A Network Model for Financial Stability Monitoring*

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Extended Abstract.
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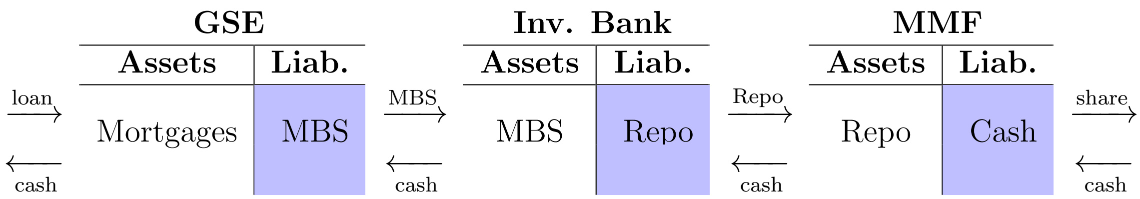
I. Introduction

Monitoring and understanding financial system "vulnerabilities"—the potential for externalities in the financial system—is a major concern for policymakers (Adrian et al., 2015). We propose a network model of financial intermediaries' interconnectedness as a theoretical background to monitoring the stability of the financial system. The amplification of shocks through our network is a financial vulnerability. The macroeconomic consequences of addressing such vulnerabilities is especially important, as policymakers consider the costs and benefits of particular policies. Our parsimonious framework includes asset prices, which allow us to obtain an overall price of risk in the economy. We derive the asset pricing implications of the tradeoff between the benefits from financial system connections and the costs associated with those connections. In general, a policymaker that maximizes the representative agent's utility prefers a more interconnected system when the volatility of shocks is small and a less-connected network when the volatility of shocks is large. However, we show that the price of risk in less-connected networks is higher during times of low volatility and lower during times of high volatility.

As a specific example of the economic connections we have in mind, consider the chain of financial intermediation that links money market funds (MMFs) to Government Sponsored Enterprises (GSEs) through investment banks, shown in Figure 1. By diversifying across investors, each of these institutions can lower the cost of intermediation associated with, for example, asymmetric information. However, the benefit of these connections comes with potentially costly vulnerabilities. Each MMF has many shareholders, attracted by the promised return on investment in commercial paper as well as the ability to withdraw their investment on demand. The MMF can handle idiosyncratic withdrawals, but is nonetheless vulnerable to runs (Schmidt et al., 2016). Similarly, an individual investment bank is also vulnerable to run risk (Copeland et al., 2014). Lastly, the GSEs may themselves be exposed to rollover risk related to the quality of the underlying collateral (Jaffee, 2010).

Figure 1. Example chain of intermediation



Here are the main characteristics of our network model. The nodes in our network are stylized representations of financial intermediaries whose payoff depends both positively and negatively on their connections to other intermediaries. Each intermediary receives a payoff that increases with the number of its connections, as a consequence of the lower cost of financial intermediation. At the same time, the payoff is decreasing in the number of connections because denser networks can propagate large shocks more swiftly.

The network of intermediaries is assumed to be random, both for simplicity and because the network vulnerabilities of the financial system are generally difficult to anticipate. The random network is a reduced form representation of the interconnectedness of the financial system. For example, two intermediaries may be connected because they trade short-term financial assets or derivatives. In an alternative interpretation, the random network could represent indirect exposures.[1]

Shocks to the financial system are amplified through the random network. In our model, a shock is defined as an initial fraction of intermediaries that enter distress and receive a zero payoff. A larger shock to the financial system corresponds to a greater fraction of intermediaries that enter distress. The size of the initial shock is drawn from an exogenous distribution. Subsequently, surviving intermediaries will also enter distress if the fraction of their connected intermediaries that are in distress exceeds a threshold. Thus, the combination of shocks and amplification through the network determines the overall price of risk in the economy.

[1] See, for example, Brunnermeier and Oehmke (2013).

To obtain the overall price of risk in the economy, we assume that a representative agent buys the intermediaries. We also refer to the intermediaries as assets because the representative agent can consume the intermediaries' payoffs. The representative agent holds all the assets, so the price for each asset can be generalized to obtain the representative agent's "price of risk" associated with the aggregate payoffs. We derive the relationship between the general price of risk and the volatility of shocks in the financial system by running comparative statics.[2]

Our paper contributes to several strands of literature. Our model is closely related to the financial stability monitoring framework described by Adrian et al. (2015). We provide a microfoundation for the relationship between the price of risk, the vulnerabilities of the financial system associated with interconnectedness, and the volatility of economic shocks.

