Skewed Business Cycles*

Sergio Salgado[†] Fatih Guvenen[‡] Nicholas Bloom[§]

February 14, 2019

Abstract

We show in panels of US Census and international firm data that employment and sales growth skewness is procyclical. In particular, during recessions they display a large left-tail of negative growth rates (and during booms a large right tail of positive growth rates). These results are extremely robust to different selection criteria, across countries, industries, and measures. We built a heterogeneous agents model in which entrepreneurs face shocks with time varying skewness risk that matches the firm-level distributions we document for the US. This model shows a negative shock to skewness (that keeps the mean and variance constant) to firms' productivity growth generates significant and persistent decreases in investment, hiring, growth and consumption. Hence, we argue, that periods of heightened left-tail risk help to drive business cycle fluctuations.

*For helpful comments and suggestions, we thank John Shea, the participants at the 11th World Congress of the Econometric Society (Montreal, 2015), and at the CESifo Conference on Macroeconomics and Survey Data (Munich, 2015).

[†]University of Minnesota; salga010@umn.edu.

[‡]University of Minnesota, FRB of Minneapolis, and NBER; guvenen@umn.edu

[§]Stanford University and NBER; nbloom@stanford.edu

1 Introduction

This paper studies the cyclicality of the distribution of the growth rate of firm-level and macroeconomic-level outcomes. In the prior literature, recessions have been characterized as a combination of a negative first-moment (mean) shock and a positive second-moment (uncertainty) shock (Bloom, 2014). In this paper we argue that recessions are also accompanied by negative third-moment (skewness) shocks which implies that, during economic downturns, a subset of firms and countries does extremely badly, leading to a left tail of large negative outcomes. Consequently, the skewness of the growth rates is procyclical.

We use firm-level panel data and macroeconomic-level data for several countries to document two main empirical facts. First, using firm-level data for the United States and more than 30 other countries, we show that the cross-sectional skewness of the distribution of the growth rate of several firm-level outcomes, such as sales growth, employment growth, and stock returns, is strongly procyclical, declining sharply during recessions. As an illustration of our first empirical result, the upper panel of figure 1 displays the distribution of firms' employment growth from the Census' Longitudinal Business Dynamics dataset (LBD). The solid red line shows the empirical density of employment growth pooling observations from two recession years, 2001 and 2008. Similarly, the dashed blue line shows the density for expansion years, in this case years 2003 to 2006 and 2010 to 2014. In the figure, both densities are adjusted to have zero mean so one can directly compare the changes in the tails of the distribution. One can clearly see that, relative to expansion periods, the distribution of employment growth during recessions has a thicker left tail, indicating that a large fraction of the dispersion is accounted for by the left tail of the distribution. In fact, the dispersion of employment growth is larger during recessions (the distance between the 90th and 10th percentiles widens from 0.40 to 0.45), but all of this increase is accounted for by a widening of the left tail whereas the right tail of the distribution shrinks.

Skewness can also be quantified using the Kelley skewness (Kelley, 1947), which is defined as the difference between the 90th-to-50th percentiles spread (a measure of dispersion in the right tail) and the 50th-to-10th percentiles spread (a measure of dispersion in the left tail) divided by the distance between the 90th-to-10th percentiles spread (a measure of the total dispersion of the distribution). For a distribution with a compressed upper half and a dispersed lower half (i.e., a left-tail skew), the Kelley skewness is negative. In the case of the upper panel of figure 1 we find a decline of the dispersion above the median from 0.22 to 0.20 from expansions to recession years whereas the dispersion below the median increases from 0.17 to 0.25. This differential change in the tails generates a decline in the Kelley skewness from 0.10 to -0.12 from expansion to recession periods. Put in a different way, a Kelley skewness of 0.10 indicates that during expansion, 45% of all the dispersion is accounted for by firms with employment growth below the median, whereas during recessions, this share increases to 56%. The bottom panel of figure 1 shows a similar pattern for the empirical density of sales growth for a sample of publicly traded firms. As in the case of employment growth, here we can also see that recessions are characterized by an increase in dispersion, but all this increase in dispersion is accounted for by a widening left tail. Hence, the skewness of the sales growth distribution also drops during recessions.

We also find that the skewness of firm-level outcomes is strongly procyclical at the industry-level. That is, within 2-digit NAIC industries the skewness of the firm-level sales growth, employment growth, and stock returns is negatively correlated with the within-industry economic cycle. To illustrate this point, the upper panel of figure 2 shows a bin scattered plot in which the x-axis is the average sales growth within an industry-year cell and the y-axis is the cross sectional Kelly skewness upper sales growth within the same group. Each dot represents a quantile of the average sales growth distribution after we have controlled for industry and time fixed effects. Hence, a positive relation indicates that periods of low economic activity at the industry-level are associated with a distribution of sales growth at the firm-level that is negatively skewed, whereas periods of high economic activity are associated with periods with a positively skewed distribution of sales growth.

Our second empirical finding is that the skewness of macroeconomic outcomes such as GDP growth, employment growth, and aggregate stock returns is also procyclical. To show this we exploit quarterly data on GDP growth and employment growth for over 40 countries and daily returns on an stock index to measure the skewness of the distribution of these outcomes over a centered window. We find that during recessions, the skewness of the distribution collapses as countries experience large negative outcomes. To illustrate this finding, the bottom panel of figure 2 shows the relation between the annual growth rate of quarterly GDP and the skewness of the same variable for the United States. Similarly to the evidence based on microeconomic data, a positive relation between the GDP and the time series skewness indicates that during recessions, the distribution of aggregate outcomes shifts to the left, increasing the likelihood of negative outcomes. Below we show this pattern is also observed across countries and for other aggregate outcomes such as employment growth and aggregate stock returns.

Motivated by the robust empirical evidence that the skewness of micro- and macro-level outcomes decreases during economic downturns, in the second part of the paper we build a heterogeneous agents model. The key feature of the model is the presence of a large number of entrepreneurs that face shocks with time-varying risk that feature both, time-varying dispersion and time-varying skewness. In order to capture the potentially non-linear responses of firms to shocks, we assume that entrepreneurs are risk averse, face a combination of convex and non-convex adjustment costs to capital, and can invest in capital and in a risk-free asset. We numerically solve the model and estimate the parameters so that our model reproduces the decline in skewness of the sales growth distribution we observed among US firms during a typical recession. Our results suggest that the presence of risk-aversion and adjustment costs, which both generate non-convexities in the decision rules of the firms, are not sufficient to reproduce the large swings in the skewness of firm-level outcomes that we see in the data. Hence, the stochastic process we estimate shows a large deviation from normality with negative skewness.

In our main quantitative exercise we analyze the aggregate effect of a pure skewness shock: a decline in the skewness of the shocks affecting firms while keeping the mean and dispersion of shocks constant. Our model predicts that a change in the skewness of the distribution of firm-level shocks has a negative effect in gross domestic output (GDP) of around 0.6%, and this decline is quite persistent as output stays below its pre-shock level even eight quarters after the shock. This is in contrast to the standard uncertainty shock studied in the literature that typically generates a sharp drop and rapid rebound in output. The significant and persistent drop in output generated by a change in the skewness is driven by a decline in capital investment which is the result of two factors: first, the presence of fixed adjustment costs creates a real options effect that reduces the incentives of firms to invest in capital when skewness declines, and second, the drop in skewness makes capital more risky, inducing a higher investment in the risk free asset. Relative to the standard uncertainty shock (a symmetric increase in dispersion), in our model a decline in skewness commands a widening of the left tail of the firm productivity distribution without a corresponding widening of the right tail (an asymmetric increase in dispersion), which acts as an additional force that further pushes down aggregate economic activity.

Related literature

This paper is related to several strands of literature. First, a growing body of research studies how macroeconomic models respond to non-Gaussian shocks. Several authors have suggested that rare disasters—presumably arising from an asymmetric distribution of shocks—are useful in explaining large fluctuations in economic activity, such as the Great Recession, and in accounting for the movement of real and financial variables. Reviving the ideas introduced first by Rietz (1988), Barro (2006), and Barro and Ursua (2011) use a panel of countries to estimate the probability of large macroeconomic disasters and argue that these low-probability events can have substantial implications for asset pricing. Based on this evidence, Gourio (2012) extends the standard dynamic stochastic general equilibrium model to include the probability of a small risk of a large negative shock. He finds that an increase in the probability of a disaster induces a contraction in output, employment, and especially, investment. Gourio (2013) finds that disaster risk can also explain the volatility and counter cyclicality of credit spreads. Kozlowski *et al.* (2016) argues that extreme rare events such as the Great Recession can have a long lasting macroeconomic impact because these rare events affect the beliefs of economic agents. Orlik and Veldkamp (2014) analyzes how uncertainty can endogenously vary over time as a consequence of changes in tail risk. Wachter (2013) shows that a small and time-varying probability of a disaster can help to account to the equity premium and the excess returns predictability whereas Kilic and Wachter (2015) study the effects of a disaster shock in the context of a search and matching model. They show that including disaster risk improves the performance of the model in terms of unemployment dispersion, without resorting to large and volatile productivity shocks. We contribute to this literature by showing that tail risk is an intrinsic part of the business cycle not only at the macroeconomic or sectoral level but also at the microeconomic level.

Second, our paper relates directly to the study of the effects of uncertainty on firms' decisions. Bloom *et al.* (2018) show that an increase in the dispersion of firms' shocks can lead to a recession. The main propagation mechanism is a real-option channel: in the presence of fixed adjustment costs or irreversibility, an uncertainty shock makes firms more cautious and less willing to invest and hire because of the irreversible cost induced by these decisions, generating a drop in aggregate economic activity. Arellano *et al.* (2010) find that an increase in uncertainty can lead to a reduction in employment and output in a model where firms are financially constrained, whereas Gilchrist *et al.* (2014) evaluate quantitatively which of these channels—financial frictions or the wait-and-see behavior generated by the adjustment costs of capital and labor—is more important in accounting for the empirical evidence. Berger *et al.* (2017) show that an increase in the realized volatility of shocks, rather than an increase in the uncertainty about future outcomes, has a significant impact on aggregate economic activity. They rationalize these facts in a model where aggregate productivity shocks are negatively skewed. Our paper contributes to this literature in two ways: first, by documenting that the surge in dispersion observed during recessions is related to an increase in the probability of large negative shocks, and second, by studying how asymmetric changes in risk could generate larger effects in economic activity than those found so far in the literature.[1]

Our results at the macro-level follow a long literature on the asymmetric nature of business cycles. Research by Neftci (1984), DeLong and Summers (1984), Sichel (1993), Kontolemis (1997), and more recently Jovanovic (2006), McKay and Reis (2008), and Ordonez (2013) have all studied the asymmetry of the distribution of growth rate of macro aggregates and how it changes over the busyness cycle. Our work is also related to Adrian *et al.* (Forthcoming) who study the empirical distribution of GDP growth and its relation with financial conditions using data of the United States in a quantile regression setup. Similar to the evidence we present in this paper, the authors report that recessions are associated to a decline in the skewness of the distribution of GDP growth whereas during expansions the distribution is symmetric. Additionally, they report that the mean and the left tail of the distribution of GDP growth exhibit strong time series variation. Our paper differs to Adrian *et al.* (Forthcoming) in that we analyze the evolution of the skewness of several micro and macro level outcomes, not only GDP growth, both for the United States and for a large sample of countries.[2]

Finally, a growing literature analyzes the behavior of the skewness and higher order moments of the distribution of economic outcomes in different contexts. For example, Guvenen *et al.* (2014) study the characteristics of individual earnings risk. They find that idiosyncratic shocks do not show any countercyclical variation in dispersion but do exhibit strong pro cyclical skewness. That is, during recessions the upper tail of the

[1]Several other papers also suggest that uncertainty shocks can have a negative impact on macroeconomic aggregates. See for instance Bachmann and Bayer (2013), Jurado *et al.* (2015), Leduc and Liu (2016), Basu and Bundick (2017), Alfaro *et al.* (2018), among others.

[2]Several other authors have studied the distribution of aggregate time series and their relation with macroeconomic performance. For instance Jurado *et al.* (2015) characterizes time-varying uncertainty by using a large set of macroeconomic and financial time series. See also McQueen and Thorley (1993), Hamilton (1989), Falk (1986), Bai and Ng (2005), Fagiolo *et al.* (2008), Bekaert and Popov (2012), Ferraro (2018), among others.

earnings growth rate distribution collapses, while the left tail becomes thicker, implying a greater probability of observing large negative shocks. Our analysis is in the same spirit as theirs but focuses on firm- and macro-level variables instead of workers' wages. Kehrig (2011) uses a panel of manufacturing firms to study the cyclicality of the dispersion of the distribution of firm-level productivity. He finds that the cross-sectional dispersion increases during recessions and that it is left tail that accounts for most of this increase. Our results are similar as we find that most of the increase in dispersion of the sales growth distribution comes from a left tail that stretches out while the right tail changes little. Ilut *et al.* (2018) study the asymmetric response of firms to news. Their analysis predicts that the distribution of growth rates of employment should be negatively skewed, which is confirmed by Census data. We find similar results; however, our focus is the variability of the skewness of different firm and macro level outcomes and how it moves during the business cycle. Decker *et al.* (2015) document the declining trend in the skewness of the firm growth rate distribution in the United States. They find that this decline is due to the drop in the number of young high-growth firms, especially during the post-2000 period. Distante *et al.* (2013) characterize the distribution of firm-level growth using a quantile regression approach. As in our paper, they find strong pro cyclical skewness, whereas changes in the dispersion are of second-order importance.

The rest of the paper is organized as follows. Section 2 described the data we use and describes the basics statistics discussed in the empirical section. Section 3 shows the main empirical results of our paper, that is, that the skewness of several firm and macro level outcomes is procyclical. Section 4 shows the model and section 5 shows our quantitative results. Section 6 concludes.

2 Data and Measurement

2.1 Data and Sample Selection

Our analysis is based on four large datasets. First, we extract panel data on employment at the firm-level from the Census Bureau's Longitudinal Business Data Base (LBD). The LBD provides high quality measures of employment, wage bill, industry, and firm age for the entire non farm business sector linked over time at the establishment level from 1976 to 2015. From the LBD we construct employment at the firm level and use it to calculate cross sectional measures of skewness at narrow firm population groups.

Second, we draw panel data information of publicly traded firms from the CRSP-Compustat merged dataset, which contains information on sales, employment, stock prices, and other firm-level outcomes. We use data on quarterly sales, daily stock prices, and annual employment from 1970 to 2017, and we restrict attention to a sample of firms with more than 10 years of data to minimize the types of compositional issues identified in Davis *et al.* (2006).

Third, we extend our firm-level evidence to other countries, both developed and developing. We consider a dataset of firm-level panel data from 44 countries with of annual sales and annual employment information between 1986 and 2016 obtained from the Bureau van Dijk's Osiris dataset.[3] To ensure that changes in the sample of firms do not bias our results, we focus on firms that are present in the sample for 10 years or more. Additionally, we restrict our sample to country/year cells with more than 100 firms, countries with at least 10 years of data, and years with 5 countries or more. We complement this dataset with information on firm-level stock prices obtained from the Global Compustat dataset which contains daily stock price information for firms in 23 countries from 1986 to 2018.

Finally, we analyze whether the patters that we observe at the firm-level are also found at the macro-level. In particular, we study a panel of countries with information on quarterly GDP growth, employment growth, and daily returns data on a stock market index. We construct an unbalanced panel dataset from 1970 to 2017. Quarterly GDP data is obtained from the OECD reports and covers a total of 45 countries. Quarterly employment data is also obtained from the OECD records. For most of the countries in the sample, quarterly employment data starts around 1995. Hence, when we consider measures of aggregate employment growth our sample is reduced to 35 countries. Finally, we collect stock price index information for a total of 30 countries.

2.2 Measures of Dispersion and Skewness

Most of our results refer to the growth rate of firm–level outcomes such as real sales, employment, and stock returns. We measure the growth rate as the log-difference between periods t and $t+k$ where t is a quarter for real sales and stock returns, and a year in the case of employment. As an alternative measure of growth we use the arc-percentage change between periods t and $t+k$. The arc-percentage change is defined as

[3] Osiris is a public company dataset published by Bureau van Dijk. Public companies are those with listed public equity.

$2\left(x_{i,t+k}-x_{i,t}\right)/\left(x_{i,t+k}+x_{i,t}\right)$. This measure has been popularized in the firm dynamics literature by Davis and Haltiwanger (1992) and has the advantage that, while it is similar to a percentage change measure, it allows for entry/exit by including both time t and $t+k$ measures in the denominator, one of which is allowed to be zero.[4]

Our main measure of dispersion is the cross-sectional spread between the 90th and 10th percentiles, denoted by $P9010_t$, where t is a quarter or a year depending on the dataset. In addition, we use the difference between the 90th and 50th percentiles, denoted by $P9050$, and the difference between the 50th and 10th percentiles, denoted by $P5010_t$, as measures of dispersion in the right and left tails of the distribution. Finally, our preferred measure of skewness is the Kelley's measure, which is defined as

$$KSK_t = \frac{(P90_t - P50_t) - (P50_t - P10_t)}{P90_t - P10_t} \qquad \in [-1, 1]. \tag{1}$$

Relative to the third standardized moment (which is another measure of skewness), this measure has the advantage of being robust to potential outliers.[5] A negative value of this measure indicates that more than 50% of the total dispersion comes from the left tail and the distribution is negatively skewed. In the same way, a positive value indicates a positive skewed distribution, with more dispersion coming from the right tail. Clearly, this measure is equal to 0 if the distribution is symmetric, such as for the Normal distribution.

At the macro level, we calculate dispersion and skewness of the growth rate of GDP per capita, employment growth, and daily stock returns over a trailing window of three years. That is, to calculate the skewness of GDP growth in quarter t we consider the growth rate of GDP between quarters t and $t-12$. Additional details on the data construction, selection criteria, and moment calculation can be found in Appendix A.

[4]Notice that, for a firm with positive value of $x_{i,t}$ which is inactive in period $t+k$, and henceforth has a value of $x_{i,t+k}$ equal to 0, the arc-percent change takes the value of -2. Similarly, for an entering firm (that is, $x_{i,t}$ is equal to 0 but $x_{i,t+k}$ is positive), the arc-percent change takes the value of 2.

[5]An important drawback of this measure of dispersion is that it is invariant to 20% of the observations in the sample (the top and bottom 10% of the distribution). Alternatively, one could write KSK_t by using the 95th and 5th percentiles of the distribution. Our main results, however, are not particularly sensitive to changes in the percentiles used to calculate KSK_t. Additional measures of skewness can be found in **?**.

3 Skewness over the Business Cycle

In this section we first show firm-level growth has a larger left tail in recessions in both the United States (section 3.1) and internationally (section 3.2), and then confirm this result holds even for industry – rather than macro – recessions (section 3.3). Finally, in section 3.4 we show that macro growth is also more left-skewed in recessions.

3.1 Firm Level Skewness: US data

The first contribution of our paper is to show that the skewness of the growth rates of firm–level outcomes varies over time and is strongly procyclical, declining substantially during recessions. We start by considering the evolution of the skewness of the distribution of the growth rate of quarterly sales for a sample of publicly traded firms drawn from Compustat. The left panel of figure 3 shows that the Kelley skewness of the cross sectional distribution of one year log-changes of quarterly sales is strongly procyclical, declining from an average of 11% at the peak of a recession to an average of -15% at the trough, that is, a drop of 25 percentage points. To calculate Kelley skewness we weight each observation by firm's sales so that our measures reflect the underlying firm-size distribution.[6] The right panel of figure 3 shows more clearly that the decline of the skewness is generated by a large increase in the dispersion below the median as the spread between the 50th and the 10th percentiles of the sales growth distribution ($P5010_t$, solid blue line) increases more during recessions than the spread between the 90th and the 50th percentiles, which does not show a cyclical pattern ($P9050_t$, dashed red line).

To have a better sense of the magnitude of the change of skewness and its relation with the cycle, the left panel of table I shows a set of time series regressions where the dependent variable is the Kelley skewness of the cross-sectional distribution of different firm-level outcomes for the United States. In all regressions, the independent variable is the growth rate of real GDP per capita. To better compare the magnitude of the coefficient of the GPD per capita across different firm-level outcomes we have normalized the time-series of GDP growth to have unitary variance so that the coefficient can be interpreted as the effect of a change in GDP per capita of one standard deviation.

