

Structural Change in Investment and Consumption: A Unified Approach*

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Abstract

Existing models of structural change typically assume that all of investment is produced in the goods sector. We show that this assumption is strongly counterfactual: in the postwar US, the share of services value added in investment expenditure has been steadily growing and now exceeds that of goods value added. We build a new model, which takes a unified approach to structural change in investment and consumption and yields to three new insights. First, for empirically plausible parameter values technological change is endogenously investment specific. Second, constant TFP growth in all sectors is inconsistent with structural change happening along aggregate balanced growth path. Third, the sector with the slowest TFP growth absorbs all resources asymptotically.

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JEL classification: O11; O14.

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1 Introduction

A growing literature has proposed extensions of the one-sector growth model in order to jointly study growth and structural change. The standard framework in the literature allows for multiple consumption goods and structural change among the sectors producing them, while abstracting from structural change within investment. In this paper we develop and analyze a new model that offers a unified treatment of structural change in both investment and consumption. We use our model to revisit three core questions studied in the structural-change literature. What properties hold along an aggregate balanced growth path? Under what conditions is there balanced growth of aggregate variables? And what determines the asymptotic rate of growth in the economy?¹

For the standard framework in the literature to address structural change in the overall economy, one needs to make an additional assumption regarding activity in the investment sector. The literature has typically assumed that all investment reflects value added in the industry sector, or more broadly the goods sector.² The starting point for our analysis is an assessment of this claim in the context of the post WWII US during 1947–2016. We first show that the standard assumption is squarely at odds with the data – the share of services value-added in investment expenditure has been steadily growing and now exceeds that of goods value added. Moreover, structural change within the investment sector features the same qualitative patterns as structural change within the consumption sector: the expenditure share of services has increased at the same time that the relative price of services has increased. We note that these statements are true even though the share of goods value added is still much higher in investment expenditure than in consumption expenditure.

Motivated by these facts, we develop a new model of growth and structural change in which both final consumption and final investment are produced by combining intermediate inputs from the goods and services sectors. Notably, we take a unified approach and treat the production of consumption and investment in a symmetric manner by assuming that each is a CES

¹We use the term aggregate balanced growth to mean that *aggregate* variables grow at constant rates or remain constant while *sectoral* variables may change in non-linear ways. This is also called generalized balanced growth.

²This practice started with the early models of growth and structural change by Echevarria (1997) and Kongsamut et al. (2001). See Herrendorf et al. (2014) for a review of the literature on structural change that has emerged since then.

aggregator of goods and services value added, with possibly different weights and elasticities of substitution. Our model could be extended to allow for further disaggregation of the final expenditure or the intermediate inputs. For example, it could be of interest to further disaggregate investment into equipment and structures as in Greenwood et al. (1997), and to disaggregate services into low and high skilled as in Buera and Kaboski (2012) and Buera et al. (2015) or low and high productivity growth as in Duernecker et al. (2017). To facilitate exposition, we nonetheless focus on the simplest framework with final investment and consumption expenditures and with intermediate goods and services inputs that in turn are produced from capital and labor. Our framework has three sources of technological change: improvements in the TFPs of the goods-producing sector, of the services-producing sector, and of the investment-producing technology. The last source of technological change captures the possibility of exogenous investment-specific technological change.³

Having developed our new model, we proceed to examine its properties. The behavior of our model differs from those of the existing literature along several key dimensions. First, for empirically plausible parameter values, our model implies that technological change is endogenously investment-specific. This comes about because empirically the goods sector has stronger TFP growth than the services sector and inputs from the goods sector are more important in investment than they are in consumption. Together, these features imply that the TFP growth of investment is larger than of consumption, even if there is no exogenous investment-specific technological change. In other words, TFP growth is endogenously investment biased.

We then proceed to characterize the properties of a Generalized Balanced Growth Path (GBGP henceforth) of our model and ask under what conditions it will exist. We find that the behavior of our model differs from those of the existing literature along two additional key dimensions. To begin with, we show that constant growth of all sectoral TFPs is inconsistent with the joint occurrence of aggregate balanced growth and structural change. For both of them to occur at the same time, a non-linear aggregate of the three TFPs must grow at a constant rate, which is impossible if all TFPs grow at constant rates. The fact that we model investment as a nonlinear aggregate of goods and services is key to this result. As noted in Herrendorf et al.

³Greenwood et al. (1997) argued for the importance of this form of technological change. Our formulation more closely follows Oulton (2007).

(2014), nonlinear aggregation in the consumption sector does not create any issues regarding balanced growth in standard two-sector models with log utility of period consumption, because balanced growth is effectively anchored by the constant TFP growth in the investment sector. One novelty of the current paper is to construct a GBGP without imposing constant TFP growth in the investment sector.

Lastly, we show that our model has distinct implications for the asymptotic growth rate along a balanced growth path. Ngai and Pissarides (2007) found that the sector with the slowest productivity growth would not take over the entire economy in a model with capital accumulation. This comes about because they assumed that investment is produced in a sector with larger than the slowest productivity growth (which they call the manufacturing sector). The result of Ngai-Pissarides no longer holds in our framework which has structural change in both investment and consumption. Despite having capital, our framework is consistent with the pessimistic view expressed by Baumol (1967) in which the sector with the slowest productivity growth would come to dominate the entire economy in the limit. The reason, of course, is that structural change in both consumption and investment implies that the services sector takes over investment as well as consumption.⁴

The CES aggregators in the consumption and investment sectors of our simple framework are admittedly somewhat specialized, and so there may be concern about how well our specification performs quantitatively. We show that it can quantitatively capture most of the salient features of structural change in the US data in the post WWII period, if we impose that there is little scope for substitution between goods and services in the production of both investment and consumption. In fact, a Leontief specification for both turns out to match the data most closely. This result extends the earlier analysis of Herrendorf et al. (2013) to the case of structural change within the investment sector. Given the findings of Herrendorf et al. (2013) that income effects play some role at generating the observed consumption value added shares, it is expected that the current model with a CES production function of consumption misses some of the structural change in the consumption sector. We nonetheless abstract from income effects in consumption here because it allows us to take a unified approach to structural change in in-

⁴Duernecker et al. (2017) offer an analysis of the implications of this so-called Baumol's disease for aggregate productivity growth.

vestment and consumption, obtain analytical solutions of our model, and focus our attention on structural change within investment. We emphasize that the resulting model captures structural change within investment remarkably well.

We also carry out an empirical assessment of our theoretical condition that is necessary for balanced growth. In particular, we estimate all three TFP growth rates using standard methods and evaluate the extent to which our theoretical condition holds and the role of changes in each of the sectoral TFPs. We find that our theoretical condition holds approximately in the data. Interestingly, however, it holds despite the fact that the growth rates of the three TFP terms vary quite dramatically over time. This suggests that, contrary to common practice, constant growth of sectoral TFPs is not a restriction that is natural to impose on the parameters in the context of balanced growth in multi-sector models.

Our work is closely related to that of Garcia-Santana et al. (2016), who also decomposed investment into the components of goods and services. They study how structural change is affected by the transition dynamics to balanced growth that are associated with growth miracles and the resulting investment booms. Considering structural change in investment is particularly important during investment booms when investment constitutes a large share of GDP. In contrast to Garcia-Santana et al. (2016), we focus on the behavior of the economy along a GBGP and show that even with aggregate balanced growth structural change in investment has important implications for investment-biased technical change, the growth rates of sectoral TFP and the asymptotic behavior of the economy.

An outline of the paper follows. In the next section we present the key facts from the US time series data. Section 3 presents our model and Sections 4 characterizes key properties of the competitive equilibrium. Section 5 studies the features of structural change and a GBGP. Section 6 derives the three insights that our unified approach yields. Section 7 examines key aspects of the model and its theoretical properties from an empirical perspective. Section 8 concludes.

