

Inside Money, Investment, and Unconventional Monetary Policy*

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Abstract

I develop a new monetarist model to analyze why an economy can fall into a liquidity trap, and what the effects of unconventional monetary policy measures such as helicopter money and negative interest rates are under these circumstances. I find that liquidity traps can be caused by a decrease in the bonds-to-money ratio, by a decrease in productivity of capital, or by an increase in demand for consumption. The model shows that, while conventional monetary policy cannot control inflation in a liquidity trap, unconventional monetary policies allow the monetary authority to regain control over the inflation rate, and that an increase in the bonds-to-money ratio is the only welfare-improving policy.

Keywords: *New monetarism, liquidity trap, helicopter money, negative interest rates, government debt, Ricardian equivalence, banking*

JEL codes: E43, E52, E63, G21, H63

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1 Introduction

After the financial crisis of 2007-2009, the conditions faced by monetary policy have changed. Interest rates on bank deposits, bonds, and policy rates have been reduced to zero or have even become negative in many developed countries. On top of this, central banks in these countries have consistently undershot their inflation targets for several years in a row. In order to meet their targets again, some of them have tried untested policies such as quantitative easing, forward guidance, and negative interest rates, and have considered implementing other policies, such as helicopter money.

Most economists argue that the reason for failing to meet the inflation target is the zero lower bound. In their opinion, the economic circumstances would demand a further decrease in the nominal interest rate, but due to the existence of cash, nominal interest rates below zero are (thought to) be infeasible. The scenario where the interest rate becomes stuck at the zero lower bound is usually referred to as a liquidity trap. However, this is not the complete picture. Although it is clear that the zero or at least the effective lower bound¹ is technically a problem for central banks, there are still important unresolved questions regarding this situation. The two principal questions that need to be answered are how the economies of developed countries became stuck in the liquidity trap, and why they have been unable to get out of it even after being in it for almost a decade. This second question is particularly important, because economic theory suggests that in the long run, inflation is determined by the quantity theory of money (Friedman, 2005); i.e. that the growth rate of money dictates the inflation rate. This idea is also applied in New Monetarist models as presented in Lagos and Wright (2005). Yet, money growth has remained at record levels since the financial crisis, but inflation rates have barely changed at all, which suggests that either a timeframe of just under a decade is not the long run, or that it is possible to become stuck in a liquidity trap even with positive money growth rates. The goal of this paper is therefore twofold: First, I want to explain how a liquidity trap can occur, how it can be prevented, and whether it is a bad situation for the economy to be in in terms of welfare. Second, I want to assess the effectiveness of unconventional monetary policies, namely helicopter money and negative interest rates, in a liquidity trap environment.

To achieve these goals, I build a model that exhibits different types of equilibria depending on parameters, with some of them sharing the features of a liquidity trap: Interest rates in the economy are zero, and conventional monetary policy, namely an increase in the growth rate of

¹The effective lower bound is the nominal interest rate at which agents prefer storing cash over holding deposits at a bank. Because of storage costs and the risk of theft, this rate can be below zero.

fiat money induced by open-market sales of reserves, is unable to affect the inflation rate. Next, I analyze the effect of unconventional monetary policies such as negative interest rates, forward guidance, and helicopter money in the liquidity trap equilibrium. I also assess the role of fiscal policy, which in terms of the model's environment mainly concerns the decision whether government spending should be financed through debt or taxes.

To obtain such an environment, I build a model in which money is essential for agents to trade. Money can either be fiat money issued by central banks, or inside money issued by private banks. If agents are allowed to use inside money, it always (weakly) dominates fiat money, because it allows agents to earn some interest. Private banks want to issue this inside money, because it allows them to attract deposits, which they can use to invest in capital, in government bonds, or hold excess reserves (fiat money) if the other assets are not attractive enough. The private banks are legally obliged to hold some fiat money due to a reserve requirement. If banks decide to hold excess reserves, conventional monetary policy is unable to affect the inflation rate. This situation can only arise if the nominal interest rate on both bonds and deposits is driven down to zero, i.e., if the economy is in a liquidity trap. Banks are more likely to invest in excess reserves and thus cause the economy to fall into the liquidity trap if there is a large supply of deposits from agents, if investment opportunities are bad, or if there is a shortage of savings instruments (government bonds) in the economy, measured by the bonds-to-money ratio. There is strong evidence in the data after the financial crisis that supports these three situations, especially the third one: The ratio of the total US government debt to the monetary base fell from a high of 11.22 in the third quarter of 2008 to a low of 4.39 in the third quarter of 2014. After that, it recovered slightly to 5.29 in the first quarter of 2017. Deposits also increased after the financial crisis, though not at higher rates than before². Caballero et al. (2017) also argue that a shortage of safe assets is the cause of the economic problems faced during and after the financial crisis, so this model fits well with their analysis.

Conventional monetary policy is defined in the model as an open market operation, i.e. the central bank buys assets with fiat money which it can print at zero cost. The distribution of newly printed currency directly to agents is referred to as helicopter money in the model. The distinction between these two monetary policy tools allows me to model a standard transmission mechanism, and helps me to capture the important role that assets, such as government bonds, play in the transmission mechanism of monetary policy. Negative interest rates are modeled in a straightforward manner, as an interest rate paid by banks on their fiat money holdings (i.e. their reserves).

²All data are taken from FRED of the St. Louis Federal Reserve Bank. For the debt-to-money ratio, the series with keywords AMBSL (monetary base) and GFDEBTN (total federal debt) have been used.

My findings show that both helicopter money and negative interest rates are tools that allow the monetary authority to regain control over the inflation rate. While helicopter money immediately affects inflation even in a liquidity trap, negative interest rates allow the economy to emerge from the liquidity trap, making conventional monetary policy effective again. Additionally, I can show that forward guidance, defined as the announcement of a future policy implementation that will help the economy get out of a liquidity trap, has real effects, but that the direction of these effects can not be determined by theory.

However, from a welfare perspective, these policies are not desirable, because an increase in inflation in a liquidity trap, as well as in all the other equilibrium cases in the model, actually decreases welfare. Under such circumstances, however, an increase in the bonds-to-money ratio is welfare-increasing for two reasons: It allows banks to reduce their reserve holdings and thus liberates the economy from the liquidity trap, and it is also a cheaper way of financing government expenditure. This is because in a liquidity trap, the real interest rate on government debt is lower than the agents' rate of time-preference; i.e., the Ricardian equivalence does not hold, so that agents prefer government expenditures to be financed by issuing more debt instead of taxes. Interestingly, in this model the former policy leads to a temporary increase in inflation (at least if this is not sterilized by the monetary authority), so that inflation is positively correlated with leaving a liquidity trap, even though inflation on its own will not suffice to extricate an economy from the liquidity trap. This paper builds on the framework developed by Lagos and Wright (2005), which can be used to model monetary economics from a micro-founded perspective. I extend their baseline model in a way similar to Williamson (2012), especially regarding the importance of government bonds in the economy. The model in this paper differs from Williamson for three main reasons: (1) because bonds are not liquid themselves, but can be used by private banks to back liquid inside money; (2) because it allows me to analyze negative interest rates and helicopter money, and (3) because the effect of monetary policy on some additional economic variables such as investment can be assessed. Investment in my paper is similar to that in Lagos and Rocheteau (2008), and I can show that many of the issues they find for commodity-based money are still prevalent in an economy with inside money and a fractional reserve system.

Two of the earliest papers that study monetary policy in a liquidity trap are Krugman et al. (1998) and Eggertsson and Woodford (2003, 2004). Krugman et al. study the issue of a liquidity trap in the context of Japan's experience in the 1990s in a variety of simple models, and find, similarly to my paper, that an expansion of the monetary base has no effect on broader monetary aggregates. The reason for this failure is different, however, as the failure to increase inflation in a

liquidity trap is linked to a credibility problem, which is not the case in my model. Similarly, Eggertsson and Woodford argue that an open-market operation at the zero lower bound is ineffective only if it does not alter expectations about future inflation. This means that a credible commitment about future policy can overcome the issues created by the zero lower bound. Werning (2012) also focuses on the role of expectations at the zero lower bound. After the financial crisis, there was a surge in articles about the liquidity trap and monetary policy at the zero lower bound. There seems to be general agreement that more government debt is beneficial in such situations; however the reasons for this being so differ. While New Keynesian papers emphasize the role of government spending (e.g., Eggertsson and Krugman (2012) or Christiano et al. (2011)) or tax policy (e.g., Correia et al. (2013)), papers such as those of Williamson (2012, 2016), Geromichalos and Herrenbrueck (2017) or Rocheteau et al. (2016) show that government debt is important, since at the zero lower bound there is a shortage of liquid assets in the economy, which an increase in government bonds could help to overcome. The findings in my paper about the causes of a liquidity trap are similar. Bacchetta et al. (2016) also obtain similar results to mine. They show that quantitative easing in a liquidity trap only worsens the problem, and that negative interest rates may help an economy to get out of a liquidity trap, but are unable to solve the underlying problem, which is asset scarcity. Recently, Guerrieri and Lorenzoni (2017) studied the liquidity trap in a model with heterogeneous agents and incomplete markets. They calibrate output responses in a liquidity trap after adverse shocks to borrowing capacities, and find that drops in output are more severe in a liquidity trap. Cochrane (2017) offers an alternative view on a liquidity trap in a new-Keynesian framework, namely that there is also an equilibrium at the zero lower bound which predicts small effects on inflation, output, and policy.

Research about unconventional monetary policy measures, especially helicopter money and negative interest rates, is still at an early stage. Kiyotaki and Moore (2012) study the effect of open market operations and helicopter money after a liquidity shock. Contrary to my paper's results, they find that open market operations have real effects, while helicopter money does not. Their contrary findings stem from the fact that there is no role for assets as investment opportunities for banks in their paper. Buiter (2014) argues that if three conditions are satisfied (i.e., fiat money is held for other reasons than its return, fiat money is irredeemable, and the price of money is positive), helicopter money can always be used to boost demand, which in turn increases inflation. All of these conditions are satisfied in my model, so my results support Buiter's claim. On the other hand, Gali (2014) shows that a money-financed fiscal stimulus (i.e., something like helicopter money) has strong effects on economic activity, but only relatively mild inflationary consequences. However, the way helicopter money affects the economy is quite different in both Buiter's and

Gali's work compared to my paper.

The research on negative interest rates has been developing mainly in Europe, as only European countries have implemented negative policy rates so far. Demiralp et al. (2017) empirically study the effects of the introduction of negative rates in the Eurozone on banks' activities and find that the reaction is different from standard rate cuts in positive-rate territory. In a theoretical paper, Dong and Wen (2017) show that negative interest rates can be useful when it is the objective of the central bank to keep nominal rates as low as possible. Rognlie (2016) shows in a New Keynesian framework that it is sometimes optimal to set rates below zero to spur demand, and that the option of doing so lowers the optimal long-run inflation target for the central bank.

To the best of my knowledge, my paper is the first to analyze unconventional monetary policy in a model with a strict micro-foundation of a liquidity trap and the transmission mechanism of monetary policy. Although some of the results are similar to the findings of other papers, I think it is crucial to analyze these policies in a model where a liquidity trap and thus the ineffectiveness of conventional monetary policies arise endogenously, as it is the case in my paper. Only a model that is able to explain why conventional policies fail to affect inflation will have the capacity to explain which unconventional policy tools could still work under these circumstances. My paper is also the first to assess several different unconventional monetary policy tools in the same framework.

The rest of the paper is organized as follows. In Section 2, the model is explained, and in Section 3, the steady-state equilibrium is defined. Section 4 discusses the welfare properties of the model and the comparative statics of steady-state policy changes. In Section 5, conventional and unconventional monetary policy are formally defined, and their effects on the economy in different equilibria are analyzed in order to answer the main research questions of this paper. Finally, Section 6 concludes.

