

How Safe are Central Counterparties in Derivatives Markets?*

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Abstract

We propose a general framework for estimating the likelihood of default by central counterparties (CCP) in derivatives markets. Unlike conventional stress testing approaches, which estimate the ability of a CCP to withstand nonpayment by its two largest counterparties, we study the direct and indirect effects of nonpayment by members and/or their clients through the full network of exposures. We illustrate the approach for the U.S. credit default swaps (CDS) market under shocks that are similar in magnitude to the Federal Reserve's stress tests. The analysis indicates that conventional stress testing approaches may underestimate the potential vulnerability of the main CCP for this market.

Keywords: Credit default swaps, central counterparties, stress testing, systemic risk, financial networks

JEL Classification Numbers: D85, G01, G17, L14

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1 Overview

Central counterparties have assumed a key role in clearing over-the-counter derivatives, as a result of regulatory reforms since the financial crisis (BCBS and IOSCO (2015)). In a centrally cleared market, parties to a derivatives contract enter into two back-to-back contracts with the central counterparty (CCP) that offset one another. There are several important advantages to this arrangement: it creates greater transparency and standardization of contracts, it offers greater potential for the netting of positions, and it shortens the length of intermediation chains, which in principle can reduce contagion (Evanoff et al. (2006)). It can also reduce the cost of allowing a primary dealer to default (Calomiris (2009)). A significant disadvantage is that the CCP increases systemic vulnerability by creating a critical counterparty whose default would have widespread consequences (Yellen (2013)). It is therefore crucial to understand whether CCPs can withstand large and sudden shocks to asset values, such as occurred in the crisis of 2007-09 and in the European debt crisis of 2011-12.

The conventional approach to stress testing CCPs is to examine whether they have sufficient funds on hand – in the form of initial margins and default fund contributions – to cover payment delinquencies if their two largest counterparties should fail to meet clearing obligations (CFTC (2016)). Several recent papers have argued that this standard is inadequate, because it considers only the direct impact of the two members’ failure to pay, and does not take into account network spillover and contagion effects that can amplify the initial payment shortfalls (Nahai-Williamson et al. (2013), Poce et al. (2016), Campbell and Ivanov (2016)).

We examine this issue for the U.S. market in credit default swaps (CDS) and its principal central counterparty. Although the paper is in the same spirit as Nahai-Williamson et al. (2013), Poce et al. (2016) and Campbell and Ivanov (2016), our approach differs in several key respects. First, our access to Depository Trust & Clearing Corporation (DTCC) data¹ means that we have a large representation of the network of CDS exposures at different points in time. The trade-level details provided by DTCC include all contractual positions in which the reference entity and/or one of the counterparties is U.S.-based. Second, we model contagion using a variant of the Eisenberg-Noe model (2001) that incorporates behavioral responses to balance sheet stress that can be institution

¹The DTCC data used in this paper are confidential in nature and are provided to the OFR by agreement.

specific. We then estimate the total payment deficiencies that would result from a given financial shock to the system. In particular we consider the impact of shocks that are similar in magnitude to the Federal Reserve’s 2015 Comprehensive Capital Analysis and Review (CCAR) shock, which was specifically designed to subject the financial markets to a severe but plausible market stress. Such a shock triggers a sudden drop in the value of credit instruments, which translates into large and sudden variation margin payments on CDS contracts. Firms that are large net sellers of protection may not be able to meet these variation margin payments, which puts increased stress on their counterparties and can lead to a systemwide cascade of payment delinquencies.²

This paper makes two contributions to the literature on stress testing and CCP evaluation. First, we introduce a modeling framework that accounts for the network spillover effects between the CCP’s counterparties, CCP members and client firms. These indirect effects are not considered in the stress tests run by U.S. regulators. The second contribution of the paper is to introduce a general method for estimating the likelihood that the CCP defaults *relative to* the probability that an average CCP member defaults under shocks of different magnitudes. We find that under a shock that is somewhat larger than the 2015 CCAR shock, the CCP would be at least as likely to default as a typical member. Combining this with the market-implied default probabilities of individual members, we can estimate the absolute probability that the CCP will default under various shock scenarios. This estimation technique provides a new tool for regulators to gauge the default risk of a CCP.

The plan of the paper is as follows. In the next section we summarize the protections that the CCP has in place to deal with defaults by its members (the default waterfall). Section 3 discusses the nature of the DTCC data and how we use it to derive variation margin payment demands and default probabilities under the CCAR shock. Section 4 analyzes the Cover 2 standard in the context of the CCAR shock. Although this analysis suggests that the CCP’s guarantee fund is adequate to cover the defaults of its two largest members, we show that a different conclusion emerges when we take into account network contagion. In Section 5 we introduce the contagion model, which traces

²Poce et al. (2016) study the Italian fixed income market instead of the derivatives market. They apply an exogenous shock to firms’ equity, estimate the impact on the assets of their counterparties using a Merton model, and then examine the impact on the CCP for the market in Italian government bonds (Cassa di Compensazione e Garanzia). Unlike the present study they do not have direct knowledge of firms’ network exposures, but must impute them. As in our study, however, they find that network contagion effects are substantial and imply a greater risk of CCP default than does the conventional Cover 2 standard.

how payment delinquencies by some firms can escalate as they cascade through the network of CDS exposures. A novel feature of this model is the treatment of stress transmission, which depends on firms' liquidity buffers (which we treat as random variables) and on their risk management policies. We demonstrate the sensitivity of the model to different values for the shock and stress transmission parameters in Section 7.

Section 8 develops a probabilistic model of firm default rates that takes into account a positive correlation in their default probabilities. The model is very general and allows us to estimate the probability that the CCP defaults *relative* to the probability that a member defaults while making minimal assumptions about the degree of correlation among member default rates. We use this model to estimate the probability that a CCP would default during a stress of similar magnitude to the CCAR shock. This estimate takes account of two distinct effects. First, the variation margin payments are sudden and large, which can lead to payment delinquencies due to *liquidity* constraints. Second, one or more parent firms may default due to *insolvency*, which would lead them to default on their variation margin payments. We estimate the impact on the CCP of these two effects in combination. Our conclusion is that, in a severe financial crisis of comparable magnitude to the CCAR shock, the CCP could be at least as likely to default as a typical member.

