + A_2\right)\left(\varepsilon_{t-2} + \varepsilon_{t-3} + \varepsilon_{t-4} + \varepsilon_{t-5}\right)\\ &+ \left(A_1^3 + A_2A_1 + A_2A_1 + A_3\right)\left(\varepsilon_{t-3} + \varepsilon_{t-4} + \varepsilon_{t-5} + \varepsilon_{t-6}\right)\end{aligned}$$

Finally,

$$Proj\left(\xi_\tau/\varepsilon_{t-3}\right) = \mathbb{E}\left[\left(\varepsilon_{t-3}^{'}\varepsilon_{t-3}\right)^{-1}\left(\varepsilon_{t-3}u_\tau\right)\right] = \Theta_0^\tau \tag{A.9}$$

A.3 Proposition III

This proof follows directly from the proof of Proposition II, which showed that the IRFs $\Theta_0^t, \Theta_1^t, ..., \Theta_{m-1}^t$ and the elements of the temporal aggregation filter $\mathcal{I}, D_1, D_2, ..., D_{m-1}$ are generated by the same generalized Fibonacci sequence $S_p\left(A_1, ..., A_p\right)$ (disregarding the initial condition B). This implies that the contemporaneous LF impact effect of the shock $\varepsilon_{t,1}$ is given by the sum of the first m elements of the Fibonacci sequence denoted by S_p^m. The effect of $\varepsilon_{t,2}$ is given by S_p^{m-1} and so on and so forth until $\varepsilon_{t,m-1}$ that impact through B. In the case of skip-sampling, $\Theta_0^\tau = S_p^m\left(A_1, ..., A_p\right)$ whereas in the case of averaging $\Theta_0^\tau = \sum_{i=1}^m BS_p^i\left(A_1, ..., A_p\right)$. In both cases, projecting ξ_τ on $\varepsilon_\tau = \varepsilon_{t,1} + \varepsilon_{t,2} + ... + \varepsilon_{t,m-1}$ recovers Θ_0^τ.

Consider that $\varepsilon_\tau = \left(I + L + ... + L^{m-1}\right)\varepsilon_t$ and $\xi_\tau = W(L)D(L)B\varepsilon_t$ where the first m elements of $D(L)$ are $I + D_1L + ... + D_{m-1}L^{m-1}$. It is straightforward to verify that

$$Proj\left(\xi_\tau/\varepsilon_\tau\right)=\mathbb{E}\left[\left(\varepsilon_\tau^{'}\varepsilon_\tau^{'}\right)^{-1}\left(\varepsilon_\tau^{'}\xi_\tau\right)\right]=W(L)B\left(I+D_1L+...+D_{m-1}L^{m-1}\right)=\Theta_0^\tau \tag{A.10}$$

applying the equality of the Fibonacci sequences generated by $D(L)$ and the recursive definition of IRFs.

B The Illustrative Example with Averaging Temporal Aggregation

This section presents the same derivations of Section 2 but when time aggregation follows an averaging scheme.

The HF VAR is given by

$$\begin{bmatrix} x_t \\ y_t \end{bmatrix}=\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}\begin{bmatrix} x_{t-1} \\ y_{t-1} \end{bmatrix}+\begin{bmatrix} b_{11} & 0 \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix} \varepsilon_t^x \\ \varepsilon_t^y \end{bmatrix} \tag{B.1}$$

which is observed at LF, under averaging time aggregation as

$$\begin{bmatrix} x_\tau \\ y_\tau \end{bmatrix}=\begin{bmatrix} a_{11}^2+a_{12}a_{21} & a_{11}a_{12}+a_{12}a_{22} \\ a_{11}a_{21}+a_{21}a_{22} & a_{12}a_{21}+a_{22}^2 \end{bmatrix}\begin{bmatrix} x_{\tau-1} \\ y_{\tau-1} \end{bmatrix}+\begin{bmatrix} \xi_\tau^x \\ \xi_\tau^y \end{bmatrix} \tag{B.2}$$

where the reduced-form residuals ξ_τ are combination of the structural ones over periods $t-1$ and t. In the case of averaging, an additional moving average component arises from the aggregation:

$$\begin{bmatrix} \xi_\tau^x \\ \xi_\tau^y \end{bmatrix}=\begin{bmatrix} b_{11}\left(\varepsilon_t^x+\varepsilon_{t-1}^x\right)+\left(a_{11}b_{11}+a_{12}b_{21}\right)\left(\varepsilon_{t-1}^x+\varepsilon_{t-2}^x\right)+a_{12}b_{22}\left(\varepsilon_{t-1}^y+\varepsilon_{t-2}^y\right) \\ b_{21}\left(\varepsilon_t^x+\varepsilon_{t-1}^x\right)+b_{22}\left(\varepsilon_t^y+\varepsilon_{t-1}^y\right)+\left(a_{21}b_{11}+a_{22}b_{21}\right)\left(\varepsilon_{t-1}^x+\varepsilon_{t-2}^x\right)+a_{22}b_{22}\left(\varepsilon_{t-1}^y+\varepsilon_{t-2}^y\right) \end{bmatrix}$$

As in Section 2, assume that identification at HF can be correctly achieved yielding z_t^y. For the averaging case, properly employing z_t^y means to use exclusively the observations corresponding to $h = 1, 3, ..., T$, i.e. the observation in the first semester of each year. We denote such a series as $z_{t,h}^y$. Once the correct filter is applied to z_t^y, such that we obtain $z_\tau^y = z_{t,h}^y$, we employ z_τ^y as an external instrument (or proxy) for the structural shock ε^y. This strategy is valid under the typical assumption in the Proxy-SVAR literature on the exogeneity of z_t^y, i.e. $\mathbb{E}\left[\left(z_t^y\right)' \varepsilon_t^x\right] = 0$, and its strength, i.e. $\mathbb{E}\left[\left(z_t^y\right)' \varepsilon_t^y\right] \neq 0$. The external information correctly aggregated recovers the ratio of impulse responses:

$$\hat{\Theta}_{22} = \mathbb{E}\left[\left(z_\tau^y\right)' z_\tau^y\right]^{-1} \mathbb{E}\left[\left(z_\tau^y\right)' \xi_\tau^y\right] = \mu\left(1 + a_{22}\right) b_{22} \tag{B.3}$$

$$\hat{\Theta}_{12} = \mathbb{E}\left[\left(\hat{\Theta}_{22}\xi_\tau^y\right)' \hat{\Theta}_{22}\xi_\tau^y\right]^{-1} \mathbb{E}\left[\left(\hat{\Theta}_{22}\xi_\tau^y\right)' \xi_\tau^x\right] = \mu a_{12} b_{22} \tag{B.4}$$

Notice that the actual average impulse response is given by the HF IRF to which the temporal aggregation filter is applied. In this case, $\Theta_{22} = b_{22}\left(1 + a_{22}\right)$ and $\Theta_{12} = b_{22}a_{12}$, whose ratio is $\frac{a_{12}}{1+a_{22}}$. Using the correct temporal aggregation scheme of the shocks, the ratio $\frac{\hat{b}_{12}}{\hat{b}_{22}} = \frac{a_{12}}{1+a_{22}}$ is correct.