On Processing Central Bank Communications:

Can We Account for Fed Watching?

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Abstract: Central bank communications need interpretation. We contribute to the communications literature by focusing on the effort expended on deciphering central bank communications. We build a model economy in which banks provide a deposit/insurance function for consumers subject to idiosyncratic liquidity shocks. Though ex ante identical, banks exhibit ex post heterogeneity by choosing different predictors that vary in terms of accuracy with respect to the expected future return on money. In the literature with heterogeneous forecasts, the modelling approach has relied on stochastic costs as the primary force accounting for the coexistence of different predictors in equilibrium. Here, model the problem as a willingness to pay for different predictors; each predictor has a different forecast accuracy with more accurate predictors resulting in higher expected utility. By the concavity of the consumer's utility function, there exists a willingness-to-pay which satisfies an indifference condition. More accurate forecast predictors correspond with greater willingness-to-pay amounts. The resources expended to obtain a more accurate forecast correspond with the bank's processing of the central bank's communications. Hence, we interpret the willingness to pay as Fed watching. The model with Fed watching exhibits local stability, while we derive conditions in which no Fed watching results in local instability. We further apply this approach to a banking economy in which the returns to one asset are subject to a fractional externality; that is, the return to one asset is negatively related to the fraction of banks holding that asset. The approach is designed to capture how herding and the regulatory settings are related to what the central bank knows (and communicates) about bank operations.

1. Introduction

Researchers have long been interested in how central bank communications affect macroeconomic outcomes. In his 2013 speech, Chairman Bernanke made central bank transparency one of his chief priorities upon beginning his term. As the financial crisis unfolded, forward guidance emerged as the term that central bankers applied to the transparency mission. With a change in central bank communications, it follows that we are interested in the effects.

Blinder, Ehrmann, Fratzcher, De Haan, and Jansen (2008) argue that there are four key mechanisms through which central bank communications matter. Nonstationarity reflects the changing nature in the communications over time. In addition, learning, some form of nonrational expectations and asymmetric information capture the means through which private agents respond to the flow of communication.

There is a vast literature on central bank announcements. Two papers focus on the most recent experience with forward guidance, asking about the empirical relationship between central bank announcements and macroeconomic outcomes. Svensson (2015) looks closely at the date of the communications, comparing what the central bank said it would do in the future (the published forecast rate) and market expectations. Svensson takes the first snap shots of how monetary policy announcements are translated into how predictable interest rates have been under forward guidance communication structure. Del Negro, Giannoni and Patterson (2015) observe that DSGE models ascribe unreasonably large responses reported to central bank announcements. After reporting evidence on the size of observed responses in the actual economy—the responses are not that large quantitatively—Del Negro et al. demonstrate that a model economy yields more measured responses by fitting a perpetual-youth variant of the canonical DSGE model.[1] The age distribution results in discounts the future so that 8-quarter ahead announcements have smaller effects than the standard DSGE models. Goy, Hommes and Mavromatis (2018) extend the work by Del Negro, et al. to consider what happens in economics in which agents form expected values of future interest rates in a New Keynesian model with Rotemberg-style price-adjustment costs.

Economic theory has also sought to characterize the channels through which monetary policy announcements affect the aggregate economy. Morris and Shin (2018), for example, deal with the thorny issue of circularity. In particular, the central bank is using communications to steer the economy by looking at movements in market prices. Yet, the market prices are simultaneously determined, hence are reflecting the central bank's communications. They solve the problem in a static game, demonstrating how the reflection problem can induce responses even the central bank does not follow through with their announced reaction function.

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[1] See also work by Piergallini (2006), Castelnuovo and Nisticò (2010) and Nisticò (2012).

In addition, Gu, Han and Wright (2018) study the relationship between central bank announcements and the nature of the dynamic response. In their paper, Gu, et al, focus on economies in which fiat money is valued in equilibrium. The experiment is clear: the central bank announces that it will implement a monetary policy action that will occur at some specified future date. Gu, et al. explain the kind of advance warning encoded in the central bank communication can result in additional volatility. Interestingly, the additional volatility may be welfare improving.

The purpose of this paper is to reconsider the central bank communication problem from the perspective of what we call Fed watching. In the central bank communication literature, there is a common thread; specifically, the models studies devote effort on describing how the announcement is constructed and use the discipline encoded in the rational agent as the sole means of processing the central bank communication. We take the non-rational expectations mechanism described by Blinder, at al. literally, studying how processing the central bank communication will affect aggregate outcomes. We construct expectations of future equilibrium outcomes in ways that are similar to Goy, et al. In addition, to the central bank relying VAR forecasts to learn the economic outcomes, they also include private agents forming forecasts of the equilibrium outcomes by assessing central bank credibility with respect to heir announcements and different forecast technologies like we do. The key difference between our study and Goy, et al. is that they use the Phillips curve to pin down the output-inflation relationship. Our model also builds on the absence of common-knowledge friction analyzed in Angeletos and Lian (2018); the information difference exists between what the central bank knows and what private agents know. To keep our illustration clear, the central bank does not use its information advantage strategically. Rather, we consider an economy in which the central bank communication is credible, allowing private citizens and banks spend effort and resources to understand what the consequences of the communication will be. The effort is now endogenized and is called Fed watching. In our first model economy, the rewards to Fed watching are that forecast accuracy improves, prices are less sticky, and welfare losses are smaller.

Despite the absence of careful measurement, we all know that Fed watching exists. Kurov (2012), for example writes: "Market participants analyze every word of Fed officials for clues of possible directions of monetary policy because monetary policy affects asset prices, particularly stock prices." (p. 175). Perhaps the most famous type of Fed watching was the briefcase indicator applied to Chairman Greenspan. When central bank communications were less overt, Fed watchers constructed a measure of policy announcements from the thickness of Chairman Greenspan's briefcase.[2] Lastly, nearly every financial institution with a newsletter will devote some space to a prediction of what the Federal Reserve

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[2] A quick Google search yielded 9,460 results for a search "Greenspan briefcase indicator. Gavin and Mandel (2000) construct a measure of the thickness and conclude that between 1994 and 2000, there is no evidence that the briefcase indicator is a good predictor of future Federal Reserve actions.

will do in the coming months. With so many resources devoted to watching the central bank, it seems worthy to build a model economy in which Fed watching is considered.

The model economy is a random-relocation version of the overlapping generations economy. The chief benefit of the random-relocation model is that it accounts for the need for liquidity by people being subject a moving shock; like Diamond and Dybvig, one does not know ex ante whether they will need the liquid asset or not. Banks enter to offer state-contingent deposit contracts that are conditioned on the moving status of the depositor. Banks play a central role in Fed watching in our model economy. In the most stripped down version of the model economy, banks consider how much they would have been willing to pay to avoid the consequences of inaccurate predictions in the previous period. This observable, regret-based measure is a measure of the value of Fed watching. An increase in willingness to pay is interpreted as the intensity of Fed watching. Because the willingness-to-pay measure is computed at the indifference point between Fed watching and not Fed watching, the bank chooses the same state contingent deposit contracts and the same allocation of deposits between money and an interest-bearing asset.

In this model economy, banks allocate deposits between fiat money and an illiquid asset. The allocation maximizes expected utility of the deposit by offering state-contingent returns where the state depends on the realization of an idiosyncratic shock. The depositor either realizes a move notice and withdraws liquidity or realizes a non-move notice withdraws later. The central bank's communication affects the expected real return. What we contribute is an investigation into the formation of the expected real return on money; more specifically, resources devoted to processing the central bank communications may have important aggregate economic consequences through their effects on the bank's optimal allocation. We interpret these resources as Fed watching, which operates through the forecast accuracy of the expected real return on money.

To make Fed watching more concrete, suppose the bank has access to two forecasting technologies, or predictors. One predictor is the perfect-foresight and is costly to use while the other predictor uses past price level observations and is free. Given the curvature of the consumer's preferences, the question is whether there exists a willingness to pay for the more expensive predictor that equates the expected utility across the two predictors. The bank solves a Hicksian-compensation regret problem; what would the bank have been willing to pay last period to be indifferent between Fed watching and no Fed watching. The regret is a way to examine the problem with circularity associated with the bank trying to forecast the forecast of the central bank and vice versa. The willingness-to-pay measure is the intensity of Fed watching.

Thus, one of the contributions is to consider why the distribution of predictors is non-degenerate. There is a large literature on economies with heterogeneous predictors. In those papers, researchers resort

to an idiosyncratic cost or preference idiosyncratic shocks to account for why the distribution of predictors used across the population does not mass at a single predictor, that is, why the cross-section distribution is not degenerate.[3] Rather than building in a shock that affects the choice problem, we use the curvature of the consumer's utility function to derive a willingness-to-pay value. With a continuum of competitive banks, each one offers a deposit contract consisting of state-contingent returns and a regret-based measure of forecast accuracy. Central to implementing a competitive deposit contract is the fact that there exists an indifference deposit contract; that is, depositors are indifferent in expected utility between say, a deposit contract with lower state-contingent returns and greater forecast accuracy and another deposit contract with higher state-contingent returns and lesser forecast accuracy. A bank computes what person would have been willing to pay to be indifferent between alternative deposit contracts based on past forecast accuracy, which is what captures the regret-based notion. In other words, the bank knows how much it would have been willing to pay in order to be indifferent between the regret-based forecast accuracy of one predictor and another predictor. Because expected utility is held constant, depositors are indifferent across the competing deposit contracts and can diversify deposits across the continuum of banks.

There are three main result presented in this paper. First, there exists a positive value at which an ex ante bank is indifferent between expending the resources to process the central bank's communications and using an inferior predictor (read less accurate) to forecast future prices. We treat the willingness to pay as the cost of the predictor technology. Then, we follow the methods developed by Brock and Hommes (1997) to construct a regret-based measure that corresponds to what the bank would have wanted to spend on Fed watching had it known the inaccuracy of its forecast. In the absence of Fed watching, forecast accuracy is greater. Moreover, the backward-looking predictor imparts stickiness on the price level. Because of the forecast inaccuracy, consumer welfare is lower compared to the rational-expectations equilibrium.

Second, the existence of Fed watching imparts some interesting properties to the local dynamics. In doing so, our findings add to our understanding of the relationship between information and stability. In addition, when some measure of banks do not Fed watch, some price stickiness is imparted into the local dynamics. We show that steady state is locally monotone-jump stable when banks use the perfect-foresight predictor. However, for banks that use a money growth-adjusted predictor, we derive conditions in which the Jury-Marden conditions fail, indicating that the low-cost predictor results in an unstable steady state. Numerical illustrations extend the conditions to demonstrate that there a wider set of conditions in which the steady state is unstable. Thus, our findings suggest that there is a relationship

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[3] See equation (1) in Branch and McGough (2016) and Brock and de Fontnouvelle (2000) for examples of the role that forecast accuracy and cost shocks play in determining what predictor a saver chooses.

between the resources expended to processing central bank communications and stability. Or, put another way, more information accounts for a stable economy and less volatility.[4] With respect to the relationship between price stickiness and stability, we are not arguing that as prices becoming increasingly sticky, the economy is less stable. The entire DSGE literature renders such a claim invalid. But, our findings do suggest that the existence of price stickiness can impart instability into a perfect-foresight model economy.

