

# Investment and Saving along the Development Path<sup>\*</sup>

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## **Abstract**

In this paper we document that the investment and saving rates follow a large and long-lived hump-shaped profile along the development path. This pattern is present both in the large panel of countries of the Penn World Tables (PWT) between 1950 and 2010 and also in the historical data recently assembled by Jordà, Schularick, and Taylor (2017). The hump of investment with development is a challenge for the standard neo-classical model of growth. We propose two simple mechanisms that can jointly explain the observed paths of investment. First, we allow for richer transitional dynamics by assuming that households value their consumption in reference to a slow-moving endogenous standard of living of the society where they live in, and show that a calibrated model with this feature alone can reproduce the large and long-lasting hump-shaped profiles of investment with development. Second, we look at the data of each country as coming from the transitional dynamics of economies whose technology does not grow at a constant rate. In particular, we extend the model to have separate consumption and investment goods sectors and feed it with the observed paths of investment-specific technical change, total factor productivity, and investment depreciation for each country. We exploit the Euler equation of consumption with a large panel of countries to estimate the preference parameters of the model as it is standard in the micro consumption literature. The estimated model allows us to understand which forces drive the investment rates observed in the data for every country and period.

*JEL classification:* E23; E21; O41

*Keywords:* Investment; Growth; Transitional Dynamics

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# 1 Introduction

- The stylized facts of growth —first stated by Kaldor (1961)— describe a set of empirical regularities associated to the growth experience of economies at or near the technological frontier. As recently emphasized by Jones (2016), these facts can be enlarged with the observation that investment rates are more or less constant in the long run. Less is known, however, about the evolution of the investment rate along the development path. For instance, Buera and Shin (2013) study the development episodes of the so-called Asian Dragons and show that the investment rates started low, increased gradually over time, and stabilized or started to decline around 15 years after the start. Also, Christiano (1989) and Chen, Imrohoroglu, and Imrohoroglu (2006) document a hump-shaped path for the saving rate in Japan between the mid 1950's and the late 1990's.
- In this paper we describe the evolution of the investment and saving rates more comprehensively. In particular, we uncover a distinct long-lived hump-shaped profile of investment and saving with income, which seems to be an inherent property of the development process. This pattern is present both in the large panel of countries of the Penn World Tables (PWT) between 1950 and 2010 and also in the historical data recently assembled by Jordà, Schularick, and Taylor (2017). In particular, the investment rate increases by 20 percentage points as income per capita grows more than 10-fold (between 700 and 8,000 1995 international dollars, which correspond to levels of development like China in 1950 and Thailand in 2005). After that, the investment rate declines by 10 percentage points as income grows more than 5-fold.<sup>1</sup>
- The hump of investment with development is a challenge for the standard neo-classical model of growth: the investment rate is predicted to be constant along the balanced growth path and monotonically increasing or decreasing in the transitional dynamics. In particular, the capital deepening process of the transitional dynamics generates a steady decline of the return to investment (the substitution effect) alongside a steady increase in output (the income effect). For standard calibrations the former effects dominates monotonically, which counterfactually reduces the saving rate along the development path. This is unfortunate as the optimal dynamic decision of consumption and saving is all what the neo-classical growth model is

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<sup>1</sup>To give a sense of the time length of this process, both Thailand and South Korea saw their investment rates increase monotonically from 12 percent of GDP in the 50's and 60's to a peak of 42 percent of GDP in 1990, when the investment rates started to decline.

about.

- In this paper we propose two simple mechanisms that can jointly explain the observed paths of investment. First, we extend the preference class of the standard model in order to allow for richer transitional dynamics. In particular, we assume that households value their consumption in reference to a slow-moving endogenous standard of living of the society where they live in, and show that a calibrated neo-classical growth model with this feature alone can reproduce the large and long-lasting hump-shaped profiles of investment with development as the result of transitional dynamics with constant technology growth. This calibration result reconciles the neo-classical growth model with the large observed humps of investment with development, but does not necessarily imply that a specific type of intertemporal preferences is the unique factor behind the investment hump. Second, we look at the data of each country as coming from the transitional dynamics of economies whose technology does not grow at a constant rate. In particular, we extend the model to have separate consumption and investment goods sectors and feed it with the observed paths of investment-specific technical change (ISTC), total factor productivity (TFP), and investment depreciation for each country.
- We exploit the Euler equation of consumption with a large panel of countries to estimate the preference parameters of the model as it is standard in the micro consumption literature. We restrict preference parameters to be common across countries but allow the evolution of technology and depreciation to be country-specific. Furthermore, we allow for country-specific investment wedges as in Chari, Kehoe, and McGrattan (2007). The estimated model allows us to understand which forces drive the investment rates observed in the data for every country and period, while the wedges inform us of which countries and periods the observed saving rate is at odds with the theory.
- Findings: TO BE WRITTEN

## 1.1 Related literature

- Barro and Sala-i-Martin (1999) show analytically that the standard neo-classical growth model in continuous time with Cobb Douglas production and CRRA utility predicts a monotonic evolution of the saving rate in transitional dynamics. Antràs (2001) extends this result by showing theoretically that the saving rate can display a hump in the transitional dynamics when either the production function features an

elasticity of substitution between capital and labor less than one or when the utility function is of the Stone-Geary type with a survival consumption term, with this last result shown previously by King and Rebelo (1993) in a numerical exercise. We show that quantitatively these two features fall very short of the observed humps in the data. Carroll, Overland, and Weil (2000) show that in a model with linear production technology one can obtain rich transitional dynamics when preferences feature habit formation, while Álvarez Cuadrado, Monteiro, and Turnovsky (2004) show numerically that a standard neo-classical growth model with either internal or external habit can produce a sizeable hump in the saving rate. In relation to this literature, we show that the external habit has the quantitative bite that other explanations lack.

- The role of the relative price of investment goods on development has already been emphasized. Caselli and Feyrer (2007) show that differences in the relative price of investment goods are important to understand cross-sectional differences in the marginal product of capital because countries with lower capital to labor ratio tend to have more expensive investment goods. Hsieh and Klenow (2007) show that there is no correlation between the investment rates at domestic prices and income per capita, but the investment rates at international prices are larger for richer countries. The reason is that richer countries face lower relative prices of investment goods, something already highlighted by Restuccia and Urrutia (2001). Our results focus instead in the within country variation, allowing for cross-country differences in investment to reflect country fixed effects in institutions, policies or preferences as well as differences in income.
- Several papers have attempted to explain investment or saving dynamics that are at odds with the standard neo-classical growth model for particular countries. Chen, Imrohoroglu, and Imrohoroglu (2006) explain the hump in Japan after the IIWW as the result of the particular TFP trajectory while Chang and Hornstein (2015) show that feeding the actual process of ISTC into the neo-classical model can account for the hump in South Korea. Song, Storesletten, and Zilibotti (2011) and Buera and Shin (2013) emphasize financial frictions for the cases of China and the Asian Tigers. Imrohoroglu and Zhao (2017) argue that changes in demographic factors are key to understand the evolution of the saving rate in China, while Chen, Imrohoroglu, and Imrohoroglu (2007) say they do not matter in Japan. We contribute to this literature by offering a unified approach to understand the behavior of the investment rate in many countries together

## 2 Investment paths in the data

The Kaldor stylized facts of growth imply that in the long run the investment rate of growing economies is more or less constant. However, in Section 2.1 below we show that while this is true for the U.S., it does not seem to be the case when looking at data from many other countries. In Section 2.2 we show that this impression is strengthened when pooling together data from many countries over the last 60 years. Indeed, we show that the investment rate displays a distinct hump-shaped profile with economic development. Finally, in Section 2.3 we show that the much longer time series from early developers are consistent with the hump-shaped profile of investment with development.

