The Long-term Debt Accelerator*

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Abstract

We introduce risky long-term debt to a standard model of firm financing and investment. This allows us to identify a novel amplification mechanism: the Long-term Debt Accelerator. A negative shock triggers an adverse feedback loop between low investment and high credit spreads. Relative to a frictionless RBC setup, the Long-term Debt Accelerator amplifies shocks by about 160%. This amplification mechanism is absent from standard models including only short-term debt. Negative shocks are more severe than positive shocks of equal size and amplification is stronger for larger shocks. If fundamental volatility is lower and firms accumulate more debt, recessions become more severe. The Long-term Debt Accelerator is in line with the empirically observed cyclical behavior of credit spreads, leverage, and debt maturity.

Keywords: business cycles, amplification, long-term debt, credit spreads.
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1. Introduction

Do credit-market imperfections matter for economic activity? Do financial frictions amplify shocks to the economy? The standard approach to studying these questions is to model debt as short-term, i.e. all debt issued in period t fully matures in period $t+1$. However, this assumption is made for tractability, not because of empirical relevance. In the U.S., the typical maturity of a business loan is 3.5 years. For corporate bonds, the average maturity is 8.3 years. About 77% of U.S. corporate debt does not mature within the next year. More than half of it does not mature within three years.[1]

In this paper, we introduce risky long-term debt (and a maturity choice) to a standard model of firm financing and investment. We show that the introduction of long-term debt substantially amplifies the model's response to fundamental shocks. In the model with long-term debt, the size of the output response to a productivity shock is about 160% of the response in a frictionless RBC setup or in a model with only short-term debt. For comparison, the financial accelerator in Bernanke, Gertler, and Gilchrist (1999) generates an amplification of about 150%.

This novel amplification mechanism works through endogenous movements in asset prices. But unlike in Kiyotaki and Moore (1997), these asset prices are credit spreads. The mechanism is the following. Firms maximize shareholder value. Through the bond market, they fully internalize the effect of their actions on the value of newly issued debt. At the same time, firms disregard the effect of their actions on the value of previously issued debt. In the model, this implies that firms accept a high risk of default whenever the ratio of previously issued debt to newly issued debt is high.[2]

After a negative shock, firms reduce investment and the amount of newly issued debt. However, the stock of previously issued debt is still high. This implies that the ratio of previously issued debt to newly issued debt is high, which induces firms to accept a higher risk of default during the downturn. Creditors respond by charging higher credit spreads. This raises firms' cost of capital which induces them to invest even less. The amount of newly issued debt drops again which drives up the ratio of previously issued debt to newly issued debt by even more. As a consequence, firms accept an even higher risk of default which again drives up credit spreads. Firms respond by investing even less and so on. There is an adverse feedback loop between low investment and high credit spreads.

We parametrize the model to generate realistic levels of factor shares, leverage, credit spreads, and debt maturity. We find that the 'Long-term Debt Accelerator' amplifies shocks by about 160%. Recessions are particularly severe if they occur at a time when

[1]See Adrian, Colla, and Shin (2013), Table 2, for the maturity of U.S. bank loans and corporate bonds. The reported long-term debt shares are calculated based on Compustat data excluding financial firms and utilities.

[2]The key feature of a model with long-term debt is that firms take decisions in the presence of previously issued debt. Jungherr and Schott (2017) show that in such a model the stock of existing debt matters for firm behavior. See Myers (1977) or Admati, DeMarzo, Hellwig, and Pfleiderer (2013) for implications in corporate finance, and Arellano and Ramanarayanan (2012), Chatterjee and Eyigungor (2012), or Hatchondo, Martinez, and Sosa-Padilla (2016) for related results in models of sovereign default.

firms have accumulated a lot of debt, e.g. because fundamental volatility is low. There are important non-linearities. Negative shocks are more severe than positive shocks of equal size and amplification is stronger for larger shocks. These non-linearities highlight the importance of non-linear solution methods.

This amplification mechanism is very general. Any shock which induces a response of firm investment is amplified in the way described above. This is true both for supply and demand shocks. Amplification occurs even though debt is used only to finance capital (i.e. there is no working capital), and even though equity issuance is completely frictionless. In the model, it is not the case that output is low during a recession because firms are *unable* to raise additional equity. Output is low because firms are *unwilling* to raise equity if this benefits existing creditors.

The 'Long-term Debt Accelerator' is in line with the empirically observed counter-cyclical behavior of credit spreads and leverage, and the pro-cyclical behavior of debt maturity and the term structure of credit spreads. As shown in Jungherr and Schott (2017), the model is also consistent with various empirical patterns in the cross-section of U.S. publicly traded firms.

2. Literature

The paper most closely related to ours is Gomes, Jermann, and Schmid (2016). Their main result is that shocks to inflation change the real burden of existing nominal long-term debt and thereby distort investment. In a setup with sticky prices, they show that their model displays relatively weak amplification. While there are several differences with respect to our model (e.g., we study a setup with flexible prices), one key difference is that Gomes et al. (2016) approximate the equilibrium using first-order perturbation methods. These linear approximations are only accurate in the immediate neighbourhood of the deterministic steady state, e.g. after very small shocks. With our fully non-linear global solution method, we find that the amount of amplification is an increasing function of the size of the shock. This may explain why Gomes et al. (2016) find little amplification while we find substantial effects.

Our paper relates to classic studies of amplification through credit markets, e.g. Kiyotaki and Moore (1997) and Bernanke et al. (1999). In flexible-price environments, the amount of amplification generated by these models often is relatively modest.[3] We contribute to this literature by proposing a previously unexplored amplification mechanism which performs well quantitatively in an environment with flexible prices. Like Brunnermeier and Sannikov (2014), we solve for the global dynamics of the model. In their model, financial frictions matter mostly during rare events which take the equilibrium variables far away from the steady state. In contrast, our 'Long-term Debt Accelerator' amplifies all shocks to the economy.

Like Cooley, Marimon, and Quadrini (2004), we study amplification in a model of long-term financing and limited commitment. However, the underlying mechanism is quite

[3]See Carlstrom and Fuerst (1997), Kocherlakota (2000), or Cordoba and Ripoll (2004). The sticky-price model in Bernanke et al. (1999) generates sizable amplification of around 150%.

different. In Cooley et al. (2004), amplification is generated because the commitment problem becomes more severe during booms. In our model, it becomes more severe during downturns (i.e. default rates are counter-cyclical). Similar to our model, the existence of previously issued short-term debt also plays a key role in Occhino and Pescatori (2015). However, they do not model long-term debt and the stock of existing debt is exogenous in their model.

Our paper also relates to the result in Covas and Den Haan (2012) that equity financing must become less attractive during downturns in order to generate counter-cyclical default rates. They show this by introducing an ad-hoc equity issuance cost which increases during downturns. We show in a model without any equity issuance costs that firms choose to raise less equity during downturns because they accept a higher default risk whenever the ratio of existing debt to total debt is high.

3. Two-period Model

We begin our analysis of the 'Long-term Debt Accelerator' in a simple two-period economy. A firm produces output using capital and labor. The firm's stock of capital is financed through equity and debt. The optimal capital structure solves a trade-off between the tax advantage of debt and the expected costs of default. Importantly, the firm decides on its scale of production and the preferred capital structure in the presence of previously issued long-term debt. This variable is exogenous in the two-period setup. It will be endogenized in the fully dynamic economy of Section 4.

In the presence of previously issued long-term debt, the firm does not fully internalize the potential costs which might result from default in the future. We will show below that this mechanism raises the risk of default, the credit spread, and thereby also the firm's cost of capital. Furthermore, we will show that this effect is stronger during downturns which renders the default rate and the credit spread counter-cyclical. The resulting counter-cyclical cost of capital can amplify fluctuations of investment, labor demand, and output.

3.1. Setup

There are two periods: $t = 1, 2$. In period 2, a firm uses capital k and labor l to produce output y using a technology with diminishing returns:

$$y \, = \, z \, \left(k^{\psi} \, l^{1-\psi}\right)^{\zeta} \, , \tag{1}$$

where z denotes productivity, and $\zeta, \psi \in (0, 1)$. The firm chooses capital and labor input in the initial period $t = 1$. At that point, productivity z is known. Firm earnings are uncertain because of an earnings shock ε. Earnings before interest and taxes in $t = 2$ are given as:

$$y - f - \delta k + \varepsilon k - wl \, , \tag{2}$$

where f is a fixed cost, δ is depreciation, and w is the wage rate. At t=1, ε is a random variable with probability density $\varphi(\varepsilon)$. There are two ways to finance capital: equity and debt.

Definition: Debt. A debt security is a promise to pay one unit of the numéraire good together with a fixed coupon payment c at the end of period 2.

In this two-period model, the firm issues new debt only once. Because we are interested in the question of how existing debt affects the firm's behavior, we introduce an exogenous variable b which denotes the quantity of bonds outstanding at the beginning of the initial period. These bonds mature at time 2 just like the one-period bonds which the firm can issue in period 1. One may think of b as long-term debt which has been issued before period 1.

Let p be the market price of a one-period bond sold by the firm in period 1. If the firm sells an amount Δ of new bonds, it raises an amount $p\Delta$ on the bond market. This brings its stock of debt to $b + \Delta = \tilde{b}$. An alternative to debt financing is equity. Let e be net equity issuance in period 1, that is, the cash flow from shareholders to the firm. If q is the stock of firm capital in place at the beginning of period 1, the firm's stock of capital is:

$$k \;=\; q + e + p\Delta \;=\; q + e + p\,(\tilde{b} - b)\,. \tag{3}$$

Firm earnings are taxed at rate τ. The firm's stock of equity (and firm assets) after production in period 2 is:

$$\tilde{q} = k - \tilde{b} + (1-\tau)[y - f - \delta k + \varepsilon k - wl - c\tilde{b}]\,. \tag{4}$$

The fact that coupon payments are tax-deductible lowers the total tax payment by the amount $\tau c\tilde{b}$. This is the benefit of debt. The downside is that the firm cannot commit to repaying its debt after production in period 2.

Definition: Limited Liability. Shareholders are protected by limited liability. They are free to default and hand over the firm's assets to creditors for liquidation. Default is costly. A fixed fraction ξ of the firm's assets is lost in this case.

The timing can be summarized as follows.

t=1 Given an existing stock of debt b and capital q, the firm chooses capital k and labor l. Capital is financed through equity issuance e and the revenue $p\,(\tilde{b} - b)$ from the sale of additional bonds.

t=2 The firm's stock of debt is $\tilde{b}$. Earnings are: $y - f - \delta k + \varepsilon k - wl$. The firm decides whether to default.