2 Evidence

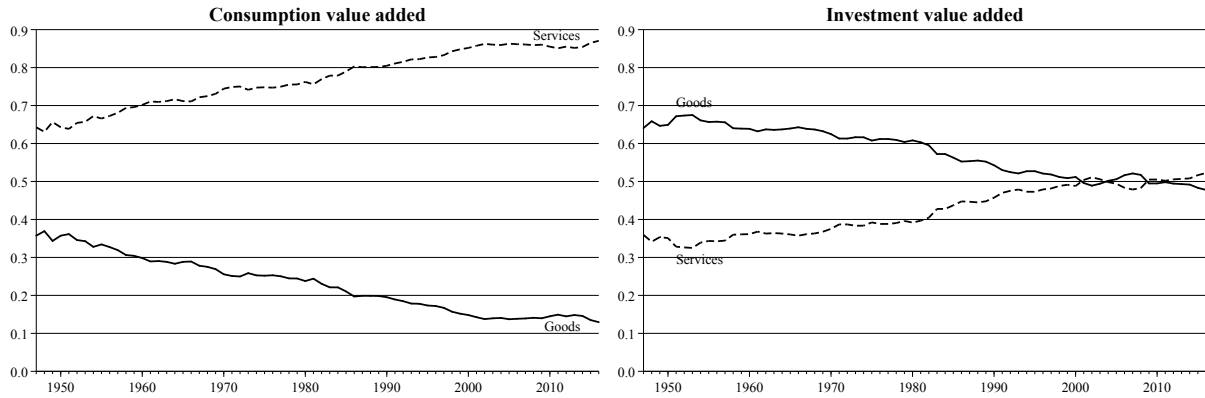
Our goal in this section is to complement existing presentations of the stylized facts of structural change and motivate the framework that we develop. We offer a unified analysis of the structural-change facts for both the final consumption and final investment sectors. We combine US industry data from WORLD KLEMS with the annual input-output tables from the BEA to decompose final expenditures into its value added components. In principle one could do this decomposition along many dimensions. But, as noted in the introduction, to facilitate exposition and focus attention on the novel implications, in what follows we consider two final expenditure categories – consumption and investment – and two value added components – goods and services.

In implementing these decompositions we define the goods sector to consist of agriculture, mining, manufacturing, public utilities and construction.⁵ Services consists of the remaining industries – wholesale and retail trade, transportation, business services, personal services and government. The analysis that we carry out in the following sections will focus on the case of a closed economy. To connect the closed economy model to the data requires that we allocate net exports between consumption and investment. Because net exports are not that large, the rule for allocating them is not of first-order significance. In what follows we allocate all of net export to our measure of consumption. The benefit of doing this is that the notion of investment and capital in the model will correspond to the notion of investment and capital as measured in the data. We decompose the final expenditure into the value added from different sectors following the methodology developed in Herrendorf et al. (2013), which involves the use of input–output relationships and total requirement matrices. Note that while in Herrendorf et al. (2013), we decomposed final consumption expenditure into the value added from agriculture, manufacturing, and services, here we decompose both final consumption and investment expenditure into the value added from goods and services.

Figure 1 shows the key facts for this two-by-two decomposition. Consumption and invest-

⁵In much of the structural change literature it is common to consider three components of consumption: agriculture, non-agricultural goods (typically referred to as manufacturing) and services. Because we focus on the post World War II US and during this period agriculture is a relatively unimportant component of investment in particular and of structural change more generally, we combine agriculture and manufacturing into a single goods producing-sector.

Figure 1: Sector shares in consumption and investment



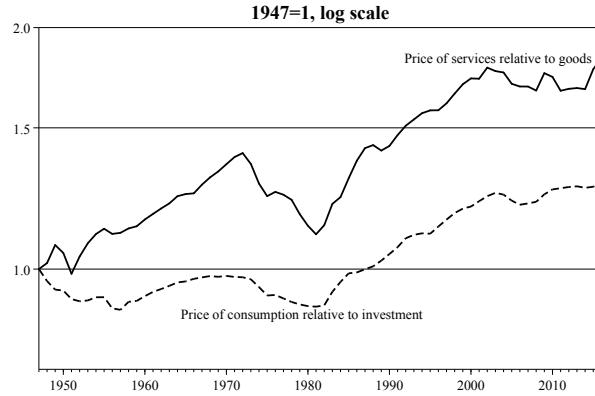
ment show a similar pattern, with an increase in the services value added share and a decrease in the goods value added share. The initial level varies significantly across expenditure categories while the percentage change are fairly similar: the services value added share in consumption begins at roughly .60 and increases to about .80, whereas for investment the increase is from around .40 to around .50.

We stress three key properties relative to the existing literature on structural change. First, assuming that investment is produced entirely by the goods sector is strongly at odds with the data; in fact, by the end of the sample the goods value added share is less than the services value added share in investment. Second, the value added shares exhibit important changes over time in both consumption and investment. This suggests that any analysis of structural change at the aggregate level needs to confront the reality that structural change occurs both within the consumption-producing sector and the investment-producing sector. Third, the value added shares differ significantly between investment and consumption, suggesting that it is important that consumption and investment be modelled separately.

Relative prices will play an important role in the analysis in later sections, in particular when we restrict the parameters of our model so that its implications are consistent with the salient features of the data. Therefore, we also present time series evidence on the relative price of goods and services, as well as the relative price of consumption to investment.

The figures reveal two key secular trends that are familiar from the existing literature, but for completeness are worth repeating here. First, there has been a marked increase in the price

Figure 2: Relative prices



of services relative to goods. This increase accelerates in the period after 1980. The somewhat unusual behavior of this relative price in the 1970s is driven by a dramatic spike in the prices of agriculture and oil during the early 1970s. While this suggests that a more detailed analysis might warrant further disaggregation, the somewhat anomalous behavior of the 1970s should not distract us from the clear secular trends over this period. Second, there is also a marked increase in the price of consumption relative to investment. Once again, the behavior is quite distinct in the pre-1970 and post 1980-period, with little trend in the first subperiod and a marked positive trend in the second subperiod. And again, the 1970s show some anomalous behavior, though perhaps less so than in the previous case.

3 Model

We build a multi-sector extension of the standard one-sector neoclassical growth model, formulated in continuous time. Motivated by the presentation in the previous section, we will view the production of goods and services from capital and labor as the two core production activities, with consumption and investment in turn produced by combining goods and services. The general approach that we follow is to start with sectoral valued-added production functions and view the production for final expenditure categories as combining value added from the underlying sectors.

Goods and services value added (denoted by Y_{gt} and Y_{st} , respectively) are each produced

according to Cobb-Douglas production functions with the same capital shares but potentially subject to different rates of technological progress:

$$Y_{jt} = A_{jt} K_{jt}^{\theta} L_{jt}^{1-\theta}, \quad j \in \{g, s\},$$

where $\theta \in (0, 1)$ and the A_{jt} represent exogenous technological progress. The assumption that the underlying production functions are Cobb-Douglas with identical capital shares is common to the literature on structural change, as it allows for the existence of a balanced growth path. Herrendorf et al. (2015) show that this case also does a reasonable job at replicating the labor allocation across sectors in the postwar US.⁶

The outputs of the goods and services sectors are in turn used as intermediate inputs to produce final consumption and final investment. We use CES aggregators in both sectors. While these specifications are analytically convenient, we will show later that they do a reasonable job at capturing the empirical patterns presented in the previous section.