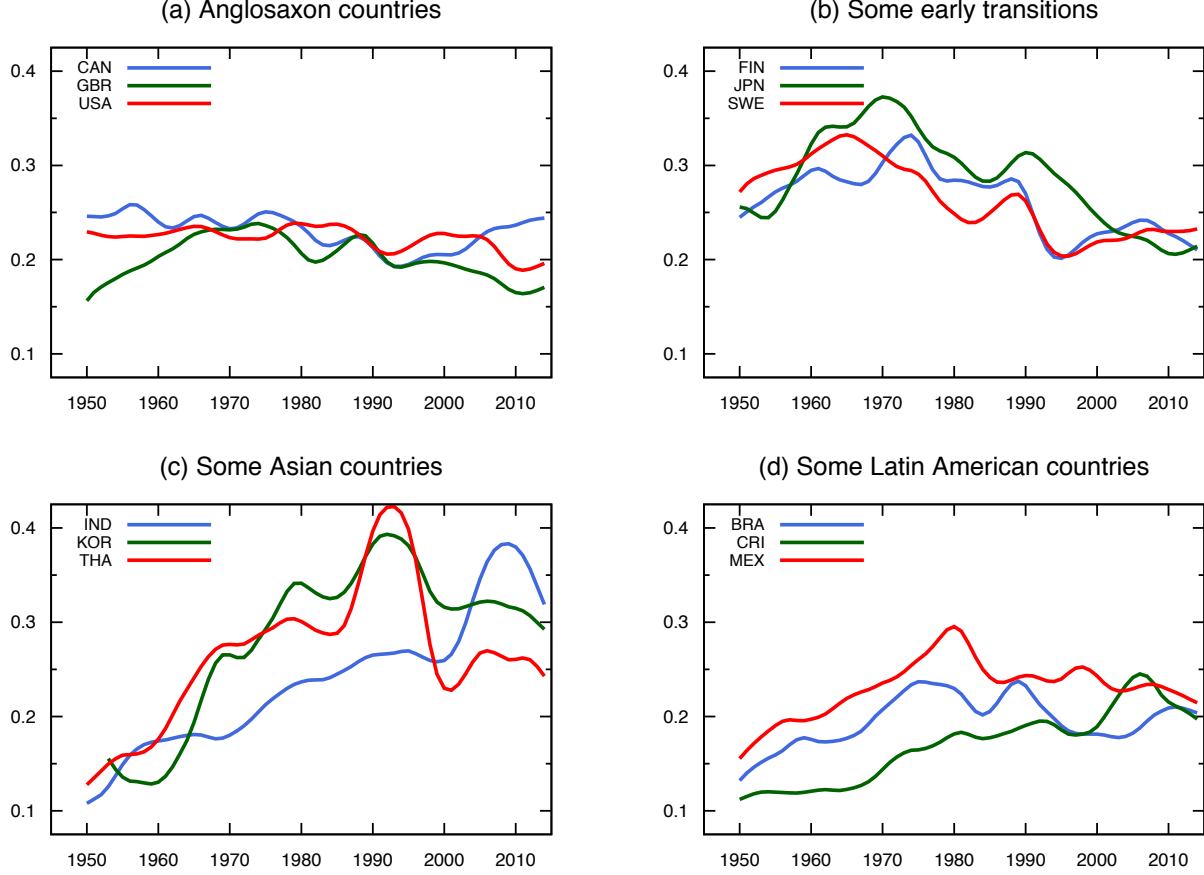
### 2.1 Some time series

Panel (a) in Figure 1 shows the stationary pattern of investment rates that is commonly associated with a balanced growth path. For Canada, UK, and the US the investment rate of the economy fluctuates little and without a trend between 1950 and 2010, with perhaps the exception of UK where the investment rate shows a mild hump over the period. Panel (b) in Figure 1 shows the investment rates for Finland, Japan, and Sweden, a group of countries that underwent economic transitions within the period. The investment rates for these countries changed substantially following a sharp hump-shaped profile over time, which peaked between the mid 60's and early 70's. The changes are big and slow: for instance, the investment rate in Japan increased from around 25% in 1950 to 38% in the mid 70's, declining monotonically afterwards down to 20% in 2010. Finally, Panels (c) and (d) in Figure 1 show the investment rates for India, South Korea, Thailand, Brazil, Costa Rica, and Mexico, a group of countries that experienced large economic transformations during the second half of the XXth Century. For these countries we see that the investment rates also change substantially over long periods of time. For instance, the investment rate in India increased from 10 to almost 40 percent between 1950 and 2010. The time series of aggregate data in countries like those in Panels (b), (c) and (d) in Figure 1 are hence better characterized by transitional dynamics than by the balanced growth path assumption, and seem at odds with the implications of the neo-classical growth model.

### 2.2 Investment rates and development

In order to visualize the data for all countries in our sample, in Figure 2 we plot the investment rate in each country-year against their level of development. We want to filter out level differences in investment rates across countries due to potential differences in

FIGURE 1: Evolution of investment rates



*Notes.* Investment rates at current prices for a selection of countries, data from PWT. Series are smoothed with HP-filter with parameter 6.25 for annual data.

institutions, policies, or preferences. For this reason, we run the following regression with country fixed effects:

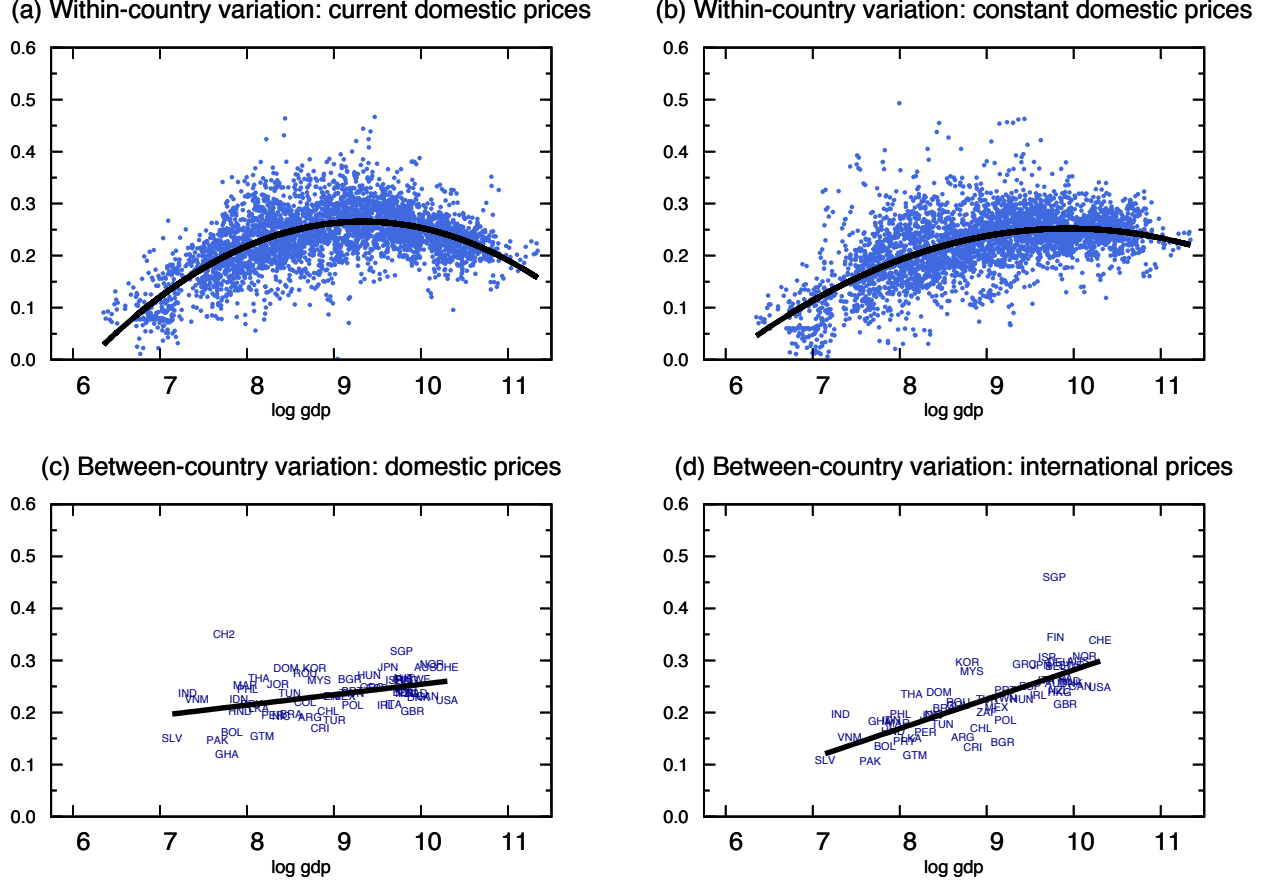
$$x_{it} = f_i + \beta_1 y_{it} + \beta_2 y_{it}^2 + \varepsilon_{it}$$

where  $x_{it}$  is the investment rate at current domestic prices of country  $i$  at time  $t$ ,  $y_{it}$  is the log of per capita GDP in PPP, and  $f_i$  is the country fixed effect. In Panel (a) we plot the within-country variation of the investment rate at current prices against the log of GDP.<sup>2</sup> What we observe is a clear hump-shaped profile of investment with the level of

<sup>2</sup>In particular, we report  $\hat{x}_{it} \equiv \frac{1}{n} \sum_i f_i + \hat{\beta}_1 y_{it} + \hat{\beta}_2 y_{it}^2$  (solid black line) and  $\hat{x}_{it} + \varepsilon_{it}$  (blue dots), where  $\hat{\beta}_1$  and  $\hat{\beta}_2$  are estimated with deviations from country means, that is, with an OLS regression of

$$x_{it} - \bar{x}_i = \beta_1 (y_{it} - \bar{y}_i) + \beta_2 (y_{it}^2 - \bar{y}_i^2) + (\varepsilon_{it} - \bar{\varepsilon}_i)$$

FIGURE 2: Investment rates and development



Notes. All data from PWT.

development. When poor, countries invest a small fraction of their output. As countries develop, the investment rate first increases about 15 percentage points and then declines about 10 percentage points. This hump is not only large, but also slow and long-lived: during the process just described GDP per capita increases about 4.5 log points, which is equivalent to a 90-fold increase.<sup>3</sup>

The evolution of the investment rate along the development path may be driven by both variation in prices and variation in quantities. In order to disentangle those two components, we also look at the within-country evolution of the investment rate at constant domestic prices, see Panel (b) of Figure 2. We find that the increasing part of the investment rate in the first half of the development process is the same at constant and

<sup>3</sup>The behavior of the investment rate with development is reproduced by its components. In particular, the hump-shaped profile is apparent for both investment in structures and non-structures.

current prices, so it is likely driven by the increase in quantities, not prices. However, after the peak the decline in the investment rate at constant prices is milder (if at all) than in the investment rate at current prices, suggesting that part of the latter decline is due to the fall in the relative price of investment goods.

The findings presented here may seem at odds with the well-known results by Hsieh and Klenow (2007). These authors show that, when looking at a cross-section of countries, there is no relationship between investment rates at domestic prices and income but there is a positive relationship between investment rates at international prices and income. These facts are consistent with the idea that the relative price of investment goods is lower in richer countries and that rich countries do invest more than poor countries in spite of not making a higher effort in terms of forgone consumption. Indeed, when looking at between-country variation with our data we obtain similar results as Hsieh and Klenow (2007). To show this, in Panel (c) we report the between-country variation of investment rates at current domestic prices.<sup>4</sup> We find only a very mild increasing pattern of investment with income. Instead, in Panel (d) of Figure 2 we plot the between-country variation of the investment rate at common international prices (the investment rate at current domestic prices has been multiplied by the relative price of investment in international dollars as reported by the PWT). Here we observe a larger slope than in Panel (c), consistent with the idea that, due to lower investment prices, richer countries invest more without spending more in investment. We emphasize that our results do not reflect cross-country but within-country variation of income over time. We think that within-country variation gives a better sense of the change of the investment rate with development because it is not contaminated by country-specific variation in preferences, institutions or policies.

