A theory of structural change that can fit the data*

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Abstract

We propose a dynamic theory that is consistent with the long-run structural change of consumption expenditure shares in agriculture, manufacturing and services over more than a century. We first document three robust features in the long-run data for the United States, the United Kingdom, Canada and Australia: (i) a monotonic decrease in agriculture; (ii) a hump shape in manufacturing; and (iii) an accelerated rise of services. Using historical panel data on sectoral prices and nominal expenditures from 1900 to 2014, we then test what demand side theory can quantitatively explain the observed structural change. We find that the standard non-homothetic preference specifications used in the literature - the generalized Stone-Geary and the Price Independent Generalized Linearity (PIGL) specification - struggle to do so. Within our intertemporal framework, we then consider the entire class of preferences that allows for the aggregation of individual Euler equations - the class of intertemporally aggregable (IA) preferences. This general class nests the standard specifications and allows for the identification of preference parameters from aggregate data. Moreover, its expenditure system is flexible enough to capture the non-monotonic relationship between sectoral expenditure shares. Despite the flexibility, the IA preference specification is parsimonious and can be used in a multi-sector general equilibrium model with steady growth and heterogeneity in individual consumption expenditures. In the empirical analysis we show that the standard specifications are rejected against the more flexible parameterizations of IA preferences and we document the importance of flexible income effects.

Keywords: Structural change, Multi-sector growth model, Non-homothetic preferences, Relative price effects, Non-monotonic expenditure shares, Aggregation.

JEL classification: O11, O14, L16, E21.

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1 Introduction

Structural transformation – the reallocation of economic activity across sectors – is a stylized fact of modern economic development. As countries develop, the consumption expenditure and value-added share of the agricultural sector tends to decline steadily, the manufacturing share first increases and then decreases, and eventually services become the dominant sector. Besides technological change on the supply side that drives relative prices and total expenditure, the process of structural change crucially depends on the strength of the relative price and income effects on the demand side. This has important implications for aggregate productivity growth and welfare. For example, the Baumol (1967) cost disease can occur when economic activity is reallocated from more to less productive sectors, which slows down aggregate growth.[1]

Although the pattern of structural change is well known and calls for a suitable theoretical framework, the empirical literature on structural change has come to different conclusions on whether existing demand side theories are consistent with the observed reallocation of economic activity - even across the broad sectors agriculture, manufacturing, and services. For example, Herrendorf, Rogerson, and Valentinyi (2013) find that the sectoral demands derived from the standard non-homothetic generalized Stone-Geary preference specification are consistent with the United States' structural change in the postwar era. On the other hand, Buera and Kaboski (2009) show for a historical sample starting in 1870, that the same specification struggles to fit the data for the United States.

In this paper we address the question what preference specification can fit the long-run data on structural change. We aim to make three contributions to the existing literature: (i) We document the evolution of sectoral final consumption expenditure shares, relative prices, and per-capita expenditure in the United States, the United Kingdom, Canada, and Australia since the beginning of the last century. This long-run perspective is crucial in order to observe the non-monotonic patterns, but the existing literature has mainly focused on consumption expenditure data

[1]If the service sector constitutes a gross complement to the rest of the economy and its relative price increases over time, more and more resources will be shifted to the service sector. An analogous argument applies if services have a higher income elasticity than manufacturing and real income per-capita is steadily increasing over time.

from the postwar era. (ii) We investigate whether preference specifications previously used in the literature on structural change can fit the historical data. We find that existing theories struggle to simultaneously match the hump-shape in manufacturing, the accelerated rise of services, and the continued decline of agriculture observed in the long-run data. (iii) We propose the class of intertemporally aggregable (IA) preferences that can fit the long-run data, allows for heterogeneity in individual expenditures, and is consistent with standard multi-sector growth models. The defining characteristic of these preferences is that they allow for intertemporal aggregation, such that we can identify the preference parameters from historical time series data on aggregates only. Furthermore, the IA preference specification nests both the PIGL and the generalized Stone-Geary as special cases, which allows us to directly test these standard preferences against more flexible parameterizations.

Using final consumption expenditure data for the USA, GBR, CAN, and AUS, we confirm the finding of the existing literature that the standard preference specification explains the observed structural transformation very well in the postwar era. Herrendorf et al. (2013) document this for the United States using the generalized Stone-Geary specification and we reach the same conclusion for the remaining three countries as well. However, once we consider long-run samples that also include data from the first half of the past century, the standard preferences struggle to fit three strong regularities that are observed in the historical data across all four countries: (i) the continued decline of the expenditure share for agriculture, (ii) the hump-shape of the manufacturing share, (iii) and the accelerated rise of the service sector. This finding confirms the result in Buera and Kaboski (2009) who show that the generalized Stone-Geary specification does not fit the historical consumption value-added shares of agriculture, manufacturing, and services. This is a striking result, because (i) - (iii) represent a robust pattern of structural change that is systematically found in data on consumption expenditure, gross output, value-added, and employment shares (see Herrendorf, Rogerson, and Valentinyi (2014) for an excellent survey). One reason for the poor fit is that – in the long-run data – the generalized Stone-Geary demands an unreasonably high subsistence level (income effect) of agricultural consumption to match the decline in its share.[2] Moreover, to

[2]The generalized Stone-Geary preferences imply subsistence levels in agriculture that are binding (even for the average household) in the early years in our sample.

generate a pronounced hump-shape in the manufacturing sector share, the data asks for a non-monotonic relationship between expenditure shares and nominal expenditure levels.[3] However, the Stone-Geary specification restricts this relationship to be monotonic when relative prices remain roughly constant.

In the microeconomic literature on consumer demand estimation, preference specifications with more flexible income effects are already established, for example the Quadratic Almost Ideal Demand (QAID) System proposed in Banks, Blundell, and Lewbel (1997). However, the QAID framework is not suitable for environments with sustained growth, as the predicted expenditure shares may turn negative once per-capita expenditure has grown sufficiently over time. Moreover, the preference parameters of the QAID cannot be identified from the aggregate market expenditure shares alone and heterogeneity of expenditures across individuals is often unobserved in the historical data. In a microeconomic analysis with cross-sectional data on individuals' expenditures this is less of a concern, but in macroeconomic time series data this is an important caveat.

To overcome the theoretical inconsistency of the QAID specification in steady growth environments, while at the same time preserving the functional flexibility of the demand system, we propose the IA preference specification which directly nests the generalized Stone-Geary and the PIGL specifications (see Muellbauer (1975, 1976)), but allows for the estimation of non-monotonic expenditure share Engel curves. This flexibility allows our IA preferences to fit the non-monotonic structural change in the long-run data much better than the standard ones. Despite its flexible form, the IA preference class allows for tractable aggregation and we can identify preferences parameters from aggregate data. We also demonstrate the specification's consistency with the benchmark multi-sector growth model proposed in Herrendorf et al. (2014).

Since the generalized Stone-Geary and PIGL preferences are IA, we can directly test the standard preferences and we find that they are rejected against more flexible parameterizations of IA preferences. We document that, in particular for the long-run data that goes back to 1900, it is crucial to have sustained and flexible income effects in order to fit the observed patterns in the expenditure shares. In contrast

[3] Intuitively, manufacturing can be a luxury good at low expenditure levels and then become more of a necessity at higher levels.

to generalized Stone-Geary preferences, for which income effects vanish as households get richer, IA preferences permit for sustained income effects like the PIGL secification. This flexibility allows us, for example, to fit the continued decline in agriculture even in the later years. Furthermore, IA preferences can also fit the non-monotonic pattern in manufacturing and the acceleration in services observed in the historical data.

IA preferences also provide an empirically and theoretically valid framework to assess the welfare effects of structural change. For example, an aggregate productivity slowdown from the Baumol (1967) cost disease could be reinforced or dampened by income effects. With non-homothetic preferences and household heterogeneity, the distributional effects of such trends are non-trivial but potentially important. Similarly, there is a prominent debate on the effects of deindustrialization, in particular in developing countries (see for example Rodrik, 2016). For the analysis of the welfare effects of such trends and of potential policies, a dynamic general equilibrium framework and the estimation of the preference parameters from a utility-based demand system are essential.

The remainder of the paper is organized as follows. Section 2 discusses the related literature. Section 3 describes the historical panel data and establishes the empirical regularities. In Section 4 we present the general theoretical framework, and in Section 5 we discuss the properties of specific preference specifications. Section 6 contains the structural estimation of preference parameters and discusses the main empirical results. Section 7 concludes. Additional figures and tables, the general parameterization of IA preferences, the proofs, a detailed description of the historical data that enter the estimations, and the discussion of alternative preference specifications are delegated to Appendixes A-E.

2 Related literature

Our paper relates to several recent developments in the macroeconomic literature on structural change and to the microeconomic literature on demand system estimation. The papers most closely related to ours are Buera and Kaboski (2009), Herrendorf et al. (2013, 2014), Boppart (2014), and Comin et al. (2015, 2017). Being close in spirit to Herrendorf et al. (2013), we focus on the implications of a given preference specification on structural transformation and take as given the relative prices and nominal per-capita expenditure of the economy in the empirical analysis. As in Boppart (2014) and Comin et al. (2015, 2017), the goal is to find a preference specification that allows for both sustained income and relative price effects in a standard multisector growth framework as presented in Herrendorf et al. (2014).[4]

Herrendorf et al. (2013) find that the generalized Stone-Geary preference specification fits the postwar data in the USA very well. They take two natural perspectives on structural transformation, categorizing the broad sectors agriculture, manufacturing, and services either in terms of final consumption expenditures or in terms of consumption in value-added. The main focus of their paper is to reconcile the differences in the estimated preference parameters that result from the two perspectives. Their consumption expenditure perspective yields estimates that suggest that income effects are empirically very important, whereas the value-added perspective suggests that relative price effects are the main driver of structural change. The main aspects in which our empirical analysis differs from Herrendorf et al. (2013) are that (i) we consider long-run data that includes the pre-war period; (ii) we complement the analysis with data for GBR, CAN, and AUS; and (iii) we test the standard preferences against the more genera IA preferences. When considering historical final consumption expenditure data from 1900 onwards, our conclusions differ in important ways from the ones of Herrendorf et al. (2013) drawn from the postwar sample: the generalized Stone-Geary specification struggles to fit the historical expenditure shares, and income effects are no longer the single main force behind structural transformation in final consumption expenditure.

Our empirical results for the long sample resemble the findings in Buera and Kaboski (2009). Using decennial value-added data from 1870 onwards, they emphasize that the generalized Stone-Geary specification cannot fit the most salient patterns of structural change. However, our data allow us to focus on the final output that is domestically consumed, while Buera and Kaboski (2009) run the non-homothetic specification over total output (where all investment is deducted from manufactur-

[4]Kongsamut et al. (2001), Ngai and Pissarides (2007), Foellmi and Zweimüller (2008), shut down either the relative price or the income effect in order to be consistent with an exact balanced growth path. Like Comin et al. (2015, 2017), we consider specifications that are consistent with an asymptotic balanced growth path, while Boppart (2014) establishes structural change along an exact balanced growth path.

ing), which also includes exports. Not only do we confirm their negative result for generalized Stone-Geary using historical final consumption expenditure data, we also resolve the concern by proposing a new preference specification that can fit the historical expenditure shares.

A theoretical drawback of the Stone-Geary specification is that its consistency with exact balanced growth depends on knife-edge parameterizations that imply mutually exclusive income effects (Kongsamut et al., 2001) or relative price effects (Ngai and Pissarides, 2007), respectively. Boppart (2014) overcomes these limitations and proposes a PIGL preference specification where structural transformation occurs along an exact balanced growth path. While Boppart (2014) considers an economy with two broad sectors for services and goods, we are splitting up the goods sector into agriculture and manufacturing as is common in the structural change literature. Our IA class of preferences also allows for more flexible income effects, which are important to fit the long-run data. Moreover, we provide empirical evidence for the importance of relative price and income effects for the entire 20th century and for four different countries, while he focuses on the postwar period in the USA.

Comin et al. (2015, 2017) apply the non-homothetic CES preference as specified by Hanoch (1975) in a dynamic general equilibrium framework and argue that such a preference specification provides a good fit to sectoral value added shares in 25 countries during the postwar era. The non-homothetic CES preferences, however, are not part of the IA class of preferences proposed in this paper. Hence, with non-homothetic CES preferences the preference parameters cannot be identified without aggregation bias. This feature is central to our application of structural change, which includes the pre-war period for which a micro consumer expenditure survey does not exist on a regular basis.

Our paper is also related to the microeconomic literature on demand system estimation. The PIGL class of preferences has been introduced by Muellbauer (1975, 1976) and yields expenditure shares that are quasi-linear in the nominal expenditure level to some power (in the special case of PIGLOG it is a quasi-linear function in the logarithm of the nominal expenditure level). Banks et al. (1997) established the QAID system that results from the generalization of the Almost Ideal Demand (AID) system, which is itself a special case of PIGLOG. Similar to our IA preferences, the

QAID specification allows for a non-monotonic relationship between expenditure shares of different sectors and the nominal expenditure level. However, in contrast to IA preferences, it is not compatible with sustained growth and mostly used for the analysis of cross-sectional data where this is less of a concern.

In the empirical literature, Buera and Kaboski (2012) document similar empirical regularities for the sectoral value-added shares for an even longer time series and a broader set of countries. However, since information on prices is not available for their historical panel data, the estimation of preference specifications is not possible with their data. They also provide a theory where the scale of technologies drives the rise of the service economy and the hump-shape of the manufacturing share. Technological progress in manufacturing leads first to the demarketization of services into home production. Later growth in the scale of services then explains the marketization of home production into services. Finally, the handbook chapter of Herrendorf et al. (2014) provides an excellent discussion of the recent empirical and theoretical literature on growth and structural transformation. It also provides detailed references to many valuable data sources for a broad range of countries.

3 Historical data on structural change

We first provide a brief description of our data and then discuss the observed patterns of structural change in the USA, GBR, CAN, and AUS. The exact data sources and the detailed categorization of the broad sectors for each country and time period are described in Appendix D.

3.1 Data description

We use nominal final consumption expenditure and price data provided by the national statistical offices of the USA, GBR, CAN, and AUS whenever they are available. These primary sources are complemented with historical data from Carter et al. (2006) for the USA, Feinstein (1972) for GBR, and Haig and Anderssen (2006) for AUS. This leaves us with an unbalanced panel of four countries, where the data cover the periods 1902 to 2014 (USA), 1900 to 2014 (GBR), 1926 to 2014 (CAN), and 1900 to 2014 (AUS).

For the ease of comparison we use the same categorization of the broad sectors as in Herrendorf et al. (2013). Roughly speaking, agriculture consists of food and beverages purchased for off-premise consumption. Manufacturing captures durable goods, clothing and footwear, gasoline and other energy goods, and other nondurable goods. Services consist of private services and in some specifications also government consumption. The latter categorization is challenging for the pre-1929 period when government consumption is not always observed and our baseline therefore focuses on private consumption. Furthermore, during the periods of World War I (WWI) and WWII, the expenditure shares in government consumption increased sharply. For this reason - and because we are mainly interested in long-run trends and not short-run fluctuations - we exclude years where each of the countries was involved in WWI and WWII from the analysis. We also drop the years when countries were affected severely by the Great Depression.

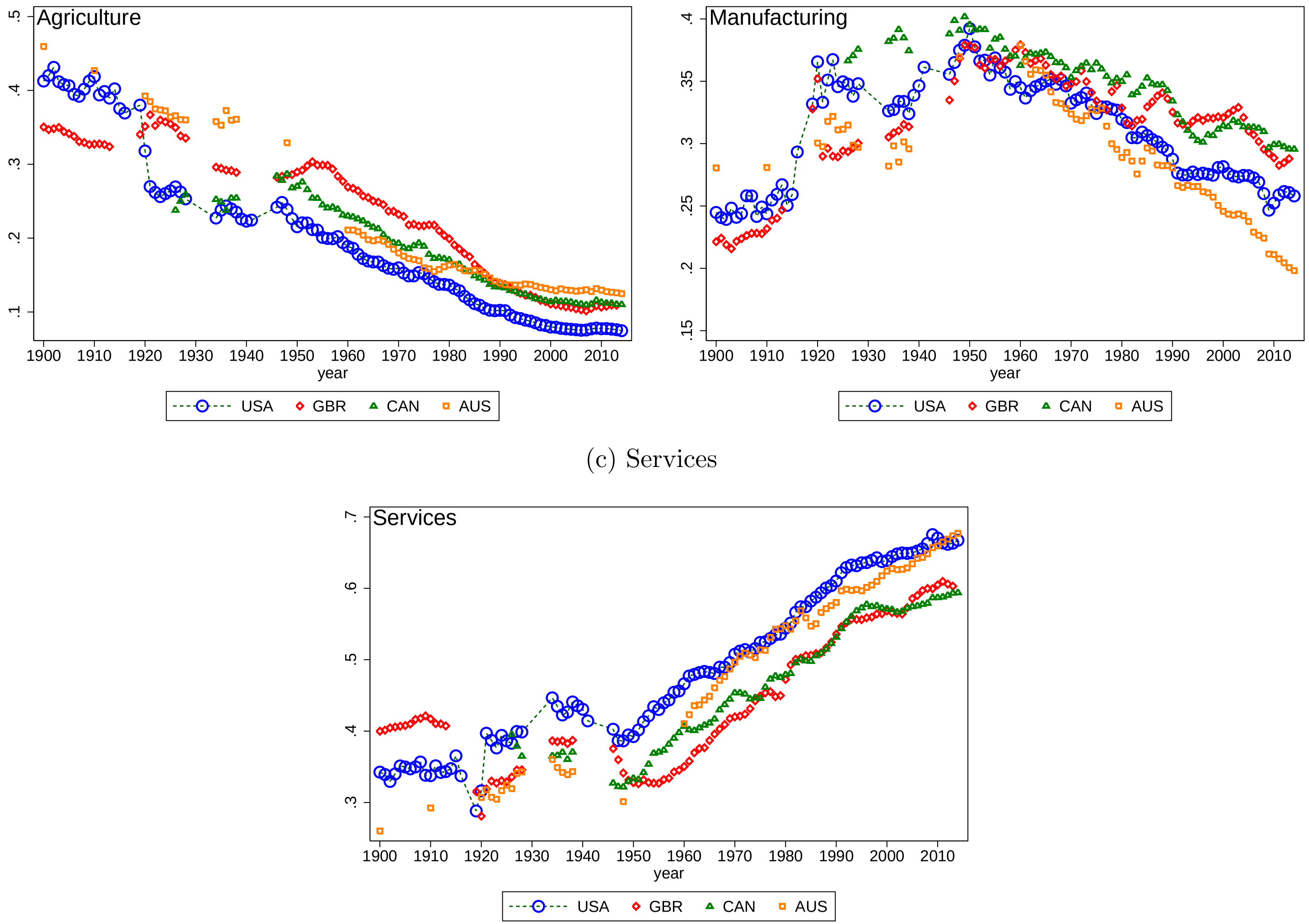
Finally, we aggregate up prices and quantities (which we obtain for the more detailed subcategories) to the three broad sectors agriculture, manufacturing, and services using Fisher indexes. The resulting sectoral price indexes are then adjusted for local currencies and purchasing power parity (PPP) according to the PPP conversion factors provided in the World Development Indicators (WDI). The details of how both procedures are implemented are explained in Appendix D.

3.2 Final consumption expenditure shares

We start the description of the historical data by presenting the final private consumption expenditure shares for each of the three broad sectors in the USA, GBR, CAN and AUS. Figure 1 illustrates three robust regularities of structural change in the four countries since the beginning of the last century: (i) there has been a steady decline in the expenditure share for agricultural consumption. Historically, agriculture used to be the largest sector, for example in the USA, starting with a share of 41% in private consumption. Over time this share has fallen substantially to only 7% in the final year 2014. (ii) the expenditure share for manufacturing consumption is hump-shaped over time. Again using the USA as an example, in 1900 the share of manufacturing was 24%, then reached its peak of 39% in 1950, and finally declined gradually to 26% by the end of the sample. (iii) there is an accelerated rise of the

(a) Agriculture

(b) Manufacturing



Notes: The figure plots the final private consumption expenditure share for the USA, GBR, CAN, and AUS by sector. The years affected by WWI, WWII, and the Great Depression are excluded. Sources: Several, see Appendix D.

Figure 1: Final private consumption expenditure shares, by sector

service sector. The share of services increased moderately between 1900 (34% in the USA) and 1950 (39%), before it increases rapidly to 67% in the second half of the sample.[5]

The same regularities hold qualitatively for all four countries and they have also been documented for further countries and complementary measures of structural changes (see Buera and Kaboski (2012), Uy et al. (2013), Herrendorf et al. (2014), and Comin et al. (2015) for recent contributions).[6] The four countries experienced similar living standards and real per-capita income growth over the considered period such that the regularities also hold up if the expenditure shares are plotted against logarithmic real per-capita GDP.[7] The fact that the expenditure share for manufacturing is hump-shaped in real per-capita income is related to the well-established results in the literature on consumer demand system estimation that addresses non-monotonic expenditure share Engel curves at the microeconomic level.[8]

3.3 Relative prices and real per-capita expenditure

The expenditure share of each sector is determined by both the price and the real quantities demanded relative to the other sectors. At the outset, the observed pattern of structural change could be completely driven by changes in relative prices. However, in this section we document the development of the three sector's relative prices over the last century and show that relative price effects alone cannot explain

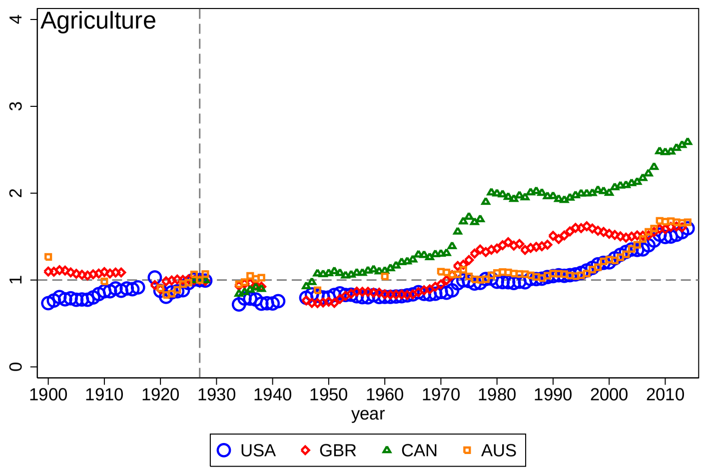
[5]Figure 7 in Appendix A shows all three shares together for the USA.

[6]The regularities are also robust to the inclusion of government consumption in the service sector. In the benchmark categorization of Herrendorf et al. (2013), government consumption belongs to services.