Final consumption C_t is a CES aggregate of consumption of goods (C_{gt}) and services (C_{st}):

$$C_t = \left(\omega_c^{\frac{1}{\varepsilon_c}} C_{gt}^{\frac{\varepsilon_c-1}{\varepsilon_c}} + (1 - \omega_c)^{\frac{1}{\varepsilon_c}} C_{st}^{\frac{\varepsilon_c-1}{\varepsilon_c}} \right)^{\frac{\varepsilon_c}{\varepsilon_c-1}}, \quad (1)$$

where $\varepsilon_c \in [0, \infty)$ is the elasticity of substitution between goods and services in the production of consumption and $\omega_c \in [0, 1]$ determines the relative weight of intermediate inputs from the goods sector into the production of consumption. Our consumption aggregator is homothetic, and so it abstracts from income effects. As we will see in Section 7 below, and consistent with the findings of the existing literature, allowing for income effects does improve the fit of the model to the data somewhat. We nonetheless adopt a homothetic aggregator for consumption to allow us to better focus on the novel implications of our formulation of investment.⁷

For our purposes it does not matter whether we think of the household as buying goods and

⁶The case in which the two technologies do not have the same capital share has previously been studied in a different context by Acemoglu and Guerrieri (2008), who show that in that setting a balanced growth path exists only asymptotically.

⁷Several papers have recently studied the role that income effects play for structural change. In addition to the early paper by Kongsamut et al. (2001), recent examples include Herrendorf et al. (2013), Boppart (2016), Comin et al. (2015), and Duernecker et al. (2017).

services in the market and combining them itself to produce C_t or alternatively assume that a firm purchases goods and services as intermediate inputs, combines them into the aggregate C_t and then sells the aggregate consumption good directly to the household. In what follows we adopt the latter formulation. We will therefore view relationship (1) as a production function for a firm that operates in the market.

We take a unified approach to structural change in investment and consumption, assuming that symmetrically with the production of final consumption, final investment (X_t) is also produced using inputs from the goods and services sectors, but with its own CES aggregator:

$$X_t = A_{xt} \left(\omega_x^{\frac{1}{\varepsilon_x}} X_{gt}^{\frac{\varepsilon_x-1}{\varepsilon_x}} + (1 - \omega_x)^{\frac{1}{\varepsilon_x}} X_{st}^{\frac{\varepsilon_x-1}{\varepsilon_x}} \right)^{\frac{\varepsilon_x}{\varepsilon_x-1}},$$

where A_{xt} represents investment-specific technological change, $\varepsilon_x \in [0, \infty)$ is the elasticity of substitution between goods and services in producing final investment, and $\omega_x \in [0, 1]$ determines the weight of goods inputs in investment production. Note that only relative sector-specific technological change matters for the analysis and that we have normalized consumption-specific technological change to one.

There is an infinitely lived representative household with preferences represented by the utility function

$$\int_0^\infty e^{-\rho t} \log(C_t) dt,$$

where $\rho > 0$ is the discount rate. The household is endowed with one unit of time at each instant, which will be supplied inelastically, and a positive initial capital stock, $K_0 > 0$.

The law of motion for capital is given by:

$$\dot{K}_t = X_t - \delta K_t,$$

where $\delta \in (0, 1]$ is the depreciation rate. The economy begins at time 0 with $K_0 > 0$ units of capital owned by the households. We assume that capital and labor are freely mobile across

sectors so that feasibility for this economy requires:

$$\begin{aligned} K_{gt} + K_{st} &\leq K_t, & L_{gt} + L_{st} &\leq 1, \\ C_{gt} + X_{gt} &\leq Y_{gt}, & C_{st} + X_{st} &\leq Y_{st}. \end{aligned}$$

4 Equilibrium

We study the competitive equilibrium for the above economy, assuming that the household accumulates capital and rents it to the firms. We assume a representative firm in each of the goods and services sectors, as well as a representative firm that produces final consumption and one that produces final investment. At each point in time there will be six markets: four markets for outputs and two markets for production factors. We choose investment as the numeraire. The rental prices for capital and labor are denoted as R_t and W_t respectively. The prices for goods and services will be denoted by P_{gt} and P_{st} , respectively. The prices of the final consumption and final investment aggregates will be denoted by P_{ct} and P_{xt} . The price of the final investment good will be normalized to one in each period. Arbitrage implies that the interest rate will equal $R_t - \delta$. Given that the equilibrium concept is standard, we do not provide a formal definition of equilibrium.⁸

In the remainder of this section, we provide a partial characterization of the equilibrium. In the first step we derive analytic expressions for the output prices relative to investment in terms of model primitives. As a by product of the first step we also generate expressions for relative expenditure shares on inputs in each of the final sectors in terms of model primitives. In the second step we use these expressions to derive an expression for a pseudo aggregate production function that expresses final output evaluated using current prices as a function of aggregate inputs and current TFP. In the third step we derive an expression to characterize the nature of structural change along the equilibrium path. Note that the derivations in the first two steps rely only on the production side of the economy. The derivations in the third step combine the

⁸The equilibrium of our model is efficient, and so we could also study the planner problem. Since the evidence on relative prices as documented in Figure 2 will be important to restrict the parameter values of our model, we study competitive equilibrium including equilibrium prices, instead of the planner problem.

production side with the household side of the economy.

4.1 Output Prices

We start with the first-order conditions for labor and capital for the two firms producing goods and services. For $j \in \{g, s\}$ these are given by:

$$\theta P_{jt} A_{jt} K_{jt}^{\theta-1} L_{jt}^{1-\theta} = R_t, \quad (2)$$

$$(1 - \theta) P_{jt} A_{jt} K_{jt}^{\theta} L_{jt}^{-\theta} = W_t. \quad (3)$$

Taking the ratio of the two first-order conditions for a given sector j and rearranging gives:

$$\frac{K_{jt}}{L_{jt}} = \frac{\theta}{1 - \theta} \frac{W_t}{R_t}. \quad (4)$$

It follows that the capital-labor ratio will be equalized across the two sectors. Given that aggregate labor is one, it follows that $K_{jt}/L_{jt} = K_t$ for all times t and $j \in \{g, s\}$. Using this fact, the two first-order conditions for capital imply

$$\frac{P_{gt}}{P_{st}} = \frac{A_{st}}{A_{gt}} \quad (5)$$

so that relative prices are the inverse of relative TFP for these two sectors. This is a standard result in the structural change literature when sector technologies differ only in their TFP growth.

Next we derive relations between the prices of goods and services and the prices of the two final outputs. Because both final-output production functions are constant returns to scale and profits must therefore equal zero in a competitive equilibrium, it follows that P_{ct} must equal the cost minimizing production cost of a unit of consumption, and similarly that P_{xt} , which is normalized to one, must equal the cost minimizing production cost for a unit of investment. It

is straightforward to show that these conditions imply:

$$P_{ct} = \left(\omega_c P_{gt}^{1-\varepsilon_c} + (1 - \omega_c) P_{st}^{1-\varepsilon_c} \right)^{\frac{1}{1-\varepsilon_c}}, \quad (6)$$

$$1 = \frac{\left(\omega_x P_{gt}^{1-\varepsilon_x} + (1 - \omega_x) P_{st}^{1-\varepsilon_x} \right)^{\frac{1}{1-\varepsilon_x}}}{A_{xt}}. \quad (7)$$

These results allow us to fully characterize relative prices in terms of primitives. Specifically, the zero-profit condition for the final investment good imposes a relation between the prices of goods and services. But we previously showed that relative TFPs in the goods and services sectors determine relative prices of goods and services, thus giving us two equations in the two unknown prices.⁹ Straightforward algebra yields:

$$P_{gt} = \frac{A_{xt} \left(\omega_x A_{gt}^{\varepsilon_x-1} + (1 - \omega_x) A_{st}^{\varepsilon_x-1} \right)^{\frac{1}{\varepsilon_x-1}}}{A_{gt}}, \quad (8)$$

$$P_{st} = \frac{A_{xt} \left(\omega_x A_{gt}^{\varepsilon_x-1} + (1 - \omega_x) A_{st}^{\varepsilon_x-1} \right)^{\frac{1}{\varepsilon_x-1}}}{A_{st}}. \quad (9)$$

