Business Formation and Economic Growth Beyond the Great Recession

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ABSTRACT

We develop a model with endogenous entry and exit in an economy subject to non-stationary shocks to aggregate total factor productivity. Firms exhibit a life-cycle consistent with microeconomic data, and our model reproduces key moments of the empirical firm age and size distribution. In this setting, persistent shocks to trend growth can drive long term reductions in business formation. The economic consequences of a persistent decline in entry grow over time and, together with 'wait-and-see' effects on aggregate capital investment, can compound and protract a productivity slowdown.

We apply our model to understanding the last decade of U.S. economic activity, an episode marked by slow GDP growth and persistently low entry rates and business fixed investment in the aftermath of the Great Recession. Firms vary in the permanent and transitory components affecting their productivity and in their capital stocks, and their capital adjustments are subject to one period time-to-build, alongside convex and nonconvex costs. Thus, the economy's aggregate state includes a distribution of firms over productivity and capital, and changes in this distribution have a persistent influence on economic activity. Epstein-Zin preferences and a time-varying relative price of capital goods combine to accommodate countercyclical risk premia capable of driving large changes in firm values. Following a persistent shock to trend growth, equilibrium movements in firms' stochastic discount factors imply long-run reductions in the value of entry. The resulting declines in new business formation propagate the shock's effects on investment and GDP, slowing aggregate recovery.

Keywords: business cycles, long run risk, entry and exit, capital adjustment costs, (S,s) policies

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1 Introduction

U.S. economic experience over a decade following the Great Recession has featured a lack of recovery in GDP and business fixed investment relative to their long-run trends, alongside reduced business dynamism and productivity slowdown, provoking extensive concerns among policymakers and researchers that we may be facing secular stagnation. Meanwhile, official growth forecasts have been downwardly revised repeatedly since 2005, creating uncertainty about the growth rate of the U.S. economy. How does such growth-rate uncertainty affect firm-level decisions in general, and new business formation and investment in particular? To what extent might the economic slowdown be compounded by these micro-level considerations?

A growing literature highlights the importance of uncertainty at the firm-level and its impact on firm dynamics and aggregate productivity (leading examples include Bloom (2009) and Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2016)). We extend the canonical heterogeneous firm framework to accommodate endogenous firm entry and exit, alongside shocks to the growth rate of aggregate total factor productivity (TFP), a time-varying relative price of capital and a departure from time-separable utility permitting countercyclical risk premia and thus adequately large changes in firm values to permit persistently low entry rates. Using this model to contemplate the possibility of secular stagnation, we assess the relevance of trend growth shocks in accounting for persistently low rates of business formation, and its role in the broader economic slowdown.

The U.S. has experienced a persistent decline in business dynamism since the 2007 recession. The birth rate of new firms was 10.8 percent in 2006, and steadily fell a full 3 percentage points over the subsequent four years. Whereas exit rates fell back to normal fairly quickly and have been comparatively low since 2010, entry rates have recovered less than 1 percentage point since 2010. Similar observations hold for the birth rate of new establishments. This is a concern not only because the contribution of start-up firms to job creation is large, but also because what has happened to new firms over this period may have long-lasting consequences for the aggregate economy.[1] For example, Sedláček and Sterk (2016) show that start-up firms incorporated during recessions remain persistently smaller even at their later stage of growth trajectory.[2] Such cohort effects can have a long-run effects on aggregate investment and employment. Examining U.S. state-level data, Gourio, Messer, and Siemer (2016) find that reductions in new business formation drive a significant and

[1] See Haltiwanger, Jarmin, and Miranda (2013).

[2] Moreira (2017) finds such cohort effects on firm size using data from the Census Bureau's Longitudinal Business Database (LBD).

persistent drop in real GDP. They argue two questions need answers: (1) what are the causes of the declining firm entry? and (2) what are the aggregate consequences?

This paper examines the extent to which uncertainty about economic growth rates discourages firm entry, and whether a persistent decline in entry rates has important implications for aggregate investment, employment and productivity. A quantitative analysis of these effects must be consistent with the rich features of firm dynamics highlighted in the studies above. The model we propose includes a unique non-Gaussian process of firm-level productivity, endogenous entry and exit decisions, and capital adjustment frictions, allowing us to capture these elements. Beyond these micro-level ingredients allowing our model reproduce key aspects of the empirical firm age and size distribution, we allow for trend growth shocks to aggregate TFP in a setting with a time-varying relative price of capital and a Epstein and Zin's (1989) recursive household preferences. To understand the patterns we have seen in business entry over the past decade, we require a theoretical setting capable of generating sharp changes in firm value; it is for that reason that we include these elements from the asset pricing literature known to deliver countercyclical stochastic discount factors.

Our inclusion of trend growth shocks to aggregate TFP is motivated by the unusually anemic recovery of the U.S. economy from the Great Recession trough. This has led to increasing pessimism about future economic growth. One clear example is the IMF's outlook for real GDP growth in the U.S. which has been downwardly revised repeatedly, indicating persistent movements in expected growth rates and uncertainty surrounding them. The canonical business cycle model is ill-suited to analyze this situation, as its fluctuations are derived from shocks to the level of productivity, not its growth rate. While focusing more expressly on long-run growth, another line of research has produced many endogenous growth models highlighting the role of firms and their innovative activity (see Aghion, Akcigit, and Howitt, 2014 for a recent survey). For example, Acemoglu, Akcigit, Bloom, and Kerr (2017) highlight the rich heterogeneity in innovation activity using micro data and build model of firm-level innovation and endogenous entry and exit to show the importance of selection in shaping aggregate productivity. To our knowledge, ours is the first business cycle model that can be used to examine a mechanism through which changes in growth expectation influence business dynamism and macroeconomic series like business investment and aggregate productivity.

A negative shock to total factor productivity growth is propagated by changes in our model's firm distribution. Lower productivity implies a long-lived decline in the value of production, leading firms to expect a persistent fall in earnings. As a result, the onset of a recession is accompanied by a rise in exit and a persistent reduction in entry. Our emphasis on matching the size and age distribution of employment across firms implies that the aggregate effects of the resulting decline in the number of firms grows over time. Any cohort of entrants increases its share of total production gradually. Entrants start small and, on average, experience lifecycle growth in their individual productivities. Given the presence of capital adjustment costs, initially low levels of firm-specific productivity imply slow growth in average firm size within any cohort of entrants. When a recession leads to several years of below-average entry, the economy fails to offset the reduction in the number of firms resulting from the increase in exit rates.