3.2. Firm Problem

The firm maximizes shareholder value. Since shareholders are risk neutral, the firm's objective is the expected present value of net cash flows from the firm to shareholders.

We can solve the firm's problem using backward induction, beginning with the default decision after the realization of firm earnings in period 2. Limited liability protects shareholders from large negative realizations of the earnings shock ε. Given a firm's stock of capital k and debt $\tilde{b}$, there is a unique threshold realization $\bar{\varepsilon}$ which sets the firm's equity stock after production $\tilde{q}$ equal to zero:

$$\bar{\varepsilon}: \quad \tilde{q} = 0 \quad \Leftrightarrow \quad k - \tilde{b} + (1-\tau)[y - f - \delta k + \bar{\varepsilon} k - wl - c\tilde{b}] = 0\,. \tag{5}$$

If ε is smaller than $\bar{\varepsilon}$, full repayment would result in negative equity $\tilde{q}$ while default provides an outside option of zero. In this case, the firm optimally defaults on its liabilities.

In period 1, the firm decides on its scale of production k, and its preferred financing mix of equity and debt. The firm anticipates that shareholders receive $\tilde{q}$ whenever $\varepsilon \geq \bar{\varepsilon}$ and zero otherwise:

$$\max_{k,l,e,\tilde{b},\bar{\varepsilon},y} \quad -e + \frac{1}{1+r}\int_{\bar{\varepsilon}}^{\infty} [k - \tilde{b} + (1-\tau)[y - f - \delta k + \varepsilon k - wl - c\tilde{b}]\;\varphi(\varepsilon)\,d\varepsilon \tag{6}$$

$$\text{subject to:} \quad y = z\left(k^{\psi}\, l^{1-\psi}\right)^{\zeta}$$

$$\bar{\varepsilon}: \quad 0 = k - \tilde{b} + (1-\tau)[y - f - \delta k + \bar{\varepsilon} k - wl - c\tilde{b}]$$

$$k = q + e + p\,(\tilde{b} - b)\,,$$

where r is the risk-free interest rate. The optimal firm policy crucially depends on the bond price p. A high bond price implies a low credit spread which reduces the firm's cost of capital. We derive the firm-specific bond price from the creditors' optimization problem.

3.3. Creditors' Problem

Creditors are risk-neutral and discount the future at the same rate $1/(1+r)$ as shareholders. They buy the firm's debt in period 1. If the firm does not default in period 2, they receive full repayment. In case of default, they receive the firm's liquidation value $(1-\xi)\,\tilde{k}$, where:

$$\tilde{k} \equiv k + (1-\tau)[y - f - \delta k + \varepsilon k - wl]\,. \tag{7}$$

Competitive creditors break even on expectation. The break-even price p of firm debt in period 1 depends on the probability $\Phi(\bar{\varepsilon})$ that the firm defaults in period 2:

$$p = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon})](1+c) + \frac{(1-\xi)}{\tilde{b}}\int_{-\infty}^{\bar{\varepsilon}} \tilde{k}\,\varphi(\varepsilon)\,d\varepsilon\right]. \tag{8}$$

If creditors expect a positive risk of default, they will charge a credit spread over the riskless rate.[4]

3.4. Equilibrium

We solve for the partial equilibrium allocation given the wage w and the risk-free rate r. In equilibrium, the firm maximizes shareholder value (6) subject to creditors' break-even condition (8). We can simplify this problem by re-writing it in terms of only two endogenous variables: the scale of production k, and the default threshold $\bar{\varepsilon}$.

3.4.1. Consolidated Problem

Given the wage rate w, a necessary and sufficient condition for optimal labor demand is:

$$l^* = \frac{\zeta(1-\psi)y}{w}. \tag{9}$$

Using this condition, we can express output net of wage payments as a function of capital only:

$$y^* - wl^* \;=\; A\,k^\alpha\,, \tag{10}$$

where A and α are functions of productivity z, the wage rate w, and the technology parameters ζ and ψ. There are diminishing returns to scale: $\alpha \in (0,1)$.[5]

We continue by expressing the stock of debt $\tilde{b}$ in terms of k and $\bar{\varepsilon}$. From the definition of $\bar{\varepsilon}$ it follows:

$$\tilde{b}\,[1+(1-\tau)c] = k+(1-\tau)[Ak^\alpha - f - \delta k + \bar{\varepsilon}k] \;\Leftrightarrow\; \tilde{b} = \frac{k+(1-\tau)[Ak^\alpha - f - \delta k + \bar{\varepsilon}k]}{1+(1-\tau)c}. \tag{12}$$

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[4]Sometimes more than one bond price satisfies creditors' break-even condition. In this case, different bond prices also imply different default probabilities. See Calvo (1988). The conditions which introduce multiplicity are described in Nicolini, Teles, Ayres, and Navarro (2015). By allowing the firm in (6) to directly select the default probability through $\bar{\varepsilon}$, we implicitly assume that the firm sells its bonds to creditors by making a take-it-or-leave-it offer specifying both a price p and a quantity $\tilde{b}$ of bonds. This implies that the firm is always able to select the preferred (i.e. the lower) default probability and there is a unique equilibrium. See also Crouzet (2016).

[5]In particular:

$$A \;\equiv\; z^{\frac{1}{1-\zeta(1-\psi)}}\,(1-\zeta(1-\psi))\left(\frac{\zeta(1-\psi)}{w}\right)^{\frac{\zeta(1-\psi)}{1-\zeta(1-\psi)}}, \text{and:} \quad \alpha \;\equiv\; \frac{\zeta\psi}{1-\zeta(1-\psi)}, \tag{11}$$

Consider first the left hand side of the first equation in (12). Creditors are entitled to a fixed payment of $\tilde{b}[1+c]$ in period 2. But effectively the firm only pays $\tilde{b}[1+(1-\tau)c]$ because it can deduct $\tilde{b}\tau c$ from its tax bill. The right hand side of the first equation states that this payment consists of two parts: the safe part of firm assets after production, $k+(1-\tau)[Ak^{\alpha}-f-\delta k]$, and a fixed promised amount of the risky part of earnings, $(1-\tau)\bar{\varepsilon}k$.

Similarly, we can express equity issuance e in terms of k, $\tilde{b}$, and p:

$$e = k - q - p\,(\tilde{b} - b)\,. \tag{13}$$

Using these two expressions, the firm's problem can be re-written as:

$$\max_{k,\tilde{b},\bar{\varepsilon},p} \quad -k\,+\,q\,+\,p\,(\tilde{b}-b)\,+\,\frac{1-\tau}{1+r}\,k\int_{\bar{\varepsilon}}^{\infty}[\varepsilon-\bar{\varepsilon}]\;\varphi(\varepsilon)\,d\varepsilon\,. \tag{14}$$

The firm's objective is to maximize shareholder value. But in (14), the firm maximizes the total return to capital k including the value of newly issued debt $p(\tilde{b}-b)$. Shareholders benefit from a high value of newly issued debt as less equity issuance e is required for a given level of capital k (see equation (13)). Because creditors break even on expectation, shareholders appropriate the entire surplus created by the investment of k.

If labor demand is chosen according to equation (9), we know from (8) that p depends on k, $\bar{\varepsilon}$, and $\tilde{b}$. By (12), $\tilde{b}$ only depends on k and $\bar{\varepsilon}$. It follows that (14) characterizes the equilibrium allocation in terms of k and $\bar{\varepsilon}$ only. The firm maximizes (14) subject to (8), (9), and (12).

3.4.2. First Order Conditions

As shown above, the equilibrium allocation is pinned down by two endogenous variables: the scale of production k, and the default threshold $\bar{\varepsilon}$. Accordingly, an interior solution is characterized by two first order conditions.

For analytical tractability, in this part we consider the special case of $\xi=1$. This means that the liquidation value of the firm is zero in case of default and the bond price in (8) only depends on $\bar{\varepsilon}$:

$$p = \frac{1+c}{1+r}[1-\Phi(\bar{\varepsilon})]\,. \tag{15}$$

The same is true for the credit spread: $(1+c)/p-(1+r) = (1+r)\,\Phi(\bar{\varepsilon})/[1-\Phi(\bar{\varepsilon})]$.

The derivative of (14) with respect to k yields a first order condition for the optimal scale of production:

$$\underbrace{-1}_{\substack{Marginal\\cost\,of\\capital}}\;+\underbrace{\frac{1+c}{1+r}[1-\Phi(\bar{\varepsilon})]\frac{1+(1-\tau)\,[A\alpha k^{\alpha-1}-\delta+\bar{\varepsilon}]}{1+(1-\tau)c}}_{\substack{Marginal\,increase\\in\,value\,of\,newly\,issued\,debt}}+\underbrace{\frac{1-\tau}{1+r}\int_{\bar{\varepsilon}}^{\infty}[\varepsilon-\bar{\varepsilon}]\;\varphi(\varepsilon)\,d\varepsilon}_{\substack{Marginal\,increase\\in\,expected\,dividend}}\;=\;0\,. \tag{16}$$

A marginal increase in k has an opportunity cost of one. The benefit consists of an increase in the value of newly issued debt and equity. Because of diminishing returns to production, the marginal increase in the value of newly issued debt is falling in k. If productivity z is high, A is high as well. This raises the firm's incentive to invest. Note that the number of previously issued bonds b does not appear in the first order condition for k. Conditional on the firm's choice of $\bar{\varepsilon}$, the existing stock of debt b does not influence investment.

A first order condition for an optimal choice of $\bar{\varepsilon}$ is:

$$\underbrace{[1-\Phi(\bar{\varepsilon})](1-\tau)k\frac{\tau c}{1+(1-\tau)c}}_{Marginal\, tax\, benefit\, of\, \bar{\varepsilon}} \quad - \quad \underbrace{\varphi(\bar{\varepsilon})(1+c)(\tilde{b}-b)}_{\substack{Marginal\, increase\, in \\ expected\, costs\, of\, default \\ internalized\, by\, the\, firm}} \;=\; 0\,. \tag{17}$$

The first term is the marginal benefit of an increase in $\bar{\varepsilon}$. It is weighted with the repayment probability $[1-\Phi(\bar{\varepsilon})]$. If default is avoided, a higher value of $\bar{\varepsilon}$ increases the fixed amount promised to creditors by $(1+c)\partial\tilde{b}/\partial\bar{\varepsilon}$, and reduces the dividend by $(1-\tau)k$.[6] Since coupon payments are tax-deductible, it costs shareholders only $1+(1-\tau)c$ to increase the promised payment to creditors by $1+c$. Because competitive creditors break even, the entire tax benefit generated from substituting equity by debt is captured by shareholders.