2 The model

Time is discrete and continues forever. There is a unit measure for the number of buyers and the number of sellers in the economy, collectively called agents. There is also a unit measure for the number of banks in the economy, as well as a monetary and a fiscal authority. Each period is divided into two subperiods, called the decentralized market (DM) and the centralized market (CM). At the beginning of a period, the DM takes place, and after it closes, the CM opens and remains open until the period ends. Each seller is able to produce a special good q in the DM, and each buyer is able to produce a general good x in the CM. Buyers gain utility from consuming the

special good in the DM and sellers gain utility from consuming the general good in the CM. In the DM, buyers and sellers are matched bilaterally at random. With probability σ , the special good q produced by the seller in a match gives utility to the buyer. In the CM, there exists a centralized market for general goods. Neither general goods nor special goods can be stored by agents. The preferences of buyers are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (u(q_t) - y_t). \quad (1)$$

Equation (1) states that buyers discount future periods by a factor $\beta \in (0, 1)$, gain utility $u(q)$ from consuming the special good in the DM, with $u(0) = 0$, $u'(q) > 0$, $u''(q) < 0$, $u'(0) = \infty$, and linear disutility y from producing the general good in the CM. The preferences of the sellers are

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t (-c(q_t) + x_t). \quad (2)$$

Sellers also discount future periods by a factor β , gain disutility $c(q)$ from producing in the DM and linear utility x from consuming in the CM, with $c(0) = 0$, $c'(0) = 0$, $c'(q) > 0$, $c''(q) > 0$, and $c(\bar{q}) = u(\bar{q})$ for some $\bar{q} > 0$. Furthermore, I define q^* as $u'(q^*) = c'(q^*)$; i.e., the socially efficient quantity.

In the DM, buyers are randomly allocated to two different kinds of meetings, namely an outside or an inside meeting. In an outside meeting, sellers accept only fiat (outside) money to settle a transaction, whereas in an inside meeting, other liquid assets are also eligible, especially bank deposits (inside money). The kind of meeting buyers are having in the next DM is already revealed in the CM of the previous period. This can be imagined as buyers knowing what kind of good they want to purchase in the next period, and whether the seller of that good accepts debit cards and checks, or only cash. In any given DM, the fraction of buyers who have an inside meeting is η , while the remaining fraction $1 - \eta$ have an outside meeting and therefore need cash to settle transactions in the DM. These fractions remain constant over time, but individual buyers will switch from one type of meeting to the other from period to period at random. It can be assumed that a fraction η of sellers own the technology that is required to use debit cards, while it would be infinitely costly for the remaining $1 - \eta$ sellers to acquire this technology.

Banks are agents that exist from the beginning of the CM of period t until the end of the CM of period $t + 1$, while a new set of banks emerge at the beginning of the CM in period $t + 1$, such that there are always two sets of banks during the CM of each period, but only one set of banks during the DM, making the lifespans of banks similar to lifespans of agents in overlapping

generations models. Banks cannot produce any goods and gain utility from consumption during their second CM only³. Banks are not anonymous in the CM and are under full commitment, so that they will always pay back their debt. Because of these features, sellers who have the respective technology are willing to accept a claim to an asset that the bank holds (an IOU, or, more precisely, a bank deposit) as a means of payment, knowing that it will allow them to obtain the asset from the bank in the following CM. Banks take prices as given and compete for deposits. In the CM of each period, each bank has access to an individual source of capital that yields $f(k)$ during the next CM, with $f'(k) > 0$, $f''(k) < 0$, and $f'(0) = \infty$ ⁴. During the CM, general goods can be transformed into capital, so banks need to acquire general goods in order to invest. The socially efficient quantity of capital k^* is given by $f'(k^*) = 1/\beta$. Capital should be thought of as the universe of investment opportunities in this economy, ordered by their profitability⁵. Because of the concavity of $f(k)$ and the market structure of the banking market, banks can make profits by investing in capital, which in turn creates the banks' demand for deposits, because they do not have any funds of their own to invest. Buyers facing an inside meeting in the DM are willing to supply these deposits as long as these earn a return that is at least as high as fiat money, because they know that they can use them as a means of payment. Deposits are nominal claims and are denoted by d . The interest paid on deposits is called i^d , and it is paid out in the CM. Banks are subject to a reserve requirement, which means they have to hold a fraction $\delta \in (0, 1)$ of the deposits d made by customers as fiat money.

The monetary authority issues fiat money M_t , which it can produce without cost. If fiat money is held by a bank, it can be considered reserves, but reserves and fiat money are the same object in this model. The monetary authority always implements its policies at the beginning of the CM. The amount of general goods that one unit of fiat money can buy in the CM of period t is denoted by ϕ_t , the inflation rate is defined as $\phi_t/\phi_{t+1} - 1 = \pi_{t+1}$, and the growth rate of fiat money from period $t - 1$ to t is $\frac{M_t}{M_{t-1}} = \gamma_t^M$. I will assume $\pi_t \geq 0 \forall t$ throughout the paper, unless stated differently. The monetary authority issues fiat money either by trading in the CM or by issuing

³These assumptions about banks greatly simplify the analysis without qualitatively changing the results. If banks were infinitely lived and could consume in every CM, they could retain some deposits for the purpose of immediate consumption, and consequently there would be more endogenous variables to keep track of.

⁴For simplicity, it is assumed that agents do not have access to capital. However, changing this assumption would only affect the production of agents in the CM, but not any of the model's other results. Apart from agents not having access to it, capital in this model is similar to the illiquid capital in Lagos and Rocheteau (2008).

⁵One rationale for the capital being individual to each bank is that banks have a regional monopoly in issuing loans, or a monopoly in issuing loans to a specific sector of the economy. Both of these monopolies could arise due to lower monitoring costs compared to other banks.

lump-sum transfers to agents. To issue the amount $M_t - M_{t-1}$ of newly created fiat money by trading, the monetary authority buys bonds with the corresponding value $\phi_t(M_t - M_{t-1})$ from either agents, banks, or directly from the fiscal authority. Equally, to withdraw fiat money, the monetary authority sells bonds to banks in exchange for fiat money. Such trades are called open-market operations. The bonds held by the central bank are denoted by b_t^M . The balance sheet of the monetary authority consists of bonds on the assets side and fiat money on the liabilities side. Note that the monetary authority will make profit if it uses open-market operations and a positive interest rate is paid on bonds. This central bank profit (in real terms) is given by $\Pi_t = \phi_t i^B b_{t-1}^M$, where i^B denotes the interest rate on bonds. This profit is transferred to the fiscal authority.

The fiscal authority is the only entity in the model which is able to levy taxes. It has to finance some spending g_t in each period, and can do so by levying lump-sum taxes τ , issuing bonds B , or using the profits earned by the monetary authority. This gives rise to the following government budget constraint:

$$\phi_t B_t + 2\tau_t + \Pi_t = \phi_t(1 + i^B)B_{t-1} + g_t. \quad (3)$$

It is assumed that the government determines exogenously how to finance its expenditure, and that it always finances some fraction of its expenditure through bonds. This implies that the quantity of bonds in the economy is positive, and that the fiscal authority has to pay the interest rate i^B which clears the bond market. The growth rate of bonds is defined as $\frac{B_t}{B_{t-1}} = \gamma_t^B$.

Next, I look at the banks' problem to determine the demand for deposits. After that, I solve the buyer's problem to determine the supply of deposits, and then turn to the equilibrium analysis in the next section.

2.1 The banks' problem

A bank has to decide on the value of deposits that it wishes to attract and how to invest these funds in the different asset classes. Its goal is to maximize consumption in the following period, which translates into maximizing the difference between the value of its assets and its liabilities in period $t + 1$. This can be summed up in the following maximization problem:

$$\begin{aligned}
\max_{d_t, \alpha_M, \alpha_B} \quad & f((1 - \alpha_M - \alpha_B)\phi_t \eta d_t) + \alpha_B \phi_{t+1}(1 + i^B)\eta d_t + \alpha_M \phi_{t+1} \eta d_t - \phi_{t+1}(1 + i^d)\eta d_t \\
s.t. \quad & \alpha_M \geq \delta \\
& \alpha_B \geq 0 \\
& \alpha_B + \alpha_M \leq 1.
\end{aligned}$$

The first term represents the value of the investment in capital; the second term represents the real value of the bonds held by the bank, with α_B being the share of assets invested in bonds; the third term represents the real value of the fiat money held as reserves, with α_M being the share of assets invested in fiat money; and the final term represents the real value of the deposits. The constraints ensure that the reserve requirement is met and that investment in all types of assets is non-negative. Note that the last constraint never binds due to the assumption that $f'(0) = \infty$. The maximization problem leads to the following first-order conditions:

$$f'((1 - \alpha_M - \alpha_B)\phi_t \eta d_t) \geq \frac{1}{1 + \pi_{t+1}} \quad (4)$$

$$f'((1 - \alpha_M - \alpha_B)\phi_t \eta d_t) \geq \frac{1 + i^B}{1 + \pi_{t+1}} \quad (5)$$

$$(1 - \alpha_M - \alpha_B)f'((1 - \alpha_M - \alpha_B)\phi_t \eta d_t) + \alpha_B \frac{1 + i^B}{1 + \pi_{t+1}} + \alpha_M \frac{1}{1 + \pi_{t+1}} = \frac{1 + i^d}{1 + \pi_{t+1}}. \quad (6)$$

The conditions (4) and (5) show that banks should invest in such a way that the real return on bonds and fiat money equals the return on capital. If the constraints are binding, the first-order conditions (4) and (5) will not hold with equality, meaning that banks will only invest what they are legally required to hold as reserves in fiat money, or will invest nothing in bonds, respectively. Condition (6) shows that banks should demand the value of deposits that allows them to equalize the weighted average marginal return on their assets to the marginal cost of deposits for given investment weights and interest rates. Note that the marginal return will either be the same for all the three assets (if none of the constraints is binding), or it will be a weighted average of δ times the marginal return on fiat money and $1 - \delta$ times the return on capital (because the return on bonds is either the same as the return on capital, or otherwise α_B is zero, as can be seen in condition (5)). The banks' demand schedule for deposits is decreasing in the interest rate paid on deposits i^d .

2.2 The buyers' problem

The buyer's problem is in general very similar to Lagos and Wright (2005), with the exception that buyers in inside meetings have a choice between deposits and fiat money. This means that for a buyer in an inside meeting, his liquid assets, i.e., the nominal amount that he can use in a DM meeting, are equal to $(1 + i^d)d + m$, while for a buyer in an outside meeting, this amount is simply m .

In the DM, it is assumed that buyers can make a take-it-or-leave-it offer⁶. The optimal choice of liquid assets transferred to the seller for buyers in inside meetings l^i is given by:

$$l^i = \begin{cases} \frac{c(q^*)}{\phi_t} & \text{if } (1 + i^d)\phi_t d + \phi_t m \geq c(q^*) \\ (1 + i^d)d + m & \text{if } (1 + i^d)\phi_t d + \phi_t m < c(q^*). \end{cases} \quad (7)$$

So buyers will spend the amount that allows them to buy the socially efficient quantity if they can, or all they have otherwise.

The optimal portfolio choice of buyers in inside meetings is then given by:

$$\Rightarrow \max_{d \geq 0, m \geq 0, b \geq 0} \left[- \left(\frac{1 + \pi_{t+1}}{\beta} - (1 + i^d) \right) \phi_{t+1} d - \left(\frac{1 + \pi_{t+1}}{\beta} - 1 \right) \phi_{t+1} m \right. \\ \left. - \left(\frac{1 + \pi_{t+1}}{\beta} - (1 + i^B) \right) \phi_{t+1} b + \sigma \max_{l^i \leq (1 + i^d)d + m} \{ u \circ c^{-1}(\phi_{t+1} l^i) - \phi_{t+1} l^i \} \right]. \quad (8)$$

In (8), the first three terms denote the cost of holding deposits, fiat money, and bonds, respectively. The final term is the surplus from the DM; i.e., the benefit of holding more liquid assets. Since bonds are not a liquid asset in this economy, agents are only willing to hold them if $\frac{1 + \pi_{t+1}}{\beta} \leq (1 + i^B)$. For the liquid assets, it is clear that deposits always strictly dominate fiat money as long as the interest rate on deposits is positive, and weakly dominate fiat money if the interest rate on deposits is zero. Buyers in inside meetings will therefore hold no fiat money if $i^d > 0$, and I assume without loss of generality that they also hold no fiat money at $i^d = 0$. If the interest rate on deposits is negative, however, no buyers are willing to hold deposits, because they can switch to fiat money instead. This creates a zero lower bound for the interest rate on deposits. Since the cost of holding deposits is decreasing in the interest rate on deposits, the supply of deposits provided by buyers is upward-sloping in the interest rate on deposits.