The entry and exit of firms adds an extensive margin absent in standard heterogeneous firm models focussing on capital adjustment such as Bloom (2009) or Khan and Thomas (2008). Given decreasing returns to scale at each individual firm, changes in the number of production locations imply endogenous movements in aggregate total factor productivity. A loss of firms, given the level of other productive resources, implies lower aggregate productivity. As the importance of a cohort of firms grows over time with their sizes and survival rates, the effects of reduced entry are felt several periods after the onset of a recession. This leads to a very slow recovery. Thus a period of low entry propagates a recession, and measured TFP remains low when the exogenous component of aggregate TFP begins to recover.

A missing generation of firms created by growth rate uncertainty is a distinct aspect of our model, and we evaluate its quantitative importance in shaping aggregate productivity dynamics.[3] The literature highlighted the relevance of uncertainty for the longer-run decisions such as investment and R&D. In fact, Barrero, Bloom and Wright (2017) find that investment and R&D are significantly more responsive to long-run uncertainty than hiring. Our results are consistent with their empirical finding; indeed, since creating new firms is so forward looking and uncertain, persistent fluctuations in expected growth rates play an extremely important role.[4]

Our inclusion of shocks to the growth rate of productivity is new in a heterogeneous firm framework.[5] This is both motivated by the anemic recovery in post- Great Recession data, and also aimed at addressing a few challenges in a quantitative macroeconomic research using a heterogeneous firm framework. First, a growing literature highlights the importance of growth-rate shocks in accounting for aggregate dynamics (see Nakamura, Sergeyev, Steinsson, 2017). Ours is the first to look at firm

[3] See Siemer (2016) for a quantitative study how financial frictions plays a role in adversely affecting firm entry and its aggregate implications.

[4] See, for example, Garcia-Macia (2017) and Schmitz (2014) for studies on the long-run consequences of the Great Recessions through the lens of investment activities in productivity-enhancing technology.

[5] See Favilukis and Lin (2013) for a model with representative firm.

dynamics as a micro-origin that plays an crucial role in amplifying growth-rate shocks in driving macroeconomic fluctuations.[6]

2 Model

Our model has three types of economic agents: a large number of identical households, a representative capital goods producer, and a large number of perfectly competitive, heterogeneous production firms supplying a common final good they make using capital and labor. Production firms are heterogeneous in their capital, k, and in their idiosyncratic productivity, ϵ. Each firm's idiosyncratic TFP is the product of ε, a persistent random variable following a Markov Chain, $\left\{\varepsilon_m, \{\pi^{\varepsilon}_{mn}\}_{n=1}^{N_\varepsilon}\right\}_{m=1}^{N_\varepsilon}$, a permanent component, $s \in \mathbf{S}$, determined just prior to the firm's entry, and a component reflecting its nearness to productive maturity, x. New firms initially operate with $x_1 < 1$; each period thereafter, any young firm faces a constant probability that its x will permanently rise to $x_2 = 1$ allowing it to reach its full productive potential.

The economy's aggregate state includes μ, the time-varying distribution of firms over idiosyncratic TFP components and capital, (ε, x, s, k), alongside the level of exogenous aggregate total factor productivity, $A = A_{-1}z$, where z is a trend growth shock following a Markov Chain, $\left\{z_i, \left\{\pi^{z}_{ij}\right\}_{j=1}^{N_z}\right\}_{i=1}^{N_z}$. The assumptions we specify below render the value functions of households and firms homogeneous of degree one in A_{-1}. Anticipating this result, we detrend the model by A_{-1} and represent the aggregate state of the detrended economy by (z, μ).

2.1 Households

The representative household values consumption, C, and inelastically supplies its time endowment as labor to production firms each period. The household owns all of the economy's firms, receives aggregate dividends from production firms, and has access to a full set of Arrow Securities. As noted above, we allow the possibility of a countercyclical risk premium by assuming Epstein-Zin (1989) preferences for this household.

The household's expected discounted lifetime utility is:

$$W = \left[(1-\beta)\, C^{1-\sigma} + \beta \Big(E[W'^{1-\gamma}] \Big)^{\frac{1-\sigma}{1-\gamma}} \right]^{\frac{1}{1-\sigma}},$$

where γ is its coefficient of relative risk aversion and $\frac{1}{\sigma}$ is its intertemporal elasticity of substitution. In parameterizing our model, we will set $\gamma > \sigma$ to ensure that the household has a preference

[6] See Kung and Schmid (2014).

for early resolution of uncertainty and so is averse to uncertainty about long-run growth rates. Denoting growth-deflated economywide dividends and redeemable contingent claims by $\pi(z_i,\mu)$ and a, respectively, and defining $h(a,z,\mu) \equiv W(a,z,A_{-1},\mu)/A_{-1}$ and $c \equiv C/A_{-1}$, we can write the household's detrended optimization problem as follows.

$$h(a,z_i,\mu) = \max_{c,\{a'_j\}_{j=1}^{N_z}} \left[(1-\beta)c^{1-\sigma} + \beta \Big(\sum_{j=1}^{N_z} \pi_{ij} [z_i h(a'_j, z_j, \mu')]^{1-\gamma} \Big)^{\frac{1-\sigma}{1-\gamma}} \right]^{\frac{1}{1-\sigma}} \tag{1}$$

subject to :

$$c + \sum_{j=1}^{N_z} z_i q_j(z_i,\mu) a'_j \leq a + w(z_i,\mu) n(z_i,\mu) + \pi(z_i,\mu), \tag{2}$$

$$c \geq 0,\ a'_j \geq 0 \text{ and } \mu' = \Gamma(z_i,\mu)$$

In recursive competitive equilibrium, consumption is fully determined as a function of the economy's aggregate state, $c = c(z,\mu)$. A second implication of our representative household assumption is that equilibrium in the markets for contingent claims requires $a'_j = 0$ for each $j = 1,\ldots,N_z$. The equilibrium prices of growth-adjusted claims achieving this are given by:

$$z_i q_j(z_i,\mu) = \frac{h(0,z_j,\mu')^{\sigma-\gamma}}{\left(\sum_{j=1}^{N_z} \pi_{ij} [h(0,z_j,\mu')]^{1-\gamma}\right)^{\frac{\sigma-\gamma}{1-\gamma}}} z_i^{1-\sigma} \beta \frac{\pi_{ij} c(z_j,\mu')^{-\sigma}}{c(z,\mu)^{-\sigma}}. \tag{3}$$

Note that equilibrium in the markets for firm shares will require that the state contingent discount factors firms use to weight their future values are $\rho(z_j,z_i,\mu) = z_i q_j(z_i,\mu)$ consistent with (3).

2.2 Investment firms

A perfectly competitive investment firm supplies new capital goods to production firms at price $p(z_i,\mu)$ at the end of each period. This firm creates new capital by combining units of investment spending (final output) together with existing capital its hires from production firms at rental price $d(z_i,\mu)$ after their production has taken place. Because its production technology $\Phi(K',K)$ has a diminishing marginal product of investment spending, the firm's marginal cost of creating new capital depends on the quantity it sells, implying a time-varying relative price of capital.