Column 1 confirms that the skewness of the sales growth distribution is procyclical.

[6] In particular, we weight sales growth of firm i in period t by the average sales growth between periods t and $t+1$, that is $\overline{S}_{i,t} = 0.5 \times (S_{i,t} + S_{i,t+1})$.

We find that one standard deviation in GDP per capita is associated with a change in the skewness of 4.8 percentage points. We also find procyclical skewness in other firm level outcomes. For instance, column 3 shows that a change in GDP of one standard deviation is associated with a 2.7 change in the skewness of the distribution of firm-level stock returns. Furthermore, recessions also coincide with a decline in the skewness of the distribution of permanent shocks to firms, measured using three-years growth rate of sales and returns. In fact, a change of 1 standard deviation in GDP growth is associated with a decline of 2.9 percentage points of the skewness of the sales growth distribution (column 2 in table I), and a decline of 4.0 percentage points of the skewness of the distribution of stock returns.[7] Hence, declines in economic activity are associated with changes in the distribution of firm-level outcomes in the short and in the long term.[8]

To evaluate whether the cyclical patterns found in publicly traded firms are also observed in the rest of the economy we draw panel data information of employment at the firm level from the Longitudinal Business Database (LBD).[9] The LBD covers all the establishments and firms in the US private sector excluding farms. Currently the data runs from 1976 to 2015 and contains detailed industry information, location, employment, payroll, and firm-age for all firms with at last one paid employee. From this dataset we construct employment at the firm-level summing up the employment across all establishments of the firm and calculate the growth rate of employment as the log-change of employment between years t and $t+k$. We then calculate different moments of the employment growth distribution.[10]All moments are weighted by firm size so as

[7]Table A.8 in appendix B shows that the pro cyclicality of skewness of firm-level outcomes remains strongly procyclical if we residualize the growth rate of sales at the firm level by firm's observable characteristics and fixed heterogeneity. Similarly, we find that the cross sectional skewness of the distribution of the growth rate of sales-per-worker – which some researchers interpret as a measure more closely related to firm's productivity – is also procyclical.

[8]Table A.7 in appendix B shows the results for a similar set of regressions where the dependent variable is the dispersion of the growth rate of different firm level outcomes measured by the 90th-to-10th percentiles spread. The results are in line with the previous literature that show that dispersion of different firm-level outcomes is countercyclical. Moreover, we find that the kurtosis of the distribution of sales growth and stock returns is also procyclical (table A.8 in appendix B).

[9]Publicly traded firms represent a sizable fraction of the employment and sales in the United States. Their cyclical patterns, however, might not reflect the evolution of the US economy as a whole. For instance, the cross sectional dispersion of the sales growth and employment growth for publicly traded firms shows an increasing trend (Comin and Philippon (2005)) which is not observed among privately held firms (Davis *et al.* (2006)).

[10]Due to disclosure requirement, the moments of the employment distribution where calculated using averages around the point estimates. For instance, the 90th percentile of the employment distribution in a given year is the employment weighted average across all observation between the 89th and the 91th percetiles. Other percentiles of the wage growth distribution are constructed in the same fashion.

the reflect the underlying distribution of workers across firms. Additional details on the data selection and moment calculation can be found in appendix A.2.

Similar to what we observe for quarterly sales, the skewness of the employment growth distribution displays strong procyclicality shown here by the blue-solid line in the left panel of figure 4. Three points are worth noticing. First, in every recession, the skewness drops from an average of 10 percent at the peak of the recession, indicating a larger fraction of the dispersion accounted for by the right tail of the employment growth distribution, to an average of -10 percentage points in the trough of the recession. This is a large shift in the distribution of employment growth. Second, the Great Recession shows the largest decline of the skewness of employment growth distribution with a drop of almost 30 percentage points from peak to trough. Third, the skewness of the employment growth in the LBD and among publicly traded firms display similar patterns, at it is show by the dashed-black line in figure 4. Columns 5 and 6 of table I further confirm our results, showing that the skewness of one-year (three years) changes in employment is strongly procyclical, declining 4.6 percentage points (1.1 percentage points) when GDP growth declines in one standard deviation.

It is also important to separate what part of the changes in skewness observed during recession periods comes from changes of the right tail from change of the left tail of the distribution. The right panel of figure 4 shows the 90th-to-50th percentiles differential and the 50th-to-10th percentiles differential. Putting aside the declining trend of the dispersion of employment growth (an issue discussed for in Decker *et al.* (2015)), it is clear that dispersion increases both at the right and left ends of the employment growth distribution, but this increase is uneven, as the 50th-to-10th spread rises much more during every recession, commanding a drop in skewness.

The large sample size of the LBD allows us to address some concerns about the evidence presented so far. The first relates to the calculation the Kelley skewness. By considering the 90th and the 10th percentiles we are effectively dropping 20 percent of the distribution, which is potentially important, as most of our results hinge on the differential response of the tails of the distribution of firm-level outcomes to aggregate economic conditions. We can modify the Kelley skewness in equation 1 to reduce the proportion of the sample left out at the tails. In particular, we calculate the skewness considering the 95th and 5th percentiles of the distribution, and, as a third measure we consider the 2.5th and 97.5th percentiles. The upper left panel of figure 5 shows that decreasing the proportion of the distribution left out of the sample does not change substantially the cyclical properties of the skewness of the distribution of employment growth.

A second concern relates the entry and exit of firms. Our main results are based on the distribution of employment growth calculated as the log-change of firm employment. If a firm exits the market due to a change in aggregate economic conditions or a new firm enters the market, our measure of growth rate, and consequently, the skewness of the distribution, will not take them into account. To solve this problem we calculate the skewness of the employment growth distribution considering the arc-percent change of employment which takes into account the entry and exit of firms. The right panel of figure 5 shows that the cyclical properties of the skewness of employment growth do not change substantially when incorporating the entry and exit of firms. Taken together, these results further confirm that cyclical patterns of the skewness of firm-level outcomes are not a feature of a small group of publicly traded firms but are observed across the entire US economy.

3.2 Firm-level Skewness Across Countries

Are the cyclical properties of the skewness of firm-level outcomes a characteristic of the US economy or are experienced across a broader set of countries? To answer this question we use firm-level data from a panel of countries to show that the skewness at a microeconomic level positively co moves with the country business cycle.

The left panel of figure 6 displays the empirical density of the distribution of the growth rate of annual real sales (in US dollars as of 2005) for a panel of firms spanning across 44 countries over the period 1986 to 2016.[11] To construct this figure, we start by pooling all the firms available in the panel and then normalize the distribution to have zero mean and unit variance. The solid red line is the density of the log change of real sales during recession periods, where a recession is defined as a year in which the annual growth rate of GDP is in the first decile of the country-specific GDP growth distribution. The dashed black line is the density of sales growth during expansion periods defined as years in which GDP growth is above the first decile of the country-specific distribution of GDP growth. The vertical solid (dashed) lines, from left to right, are the 10th and 90th percentiles of the distribution of sales growth during recession (expansion) periods.

[11] Table A.3 in Appendix A shows the number of years and firm-level data available for each of the countries in the sample.

Similar to the results in figure 1, the dispersion of the sales growth distribution increases as the difference between the 90th and the 10th percentiles of the distribution widens from 1.73 to 1.95, an increase of 22 log points. This increase, however, is mostly attributable to a change in the left tail of the distribution that stretches out, with a corresponding increase in the spread between the 50th and 10th percentiles from 0.77 to 0.94, or an increase of 17 log points, which is almost three times as large as the increase in the spread between the 90th and 50th percentiles which increases from 0.96 to 1.02, or 6 log points. Hence, the Kelley skewness drops from 0.10 to 0.04.

We also find procyclical skewness in other firm-level outcomes such as employment growth and stock returns. The right panel of figure 6 summarizes this result. Here, each country-period is placed into a bin based on the deciles of the country-specific distribution of the growth rate of annual GDP with bins, from 1 to 10, where 1 is the lowest decile of growth and 10 is the highest decile. So, for example, for the United States, bin 1 is for growth rates below –1.2%, bin 2 is for growth rates between –1.2% and 0.1% and so on, whereas for the United Kingdom, the first bin is for growth rates below –1.1%, the second bin is for growth rates between –1.1% and 0.2%, and so on. The skewness measures plotted for each bin are averages over each country-period in the bin. In each decile, we plot the Kelley skewness for three different distributions—with each measure normalized to a mean 0 and standard deviation of 1: the within-country cross-sectional distribution of firm-level real sales growth, the within-country distribution of employment growth, and the within-country cross-sectional distribution of firm-level daily stock returns. Here, a positive trend from low to high deciles of the GDP growth distribution indicates strong pro cyclicality of the skewness at the micro-level. In other words, the skewness is low when the growth rate of GDP is in its lowest decile, which is typically during a recession.

The right panel of table I exploits our cross-country data to evaluate more systematically the relation between micro skewness and aggregate economic conditions. In particular, column 7 shows a country panel regression in which the dependent variable is the cross sectional skewness of sales growth across all firms within the country. The business cycle is captured by the growth rate of GDP per capita which is again normalized to have unit variance so the results can be directly compared with those obtained using US data only. The regression also includes a full set of time and country fixed effects to control for aggregate economic conditions that might affect all countries – such as the Great Recession – or fixed differences across countries. Similarly to our results for the United States, we find strong procyclical skewness for sales growth, employment growth, and firm's stock results. This further confirms that the decline in the skewness of firm-level outcomes is a robust feature of the business cycles.[12]

3.3 Within Industry Cycles and Skewness

In table II we show an additional set of results which disaggregate our measures of skewness down to the industry level, finding again that it is positively correlated with economic activity within industries, at different horizons, and for different firm-level outcomes such as real sales growth, employment growth, and stock returns. To do this, we first use firm–level data from a sample of firms from Compustat to examine the relation of the industry cycle and the skewness of sales growth, employment growth, and quarterly returns within NAIC 2-digit industry-period cells.[13] Table II displays a series of industry panel regression in which the dependent variable is the Kelley skewness of the growth rate of different firm-level outcomes across all firms within a industry-period cell. The specification we run is,

$$KSK_{j,t} = \alpha_j + \gamma_t + \beta\overline{\Delta S}_{j,t} + \epsilon_{j,t}, \tag{2}$$

where $KSK_{j,t}$ is the Kelley skewness of annual growth of quarterly sales (columns 1 and 2), annual growth employment (columns 3 and 4), and annual returns of stock returns (columns 5 and 6) in sector j in period t, α_j is an industry fixed effect, and γ_t is a period fixed effect. The main explanatory variable in each regression, $\overline{\Delta S}_{j,t}$, is the average of the real sales growth within industry j in period t. Here we have re scaled the real sales growth within each sector to have a variance of 1 so that the regression coefficient should be interpreted as the effect of a change in the within industry sales growth of one standard deviation and can be directly compared to the coefficients in table I.

Column 1 shows that the skewness of sales growth is significantly lower during industry slowdowns. Specifically, a one standard deviation decline in the within industry sales growth is correlated with a decline in the skewness of 14 percentage points. This is almost three times higher than the effect of a change in one standard deviation in GDP

[12] In appendix B we show that the decline in the skewness of firm-level outcomes across countries is also generated by a left tail that is strongly counter cyclical (figure A.1). Furthermore figure A.4 shows that countries experience a decline in the skewness of the distribution of firm-level outcomes independently on their level of development.

[13] Table A.9 in appendix B shows the distribution of observations across industries.

growth on the skewness of sales growth across all firms in the economy (column 1 in table I). Similarly, a one standard deviation decline of the within-industry sales growth is correlated with a 8.8 percentage points decline in the skewness of employment growth and 1.6 percentage points decline in the skewness of the distribution of stock returns. Importantly, since these regressions include a full set of period and industry dummies, the relation between skewness and the business cycle is independent of the aggregate economic conditions.[14]

The effect of the within-industry economic cycle has an persistent effect on the skewness of the distribution of firms' outcomes. To see this we switch the dependent variable in equation 2 for the skewness of three-years growth while keeping the right hand side the same. Hence, the value of β captures the change of the skewness of long-term growth of the firm associated to a short-term change in industry economic conditions. The effect of industry cycle on the skewness of the distribution of long-term growth rates is positive and statistically significative: the skewness of the growth rate of sales at three years drops 5.4 percentage points when the within-industry average sales growth drops by one standard deviation (column 2 of table 2). The results are similarly significant for the employment growth (column 4) and stock returns distributions (column 6).[15] In summary, both at the aggregate and industry-level, slowdowns in growth are associated sharp drops in the cross-sectional skewness of real sales, employment growth, and stock returns.[16]

In the Census LBD data we follow the same approach as in table II and run a series of industry panel regressions where the dependent variables are moments of the employment growth distribution. We measure the within-industry business cycle by the average employment growth within each industry (denoted by $\overline{\Delta E_{j,t}}$). We also include industry and time fixed effects so as to control for fixed differences across industries and aggregate fluctuations affecting all industries. Confirming our results from previous sections, column 1 of table III shows that within-industry skewness is highly procyclical: A one percent decrease in the within industry employment growth is associated to a

[14]We have explored how skewness changes over the business cycles within other firm groups. For instance, figure A.3 in appendix B shows that the skewness of sales growth and stock returns is procyclical within employment size groups.

[15]Table A.6 in appendix B shows a similar set of regressions for the dispersion of firm-level outcomes. The results are broadly consistent with the previous literature in that dispersion is counter cyclical.

[16]The effects of the industry cycle on the distribution of firm outcomes are persistent and economically significant even at longer horizons. For instance, a one standard deviation decline of the within industry median sales growth is associated to a 4.8 points decline of the distribution of five-years sales growth.

decline of 1.0 percentage point in the skewness of the distribution. This decline is the result of the uneven response of the dispersion below the median relative to the response of the dispersion above the median. In fact, as it is shown in column (4) and (5) of table III, the 50th-to-10th percentiles spread of the within industry distribution of employment growth is more than twice as responsive to changes in the average industry condition as the 90th-to-50th percentiles spread.[17] We also find counter cyclical dispersion – column (6) – which, is driven mostly by a left tail that becomes fatter during recessions.

3.4 Macroeconomic Skewness

The second empirical contribution of our paper is to show that during recessions the distribution of macroeconomic outcomes such as GDP growth, employment growth, and aggregate stock returns also declines. To see this, we exploit macroeconomic time series for a sample of more than 40 countries and we construct time-series measures of skewness for each country in a moving window. Figure 7 summarizes our main point by plotting the skewness of different macroeconomic outcomes across deciles of the GDP growth distribution for the United States only and for the (average) value of the skewness across all the countries in the sample. As before, a positive relation between the skewness and the business cycle indicates that recessions are characterized by a decline in the skewness, in this case, of macroeconomic outcomes.

Table IV evaluates the relationship between our macro measures of skewness in the economic activity across countries. In particular, we run the following specification,

$$KSK_{i,t}\left(x\right) = \alpha_i + \gamma_t + \beta \Delta GDP_{i,t} + \epsilon_{j,t},$$

where $KSK_{i,t}\left(x\right)$ is the skewness the a macro–level outcome, x, for country i in period t, α_i is a country fixed effect to control for fixed different in skewness across countries, and γ_t is a year-fixed effect to control for aggregate, cross-country, conditions. The left panel of table IV shows the results of this regression for the United States (fixed effects not included here) and for our sample of countries. We consider three main variables: the growth rate of GDP per capita, aggregate employment growth, and the stock returns of country specific stock market index. As in the previous section, we have re scaled the within–country GDP per capita to have unitary variance so each coefficient can be

[17] Appendix figure 5 shows the time series of the P9050 and the P5010 measures of the employment growth distribution. The cyclical patterns at the more aggregate level confirm regression results presented in table III.

interpreted as the effect of a change of one standard deviation of the within-country GDP growth.

Columns 1 to 3 of table IV show that within the United States, periods of low economic activity are associated with a significant decline in the skewness of the distribution of macroeconomic outcomes. In other words, recessions are also accompanied by a shift of the distribution of macroeconomic outcomes. In terms of magnitudes, we find that a decline of the GDP growth of one standard deviation implies a drop in the skewness of almost 7 percentage points in the GDP growth distribution and 10 percentage points decline in the employment growth distribution, which is twice as large as the coefficient we found when looking at firm-level skewness. Columns 4 to 6 confirm these results when considering the cross section of countries.

In summary, we have shown that slowdowns in economic activity are accompanied by large declines in the skewness of the distribution of microeconomic and macroeconomic outcomes, both in the United States, within different industries, and across different countries.

4 Model

In this section we analyze the quantitative impact of a variation in the skewness on firm-level shocks in the context of an heterogeneous agents model. Specifically, we consider an economy populated by a large number of households/entrepreneurs that have access to a technology that uses capital and labor to produce an homogeneous good. Entrepreneurs can save in capital and in a risk free assets that pays a fixed return. Crucially, the shape of the distribution of idiosyncratic shocks changes with aggregate business conditions so as to reflect the time-varying nature of the distribution of firm-level outcomes. Furthermore, in order to capture potential non-linearities in the response of entrepreneurs to aggregate and idiosyncratic shocks we assume that entrepreneurs are subject to capital adjustment costs.

4.1 Entrepreneurs

4.1.1 Production Technology

The economy is populated by a large number of infinitely lived heterogeneous households that use capital and labor inputs to produce an homogeneous good by means of a decreasing returns to scale production function. Specifically, each entrepreneur j produces output according to,

$$y_{j,t} = A_t e_{jt} k_{j,t}^{\alpha} n_{j,t}^{\nu}, \text{ with } \alpha + \nu < 1.$$

The aggregate productivity shocks, denoted by A_t, follows a standard first-order autoregressive process, given by

$$\log A_t = \rho_A \log A_{t-1} + \sigma_\nu \nu_t,$$

where ν_t is a Gaussian innovation with zero mean and unitary variance. We assume the idiosyncratic process $e_{j,t}$ is given by,

$$e_{j,t} = \rho e_{j,t-1} + \epsilon_{j,t}, \tag{3}$$

where the innovation $\epsilon_{j,t}$ is assumed to have zero mean, time varying variance, denoted by $\sigma_{\epsilon,t-1}$ and, time varying skewness, denoted by $\gamma_{\epsilon,t-1}$. Notice we are assuming that the distribution of innovations in period t depends on the values of the variance and skewness observed in period $t-1$. Hence, an increase in risk, in the form an increase in dispersion or a decrease in the skewness in period t, represents news about the future realization of $\epsilon_{j,t}$ and not a current change in the realizations of $\epsilon_{j,t}$.