Now we turn to the portfolio choice of buyers preparing for outside meetings. These buyers' optimal choice in the DM is:

⁶The effect of different bargaining protocols and powers has been analyzed by Lagos and Wright (2005), and the effect of different market structures in the DM has been analyzed by Rocheteau and Wright (2005).

$$l^o = \begin{cases} \frac{c(q^*)}{\phi_t} & \text{if } \phi_t m \geq c(q^*) \\ m & \text{if } \phi_t m < c(q^*). \end{cases} \quad (9)$$

This is similar to equation (7), except that fiat money is the only liquid asset for these buyers. The optimal portfolio choice of buyers in outside meetings is given by:

$$\Rightarrow \max_{d \geq 0, m \geq 0, b \geq 0} \left[- \left(\frac{1 + \pi_{t+1}}{\beta} - (1 + i^d) \right) \phi_{t+1} d - \left(\frac{1 + \pi_{t+1}}{\beta} - 1 \right) \phi_{t+1} m \right. \\ \left. - \left(\frac{1 + \pi_{t+1}}{\beta} - (1 + i^B) \right) \phi_{t+1} b + \sigma \max_{l^o \leq m} \{ u \circ c^{-1}(\phi_{t+1} l^o) - \phi_{t+1} l^o \} \right]. \quad (10)$$

This is the same as in equation (8), but with a different constraint for the inner maximization problem. Buyers in outside meetings can only use fiat money in transactions, so they face a trade-off regarding how much fiat money to hold, but there is no trade-off associated with the deposits and bonds that they hold. Therefore, bonds and deposits will only be held by these buyers if it is costless or even beneficial for them to hold these assets. In what follows, I will denote the quantities traded in inside meetings as q^i and in outside meetings as q^o .

3 Equilibrium

We analyzed the behavior of buyers and banks in the previous section. To solve for equilibrium, we also have to account for the market clearing of assets, namely bonds, fiat money, and deposits.

3.1 Bond market clearing

From the government's budget constraint (equation (3)), we know that there is some amount of bonds B_t in the economy. From the buyer's maximization problems (equations (8) and (10)), we know that buyers will only hold bonds if there is no cost to hold them. The same is true for sellers. This means that agents only hold bonds if $1 + i^B \geq \frac{1 + \pi_{t+1}}{\beta}$. However, if $1 + i^B > \frac{1 + \pi_{t+1}}{\beta}$, agents want to hold an infinite amount of bonds. Since the supply of bonds is finite, the interest rate on bonds will be driven down until $1 + i^B = \frac{1 + \pi_{t+1}}{\beta}$, which is usually referred to as the Fisher equation (Fisher, 1930), and is determined by the natural real interest rate $1 + r = \frac{1}{\beta}$ and the inflation rate⁷. This creates an upper bound on the interest rate on bonds. The amount of bonds held by an individual agent is called b_t , so the total amount of bonds demanded by agents is $2b_t$.

⁷Note that in the model, there is also a time-varying real interest rate given by $f'(k_t) = 1 + r_t$. However when I use the term Fisher equation, I always refer to the nominal interest rate determined by the natural real interest rate and the inflation rate.

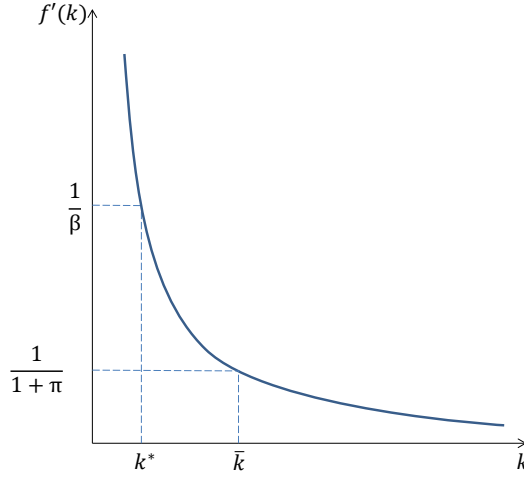


Figure 1: Marginal return on capital

Since holding bonds allows banks to issue more deposits at a given interest rate, it is possible that they are willing to hold bonds even if the rate of return is lower than the rate required by the Fisher equation. However if that is the case, all bonds have to be held by banks, because as we saw, agents are not willing to hold bonds if they pay an interest rate that does not fully compensate them for inflation and discounting. The banks' nominal demand for bonds is denoted by $\alpha_B \eta d_t$, and it is determined by equation (5). This allows us to state the market clearing condition for bonds:

$$\alpha_B \eta d_t + 2b_t = B_t - b_t^M \quad (11)$$

with $b_t = 0$ if $1 + i^B < \frac{1 + \pi_{t+1}}{\beta}$, and $2b_t = B_t - b_t^M - \alpha_B \eta d_t$, otherwise.

Since b_t^M denotes the bond holdings of the monetary authority, $B_t - b_t^M$ is the supply of bonds that are publicly available, and this supply has to be equal to the private demand for bonds (demand from banks and agents). The bonds held by agents $2b_t$ can take any non-negative value if bonds are priced at the Fisher equation, so if the demand of banks for bonds at the Fisher equation is less than the supply of bonds, $2b_t$ will be equal to the difference between the supply of bonds and the banks' demand for bonds. If the interest rate on bonds is lower than the rate required by the Fisher equation, the banks' demand for bonds must equal the supply of bonds. Since the banks' demand for bonds is downward sloping with respect to the interest rate i^B , the interest rate will decrease if the demand for bonds is higher than the supply of bonds at a specific interest rate. If the monetary authority does not make use of helicopter money, it will have to hold the same value of bonds as it has in fiat money outstanding, so that $b_t^M = M_t$. I will assume this for the rest of

the paper, unless stated differently.

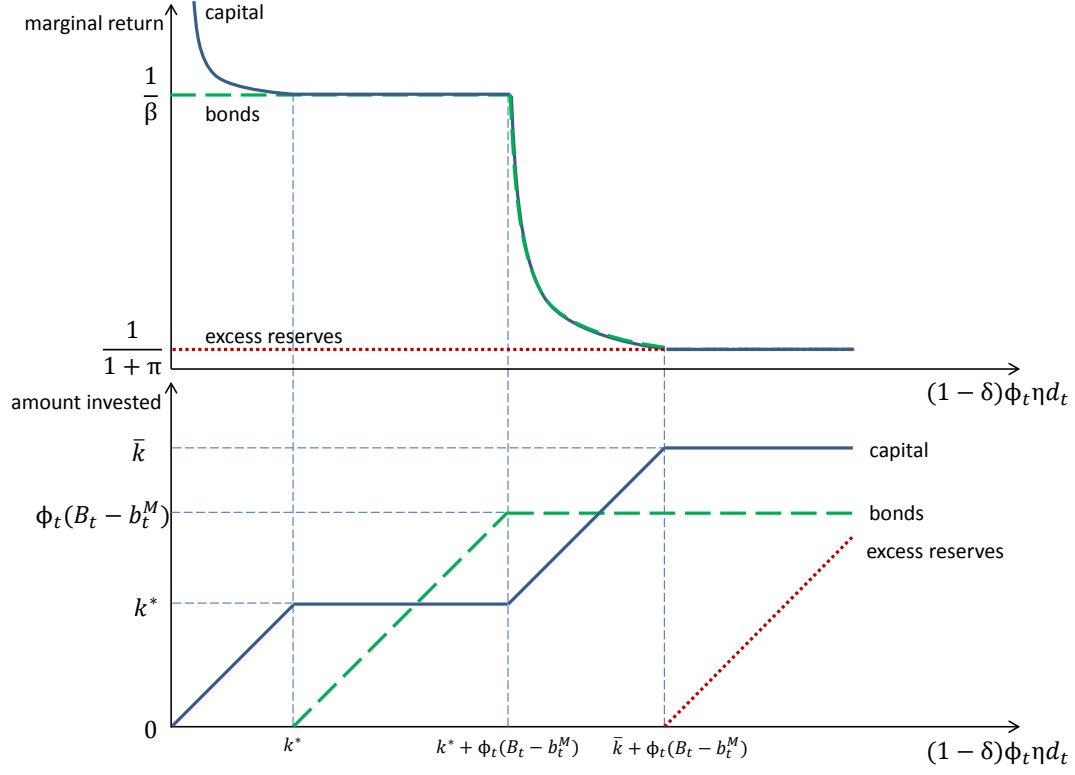


Figure 2: The first panel shows the real return on the three different assets as a function of total investment. The second panel shows the amounts invested in the separate assets as a function of total investment.

Figure 1 shows the marginal return on capital and illustrates the two quantities k^* and $\bar{k}$. k^* , which I already defined as the socially efficient quantity of investment, is the quantity that, if invested in capital, pays the same real return as a bond priced at the Fisher equation, while $\bar{k}$ is the quantity that pays the same real return as fiat money. With the help of these two quantities, I can now define the different cases for the interest rate on bonds, which also affect the other investment decisions of the banks. These different cases can also be seen in Figure 2, which shows the total real returns on the different assets that are available to banks in the first panel, and the investment by asset type in the second panel, both as a function of the funds that are available to banks for investment ($1 - \delta$ times the deposits the banks attract).

Case 1 $k^* \geq (1 - \delta)\phi_t \eta d_t$: In this case, the banks' investment demand can be fully satisfied by capital. Therefore, banks do not hold any bonds, and the interest rate on bonds is $1 + i^B = \frac{1 + \pi_t + 1}{\beta}$.

Case 2 $k^* < (1 - \delta)\phi_t \eta d_t < k^* + \phi_t(B_t - b_t^M)$: In this case, the banks' investment demand is larger than the quantity k^* . Thus, banks also hold some bonds. However, their investment demand is still less than k^* plus the real value of all publicly available bonds, so some bonds are still held by agents and the interest rate on bonds remains at $1 + i^B = \frac{1 + \pi_{t+1}}{\beta}$.

Case 3 $k^* + \phi_t(B_t - b_t^M) < (1 - \delta)\phi_t \eta d_t < \bar{k} + \phi_t(B_t - b_t^M)$: If this situation holds, the banks' investment demand is not satisfied even when they invest k^* and hold all the publicly available bonds. However, the banks' investment demand is less than the sum of investment amount $\bar{k}$ and the real value of all publicly available bonds. This means that at $1 + i^B = \frac{1 + \pi_{t+1}}{\beta}$, the demand for bonds is higher than the supply, which in turn means that the price has to adjust for the bond market to clear, so i^B decreases. As a result, banks can invest more in capital, because investments with lower return now also become attractive.

Case 4 $\bar{k} + \phi_t(B_t - b_t^M) < (1 - \delta)\phi_t \eta d_t$: As explained above, if i^B falls, banks will invest more in capital. This process will continue until the amount invested reaches $\bar{k}$. If the banks' demand for investment cannot be satisfied even by the real value of all publicly available bonds and $\bar{k}$, the interest rate on bonds will be driven all the way down to zero, so that bonds and fiat money become perfect substitutes. Therefore, although at $i^B = 0$ the banks' demand for bonds is still higher than the supply of bonds, the interest rate will not decrease further, and instead the banks will begin to hold excess reserves.

Note that in cases 3 and 4, investment goes beyond the socially optimal quantity k^* . We will see in Section 3.3 that such over-investment can occur in equilibrium. On the other hand, there is too little investment in case 1. We will see that this can occur in equilibrium due to the reserve requirement.