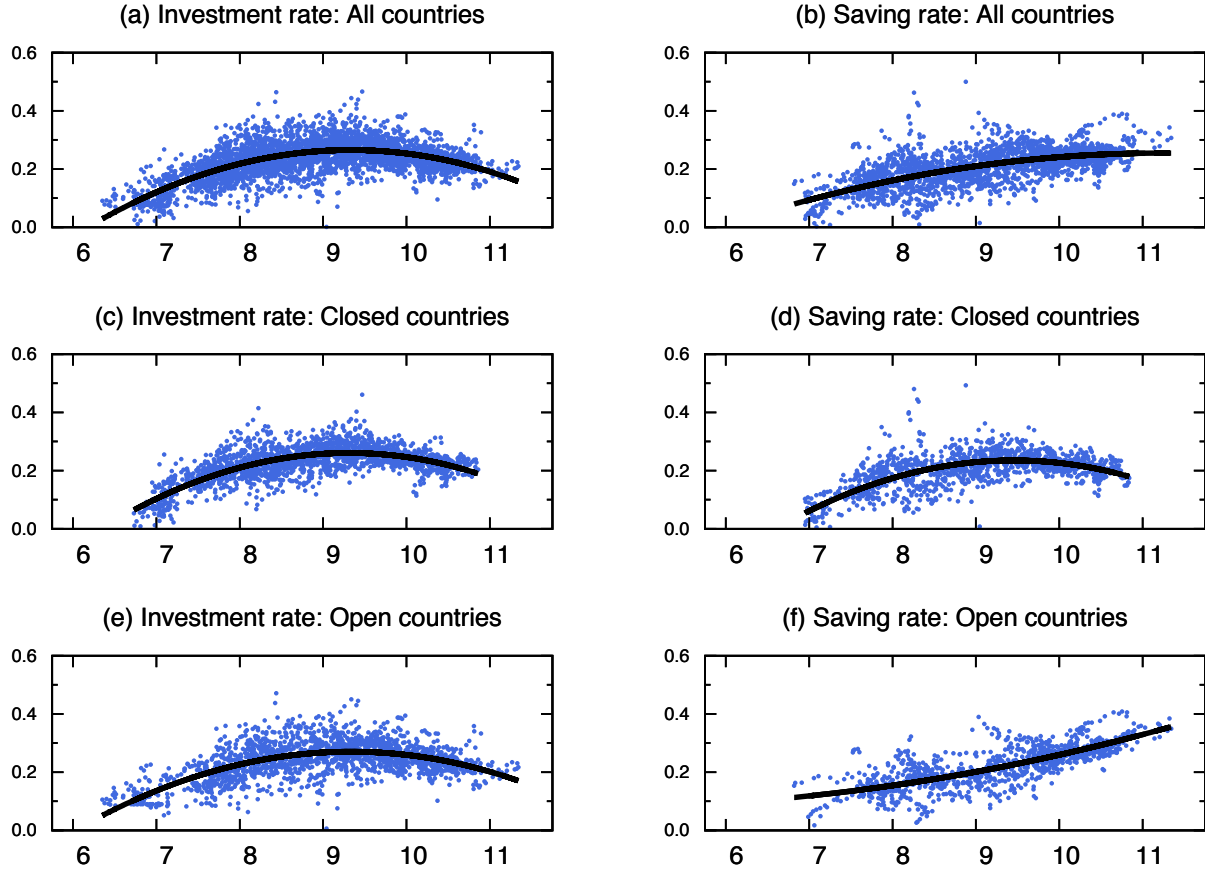
Before finishing this Section we want to show one more piece of evidence: the behavior of the saving rate with development goes in parallel with the evolution of the investment rate, at least in the first half of the process. In Panel (a) of Figure 3 we reproduce again the within-country variation of the investment rate at current domestic prices for all countries. In Panel (b) we plot the within-country variation of the saving rate. The saving rate displays a similar increase in the early stages of development, although the latter decline is milder. To look further into the behavior of the saving rate, we divide our sample of countries in two groups: those above and those below the median level

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<sup>4</sup>In particular, we report  $\tilde{x}_i \equiv \tilde{f}_i + \tilde{\beta}_1 \tilde{y}_i + \tilde{\beta}_2 \tilde{y}_i^2$  (solid black line) and  $\tilde{x}_i + \tilde{\varepsilon}_i$  (blue dots), where  $\tilde{\beta}_1$  and  $\tilde{\beta}_2$  are estimated with country means, that is, with an OLS regression of

$$\tilde{x}_i = \tilde{f}_i + \beta_1 \tilde{y}_i + \beta_2 \tilde{y}_i^2 + \tilde{\varepsilon}_i$$

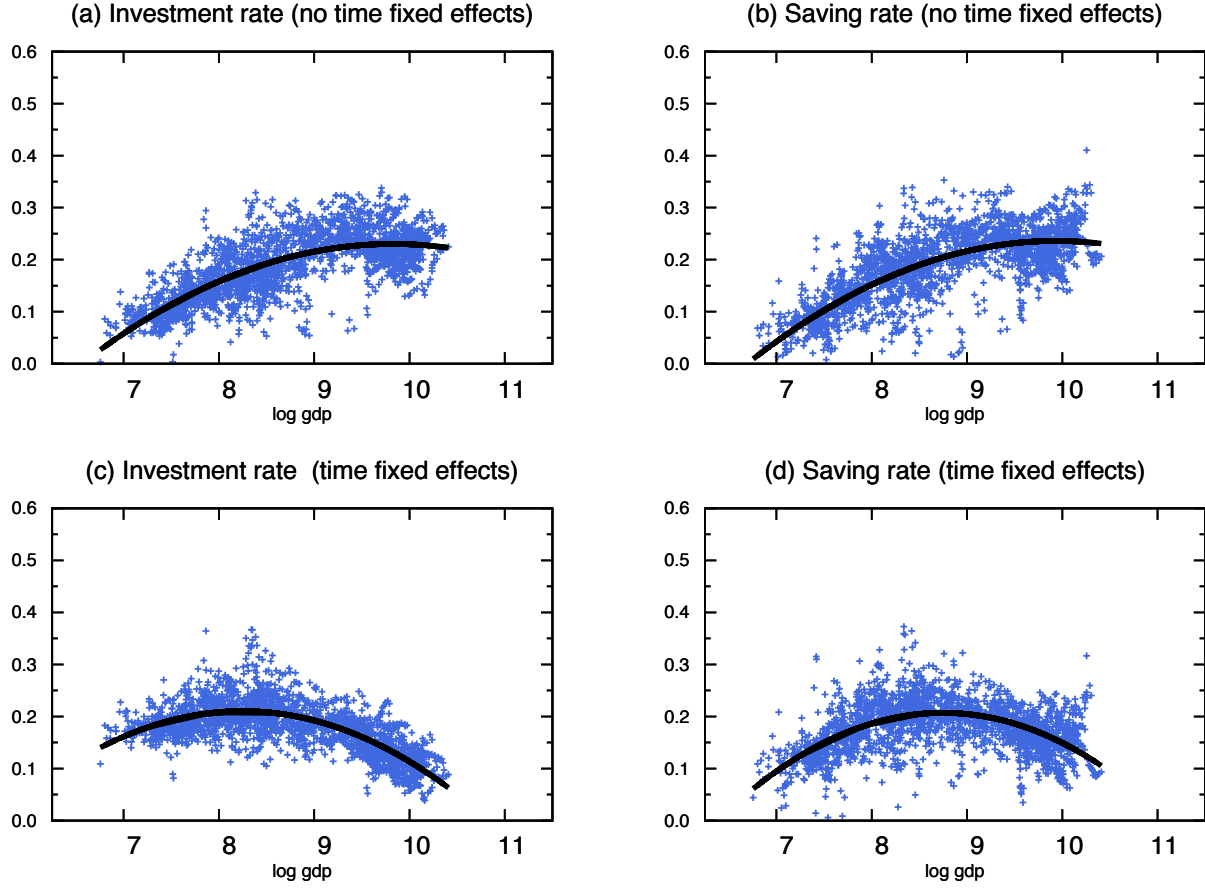
FIGURE 3: Investment and saving rates and development



*Notes.* Investment data from PWT, saving data from WDI. The samples for investment data and saving data are slightly different due to the lack of saving rate data for some countries and years. Open countries are those whose sum of imports and exports over GDP is above the sample median, while closed countries are the rest.

of openness (imports plus exports over GDP). We find that the hump-shaped profile of the investment rate is the same in both groups, see Panels (c) and (e). However, the profiles of savings are different. The evolution of the saving rates tracks the evolution of the investment rates for the less open economies (which include for instance India, China, South Korea, Japan, Canada or the US), but is monotonically increasing for the more open economies (which include El Salvador, Vietnam, Malaysia, Singapore, Finland or Switzerland), see Panels (d) and (f). This partition between open and closed economies is of course strongly correlated with country size: average population is 141 millions for the closed economies and is 11 millions for the open ones. Hence, we conclude that capital inflows and outflows are not critical to understand the behavior of the investment rate along the development path, although they may play a role for the latter stages of

FIGURE 4: Investment and saving rates



*Notes.* All data from Jordà, Schularick, and Taylor (2017).

development in small open economies.