Third, we extend the model economy to consider multiple illiquid assets. Our goal is to examine the role that central bank communications play in an economy in which there is a negative externality associated with the fraction of bank asset held in one of the assets. Here, the central bank communicates the aggregate fraction of deposits allocated between the two illiquid assets. Without constructing a model economy with risky assets, we use this setup to study how the bank's portfolio allocations can result in lower returns for depositors. Banks are not necessarily ignorant that an externality exists, they cannot internalize the social costs unless they are willing to expend resources to process what the central bank knows about both an aggregate state variable and the efficient aggregate fraction of deposits allocated to the externality-riddled illiquid asset. By computing its willingness to pay, we can infer how valuable it is process the central bank's communication regarding the socially efficient allocation of illiquid assets. Here, we show that the bank's willingness to pay depends positively on distance between the past, observed welfare costs.

The last set of results can potentially held shed light on the cycles in regulatory reform. Suppose we interpret the aggregate state variable as a measure of productivity. If productivity increased at date t, for example, the gap between the free predictor of welfare and the socially-efficient level widens As the gap widens, regret-based approach indicates that the intensity of processing the central bank communication increases; it would have been more valuable to Fed watch. As the intensity increases, the social cost of the externality decreases since banks are processing the central bank communications means they internalizing those communications. There is less incentive to implement costly regulations on banks because the bank is not overinvesting in the illiquid asset subject to the externality. Consider a case in which aggregate state variable declines in the date $t+1$. Now, banks reduce the intensity of Fed watching; correspondingly, more banks will overinvest and the return to bank deposits decline and the welfare loss of the externality increases. The implication is that regulation is most valuable to an economy after the

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[4] The relationship between information and stability goes back to Hirschleifer (1971). Our results are not directly comparable to that line of research because our results are built on the notion that the social value of the predictor (and its information) is socially beneficial. Gu, et al. demonstrate that advance warning in the form of central bank announcements can result in greater economic volatility. But our central bank communications are very simple and banks are perfectly transparent.

negative productivity shock as hit. In other words, the regulatory environment changes to banking conditions with a lag.

In both model specifications, it is important to note that is substantial overlap in terms of the underlying economics. Because it is costly to process information, there is a welfare loss in equilibrium compared to the economies in which rational expectations are costless. Depositors want to get the highest state-contingent returns. However, costly information means that the bank's portfolio allocation is adversely affected; banks can Fed watch taking some of the resources deposited or not Fed watch and inaccurately forecast the return on money. In the externality case, banks can process the central bank communication—a kind of Fed watching—and reduce returns to depositors or not Fed watch and overinvest in "bad" illiquid asset. In both cases, there is a welfare loss to depositors owing to the costly information. We derive a class of second-best equilibrium that arise because it costly to implement the first best.

The paper outline is as follows. Section 2 describes the basic random relocation model. We introduce two predictors for the future price level and describe the equilibrium for each predictor, including the stability conditions and some comparative static results in Section 3. Because of the predictors is so naïve, we consider a predictor that takes the money growth into account and consider the stability conditions in Section 4. Section 5 treats Fed watching as a two-step problem that involves the bank's willingness to pay to identify the intensity of Fed watching. Section 6 extends the model economy to consider the two types of illiquid assets. A brief summary and conclusion is presented in Section 7.

2. The model economy

Here, we consider a modified version of the random-relocation economy developed in Bencivenga and Smith (1991) and presented in Haslag and Martin (2006). Accordingly, we provide a succinct description of the basic economy, pointing the interested reader to either Bencivenga and Smith or Haslag and Martin for greater detail.[5]

In the economy, there are an infinite number of identical, discrete time periods. Let $t=0,1,2,...$ denote the indexed value for each date. There are two spatially-separated islands. On each island, there is a continuum of agents of unit mass born each period. Agents live for two periods, receiving an endowment of y units of the single, perishable consumption good when young and nothing when old. At

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[5] In our case, banks are operating as the most efficient insurers for old-age consumption. See Dang, Gorton, Holmstrom and Ordonez (2017) for a model economy in which banks can hide information from depositors.

date $t = 0$, there is a continuum of agents of measure one on each island that live for one period. We refer to this group as the initial old. We assume that the initial old are endowed with m_0 units of fiat money.

At each date $t \geq 0$, there is a continuum of measure one of two-period lived agents born on each island. In addition, there is a one generation, labelled the initial old who live on each island for one period but only exist at date $t = 0$. The initial old also consists of a continuum of measure one of agents. There is a single, perishable consumption good. The consumption good can be transformed into a storage good at a one-for-one rate. For each consumption good stored at date t, there are x units of the date-$t+1$ consumption good. Young agents value consumption when young and when old. We assume that preferences are time-additively separable. Agents receive an endowment of y units of the consumption good at the beginning of the period when young and nothing when old.

Two-period lived agents want to consume when young and when old. Preferences are captured by additively separable utility functions, given by $U\left(c_t^1\right) + V\left(c_{t+1}^2\right)$, where $c_t^1\left(c_{t+1}^2\right)$ denotes consumption by a young person born at date t (consumption by an old person born at date t).

Each young person faces an idiosyncratic shock. The relocation shock is realized at the end of each period. Let each young person receive a message of whether they are going to be relocated or not. The probability of being relocated is represented by α, $where\ 0 < \alpha < 1$. For now, suppose the value of the relocation probability is constant over time.[6] By the law of large numbers, α represents the measure of young agents who are relocated. To ensure symmetry, we assume that α is the same on both islands. The key friction in this economy is that goods cannot be transported across locations. In addition, claims against goods can be verified across islands.[7] Only fiat money is recognizable across the two locations. This is basically a Diamond-Dybvig liquidity shock that is realized by each person. As Bencivenga and Smith (1991) show, expected lifetime utility is maximized by depositing the unconsumed portion of the endowment into a bank. As a Nash competitor, the bank accepts deposits, offering state-contingent returns to those relocated, hereafter called movers, and to those staying on their home island both periods, hereafter called non-movers. Each bank's problem is to allocate deposits between two assets. One is the stock of fiat money, while the other is storage technology that yields x units of the date-$t + 1$ consumption good for every consumption good stored at date t. Now, we can how that the nature of the friction is clear. Goods put into storage by the bank on one island cannot be used by those relocated when

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[6] We use a modified version of the Champ, Smith and Williamson model. See Haslag and Martin (2007) for description of the random-relocation model.
[7] Note that this unrecognizability assumption is a form of imperfect record-keeping.

young on the other island. Claims that such goods exist cannot be verified and the storage goods cannot be shipped to the other island.

There exists a central bank responsible for the quantity of fiat money circulating in the economy. In each island, the central bank follows a simple rule: each young person is taxed (transferred) a fixed number of goods that are paid for by contracting (expanding) the money supply.[8] For $t \geq 0$, the government budget constraint is represented by $M_t - M_{t-1} = p_t \tau_t$, where τ_t is value of the lump-sum transfer (tax) that applies to each young person. Overall, the central bank is capable of distinguishing between young and old agents. For our purposes, we assume the central bank implements the change in the money supply prior to young depositing goods in the bank. Moreover, the central bank can costlessly compute the perfect-foresight rational expectations equilibrium.

In this setup, the bank serves as an agent to collect deposits, apply the law of large numbers, and offer state-contingent returns. The typical assumption is that the bank operates as a Nash competitor; in other words, the bank is maximizing expected utility of the depositor. To make things simpler, we treat the consumer as if each one is directing the bank how to allocate deposits on the consumer's behalf. The bank's problem is designed to achieve the same goal; that is, to maximize the expected utility of the representative consumer.

More formally, suppose there is a continuum of measure one of banks. In practice, a young consumer takes after-transfer resources and divides between consumption and deposits, dividing deposits across all the banks. With the deposits, each bank applies the law of large numbers to allocate the deposits between real balances and storage. Those realizing the liquidity shock will then withdraw before moving, taking the amount of real balances, and purchasing consumption goods when old. For non-movers, they withdraw the proceeds from the storage technology when old and consume.

The young consumer's maximization problem can be written as:

$$U\left(c_t^1\right) + \alpha V\left(c_{t+1}^{2*}\right) + (1-\alpha)V\left(c_{t+1}^2\right) \tag{1}$$

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[8] Here, we assume the central bank is passively responding to a fixed level of government spending or taxation. The alternative is an active central bank that sets its law of motion, picking the money growth rate each period to execute either the transfer or the tax. For example, $M_t = (1+\theta_t)M_{t-1}$, corresponds to an active central bank setting the money growth rate, denoted by θ with a passive fiscal authority solving for the transfers or taxes that are affordable for a given money growth rate.

where the superscript "*" denotes old-age consumption by movers and the absence of a superscript denotes the old-age consumption by non-movers.

Note that banks accept deposits from consumers, facing a balance-sheet constraint and two payoff constraints: one that applies to movers and the other that applies to non-movers.[9] Formally,

$$d_t = M_t^d \big/ p_t + s_t \tag{2}$$

$$\alpha r_{t+1}^* d_t = \frac{M_t}{p_{t+1}} \tag{3}$$

$$(1-\alpha) r_{t+1} d_t = x s_t. \tag{4}$$

where d_t is the quantity of goods deposited in the bank, $M_t^d \big/ p_t$ is the quantity of deposits held in the form of real balances, s_t is the quantity of deposits held in the storage technology, r_{t+1}^* is the rate that the bank offers to movers and r_{t+1} is the return the bank offers to non-movers.

Throughout our analysis, we assume that money supply increases at a constant rate. Let $M_t^s = (1+\theta) M_{t-1}^s$, where M_t^s denotes the nominal quantity of money supplied at date t, given initial money stock, M_{-1}. From the consumer's budget constraint, we know deposits are equal to the difference between income and consumption when young; therefore, we can write $y + \theta M_{t-1}^s \big/ p_t = c_{1,t} + d_t$. Let $m_t^s = M_t^s \big/ p_t$ and $m_t^d = M_t^d \big/ p_t$. Then, after substituting for consumption by young, old movers and old non-movers, we can write the consumer's problem as

$$\max_{d_t, M_t^d} U\left(y + \theta M_t^s \big/ p_t - d_t \right) + \alpha V\left[M_t^d \big/ \alpha p_{t+1}^e \right] + (1-\alpha) V\left[x s_t \big/ (1-\alpha) \right]$$

$$\text{s.t. } d_t - M_t^d \big/ p_t = s_t.$$

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[9] In this model economy, the bank acquires money balances by selling units of the consumption good to old movers. As such, the bank serves as both a depository institution, which is why we use the terms "bank", and "dry-goods store."