Let m represent current detrended capital rental, $m = K/A_{-1}$ in equilibrium, and let $\psi(z_i m', m)$ be the investment of final goods required to produce $z_i m' - (1-\delta)m$ units of the capital good through the technology Φ, where $\delta \in (0,1)$ is the rate of capital depreciation. The investment firm solves the following static profit maximization problem.

$$\max_{m',m} \Big[p(z,\mu)\left[z_i m' - (1-\delta)m\right] - \psi(z_i m', m) - d(z,\mu) m \Big] \tag{4}$$

Its first order conditions restrict the capital sale price and rental rate as follow.

$$p(z,\mu) = D_1\psi\left(zm', m\right) \tag{5}$$

$$d(z,\mu) = -(1-\delta)\,p(z,\mu) - D_2\psi\left(zm', m\right) \tag{6}$$

We specify Φ below so that the cost function $\psi\left(zm', m\right)$ is homogeneous of degree 1. Given the price restrictions above, this ensures the investment firm makes zero profits in equilibrium.

2.3 Production firms

Heterogeneous production firms (hereafter, *firms*) supply final output, y, using capital, k, and labor, n, in a decreasing returns to scale technology; $y = (z\epsilon)^{1-\alpha}k^{\alpha}n^{\nu}$. Recall from above that z is the growth rate shock to aggregate exogenous total factor productivity and $\epsilon = \varepsilon x s$ is firm-specific productivity, where ε is a persistent, stationary TFP shock, s is a permanent productivity component the firm realizes prior to its entry decision, and $x \in (0,1]$ reflects its proximity to productive maturity when the full s is achieved. The presence of s allows our model to address moments from the firm size distribution, x is included for consistency with the firm age distribution, and ε allows for volatility conditional on age and size. We assume all new firms begin with $x_1 < 1$ and face a constant probability $\lambda \in (0,1)$ of maturing to $x_2 = 1$ in each subsequent date, though it is straightforward to generalize this to accommodate a smoother lifecycle for individual firms.

The timing of existing firms' decisions is as follows. Each firm enters the period identified by its current firm-level TFP, ε, and the capital stock it selected in the previous period, k. It observes the current aggregate state, (z,μ), then hires a chosen level of labor from households at real wage $w(z,\mu)$, sells the output it produces and pays its wage bill. We assume that each existing firm can productively employ only $n \leq n\left(\varepsilon x s, k, z\right)$; however, we will show below that households' inelastic labor supply permits equilibrium real wages at which no firm's capacity constraint strictly binds and aggregate employment is procyclical.

Each firm draws a random continuation cost, $\xi_c \sim \vartheta_c\left(\xi_c\right)$ after production. This cost is denominated in units of labor and scaled by a function of the firm's long-run productivity, $b(s)$; thus, the firm must pay $w(z,\mu)\xi_c b(s)$ units of output to continue into the following period. If it chooses to avoid its ξ_c, it sells its remaining capital, paying the proceeds and its current flow profits as dividends to households as it exits the economy. Otherwise, conditional on continuing, the firm draws a fixed capital adjustment cost, $\xi_k \sim \vartheta_k\left(\xi_k\right)$, also denominated in labor units and scaled by $b(s)$, and then chooses its capital for the next period.

Nonadjusted capital physically depreciates at rate $\delta \in (0, 1)$. A continuing firm can select any $k' \geq 0$ it wishes by purchasing the requisite capital goods, $zk' - (1-\delta)k$, at price $p(z, \mu)$ and *either* hiring the labor necessary to cover its fixed capital adjustment costs $\xi_k b(s)$ or instead forfeiting an output-denominated convex adjustment cost $\phi(k', k)$. By choosing the latter and setting $k' = \frac{1-\delta}{z}k$, it can avoid adjustment costs.

As existing firms are making their continuation decisions after production each period, a time-invariant number of potential entrants of each permanent productivity type, $\omega(s)$, comes online. Each observes the current aggregate state and its s level, and each realizes its own draw of a fixed entry cost ξ_e from time-invariant distribution $\vartheta_e(\xi_e)$. As with the fixed costs facing existing firms, entry costs are labor-denominated and scaled by $b(s)$. Thus, a potential entrant of type s drawing ξ_e may pay $w(z, \mu)\xi_e b(s)$ units of output to select and purchase a capital stock with which it will produce as a new firm in the next period, or it may forfeit its s and disappear. Given this timing, the distribution μ' comprising the economy's endogenous state at the start of the next period will include incumbent firms that opt to continue beyond their production in the current period, alongside entrants that at the same time choose to pay their entry costs and select initial capital stocks.

We list the problems facing existing firms and potential entrants across the next two subsections. Three sets of transition probabilities will be relevant to those problems. We let π^z_{ij} denote the probability of aggregate growth shock $z' = z_j$ being realized conditional on $z = z_j$, and assign $\rho(z_j, z_i, \mu) = z_i q(z_j, z_i, \mu)$ as the associated state contingent discount factor. We denote a firm's probability of realizing $\varepsilon' = \varepsilon_n$ conditional on $\varepsilon = \varepsilon_m$ by π^ε_{mn}, and we let π^x_{ab} drawn from the transition matrix $[1-\lambda \;\; \lambda; 0 \;\; 1]$ be the probability of $x' = x_b$ conditional on $x = x_a$.

2.4 Firm's problem

Let v_0 represent the start of period value of a firm of type $(\varepsilon_i, x_a, s, k)$ given aggregate state (z_i, μ), let v_1 be this firm's value on realizing its labor-denominated fixed continuation cost ξ_c, and let v_2 be its expected post-production value conditional on paying that cost, just before it sees its labor-denominated fixed capital adjustment cost draw. Given this notation and using v_f to represent the firm's end-of-period expected future value conditional on its k' choice, we can write the growth deflated problem of any existing firm as follows. For expositional convenience, we suppress idiosyncratic and aggregate state indices everywhere possible below, listing them only where they are essential to clarify transition probabilities.