3.2 Money market clearing

Next, we can state the market clearing condition for fiat money:

$$(1 - \eta)z^m + \alpha_M \eta z^d = \phi_t M_t. \quad (12)$$

Here, $z^m = \phi_t m_t$ and $z^d = \phi_t d_t$, so the left-hand side denotes the total real demand for fiat money, given by real balances of buyers in monetary meetings, and the real money holdings of banks. This demand has to equal the supply of fiat money. z^m results from (10). For any $\pi_{t+1} > \beta - 1$, buyers in outside meetings spend all their money holdings, so the first-order condition of (10) can be written as:

$$\frac{u' \circ c^{-1}(\phi_{t+1}m_t)}{c' \circ c^{-1}(\phi_{t+1}m_t)} = 1 + \frac{1}{\sigma} \cdot \frac{1 + \pi_{t+1} - \beta}{\beta}. \quad (13)$$

This shows that the real amount of buyers' money holdings negatively depends on inflation. z^d is the real value of the equilibrium-level of deposits, which is given by the level of deposits at market clearing. For a constant bonds-to-money ratio, z^d is also decreasing with regard to inflation.

3.3 Deposits market clearing

The analysis of bond market clearing showed that there are four different cases regarding the banks' investment in bonds, and these cases lead to non-monotonicities in the banks' demand schedule for deposits. In the cases where additional deposits are invested in capital, namely in cases 1 and 3, the banks' demand is downward sloping in regards to the interest rate on deposits, since any additional investment leads to lower marginal returns. In cases 2 and 4, however, additional investment demand is met by higher investment in bonds and fiat money, respectively, and since returns on these assets are constant at the margin, the marginal return of the banks also remain constant, which in turn means their deposit demand is flat in these regions.

The buyers' problem (8) is also non-monotonic, so we have to distinguish between three cases, which depend on the cost of holding deposits:

Case a $1 + i^d = \frac{1+\pi_{t+1}}{\beta}$: In this case, buyers are completely compensated for inflation and discounting by the interest rate, so that the interest rate on deposits satisfies the Fisher equation. In this situation, the supply of deposits is any value larger or equal to the value of deposits needed to pay for q^* in the DM, which is $d^* = \frac{c(q^*)}{(1+i^d)\phi_t}$. So to determine whether $1 + i^d = \frac{1+\pi_{t+1}}{\beta}$ constitutes an equilibrium, we have to check whether the banks' demand for deposits at this interest rate is at least d^* . From equation (6) we know that this will hold if:

$$(1 - \delta)f' \left((1 - \delta)\eta c(q^*) \frac{\beta}{1 + \pi_{t+1}} \right) + \delta \frac{1}{1 + \pi_{t+1}} \geq \frac{1}{\beta}; \quad (14)$$

i.e., if the marginal real return on assets at d^* and for $1 + i^d = \frac{1+\pi_{t+1}}{\beta}$ is higher than or equal to the marginal real cost of deposits, which is $\frac{1}{\beta}$ in this case⁸. If condition (14) holds, the efficient quantity q^* is traded in the DM, the equilibrium-level of deposits equals the demand for deposits given by equation (6), and the equilibrium interest rate on deposits is $i^d = \frac{1+\pi_{t+1}}{\beta} - 1$.

Case b $1 + i^d < \frac{1+\pi_{t+1}}{\beta}$: From the considerations above, we know that this can constitute an equilibrium only if (14) does not hold. In this case, deposits are costly to hold, which means that

⁸Note that I made use of the fact that banks can only pay the Fisher equation interest rate on deposits for $\alpha_B = 0$ and $\alpha_M = \delta$ for $\pi_{t+1} \geq 0$ and $\delta > 0$.

buyers will not hold a higher value of deposits than they want to spend in the DM, so $l^i = (1 + i^d)d$. Then, the solution to the maximization problem in (8) becomes:

$$\frac{u' \circ c^{-1}(\phi_{t+1}(1 + i^d)d)}{c' \circ c^{-1}(\phi_{t+1}(1 + i^d)d)} = 1 + \frac{1}{\sigma} \cdot \frac{1 + \pi_{t+1} - \beta(1 + i^d)}{\beta(1 + i^d)}. \quad (15)$$

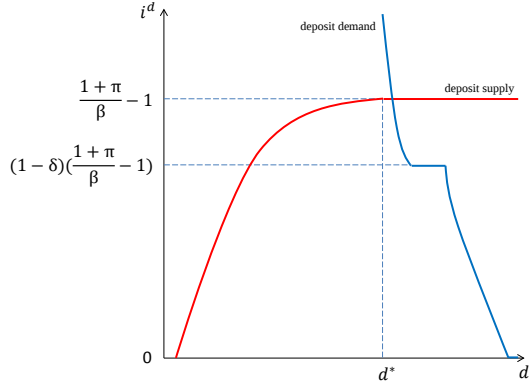
Case c $1 + i^d > \frac{1 + \pi_{t+1}}{\beta}$: In this situation, the cost of holding deposits is negative, so buyers want to supply an infinite amount of deposits. However, since demand for deposits is well-defined for any i^d and decreasing with regard to the interest rate, we can conclude that in this case the supply of deposits is higher than demand, which will drive down the interest rate. Therefore, this case cannot be an equilibrium.

So to sum up, the deposit demand side gives rise to four cases that are potentially part of an equilibrium as explained in Section 3.1, while the deposit supply side gives rise to two cases that are potentially part of an equilibrium as explained above. In principle, this would permit 8 different combinations in equilibrium. Luckily, **case a** can only occur if the bond market clears such that **case 1** prevails. This is because it is necessary for the banks to obtain an average marginal return of $\frac{1 + \pi_{t+1}}{\beta}$, as formulated in equation (6) in order to reach the interest rate on deposits that is required to set the cost of holding deposits to zero. But if banks hold bonds, their marginal return on bonds and capital is at most equal to $\frac{1 + \pi_{t+1}}{\beta}$, and due to the reserve requirement, their average marginal return will be lower⁹. **Case b**, however, is feasible in combination with any of the four cases for bond market clearing, so there are five equilibrium cases in total. The equilibrium where buyers in inside meetings are able to purchase q^* in the DM will be called **case 1a**, while the situation where banks hold no bonds, but buyers in inside meetings purchase less than q^* will be called **case 1b**. **Cases 2-4** are characterized by the banks' investment decisions as explained in Section 3.1 on bond market clearing, and equation (15) from **case b**¹⁰.

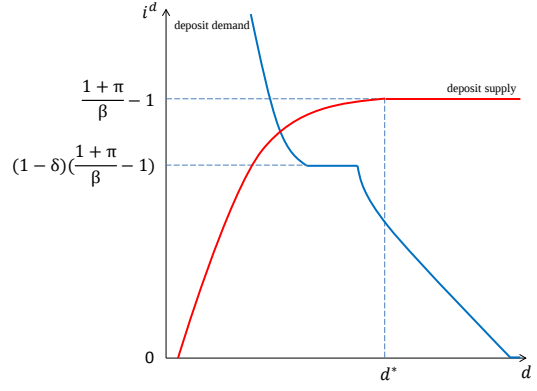
Figure 3 shows the demand and supply curves for deposits. The five graphs depict the five different equilibrium cases. The banks' demand curve for deposits consists of four segments, which correspond to the four cases that are prevalent on the bond market: the upper decreasing segment

⁹This is not true if $1 + \pi_{t+1} = \beta$. If $1 + \pi_{t+1} = \beta$, $k^* = \bar{k}$, and thus the cases 3 and 4 from bond market clearing collapse into case 2, and the interest rate on deposits in cases 1 and 2 would be equal. To rule this out, I only consider cases where $\pi_{t+1} \geq 0$.

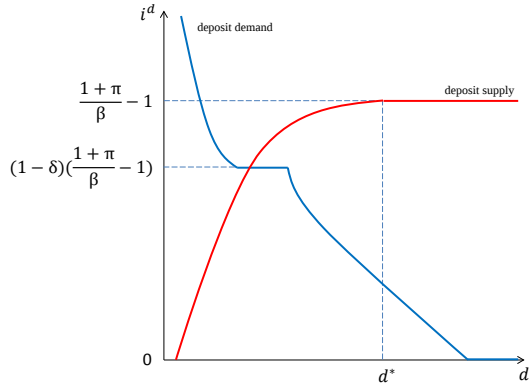
¹⁰To be consistent, these cases should be labeled 2b, 3b, and 4b. However, since **case a** cannot occur when **cases 2, 3 or 4** on the bond market are prevalent, I will use the simpler notation of just referring to the prevalent bond market case.



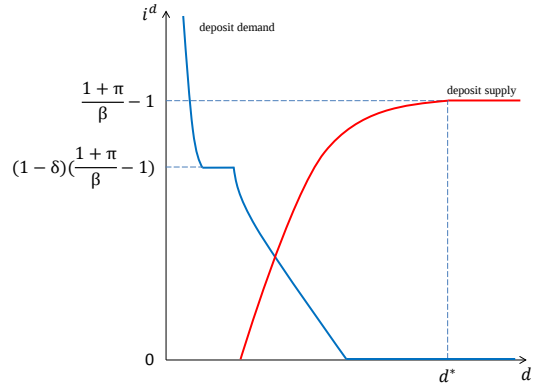
(a) Equilibrium case 1a



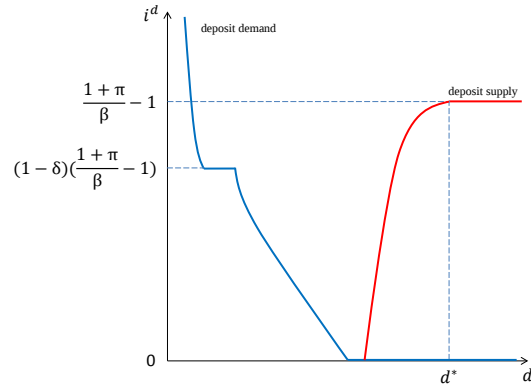
(b) Equilibrium case 1b



(c) Equilibrium case 2



(d) Equilibrium case 3



(e) Equilibrium case 4

Figure 3: The five equilibrium cases.

(case 1), the upper flat segment (case 2), the lower decreasing segment (case 3), and the lower flat segment (case 4). As explained above, the banks' demand for deposits is decreasing with respect to the interest rate on deposits when an increase in deposits leads to a decrease in their marginal return, which happens in cases 1 and 3. In cases 2 and 4, banks can react to an increase in deposits by holding more bonds (case 2) or more fiat money (case 4), which leaves their marginal return unchanged, so that their demand for deposits is not decreasing with regard to i^d in these regions. The cases a and b resulting from the buyers' problem are reflected by the flat segment (case a) and the increasing segment (case b) of the buyers' supply curve, respectively. As shown in graph 3a, the flat segment of the buyers' supply curve can only intersect the upper decreasing segment of the banks' demand curve. This is because the flat segment of the buyers' supply curve lies above the flat segment of the banks' demand curve for any $\delta > 0$. In all other cases, the intersection occurs in the increasing segment of the buyers' supply curve, so the equilibrium case is determined solely by the banks' demand curve.

One of the goals of this paper is to describe the circumstances that cause an economy become stuck in a liquidity trap, where a liquidity trap is a situation where interest rates are at the zero lower bound and conventional monetary policy is ineffective. Among all the equilibrium cases that I identified in the model, case 4 is the only one that satisfies the first part of this definition. As I will formally show in Section 5, equilibrium case 4 also satisfies the second part of the definition; i.e., it corresponds to a liquidity trap.

Equilibrium case 4 is more likely to occur in the following three situations: (1) when the banks' demand curve for deposits is relatively steep, i.e., when there is a sharp fall in demand for small increases in the interest rate on deposits, which happens if the return on capital is not very high; (2) when the upper flat segment in the banks' demand curve is relatively short, where its length is given by the bonds-to-money ratio, and (3) if d^* is relatively large and / or if the buyers' supply schedule for deposits is relatively steep, which happens when there are big gains from trade to be made in the DM and when buyers are not very sensitive to the cost of holding money. To sum up, the drivers increasing the likelihood of a liquidity trap occurring are the specific forms of the functions $f(k)$, $u(q)$, and $c(q)$, as well as the bonds-to-money ratio. Out of these drivers, only the bonds-to-money ratio is a policy variable, while the others are fundamentals.