The first-order conditions for this problem are

$$U'\left(y + \theta M_t^s \Big/ p_t - d_t\right) = xV'\left(xs_t \Big/ (1-\alpha)\right) \qquad (5)$$

$$\left(p_t \Big/ p_{t+1}^e\right) V'\left(M_t^d \Big/ \alpha p_{t+1}^e\right) = xV'\left(xs_t \Big/ (1-\alpha)\right). \qquad (6)$$

Equation (5) depicts the tradeoff between consumption today and deposits. The lhs term is the marginal foregone utility of giving up consumption when young and the right-hand-side term is the marginal gain in utility from the return to deposits put into storage. Equation (6) is the tradeoff between deposits placed into money and those placed in storage. By equation (6), we obtain the following lemma.

Lemma 1: Perfect consumption smoothing across old consumers is realized if and only if return on money is equal to the return on storage.

Proof: In equation (6), suppose $p_t \Big/ p_{t+1}^e = x$. Therefore, the FONC can only be satisfied if

$V'\left(M_t^d \Big/ \alpha p_{t+1}^e\right) = V'\left(xs_t \Big/ (1-\alpha)\right)$. This last expression if the marginal utility of old consumers that

move and the marginal utility of old consumers that do not move. In other words, $c_{t+1}^{2*} = c_{t+1}^2$. Note that

the difference, $c_{t+1}^2 - c_{t+1}^{2*}$ is increasing, for example, as the gap between the return on money and the

return on storage increases.■

Before we proceed with the equilibrium analysis, we offer some change of variables that will make the derivations more straightforward. In particular, we use $M_t^s = (1+\theta)^t M_{-1}$ and $p_t = (1+\theta)^t q_t$ where q_t is interpreted as the money-growth-rate adjusted price of the consumption good. With the definitions, we rewrite equation (5) as

$$U'\left(y + \frac{\theta M_{-1}}{q_t} - d_t\right) = xV'\left[\frac{x\left(d_t - M_{-1} \Big/ q_t\right)}{(1-\alpha)}\right] \qquad (5')$$

The representation of the other first-order condition will be examined for two special cases. Often, it will be more convenient to use the stationary value of the price level, q. Suppose for example, that the expected future price level is based on the previous period; that is, $p_{t+1}^e = p_{t-1}$, then equation (6) can be expressed as

$$\left[\frac{q_t(1+\theta)}{q_{t-1}}\right]V'\left[\frac{(1+\theta)M_{-1}}{\alpha q_{t-1}}\right] = xV'\left[\frac{x\left(d_t - \frac{M_{-1}}{q_t}\right)}{(1-\alpha)}\right]. \tag{7}$$

Alternatively, with $p_{t+1}^e = p_{t+1}$, we obtain

$$\left[\frac{q_t}{(1+\theta)q_{t+1}}\right]V'\left[\frac{M_{-1}}{(1+\theta)\alpha q_{t+1}}\right] = xV'\left[\frac{x\left(d_t - \frac{M_{-1}}{q_t}\right)}{(1-\alpha)}\right]. \tag{8}$$

Note that either the combination of equations (5') and (7) or the combination of equations (5') and (8) will serve as the pair of necessary and sufficient conditions for the bank's optimizing problem. It is how the bank processes the communication from the central bank that will determine which combination applies. Once we have established what occurs in the two special cases, we consider the intensive margin in which some combination of the two processes will apply.

3. Equilibrium

Consider a special case in which the utility functions are power functions. Formally, let $U(c) = V(c) = (c^{1-\rho} - 1)/1 - \rho.$ For this specific functional form, equation (5') is written as

$$\left(y + \frac{\theta M_{t-1}^s}{p_t} - d_t\right)^{-\rho} = x\left[\frac{xs_t}{(1-\alpha)}\right]^{-\rho}. \tag{9}$$

We derive the closed-form solution for the level of deposits by solving equation (9) for d_t, yielding

$$d_t = \left.\left\{y + \frac{\theta M_{t-1}^s}{p_t} + \left(x^{((\rho-1)/\rho)}(1-\alpha)^{-1}\right)\left(\frac{M_t^d}{p_t}\right)\right\}\right/\left[1 + x^{((\rho-1)/\rho)}(1-\alpha)^{-1}\right]. \tag{10}$$

In order to obtain an expression for optimal storage, we subtract $M_t^d \big/ p_t$ from both sides of equation (10).

In terms of q, we obtain

$$d - M_{-1} \big/ q_t = \frac{\left\{ y + \dfrac{\theta M_{-1} \big/ (1+\theta) q_t - \left(M_{-1} \big/ q_t \right)}{} \right\}}{\left[1 + x^{((\rho-1)/\rho)} (1-\alpha)^{-1} \right]}. \tag{11}$$

A perfect-foresight competitive equilibrium is: (i) agents choosing deposits and consumption when young and when old to maximize expected lifetime utility, taking prices and the central bank's policies as given, (ii) banks allocate the deposits between real balances and storage and offer state-contingent returns to movers and nonmovers to maximize expected lifetime utility, taking the central bank's actions as given, and (iii) the market for the consumption good, deposits, the storage good, and money clear combined with the government budget constraint being satisfied. With perfect foresight, the expected value of the date-$t+1$ price is equal to the actual value. In other words, $p_{t+1}^e = p_{t+1}$. In the following derivations, we continue to use the general notation, which will be helpful in following sections of the paper.

The money market clearing condition is given by $M_t^s = p_t \gamma_t d_t$, where $\gamma_t \equiv M_t^d \big/ p_t \div d_t$. In q units, equilibrium real money balances are represented by $M_{-1} \big/ q_t$. Here, it is will be more convenient to express real money balances by adjusting for the money growth rate. The math yields an expression in which real balances at each date t can be characterized by the initial stock of nominal balances divided by the price level adjusted by inflation. By substituting the equilibrium real balances into equation (11) and using the law of motion for nominal money supply, we obtain

$$d_t - M_{-1} \big/ q_t = \frac{\left\{ y - \left(1 \big/ 1+\theta \right) \left(M_{-1} \big/ q_t \right) \right\}}{\left[1 + x^{(\rho-1)/\rho} (1-\alpha)^{-1} \right]}. \tag{12}$$

Thus, we have the equilibrium expression for optimal storage. Note that equation (12) expresses the optimal storage level in equilibrium as being linear in real money balances.

3.1 Equilibrium law of motion and the two forecasts

Next, we derive the equilibrium law of motion. We use equation (6). The law of motion clearly depends on how the bank forms the expected future price level. This seems like the appropriate place to present the two different technologies that are used to form the expected future price level. One is the backward-looking technology that sets the future expected price level equal to the previous observed price level. The second technology set the future expected price level equal to the actual future price level.[10]

We present an equilibrium law for each different technology. In each case technology, the equilibrium is stated in terms of the theta-adjusted price level. With $p_{t+1}^e = p_{t-1}$, the equilibrium law of motion can be written as

$$\left[\frac{(1+\theta)q_t}{q_{t-1}}\right]\left[\frac{(1+\theta)M_{-1}}{\alpha q_{t-1}}\right]^{-\rho} = k\left[y - M_{-1}\Big/(1+\theta)q_t\right]^{-\rho} \tag{13}$$

where $k \equiv x^{1-\rho}\left[(1-\alpha) + x^{(\rho-1)/\rho}\right]^{\rho}$.

Alternatively, with $p_{t+1}^e = p_{t+1}$, we derive the following equilibrium law of motion,

$$\left[\frac{q_t}{(1+\theta)q_{t+1}}\right]\left[\frac{M_{-1}}{\alpha(1+\theta)q_{t+1}}\right]^{-\rho} = k\left[y - M_{-1}\Big/(1+\theta)q_t\right]^{-\rho}. \tag{14}$$

Define $q_{t+1} = q_t = q^{*P}$ for the steady state in the perfect-foresight economy and $q_{t+1} = q_t = q^{*B}$ for the steady state in the backward-looking economy. Substitute the steady state value into equations (13) and (14). The resulting expressions are

$$(1+\theta)\left[\frac{M_{-1}}{\alpha(1+\theta)q^{*B}}\right]^{-\rho} = k\left[y - \frac{M_{-1}}{(1+\theta)q^{*B}}\right]^{-\rho}. \tag{15}$$

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[10] Here is the critical part of the story in the sense that the bank knows its allocation and hence the demand for money, but cannot compute the aggregate demand for money without expending resources. This assumption is central to the disconnect between perfect foresight and other means of predicting the future price level.

$$\left(\frac{1}{1+\theta}\right)\left[\frac{M_{-1}}{\alpha(1+\theta)q^{*P}}\right]^{-\rho}=k\left[y-\frac{M_{-1}}{(1+\theta)q^{*P}}\right]^{-\rho}. \tag{16}$$

By comparing equations (15) and (16), it is easy to see that the steady state obtained in the perfect-foresight case is different than the steady state in the backward-looking case. In other words, the forecasting technology affects the equilibrium outcome. To illustrate this, let

$L^{B}\left(M/q\right)=(1+\theta)\left[\dfrac{M_{-1}}{\alpha(1+\theta)q^{*B}}\right]^{-\rho}$ and $L^{P}\left(M/q\right)=\left(\dfrac{1}{1+\theta}\right)\left[\dfrac{M_{-1}}{\alpha(1+\theta)q^{*P}}\right]^{-\rho}$ correspond to the left-

hand-side of equations (15) and (16), respectively. In addition, let $R\left(M/q\right)=k\left[y-\dfrac{M_{-1}}{(1+\theta)q^{*,i}}\right]^{-\rho}$,

where $i=B,P$ to correspond to the backward-looking and perfect-foresight economies. Both the $L^{B}(.)$ and $L^{P}(.)$ functions are strictly decreasing in the level of theta-adjusted, real money balances with the vertical axis serving as an asymptote. Meanwhile, the $R(.)$ is strictly increasing in theta-adjusted real money balances with an asymptote at $(1+\theta)y$. Though not a proof, Figure 1 provides a plot of the resulting steady state values if they exist. Figure 1 depicts the case in which the money growth rate is positive. As Figure 1 shows, $\left(M/q\right)^{B}>\left(M/q\right)^{P}$.

The economic intuition for this result is straightforward. With positive money growth, the two forecasting technologies produce different forecasts of the rate of change in prices. In the backward-looking case, the impact of positive money growth is completely downplayed, resulting in bank's forming an expected real return on money balances that is greater than the expected real return that is computed in the perfect-foresight case. Therefore, it is not surprising that the bank would hold larger quantities of money balances in the backward-looking case. Indeed, this is what Figure 1 depicts, the $L^{B}(.)$ function intersects the $R(.)$ function at a point to the right of where the $L^{P}(.)$ function intersects the $R(.)$ function. It is not surprising that different forecasting technologies would yield different expected returns to money. Nor it is surprising that banks will hold larger quantities of money balances when using the technology that yields a higher expected return to money.