Substituting these equations into the expression for the price of the final consumption good yields:

$$P_{ct} = \frac{A_{xt} \left(\omega_x A_{gt}^{\varepsilon_x-1} + (1 - \omega_x) A_{st}^{\varepsilon_x-1} \right)^{\frac{1}{\varepsilon_x-1}}}{\left(\omega_c A_{gt}^{\varepsilon_c-1} + (1 - \omega_c) A_{st}^{\varepsilon_c-1} \right)^{\frac{1}{\varepsilon_c-1}}}. \quad (10)$$

The calculations that yield unit production costs as a function of input prices also generate standard expressions for relative input expenditure in each of the two final output sectors:

$$\frac{P_{gt} C_{gt}}{P_{st} C_{st}} = \frac{\omega_c}{1 - \omega_c} \left(\frac{P_{gt}}{P_{st}} \right)^{1-\varepsilon_c}, \quad (11)$$

$$\frac{P_{gt} X_{gt}}{P_{st} X_{st}} = \frac{\omega_x}{1 - \omega_x} \left(\frac{P_{gt}}{P_{st}} \right)^{1-\varepsilon_x}. \quad (12)$$

These relationship determine the directions of structural change within consumption and investment depending on the behavior of the relative price of goods to services and the elasticities of

⁹Alternatively, the zero profit condition for the investment sector can be rewritten as an expression linking the price of goods and the relative price of goods to services. Since the relative prices is determined by relative TFPs, this equation gives us the price of goods in terms of primitives.

substitution.

4.2 Aggregate Output

We define aggregate final output as measured in units of final investment as $Y_t = X_t + P_{ct}C_t$. It is useful to derive a pseudo aggregate production function for Y_t . Because payments to inputs will exhaust income in each of the final-output producing sectors, we can also write:

$$Y_t = P_{gt}X_{gt} + P_{st}X_{st} + P_{gt}C_{gt} + P_{st}C_{st} = P_{gt}(X_{gt} + C_{gt}) + P_{st}(C_{st} + X_{st}).$$

Feasibility in turn implies that $X_{gt} + C_{gt} = Y_{gt}$ and $X_{st} + C_{st} = Y_{st}$. Substituting these equations and using the fact that $P_{gt}A_{gt} = P_{st}A_{st}$ gives:

$$\begin{aligned} Y_t &= P_{gt}A_{gt}K_{gt}^\theta L_{gt}^{1-\theta} + P_{st}A_{st}K_{st}^\theta L_{st}^{1-\theta} \\ &= P_{gt}A_{gt} \left(K_{gt}^\theta L_{gt}^{1-\theta} + K_{st}^\theta L_{st}^{1-\theta} \right). \end{aligned}$$

Using the fact that the capital-to-labor ratio in each sector is equal to the aggregate capital-to-labor ratio while aggregate labor equals one, we have:

$$\begin{aligned} Y_t &= P_{gt}A_{gt} \left[L_{gt} \left(\frac{K_{gt}}{L_{gt}} \right)^\theta + L_{st} \left(\frac{K_{st}}{L_{st}} \right)^\theta \right] \\ &= P_{gt}A_{gt} (L_{gt}K_t^\theta + L_{st}K_t^\theta) \\ &= P_{gt}A_{gt}K_t^\theta. \end{aligned}$$

Note that since $P_{gt}A_{gt} = P_{st}A_{st}$ the last part could also be written using the price of services and TFP in the service sector. Either way, using our previously derived expression for P_{gt} or P_{st} in terms of primitives this last expression can in turn be written as:

$$Y_t = A_{xt} \left(\omega_x A_{gt}^{\varepsilon_x - 1} + (1 - \omega_x) A_{st}^{\varepsilon_x - 1} \right)^{\frac{1}{\varepsilon_x - 1}} K_t^\theta. \quad (13)$$

This relation expresses aggregate output as a function of the aggregate inputs capital and labor and of an aggregator of the sectoral TFPs. Note that the aggregate input of labor does not

feature explicitly because it equals one.

5 Structural Change and Balanced Growth

We are now ready to explore the implications of our model for structural change and for aggregate balanced growth. As mentioned before, when we talk about aggregate balanced growth we are referring to aggregate variables growing at constant rates while structural change is taking place underneath. This is often called a generalized balanced growth path (GBGP). Throughout this section, we will assume that a GBGP exists and characterize its properties. We will postpone the derivation of an existence condition to the next section where we present its implications as one of the three new insights that we obtain from taking a unified approach to structural change in investment and consumption.

5.1 Structural Change

The key aspect of structural change that concerns us is the composition of aggregate economic activity across goods and services. Structural change can be measured either as changes in sectoral value added shares or changes in sectoral employment shares. We first show that our model has the common feature in the literature on structural change that these two measures are identical in equilibrium, and hence it does not matter which one we focus on. To see this, we note that the nominal value added shares for goods and services, which we denote as V_{jt} ($j \in \{g, s\}$), is given by:

$$V_{jt} \equiv \frac{P_{jt}Y_{jt}}{Y_t} = \frac{P_{jt}Y_{jt}}{P_{gt}Y_{gt} + P_{st}Y_{st}},$$

where, as we argued above, the second equality again derives from the fact that with constant returns to scale, payments to factors of production exhaust all income. Substituting the sectoral production functions and using that in equilibrium $P_{gt}A_{gt} = P_{st}A_{st}$ and the capital-to-labor ratio in each sector is the same as the aggregate capital-to-labor ratio, we have:

$$V_{jt} = \frac{P_{jt}A_{jt}K_t^\theta L_{jt}}{P_{gt}A_{gt}K_t^\theta L_{gt} + P_{st}A_{st}K_t^\theta L_{st}} = \frac{L_{jt}}{L_{gt} + L_{st}} = L_{jt}.$$

In other words, sectoral nominal value added shares equal sectoral employment shares and we can restrict our attention to characterizing the former.

In what follows we focus on the behavior of

$$V_{st} = \frac{P_{st}C_{st} + P_{st}X_{st}}{Y_t},$$

where we used that $Y_{st} = C_{st} + X_{st}$. Focusing on V_{st} is sufficient because $V_{gt} = 1 - V_{st}$. Carrying out some simple manipulations gives:

$$\begin{aligned} V_{st} &= \frac{P_{st}C_{st}}{P_{ct}C_t} \frac{P_{ct}C_t}{Y_t} + \frac{P_{st}X_{st}}{X_t} \frac{X_t}{Y_t} \\ &= \frac{P_{st}C_{st}}{P_{gt}C_{gt} + P_{st}C_{st}} \frac{P_{ct}C_t}{Y_t} + \frac{P_{st}X_{st}}{P_{gt}X_{gt} + P_{st}X_{st}} \frac{X_t}{Y_t} \\ &= \frac{1}{(P_{gt}C_{gt})/(P_{st}C_{st}) + 1} \frac{P_{ct}C_t}{Y_t} + \frac{1}{(P_{gt}X_{gt})/(P_{st}X_{st}) + 1} \frac{X_t}{Y_t}. \end{aligned}$$

We previously solved for expenditure shares of goods relative to services in terms of relative prices. Using that the price of goods relative to services is the inverse of the TFP in goods relative to services, we obtain the equilibrium ratio of the expenditure shares in each of the final output sectors as a function of primitives:

$$\frac{P_{gt}C_{gt}}{P_{st}C_{st}} = \frac{\omega_c}{1 - \omega_c} \left(\frac{A_{st}}{A_{gt}} \right)^{1-\varepsilon_c}, \quad (14)$$

$$\frac{P_{gt}X_{gt}}{P_{st}X_{st}} = \frac{\omega_x}{1 - \omega_x} \left(\frac{A_{st}}{A_{gt}} \right)^{1-\varepsilon_x}. \quad (15)$$

Lemma 1 *If $\widehat{A}_{gt} > \widehat{A}_{st}$ and $\varepsilon_c, \varepsilon_x \in [0, 1)$, then $\widehat{P}_{gt}C_{gt} < \widehat{P}_{st}C_{st}$ and $\widehat{P}_{gt}X_{gt} < \widehat{P}_{st}X_{st}$.*

In other words, within each final output sector we have the standard patterns emphasized in the literature on structural change in the presence of uneven technological change: the expenditure share on goods is decreasing relative to the expenditure share on services in both final consumption and final investment. However, from the perspective of the overall economy, structural change is also potentially influenced by changes in the composition of final output between investment and consumption. If the final output shares are constant over time, then the above effects will imply that labor is moving systematically from the goods-producing sec-

tor to the services-producing sector. But if there is a change in final output shares between consumption and investment, then one cannot infer the nature of structural change without additional assumptions. The reason for this is that labor could be reallocated to the investment sector, which has the higher goods share, sufficiently fast that the overall goods share rises even though the goods shares of both sectors decline.