Our paper contributes to the extensive literature on networks and financial stability, including Acemoglu et al. (2015), Castiglionesi et al. (2017), Ramirez (2017), and the models of cascading failures of financial institutions in Elliott et al. (2014) and Cabrales et al. (2014). A key contribution of our paper is the overall price of risk in the economy. As such, we also contribute to the literature on asset pricing in a network model, including Martin (2013).

II. Baseline Model

A. The environment

Consider a one-period endowment economy populated by financial intermediaries and investors with a single consumption good. We assume there is a large number of identical individuals whose decisions can be aggregated into a representative investor. The representative investor has preferences over the consumption good that are given by $u(\cdot)$, where $u(\cdot)$ is strictly increasing, strictly concave, twice continuously differentiable, and satisfies the Inada

[2] We denote as "good times" the periods when the volatility of shocks is low and as "bad times" the periods when the volatility of shocks is high.

conditions.

There are N intermediaries, indexed by $i = 1, \cdots, N$. Each intermediary is connected to other intermediaries, indexed by $j = 1, \cdots, J$. Let $\widetilde{x}_i$ denote the random end-of-period payoff to holding a claim on intermediary i (in units of the consumption good), with $\widetilde{x}_i = \sum_{j=1}^{J} \mathbb{1}(\widetilde{x}_j > 0)$ if i does not enter distress, and 0 otherwise. The randomness in the payoff stems from both the unknown network and from the shocks that are drawn from a distribution.

The connections among intermediaries are determined at random. These connections increase the payoff to the intermediary and also determine whether or not distress at individual intermediaries propagates elsewhere in the financial system. We assume the network structure follows a Bernoulli process, for simplicity. The probability that two intermediaries are connected is a constant $p \in (0, 1)$.

The propagation of distress is determined by the following stochastic process. First, a random fraction q of intermediaries initially enters distress. We assume that the fraction follows a beta-distribution, $q \sim \{\alpha;\ \beta\}$. To ensure that the variance is the driving factor for our results, we restrict our distribution of shocks to those with a constant mean: $E(q) = \nu$. We can relax this assumption after we are sure that our intuition carries through the variance.

Second, distress spreads via vulnerabilities. Intermediary i enters distress if the fraction of intermediaries that are directly connected to i are in distress. Let ϕ denote the fraction of intermediaries directly connected to i that are in distress and let $\bar{\phi}$ be the threshold fraction of intermediaries in distress that results in intermediary i also entering distress. The payoff to holding a claim on the intermediary i can be rewritten as:

$$\widetilde{x}_i = \begin{cases} \sum_{j=1}^{J} \mathbb{1}(\widetilde{x}_j > 0) & \text{if } \phi < \bar{\phi} \text{ and } i \text{ did not randomly enter distress,} \\ 0 & \text{otherwise.} \end{cases}$$

where $\bar{\phi} = E(q) + (Var(q))^{\frac{1}{2}}$, which is the mean plus one standard deviation of the distribution of intermediaries that enter distress. One interpretation of this assumption is that intermediary i has insulated itself from a range of shocks to its network of intermediaries, but nonetheless remains vulnerable to large shocks. An alternative interpretation is that intermediary i faces microprudential risk management on, say, its value at risk.[3]

B. *Equilibrium Asset Pricing*

Let $p_i \geq 0$ denote asset i's beginning-of-period equilibrium price that pays out $\widetilde{x}_i$ at the end of the period. If $p_i > 0$, the gross return of asset i is defined as:

$$\widetilde{R}_i = \frac{\widetilde{x}_i}{p_i}. \tag{1}$$

The representative investor owns all assets in the economy and consumes all the payoffs. There is a single-factor pricing model in equilibrium with $u'\left(\sum_{i=1}^N \widetilde{x}_i\right)$ as the factor. As a consequence, the expected gross return of asset i satisfies

$$\mathbb{E}[\widetilde{R}_i] \quad = \quad R_f + \lambda \frac{\mathrm{Cov}\left(\widetilde{R}_i, u'\left(\sum_{i=1}^N \widetilde{x}_i\right)\right)}{\mathrm{Var}\left(u'\left(\sum_{i=1}^N \widetilde{x}_i\right)\right)}, \tag{2}$$

where

$$R_f \equiv \frac{1}{\mathbb{E}\left(u'\left(\sum_{i=1}^N \widetilde{x}_i\right)\right)} \quad \text{and} \quad \lambda \equiv -\frac{\mathrm{Var}\left(u'\left(\sum_{i=1}^N \widetilde{x}_i\right)\right))}{\mathbb{E}\left(u'\left(\sum_{i=1}^N \widetilde{x}_i\right)\right)},$$

and R_f denotes the gross return of the risk-free asset, which pays one unit of the consumption good with certainty at the end of the period.