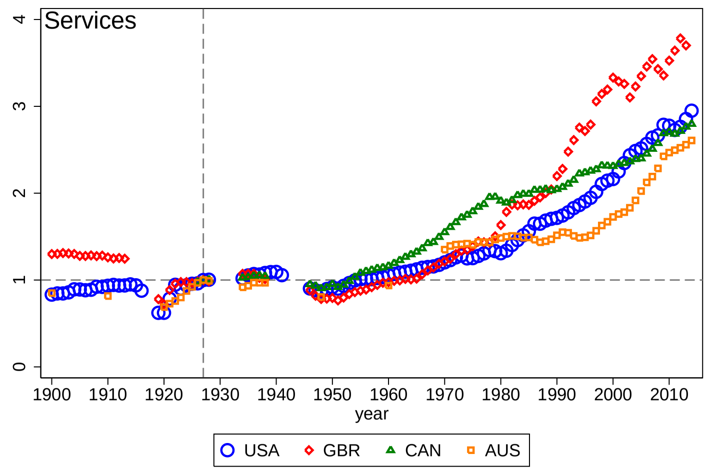
[7]This is illustrated in Figure 9 of Appendix A where we plot the expenditure shares over the logarithmic real per-capita income taken from the Maddison Project (2013). In order to document the patterns more formally, we also run regressions of the sector shares on log GDP per capita. Following Buera and Kaboski (2012), we split the sample at the GDP per capita level that corresponds to the peak in manufacturing, which is \$8,515 (international dollars, PPP). The regression of the manufacturing share on log GDP per-capita for the sample below \$8,515 yields a coefficient of 0.182 (robust standard error 0.023). In the sample above \$8,515, the coefficient is -0.072 (0.004), suggesting a hump-shape in the share of the manufacturing sector. Using the same sample definitions for the share of services yields coefficients of -0.002 (0.023) and 0.215 (0.003), suggesting a rise of the service sector in the second sample. In the agricultural sector, the same samples lead to estimates of -0.17 (0.032) and -0.141 (0.003), suggesting a continued fall in both samples.

[8] See Banks et al. (1997) for the foundations of the established estimation of quadratic almost ideal consumer demand systems.

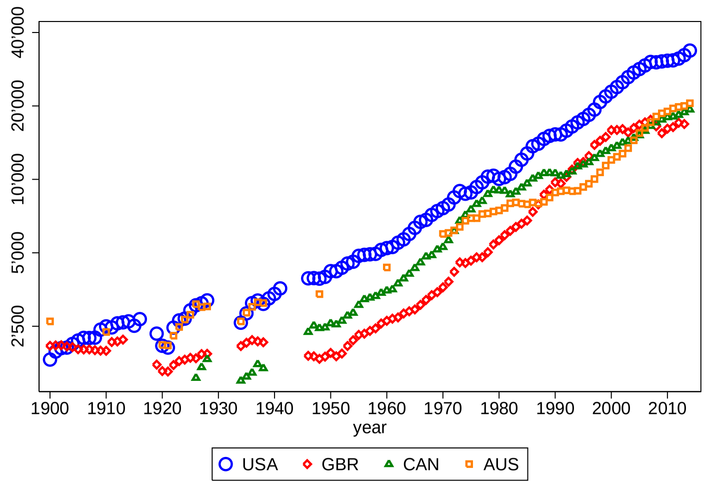
(a) Relative prices, Agriculture



(b) Relative prices, Services



(c) Relative per-capita expenditure



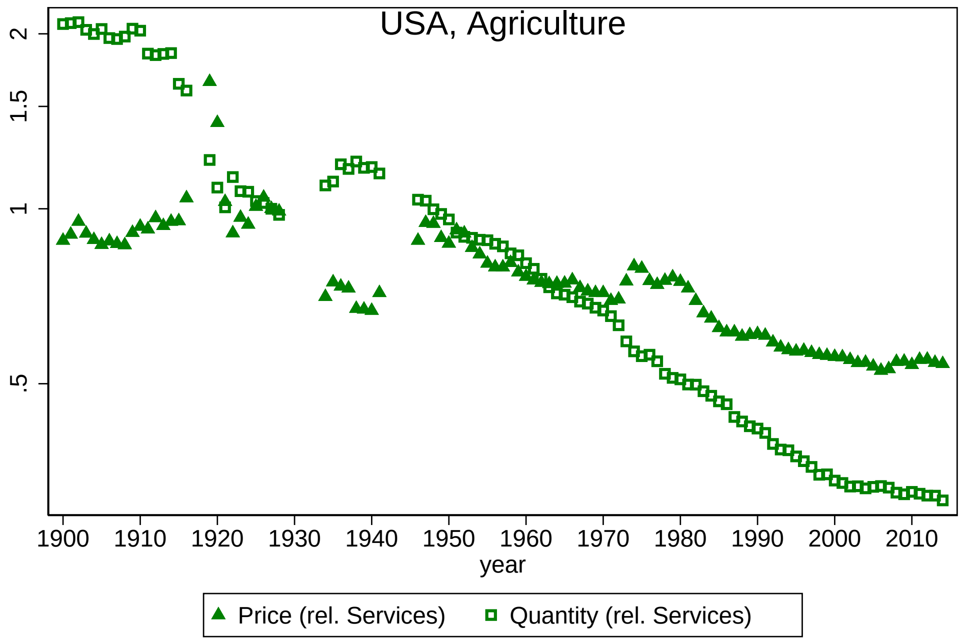
Notes: The figure plots in panel (a) and (b) the prices of agriculture and services relative to manufacturing, and in panel (c) the nominal per-capita expenditure relative to the manufacturing price in the USA, GBR, CAN, and AUS. All nominal variables are based on final private consumption expenditure and expressed in PPP-adjusted 1990 international \$. In panel (a) and (b) relative prices are normalized to unity in 1927, and in panel (c) real per-capita expenditure is plotted on a ratio scale. The years affected by WWI, WWII, and the Great Depression are excluded. Sources: Several, see Appendix D.

Figure 2: Relative prices and private per-capita consumption expenditure

the structural transformation observed in the USA, GBR, CAN, and AUS over the last century.

Panels (a) and (b) of Figure 2 plot the price of the agricultural sector and the private service sector relative to manufacturing. Prices are expressed in PPP-adjusted 1990 international $ in order to compare the four countries. We see that the sectoral prices relative to manufacturing have remained relatively stable in the first half of the century and then started to increase around 1950. The price increase is more pronounced for the service sector than for agriculture, and - if substitution effects are strong enough - the relative price alone could explain the late rise of the service sector. However, for the agricultural sector both the price and the real consumption relative to services are falling over time, in particular since 1950.[9] Such a positive relationship cannot be explained with a well-behaved substitution elasticity and shows that in addition to relative price effects also income effects are needed to explain the pattern of structural change observed in the historical data.

Finally, panel (c) of Figure 2 illustrates the there has been substantial real per-capita expenditure growth over the last century in the USA, GBR, CAN, and AUS.[10] It shows that there has been sustained growth in all four countries (with the exception of the GBR and AUS between 1900 and 1920 where per-capita expenditure is roughly stagnating). Note that per-capita expenditure is plotted on a ratio scale, thus the slope approximates the yearly growth rate. For the USA for example, real per-capita expenditure has increased by more than a factor of 18 between the first observation in 1900 and the last in 2014. Thus, if income effects are at work, they must have an important role in explaining the pattern of structural change over the last century.



Notes: The figure plots the prices of agriculture and services relative to manufacturing in the USA. The prices are based on final private consumption expenditure and normalized to unity in 1927. The years affected by WWI, WWII, and the Great Depression are excluded. Sources: BEA and Carter et al. (2006) for years pre-1929.

Figure 3: Prices and quantities of agriculture relative to the service sector, USA

3.4 Why flexible and sustained income effects are needed

In Figure 3 we show the dynamics for the relative prices and quantities of agriculture relative to the service sector. We focus on the USA, but similar conclusions can be drawn from the other countries. The figure illustrates that in the second half of the sample starting in 1950 relative prices and quantities of agriculture are both falling over time. A preference specification without income effects (for example, a simple

[9] This is illustrated in Figure 3, which plots prices and quantities of agriculture relative to the service sector in the USA. Furthermore, Figure 8 shows for the USA the relative prices of both agriculture and services together.

[10] Note that private real expenditure is not observed in the data. Given the sectoral prices, the model-consistent aggregate price index depends on the considered preference specification and its parameter values, which are unknown prior to any estimation. Thus, in the figure, we proxy real expenditure by expressing nominal expenditure relative to the price of manufacturing. The qualitative conclusions from the figure remained unchanged if we used, for example, a Fisher-index over the sectoral prices to deflate the nominal expenditure.

CES utility function over the three sectoral goods) cannot explain this pattern with a well-behaved price elasticity, since a fall in the relative price has to be accompanied by an increase in the relative quantities demanded. Thus, to fit the postwar data a non-homothetic preference specification where agriculture has a substantially lower income elasticity of demand compared to the service sector is needed. In the postwar sample, agriculture is typically estimated to be a necessity with an income elasticity of demand below one, while services have a luxury character with an income elasticity of demand above one.

Figure 3 also illustrates that the relative price and quantity of agriculture fall together for more than 60 years. Since per-capita expenditure is growing at the same time, this suggests that a sustained income effect is needed to fit the continued decline of the relative demand for agricultural goods. Finally, the figure shows that in the first half of the sample until roughly 1950, relative prices and quantities of agriculture are negatively related. This qualitative pattern could be captured with a relative price effect in the demand system alone and suggests that flexible income effects are required to fit the dynamics of the agricultural sector relative to services in the long-run data, i.e., income effects that are stronger towards the end compared to the beginning of the historical sample.

4 Theoretical framework

In this section, we present the general theoretical framework to consider structural change in a multi-sector growth model. Section 5 then discusses the specific preference specifications.

4.1 Economic environment

We consider an infinite horizon, closed economy framework in discrete time with four production sectors. Our main focus will be on three sectors that produce the consumption goods called agriculture A, manufacturing M, and services S, but we also explicitly model a fourth sector that produces an investment good X. In each sector $j \in J_+ \equiv \{A, M, S, X\}$ output $y_{j,t}$ is competitively produced according to the following Cobb-Douglas technology

$$y_{j,t} = k_{j,t}^{\alpha} \left(\gamma_j^t n_{j,t}\right)^{1-\alpha}. \tag{1}$$

Here, $k_{j,t}$ and $n_{j,t}$ denotes capital and labor used in sector j and γ_j^t is a time-varying Harrod neutral technology term (where t is a time index). The labor-augmenting technology term is normalized to one in all sectors in period $t = 0$. We have $\alpha \in (0, 1)$, and $\gamma_j \geq 1$, $\forall j$.[11] The firms in all sectors take the rental rate, $R_t = r_t + \delta$, the wage rate, w_t, and the output price $p_{j,t}$ as given and then choose their capital and labor input to maximize profits. The capital and labor market clearing conditions are

$$\sum_{j \in J_+} k_{j,t} = k_t, \text{ and } \sum_{j \in J_+} n_{j,t} = n, \tag{2}$$

where k_t and n denotes total capital and labor in the economy.

The output of agriculture, manufacturing and services is all consumed, whereas the output of sector X is invested. There is an interval of infinitely lived households indexed by $i \in [0, N]$ with the following preferences

$$\mathcal{U}_{i,0} = \sum_{t=0}^{\infty} \beta^t v(e_{i,t}, P_t), \quad P_t \equiv (p_{A,t}, p_{M,t}, p_{S,t}). \tag{3}$$

The period utility function $v(e_{i,t}, P_t)$ is given in *indirect form*, i.e., it is defined over nominal expenditure $e_{i,t}$ and the vector P_t of prices $p_{j,t}$ of all the consumption goods $j \in J \equiv \{A, M, S\}$. The parameter $\beta \in (0, 1)$ denotes the discount factor. We explicitly allow for household heterogeneity in factor endowments. Household i supplies inelastically $n_i \geq 0$ hours to the labor market and her initial wealth $a_{i,0}$ is exogenously given. Since preferences are additively separable in time, the household's problem can be split up into an intertemporal and an intratemporal problem. The intertemporal problem deals with the optimal saving/spending decision where household i chooses a sequence $\{e_{i,t}, a_{i,t+1}\}_{t=0}^{\infty}$ to maximize (3) subject to a period

[11] Furthermore, we assume that $\gamma_X > 1$ such that capital can be accumulated at a sustained positive rate. Since capital is used with a unitary elasticity and there is not technological regress in the consumption sectors this implies that output of all sectors can grow at a steady positive rate too.

budget constraint

$$a_{i,t+1} = a_{i,t}(1 + r_t) + w_t n_i - e_{i,t}, \tag{4}$$

and a standard no-Ponzi game condition.[12] In the intra-temporal problem we can simply apply Roy's identity to find how nominal expenditure $e_{i,t}$ is spent on the three different sectors to find the individual Marshallian demand $c_{i,j,t},\ j \in J$ for each household i.

In the following we choose the price of the investment good as a numéraire, $p_{X,t} = 1$. Consequently, we can write the asset market clearing condition as

$$\int_0^N a_{i,t}\, di = k_t, \tag{5}$$

and the law of motion of the aggregate capital stock becomes

$$k_{t+1} = k_t(1 - \delta) + y_{X,t}, \tag{6}$$

where $\delta \in [0, 1]$ is the depreciation rate. Furthermore, we have

$$\int_0^N n_i\, di = n, \tag{7}$$

and market clearing in the three consumption sectors imply

$$\int_0^N c_{i,j,t}\, di = y_{j,t},\ \ \forall j \in J. \tag{8}$$

In macroeconomic theory it is more common to work with direct preference specifications defined over real consumption commodities instead of the indirect formulation used here. However, as we will see below, the indirect formulation allows us to characterize the optimal saving decision as simple as in a one-sector economy. This enables us to highlight the additional restrictions that the existence of a balanced growth path imposes on preferences.[13] Furthermore, for some of the

[12]The no-Ponzi game condition can be expressed as $\lim_{T\to\infty} a_{i,T+1} \prod_{s=1}^{T}(1 + r_s)^{-1} \geq 0$.

[13]Note that we use the same generalized balanced growth definition as in Kongsamut, Rebelo, and Xie (2001) and Herrendorf et al. (2014). The word *generalized* refers to the feature that - at the sectoral level - endogenous variables are not required to grow at constant rates. Henceforth, we refer to their generalized equilibrium concept simply as balanced growth.

preferences specifications that we will evaluate below, a closed form utility function only exists in the indirect formulation. For these reasons, we prefer the indirect formulation here. In cases where a closed form solution for the direct formulation exists, we will also state this direct utility function. Note, however, that in the end the main object of interest is the implied demand system, which is identical for both the direct or indirect formulation.

Although we are interested in structural change between different consumption good sectors, we nevertheless model the investment good as a separate sector.[14] In this respect the theory is relatively general and could also accommodate investment specific technical change.

4.2 Equilibrium definition and discussion

We define an equilibrium in this economy in the standard way.

Definition 1. *An equilibrium is a sequence of prices and quantities that is jointly consistent with utility maximization of all households, profit maximization (and perfect competition) of all firms as well as all the market clearing conditions* (5)–(8).

To analyze the historical consumption expenditure data presented in Section 3.2, we will in the following focus on the competitive outcome of this dynamic general equilibrium framework as presented in Section 4.1. As we will argue below, this is a useful benchmark model to guide our data analysis. The framework is flexible enough to allow relative prices between sectors to change. Furthermore, by allowing for capital accumulation, the model includes the intertemporal margin, which is essential to discuss consistency with sustained growth in income. Although the framework is in some sense very standard, it seems nevertheless relevant to comment here on its generality.[15]

First, note that we focus on the decentralized market equilibrium. However, the welfare theorems apply and the decentralized market outcome is Pareto efficient

[14] See García-Santana et al. (2016) and Herrendorf et al. (2018) for theories of structural change induced by the investment good sector.

[15] For instance, it coincides with the "benchmark model" proposed in the handbook chapter by Herrendorf et al. (2014), except for our generality in terms of instantaneous preferences and household heterogeneity.

(and could also be characterized as the solution to a planner's problem). Second, the imposed restrictions on the preference side, like additive time separability and standard discounting, are relatively mild and standard and we keep full flexibility with respect to the period utility function. In the remainder of the paper, we will analyze the prediction of this framework under different functional form assumption of $v(e_i, P)$. On the production side, however, the framework puts some simplifying structure, most importantly it assumes identical output elasticities of capital α across the three consumption good sectors (as well as the investment good sector). This assumption implies that relative price changes are completely driven by differences in the productivity growth rates across sectors. It, however, precludes factor intensity differences as a source of relative price changes (à la Acemoglu and Guerrieri (2008)), and that shifts in the demand structure due to income effects have an impact on relative prices (see Caselli and Coleman (2001)). The Cobb-Douglas functional form of production could be relaxed and the output elasticity of capital in the investment good sector could be allowed to differ from the consumption sectors as well. These generalizations would not affect the model's main predictions.

4.3 Equilibrium implications

In equilibrium, the production side of the economy can be characterized by the following lemma.

Lemma 1. *The capital-labor ratio is equalized across all sectors, i.e.,*

$$\frac{k_{j,t}}{n_{j,t}} = \frac{k_t}{n}, \ \forall t, j \in J_+. \tag{9}$$

Furthermore, the prices are given by

$$p_{j,t} = \gamma_j^{(1-\alpha)t} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t + \delta}{\alpha}\right)^{\alpha} = \left(\frac{\gamma_X}{\gamma_j}\right)^{(1-\alpha)t}, \ \forall j \in J, \tag{10}$$

where the choice of numéraire $p_{X,t} = 1 = \gamma_X^{(1-\alpha)t} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \left(\frac{r_t+\delta}{\alpha}\right)^{\alpha}$ *has been used for* *the second equality. The equilibrium rental rate and wage rate are given by*

$$r_t + \delta = \alpha \left(\frac{\gamma_X^t n}{k_t} \right)^{1-\alpha}, \tag{11}$$

and

$$w_t = (1-\alpha)\gamma_X^t \left(\frac{k_t}{\gamma_X^t n} \right)^{\alpha}. \tag{12}$$

Finally, under optimal production, output can be expressed as

$$y_{j,t} = \gamma_j^{(1-\alpha)t} \left(\frac{k_t}{n} \right)^{\alpha} n_{j,t}, \ \forall j \in J_+. \tag{13}$$

Proof. In each period t, the representative firm in each sector $j \in J_+$ solves

$$\min_{k_{j,t}, n_{j,t}} k_{j,t}(r_t + \delta) + n_{j,t} w_t,$$

subject to an exogenously given output level $\bar{y}_{j,t} = k_{j,t}^{\alpha} \left(\gamma_j^t n_{j,t} \right)^{1-\alpha}$. The first-order conditions of the firms' problems are

$$\lambda_{j,t} \alpha \bar{y}_{j,t} / k_{j,t} = r_t + \delta,$$

and

$$\lambda_{j,t} (1-\alpha) \bar{y}_{j,t} / n_{j,t} = w_t,$$

where $\lambda_{j,t}$ denotes the multiplier attached to the constraint. These first-order conditions directly imply

$$\frac{k_{j,t}}{n_{j,t}} = \frac{w_t}{r_t + \delta} \cdot \frac{\alpha}{1-\alpha}, \tag{14}$$

which together with (5) and (7) implies (9). Furthermore, this allows us to write output as (13). Note that $\lambda_{j,t}$ can be interpreted as marginal cost and will be equal to the sectoral price $p_{j,t}$. Solving the first-order conditions for $\lambda_{j,t}$ and combining them with (14) gives (10). Finally, with our choice of the numéraire the first-order conditions of the investment sector imply (11) and (12) and establishe the lemma. ■

Lemma 1 is a direct consequence of the assumption that the output elasticity of capital is identical across sectors, which leads to equalized capital intensities across sectors (see (9)). Furthermore, since the production functions then only differ in the labor-augmenting technology terms γ_j^t, the marginal rate of transformation is constant in any given point in time and solely pinned down by the technology side, as shown in equation (10). Hence, shifts in the demand structure will not affect relative prices. Finally, since the capital intensity is identical across sectors, as shown by (13), output in each sector can be written as a linear function of labor used in this sector. Note again, that we could allow the capital intensity in the investment good sectors to differ from the consumption good sectors (since along the balanced growth path the saving and investment rate will be constant). However, for the consumption sectors for which the expenditure shares will vary over time, the assumption of identical capital intensities is crucial, since otherwise already the technology side of the economy would exclude the coexistence of structural change with an exact balanced growth path.[16] Consequently, the assumption of identical factor shares is a restriction that we impose for theoretical reasons, but there is empirical support for this assumption as well. It is important to first note that the specified production functions are written in terms of gross final output and data on factor intensities is only readily available at the industry value-add level. Hence, in order to measure the capital share α of a final consumption sector, the entire input-output structure needs to be taken into account. Valentinyi and Herrendorf (2008) provide such estimates for the U.S. for broadly defined sectors at the gross final output level. The results show that the shares of capital and labor vary surprisingly little and the assumption of identical factor shares across consumption sectors seems to be a reasonable approximation.[17] Note also that, at the value-added level, Herrendorf et al. (2015) estimate the production functions of our three sectors for the post-war U.S. and find that the formulation above with Cobb-Douglas technologies with identical output elasticities of capital, but different TFP growth, capture the main technological forces behind structural change. It is important to note that these

[16] Note that the issue arises from the intensity differences in the factor that can be accumulated, i.e., here physical capital. In a formulation with human capital that can be accumulated, the relevant assumption would be that the intensity of human plus physical capital is identical across all the sectors with changing nominal output shares.

[17] For instance, in the service sector the labor income share was in the U.S. in the year 1997 0.34, which is only marginally small then the 0.35 found for total consumption.

results are for the capital-labor margin. If other production factors like land or the division into skilled and unskilled labor were considered, then we expect factor intensity differences to be more relevant.

The assumption of Harrod-neutral technical change at constant rates is also made for theoretical reasons. A non-constant rate of technical change, in particular in the investment sector, would ex ante rule out the existence of a balanced growth path. Importantly, however, we allow the rate of technical change to be sector specific but restrict it to be constant over time. This assumption implies that relative prices will change over time, but do so at constant rates. As our data on relative prices in Section 3.3 shows, this seems to be a good first approximation of the data, where we indeed see systematic changes in relative prices in the long run. However, the data also highlights that there may be some structural break in the relative price growth around WW II. To generate such a structural break in our theoretical framework, an exogenous switch of the γ_j parameters is required.

The optimal saving behavior of a household i is characterized by the following Euler equation.