The firm's start of period value is:

$$v_0(\varepsilon, x, s, k; z, \mu) = \int_0^\infty v_1(\xi_c, \varepsilon, x, s, k; z, \mu)\, \vartheta_c(d\xi_c), \tag{7}$$

with its value on seeing its continuation cost given by:

$$\begin{aligned} v_1(\xi_c, \varepsilon, x, s, k; z, \mu) = & \max_{n \leq n(\varepsilon x s, k, z)} \left[(z\varepsilon x s)^{1-\alpha} k^\alpha n^\nu - w(z,\mu) n \right] + d(z,\mu) k \\ & + \max \Big\{ [v_2(\varepsilon, x, s, k; z, \mu) - w(z,\mu) b(s) \xi_c],\ p(z,\mu)(1-\delta) k \Big\}, \end{aligned} \tag{8}$$

where the binary max operator reflects its continuation choice. The firm will continue so long as its ξ_c draw does not exceed a threshold cost, $\xi_c^T(\varepsilon, x, s, k; z, \mu)$, satisfying:

$$\xi_c^T = [v_2(\varepsilon, x, s, k; z, \mu) - p(z,\mu)(1-\delta) k] / w(z,\mu) b(s). \tag{9}$$

Continuing firms use v_f to value future capitals as they choose whether to incur nonconvex or convex capital adjustment costs. Given that function, we can write any such firm's expected continuation value after paying its ξ_c, but before drawing its fixed adjustment cost ξ_k, as:

$$\begin{aligned} & v_2(\varepsilon, x, s, k, z, \mu) = p(z,\mu)(1-\delta) k + \\ & \int_0^\infty \max \Big\{ -w(z,\mu) b(s) \xi_k + \max_{k'} \left[-p(z,\mu) z k' + v_f(\varepsilon, x, s, k', z, \mu') \right], \\ & \max_{k'} \left[-\phi(zk', k) - p(z,\mu) z k' + v_f(\varepsilon, x, s, k', z, \mu') \right] \Big\} \vartheta_k(d\xi_k), \end{aligned} \tag{10}$$

given $\mu' = \Gamma(z,\mu)$. Finally, we use the transition probabilities specified above, the start-of-period value function v_0 defined in (7) and the firm's knowledge of the future endogenous aggregate state μ' to define the expected discounted future value of a firm with current idiosyncratic state (ε_m, x_a, s) that has chosen the capital stock k' for the next period:

$$v_f(\varepsilon_m, x_a, s, k'; z_i, \mu') \equiv \sum_{j=1}^{N_z} \pi_{ij} z_i q(z_j, z_i, \mu) \sum_{n=1}^{N_\varepsilon} \sum_{b=1}^{2} \pi_{mn}^\varepsilon \pi_{ab}^x v_0(\varepsilon_n, x_b, s, k'; z_j, \mu'). \tag{11}$$

Let $k_f^* \equiv G_f(\varepsilon, x, s; z, \mu)$ denote the continuing firm's optimal capital choice conditional on it paying its fixed capital adjustment cost, the k' solving the bracketed problem in line 2 of (10), and let $k_c^* \equiv G_c(\varepsilon, x, s, k; z, \mu)$ be its optimal capital choice otherwise, the k' solving the bracketed problem in line 3. The firm will opt to pay the fixed capital adjustment cost and adopt $k' = k_f^*$, rather than pay convex costs and adopt $k' = k_c^*$, so long as its ξ_k draw does not exceed a threshold cost, $\xi_k^T(\varepsilon, x, s, k; z, \mu)$, satisfying:

$$\xi_k^T = \left[v_f(\varepsilon, x, s, k_f^*, z, \mu') - v_f(\varepsilon, x, s, k_c^*, z, \mu') - p(z,\mu) z (k_f^* - k_c^*) + \phi(z k_c^*, k) \right] / w(z,\mu) b(s). \tag{12}$$

In specifying the convex adjustment cost function below, we assume $\phi\left(z\frac{(1-\delta)k}{z}, k\right) = 0$. Thus, a continuing firm can always choose to undertake zero investment and pay no adjustment costs.

2.5 Potential entrant's problem

The problem of a potential entrant with permanent productivity type s and fixed entry cost ξ_e is similar to the problem of an existing firm deciding whether to continue in (10) above. However, its x for the next period is known to be $x_1 < 1$, its ε for the next period will come from the invariant distribution with probability weights π_n^0, for $n = 1, ..., N_\varepsilon$, and it chooses its initial capital stock free of adjustment costs. We use v^e to represent its value conditional of the payment of its entry cost and the purchase of an initial capital stock k', the analogue to (11) above.

A potential entrant of type s realizing entry cost ξ_e solves:

$$v_0^e(s, \xi_e; z, \mu) = \max\left\{-w(z,\mu)\xi_e b(s) + \max_{k'}\left[-p\left(z,\mu\right)zk' + v^e\left(s, k', z, \mu'\right)\right], 0\right\}, \tag{13}$$

where

$$v^e\left(s, k', z_i, \mu'\right) \equiv \sum_{j=1}^{N_z} \pi_{ij} z_i q\left(z_j, z_i, \mu\right) \sum_{n=1}^{N_\varepsilon} \pi_n^0 v\left(\varepsilon_n, x_1, s, k'; z_j, \mu'\right). \tag{14}$$

Let $k_e^* \equiv G_e(s; z, \mu)$ denote the optimal capital choice conditional on entry solving the problem in (13). The potential entrant will invest to become a new firm so long as its ξ_e does not exceed a threshold cost, $\xi_e^T\left(s; z, \mu\right)$, satisfying:

$$\xi_e^T = \left[-p\left(z,\mu\right)zk_e^* + v^e(s, k_e^*, z, \mu')\right]/w(z,\mu)b(s). \tag{15}$$

Given the distribution of entry costs, ϑ_e, and the threshold costs from (15), we can easily compute the total number of new firms (entrants) arriving at the start of the next period:

$$\omega_e(z,\mu) = \sum_{j=1}^{N_s} \vartheta_e\left(\xi_e^T\left(s; z, \mu\right)\right)\omega(s). \tag{16}$$

Assuming the entry cost distribution is either unbounded above or bounded by a large finite number, the number of entrants of each type s will always be less than $\omega(s)$, and $\omega_e(z,\mu) < \sum_{j=1}^{N_s}\omega(s)$.

2.6 Equilibrium

We begin our discussion of recursive competitive equilibrium by explaining the mechanics of the real wage and firms' employment capacity. Recall that every firm in the distribution μ produces, since there are no fixed costs associated with production. Each firm of type (ε, x, s, k) determines its current employment and thus output by solving the bracketed problem listed on line 1 of (8) in section 2.4, which is subject to a capacity constraint $n \leq n(\varepsilon xs, k, z)$.

Let $n^*(\varepsilon xs, k, z)$ be the firm's target employment level, the labor input it would choose in the absence of its capacity constraint:

$$n^*(\varepsilon xs, k, z) = \left(\frac{\nu k^{\alpha}(z\varepsilon xs)^{1-\alpha}}{w(z,\mu)}\right)^{\frac{1}{1-\nu}}. \tag{17}$$

We assume capacity constraints take a form proportional to firms' target employment demand:

$$n(\varepsilon xs, k, z) = \left(\frac{\nu k^{\alpha}(z\varepsilon xs)^{1-\alpha}}{\chi_0 z^{\chi_1}}\right)^{\frac{1}{1-\nu}}, \tag{18}$$

with $\chi_0 > 0$ and $\chi_1 \in \mathbb{R}$. This assumption implies that either all firms' constraints bind (when $w(z,\mu) < \chi_0 z^{\chi_1}$) or none do.