The price of risk in our economy is λ. Intuitively, it corresponds to the value that the representative agent places on assets that covary (negatively) with the marginal utility of the consumption process, determined here by the aggregate payoff to financial intermediaries.

[3] The effect of financial intermediaries' risk capital on asset prices has also been explored by Gabaix et al. (2007), Garleanu et al. (2009), and Greenwood and Vayanos (2014).

C. *Optimization problem*

The optimization problem of our model has two steps. In the first step, the representative investor maximizes their utility while taking the processes that generate the network and the shocks as given:

$$\max_{c} \mathbb{E}\left[U(c) \mid p^{*},\ \alpha,\ \beta\right] \tag{3}$$

where $c = \sum_i \widetilde{x}_i$ and p^* is the solution to the policymaker's problem. The equilibrium asset price derived above is obtained from the optimization problem of the representative investor.

In the second step, the policymaker chooses the Bernoulli parameter for the network process (p) to maximize the representative investor's utility:

$$\max_{p} \left\{ \max_{c} \mathbb{E}\left[U(c) \mid p^{*},\ \alpha,\ \beta\right] | \alpha, \beta \right\}. \tag{4}$$

D. *Numerical solution*

We are interested in how the volatility of shocks affects the overall price of risk in the economy and the policymaker's optimal choice of linkages between intermediaries. Figure 3 shows the numerical solution to the model for a range of parametric values of the variance of the distribution of shocks. We fix the mean of the distribution of shocks to $E(q) = \nu = 0.3$ so that all the action is driven by the variance. The upper-left panel shows three example distributions for $\alpha = 1,\ 1.3,$ and 1.5. The variance of the beta distribution ($V[q]$), shown in the upper-right panel, declines monotonically over the range of $\alpha \in [1, 1.5]$. The monotonic relationship means that we can refer to the volatility of shocks and α interchangeably.

The middle panels show the representative agent's expected utility as a function of the variance and the network density parameter p. The three-dimensional plot in the middle-left panel indicates that the network density that maximizes the agent's expected utility depends importantly on the variance of the shock distribution.

The bottom panels show the representative consumer's price of risk and the policymaker's optimal network density as a function of volatility. The bottom-left panel indicates that a more resilient network (lower p) corresponds to a flatter price of risk as a function of volatility, with a higher price when the volatility of shocks are low, as in Adrian et al. (2015) and shown in Figure 2. The optimal policy, as shown in the bottom-right panel, is to have a high p^* when the volatility of shocks is low, and a low p^* when the volatility of shocks is high[4]

All of the ingredients of the model are necessary to obtain these results. The sharp increase in expected utility at low levels of p arises from the positive effects on the payoff function when consumption is low (Inada condition). The dependence of the $\bar{\phi}$ propagation threshold on the $q-$distribution of shocks generates variation in the local maximum of expected utility when p is low. Beyond this local maximum, raising p will initially lower expected utility by increasing propagation, and subsequently increase expected utility as propagation occurs for (almost) all shocks while the agent can benefit from higher payoffs when shocks are small.

E. Further work

- Imagine the economy is (Markov) switching between states of low and high volatility. If the current state were unknown, what would be the optimal policy?
- Calibrate the model to a financial (sub)network and obtain the optimal network density.
- What would happen if the network structure followed a Weibull process (power law) rather than a Bernoulli process?
- Are there tradeoffs for the policymaker considering $\bar{\phi}$ (microprudential regulation) or p (macroprudential regulation)?
- What is the microfoundation for the payoff function ($\widetilde{x}$)? Could the intermediaries choose $\bar{\phi}$ as a function of p as well as q?

[4]We think that the jump in optimal policy at $V[q] \approx 0.04$ (corresponding with $\alpha \approx 1.3$) occurs because our approximation of the q-distribution suddenly shifts more weight to the very low end of shocks (q almost zero). We need to confirm this intuition.