In the next subsection we will show that along a GBGP in which the real interest rate is constant, the final output shares are indeed constant so that the nature of structural change along such a path can be inferred without additional assumptions.

5.2 Generalized Balanced Growth Path

Having displayed some of the properties that will hold along any equilibrium path, in this subsection we examine under the properties of our model under a generalized balanced growth path (GBGP). As mentioned in the introduction, balanced growth is too strict an equilibrium concept to impose in our context where balanced growth may happen only at the aggregate level while structural change leads to non-linear changes in the sectoral employment shares. The term *generalized* balanced growth relaxes balanced growth by requiring that *aggregate* variables grow at constant rates or remain constant while *sectoral* variables are allowed to change in non-linear ways. We adopt the definition by Kongsamut et al. (2001), who defined a GBGP as an equilibrium path along which the real interest rate is constant. We emphasize that it is more challenging to study GBGP in our framework than it usually is, because we have structural change in both the consumption sector and the investment sector. In contrast, standard models of structural change abstract from structural change within the investment sector. Doing this has the advantage that it “anchors” the economy and makes it fairly straightforward to obtain a constant real interest rate by imposing that TFP growth in investment, which is a primitive of the usual models, is constant. Things are rather different in our framework in which investment TFP is an endogenous, non-linear aggregator of the other TFPs.

To characterize the properties of a GBGP, we also need the equilibrium conditions that arise from the household side of the equilibrium. The household seeks to maximize the present

discounted integral of utility subject to its budget equation:

$$E_t + \dot{K}_t + \delta K_t = R_t K_t + W_t, \quad (16)$$

where $E_t \equiv P_{ct} C_t$ denotes consumption expenditure. Letting “hats” denote growth rate, the Euler equation and the transversality condition for the household can be written as:

$$\gamma_t \equiv \widehat{E}_t = R_t - \delta - \rho, \quad (17)$$

$$\lim_{t \rightarrow \infty} e^{-\rho t} \frac{K_t}{E_t} = 0. \quad (18)$$

The first proposition establishes that if a GBGP exists, it must be the case that E_t , Y_t , K_t , and X_t all grow at the same constant rate, which is determined by the right-hand side of the Euler equation. Note that whereas both K_t and X_t are physical quantities, the other two terms – E_t and Y_t – include both prices and quantities.

Lemma 2 *If a GBGP exists, then it must be that*

$$\widehat{E}_t = \widehat{Y}_t = \widehat{K}_t = \widehat{X}_t = \gamma \equiv R - \delta - \rho.$$

Proof. See the Appendix.

Having established the properties of a GBGP in Proposition 2, we can now revisit the issue of structural change along the GBGP. As noted in the previous section, although knowing the direction of relative price changes for goods and services together with the elasticity of substitution between goods and services in production is sufficient to determine the nature of structural change for the production of consumption and investment individually, changes in the composition of final output between consumption and investment could create an opposing effect. But what we have just shown is that along a GBGP the composition of final output is constant, and thus there is no opposing effect from composition changes. It follows:

Lemma 3 *If a GBGP exists, $\varepsilon_x, \varepsilon_c \in (0, 1)$, and $\widehat{A}_{gt} > \widehat{A}_{st}$, then the shares of the services sector in total value added and total employment will increase over time.*

6 Three New Insights from the Unified Approach

In this section, we establish that three new insights arise from taking a unified approach to structural change in investment and consumption. As mentioned in the introduction, these are: that for empirically plausible parameter values technological change is endogenously investment specific; that constant TFP growth in all sectors is inconsistent with structural change and aggregate balanced growth; and that the sector with the slowest TFP growth absorbs all resources asymptotically.

6.1 Insight 1: Investment-Biased Technological Change

We start with the implications of our model for sector-biased technological change. As a first step it is useful to derive an alternative representation of the production structure in our economy. Previously we started with value-added production functions for both goods and services and then expressed the outputs of consumption and investment in terms of goods and services. But given this production structure, in equilibrium one can also write the production structure as two value added production functions for final consumption and final investment, i.e., relations that express each of final consumption and final investment in terms of capital and labor inputs in each sector and a sector-specific TFP term. Loosely speaking, this amounts to taking the production functions for each of consumption and investment and substituting for the inputs of goods and services in terms of the labor and capital that are used to produce them. The next proposition summarizes the result of this exercise.

Proposition 1 *Along any equilibrium path, investment and consumption are produced according to the following pseudo sectoral production functions:*

$$X_t = \mathcal{A}_{xt} K_{xt}^\theta L_{xt}^{1-\theta},$$

$$C_t = \mathcal{A}_{ct} K_{ct}^\theta L_{ct}^{1-\theta},$$

where

$$\begin{aligned}\mathcal{A}_{xt} &\equiv A_{xt} \left(\omega_x A_{gt}^{\varepsilon_x-1} + (1 - \omega_x) A_{st}^{\varepsilon_x-1} \right)^{\frac{1}{\varepsilon_x-1}}, & L_{xt} &\equiv \frac{X_{gt}}{A_{gt} K_t^\theta} + \frac{X_{st}}{A_{st} K_t^\theta}, & K_{xt} &= K_t L_{xt}, \\ \mathcal{A}_{ct} &\equiv \left(\omega_c A_{gt}^{\varepsilon_c-1} + (1 - \omega_c) A_{st}^{\varepsilon_c-1} \right)^{\frac{1}{\varepsilon_c-1}}, & L_{ct} &\equiv \frac{C_{gt}}{A_{gt} K_t^\theta} + \frac{C_{st}}{A_{st} K_t^\theta}, & K_{ct} &\equiv K_t L_{ct}.\end{aligned}$$

Proof. See the Appendix.

Equation (10) implies that our model has an additional source of sector-biased technological change. In a model that abstracted from any asymmetries between the production of final consumption and final investment, we would have $\omega_x = \omega_c$ and $\varepsilon_x = \varepsilon_c$. In this case we would have:

$$\frac{\mathcal{A}_{xt}}{\mathcal{A}_{ct}} = A_{xt}.$$

which is a standard result in the literature on investment specific technical change. Implicitly, the term $\left(\omega_x A_{gt}^{\varepsilon_x-1} + (1 - \omega_x) A_{st}^{\varepsilon_x-1} \right)^{\frac{1}{\varepsilon_x-1}}$ would capture TFP growth that is common to both sectors, and relative TFP growth would be captured by A_{xt} .