4.1.2 Adjustment Costs

It is possible that the distribution of firm-level outcomes changes asymmetrically during the business cycles because of the existence of adjustment cost or other rigidities that distort firms' responses to shocks. In other words, the distribution of firm-level outcomes, such as sales growth or employment growth, can change asymmetrically because of the endogenous response of firm to symmetric – gaussian – shocks. Hence, in order to capture the asymmetric response of firms to shocks we consider combination of convex and non convex adjustment cost to capital. In particular, we assume that physical capital depreciates at the rate δ^k and adjustment costs are equal to the sum of a fixed disruption cost, ϕ_1, which the entrepreneur pays for any investment or disinvestment, a quadratic adjustment cost, ϕ_2, and a resale cost for disinvestment, ϕ_3. Hence, the adjustment cost function for capital input is given by,

$$\phi\left(k_{j,t+1}, k_{j,t}\right) = \phi_1 \mathbb{I}_{|i_{j,t}|>0} y_{j,t} + \frac{\phi_2}{2}\left(\frac{i_{j,t}}{k_{j,t-1}}\right)^2 + (1-\phi_3)\left|i_{j,t}\right| \mathbb{I}_{i_{j,t}<0}, \tag{4}$$

where $i_{j,t}$ is the entrepreneur's investment in capital given by

$$i_{j,t} = k_{j,t+1} - \left(1 - \delta^k\right) k_{j,t}. \tag{5}$$

4.1.3 The Problem of the Entrepreneur

Denote entrepreneur's consumption by $c_{j,t}$ which is valued by an iso-elastic utility function given by $u\left(c_{j,t}\right) = \left(c_{j,t}\right)^{1-\sigma} / \left(1-\sigma\right)$. Entrepreneurs do not value leisure and supply one unit of labor which they use in running their own firm (they cannot work for someone else's firm). They can save in capital and in a risk free asset that pays an interest rate r_t. Denote the entrepreneur's value function by $V\left(k_{j,t}, a_{j,t}, e_{j,t}; A_t, \sigma_{t-1}, \gamma_{t-1}, \mu_t\right)$ where $k_{j,t}$ is the entrepreneur's capital stock, $a_{j,t}$ is the beginning of the period asset holdings, $e_{j,t}$ is the level of the idiosyncratic productivity, A_t is the level aggregate productivity, σ_{t-1} and γ_{t-1} are the variance and skewness of the idiosyncratic shocks respectively, and μ_t is the distribution of entrepreneurs over idiosyncratic states. Then, we can write the problem of the entrepreneur as,

$$\begin{aligned} & V\left(k_{j,t}, a_{j,t}, e_{j,t}; A_t, \sigma_{t-1}, \gamma_{t-1}, \mu_t\right) = \\ & \qquad \max_{c_{j,t}, k_{j,t+1}, a_{j,t+1}, n_{j,t}} \left\{ \frac{c_{j,t}^{1-\sigma}}{1-\sigma} + \beta \mathbb{E}\left[V\left(k_{j,t+1}, a_{j,t+1}, e_{j,t+1}; A_t, \sigma_t, \gamma_t, \mu_{t+1}\right)\right] \right\} \end{aligned} \tag{6}$$

subject to,

$$\begin{gathered} c_{j,t} + i_{j,t} + a_{i,t+1} \leq y_{j,t} - w\left(A_t, \mu_t\right) n_{j,t} - \phi\left(k_{j,t+1}, k_{j,t}\right) + \left(1 + r_t\left(A_t, \mu_t\right)\right) a_{i,t}, \\ i_{j,t} = k_{j,t+1} - \left(1 - \delta^k\right) k_{j,t}, \\ \mu_{t+1}\left(k_{j,t+1}, a_{j,t+1}, e_{j,t+1}\right) = \Gamma\left(\mu_t, A_t\right), \end{gathered}$$

given the law of motion for the aggregate shock A_t. The term $w\left(A_t, \mu_t\right)$ denotes the wage rate in the economy. In what follows, we assume the interest rate on the risk-free asset is fixed, that is $r_t\left(A_t, \mu_t\right) = r$.[18] Let $C^e\left(k_{j,t}, a_{j,t}, e_{j,t}; A_t, \sigma_{t-1}, \gamma_{t-1}, \mu_t\right)$, $K^e\left(k_{j,t}, a_{j,t}, e_{j,t}; A_t, \sigma_{t-1}, \gamma_{t-1}, \mu_t\right)$, $N^e\left(k_{j,t}, a_{j,t}, e_{j,t}; A_t, \sigma_{t-1}, \gamma_{t-1}, \mu_t\right)$, and

[18] This imply that we will not solve the interest rate in equilibrium. The wage rate, however, is such that the labor market clear.

$A^e\left(k_{j,t}, a_{j,t}, e_{j,t}; A_t, \sigma_{t-1}, \gamma_{t-1}, \mu_t\right)$, denote the policy rules of consumption, next's period capital, current period labor, and risk-free asset for the entrepreneurs.

4.2 Non Entrepreneurial Households

The economy is populated by large number of identical hand-to-mouth household that consume C_t units of the homogeneous good and supply labor elastically which we denote by N_t. In concrete, we assume that non entrepreneurial households solve the following static problem,

$$U\left(C_t, L_t\right) = \max_{C_t, L_t}\left\{\frac{C_t^{1-\sigma}}{1-\sigma} - \psi\frac{N_t^{1-\gamma}}{1-\gamma}\right\}, \tag{7}$$
$$C_t \leq w\left(A_t, \mu_t\right) L_t,$$

given the law of motion for the aggregate shock A_t and the evolution the joint distribution of idiosyncratic productivity, capital, and assets. The solution of the non entrepreneurial sector implies that

$$\begin{aligned} w\left(A_t, \mu_t\right)\left(w\left(A_t, \mu_t\right) N_t\right)^{-\sigma} - \psi N_t^{-\gamma} &= 0, \\ w\left(A_t, \mu_t\right)^{1-\sigma} &= \psi N_t^{\sigma-\gamma}, \\ \left(\frac{\left[w\left(A_t, \mu_t\right)\right]^{1-\sigma}}{\psi}\right)^{\frac{1}{\sigma-\gamma}} &= N_t, \end{aligned}$$

which can be directly used to calculate employment supply for a given wage rate. Denote by $C\left(A_t, \mu_t\right)$ and $N\left(A_t, \mu_t\right)$ the optimal choices of consumption and labor for the non entrepreneurial household.

4.3 Recursive Competitive Equilibrium

Given the exogenous process for the aggregate productivity, A, an interest rate of the risk free asset, r, and the evolution of the idiosyncratic productivity processes for the entrepreneurs, $\left\{e_j\right\}_{j\in J}$, a recursive competitive equilibrium for this economy is given by a set of policy functions $\left\{\left\{C_j^e, K_j^e, N_j^e, A_{j,}^e\right\}_{j\in J}, C, N\right\}_{t=0}^{\infty}$, a wage function $\{w\}$, and value functions $\{V, U\}$ such that i) the policy and value functions solve (6) and (7) respectively,

ii) labor market clears, that is

$$\int N^e\left(k_j, a_j, e_j; A_t, \sigma_{-1}, \gamma_{-1}, \mu\right) d\mu\left(k_j, a_j, e_j\right) = N\left(A, \mu\right),$$

and iv) the mapping $\Gamma(\mu, A)$ that determines the evolution of the joint distribution of e_j, k_j, and a_j is consistent with the policy function and the stochastic evolution of the aggregate productivity process.

4.4 Parameters, Estimation, and Model Fit

In this section, we describe the quantitative specification of our modeled economy. To solve the entrepreneurs' problem we employ non-linear methods similar to Krusell and Smith (1998). Most our parameters are standard in the macro literature and we take them from the existing estimates when possible. However, the parameters governing the stochastic process of productivity are novel to our analysis and we use a simulated method of moments approach to estimate them.

Frequency and Preferences

We set the time period to a quarter. We assume that σ, the parameter governing the risk aversion, is equal to 2. For the labor supply of non entrepreneurial households, we fix a value of γ to 1.5 and we choose ψ so that they spend an average of 33% of their time working. The household's discount rate, β, is set $0.95^{0.25}$, whereas the interest rate on the risk free asset is set to match an annual return of 2%.

Production Technology and Adjustment Costs

The exponents of the capital and labor in the entrepreneur's technology are set to $\alpha = 0.25$ and $\nu = 0.5$. The capital depreciation rate, δ^k, is set to match a 10% of annual depreciation. As for the adjustment cost parameters we consider a fixed adjustment cost of capital, ϕ_1, equal to 1.5%, a quadratic adjustment cost, ϕ_2, equal to 7.0, and a resale cost, ϕ_3, equal to 34%.

Aggregate Productivity Shocks

We assume that the aggregate productivity follows a standard AR1 process with first order autocorrelation of 0.95 and normally distributed innovations with mean 0 and standard deviation of 0.75%, similar to the quarterly values used by Khan and Thomas (2008). Table V summarizes the set of calibrated parameters.

Idiosyncratic Shocks

To capture the time-varying nature of risk we assume that economy can be in two states. The first state, which we denote as low risk episodes (typically during an economic expansion), corresponds to periods where the variance the innovations of the idiosyncratic shocks affecting firms is low, $\sigma_t = \sigma_L$, and the skewness of those innovation is high, $\gamma_t = \gamma_H$. The second state, or high risk episodes (typically during a recession), corresponds to periods of high dispersion, $\sigma_t = \sigma_H$, and low skewness, $\gamma_t = \gamma_L$. Low and high risk states alternate following a first-order Markov process. To capture the potential non-gaussian nature of the idiosyncratic shocks we assume that, conditional on the values of σ_t and γ_t, the innovations in 3 are drawn from a mixture of two normally distributed random variables, that is,

$$\epsilon_{j,t} \sim \begin{cases} N\left(\mu^s, \sigma_1^s\right) & \text{with prob } p^s, \\ N\left(-\frac{p^s}{1-p^s}\mu^s, \sigma_2^s\right) & \text{with prob } 1-p^s, \end{cases} \tag{8}$$

where s can be a high or low risk period. This implies we need to find ten parameters to fully characterize the stochastic process of idiosyncratic productivity, namely, $\{\mu^s, \sigma_1^s, \sigma_2^s, p^s\}$ with $s \in \{H, S\}$, and the parameters governing the transition probabilities between low and high risk periods, denoted by π_L and π_H respectively.

Since we do not observe directly the productivity process faced by the firms, we estimate the parameters of their stochastic process using Simulated Methods of Moments (SMM) so as our model reproduces the main features of the US data described in the empirical section of the paper. In particular, we take data of quarterly sales growth from Compustat, and we search for parameters of the stochastic process so that the cross sectional distribution of sales growth derived from the model matches the observed average values of the 90th-to-50th percentiles spread, the 50th-to-10th percentiles spread, the Kelley Skewness, and the 90th-to-10th percentiles spread during expansion periods and the same set of moments for recession periods for a total of eight moments of the quarterly sales growth distribution. The transition probability for the risky state, π_H, is set so that the fraction recession quarters that follow from another recession quarter is 0.78, whereas the transition probability of the low risk state, π_L, is set to that the share of expansion quarters following another expansion quarter is 0.95, where recession and expansion periods in the data correspond to the recession quarters defined by the NBER from 1970 to 2014. Table VIII displays the our estimates for the productivity process and the targeted and model-simulated moments in table VI.

5 Quantitative Results

In this section we study the quantitive implication of our model. We first analyze standard business cycle statistics. Then, we study the response of our modeled economy to a shock that increases risk by reducing the skewness of idiosyncratic productivity while keeping the mean and variance constant. Finally, we analyze the differential response of our model to a standard uncertainty shocks (i.e. a symmetric increase in dispersion) relative to a negative skewness shocks (i.e. an asymmetric increase in dispersion), and to a combined shock of dispersion and skewness (i.e., a change in risk that resembles what happens a in typical recession).

5.1 Business Cycle Statistics

Table 8 shows a set of the standard business cycle statistics generated from our modeled economy. To obtain these statistics we simulate the economy for 5000 periods and we discard the first 500. We then calculate the standard deviation and correlation with output of several aggregate time series. All statistics are in the neighborhood of what is observed in the data: investment is more volatile than output whereas consumption is less volatile. Labor is less volatile than output due to low value of the Frisch elasticity considered in our simulations. Our model generates an average annual risk premium of 3.19%, which is in line with the empirical estimates based on stock returns. We conclude that our model is consistent with the standard business cycle statistics found in the literature.

5.2 Idiosyncratic Shocks and Sales Growth

To evaluate the effects of a change in risk and the importance of changes in the skewness of firm-level shocks we independently simulate 1000 economies, each of 300 quarters length. For the first 150 periods the economy evolves normally, then, all economies are hit by an aggregate change in the level of risk (i.e a decrease in the skewness of firm-level shocks, an increase in dispersion of firm-level shocks, or both at the same time). From that period on, all economies evolve normally. We then average different macroeconomic outcomes across all simulated economies and we calculate the impact of the change in risk as the percentage deviation of a given variable relative to its value in the period previous the shock.

Before analyzing the effect on the macroeconomic aggregates it is important to study the evolution of the distribution of idiosyncratic shocks experienced by the firms in our model and the impact of a change in dispersion and skewness on the sales growth distribution generated by our model. In particular, we must make sure that our model can separate a change in dispersion from a change in the skewness of shocks without impacting the average productivity of the firms, so that our results are not driven by changes in the first moment of the productivity distribution, but only by changes in either the its dispersion and or its skewness.

To see how the productivity distribution evolves after a change in risk, figure 9 displays moments of the annual change in the idiosyncratic productivity, $\Delta e_{i,t} = e_{i,t} - e_{i,t-4}$ for three cases, a first case in which an increase in risk leads to an increase in dispersion and a decrease in skewness (blue line with circles), a second case in which an increase in risk leads only to a decrease in skewness (black line with diamonds), and a third case in which an increase in risk leads to an increase in dispersion only (red line with triangles).[19] The first thing to notice in figure 9 is that the typical average firm in our model does not experience a change in productivity when risk changes. This is important as it ensures that our results are not driven by a change in average firm productivity. Second, comparing the black line in the middle and left panels one can see that our model is able to generate a pure change in the skewness, that is, a change in the productivity distribution that reflect only a decrease in skewness but muted changed in the average and the dispersion of the distribution.[20] Similarly, our model also generates a pure dispersion shock (the red line with triangles across the panels). Comparing the effect of the impact of a change in risk that combines dispersion and skewness to a case in which either skewness or dispersion change is key for the quantitative analysis we perform in the next section.

It is also important to analyze the impact of the change in risk on the sales growth distribution. Figure 10 shows the average, the dispersion and, and the skewness of the annual change in quarterly sales implied by the model. It is not surprising that our baseline simulation – that considers a simultaneous increase in the variance and a decrease

[19] To make this comparison we reestimate the parameters of the stochastic process in 8 to separate the changes in dispersion (a symmetric increase in risk) from changes in dispersion and skewness (an asymmetric increase in risk). Table VII shows the estimation targets for each case.

[20] The median firm, however, receives a positive productivity shock after a decline in the skewness that keeps the mean and variance constant. This increase in productivity goes against our results as our model predict a negative aggregate response of the economy to a drop in skewness.

in the skewness of firm-level productivity – generates a corresponding increase in the cross sectional dispersion of sales growth (middle plot of figure 10), and a large decrease in skewness (right plot of figure 10). Comparing the case in which only dispersion changes – which is the typical uncertainty shock – to the case in which only the skewness changes – the case we discuss in the following section – one can see that by considering a shock with time-varying skewness the model is able to capture the asymmetric response of the tails of the sales growth distribution. Moreover, the model generates a drop in skewness which is in line with the observed drop of the Kelley skewness during recession periods in the United States. This is the first results of our quantitive analysis: in the context of a model with adjustment cost to capital and risk averse entrepreneurs, a pure second moment shock does not reproduce the asymmetric changes in the sales growth distribution that we document in this paper. Notice also that the average sales growth greatly respond to a change in the risk conditions in the economy (left panel of figure 10) but this response is only driven by the endogenous response of firms to a change in the risk conditions as the average productivity growth is unaltered.

5.3 The Macroeconomic Effect of a Skewness Shock

In this section we analyze the macroeconomic effect of a decrease in the skewness of firm-level productivity. For doing that we shock the economy with a change in the skewness of the innovations of $e_{j,t}$ and we calculate the response of different macroeconomic aggregates as the percentage change relative to their value prior the shock. In our exercise, when the economy receives a skewness shock that drives the skewness from γ_H to γ_L, we keep the mean and variance of the idiosyncratic productivity constant at their low-risk level so our results reflect a pure change in the skewness of the distribution. Moreover, our timing assumption implies that in the period when the shock arrives, the change in the skewness only represents news about the future economic conditions as the realizations of the productivity process that firm experiences are drawn from a distribution with skewness equal to its pre recession values.

Figure 11 shows that output declines 0.6% after a skewness shock. This is a significant decline in aggregate economic activity considering that only the shape of the distribution of firm-level shocks has changed. Moreover, the decline in output is quite persistent, staying below its pre-shock level even after twelve periods after the shock. This is in contrast with the typical uncertainty shock that generates a decrease in output and a rapid rebound few quarters after the shock. As we discuss in the next section our model also generates such rebound when we consider an increase in dispersion paired with the decrease in skewness.

In our model, the drop in output is due to the rapid and persistent decline in capital investment after a change in skewness. The top left panel of figure 12 shows that capital investment drops around 2% during the first quarter after an skewness shocks and stays below the pre-shock period for at least eight quarters. Labor does not drop in the first period after the shock mainly because labor is fully flexible and news about future do not change firms' hiring decisions. Consumption instead declines rapidly in response to the decrease in the skewness of the distribution. Importantly, in the first quarter after the shock the response of investment and consumption is not driven by a change in the skewness of the realization of $e_{j,t}$ received by the firms – recall our timing assumption in equation 3 – but by a change in the perception about the risk in the economy: at the moment of the shock individuals receive news that in the future the distribution of $e_{j,t}$ will be left skewed and their endogenous responses drive a decline in investment and consumption. A decrease in skewness triggers a precautionary increase on entrepreneur's savings, but since capital is more risky, investment in the risk free asset surges as it is shown in the bottom right panel of figure 12. This decline in consumption and investment and contemporaneous increase in precautionary savings is similar to the effect of uncertainty in the model of Fernandez-Villaverde *et al.* (2011). We conclude that a decline in the skewness of the distribution of idiosyncratic shocks can by itself generate a persistent drop in aggregate economic activity.

5.4 Variance and Skewness Shocks

Our empirical evidence indicates that a typical recession is characterized by an asymmetric increase in dispersion which leads to a decline in the skewness. Hence, in this section we evaluate the response of our modeled economy to a pure change in the dispersion of of firm-level shocks and to a change in risk that combines both, an increase in the variance and a decrease in the skewness of firm-level shocks. This is displayed in figure 13, where we plot the evolution of several economic aggregates after a shock that combines variance and skewness (blue line with circles), a pure skewness shock (black line with diamond), and pure variance shock (red line with triangles).

Starting with the effects of a pure uncertainty shock, we see that an increase in dispersion of idiosyncratic productivity generates an increase in output, labor, and consumption, while investment declines. The difference with respect to a pure skewness shock is due to the Oi (1961), Hartman (1972), and Abel (1983) affect: in the absence of adjustment cost to labor, a symmetric increase in dispersion pushes up the productivity of some firms at the top of the distribution, driving labor and output up. This increase in productivity of firms at the top more than compensates the decrease in productivity and labor demand from firms at the bottom of the productivity distribution, increasing aggregate output. Investment, on the other hand, responds negatively to an uncertainty shock (bottom left panel of figure 13) mainly because of the real options effects generated by the fixed adjustment cost. In contrast, a pure skewness shock does not generate an increase in output: a decrease in the skewness implies that the left tail of the productivity distribution widens, generating a decrease in investment due to the real option effects and a muted Oi-Hartman-Abel as the right tail of the productivity distribution does not change.

The overall effect of an increase in risk that combines both dispersion and skewness depends on the strengths of the Oi-Harman-Abel effect, the real option channel, and the relative change in the tails of the productivity distribution. For our model to match the asymmetric increase in dispersion of the sales growth distribution that we observe in the data, an increase in dispersion is mostly due to widening of the left tail of the distribution of firm-level shocks distribution without a parallel widening of the right tail of the distribution, which commands a decline in the skewness in productivity and sales growth.[21] Consequently, the combined effect of dispersion and skewness is followed by a significant and persistent decline in aggregate economic activity as it is shown by the blue line with circles in figure 13.

5.5 Employment and Persistence

In this section we discuss two additional results regarding the effect of skewness shocks. The first relates to the impact of a skewness shock on the distribution of employment growth. As we discussed in our empirical section, the employment growth

[21] The asymmetric increase in dispersion generated by the model can be appreciated by comparing the response of the 90th-to-50th and the 50th-to-10th percentiles spreads generated by the model. Figure 14 displays the evolution of these moments for the three cases we have discussed. In the case of a pure variance shock – red line with triangles – both tails of the distribution expand symmetrically (compare the 50th-to-10th percentile spread to the 90th-to-50th percentiles spread), but in the case of an increase in dispersion that is accompanied by a decrease in the skewness it is only the left tail that expands (measured by the 50th), whereas the dispersion of the right tail almost does not change – blue line with circles. This is exactly what we observe in the data when we compare periods of high and low risk in our model (see table VI) and in the data during recessions periods (see figure 3).

distribution also shows wide fluctuation in the skewness much in line with fluctuation on the sales growth distribution. Our model is also able to account for the decline in the skewness of the employment growth. In fact, as it is shown in the left panel of figure 15 the skewness of the distribution of one-year employment growth drops almost 10 percentage points when the economy receives a shock that reduces the skewness of firm-level productivity. This decline is almost two-fifths of the decline of the skewness of the sales growth distribution predicted by the model and around half of the decline we find in the data (see figure 4).