REFERENCES

Acemoglu, Daron, Asuman Ozdaglar, and Alireza Tahbaz-Salehi, 2015, Systemic risk and stability in financial networks, *American Economic Review* 105, 564–608.

Adrian, Tobias, Daniel Covitz, and Nellie Liang, 2015, Financial stability monitoring, *Annual Review of Financial Economics* 7, 357–395.

Brunnermeier, Markus K, and Martin Oehmke, 2013, Bubbles, financial crises, and systemic risk, in *Handbook of the Economics of Finance*, volume 2, 1221–1288 (Elsevier).

Cabrales, Antonio, Piero Gottardi, and Fernando Vega-Redondo, 2014, Risk sharing and contagion in networks, *CESifo Working Paper* .

Castiglionesi, Fabio, Fabio Feriozzi, and Guido Lorenzoni, 2017, Financial integration and liquidity crises, *Forthcoming Management Science* .

Copeland, Adam, Antoine Martin, and Michael Walker, 2014, Repo Runs: Evidence from the Tri-Party Repo Market, *The Journal of Finance* 69, 2343–2380.

Elliott, Matthew, Benjamin Golub, and Matthew Jackson, 2014, Financial networks and contagion, *American Economic Review* 104, 3115–3153.

Gabaix, Xavier, Arvind Krishnamurthy, and Olivier Vigneron, 2007, Limits of Arbitrage: Theory and Evidence from the Mortgage-Backed Securities Market, *Journal of Finance* 62, 557–595.

Garleanu, Nicolae, Lasse Heje Pedersen, and Allen M Poteshman, 2009, Demand-based option pricing, *The Review of Financial Studies* 22, 4259–4299.

Greenwood, Robin, and Dimitri Vayanos, 2014, Bond supply and excess bond returns, *The Review of Financial Studies* 27, 663–713.

Jaffee, Dwight M, 2010, The role of the GSEs and housing policy in the financial crisis, *Testimony before the Financial Crisis Inquiry Commission, February* 25, 2010.

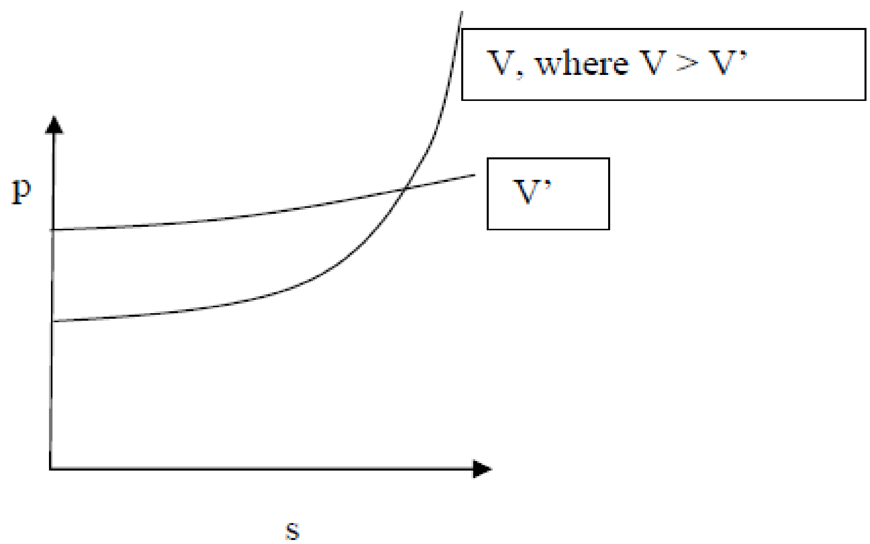
Martin, Ian, 2013, The Lucas Orchard, *Econometrica* 81, 55–111.

Ramirez, Carlos, 2017, Designing Resilient Financial Systems, *Mimeo* .

Schmidt, Lawrence, Allan Timmermann, and Russ Wermers, 2016, Runs on money market mutual funds, *American Economic Review* 106, 2625–57.

Appendix A. Figures

Figure 2. Financial Sector Vulnerability to Shocks and Pricing of Risk, from Adrian et al. (2015).



Note: p denotes the price of risk, s the size of the shock, and V is the vulnerability of the financial system. V' corresponds to a financial system with tighter regulation than V.

Figure 3. Model Computational Results with $E(q) = \nu = 0.3$. Other parameters: $\gamma = 2$, $N = 10$.