2 The central counterparty waterfall

A CCP represents a nexus of contracts in which its clearing members net and mutualize their counterparty default risk (Duffie et al. (2015)). Beyond the fees the CCP collects per transaction, the incoming and outgoing payment obligations effect each other due to its matched book. In the event that some payments are not received, the CCP has a series of risk mitigation mechanisms that it can draw on, known as the *default waterfall*. In this paper we shall focus on the major CCP for the CDS market in the United States, ICE Clear Credit. This is a privately held, for-profit company that cleared more than 97 percent of the notional value of CDS contracts on the date of our study in October of 2014.³

At the time of our study, 30 member firms were empowered (though not required) to clear their CDS contracts through the CCP. Contracts by nonmembers are permitted to cleared, but in such

³The only other CCP in this market is CME Clearing, which in 2014 cleared less than 3 percent of the contracts and has since announced its exit from the market.

cases there must be a member who acts as intermediary and fully guarantees all payments due from the nonmember *client* to the CCP. Members' balance sheets are subject to scrutiny by the CCP, they must post initial margin against their contracts according to rigorous criteria established by the CCP, and they must contribute to a common guarantee fund that can be drawn on if some members default on their payments. Indeed, there is a whole series of procedures and safeguards designed to protect the CCP in case one or more members default. The principal elements of the waterfall structure are shown in Table 1.

Table 1: Principal Elements of the Waterfall Structure of ICE Clear Credit as of December 2014.

Tranche	Total Amount
Initial Margins	\$14.1 billion
Guarantee Fund	\$2.4 billion
CCP Capital	\$50 million
Member Assessments	Up to 3 times nondefaulting members' GF contributions

Note: Initial margins and the guarantee fund are made up of U.S. Treasuries and cash (USD, CAD, EUR, GBP, JPY).

Source: SEC EDGAR 10-K Filing.

Each member's initial margin is held in a segregated account at the CCP and can only be used to cover losses generated by that member should it default. Clients also post initial margin with the CCP, and any losses they generate (including those in excess of the initial margin) are supposed to be fully covered by the member who acts as guarantor. The guarantee fund (GF) is funded solely by the members and is held in a common account to cover losses that exceed initial margins in the segregated accounts.

If a member defaults, the CCP auctions the member's portfolio of contracts, or it may transfer the contracts to non-defaulting members at mutually agreed prices. The initial margin (IM) posted by the defaulting member is applied to any losses that result from this process. In particular the IM is applied to the delinquent variation margin (VM) payments plus any further losses that may result from the auction or transfer process. (The latter may take several days to complete and will typically occur in highly stressed market conditions, hence the losses incurred in this novation process may be substantial.) To the extent that the IM is insufficient to cover the losses, the CCP

draws on the GF. If this also proves insufficient, it taps the shareholders' paid-in capital, which as Table 1 shows is very small relative to the other parts of the waterfall. If all of these sources are still insufficient, the CCP is empowered to assess the non-defaulting members by up to three times the original amount they contributed to the GF. Although this assessment power would appear to offer a substantial line of defense, in practice it is not very helpful (France and Kahn (2016)). The difficulty is that when the GF is exhausted, many of the members will already be in default and unable to pay the assessments.

3 The DTCC data and the CCAR shock

We conduct our analysis using detailed data provided to the Office of Financial Research by the Depository Trust & Clearing Corporation (DTCC). The data include all CDS transactions reported to DTCC in which at least one of the counterparties or the reference entity is a U.S. entity. We have a detailed picture of counterparty exposures for a large segment of the CDS market, including exposures between banks, dealers, hedge funds, asset managers, and insurance companies. We can apply a hypothetical credit shock and compute the value of the payment and premium legs of each CDS position as a function of spread, duration, and underlying reference entity.⁴

We focus on the change in value of each contract, and the resulting VM payment owed to each counterparty, under of the Federal Reserve's 2015 Comprehensive Capital Analysis and Review (CCAR) shock. This shock was designed to test the robustness of the financial system under a large and sudden change in asset values. The date of the shock was October 6, 2014. The shock causes a sudden decrease in the value of corporate and sovereign debt instruments, which results in large and sudden demands for VM on CDS contracts. The question this paper examines is how likely it is that the CCP could withstand a shock of approximately this magnitude. Specifically, we study the potential resilience of the CCP under shocks ranging from 0.5 to 1.5 times the magnitude of the CCAR shock.

The first step is to estimate the probability that a given member defaults under the CCAR shock. We assume that subsidiaries of the same parent firm are likely to default if and only if the parent defaults. Hence we group the members at the bank holding company (BHC) level and view

⁴For more detail on the methodology underpinning these computations see Paddrik et al. (2016).

Table 2: Members of ICE Clear Credit as of December 2014.

ICE Members			
1.a	Bank of America, N.A.	8.a	HSBC Bank USA, N.A.
.b	Merrill Lynch, Pierce, Fenner & Smith Inc.	.b	HSBC Bank plc
.c	Merrill Lynch International	.c	HSBC Securities (USA) Inc.
2.a	Barclays Bank PLC	9.a	JPMorgan Chase Bank, N.A.
.b	Barclays Capital Inc.	.b	J.P. Morgan Securities LLC
3.a	BNP Paribas	10.a	Morgan Stanley Capital Services LLC
.b	BNP Paribas Securities Corp.	.b	Morgan Stanley & Co. LLC
4.a	Citibank N.A.	11.a	Nomura International PLC
.b	Citigroup Global Markets Inc.	.b	Nomura Securities International, Inc.
5.a	Credit Suisse International	12.a	Société Générale
.b	Credit Suisse Securities (USA) LLC	.b	SG Americas Securities, LLC
6.a	Deutsche Bank AG, London Branch	13.a	The Bank of Nova Scotia
.b	Deutsche Bank Securities Inc.	14.a	UBS AG, London Branch
7.a	Goldman, Sachs & Co.	.b	UBS Securities LLC
.b	Goldman Sachs International	15.a	Wells Fargo Securities, LLC