Second, in our empirical analysis we show that recessions are also associated to a decline in the skewness of sales growth at one and three year horizons. This is important as growth rates at longer horizons are more closely related to permanent shocks to firms. Our model correctly predicts that the distribution of three-years sales growth substantially declines after a decrease in the skewness of firm-level productivity as it is shown in the right panel of figure 15. It also predicts a more moderated impact on the distribution of five-year growth rates. Importantly, in our estimation we did not target the skewness of the employment growth distribution or the skewness of sales growth at longer horizons, indicating that our model is well suited to match several additional aspects of the dynamic behavior of micro and macro aggregates. Introducing adjustment cost to labor or a permanent component in the firms' exogenous productivity might help to better account for the evolution of the employment and sales growth.

6 Conclusions

This paper studies how the distribution of the growth rate of macro- and micro-level variables changes over the business cycle. At the micro level, we use firm panel data for more than 30 countries to show that skewness is strongly procyclical, driven by a large left tail of negative growth rates during recessions. At the macro level, analyzing the growth rates of GDP and stock market returns, we find a similar phenomenon of procyclical skewness. These results are robust to different selection criteria, across countries, industries, and measures, suggesting that a widening left tail—and, consequently, a more negative skewness—is a basic stylized fact of business cycles. Given this empirical evidence, in the second part of our paper we analyze the aggregate impact of changes in the skewness of the firm-level productivity distribution in the context of an heterogeneous agents model. We assume that the exogenous idiosyncratic productivity process faced by firms subject to time-varying dispersion and time-varying skewness and we estimate the parameters of this model to match the evolution of the dispersion and skewness of the sales growth distribution we observed in the United States. Our results suggests that a change in the skewness of the firm-level productivity distribution can by itself generate a significant decline in aggregate economic even though the mean and variance of firm-level productivity is held constant.

References

Abel, A. B. (1983). Optimal investment under uncertainty. *The American Economic Review*, **73** (1), 228–233.

Adrian, T., Boyarchenko, N. and Giannone, D. (Forthcoming). Vulnerable growth. *American Economic Review*.

Alfaro, I., Bloom, N. and Lin, X. (2018). *The Finance Uncertainty Multiplier*. Tech. rep., National Bureau of Economic Research.

Arellano, C., Bai, Y. and Kehoe, P. (2010). Financial markets and fluctuations in uncertainty. *Federal Reserve Bank of Minneapolis Staff Report*.

Bachmann, R. and Bayer, C. (2013). 'wait-and-see'business cycles? *Journal of Monetary Economics*, **60** (6), 704–719.

Bai, J. and Ng, S. (2005). Tests for skewness, kurtosis, and normality for time series data. *Journal of Business & Economic Statistics*, **23** (1), 49–60.

Barro, R. and Ursua, J. (2011). *Rare Macroeconomic Disasters*. NBER Working Papers 17328, National Bureau of Economic Research, Inc.

Barro, R. J. (2006). Rare disasters and asset markets in the twentieth century. *Quarterly Journal of Economics*, **121** (3), 823–866.

Basu, S. and Bundick, B. (2017). Uncertainty shocks in a model of effective demand. *Econometrica*, **85** (3), 937–958.

Bekaert, G. and Popov, A. (2012). *On the Link between the Volatility and Skewness of Growth*. Tech. rep., National Bureau of Economic Research.

Berger, D., Dew-Becker, I. and Giglio, S. (2017). *Uncertainty shocks as second-moment news shocks*. Tech. rep., National Bureau of Economic Research.

Bloom, N. (2014). Fluctuations in uncertainty. *Journal of Economic Perspectives*, **28** (2), 153–76.

—, Floetotto, M., Jaimovich, N., Saporta-Eksten, I. and Terry, S. J. (2018). Really uncertain business cycles. *Econometrica*, **86** (3), 1031–1065.

Comin, D. and Philippon, T. (2005). The rise in firm-level volatility: Causes and consequences. *NBER macroeconomics annual*, **20**, 167–201.

Davis, S. J. and Haltiwanger, J. (1992). Gross job creation, gross job destruction, and employment reallocation. *Quarterly Journal of Economics*, **107** (3), 819–863.

—, —, Jarmin, R., Miranda, J., Foote, C. and Nagypal, E. (2006). Volatility and dispersion in business growth rates: Publicly traded versus privately held firms [with comments and discussion]. *NBER macroeconomics annual*, **21**, 107–179.

Decker, R., Haltiwanger, J., Jarmin, R. and Miranda, J. (2015). Where has all the skewness gone? the decline in highgrowth (young) firms in the u.s. *Working Paper*.

DeLong, J. B. and Summers, L. H. (1984). Are business cycles symmetric?

Distante, R., Petrella, I. and Santoro, E. (2013). Asymmetry reversals and the business cycle. *Working Paper*.

Fagiolo, G., Napoletano, M. and Roventini, A. (2008). Are output growth-rate distributions fat-tailed? some evidence from oecd countries. *Journal of Applied Econometrics*, **23** (5), 639–669.

Falk, B. (1986). Further evidence on the asymmetric behavior of economic time series over the business cycle. *Journal of Political Economy*, **94** (5), 1096–1109.

Fernandez-Villaverde, J., Guerron-Quintana, P., Rubio-Ramirez, J. F. and Uribe, M. (2011). Risk Matters: The Real Effects of Volatility Shocks. *American Economic Review*, **101** (6), 2530–61.

Ferraro, D. (2018). The asymmetric cyclical behavior of the us labor market. *Review of Economic Dynamics*.

Gilchrist, S., Sim, J. W. and Zakrajšek, E. (2014). *Uncertainty, financial frictions, and investment dynamics*. Tech. rep., National Bureau of Economic Research.

Gourio, F. (2012). Disaster risk and business cycle. *American Economic Review*, **102** (6), 2734–2766.

— (2013). Credit risk and disaster risk. *American Economic Journal: Macroeconomics*, **5** (3), 1–34.

Guvenen, F., Ozkan, S. and Song, J. (2014). The nature of countercyclical income risk. *Journal of Political Economy.*

Hamilton, J. D. (1989). A new approach to the economic analysis of nonstationary time series and the business cycle. *Econometrica: Journal of the Econometric Society*, pp. 357–384.

Hartman, R. (1972). The effects of price and cost uncertainty on investment. *Journal of economic theory*, **5** (2), 258–266.

Ilut, C., Kehrig, M. and Schneider, M. (2018). Slow to hire, quick to fire: Employment dynamics with asymmetric responses to news. *Journal of Political Economy*, **126** (5), 000–000.

Jovanovic, B. (2006). Asymmetric cycles. *The Review of Economic Studies*, **73** (1), 145–162.

Jurado, K., Ludvigson, S. C. and Ng, S. (2015). Measuring uncertainty. *American Economic Review*, **105** (3), 1177–1216.

Kehrig, M. (2011). *The Cyclicality of Productivity Dispersion.* Working Papers 11-15, Center for Economic Studies, U.S. Census Bureau.

Kelley, T. L. (1947). *Fundamentals of Statistics.* Harvard University Press.

Khan, A. and Thomas, J. K. (2008). Idiosyncratic Shocks and the Role of Nonconvexities in Plant and Aggregate Investment Dynamics. *Econometrica*, **76** (2), 395–436.

Kilic, M. and Wachter, J. A. (2015). *Risk, Unemployment, and the Stock Market: A Rare-Event-Based Explanation of Labor Market Volatility.* NBER Working Paper 21575.

Kontolemis, Z. G. (1997). Does growth vary over the business cycle? some evidence from the g7 countries. *Economica*, **64** (255), 441–460.

Kozlowski, J., Veldkamp, L. and Venkateswaran, V. (2016). The tail that wags the economy: Belief-driven business cycles and persistent stagnation.

Krusell, P. and Smith, A. A. (1998). Income and Wealth Heterogeneity in the Macroeconomy. *Journal of Political Economy*, **106** (5), 867–896.

Leduc, S. and Liu, Z. (2016). Uncertainty shocks are aggregate demand shocks. *Journal of Monetary Economics*, **82**, 20–35.

McKay, A. and Reis, R. (2008). The brevity and violence of contractions and expansions. *Journal of Monetary Economics*, **55** (4), 738–751.

McQueen, G. and Thorley, S. (1993). Asymmetric business cycle turning points. *Journal of Monetary Economics*, **31** (3), 341–362.

Neftci, S. N. (1984). Are economic time series asymmetric over the business cycle? *Journal of Political Economy*, **92** (2), 307–328.

Oi, W. Y. (1961). The desirability of price instability under perfect competition. *Econometrica: journal of the Econometric Society*, pp. 58–64.

Ordonez, G. (2013). The asymmetric effects of financial frictions. *Journal of Political Economy*, **121** (5), 844–895.

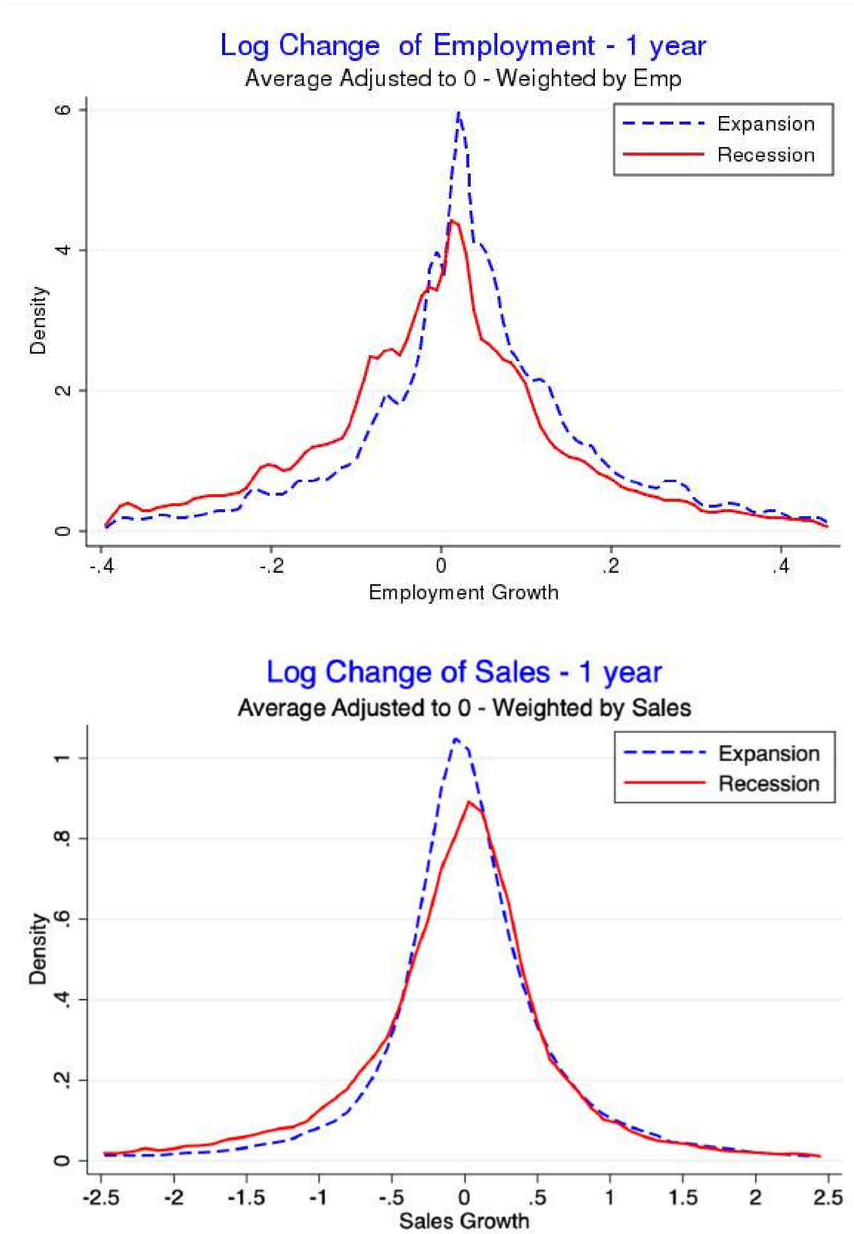
Orlik, A. and Veldkamp, L. (2014). *Understanding uncertainty shocks and the role of black swans*. Tech. rep., National Bureau of Economic Research.

Rietz, T. A. (1988). The equity risk premium: a solution. *Journal of Monetary Economics*, **22** (1), 117–131.

Sichel, D. E. (1993). Business cycle asymmetry: a deeper look. *Economic Inquiry*, **31** (2), 224–236.

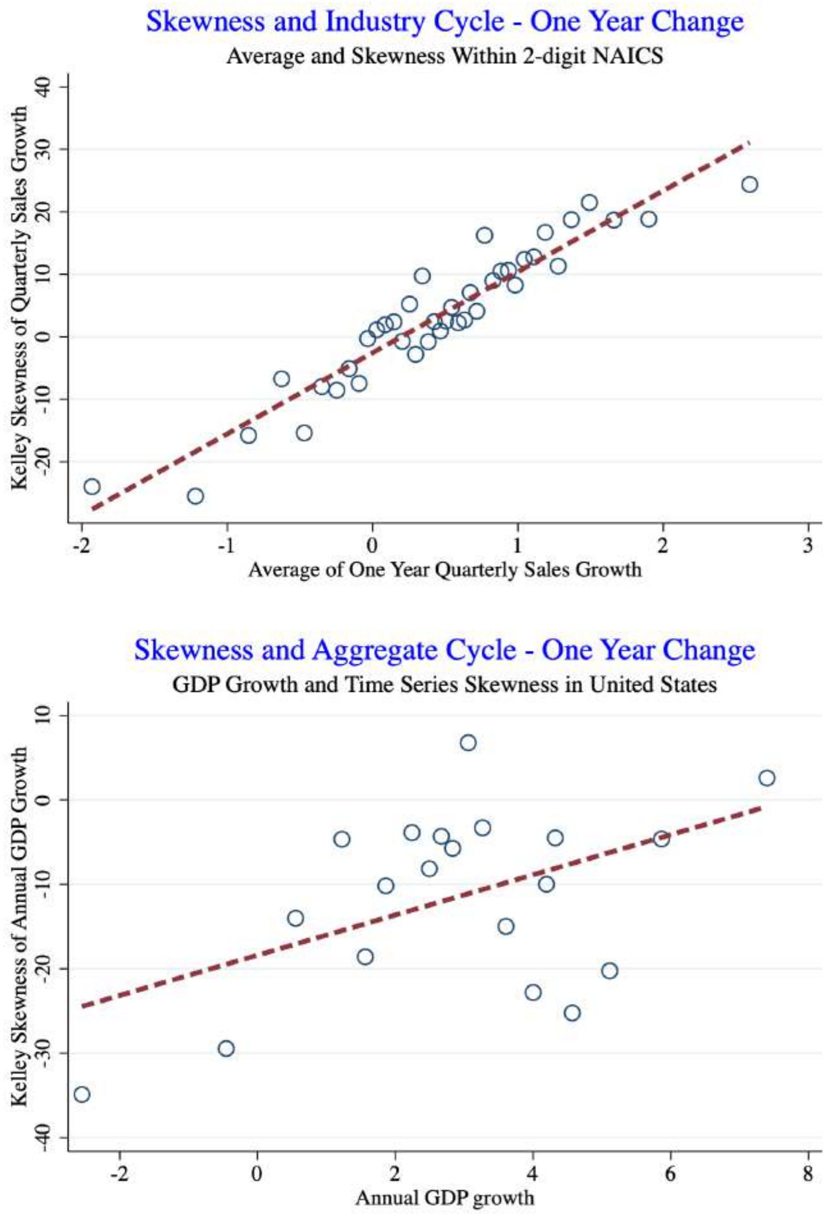
Wachter, J. A. (2013). Can Time-Varying Risk of Rare Disasters Explain Aggregate Stock Market Volatility? *Journal of Finance*, **68** (3), 987–1035.

Figure 1 – The Skewness of Firm-level Outcomes Declines During Recessions



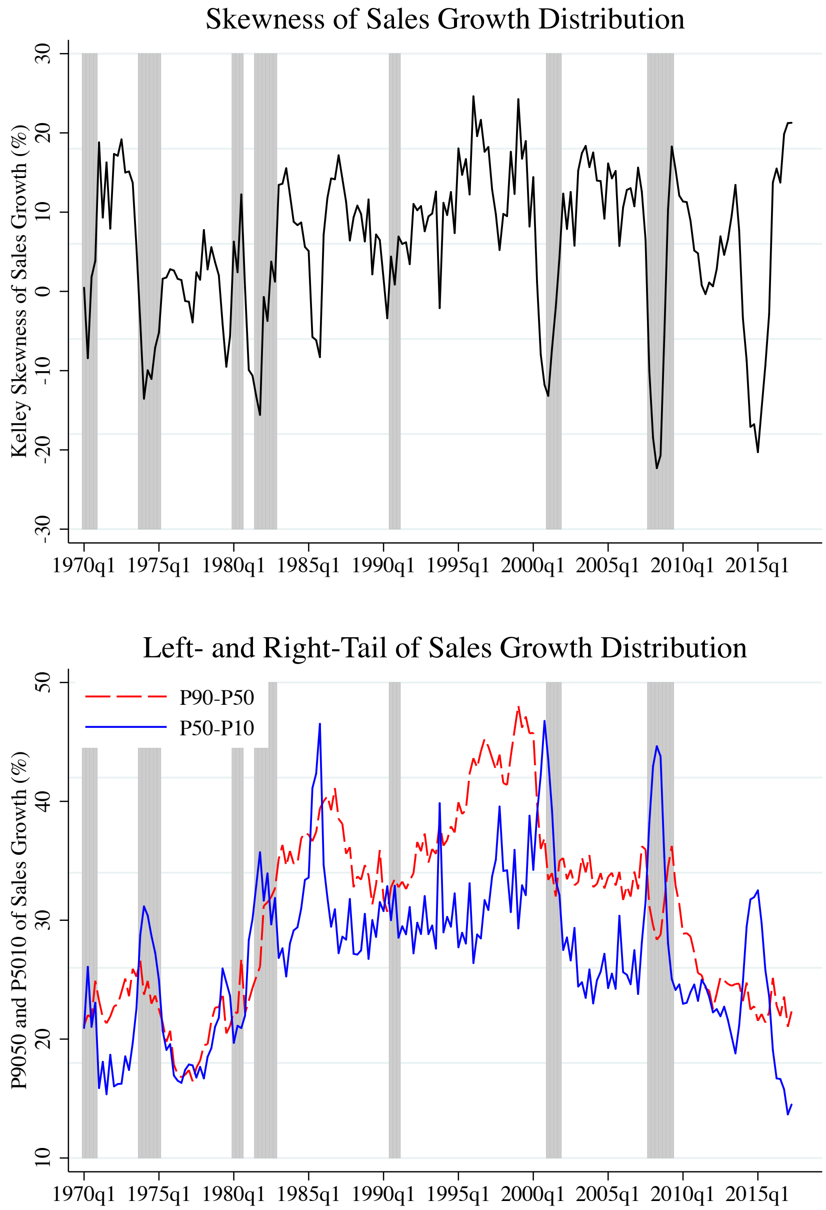
Note: The upper panel of figure 1 shows the employment weighted empirical density of the distribution of one year log-change of employment for a sample of firms from LBD. The lower panel shows the sales weighted empirical density of the distribution of one year long-change of real quarterly sales in US dollars over a balanced panel of publicly traded firms from the Compustat/CRSP merged database. In each plot the blue-dashed line correspond to the density for a pooled sample of expansion years (2003 to 2006 and 2010 to 2014) whereas the red-solid line correspond to the density for a pooled sample of recession years (2001 and 2008). In the upper panel, the 10th percentile of the employment growth distribution during expansion (recession) periods is -16.5% (-26.9%), the 50th percentile is 1.29% (-1.75%), and the 90th percentile is 23.3% (18.0%). For the sales growth distribution in the bottom panel the corresponding moments are -15.9% (-34.4%), 4.0% (1.5%), and 28.0% (24.4).