The second term in (17) plays a key role in this model. It is the marginal cost of an increase in $\bar{\varepsilon}$. The probability of default increases by $\varphi(\bar{\varepsilon})$ and creditors lose the entire amount of $(1+c)\tilde{b}$ in this case. While the firm internalizes the tax benefit of the entire stock of debt $\tilde{b}$, it does not internalize all associated costs. The firm takes into account that an increase of $\bar{\varepsilon}$ lowers the value of newly issued bonds $p(\tilde{b}-b)$. But it disregards the fact that this also lowers the value of previously issued debt pb. If $b > 0$, the firm does not fully internalize all expected costs of default.

3.4.3. Cyclical Properties

We are interested in the question whether changes in fundamentals are amplified by the credit market. In this two-period model, high values of profitability A create a boom with high values of investment, labor demand, and output; low values of A generate the opposite. Throughout this section, we assume that the firm's problem described above has an interior solution and we consider the special case of $\xi = 1$.

Proposition 3.1. ***The Credit Spread & Existing Debt:*** *The default rate $\Phi(\bar{\varepsilon})$ and the credit spread $(1+r)\,\Phi(\bar{\varepsilon})/[1-\Phi(\bar{\varepsilon})]$ are increasing in the stock of existing debt b.*

[6] The marginal tax benefit of $\bar{\varepsilon}$ can be written as:

$$[1-\Phi(\bar{\varepsilon})](1-\tau)k\frac{\tau c}{1+(1-\tau)c} = [1-\Phi(\bar{\varepsilon})]\left[(1+c)\frac{\partial\tilde{b}}{\partial\bar{\varepsilon}} - (1-\tau)k\right].$$

All proofs are deferred to Appendix A. The amount of existing debt b affects firm behavior through the first order condition for $\bar{\varepsilon}$ in (17). If $b = 0$, the entire stock of debt $\tilde{b}$ is issued in period 1 and the firm fully internalizes all expected costs of default through the break-even price of debt. But with positive b, a part of the expected costs of default is not borne by the firm but by the holders of previously issued debt. This allows the firm to enjoy a given amount of the tax benefits of debt at a lower cost. As a result, the firm optimally decides to utilize the tax benefit of debt more intensively by raising $\bar{\varepsilon}$. Creditors respond to the increase in $\bar{\varepsilon}$ by charging a higher credit spread on newly issued debt $\tilde{b} - b$.

Proposition 3.1 describes how the stock of existing debt b affects the default rate *for any given value* of profitability A. Proposition 3.2 states that this mechanism has important implications for the response of the default rate to *changes* in A.

Proposition 3.2. ***Counter-cyclical Credit Spread:*** *If $b = 0$, the default rate $\Phi(\bar{\varepsilon})$, the credit spread $(1+r)\,\Phi(\bar{\varepsilon})/[1-\Phi(\bar{\varepsilon})]$, and leverage $\tilde{b}/k$ are all constant in profitability A. If $b > 0$, the default rate and the credit spread are decreasing functions of A.*

This is an interesting result. Consider first the case that $b = 0$. Even though the model is not scale-invariant because of decreasing returns to scale, with $b = 0$ the firm's optimal choice of $\bar{\varepsilon}$ is invariant to the scale of production. One implication of this result is that the optimal capital structure does not move either: leverage $\tilde{b}/k$ is constant in A.

Consider now the case of a positive stock of existing debt. With $b > 0$, an increase in A reduces the firm's equilibrium choice of $\bar{\varepsilon}$. So why does a positive stock of existing debt render the default rate and the credit spread counter-cyclical? We know from equation (17) that the benefit of a marginal increase in $\bar{\varepsilon}$ is proportional to k. The cost is proportional to $\tilde{b} - b$. Dividing the first order condition (17) by k yields:

$$\underbrace{[1-\Phi(\bar{\varepsilon})](1-\tau)\frac{\tau c}{1+(1-\tau)c}}_{Marginal\,tax\,benefit\,of\,\bar{\varepsilon}} \quad - \quad \underbrace{\varphi(\bar{\varepsilon})(1+c)\frac{\tilde{b}-b}{k}}_{\substack{Marginal\,increase\,in\\ expected\,costs\,of\,default\\ internalized\,by\,the\,firm}} \;=\; 0\,. \tag{18}$$

As stated in Proposition 3.2, with $b = 0$ the firm optimally keeps leverage constant in A. Changes in capital k are proportional to changes in total debt $\tilde{b}$. With $b = 0$, leverage $\tilde{b}/k$ does not change with A. It follows from (18) that $\bar{\varepsilon}$ does not change either.

With $b > 0$, there is an additional term added to the marginal benefit of $\bar{\varepsilon}$ in (18):

$$\varphi(\bar{\varepsilon})\,(1+c)\,\frac{b}{k}\,. \tag{19}$$

The firm disregards a fraction b/k of the expected costs of default. If k is large relative to b, this term is small. But if k is small relative to b, the firm disregards a large fraction of the total costs of increasing $\bar{\varepsilon}$. This increases its equilibrium choice of $\bar{\varepsilon}$. Since k is low if A is low, and b cannot respond to changes in A, low values of A imply high values of b/k. If profitability A and investment k are low, the firm disregards a large fraction of the total expected costs of default: the default rate and the credit spread become counter-cyclical.

A counter-cyclical credit spread implies that the firm's cost of capital increases during downturns. Proposition 3.3 states that through this mechanism the credit market can amplify fluctuations of investment, labor demand, and output.

Proposition 3.3. ***The Long-term Debt Accelerator:*** *Assume $b > 0$ and consider a fall in profitability A. Proposition 3.2 states that the default rate and the credit spread increase. If α is sufficiently close to one, this amplifies the negative response of capital k, labor demand l, and output y.*

In principle, an increase in $\bar{\varepsilon}$ has an ambiguous effect on firm output. A higher value of $\bar{\varepsilon}$ implies that a larger share of firm earnings is paid out to creditors in the form of tax-deductible debt coupons. This reduces the effective tax rate and encourages investment. The downside is that the bond price p in (15) falls in $\bar{\varepsilon}$ which raises the cost of capital and discourages investment. If the curvature in the firm's production technology is not too high (i.e. α is sufficiently close to one), the latter effect dominates and the fact that the credit spread rises during a downturn amplifies the reduction in capital k, labor demand l, and output y.

By now it has become clear that the initial fall in profitability A can trigger an adverse feedback loop between low investment and high credit spreads. The fall in A reduces the firm's incentive to invest. For a given positive stock of existing debt b, a lower choice of k increases the share b/k of the expected costs of default which is disregarded by the firm. The equilibrium default rate increases (Proposition 3.2). If α is sufficiently close to one, the resulting increase in the credit spread depresses investment further (Proposition 3.3). The firm's choice of k falls by even more and thereby again drives up the fraction b/k. The firm disregards an even larger share of the expected costs of default, the default rate and the credit spread increase (Proposition 3.2), which depresses investment even more (Proposition 3.3), and so on.

Importantly, the described Long-term Debt Accelerator is a very general mechanism. Any initial movement in k induced by any kind of demand and supply shock is amplified in this way. Two important questions remain open at this point. (1.) In the two-period model, the stock of existing debt b is an exogenous variable. It is important to endogenize firms' choice of b in a fully dynamic model. (2.) In order to asses the quantitative significance of the Long-term Debt Accelerator, we have to proceed to a numerical analysis.

4. Dynamic Model

The main additional feature of the dynamic model with respect to the two-period setup is that firms have a maturity choice. They can sell short-term bonds and long-term bonds. The amount of long-term debt issued today determines the stock of existing debt tomorrow.

The issuance of bonds is costly. Short-term debt needs to be constantly rolled over which implies high issuance costs. Long-term debt allows to maintain a given stock of debt at a lower level of bond issuance and thereby saves roll-over costs. The downside of long-term debt is that it raises the risk of default in the future.

4.1. Setup

There is a unit mass of firms. As in the two-period economy, a firm i uses capital k_{it} and labor l_{it} to produce output y_{it} using a technology with diminishing returns:

$$y_{it} = z_t \left(k_{it}^{\psi} \, l_{it}^{1-\psi}\right)^{\zeta}, \quad \zeta, \psi \in (0,1). \tag{20}$$

Aggregate productivity z_t follows an AR(1)-process. It is known at the time when the firm chooses capital k_{it} and labor l_{it}. Earnings before interest and taxes are given as:

$$y_{it} - f - \delta k_{it} + \varepsilon_{it} k_{it} - w l_{it}. \tag{21}$$

The firm-specific earnings shock ε_{it} is i.i.d. and follows a probability distribution $\varphi(\varepsilon)$.

In contrast to the two-period economy, the firm can now choose between short-term debt and long-term debt.

Definition: Short-term Debt. A short-term bond issued at the end of period $t-1$ is a promise to pay one unit of the numéraire good together with a fixed coupon payment c in period t. The quantity of these short-term bonds sold by firm i at the end of period $t-1$ is $\tilde{b}_{it}^S$.

Definition: Long-term Debt. A long-term bond issued at the end of period $t-1$ is a promise to pay a fixed coupon payment c in period t. In addition, the firm repays a fraction $\gamma \in (0,1)$ of the principal in period t. In period $t+1$, a fraction $1-\gamma$ of the bond remains outstanding. The firm pays a coupon payment $(1-\gamma)c$ and repays the fraction γ of the remaining principal: $(1-\gamma)\gamma$. In this manner, payments decay geometrically over time. The maturity parameter γ controls the speed of decay. The quantity of long-term bonds chosen by the firm at the end of period $t-1$ is $\tilde{b}_{it}^L$.

This computationally tractable specification of long-term debt goes back to Leland (1994). Short-term debt and long-term debt are of equal seniority.

Definition: Floatation cost. The firm pays an amount η for each bond sold (or re-purchased). The total floatation cost $H(\tilde{b}_{it}^S, \tilde{b}_{it}^L, b_{it})$ is therefore:

$$H(\tilde{b}_{it}^S, \tilde{b}_{it}^L, b_{it}) = \eta(\,\tilde{b}_{it}^S + |\tilde{b}_{it}^L - b_{it}|\,), \tag{22}$$

where b_{it} is the stock of previously issued long-term bonds outstanding before the firm decides on its investment and financing policy at the end of period $t-1$.