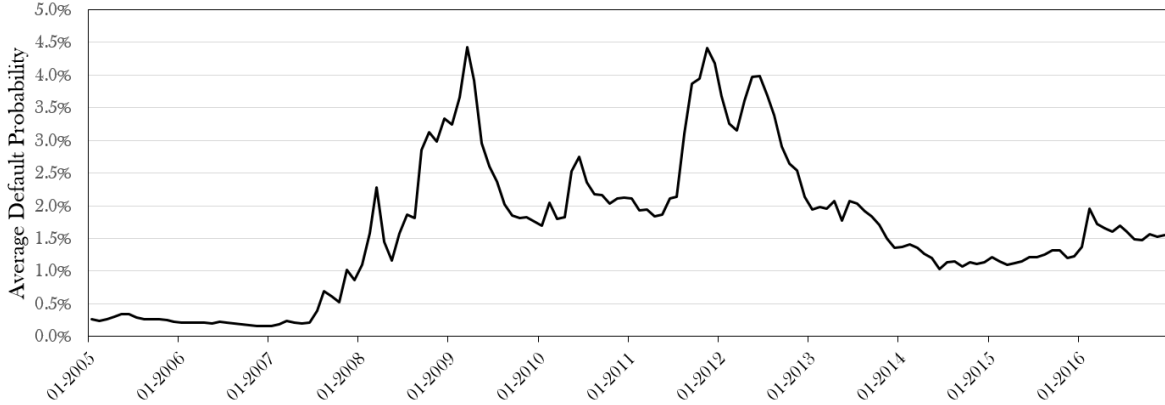
Note: Members with the same numeric value belong to the same holding company and will be treated as a single defaulting firm in our data set.

Source: ICE Clear Credit

these 15 firms as the entities that are subject to default (see Table 2).⁵ Such a shock implies a widening of credit spreads on the BHCs, from which we can infer the annual default probabilities using the methodology described in Luo (2005). On the target date of the shock the implied default probabilities among the 15 member BHCs ranged from 1.7 percent to 3.5 percent, with an average of 2.5 percent per annum. These numbers are comparable to the default probabilities implied by CDS spreads during the financial crisis of 2007-09, as shown in Figure 1.

⁵This approach is consistent with CFTC Regulation 39.33(a) on the implementation of the Cover 2 standard, which assumes that the two largest BHCs default.

Figure 1: Annual Default Probabilities Implied by CDS Spreads for the 15 BHCs.



Source: Authors' calculations, which use data provided by Markit Group Ltd.

4 Conventional risk analysis for the CCP

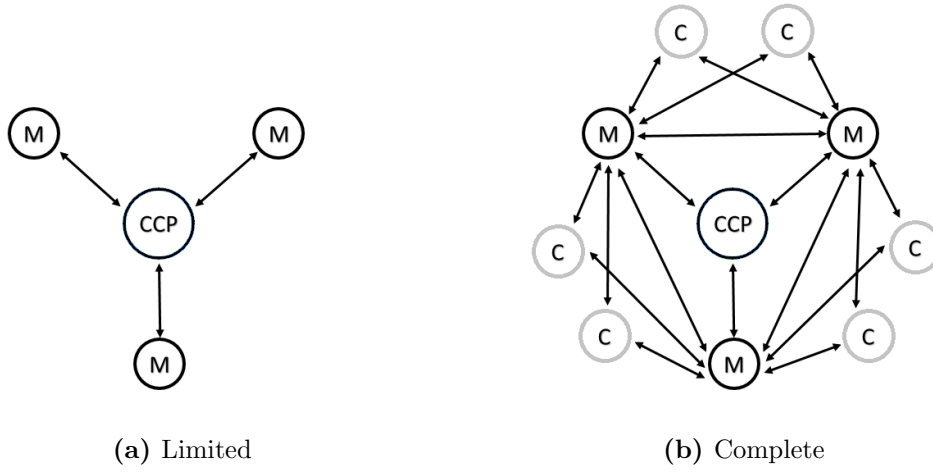
The global standard of evaluating potential risk to the CCP is to determine whether it has enough cash or highly liquid assets in its GF to cover its obligations when two of its members default simultaneously (CFTC (2016); Cont and Minca (2016); Ghamami and Glasserman (2017)). This 'Cover 2' standard is typically applied to a scenario where the two defaulting members are assumed to be those with the largest net VM obligations to the CCP.

Under the CCAR shock there are eight members that have non-negligible obligations to the CCP. Assuming that any one of them were to default, the IM collected from that member would be sufficient to cover the shortfall except in one case in which the GF would be more than adequate to absorb the remaining shortfall. (It will be recalled that the GF as of the CCAR shock date was about \$2.4 billion.) The same conclusion holds if the two largest members default simultaneously. It would therefore appear that the CCP is well-protected against defaults by its members even in a highly stressed environment.

We assert that this conclusion is overly optimistic, because it does not account for the amplification that can occur through network contagion. The left panel of Figure 2 depicts how conventional stress testing limits the channels of stress to the direct impact of a shock on the CCP. The right panel illustrates how stress can become amplified through the complete network of exposures. We shall show that when two members default simultaneously and network effects are taken

into account, there is a non-negligible probability that the CCP's GF will be insufficient to cover delinquent payments by the members. We also argue that the CCP's ability to tap its members for additional assessments will be severely limited for two reasons. First, the funds will be needed in a very short time period (typically a few hours) and the assessments may be contested. Second, many of the members will already be under severe stress and unable to pay the additional assessments.

Figure 2: Network Depiction of CCP Stress Testing



Note: M stands for member and C stands for client in the network relationships.
Source: Authors' analysis.

5 The network contagion model

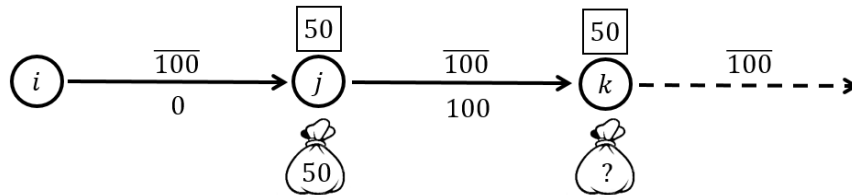
The network contagion model is a variant of Glasserman and Young (2015) that builds on the benchmark model of Eisenberg and Noe (2001).⁶ The key contribution of Eisenberg-Noe is to show how to define a consistent set of payments when firms have credit obligations to one another through interlocking balance sheets. If the assets of some firms suffer an exogenous shock, the Eisenberg-Noe framework allows one to compute the extent to which the initial loss in asset values cascades through the system, possibly leading to further defaults.