3.2 Stability conditions

We check the stability conditions for the model economy under each of the two forecasting technologies. We start with the case in which the perfect-foresight technology is used. The result is presented in the following lemma.

Lemma 2: With perfect foresight, the local dynamics exhibit monotone jump stability.

Proof: See Appendix.

For the case in which $p_{t+1}^e = p_{t-1}$, the local dynamics are characterized by differentiating equation (13). We derive the following expression:

$$\frac{dq_t}{dq_{t-1}} = \frac{1-\rho}{\left(1+\theta/q^{*B}\right)(1+\theta)^{-\rho} + \left(\alpha\rho/1+\theta\right)\left[k(1+\theta)\right]^{-1/\rho}(1+\theta)^{\rho}} \qquad (17)$$

Equation (17) is strictly positive for $0 < \rho < 1$. Note that with ρ inside the positive unit interval, the substitution effect dominates in the consumer's maximization problem. Consider a special case in which the gross real return to storage is equal to the gross real return to money at the steady state with $x = 1$ and $\theta = 0$. For these parameter settings, the local dynamics exhibit monotone convergence.

3.3 Comparative Statics

Consider the effect that changes in some of the key exogenous parameters would have on the steady state level of real money balances. In our analysis, we focus on cases in which the substitution effect dominates. Note that the qualitative results are invariant to the predictor used. For the comparative static analysis, we use the notation $i = B, P$. We summarize these effects in the following proposition.

Proposition 1: Steady state real money balances-- $\dfrac{M}{q}$ -- is (i) positively related to the fraction of movers; (ii) negatively related to the gross real return on capital (iii) negatively related to the growth rate of the money supply.

Proof: (i) A change in the fraction of movers is captured by the expression $\dfrac{d\left(M\big/q\right)^i}{d\alpha} = \dfrac{R_\alpha - L_\alpha}{L_{M/q} - R_{M/q}}$.

The second-order condition ensures that $L_{M/q} - R_{M/q} < 0$. It is straightforward to show that the

denominator is negative. With $R_\alpha < 0$ and $L_\alpha > 0$, it follows that $\dfrac{d\left(M\big/q\right)^i}{d\alpha} > 0$. The intuition is also

straightforward. With an increase in the fraction of young people moving, the steady state level of real balances will increase. Real money balances are the means by which the bank supplies liquidity to its depositors and the fraction of movers serves as a kind of anticipated liquidity shock.

(ii) With $\dfrac{d\left(M\big/q\right)^i}{dx} = \dfrac{R_x}{L_{M/q} - R_{M/q}}$, the sign of the derivative of real balances with respect to the gross

return to storage is opposite of the sign of the numerator. With $0 < \rho < 1$, $R_x = k'(x)\left(\Delta_0\right)^{-\rho}$, where

$\Delta_0 = y - \dfrac{M}{(1+\theta)q}$. With $k'(x) > 0$, it follows that $R_x > 0$. Hence, steady state real balances are

negatively correlated with the return to storage. (iii) With $\dfrac{d\left(M\big/q\right)^i}{d\theta} = \dfrac{R_\theta - L_\theta}{L_{M/q} - R_{M/q}}$, we are interested in

the sign of the numerator. The numerator is positive, implying that $\dfrac{d\left(M\big/q\right)^i}{d\theta} < 0.$ With respect to an

increase in the money growth rate, the real return on money is inversely related to the money growth rate in steady state. Consequently, an increase in the money growth rate reduces the steady state demand for real balances as the banks minimize their money holdings, for example, in response to a lower return on money.∎

4. More general forecasts

In this section, we introduce a more general predictor that relies on the observed price level and takes into account the money supply rule. It is not perfect foresight, but is a combination forecast that

relies on the most recent observation and knowledge of the monetary policy being implemented. The backward-looking forecast is useful as a means of illustrating two different forecasting technologies and the resulting equilibrium outcomes. But it is based on a processing only one piece of free information. Here, we allow the bank to forecast next period's price using both the last period's observation and the money growth rate. We refer to the second kind as a combination predictor since it relies on past price level observations combined with the acceptance that the central bank is committed to a path with constant money growth.

Consider an economy in which the forecast technology available to the bank is written as

$$p_{t+1}^e = (1+\theta)^2 p_{t-1} + g\left[(1+\theta)^2 \left(p_{t-1} - (1+\theta) p_{t-2}\right)\right]. \tag{18}$$

Equation (18) specifies that the forecast value for the future price level is a direct theta-adjustment that takes the observed price level in the previous period and adjusts it for the compounded gross rate of money supply growth. The second term is similar to an error correction. Given the most recently observed error in theta-adjusted price level represented by $p_{t-1} - (1+\theta) p_{t-2}$, the technology corrects for that error by the parameter value g. For example, we refer to the case in which $0 < g < 1$ as a weak trend chaser. Here, the weakness refers to the response to the most recent observed error in trend prices. Alternatively, a strong trend chaser applies to cases in which $g > 1$.

We construct the stationary representation of the price level using $q_t \equiv \dfrac{p_t}{(1+\theta)^t}$ and express the gross real return on money balances. By definition of q_t and equation (18), substitution and simplifying yields the following expression:

$$\frac{p_t}{p_{t+1}^e} = \frac{q_t}{\left\{(1+\theta)\left[q_{t-1} + g\left(q_{t-1} - q_{t-2}\right)\right]\right\}}. \tag{19}$$

We proceed to derive the steady value of real money balances for this forecast technology. Let $q^* = q_t = q_{t-1} \ \forall t$. By equation (19), it is clear that in the stationary equilibrium, $p_{t+1}^e = (1+\theta) p_t$. To obtain the equilibrium law of motion, we use the expression

$$\left(\frac{p_t}{p_{t+1}^e}\right)\left(\frac{M_t^s}{\alpha p_{t+1}^e}\right) = (1-\alpha)^\rho x^{1-\rho}\left(d - \frac{M_{-1}}{q_t}\right)^{-\rho}$$

which can simplified to

$$\frac{q_t\left(M_{-1}\right)^{-\rho}\alpha^{\rho}}{\left\{\left(1+\theta\right)\left[q_{t-1}+g\left(q_{t-1}-q_{t-2}\right)\right]\right\}^{1-\rho}} = \left(1-\alpha\right)^{\rho}x^{1-\rho}\left(d-\frac{M_{-1}}{q_t}\right)^{-\rho}. \qquad (20)$$

After differentiating equation (20) and evaluating at steady state, we have a second-order nonlinear difference equation in the price, q. We define steady state for this model economy by $q_t = q_{t+1} = q^{*\theta}$. The local dynamics are represented by

$$\left\{\left(M_{-1}\right)^{\rho}\alpha^{\rho}\left[\left(1+\theta\right)q^{*\theta}\right]^{\rho-1}+\left(1-\alpha\right)^{\rho}x^{1-\rho}\rho\left(d-\frac{M_{-1}}{q^{*\theta}}\right)^{-\rho-1}\frac{M_{-1}}{\left(q^{*\theta}\right)^2}\right\}dq_t$$

$$+\left\{q^{*\theta}\left(M_{-1}\right)^{-\rho}\alpha^{\rho}\left(\rho-1\right)\left[\left(1+\theta\right)q^{*\theta}\right]^{\rho-2}\left(1+\theta+g\right)\right\}dq_{t-1} \qquad (21)$$

$$-\left\{q^{*\theta}\left(M_{-1}\right)^{-\rho}\alpha^{\rho}\left(\rho-1\right)\left[\left(1+\theta\right)q^{*\theta}\right]^{\rho-2}g\right\}dq_{t-2} = 0.$$

After collecting terms and simplifying, equation (21) takes the form

$$a_o dq_t + a_1 dq_{t-1} + a_2 dq_{t-2} = 0 \qquad (22)$$

where

$$a_0 \equiv \left\{\left(M_{-1}\right)^{\rho}\alpha^{\rho}\left[\left(1+\theta\right)q^{*\theta}\right]^{\rho-1}+\left(1-\alpha\right)^{\rho}x^{1-\rho}\rho\left(d-\frac{M_{-1}}{q^{*\theta}}\right)^{-\rho-1}\frac{M_{-1}}{\left(q^{*\theta}\right)^2}\right\}$$

$$a_1 \equiv \left\{q^{*\theta}\left(M_{-1}\right)^{-\rho}\alpha^{\rho}\left(\rho-1\right)\left[\left(1+\theta\right)q^{*\theta}\right]^{\rho-2}\left(1+\theta+g\right)\right\}$$

$$a_2 \equiv -\left\{q^{*\theta}\left(M_{-1}\right)^{-\rho}\alpha^{\rho}\left(\rho-1\right)\left[\left(1+\theta\right)q^{*\theta}\right]^{\rho-2}g\right\}.$$

The Jury-Marden test allows one to derive conditions for local stability of discrete-time models. We form the characteristic equation of the form $D\left(z\right) = a_0 z^2 + a_1 z + a_2 = 0$. Then evaluate the characteristic equation at $z=1$ and $z=-1$. For the roots to be inside the unit circle, the conditions for a second-order difference equation are:

$$D(1) = a_0 + a_1 + a_2 > 0$$
$$D(-1) = a_0 - a_1 + a_2 > 0$$
$$a_0 > |a_2|.$$

Example: We use the Jury-Marden conditions to examine the impact that the coefficient on the error-correction term has on the local dynamics. Suppose the money stock is constant over time. Further, we assume that there is no rate of return dominance; that is, $x = 1$ and $\alpha = 0.5$. It follows that $q^{*\theta} = \dfrac{2M_{-1}}{d}$.

We consider all cases with $0 < \rho < 1$. We will focus on the last condition, namely, $a_0 > |a_2|$. In this case, substitution yields $a_0 = (0.5)^\rho \left(q^*\right)^{\rho-1} \left(M_{-1}\right)^{-\rho} (1+\rho)$ and $a_2 = (0.1)^\rho \left(q^*\right)^{\rho-1} \left(M_{-1}\right)^{-\rho} g(1+\rho)$.

Thus, the Jury-Marden condition, $a_0 > |a_2|$, fails if and only if $g > \dfrac{1+\rho}{1-\rho}$. ∎

Thus, this simple example demonstrates that the coefficient on the error-correction term matters. Indeed, the numerator is unambiguously greater than one in this example. But it shows that the local dynamics exhibit explosive behavior when g is large enough.

We next consider numerical methods that look at how the money growth rate affects stability. Table 1 reports the results of the fixed point problem in the steady state expression represented by equation (20). Once we have the state level $q^{*\theta}$, we compute the Jury-Marden conditions for stability.