Because households do not value leisure, we have some flexibility in selecting a real wage function to clear the labor market. By selecting the wage function:

$$w(z,\mu) = \chi_0 z^{\chi_1}, \tag{19}$$

we ensure that capacity is neither strictly binding nor slack for any firm, so all firms choose their target employments, $n^*(\varepsilon xs, k, z)$, and produce output $y^*(\varepsilon xs, k, z) = (z\varepsilon xs)^{1-\alpha} k^a n^*(\varepsilon xs, k, z)^{1-\nu}$. Notice this has a substantial computational benefit in delivering a market clearing real wage independent of the distribution μ. Moreover, it easily ensures wages are procyclical, and allows us to determine to what extent, by choice of $\chi_1 > 0$. Finally, so long as $\chi_1 < 1-\alpha$, it implies that total employment used in production is procyclical.[7]

Before we describe equilibrium, it is useful to define some shorthand (growth-deflated) variables, beginning with current aggregate capital and final output production, $K(\mu) \equiv \int k d\mu(\varepsilon, x, s, k)$ and $Y(z,\mu) \equiv \int y^*(\varepsilon xs, k, z)\, d\mu(\varepsilon, x, s, k)$. Define the total future capital stocks selected across entering firms, $K'_e(z,\mu)$, continuing firms paying fixed capital adjustment costs, $K'_f(z,\mu)$, and those avoiding capital adjustment costs, $K'_c(z,\mu)$, as follow.

$$K'_e(z,\mu) \equiv \sum_{j=1}^{N_s} G_e(s;z,\mu)\vartheta_e\Big(\xi_e^T(s;z,\mu)\Big)\omega(s)$$

$$K'_f(z,\mu) \equiv \int \vartheta_c\Big(\xi_c^T(\varepsilon,x,s,k;z,\mu)\Big)\Big[G_f(\varepsilon,x,s;z,\mu)\int_{\underline{\xi}_k}^{\xi_k^T(\varepsilon,x,s,k;z,\mu)} \vartheta_k(d\xi_k)\Big] d\mu(\varepsilon,x,s,k)$$

[7]This will be sufficient to render aggregate employment procyclical if the threshold fixed costs associated with continuation and entry are procyclical (a foregone conclusion) and the threshold fixed costs associated with capital adjustment are either procyclical (impossible to establish theoretically) or not too countercyclically volatile.

$$K_c'(z,\mu) \equiv \int \vartheta_c\Big(\xi_c^T(\varepsilon,x,s,k;z,\mu)\Big)\Big[1-\xi_k^T(\varepsilon,x,s,k;z,\mu)\Big]G_c(\varepsilon,x,s,k;z,\mu)d\mu(\varepsilon,x,s,k)$$

Given these terms, we have the total frontier capital stock required for the next period,

$$K'(z,\mu) \equiv K_e'(z,\mu) + K_f'(z,\mu) + K_c'(z,\mu),$$

the total goods the investment firm must use to deliver it, $I(z,\mu) = \psi\Big(zK'(z,\mu), K(\mu)\Big)$, and the total goods production firms require to cover their convex capital adjustment costs,

$$\Theta_c(z,\mu) \equiv \int \vartheta_c\Big(\xi_c^T(\varepsilon,x,s,k;z,\mu)\Big)\Big[1-\xi_k^T(\varepsilon,x,s,k;z,\mu)\Big]\phi\Big(zG_c(\varepsilon,x,s,k;z,\mu),k\Big)d\mu(\varepsilon,x,s,k).$$

Define the total demand for labor in production as $N_p(z,\mu) \equiv \int n^*(\varepsilon xs,k,z)\,d\mu(\varepsilon,x,s,k)$, and next define that demanded to cover firms' fixed entry and continuation costs, $N_c(z,\mu)$ and $N_e(z,\mu)$, and that used to cover continuing firms' fixed capital adjustment costs, $N_f(z,\mu)$, as follow.

$$N_e(z,\mu) \equiv \sum_{j=1}^{N_s}\int_{\underline{\xi}_e}^{\xi_e^T(s;z,\mu)} \xi_e\vartheta_e(d\xi_e)b(s)\omega(s)$$

$$N_c(z,\mu) \equiv \int\int_{\underline{\xi}_c}^{\xi_c^T(\varepsilon,x,s,k;z,\mu)} \xi_c\vartheta_c(d\xi_c)b(s)d\mu(\varepsilon,x,s,k)$$

$$N_f(z,\mu) \equiv \int \vartheta_c\Big(\xi_c^T(\varepsilon,x,s,k;z,\mu)\Big)\Big[\int_{\underline{\xi}_k}^{\xi_k^T(\varepsilon,x,s,k;z,\mu)} \xi_k\vartheta_k(d\xi_k)\Big]b(s)d\mu(\varepsilon,x,s,k)$$

Using these terms, we can write aggregate employment demand as: $N(z,\mu) = N_p(z,\mu) + N_e(z,\mu) + N_c(z,\mu) + N_f(z,\mu)$. Finally, purely for the purpose of computing entry and exit rates in our model as they are computed in the Business Dynamics Statistics database, define the total number of incumbent firms continuing into the next period as:

$$\omega_I(z,\mu) \equiv \int \vartheta_c\Big(\xi_c^T(\varepsilon,x,s,k;z,\mu)\Big)d\mu(\varepsilon,x,s,k),$$

and recall the number of entrants for next period is $\omega_e(z,\mu) \equiv \sum_{j=1}^{N_s}\vartheta_e\Big(\xi_e^T(s;z,\mu)\Big)\omega(s)$. Using these terms and the current distribution, we can pre-compute and store $\Upsilon(z,\mu)$ representing the average number of firms between this and next period,

$$\Upsilon(z,\mu) = \frac{1}{2}\int d\mu(\varepsilon,x,s,k) + \frac{1}{2}\Big[\omega_I(z,\mu) + \omega_e(z,\mu)\Big].$$

Next period's entry rate is then $\omega_e(z,\mu)/\Upsilon(z,\mu)$, and we have the exit rate once next period's trend growth shock is realized: $\Big[\int d\mu'(\varepsilon,x,s,k) - \omega_I(z',\mu')\Big]/\Upsilon(z,\mu)$.