Figure 2 – The Skewness is Procyclical at Industry- and Macro-Level



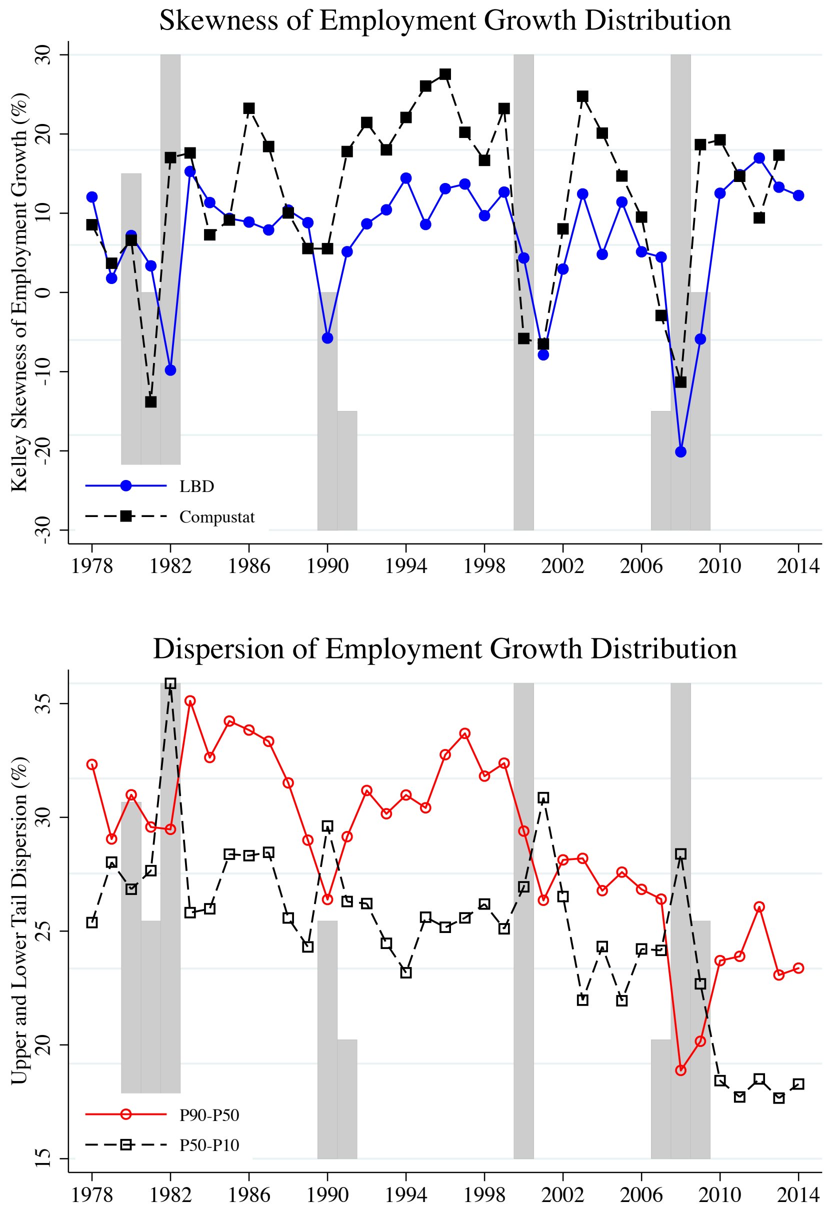
Note: The upper panel of figure 2 displays a bin scattered plot showing the relation between the within-industry business cycle, measured by the average growth rate of sales, and the within-industry skewness, measured by the Kelley skewness of sales growth for a sample of Compustat firms. Each dot is a quantile of the industry-year distribution of average sales growth. The lower panel of figure 2 displays a bin scattered plot showing the relation between the US GDP growth and the skewness of GDP growth measured over a moving window.

FIGURE 3 – SKEWNESS OF SALES GROWTH IS STRONGLY PROCYCLICAL



Note: Figure 3 shows the time series of the skewness (upper panel) and dispersion ($P9050_t$ and $P5010_t$ in the lower panel) of the cross-sectional distribution of the annual growth rate of quarterly sales for a sample of publicly traded firms from the Compustat/CRSP database. Gray bars are NBER recession periods.

Figure 4 – The Skewness of the Employment Growth Distribution is Procyclical – Census Data



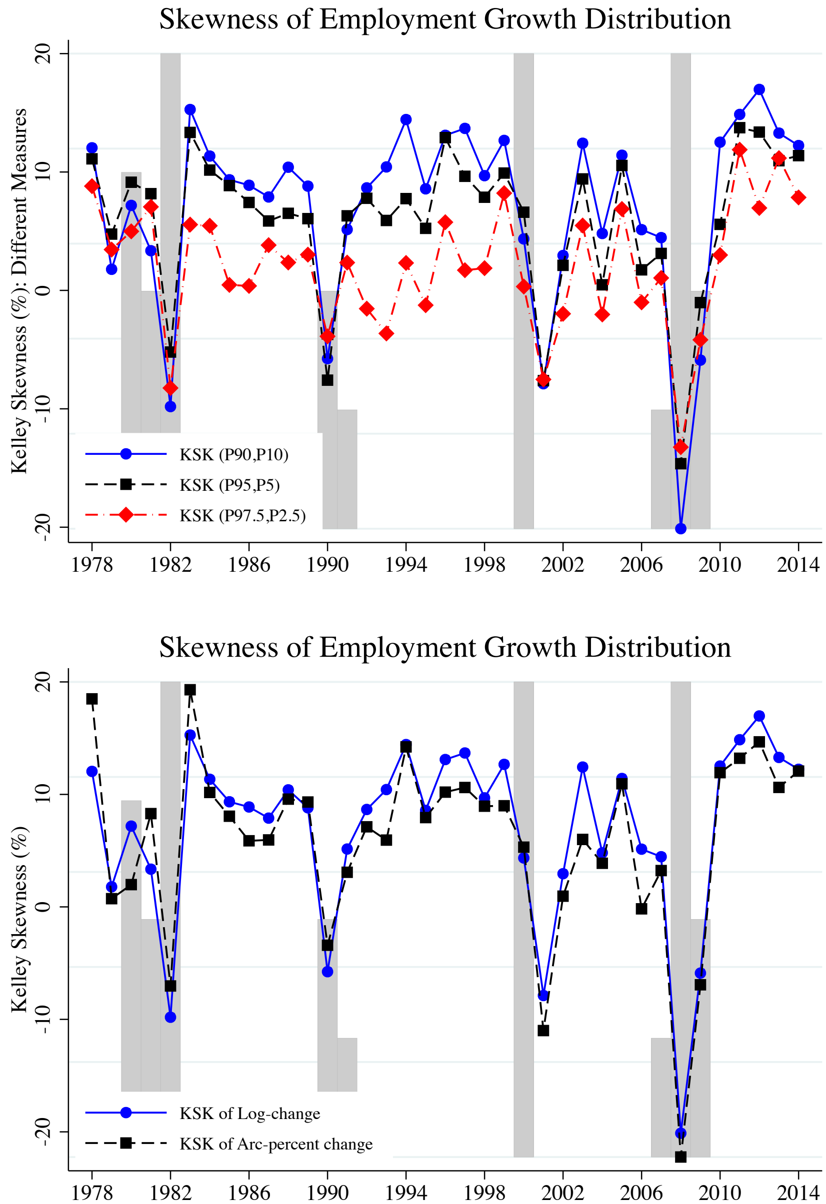
Note: The upper panel of figure 4 shows the time-series of the employment weighted Kelley skewness of the distribution of employment growth for a sample of firms from LBD (solid blue line with circles) and a sample of publicly traded firms from Compustat (dashed black line with squares). The bottom panel shows the dispersion of the employment growth distribution measured by the 90th-to-50th percentiles differential (red line with circles) and by the 50-to-10th percentiles differential (black line with squares) for a sample of firms from LBD. In both plots, the shaded bars represent the share of the year – in quarters – that were declared as recession by the NBER (a full bar represent an entire year of recession). See appendix A.2 for additional details on the sample construction and moment calculations in the LBD.

Table I – The Skewness of Firms Outcomes is Lower During Recessions

	Kelley Skewness of the Growth Rate of Firm Level Outcomes								
	United States						Cross-Country		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Firm Sales		Stock Returns		Firm Emp		Firm Sales	Firm Stock	Firm Emp.
	One Year	Three Years	One Years	Three Years	One Year	Three Years	Growth	Returns	Growth
$\Delta GDP_{i,t}$	4.79**	2.89**	2.65**	3.96***	4.64***	1.14**	2.17***	1.27***	4.10***
	(2.15)	(1.32)	(1.41)	(1.12)	(1.45)	(0.50)	(0.688)	(0.241)	(0.953)
R^2	0.01	0.01	0.03	0.10	0.32	0.15	0.37	0.40	0.26
N	184	182	180	180	39	35	838	4,306	824
Freq.	Qtr	Qtr	Qtr	Qtr	Yr	Yr	Yr	Yr	Yr
F.E.	N	N	N	N	N	N	Yr./Ctry.	Qtr./Ctry.	Yr./Ctry.
Sample	640K	640K	650K	650K	-	-	633K	5,800K	357K
Source	CSTAT	CSTAT	CSTAT	CSTAT	LBD	LBD	BVD	GCSTAT	BVD

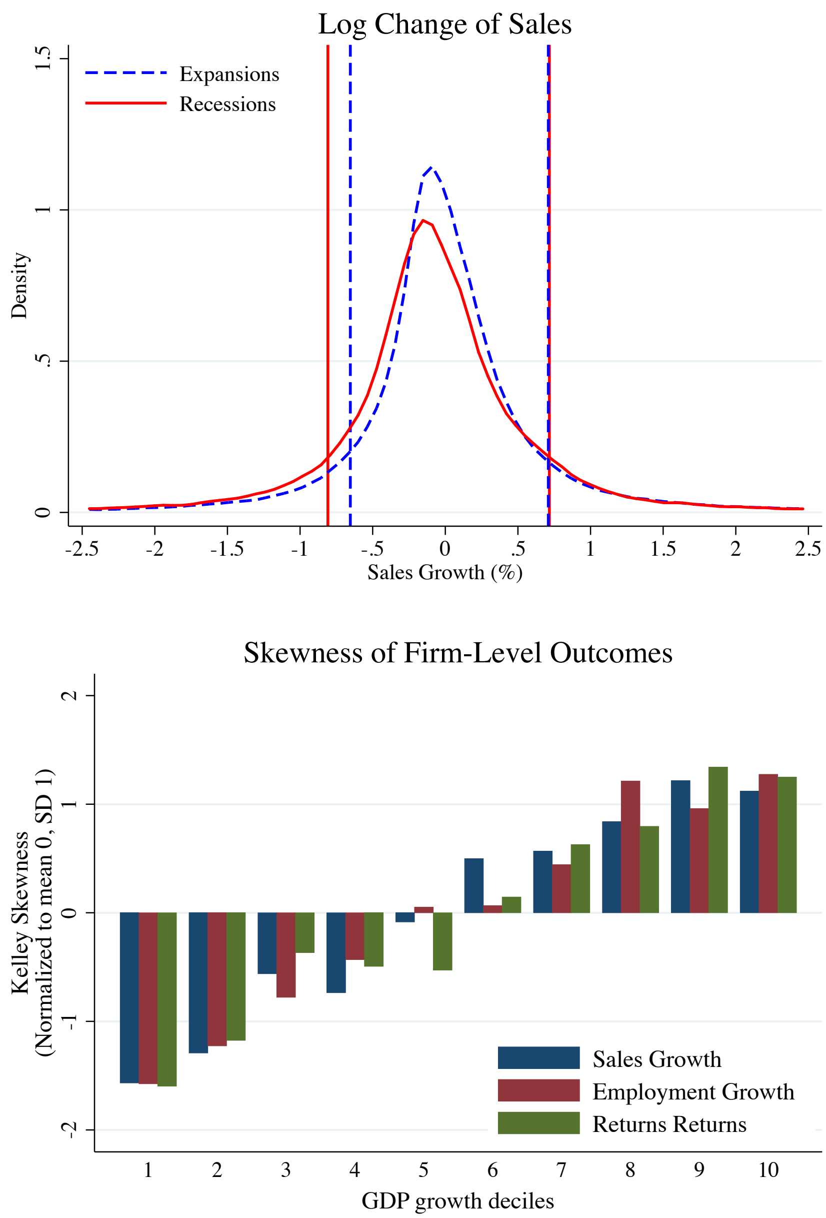
Note: The left panel of table I shows a series of times series regressions for the United States in which the dependent variable is the Kelley Skewness of the one-year and three-years growth rate of sales growth (columns 1 and 2), stock returns (columns 3 and 4), and employment growth (columns 5 and 6) for a sample of firms from Compustat (CSTAT) (columns 1 to 4) and the Longitudinal Business Database (LBD) (columns 5 and 6). CSTAT data covers the period 1970 to 2017 whereas LBD data covers the period 1976 to 2015. In each regression the independent variable is the annual growth rate of quarterly GDP per capita. All firm level moments were calculated weighting growth rate observations by firm size measured by the average sales of the firm between periods t and $t+k$. All regressions include a linear trend. Newey-West standard errors in parentheses below the point estimates. The right panel of table I shows a series of country-panel regressions where the dependent variable is the within country Kelley skewness of firm level sales growth, stock returns, and employment growth. The independent variable is the growth rate of GDP per capita at the country level. Sales and employment data is obtained from the Bureau Van Dijk's Osiris (BVD) database whereas stocks returns are obtained from Global Compustat (GCSTAT). All cross sectional moments where calculated weighting growth rate observations by firm size. All regressions consider a full set of time and country fixed effects. The raw labeled Sample shows the underlying sample of firms used to calculate the cross sectional moments. LBD data on sample size are not disclosed. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Figure 5 – Skewness of Employment Growth Distribution – Census Data



Note: Figure 5 is based on Longitudinal Business Database, LBD. The upper panel of figure 5 shows different measures of the Kelley skewness, KSK, based on different percentiles of the distribution. KSK (P90,P10) is calculated as in equation 1. KSK (P95,P5) is calculated as $\frac{(P95_t - P5_t) - (P50_t - P5_t)}{P95_t - P5_t}$. KSK (P97.5,P2.5) is similarly calculated by changing the corresponding upper and lower ends of the distribution. The upper right panel shows the Kelley skewness for two different measures of employment growth, the log-change of employment and the arc-percent change defined as $(x_{t+1} - x_t)/0.5 \times (x_{t+1} + x_t)$. The lower left panel shows the evolution of the Kelley skewness of the distribution of one and five years employment growth. Finally, the lower right panel shows the evolution of the P9050 and P5010 of the distribution of employment growth. In each plot the shaded bar represent the share of the year – in quarter – that where declared as recession years by the NBER (a full bar represent an entire year of recession). See appendix A.2 for additional details on the sample construction and moment calculations in the LBD.

Figure 6 – The Skewness of Sales Growth Distribution Declines During Recessions



Note: The upper panel of figure 6 shows the empirical density of the growth rate of annual sales in US dollars over a panel of firms in 44 countries between 1986 and 2013. To construct the figure, we first adjust the sales growth distribution within each country to have mean zero and unit variance. The red solid line is the empirical density over all the observations of firms during recession periods, defined as periods in which the country is in the first decile of the country-specific distribution of the growth rate of GDP per capita (77,137 observations). The blue dashed line is the empirical density over all the observations of firms during non-recession periods (418,256 observations). The 10th percentile of the sales growth distribution during expansion (recession) periods is -65.4% (-80.0%), the 50th percentile is 3.5% (-9.0%), and the 90th percentile is 70.9% (71.6%). The lower panel shows the average of the within-country skewness for different firm-level outcomes for different deciles of the country-specific distribution of the growth rate of GDP per capita.

TABLE II – SKEWNESS IS LOWER DURING INDUSTRY RECESSIONS

	Kelley Skewness of Firm's Outcomes					
	(1)	(2)	(3)	(4)	(5)	(6)
	Real Sales Growth		Employment Growth		Stock Returns	
	One Year	Three Years	One Year	Three Years	One Year	Three Years
$\overline{\Delta S}_{j,t}$	12.99***	5.92***	8.84***	6.02***	1.88*	0.70
	(2.04)	(1.60)	(1.42)	(1.67)	(0.99)	(1.17)
R^2	0.16	0.12	0.14	0.11	0.12	0.09
N	3,698	3,652	969	930	3,646	3,602
F.E.	Qtr/Ind	Qtr/Ind	Yr/Ind	Yr/Ind	Qtr/Ind	Qtr/Ind
Freq.	Qtr	Qtr	Yr	Yr	Yr	Yr
Sample	802K	780K	231K	193K	733K	651K

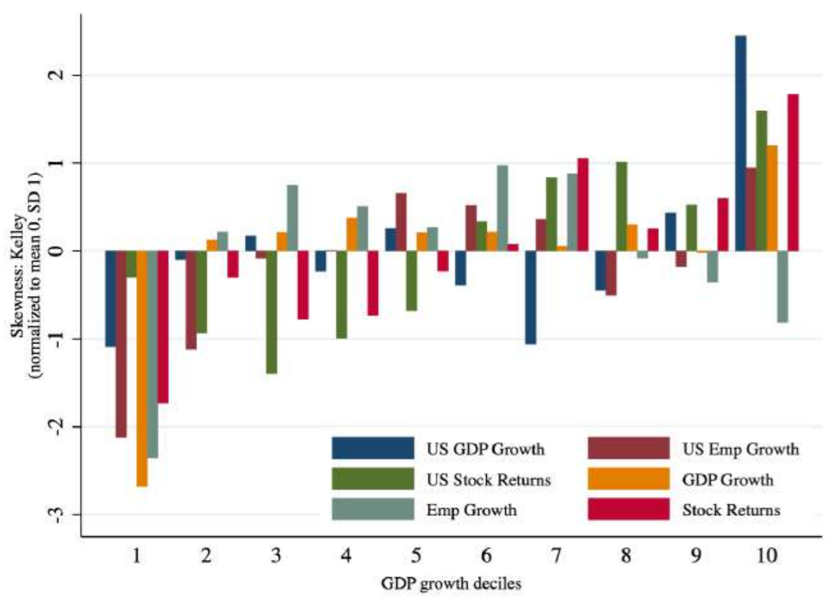
Note: Table II shows a series of industry-level panel regressions. In each column the depend variables are cross sectional Kelley skewness of the growth rates of real quarterly sales, annual employment growth, and quarterly stock returns distribution within period-industry cells defined by 2-digit NAICS (total of 22 industries) for a sample of publicly traded firms from the Compustat/CRSP dataset. The independent variable, $\overline{\Delta S}_{j,t}$, is the average of the sales growth distribution within the period-industry cell. All cross sectional moments were calculated weighting by firm-size within each industry. In all regressions the sample period is 1970 to 2017 and consider a full set of period and industry fixed effects. Row labeled Sample corresponds to the total firm-period observations used to calculate the cross sectional moments. N corresponds to the number of period-industry observations used in the regressions. Standard errors in parentheses below the point estimates are clustered at the NAIC-2 industry level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table III – Employment Growth Moments Within Industries – Census Data

	(1) Kelley Skewness	(2)	(3) Coef. of Skewness	(4) 50th to 10th Prtle. Spread	(5) 90th to 50th Prtle. Spread	(6) 90th to 10th Prtle. Spread
$\overline{\Delta E}_{j,t}$	1.06***	0.92***	8.15**	−2.47***	1.09***	−1.38***
	(0.21)	(0.16)	(4.05)	(0.42)	(0.08)	(0.34)
R^2	0.43	0.55	0.31	0.66	0.52	0.58
N	900	900	900	900	900	900
F.E	N	Yr/Ind	Yr/Ind	Yr/Ind	Yr/Ind	Yr/Ind
Freq.	Yr	Yr	Yr	Yr	Yr	Yr

Note: Table III shows a series of industry-level panel regressions using data from the Longitudinal Business Database, LBD. In each column the dependent variable is a moment of the distribution of log-change of employment at the firm level. In columns (1) and (2) the dependent variable is the Kelley's measure of skewness; column (3) is the coefficient of skewness (third standardized moment); column (4) is the spread between the 50th and 10th percentiles of the employment growth distribution; column (5) is the 90th to 50th percentiles spread; column (6) is the 90th to 10th percentiles spread. In each column the independent variable, $\overline{\Delta E_{j,t}}$, is the average employment growth within the sector. All cross sectional moments were calculated weighting by firm-size within each industry. Firm level data comes from the LBD. All regression consider a full set of year and industry fixed effects. See appendix A.2 for additional details on sample selection and moment calculations. Standard errors in parentheses below the point estimates are clustered at the industry level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Figure 7 – The Skewness of Macroeconomic Variables Declines During Recessions



Note: Figure 7 shows the average of the within-country skewness for different macro-level outcomes for different deciles of the country-specific distribution of the growth rate of GDP per capita.