For out numerical analysis, we specify the following parameter values. Let $M_{-1} = 1,000$, $\rho = 0.6$, the given level of deposits is $d = 10$, $x = 1.2$, and $\alpha = 0.1$. We consider the error-correction parameter set at $g = 0.7$. We are most interested in assessing the impact that changes in the money growth rate have on the steady level and on the stability of the economy. With $g = 0.7$, Table 1 shows two things. First, the steady state level of $q^{*\theta}$ is strictly increasing in the money growth rate. This implies that real balances are decreasing monotonically in response to decreases in the gross real return to money. Second, the Jury-Marden conditions are satisfied and the economy exhibits local stability. However, with $g = 3$, the bottom panel of Table 1 shows that the Jury-Marden conditions fail, implying that the local dynamics are not stable for the model economy with past price observations adjusted for the money growth rate and a strong error-correction mechanism. The main point is that derive numerical conditions in which the economy is not locally stable in the absence of Fed watching.

Thus, the numerical results offer some insight into the possible role of Fed watching. We apply a definition of Fed watching that focuses on the resources expended in order to improve the forecast accuracy of aggregate economic outcomes. We will offer an explicit model of Fed watching in the next section. But, for now, suppose that a bank—the entity that actually Fed watches—has two different forecast technologies. One technology is perfect foresight while the other technology is a money-growth-rate adjustment applied to the most recent (past) price level observation. Suppose that the perfect-foresight technology can be adopted, but it takes more resources than the combination predictor.

Proposition 2: Fed watching guarantees in the model economy exhibiting stable local dynamics in the neighborhood of the steady state.

In our current modelling approach, we have what amounts to a comparison between two pure strategies. One strategy is the Fed watching economy in which banks expend resources to implement the perfect-foresight forecasting technology. The other pure strategy is the economy in which banks implement the forecasting technology that combines observed history and the (credible) communication/announcement of the central banks money growth rate. [11] In the perfect-foresight economy, the steady state is locally monotone jump stable. In contrast, the numerical results indicate that in the economy in which the combination predictor is implemented, the steady state is locally unstable. By relying on the past observation, the numerical analysis indicates that relying on the past observations induces a repelling response to the steady state. Our intuition is based on the assumption that fiat money is intrinsically useless; consequently, predictors based on past price level observations induce errors that repel the local dynamics away from the steady state. For our purposes, the comparison across the economies with forward-looking and backward-looking predictors shows the important role that the forecasting technology plays. Indeed, the results indicate that Fed watching plays a role in stabilizing the economy in the sense that local dynamics exhibit stability when the perfect-foresight technology is implemented and the local dynamics are unstable when the combination predictor is implemented.

The stability analysis establishes a link between the effort devoted to processing information and convergence to a steady state with positive real money balances.[12] Our findings address the inherent

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[11] In the model economy in which every bank implements the perfect-foresight equilibrium, one might wonder how we have incorporated the cost of Fed watching. A simple way to introduce such costs within our modelling approach is to introduce new notation; let y^P denote the endowment for consumers net of Fed watching costs. Because y^P is taken as given in the consumer's problem and the bank's problem, the analysis of the local dynamics will not be affected. In other words, the difference in endowments—that is, $y - y^P > 0$--does not qualitatively affect the stability analysis.

[12] See Beaudry, Galizia and Portier (2017) for a discussion on the value of convergence in dynamic aggregate models. Their contribution lies in asking whether linear time series methods are sufficient for assessing

stability of monetary economy as conditional. The condition we identify is that the predictor used in the economy plays a critical role in determining whether shocks to monetary economy converge to the new (monetary) steady state or potentially diverges to a possibly non-monetary steady state. Insofar as the instability corresponds to greater volatility in the aggregate economy, our results add to the extensive literature on information impacts on equilibrium outcomes.[13] Our findings indicate that by processing more information—that is, using the available predictor technology—the aggregate economy will converge while less efficient information processing can result in unstable, more volatile aggregate swings.

In the next section, the paper formalizes the notion of Fed watching. In particular, we develop a modelling approach in which there is a tradeoff between forecast accuracy and consumption. An improvement in forecast accuracy uses up resources that could otherwise have been used to increase lifetime consumption. Using a regret-based measure of the costs of reduced forecast accuracy, we compute a willingness-to-pay measure that represents the quantity of resources that a bank would be willing to expend to be indifferent between adopting the high-cost, perfect-foresight technology and adopting the low-cost forecast technology. For the ease of implementation, we begin with the backward-looking forecast technology to illustrate how the decision problem would be formulated.

5. A model with Fed watching

In this section, we modify the basic model to consider two technologies. One produces a forecast of the next period's price level that is directly related to the observed history of the price level. The other technology produces a forecast of the next period's price level that is prefect-foresight predictor.

We approach this problem from the perspective of a bank that seeks to maximize the expected utility of a depositor, taking deposits as given. As before the bank will allocate resources between money balances and capital, offering state-contingent returns that depend on when the depositor withdraws from the bank. Because a bank makes this allocation decision based on the forecasted price level next period, the technology used to forecast that price level is important. In our setup, the two technologies differ in terms of the forecast accuracy.

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the stability of the aggregate economic variables. In this theoretical structure, we apply linear methods to assessing the local dynamics.

[13] See, for example, the work by Danthine and Moresi (1993) and more recently, papers by Morris and Shin (1998, 2001) and Angeletos and Werning (2006).

There is an important modeling choice in terms of how to describe the loss function for the bank. Note that this is the point of contact at which expectations are formed. As such, the modeling choice bears directly on what Morris and Shin (2018) dub the reflection problem. Specifically, if the bank measures forecast error accuracy contemporaneously, which may seem like the most natural way to construct the loss function, then it follows that the aggregate demand for money depends on what the central bank communicates to banks in terms of the selected money growth rate. For a given actual money growth rate, the price level is determined. Insofar as the central bank responds to the price level movements in selecting the money growth rate, then a reflection problem is present. Put in a more mechanical way, the Morris-Shin reflection problem is a fixed-point type of setting such that if the CB tries to exploit it for policy then it becomes weaker in their informativeness since but they still can find an optimal choice for the central bank's policy parameter.

Our goal is to concentrate on the role that the processing expectations is operationalized. As such, the simultaneity that lies at the heart of the reflection problem is sidestepped by forming a regrets-based approach. In practice, we will form the loss function for the bank by looking backward at the forecast error measurement.

5.1 Willingness to pay

Our first goal is to represent a regrets-based approach to the bank's willingness to pay for the perfect-foresight technology.[14] Rather than relying on an idiosyncratic shock to identify what predictor is most desirable, we use the regrets-based approach to construct an observable proxy for the willingness-to-pay. Willingness-to-ay is derived from an indifference condition; namely, that a deposit contract offered with a less accurate and less costly predictor offers state-contingent returns to depositors that yield equal expected utility to a deposit contract in which a more accurate and more expensive predictor is combined with state-contingent returns. To avoid any logical circularity, we create an observable proxy for the

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[14] There is potential logical circularity present if willingness-to-pay were not based on regret. Consider, for example, the bank's decision on the quantity of Fed watching. Suppose we start with the notion that consumers want to compute the optimal amount of resources they spend on learning what the central bank will do. That notion presumes the consumer knows something about the relationship between what the central banks actually ends up doing—that is, the central bank's best-response function—and the resources needed to figure out what the central bank will do. By knowing the central bank's best response, we are effectively treating the problem as if learning what the money growth rate is going to be. The best response, therefore, embeds a relationship between what money growth rate the Fed chooses and the quantity of Fed watching the bank spent on figuring out what the Fed was going to do. Herein lies the circularity: it is presumed that the bank knows the money growth rate the Fed is going to choose. With this presumption, it is natural ask: Why would the bank pay any positive amount for information regarding what the Fed is going to do when it already knows what the Fed is going to do for free?

unobservable willingness-to-pay by specifying what the bank would have been willing to pay using past observable forecast errors.

To illustrate how regret would work, consider the date-t decision in which the expected next-period price level is represented as $p_{t+1}^e = p_{t-1}$. How much would a consumer be willing to pay out of their deposits to obtain the perfect-foresight forecast of the next-period price level? The bank will allocate deposits to maximize the expected welfare of the old consumer. With the backward-looking forecast, the optimal demand for money balances, taking deposits as given, is represented by the following problem:

$$M_t^{d*}(p_{t-1}) = \arg\max\left\{\alpha\left[M_t^d \Big/ \alpha p_{t-1}\right]^{1-\rho} \div(1-\rho)\right\} + (1-\alpha)\left[xs_t \Big/ (1-\alpha)\right]^{1-\rho} \div(1-\rho) \qquad (23)$$

Because the lagged value of the price level is an inaccurate forecast of the next-period price level, it follows that old-age utility with the perfect-foresight value of the next-period price level cannot be less than utility with the solution to equation (23). If we define $M_t^{d*}(p_{t+1})$ in an analogous way to equation (23) with $p_{t+1}^e = p_{t+1}$, then formally, we have[15]

$$\left\{\alpha\left[M_t^{d*}(p_{t-1})\Big/ \alpha p_{t-1}\right]^{1-\rho} \div(1-\rho)\right\} + (1-\alpha)\left[x\left(d_t - M_t^{d*}(p_{t-1})\right)\Big/ (1-\alpha)\right]^{1-\rho} \div(1-\rho)$$
$$\leq \left\{\alpha\left[M_t^{d*}(p_{t+1})\Big/ \alpha p_{t+1}\right]^{\rho} \div(1-\rho)\right\} + (1-\alpha)\left[x\left(d_t - M_t^{d*}(p_{t+1})\right)\Big/ (1-\alpha)\right]^{\rho} \div(1-\rho) \qquad (24)$$

Equation (24) is our starting point for characterizing a regrets-based (read observable) willingness to pay. The intuition is straightforward. Let willingness to pay, hereafter denoted *WTP*, be the resources expended such that the left-hand-side of equation (24) is equal to the right-hand-side. Therefore, for a particular bank,

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[15] We define the optimal money demand for the perfect-foresight price level as follows:

$$M_t^{d*}(p_{t+1}) = \arg\max\left\{\alpha\left[M_t^d \Big/ \alpha p_{t+1}\right]^{1-\rho} \div(1-\rho)\right\} + (1-\alpha)\left[xs_t \Big/ (1-\alpha)\right]^{1-\rho} \div(1-\rho)$$

$$\left\{\alpha\left[M_t^{d*}(p_{t-1})\Big/\alpha\,p_{t-1}\right]^{1-\rho}\div(1-\rho)\right\}+(1-\alpha)\left[x\big(d_t-M_t^{d*}(p_{t-1})\big)\Big/(1-\alpha)\right]^{1-\rho}\div(1-\rho)$$

$$=\left\{\alpha\left[M_t^{d*}(p_{t+1})\Big/\alpha\,p_{t+1}\right]^{1-\rho}\div(1-\rho)\right\}+(1-\alpha)\left[x\big(d_t-WTP_{t,i}-M_t^{d*}(p_{t+1})\big)\Big/(1-\alpha)\right]^{1-\rho}\div(1-\rho)$$