Lemma 2. *Solving the intertemporal household problem gives rise to the following Euler equation*

$$\frac{v_e(e_{i,t}, P_t)}{v_e(e_{i,t+1}, P_{t+1})} = \beta(1 + r_{t+1}), \tag{15}$$

where $v_e(e_{i,t}, P_t)$ is the indirect marginal (momentary) utility of nominal consumption expenditure in a given period.

Proof. The Lagrangian of the household problem can be written as

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t v(e_{i,t}, P_t) + \sum_{t=0}^{\infty} \lambda_{i,t} \beta^t \left(a_{i,t}(1 + r_t) + w_t n_i - e_{i,t} - a_{i,t+1} \right). \tag{16}$$

The first-order conditions are then given by

$$v_e(e_{i,t}, P_t) = \lambda_{i,t},$$

$$\lambda_{i,t} = \lambda_{i,t+1} \beta \left(1 + r_{t+1}\right),$$

and

$$a_{i,t}(1+r_t) + w_t n_i - e_{i,t} = a_{i,t+1}.$$

Combining the first two first-order conditions establish the lemma. ■

Jointly with the household budget constraint, (4), the standard transversality condition, as well as with the initial wealth $a_{i,0}$, this fully characterizes the household's spending and saving behavior. Aggregating all the household budget constraints and combining it with (5) and (7) gives

$$k_{t+1} = k_t(1-\delta) + k_t^{\alpha}\left(\gamma_X^t n\right)^{1-\alpha} - E_t, \tag{17}$$

where $E_t \equiv \int_0^N e_{i,t} di$ is aggregate nominal consumption expenditure. This allows us to characterize the dynamics of the capital stock and finally solve the model.

In the following we are interested in the long-run properties of the equilibrium path. To this aim, we next define the concept of balanced growth.

Definition 2. *A balanced growth path is an equilibrium path along which the aggregate physical capital stock k_t grows at a constant positive rate. If such a balanced growth path can be reached in finite time, then we call it an exact balanced growth path. If the balanced growth path only exists as time goes to infinity, then we call it an asymptotic balanced growth path.*

The production side of the economy is potentially in line with an (exact) balanced growth path. Hence, if a balanced growth path exists, then its dynamics are fully determined by the exogenous rate of technical change. This is summarized in the next proposition.

Proposition 1. *Along a balanced growth path, the aggregate capital stock, k_t, aggregate nominal output, $y_t = k_t^{\alpha}\left(\gamma_X^t n\right)^{1-\alpha}$, aggregate nominal consumption, E_t, and the nominal wage rate w_t all grow at constant gross rate γ_X, and the nominal interest rate, r_t, is constant.*

Proof. Positive capital growth requires positive savings and investments. Hence, along a balanced growth path we must have $k_t^{\alpha}\left(\gamma_X^t n\right)^{1-\alpha} > E_t$. Then, the resource constraint (17) implies that a constant capital growth rate requires $\frac{k_{t+1}}{k_t} = \gamma_X$. It is then straightforward to see that along this path nominal output and nominal expenditure grow at the same gross rate γ_X. Finally, (11) and (12) imply that the interest rate is constant and that the wage rate grows at gross rate γ_X as well. ■

Whether the economy admits a balanced growth path depends on the specified period utility function. This (as highlighted in Lemma 2) essentially boils down to the question of whether the Euler equation (15) is jointly consistent with a constant interest rate, r_{t+1}, and a constant gross nominal expenditure growth rate, $\frac{e_{i,t+1}}{e_{i,t}}$, either asymptotically or in finite time. We discuss this aspect of preferences below and in Section 5.

4.4 Aspects of preferences

In addition to the intertemporal Euler equation, applying Roy's identity gives the Marshallian demands, i.e., how a given expenditure level is optimally spent on the different sectoral output. The functional form of this demand system depends on the precise formulation of $v(e_{i,t}, P_t)$. In Section 5, we present a broad class of preferences that accommodates as special cases frequently used formulations, such as the homothetic constant elasticity of substitution form, a generalized Stone-Geary utility function, as well as the price independent generalized linearity class (PIGL). We discuss the special cases within our broader class and consider their properties in terms of the following dimensions: (i) allowing for aggregation, (ii) implications for intertemporal optimization, and (iii) their flexibility of the implied demand system. We outline the general properties below and then discuss their applicability to specific preference specifications in Section 5.

Aggregation In order to ensure tractable aggregation properties, the most narrowly defined concept is the class of homothetic preferences. In this homothetic class of preferences, the indirect form utility can be written as a function of one argument only, which is nominal expenditure divided by a price index. Then, the economy admits a representative household and aggregate variables can be used for welfare statements. However, the class of homothetic preferences is clearly too rigid for our purpose, since this class of preferences excludes income effects in the demand structure. The challenge is then to find a demand system that is flexible enough and at the same time preserves some aggregation properties, i.e., a demand system that allows – even in the presence of income heterogeneity – to identify the true preference parameters from aggregate data in the theoretical framework presented above. In a narrow sense, the most general such class is the Gorman class of preferences, in which the aggregate demand structure is only a function of prices and the average per-capita expenditure level (see Gorman (1981)). In the framework above, the intertemporal optimization helps us to broaden this class, since the Euler equation (15) guarantees that the marginal utility of nominal expenditure grows at any given point in time at the identical rate for all individuals. Hence, we obtain a more general aggregation result, which, however, hinges on the framework in Section 4.1. In particular, it relies on intertemporal optimization and complete markets.

Intertemporal choice The final goal is to use the preference specification in the dynamic general equilibrium model specified in Section 4.1. Consequently, the implication of the preferences for the intertemporal Euler equation deserves some special attention. We will see that, as long as preferences are well specified, *asymptotic balanced growth* is generally fulfilled. The reason for this is that asymptotically all the expenditure shares have already converged to their asymptotic values. Consequently, asymptotically preferences look like a Cobb-Douglas with constant expenditure shares (in some case, even as a one sector economy), which is a well known special case that supports balanced growth in the above framework. However, it is crucial to note that the criterion that preferences are asymptotically well specified puts some important restrictions on the preference class. Some utility functions, like the Almost Ideal Demand (AID) System, would imply non-positive demand for some commodities as income and relative prices grow indefinitely.

In addition, we also analyze under which restrictions *exact balanced growth* is feasible, i.e., a balanced growth path exists in finite time. One way to get exact balanced growth is to restrict preferences such that the intertemporal elasticity of substitution is equal to a fixed parameter value that is identical and constant in the cross-section as well as over time. This restricts the preferences to be of the PIGL/PIGLOG form (see Crossley and Low (2011)). This PIGL/PIGLOG form however puts important restrictions on the flexibility of the demand system.

Flexibility In order to give rise to a good fit of the expenditure data, we will see that a relatively flexible functional form of the income effects is required. First, as argued in Section 5.4.2, the data requires a prolonged income effect, i.e., income elasticities that do not converge to unity asymptotically. Second, it is important to have the flexibility that a commodity can first be a luxury (with an expenditure elasticity larger than one) and then over the course of development turn into a necessity (with an expenditure elasticity smaller than one). This is particularly important to fit the share spent on manufacturing, which is non-monotonic in the long sample. One indicator of the flexibility of the income effects of a demand system is the "rank" (see Lewbel (1991)) which is the minimal number of additive terms that are different functions of expenditure needed to express the demand Marshallian demand function. All homothetic preferences imply a rank of the demand system of one, whereas matching micro data usually requires a rank of (at least) three (see, e.g., Banks et al. (1997)). There is, however, a trade-off between a high flexibility (and a high rank) of a demand system and aggregation properties. For instance, the theorem in Gorman (1981) states that a rational and "exactly aggregable" demand system can achieve a maximum rank of 3.[18]

In the rest of the paper we focus on the demand side, where we consider different preference specifications and analyze how they perform in terms of fitting the historical expenditure data. Note that for this estimation of the demand system, no dynamic general equilibrium framework would be required. We could also assume a static partial equilibrium in which a representative agent spends $e_{i,t} = e_t$, derive the predicted expenditure structure for given preferences $v(e_t, P_t)$, and then ask what parameters give rise to a good fit of the historical market expenditure data. However, the aspects described above show that the dynamic environment is crucial to be able to identify preference parameters from aggregate data when households are heterogeneous. We therefore discuss the theoretical properties of the different instantaneous utility functions in the infinite horizon general equilibrium model stated in Section 4.1. We will specifically address potential aggregation problems and discuss the asymptotic dynamics and whether an exact balanced growth path exists under various preference specifications.

[18]Exactly aggregable requires the demand system to be written as an additive function of products of the nominal expenditure level times an arbitrary function of the prices.

5 Specific preference specifications

The consensus in the structural change literature is that both relative price effects and income effects are important demand side drivers of structural transformation. Thus, it is natural to focus the analysis on non-homothetic preferences that allow for both effects. Moreover, in light of the steady per-capita income growth that today's advanced economies have experienced over the last century, we aim to incorporate structural transformation and preferences with flexible income effects into the canonical dynamic multi-sector growth model of Section 4. At the same time, the general preference structure outlined below will preserve the balanced growth properties and the tractable aggregation of standard models in the literature.[19]

5.1 Restricting Preferences: Intertemporal Aggregation

In the historical data we mostly observe aggregate variables and cannot rely on annual distributional data to identify the individual preference parameters. Therefore, it is crucial to work with a flexible preference specification that nevertheless allows for identification without aggregation bias. To this aim, we define below the class of intertemporally aggregable (IA) preferences.

Definition 3. *Time-additive preferences* $\mathcal{U}_{i,0} = \sum_{t=0}^{\infty} \beta^t v(e_{i,t}, P_t)$ *are intertemporally aggregable if per-capita expenditure* E_t/N *satisfies the individual Euler equation*

$$\frac{v_e(E_t/N, P_t)}{v_e(E_{t+1}/N, P_{t+1})} = \beta(1 + r_{t+1}), \quad \forall P_t, P_{t+1}, r_{t+1}. \tag{18}$$

Definition 3 allows individual expenditure to change with different scale relative to the per-capita expenditure over time. However, $v_e(e_{i,t}/N, P_t)$ is required to change with the same scale as $v_e(E_t/N, P_t)$. Thus, IA is a weaker restriction than for example mean-scaling as discussed in Lewbel (1989). In the next proposition we show that within our intertemporal framework, Equation (18) imposes a restriction on the period utility function that allows to derive a simple and explicit representation of $v(e, P)$ such that the preferences $\mathcal{U}_{i,0}$ are IA. Furthermore, as we will establish

[19] See Herrendorf et al. (2014) for a recent overview of the discussion on growth and structural transformation.

below, the Marshallian demand system associated with IA preferences will allow for identification of preference parameters from aggregagte data.

Proposition 2. *Let $v(e,P)$ be twice continuously differentiable in e with $v_e(e,P) > 0$ and $v_{ee}(e,P) < 0$. Then, $\mathcal{U}_{i,0}$ is IA if and only if $v(e,P)$ is of the form*

$$v(e,P) = \frac{1}{\epsilon}\left(\frac{e}{\mathbf{B}(P)} - \mathbf{A}(P)\right)^{\epsilon} - \frac{1}{\epsilon} - \mathbf{D}(P), \quad \epsilon \in (-\infty, 0) \cup (0,1), \tag{19}$$

including the limit case

$$\lim_{\epsilon \to 0} v(e,P) = \log\left(\frac{e}{\mathbf{B}(P)} - \mathbf{A}(P)\right) - \mathbf{D}(P), \tag{20}$$

where $\mathbf{B}(P)$ is linear homogenous, and $\mathbf{A}(P)$, $\mathbf{D}(P)$ are zero homogenous in the price vector P.

Proof. In Section C.1 of the Appendix. ■

With slight abuse of terminology, we will henceforth refer to the indirect period utility function in (19) as the IA preference specification. Given the general restriction in 18, the resulting period utility function (19) is fairly parsimonious, flexible, and general. It has three non-redundant price functions such that the associated Marshallian demand system has rank 3 (which is the maximum rank in a three-sector model) and it encompasses the homothetic, the quasi-homothetic, the PIGL, and the quadratic demand systems as special cases. This can easily be verified from Theorem 1 in Lewbel (1987) and the following proposition.

Proposition 3. *The Marshallian demand of each commodity j is given by*

$$c_{i,j,t} = \mathbf{A}_j(P_t)\mathbf{B}(P_t) + \frac{\mathbf{B}_j(P_t)}{\mathbf{B}(P_t)} \cdot e_{i,t} + \frac{\mathbf{D}_j(P_t)\mathbf{B}(P_t)}{v_e\left(e_{i,t}, P_t\right)}, \tag{21}$$

where $\mathbf{A}_j(P_t)$, $\mathbf{B}_j(P_t)$, and $\mathbf{D}_j(P_t)$ denote derivatives of the corresponding functions with respect to $p_{j,t}$. In per-capita terms, the Marshallian demand of each commodity is given by

$$C_{j,t}/N = \mathbf{A}_j(P_t)\mathbf{B}(P_t) + \frac{\mathbf{B}_j(P_t)}{\mathbf{B}(P_t)} \cdot E_t/N + \frac{\mathbf{D}_j(P_t)\mathbf{B}(P_t)}{v_e\left(E_t/N, P_t\right)}\kappa/N, \tag{22}$$

where $\kappa = \int_0^N \frac{v_e(E_t/N, P_t)}{v_e(e_{i,t}, P_t)} di$ *is a constant aggregation factor.*

Proof. The Marshallian demand (21) follows immediately from applying Roy's identity to (19). Equation (22) is derived by aggregating (21) over all indviduals and rearranging terms. Finally, the aggregation factor κ is constant because IA preferences imply that both $v_e(e_{i,t}, P_t)$ and $v_e(E_t/N, P_t)$ growth with the same gross rate $\beta(1 + r_{t+1})$ over time for all individuals i. ■

The market demand in (22) establishes that IA preferences generalize in a straightforward manner the aggregation properties of the PIGL demand derived in Muellbauer (1975), i.e., the market demand in (22) allows us to identify the preference parameters with minimal distributional information. The following corollary generalizes Theorem 7 in Muellbauer (1975) to the case where $\mathbf{A}(P)$ is non-zero.

Corollary 1. *If the distribution of* $v_e(e_{i,t}, P_t)/v_e(E_t/N, P_t)$ *is constant over time, then IA is the most general preference specification for which, given knowledge of the distribution* $e_{i,t}$ *at some point in the data period, there is no aggregation bias from using per-capita expenditure* E_t/N *as the relevant expenditure variable.*

Proof. Since $v_e(e_{i,t}, P_t)$ satisfies the individual Euler, the distribution of relative marginal utilities $v_e(e_{i,t}, P_t)/v_e(E_t/N, P_t)$ is constant if and only if $v(e, P)$ is IA. With aggregate data on per-capita expenditure only, then (22) allows to identify all parameters of the IA preferences up to the scale of the function $\mathbf{D}(P)$ (note that we can assume without loss of generality that $\mathbf{B}(P)$ has no price-independent scalar and $\mathbf{D}(P)$ no price-independent additive term since $v(e, P)$ is only determined up to price-independent affine-linear transformations). Furthermore, when distributional data for $e_{i,t}$ is available at some point in the data period, then the scale of $\mathbf{D}(P)$ can easily be identified by separating it from the associated aggregation factor κ based on the estimated parameters for $\mathbf{A}(P)$ and $\mathbf{B}(P)$ from (22). ■

The key implication of Propostion 3 and Corollary 1 is that the per-capita demand can be expressed as a function of the prices, nominal per-capita expenditure, as well as a constant inequality index of relative marginal utilities. Thus, up to the constant aggregation factor κ, the individual demand (21) and the market demand (22) have identical structure, which allows us to empirically identify all the preference parameters from aggregate data except for the scalar of $\mathbf{D}(P)$. Note that this result holds true although the preference class is more general than the Gorman and the PIGL form. The result crucially hinges on the intertemporal (Euler) equation and would not apply if some of the households were for instance credit constraint. If the goal is to retrieve preference parameters from aggregate data on structural change, then the preference class specified by (19) is a natural starting point. Finally, Corollary 1 shows that after having estimated preferences from (22) (or, the corresponding expenditure share), the unknown aggregation factor κ can simply be calculated from distributional data for only one period.

One specification that is not part of the IA class defined in (19) is the non-homothetic CES as specified in Hanoch (1975) and applied in the context of structural change in Comin et al. (2015, 2017). These preferences do not allow to express the Marshallian demand for commodity j by a single equation. Consequently, the system cannot straightforwardly be estimated. Furthermore, for non-homothetic CES the aggregation result of Proposition 3 above does not hold (except for the trivial homothetic case). This implies that one cannot identify the preference parameters from aggregate data and this expenditure system is consequently not suitable for our purpose. Another subclass that is not part of (19) is the Quadratic Almost Ideal Demand (QAID) system. However, as we will see below, these preferences (and the entire AID system) are not compatible with sustained growth and therefore cannot be used in our context.

5.2 Flexibility and Sustained Growth

From (22), the market expenditure shares associated with IA preferences can be obtained as

$$\begin{aligned}\eta_{j,t} \equiv \frac{p_{j,t}c_{j,t}}{e_t} = \mathbf{A}_j(P_t)p_{j,t}\left(\frac{e_t}{\mathbf{B}(P_t)}\right)^{-1} + \frac{\mathbf{B}_j(P_t)p_{j,t}}{\mathbf{B}(P_t)}& \\ + \mathbf{D}_j(P_t)p_{j,t}\left(\frac{e_t}{\mathbf{B}(P_t)} - \mathbf{A}(P_t)\right)^{1-\epsilon}\left(\frac{e_t}{\mathbf{B}(P_t)}\right)^{-1}\kappa,& \qquad (23)\end{aligned}$$

where the lower case variables $c_{j,t} \equiv C_{j,t}/N$ and $e_t \equiv E_t/N$ denote per-capita consumption and expenditure, respectively. The expenditure shares in (23) consist of three non-redundant additive terms, thus the IA preference specification yields a rank 3 expenditure system, which corresponds to the maximum rank in the three sector model that we consider in the empirical analysis below. It is worthwhile noting that this is one rank more flexible than the generalized Stone-Geary or the PIGL specifications, which are missing either the last term ($\mathbf{D}(P) = 0$), or the first term ($\mathbf{A}(P) = 0$), respectively.

The Euler equation implied by the IA preference specification (19) can be written as

$$\left(\frac{(e_t/\mathbf{B}(P_t) - \mathbf{A}(P_t))}{(e_{t+1}/\mathbf{B}(P_{t+1}) - \mathbf{A}(P_{t+1}))} \right)^{\epsilon - 1} \left(\frac{\mathbf{B}(P_t)}{\mathbf{B}(P_{t+1})} \right)^{-1} = \beta(1 + r_{t+1}). \tag{24}$$

Asymptotically, (24) supports at the same time a constant interest rate and a constant growth rate of per-capita consumption expenditure if the price function $\mathbf{A}(P_t)$ grows slower than $e_t/\mathbf{B}(P_t)$,

$$\lim_{t \to +\infty} \mathbf{A}(P_t) \left(\frac{e_t}{\mathbf{B}(P_t)} \right)^{-1} = 0, \quad \forall j \in J, \tag{25}$$

and either $\epsilon \to 0$, or if $\epsilon \neq 0$ when the price function $\mathbf{B}(P_t)$ approaches a Cobb-Douglas of prices

$$\lim_{t \to +\infty} \mathbf{B}(P_t) = \prod_{j \in J} p_{j,t}^{\phi_j}, \quad \sum_{j \in J} \phi_j = 1, \quad \phi_j \geq 0, \tag{26}$$

such that $\mathbf{B}(P_t)$ grows at the constant rate $\prod_{j \in J} (\gamma_X/\gamma_j)^{(1-\alpha)\phi_j}$. Finally, the expenditure shares in equation (23) stay within the unit interval, $\eta_{j,t} \in [0, 1]$, if (25) and

$$\lim_{t \to +\infty} \mathbf{D}_j(P_t) p_{j,t} \left(\frac{e_t}{\mathbf{B}(P_t)} \right)^{-\epsilon} = 0, \quad \forall j \in J, \tag{27}$$

are jointly satisfied.

To sum up, the IA preference specification yields an expenditure system that is more flexible than, for example, the generalized Stone-Geary or the PIGL. Nevertheless, the specification remains consistent with an asymptotic balanced growth path and non-negative demand under appropriate parameter restrictions.

5.3 Other preference specifications

The IA preference specification allows for tractable aggregation and sustained growth in per-capita expenditure. Furthermore, the preferences have rank 3 and thus a high flexibility. In this section we briefly discuss other preference specifications—the generalized PIGLOG (QAID) and the non-homothetic CES—that are not IA. These preferences also have a high flexibility, but we will see that neither of them allows for tractable aggregation. Furthermore, generalized PIGLOG does not allow for sustained growth. We focus here on the key aspects of these preferences and the details are delegated to Appendix E.

Generalized PIGLOG (QAID) The Quadratic Almost Ideal Demand (QAID) system in Banks et al. (1997) has been widely used in the microeconomic literature on consumer demand estimation. The demand system results from the generalized PIGLOG preference specification and allows the Engel curves to depend on quadratic terms of the logarithm of expenditure. Banks et al. (1997) show that this exactly aggregable system has the maximum rank of 3 and that this flexibility is necessary to fit the data. As in our case, their preference specification allows a good to be a luxury at some income levels and a necessity at other levels.

While we share with QAID the feature of flexibility to fit the data, there are two important differences between our IA preference specification and their QAID system. Firstly, while they nest cases that are exactly aggregable such as the Translog model, their general specification does not allow for constant aggregation factors as discussed in Blundell et al. (1993). In contrast, our dynamic framework has constant aggregation factors and allows for the identification of preference parameters from aggregate data. Second, the QAID system is not consistent with sustained growth, because the budget shares can go outside of the unit interval as expenditures grow over time. Since steady growth is a stylized fact of long run economic development, this is an essential feature for our analysis of structural change over more than a century.

Non-homothetic CES Comin et al. (2015, 2017) propose using the non-homothetic CES preference specification from Hanoch (1975) in a multi-sector growth model to study structural change. An important feature of these preferences is that they allow for sector-specific income elasticities that do not vanish as income grows. However, the preferences do not allow for tractable aggregation of individual demand and the preference parameters cannot be identified from aggregate data with heterogeneity in household expenditures. Furthermore, non-homothetic CES does not have a closed form for the Marshallian demand system and it therefore requires a more complicated estimation procedure that takes into account the implicit structure of real income and the preference parameters to be estimated.

5.4 Parameterizations

In this section we make explicit that the generalized Stone-Geary and the PIGL preferences are IA and that they are nested in (19). In the empirical analysis of Section 6, we then propose two simple but more flexible parameterizations if IA preferences, which allows us to directly test the above standard specificiations against the more flexible IA specifications in the historical panel data.