3.4 Steady-state equilibrium

In a steady-state equilibrium, the inflation rate is usually pinned down by the money growth rate in New Monetarist models. That is also true in this model, but because of the importance of

government bonds for monetary policy, additional requirements have to be met for a steady state:

Definition 1. *In a steady state, the growth rates of fiat money and bonds have to be equal and jointly define the inflation rate: $\gamma_t^M = \gamma_t^B = 1 + \pi_t$.*

This relation is driven by the market clearing condition for bonds (equation (11)), as in this equation real variables can only stay constant over time if the growth rates of fiat money and bonds are equal¹¹.

With the help of this steady state definition, we are now ready to define a steady-state equilibrium:

Definition 2. *An equilibrium is a sequence of prices i^B, i^d, π_{t+1} , quantities z_t^d, z_t^m, b_t, l_t^i , and ratios α_M , and α_B that simultaneously solve the equations (4), (5), (6), (7), (8), (13), (11) and satisfy the corresponding complementary slackness condition on agents' bond holdings, and the condition from definition 1 $\forall t$.*

With the help of this definition, I can also formally define the five equilibrium cases characterized in Section 3.3. In cases 1a and 1b, $\alpha_M = \delta$ and $\alpha_B = 0$, such that both (4) and (5) do not hold with equality. Further, from (11), $b_t > 0$, such that $1 + i^B = \frac{1+\pi_{t+1}}{\beta}$. In case 1a, from (7), $l^i = \frac{c(q)}{\phi_t}$, while in case 1b, $l^i = (1 + i^d)\phi_t d_t + \phi_t m_t$ and thus (8) reduces to (15), which in fact holds in all cases except 1a. In cases 2 and 3, $\alpha_M = \delta$, but $\alpha_B > 0$, and so (4) still does not hold with equality, but (5) does. In case 2, from (11), $b_t > 0$, so $1 + i^B = \frac{1+\pi_{t+1}}{\beta}$, while in case 3, $b_t = 0$, and so $\alpha_B \eta d_t = B_t - b_t^M$. Finally, in case 4, also $\alpha_M > \delta$, and so (4) holds with equality too. Everything else is the same as in case 3.

4 Welfare and steady-state policy changes

In this section, I want to analyze the welfare properties of the different equilibrium cases and how steady-state policy changes affect welfare and economic variables in general. In the model, three variables can be considered policy tools, namely the reserve requirement δ , the inflation rate π_{t+1} , and the quantity of publicly available bonds, or, more precisely, the bonds-to-money ratio $\frac{B_t - b_t^M}{M_t}$. It is clear that the optimal reserve requirement in this model is zero, since the reserve requirement

¹¹In this paper, γ_t^B is a policy choice, so inflation can still be defined by the monetary and fiscal authorities. However, the model would not change if there were also private bonds that could be used by banks to back deposits. Inflation would then be defined by the growth rate of all bonds, and if the amount of private bonds is substantially larger than the amount of government bonds in the economy, this growth rate would be hard to control for the fiscal authority. This, in turn, would mean that inflation is driven by factors exogenous to monetary and fiscal policy in the long run in an economy with a substantial quantity of private bonds.

forces the banks to hold assets with a low rate of return, thus lowering their marginal return on assets, which in turn lowers the equilibrium interest rate on deposits. The higher the deposit interest rate is, however, the closer q^i may be to q^* . Despite this result, reserve requirements are observed in many developed countries. Monnet and Sanches (2015) show that a positive reserve requirement is actually welfare improving in an economy with stochastic returns, and Bhattacharya et al. (1997) show that reserve requirements can help to rule out undesirable equilibria. Therefore, I will treat the reserve requirement not as a policy variable, but as an exogenous parameter with a positive value¹². This leaves only the inflation rate and the bonds-to-money ratio as policy tools for the monetary and fiscal authorities. As explained in definition 1, the steady-state inflation rate can only be controlled jointly by the fiscal and monetary authority. In this section, I will consider such a joint determination. In Section 5, I will analyze what happens if the monetary authority unilaterally tries to control the inflation rate.

4.1 Welfare properties of the equilibrium cases

The first-best allocation is defined by the quantities $q^i = q^o = q^*$ and $k = k^*$. From equation (13), we see that $q^o = q^*$ can only be achieved at the Friedman rule; i.e., for $1 + \pi = \beta$. At the Friedman rule, the Fisher equation for bonds and deposits is satisfied for the interest rates $i^d = i^B = 0$. If this is the case, all the equilibrium cases collapse into one, and banks invest up to k^* . This means that all three assets a bank can invest in have the same real marginal rate of return ($\frac{1}{\beta}$), and thus banks are indifferent about their reserve and bond holdings, allowing them to always provide at least d^* to buyers in inside meetings so that they can consume q^* . In short, this means that the Friedman rule allows this economy to achieve the first-best, regardless of other variables.

To allow for the different equilibrium cases, I only consider non-negative inflation rates. Note that for non-negative inflation rates and a positive reserve requirement, it is not possible to achieve $q^i = q^*$ and $k = k^*$ simultaneously. This is because equilibrium case 1a is required in order to obtain $q^i = q^*$, while equilibrium case 2 is required to obtain $k = k^*$. Thus, equilibrium cases 1a, 1b and 2 are relatively efficient regarding k and q^i , while equilibrium cases 3 and 4 clearly are not efficient in that regard as we simultaneously have $k > k^*$ and $q^i < q^*$ in these cases. Comparing total welfare among different equilibrium cases is only straightforward for a given inflation rate, as an increase in inflation always lowers q^o , which reduces welfare. Thus, we can conclude that for a given inflation rate, equilibrium cases 3 and 4 are inefficient.

¹²Note that setting $\delta = 0$ would not fundamentally change any results of the paper as long as there is still positive demand for fiat money; i.e., as long as $\eta < 1$. With $\delta = 0$, the equilibrium cases 1a, 1b, and 2 collapse into a single case where $q^i = q^*$ and $k = k^*$ hold simultaneously.

4.2 Changes in the bonds-to-money ratio

From the equilibrium analysis, we learned that bonds are scarce in the equilibrium cases 3 and 4. I showed in Section 4.1 that for a given inflation rate, equilibrium cases 3 and 4 are dominated by equilibrium case 2. Therefore, it is obvious that an increase in the bonds-to-money ratio has positive effects on welfare if the economy is in either equilibrium case 3 or 4¹³. This is because such a policy change increases the interest rate on bonds, which in turn increases the interest rate on deposits and allows buyers in inside meetings to consume more. Simultaneously, it also reduces over-investment, because investment into k becomes less attractive if the interest rate on bonds increases. In the steady state, all of this happens without affecting the inflation rate, since the steady-state inflation rate is determined by the growth rate of money and bonds, but not by one-time changes in the bonds-to-money ratio.

In equilibrium cases 1a, 1b, and 2, an increase in the bonds-to-money ratio has no effect on welfare, since it only increases the bond holdings of agents, without affecting those of banks. Since agents are indifferent about the quantity of bonds that they hold in these equilibrium cases, such a policy change has no real effect. A small increase in the bonds-to-money ratio also has no effect in equilibrium case 4, since the interest rate on bonds only starts increasing in the bonds-to-money ratio once the change is sufficiently large to reach equilibrium case 3.

This analysis allows us to make a statement about the optimal debt level:

Proposition 1. *In this economy, there is an optimal minimal level of government debt which requires that $B_t - b_t^M \geq \alpha_B \eta d_t$ at the interest rate $1 + i^B = \frac{1+\pi}{\beta}$. If the government debt is below that threshold, the Ricardian equivalence does not hold, and hence it is cheaper for the government to issue debt than to raise taxes. Above that threshold, Ricardian equivalence holds, and thus welfare is not affected by the level of government debt.*

If the government does not issue sufficient debt, bonds pay a liquidity premium, which reduces the interest rates in the economy and thus makes holding deposits more costly, which means that agents consume less in equilibrium. However, one might object that issuing more bonds is costly for the fiscal authority, because it increases the interest rate on bonds. But from the budget constraint of the fiscal authority (equation (3)) we know that the fiscal authority always has the option to either tax the agents now or to issue bonds and tax them later to pay back the bonds, including the interest rate. An agent always prefers the government to do what is less costly, and because of discounting, issuing debt is less costly if $\frac{1+i^B}{1+\pi} < \frac{1}{\beta}$, which is the case when bonds are

¹³In equilibrium case 4, only a sufficiently large change in the bonds-to-money ratio, i.e., one that is large enough to move an economy from case 4 to case 3, has a positive welfare effect, while a small one does not affect any real variables. In equilibrium case 3, however, any increase in the bonds-to-money ratio has positive welfare effects.

scarce; i.e., as long as the amount of government debt is below the optimal debt level defined above. Thus, it is both socially beneficial and cheaper for the fiscal authority to issue more bonds if the economy is in a case 3 or 4 equilibrium, which shows that the Ricardian equivalence does not hold in these equilibrium cases. And since agents have linear preferences in the CM, a higher level of government debt has no negative effect on their utility (i.e., agents have no preferences about consumption smoothing across the centralized markets of different periods).

4.3 Changes in the steady-state inflation rate

As explained above, an increase in inflation has a welfare-reducing effect, because it unambiguously reduces q^o . However, if an increase in inflation were to have a welfare-improving effect on the variables k or q^i , the total effect of an increase in inflation would be potentially welfare-improving. The effect of a change in inflation on welfare might be different in any of the five equilibrium cases, thus we need to analyze its effect separately for each case. However, such an analysis shows that an increase inflation is bad for welfare in all equilibrium cases.

Proposition 2. *An increase in inflation has a negative effect on aggregate welfare in all possible equilibrium cases. q^o is reduced with an increase in inflation in all equilibrium cases. q^i is reduced with an increase in inflation in equilibrium cases 1b, 2, and 4, stays unchanged in equilibrium case 1a, and either stays unchanged or is reduced in equilibrium case 3. k is reduced with an increase in inflation in equilibrium cases 1a and 1b, increases with inflation in equilibrium case 4, stays unchanged in equilibrium case 2, and either stays unchanged or increases in equilibrium case 3.*

The proof of Proposition 2 can be found in the appendix. To sum up, inflation never has a welfare-improving effect on any variable, so the aggregate effect is clearly negative in all equilibrium cases.

4.4 Escaping the liquidity trap

Equilibrium case 4 corresponds to a liquidity trap, which I will formally define in Section 5. Thus, the analysis in Section 4.2 shows that a change in the bonds-to-money ratio can get the economy out of the liquidity trap. This change can only be achieved by the fiscal authority however, which is why I will analyze what the monetary authority can do unilaterally in a liquidity trap in Section 5. Note also that during the transition out of a liquidity trap, banks will release their excess reserves. This means that inflation increases for one period, and if the increase in bonds was announced in the period prior to the implementation, the inflation will be expected and thus also have real

effects. This means that the policy that is able to increase welfare - an increase in the bonds-to-money ratio - will also increase inflation temporarily. This is interesting because according to conventional wisdom, central banks should try to increase inflation in a liquidity trap. According to this model, inflation will indeed increase temporarily when an economy leaves the liquidity trap, but only as a side effect of the optimal policy, which is an increase in the bonds-to-money ratio. This gives rise to a positive correlation between increases in inflation and economies leaving a liquidity trap, which could mislead an econometrician to think that there is a causal relationship between increasing inflation and an economy leaving the liquidity trap. However, the increase in inflation can also be prevented if the monetary authority announces that it is going to withdraw the fiat money that the banks are trying to get rid of by lowering their reserve holdings.