(25)

where $WTP_{t,i}$ represents the resources that bank i would have been willing to pay at date t to have a more accurate predictor of the next-period price level. Equation (25) ignores the circularity problem; if the bank knows what money demand would be under the perfect-foresight case, the underlying presumption is that the bank knows the perfect-foresight forecast. Why would the bank be willing to pay anything if it knew what the best-response money demand would be in the perfect-foresight setting? Such a hypothetical is taken off the table to avoid any circularity.[16]

In order to obtain something that is observable, we consider the following:

$$\left\{\alpha\frac{\left[M_t^{d*}(p_{t-3})\Big/\alpha\,p_{t-3}\right]^{1-\rho}}{1-\rho}\div(1-\rho)\right\}+(1-\alpha)\left[\frac{x\big(d_{t-2}-M_t^{d*}(p_{t-3})\big)\Big/(1-\alpha)}{1-\rho}\right]^{1-\rho}\div(1-\rho)$$

$$=\left\{\alpha\frac{\left[M_t^{d*}(p_{t-1})\Big/\alpha\,p_{t-1}\right]^{1-\rho}}{(1-\rho)}\div(1-\rho)\right\}+(1-\alpha)\frac{\left[x\big(d_{t-2}-WTP_{t-2,i}-M_t^{d*}(p_{t-1})\big)\Big/(1-\alpha)\right]^{1-\rho}}{1-\rho}\div(1-\rho)$$

(26)

Equation (26) proposes the following experiment. In date t, a banker observes the demand for money conditioned on two forecasts of the price level. On the left-hand-side, the bank uses the free, backward-looking forecasting technology and computes the expected utility for the depositor. On the right-hand-side

[16] The key point is that there is a strong version of intertemporal substitutability present in the overlapping generations model. Because the old die, the bank's contract depends only on maximizing the expected utility of depositors with one period of life left. In an infinite horizon setting, current and an infinite number of future price levels could potentially affect the predictor choice; the continuation value in the Bellman equation is built on the notion that future predictors enter. See Branch and McGough (2016) for an example of the strong intertemporal restrictions present in the infinite-horizon setting in terms of the predictor choice.

of equation (26), the banker can observe what expected utility would have been with the perfect-foresight forecasting technology. With only one unknown, the banker can compute the willingness-to-pay amount that equates expected utility for the two alternative forecasting technologies. The last time the banker would have had the information set comprised of the two alternative forecasting technologies is two period previously since we need to know what the price level was, and the most recent observation was one period ago.

Thus, equation (26) expresses a form of regret. Indeed, $WTP_{t-2,i}$ can be interpreted as what the bank would have been willing to pay to use the perfect-foresight predictor at date *t-2*. The regret comes from looking backward at forecast accuracy when using p_{t-3} as p_{t-1}^e instead of using the perfect-foresight predictor. Because the regret-based approach yields an observable measure of forecast accuracy without the conundrum of knowing (i) what the impact that Fed watching would have (ii) without assuming you know what the Fed is going to do. Without this reflection problem, we can compute a measure of willingness to pay.

We also present the following proposition that characterizes the relationship between risk aversion and willingness to pay.

Proposition 3: Assume, $\left| \dfrac{dM^{d*}(p_{t-1})}{d\rho} \right| > \dfrac{dM^{d*}(p_{t-3})}{d\rho}$ **, then an increase in risk aversion will result in a larger willingness to pay.**

Proof: Equation (26) is the starting point for this proof. Totally differentiate both sides, setting $d\rho, dWTP \neq 0$. After collecting terms, the condition in Lemma 3 is sufficient to guarantee that

$$\frac{dWTP}{d\rho} > 0. \ \blacksquare$$

The intuition for Proposition 3 is straightforward. The more risk averse a person is, the larger optimal real balances are. Provided the impact on real balances is quantitatively larger (in absolute value) for consumers with perfect foresight than for consumers with the backward-looking technology, then the bank's willingness to pay for the perfect-foresight technology increases with an increase in the consumer's risk aversion.

The objective is to form a problem that allows us to compute a bank's willingness to pay for a forecasting technology that is more accurate than a forecast technology that requires less resources to

implement. By continuity and curvature of the utility function, we exploit the intermediate value theorem to identify what resources the bank would be willing to pay so that the depositor is indifferent between the two alternative deposit contracts. Given that all banks have the same objective function, the willingness-to-pay calculation is unique.[17]

We have applied a Hicksian compensation approach to identify whether a bank (and consumer) would benefit from expending any resources to use the more accurate predictor and the answer is yes.

Proposition 4: The intensity of Fed watching is positively related to the money growth rate.

Proof: Equations (26) serves as the basis for the comparative statics presented in Proposition 4. As the money growth rate increases, the steady state rate of inflation rate increases by the quantity theory. With the backward-looking predictor, the return on money is greater than in the perfect-foresight setting. The greater the forecast error, measured by the predicted return and the actual return, the greater the bank's willingness to pay. Thus, by equation (26), the intensity of Fed watching increases, for example, when the money growth rate increases.∎

The heterogeneous-predictor equilibrium concept is characterized as follows:

1. Consumers maximize expected lifetime utility, choosing consumption when young and consumption when old, taking the current price level and the state contingent returns on deposits as given;

2. Banks take what the central bank does as given and determines the deposit contract, consisting of the state-contingent returns offered and the predictor. The bank maximizes expected old-age utility offering a deposit contract by expending resources such that the expected utility of every possible deposit contract is equal across the set of predictors. In addition the bank chooses the allocation of deposits between storage and money balances that

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[17] There is one additional consideration. The willingness-to-pay measure captures the intensity with which a bank would have wanted to implement the perfect-foresight technology. The measure of banks that would choose the perfect-foresight predictor is indeterminate in the [0,1] interval. One way to address this concern is to assume there exists a function, $\Upsilon\left(WTP_{t-2}\right)$ which is a mapping from willing-to-pay to the unit interval. More precisely, let $\Upsilon(0)=0$, $\Upsilon(\infty)=1$, and $\Upsilon'(WTP)>0$. The mapping in interpreted as the measure of banks that implement the perfect-foresight predictor for a given intensity of Fed watching. Because willingness-to-pay is unique and based on an indifference calculation, the function, $\Upsilon(.)$ determines the measure of banks using one predictor. In an economy with two predictors, it follows that $1-\Upsilon\left(WTP_{t-2}\right)$ is the measure of banks implementing the backward-looking predictor.

will makes the deposit contract feasible, taking the price level and the quantity of deposits as given;

3. Markets clear.

There are two key results in this section. First, with continuous utility functions, the intermediate value theorem assures that there exists a willingness to pay—the resources expended by the bank—that satisfies expected utility equality with and without Fed watching.

In equilibrium, the distribution of bank implementing the set of predictors is non-degenerate for any positive measure of banks using any predictor other than perfect-foresight

5.2 Discussion

In this paper, Fed watching is an equilibrium response to the given cost of processing central bank communications. As we have modelled the problem, the central bank communications are pretty straightforward: the central bank communicates a fixed money growth rate. In addition, the central bank can compute the perfect-foresight equilibrium values. We define Fed watching as the resources expended by banks to process the available information. Here, the processing costs are expended in order for private banks to compute the perfect-foresight equilibrium values. This is what Blinder, et al. refer to when they talk about non-rational expectations.

Perhaps, one would argue that our model is not about Fed watching, but about a form of rational inattention or an application of robust control.[18] The basis for this view is that central bank communications are noisy; Fed watching therefore is about the resources expended to uncover the central bank's actions. In contrast, the central bank's communications are not noisy, but processing them is. In doing so, future economic outcomes are predicted more accurately. As such, our modeling approach simplifies the nature of the communication and focuses processing part. By adding more bells and whistles to the model economy, it might look more Fed watching, but we contend that our approach is solving the part of the problem that has gone ignored to date.

Under our definition of Fed watching, the cost of processing the central bank communications is like a ceteris paribus experiment, focusing on the intensive margin. In other words, consider the quantity of information from the central bank communication is constant, the question is, how intensely do banks Fed watch? Our answer is that Fed watching intensity depends critically on how costly it is for the bank to

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[18] Indeed, the model economy presented in Hansen and Sargent (2007) has a similar structure to ours in the sense the consumer can observe the state perfectly while we assume the central bank communication is not noisy. The approach taken by Hansen and Sargent is that model uncertainty exists. In this paper, we take the approach that a rational agent cannot accurately forecast future equilibrium prices without expending resources. Forecast inaccuracy is free, reflecting some kind of processing problem. We do not take a stand on the exact nature of the processing problem, we simply assume it exists.

not process the communication. The tradeoff is between the cost of Fed watching compared against the cost of inaccurate forecasts. If you think of the central bank as setting the cost, this is another policy lever available. Indeed, our results indicate that the central bank can vary the intensity by raising or lowering the cost since Proposition 4 shows us that Fed watching intensity is positively related to other central bank activities, like setting the money growth rate.

Why is this intensity interesting? In Brock and Haslag (2018), the authors demonstrate how a backward-looking predictor will induce price stickiness into the model economy. In that paper, one can think of price stickiness as capturing the rate of convergence at which prices adjust from one steady state to another. Moreover, Brock and Haslag show that as the measure of consumers implementing the backward-looking predictor increases, for example, prices become more sticky. Insofar as Fed watching intensity is monotonically related to the banking industry's reliance on the perfect-foresight predictor, our results suggest that the central bank has some ability to influence the degree to which prices are sticky through its inflation forecast accuracy. If the inflation forecast, for example, is "close" to the actual inflation rate, banks are not willing to pay much for an improved forecast. With a monotonic relationship of intensity and the reliance on the perfect-foresight predictor, prices would actually be stickier in an economy with nearly accurate inflation forecasts. Alternatively, if inflation forecast are not accurate, for example, the willingness-to-pay increases and prices become less sticky as the banking industry relies on the perfect-foresight predictor more heavily.

In this section, we derive the primary results regarding the intensity of Fed watching. The first result is to demonstrate that there exists a positive willingness to pay for a more accurate predictor. The second result uses the first result to show that an interior solution for Fed watching exists. We discuss the interpretation of the results and the meaning for sticky price dynamics at the end of the section.

6. A Case with Externalities

In this section, we consider an alternative version of the model economy in which banks are processing a different set of central bank communications. Indeed, we examine how a regulatory changes can impact the bank's willingness to pay. The regulatory consideration reflects an externality that is present in the economy because there are now two illiquid assets.