The firm finances its capital stock by injecting equity and selling new short- and long-term bonds. Let q_{it-1} be the the stock of assets in place before the firm decides on equity and debt issuance, and let e_{it} denote net equity issuance at the end of period $t-1$. A positive value of e_{it} indicates an injection of new equity into the firm, e.g. through the sale of new shares or the repurchase of existing ones. A negative value indicates a net dividend payment from the firm to shareholders. It follows for the new stock of capital in period t:

$$k_{it} = q_{it-1} + e_{it} + p^S_{it}\tilde{b}^S_{it} + p^L_{it}(\tilde{b}^L_{it} - b_{it}) - H(\tilde{b}^S_{it}, \tilde{b}^L_{it}, b_{it})\,. \tag{23}$$

The stock of firm assets after production in period t is:

$$q_{it} = k_{it} - \tilde{b}^S_{it} - \gamma\tilde{b}^L_{it} + (1-\tau)[y_{it} - f - \delta k_{it} + \varepsilon_{it}k_{it} - wl_{it} - c(\tilde{b}^S_{it} + \tilde{b}^L_{it})]\,. \tag{24}$$

Definition: Limited Liability. Shareholders are protected by limited liability. They are free to default and hand over the firm's assets to creditors for liquidation. A fixed fraction ξ of the firm's assets is lost in this case.

We assume that a defaulting firm is replaced by a new firm with zero debt and zero assets.

Timing

End of period $t-1$: Firm i has an amount b_{it} of long-term debt outstanding and a stock of assets q_{it-1}. The firm observes aggregate productivity z_t. Given b_{it}, q_{it-1}, and z_t, the firm chooses capital k_{it} and labor l_{it}. Capital is financed through equity issuance e_{it}, and by selling short-term bonds $\tilde{b}^S_{it}$ and additional long-term bonds $\tilde{b}^L_{it} - b_{it}$.

Beginning of period t: The firm draws the realization ε_{it}. This determines firm earnings. The firm decides whether to default. If it decides not to default, it pays corporate income tax on its earnings net of depreciation and coupon payments. This leaves the firm with a stock of assets after production of q_{it}. Next period's amount of long-term debt is $b_{it+1} = (1-\gamma)\tilde{b}^L_{it}$.

4.2. Firm Problem

As in the two-period economy, the firm maximizes expected shareholder value. Since shareholders are risk neutral, have deep pockets, and since there are no equity issuance costs, the amount of assets in place q_{it-1} has no influence on the optimal firm policy. In contrast, the stock of existing debt b_{it} and aggregate productivity z_t will matter for firm behavior. We can express shareholder value at the end of period $t-1$ as the sum of a part which does and one which does not depend on firm behavior: $q_{it-1} + V_{t-1}(\, b_{it}\,,\, z_t\,)$.

The amount of assets after production q_{it} in (24) is an increasing function of ε_{it}. There is a unique threshold realization $\bar{\varepsilon}_{it}$ which sets shareholder value at the end of period t

(before the realization of z_{t+1}) equal to zero:

$$\bar{\varepsilon}_{it}: \quad q_{it} + \mathbb{E}_{z_{t+1}|z_t} V_t\Big((1-\gamma)\tilde{b}^L_{it}\,,\, z_{t+1} \Big) \;=\; 0\,. \tag{25}$$

If ε_{it} is smaller than $\bar{\varepsilon}_{it}$, the firm optimally decides to default.

We assume that the firm has no ability to commit to future actions. This lack of commitment not only affects the firm's default choice, but also its decision of how much to produce and how to finance capital. The firm must therefore take its own future behavior as given. The only way in which it can influence $V_t((1-\gamma)\tilde{b}^L_{it}, z_{t+1})$ is through today's choice of long-term debt $\tilde{b}^L_{it}$.

Given a stock of assets q_{it-1}, existing debt b_{it}, and productivity z_t, the firm solves at the end of period $t-1$:

$$\max_{\substack{k_{it}, l_{it}, e_{it} \geq \underline{e}, \\ \tilde{b}^S_{it}, \tilde{b}^L_{it}, \bar{\varepsilon}_{it}, y_{it}, q_{it}}} \quad -e_{it} + \frac{1}{1+r}\int_{\bar{\varepsilon}_{it}}^{\infty} \Big[q_{it} + \mathbb{E}_{z_{t+1}|z_t} V_t\Big((1-\gamma)\tilde{b}^L_{it}, z_{t+1}\Big) \Big] \varphi(\varepsilon) d\varepsilon \tag{26}$$

$$\begin{aligned} \text{subject to:} \quad & q_{it} = k_{it} - \tilde{b}^S_{it} - \gamma \tilde{b}^L_{it} + (1-\tau)[y_{it} - f - \delta k_{it} + \varepsilon_{it} k_{it} - w l_{it} - c(\tilde{b}^S_{it} + \tilde{b}^L_{it})] \\ & y_{it} \;=\; z_t \left(k^{\psi}_{it}\, l^{1-\psi}_{it} \right)^{\zeta} \\ & \bar{\varepsilon}_{it}: \quad q_{it} + \mathbb{E}_{z_{t+1}|z_t} V_t\Big((1-\gamma)\tilde{b}^L_{it}, z_{t+1}\Big) = 0 \\ & k_{it} = q_{it-1} + e_{it} + p^S_{it}\tilde{b}^S_{it} + p^L_{it}(\tilde{b}^L_{it} - b_{it}) - H(\tilde{b}^S_{it}, \tilde{b}^L_{it}, b_{it})\,. \end{aligned}$$

We introduce a lower bound $e_{it} \geq \underline{e} < 0$. This is an upper limit for net dividend payments (recall that a negative choice of e_{it} implies a net dividend payment from the firm to shareholders). If the stock of existing debt b_{it} is sufficiently large, the firm may find it optimal to choose a corner solution and pay out the entire asset value of the firm as dividend: $e_{it} = -q_{it-1}$. In practice, it is illegal to pay dividends which substantially exceed firm earnings and deplete a firm's capital stock. We choose the value of the lower bound $\underline{e}$ such that it rules out the corner solution but is not binding in equilibrium. The exact value of $\underline{e}$ does not affect equilibrium variables.

4.3. Creditors' Problem

As in the two-period setup, the optimal firm policy crucially depends on the two bond prices p^S_{it} and p^L_{it}. Risk-neutral and competitive creditors break even on expectation. In case of default, the value of the firm's assets is:

$$\tilde{k}_{it} \equiv k_{it} + (1-\tau)[y_{it} - f - \delta k_{it} + \varepsilon_{it} k_{it} - w l_{it}]\,. \tag{27}$$

At this point, creditors liquidate the firm's assets and receive $(1-\xi)\tilde{k}_{it}$. Since short-term debt and long-term debt have equal seniority, the price of short-term debt is:

$$p_{it}^{S} = \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon}_{it})]\,(1+c) \, + \, \frac{(1-\xi)}{\tilde{b}_{it}^{S}+\tilde{b}_{it}^{L}}\int_{-\infty}^{\bar{\varepsilon}}\tilde{k}_{it}\,\varphi(\varepsilon)\,d\varepsilon\right]. \tag{28}$$

The price of long-term debt p_{it}^{L} not only depends on the firm's choices today, but also on the future price of long-term debt p_{it+1}^{L}:

$$p_{it}^{L} = \frac{1}{1+r}\Bigg[[1-\Phi(\bar{\varepsilon}_{it})]\,\Big(\gamma + c + (1-\gamma)\,\mathbb{E}_{z_{t+1}|z_t}\,p_{it+1}^{L}\Big((1-\gamma)\tilde{b}_{it}^{L}\,,\,z_{t+1}\Big)\Big) \\ + \, \frac{(1-\xi)}{\tilde{b}_{it}^{S}+\tilde{b}_{it}^{L}}\int_{-\infty}^{\bar{\varepsilon}}\tilde{k}_{it}\,\varphi(\varepsilon)\,d\varepsilon\Bigg]. \tag{29}$$

Since the future price of long-term debt depends on future firm behavior, it is a function of the future state of the firm: $p_{it+1}^{L}((1-\gamma)\tilde{b}_{it}^{L}, z_{t+1})$. As the firm today cannot directly control future firm behavior, the only way in which it can influence the future bond price is through today's choice of long-term debt $\tilde{b}_{it}^{L}$.

When the firm sells long-term debt to creditors, it would like to promise to maintain low levels of leverage and default risk in the future regardless of the future stock of existing debt b_{it}. Such behavior would justify a high bond price p_{it}^{L} and therefore a low credit spread. Unfortunately, this promise is not credible. Once the firm has sold long-term debt and raised the associated revenue on the bond market, in the future the firm has no incentive ex-post to take into account the effects of its actions on the value of previously issued debt. Creditors anticipate this behavior and pay a low bond price p_{it}^{L}.

4.4. Markov Perfect Equilibrium

As in the two-period economy, we solve for the partial equilibrium allocation given the wage w and the risk-free rate r. In equilibrium, a firm maximizes shareholder value (26) subject to creditors' two break-even conditions (28) and (29). Because we assume that the firm has no ability to commit to future actions, it plays a game against its future selves. We restrict attention to the Markov Perfect equilibrium, i.e. we consider strategies which are functions of the current state of the firm.