### 2.3 Historical data

The evolution of the investment rate with economic development can also be explored by use of the historical data put together by Jordà, Schularick, and Taylor (2017), which offers data from 17 countries between 1870 and 2013. In Figure 4 we plot the within-country variation in investment rates and saving rates as constructed in Section 2.2 above, see Panel (a) and (b) respectively. We observe the same type of hump-shaped profile as in the data of the PWT, although without a clear fall at the end of the process. However, given the long time series dimension of these data, we may also want to filter out year fixed effects capturing events like World wars or ups and downs in international trade. Hence, Panel (c) and (d) plot the within-country variation in investment rates and saving

rate when we filter our country and year fixed effects. The results show a sharper hump in the sense that the decline of the investment and saving rate with at the latest stages of development is more pronounced.

### 3 Model

Our model economy extends the standard neo-classical growth model in two directions. First, following Greenwood, Hercowitz, and Krusell (1997), Caselli and Feyrer (2007), Hsieh and Klenow (2007) or Karabarbounis and Neiman (2014) it distinguishes between consumption and investment good sectors in order to allow for investment-specific technical change. Second, it allows for preferences with an endogenously-changing survival level of consumption. We are going to model a closed economy where investment equals savings. While this equality does not hold in the data at every period in time, we argue that it is a reasonable approximation. First, Aizenman, Pinto, and Radziwill (2007) show that capital accumulation of developing economies is mainly self-financed through internal savings. Second, Faltermeier (2017) shows that the decline of the marginal product of capital with development is unrelated to capital flows. And third, our own evidence in Section 2.2 shows that the paths of investment and saving only diverge in the later stages of development for small open economies.

#### 3.1 Setup

The economy consists of two different goods: consumption  $C_t$  and investment  $X_t$ . An infinitely-lived representative households of size  $N_t$  rents capital  $K_t$ , inelastically supplies labor  $N_t$  to firms, and decides how much to buy of each good. The household budget constraint in per capita terms is given by:

$$w_t + r_t k_t + e_t = p_{ct} c_t + (1 + \tau_{xt}) p_{xt} x_t \quad (1)$$

where  $p_{ct}$  and  $p_{xt}$  are the price of each good at time  $t$ ,  $w_t$  is the wage rate at time  $t$ , and  $r_t$  is the rental rate of capital between  $t - 1$  and  $t$ . Following Chari, Kehoe, and McGrattan (2007), we introduce the investment wedge  $\tau_{xt}$  and the lump sum transfers  $e_t$  aimed at capturing deviations of the data from the Euler equation (see Section 5).

Capital accumulates with the law of motion

$$(1 + n_{t+1}) k_{t+1} = (1 - \delta_{kt}) k_t + x_t \quad (2)$$

where  $0 < \delta_{kt} < 1$  is the depreciation rate of capital held between  $t - 1$  and  $t$  and  $n_{t+1}$  is the rate of population growth between  $t$  and  $t + 1$

Finally, we need to describe preferences. Households have a CRRA period utility function over the consumption basket  $c_t$  above a minimal standard of living  $\phi h_t$ ,

$$u(c_t - \phi h_t) = \frac{(c_t - \phi h_t)^{1-\sigma}}{1-\sigma} \quad (3)$$

where we define  $h_t$  as follows,

$$h_t = (1 - \delta_h) h_{t-1} + \delta_h c_{t-1} \quad (4)$$

We think of  $h_t$  as an external habit. Households value consumption flow  $c_t$  in relation to the standard of living  $h_t$  that they are used to, but because they are atomistic they do not internalize the changes in  $h_{t+1}$  when choosing their own consumption flow  $c_t$ . The parameter  $\phi$  drives the strength of the external habit and the parameter  $\delta_h$  its persistence. We introduce this feature to match the evolution of the saving rate as economies get richer. A standard CRRA utility function in the consumption level  $c_t$  only would feature either a monotonically increasing or a monotonically decreasing saving rate along the development path. The minimal consumption term allows to produce hump-shaped profiles of the saving rate with development because it makes the income effect very strong at the start of the development process and very weak at the end, see Christiano (1989) and King and Rebelo (1993)). Modelling the minimal consumption as an external habit gives both an economic interpretation of this mechanism and a flexible framework for quantitative work.

### 3.2 Household problem

With all these elements in place the optimal household plan is the sequence of consumption and investment choices that maximizes the discounted infinite sum of utilities. We can write the Lagrangian as,

$$\sum_{t=0}^{\infty} N_t \beta^t \left\{ u(c_t - \phi h_t) + \lambda_t \left[ w_t + r_t k_t + e_t - p_{ct} c_t - (1 + \tau_{xt}) p_{xt} x_t \right] + \eta_t \left[ (1 - \delta_{kt}) k_t + x_t - (1 + n_{t+1}) k_{t+1} \right] \right\}$$

where  $\lambda_t$  and  $\eta_t$  are the shadow values at time  $t$  of the budget constraint and the law of motion of capital respectively. Taking prices as given we get the Euler equation,

$$u'(c_t - \phi h_t)(1 + \tau_{xt}) \frac{p_{xt}}{p_{ct}} = \beta u'(c_{t+1} - \phi h_{t+1}) \frac{1}{p_{ct+1}} [r_{t+1} + (1 + \tau_{xt+1}) p_{xt+1} (1 - \delta_{kt+1})]$$

that can also be written as,

$$u'(c_t - \phi h_t) = \beta u'(c_{t+1} - \phi h_{t+1}) \frac{(1 + \tau_{xt+1}) p_{xt+1} p_{ct}}{(1 + \tau_{xt}) p_{ct+1} p_{xt}} \left[ \frac{r_{t+1}}{p_{xt+1}} + (1 - \delta_{kt+1}) \right] \quad (5)$$

The term in square brackets is the investment return in units of the investment good. When multiplied by the increase in the relative price of investment it becomes the investment returns in units of the consumption good, which is the relevant one for the Euler equation.

### 3.3 Production

There is a representative firm in each sector  $i = \{c, x\}$  combining capital  $k_{it}$  and labor  $l_{it}$  to produce the amount  $y_{it}$  of the final good  $i$ . The production functions are Cobb-Douglas with equal capital shares and different technology level  $B_{it}$ ,

$$y_{it} = k_{it}^\alpha (B_{it} l_{it})^{1-\alpha}$$

The objective function of each firm is given by,

$$\max_{k_{it}, l_{it}} \{p_{it} y_{it} - r_t k_{it} - w_t l_{it}\}$$

Leading to the standard FOC,

$$r_t = \alpha p_{it} \left( \frac{k_{it}}{B_{it} l_{it}} \right)^{\alpha-1} \quad (6)$$

$$w_t = (1 - \alpha) B_{it} p_{it} \left( \frac{k_{it}}{B_{it} l_{it}} \right)^\alpha \quad (7)$$

For convenience, let's define

$$\begin{aligned} B_t &\equiv B_{ct} \\ \chi_t &\equiv \left( \frac{B_{xt}}{B_{ct}} \right)^{1-\alpha} \end{aligned}$$

where  $B_t$  represents the common TFP affecting the production of both consumption and investment goods, and  $\chi_t$  represents the excess of TFP in the investment sector or the investment-specific technical level.

### 3.4 Equilibrium

An equilibrium for this economy is a sequence of exogenous paths  $\{B_t, \chi_t, N_t, \delta_{kt}, \tau_{xt}\}_{t=1}^{\infty}$  a sequence of allocations  $\{c_t, x_t, e_t, k_t, k_{ct}, k_{xt}, l_{ct}, l_{xt}, y_{ct}, y_{xt}\}_{t=1}^{\infty}$ , and a sequence of equilibrium prices  $\{r_t, w_t, p_{xt}, p_{ct}\}_{t=1}^{\infty}$  such that

- Households optimize: equations (1), (2), (4), (5) hold
- Firms optimize: equations (6), (7) hold
- All markets clear:  $k_{ct} + k_{xt} = k_t$ ,  $l_{ct} + l_{xt} = 1$ ,  $y_{ct} = c_t$  and  $y_{xt} = x_t$
- Transfers equal taxes:  $e_t = \tau_{xt} p_{xt} x_t$

Note that in equilibrium the FOC of the firms imply that the capital to labor ratio is the same for both goods and equal to the capital to labor ratio in the economy  $\frac{k_{ct}}{l_{ct}} = \frac{k_{xt}}{l_{xt}} = k_t$ , with

$$k_t = \frac{\alpha}{1 - \alpha} \frac{w_t}{r_t} \quad (8)$$

and that relative prices are given by

$$\frac{p_{xt}}{p_{ct}} = \frac{1}{\chi_t} \quad (9)$$

We can write the interest rate  $r_t$  and total output  $y_t \equiv p_{ct} y_{ct} + p_{xt} y_{xt}$  in units of the consumption good as a function of capital per capita in the economy,

$$\begin{aligned} r_t/p_{ct} &= \alpha B_t^{1-\alpha} k_t^{\alpha-1} \\ y_t/p_{ct} &= B_t^{1-\alpha} k_t^{\alpha} \end{aligned}$$

and likewise we can write the expressions for the interest rate and total output in units of the investment good as follows:

$$\begin{aligned} r_t/p_{xt} &= \alpha \chi_t B_t^{1-\alpha} k_t^{\alpha-1} \\ y_t/p_{xt} &= \chi_t B_t^{1-\alpha} k_t^{\alpha} \end{aligned}$$

Finally, we can characterize the equilibrium aggregate dynamics of this economy with the laws of motion for  $c_t$  and  $k_t$

$$\left( \frac{c_{t+1} - \phi h_{t+1}}{c_t - \phi h_t} \right)^\sigma = \beta \frac{(1 + \tau_{xt+1})}{(1 + \tau_{xt})} \frac{\chi_t}{\chi_{t+1}} \left[ \alpha \chi_{t+1} \left( \frac{k_{t+1}}{B_{t+1}} \right)^{\alpha-1} + (1 - \delta_{kt+1}) \right] \quad (10)$$

$$(1 + n_{t+1}) k_{t+1} = (1 - \delta_k) k_t + \chi_t (y_t - c_t) \quad (11)$$

and the law of motion for habits  $h_t$  given by equation (4).