Consider an economy in which there are two illiquid—that is, immovable and unverifiable—assets. Let s_1 and s_2 denote the two illiquid assets with the return on the two assets are represented by x_1 and x_2, respectively. To formalize the problem, let $\hat{x} = \phi x_1 + (1 - \phi) x_2$ with $0 \leq \gamma \leq 1$. In this analysis, we assume that $x_2 = x_1 + \eta$, $\eta > 0$. With this notation in place, the bank's problem can be written as

$$\alpha \left[\frac{M^{d*}\left(p_{t+1}\right)}{\alpha\,p_{t+1}} \right]^{1-\rho} \div \left(1-\rho\right) + \left(1-\alpha\right) \left[\frac{\hat{x}\left(d_t - M^{d*}\left(p_{t+1}\right)\right)}{1-\alpha} \right]^{1-\rho} \div \left(1-\rho\right). \tag{27}$$

Equation (27) specifies the bank's choosing the level of real money balances to maximize the expected welfare of depositors. Note that in equation (31), we assume that the bank costlessly knows the perfect-foresight value of the price level. We assume that return to the weighted sum of the two illiquid assets depends on the portfolio allocation applied to the second asset. As presented, the problem is straightforward. Because of the return-dominance, the competitive, atomistic bank will set $\phi = 0$, and $\hat{x} = x_2$.

To make things more interesting, we introduce an externality. We assume that as the aggregate of banks allocate a larger fraction of deposits in s_2, the return to this illiquid asset is adversely affected. Our idea is to use the negative externality as a way to capture the exposure to low realized returns on an asset.[19] More specifically, our aim is to incorporate some of the ideas put forward by Admati (2016) and by Aikman, Haldane, Hinterschweiger, and Kapadia (2018). Admati reiterates the role played by a regulatory environment; regulations address distortions and externalities. Admati (2014) focuses on bank's decision to use the bank's liabilities to acquire potentially risky assets. Because leveraging is a means to the high-return end, Admati puts forward the idea that such leveraging is a candidate for regulation. Aikman, et al. build on this concern, presenting a historical review in which regulatory reforms have waxed and waned over time. As regulations change over time, so does the riskiness of returns to banks. In both papers, the central message is that risk and leveraging have adverse consequences that are tied to the regulatory environment.

Here, we deviate from the Admati's model economy in two specific ways. First, she is interested in the equity-liability regulatory structure whereas we take the financing option for banks as given; indeed, banks are completely leveraged in our model economy with no equity. Instead we focus on the asset side of the bank's portfolio. Second, we are trying capture the idea, in a deterministic way, that there can be too much of a good thing in terms of banks holding certain kinds of assets.[20]

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[19] Though we do not build the machinery of risky and non-risky assets, one can think of s_2 as capturing the some of the key features associated with asset-backed securities in 2007 and 2008. As banks allocate a larger fraction to asset-backed securities or derivatives, total returns can be adversely affected.

[20] Sanches (2014) provides a very nice overview of the problems that emerged in the shadow banking industry as the values of asset-backed securities declined. For example, Merrill Lynch took two large write downs in the last quarter of 2007 and January 2008. In both cases, the write downs were reported as being tied to sharp

To complete the description of the model economy, we assume that banks allocate deposits between money balances needed to meet the liquidity needs of movers, the externality-free illiquid asset (s_1) and the externality-riddled illiquid asset (s_2). Here, the central bank is given a different policy tool; instead of changing the money growth rate, the central bank acts a benevolent planner, choosing the fraction that is socially efficient.

Here, we assume the externality is state-dependent. Formally, let

$f(\phi, h_t)\hat{x} = f(\phi, h_t)\left[\phi x_1 + (1-\phi)x_2\right]$, where $h_t \in \left[h_1, ..., h_n\right]$ denotes an aggregate state variable that affects the size of the externality. We assume that n is a finite integer and $h_j > h_{j-1}$ for all $j = 2, ..., n$. The properties of the function are as follows: $f_1(\phi, h) > 0$, $\lim_{\phi \to 1} f(\phi, h)\hat{x} = x_2 = x_1 + \eta$ for all h,

$\lim_{\phi \to 0} f(\phi, h)\hat{x} < x_1$ for all h, and $f(\tilde{\phi}, h_j) > f(\tilde{\phi}, h_{j-1})$ for all $j = 2, ..., n$ and $\tilde{\phi}$ fixed. Overall, the assumptions indicate that the externality induces a negative relationship between the return on bank's holdings of illiquid assets and the aggregate fraction of illiquid assets held in the form of s_2. By assumption, the return falls on s_2 is less than the return on s_1 as the aggregate fraction approaches 1. In addition, the aggregate state variable affects the size of the negative externality. As "better" states are reported, the negative impact that the fraction has on the combined returns is smaller in absolute value.

Here, the central bank computes the socially efficient level by solving the following utility maximization problem:

$$\alpha\left[\frac{M^{d*}(p_{t+1})}{\alpha p_{t+1}}\right]^{1-\rho} \div (1-\rho) + (1-\alpha)\left[\frac{f(\phi^*, h_t)\hat{x}(d_t - M^{d*}(p_{t+1}))}{1-\alpha}\right]^{1-\rho} \div (1-\rho). \qquad (28)$$

Let ϕ^* denote the fraction of assets in the s_2 type of illiquid assets that solves equation (28) for a given realization of the exogenous state variable. On the margin, the socially efficient level is to choose the aggregate fraction of illiquid assets in s_2 such that $f(\phi^*, h_t)\hat{x} = x_1$; banks will allocate deposits into the

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reductions in the value of Lynch's holdings of collateralized debt obligations and subprime mortgages, which caused declines in the value of mortgage-backed securities.

externality asset up to the point where the marginal return on s_2 is equal to the (marginal, fixed) return on s_1.

Would any banks be willing to pay to learn the value of the aggregate state variable and the value of the externality? To answer this question, we develop an approach that mimics willingness-to-pay in the perfect-foresight problem. We assume that banks have partial information about the externality. But, in the absence of expending these resources, the individual bank would act like an atomistic agent, treating the aggregate fraction of assets put into s_2 as given and knowing little about the aggregate state variable.. For our purposes, we assume that each bank takes $\phi_{t-1} = \underline{\phi} < \phi^*\left(h_t\right)$ where $\phi^*\left(h_{t-1}\right)$ is used to indicate that the efficient aggregate fraction is conditional on the aggregate state variable. For an atomistic bank, the existing impact of the externality is encoded in $\underline{\phi}$ and we assume that every bank costlessly receives the signal that $h_t = h_1$. Lastly, we deal with the potentially circularity problem by assuming that the bank considers the loss based on regret; that is, we ask what the bank would have been willing to pay given the most recent past observation of the cost associated with not paying. For a two-period lived person, bank i where $i \in [0,1]$ looks at $WTP_{t-1,i}$. Note that willingness-to-pay can be interpreted as what the bank would have been willing to pay to learn the efficient aggregate fraction of deposits allocated to s_2 at date t-2. The regret comes is the reliance on a past observed level of welfare loss associated with realizing

$$\alpha\left[\frac{M^{d*}\left(p_{t-1}\right)}{\alpha p_{t-1}}\right]^{1-\rho} \div\left(1-\rho\right)+\left(1-\alpha\right)\left[\frac{f\left(\underline{\phi},h_1\right)\hat{x}\left(d_{t-1}-M^{d*}\left(p_{t-1}\right)\right)}{1-\alpha}\right]^{1-\rho} \div\left(1-\rho\right) \text{ instead of using}$$

$$\alpha\left[\frac{M^{d*}\left(p_{t-1}\right)}{\alpha p_{t-1}}\right]^{1-\rho} \div\left(1-\rho\right)+\left(1-\alpha\right)\left[\frac{f\left(\phi_{t-1}^*,h_{t-1}\right)\hat{x}\left(d_{t-1}-M^{d*}\left(p_{t-1}\right)\right)}{1-\alpha}\right]^{1-\rho} \div\left(1-\rho\right).^{21} \text{ Because the}$$

regret-based approach yields an observable measure of welfare loss without the conundrum of knowing what impact that Fed watching would have without assuming you know what the Fed is going to do, we can compute a measure of willingness to pay.

So, willingness to pay for the extra information satisfies the following indifference expression:

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[21] Note that in this expression, $M^{d*}\left(p_{t-1}\right)$ is the demand for real money balances for a bank in which the actual price level is equal to the predicted price level at date $t-1$.

$$\alpha \left[\frac{M^{d*}(p_{t-1})}{\alpha p_{t-1}} \right]^{1-\rho} \div (1-\rho) + (1-\alpha) \left[\frac{f(\underline{\phi}, h_1) \hat{x}(d_{t-1} - M^{d*}(p_{t-1}))}{1-\alpha} \right]^{1-\rho} \div (1-\rho)$$

$$= \alpha \left[\frac{M^{d*}(p_{t-1})}{\alpha p_{t-1}} \right]^{\rho} + (1-\alpha) \left[\frac{f(\phi_{t-1}^*, h_{t-1}) \hat{x}(d_{t-1} - WTP_{t-1} - M^{d*}(p_t))}{1-\alpha} \right]^{\rho}$$

(29)

where $f(\underline{\phi}, h_1)$ denotes the aggregate fraction of deposits allocated to s_2 and the free value of the aggregate state variable.

The intuition here could be challenging. What seems unnatural is the notion that an otherwise atomistic bank would be capable of paying to internalize the welfare costs of the overinvesting in the externality-riddled asset. With the regret based approach, the bank looks at the past and sees that something is driving a wedge between the expected and realized return on s_2. In other words, the welfare loss is observable. The regret-based approach takes the stand that if the bank would have known about the overinvestment, they would have regretted participating in the outcome. Because the aggregate state variable is not observable without paying to obtain what the central bank knows, the atomistic deposit bank sees the externality through the veil of missing information.[22] At the socially efficient level, the return on illiquid assets is the same as the return on x_1. In this model economy, each bank looks back to see what the realized return on the combination of illiquid assets was. If the weighted return is less than x_1 then each bank knows that the aggregate fraction of deposits invested in s_2 was excessive. So, the regret-based question is: What would a bank have been willing to pay to avoid the welfare cost of the externality? Accordingly, the right-hand-side of equation (33) represents the Hicksian compensation necessary to make the bank indifferent between the socially efficient fraction and the known values. As we observed in the price-forecasting version of the model economy, there will still be some efficiency lost in the externality-version of the model economy relative to the planner allocation. Insofar as some positive measure of banks are willing to pay, the welfare loss will be smaller. More concretely, as long as $f(\phi^*, h_t) < f(\underline{\phi}, h_1)$, there is an incentive for an individual bank to unilaterally deviate from the efficient level.

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[22] Canzoneri (1985) and Cukierman and Meltzer (1986) describe economies in which the central bank has knowledge about the aggregate economy that is unknown by the general public. In these papers, the veil is what allows the central bank to keep the actual monetary policy from being inferred. Their aim is to examine the inflationary bias in a monetary policy game and how such bias could be mitigated by asymmetric information.

Overall, we derive the following proposition.

Proposition 5: A bank's willingness to pay for the information combined in the efficient aggregate fraction and the aggregate state variable is positively related to the difference between $f\left(\underline{\phi}, h_1\right)$ and $f\left(\phi^*, h_t\right)$.