Since the amount of assets in place q_{it-1} has no influence on the optimal firm policy, we can compute shareholder value as the sum of assets in place and a part which depends on firm behavior: $q_{it-1}+V_{t-1}(b_{it}, z_t)$. The value $V_{t-1}(b_{it}, z_t)$ can be computed recursively. It is convenient to define the sum of q_{it-1} and e_{it} as a new choice variable: $\tilde{e}_{it} \equiv q_{it-1}+e_{it}$. In each period, the firm chooses a policy vector $\phi(b, z) = \{k, l, \tilde{e}, \tilde{b}^{S}, \tilde{b}^{L}, \bar{\varepsilon}, y, \tilde{q}\}$ which solves:

$$V\big(b\,,\,z\big) = \max_{\phi(b,z)=\left\{\substack{k,l,\tilde{e}\geq\underline{\tilde{e}},\\ \tilde{b}^S,\tilde{b}^L,\bar{\varepsilon},y,\tilde{q}}\right\}} \quad -\tilde{e} + \frac{1}{1+r}\int_{\bar{\varepsilon}}^{\infty}\Big[\tilde{q} + \mathbb{E}_{z'|z}V\big((1-\gamma)\tilde{b}^L\,,\,z'\big)\Big]\varphi(\varepsilon)d\varepsilon \tag{30}$$

$$\begin{aligned}
\text{subject to:}\quad \tilde{q} \;&=\; k - \tilde{b}^S - \gamma\tilde{b}^L + (1-\tau)\big[y - f - \delta k + \varepsilon k - wl - c(\tilde{b}^S + \tilde{b}^L)\big] \\
y \;&=\; z\,\left(k^{\psi}\,l^{1-\psi}\right)^{\zeta} \\
\bar{\varepsilon}:\quad & \tilde{q} \,+\, \mathbb{E}_{z'|z}\,V\big((1-\gamma)\tilde{b}^L\,,\,z'\big) \;=\; 0 \\
k \;&=\; \tilde{e} \,+\, p^S(b,z)\,\tilde{b}^S \,+\, p^L(b,z)\,(\tilde{b}^L - b) \;-\; H(\tilde{b}^S,\tilde{b}^L,b) \\
p^S(b,z) \;&=\; \frac{1}{1+r}\left[[1-\Phi(\bar{\varepsilon})]\,(1+c) \,+\, \frac{(1-\xi)}{\tilde{b}^S + \tilde{b}^L}\int_{-\infty}^{\bar{\varepsilon}}\tilde{k}\,\varphi(\varepsilon)\,d\varepsilon\right] \\
p^L(b,z) \;&=\; \frac{1}{1+r}\Bigg[[1-\Phi(\bar{\varepsilon})]\,\Big(\gamma + c + (1-\gamma)\,\mathbb{E}_{z'|z}\,p^L\Big((1-\gamma)\tilde{b}^L\,,\,z'\Big)\Big) \\
&\qquad\qquad + \frac{(1-\xi)}{\tilde{b}^S + \tilde{b}^L}\int_{-\infty}^{\bar{\varepsilon}}\tilde{k}\,\varphi(\varepsilon)\,d\varepsilon\Bigg].
\end{aligned}$$

Since a firm's policy $\phi(b,z) = \{k, l, e, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon}, y, \tilde{q}\}$ only depends on its state (b,z) and since the probability distribution over the firm's future state only depends on its current state, the equilibrium bond prices $p^S(b,z)$ and $p^L(b,z)$ likewise only depend on b and z.

4.5. Quantitative Analysis

The Markov Perfect equilibrium in (30) can only be computed using numerical methods. Before choosing parameter values, we briefly describe our solution method.

4.5.1. Solution Method

Following the literature on sovereign default with long-term debt (e.g. Hatchondo and Martinez (2009)), we compute the equilibrium allocation of a finite-horizon economy. Starting from the final date, we iterate backward in time until the firm's value function and the two bond prices have converged. We then use the first-period equilibrium functions as the infinite-horizon-economy equilibrium.

Common practice in the literature on risky debt is to compute the complete bond price schedules for all possible actions: $p^S(k, l, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon})$ and $p^L(k, l, \tilde{b}^S, \tilde{b}^L, \bar{\varepsilon})$. These price schedules from the 'outer loop' are then used to compute the optimal policy in an 'inner loop'. We find this 'inner-loop-outer-loop' procedure to be costly in terms of computing time. The 'outer loop' for the bond price schedules needs to be highly precise in order to get meaningful results from the 'inner loop' which computes the optimal firm policy.

For this reason, we resort to an alternative solution method. Similar to the approach used in the consolidated problem of Section 3.4.1, we express equilibrium bond prices as a function of today's choice variables. Given the firm's future policy, both bond prices are pinned down by the firm's choices today. This allows us to compute equilibrium bond prices and today's firm policy in a single step. This reduces the number of necessary computations and allows for a faster and more precise solution.

4.5.2. Parametrization

The model period is one year. This is a conservative modeling choice. Had we chosen a quarter as model period, firms would be able to adjust their policy in each quarter. A yearly parametrization provides firms with more commitment than a quarterly parametrization. Since the Long-term Debt Accelerator is generated by firms' lack of commitment to future actions, we prefer a yearly parametrization in order not to overstate its quantitative significance.

The annual rate of return on a riskless asset is set to $r = 3.09\%$. We also specify $c = r$, which implies that the price of a riskless short-term bond and a riskless long-term bond are both equal to one. We set δ to match a 10% annual capital depreciation rate. The fixed cost f is chosen such that shareholder value of a new entrant is zero on average: $\mathbb{E}_z V(0, z) = 0$.

The repayment rate of long-term debt γ is an important parameter. We set it to match the Macaulay duration of U.S. corporate bonds with remaining term to maturity above one year. Gilchrist and Zakrajsek (2012) calculate an average duration of 6.47 years. This implies: $\gamma = 0.1284$. We follow Gomes et al. (2016) in setting the tax rate $\tau = 0.4$.[7]

The long-run average of aggregate productivity $\mathbb{E}[z]$ and the exogenous wage rate w are not identified. We normalize $\mathbb{E}[z] = 1$ and set w such that the long-run average of profitability $\mathbb{E}[A] = 1$.[8] Aggregate productivity z_t follows an AR(1)-process:

$$\log z_t = \rho \log z_{t-1} + \epsilon_t^z \,, \tag{31}$$

where ϵ_t^z is white noise with standard deviation σ_z. We construct measures of the Solow residual from U.S. data on GDP, investment, and hours worked 1984-2016. We apply the methodology used in King and Rebelo (1999) and Gomes et al. (2016) and estimate: $\rho = 0.965$.

The firm-specific earnings shock is Normal with zero mean and standard deviation σ_ε. This leaves us with six unspecified parameters: ζ, ψ, σ_ε, η, ξ, and σ_z. We choose these

[7]Hennessy and Whited (2005) suggest a value of 0.3. Gomes et al. (2016) argue that τ should be thought of as capturing additional relative benefits of using debt rather than equity (e.g. equity issuance costs).

[8]Recall that profitability A is defined as in equation (10):

$$A\, k^\alpha \;=\; y^* - wl^* \,, \quad \text{with:} \quad l^* = \zeta(1-\psi)y/w \,.$$

Table 1: Empirical Moments

	Data	Model
1984-2015:		
Capital-output ratio	1.81	1.81
Labor share	61.8%	61.8%
Leverage: Debt / Assets	29.3%	29.3%
Long-term debt share	75.5%	75.4%
Standard deviation log GDP	0.01	0.01
1998-2010:		
Credit spread	2.30%	2.32%

Note: The capital-output ratio and the labor share are from the Flow of Funds for the aggregate of non-financial corporate businesses. The labor share is compensation of employees divided by revenue from sales of goods and services. The capital-output ratio is non-financial assets (marked-to-market, excluding inventories) divided by revenues. We prefer the Flow of Funds for data on assets because Compustat measures assets at historical cost. Leverage and the long-term debt share are from Compustat (excluding financial firms and utilities). Leverage is the ratio of aggregate book value of debt to aggregate book value of assets. The long-term debt share is the ratio of aggregate debt due more than one year from today to aggregate debt. We calculate the standard deviation of detrended log real GDP. We use the Hodrick-Prescott filter with a smoothing parameter of 6.25 as recommended by Ravn and Uhlig (2002) for yearly data. The credit spread is computed based on Adrian et al. (2013). It is the amount-weighted average of the credit spread for loan and bond issuance. The model counterpart is the amount-weighted average of the credit spread for short-term debt issuance and long-term debt issuance.

values to replicate six key statistics given in Table 1: the capital-output ratio, the labor share, leverage, the long-term debt share, the average credit spread, and volatility of detrended log GDP. Model counterparts of empirical moments are derived in Appendix B.

In our model, firms differ with respect to the stock of existing debt b. Given the equilibrium distribution of firms over b, aggregate variables are constructed as weighted averages of firm policies. Table 2 reports our choice for the full set of parameter values.[9]

Untargeted Moments

To assess our parametrization, we use untargeted empirical moments. The parameter ζ controls the degree of diminishing returns. Empirical estimates by Blundell and Bond (2000) suggest a value close to one. The chosen value $\zeta = 0.895$ is broadly in line with these estimates. Altınkılıç and Hansen (2000) provide micro-evience for the floatation cost η. They calculate an average floatation cost of 0.0109. This is very close to our value of $\eta = 0.008825$.

The annual default rate generated by the model is 2.52%. This is high compared to an empirical value of around 1.05% reported by Duan, Sun, and Wang (2012). It is well known that empirical credit spreads are not fully explained by realized default risk (e.g. Elton, Gruber, Agrawal, and Mann (2001)). In our model, credit spreads are driven exclusively by default risk. This means that we have to decide whether we want

[9]The specified standard deviation of the earnings shock σ_ε is high. Given the stylized nature of our model with iid shocks and no costs to equity issuance, large shocks are necessary to generate a positive default rate for realistic levels of leverage.

Table 2: Parametrization

Par.	Description	Value	Target/Source
r	riskless rate	0.0309	
c	debt coupon	r	
δ	depreciation	0.1	
f	fixed cost		$\mathbb{E}_z V(0, z) = 0$
γ	repayment rate long-term debt	0.1284	Gilchrist and Zakrajsek (2012)
τ	corporate income tax rate	0.4	Gomes et al. (2016)
w	wage rate	0.3413	Normalization
ρ	persistence aggr. productivity	0.965	Estimate U.S. data
σ_z	st. dev. error agg. productiv.	0.00031	Std. dev. log GDP 0.01
ζ	technology parameter	0.895	Capital-output ratio 1.81
ψ	technology parameter	0.309	Labor share 61.8%
σ_ε	st. dev. firm earnings	0.6762	Leverage 29.3%
η	marginal floatation cost	0.008825	Long-term debt share 75.5%
ξ	default cost parameter	0.6	Credit spread 2.30%

our model to match the default rate and generate unrealistically low credit spreads, or if want to match credit spreads at the cost of generating unrealistically high default rates. In our model, the bond price schedule is key to understanding firm behavior. We therefore choose to match the average credit spread rather than the default rate.

4.6. Results

We now describe the numerical solution to the equilibrium of the fully dynamic model described above. The intuition that was developed with the help of the two-period model continues to apply here. In contrast to the two-period model, however, the stock of long-term debt b is now an endogenous state-variable. In addition, the firm has the option of issuing both long- and short-term debt. Because of our global solution method, we are able to define policy functions over the entire state space. These are shown below. Next, we describe the effects of a shock to aggregate productivity z and discuss the amplification result.

4.6.1. Policy Functions

We begin by describing the firm's policy functions in Figure 1 and 2. The common x-axes shows the endogenous state variable b, the stock of outstanding long-term debt, normalized by the frictionless capital stock $\bar{k}^*$.[10] Where convenient, the y-axes have been normalized by the frictionless values of the variable under consideration.

The first panel of Figure 1 shows firm output as a function of outstanding long-term debt for three different levels of aggregate productivity z. Output is increasing in z and

[10]The frictionless capital stock is computed for a level of aggregate productivity equal to its unconditional mean. We assume no taxes or flotation costs in the frictionless economy.