5.4.1 PIGL preferences

The PIGL class of preferences defined in Muellbauer (1975, 1976) is nested in (19) when $\mathbf{A}(P) = 0$. In the following, we parametrize the price function $\mathbf{B}(P)$ with the CES aggregator

$$\mathbf{B}(P) \equiv \begin{cases} \left(\sum_{j\in J} \phi_j p_j^{1-\sigma}\right)^{\frac{1}{1-\sigma}}, & \sigma \neq 1 \\ \prod_{j\in J} p_j^{\phi}, & \sigma = 1, \end{cases} \tag{28}$$

where $\sigma > 0$, $\sum_{j\in J} \phi_j = 1$, and $\phi_j \geq 0$. When $\sigma = 1$, then the price function $\mathbf{B}(P)$ is of the Cobb-Douglas form outlined in (26) and the PIGL specification is consistent with an exact balanced growth path.[20] Henceforth, we will refer to the PIGL specification as the following exact balanced growth compatible formulation

[20] See Boppart (2014) for an exact balanced growth consistent PIGL specification with two sectors.

of the indirect utility function

$$v(e,P)=\frac{1}{\epsilon}\left(\frac{e}{\prod_{j\in J}p_j^{\phi_j}}\right)^{\epsilon}-\frac{1}{\epsilon}-\mathbf{D}(P),\quad \epsilon\in(-\infty,0)\cup(0,1). \tag{29}$$

The aggregate expenditure shares are then given by

$$\eta_{j,t}=\phi_j+\mathbf{D}_j(P_t)p_{j,t}\left(\frac{e_t}{\prod_{l\in J}p_l^{\phi_l}}\right)^{-\epsilon}\kappa,\quad \kappa=\int_0^N(e_t/e_{i,t})^{\epsilon-1}\,di. \tag{30}$$

Given that $\mathcal{A}(P)=0$, the flexibility of the PIGL expenditure system is reduced by one additive term to rank 2 compared to (23). For the empirical estimation, this is an important limitation relative to the flexible IA class of preferences. On the other hand, since conditions (25) and (26) are met at any point in time, the PIGL specification in (29) is consistent with an exact balanced growth path, while generally IA preferences only allow for an asymptotic balanced growth path. Finally, the constant aggregation factor κ becomes independent of prices and only the parameter ϵ is required to determine the aggregation bias.

5.4.2 Generalized Stone-Geary preferences

The generalized Stone-Geary specification is nested in (19) when $\mathbf{D}(P)=0$, $\mathbf{A}(P)=\mathbf{B}(P)^{-1}\sum_{j\in J}\bar{c}_j p_j$, and the weights of the price function $\mathbf{B}(P)$ are strictly positive, $\phi_j\equiv\omega_j>0$, $\forall j\in J$. The indirect utility function then reads

$$v(e,P)=\frac{1}{\epsilon}\left(\frac{e-\sum_{j\in J}p_j\bar{c}_j}{\left(\sum_{j\in J}\omega_j p_j^{1-\sigma}\right)^{\frac{1}{1-\sigma}}}\right)^{\epsilon}-\frac{1}{\epsilon},\quad \epsilon\in(-\infty,0)\cup(0,1). \tag{31}$$

Note that the validity of this indirect utility function hinges on the assumption that each individual i has enough budget to cover the subsistence consumption $\bar{c}_j$ for each good,

$$p_j(c_{i,j}-\bar{c}_j)\geq 0,\ \forall j\in J. \tag{32}$$

This restriction follows from the formulation of the underlying generalized Stone-Geary utility function.[21] The aggregate expenditure shares are given by

$$\eta_{j,t} = \frac{\omega_j p_{j,t}^{1-\sigma}}{\sum_{l\in J}\omega_l p_{l,t}^{1-\sigma}} + \left[p_{j,t}\bar{c}_j - \frac{\omega_j p_{j,t}^{1-\sigma}}{\sum_{l\in J}\omega_l p_{l,t}^{1-\sigma}}\sum_{l\in J} p_{l,t}\bar{c}_l\right] e_t^{-1}. \tag{34}$$

This is often referred to as the Linear Expenditure System (LES), because the system allows for exact linear aggregation such that market expenditure shares are functions of per-capita expenditure only, and other moments of the expenditure distribution are irrelevant.

The functional form of Equation (34) shows that the parameter σ is an important determinant of the expenditure shares' price elasticity, i.e., when $\sigma = 1$ prices do not effect the expenditure shares other than through the terms that contain the subsistence consumption. On the other hand, it is easy to see that the subsistence levels $\bar{c}_j$ are driving the income effect. However, when in the long-run per-capita consumption expenditure outgrow prices as suggested by the data, i.e., $\lim_{t\to+\infty} p_{j,t}/e_t = 0$, then all the terms involving the subsistence levels $\bar{c}_j$ asymptotically become irrelevant. Therefore, when $\sigma < 1$, the generalized Stone-Geary preference specification implies that over time structural change must be driven more and more by relative price effects. Finally, note that the parameter ϵ, which drives the intertemporal elasticity of substitution, drops out of the expenditure system and cannot be identified from the expenditure shares.

Herrendorf et al. (2013) (henceforth referred to as HRV 2013) demonstrate that the budget share system in Equation (34) predicts an accurate fit of the USA's structural transformation in the postwar period.[22] However, a direct consequence of

[21] The indirect utility function of the generalized Stone-Geary specification follows from maximizing the utility function

$$u = \frac{1}{\epsilon}\left[\left(\sum_{j\in J}\omega_j^{\frac{1}{\sigma}}\left(c_j^i - \bar{c}_j\right)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}}\right]^{\epsilon} - \frac{1}{\epsilon}, \quad \epsilon \in (-\infty, 0)\cup(0,1), \tag{33}$$

subject to the intratemporal budget restriction $\sum_{j\in J} p_j c_j^i \leq e_i$ and the regularity constraints $c_{i,j} - \bar{c}_j \geq 0$, $\forall j \in J$.

[22] Following the previous literature, HRV 2013 restrict the subsistence level of manufacturing consumption to zero, $\bar{c}_M = 0$.

the generalized Stone-Geary's functional form discussed above is that the resulting estimates for the parameters that govern relative price and income effects will depend on the time period covered by the sample (or, the dispersion of income in cross-sectional data). In particular, generating significant income effects towards the end of a long sample requires very high subsistence or endowment levels. However, the income effects will then be even stronger at earlier stages of the sample when per-capita expenditures are lower. Thus, the generalized Stone-Geary specification will struggle when estimating parameters from data for the period before WW II. This has already been emphasized earlier by Buera and Kaboski (2009) with historical sectoral value-added consumption shares in the USA. We will show in Section 6 that this finding also holds true in historical final consumption expenditure data and not only for the USA, but also for GBR, CAN and AUS.

Moreover, note again that the aggregate budget shares in Equation (34) are only valid if the subsistence expenditure for each good j does not exceed the consumption expenditure of any individual as stated in inequality (32).[23] The empirical structural change literature has mostly ignored this regularity constraint by directly working under the representative (average) agent assumption.

On the theoretical side, the existing literature has emphasized that generalized Stone-Geary preferences are consistent with exact balanced growth for a narrow set of parameterizations. Kongsamut et al. (2001) consider the special case without relative price effects where $\sigma = 1$ and $\sum_{j \in J} p_{j,t}\bar{c}_j = 0$. Ngai and Pissarides (2007) on the other hand assume that there are no income effects, $\bar{c}_j = 0$ and $\epsilon \to 0$.[24] We will show in Section 6 that the parameters estimated from the historical data suggest that both relative price and income effects are needed to fit the data. Thus, the corresponding parameterizations of the Stone-Geary preferences will be generally inconsistent with exact balanced growth. However, if the income effects vanish

[23]Suppose that the subsistence expenditure for agricultural per-capita consumption $p_A\bar{c}_A$ were higher than the per-capita consumption expenditure p_Ac_A measured in the macroeconomic data. Then the generalized Stone-Geary utility function in is no longer well-defined, even for the consumer with the average agricultural consumption expenditure. Such a high subsistence level would imply that the average consumer (and every individual with less expenditure) in the economy were to starve in that period.

[24] It is straightforward to see from the indirect utility function in (31) that both of these parameter restrictions nest the PIGL form in (29).

asymptotically,

$$\lim_{t\to+\infty} p_{j,t}/e_{i,t} = 0, \tag{35}$$

and $\sigma \to 1$ or $\epsilon \to 0$, then the generalized Stone-Geary specification is consistent with an asymptotic balanced growth path. Moreover, the same conditions also guarantee that the expenditure shares stay in the unit interval with

$$\lim_{t\to+\infty} \eta_{j,t} = \frac{\omega_j p_{j,t}^{1-\sigma}}{\sum_{k\in J} \omega_k p_{k,t}^{1-\sigma}}, \quad \forall j \in J.$$

Finally, similar to the PIGL demand system in (30), the demand system in (34) has rank 2. Thus, it's functional flexibility remains below the maximum possible rank with three sectors.

5.4.3 General parameterization

In Section B.1 of the Appendix we outline a general parameterization of the IA preferences that nests at the same time the generalized Stone-Geary *and* the PIGL specification. In the empirical analysis of Section 6 below, for simplicity, we focus on two simpler cases of the general parameterization: one that directly nests the Generalized Stone-Geary and another that nests directly the PIGL specification. However, in Section B.1 we complement the estimation results reported in the main text with further evidence from the more general parameterization of the IA preferences.

6 Empirical results

In this section we estimate and test the expenditure systems of the IA preference specifications presented in Section 5.4. In particular, we use parameterizations that nest the PIGL and generalized Stone-Geary preferences, which allows us to test whether these standard specifications are rejected against the more flexible ones.

For the identification of the underlying preference parameters, we use the variation in the historical data on sectoral prices and nominal final consumption expenditure per-capita for the USA, GBR, CAN, and AUS over the period 1900 to 2014. Following HRV 2013, we report the feasible generalized nonlinear least squares

(FGNLS) estimator with robust standard errors for the parameters of each specification.[25] As the expenditure shares for the three sectors add up to unity, it is sufficient to estimate the parameters of the expenditure shares for the manufacturing and the service sector. The point estimates and standard errors of any remaining parameters can then be derived from the appropriate parameter restrictions.

The main finding of this section is that the generalized Stone-Geary and the PIGL preference specifications are rejected against more flexible parameterizations of the IA preferences. This result applies to both the full time period starting in 1900 and the more recent postwar period. In particular for the longest sample, the standard specifications struggle to fit simultaneously the pronounced decline of agriculture, the hump-shape of manufacturing, and the accelerated rise of services that we document in Section 3.

6.1 Comparison with generalized Stone-Geary

We start the empirical analysis by testing the generalized Stone-Geary specification (31) against the following parameterization of (19),

$$v(e,P) = \frac{1}{\epsilon}\left(\frac{e}{\mathbf{B}(P)} - \frac{\sum_{j\in\{A,S\}}\bar{c}_j p_j}{\mathbf{B}(P)}\right)^{\epsilon} - \frac{1}{\epsilon} - \frac{\overline{D}}{\kappa}\sum_{j\in\{M,S\}}\nu_j \log\left(\frac{p_j}{p_A}\right), \tag{36}$$

which directly nests the generalized Stone-Geary specification under the simple restriction that $\overline{D}=0$. We have chosen the price function $\mathbf{D}(P)$ to be a simple sum of logarithmic relative prices with weights $\overline{D}\nu_M\kappa^{-1}$ and $\overline{D}\nu_S\kappa^{-1}$, where $\sum_{j\in J}\nu_j = 1$. Compared to the generalized Stone-Geary in (31), the parameterization in (36) has only two additional parameters. Moreover, adding the price function $\mathbf{D}(P)$ will allow us to identify the parameter ϵ which is not possible with the LES that results from the generalized Stone-Geary specification. Finally, we follow the convention in the existing literature that agricultural consumption is restricted to be a necessity, $\min\{c_{A,t}\} \geq \bar{c}_A \geq 0$, manufacturing neutral, $\bar{c}_M = 0$, and services a luxury, $\bar{c}_S \leq 0$, in the term that involves the subsistence consumption levels.

Table 1 reports the estimation results when we consider final private consumption

[25] This estimator is implemented by Stata's command *nlsur* and can be applied to estimate the parameters of nonlinear systems of equations.

expenditure shares. Columns (1) to (3) show the results for the generalized Stone-Geary specification, i.e., when we impose the restriction $\overline{D} = 0$ in (36). Columns (4) to (6) show the estimation results when we relax this restriction. In both cases we pool the data of all four countries and consider different sample periods starting in 1900, 1929, and 1947.[26]

6.1.1 Generalized Stone-Geary

We start the discussion with the estimation results for the generalized Stone-Geary specification reported in columns (1) to (3) of Table 1. In the first row, the point estimate for the price elasticity parameter σ is precisely estimated and ranges between 0.33 and 0.74 across the three considered samples. The point estimate is decreasing with the sample length and remains significantly smaller than 1 as indicated by the robust standard errors reported in parenthesis. This shows that the generalized Stone-Geary specification attributes a substantial fraction of the variation in the expenditure shares to relative price effects, in particular in the longest sample.

The second row in Table 1 shows that the estimated subsistence level of agricultural consumption hits the upper bound $\bar{c}_A = \min\{c_{A,t}\}$ for all subsamples. This means that the model fit would improve if the subsistence level of agricultural goods were allowed to be higher than the minimum per-capita consumption measured in the data. The reason is that the wider the dispersion of per-capita expenditure in the data, the higher subsistence levels are required to generate strong enough income effects at the end of the sample where per-capita expenditures are high. As a consequence, the fall in the expenditure share for agriculture predicted by the generalized Stone-Geary is generally not steep enough to fit the long-run data, as can be seen in panel (a) of Figure 4, for example.[27]

The remaining estimates in columns (1) to (3) of Table 1 are relatively standard.

[26] The subsample starting in 1947 is considered because HRV 2013 estimate the generalized Stone-Geary specification for the USA in the postwar period from 1947 to 2010. Moreover, structural change in the postwar period has been remarkably monotone in the considered countries. The other threshold year in 1929 is chosen because the BEA reports harmonized consumption expenditure and price data for the USA from 1929 onwards. Observations from earlier periods are based on historical data from Carter et al. (2006). The details of the data construction are documented in Appendix D.

[27] As we see below, the subsistence level of services in column (1) of Table 1 also hits the non-negativity bound, thus the fit would improve if services were allowed to be a necessity.

	Generalized Stone-Geary			IA Preferences (36)		
	1900-2014	1929-2014	1947-2014	1900-2014	1929-2014	1947-2014
	(1)	(2)	(3)	(4)	(5)	(6)
σ	.339***	.647***	.737***	.334***	.518***	.507***
	(.020)	(.030)	(.038)	(.035)	(.030)	(.037)
$\bar{c}_A$	709.504	758.941	947.227	709.504	758.941	947.227
	(.)	(.)	(.)	(.)	(.)	(.)
$\bar{c}_S$	-.000	-918.347***	-2351.917***	-.000	-.000	-.000
	(.)	(124.850)	(274.803)	(.)	(.)	(.)
ω_A	.096***	.072***	.043***	.000	.000	.000
	(.002)	(.002)	(.001)	(.)	(.)	(.)
ω_M	.325***	.305***	.271***	.382***	.277***	.248***
	(.002)	(.004)	(.006)	(.011)	(.014)	(.011)
ω_S	.580***	.623***	.686***	.618***	.723***	.752***
	(.003)	(.005)	(.007)	(.011)	(.014)	(.011)
ϵ				.466***	.429***	.472***
				(.031)	(.044)	(.030)
$\overline{D}\nu_M$				-3.504***	2.913***	7.009***
				(1.176)	(.855)	(1.939)
$\overline{D}\nu_S$				-1.952**	-6.550***	-10.831***
				(.921)	(2.160)	(2.906)
N	341	276	250	341	276	250
AIC	-2719	-2537	-2536	-2974	-2683	-2649
$\mathrm{E}[\hat{\eta}_{A,t} - \hat{\eta}_{A,t\|\overline{D}=0}] = 0$				0.000	0.000	0.000
$\mathrm{E}[\hat{\eta}_{M,t} - \hat{\eta}_{M,t\|\overline{D}=0}] = 0$				0.000	0.000	0.000
$\mathrm{E}[\hat{\eta}_{S,t} - \hat{\eta}_{S,t\|\overline{D}=0}] = 0$				0.007	0.000	0.000

Table 1: Testing Generalized Stone-Geary, Private consumption

Note: Years where the USA was involved in WWI (1917-1918), WWII (1942-1945), or affected by the Great Depression (1929-1933), and years where AUS, CAN, and GBR where involved in WWI (1914-1918), WWII (1939-1945), or affected by the Great Depression (1929-1933) are excluded from all estimations. All variables are based on final private consumption expenditure. Robust standard errors are reported in parenthesis. ***, **, * is significant at the 1%, 5%, and 10% level, respectively.

In the third row, consumption of services is estimated to be a luxury with a substantial endowment level $\bar{c}_S < 0$ in the post-1929 samples. The share parameters ω_j stay between zero and one as required by the model specification. Finally, note that due to the relevance of both relative price effects and income effects, as discussed above, the estimation results for the parameters in columns (1) to (3) of Table 1 reject parameterizations of the generalized Stone-Geary specification that are consistent with exact balanced growth (see Section 5.4). In particular, the subsistence levels of consumption are nonzero and the price elasticity parameter σ is smaller than unity.

In Table 3 of Appendix A we additionally report the estimation results of the specification in (36), when we consider final total consumption expenditure that includes government expenditure.[28] The results are very similar, in particular the parameter estimate of σ is significantly below 1 and falls with the length of the sample, and the agricultural subsistence consumption hits again the upper bound.

Finally, for comparison with the results of HRV 2013, we report in Table 5 of Appendix A the estimation results for the generalized Stone-Geary specification when we restrict the sample to the USA. Columns (1) to (3) show the estimation results for private consumption expenditure, while columns (4) to (6) show the results for total consumption expenditure that includes government expenditure. In the postwar sample reported in column (6) of Table 5, relative price effects are estimated to be less important for total consumption expenditure shares (the point estimate for σ is 0.85), which is consistent with the estimate in Table 1 of HRV 2013. However, once longer samples for the USA are considered in columns (4) and (5), the point estimate for σ becomes significantly smaller. This confirms the earlier conjecture that the relative importance of price and income effects predicted by the generalized Stone-Geary specification varies with the length of the sample. The more dispersed the observed per-capita expenditure levels, the more important are the relative price effects in comparison to the income effects predicted by the Stone-Geary specification.[29] For private consumption expenditure, as reported in

[28] Due to the limited data availability of government expenditure for the USA prior to 1929 (Carter et al. (2006) reports numbers for 1902, 1913, 1922, 1927), the number of data points reduces from 341 to 318 when we consider final total consumption expenditure.

[29] For an even longer sample with decennial value-added data of the United States (1870-2000), Buera and Kaboski (2009) conclude that the best fit of the generalized Stone-Geary specification is achieved when the relative price effects correspond to a Leontief specification, $\sigma \to 0$, i.e., relative real quantities are inelastic to changes in relative prices such that the effect on the nominal per-

column (1) of Table 5, the point estimate for σ is as low as 0.13 in the long-run data.

6.1.2 IA Preferences

Table 1 shows in columns (4) to (6) the results when we relax the restriction that $\overline{D} = 0$ in order to estimate the flexible IA preference specification in (36). Before discussing the point estimates reported in the table, we note that the substantially lower Akaike information criterion (AIC) reported at the bottom of columns (4) to (6) suggests that the fit of the historical expenditure shares improves substantially when we consider the unrestricted specification. For instance, in the long-run sample reported in columns (1) and (4), the AIC decreases from -2719 to -2974. However, since we are estimating parameters with robust standard errors, we cannot employ the log-likelihood ratio statistic to test the nested generalized Stone-Geary against the unrestricted IA specification. Instead, we will use a simple Wald test to reject the nested specification.

More specifically, let $\hat{\eta}_{j,t}$ denote the nonlinear prediction of the sectoral expenditure shares based on the estimates reported in columns (4) to (6) of Table 1. Let $\hat{\eta}_{j,t|\overline{D}=0}$ stand for the same prediction, but under the hypothesis that $\overline{D} = 0$. Formally, we then test whether the expected difference between the two predictions is zero for each sector

$$\mathrm{E}\left[\hat{\eta}_{j,t} - \hat{\eta}_{j,t|\overline{D}=0}\right] = 0, \quad \forall j \in J,$$

and report the corresponding p-values at the bottom of Table 1. For example, column (4) suggests that the expected difference in the predicted expenditure shares is significantly different from zero at the 1% level for all sectors. In the shorter samples reported in column (5) and (6), the same significance of the expected difference applies. Thus, the generalized Stone-Geary (31) specification is rejected against (36) at the 1% level of confidence in all subsamples reported in Table 1. The generalized Stone-Geary is also rejected at the 1% level when we consider total consumption expenditure shares, as can be seen from the reported p-values at the bottom of Table 3 in Appendix A.

capita expenditure is maximal. However, even for this extreme case, Buera and Kaboski (2009) found the overall fit of the generalized Stone-Geary specification to be questionable.

While the IA preference specification in Equation (36) improves the fit of the historical data compared to the generalized Stone-Geary significantly, the specification hits corners with respect to both subsistence levels $\bar{c}_A$ and $\bar{c}_S$, as can be seen in columns (4) to (6) of Table 1. As a consequence, relative price effects become more important and the point estimates of the price elasticity parameter σ remain below the ones of the generalized Stone-Geary. The share parameter for the agricultural sector ω_A is estimated to be zero in columns (4) to (6) as well (which is unproblematic for the unrestricted specification). Finally, the intertemporal substitution elasticity parameter ϵ is identified and precisely estimated between 0.42 and 0.48. This confirms the earlier discussion that sustained income effects are indeed important to fit the historical panel data.