Proposition 5 tells us that a bank would have been willing to pay a smaller amount when the difference between the free-information aggregate fraction and the past socially efficient fraction is small. The proof is a straightforward extension of the diminishing marginal utility assumption. Note that when the willingness-to-pay is a small value.

There are two main insights that can be drawn from the model economy with externalities. First, our results show how the idea of Fed watching can be extended to deal with the central bank's regulatory function. Our results suggest that with the willingness-to-pay calculation, there is a relationship between the welfare loss and the banking industry's intensity to internalize the externality. In short, the banking industry has an incentive to self-police by more intensely watching the central bank's knowledge of the aggregate state variable. Note that the existence of willingness-to-pay does not justify regulatory inaction. It does seem important to recognize that banks do have some incentive to look backward and assess the value of paying some resources to avoid low returns.

Second, Proposition 5 can be used to shed some light on why the evolution of regulatory practices over time. Consider a modification to our model economy in which it is costly to implement bank regulations. Now, consider a setting in which the date *t-1* aggregate state variable is close to the free value that is given to the atomistic bank such that $f\left(\underline{\phi}, h_1\right) \rightarrow f\left(\phi^*, h_{t-1}\right)$, we have the limiting case in which the information difference is vanishing; in other words, willingness-to-pay for what the central bank knows about the welfare loss last period would have been close to zero. Let $h_t = h_n > h_{t-1}$. In this case, the date *t* willingness-to-pay occurs in a period in which the welfare loss is large. The illustration describes an economy in which the regret-based Fed watching intensity may not be synched with the welfare loss. Because Fed watching intensity is low, the banking industry is marked by overinvesting in the illiquid asset subject to the externality. In such a case, the incentive to regulate banks is great just after the welfare loss is greatest. It is also the time at which banks would begin self-policing since h_t induced a large welfare loss, Fed watching intensity would also increase. If, for example, $h_{t+1} < h_t$, Fed watching intensity would have increased at a time when the welfare loss was declining because of the regulatory

environment and because of the regret. Accordingly, returns on the illiquid asset would have been like an excessive return for banks.[23]

Overall, the externality permits us to take a rather nuanced approach to regulatory action that is missing in the reforms put forward by Admati and Aikman, et al. The calls for regulation, for example, do not consider an economy in which the banks have a good estimate of the aggregate fraction and the aggregate state variable. In other words, the value of the externality cannot internalized in their setups. Here, however, regrets may induce a fraction of individual banks to expend the resources to internalize the costs of so that unilaterally acquiring more s_2 has zero private marginal benefit. When a large measure of banks act to internalize the externality and thus avoid the welfare loss, we see that there is no justification for implementing a regulatory setting with respect to the bank's portfolio. Our conclusion is based on the premise that regulating banking asset holdings are costly to implement. So, when there is zero social marginal benefits, the efficient outcome is to not regulate. Our framework is flexible enough to see that when it is expensive to internalize the costs, then a large fraction of assets in will be placed in the externality-riddled asset. Under these conditions, the policy prescriptions put forward by Admati and Aikman, et al are justified. When the externality is not taken into account, the insufficient regulations will allow banks to exacerbate the problem by overinvesting in the problematic illiquid asset.

7. Summary and conclusion

In this paper, we consider an economy in which it is costly to process central bank communications. Fed watching is built on the notion that the central banks knows things that bank and consumers do not know. To be clear, this knowledge is not used strategically by the central bank; there is no policy reaction functions that are built into the model. Rather, we focus on two costs: one is the direct cost of processing the central bank communication and the other is the welfare cost of not processing the central bank communication.

We specify two model economies to study the balancing act between these two costs. In the first model economy, Fed watching improves forecast accuracy. Banks maximize expected welfare for depositors needing to predict next period's price level, or alternatively, the return to money. There is a free predictor that suffers from forecast error and a costly one in which forecast error is absent. We determine the cost of the accurate predictor by forming a regret-based problem that computes what the

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[23] Lambert (2017) provides a thorough discussion on the tradeoff between costs and benefits of implementing regulation. As such, he argues that quantifying the cost and benefits when considering what regulations are worth it.

bank would have been willing to pay to have avoided past forecast errors. When forecasts errors are large, for example, the bank's Fed watching cost—which is measured by the willingness to pay—are greater. So, we get this result that the measure of banks Fed watching declines. In other words, the binary choice problem says that Fed watching is less intense when forecasts are less accurate, resulting in less accurate forecasts.

In the second model, we modify the economy to consider two illiquid assets. One of the illiquid assets is subject to a negative externality. Here, the welfare cost owes to the bank's inability to internalize the externality. We allow for a bank to expend resources—a willingness to-pay—to have avoided past welfare losses. Here, the central bank knows the value of the aggregate state variable and banks are permitted to process central bank communications; the processing costs allow the bank to observe the true aggregate state variable. Banks, therefore, are balancing between not internalizing the welfare costs of the negative externality against the cost of processing what the central bank knows. The cost of processing are increasing in the distance of past welfare costs. So, the measure of banks processing the central bank communication is decreasing when the aggregate state variable is increasing. Thus, with fewer banks processing information, the overinvestment in the "bad" illiquid asset might be more intense. This setup can explain why regulation needs to be implemented when economic conditions are good, not when they are bad.

With this structure in place, there are a number of extensions that need to be considered. We examined economies in which central bank communications are not subject to any uncertainty. A natural next step is to incorporate uncertainty into the model economy. In addition, there is no strategic aspects to the central bank's communications. Hence, issues associated with credibility and ambiguity need to be addressed.

Appendix

Proof of Lemma 2:

Differentiate equation (14), obtaining

$$\left(\frac{1}{(1+\theta)q_{t+1}}\right)\left[\frac{M}{\alpha(1+\theta)q_{t+1}}\right]^{-\rho}dq_t - \left[\frac{q_t(1+\theta)}{\left[(1+\theta)q_{t+1}\right]^2}\right]\left[\frac{M}{\alpha(1+\theta)q_{t+1}}\right]^{-\rho}dq_{t+1}$$

$$+\rho\left(\frac{q_t}{(1+\theta)q_{t+1}}\right)\left[\frac{M}{\alpha(1+\theta)q_{t+1}}\right]^{-\rho-1}\left[\frac{M(1+\theta)\alpha}{\left[\alpha(1+\theta)q_{t+1}\right]^2}\right]dq_{t+1} \qquad (A.1)$$

$$=-\rho k\left[y-\frac{M}{(1+\theta)q_t}\right]^{-\rho-1}\left[\frac{(1+\theta)M}{\left[(1+\theta)q_{t+1}\right]^2}\right]dq_t$$

We evaluate equation (A.1) at steady state, factoring out $\left(\frac{1}{(1+\theta)q}\right)\left[\frac{M}{\alpha(1+\theta)q}\right]^{-\rho}$, using equation

(16), which implies $\left[y-\frac{M}{(1+\theta)q}\right]^{-\rho-1}=\left[k(1+\theta)\right]^{\frac{-(1+\rho)}{\rho}}\left[\frac{M}{\alpha(1+\theta)q}\right]^{-\rho-1}$, and simplifying to obtain

$$\left(dq_t-dq_{t+1}\right)+\frac{\rho}{(1+\theta)}\left(\frac{M}{\alpha q}\right)\left[\frac{M}{\alpha(1+\theta)q}\right]^{-1}dq_{t+1}$$

$$-\rho k\left[k(1+\theta)\right]^{\frac{-(1+\rho)}{\rho}}\left(\frac{M}{q}\right)\left[\frac{M}{\alpha(1+\theta)q}\right]^{-1}dq_t=0 \qquad (A.2)$$

From here, it is straightforward to cancel out q and $1+\theta$ in equation (A.2) collect terms, yielding

$$\frac{dq_{t+1}}{dq_t}=\frac{1+\alpha\rho\left[k(1+\theta)\right]^{-\frac{1}{\rho}}}{1-\rho}. \qquad (A.3)$$

The condition for monotone jump stability is that the right hand side of equation (A.3) is strictly greater

than one. Or, $1+\alpha\rho\left[k(1+\theta)\right]^{-\frac{1}{\rho}}>1-\rho$. Recall that we are focusing on cases in which $0<\rho<1$, so

the denominator in equation (A.3) is positive. The numerator in equation (A.3) is also strictly positive.

We can simplify the condition monotone jump stability to be $\alpha\left[k(1+\theta)\right]^{-\frac{1}{\rho}}>-1$. Because the right-

hand-side of this condition is strictly positive, the condition is satisfied and the local dynamics exhibit

monotone jump stability. ∎

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Figure 1

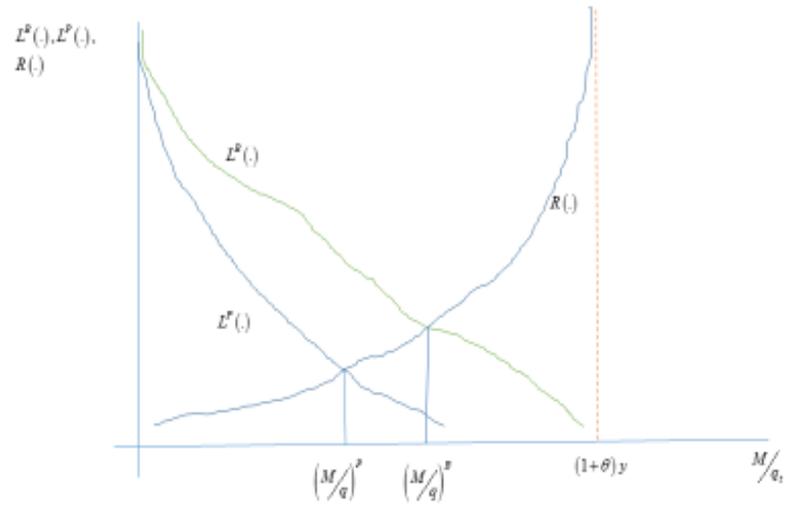**Steady state values for perfect-foresight**

And backward-looking expectations

Table 1

Numerical Results for Steady States and Jury-Marden Conditions: g = 0.7

g = 0.7		
$\underline{\theta}$	$\underline{q}^{*}$	**Jury-Marden conditions**
0.01	1123.1	Satisfied
0.02	1129.82	Satisfied
0.03	1136.5	Satisfied
0.05	1149.9	Satisfied
0.1	1247.7	Satisfied
0.2	1431.7	Satisfied $a_0 + a_1 + a_2 < 0$

g = 3	
$\underline{\theta}$	**Jury-Marden conditions**
0.01	Failed: $a_0 < \mid a_2 \mid$
0.02	Failed: $a_0 < \mid a_2 \mid$
0.03	Failed: $a_0 < \mid a_2 \mid$
0.05	Failed: $a_0 < \mid a_2 \mid$
0.1	Failed: $a_0 < \mid a_2 \mid$
0.2	Failed: $a_0 < \mid a_2 \mid$