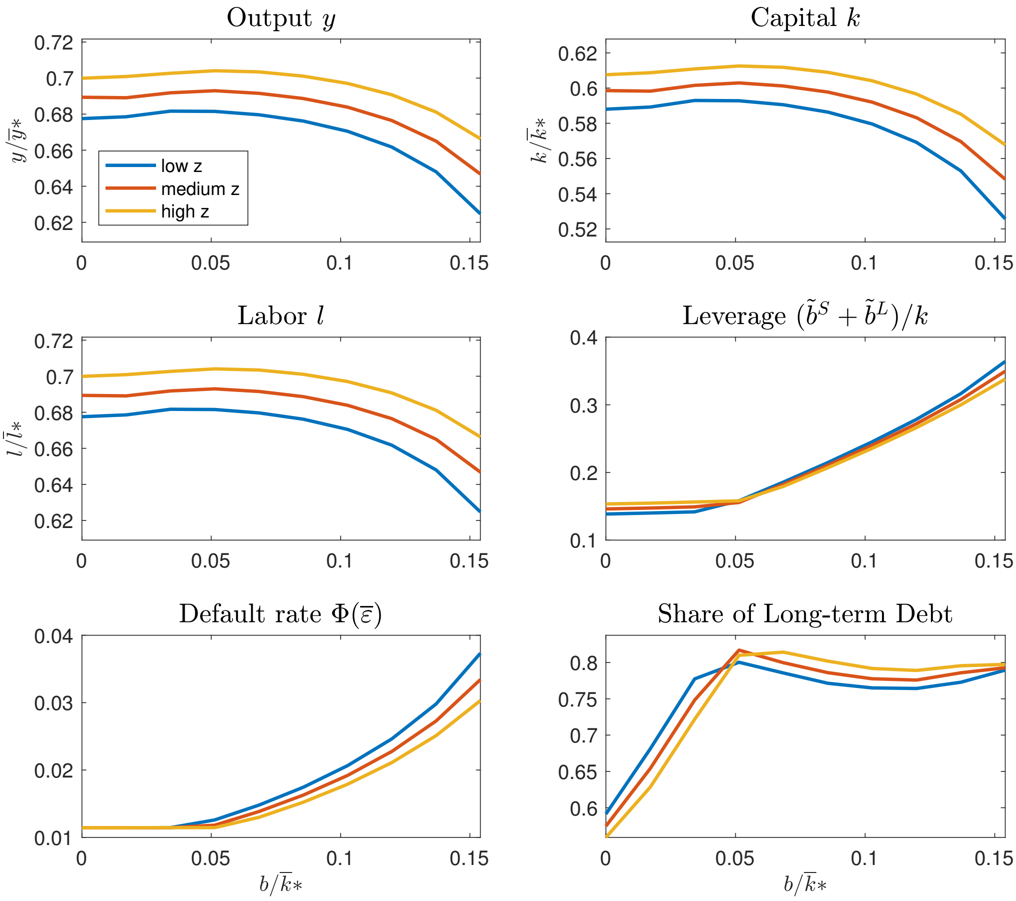

Figure 1: Policy Functions Part I

Outpt, labor, and capital are normalized by their respective average frictionless values $\overline{y}^*$, $\overline{l}^*$, and $\overline{k}^*$. The x-axes show the existing stock of debt b as a fraction of $\overline{k}^*$.

decreasing in b. The negative effect of outstanding long-term debt on output arises via the effect of long-term debt on factor inputs, labor and capital. Those are shown in the second and third panels of Figure 1. With a larger stock of outstanding long-term debt b, the costs of capital increase as a direct result of Proposition 3.1, a higher default probability (panel 5 of Figure 1) and consequently higher credit spreads (panels 5 and 6 of Figure 2).[11] The last panel of Figure 1 shows the maturity choice, i.e. the firms' optimal mix between short-term, and long-term debt. By issuing primarily short-term debt, a firm can reduce the future stock of outstanding debt and thereby minimize the future costs of long-term debt. The non-monotonicity in the maturity choice is a result of the various factors that determine the optimal trade-off between short- and long-term debt.

[11] As long as the default rates are not affected by additional long-term debt, the effect of b on capital (and hence labor and output) is positive. This effect is due to the fact that the firm can use the tax benefit more intensely. See Jungherr and Schott (2017).

First, the existence of floatation costs makes long-term debt preferable to short-term debt, as a given level of leverage can be maintained at a lower cost. Second, relatively more long-term debt entails a higher future ratio of b/k, worsening the commitment problem.

In Figure 2 we show the behavior of the end-of-period stock of long-term debt (panel 1), as well as the issuance of new short- and long-term debt (panels 2 and 3). In the first panel we have included a dashed line which represents the amount of end-of-period long term debt $\tilde{b}^L$ necessary to keep b constant. While issuance of long-term debt is monotonically decreasing in outstanding debt, the issuance of short-term debt $\tilde{b}^s$ is non-monotonic, as explained for the optimal maturity choice above. The last two panels of Figure 2 show the credit spreads for short- and long-term debt. They illustrate Propositions 3.2 and 3.3, which stated that credit spreads increase in b and fall in profitability A.

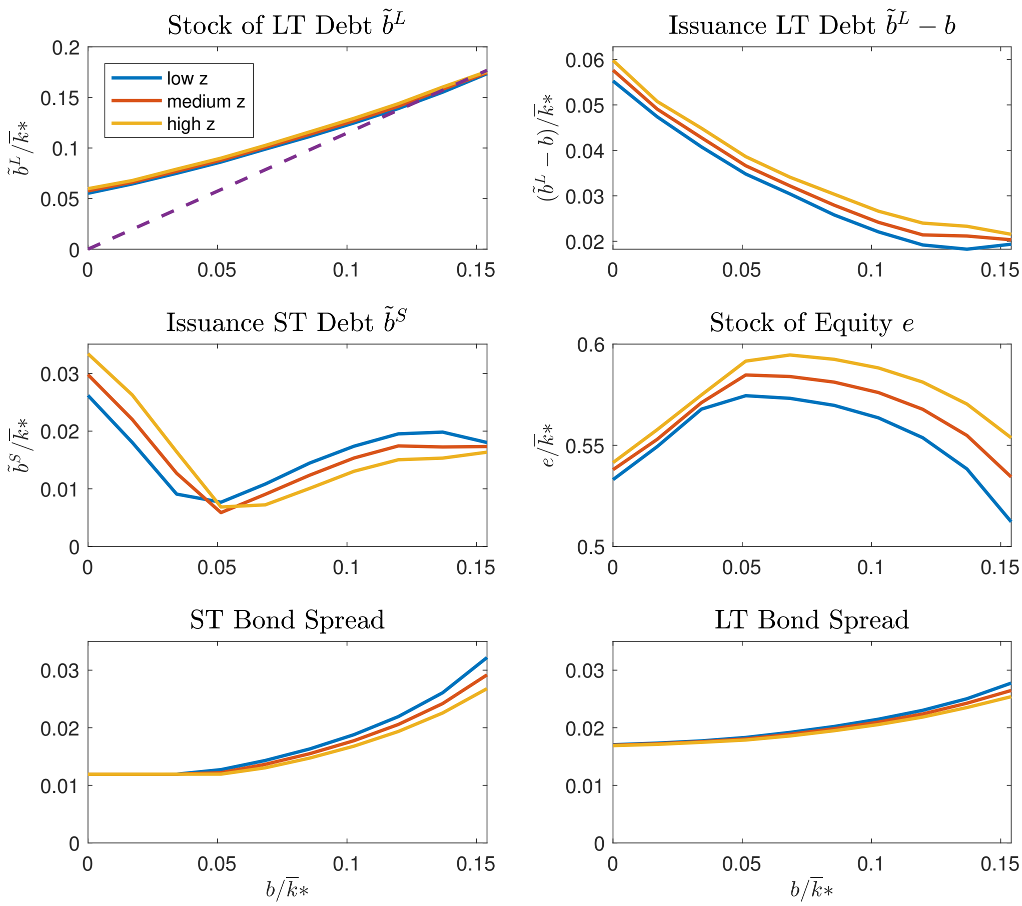

Figure 2: Policy Functions Part II

Debt and Equity are normalized by the average frictionless capital stock $\bar{k}^*$. The x-axes show the existing stock of debt b as a fraction of $\bar{k}^*$.

4.6.2. Impulse Response Functions

Figure 3 shows the response of the economy after a persistent negative shock to total factor productivity z.[12] The shock occurs in period 10. The drop in z has a direct effect on the marginal product of capital and labor and thus acts to decrease investment. As a result capital and labor inputs fall and output declines.

A second effect arises due to the feedback loop created by the financial variables. As the second to last panel indicates, during a recession the stock of outstanding debt relative to capital, b/k, falls. This is because b is a state variable that was determined in the period prior to the recession, while the optimal scale of production k was determined after z had been realized. As we showed above, the fraction b/k represents the fraction of the expected costs of default, that are disregarded by firms. When a larger fraction of default costs are not fully internalized, firms react by increasing the probability of default through an increase in leverage. This causes credit spreads for both short- and long-term debt to go up. The effect on the short-term bond spreads is stronger due to mean reversion in the productivity shock and because the firm reacts to the negative productivity shock by increasing the share of short-term to long-term debt. This generates a counter-cyclical term structure. The increase in bond spreads implies a higher cost of raising capital, further lowering investment beyond the direct effect of a fall in z. This is the adverse feedback loop referred to earlier. The additional decline in investment further decreases the ratio b/k, thereby causing an additional increase in the default probability, and credit spreads.

To summarize, due to the existence of the commitment problem associated with long-term debt, the default rate, leverage, and bond spreads increase after a negative shock to aggregate revenue productivity z. As we showed in Proposition 3.2 if all debt was short-term, the ratio b/k would be zero at all times, and the additional feedback loop would be absent.[13]

Note that the unconditional level of long-term spreads is slightly below the level of short-term spreads. In case of default, short-term creditors lose a promised payment of $1 + c$. Long-term creditors lose $\gamma + c$ in addition to future payments of value $(1 - \gamma)p^L$. Since the equilibrium price of risky long-term debt p^L is smaller than 1, long-term creditors lose less than short-term creditors. However, both of them are entitled to an identical claim on the liquidation value of the firm's assets $(1 - \xi)\tilde{k}$. For this reason, long-term creditors demand a smaller average spread in our model than short-term creditors.[14]

[12] The size of the shock is equal to one standard deviation.