### 3.5 Balanced growth path

Assume that  $N_t$ ,  $\chi_t$ ,  $B_t$  grow at constant rates  $n$ ,  $\gamma_\chi$ ,  $\gamma_B$ , that  $\tau_t$  and  $\delta_{kt}$  are constant and equal to  $\tau$  and  $\delta_k$ , and that there is no investment wedge,  $\tau_{xt} = 0$ . We define the Balanced Growth Path (BGP) as an equilibrium in which all per capita variables grow at constant rates. To characterize the BGP first note that the law of motion for habits, equation (4), can be written as,

$$\frac{h_{t+1}}{c_{t+1}} (1 + \gamma_c) = (1 - \delta_h) \frac{h_t}{c_t} + \delta_h$$

where  $\gamma_c$  is the rate of growth of consumption. This law of motion has a fixed point

$$\frac{h}{c} = \frac{\delta_h}{\gamma_c + \delta_h}$$

to which the economy converges in BGP if  $\frac{(1-\delta_h)}{(1+\gamma)} < 1$ . Hence, the Euler equation of consumption in BGP becomes,

$$(1 + \gamma_c)^\sigma = \beta \frac{1}{1 + \gamma_\chi} \left[ \alpha \chi_{t+1} \left( \frac{k_{t+1}}{B_{t+1}} \right)^{\alpha-1} + (1 - \delta_k) \right]$$

For the Euler equation to be consistent with the BGP the MPK has to be constant, that is,  $(1 + \gamma_k) = (1 + \gamma_B) (1 + \gamma_\chi)^{\frac{1}{1-\alpha}}$ . Then the production function implies that  $(1 + \gamma_y) = (1 + \gamma_B) (1 + \gamma_\chi)^{\frac{\alpha}{1-\alpha}}$ . Finally, the law of motion for capital (11) becomes,

$$(1 + n_{t+1}) (1 + \gamma_k) = (1 - \delta_k) + \chi_t \frac{y_t}{k_t} - \chi_t \frac{c_t}{k_t}$$

Given their respective rates of growth the term  $\chi_t \frac{y_t}{k_t}$  is constant in BGP, so this expression requires  $(1 + \gamma_c) = (1 + \gamma_k) / (1 + \gamma_\chi) = (1 + \gamma_y)$ . Finally, the aggregate resource constraint  $y_t = c_t + x_t / \chi_t$  requires  $(1 + \gamma_x) = (1 + \gamma_y) (1 + \gamma_\chi) = (1 + \gamma_k)$ . Therefore, per

capita variables grow as follows,

$$\begin{aligned}
(1 + \gamma_y) &= (1 + \gamma_B) (1 + \gamma_\chi)^{\frac{\alpha}{1-\alpha}} \\
(1 + \gamma_c) &= (1 + \gamma_B) (1 + \gamma_\chi)^{\frac{\alpha}{1-\alpha}} \\
(1 + \gamma_h) &= (1 + \gamma_B) (1 + \gamma_\chi)^{\frac{\alpha}{1-\alpha}} \\
(1 + \gamma_k) &= (1 + \gamma_B) (1 + \gamma_\chi)^{\frac{1}{1-\alpha}} \\
(1 + \gamma_x) &= (1 + \gamma_B) (1 + \gamma_\chi)^{\frac{1}{1-\alpha}}
\end{aligned}$$

To write the model in terms of stationary variables we can define the transformed variables in units of consumption good  $\hat{y}_t$ ,  $\hat{c}_t$ , and  $\hat{h}_t$  as the original ones divided by  $B_t \chi_t^{\frac{\alpha}{1-\alpha}}$  and the transformed variables in units of the investment good  $\hat{k}_t$  and  $\hat{x}_t$  as the original ones divided by  $B_t \chi_t^{\frac{1}{1-\alpha}}$ . Then, the dynamic system in terms of detrended variables can be written as,

$$\frac{\hat{c}_{t+1} - \phi \hat{h}_{t+1}}{\hat{c}_t - \phi \hat{h}_t} (1 + \gamma_{Bt+1}) (1 + \gamma_{\chi t+1})^{\frac{\alpha}{1-\alpha}} = \beta^{\frac{1}{\sigma}} \left[ \frac{1}{1 + \gamma_{\chi t+1}} \right]^{\frac{1}{\sigma}} \left[ \alpha \hat{k}_{t+1}^{\alpha-1} + (1 - \delta_{kt+1}) \right]^{\frac{1}{\sigma}} \quad (12)$$

$$(1 + n_{t+1}) \frac{\hat{k}_{t+1}}{\hat{k}_t} (1 + \gamma_{Bt+1}) (1 + \gamma_{\chi t+1})^{\frac{1}{1-\alpha}} = (1 - \delta_{kt}) + \hat{k}_t^{\alpha-1} - \frac{\hat{c}_t}{\hat{k}_t} \quad (13)$$

$$\frac{\hat{h}_{t+1}}{\hat{h}_t} (1 + \gamma_{Bt+1}) (1 + \gamma_{\chi t+1})^{\frac{\alpha}{1-\alpha}} = (1 - \delta_h) + \delta_h \frac{\hat{c}_t}{\hat{h}_t} \quad (14)$$

where we allow for transitional dynamics to be enriched by changes in the exogenous paths of  $N_t$ ,  $\chi_t$ ,  $B_t$ ,  $\tau_t$  and  $\delta_{kt}$ .

## 4 Reproducing the hump as transitional dynamics

- The standard neo-classical growth model cannot generate a hump-shaped profile of investment with development if we constraint technology to grow at a constant rate. The capital deepening process associated to economic development generates a steady decline of the marginal product of capital (the substitution effect) alongside the steady increase in output (the income effect). Barro and Sala-i-Martin (1999) show analytically that whichever force dominates it does so for whole the transition. Hence, the model predicts a monotonic evolution of the saving rate in transitional dynamics. Indeed, for standard calibrations the former effects dominates monotonically and reduces the saving rate along the development path. Antràs (2001) extends this result by showing that the saving rate can display a hump in the transitional

dynamics when either the production function features an elasticity of substitution between capital and labor less than one or when the utility function is of the Stone-Geary type with a survival consumption term.

- In Sections 4.1 and 4.2 below we show that these two explanations generate very small humps compared to the data. However, in Section 4.3 we show that an external habit has the potential of generating a long-lasting hump because it allows for a strength of the income effect that varies with development.
- We focus on a version of the model in Section 3 with constant capital depreciation, constant population growth, constant TFP growth, constant investment-specific technical change, and no wedges. That is, we set  $\delta_{kt} = \delta_k$ ,  $n_t = n$ ,  $\gamma_{Bt} = \gamma_B$ ,  $\chi_t = 1$ , and  $\tau_{xt} = 0 \forall t$

#### 4.1 Non Cobb-Douglas production

If we set  $\phi = 0$  we restrict the model to the case with time-separable preferences. We can upgrade the production function to a CES

$$Y_t = F(K_t, B_t N_t) = A [\alpha K_t^\rho + (1 - \alpha) (B_t N_t)^\rho]^\frac{1}{\rho}$$

with elasticity of substitution between capital and labor  $\frac{1}{1-\rho}$ . We calibrate the economy to BGP targets. The rates of growth of technology and population  $\gamma_B$  and  $n$  can be taken off the shelves to equal values around 2% and 1%. The depreciation rate  $\delta_k$  can be obtained from the resource constraint in BGP for given capital to output  $k/y$  and investment to output ratios  $x/y$ :

$$(1 + n) (1 + \gamma_B) \hat{k} = \hat{x} + (1 - \delta_k) \hat{k} \Rightarrow (1 - \delta_k) = (1 + n) (1 + \gamma_B) - \frac{x}{y} \frac{y}{k}$$

The discount factor  $\beta$  can be chosen to achieve certain capital to output ratio  $\frac{k}{y}$  and certain capital share  $s_k$  given the Euler equation in BGP,

$$\beta (1 + \gamma_B)^{-\sigma} \left[ s_k \frac{y}{k} + (1 - \delta_k) \right] = 1$$

The normalization level constant  $A$  can be chosen such that output  $\hat{y}$  in BGP is equal to one for given capital to output ratio  $k/y$ . However, when  $\rho \neq 0$  the capital share

parameter  $\alpha$  also enters this expression, which gives two parameters and one equation:

$$\hat{y} = A[\alpha \hat{k}^\rho + (1 - \alpha)]^{\frac{1}{\rho}} \Rightarrow 1 = A \left[ \alpha \left( \frac{\hat{k}}{\hat{y}} \right)^\rho + (1 - \alpha) \right]^{\frac{1}{\rho}} \Rightarrow 1 = \alpha A^\rho \left( \frac{k}{y} \right)^\rho + (1 - \alpha) A^\rho$$

We can get a second equation to solve for the share parameter  $\alpha$  given by the capital share:

$$s_k = \alpha A^\rho \left( \frac{k}{y} \right)^\rho$$

which also depends on the level constant  $A$ . Substituting the latter into the former we get

$$A^\rho = \frac{1 - s_k}{1 - \alpha}$$

which can be plugged back into the capital share expression to obtain an expression for  $\alpha$  as function of the capital share  $s_k$  and the capital to output ratio  $k/y$

$$\frac{\alpha}{1 - \alpha} = \frac{s_k}{1 - s_k} \left( \frac{k}{y} \right)^{-\rho}$$

Finally, the curvature of the utility function  $\sigma$  (driving the strength of the income effect) and the CES parameter  $\rho$  (driving the elasticity of substitution between capital and labor and hence the strength of the substitution effect) are not easily calibrated, so we look at different values. We try values for  $\sigma$  equal to 2, 3, 5, and 6.67. The first three values are in the range of values used in the literature, while the fourth one corresponds to the case for which the investment rate should be flat in the continuous time version of the model.<sup>5</sup> We try values of  $\rho$  equal to 0, -1/4, and -1/2, which correspond to elasticities of substitution of 1, 0.80, and 0.67. The first case is the Cobb-Douglas and the other two cases fall into the range of values estimated in the literature.<sup>6</sup> All the values for the calibration are listed in Table 1.