In the present context, the set-up is somewhat different: an exogenous shock to credit instruments determines the intra-day VM payment obligations between firms on their CDS contracts. These payments must be made within a very short time frame. If the VM owed by a given firm

⁶A similar model is used in Paddrik, Rajan and Young (2016) to analyze the extent to which the CCP contributes to network contagion.

exceeds the amount it is owed, the firm experiences short-term stress. This stress can be relieved by drawing on cash and cash equivalents held by the parent institution, but if the stress is large, these funds may be inadequate. In that case, the firm may either delay payments, make some payments in illiquid collateral instead of in cash, or default completely. Any of these responses will exacerbate the stress on its counterparties, leading to systemwide contagion.

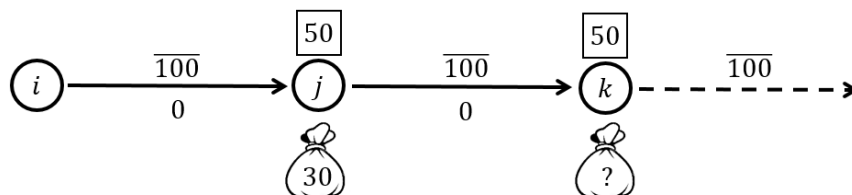
Figure 3: Payment Chain



Source: Authors' analysis.

To illustrate, consider a hypothetical situation involving three firms (i, j, k) as shown in Figure 3. Firm i owes 100 to firm j , which owes 100 to firm k , shown above the arrows. We suppose that these VM payments are triggered by a sudden exogenous shock and are due within a few hours. Suppose that i defaults completely, meaning the realized payment, shown below the arrow, is zero. Then j seizes the initial margin it collected from i (50 in the square box), but this is not enough to cover its obligations to k , which are due immediately. Hence j dips into the firm's treasury (50 in the safe bag) and meets its payment of 100. It could happen, however, that the treasury only contains 30 in liquid assets, as shown in Figure 4. In this case, j would default in its payment to k , which could cause k to default to its counterparties, depending on the amount in k 's treasury.

Figure 4: Payment Deficit Contagion



Source: Authors' analysis.

This example shows that the transmission of payment shortfalls is subject to considerable uncertainty. It depends on the amount of cash available in a firm's treasury not claimed by other sources, its non-cash assets on hand, and its relationships with its counterparties. In our network model, we shall treat these factors as random variables. The approach differs from models based on Eisenberg and Noe (2001), which treat default as a deterministic event that is triggered when the default boundary is breached.

We now describe the network model in full. Given a shock to the reference entities on which CDS contracts are written, we shall represent the induced VM payment obligations by a matrix $[\bar{p}_{ij}]$, where $\bar{p}_{ij}$ is the *net* amount of VM owed by node i to node j in the aftermath of the shock. (Thus, not both $\bar{p}_{ij}$ and $\bar{p}_{ji}$ are positive because they are bilaterally netted.) Let $\bar{p}_i = \sum_{j \neq i} \bar{p}_{ij}$ be the total payment obligation of i to all other nodes. We shall restrict attention to the nodes i such that $\bar{p}_i > 0$; the others represent firms that are solely buyers of protection and have no VM obligations. Let $i = 0, 1, 2, \dots, n$ index the nodes with positive payment obligations and let '0' represent the CCP.

The *relative liability* of node i to node j is

$$a_{ij} = \bar{p}_{ij} / \bar{p}_i. \quad (1)$$

The *relative liability matrix* $A = (a_{ij})$ is row substochastic, that is for every i , $\sum_{j \neq i} a_{ij} \leq 1$.

For each node i , let c_{ki} denote the amount of initial margin i collects from counterparty k . The purpose of the IM is to cover possible payment delinquencies. In particular, if counterparty k fails to pay VM to i in a timely manner, the position may be closed out and the IM will be applied to any losses that are incurred between the time of the counterparty's default and the time it takes to close out the position. Alternatively, i may accept partial payment by k and not close out the position, but seize the IM as security until the balance is paid. (Of course this is risky because the value of the contract to i might deteriorate further in the interim.)

Let $p_{ki} \leq \bar{p}_{ki}$ denote the realized current payment from k to i . If $p_{ki} < \bar{p}_{ki}$ the difference will be made up out of the initial margin sitting in k 's account at firm i provided $\bar{p}_{ki} - p_{ki} \leq c_{ki}$. If $\bar{p}_{ki} - p_{ki} > c_{ki}$, then the difference $\bar{p}_{ki} - (p_{ki} + c_{ki})$ must be borne by i . We define the *stress* at i , s_i , to be the amount by which i 's payment obligations exceed the incoming payments from i 's

counterparties buttressed by the initial margins, that is,

$$s_i = \left[\sum_{k \neq i} \bar{p}_{ik} - \sum_{k \neq i} ((p_{ki} + c_{ki}) \wedge \bar{p}_{ki}) \right]_+ .^7 \quad (2)$$

Note that when all of i 's counterparties pay in full, that is $p_{ki} = \bar{p}_{ki}$ for all k , then there is no stress at i ($s_i = 0$).

6 Stress transmission

To complete the model we need to specify how firms respond to balance sheet stress, that is, how much they actually pay their counterparties when they are under stress. The answer depends on a variety of factors, including a firm's cash reserves, its short-term lines of credit, the non-CDS assets on the balance sheet and its relationships with its counterparties. For most firms we do not have enough information to model these factors accurately. Instead we shall adopt a reduced-form approach in which we estimate the expected payments to counterparties as a function of balance sheet stress.