But in the more general case in which production of final consumption and final investment is not symmetric, there is an additional term that shows up in the expression for relative TFP growth. Although this additional term cannot be signed in general, it can be signed for what we will later show is an empirically relevant case. In particular, suppose that $\varepsilon_x = \varepsilon_c \in [0, 1)$, that $0 < \omega_c < \omega_x < 1$, and that $\widehat{A}_{gt} > \widehat{A}_{st}$.¹⁰ Then it is easy to show that $\widehat{\mathcal{A}}_{xt} - \widehat{\mathcal{A}}_{ct} > \widehat{A}_{xt}$. In particular, we would have $\widehat{P}_{ct} > 0$ even if $\widehat{A}_{xt} = 0$. Importantly, the relative price between final consumption and final investment is no longer solely a function of A_x .

6.2 Insight 2: Sectoral TFP Growth and GBGP

Lemma 2 above characterized the properties that would hold along a GBGP. But under what conditions will such a path exist? This is the question that we take up in this subsection.

¹⁰A “hat” again denotes growth rates. We show later that these conditions are all found to hold when we connect our model to the data.

Proposition 2 *A GBGP exists if and only if*

$$A_{xt} \left(\omega_x A_{gt}^{\varepsilon_x - 1} + (1 - \omega_x) A_{st}^{\varepsilon_x - 1} \right)^{\frac{1}{\varepsilon_x - 1}} \theta [K_0 \exp(\gamma t)]^{\theta - 1} \text{ is constant } \forall t \in [0, \infty). \quad (19)$$

If $\varepsilon_x \neq 1$, this implies that for a GBGP and structural change to coexist, at least one of the growth rates $\widehat{A}_{xt}$, $\widehat{A}_{gt}$, $\widehat{A}_{st}$ must not be constant.

To show the claims of the proposition, we start by recalling the pseudo aggregate production function:

$$Y_t = A_{xt} \left(\omega_x A_{gt}^{\varepsilon_x - 1} + (1 - \omega_x) A_{st}^{\varepsilon_x - 1} \right)^{\frac{1}{\varepsilon_x - 1}} K_t^\theta.$$

Taking the derivative with respect to capital and using the fact that if there is a GBGP then the real interest rate is constant and K_t grows at rate γ , we obtain that condition (19) is equivalent to having a constant real interest rate. For condition (19) to hold,

$$A_{xt} (\omega_x A_{gt}^{\varepsilon_x - 1} + (1 - \omega_x) A_{st}^{\varepsilon_x - 1})^{\frac{1}{\varepsilon_x - 1}}$$

must grow at a constant rate. A necessary condition for structural change to occur is that $\varepsilon_x \neq 1$. It then follows that if structural change occurs along a GBGP, then at most two of the three growth rates of A_{xt} , A_{gt} , and A_{st} can be constant in general. Specifically, if we were to restrict attention to constant rates of investment-specific technological change and of technological change in both the goods and services sector, this condition could only hold if both the goods and services sectors experienced the same rate of technological progress. But if this was the case then there would be no changes in the relative price of goods and services and hence there would be no structural change.

While the literature on structural change and balanced growth has tended to focus on the case in which sectoral TFPs grow at constant rates, there is no natural reason to favor this case. In a one-sector model, constant growth at the aggregate level necessarily requires constant growth in aggregate TFP. But in a multi-sector model with non-linear aggregators, the fact that certain aggregates grow at (approximately) constant rates does not translate to (approximately) constant rates of growth in sectoral TFPs. In fact, from the previous subsection, along a GBGP

both X_t and E_t will grow at the same constant rate. If these conditions hold and structural change is occurring, then it follows that sectoral TFPs *cannot* all be growing at constant rates. These conditions can be checked in the data, which we will do in the next section.

6.3 Insight 3: Asymptotic Behavior

Baumol (1967) was the first to note that if sectors experience differential productivity growth and if low substitutability implies that resources systematically move from sectors with high productivity growth to sectors with low productivity growth, then the economy will exhibit a secular decline in aggregate productivity growth. Taken to the extreme, this argument suggested that if there is one sector with low, or even zero, productivity growth and without a close substitute, then it will eventually dominate the entire economy. Importantly, Baumol's formal analysis abstracted from capital.

In contrast, Ngai and Pissarides (2007) built a model of structural change with capital and showed that it has a different asymptotic implication. Among the many sectors of their model, just one is assumed to produce capital, along with consumption. Since the capital-to-output ratio is constant along a GBGP, the capital-producing sector does not disappear, even asymptotically. Assuming that the capital-producing sector is manufacturing, which does not have the slowest productivity growth, the sector with the slowest productivity growth will dominate consumption, but not the entire economy. To translate this result to our setting, recall that for simplicity we only have the goods and services sectors. A version of their result obtains in our model if we assume that investment is produced only with goods, that goods and services are complements in the production of consumption, that TFP grows more strongly in the goods sector, and that in the limit the growth rate of goods TFP is bounded away from the growth rate of services TFP. While the service sector then takes over consumption asymptotically, the goods sector continues to produce investment. Since along the GBGP, investment constitutes a constant fraction of GDP, the share of the goods sector will converge to that of investment, and so the goods sector will not disappear asymptotically.

Consider instead our more general specification in which both goods and services are used to produce investment. Additionally, assume again that goods and services are complements

in both investment and consumption production, that TFP grows more strongly in the goods sector, and that in the limit the growth rate of goods TFP is bounded away from the growth rate of services TFP. For concreteness, also assume that investment-specific technological change does not play a role. Under these assumptions, it remains true that investment will be a constant fraction of output, but the value added share of services will approach one for both consumption and investment. Therefore, all labor will asymptotically be in the services sector, which in our model is the sector with the slowest TFP growth. The next proposition summarizes this result:

Proposition 3 *If the parameters are such that a GBGP exists, $\varepsilon_x, \varepsilon_c \in [0, 1)$, $\widehat{A}_{gt} > \widehat{A}_{st}$, $\lim_{t \rightarrow \infty} \widehat{A}_{gt} > \lim_{t \rightarrow \infty} \widehat{A}_{st} + \varepsilon_1$ for $\varepsilon_1 > 0$, then there are two cases*

- $\omega_x = 1$: $\lim_{t \rightarrow \infty} \frac{P_{st} Y_{st}}{Y_t} = \frac{C}{Y}$ (standard result in the literature);
- $\omega_x \in [0, 1)$: $\lim_{t \rightarrow \infty} \frac{P_{st} Y_{st}}{Y_t} = 1$ (novel result of our unified approach).

The proposition implies that if one takes structural change in investment into account, then the sector with the slowest productivity growth will take over the economy asymptotically. This result is important when one seeks to understand the role that structural change plays for the productivity slowdown. In related work, Duernecker et al. (2017) show that the sector with the slowest productivity growth may take over the economy asymptotically when one disaggregates further and distinguishes between services with fast and slow productivity growth.

7 Empirical Analysis

In this section we complement the preceding theoretical analysis of the model of Section 3 with an empirical assessment of two questions. First, to what extent are the CES aggregators in our formulation able to capture the secular trends in the post WWII US data? Second, to the extent that we observe something that looks like approximate balanced growth in the data, what is the nature of the technological change that accounts for this. Put somewhat differently, what is the behavior of the various terms in the theoretical condition necessary for generalized balanced growth?