Recursive competitive equilibrium satisfies the following requirements. (i) Households' value functions h solve (1)-(2) with consumption decision rule $c(z,\mu)$. (ii) The investment firm solves (4) given equilibrium capital sale and rental prices $p(z,\mu)$ and $d(z,\mu)$ satisfying (5) - (6). (iii) Production firms' value functions (v_0, v_1, v_2 and v_f) solve (7) - (8) and (10) - (11) given $p(z,\mu)$ and $d(z,\mu)$, the equilibrium wage $w(z,\mu)$ identified in (19) and state-contingent discount factors $z_i q(z_j; z_i, \mu)$ consistent with (3), yielding the decision rules n^*, ξ_c^T, G_f, G_c and ξ_k^T. (iv) Potential entrants' value functions (v_0^e and v^e) solve (13) - (14) given the prices $p(z,\mu)$, $w(z,\mu)$ and $z_i q(z_j; z_i, \mu)$, yielding decision rules G_e and ξ_e^T. (v) The net supply of consumption goods, $Y(z,\mu) - I(z,\mu) - \Theta_c(z,\mu)$, satisfies the household decision rule $c(z,\mu)$. (vi) The evolution of the distribution taken as given by households and firms, $\mu' = \Gamma(z,\mu)$, is consistent with the decision rules ξ_c^T, G_f, G_c, ξ_k^T, G_e and ξ_e^T.

3 Solution

Three aspects of our model complicate the search for recursive competitive equilibrium. First, our firms are confronted with three sets of binary choices involving their entry, their continuation and whether to pay fixed capital adjustment costs or convex ones. Assuming continuous distributions for ϑ_e, ϑ_c and ϑ_k, we simplify these (S,s) decision problems by determining the thresholds $\xi_e^T(s; z, \mu)$, $\xi_c^T(\varepsilon, x, s, k; z, \mu)$ and $\xi_k^T(\varepsilon, x, s, k; z, \mu)$ derived in sections 2.5 and 2.4 above. This allows firms to compute state-contingent probabilities and conditional expectations regarding their future binary choices in solving their optimization problems, and facilitates the aggregation in section 2.6.

The second challenge is the fact that our economy's aggregate state includes a high-dimensional object, the distribution of firms over idiosyncratic productivity components and capital, $\mu(\varepsilon, x, s, k)$. It is well understood that such models must be solved numerically using some type of aggregate state space approximation. Here, we proxy for the distribution with a one-dimensional object as is common in the quantitative analysis of dynamic stochastic equilibrium in heterogeneous agent economies; in solving firm and household decision problems, we replace $\mu(\varepsilon, x, s, k)$ with $m \equiv \int k d\mu(\varepsilon x s, k)$. However, because any reasonable facsimile of the U.S. firm size distribution requires extreme skewness in μ, we abandon the conventional Khan and Thomas (2003, 2008) approach of adapting the Krusell and Smith (1998) to complete markets models with a distribution of firms in favor of a new approach adapted from Reiter's (2010) Backwards Induction method. By doing so, we avoid repeatedly solving the impractically long outer-loop equilibrium simulation that would be both necessary to isolate accurate forecasting rules and more involved than usual given our household preferences.

Third, to assess the quantitative effects on entry arising from persistent reductions in productivity growth and increased uncertainty, our setting must accommodate realistically large shifts in firm values and hence countercyclical risk premia. To achieve that, we have abandoned time-separable utility in favor of Epstein-Zin preferences, which introduces household future expected value into the stochastic discount factor (equation 3 in section 2.1). Thus, we lose the convenience of expressing firm values in units of current marginal utility to handle the state-contingent aspect of their discounting in deriving their decision rules; in other words, we cannot side-step the problem of solving for $q(z_j, z_i, m)$, which requires solving for $h(0, z_i, m)$ using $c(z, m)$. To offset some of the numerical complication that implies, we have taken care to ensure that our inclusion of procyclical employment fluctuations is achieved by an equilibrium wage function depending only on the economy's exogenous state, as described in the text surrounding equations 17 - 19 of section 2.6.

The Backwards Induction method combines aggregate state space approximation with a non-parametric law of motion agents use to forecast the future proxy aggregate state. The second point of departure from a Krusell and Smith approach is that the aggregate law of motion and individual decision rules are solved simultaneously with no distinct Monte Carlo simulation. Over a grid on the approximate aggregate state (z, m), we forecast a future state pointwise, $m'_0 = g(z, m)$. Given this forecast, we solve for household and firm value functions. This implies an actual end of period distribution and an actual future approximate aggregate state, m'_1 for each (z, m). We iterate on the forecast until $m'_0 = m'_1$. This requires a proxy distribution $\mu(\varepsilon xs, k; z, m)$ at each point (z, m); given the proxy distributions we can aggregate individual decision rules into an implied m'.[8]

Our solution algorithm retains an aspect of Krusell and Smith's method that we find useful for speed and stability. We avoid direct iterations between $\Gamma^l(z, m)$ and v_0 updating that would otherwise imply revisions in price expectations within an iteration. Instead, we iterate on $c^l(z, m)$ and $m' = \Gamma^l(z, m)$, recovering the implied $h^l(0, z, m)$ and then computing the implied $q^l(z_j, z_i, m)$, $p^l(z, m)$ and $d^l(z, m)$ and sending these to an inner core that solves the implied $v_0^l(\varepsilon, x, s, k; z, m)$ and associated decision rules. Combining these with the known proxy distributions $\mu(\varepsilon xs, k; z, m)$, we compute the implied $c(z, m; l)$ and $m'(z, m; l)$ and use these to determine $c^{l+1}(z, m)$ and $\Gamma^{l+1}(z, m)$.

4 Parameterization

We are currently in the process of calibrating our annual model to match conventional moments drawn from postwar averages of macroeconomic quantity series, the average risk free interest rate and

[8] Proxy distributions are refined using model simulations and must be moment consistent; that is, the distribution of firms associated with (z, m) has $m = \int k\mu(d[\varepsilon xs \times k]; z, m)$.

equity premium, alongside several microeconomic investment rate moments from the Longitudinal Research database reported in Cooper and Haltiwanger (2006) and a large set firm age-size distribution moments drawn from the Business Dynamics Statistics database. For the results that follow, we choose a set of parameter values that illustrate the model's ability to replicate observed patterns in the age and size distributions. This particular example has just one permanent productivity type for firms, $\overline{s} = 3.5$, and assumes CRRA utility with $\sigma = \gamma = 2$. The calibration in progress assumes $N_s = 5$ and sets $\sigma = 0.67$ and $\gamma = 5$, which implies a Sharpe ratio similar to the empirical value in a representative agent version of the otherwise same model. Given these values for the inverse of the elasticity of intertemporal substitution and the coefficient of relative risk aversion, we choose $\beta = 0.96$ to imply a 4 percent annual risk free real interest rate in the steady state where TFP grows at 1.6 percent per year, and select $\alpha = 0.27$ and $\delta = .069$ to yield an annual capital-to-output ratio of 2.37 and investment-to-capital ratio of 0.069. We set $\chi_0 = 3.56$, $\chi_1 = 0.15$ and $\nu = 0.6$; taken alongside the remaining parameters, these imply a steady state labor's share of 0.70.