6.2 Comparison with PIGL

In the previous section, we have established that a simple parameterization of the IA preferences fits the data better than the generalized Stone-Geary. In this section, we allow the IA preferences to have more flexibility in the price function $\mathbf{D}(P)$. Furthermore, we test whether a simple parameterization of the function $\mathbf{A}(P)$ fits the data better than the nested PIGL specification that does not have the $\mathbf{A}(P)$ term. More specifically, in Table 2 we report the estimation results for the following parameterization of the IA preferences

$$v(e,P) = \frac{1}{\epsilon}\left(\frac{e}{\prod_{j\in J} p_j^{\phi_j}} - \overline{A}\prod_{j\in J} p_j^{\mu_j-\phi_j}\right)^{\epsilon} - \frac{1}{\epsilon} - \frac{\overline{D}}{\kappa}\sum_{j\in\{M,S\}}\frac{\nu_j}{\psi_j}\left(\frac{p_j}{p_A}\right)^{\psi_j} - \frac{\nu_j}{\psi_j}, \quad (37)$$

which directly nests the exact balanced growth consistent PIGL specification when $\overline{A} = 0$. The price function $\mathbf{A}(P)$ is a simple Cobb-Douglas aggregate of prices, with $\sum_{j\in J}\mu_j = 1$, and $\mathbf{D}(P)$ is a weighted sum of power functions in relative prices, with $\psi_j \neq 0$. Analogous to the previous section, since the PIGL is directly nested, we can again test it against the unrestricted IA preference specification in (37) using a Wald statistic. More formally, we test the hypothesis that

$$\mathrm{E}\left[\hat{\eta}_{j,t} - \hat{\eta}_{j,t|\overline{A}=0}\right] = 0, \quad \forall j \in J, \quad (38)$$

	PIGL			IA Preferences (37)		
	1900-2014	1929-2014	1947-2014	1900-2014	1929-2014	1947-2014
	(1)	(2)	(3)	(4)	(5)	(6)
ϵ	.708***	.684***	.758***	.594***	.438***	.482***
	(.016)	(.031)	(.025)	(.064)	(.037)	(.040)
ϕ_M	.306***	.263***	.222***	.119**	.161***	.131***
	(.003)	(.011)	(.009)	(.052)	(.035)	(.038)
ϕ_S	.694***	.737***	.778***	.881***	.839***	.869***
	(.003)	(.011)	(.009)	(.052)	(.035)	(.038)
$\overline{D}\nu_M$	.777*	25.049***	94.531***	92.581**	16.046***	24.752***
	(.456)	(5.946)	(15.149)	(40.020)	(3.780)	(5.704)
$\overline{D}\nu_S$	-98.153***	-102.797***	-247.275***	-102.567**	-18.847***	-30.483***
	(14.325)	(25.861)	(49.919)	(47.045)	(4.562)	(7.922)
ψ_M	4.321***	.363**	.014	-.076**	-.517***	-.524***
	(.710)	(.183)	(.073)	(.030)	(.049)	(.036)
ψ_S	-.496***	-.482***	-.265***	-.331***	-.463***	-.339***
	(.045)	(.043)	(.023)	(.024)	(.046)	(.036)
$\overline{A}$				2630.111***	632.368***	483.290***
				(204.023)	(73.131)	(50.714)
μ_M				-.456***	-1.772***	-1.965***
				(.100)	(.079)	(.081)
μ_S				1.048***	1.304***	1.445***
				(.052)	(.071)	(.067)
N	341	276	250	341	276	250
AIC	-2781	-2618	-2617	-3032	-2793	-2792
$\mathrm{E}[\hat{\eta}_{A,t} - \hat{\eta}_{A,t\vert\overline{A}=0}] = 0$				0.000	0.000	0.000
$\mathrm{E}[\hat{\eta}_{M,t} - \hat{\eta}_{M,t\vert\overline{A}=0}] = 0$				0.000	0.000	0.000
$\mathrm{E}[\hat{\eta}_{S,t} - \hat{\eta}_{S,t\vert\overline{A}=0}] = 0$				0.000	0.000	0.000

Table 2: Testing PIGL, Private consumption

Note: Years where the USA was involved in WWI (1917-1918), WWII (1942-1945), or affected by the Great Depression (1929-1933), and years where AUS, CAN, and GBR where involved in WWI (1914-1918), WWII (1939-1945), or affected by the Great Depression (1929-1933) are excluded from all estimations. All variables are based on final private consumption expenditure. Robust standard errors are reported in parenthesis. ***, **, * is significant at the 1%, 5%, and 10% level, respectively.

where $\hat{\eta}_{j,t|\overline{A}=0}$ denotes again the predicted expenditure share, but under the hypothesis that $\overline{A} = 0$.

6.2.1 PIGL

We start the discussion with columns (1) to (3) of Table 2 that show the estimation results for the PIGL specification, which is consistent with exact balanced growth, i.e., when we restrict $\bar{A} = 0$ in (37). The intertemporal elasticity parameter ϵ is estimated very precisely with a point estimate between 0.68 and 0.76, which suggests a more sustained income effect compared to the generalized Stone-Geary. The parameters are well identified and the AIC of -2781 reported in column (1) at the bottom of Table 2 suggests that the fit of the PIGL is slightly better than the generalized Stone-Geary (AIC of -2719) in the pooled long-run sample for private consumption expenditure. Moreover, the PIGL specification is consistent with an exact balanced growth for the estimated parameter values, while the generalized Stone-Geary is not. In Table 4 in Appendix A, we show that the above results are robust to considering consumption expenditure shares including government consumption.

6.2.2 IA Preferences

In columns (4) to (6) of Table 2, we report the estimation results for the unrestricted IA preference specification in (37). The parameter ϵ remains precisely estimated for all reported samples and varies between 0.44 and 0.59, which suggests again that sustained income effects are important to fit the historical data. The additional parameters $\overline{A}$, μ_M and μ_S that enter the function $\mathbf{A}(P)$ are all precisely estimated, which already points to the fact that the flexibility of the IA class of preferences is important to fit the historical expenditure shares.

At the bottom of Table 2, we report the p-values of the Wald statistic associated with the hypothesis in (38). The PIGL is rejected against the more flexible IA specification (37) at the 1% level of significance for all three sectors and all samples. This result is again reflected in the substantially lower AIC values reported at the bottom of columns (4) to (6) compared to (1) to (3), which is another indicator that the generalization is preferred over the standard PIGL.

Finally, the improved fit of the IA preference specification in the historical data is confirmed if we consider total consumption expenditure (Table 4 in Appendix A) or restrict the sample to individual countries. For example, Table 6 in Appendix A reports the estimation results for specification (37) when we restrict the sample to the USA. The AIC criterion improves substantially in columns (4) and (5) compared to the one in columns (1) and (2), and the hypothesis in (38) is rejected at the 1% of confidence level. Note that in column (6) we had to exogenously set $\overline{A} = 1$ for the IA preferences, because the parameter is not well identified in the postwar sample. Still, in all specifications, the parameter ϵ is precisely estimated and below 1 in all subsamples.

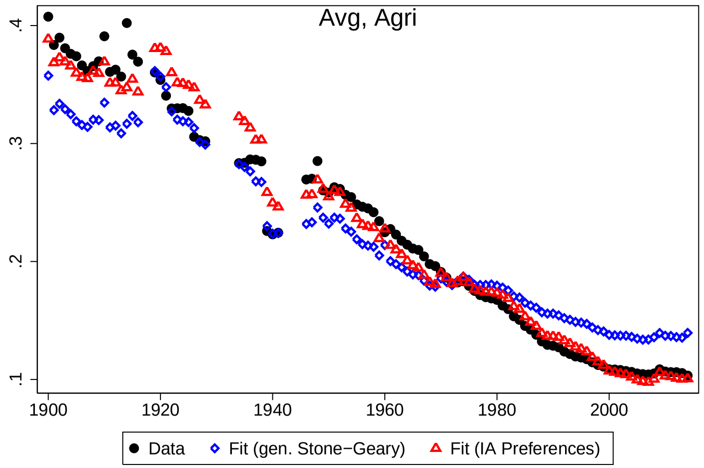
6.3 Predicted Nominal Expenditure Shares

In this section we visually compare the fit of the predicted expenditure shares to the data over the considered specifications. We focus on the comparison of the generalized Stone-Geary specification and our preferred IA specification in (37). Moreover, we consider the longest sample in the historical panel data.[30]

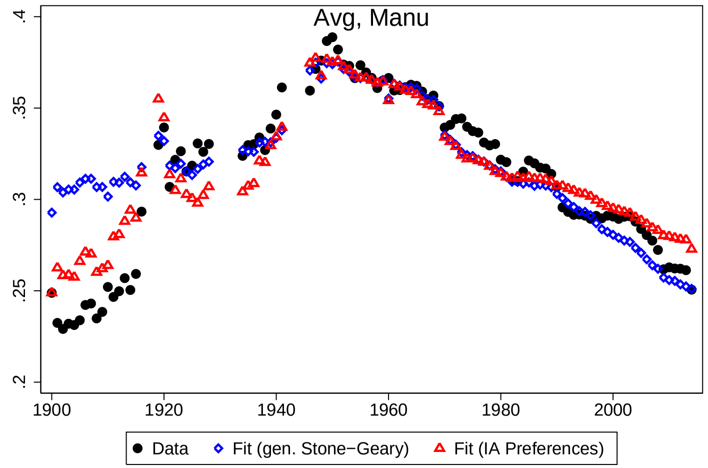
Panel (a) of Figure 4 shows that - averaging across countries - the generalized Stone-Geary specification underpredicts the decline of the agricultural expenditure share over the last century substantially. Between 1900 and 2014, the decline of the agricultural share is predicted to be only 21.8 percentage points (pp) (from 35.8% to 14.0%), while the actual data shows a decline of 30.4 pp from (40.7% to 10.3%). This shows that the estimated level of the agricultural subsistence consumption in the generalized Stone-Geary preference specification generates too strong income effects at the beginning of the sample, but too weak income effects at the end of the sample. The IA preferences on the other hand predict the agricultural share to fall by 28.8 pp (from 38.9% to 10.0%), which is much closer to the pronounced decline observed in the data. A similar pattern occurs for the fit of the initial increase of the manufacturing expenditure share, which is shown in panel (b). Between 1900 and 1950, the generalized Stone-Geary predicts an increase by 8.1 pp (from 29.3% to 37.4%) compared to the actual increase of 14.0 pp (from 24.9% to 38.9%)

[30]Figure 10 in Appendix A additionally shows the fit of the IA specification (37) in column 4 of Table 1 and the PIGL specification in column 1 of Table 2. The figure shows that our preferred IA specification (column 4 of Table 2) provides the best fit of the data.

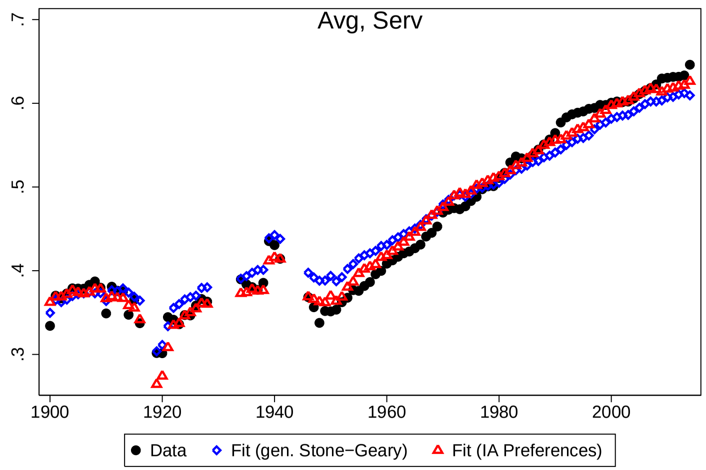
(a) Agriculture



(b) Manufacturing



(c) Services



Notes: The figure plots the average prediction and the average final private nominal consumption expenditure shares across the four countries by each sector. The blue dots show the fit of the generalized Stone-Geary specification estimated in column (1) of Table 1 and the red dots show the fit of the IA preference specification estimated in column (4) of Table 2.

Figure 4: Predicted final private nominal consumption expenditure shares, Full Sample, Averages

in the data. On the other hand, the IA preferences predict an increase of the manufacturing sector of 12.6 pp (from 24.9% to 37.5%). Finally, panel (c) shows that the accelerated increases of the service sector from around 1950 is predicted similarly by the generalized Stone-Geary and the IA specification (37).

6.4 Predicted Real Expenditure Shares

An alternative to the nominal expenditure shares is to visualize the data and the predictions as "real shares", as for instance suggested by Herrendorf et al. (2014). To this aim, we normalize all the prices to 1 in the year 1990, calculate real quantities of agriculture, manufacturing, and services in the data as well as from the predictions, and express each sector's quantity as a share of the sum over all sectors' quantities.[31] These real shares are shown in Figure 5. The normalization of prices implies that all nominal and real shares coincide in the year 1990.

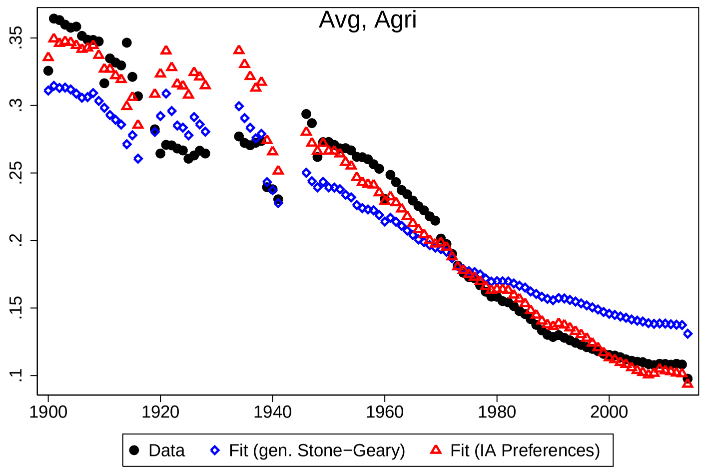
Panel (a) in Figure 5 shows (similar to the predicted nominal shares in Figure 4) that the generalized Stone-Geary specification substantially underpredicts the decline in agriculture. The initial share of agriculture is predicted to be too low, while the share towards the end of the sample is too high. In contrast, the IA specification is able to match the decline relatively well. Panel (c) illustrates that generalized Stone-Geary struggles to generate the hump-shape in the services sector and the IA preference specification can, due to the more flexible income effects, generate this non-monotonicity better. We document the role and importance of the flexible income effects in more detail in the next section and in Figure 6.

6.5 Predicted income elasticities

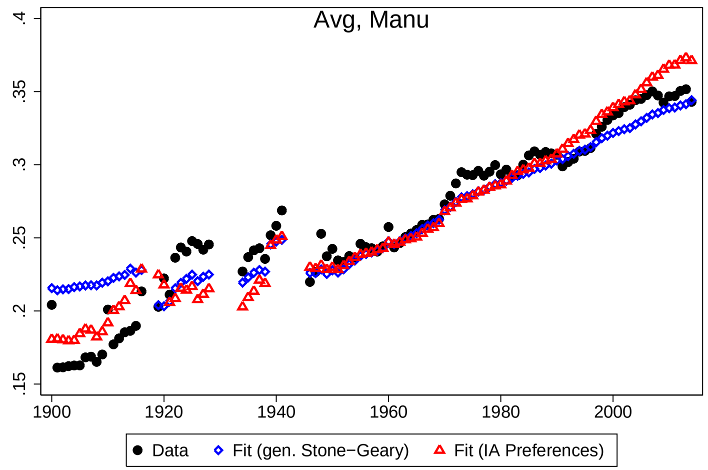
In all of the considered specifications, the income effects on the sectoral expenditure shares depend on the nominal per-capita expenditure and the sectoral prices, and therefore change over time. In this section we predict the corresponding income elasticity of the expenditure shares for the generalized Stone-Geary and the IA

[31] More precisely, the share of real consumption of good j is expressed as a share of the sum of real consumption across all goods, i.e. $c_j/(c_A + c_M + c_S)$, for $j = A, M, S$. An alternative would be to use Fisher aggregation, but the advantage of this visualization is that the shares add up to 1 like for the nominal shares.

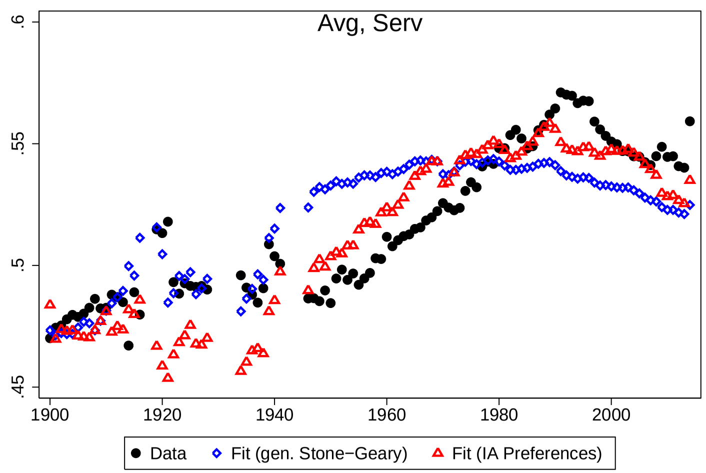
(a) Agriculture



(b) Manufacturing



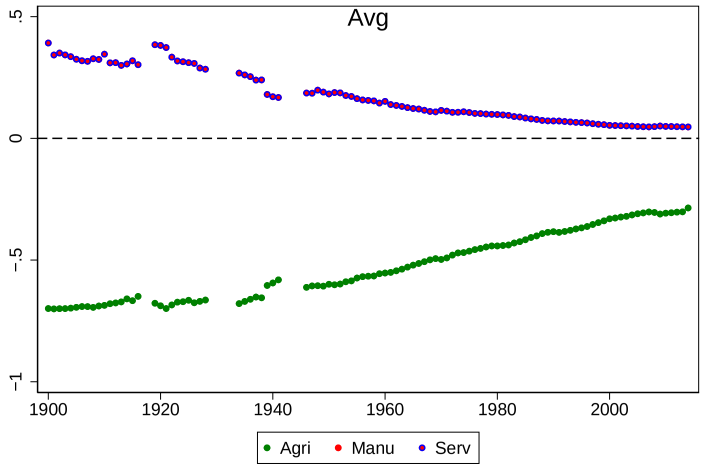
(c) Services



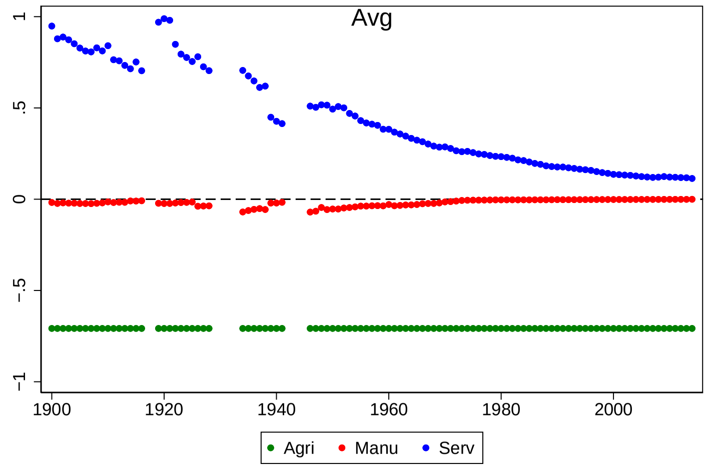
Notes: The figure plots the prediction and the actual final private consumption expenditure quantities for each sector, relative to the sum of predicted quantities. The predictions are averaged over all countries.

Figure 5: Predicted final private consumption expenditure quantities (relative to sum of predicted quantities), Full Sample, average across countries

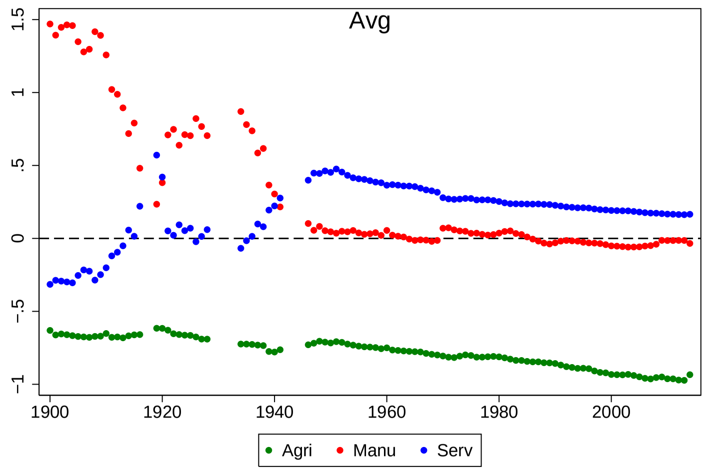
(a) Generalized Stone-Geary



(b) PIGL



(c) IA Preferences (37)



Notes: The figure plots the predicted income elasticities of the sectoral expenditure shares averaged across the four countries in each period. Panel (a) shows the elasticities predicted by the generalized Stone-Geary specification estimated in column (1) of Table 1, and panel (b) and (c) show the elasticities predicted by the specifications estimated in column (1) and (4) of Table 2, respectively.

Figure 6: Predicted income elasticities of the expenditure shares

specification using again the point estimates of the preference parameters.[32] When the elasticity of the expenditure share is positive, the corresponding sector has a luxury character. Sectors with a negative elasticity have the character of a necessity.

Figure 6 illustrates the average predicted income elasticities of different specifications in the pooled sample. Panel (a) shows that generalized Stone-Geary predicts income effects that are very monotone and become weaker over time as per-capita expenditure increases. This makes it difficult to match the continued decline in the agricultural sector towards the end of the sample. Moreover, note that since the subsistence levels of manufacturing and services are estimated to be zero in column (1) of Table 1, the income elasticity of the two sectors must be the same.

For the IA preference specification shown in panel (c), the predicted income effects are more flexible and sustained. We consider first the agricultural sector. Its income elasticity becomes gradually more negative over the considered period, which is essential to fit the continued decline of the agricultural share as discussed previously in panel (a) of Figure 4. The manufacturing sector starts out as a luxury good with a high income elasticity in order to generate the increasing part of it's hump-shaped expenditure share. After 1950, the income elasticity of the manufacturing sector decreases to around zero and mostly relative price effects are relevant to fit its expenditure share. Finally, the service expenditure share's income elasticity is hump-shaped and predicted to be a necessity at the beginning and a luxury at the end of the sample.

Finally, panel (b) shows that the flexible IA class of preferences is necessary to generate the non-monotonic income effects. Although the PIGL is able to generate a sustained income effect for the agricultural sector, the income elasticities generally remain very monotone. In particular, the income elasticity of the manufacturing share remains always around zero, which makes it very difficult to predict a pronounced hump-shape of its expenditure share.