Table 1: Preference and technology parameters

ε_c	ω_c	ε_x	ω_x
0.00	0.19	0.00	0.52

We begin with the first question. Our analysis focuses on the expressions for goods and services expenditure shares by final expenditure sector that we previously derived, which we repeat here for convenience:

$$\frac{P_{gt}C_{gt}}{P_{st}C_{st}} = \frac{\omega_c}{1 - \omega_c} \left(\frac{P_{gt}}{P_{st}} \right)^{1-\varepsilon_c},$$

$$\frac{P_{gt}X_{gt}}{P_{st}X_{st}} = \frac{\omega_x}{1 - \omega_x} \left(\frac{P_{gt}}{P_{st}} \right)^{1-\varepsilon_x}.$$

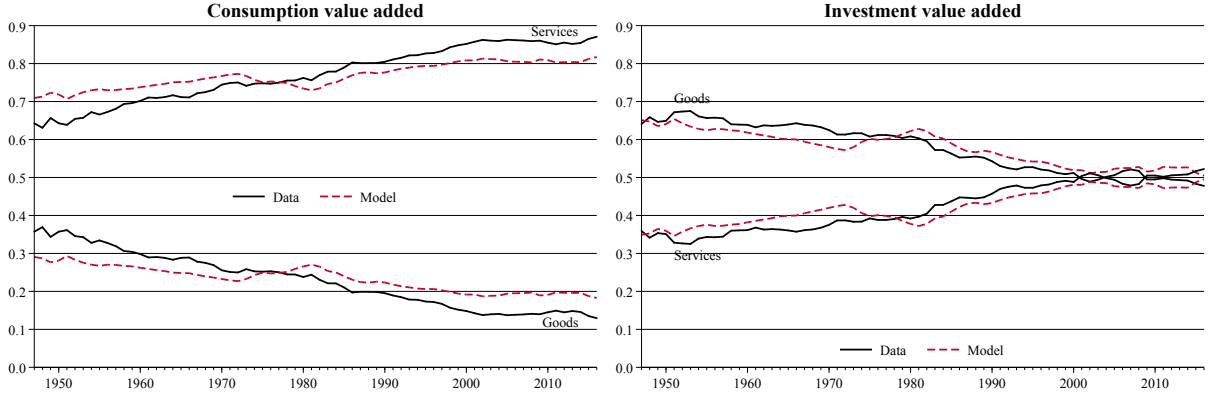
To assess whether the assumed CES structure is empirically reasonable for our purposes we ask whether there are values for the share and elasticity parameters (the ω_j and ε_j) such that when taking relative prices as given by the data, we are able to capture the key secular changes in expenditure shares. To implement this we use the data as presented in Section 2 and estimate the share and elasticity parameters via iterated generalized least squares. This is the same procedure that we used in Herrendorf et al. (2013). The estimated values are given in Table 1.

The striking result is that both elasticity parameters are estimated to be zero, implying that both aggregators are Leontief. The result that $\varepsilon_c = 0$ is not surprising given the results obtained earlier in Herrendorf et al. (2013). What is surprising to us is that the estimation implies that the aggregator in the investment sector features a similarly low degree of substitutability.¹¹ Figures 3 show the fitted values for the expenditure shares along with their corresponding values in the data.

The fitted values track the secular change within the investment sector very well, whereas the fitted values for consumption explain only about half of the change in expenditure shares

¹¹We note that the estimation procedure moves to the lower boundary of the allowable parameter space for both elasticity parameters and so standard errors are not available.

Figure 3: Expenditure shares – data versus model



within consumption. As we noted earlier, the latter result is expected since we have abstracted from income effects by virtue of considering a homothetic aggregator and since our earlier work that focused solely on consumption did find a role for income effects.¹² Additionally, recall that we have aggregated agriculture and manufacturing into a single goods sector and that nonhomotheticities are typically found to be important for the decline of agriculture. We are not too worried about missing an important part of the structural change in consumption here, because the fit for investment is of greater interest in the current context. After all, this is the novel aspect of our framework, and the figures show that our CES aggregator does a very good job of capturing the secular change in the US over this time period.

Next we consider the necessary condition for balanced growth – that the TFP in the investment sector, $\mathcal{A}_{xt}$, grow at a constant rate – from an empirical perspective. To do this we use the estimated values for ω_x from the previous exercise and compute series for the A_{jt} 's using standard growth accounting methods as in Solow (1957). To solve for A_{gt} and A_{st} we use labor services in efficiency units, capital services, and factor shares from WORLD KLEMS. To estimate TFP growth in the investment sector we use:¹³

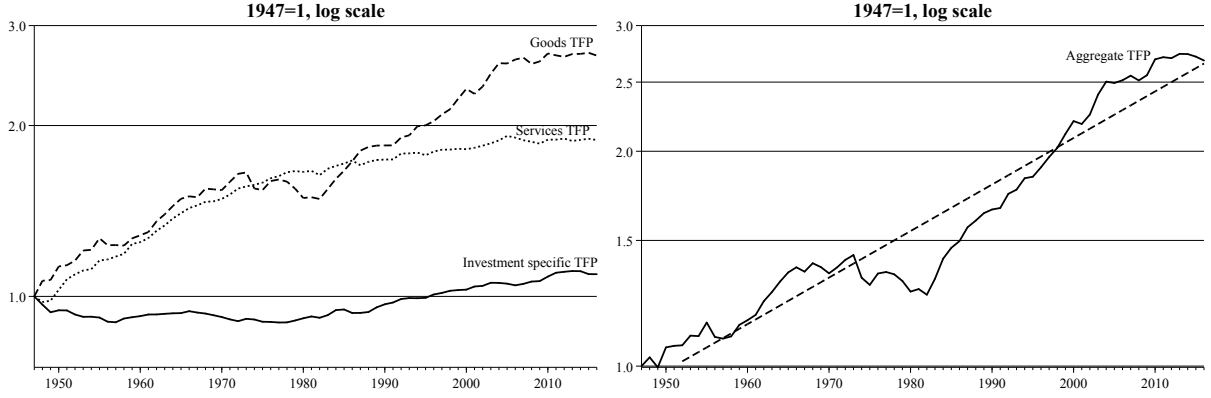
$$\frac{\dot{X}_t}{X_t} = \frac{\dot{A}_{xt}}{A_{xt}} + \frac{P_{gt}X_{gt}}{X_t} \frac{\dot{X}_{gt}}{X_{gt}} + \frac{P_{st}X_{st}}{X_t} \frac{\dot{X}_{st}}{X_{st}},$$

from which we compute the implied growth rate series for A_{xt} . Note that we do not impose our

¹²We note that adding net exports to consumption tends to worsen the fit of the model.

¹³The continuous-time divisia index was approximated with the Törnqvist index in the data.

Figure 4: Sectoral and Aggregate TFPs



functional forms when computing the TFP series.

The left panel of Figure 4 shows the implied series for the level of each TFP term. The graph has the interesting implication that there is little purely investment-specific technological change. Instead, the main reason for the higher productivity growth of the investment sector is that, as we explained in Insight 1, technological progress is endogenously investment biased in our model because the investment aggregator puts a higher relative weight on goods which have higher TFP growth than services.

Given our estimates of ω_{xj} , $j = g, s$ and ε_x , plus our estimated series for the A_{jt} , $j = x, g, s$, we evaluate the value of $A_{xt} \left(\omega_x A_{gt}^{\varepsilon_x - 1} + \omega_{xs} A_{st}^{\varepsilon_x - 1} \right)^{\frac{1}{\varepsilon_x - 1}}$. Recall that constant growth in this quantity is necessary in order to have a GBGP. The right panel of Figure 4 plots the log of this value over time. The figure shows that this term indeed grows on average at a constant rate over the postwar period. We emphasize two points. First, the construction of this figure did not directly use the information that the average growth of chained GDP in the US data is approximately constant. Second, although this term does grow at an approximately constant rate over the time period being studied, none of the individual TFP terms exhibits approximate constant growth. In particular, while the growth in A_{st} declines over time, the growth in A_{xt} actually increases over time.

8 Conclusion

In this paper, we have proposed a new framework for studying structural change and growth that treats consumption and investment in a symmetric manner. In particular, we have modelled production at the level of sector value added and have treated both final consumption and final investment as aggregates of the underlying sectoral value added. We have studied a simple form of this framework with two final demand sectors – consumption and investment – and two underlying sectors producing value added – goods and services. We have focused on this simple framework to best able to illustrate its distinctive features. First, for empirically plausible parameter values technological change is endogenously investment specific. Second, constant growth in all sectoral TFPs is generically inconsistent with aggregate balanced growth. Third, the sector with the lowest productivity growth asymptotically dominates the entire economy. Lastly, we have shown that a version of the model with a CES aggregator in the investment sector can account for the salient trends in the value-added composition of final investment.