Recall that the representative investment firm faces a diminishing marginal product of investment in producing capital goods. We specify its production function as:

$$\Phi\left(\frac{zm' - (1-\delta)\,m}{m}\right)m = a_0 + \frac{a_1}{\omega}\left(\frac{zm' - (1-\delta)\,m}{m}\right)^{\omega} m,$$

and assign $\omega = 0.5$. Given ω, δ and the economy's average growth rate, we set a_0 and a_1 to imply that the relative price of capital goods is $p(z,\mu) = 1$, and there are no marginal adjustment costs, $d(z,\mu) = 0$, in the model's steady state.

We assume the fixed cost distributions facing production firms, ϑ_e, ϑ_c and ϑ_k, are all log-normal, with mean and variance $(-2.8, 1)$, $(-2.1, 1)$ and $(-6.9, 4)$, respectively, and the convex cost function facing firms avoiding fixed capital adjustment costs is: $\phi(zk', k) = \left(\frac{zk'}{k} - 1\right)^2 k$. We set $b(s) = as^b$ with $a = 0.03$ and $b = 0.72$. In our general case of $N_s = 5$, this form prevents more productive firms from effectively outgrowing the consequences of fixed costs; in our current example with a common s, it implies a labor-denominated cost of 0.2 per unit ξ.

Entrants' initial x values (nearness to productive maturity) are $x_1 = 0.71$, and they mature each period to $x_2 = 1$ with probability $\lambda = 0.1$. Thus, the typical member of a cohort takes roughly 10 years to reach mature size in the model's steady state. Finally, the persistent idiosyncratic shock ε is log-normal with mean 0, persistence 0.757 and a standard deviation of innovations of 0.05, which we discretize with $N_\varepsilon = 7$ values. Thus, the composite idiosyncratic state εxs has 14 values.

5 Results

We begin this section by comparing some pertinent microeconomics aspects of our model's steady state to their empirical counterparts. Table 1 presents exit rates and employment size over firm age groups drawn from the Bureau of Labor Statistics' Business Dynamics Statistics database, averaged over 1984-2006 and the corresponding distribution in the version of our model with one permanent productivity level, s, where we have selected the number of potential entrants to imply a unit measure of firms in the the stationary distribution.

Our model economy's overall exit rate matches the 8.6 percent exit rate in the data, comparing the 'all firms' entry in the two exit rate rows of Table 1. In each of columns '1' through '5', we list what percent of firms in that age group do not survive to the next year (exit rate), the average employment size among firms of that age relative to total employment (avg. rel. size) and their combined share of aggregate employment (group emp. share). The '0 - 5' column considers young firms as a whole, reporting what percent do not survive beyond youth, the average relative size of a young firm and the share of aggregate employment attributable to young firms.

TABLE 1. Distribution of Firms by Age

age group	0	1	2	3	4	5	0 - 5	all firms
A. BDS 1984-2006								
exit rate		21.03	15.85	13.36	11.61	10.50	54.45	8.58
avg. rel. size	28.26	36.97	40.08	43.51	46.34	49.11	38.78	100.00
group emp. share	2.66	2.56	2.39	2.27	2.18	2.14	14.41	100.00
B. Model								
exit rate		30.83	25.93	20.96	16.29	12.26	70.26	8.57
avg. rel. size	20.51	26.34	40.58	56.96	72.99	86.99	38.77	100.00
group emp. share	1.76	1.45	1.55	1.61	1.63	1.63	9.63	100.00

Our model emulates the essential lifecycle aspects of firms, predicting that: a typical entrant is small relative to other firms, exit rates are initially high and fall with age, and, conditional on survival, the average firm in a cohort grows over its first five years in production. Our young firms are smaller, more likely to fail and, conditional on survival, grow faster than other firms, as is true in the data. The relative size of an entrant in our model example is lower than in the BDS (21 versus 28 percent). However, the average relative size of firms aged 0-5 matches closely at 38 percent, because firms in the data grow more gradually than our model predicts. At the same time, young U.S. firms are responsible for 14 percent of overall employment, versus 10 percent in the model, because our model's failure rates in youth are exaggerated. These observations suggest that a tight quantitative fit to the age-size distribution will require a lower value of λ (maturing rate), a value of x_1 a bit nearer to 1 (and perhaps another intermediate x value), and an increased mean entry cost.

Table 2 compares our model's stationary investment rate distribution to Cooper and Haltiwanger's (2006) summary of the time-averaged investment rate distribution among large, continuing firms in the Longitudinal Research Database data over 1971-2005. Cooper and Haltiwanger define the following investment categories: inaction is $\left|\frac{i}{k}\right| < 0.01$, (active) positive and negative investment are $\frac{i}{k} > 0.01$ and $\frac{i}{k} < -0.01$, and positive and negative spikes are $\frac{i}{k} > 0.20$ and $\frac{i}{k} < -0.20$. The LRD row lists what fraction of firm observations fall into each category in the average year. The model row shows the results from a comparable sample of firms simulated in our model's steady state. Note that our model closely reproduces the 18.6 percent of observations in a typical year that are positive spikes and the 81.5 percent that are positive investments, and it does reasonably well in matching the sharp asymmetry between positive and negative spikes and between positive and negative investment. The overall share of investment attributable to firms exhibiting positive spikes in our model, 22.3 percent, also lines up well against its empirical counterpart.

TABLE 2. Distribution of Investment Rates

investment category	lumpy share	inaction	pos. spike	neg. spike	pos. inv.	neg. inv.
LRD 1971-2005	20.4	8.1	18.6	1.8	81.5	10.4
Model	22.3	0.0	18.0	4.0	81.7	18.2

On balance, these results indicate that our idiosyncratic productivity shock distribution is well suited to the capital adjustment frictions we have in the model. Our investment spikes and asymmetries stem from the fixed cost distribution and the downward pull of physical and economic depreciation. On their own, with no exemptions for small investment rates, the nonconvex adjustment costs would imply far too much inaction relative to the 8.1 percent in the data. We avoid this problem by giving firms the choice of paying convex costs instead, which allows for low inaction rates and persistence in firm-level investment. Our understated inaction rate indicates the need for a larger coefficient anchoring the $\phi(zk', k)$ function.