[13] In Section 3 we have derived results for the case of $\xi = 0$. In the quantitative model $\xi \neq 0$, which implies that the default rate is counter-cyclical. This is because default becomes relatively more attractive during a recession, as the outside option is always equal to zero.

[14] This preferential treatment of long-term bonds also influences firms' maturity choice. In our parametrization, the quantitative significance of this channel turns out to be small. Without the transaction cost on the bond market ($\eta = 0$), firms barely issue any long-term debt.

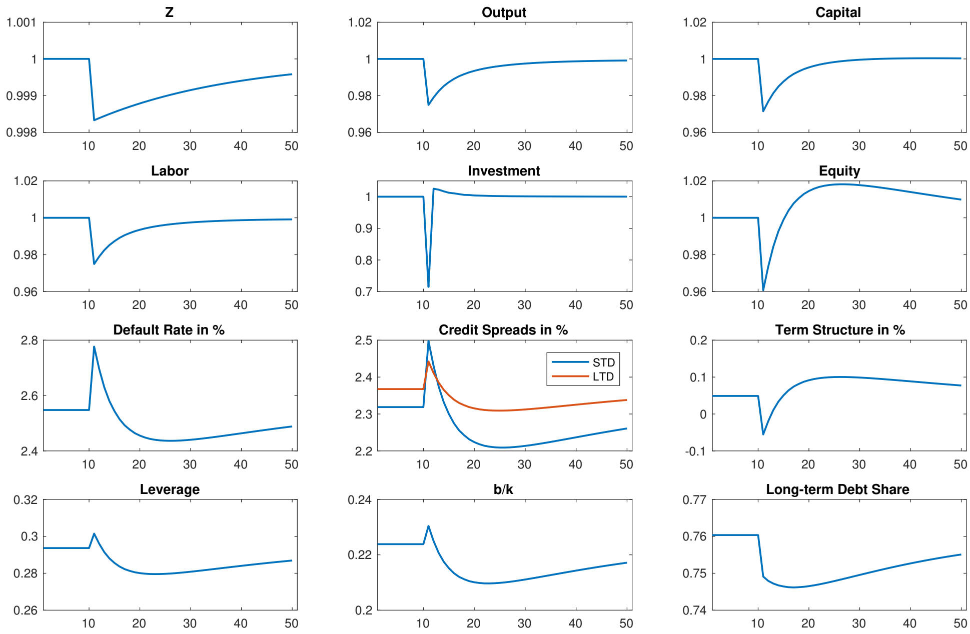

Figure 3: Impulse Response

4.6.3. Amplification

Figure 4 focuses on the response of output in Figure 3 and compares it to the output response of two alternative models following a same-sized shock to total factor productivity. The first alternative is an economy without long-term debt, i.e. an economy with $\gamma = 1$, where firms can only issue short-term debt. The second alternative we consider is an economy with frictionless capital markets and zero taxation.

The blue line in Figure 4 denoted as 'LTD' corresponds to our benchmark economy with long-term and short-term debt. In response to the negative shock to z, output falls by 2.51%. The red and yellow lines in Figure 4, respectively denoted as 'STD' and 'FL', show the response of the economy with only short-term debt and the frictionless economy. Following the negative shock to z, output falls by 1.58% in the model with only short-term debt and by 1.56% in the frictionless model. **This implies that the model with long-term debt generates an amplification of productivity shocks of 160.3% of the response of the frictionless model.** The size of the amplification is 158.5% of the response of the model where only short-term debt can be issued.

The responses of output in our benchmark economy with long-term debt is so much more pronounced than in the frictionless case or in the case with only short-term debt because of the existence of the 'Long-term Debt Accelerator'. The adverse feedback loop from a decrease in z and k to higher credit spreads works through the endogenous ratio b/k. Rising spreads feed back into low investment and less borrowing. This increases the ratio of previously issued debt to newly issued debt further, making additional debt

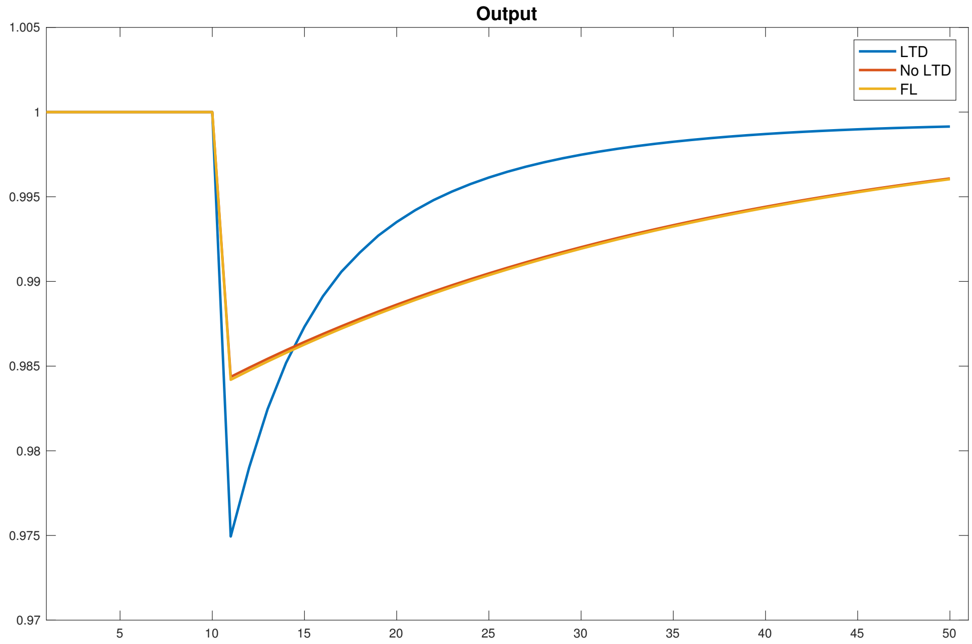

Figure 4: Amplification of a negative shock to z in the benchmark model, a model with only short-term debt, and a model with frictionless capital markets and zero taxation.

even more attractive. In models without long-term debt this mechanism is absent.

There are important non-linearities in the economy's response to shocks that arise from the setup described here. First, note that the 'Long-term Debt Accelerator' implies that larger shocks are amplified more strongly than milder shocks. This is because of the fact that larger shocks generate a larger initial decrease in k and hence a larger increase in the b/k ratio than smaller shocks. This amplifies the negative feedback loop discussed above. As a case in point, if we consider a negative shock to z equal to two standard deviations (instead of one, as above), the output effect generated by the benchmark model is 170.4% of the response of the frictionless model after a same-sized shock.[15]

Secondly, the transmission of positive and negative shocks occurs asymmetrically. This is again due to the fact that a positive shock to z will imply that the commitment problem becomes *less* severe, as the ratio b/k *falls*. A positive shock to z generates a response in output in the benchmark model that is 147.7% larger than the corresponding response in the frictionless economy.

One implication of these non-linearities is that if the fundamental volatility of the economy is lower, for example because recessions caused by negative productivity shocks occur less frequently, this can cause eventually occurring recessions to be *more severe*

[15] In this case the size of the amplification is 168.6% of the response of the model where only short-term debt can be issued.

events (see Brunnermeier and Sannikov (2014)). This is due to the fact that firms build up larger stock of debt during good times. Once a recession occurs, this increases the severity of the feedback loop arising from the existence of long-term debt.

We conclude this section by noting that the amplification mechanism discussed here is very general and is not limited to productivity shocks. Any supply or demand shock which induces a response of firm investment is amplified in the way described above.

5. Conclusion

We have introduced risky long-term debt into a standard model of firm financing and investment. This allowed us to identify a novel amplification mechanism: the Long-term Debt Accelerator. A negative shock can trigger an adverse feedback loop between low investment and high credit spreads. Relative to a frictionless RBC setup, the Long-term Debt Accelerator amplifies shocks by about 160%. This amplification mechanism is absent from standard models including only short-term debt. Furthermore, the cyclical patterns generated by the model's response to productivity shocks are in line with the empirical cyclicality of credit spreads, leverage, and debt maturity.

The model generates amplification even without the need for 'working capital' or frictions in equity issuance.

The model generates important non-linearities in the response to shocks. First, because recession worsen the commitment problem associated with long-term debt, negative shocks are amplified more strongly that positive shocks of equal magnitude. Secondly, because firms build up larger amounts of debt in booms, this sows the seeds of larger recessions in the future. Higher levels of debt imply higher b/k ratios during an eventual recession, making the commitment problem associated with long-term debt more severe and generating larger feedbacks between credit spreads, investment, and output. Even when the fundamental volatility of productivity or demand shocks decreases, the realized volatility in output can increase.

A. Proofs of Analytical Results

Proof of Proposition 3.1

The first order condition (17) associated to $\bar{\varepsilon}$ is:

$$[1-\Phi(\bar{\varepsilon})](1-\tau)k\frac{\tau c}{1+(1-\tau)c} - \varphi(\bar{\varepsilon})(1+c)\left(\frac{k+(1-\tau)\left[Ak^{\alpha}-\delta k+\bar{\varepsilon}k\right]}{1+(1-\tau)c}-b\right) = 0\,. \tag{32}$$

The marginal benefit of $\bar{\varepsilon}$ is increasing in b. As long as $\bar{\varepsilon}$ has an interior solution, it follows that this solution increases in b.

Proof of Proposition 3.2

We divide the first order condition associated to $\bar{\varepsilon}$ by k:

$$[1-\Phi(\bar{\varepsilon})](1-\tau)\frac{\tau c}{1+(1-\tau)c} - \varphi(\bar{\varepsilon})(1+c)\left(\frac{1+(1-\tau)\left[Ak^{\alpha-1}-\delta+\bar{\varepsilon}\right]}{1+(1-\tau)c}-\frac{b}{k}\right) = 0\,.$$

There are two cases to consider: (1.) $b = 0$, and (2.) $b > 0$.