Table IV – Skewness of Growth Rates is Procyclical Across Countries

	Kelley Skewness					
	United States			Cross-Country		
	(1)	(2)	(3)	(4)	(5)	(6)
	GDP growth	Agg. Emp. Growth	Agg. Stock Returns	GDP growth	Agg. Emp. Growth	Agg. Stock Returns
$\Delta GDP_{i,t}$	6.53**	9.48***	3.04***	2.53***	3.55*	0.17
	(2.55)	(2.65)	(0.38)	(0.80)	(1.86)	(0.15)
R^2				0.33	0.38	0.62
N	183	173	180	6,175	2,893	3,139
Freq.	Quarter	Quarter	Quarter	Quarter	Quarter	Quarter
F.E.	N	**N**	N	Qtr/Ctry	Qtr/Ctry	Qtr/Ctry
Period	1970-2017	1970-2017	1970-2014	1970-2017	1970-2017	1970-2014

Note: The left panel of table IV shows a series of time-series regressions for the United States where the dependent variable is the Kelley skewness of three macroeconomic outcomes. Skewness is calculated over a moving window of 12 quarters. Newey-West standard errors in parentheses below the point estimates. The right panel shows the results of a set of country-period OLS panel regressions. In each column the dependent variables is the Kelley skewness of the growth rate of different macro level outcomes. Standard errors in parentheses below the point estimates are clustered at the country-level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

Figure 8 – Business Cycle Statistics

	Data			Model		
	$\sigma(x)$	$\sigma(y)/\sigma(x)$	$\rho(x,y)$	$\sigma(x)$	$\sigma(y)/\sigma(x)$	$\rho(x,y)$
Output	1.47	1.00	1.00	1.76	1.00	1.00
Capital Investment	6.86	4.64	0.91	6.93	3.94	0.66
Consumption	1.21	0.82	0.87	1.10	0.63	0.96
Hours	1.89	1.28	0.87	1.17	0.66	1.00

Note: The left panel of table 8 contains busyness cycles statistics for quarterly US data covering 1970Q1 to 2017Q4. The column $\sigma(x)$ is the standard deviation of the log variable in the first column. The column $\sigma(y)/\sigma(x)$ is the standard deviation of the variable relative to the standard deviation of log output. All business cycle data are current as February, 03, 2019. Output is real gross domestic product (FRED GDPC1), investment is real gross private domestic investment (GPDIC1), consumption is real personal consumption expenditures (PCECC96), and hours is total nonfarm business sector hours (HOANBS). The second panel contains business cycle statistics from unconditional simulation of the estimated model, computed from a 5000-quarter simulation with the first 500 periods discarded. All series are HP-filtered with smoothing parameter 1600, in logs expressed as percentages.

Table VI – Risk Process Moments

	$P90-P10$	$P90-P50$	$P50-P10$	KSK	Yrs
Data					
Low Risk	0.54	0.30	0.24	0.10	03-06;10-14
High Risk	0.70	0.31	0.39	-0.11	01,08
$\Delta\,(H-L)$	0.16	0.01	0.15	-0.20	-
Model					
Low Risk	0.48	0.27	0.20	0.15	-
High Risk	0.58	0.26	0.32	-0.10	-
$\Delta\,(H-L)$	0.10	-0.01	0.12	-0.15	-

Note: The upper panel of table VI shows cross sectional moments of the annual growth rate of quarterly sales from Compustat for low risk periods – quarters in years 2003 to 2006 and quarters in years 2010 to 2014 – and high risk periods – quarters in years 2001 and 2008. Quarters in years 2002 and 2009 are discarded for not representing full recession years. The model moments, shown in the lower panel of table VI, are calculated from a 5000-quarters simulation with the first 500 periods discarded.

Table V – Calibrated Parameters

Preferences and Technology		
γ	0.45	Frisch elasticity of labor supply
ψ	2.5	Leisure preference, non-entrepreneurs spend 1/3 time working
σ	2.0	Risk aversion of 2
β	$0.95^{0.25}$	Annual discount factor of 95%
r	0.005	Annual return of risk free asset of 2%
α	0.25	CRS production, markup of 33%
ν	0.50	CRS labor share of 2/3, capital share of 1/3
δ^k	0.026	Annual depreciation of capital stock fo 10%
ρ_a	0.95	Quarterly Persistent of Aggregate Productivity
σ_a	0.075	Dispersion of Innovation of Aggregate Productivity
ρ	0.95	Quarterly persistence of Idiosyncratic Productivity
Adjustment costs		
ϕ_1	0.34	Fixed cost of changing capital stock
ϕ_2	7.0	Quadratic cost of changing capital stock
ϕ_3	0.015×4	Resale loss of capital

Note: Table V shows the calibrated parameters referring to preferences, technology, and adjustment costs.

Table VII – Targeted Moments for Numerical Comparison

	$P9010$	$P9050$	$P5010$	KSK
Low Risk	0.54	0.30	0.24	0.10
High Risk	0.70	0.31	0.39	-0.10
Only Skewness	0.54	0.243	0.297	-0.10
Only Variance	0.70	0.39	0.31	0.10

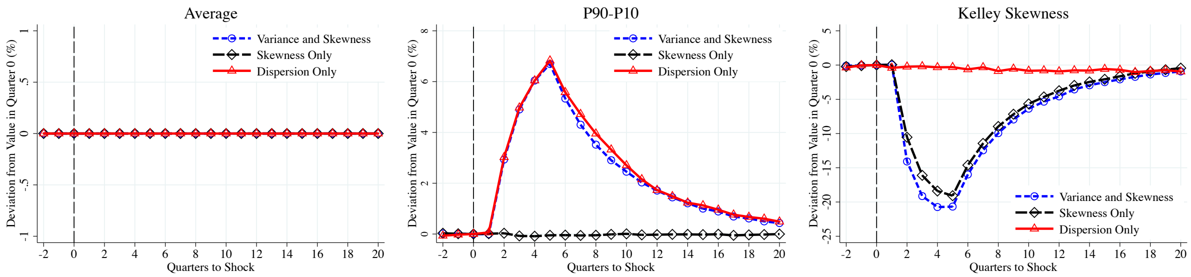
Note: Table VII shows the target used in the estimation of the firm-level productivity process. Rows labelled "Low Risk" and "High Risk" are used in the baseline estimation. The values for "Only Skewness" are used to estimate the parameters when the economy is shocked with a change in the skewness only. Similarly, the values for "Only Variance" are used to estimate the parameters when the economy is assumed to be shocked only by a changes in dispersion keeping the skewness constant.

Table VIII – Estimated Parameters of the Stochastic Process

Parameter Estimates of Idiosyncratic Stochastic Process		
σ_1^L	-0.92	Standard deviation of first mixture in low risk periods (%)
σ_2^L	1.45	Standard deviation of second mixture in low risk periods (%)
μ^L	7.55	Mean of first mixture in low risk periods (%)
p^L	63.67	Probability of first mixture in low risk periods (%)
σ_1^H	4.37	Standard deviation of first mixture in high risk periods (%)
σ_2^H	9.06	Standard deviation of second mixture in high risk periods (%)
μ^H	1.98	Mean of first mixture in high risk periods (%)
p^H	78.28	probability of first mixture in high risk periods (%)
Transition Probabilities of Risk States		
π_L	0.97	Quarterly probability of remaining in low risk state
π_H	0.79	Quarterly probability of remaining in high risk state

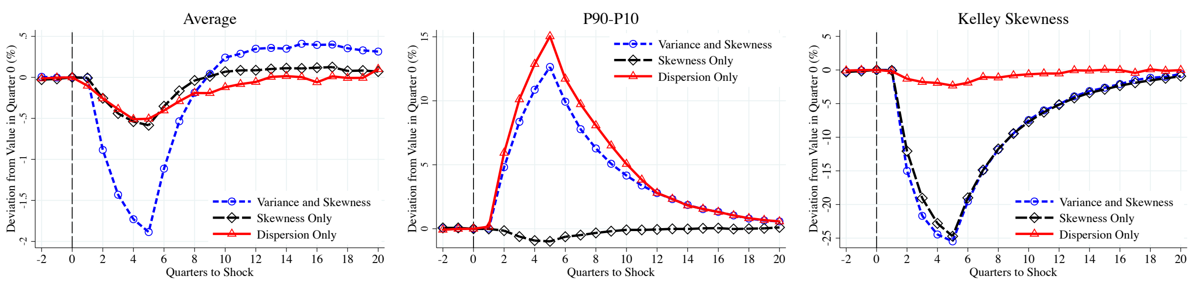
Note: The upper panel of table VIII shows the parameters of the stochastic process of firm-level productivity using Simulated Method of Moments. The estimations process targets moments of the annual change of quarterly sales in Compustat. The parameters for low-risk periods (denoted by an upper script L) are estimated targeting the P90-P10, P90-P50, P50-P10, and Kelley Skewness of the sales growth distribution for the all the full expansion years between 2000 and 2014. The parameters for high-risk periods (denoted by an upper script H) are estimated targeting the same set of moments for years 2001 and 2008 (full recession years). The transition probability π_L is calculated as the share of expansion quarters that where followed by another expansion quarter whereas π_H is calculated as the share of recession quarters that where followed by another recession quarter using data from 1970 to 2014. Recession quarters are taken from the NBER dates.

Figure 9 – Average, Dispersion, and Skewness of Productivity Growth



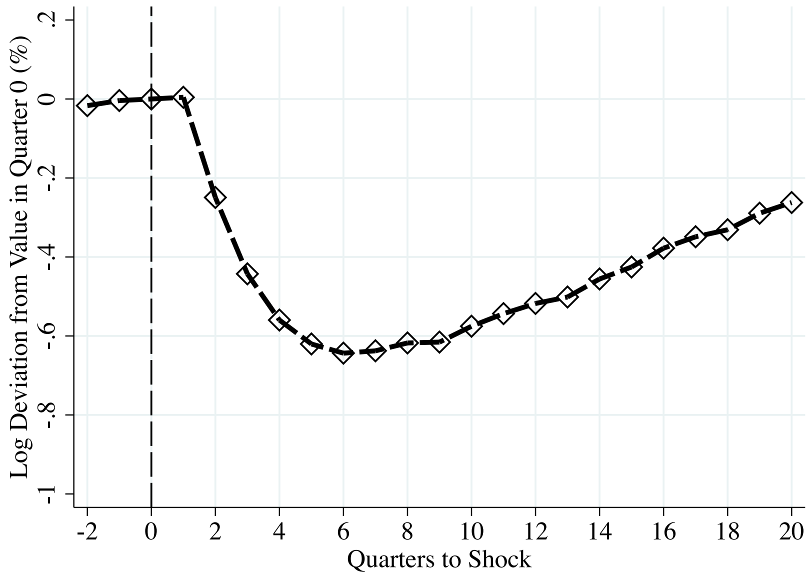
Note: Figure 9 shows moments of the one-year productivity growth distribution ($\Delta e_{j,t} = e_{j,t} - e_{j,t-4}$). Left panel shows the cross sectional average, the middle panel shows the 90th-to-10th percentile differential, and the right panel shows the Kelley skewness. Each plot display the evolution of the corresponding moment after an increase in variance paired with a decrease in skewness (blue line with circles), an increase in dispersion only (red line with triangles), and a decrease in skewness only (black line with diamonds). Each plot shows the average across the independent simulations of 1000 economies for 300-quarter length. We impose a decline in the skewness in quarter 1, allowing normal evolution of the economy afterwards. We display the deviation of the corresponding moment with respect to its value in quarter 0.

Figure 10 – Average, Dispersion, and Skewness of Sales Growth



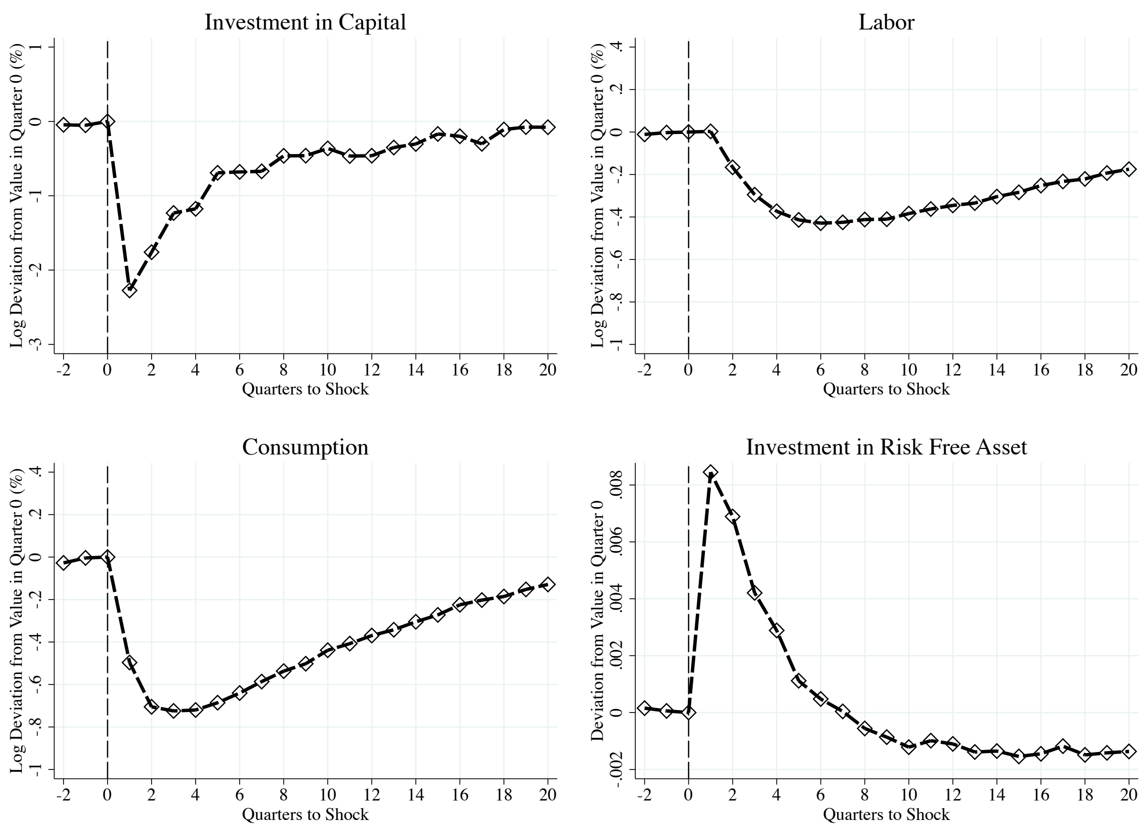
Note: Figure 10 shows moments of the one-year sales growth distribution ($\Delta y_{j,t} = \log y_{j,t} - \log y_{j,t-4}$). Left panel shows the cross sectional average, the middle panel shows the 90th-to-10th percentile differential, and the right panel shows the Kelley skewness. Each plot display the evolution of the corresponding moment after an increase in variance paired with a decrease in skewness (blue line with circles), an increase in dispersion only (red line with triangles), and a decrease in skewness only (black line with diamonds). Each plot shows the average across the independent simulations of 1000 economies for 300-quarter length. We impose a decline in the skewness, increase in dispersion, or both, in quarter 1, allowing normal evolution of the economy afterwards. We display the deviation of the corresponding moment with respect to its value in quarter 0.

Figure 11 – Effect of Skewness Shock in Output



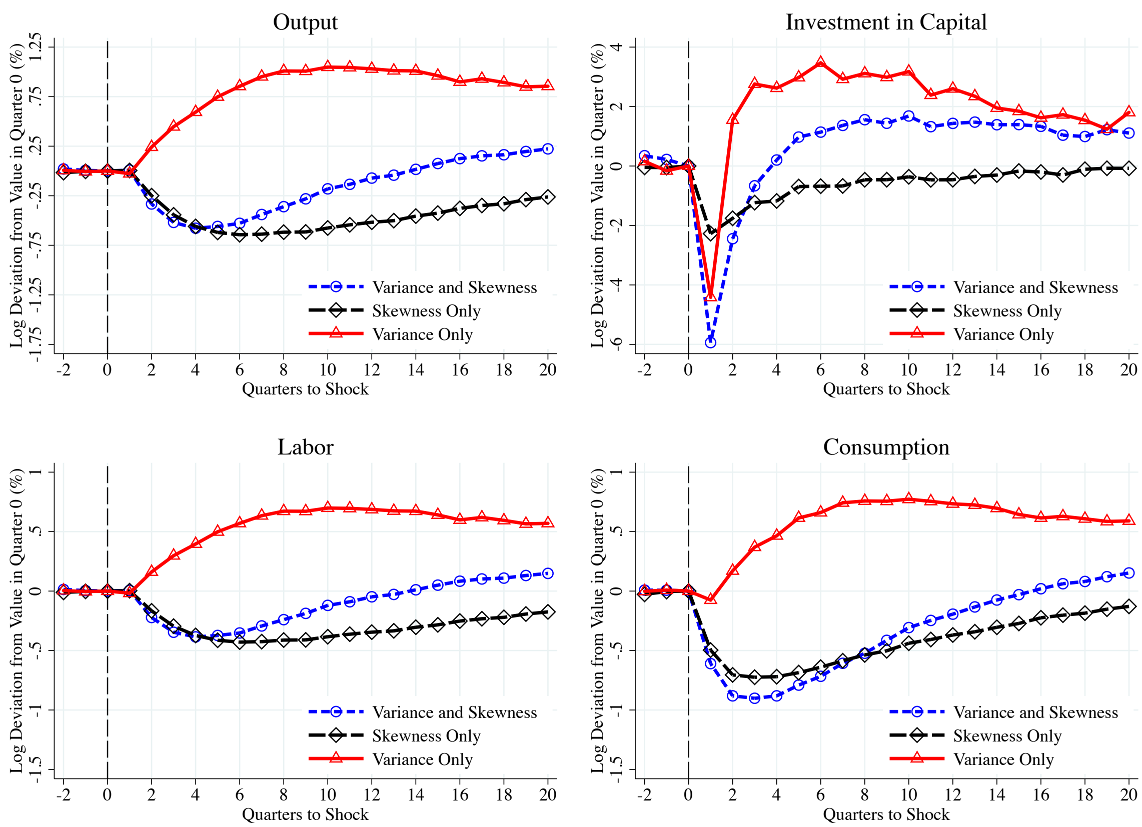
Note: Figure 11 shows the effect of a decline in the skewness of idiosyncratic shocks based in independent simulations for 1000 economies for 300-quarter length. We impose a decline in the skewness in quarter 1, allowing normal evolution of the economy afterwards. We plot the percentage deviation of Output from its value in quarter 0.

Figure 12 – Effect of Skewness Shock on Macro Aggregates



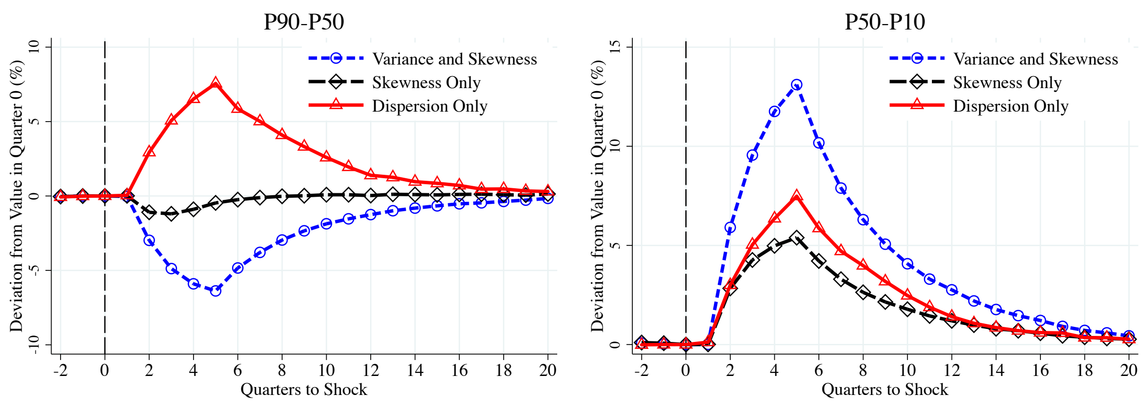
Note: Figure 12 shows the effect of a decline in the skewness of idiosyncratic shocks based in independent simulations for 1000 economies for 300-quarter length. We impose a decline in the skewness in quarter 1, allowing normal evolution of the economy afterwards. We plot the percentage deviation of each macroeconomic aggregate from its value in quarter 0.