Before describing the approach, however, let us observe that for the CCP itself we do have enough information to model its payments to counterparties under different levels of stress. In particular we know how much liquidity reserves are held in its GF, which is available to cover residual losses when defaulting members' initial margins are insufficient. We also know that the balance sheet consists entirely of CDS assets, CDS liabilities, and cash reserves; there is no hedging from non-CDS positions. Finally, we know that the CCP is contractually obligated to distribute any losses pro rata among its members (a practice known as *variation margin gains haircutting*)

The difference between the CCP's VM obligations and the resources it has to meet them is given by the expression

$$s_0 = \left[\sum_{k \neq 0} \bar{p}_{0k} - \sum_{k \neq 0} ((p_{k0} + c_{k0}) \wedge \bar{p}_{k0}) \right]_+ , \quad (3)$$

where k ranges over the CCP's members. Let b_0 be the amount in the CCP's GF.⁸ Our assumption

⁷In general, $x \wedge y$ denotes the minimum of two real numbers x and y .

⁸As of December 2014, b_0 was approximately \$2.4 billion, as shown in Table 1.

is that if $s_0 > b_0$ than the CCP pro-rates the shortfall $s_0 - b_0$ over the payment obligations to its members as follows

$$\forall j, \quad p_{0j} = \bar{p}_{0j} - a_{0j}(s_0 - b_0)_+. \quad (4)$$

For firms other than the CCP we shall estimate the expected payment to counterparties using a different approach. Specifically, we shall estimate a *transmission factor*, $\tau_i \geq 0$, such that firm i 's expected shortfall in payments $\bar{p}_i - p_i$ is proportional to the level of stress s_i :

$$\forall i, \quad \bar{p}_i - p_i = \tau_i s_i \quad (5)$$

Assuming that the shortfall is apportioned pro-rata among i 's counterparties, we obtain the mapping

$$\forall i \neq 0, \forall j, \quad p_{ij} = \Phi(p)_{ij} = \left[\bar{p}_{ij} - \tau_i a_{ij} \left[\sum_{k \neq i} \bar{p}_{ik} - \sum_{k \neq i} ((p_{ki} + c_{ki}) \wedge \bar{p}_{ki}) \right] \right]_{+} \Big|_{+}. \quad (6)$$

The CCP's payments are given by

$$\forall j, \quad p_{0j} = \Phi(p)_{0j} = \left[\bar{p}_{0j} - a_{0j} \left[\sum_{k \neq 0} \bar{p}_{0k} - \sum_{k \neq 0} ((p_{k0} + c_{k0}) \wedge \bar{p}_{k0}) - b_0 \right] \right]_{+} \Big|_{+}. \quad (7)$$

The function $\Phi(p)$ defined by (6) and (7) is monotone and bounded, hence by Tarski's Theorem it has at least one fixed point (Tarski (1955)). In the empirical applications we shall specify a common value τ for the transmission factors, and then recursively compute the greatest fixed point of this system by taking the limit of the sequence $p^1 = \Phi(\bar{p}), p^2 = \Phi(p^1), \dots$

We now describe two approaches to estimating the transmission factor τ_i . The value of τ_i will depend on the level of cash that a firm can draw upon to cope with a given level of stress, and how much it pays its counterparties if its reserves are inadequate, i.e. if it cannot meet its payment obligations in full.

Let b_i denote the size of firm i 's liquid reserves. Then i can pay in full if $b_i \geq s_i$. If $b_i < s_i$ we consider two scenarios.

$$\text{Hard Default: } b_i < s_i \Rightarrow p_i = 0 \quad (8)$$

$$\text{Soft Default: } b_i < s_i \Rightarrow p_i = \bar{p}_i - s_i + b_i \quad (9)$$

Let us view the buffer b_i as the realization of a random variable B_i . This reflects the fact that the value of b_i in a crisis both depend on a variety of unknown factors such as the firm's exposure to other assets and its general risk management policies. In the hard default scenario, the expected shortfall in i 's VM payments equals

$$d_i = \bar{p}_i P(b_i < s_i). \quad (10)$$

We can make a rough estimate of the latter as follows. Since the net amount owed, $\bar{p}_i$, arises in a highly stressed environment, it is unlikely that the firm's cash reserves are larger than $\bar{p}_i$. Let us therefore assume that the support of B_i is $[0, \bar{p}_i]$. Let us further suppose that B_i has a density $g_i(b_i)$ that is nonincreasing, that is, smaller buffers are at least as likely as larger buffers. It follows that for every realized level of stress s_i ,

$$P(b_i < s_i) \geq s_i / \bar{p}_i. \quad (11)$$

Together, (10) and (11) imply

$$d_i \geq s_i, \quad (12)$$

that is, the transmission factor is at least one.

By contrast, in the soft default scenario i 's expected shortfall in payments can be expressed as

$$d_i = E[s_i - b_i | b_i < s_i] P(b_i < s_i). \quad (13)$$

Given the above assumption on $g(b_i)$ this leads to an estimated transmission factor that is less than one.

An alternative approach to estimating the transmission factor is to estimate the liquidity buffers

b_i directly from the data and then to solve the model under various assumption about the size of the shock and how firms respond to stress. From the DTCC data we can estimate, for each firm i , the minimum amount of b_i of cash reserves that i *would have been needed* to avoid suffering a shortfall in its CDS payments on any given day in the five years prior to the shock date. Specifically, for each firm i , the DTCC data allows us to infer the change in value of i 's CDS contracts on any given day. From this we can deduce the payments due *from* all of i 's counterparties, as well as the payments due from i *to* each of its counterparties and hence the *net* VM payment owed by i on that day. We choose b_i to be the smallest value that exceeds the maximum net payment due over the five year period prior to October 6, 2014.

Given a shock of size α on the CCAR shock date, we compute the greatest payment equilibrium under two stress response scenarios: soft default and hard default. In the soft default scenario we find the maximal fixed point of the system

$$\forall i \neq 0, \forall j, \quad p_{ij} = \Phi(p)_{ij} = \left[\bar{p}_{ij} - a_{ij} \left[\sum_{k \neq i} \bar{p}_{ik} - \sum_{k \neq i} ((p_{ki} + c_{ki}) \wedge \bar{p}_{ki}) - b_i \right] \right]_{+} \quad (14)$$

$$\forall i, \quad p_{0j} = \Phi(p)_{0j} = \left[\bar{p}_{0j} - a_{0j} \left[\sum_{k \neq 0} \bar{p}_{0k} - \sum_{k \neq 0} ((p_{k0} + c_{k0}) \wedge \bar{p}_{k0}) - b_0 \right] \right]_{+} \quad (15)$$