We believe that richer versions of this model will prove useful in refining our view of the nature of structural change and the forces that drive it. In particular, we think it will be of interest to separately model structures and equipment within the investment sector. To the extent that goods represent a higher share of investment in equipment than they do in final consumption, it is interesting to ask whether the higher TFP growth in equipment is truly specific to equipment or if it instead reflects higher TFP growth in goods combined with a higher share of goods. Similarly, we think it is of interest to consider further decompositions of the service sector. We believe this framework will help us better isolate the underlying sources of TFP growth.

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Appendix

Proof of Lemma 3

Note first that if R_t is constant, the household’s Euler equation (17) implies that E_t must grow at a constant rate. Denoting this rate by γ , we have that γ satisfies:

$$\gamma = R - \delta - \rho.$$

Second, because both the goods and services sectors have Cobb-Douglas production functions with the same capital share, total payments to capital will be a fraction θ of total output: $RK_t = \theta Y_t$. A constant value for R thus also implies that Y_t/K_t is constant. Using this expression to substitute for R in the Euler equation gives:

$$\frac{Y_t}{K_t} = \frac{\gamma + \delta + \rho}{\theta}. \quad (20)$$

The fact that Y_t and K_t grow at the same rate does not necessarily imply that they grow at constant rates. But we next show that along a GBGP it is indeed the case that K_t grows at a constant rate, and furthermore that this rate is γ . Using $\dot{K}_t = X_t - \delta K_t$ with the resource

constraint $X_t = Y_t - E_t$ gives:

$$\widehat{K}_t = \frac{Y_t}{K_t} - \delta - \frac{E_t}{K_t}.$$

Substituting for Y_t/K_t using equation (20) gives:

$$\widehat{K}_t = \frac{\gamma + \delta + \rho}{\theta} - \delta - \frac{E_t}{K_t}. \quad (21)$$

To show that $\widehat{K}_t$ is constant and equal to γ , we argue by way of contradiction. First, assume that at some time T $\widehat{K}_t > \gamma$. Since E_t grows at rate γ this implies that E_t/K_t is decreasing. Equation (21) then implies that $\widehat{K}_t$ is increasing, so that $\widehat{K}_t$ will exceed γ for all t greater than T . It follows that the limit of E_t/K_t must be zero and hence that:

$$\lim_{t \rightarrow \infty} \widehat{K}_t = \frac{\gamma + \delta + \rho}{\theta} - \delta = \frac{\gamma + \rho}{\theta} + \frac{1 - \theta}{\theta} \delta > \gamma + \rho.$$

But this implies that the transversality condition, which requires that $\widehat{K}_t < \gamma + \rho$, is violated.

Suppose alternatively that at some time T we have $\widehat{K}_t < \gamma$. Arguing as above, E_t/K_t is now increasing, implying that $\widehat{K}_t$ must be decreasing, so that $\widehat{K}_t$ will be less than γ for all $t > T$. This implies that E_t/K_t will tend to infinity and hence that:

$$\lim_{t \rightarrow \infty} \widehat{K}_t = -\infty.$$

It follows that the growth rate of K_t is negative and bounded away from zero beyond some finite date, implying that K_t must become negative in finite time. This violates feasibility.

We conclude that K_t grows at the constant rate γ . Since Y_t grows at the same rate as K_t it follows that Y_t also grows at the same constant rate γ . It remains to show that X_t grows at rate γ . Combining $\dot{K}_t = X_t - \delta K_t$ with the fact that K_t grows at the constant rate γ gives:

$$\frac{X_t}{K_t} = \gamma + \delta,$$

so that X/K is constant. It follows that it must also be that $\widehat{X}_t = \gamma$. **QED**

Proof of Proposition 1

We only show the claim for X_t . The proof for C_t follows the exact same steps. We start by rewriting the production function of X_t :

$$\begin{aligned} X_t &= A_{xt} \left(\omega_x^{\frac{1}{\varepsilon_x}} + (1 - \omega_x)^{\frac{1}{\varepsilon_x}} \left(\frac{X_{st}}{X_{gt}} \right)^{\frac{\varepsilon_x - 1}{\varepsilon_x}} \right)^{\frac{\varepsilon_x}{\varepsilon_x - 1}} X_{gt}, \\ X_t &= A_{xt} \left(\omega_x^{\frac{1}{\varepsilon_x}} \left(\frac{X_{gt}}{X_{st}} \right)^{\frac{\varepsilon_x - 1}{\varepsilon_x}} + (1 - \omega_x)^{\frac{1}{\varepsilon_x}} \right)^{\frac{\varepsilon_x}{\varepsilon_x - 1}} X_{st}. \end{aligned}$$

Equation (15) implies that:

$$\frac{X_{st}}{X_{gt}} = \frac{1 - \omega_x}{\omega_x} \left(\frac{A_{st}}{A_{gt}} \right)^{\varepsilon_x} \quad \text{and} \quad \frac{X_{gt}}{X_{st}} = \frac{\omega_x}{1 - \omega_x} \left(\frac{A_{gt}}{A_{st}} \right)^{\varepsilon_x}.$$

Combining the last two sets of equations, we obtain:

$$\begin{aligned} X_t &= A_{xt} \left(\omega_x^{\frac{1}{\varepsilon_x}} + \frac{1 - \omega_x}{\omega_x^{\frac{\varepsilon_x - 1}{\varepsilon_x}}} \left(\frac{A_{st}}{A_{gt}} \right)^{\varepsilon_x - 1} \right)^{\frac{\varepsilon_x}{\varepsilon_x - 1}} X_{gt}, \\ X_t &= A_{xt} \left(\frac{\omega_x}{(1 - \omega_x)^{\frac{\varepsilon_x - 1}{\varepsilon_x}}} \left(\frac{A_{gt}}{A_{st}} \right)^{\varepsilon_x - 1} + (1 - \omega_x)^{\frac{1}{\varepsilon_x}} \right)^{\frac{\varepsilon_x}{\varepsilon_x - 1}} X_{st}. \end{aligned}$$

Rearranging gives:

$$\begin{aligned} \omega_x A_{gt}^{\varepsilon_x - 1} X_t &= A_{xt} \left(\omega_x A_{gt}^{\varepsilon_x - 1} + (1 - \omega_x) A_{st}^{\varepsilon_x - 1} \right)^{\frac{\varepsilon_x}{\varepsilon_x - 1}} \frac{X_{gt}}{A_{gt}}, \\ (1 - \omega_x) A_{st}^{\varepsilon_x - 1} X_t &= A_{xt} \left(\omega_x A_{gt}^{\varepsilon_x - 1} + (1 - \omega_x) A_{st}^{\varepsilon_x - 1} \right)^{\frac{\varepsilon_x}{\varepsilon_x - 1}} \frac{X_{st}}{A_{st}}. \end{aligned}$$

Adding the two equations gives:

$$\left(\omega_x A_{gt}^{\varepsilon_x - 1} + (1 - \omega_x) A_{st}^{\varepsilon_x - 1} \right) X_t = A_{xt} \left(\omega_x A_{gt}^{\varepsilon_x - 1} + (1 - \omega_x) A_{st}^{\varepsilon_x - 1} \right)^{1 + \frac{1}{\varepsilon_x - 1}} \left(\frac{X_{gt}}{A_{gt}} + \frac{X_{st}}{A_{st}} \right)$$

Dividing through the first term on the left-hand side and using the definitions of $\mathcal{A}_{xt}$, K_{xt} , and L_{xt} proves the claim. **QED**