1. $b = 0$. In case $b = 0$, the first order condition associated to $\bar{\varepsilon}$ is:

   $$[1-\Phi(\bar{\varepsilon})](1-\tau)\frac{\tau c}{1+(1-\tau)c} - \varphi(\bar{\varepsilon})(1+c)\left(\frac{1+(1-\tau)\left[Ak^{\alpha-1}-\delta+\bar{\varepsilon}\right]}{1+(1-\tau)c}\right) = 0\,.$$

   The first order condition associated to k can be re-arranged as:

   $$\begin{aligned} -1+\frac{1+c}{1+r}[1-\Phi(\bar{\varepsilon})]\frac{1+(1-\tau)[-\delta+\bar{\varepsilon}]}{1+(1-\tau)c}+\frac{1-\tau}{1+r}\int_{\bar{\varepsilon}}^{\infty}[\varepsilon-\bar{\varepsilon}]\,\varphi(\varepsilon)\,d\varepsilon \\ = -\frac{1+c}{1+r}[1-\Phi(\bar{\varepsilon})]\frac{(1-\tau)}{1+(1-\tau)c}\,A\alpha k^{\alpha-1}\,. \end{aligned}$$

   The two first order conditions can be combined by substituting out $Ak^{\alpha-1}$. We are left with a single equation with $\bar{\varepsilon}$ as the only endogenous variable. Neither profitability A nor capital k appear in this equation. It follows that the optimal choice of $\bar{\varepsilon}$ and the default rate $\Phi(\bar{\varepsilon})$ do not depend on A if $b = 0$. This implies that the bond price p and the credit spread are constant as well.

2. $b > 0$. The first order condition of k remains unchanged. With $b > 0$, there is an additional term added to the marginal benefit of $\bar{\varepsilon}$. This term is equal to:

   $$\frac{b}{k}\,\varphi(\bar{\varepsilon})\,(1+c)\,.$$

   This term is positive and increasing in b. By adding this term to the firm's first order condition, the solution for $\bar{\varepsilon}$ is increased. The optimal choice of $\bar{\varepsilon}$ is raised by less if k is higher:

$$\frac{\partial}{\partial k}\left[\frac{b}{k}\,\varphi(\bar{\varepsilon})\,(1+c)\right] \;=\; -\,\frac{b}{k^2}\varphi(\bar{\varepsilon})\,(1+c)\,.$$

Since the optimal choice of k is increasing in A, it follows that a high value of A implies a low value of $\bar{\varepsilon}$ if $b > 0$. It follows that $\bar{\varepsilon}$ and the default rate $\Phi(\bar{\varepsilon})$ are falling in A if $b > 0$. This implies that the bond price p rises and the credit spread falls in A.

Proof of Proposition 3.3

Consider the marginal benefit of capital MPK as given on the left hand side of first order condition (16). An increase in A affects this expression by:

$$\frac{\partial MPK}{\partial A} \;=\; \frac{1+c}{1+r}[1-\Phi(\bar{\varepsilon})]\frac{1-\tau}{1+(1-\tau)c}\alpha k^{\alpha-1} + \frac{\partial\bar{\varepsilon}}{\partial A}\,\frac{\partial MPK}{\partial\bar{\varepsilon}}\,. \tag{33}$$

From Proposition 3.2 we know that $\bar{\varepsilon}$ falls in A. A marginal increase of $\bar{\varepsilon}$ affects MPK according to:

$$\frac{\partial MPK}{\partial\bar{\varepsilon}} = -\frac{1+c}{1+r}\varphi(\bar{\varepsilon})\frac{1+(1-\tau)[A\alpha k^{\alpha-1}-\delta+\bar{\varepsilon}]}{1+(1-\tau)c} + [1-\Phi(\bar{\varepsilon})]\frac{1-\tau}{1+r}\frac{\tau c}{1+(1-\tau)c}\,. \tag{34}$$

We consider a drop in $\bar{\varepsilon}$ which is caused by an increase in A. Since $\bar{\varepsilon}$ is chosen optimally, the first order condition (17) holds:

$$[1-\Phi(\bar{\varepsilon})]\frac{1-\tau}{1+r}\frac{\tau c}{1+(1-\tau)c} \;=\; \varphi(\bar{\varepsilon})(1+c)\frac{\tilde{b}-b}{k(1+r)}\,. \tag{35}$$

A rise in $\bar{\varepsilon}$ implies a change in the marginal benefit of k of:

$$\frac{\partial MPK}{\partial\bar{\varepsilon}} = -\frac{1+c}{1+r}\varphi(\bar{\varepsilon})\frac{1+(1-\tau)\,[A\alpha k^{\alpha-1}-\delta+\bar{\varepsilon}]}{1+(1-\tau)c} + \varphi(\bar{\varepsilon})(1+c)\frac{\tilde{b}-b}{k(1+r)}\,. \tag{36}$$

It follows that a fall in $\bar{\varepsilon}$ raises the benefit of increasing k if and only if the expression above is negative:

$$\frac{1+c}{1+r}\varphi(\bar{\varepsilon})\left[\frac{1+(1-\tau)\,[Ak^{\alpha-1}-\delta+\bar{\varepsilon}]}{1+(1-\tau)c} - \frac{b}{k} - \frac{1+(1-\tau)[A\alpha k^{\alpha-1}-\delta+\bar{\varepsilon}]}{1+(1-\tau)c}\right] < 0\,. \tag{37}$$

This is the case if and only if:

$$\frac{(1-\tau)Ak^{\alpha-1}\,[1-\alpha]}{1+(1-\tau)c} < \frac{b}{k}\,. \tag{38}$$

For any positive value of b, there is a value of α sufficiently close to one such that the inequality above holds. In this case, the fact that $\bar{\varepsilon}$ falls in A amplifies the positive response of k with respect to A. Through the same mechanism, the negative output response to a fall in A is amplified by the resulting increase in $\bar{\varepsilon}$. The last step is to recall that optimal labor demand l is an increasing function of k, and output y is an increasing function of k and l.

B. Derivation of Model Variables

Table 3 defines key model variabels. In the following, we derive some of the expressions from Table 3 in more detail.

The total amount of debt is the present value of future debt payments:

$$\begin{aligned} D &= \frac{1+c}{1+r}\,\tilde{b}^S + \frac{\gamma+c}{1+r}\,\tilde{b}^L + (1-\gamma)\frac{\gamma+c}{(1+r)^2}\,\tilde{b}^L + (1-\gamma)^2\frac{\gamma+c}{(1+r)^3}\,\tilde{b}^L + ... \\ &= \frac{1+c}{1+r}\,\tilde{b}^S + \frac{\gamma+c}{1+r}\,\tilde{b}^L \sum_{j=0}^{\infty}\left(\frac{1-\gamma}{1+r}\right)^j = \frac{1+c}{1+r}\,\tilde{b}^S + \frac{\gamma+c}{\gamma+r}\,\tilde{b}^L\,. \end{aligned} \tag{39}$$

The long-term debt share is the present value of debt payments due more than one year from today divided by D:

$$\begin{aligned} &\frac{1}{D}\left((1-\gamma)\frac{\gamma+c}{(1+r)^2}\,\tilde{b}^L + (1-\gamma)^2\frac{\gamma+c}{(1+r)^3}\,\tilde{b}^L + ...\right) \\ &= \frac{1}{D}\,\frac{\gamma+c}{\gamma+r}\,\frac{1-\gamma}{1+r}\,\tilde{b}^L\,. \end{aligned} \tag{40}$$

The short-term spread compares the gross return (in the absence of default) from buying a short-term bond with the riskless rate:

$$\frac{1+c}{p^S} - (1+r)\,. \tag{41}$$

The long-term spread compares the gross return (in the absence of default and assuming p^L is constant) from buying a long-term bond with the riskless rate:

$$\frac{\gamma+c+(1-\gamma)p^L}{p^L} - (1+r) = \frac{\gamma+c}{p^L} + 1 - \gamma - (1+r)\,. \tag{42}$$

The Macaulay duration is the weighted average term to maturity of the cash flows from a bond divided by the price:

$$\mu = \frac{1}{p^L_r}\sum_{j=1}^{\infty} j\,(1-\gamma)^{j-1}\,\frac{c+\gamma}{(1+r)^j} \;=\; \frac{c+\gamma}{p^L_r}\,\frac{1+r}{(\gamma+r)^2}\,, \tag{43}$$

Table 3: Key Variables

Capital-output ratio	$\frac{k}{y}$
Total Debt	$D \equiv \frac{1+c}{1+r}\,\tilde{b}^S + \frac{\gamma+c}{\gamma+r}\,\tilde{b}^L$
Leverage: Debt / Assets	$\frac{D}{k}$
Default Rate	$\Phi(\bar{\varepsilon})$
Stock of Long-term Debt	$\frac{\gamma+c}{\gamma+r}\,\tilde{b}^L$
Share of Long-term Debt	$\frac{1}{D}\,\frac{\gamma+c}{\gamma+r}\,\frac{1-\gamma}{1+r}\,\tilde{b}^L$
Issuance Long-term Debt	$\frac{\gamma+c}{\gamma+r}\,(\tilde{b}^L - b)$
Share of Old Debt	$\frac{\gamma+c}{\gamma+r}\,\frac{b}{D}$
Labor Share	$\frac{wl}{y}$
Short-term Spread	$\frac{1+c}{p^S} - (1+r)$
Long-term Spread	$\frac{\gamma+c}{p^L} + 1 - \gamma - (1+r)$
Macaulay Duration	$\frac{1+r}{\gamma+r}$

where p_r^L is the price of a riskless long-term bond:

$$p_r^L = \sum_{j=1}^{\infty} (1-\gamma)^{j-1} \frac{c+\gamma}{(1+r)^j} = \frac{c+\gamma}{r+\gamma}\,. \tag{44}$$

It follows for the Macaulay duration:

$$\mu = \frac{1+r}{\gamma+r}\,. \tag{45}$$

C. Additional Quantitative Results

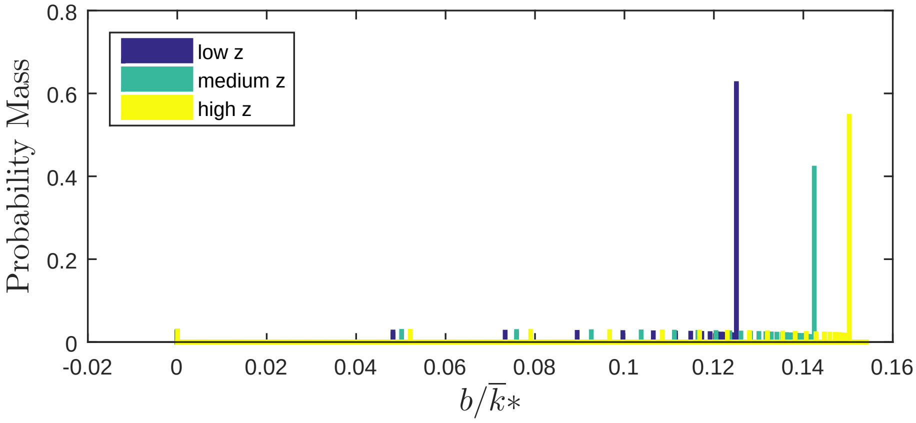

Figure 5: Firm Distribution

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