With this calibration we perform a series of exercises in the spirit of King and Rebelo (1993) to see if the transitional dynamics of the model can reproduce a hump of investment with development as described in Section 2. In particular, we solve for a trajectory of an economy whose GDP per capita increases 90-fold (4.5 log points). We assume that (a) at

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<sup>5</sup>In particular, the continuous time version of the model predicts increasing investment rate along the transition if  $\sigma > (x/y)^{-1}$ , decreasing rate if  $\sigma < (x/y)^{-1}$  and constant investment rate if  $\sigma = (x/y)^{-1}$ , see Barro and Sala-i-Martin (1999, Appendix 2B). In our calibration  $x/y = 0.15$  and  $(x/y)^{-1} = 6.67$

<sup>6</sup>For instance, Oberfield and Raval (2014) estimate an elasticity of 0.7 in the US, slightly over 0.8 in Colombia and Chile, and of 1.15 in India. Villacorta (2017) reports estimates for several countries, with an average elasticity of 0.9.

TABLE 1: Calibration: CES economy

| Parameter   | Description                | Value            | Target                     |
|-------------|----------------------------|------------------|----------------------------|
| $n$         | Population growth          | 1%               | standard                   |
| $\gamma_B$  | Technology growth          | 2%               | standard                   |
| $\sigma$    | Curvature utility          | 2, 3, 5, 6.67    |                            |
| $\rho$      | Elasticity substitution    | 0, -0.25, -0.50  |                            |
| $\phi$      | Strength of habit          | 0, 0.75          |                            |
| $\delta_h$  | Depreciation rate of habit | 0.05, 0.10, 0.25 |                            |
| $\beta$     | Discount factor            | several          | $k/y = 3.0$                |
| $\delta_k$  | Depreciation rate          | several          | $x/y = 0.15$               |
| $\alpha$    | capital share              | several          | $rk/y = 0.33$              |
| $\hat{k}_0$ | initial capital stock      | several          | see text                   |
| $\hat{h}_0$ | initial habit stock        | several          | $\hat{h}_0 = 0.5\hat{y}_0$ |

Notes:

the end of the trajectory the economy is in BGP and (b) half of the GDP increase is due to transitional dynamics. This implies a choice of the initial capital  $\hat{k}_0$ .<sup>7</sup>

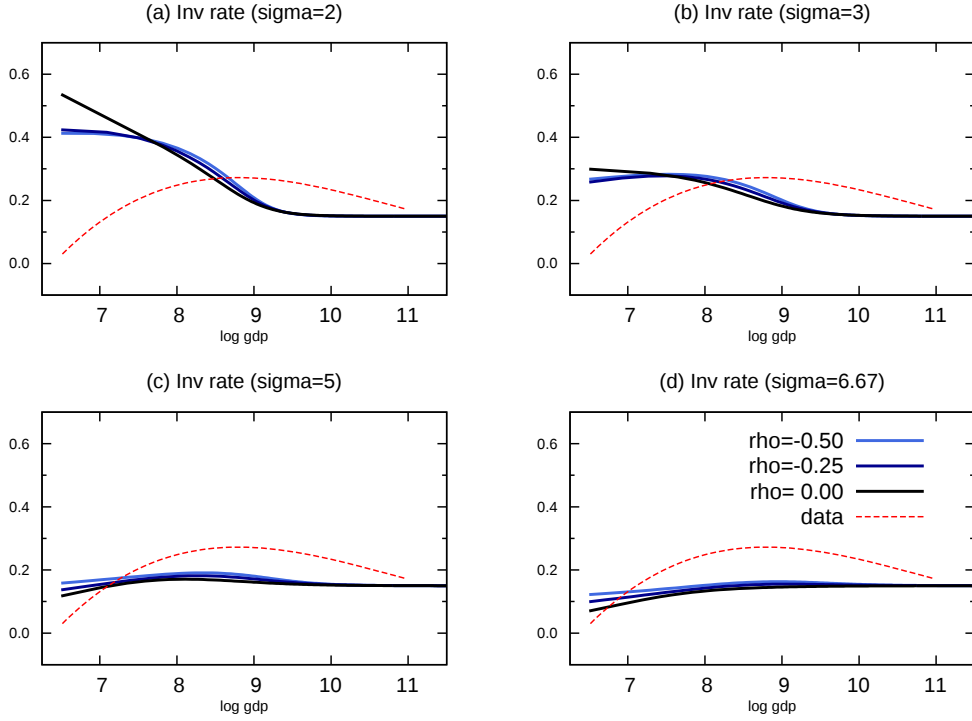
The results of these transitions are reported in Figure 5. Each panel corresponds to a different  $\sigma$  and each solid line to a different  $\rho$ . The dotted line reproduces the hump in the data. We see that low elasticities of substitution can generate a very mild hump for  $\sigma = 3$  and  $\sigma = 5$ , but none of the simulated trajectories is close to produce a large and long-lasting hump of the investment rate as in the data.

## 4.2 Minimal consumption

A different approach to make the income effect evolve in a non-monotonic way is to think about a Stone-Geary type of utility function with a survival consumption. The logic is that at the start of the transition income is too close to the survival level of consumption for household to exploit the large interest rates, so the investment rate is low. As the economy

<sup>7</sup>The GDP growth between two dates can be decomposed between the growth of technology and the growth of the economy in transitional dynamics,  $y_t/y_0 = (1 + \gamma_B)^t (\hat{y}_t/\hat{y}_0)$ . We are setting  $y^*/y_0 = 90$  and  $\hat{y}^*/\hat{y}_0 = (y^*/y_0)^{1/2}$ , which states that  $\hat{y}_0 = \hat{y}^* 90^{-1/2}$ . The initial  $\hat{k}_0$  follows from the production function. This implies that the initial capital in efficiency units is low compared to BGP, the initial MPK is high compared to BGP, and that it takes 113 years for the economy to complete the trajectory.

FIGURE 5: Transitional dynamics of the CES economy



gets richer income moves away from survival and households invest more up to a point in which the decline of the interest rate dominates. However, King and Rebelo (1993) show that this mechanism is very short-lived if there is exogenous technological growth as the survival consumption relative to current income shrinks very fast. Christiano (1989) shows that if the survival consumption is made proportional to the technology level, then the transitional dynamics can account for the observed hump in the savings rate of Japan after WWII.

We analyze here an economy like Christiano (1989) by setting  $h_t = B_t$  with  $\phi = 0.50\hat{y}_0$  and  $\phi = 0.75\hat{y}_0$ , see Figures 6 and 7 respectively.

### 4.3 External habit

If we allow for  $\phi \neq 0$  we have the standard one-sector neoclassical model extended with an external habit. We want to show that the presence of a slow-moving external habit can generate interesting dynamics in the evolution of the strength of the income effect, leading to a long hump of the investment rate with development. To do so we set  $\phi = 0.75$ ,  $\hat{h}_0 = 0.5\hat{y}_0$  and allow for different values of  $\delta_h$  —in particular 1, 0.25, 0.10, and 0.05— as well as for the different values of  $\sigma$  and  $\rho$  already tried in Section 4.1 above. The results are reported in Figure 8-11 respectively. We see that a living standard with very short