Let p^* denote this greatest equilibrium. We then compute the total payment deficiency $d^* = \sum_{0 \leq i, j \leq n} (\bar{p}_{ij} - p_{ij}^*)$ and the total stress $s_i^* = \sum_{i=0}^n s_i^*$, where

$$\forall i, \quad s_i^* = \left[\sum_{k \neq i} \bar{p}_{ik} - \sum_{k \neq i} ((p_{ki}^* + c_{ki}) \wedge \bar{p}_{ki}) \right]_{+} \quad (16)$$

The ratio

$$\tau^* = d^*/s^* \quad (17)$$

is an estimate of the average rate of stress transmission over all nodes in the system in the soft default scenario.⁹

Similarly, in the hard default scenario, we compute a maximal fixed point of the system

⁹Alternatively we could estimate the average transmission factor by the expression $(1/n) \sum_{i=0}^n (\bar{p}_i - p_i^*)/s_i^*$. It can be shown that this yields a value at least as large as τ^* in (17).

$$\forall i \neq 0, \forall j, \quad p_{ij} = \tilde{\Phi}(p)_{ij} = \begin{cases} \bar{p}_{ij} & \text{if } \sum_{k \neq i} \bar{p}_{ik} \leq \sum_{k \neq i} ((p_{ki} + c_{ki}) \wedge \bar{p}_{ki}) + b_i \\ 0 & \text{otherwise,} \end{cases} \quad (18)$$

$$\forall j, \quad p_{0j} = \tilde{\Phi}(p)_{0j} = \left[\bar{p}_{0j} - a_{0j} \left[\sum_{k \neq 0} \bar{p}_{0k} - \sum_{k \neq 0} ((p_{k0} + c_{k0}) \wedge \bar{p}_{k0}) - b_0 \right] \right]_{+}^{10} \quad (19)$$

Let p^{**} denote the greatest equilibrium of this system. As before we compute the total payment deficiency $d^{**} = \sum_{0 \leq i, j \leq n} (\bar{p}_{ij} - p_{ij}^{**})$ and the total stress $s_i^{**} = \sum_{i=0}^n s_i^{**}$, where

$$\forall i, \quad s_i^{**} = \left[\sum_{k \neq i} \bar{p}_{ik} - \sum_{k \neq i} ((p_{ki}^{**} + c_{ki}) \wedge \bar{p}_{ki}) \right]_{+} \quad (20)$$

The estimate of τ in this case is

$$\tau^{**} = d^{**} / s^{**}. \quad (21)$$

Table 3 shows the resulting estimates for τ under a range of α values and hard vs. soft default. Not surprisingly, the average value of τ under soft default is somewhat smaller than it is under hard default. In the latter scenario the value is somewhat in excess of one, which is consistent with our earlier estimate in (12).

Table 3: Average value of τ over all firms when liquidity buffers are estimated from the DTCC data.

	α		
	0.75	1.00	1.25
Hard Default:	1.11	1.11	1.07
Soft Default:	0.54	0.62	0.69

Source: Authors' calculations which use data provided to the OFR by The Depository Trust & Clearing Corporation and Markit Group Ltd.

¹⁰We always assume that the CCP engages in variation margin gains haircutting (i.e., soft default) even when all other firms engage in hard default, due to the CCP's contractual obligations to its members.

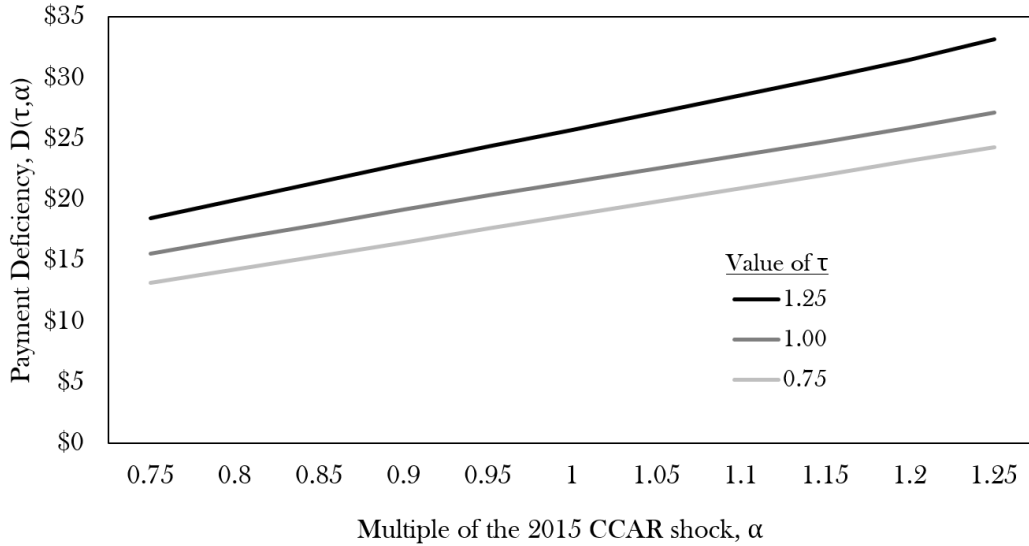
7 Sensitivity analysis

We now apply this framework to evaluate the potential amount of contagion in the CDS market that would be produced by a shock of size α over a range of values for τ .¹¹ We define the total *impact* of the shock to be the total deficiency in VM payments summed over all directed edges in the network. In our notation the total deficiency can be expressed as follows

$$D = D(\tau, \alpha) = \sum_i d_i, \quad (22)$$

where $d_i = \bar{p}_i - p_i$ and $(p_0, \dots, p_n)$ is the equilibrium payment vector representing the greatest fixed point of the mapping $\Phi(p)$. For various values of τ and α we compute the payment deficiency $D = D(\tau, \alpha)$; the results are shown in Figure 5. Figure 5 shows that the total deficiency $D(\tau, \alpha)$ is increasing in both τ and α , and is nearly linear in α over the range of values under consideration.

Figure 5: Payment Deficiency, $D(\tau, \alpha)$



Source: Authors' calculations which use data provided to the OFR by The Depository Trust & Clearing Corporation and Markit Group Ltd.