5 Monetary policy

As explained in the introduction, one of the goals of this paper is to understand why the monetary authority is sometimes unable to affect the inflation rate, and whether there are unconventional tools that could be used to regain control over inflation in such situations. The goal of this section is to answer these questions. Therefore, I will first formally define conventional monetary policy as well as the unconventional monetary policy tools I am considering. All of these policies are unilaterally implemented by the monetary authority; i.e., it is assumed that the fiscal authority adheres to its policy without taking monetary effects into account. After defining the policy tools, I will analyze the effect of conventional monetary policy on inflation in the different equilibrium cases described in Section 3.3 to formally show that equilibrium case 4 corresponds to a liquidity trap environment in the model, i.e., an environment where conventional monetary policy is powerless. Finally, I will consider the effect on inflation of the unconventional monetary policy tools in the liquidity trap environment. Note that I already showed in Section 4.3 that an increase in inflation is never welfare-improving in this model. Nevertheless, central banks think that a short-term increase in the inflation rate has positive effects on economic growth. The goal of this section is therefore not to define a welfare-improving policy, but rather to mechanically understand under which circumstances it may or may not be possible for the monetary authority to influence the inflation rate.

As shown in definition 1, in the long run the growth rates of fiat money and bonds have to be equal. This means that the monetary authority is only able to change fiat money growth rates unilaterally for a restricted period of time. Therefore, I analyze short-term deviations from the steady state in this section, namely short periods during which the growth rate of fiat money differs from the growth rate of bonds. Specifically, I want to analyze whether such a policy change actually

affects the inflation rate, in order to determine whether the monetary authority is able to influence inflation. I assume that the monetary authority deviates for one period, before reverting to the steady state level, but the analysis can be generalized to deviations that are longer than one period. Additionally, I assume that this deviation is announced one period prior to the implementation; i.e., if the monetary authority plans to set γ_{t+1}^M above the steady state level, it will announce that it will do so in period t . The timing of events is important, since this policy only has real effects due to the announcement¹⁴. I will then solve for the equilibrium in which the economy returns most quickly to the steady state. Because there are typically multiple non-steady state equilibria in a monetary model, I will only prove existence, not uniqueness.

The change in the growth rate of fiat money described above can be induced by both open-market operations, which I call conventional monetary policy, and helicopter money, which I define as one of the unconventional monetary policy tools. Besides analyzing the change in the growth rate of fiat money, I also analyze the effect of the introduction of negative interest rates, which I define as another unconventional monetary policy tool. At the end of this section, I also consider the effects of forward guidance.

5.1 Conventional monetary policy

Conventional monetary policy in this model is defined as an open market operation; i.e., the central bank buys bonds for newly printed fiat money. This means that in reaction to conventional monetary policy, M_t increases by exactly the same amount as b_t^M increases, which in turn means that the amount of publicly available bonds $B_t - b_t^M$ decreases by the same amount.

5.2 Helicopter money

One of the unconventional monetary policy measures that I consider is helicopter money. Helicopter money differs from conventional monetary policy in its effect on b_t^M . Instead of issuing fiat money by buying assets and thus increasing b_t^M , here the monetary authority simply distributes the newly printed money to agents, thus leaving b_t^M unchanged. This corresponds to lump-sum transfers, which are often used as the standard transmission mechanism for monetary policy in relatively simple models.

It is possible that a central bank is restricted by law from making such transfers to agents. While these lump-sum transfers are the most straightforward way to implement helicopter money in this

¹⁴This is similar to the effects in Gu et al. (2016), or Berentsen and Waller (2011), and Berentsen and Waller (2015).

model, there are other methods that have the same effect. What is needed in general is that the fiat money reaches the CM goods market, and that the quantity of outstanding bonds $B_t - b_t^M$ is unaffected. Specifically, the following methods have the same effect as a lump-sum transfer to agents: The monetary authority could buy CM goods from the agents with the newly-printed fiat money and then simply consume these goods. The monetary authority could also transfer either the newly printed fiat money or goods acquired with that fiat money to the fiscal authority. Then, if the fiscal authority either increases spending g_t or lowers taxes τ_t as a reaction to this transfer from the monetary authority, the policy tool still has the same effect as a direct transfer to agents. The tool does not work, however, if the fiscal authority instead reduces its debt as a reaction to the transfer, because then helicopter money essentially becomes equivalent to an open-market operation.

5.3 Negative interest rates

In addition to helicopter money, I also analyze negative interest rates. To do so, I first have to introduce an interest rate that is earned on reserves. With interest on reserves, the banks' first order conditions (4) and (6) become:

$$f'((1 - \alpha_M - \alpha_B)\phi_t\eta d_t) \geq \frac{1 + i^R}{1 + \pi_{t+1}}, \quad \text{and} \quad (16)$$

$$(1 - \alpha_M - \alpha_B)f'((1 - \alpha_M - \alpha_B)\phi_t\eta d_t) + \alpha_B \frac{1 + i^B}{1 + \pi_{t+1}} + \alpha_M \frac{1 + i^R}{1 + \pi_{t+1}} = \frac{1 + i^d}{1 + \pi_{t+1}}. \quad (17)$$

Equation (17) is valid if interest rates on reserves are paid (or raised in the case of negative rates) on all the reserves that the banks are holding; i.e., required and excess reserves. If, instead, the interest rate on reserves only applies to excess reserves, the equation becomes:

$$(1 - \alpha_M - \alpha_B)f'((1 - \alpha_M - \alpha_B)\phi_t\eta d_t) + \alpha_B \frac{1 + i^B}{1 + \pi_{t+1}} + (\alpha_M - \delta) \frac{1 + i^R}{1 + \pi_{t+1}} + \delta \frac{1}{1 + \pi_{t+1}} = \frac{1 + i^d}{1 + \pi_{t+1}}. \quad (18)$$

Negative interest rates are then simply defined as $i^R < 0$. These are the only equations from the equilibrium definition that change due to this policy. I will separately consider the two cases of either raising negative interest rates on all reserves, or only raising them on excess reserves, when analyzing this policy.

5.4 Environments for monetary policy

In Section 3, I have shown that there are five equilibrium cases, given positive values of δ and π_{t+1} . These different cases can be summed up into three larger groups. In each of these environments, conventional monetary policy as defined in Section 5.1 has different effects on the inflation rate. The three environments can be summed up by the interest rate on bonds, which has to lie between 1 and $\frac{1+\pi}{\beta}$:

$$1 < 1 + i^B = \frac{1+\pi}{\beta}: \textbf{No-scarcity environment}$$

This environment entails the equilibrium cases 1a, 1b, and 2. What these three cases have in common is that at least some of the bonds are held by agents, so that $b_t > 0$. This means that the market clearing condition for bonds is essentially slack, because the bonds that are not held by banks are absorbed by agents, and the interest rate on bonds is determined by the Fisher equation. Because $i^d > 0$ in these cases, we have $q^o < q^i \leq q^*$ ¹⁵. Since the interest rate on bonds is positive, banks do not hold any excess reserves, so that $\alpha_M = \delta$. The interest rate on deposits is at least $(1 - \delta)i^B$.

Proposition 3. *In a no-scarcity environment, conventional monetary policy can directly affect the inflation rate in the short run, which means that an increase in γ_t^M induced by conventional monetary policy leads to an increase in π_{t+1} .*

The proof to this proposition can be found in the Appendix. The logic behind it is straightforward, however. Because agents hold some of the bonds in this environment, the growth rate of bonds does not restrict monetary policy in the short run, and the effects of a temporary increase in γ_t^M are the same as in Lagos and Wright (2005).

$$1 < 1 + i^B < \frac{1+\pi}{\beta}: \textbf{Scarce investment opportunities environment}$$

This equilibrium exists if the case 3 regarding the bonds' returns is prevalent, and it implies that bonds are scarce and therefore pay a liquidity premium. This implies that $b_t = 0$. However, the interest rate on bonds is still positive, so that $\alpha_M = \delta$. The interest rate on deposits has to be $(1 - \delta)i^B$, so that both money and deposits are costly to hold, which means $q^o < q^i < q^*$. Note that this situation will lead to over-investment, since the amount invested is larger than k^* , the socially efficient investment quantity.

Proposition 4. *Conventional monetary policy can still affect the inflation rate in a scarce investment opportunities environment, but its effect is dampened compared to the no scarcity environ-*

¹⁵ q^i is equal to q^* in a case 1a equilibrium, but below q^* in equilibrium cases 1b and 2.

ment. This is because in a scarce investment opportunities environment, conventional monetary policy makes bonds even more scarce, which drives down the interest rate on both bonds and deposits.

For a sketch of the proof of this proposition, see the Appendix. Again, the logic is straightforward. Conventional monetary policy increases the supply of fiat money, which typically leads to inflation. However, it also makes the amount of bonds available in the economy more scarce. This shifts the banks' demand schedule for deposits, and makes them demand fewer deposits for any interest rate in period t . In equilibrium, this lowers both the interest rate and the amount of deposited funds. This means that the banks already hold less fiat money in period t , and thus the value of money already decreases in period t , which in turn means that the change in the value of fiat money from period t to period $t + 1$ is lower compared to what it would be in the no-scarcity environment.

$$1 = 1 + i^B < \frac{1+\pi}{\beta}: \textbf{Liquidity trap environment}$$

In this situation, the interest rate on bonds is driven down to zero because of the scarcity of bonds and profitable capital. In turn, this also drives the interest rate on deposits down to zero, so that deposits and fiat money become perfect substitutes. This implies that $q^o = q^i < q^*$, and $\alpha_M > \delta$, so that banks hold excess reserves and there is over-investment up to $\bar{k}$. As already mentioned earlier, the liquidity trap environment corresponds to the equilibrium case 4. To formally show that this really is a liquidity trap environment, we need to show that inflation cannot be controlled by conventional monetary policy in this situation, which is what proposition 5 states:

Proposition 5. *In a liquidity trap environment, conventional monetary policy is unable to affect π_{t+1} . An increase in γ_t^M only leads to changes in α_M and α_B , but none of the real variables are affected, and therefore inflation also does not change.*

The proof to Proposition 5 can be found in the Appendix. The intuition behind it is as follows: In a liquidity trap, fiat money and bonds are perfect substitutes for banks, and all available bonds are held by banks. Since an open market operation by the monetary authority reduces the supply of bonds by the same amount as it increases the supply of fiat money, the total amount of these assets in the economy remains constant. Thus, the banks absorb all the newly issued money to replace the bonds bought by the central bank, and therefore the amount of money in the goods market remains unaltered, and, consequently, the newly issued fiat money has no inflationary effect.

5.5 Helicopter money in a liquidity trap environment

Section 5.4 shows that conventional monetary policy is normally a useful tool for controlling inflation, but it is powerless in a liquidity trap. The next question to answer is whether helicopter money can be used instead.

It is quite clear that helicopter money has a different effect in a liquidity trap environment. From the proof of proposition 5, we see that the powerlessness of conventional monetary policy comes from the simultaneous effect of conventional monetary policy on bonds and fiat money. Since helicopter money does not affect bonds, its effect on inflation should be different from the effect of conventional monetary policy.

Proposition 6. *In a liquidity trap environment, helicopter money is able to affect π_{t+1} . An increase in γ_t^M induced by helicopter money affects real variables and increases inflation.*

Proposition 6 states that helicopter money enables the monetary authority to control inflation in a liquidity trap in the short run, which makes it a useful tool for central banks if they want to increase inflation. The proof to this proposition can be found in the appendix. The intuition for this result is that with helicopter money, banks have no incentive to absorb the additional fiat money issued, so inflation has to increase.