FIGURE 6: Transitional dynamics of survival consumption:  $\phi = 0.5\hat{y}_0$

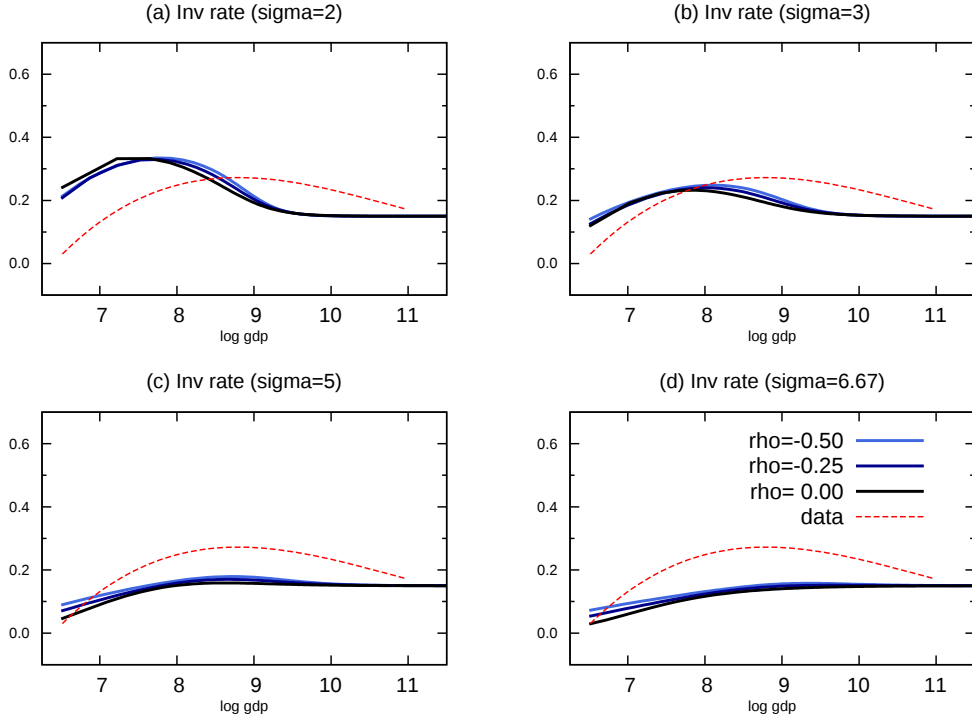
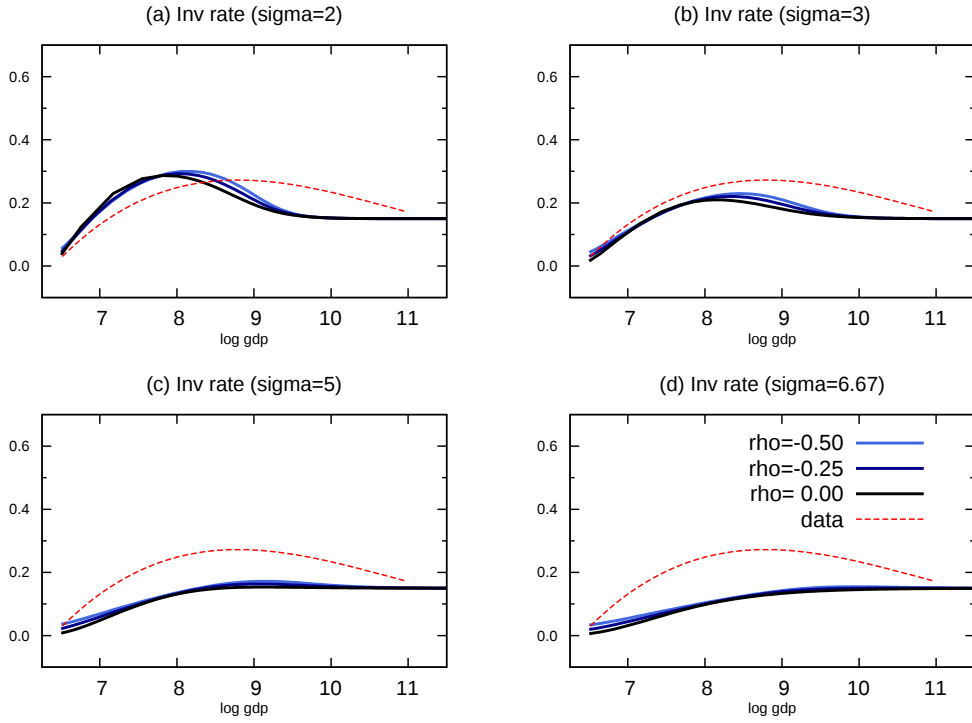
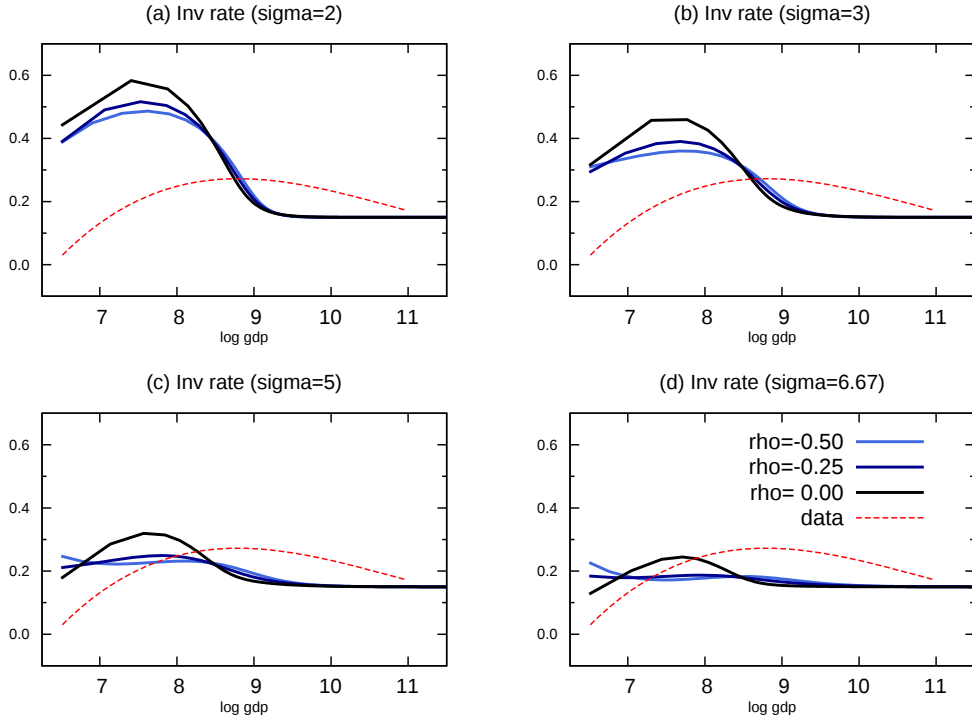


FIGURE 7: Transitional dynamics of survival consumption:  $\phi = 0.9\hat{y}_0$



memory ( $\delta_h = 1$ ) can generate sharp humps by very short-lived. Instead, as we allow for slow-moving habits ( $\delta_h = 0.25, 0.1, 0.05$ ) we see that the humps are flatter and longer-lived. In particular, with  $\delta_h = 0.1$ ,  $\sigma = 5$  and Cobb-Douglas production we have a reasonable approximation to the observed hump (see Panel (c) in Figure 10). Other parameter combinations, like for instance  $\delta_h = 0.25$ ,  $\sigma = 6.67$  and Cobb-Douglas production (see Panel (d) in Figure 9), deliver similar transitions, which show the potential problem in trying to identify preference parameters in the data from investment trajectories alone.

FIGURE 8: Transitional dynamics of the habit economy:  $\delta_h = 1.00$



## 5 Understanding the investment path country by country

We want to measure if the neo-classical model described in Section 3 is consistent with the observed paths of the investment rate in each country ones the proper country-specific paths for  $N_t$ ,  $B_t$ ,  $\chi_t$ ,  $\delta_{kt}$  are fed into the model. For this purpose, we use the Euler equation to estimate the preference parameters  $\sigma$ ,  $\phi$ ,  $\delta_h$  and the initial habit stock  $h_1$  and to recover the path for the wedge  $\tau_{xt}$  distorting the investment decision. Following Chari, Kehoe, and McGrattan (2007), this wedge measures the deviations of the Euler