Figure 13 – Effect of Skewness and variance Shock on Macro Aggregates



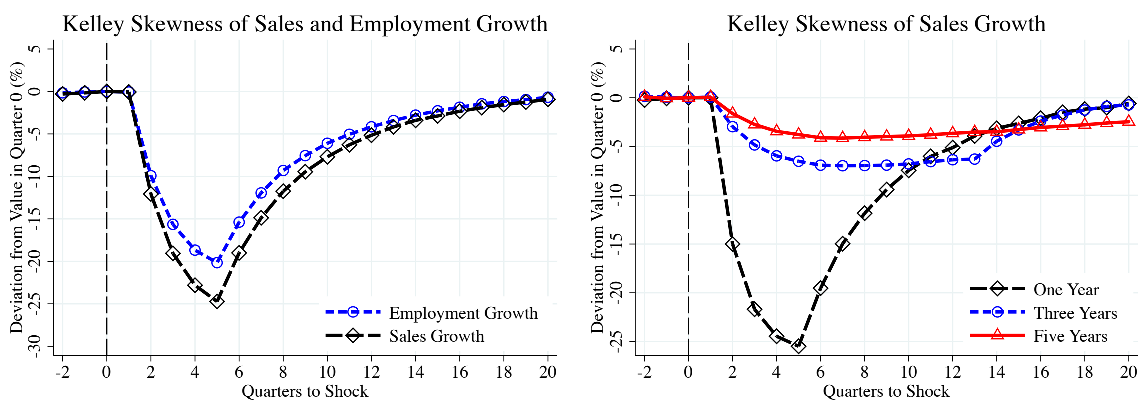
Note: Figure 13 shows the effect of an increase in risk in the form of a decline in skewness of idiosyncratic shocks (black line with diamonds), an increase in dispersion of idiosyncratic shocks (red line with squares), and a decrease in skewness paired with an increase in dispersion of idiosyncratic shocks (blue line with circles) for different macroeconomic outcomes. Each plot is based in independent simulations for 1000 economies for 300-quarter length. We impose a drop in the skewness, increase in dispersion, or both, in quarter 1, allowing normal evolution of the economy afterwards. We plot the percentage deviation of each macroeconomic aggregate from its value in quarter 0.

Figure 14 – Right and Left Tail Dispersion of Sales Growth



Note: Figure 14 shows moments of the one-year sales growth distribution ($\Delta y_{j,t} = \log y_{j,t} - \log y_{j,t-4}$). Left panel shows the cross sectional 90th-to-50th percentiles spread and the right panel shows the 50th-to-10th percentiles spread. Each plot display the evolution of the corresponding moment after an increase in variance paired with a decrease in skewness (blue line with circles), an increase in dispersion only (red line with triangles), and a decrease in skewness only (black line with diamonds). We impose a decline in the skewness, increase in dispersion, or both, in quarter 1, allowing normal evolution of the economy afterwards. Each plot shows the average across the independent simulation of 1000 economies for 300-quarter length. We display the deviation of the corresponding moment with respect to its value in quarter 0.

Figure 15 – Skewness of Employment and Sales Growth



Note: The left panel of figure 15 shows the Kelley skewness of the one-year sales growth distribution ($\Delta y_{j,t} = \log y_{j,t} - \log y_{j,t-4}$, black line with diamonds) and the one-year employment growth distribution ($\Delta n_{j,t} = \log n_{j,t} - \log n_{j,t-4}$, blue line with circles). The right panel shows the Kelley skewness of the one-, three-, and five-years sales growth distribution. We impose a decline in the skewness, increase in dispersion, or both, in quarter 1, allowing normal evolution of the economy afterwards. Each plot shows the average across the independent simulation of 1000 economies for 300-quarter length. We display the deviation of the corresponding moment with respect to its value in quarter 0.

A Data Sources and Variable Construction

This appendix describes the data sources and sample selection. Firm-level data for the United States come from the CRSP/Compustat merged data files and the Census Bureau's Longitudinal Business Statistics (LBD). For the cross-country comparison, we use firm-level data available in the Bureau van Dijk's Osiris database and Global Compustat. Finally, the panel data of macroeconomic series is constructed by using time series of the quarterly GDP from the OECD databases and stock market indexes, retrieved from the corresponding official websites for each country in our sample. In this appendix we explain in detail the data sources, sample selection, and construction of the different moments of the distribution of micro- and macro-level output used in the main body of the text.

A.1 Firm-Level Data for the US Sample

For the United States, we construct time series of cross-sectional dispersion and skewness of the sales growth distribution and the distribution of daily stock returns. To construct the time series of the cross sectional moments of sales growth we proceed as follows. We begin by retrieving firm-level data of net sales, inventories, and cost of sold goods at a quarterly frequency, and employment at an annual frequency, from Compustat from 1964q1 to 2017q4 available at WRDS database. The raw data of sales contain more than 930,000 quarter-firm observations with an average of approximately 4,660 firms per quarter. From here we drop all observations with negative sales and repeated observations. We also drop all observations that do not have a SIC classification or where the classification is above 90. Then, we transform nominal sales into real sales dividing by the CPI, and we calculate the growth rate of sales as the log difference and the arc percentage change between quarter t and $t-4$. This leaves us with 815,990 sales growth (log difference) observations. For our main results, we consider firms with at least 25 years of data on quarterly sales (100 quarters, not necessarily continuous), which further reduces the sample to 266,485 observations, with an average of 1,336 firms per quarter. Finally, in each quarter we calculate the different cross-sectional moments discussed in the main body of this document. For robustness, we provide additional results in which we relax these restrictions by extending the sample to firms with at least 10 years of data (40 quarter) and one year of data (4 quarters). Table A.1 shows the number of observations for each of these samples as well as some cross-sectional moments of the sales growth distribution. When accounting for entry and exit of firms using the arc percentage change, for each we add an observation upon entry (equal to 2) and one additional observation upon exit (equal -2) under the assumption that before and after exit, the firm would have a value of sales equal to 0. We consider entry firms as newly listed firms, while exiting firms are those delisted in a particular period, independent of the reason (M&A, bankruptcy, or any other).

Table A.1 – Sample Size and Cross-Sectional Moments of the Sales Growth Distribution

	N	Mean	S.D.	Min	Q1	Q2	Q3	Max
25 years +	266,485	0.08	0.34	-8.28	-0.02	0.08	0.18	8.84
10 years +	642,813	0.09	0.44	-12.32	-0.03	0.08	0.2	12.43
2 years +	812,912	0.09	0.50	-12.32	-0.04	0.08	0.22	12.43
1 year +	815,990	0.09	0.50	-12.32	-0.04	0.08	0.22	12.43

To construct the quarterly time series of the moments of the distribution of the daily stock price returns we start by downloading daily stock price data from the CRSP/Compustat merged database from 1964 to 2014. The raw data contain more than 75 million day-firm observations. To keep the results as comparable as possible with the sample of sales growth, we restrict the same to firms with 25 or more years of data on the stock price variable (that is, firms with at least 200×25 observations, where 200 is an approximate number of trading days). Then, for each firm we calculate daily returns as the log difference between two consecutive trading days. This leaves us with a sample of 31,230,036 observations, with roughly 153,000 observations per quarter. Then, we calculate different moments of the cross-sectional distribution over all the observations in each quarter. Table A.2 shows the number of observations in each sample and some cross-sectional moments of daily stock returns.

Table A.2 – Sample Size and Cross-Sectional Moments of the Daily Stock Returns

	N	Mean	S.D.	Min	Q1	Q2	Q3	Max
25 years +	31,230,036	-0.00	0.04	-6.32	-0.01	0.00	0.01	6.17
10 years +	73,670,235	-0.00	0.04	-6.32	-0.01	0.00	0.01	6.17

A.2 LBD: Sample selection and moments calculation

We construct measures of employment growth at the firm level using the Census Bureau's Longitudinal Business Database (LBD). The LBD covers the universe of establishment in the non-farm private sector in the United States from 1976 to 2015. It provides detailed establishment and firm-level information on employment, payroll, location, firm age, industry, etc.. Crucially, the LBD allows following firms over time, allow us to construct measures of employment growth at different time horizons. From the LBD we select a sample of establishments that, in a given year, have nonnegative, non missing employment and payroll and have valid industry data. We then sum up the establishment employment within the same firm to construct an annual measure of employment. We measure the growth rate of employment of firm j in period t as the log-difference between periods t and $t+k$, $g^e_{j,t} = \log E_{j,t+k} - \log E_{j,t}$ where $k \in \{1, 3, 5\}$. This measure does not allow us to include firms entering and exiting the market, henceforth, we also calculate employment growth using the arc-percent change.

Our preferred measure of skewness of the employment growth distribution is the Kelley measure of skewness calculated as in equation (1). This measure requires the calculation different percentiles of the distribution. Notice that a percentile provides information of a particular firm in the distribution, and as such, violates the disclosure criteria imposed by the Census. Hence, to avoid the disclosure of any sensitive information we calculate the *pth* percentile of the employment growth distribution as the employment-weighted average around a band of +1 and -1 percent. Hence, the 90th percentiles of the distribution is the weighted average of the employment growth across all observations between the percentiles 89th and 91th of the distribution, both ends included. We proceed in the same way to construct the 10th and 50th percentiles of the distribution and use these values to calculate the Kelley skewness. All measures are weighted by the average employment of the firm between periods t and $t+k$, that is $\overline{E}_{j,t} = 0.5 \times (E_{j,t+k} + E_{i,t})$. The massive sample size of the LBD ensures that the sample used to calculate each of the percentile is large enough to have a an accurate approximation to the actual percentiles of the distribution. Within industry moments used in the regression results presented in table III where calculated in the same way.

We also use the LBD to compare the distribution of employment growth between recessions and expansions years using kernel density estimation. The sample selection is the same used in the rest of our results, however, Census guidances for calculating kernel densities require to drop the bottom and top 5% of the distribution. The kernel densities presented in figure 4 where calculated over the remaining sample.

A.3 Firm-Level Data for the Cross-Country Sample

Here we describe the construction of the cross-sectional moments of the sales growth distribution and daily stock returns for the panel of countries. Sales data come from the Bureau van Dijk's Osiris database. Osiris is a database of globally listed public companies, commodity producing firms, banks, and insurance companies from over 190 countries. The combined industrial company data set contains standardized and as reported financial information, including restated accounts, for up to 20 years over 80,000 companies. However, we focus on the industrial data set only and we do not perform any analysis using the data on banks or other financial institutions. The raw data contain 873,882 country/firm/year observations from 1982 to 2014 over 148 countries. Then we drop all observations with missing or negative sales and we clean all duplicated observations. We transform all observations to US dollars using the exchange rate reported in the same database. Then, we transform sales into real sales using annual CPI and calculate the growth rate of real sales as the log change and arc percentage change between years t and $t+1$. This leaves us with 858,915 observations. We further restrict the sample to country/year cells with more than 100 observations, countries with more than 10 years of data, and years with more than 5 countries. This sample selection reduces the total number of observations to 619,918 in 44 countries. Table A.3 shows the countries in the sample, the number of years and observations available for each of them, and some cross-sectional statistics of the sales growth distribution. We complement this data with real GDP in US dollars from World Bank's World Development Indicators database.

Table A.3 – Countries and Cross-Sectional Moments of Sales Growth Distribution

Country	Start	End	N	Mean	S.D.	Min	Max	Q1	Q2	Q3
ARG	2000	2012	2,326	0.05	0.59	-9.19	4.19	-0.07	0.07	0.21
AUS	1985	2013	14,476	0.15	0.62	-3.39	4.57	-0.09	0.10	0.32
BEL	1997	2012	2,512	0.07	0.42	-4.74	7.13	-0.08	0.05	0.20
BMU	1993	2013	9,750	0.04	0.55	-2.89	3.81	-0.14	0.05	0.23
BRA	1995	2012	8,057	0.09	0.37	-3.17	3.24	-0.11	0.09	0.27
CAN	1985	2013	37,649	0.16	0.62	-4.02	3.92	-0.08	0.10	0.34
CHE	1991	2012	4,062	0.05	0.28	-2.92	4.06	-0.07	0.05	0.16
CHL	1992	2012	7,643	0.06	0.39	-3.98	5.19	-0.07	0.07	0.19
CHN	1998	2012	23,188	0.16	0.30	-2.67	2.15	0.01	0.15	0.30
COL	2002	2011	1,586	0.06	0.42	-3.36	3.60	-0.06	0.08	0.19
CYM	1999	2013	7,868	0.19	0.57	-3.90	6.69	-0.04	0.17	0.38
DEU	1985	2012	12,667	0.06	0.34	-2.85	2.32	-0.09	0.05	0.18
DNK	1993	2012	2,625	0.06	0.34	-4.51	3.06	-0.08	0.05	0.18
EGY	2002	2012	2,919	0.05	0.51	-7.02	6.66	-0.12	0.05	0.22
ESP	1998	2012	2,472	0.07	0.50	-6.80	6.71	-0.08	0.05	0.20
FIN	1999	2012	2,407	0.06	0.27	-1.52	2.54	-0.08	0.04	0.18
FRA	1985	2012	13,714	0.07	0.27	-3.89	5.13	-0.07	0.06	0.18
GBR	1985	2013	31,839	0.11	0.39	-2.63	3.26	-0.07	0.07	0.23
GRC	1999	2012	2,593	0.03	0.35	-5.76	3.59	-0.13	0.03	0.20
HKG	1994	2012	4,006	0.06	0.52	-4.06	4.61	-0.11	0.05	0.22
IDN	2002	2012	3,401	0.09	0.40	-2.15	4.30	-0.06	0.09	0.24
IND	1998	2013	32,062	0.05	0.58	-3.42	3.67	-0.14	0.06	0.26
IRN	2002	2013	2,005	-0.01	0.44	-1.81	3.45	-0.13	0.06	0.21
ISR	1996	2012	6,064	0.10	0.50	-4.27	4.01	-0.08	0.08	0.24
ITA	1996	2012	3,471	0.06	0.38	-5.39	7.01	-0.09	0.04	0.18
JOR	2002	2012	1,633	0.04	0.63	-4.88	5.38	-0.14	0.04	0.20
JPN	1990	2013	71,168	0.01	0.18	-1.24	1.45	-0.10	0.00	0.11
KOR	1990	2012	33,721	0.09	0.33	-4.56	4.34	-0.06	0.08	0.23
MEX	1995	2012	3,560	0.05	0.51	-8.20	7.92	-0.07	0.06	0.16
MYS	1985	2013	14,673	0.04	0.38	-2.96	2.76	-0.12	0.05	0.20
NLD	1988	2012	4,231	0.07	0.28	-1.73	2.20	-0.07	0.05	0.19
NOR	1995	2012	2,919	0.11	0.53	-5.14	6.28	-0.07	0.08	0.25
NZL	2002	2013	1,873	0.13	0.48	-4.27	6.34	-0.05	0.08	0.22
PAK	1998	2012	2,727	0.05	0.39	-6.74	3.79	-0.10	0.05	0.20
PER	1997	2012	2,487	0.07	0.38	-4.17	2.73	-0.07	0.07	0.20
PHL	2000	2012	2,125	0.07	0.57	-4.62	6.23	-0.10	0.06	0.22
RUS	2003	2012	3,212	0.09	0.40	-2.53	4.29	-0.07	0.09	0.23
SGP	1987	2013	8,774	0.08	0.37	-4.25	2.87	-0.10	0.07	0.23
SWE	1993	2012	5,862	0.11	0.46	-3.33	4.28	-0.08	0.08	0.23
THA	1995	2012	6,552	0.09	0.34	-1.90	3.08	-0.05	0.08	0.23
TUR	2003	2012	2,271	0.07	0.69	-11.32	14.09	-0.13	0.07	0.23
TWN	1997	2012	24,795	0.03	0.28	-1.17	1.81	-0.11	0.02	0.18
USA	1985	2013	127,538	0.14	0.48	-2.35	3.24	-0.05	0.07	0.25
ZAF	1996	2013	4,147	0.08	0.47	-3.21	12.06	-0.10	0.05	0.22

Table A.4 – Countries and Cross-Sectional Moments of the Daily Returns Distribution

Country	Start	End	N	Mean	S.D.	Q1	Q2	Q2
AUS	1988q4	2014q4	2,507,095	0.00	0.19	-0.01	0.00	0.01
BEL	2001q3	2013q2	412,435	0.00	1.34	-0.01	0.00	0.01
BRA	2001q1	2014q4	429,362	0.00	0.26	-0.02	0.00	0.02
CHE	1993q3	2014q4	833,941	0.00	0.39	-0.01	0.00	0.01
DEU	1988q3	2014q4	2,606,624	0.00	0.19	-0.01	0.00	0.01
DNK	2001q2	2014q4	418,823	0.00	0.21	-0.01	0.00	0.01
ESP	1997q2	2011q3	526,412	0.00	0.13	-0.01	0.00	0.01
FIN	2001q4	2014q4	392,801	0.00	0.28	-0.01	0.00	0.01
FRA	1989q1	2014q4	2,023,102	0.00	0.31	-0.01	0.00	0.01
GBR	1986q1	2014q4	4,836,660	0.00	0.35	-0.01	0.00	0.01
GRC	1999q2	2014q4	765,160	0.00	0.09	-0.01	0.00	0.01
IDN	1994q3	2014q4	661,664	0.00	0.83	-0.01	0.00	0.01
IND	1996q2	2014q4	2,555,536	0.00	0.37	-0.02	0.00	0.02
ISR	1999q4	2014q4	621,181	0.00	0.18	-0.01	0.00	0.01
ITA	1990q3	2014q4	955,105	0.00	0.27	-0.01	0.00	0.01
JPN	1986q1	2014q4	13,800,000	0.00	0.13	-0.01	0.00	0.01
KOR	1989q1	2014q4	3,736,246	0.00	0.41	-0.02	0.00	0.02
NLD	1992q3	2014q4	679,452	0.00	0.34	-0.01	0.00	0.01
POL	1998q4	2014q4	521,611	0.00	0.11	-0.01	0.00	0.01
SWE	1998q1	2014q4	780,881	0.00	0.36	-0.01	0.00	0.01
TUR	1996q1	2014q4	923,139	0.00	0.34	-0.02	0.00	0.02
USA	1986q1	2014q4	31,100,000	0.00	0.03	-0.01	0.00	0.01
ZAF	1995q3	2014q4	712,133	0.00	0.48	-0.01	0.00	0.01

The data on daily stock prices come from the Global Compustat database, which provides standardized information on publicly traded firms for several countries at annual, quarterly, and daily frequencies. The raw data contain firm-level observations of daily stock prices between 1986 and 2014 for 48 countries. We drop all duplicated observations and drop all firms with less than 2000 observations (approximately 10 years of data depending on the number of trading days) . Then we calculate daily price returns as the log difference of the stock price between two consecutive trading days. We further restrict our sample to quarter/country periods with more than 100 firms. This produces reduces the sample to 24 countries. Then, within each quarter, we calculate different cross-sectional moments of the daily stock price distribution. Table A.4 shows the number of observations per country, the period that our sample covers, and some cross sectional moments of the daily stock price distribution within each country.

A.4 Macro-Level Data

To construct our measures of macroeconomic dispersion and skewness, we construct a panel of countries for which we collect information on quarterly GDP growth and daily prices of the main stock price index of the corresponding country. Real GDP is obtained from the quarterly national accounts in the OECD data-base (historical GDP expenditure approach). The raw data contain information for about 66 countries starting in different points at time. To keep the sample as homogeneous as possible, we only consider observations between 1970 and 2014. This gives us a panel of 44 countries with an average of 130 observations per country. Columns (1) to (4) of Table A.5 show the countries and periods for which we have quarterly GDP data. We calculate the growth rate as the log difference of the real GDP between quarter t and the same quarter of the following year. Then, for each country i, we calculate a time series of the different moments of the GDP growth distribution over a trailing window of 13 quarters (the corresponding quarter and the observations in the previous 3 years).