[32]The income elasticity of the expenditure share for the IA specification is $\frac{\partial \eta_{j,t}/\eta_{j,t}}{\partial e_t/e_t} = -\mathbf{A}_j(P_t)p_{j,t}\left(\frac{e_t}{\mathbf{B}(P_t)}\right)^{-1}\frac{1}{\eta_{j,t}} + \mathbf{D}_j(P_t)p_{j,t}\left[\mathbf{A}(P_t)\left(\frac{e_t}{\mathbf{B}_t}\right)^{-1} - \epsilon\right]\frac{\kappa}{\eta_{j,t}}\left(\frac{e_t}{\mathbf{B}(P_t)} - \mathbf{A}(P_t)\right)^{-\epsilon}$. The expressions for the nested standard preferences are easily obtained with the appropriate parameterizations

7 Conclusion

Structural transformation is a stylized fact of modern economic development, but the existing literature has struggled to provide a theory of consumer demand that can explain the observed reallocation across sectors and is also consistent with sustained growth in per capita expenditure. We contribute to this literature in three ways. First, we document the reallocation of consumption shares across the three broad sectors agriculture, manufacturing, and services using historical final consumption expenditure data for more than one century in the United States, the United Kingdom, Canada and Australia. The data allows us to analyze relative prices and consumption shares for a larger sample and a longer time period than existing studies.

Second, we analyze the features of the data that make it difficult for existing demand theories to match the observed patterns. Generalized Stone-Geary preferences, which are often used in the structural change literature, face the challenges that (i) the estimates imply an unreasonably high subsistence level in agriculture, (ii) the income effects are not sustainable in long samples, and (iii) it is difficult to generate the non-monotonic sectoral expenditure shares observed in the long-run data.

Our third contribution is to propose an intertemporally aggregable class of preferences that fits the historical data much better, allows for tractable aggregation, and is also consistent with sustained growth environments. IA preferences are a more fleixble form of the PIGL class which also allows for flexible and sustained income effects. Since the IA preferences nests the standard preferences, we can directly test the different parameterizations. We show that the flexble IA preferences provide a better fit for the historical data on consumption expenditures than existing theories when considering data for more than a century in the United States, the United Kingdom, Canada and Australia.

We believe that our findings are important in a number of ways. The existing literature has come to different conclusions regarding whether the generalized Stone-Geary preferences can fit the observed patterns of structural change. Our analysis of historical consumption expenditure data over more than 100 years and across four countries confirms the view that the standard preference specifications struggle to fit the long-run data. Our findings also have important implications for the external validity of structural transformation in the development process. The observation that the generalized Stone-Geary preferences imply subsistence levels in agriculture that are binding for (not unreasonably) low income levels limits the ability to apply it to contexts with large variation in incomes. We expect that IA preferences that avoid this problem will provide a basis for the analysis of structural change in a wide development context. We therefore plan to consider in future work a broader sample of countries.

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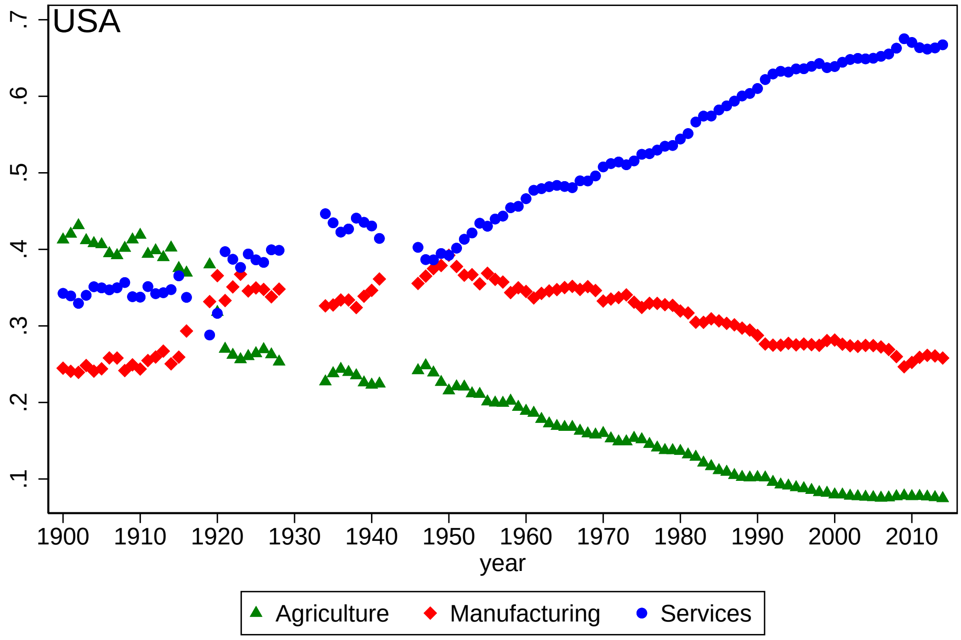
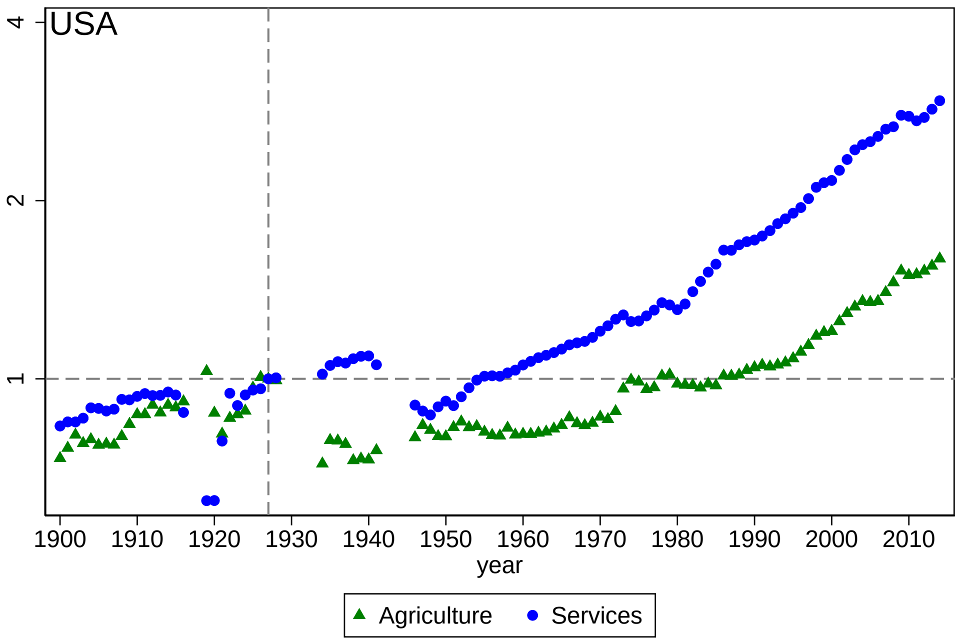
A Additional figures and tables



Notes: The figure plots the final private consumption expenditure shares of agriculture, manufacturing and services in the USA. The years affected by WWI, WWII, and the Great Depression are excluded. Sources: Bureau of Economic Analysis (BEA) and Carter et al. (2006) for the years pre-1929.

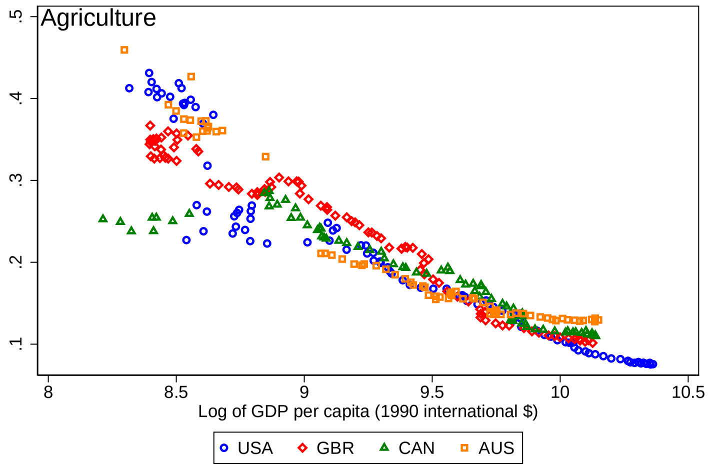
Figure 7: Final private consumption expenditure shares, USA



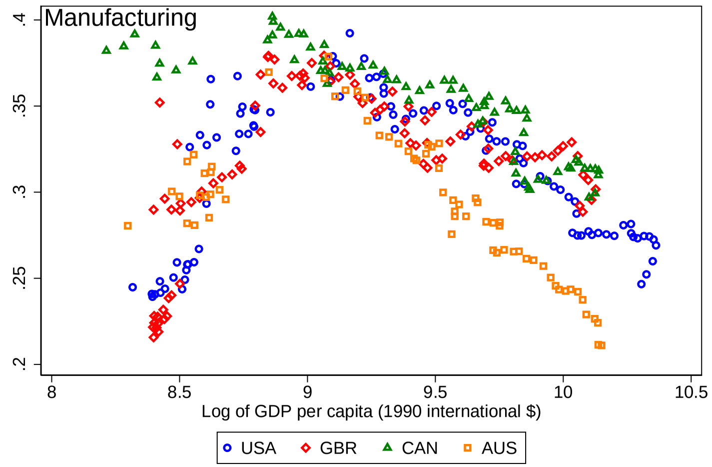
Notes: The figure plots the prices of agriculture and services relative to manufacturing in the USA. The prices are based on final private consumption expenditure and normalized to unity in 1927. The years affected by WWI, WWII, and the Great Depression are excluded. Sources: BEA and Carter et al. (2006) for years pre-1929.

Figure 8: Prices relative to manufacturing, USA

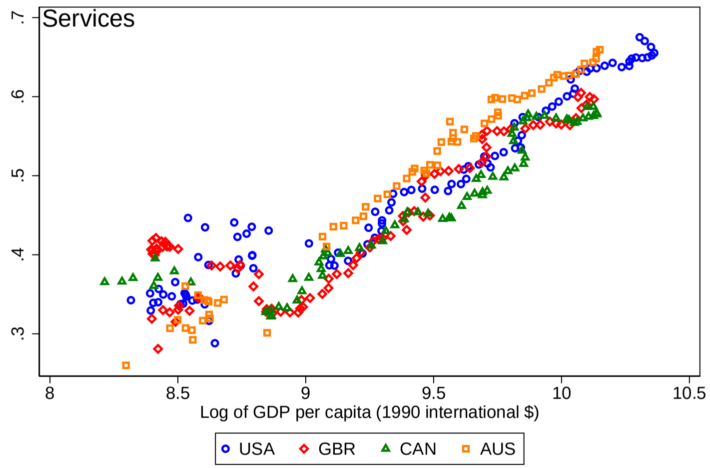
(a) Agriculture



(b) Manufacturing



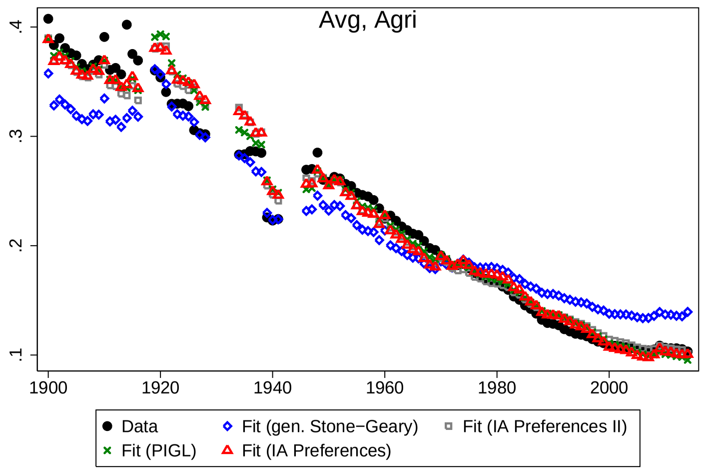
(c) Services



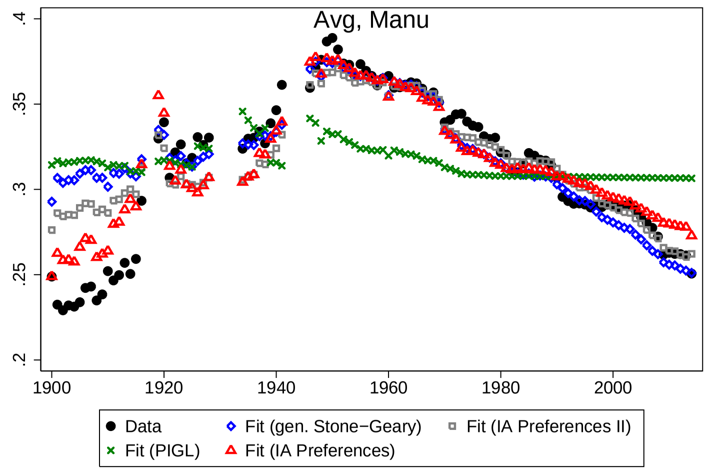
Notes: The figure plots the final private consumption expenditure share over log of GDP per capita (1990 international $) for the USA, GBR, CAN, and AUS by sector. The years affected by WWI, WWII, and the Great Depression are excluded. Source: Expenditure shares (see Appendix D), GDP per capita (Maddison Project, 2013).

Figure 9: Final private consumption expenditure shares

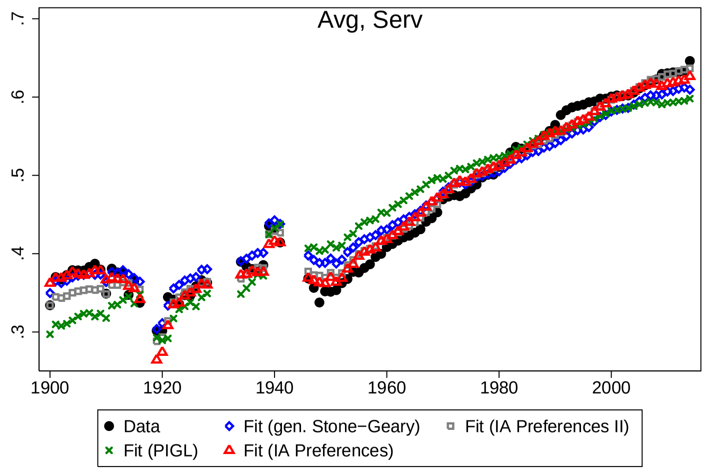
(a) Agriculture



(b) Manufacturing



(c) Services



Notes: The figure plots the average prediction and the average final private nominal consumption expenditure shares across the four countries by each sector. The blue diamonds show the fit of the generalized Stone-Geary specification estimated in column (1) of Table 1. The gray squares show the fit of the IA preference specification that nests generalized Stone-Geary, as estimated in column (4) of Table 1. The green crosses show the fit of the PIGL specification estimated in column (1) of Table 2. The red triangles show the fit of the IA preference specification estimated in column (4) of Table 2.

Figure 10: Predicted final private nominal consumption expenditure shares, Full Sample, Averages

	Generalized Stone-Geary			IA Preferences (36)		
	1900-2014	1929-2014	1947-2014	1900-2014	1929-2014	1947-2014
	(1)	(2)	(3)	(4)	(5)	(6)
σ	.434***	.632***	.665***	.425***	.503***	.548***
	(.029)	(.028)	(.037)	(.029)	(.062)	(.061)
$\bar{c}_A$	709.504	758.941	947.227	635.358***	575.158***	947.227
	(.)	(.)	(.)	(75.967)	(152.342)	(.)
$\bar{c}_S$	-822.076***	-1868.867***	-2479.691***	-.000	-6802.571**	-6067.269***
	(135.706)	(225.648)	(520.973)	(.)	(3013.138)	(886.264)
ω_A	.061***	.050***	.033***	.000	.000	.000
	(.002)	(.001)	(.001)	(.)	(.)	(.)
ω_M	.225***	.216***	.210***	.253***	.578***	.567***
	(.002)	(.003)	(.007)	(.010)	(.088)	(.101)
ω_S	.714***	.735***	.756***	.747***	.422***	.433***
	(.003)	(.004)	(.008)	(.010)	(.088)	(.101)
ϵ				.501***	.223**	.193***
				(.091)	(.088)	(.055)
$\overline{D}\nu_M$				-.871	-3.769	-2.641***
				(1.647)	(2.911)	(.801)
$\overline{D}\nu_S$				-6.363	3.369	2.472***
				(5.790)	(2.547)	(.712)
N	318	276	250	318	276	250
AIC	-2904	-2764	-2727	-3020	-2829	-2817
$\mathrm{E}[\hat{\eta}_{A,t} - \hat{\eta}_{A,t\vert\overline{D}=0}] = 0$				0.000	0.000	0.000
$\mathrm{E}[\hat{\eta}_{M,t} - \hat{\eta}_{M,t\vert\overline{D}=0}] = 0$				0.482	0.000	0.001
$\mathrm{E}[\hat{\eta}_{S,t} - \hat{\eta}_{S,t\vert\overline{D}=0}] = 0$				0.000	0.000	0.001

Table 3: Testing generalized Stone-Geary, Total consumption

Note: Years where the USA was involved in WWI (1917-1918), WWII (1942-1945), or affected by the Great Depression (1929-1933), and years where AUS, CAN, and GBR where involved in WWI (1914-1918), WWII (1939-1945), or affected by the Great Depression (1929-1933) are excluded from all estimations. All variables are based on final total consumption expenditure (incl. government consumption). Robust standard errors are reported in parenthesis. ***, **, * is significant at the 1%, 5%, and 10% level, respectively.

	PIGL			IA Preferences (37)		
	1900-2014	1929-2014	1947-2014	1900-2014	1929-2014	1947-2014
	(1)	(2)	(3)	(4)	(5)	(6)
ϵ	.563***	.542***	.727***	.558***	.339***	.385***
	(.033)	(.028)	(.030)	(.045)	(.035)	(.035)
ϕ_M	.168***	.117***	.135***	.245***	.060	.048
	(.015)	(.012)	(.012)	(.018)	(.047)	(.037)
ϕ_S	.832***	.883***	.865***	.755***	.940***	.941***
	(.015)	(.012)	(.012)	(.018)	(.047)	(.036)
$\overline{D}\nu_M$	15.533***	21.575***	105.473***	12.375***	6.903***	11.475***
	(2.593)	(4.353)	(22.321)	(2.763)	(1.118)	(3.882)
$\overline{D}\nu_S$	-37.672***	-40.099***	-212.538***	-28.708***	-8.487***	-13.094***
	(9.073)	(9.169)	(52.061)	(9.460)	(1.641)	(3.214)
ψ_M	-.210***	-.143***	-.131***	-.897***	-.385***	-.441***
	(.057)	(.027)	(.041)	(.179)	(.054)	(.048)
ψ_S	-.494***	-.359***	-.271***	-.614***	-.374***	-.335***
	(.033)	(.025)	(.021)	(.040)	(.039)	(.034)
$\overline{A}$				346.245***	563.683***	625.382***
				(79.236)	(65.261)	(153.905)
μ_M				-2.236***	-1.984***	-1.957***
				(.189)	(.066)	(.146)
μ_S				2.358***	1.959***	1.819***
				(.246)	(.055)	(.169)
N	318	276	250	318	276	250
AIC	-2913	-2910	-2810	-3066	-3193	-3071
$\mathrm{E}[\hat{\eta}_{A,t} - \hat{\eta}_{A,t\mid\overline{A}=0}] = 0$				0.000	0.000	0.000
$\mathrm{E}[\hat{\eta}_{M,t} - \hat{\eta}_{M,t\mid\overline{A}=0}] = 0$				0.067	0.000	0.000
$\mathrm{E}[\hat{\eta}_{S,t} - \hat{\eta}_{S,t\mid\overline{A}=0}] = 0$				0.000	0.000	0.000

Table 4: Testing PIGL, Total consumption

Note: Years where the USA was involved in WWI (1917-1918), WWII (1942-1945), or affected by the Great Depression (1929-1933), and years where AUS, CAN, and GBR where involved in WWI (1914-1918), WWII (1939-1945), or affected by the Great Depression (1929-1933) are excluded from all estimations. All variables are based on final total consumption expenditure (incl. government consumption). Robust standard errors are reported in parenthesis. ***, **, * is significant at the 1%, 5%, and 10% level, respectively.

	Private consumption			Total consumption		
	1900-2014	1929-2014	1947-2014	1900-2014	1929-2014	1947-2014
	(1)	(2)	(3)	(4)	(5)	(6)
σ	.132***	.576***	.687***	.498***	.678***	.849***
	(.023)	(.028)	(.040)	(.029)	(.027)	(.053)
$\bar{c}_A$	714.170	854.099	1019.629***	786.821	854.099	991.665***
	(.)	(.)	(18.957)	(.)	(.)	(22.044)
$\bar{c}_S$	-.000	-1210.932***	-3546.370***	-1059.533***	-2028.498***	-6618.772***
	(.)	(146.390)	(500.090)	(165.900)	(204.559)	(1556.217)
ω_A	.083***	.048***	.029***	.046***	.036***	.022***
	(.004)	(.002)	(.001)	(.002)	(.001)	(.001)
ω_M	.303***	.295***	.254***	.235***	.226***	.181***
	(.004)	(.004)	(.008)	(.003)	(.003)	(.012)
ω_S	.614***	.657***	.717***	.720***	.738***	.797***
	(.002)	(.005)	(.009)	(.003)	(.003)	(.013)
N	104	77	68	81	77	68
AIC	-954	-921	-949	-923	-987	-1009

Table 5: Estimation Generalized Stone-Geary, USA

Note: Years where the USA was involved in WWI (1917-1918), WWII (1942-1945), or affected by the Great Depression (1929-1933) are excluded from all estimations. Variables in columns (1) to (3) are based on final private consumption expenditure, variables in columns (4) to (6) are based on final total consumption (incl. government expenditure). Robust standard errors are reported in parenthesis. ***, **, * is significant at the 1%, 5%, and 10% level, respectively.

	PIGL			IA Preferences (37)		
	1900-2014	1929-2014	1947-2014	1900-2014	1929-2014	1947-2014
	(1)	(2)	(3)	(4)	(5)	(6)
ϵ	.712***	.584***	.629***	.569***	.834***	.564***
	(.027)	(.032)	(.019)	(.073)	(.036)	(.022)
ϕ_M	.310***	.225***	.156***	.019	.230***	.134***
	(.004)	(.012)	(.012)	(.102)	(.033)	(.020)
ϕ_S	.690***	.775***	.844***	.981***	.770***	.866***
	(.004)	(.012)	(.012)	(.102)	(.033)	(.020)
$\overline{D}\nu_M$	-.490	20.227***	57.855***	102.069**	554.860***	36.876***
	(1.233)	(6.645)	(7.388)	(47.895)	(105.487)	(4.519)
$\overline{D}\nu_S$	-95.789***	-48.861***	-101.653***	-107.137**	-947.275***	-60.120***
	(24.195)	(15.033)	(15.044)	(51.683)	(303.832)	(9.082)
ψ_M	-9.710	.013	-.017	-.077**	.095	-.136***
	(6.280)	(.079)	(.035)	(.037)	(.091)	(.036)
ψ_S	-.540***	-.474***	-.378***	-.293***	-.185***	-.375***
	(.082)	(.040)	(.024)	(.040)	(.061)	(.027)
$\overline{A}$				3094.630***	4137.558***	1.000
				(181.668)	(101.349)	(.)
μ_M				-.497***	-.168***	-8.384***
				(.085)	(.023)	(.407)
μ_S				1.111***	1.238***	6.097***
				(.055)	(.066)	(.950)
N	104	77	68	104	77	68
AIC	-821	-938	-992	-1049	-1127	-1010
$\mathrm{E}[\hat{\eta}_{A,t} - \hat{\eta}_{A,t\|\overline{A}=0}] = 0$				0.000	0.000	0.000
$\mathrm{E}[\hat{\eta}_{M,t} - \hat{\eta}_{M,t\|\overline{A}=0}] = 0$				0.000	0.000	0.000
$\mathrm{E}[\hat{\eta}_{S,t} - \hat{\eta}_{S,t\|\overline{A}=0}] = 0$				0.000	0.000	0.000

Table 6: Testing PIGL, USA

Note: Years where the USA was involved in WWI (1917-1918), WWII (1942-1945), or affected by the Great Depression (1929-1933) are excluded from all estimations. All variables are based on final private consumption expenditure. Robust standard errors are reported in parenthesis. ***, **, * is significant at the 1%, 5%, and 10% level, respectively.