The application of a common value τ to all nodes in the system is purely for illustrative purposes. In reality, the values τ_i will depend on the risk management policies and liquidity buffers of

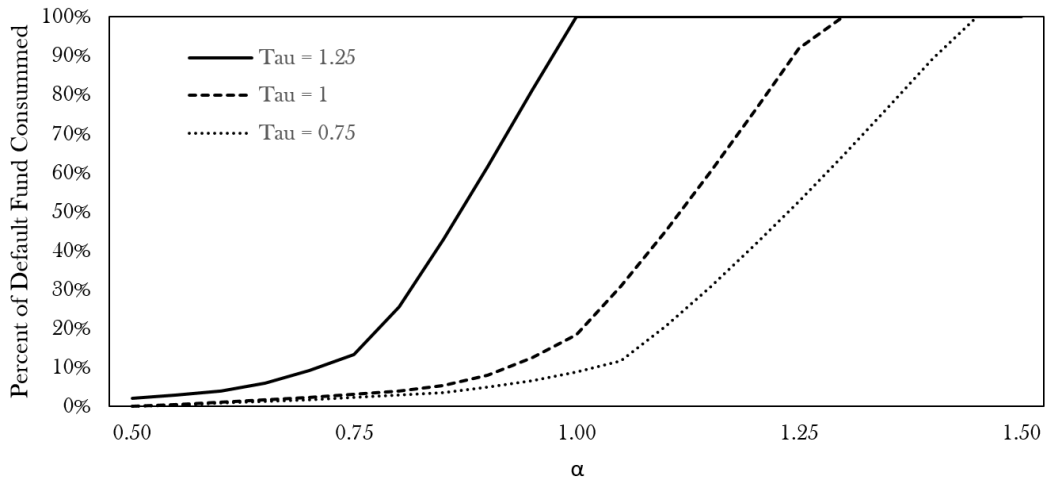
¹¹For simplicity we assume here that all transmission factors have the same value τ , except for the CCP where transmission factor is $\tau_0 = 1$.

particular firms. We do not have enough information to pin down these values precisely but the curves in Figure 5 give some idea of the magnitude of the impact for different levels of τ and α .

8 Evaluating the effect of member defaults from other causes

The preceding results suggest that the CCP may be reasonably well protected against shocks that are comparable in magnitude to the Federal Reserve’s 2015 global trading book shock. However, this analysis is based solely on payment obligations arising from the CDS contracts, and neglects the possibility that, in a severe financial crisis, member firms might default due to stresses on *other* parts of their balance sheets. In the section we consider the effects of such exogenous failures without attempting to model their causes explicitly. Instead we shall ask how many defaults of member firms would cause the CCP to default given that it is already under stress due to CDS payment demands.

Figure 6: CCP of Guarantee Fund Usage when τ is Varied.



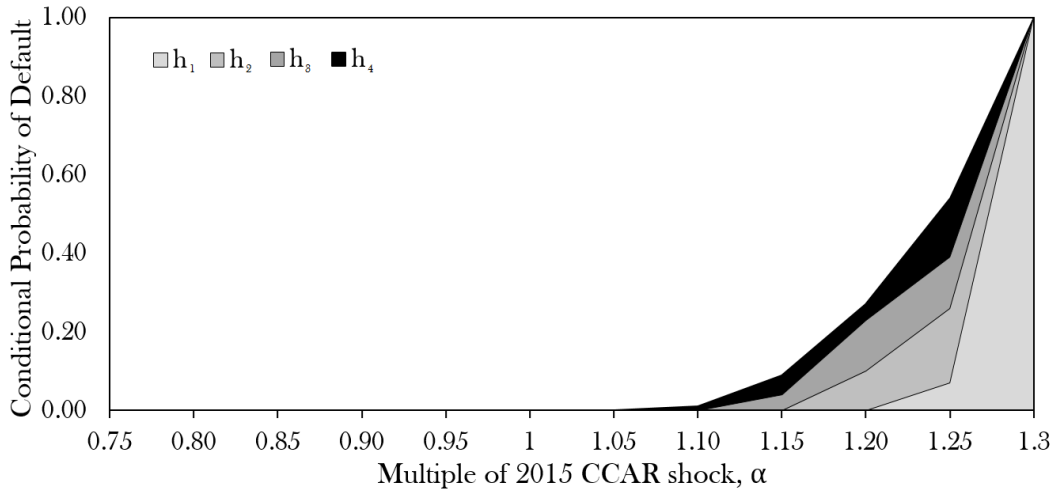
Source: Authors’ calculations, which use data provided to the OFR by The Depository Trust & Clearing Corporation and Markit Group Ltd.

Figure 7 shows the impact on the CCP when zero, one, two, three, and four member firms are drawn at random and assumed to default completely on their VM payments. We assume here that the transmission factor $\tau = 1$ for all firms, but the same approach can be applied with other values of τ including values that vary among firms. Let us fix the value of α . For each integer $k = 0, 1, 2, 3$, or 4 , draw k firms at random from the 15 BHC member firms, and assume that these firms

fail due to unmodelled exogenous causes. For each such draw we compute the greatest payment equilibrium under the assumption that these k firms default completely on their VM payments, while all other firms have a transmission factor $\tau = 1$. If the CCP guarantee fund is used up we say that it defaults for this random draw. We then count the proportion of $\binom{15}{2}$ draws in which the CCP defaults, which yields the curve $h_k(\alpha)$.

Figure 7 shows that when the scale of the shock is less than 1.1, the CCP does not default even when four of its members fail exogenously. This finding is reassuring. For the larger shocks, however, exogenous member failures can push the CCP into default. A priori we do not know how likely such exogenous failures might be; nevertheless we can estimate the probability that the CCP defaults relative to the failure probability of an average member using the following argument.

Figure 7: Conditional Probability of CCP Default when $\tau = 1$.



Note: Conditional probability of the CCP suffering a default given one, two, three or four BHC members default, assuming the transmission factor $\tau = 1$.

Source: Authors' calculations, which use data provided to the OFR by The Depository Trust & Clearing Corporation and Markit Group Ltd.