5.6 Negative interest rates in a liquidity trap environment

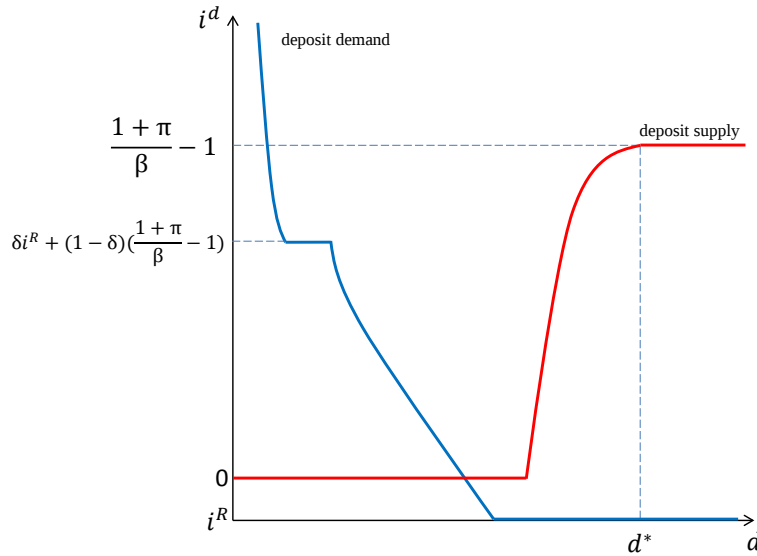


Figure 4: Deposit market clearing with negative interest rates.

With negative interest rates, the analysis is slightly different from that for helicopter money.

While helicopter money is a method for increasing γ^M directly, negative interest rates do not affect the growth rate of fiat money in the steady state. The question is thus whether the permanent introduction of negative interest rates changes the effectiveness of conventional monetary policy in a liquidity trap environment. However, it is still interesting as a first step to analyze how the introduction of negative interest rates affects the economy. Figure 4 shows deposit market clearing after the introduction of negative interest rates on all reserves. There are two important changes to note: First, the introduction of negative interest rates moves the lower flat part of the banks' demand curve below zero. Second, because fiat money dominates deposits in inside meetings if the interest rate on deposits is negative, we will never observe negative interest rates on deposits in equilibrium, which is shown by the flat segment of the supply curve at zero. From equation (17), we learn that after the introduction of negative interest rates on reserves, the return on bonds and real investment (which is equal in equilibrium cases 2-4) has to increase in order to keep the average marginal return of the banks constant. This means that if the economy is in a liquidity trap when negative interest rates are introduced, the equilibrium amount of deposited funds decreases, the interest rate on deposits remains at zero, and the return that banks require from real investment and bonds increases¹⁶. Note that compared to a situation without negative rates on reserves, the deposit demand curve moved slightly to the left (at any interest rate, banks' profits are lower because of the lower return of required reserves), and the upper flat segment of the deposit demand curve moved down. If instead negative interest rates are only levied on excess reserves, the demand schedule is unchanged for positive deposit interest rates. The resulting equilibrium has higher deposits and thus higher capital investment than would be the case if negative rates are charged on all reserves.

Now we can analyze how the environment for monetary policy changes in reaction to the introduction of negative interest rates on reserves. From Figure 4, it is obvious that the environment changes in a way beneficial to the monetary authority. Remember that a liquidity trap environment occurs whenever the equilibrium in the deposit market happens to occur in the lower flat segment of the banks' demand schedule. But with negative interest rates on reserves, this is impossible, because the lower flat segment now lies in a region where the supply of deposits from buyers is always zero. This means that a case 4 equilibrium and thus a liquidity trap is no longer possible, and only equilibrium cases 1-3 remain. I have already shown that conventional monetary policy is effective in these cases, so we can conclude that the introduction of negative interest rates on

¹⁶This effect was observable after the introduction of negative interest rates in Switzerland. As a result of the introduction of negative interest rates on reserves, the interest rates on mortgages increased. Compare for example <https://ftalphaville.ft.com/2016/03/07/2155458/the-swiss-banking-response-to-nirp-increase-interest-rates/>.

reserves allows the monetary authority to regain control of the inflation rate.

However, there are at least two caveats to this result. First, it is not possible in the model for the interest rate on bonds to become negative, although we observed this in reality. As the conclusion that the introduction of negative rates allows the monetary authority to control inflation via conventional monetary policy depends crucially on the fact that bonds and excess reserves have different rates of return after the introduction of negative rates, it is clear that the mechanism proposed here does not work in practice if bond rates become negative. A possible reconciliation of the model with reality might be the presence of disutility from using cash, which prevents agents from switching to fiat money. This would then mean that the negative interest rates on reserves observed in reality are not high enough.

Second, in the model banks immediately get rid of excess reserves after the introduction of negative rates. While this is simple in the model, it is much more difficult to get rid of reserves in reality, since banks would have to force clients to withdraw some of their deposits in cash to achieve this.

5.7 Forward guidance

As explained in Section 4.4, exiting the liquidity trap leads to inflation, because banks get rid of their excess reserves. From Sections 4.2 and 5.6, we know that a large increase in bonds or the introduction of negative interest rates on reserves can get an economy out of the liquidity trap equilibrium. But if either one of these policies is announced n periods before they are actually implemented, agents are aware that inflation will increase in n periods, whereby the size of the increase in inflation at $t + n$ depends on the amount of excess reserves that banks are holding. The announcement of a policy that leads an economy out of the liquidity trap implemented in period $t + n$ can be considered forward guidance. From Gu et al. (2016), it is known that expected increases in inflation in the future have real effects on current variables. However, if the expected increase occurs more than one period in the future (i.e., for $n > 1$), many dynamics are possible, so that it is unclear whether the effect on current inflation and thus real variables will be positive or negative. This suggests that forward guidance is not a very useful tool for monetary policy, unless the central bank is somehow able to stabilize the path of inflation in the periods before the anticipated event occurs.

6 Conclusion

This paper shows that a liquidity trap can exist at positive inflation rates, and that an economy can fall into a liquidity trap because of preference or production parameters, or because of a scarcity of bonds. Therefore bonds, or saving assets in general, are crucial in determining the effectiveness of conventional monetary policy. This is due to the nature of the transmission mechanism of monetary policy. In particular, the paper shows that conventional monetary policy cannot affect inflation rates in a liquidity trap, i.e., when the interest rate on bonds is zero and banks hold excess reserves. However, the use of helicopter money or the introduction of negative interest rates will allow the monetary authority to regain control over the inflation rate in a liquidity trap equilibrium. Forward guidance also has real effects, but the direction of these effects is a priori unclear.

A shortage of government bonds is in several ways harmful to an economy, because it leads to too little trade, consumption and production, and also to over-investment. Thus the fiscal authority can increase welfare and prevent a liquidity trap by issuing at least as many bonds as required by the optimal minimal level of government debt. Since the Friedman rule delivers the first-best allocation, inflation is bad for welfare in this economy.

Bibliography

- Bacchetta, P., Benhima, K., and Kalantzis, Y. (2016). Money and capital in a persistent liquidity trap. CEPR Discussion Paper No. DP11369.
- Berentsen, A. and Waller, C. (2011). Price level targeting and stabilization policy. *Journal of Money, Credit and Banking*, 43(2):559–580.
- Berentsen, A. and Waller, C. (2015). Optimal stabilization policy with search externalities. *Macroeconomic Dynamics*, 19(3):669–700.
- Bhattacharya, J., Guzman, M. G., Huybens, E., and Smith, B. D. (1997). Monetary, fiscal, and reserve requirement policy in a simple monetary growth model. *International Economic Review*, 38(2):321–350.
- Buiter, W. H. (2014). The simple analytics of helicopter money: Why it works - always. *Economics: The Open-Access, Open-Assessment E-Journal*, 8(28).
- Caballero, R. J., Farhi, E., and Gourinchas, P.-O. (2017). The safe asset shortage conundrum. *Journal of Economic Perspectives*, 31(3):29–46.

- Christiano, L., Eichenbaum, M., and Rebelo, S. (2011). When is the government spending multiplier large? *Journal of Political Economy*, 119(1):78–121.
- Cochrane, J. H. (2017). The new-keynesian liquidity trap. *Journal of Monetary Economics*, 92:47–63.
- Correia, I., Farhi, E., Nicolini, J. P., and Teles, P. (2013). Unconventional fiscal policy at the zero bound. *American Economic Review*, 103(4):1172–1211.
- Demiralp, S., Eisenschmidt, J., and Vlassopoulos, T. (2017). Negative interest rates, excess liquidity and bank business model: Banks’ reaction to uncoventional monetary policy in the euro area. *Koc University - Tusiad Economic Research Forum Working Paper Series*.
- Dong, F. and Wen, Y. (2017). Optimal monetary policy under negative interest rate. *Federal Reserve Bank of St. Louis Working Paper 2017-019A*.
- Eggertsson, G. B. and Krugman, P. R. (2012). Debt, deleveraging, and the liquidity trap: A fisher-minsky-koo approach. *The Quarterly Journal of Economics*, 127(3):1469–1513.
- Eggertsson, G. B. and Woodford, M. (2003). The zero bound on interest rates and optimal monetary policy. *Brookings Papers on Economic Activity*, pages 139–233.
- Eggertsson, G. B. and Woodford, M. (2004). Policy options in a liquidity trap. *American Economic Review*, 94(2):76–79.
- Fisher, I. (1930). *The Theory of Interest*. The Macmillan Company, New York.
- Friedman, M. (2005). *The Optimum Quantity of Money*, chapter The Quantity Theory of Money, pages 51–67. Transaction Publishers.
- Gali, J. (2014). The effects of a money-financed fiscal stimulus. CEPR Discussion Paper No. DP10165.
- Geromichalos, A. and Herrenbrueck, L. (2017). The liquidity-augmented model of macroeconomic aggregates. *mimeo, Simon Fraser University*.
- Gu, C., Han, H., and Wright, R. (2016). The effects of monetary policy and other announcements. *mimeo*.
- Guerrieri, V. and Lorenzoni, G. (2017). Credit crises, precautionary savings, and the liquidity trap. *The Quarterly Journal of Economics*, 132(3):1427–1467.

- Kiyotaki, N. and Moore, J. (2012). Liquidity, business cycles, and monetary policy. NBER Working Paper 17934.
- Krugman, P. R., Dominguez, K. M., and Rogoff, K. (1998). It's baaack: Japan's slump and the return of the liquidity trap. *Brookings Papers on Economic Activity*, 1998(2):137–205.
- Lagos, R. and Rocheteau, G. (2008). Money and capital as competing media of exchange. *Journal of Economic Theory*, 142:247–258.
- Lagos, R. and Wright, R. (2005). A unified framework for monetary theory and policy analysis. *Journal of Political Economy*, 113 (3):463–484.
- Monnet, C. and Sanches, D. R. (2015). Private money and banking regulation. *Journal of Money, Credit and Banking*, 47(6):1031–1062.
- Rocheteau, G. and Wright, R. (2005). Money in search equilibrium, in competitive equilibrium, and in competitive search equilibrium. *Econometrica*, 73(1).
- Rocheteau, G., Wright, R., and Xiao, S. X. (2016). Open market operations. *mimeo*.
- Rognlie, M. (2016). What lower bound? monetary policy with negative interest rates. *mimeo*.
- Werning, I. (2012). Managing a liquidity trap: Monetary and fiscal policy. *MIT mimeo*.
- Williamson, S. (2012). Liquidity, monetary policy, and the financial crisis: A new monetarist approach. *American Economic Review*, 102 (6):2570–2605.
- Williamson, S. (2016). Scarce collateral, the term premium, and quantitative easing. *Journal of Economic Theory*, 164:136–165.

Appendix A

A.1 Proof of Proposition 2

Proposition 2 states that an increase in inflation is welfare-reducing in all equilibrium cases. In the following proof, I will go through each case separately to show this.

Proof. In equilibrium case 4, the interest rate on deposits is zero. Since I am only considering small changes in inflation that keep the economy in the same equilibrium case, I can conclude that the change in inflation will not affect the interest rate on deposits. But from equation (15), we know that buyers will reduce their supply of deposits given an increase in inflation without an offsetting change in the interest rate on deposits. Furthermore, because the banks' investment in equilibrium case 4 equals $\bar{k}$ s.t. $f'(\bar{k}) = \frac{1}{1+\pi}$, over-investment will also increase as a reaction to an increase in inflation for an economy in equilibrium case 4.