B General Parameterization

B.1 General Parameterization of IA Preferences

We have already parameterized the function $\mathbf{B}(P)$ in (28). For $\mathbf{A}(P)$ and $\mathbf{D}(P)$ any zero homogenous function of prices is possible. A fairly general parameterization is achieved by defining the price functions as

$$\mathbf{A}(P) \equiv \begin{cases} \mathbf{B}(P)^{-1}\overline{A}\left(\sum_{j\in J}\mu_l p_j^{1-\theta}\right)^{\frac{1}{1-\theta}}, & \theta \in [0,1)\cup(1,\infty) \\ \mathbf{B}(P)^{-1}\overline{A}\prod_{j\in J} p_j^{\mu_j}, & \theta \to 1, \end{cases} \tag{39}$$

where $\sum_{j\in J}\mu_j = 1$, and

$$\mathbf{D}(P) \equiv \begin{cases} \overline{D}\kappa^{-1}\sum_{j\in J}\frac{\nu_j}{\psi_j}\left(\frac{p_j}{\mathbf{F}(P)}\right)^{\psi_j} - \frac{\nu_j}{\psi_j}, & \psi_j \in (-\infty,0)\cup(0,\infty), \\ \overline{D}\kappa^{-1}\sum_{j\in J}\nu_j \log\left(\frac{p_j}{\mathbf{F}(P)}\right), & \psi_j \to 0, \end{cases} \tag{40}$$

where $\sum_{j\in J}\nu_j = 1$ and the price function $\mathbf{F}(P)$ is linear homogeneous in the price vector. Note that the scalar κ^{-1} corresponds again to the constant inequality index defined in (22). It is straightforward to see that a subset of exact balanced growth compatible PIGL specifications is nested if $\overline{A} = 0$ (such that $\mathbf{A}(P) = 0$ independent of θ) and $\sigma \to 1$.[33] The generalized Stone-Geary is nested if $\overline{D} = 0$, $\theta = 0$, $\overline{A}\mu_j \equiv \bar{c}_j$, $\mu_M = 0$, and $\phi_j \equiv \omega_j > 0$. With the general parameterization of the price functions in (28), (39), and (40), the relevant additive terms that enter the

[33] In particular, Boppart (2014) studies a special case of the above specification with only two sectors, $J = \{1,2\}$, interpreted as goods and services using the parameterization $\overline{A} = 0$, $\sigma \to 1$, $\phi_1 = 1$, $\mathbf{F}(P) = p_1$, $\nu_1 = 0$, $\overline{D}\nu_2 = \nu$, and $\psi_2 = \gamma$.

aggregate expenditure shares of the IA preferences in (23) are then given by

$$\begin{aligned}
\mathbf{A}_j(P_t)p_{j,t} &= \mathbf{A}(P_t)\left[\frac{\mu_j p_{j,t}^{1-\theta}}{\sum_{l\in J}\mu_l p_{l,t}^{1-\theta}} - \frac{\mathbf{B}_j(P_t)p_{j,t}}{\mathbf{B}(P_t)}\right]. \\
\mathbf{B}_j(P_t)p_{j,t} &= \frac{\phi_j p_{j,t}^{1-\sigma}}{\mathbf{B}(P_t)^{1-\sigma}} \\
\mathbf{D}_j(P_t)p_{j,t} &= \overline{D}\left[\nu_j\left(\frac{p_{j,t}}{\mathbf{F}(P_t)}\right)^{\psi_j} - \frac{\mathbf{F}_j(P_t)p_{j,t}}{\mathbf{F}(P_t)}\sum_{l\in J}\nu_l\left(\frac{p_{l,t}}{\mathbf{F}(P_t)}\right)^{\psi_l}\right].
\end{aligned}$$

C Proofs

C.1 Proof of Proposition 2

Since $v(e, P)$ is homogenous of degree 0, then $v_e(e, P)$ must be homogenous of degree -1, $v(e, P) = v(\lambda e, \lambda P) \Leftrightarrow \lambda^{-1} v_e(e, P) = v_e(\lambda e, \lambda P)$. Thus, the individual Euler equation can be expressed as

$$v_e(e_{i,t}, P_t) = \beta(1 + r_{t+1}) v_e(e_{i,t+1}, P_{t+1}) = v_e(e_{i,t+1} x_{t+1}, P_{t+1} x_{t+1}), \tag{41}$$

where $x_{t+1} \equiv [\beta(1 + r_{t+1})]^{-1}$. Equation (41) is equivalent to Equation (18) stated in the Proposition if and only of there exists a monotone transformation $g(\cdot)$ such that both sides of the Euler equation become linear functions of $e_{i,t}$ and $e_{i,t+1}$, respectively:[34]

$$\begin{aligned} & g(v_e(e_{i,t}, P_t)) = g(v_e(e_{i,t+1} x_{t+1}, P_{t+1} x_{t+1})) \\ \Leftrightarrow \quad & \hat{e}_{i,t} \equiv \frac{e_{i,t}}{\mathcal{B}(P_t)} - \mathcal{A}(P_t) = \frac{e_{i,t+1} x_{t+1}}{\mathcal{B}(P_{t+1} x_{t+1})} - \mathcal{A}(P_{t+1} x_{t+1}) \equiv \hat{e}_{i,t+1}, \end{aligned} \tag{42}$$

where $\mathcal{B}(P)$ and $\mathcal{A}(P)$ are any functions of prices. Aggregating (42) over all individuals $i \in [0, N]$ and dividing by N then yields:

$$\hat{e}_t \equiv \frac{(E_t/N)}{\mathcal{B}(P_t)} - \mathcal{A}(P_t) = \frac{(E_{t+1}/N) x_{t+1}}{\mathcal{B}(P_{t+1} x_{t+1})} - \mathcal{A}(P_{t+1} x_{t+1}) \equiv \hat{e}_{t+1}. \tag{43}$$

It is easy to verify that Equation (18) holds when applying the inverse transformation $g^{-1}(\cdot)$ to both sides of (43). Note that Equations (42) and (43) imply that $\hat{e}_{i,t} \equiv \hat{e}_i$ $\forall i$ and $\hat{e}_t \equiv \hat{e}$ remain constant over time.

Next, define $\lambda_i \equiv \hat{e}_j / \hat{e}_i \, \forall i$. Without loss of generality, we can write the marginal

[34] It is easy to verify that the indirect utility function stated in the proposition yields the transformations $g(v) = v^{1/(\epsilon-1)}$. Since P is a vector and x is a scalar, the right-hand side of (42) can be written as

$$\begin{aligned} x^{-1/(\epsilon-1)} \left(\frac{e}{\mathbf{B}(P)} - \mathbf{A}(P) \right) \mathbf{B}(P)^{-1/(\epsilon-1)} &= \frac{ex}{\mathbf{B}(Px)\mathbf{B}(Px)^{1/(\epsilon-1)}} - \mathbf{A}(Px)\mathbf{B}(Px)^{-1/(\epsilon-1)} \\ &\equiv \frac{ex}{\mathcal{B}(Px)} - \mathcal{A}(Px), \end{aligned}$$

since $\mathbf{A}(P)$ is homogenous of degree 0 and $\mathbf{B}(P)$ homogenous of degree 1.

indirect utility function as a function of of $\hat{e}_j$ instead of e_j for all $j \in [0, N]$.

$$v(e_j, P) \equiv f(\hat{e}_j, P) \quad \Leftrightarrow \quad v_e(e_j, P) = f_{\hat{e}}(\hat{e}_j, P)\mathcal{B}(P)^{-1}. \tag{44}$$

Equations (41) and (44) then imply

$$\frac{f_{\hat{e}}(\lambda_i \hat{e}_i, P_t)}{f_{\hat{e}}(\lambda_i \hat{e}_i, P_{t+1})} = \frac{\mathcal{B}(P_t)}{\mathcal{B}(P_{t+1})x_{t+1}} \quad \forall \lambda_i, P_t, P_{t+1}, x_{t+1}. \tag{45}$$

Take logs on both sides, differentiate with respect to λ_i, and multiply by $-\lambda_i$ to yield

$$\begin{aligned} -\frac{f_{\hat{e}\hat{e}}(\lambda_i \hat{e}_i, P_t)\lambda_i \hat{e}_i}{f_{\hat{e}}(\lambda_i \hat{e}_i, P_t)} &= -\frac{f_{\hat{e}\hat{e}}(\lambda_i \hat{e}_i, P_{t+1})\lambda_i \hat{e}_i}{f_{\hat{e}}(\lambda_i \hat{e}_i, P_{t+1})} \\ &\equiv h(\hat{e})\hat{e}, \quad \forall \lambda^i, P_t, P_{t+1}, \end{aligned} \tag{46}$$

where $h(\hat{e}) \neq 0$ denotes the absolute risk aversion (ARA) as will become clear below. Next consider λ_i such that $\lambda_i \hat{e}_i = \hat{e}$. Multiply both sides of Equation (46) by $\hat{e}^{-1}$ to yield

$$-\frac{f_{\hat{e}\hat{e}}(\hat{e}, P_t)}{f_{\hat{e}}(\hat{e}, P_t)} = -\frac{f_{\hat{e}\hat{e}}(\hat{e}, P_{t+1})}{f_{\hat{e}}(\hat{e}, P_{t+1})} = h(\hat{e}), \quad \forall P_t, P_{t+1}. \tag{47}$$

The last equality implies that

$$\begin{aligned} v_e(e, P) &= f_{\hat{e}}(\hat{e}, P)\mathcal{B}(P)^{-1} \\ &= \mathcal{G}(P) \exp\left(\int h(\hat{e}) d\hat{e} \right) \mathcal{B}(P)^{-1}, \end{aligned}$$

where the function $\mathcal{G}(P)$ will be determined below. Thus, the aggregation condition $g(v_e(e, P)) = \hat{e}$ is satisfied if and only if $h(\hat{e})$ is either a constant or hyperbolic in $\hat{e}$ such that $\int h(\hat{e}) d\hat{e}$ is of the linear or logarithmic form, respectively. Henceforth, we parameterize the function h with the standard hyperbolic absolute risk aversion (HARA) form $h(\hat{e}) = (\hat{e}/(1-\epsilon) + b/a)^{-1}$, where the parameters ϵ, a, and b will be determined below. It then follows immediately that f must be of the HARA form

$$f(\hat{e},P) = v(e,P) = \mathcal{G}(P)\frac{1-\epsilon}{\epsilon}\left(\frac{a\hat{e}}{1-\epsilon}+b\right)^{\epsilon} - \frac{1}{\epsilon} - \mathcal{D}(P)$$
$$= \mathcal{G}(P)\frac{1-\epsilon}{\epsilon}\left(\frac{a}{1-\epsilon}\left(\frac{e}{\mathcal{B}(P)} - \mathcal{A}(P)\right)+b\right)^{\epsilon} - \frac{1}{\epsilon} - \mathcal{D}(P), \quad (48)$$

where $\epsilon \neq 0$. Note that Equation (48) includes the following limit case when $b = 1$:

$$\lim_{\epsilon\to-\infty} v(e,P) = -\mathcal{G}(P)\exp\left(-a\left(\frac{e}{\mathcal{B}(P)} - \mathcal{A}(P)\right)\right) - \mathcal{D}(P).$$

Finally, let us determine the unknown function $\mathcal{G}(P)$ and the parameters a and b. When $\epsilon > -\infty$, derive $v_e(e,P)$ from (48) and apply the transformation $g(v) = v^{1/(\epsilon-1)}$ to yield

$$\begin{aligned} g\left(v_e(e,P)\right) &= \mathcal{G}(P)^{1/(\epsilon-1)}\left(\frac{a}{1-\epsilon}\left(\frac{e}{\mathcal{B}(P)} - \mathcal{A}(P)\right)+b\right)\left[a\mathcal{B}(P)^{-1}\right]^{1/(\epsilon-1)} \\ &= \frac{e}{\mathcal{B}(P)a^{-1}(1-\epsilon)\left[a\mathcal{B}(P)^{-1}\mathcal{G}(P)\right]^{-1/(\epsilon-1)}} \\ &\quad - \left(\mathcal{A}(P)-b\right)a(1-\epsilon)^{-1}\left[a\mathcal{B}(P)^{-1}\mathcal{G}(P)\right]^{1/(\epsilon-1)} \\ &\equiv \frac{e}{\mathcal{B}(P)} - \mathcal{A}(P). \end{aligned}$$

The first term determines $\mathcal{G}(P)$

$$a^{-1}(1-\epsilon)\left[a\mathcal{B}(P)^{-1}\mathcal{G}(P)\right]^{-1/(\epsilon-1)} = 1 \quad \Leftrightarrow \quad \mathcal{G}(P) = a^{-\epsilon}(1-\epsilon)^{\epsilon-1}\mathcal{B}(P).$$

The second term than determines that $b = 0$ since $\mathcal{A}(P) = \mathcal{A}(P) - b$. On the other hand, when $\epsilon \to -\infty$ and $b = 1$, we apply the transformation $g(v) = \log(v)$ to yield

$$\begin{aligned} g(v_e(e,P)) &= \log\left(\mathcal{G}(P)\right) - a\left(\frac{e}{\mathcal{B}(P)} - \mathcal{A}(P)\right) + \log\left(a\mathcal{B}(P)^{-1}\right) \\ &= \frac{e}{(-1)a^{-1}\mathcal{B}(P)} - \left(\mathcal{A}(P)(-1)a - \log\left(a\mathcal{B}(P)^{-1}\mathcal{G}(P)\right)\right) \\ &\equiv \frac{e}{\mathcal{B}(P)} - \mathcal{A}(P). \end{aligned}$$

The first term determines that $a = -1$. The second term then implies that

$$\log\left(-\mathcal{B}(P)^{-1}\mathcal{G}(P)\right) = 0 \quad \Leftrightarrow \quad \mathcal{G}(P) = -\mathcal{B}(P).$$

Thus, the undetermined HARA form derived in (48) can be restricted to

$$\begin{aligned} v(e,P) &= \frac{1}{\epsilon}\left(\frac{1-\epsilon}{a}\right)^{\epsilon}\left(\frac{a}{1-\epsilon}\left(\frac{e}{\mathcal{B}(P)\mathcal{B}(P)^{-1/\epsilon}} - \mathcal{A}(P)\mathcal{B}(P)^{1/\epsilon}\right)\right)^{\epsilon} - \frac{1}{\epsilon} + \mathcal{D}(P) \\ &\equiv \frac{1}{\epsilon}\left(\frac{e}{\mathbf{B}(P)} - \mathbf{A}(P)\right)^{\epsilon} - \frac{1}{\epsilon} - \mathbf{D}(P), \quad \epsilon \in (-\infty, 0) \cup (0, \infty) \end{aligned} \tag{49}$$

and

$$v(e,P) = \mathcal{B}(P)\exp\left(\frac{e}{\mathcal{B}(P)} - \mathcal{A}(P)\right) - \mathcal{D}(P), \quad \epsilon \to -\infty,\, b = 1 \tag{50}$$

The limit case can be disregarded, because there exists no function $\mathcal{B}(P)$ such that (50) is homogenous of degree zero in e and P. Finally, (49) satisfies the properties stated in the proposition if $\epsilon < 1$ (such that $v_{ee}(e,P) < 0$), $\mathbf{B}(P)$ is linear homogenous, and $\mathbf{A}(P)$, $\mathbf{D}(P)$ are zero homogenous in the price vector P. Finally, note that (49) includes the limit case

$$\lim_{\epsilon \to 0} v(e,P) = \log\left(\frac{e}{\mathbf{B}(P)} - \mathbf{A}(P)\right) - \mathbf{D}(P).$$

This concludes the proof of the proposition.

D Data description

D.1 Sources

D.1.1 United States

Household and government consumption expenditures, 1929-2014

We construct the broad consumption expenditure categories from the historical National Income and Product Account (NIPA) tables provided by the Bureau of Economic Analysis (BEA). The series on annual real and nominal private consumption expenditures are taken from:

- Table 2.4.3. Real Personal Consumption Expenditures by Type of Product, Quantity Indexes
- Table 2.4.5. Personal Consumption Expenditures by Type of Product

The corresponding government consumption expenditures are taken from:

- Table 3.10.3. Real Government Consumption Expenditures and General Government Gross Output, Quantity Indexes
- Table 3.10.5. Government Consumption Expenditures and General Government Gross Output

We use the same categorization of the broad sectors as in Herrendorf et al. (2013) by aggregating the following subcategories of the NIPA tables (which are themselves classified in even finer categories):

- Agriculture: Food and beverages purchased for off-premise consumption.
- Manufacturing: Durable goods, Clothing and footwear, Gasoline and other energy goods, other nondurable goods.
- Services: Services, Government consumption expenditures.

Household and government consumption expenditures 1900-1929

For periods earlier than 1929, we use the historical consumption expenditure data from Carter et al. (2006). The series on annual real and nominal private consumption expenditures are taken from:

- Tables Cd78-152. Consumption expenditures, by type: 1900-1929 [1987 constant $].
- Tables Cd1-77. Consumption expenditures, by type: 1900-1929.

The corresponding nominal government consumption expenditures are taken from:

- Tables Ea24-51. Total government revenue, by source: 1902-1995.
- Tables Ea52-60. Total government expenditure, by character and object: 1902-1995.

Since the subcategories available for earlier periods from Carter et al. (2006) are not as detailed as the more recent NIPA tables, we list the exact categorization of the broad categories as shown below (column codes from Carter et al. (2006) in brackets):

- Agriculture: Purchased food and meals without alcohol (Cd3), Food to employees (Cd4), Food consumed on farms (Cd5), Alcohol (Cd6)
- Manufacturing: Tobacco (Cd7), Shoes (Cd9), Civilian clothing (Cd10), Military clothing (Cd13), Jewelry (Cd14), Toilet articles (Cd17), Furniture and mattresses (Cd25), Kitchen appliances (Cd26), China (Cd27), Furnishings, other durables (Cd28), Furnishings, semidurables (Cd29), Cleaners and polishes (Cd30), Stationery (Cd31), Wood, gas, and coal (Cd35), Drugs (Cd40), Motor vehicles and wagons (Cd53), Tires and accessories (Cd54), Gasoline and oil (Cd56), Books and maps (Cd61), Magazines and newspapers (Cd62), Nondurable toys (Cd63, Durable toys and wheel goods (Cd64), Music, radio, and television (Cd65), Flowers and plants (Cd66)

- Services: Clothing services (Cd15), Barber and beauty (Cd18), Owner occupied housing (Cd20), Tenant occupied housing (Cd21), Rent of farmhouse (Cd22), Other housing (Cd23), Electricity (Cd32), Gas (Cd33), Water (Cd34), Telephone and telegraph (Cd36)[35], Domestic services (Cd37), Other household operations (Cd38), Ophthalmology (Cd41), Medical, dental, and other professional services (Cd42), Hospitals (Cd43), Health insurance (Cd44), Brokerage (Cd46), Banking and financial services (Cd47), Life insurance (Cd48), Legal services (Cd49), Funeral (Cd50), Other personal business (Cd51), Automobile repair (Cd55), Auto insurance and tolls (Cd57), Local purchased transportation (Cd58), Intercity purchased transportation (Cd59), Recreational services (Cd67), Higher education (Cd69), Elementary education (Cd70), Other education and research (Cd71), Religion (Cd73), Welfare (Cd74), Net foreign travel (Cd75)

The composition of the broad categories is chosen such that in the overlapping year 1929 the sectoral expenditure shares derived from the historical data from Carter et al. (2006) and the NIPA tables are consistent. For private consumption expenditures, we obtain a complete price and nominal expenditure series for agriculture, manufacturing, and services with yearly observations from 1900 to 1929.[36] For total consumption (including government consumption), we obtain four additional observations (1902, 1913, 1922, 1927) before 1929.

To construct government consumption expenditure, we subtract from the current operation series (obtained from the table by character and object) the unemployment benefits and social security expenditures (obtained from the table by source) in order to make it consistent with the definition of government consumption reported in the NIPA tables of the BEA. Nominal government consumption expenditure is only available at roughly ten year intervals (1902, 1913, 1922, 1927) before 1929 and price data is missing over the entire historical period. Thus, we are using the

[35]We classify this as services but this category could be ambiguous. However, it is a small share of total expenditures (less than 1% in both 1900 and 1929).

[36]Whenever possible, we take the finest categories reported in the tables. We then calculate the Fisher aggregates based on these categories. For civilian clothing, the data in constant prices is only available for the total clothing, while the nominal data is reported separately for women and men civilian clothing. We therefore rely on total civilian clothing to calculate prices and Fisher aggregates.

same deflator for government expenditure as for private services to approximate the historical price index for the total service sector including government services.[37]

Population

The population size from 1929 onward is taken from the NIPA Table 7.1: "Selected Per Capita Product and Income Series in Current and Chained Dollars." For earlier years, we obtain the population size from table Aa6-8. "Population: 1790-2000 [Annual estimates]" in Carter et al. (2006).

D.1.2 United Kingdom

Household and government consumption expenditures

We obtain the annual nominal and real household consumption expenditure data from three sources:

1995-2013 We obtain the consumption expenditure series from EuroStat (2015), Series nama_co3_c (Final consumption expenditure of households by consumption purpose - COICOP 3 digit - aggregates at current prices), Series nama_co3_k (Final consumption expenditure of households by consumption purpose - COICOP 3 digit - volumes).

We classify consumption expenditures as follows:

- Agriculture: Food, Non-alcoholic beverages, Alcoholic beverages
- Manufacturing: Tobacco, Clothing, Footwear, Furniture & carpets, Household textiles, Household appliances, Glass & table utensils, Tools & equipment, Goods and services for maintenance[38], Medical products & appliances, Vehicle purchases, Telecommunication equipment, Audio and visual equipment, Durable recreational goods, Newspapers and books, Personal care, Personal effects.