Fix a value of α (the scale of the shock), and let $q_k(\alpha)$ be the probability that exactly k out of the 15 member fail under a shock of this magnitude. As above let $h_k(\alpha)$ be the conditional probability that the CCP defaults given the default of k of its member drawn at random. The average probability that a given member defaults can be written

$$p(\alpha) = \sum_{k=0}^{15} k q_k / 15. \quad (23)$$

The probability that the CCP defaults is

$$q(\alpha) = \sum_{k=0}^{15} h_k(\alpha) q_k(\alpha). \quad (24)$$

We wish to estimate the ratio $q(\alpha)/p(\alpha)$; in particular we shall bound it from above and below under mild restrictions on the probabilities $q_k(\alpha)$. Specifically let us assume that they are nonincreasing in k ($q_1(\alpha) \geq q_2(\alpha) \geq \dots \geq q_{15}(\alpha)$), and for sufficiently large k the $q_k(\alpha)$'s are close to zero.¹²

To bound q/p from below we solve the optimization problem

$$\min q(\alpha)/p(\alpha) = \frac{15 \sum_{k=0}^{15} h_k(\alpha) q_k(\alpha)}{\sum_{k=0}^{15} k q_k(\alpha)}, \quad (25)$$

subject to

$$\begin{aligned} 1 \geq q_0(\alpha) \geq \dots \geq q_{15}(\alpha) \geq 0 \\ \forall k \geq \bar{k}, q_k = 0 \end{aligned} \quad (26)$$

An upper bound on $q(\alpha)/p(\alpha)$ is found by solving the corresponding maximum problem. To illustrate the approach in a concrete case, let $\alpha = 1.25$ (which we omit from the notation below). The conditional default probabilities are

$$h_0 = 0.00, \quad h_1 = 0.07, \quad h_2 = 0.26, \quad h_3 = 0.39, \quad h_4 = 0.54.$$

Let us further assume that $q_k = 0$ for all $k \geq \bar{k} = 4$. (This is a conservative assumption; in fact the U.S. authorities did not allow more than two large institutions to default during the recent financial crisis.)¹³ By solving the corresponding optimization problem in (25) - (26) we find that

¹²One of the few attempts to estimate joint failure probabilities of BHC's is discussed in Giglio (2011). His data suggest that during the 2007-09 crisis, q_2 was almost an order of magnitude smaller than q_1 , and q_3 was about half the size of q_2 . (These estimates are based on Figure 10 in Giglio (2011) over the period of July 2008 through June 2009.) The inequalities in (26) are broadly consistent with those results

¹³Two large firms defaulted (Lehman and Bear Stearns) and two others (Merrill Lynch and AIG) came close to default but were rescued by the authorities. The evidence from the crisis suggests that it is highly unlikely that three or more large firms would be allowed to default. Moreover the market implied default rates during the crisis also

$q(\alpha)/p(\alpha)$ satisfies the bounds

$$1.05 \leq q/p \leq 1.89. \quad (27)$$

These bounds are not particularly sensitive to the cut-off value $\bar{k}$. For example, suppose that $q_k = 0$ for all $k \geq 5$. It can be shown that in this case $q(\alpha)/p(\alpha)$ satisfies precisely the same bounds as in (27). If we assume that $q_k = 0$ for all $k \geq 6$, we obtain the bounds $1.05 \leq q/p \leq 2.11$.

Table 4 shows the estimated bounds for $q(\alpha)/p(\alpha)$ under different combinations of τ and α , assuming that $q_k = 0$ for all $k \geq 4$. Note that when either τ or α is 0.75 or less, the CCP is unlikely to default. However if the shock is somewhat larger than the CCAR shock ($\alpha \geq 1.25$) and if $\tau \geq 1.25$, the CCP would be at greater risk of defaulting than the average member firm.

Table 4: Estimated Bounds of $q(\alpha)/p(\alpha)$

		α		
		0.75	1.00	1.25
τ	0.75	0.0	0.0	0.0-0.3
	1.00	0.0	0.0	1.1-1.9
	1.25	-	7.5-15	7.5-15

Source: Authors' calculations, which use data provided to the OFR by The Depository Trust & Clearing Corporation and Markit Group Ltd.

9 Conclusion

In this paper we have proposed a general framework for assessing the ability of a central counterparty (CCP) to withstand a severe credit shock. The framework differs from conventional stress testing of CCPs in several key respects. First, we consider the possible impact of one or more members default, not just the two members with the largest net exposures to the CCP. The default of the largest two members is an event that may have a large direct impact but a smaller probability than an event in which one or more members default at random, hence the expected default rate may be underestimated. Second, we consider the impact of the defaulting member(s) on all their counterparties, including other members of the CCP as well as non-members. This is crucial because payment deficiencies by the defaulting member will be transmitted and amplified through

assigned a low probability to four or more defaults (Giglio (2011)).

the network of exposures, increasing the ultimate impact on the CCP. Third, we propose a novel estimation methodology that allows us to place a lower bound on the amount of network contagion in the absence of detailed information about the liquid reserves of individual firms. In this respect the model differs markedly from conventional contagion models such as Eisenberg and Noe, which require detailed knowledge of the firms' balance sheets to determine the ultimate impact of a shock. Finally, we show how to estimate bounds on the probability that the CCP will default *relative* to the probability that an individual member will default, with minimal information about the degree of correlation among member defaults, which in practice is difficult to estimate empirically.

Overall, our results suggest that conventional stress testing approaches may underestimate the potential vulnerability of the main CCP for this market. The results do not include several channels that could further increase the amount of contagion and the concomitant risk of CCP default. One such channel is increased demand for initial margin in times of financial stress when firms may demand that their counterparties increase the amount of initial margin they post. These demands have the effect of placing the counterparties under even greater stress. A second channel consists of single-name CDS contracts in which the reference entity is a bank holding company (BHC). The stress induced by demands for large variation margin payments may push some of these BHCs closer to (or into) default, which will trigger large variation margin payments by the sellers of CDS on these companies. A third channel is fire sales: when a seller of protection defaults on its variation margin payments, the counterparty will try to find another (solvent) firm that is willing to assume the position of protection seller. This expansion of supply in the face of weak demand will tend to increase the cost of novating the contracts and lead to potential losses that are not covered by the defaulting party's initial margin. All of these effects can, in principle, be incorporated into the model, and will exacerbate the amount of contagion in the system. The framework we have presented is a simple way of estimating a *lower* bound on the amount of contagion and its potential impact on central counterparties.

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