The moments of the stock price index returns are constructed in a similar fashion. First we collect daily price index values for several countries. Stock prices are not readily available in a particular data set, especially for developing countries, and therefore, we took the data directly from the official sources when available. Columns (5) to (8) of Table A.5 show the set of countries and period of time for which have collected stock price data. Then, for each country i we calculate daily returns as the log difference of the index between two consecutive trading days. Finally, we calculate the moments of the stock returns distribution for a particular country i in quarter t over a trailing window that contains the corresponding quarter and the 3 previous years to make a total of 13 quarters. This generates a sample of 30 countries with an average sample size of 96 observations.

Table A.5 – Countries, Sample Size, and Cross-Sectional Moments of the Macro Time Series

Real GDP								Stock Price Index			
Country	Start	End	N	Country	Start	End	N	Country	Start	End	N
(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Argentina	1997q1	2014q4	71	Netherlands	1971q3	2014q4	173	Australia	1986q1	2014q4	115
Australia	1971q3	2014q4	173	New Zealand	1971q3	2014q4	173	Austria	1994q2	2014q4	82
Austria	1971q3	2014q4	173	Norway	1971q3	2014q4	173	Belgium	1992q4	2014q4	88
Belgium	1971q3	2014q4	173	Poland	1999q1	2014q4	63	Canada	1979q3	2014q4	141
Brazil	2000q1	2014q4	59	Portugal	1971q3	2014q4	173	Chile	2003q3	2014q4	45
Canada	1971q3	2014q4	173	Russia	1999q1	2014q4	63	Denmark	1991q2	2014q4	94
Chile	1999q1	2014q4	63	Slovak Republic	1997q1	2014q4	71	Finland	1988q3	2014q4	105
Colombia	2004q1	2014q4	43	Slovenia	2000q1	2014q4	59	France	1991q3	2014q4	93
Costa Rica	1995q1	2014q4	79	South Africa	1971q3	2014q4	173	Germany	1992q2	2014q4	90
Czech Republic	1999q1	2014q4	63	Spain	1971q3	2014q4	173	Hungary	1996q3	2014q4	73
Denmark	1971q3	2014q4	173	Sweden	1971q3	2014q4	173	Indonesia	1991q3	2014q4	93
Estonia	1999q1	2014q4	63	Switzerland	1971q3	2014q4	173	Ireland	1984q3	2014q4	121
Finland	1971q3	2014q4	173	Turkey	1971q3	2014q4	173	Israel	1996q3	2014q4	73
France	1971q3	2014q4	173	United Kingdom	1971q3	2014q4	173	Italy	1999q3	2014q4	61
Germany	1971q3	2014q4	173	United States	1971q3	2014q4	173	Japan	1985q3	2014q4	117
Greece	1971q3	2014q4	173					Korea	1981q3	2014q4	133
Hungary	1999q1	2014q4	63					Luxembourg	2000q3	2014q4	57
Iceland	1971q3	2014q4	173					Mexico	1995q3	2014q4	77
India	2000q2	2014q4	58					Netherlands	1984q3	2014q4	121
Indonesia	1994q1	2014q4	83					New Zealand	1993q3	2014q4	85
Ireland	1971q3	2014q4	173					Norway	1997q3	2014q4	69
Israel	1999q1	2014q4	63					Poland	1996q3	2014q4	73
Italy	1971q3	2014q4	173					Portugal	1989q3	2014q4	101
Japan	1971q3	2014q4	173					South Africa	1997q1	2014q4	71
Korea	1974q1	2014q4	163					Spain	1988q3	2014q4	105
Latvia	1999q1	2014q4	63					Sweden	1988q2	2014q4	106
Lithuania	1999q1	2014q4	63					Switzerland	1990q1	2014q4	99
Luxembourg	1971q3	2014q4	173					Turkey	1989q3	2014q4	101
Mexico	1971q3	2014q4	173					United Kingdom	1985q3	2014q4	117

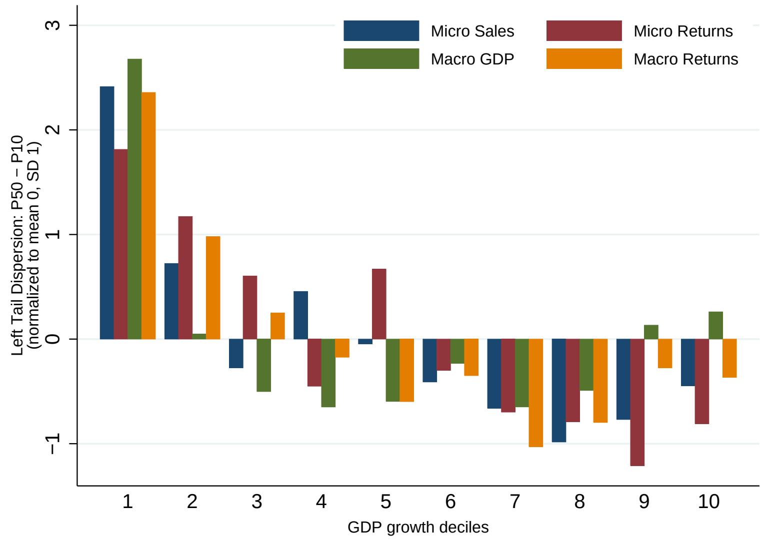
Table A.6 – Dispersion is Higher at the Industry Level During Low Industry Growth

	90th-to-10th Percentiles Differential of Firm's Outcomes					
	(1)	(2)	(3)	(4)	(5)	(6)
	Real Sales Growth		Employment Growth		Stock Returns	
	One Year	Three Year	One Year	Three Years	One Year	Three Years
$\overline{\Delta S}_{j,t}$	-3.86***	-4.43*	-0.66	0.76	-3.70**	-5.44**
	(1.243)	(2.185)	(1.583)	(1.380)	(1.509)	(2.089)
R^2	0.28	0.29	0.28	0.31	0.34	0.30
N	3,698	3,652	969	930	3,646	3,602
F.E.	Qtr/Ind	Qtr/Ind	Yr/Ind	Yr/Ind	Qtr/Ind	Qtr/Ind
Freq.	Qtr	Qtr	Yr	Yr	Yr	Yr
Sample	802K	780K	231K	193K	733K	651K

Note: Table A.6 shows a series of industry-level panel regressions. In each column the depend variables are cross sectional dispersion – measured by the 90th-to-10th percentiles spread – of the growth rates of real quarterly sales, annual employment growth, and quarterly stock returns distribution within period-industry cells defined by 2-digit NAICS (total of 22 industries) for a sample of publicly traded firms from the Compustat/CRSP dataset. The independent variable, $\overline{\Delta g}_{j,t}$, is the average of the sales growth distribution within the period-industry cell. All cross sectional moments were calculated weighting by firm-size within each industry. In all regressions the sample period is 1970 to 2017 and consider a full set of period and industry fixed effects. Row labeled Sample corresponds to the total firm-period observations used to calculate the cross sectional moments. N corresponds to the number of period-industry observations used in the regressions. Standard errors in parentheses below the point estimates are clustered at the NAIC-2 industry level. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

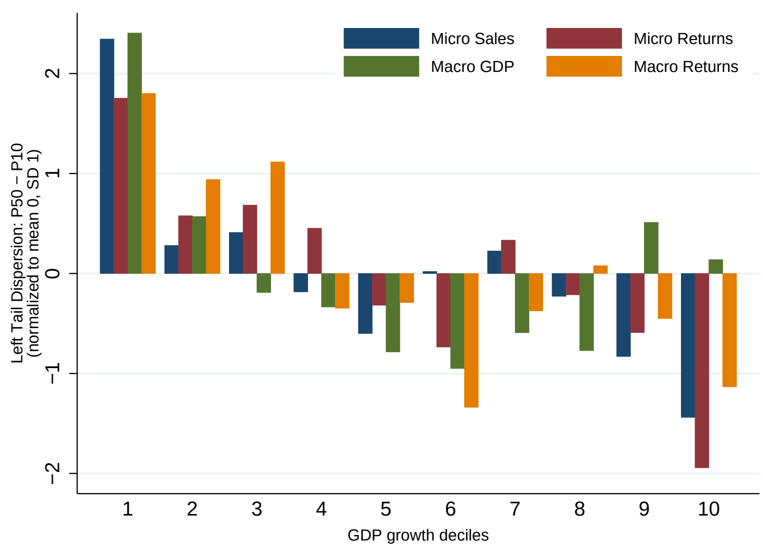
B Additional Results

Figure A.1 – Left-Tail Dispersion is Countercyclical Across Countries



Note: Figure A.1 is based on data for 44 developed and developing countries over the period 1970 to 2014. Each decile shows four measures of left-tail dispersion, two macro (the $P5010$ of the distribution of the growth rate of GDP and the $P5010$ of the distribution of daily returns of a stock pride index) and two micro (the $P5010$ of the distribution of annual sales growth and the $P5010$ of the distribution of daily stock returns). See notes in Figure **??** for additional details.

Figure A.2 – Left-Tail Dispersion is Countercyclical in the United States



Note: Figure A.2 is based on quarterly sales data, daily stock returns, and quarterly GDP growth for the United States over the period 1970 to 2014. Each decile shows four measures of left-tail dispersion, two macro (the $P5010$ of the distribution of the growth rate of GDP and the $P5010$ of the distribution of daily returns of the S&P500) and two micro (the $P5010$ of the distribution of quarterly sales growth and the $P5010$ of the distribution of daily stock returns). See notes in Figure **??** for additional details.

TABLE A.7 – DISPERSION OF FIRM'S OUTCOMES IS LOWER DURING RECESSIONS

	Dispersion of the Growth Rate of Firm Level Outcomes							
	United States					Cross -Country		
	(1)	(2)	(3)	(4)	(5)	(7)	(8)	(9)
	Firm Sales		Stock Returns		Firm Emp.	Firm Sales	Firm Stock	Firm Emp.
	One Year	Three Year	One Year	Three Year	One Year	Growth	Returns	Growth
$\Delta GDP_{i,t}$	-3.912***	2.548**	-3.929**	-4.781***	0.934*	-0.792	-1.838	-0.755
	(1.144)	(0.989)	(1.622)	(1.781)	(0.496)	(0.591)	(1.786)	(0.736)
R^2						0.371	0.405	0.258
N	184	182	180	180	39	838	4306	824
Freq.	Qtr	Qtr	Qtr	Qtr	Yr	Yr	Qtr	Yr
F.E.	N	N	N	N	N	Yr./Ctry.	Qtr./Ctry.	Yr./Ctry.
Sample								
Source	CSTAT	CSTAT	CSTAT	CSTAT	LBD	Macro	Macro	Macro

Note: The left panel of table A.7 shows a series of times series regressions in which the dependent variable are the 90th-to-10th percentiles spread of the one-year and three-years growth rate of sales growth (columns 1 and 2), stock returns (columns 3 and 4), and employment growth (columns 5 and 6) for a sample of firms from Compustat/CRSP (columns 1 to 4) and the LBD (columns 5 and 6). Compustat data covers the period 1970 to 2017 whereas LBD data covers the period 1976 to 2015. In each regression the independent variable is the annual growth rate of quarterly GDP per capita. All cross firm level moments were calculated weighting growth rate observations by firm size. All regressions include a linear trend. Newey-West standard errors in parentheses below the point estimates. The right panel of table A.7 shows a series of country-panel regressions where the dependent variable is the within country P90-P10 spread of firm level sales growth, stock returns, or employment growth. The independent variable is the growth rate of GDP per capita at the country level. Sales and employment data is obtained from the BVD Osiris data base whereas stocks returns are obtained from Global Compustat. All cross sectional moments where calculated weighting growth rate observations by firm size. All regressions consider a full set of time and country fixed effects. The raw labeled Sample shows the underlying sample of firms used to calculate the cross sectional moments. LBD data on sample size are not disclosed. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table A.8 – Skewness of Firm's Outcomes is Lower During Recessions

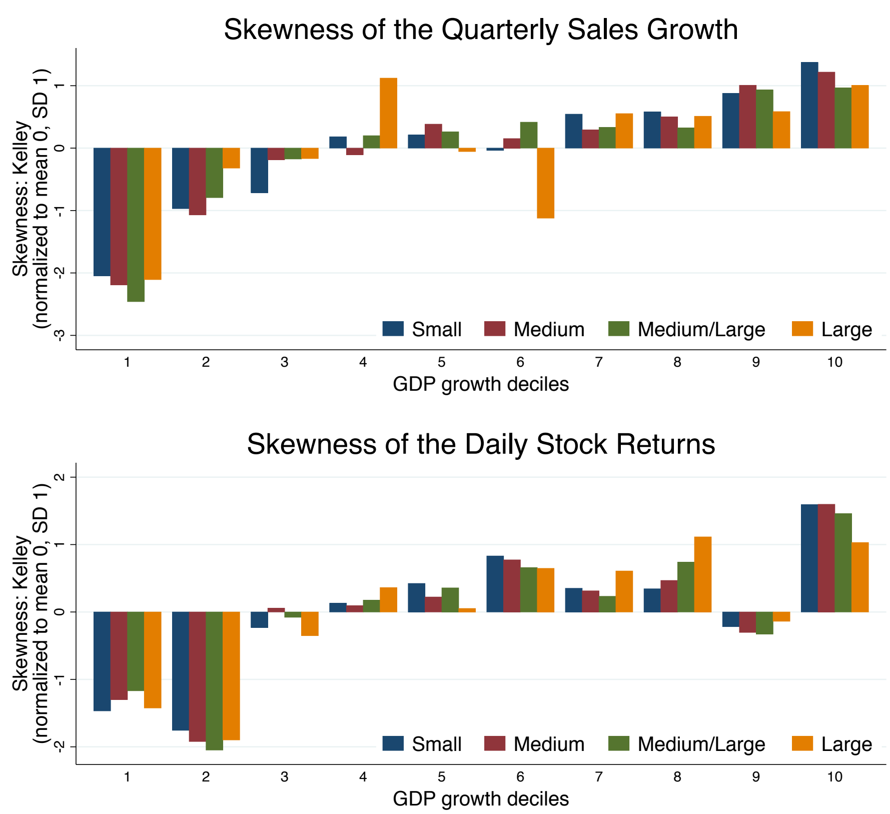
	Kelley Skewness and Kurtosis of the Growth Rate of Firm Level Outcomes								
	Kelley Skewness					Crow-Siddiqui Kurtosis			
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
	Residual Sales		Sales per Employee		Sales Deviation	Firm Sales		Stock Returns	
	One Year	Three Years	One Year	Three Years		One Year	Three Years	One Year	Three Years
$\Delta GDP_{i,t}$	3.80***	1.480	3.01**	4.17***	1.46**	0.36***	−0.19	0.16***	−0.21*
	(1.42)	(0.91)	(1.23)	(0.97)	(0.58)	(0.08)	(0.12)	(0.06)	(0.13)
N	178	178	174	166	47	184	182	180	180
Freq.	Qtr	Qtr	Qtr	Qtr	Yr	Qtr	Qtr	Qtr	Qtr
Sample	500K	500K	500K	500K	113K	640K	640K	650K	650K
Source	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT	CSTAT

Note: The left panel of table A.8 shows a series of times series regressions for the United States in which the dependent variable are the Kelley Skewness of the one-year and three-years growth rate of residualized sales growth (columns 1 and 2) and the growth rate of sales-per-employee (columns 3 and 4) for a sample of firms from Compustat/CRSP. In columns (1) and (2) the we have orthogonalized the growth rates of sales from time fixed effects, firm-fixed effect, size, and other fir-level observable characteristics. Column (5) shows the correlation of GDP growth and the cross sectional skewness of the deviation of annual firms' sales from a HP trend. Compustat data covers the period 1970 to 2017. The dependent variable in columns (6) to (9) is the Crow-Siddiqui measure of Kurtosis defined as $CKU_t = \frac{P97.5_t - P02.5_t}{P75_t - P25_t}$. In each regression the independent variable is the annual growth rate of quarterly GDP per capita. All firm level moments were calculated weighting growth rate observations by firm size measured by the average sales of the firm between periods t and $t+k$. All regressions include a linear trend. Newey-West standard errors in parentheses below the point estimates. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$.

Table A.9 – Distribution of Observations by Industry

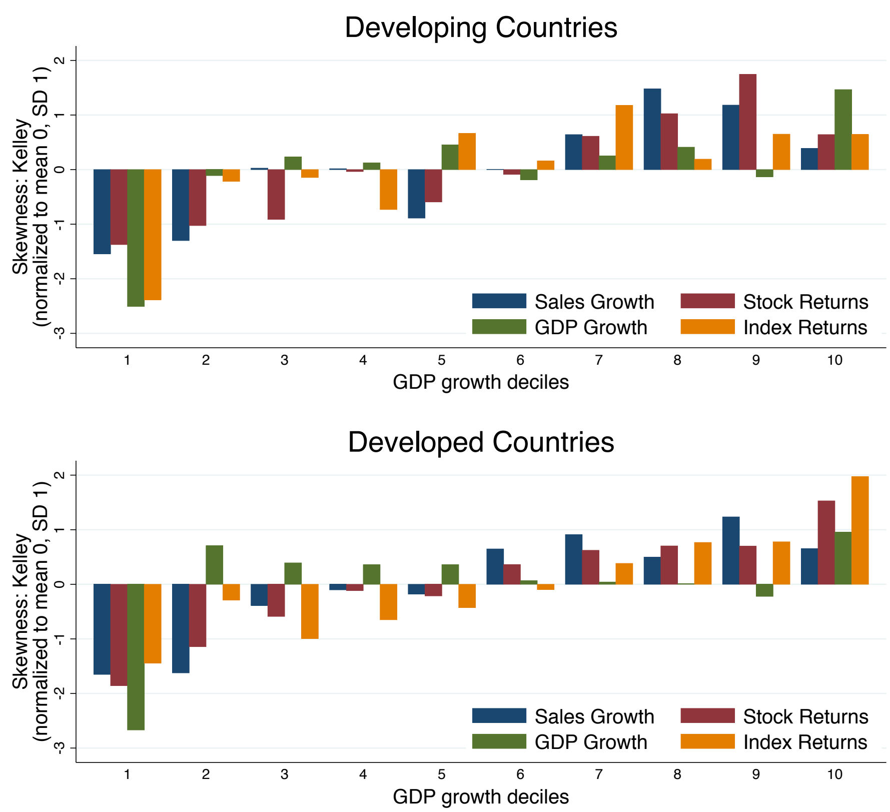
	Observations	Mean	S.D.	Min	Q1	Q2	Q3	Max
Agriculture and Extraction	13,863	0.13	0.44	-2.0	-0.05	0.13	0.32	2.0
Construction	3,952	0.13	0.34	-1.98	-0.03	0.15	0.31	2.0
Manufacturing	137,453	0.11	0.28	-2.0	0.01	0.11	0.22	2.0
Transportation and Utilities	38,428	0.12	0.24	-2.0	0.04	0.11	0.2	2.0
Trade	25,445	0.12	0.25	-2.0	0.04	0.12	0.21	2.0
FIRE	31,320	0.11	0.36	-2.0	0.01	0.11	0.23	2.0
Services	22,387	0.13	0.33	-2.0	0.02	0.12	0.23	2.0

Figure A.3 – Skewness is Procyclical for different firm-size groups



Note: The figure is based on quarterly sales data and daily stock returns data for the United States over the period 1970 to 2014. Each quarter is placed into a bin based on the decile of the annual growth rate of quarterly GDP, with bins from 1 to 10, where 1 is the lowest decile growth and 10 is the highest. Size classes are constant within a year and are defined by the quartiles of the industry-specific distribution of average employment defined by the average employment between period $t-1$ and $t-3$. Industries are defined as in Figures **??** and **??**. Both the cross-sectional moments and the industry-specific size distribution are calculated over a sample of publicly traded firms from the CRSP/Compustat data set with 10 or more years of data (40 quarters in the case of sales data and 2000 trading days in the case of stock prices).

Figure A.4 – Skewness is Procyclical Within Different Groups of Countries



Note: Each panel of Figure A.4 is based on annual, quarterly, and daily data for a sample of developed and developing countries over the period 1986 to 2014. Developing countries are those in the upper half of the distribution of GDP per capita in the year 2000 (i.e. Germany, United States, etc.) while developing countries are those in the lower half of the distribution (i.e. Argentina, Chile, etc.).