We include Table 3 below only to demonstrate why our model allows for differences in the permanent component of firm productivities, and why the s distribution potential entrants will draw from in our calibrated model must be a highly skewed. Examining the empirical firm (employment) size distribution, it is clear that such an extreme right tail would be impossible to achieve in our model using only a conventional log-normally distributed productivity shock process chosen to reproduce the volatility of firm level output or investment.

TABLE 3. Distribution of Firms by Size: BDS 1984-2006

size group (number of employees)	small (1 - 19)	medium (20 - 499)	large (500 +)
population share	88.27	11.37	0.36
employment share	20.13	32.34	47.53

We now consider the consequences of aggregate shocks in our model. For the moment, we assume stationary aggregate TFP shocks, and we begin with a series of results drawn from an example featuring CRRA preferences. As our primary interest is to understand the link between the sustained episode of slow GDP growth and unusually weak business formation coming out of the last U.S. recession, we take it as a prerequisite that our model must generate empirically plausible cyclical fluctuations in entry and exit rates. Table 4 reports HP-filtered ($\lambda = 400$) annual data on entry and exit rates drawn from the BDS 1977-2015 and from a 500 period stationary shock simulation of our model; in both cases, GDP is reported as percent standard deviation.

TABLE 4. Cyclicality of Entry and Exit Rates

	GDP	entry rate	exit rate
A. relative standard deviations			
LRD (1977-2015)	(1.97)	0.32	0.26
model	(2.10)	0.08	0.16
B. contemporaneous correlations w/GDP			
LRD (1977-2015)	1.0	0.45	0.15
model	1.0	0.53	-0.49

Our model succeeds in reproducing the procyclicality of the entry rate. However, it only generates about 25 percent of the observed volatility. Model exit is countercyclical, and about twice as volatile as entry. We expect persistent size effects and increased dispersion in ϑ_c will dampen some of the excessive volatility in exit. It remains to be seen whether long run risk can balance against this to increase the volatility in firm values and entry rates without inducing more countercyclical exit.

TABLE 5. Aggregate Quantity Dynamics

	# firms	consunp.	invest.	hours	msd. TFP
A. relative standard deviations					
with entry and exit	0.34	0.30	5.12	0.36	0.24
without entry and exit	0.00	0.29	4.89	0.33	0.23
B. contemporaneous correlations w/GDP					
with entry and exit	0.76	0.77	0.97	0.95	0.99
without entry and exit	n/a	0.85	0.98	0.98	1.00

Table 5 compares HP-filtered moments of aggregate quantity series from the model simulation reported in Table 4 with those from an otherwise same model simulation where entry and continuation costs are eliminated and the number of potential entrants adjusted downward to imply a constant unit measure of firms matching the steady state of the full model. For reference, the percent standard deviation of GDP in the 'without entry and exit' model is 2.17, versus 2.10 in the full model.

Entry and exit drive strong procyclicality in the number of firms. However, based on Table 5, these changes appear to have modest effects on aggregate quantity dynamics when fluctuations are driven by ordinary TFP shocks, with the exception of a considerably more variable investment series and reduced comovement between GDP, consumption and investment. However, examining the output responses below, we see a clear consequence for persistence.

FIGURE 1: GDP Response with Entry and Exit

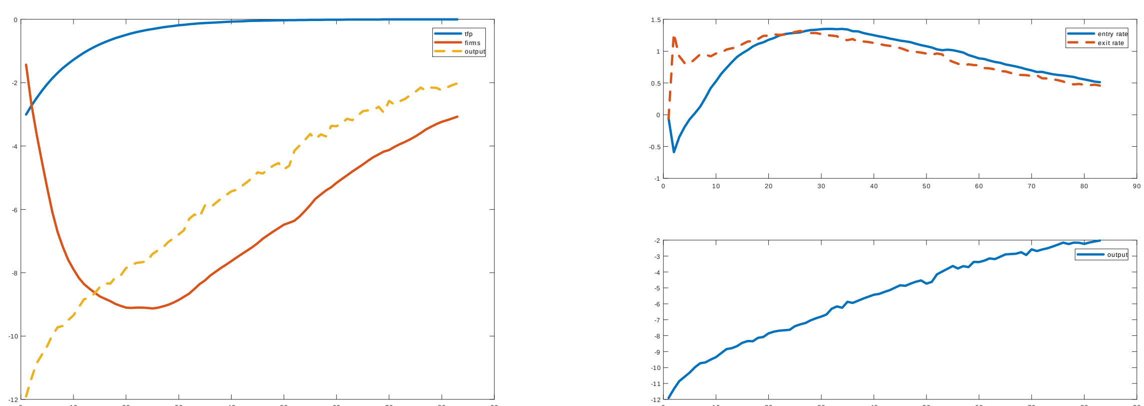
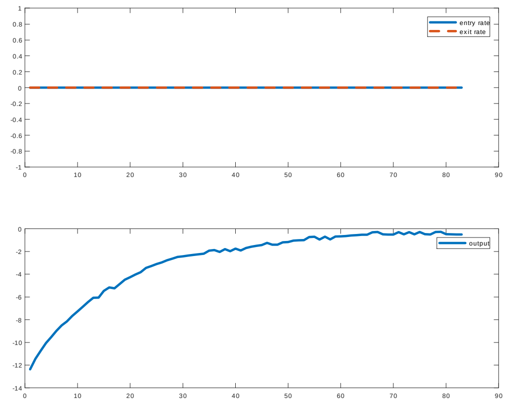


FIGURE 2: GDP Response without Entry and Exit



The left panel of Figure 1 shows a persistent and ultimately large decline in the number of firms in response to a persistent negative TFP shock. This large destruction to the stock of firms comes in roughly equal measure from a persistent fall in entry and a large rise in exit. The exit rate is more volatile, rising about 1.2 percentage points from its usual 8.7 percent, suggesting a need to adjust the distribution of continuation costs. Entry is more persistent, as it should be. Together, these changes drive larger and more persistent reductions in aggregate capital that happens in their absence, alongside a worsened allocation of that stock. As a result, GDP recovers very slowly, with a half-life of about 35 periods. This half-life is 63 percent reduced in Figure 2, where entry and exit are suppressed so that the measure of firms is fixed at 1.

The example above is extreme even relative to the large changes we have observed in recent data; however, it suggests a potentially relevant contributing factor as we consider a decade of anemic U.S. growth following the 2007 recession. Once we have succeeded in selecting distributions of entry and continuation costs to reproduce the usual volatility and cyclicality of entry and exit rates, and we reintroduce the trend shocks and non-separable utility to accommodate countercyclical risk premia, we will be poised to explore whether growth shocks and uncertainty might explain the long episode of depressed entry we have observed in the U.S. and the extent to which that may have itself contributed to the GDP and investment growth slowdown.

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