In equilibrium case 3, the equations (5), (6), (11), (12), (13), and (15) all hold with equality, with $b_t = 0$. Inflation enters these equilibrium conditions in a number of ways. From equation (15), we learn that the real amount of deposits held in equilibrium (and thus the quantities consumed in inside meetings) decreases in inflation *unless* the interest rate on deposits increases more than one-to-one with an increase in inflation. Suppose this is the case. Then, in equation (6), the right-hand side is increasing, which requires the left-hand side to increase too. It is clear that $\frac{\delta}{1+\pi}$ is decreasing in inflation. However, $f'((1 - \delta - \alpha_B)\phi_t\eta d_t)$ is also decreasing, since we assumed real deposits are increasing (and that forces α_B to decrease too, since banks already hold all bonds, so bonds as a fraction of real deposits have to decrease), so by the concavity of $f(k)$, the whole term has to decrease. Also $(1 - \delta - \alpha_B)$ decreases, so we can conclude that the first term on the left-hand side of equation (6) is decreasing given our assumption. But since (5) holds at equality, this means that also the second term has to decrease, which means that all three terms on the left-hand side are decreasing, which leads to a contradiction.

This shows that at best, real deposits and investment remain unchanged after an increase in inflation in equilibrium case 3, but since an increase in inflation has a negative welfare effect on q^o , this is enough to conclude that an increase in inflation is welfare-decreasing in equilibrium case 3.

In equilibrium case 2, the interest rate on deposits is $i^d = (1 - \delta) \left(\frac{1+\pi}{\beta} - 1 \right)$. This shows that the interest rate increases linearly with inflation, but not one-to-one. Therefore, an increase in inflation is not fully compensated by an increase in the interest rate on deposits in this case, thereby making deposits more costly to hold, which in turn means that the real deposits held in

equilibrium are decreasing with respect to inflation in equilibrium case 2. Since $k = k^*$ in equilibrium case 2 for any inflation rate, there is no offsetting welfare effect from a change in the banks' investment, which allows the conclusion above. The reduction in deposits is met by a reduction in bond holdings by banks.

In equilibrium case 1b, the equilibrium in the deposit market is determined by equations (6) and (15), which collapse to the following expressions (for simplicity, I set $\sigma = 1$ for this proof, but this is without loss of generality):

$$(1 - \delta)f'((1 - \delta)\eta z^d) + \frac{\delta}{1 + \pi} = \frac{1 + i^d}{1 + \pi}$$

$$\frac{u'(q(z^d))}{c'(q(z^d))} = \frac{1}{\beta} \cdot \frac{1 + \pi}{1 + i^d}.$$

Combining these two equations gives:

$$(1 - \delta)f'((1 - \delta)\eta z^d) + \frac{\delta}{1 + \pi} = \frac{1}{\beta} \cdot \frac{c'(q(z^d))}{u'(q(z^d))}.$$

Now I can take the total derivative to find the effect of inflation on the real amount of deposits z^d , which is:

$$\frac{\partial z^d}{\partial \pi} = -\frac{\delta}{(1 + \pi)^2} \cdot \frac{1}{a - h} < 0,$$

with $a = \frac{q'(z^d)[c''(q(z^d))u'(q(z^d)) - u''(q(z^d))c'(q(z^d))]}{[u'(q(z^d))]^2} > 0$, and $h = \beta(1 - \delta)^2\eta f''((1 - \delta)\eta z^d) < 0$.

This shows that an increase in inflation in equilibrium case 1b leads to a decrease in the real amount of deposits, which in turn also leads to a decrease in the investment by banks.

In equilibrium case 1a, the interest rate on deposits is $i^d = \left(\frac{1 + \pi}{\beta} - 1\right)$, thus it always compensates buyers fully for discounting and inflation. This means that a change in inflation has no effect on q^i in this case. However, the banks' decision about the amount of deposits, equation (6), reduces to the following expression in equilibrium case 1a:

$$(1 - \delta)f'((1 - \delta)\eta z^d) + \frac{\delta}{1 + \pi} = \frac{1}{\beta}.$$

An increase in inflation decreases the second term on the left-hand side, so an increase in the first term on the left-hand side is required. Since $f'(k)$ is decreasing with respect to k , this requires a reduction in k , so investment, which is already below the efficient level, decreases further. ■

A.2 Proof of Proposition 3

Consider an announcement by the monetary authority at time t that $\gamma_{t+1}^M > \gamma^M$ through conventional monetary policy, where γ^M is the steady state value and thus equals steady state inflation $1 + \pi$. For simplicity, assume that $\pi = 0$, so that $\gamma^M = 1$, but the proof is also valid for positive inflation rates. Assume additionally that we are in equilibrium case 2, because that means banks' demand for deposits is completely elastic with regard to small changes in supply, so we do not have to consider the banks' decisions for this analysis. We are trying to find an equilibrium where the economy returns to the steady state as quickly as possible, so we assume that the economy is back in the steady state in period $t + 2$. Only two equilibrium conditions are directly affected by conventional monetary policy, namely the money market clearing condition (equation (12)) and the bond market clearing condition (equation (11)). However, since we are in a no-scarcity environment, any changes in the amount of publicly available bonds $B_t - b_t^M$ is countered by an equivalent change in bonds held by agents b_t , since agents are indifferent between holding zero bonds and an infinite number of bonds in this environment. In period $t + 1$, the money market clearing condition is:

$$\phi_{t+1}\gamma_{t+1}^M M_t = (1 - \eta)z_{t+1}^m + \alpha_M \eta z_{t+1}^d.$$

From equations (13) and (15), we know that both z_{t+1}^m and z_{t+1}^d are decreasing functions of π_{t+2} . Since we assumed that the economy returns to the steady state in $t + 2$, $\pi_{t+2} = 0$ and thus z_{t+1}^m and z_{t+1}^d are also equal to their steady state values. But since $\gamma_{t+1}^M > 1$, ϕ_{t+1} has to be lower than the previous steady-state value of money, so that $\phi_{t+1} < \phi$ is required. In period t , the money market clearing condition is:

$$\phi_t M_t = (1 - \eta)z_t^m + \alpha_M \eta z_t^d.$$

Suppose that ϕ_t is equal to its steady state value ϕ . This requires $\pi_{t+1} > 0$, since $1 + \pi_{t+1} = \frac{\phi_t}{\phi_{t+1}}$. But since z_t^m and z_t^d are decreasing in π_{t+1} , they are below their steady state values, which in turn means that ϕ_t also has to be below its steady state value given the equation above, so this leads to a contradiction. Now suppose instead that $\phi_t = \phi_{t+1} = \phi'$; i.e., the value of money immediately drops to the new steady-state value when the policy is announced. However, this implies $\pi_{t+1} = 0$, which in turn means that z_t^m and z_t^d stay at their steady-state values. But given the equation above, this requires $\phi_t = \phi$, which also leads to contradiction. This leaves as a solution only $\phi > \phi_t > \phi_{t+1}$ and $\gamma_{t+1}^M > 1 + \pi_{t+1} > 1$. This shows that inflation is increasing as a reaction to the increase in money growth, but not one-to-one. ■

A.3 Sketch of proof of Proposition 4

Consider again an announcement by the monetary authority at time t that $\gamma_{t+1}^M > \gamma^M$ through conventional monetary policy, where γ^M is the steady state value and thus equals steady state inflation $1 + \pi$. For simplicity, assume that $\pi = 0$, so $\gamma^M = 1$, but the proof is also valid for positive inflation rates. The reasoning for the buyer's side is similar to the proof of Proposition 3. However, because bonds are scarce in this case, also the bond market clearing condition (equation (11)) is affected. The amount of bonds available for banks decreases, which means that banks offer a lower interest rate for a given amount of deposits. Since z_t^d is increasing with regard to the interest rate, it is now lower for any level of inflation. Looking again at the money market clearing condition for period t ,

$$\phi_t M_t = (1 - \eta)z_t^m + \alpha_M \eta z_t^d.$$

We see that ϕ_t decreases by more than in a no-scarcity environment. Because ϕ_{t+1} is not affected by this (it is pinned down by the steady state values of z^m and z^d), we can conclude that π_{t+1} increases by less than it would in a no-scarcity environment in reaction to conventional monetary policy. ■

A.4 Proof of Proposition 5

Proof. Consider an announcement by the monetary authority at time t that $\gamma_{t+1}^M > \gamma^M$ through conventional monetary policy, where γ^M is the steady state value and thus equals steady state inflation $1 + \pi$. For simplicity, assume that $\pi = 0$, such that $\gamma^M = 1$, but the proof is also valid for positive inflation rates¹⁷. For this proof, I will posit that, as a reaction to this policy, only α_M and α_B change, but the real variables do not; Then, I will show that this indeed constitutes an equilibrium.

From the money market clearing condition (equation (12)), supposing that all changes in money growth are represented in the new value α'_M , we obtain the following equation for period $t + 1$ (All variables without time subscripts denote steady-state values):

$$\begin{aligned} \gamma_{t+1}^M \phi M &= (1 - \eta)z^m + \alpha'_M \eta z^d \\ \alpha'_M &= \frac{\gamma_{t+1}^M \phi M - (1 - \eta)z^m}{\eta z^d}. \end{aligned}$$

¹⁷Thanks to this assumption, also nominal variables such as M_t and B_t have a steady state value, while for non-zero inflation, only their real counterparts $\phi_t M_t$ and $\phi_t B_t$ do. This assumption substantially simplifies the notation in this proof.

From the market clearing condition for bonds (equation (11), multiplied by ϕ on both sides), we obtain the following equation in period $t + 1$:

$$\begin{aligned}\alpha'_B \eta z^d &= \phi(B - b_{t+1}^M) = \phi(B - \gamma_{t+1}^M M_t) \\ \alpha'_B &= \frac{\phi(B - \gamma_{t+1}^M M)}{\eta z^d},\end{aligned}$$

where again I supposed that only α_B changes. The replacement of b_{t+1}^M is possible if we assume that the monetary authority never used helicopter money in the past, but the proof also generalizes to cases where it did.

Now, adding α'_M and α'_B yields:

$$\begin{aligned}\alpha'_B + \alpha'_M &= \frac{\phi(B - \gamma_{t+1}^M M) + \gamma_{t+1}^M \phi_t M - (1 - \eta)z^m}{\eta z^d} \\ &= \frac{\phi_t B - (1 - \eta)z^m}{\eta z^d} \\ &= \alpha_B + \alpha_M.\end{aligned}$$

This shows that if I posit that the changes in the money growth rates are all absorbed by changes in α_B and α_M , the sum of these two variables does not change. But since bonds and reserves are perfect substitutes for banks in this equilibrium case, this implies that no real variables change. Especially, from equation (6), real investment and demand for deposits stay the same. Thus, I have shown that there is an equilibrium where only α_B and α_M change in reaction to a change in the money growth rate. ■

A.5 Proof of Proposition 6

Proof. I can prove this by trying to replicate the proof for proposition 5 and showing that this leads to a contradiction. Consider an announcement by the monetary authority at time t that $\gamma_{t+1}^M > \gamma^M$ as a result of helicopter money, where γ^M is the steady state value and thus equals steady state inflation $1 + \pi$. For simplicity, assume that $\pi = 0$, so $\gamma^M = 1$, but the proof is also valid for positive inflation rates. Suppose again that in the money market clearing condition (equation (12)), only α_M changes. Since the bond market clearing condition (equation (11)) is unaffected by helicopter money, the increase in α_M is not countered by a decrease in α_B , so the sum $\alpha_M + \alpha_B$ has to increase. But from equation (4), this cannot be optimal without a change in the amount of deposits, because it would drive the return on capital below the return on fiat money. Thus, the increased growth rate cannot lead to only a change in α_M and α_B . But since the left-hand side of

equation (12) is changing, we know that some other variables in this equation have to change too, and since all variables apart from α_M in the equation are real, it is clear that real variables have to change. To prove that inflation has to increase in this situation, we can then reason analogous to the proof of proposition 3. ■