[37] When we consider the same approximation of the price for total services at later points in time where actual price data is available, we find that the difference between the approximation and the correct Fisher price index are very small. This suggests that the approximation works well.

[38] This category is ambiguous and we classify it as manufacturing.

- Services: Actual rent, imputed rent, house maintenance & repair, Water & miscellaneous services, Electricity & gas & fuel, Outpatient services, Hospital services, Transport operation, Transport services, Postal services, Telecommunication services, Recreational services, Holidays, Primary education, Secondary Education, Post secondary education, Tertiary education, Undefined education, Catering services, Accommodation services, Prostitution, Social protection, Insurance, Financial services, Other services.

1963-2011 For this earlier period, we obtain the consumption expenditure series from Office for National Statistics (2015b).[39]

For the period from 1966 to 1995 the Office for National Statistics (2015b) only provides a real consumption index for the broad categories goods, services, and aggregate private household consumption. There is no separate price index for agriculture and manufacturing. However, using a historical food price index from the retail price index for food (series CCYY) in Office for National Statistics (2015a) as the price for agriculture, we can solve for the real quantity sequence of manufacturing that - after aggregating up the three sectoral quantities (agriculture, manufacturing, and services) according to the Fisher procedure stated in Equation (51) of Appendix D - is consistent with the real quantity index of aggregate private consumption in the Office for National Statistics (2015b) data.

1900-1965 For the period going back to 1900, we obtain the consumption expenditure series from Feinstein (1972), Table 24 (current prices, 1900-65) and Table 25 (constant prices, 1900-65).

We classify the broad sectors agriculture (with and without alcohol), manufacturing, and services as follows:[40]

[39]The Office for national Statistics published this data as “ad hoc data and analysis” on August 21, 2015.

[40]For some of the periods, the categories are further aggregated. In particular for the period 1957 - 1962, the following manufacturing goods are only available as a total for the nominal expenditures: furniture, floor coverings, electrical, household textiles, hardware, matches, cleaning materials, books, miscellaneous recreational goods, chemists’ and other goods. For the same period, the following services are only available as a total: domestic service, catering, other services. There are a few further instances that involve fewer categories, but in all cases the classification into the three broad sectors is unambiguous. The real consumption expenditures have a similar feature and in some cases slightly different sub-categories than the nominal expenditures, in which case

- Agriculture: food and alcohol.

- Manufacturing: tobacco, clothing, motor cars & motor cycles, furniture & floor coverings & electrical, household textiles & hardware, matches & cleaning material, books & miscellaneous recreational goods, chemists' & other goods.

- Services: housing, fuel & light, public travel & communication, vehicle running costs, domestic services, catering (meals & accommodation), other services, and government consumption.

Government consumption Annual real and nominal government consumption for the period 1830 to 2009 is available from the Bank of England (2015) and for the more recent period 1948 to 2014 from the Office for National Statistics (2015c).[41]

Population

The population size is obtained from Office for National Statistics (2014) where data is provided from 1851 onwards.

D.1.3 Canada

Household nominal consumption expenditures, 1926-1986

We obtain the detailed subcategories of nominal household consumption expenditures for the years 1926 to 1986 from Table 380-0565. "Personal expenditure on consumer goods and services, 1968 System of National Accounts (SNA), annual (dollars)" provided by Statistics Canada (1988). The categorization is as follows:

- Agriculture: Food & non-alcoholic beverages, Alcoholic beverages

- Manufacturing: Tobacco, Mens & boys clothing, Women & children clothing, Footwear, Furniture & carpets, Household appliances, Semi-durable household furnishings, Non-durable household furnishings, Drugs, Other health goods, Vehicle purchases, Motor vehicle repair parts, Motor vehicle fuel, Other fuels,

we aggregated the nominal expenditures to match the categories in the real expenditures.

[41]The relevant ONS Series are NMRP (nominal values) and NMRY (real quantities).

Reading & entertainment supplies, Jewelry & watch, Toilet articles & cosmetics, Personal care

- Services: Rent by tenant, Rent imputed, Other occupant charges, Electricity, Gas, Laundry & dry cleaning, Domestic child care, Other health services, Physician services, Hospital care, Other automobile services, Purchase of transportation, Communication, Recreational equipment & services[42], Education & cultural services, Restaurants & hotels, Financial & legal services.

Household real consumption expenditures, 1926-1986

We obtain the nominal and real consumption expenditures of durable goods, semi-durable goods, non-durable goods, and services for the period 1926-1986 from Statistics Canada (2016c). We can calculate the prices of services directly from this series for the entire period. Agricultural goods are not separately listed and are part of non-durable goods. Similar to the case of the United Kingdom, we compute real quantities based on the nominal expenditures and the consumer price index (CPI) for food that we obtain from Statistics Canada (2014, 2016a). We then separate real quantities of manufacturing goods from the non-durable goods with a root finding procedure to find the missing sectoral quantities and prices that make the aggregate real consumption consistent with the index implied by the Fisher aggregation.

Household consumption expenditures, 1981-2014

We obtain the series for annual nominal and real household consumption expenditures for the years 1981-2014 from Statistics Canada (2016f). The classification in the three broad categories is as follows:

- Agriculture: Food & non-alcoholic beverages, Alcohol
- Manufacturing: Tobacco, Clothing, Garment, Footwear, Maintenance & repair goods, Furniture, Carpets, Textiles, Major household appliances, Small household appliances, Major tools & equipment, Small tools & miscellaneous accessories, Other semi-durable household goods, Other non-durable household

[42]This category is ambiguous and we classify it as services.

goods, Pharmaceutical & medical products, Medical products & equipment, Car purchases, Truck & van purchases, Fuel, Telecommunication equipment, Audio and visual equipment, Information processing equipment, Recording & media, Major recreational goods, Indoor recreational goods, Games & toys, Sport and camping equipment, Garden products, Pet & pet food products, Books, Newspaper, Miscellaneous print, Personal goods, Other personal goods, Jewelry & clocks & watches, Other personal effects.

- Services: Clothing services, Actual house rental, imputed house rental, Household maintenance & repair services, Water, Electricity, Gas, Repair of personal and household goods, Rent of household goods, Other property services, Outpatient services, Hospital services, Vehicle maintenance & repair, Parking, Passenger vehicle rent, Transport operation, Bus transport, Taxi & limo transport, Air transport, Rail & water transport, Other transport services, Postal services, Telephone & fax services, Veterinary services, Recreational & sport services, Cable & satellite services, Cinema, Photo services, Other cultural services, Chance & games, University education, Other education, Food & beverages services, Liquor services, Accommodation services, Insurance & financial services, Life insurance, Property insurance, Health insurance, Transport insurance, Personal services, Childcare outside services, Childcare inside services, Other social & funeral services, Legal & other services.

Government consumption

We obtain annual nominal and real government consumption for the years 1926-1986 from Statistics Canada (2016c) and for the years 1981-2015 from Statistics Canada (2016d).

Population data We get the population data until 1977 from Statistics Canada (2016e) and the more recent series are obtained from Statistics Canada (2016b).

D.1.4 Australia

D.1.5 Household consumption expenditures, 1900-1948

The nominal household consumption expenditures for the period 1900-1948 (with some gaps) is obtained from Haig and Anderssen (2006). For some subcategories we have missing observations in 1900 and 1910 that we need to approximate based on more aggregated series. In particular, we assume that car purchases in 1900 and 1910 were the same proportion of total expenditures on travel as in 1920. We apply the same strategy for private transport services. Furthermore, we calculate public transport services prior to 1920 by subtracting private transport services and vehicle purchases from total transport expenditures. The real household consumption expenditures are available from the same source.[43]

We calculate prices for each category that is available in the real and nominal series. For some categories such as private transport services we needed to calculate the nominal expenditures by subtracting subcategories from aggregates. In order to find the real quantities and prices for these subcategories, we apply a root finding procedure that inverses the Fisher aggregation as describe on earlier occasions for GBR and CAN.

We then construct aggregate quantities and prices for the three broad sectors using Fisher aggregation with the following classification:

- Agriculture: Food and non-alcoholic beverages, Alcoholic beverages
- Manufacturing: tobacco, clothing & footwear, durable goods, toys & sports goods, newspapers & books, electricity & gas & fuel[44], other goods
- Services: Rent, communication, entertainment, education, health, other services.

Since some of the subcategories like private transport had to be constructed for the early years, we need to make some assumptions in order to link the series for

[43]We corrected some obvious typos in the real data. For example, the columns for health and education were incorrectly labeled and some decimal places were shifted.

[44]This category is ambiguous because it is partly manufacturing (fuel) and partly services (electricity and gas). We allocate it to manufacturing because data from other countries suggest that fuel is the larger part. For example, in the US in 1929, expenditures on gasoline and oil were $1,814 and expenditures on electricity and gas were $1,158 (in million USD), see Carter et al. (2006).

prices and quantities over time. In particular, we use the growth rates in prices and quantities in manufacturing and services without transport in order to link the series before and after 1920.

In order to link the series prior to 1948 to the new series starting in 1970 (discussed below), we exploit the fact that the earlier series reported both nominal and real data for most major categories for the year 1948 in 1938 and 1979 prices.[45] We rely on these categories as proxies for the price and quantity growth of each broad category spanning across both time periods. We then use these proxies to link the more detailed time series for prices and quantities with a common base year 1979.

D.1.6 Household consumption expenditures, 1970-2015

We obtain household final consumption expenditure for the period 1970 to 2015 from Table 42 of the Australian System of National Accounts (Australian Bureau of Statistics (2015)).[46] The table includes expenditures in current and constant prices for 27 expenditure categories and we calculate prices for each category. The quantities for recreational goods, recreational services, and books are missing for the years prior to 1986, but we have the total expenditures for recreation. We therefore assume that price growth in each of the missing subcategories is the same as in total recreation.[47]

We then use Fisher aggregation in order to compute quantity and price growth for the broad sectors agriculture (with and without alcohol), manufacturing, and services. The subcategories are classified as follows:

- Agriculture: food and alcohol.

[45] The real quantities of the following subcategories are observed in 1948 in both 1938 and 1979 prices: food, clothing, rent, tobacco, liquor, durable goods, fuel & light, public transport, private transport, communication. We assign these subcategories to the three broad sectors as defined above and use Fisher aggregation to obtain price and quantity growth rates. It should be noted that the two observations for each real series in 1948 are not only showing different price base years, but also different currencies (Pounds and Australian Dollars). The exchange rate is obtained from the nominal series which show the value in 1948 in each currency.

[46] The table goes back to 1960, but a number of sub-categories are missing.

[47] This appears the best available approximation, but it should be noted that some sub categories combine services and other goods.

- Manufacturing: tobacco, clothing, furnishings and household equipment, vehicle purchases, goods for recreation and culture, books, papers, stationery and artists goods, electricity & gas & fuel[48].
- Services: rent, health, vehicle operation, transport services, communication, recreational services, education services, hotel & restaurant services, insurance & financial services, and government consumption.

Quantities for insurance and financial services are not available for the years prior to 1980 and we therefore approximate their growth for the early years based on the remaining services. The series from 1900 - 1948 and 1970 - 2015 are linked together as described above, with a common base year 1979.

Government consumption

The nominal and real series for government consumption for the years 1960 - 2014 are obtained from the World Bank Development Indicators (World Bank (2016)). The nominal and real series for government consumption for the years 1900 - 1970 are obtained from tables 22 and 23 of the Source Papers in Economic History Australian National University (1985). We use Fisher aggregation in order to combine government consumption with private household consumption.

Population

The population statistics from 1900 to 2010 are obtained from the Australian Historical Population Statistics (Australian Bureau of Statistics (2014)). The population statistics for the years 1981 to 2015 are available from the Australian Demographic Statistics (Australian Bureau of Statistics (2016)). We use these population series to obtain our per capita measures.

D.2 Price aggregation

We collect data on prices (or, real quantities) and nominal expenditure for the finest subcategories available in each country and time period. By dividing the

[48]This category is ambiguous and is assigned to manufacturing. See footnote 44 for details.

the nominal expenditure for each subcategory by the price (quantity) index we obtain the associated quantity (price) index. The real quantities are aggregated from the subcategories to the sectoral level according to a chained Fisher index. Fisher aggregation is also the standard procedure for the most recent NIPA tables provided by the BEA) in the USA (see Whelan (2002)). More formally, let the total real consumption index at the sectoral level (for example, manufacturing) be denoted by $c_{j,t}$. The real growth rate of the index satisfies the equation

$$\frac{c_{j,t}}{c_{j,t-1}} = \sqrt{\frac{\sum_{s=1}^{S} p_{s,t} c_{s,t}}{\sum_{s=1}^{S} p_{s,t} x_{s,t-1}} \times \frac{\sum_{s=1}^{S} p_{s,t-1} x_{s,t}}{\sum_{s=1}^{S} p_{s,t-1} x_{s,t-1}}}, \tag{51}$$

where $p_{s,t}$ is the price of subcategory $s = 1, \ldots, S$ (for example, clothing and footwear) and $c_{s,t}$ denotes the associated quantity index such that nominal expenditure on subcategory s corresponds to the product, $p_{s,t}c_{s,t}$. After choosing a base year for the sectoral quantity index $c_{j,t}$ we use the the growth rate implied by Equation (51) to chain the index backward and forward. The sectoral Fisher price index is then simply given by the $p_{j,t}$ such that the product $p_{j,t}c_{j,t}$ corresponds to the total nominal expenditure for sector j in period t.

The aggregation procedure in Equation (51) can also be applied to a more aggregate level. In particular, subcategories s can be substituted by sectors i and the left-hand side by aggregate real consumption growth. This is useful whenever we cannot directly aggregate the price indexes of subcategories to the sectoral level because of missing data. In these cases, however, when we have available an aggregate consumption price index and the other two sectoral price indexes, we are using a root-finding procedure in combination with a more aggregate version of Equation (51 to solve for the unknown sectoral price index. Or, in other words, we are solving for the unknown sectoral price index which makes the aggregate consumption price index consistent with the Fisher index that results from aggregating up the sectoral prices. This is the case for GBR in the period from 1966 to 1995, and for CAN between 1926 and 1986. The data availability of nominal expenditures for the subcategories on the other hand is always complete for the considered time periods and countries.

D.3 Purchasing power parity adjustment

We adjust the sectoral price and quantity indexes measured in local currency units for the power purchasing parity (PPP) relative to the USA for each of the other countries. We follow the Maddison Project (2013) and choose 1990 as the base year. Formally, we set the PPP-adjusted price index in 1990 for the USA to unity and for the remaining countries the price in the same period corresponds to the PPP-adjusted exchange rate reported in the World Development Indicators (World Bank (2016), series PA.NUS.PRVT.PP) in 1990. Then, we chain the PPP-adjusted prices in the base year forward and backward with the inflation rate of the non-adjusted sectoral prices. Thus, all prices can be interpreted in terms of 1990 international \$. The same procedure is applied to the sectoral real consumption indexes, except that in the base year real consumption corresponds to nominal consumption expressed in 1990 international \$. The PPP-adjusted nominal sectoral expenditure is then derived by simply multiplying the PPP-adjusted price and quantity indexes for each sector.

E Other preference specifications

In this section we briefly discuss two further preference specifications used in the literature—the generalized PIGLOG and the non-homothetic CES—which fall outside of the IA class of preferences. As a consequence, they do not allow for tractable aggregation of individual demand systems and the preference parameters cannot be identified from aggregate data without bias.

E.1 Generalized PIGLOG (QAID)

In the microeconomic literature on consumer demand systems, Banks et al. (1997) have established the estimation of the Quadratic Almost Ideal Demand (QAID) system, which results from the generalized PIGLOG preference specification

$$v(e_{i,t}, P_t) = \left\{ \left[\frac{\log(e_{i,t}) - \log(\Pi(P_t))}{\Xi(P_t)} \right]^{-1} + \Lambda(P_t) \right\}^{-1}, \tag{52}$$

where $\Xi(P_t) = \prod_{j \in J} p_{j,t}^{\xi_j}$, total expenditure $e_{i,t}$ is deflated with the aggregate price index

$$\log(\Pi(P_t)) = \pi_0 + \sum_{j \in J} \pi_{j,t} \log(p_{j,t}) + 1/2 \sum_{j \in J} \sum_{l \in J} \varphi_{jl} \log(p_{j,t}) \log(p_{l,t}),$$

and $\Lambda(P_t) = \sum_{i \in J} \lambda_j \log(p_{j,t})$. The preference specification in (52) is an extension of the PIGLOG class of preferences, which corresponds to the special case when $\Lambda_t = 0$. Applying Roy's identity to the indirect utility function in Equation (52), the resulting budget shares are quadratic in the logarithm of total expenditure $\log(e_{i,t})$,

$$\eta_{i,j,t} = \pi_j + \sum_{l \in J} \varphi_{jl} \log(p_{l,t}) + \xi_j \log(e_{i,t}/\Pi(P_t)) + \lambda_j / \Xi(P_t) \left[\log(e_{i,t}/\Pi(P_t)) \right]^2. \tag{53}$$

The key parameters of this specification are the λ_js, which regulate the impact of the quadratic terms in the expenditure system. The preferences are well-behaved if $\sum_{j \in J} \varphi_{jk} = \sum_{l \in J} \varphi_{jl} = \sum_{j \in J} \xi_j = \sum_{j \in J} \lambda_j = 0$ and $\sum_{j \in J} \pi_j = 1$. It is also common to impose symmetry of price effects, $\varphi_{jl} = \varphi_{lj}$, which allows to further reduce the number of parameters. Finally, it is standard to set π_0 to a fixed value because it is hard to identify empirically. Banks et al. (1997) set π_0 such that the real expenditure implied by the model parameters remains always positive.

It is straightforward to see from Equation (53) that the QAID system has the maximum rank 3. However, when either ξ_j or $\lambda_j/\Xi(P_t)$ are nonzero for any j, then the QAID will necessarily violate the restriction that the budget shares have to remain between 0 and 1 in the long-run. To see this, consider that the logarithm of deflated expenditures, $\log(e_{i,t}/\Pi(P_t))$, is steadily increasing in the data. Thus, it is only a matter of time until the linear or the quadratic term in Equation (52) will cause the expenditure share to violate the unit interval restriction.

Finally, aggregating (53) over all individuals i yields the market expenditure shares

$$\eta_{j,t} = \pi_j + \sum_{k\in J} \varphi_{jk} \log(p_{k,t}) + \widehat{\xi}_{j,t} \log(e_t/\Pi(P_t)) + \widehat{\lambda}_{j,t}/\widehat{\Xi}(P_t) \left[\log(e_t/\Pi(P_t))\right]^2, \quad (54)$$

where

$$\widehat{\xi}_{j,t} = \frac{\xi_j}{N} \int_0^N \frac{\log(e_{i,t}/\Pi(P_t))}{\log(e_t/\Pi(P_t))}(e_{i,t}/e_t)di, \quad \widehat{\Xi}_t(P_t) = \frac{\Xi(P_t)}{N} \int_0^N \frac{\log(e_{i,t}/\Pi(P_t))}{\log(e_t/\Pi(P_t))}(e_{i,t}/e_t)di,$$
$$\widehat{\lambda}_{j,t} = \frac{\lambda_j}{N} \int_0^N \left[\frac{\log(e_{i,t}/\Pi(P_t))}{\log(e_t/\Pi(P_t))}\right]^2 (e_{i,t}/e_t)di.$$

Note that $\widehat{\xi}_{j,t}$, $\widehat{\Xi}_t(P_t)$, and $\widehat{\lambda}_{j,t}$ depend on a time-varying inequality index, such that the preference parameters ξ_j and λ_j cannot be identified from aggregate expenditure shares unless heterogeneity in individual expenditures is assumed away.

E.2 Non-homothetic CES

Comin et al. (2015, 2017) consider the non-homothetic CES preference specification proposed in Hanoch (1975)

$$v(e_{i,t}, P_t) = \frac{1}{\epsilon}\tilde{v}(e_{i,t}, P_t)^\epsilon - \frac{1}{\epsilon}, \quad \tilde{v}(e_{i,t}, P_t) \equiv \frac{e_{i,t}}{\mathbf{P}(e_{i,t}, P_t)},$$

where the price function $\mathbf{P}(e_{i,t}, P_t)$ depends again on the expenditure level and is implicitly given by

$$\mathbf{P}(e_{i,t}, P_t) = \frac{1}{\tilde{v}(e_{i,t}, P_t)} \left(\sum_{j \in J} \omega_j p_{j,t}^{1-\sigma} \tilde{v}(e_{i,t}, P_t)^{\epsilon_j - \sigma} \right)^{\frac{1}{1-\sigma}}.$$

It follows immediately that the expenditure function can be expressed as

$$e_{i,t} = \mathbf{P}(e_{i,t}, P_t) \tilde{v}(e_{i,t}, P_t) = \left(\sum_{j \in J} \omega_j p_{j,t}^{1-\sigma} \tilde{v}(e_{i,t}, P_t)^{\epsilon_j - \sigma} \right)^{\frac{1}{1-\sigma}}. \quad (55)$$

The parameters ω_j remains positive weights that add up to one, $\sum_{j \in J} \omega_j = 1$, and preferences are well-behaved if $\omega_j > 0$, $\sigma > 0$, and $\epsilon_i > \sigma$ if $\sigma \in (0, 1)$ or $\epsilon_j < \sigma$ if $\sigma > 1$. Moreover, Comin et al. (2015, 2017) restrict the parameter space to $\sigma \in (0, 1)$ and $\epsilon_j \geq 1$.

Note that while the expenditure function of the non-homothetic specification has a closed form solution, the indirect utility function $v(e_{i,t}, P_t)$ can only be expressed in implicit form. Consequently, the Hicksian demand system will have a closed form, while the Marshallian demand system can only be expressed in implicit form. This is an important limitation in the estimation of preference parameters. Since utility $\tilde{v}(e_{i,t}, P_t)$ is unobserved, the simple Hicksian demand system cannot be used to identify the preference parameters.[49] Instead, a more complicated estimation procedure that takes the implicit structure of the Marshallian demand system into account would be needed to estimate the preference parameters. Finally, since the non-homothetic CES is not nested in (19), Corollary 1 does not apply. Therefore, the parameters of non-homothetic CES preferences cannot be identified from aggregate budget shares without aggregation bias.

[49] Hanoch (1975) proposes to substitute for utility with one sectoral expenditure share to overcome this problem. However, with three sectoral expenditure shares, after substitution there is only one linearly independent estimation equation left and not all share ω_j and elasticity parameters ϵ_j can be identified.