FIGURE 9: Transitional dynamics of the habit economy:  $\delta_h = 0.25$

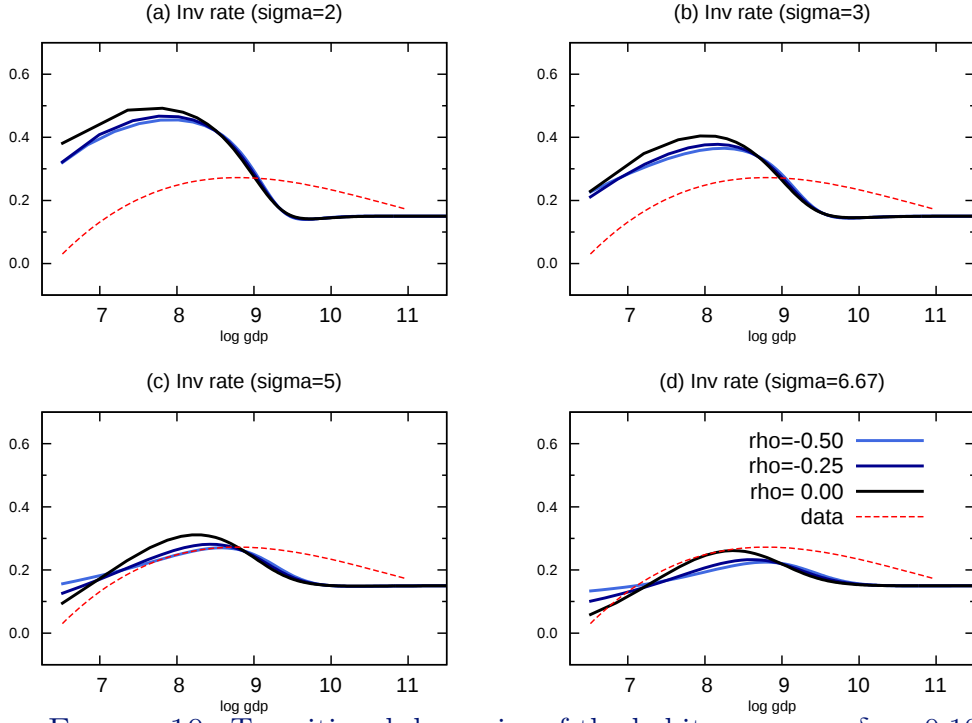


FIGURE 10: Transitional dynamics of the habit economy:  $\delta_h = 0.10$

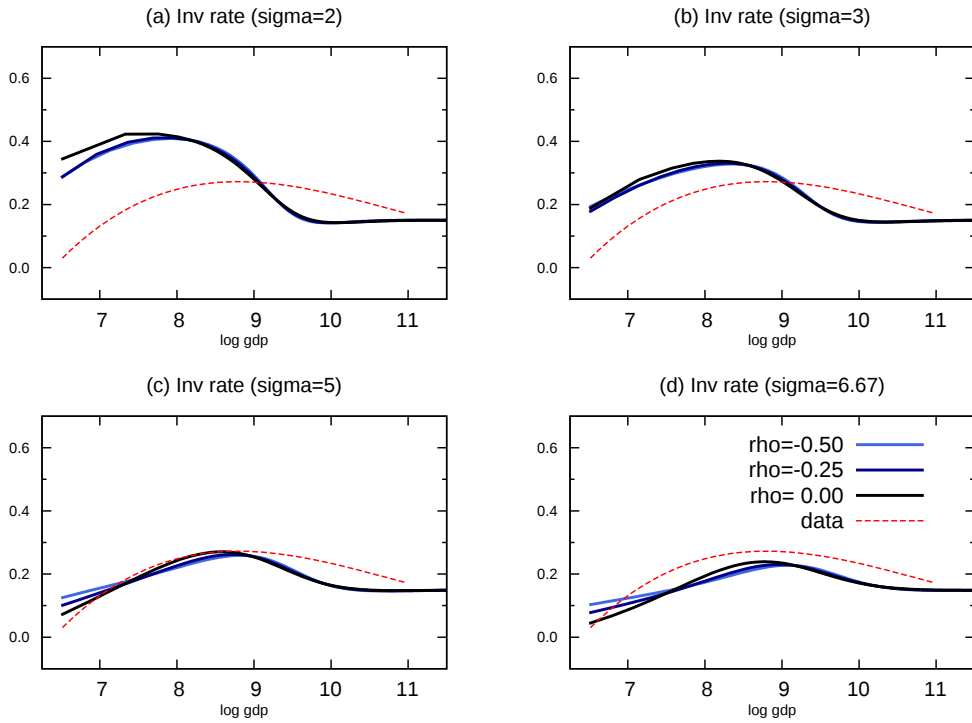
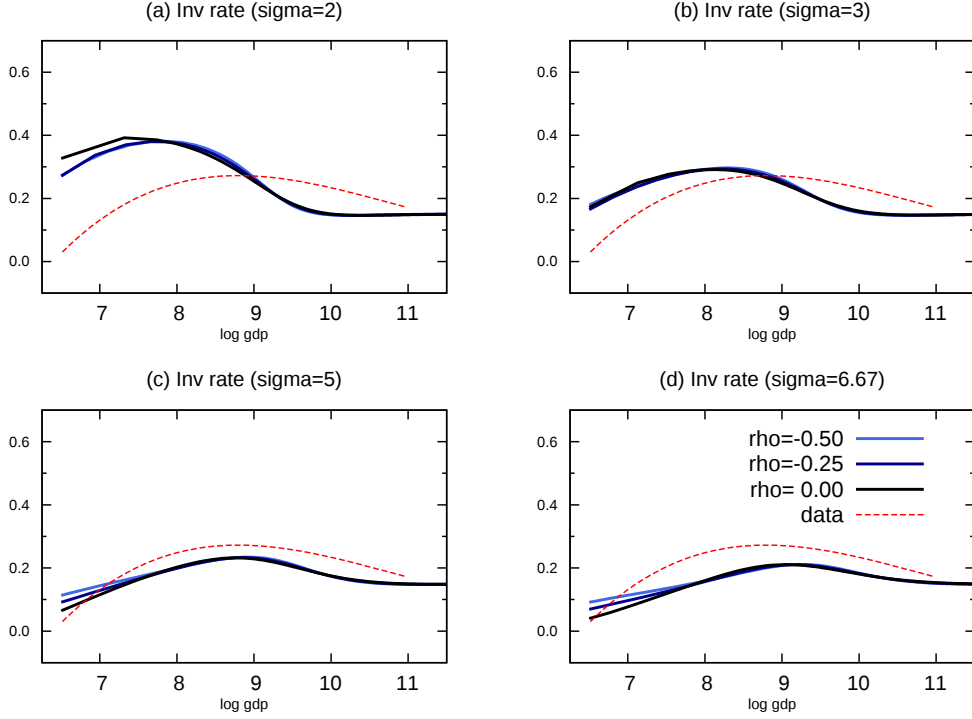


FIGURE 11: Transitional dynamics of the habit economy:  $\delta_h = 0.05$



equation from the data and signal in which periods and countries the standard model cannot account for the investment rate trajectory seen in the data. The wedge may capture for instance liquidity constraints of the households, credit market frictions of the firms (like in Song, Storesletten, and Zilibotti (2011) or Buera and Shin (2013)), capital adjustment costs, risk of expropriation, or time-changing discount factors that proxy for demographic developments. We could also have modelled the wedge as a capital income tax and the results would not be different. Instead, distortions that affect the production of investment goods (like in Restuccia and Urrutia (2001), Schmitz (2001) or García-Santana and Pijoan-Mas (2014)) should appear in the relative price of investment goods directly.

## 5.1 Estimation of preference parameters

In order to estimate the preference parameters  $\sigma$ ,  $\phi$ ,  $\delta_h$  and the initial habit stock  $h_1$  that determine the behaviour of the saving rate we use the Euler equation of consumption of the household problem, equation (5):

$$c_{t+1} = \left( \beta \frac{1 + \tau_{xt+1}}{1 + \tau_{xt}} \right)^{\frac{1}{\sigma}} \left[ \frac{p_{xt+1}/p_{ct+1}}{p_{xt}/p_{ct}} \alpha \frac{y_{t+1}}{p_{xt+1}k_{t+1}} + (1 - \delta_{kt+1}) \right]^{\frac{1}{\sigma}} (c_t - \phi h_t) + \phi h_{t+1} \quad (15)$$

The parameter  $\alpha$  is calibrated to the standard value of  $1/3$  for all countries. With  $\phi = 0$ , it is clear that  $\sigma$  is identified through the comovement of consumption growth and the relative price of future and present consumption (the term in square brackets), as it is standard in the consumption literature.<sup>8</sup> The habit parameters  $\phi$  and  $\delta_h$  are identified through the dependencies of  $c_{t+1}$  on various lags of consumption. For every country we have data from  $t = 1, \dots, T$  (with  $T$  differing across countries). We estimate this expression for  $t = 1, \dots, T - 1$ . The time series that we need for each country are GDP, capital, consumption and investment expenditure in local currency units (which correspond to  $y_t$ ,  $p_{xt}k_t$ ,  $p_{ct}c_t$ , and  $p_{xt}x_t$  in the model) and the implicit price deflators of investment and consumption (which correspond to  $p_{xt}$  and  $p_{ct}$  in the model). We use the law of motion for capital, equation (2), to obtain  $\delta_{kt}$  from the series of capital and investment in each country and the law of motion for habits, equation (4), to obtain the series of habits.

## 5.2 Obtaining the paths of exogenous variables

To obtain a full solution of the model and any counterfactual exercise we need to specify the evolution of all exogenous variables for  $t \in [1, \infty)$ . We use our data to obtain the paths for  $t \in [1, T]$ . In particular, the path of  $N_t$  corresponds to the observed population, the path of  $\delta_{kt}$  is obtained using the law of motion for capital (see above), the path of  $\tau_{xt}$  is obtained from the estimation of the Euler equation (see above) and the paths for technology levels  $B_t$  and  $\chi_t$  come from the observed output, capital, and relative prices using the relationships

$$\begin{aligned} B_t &= \left( \frac{y_t/p_{ct}}{k_t^\alpha} \right)^{\frac{1}{1-\alpha}} \\ \chi_t &= p_{ct}/p_{xt} \end{aligned}$$

We establish the evolution of the paths of exogenous variables for  $t \in [T + 1, \infty)$  by use of four conditions. First, the future evolution of all the exogenous variables has to be such that the economy eventually reaches a BGP. For this reason, we impose that in the long run technology and population grow at constant rates  $\gamma_B^*$ ,  $\gamma_\chi^*$ , and  $n^*$ , that the rate of capital depreciation is constant and equal to  $\delta_k^*$ , and that the investment wedge is

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<sup>8</sup>In the consumption literature the intertemporal price of consumption is typically given by a stochastic process for the interest rate, see for instance Hansen and Singleton (1983) or Attanasio and Weber (1995). In here it is the marginal product of capital in consumption units, which consists of two parts. First, there is the growth of the relative price of investment goods, which is driven by the exogenous level of technology  $\chi_t$ , see equation (9). Second, there is the the marginal product of capital in units of the investment good, which is determined by the actual choices of capital and the TFP term  $B_t$ .

constant and equal to  $\tau_x^*$ . Let  $\Theta^*$  be the vector of these constants. Second, transitions of all exogenous variables from the observed values in  $t = T$  to their long run values in  $\Theta^*$  is linear and takes 100 years. Third, the long run values in  $\Theta^*$  are given by  $\gamma_B^* = v \overline{\gamma_{Bt}}$  and  $\gamma_\chi^* = v \overline{\gamma_{\chi t}}$ ,  $\tau_x^* = 0$ ,  $\delta_k^* = \overline{\delta_{kt}}$ ,  $n^* = \overline{n_t}$  where the bar operator indicates the constant rate that delivers the same total growth as in each country time series and  $v \geq 0$  is a free constant. And fourth, we choose  $v$  for each country such that the initial consumption level at  $t = 1$  is consistent with the country balanced growth path. Notice that the estimation strategy in Section 5.1 based on the Euler equation makes the consumption growth series consistent with the model but it is silent about the level of the consumption series. Given the saddle path stability of this economy, there is only one initial consumption that puts the economy in the stable arm towards the BGP for a given vector  $\Theta^*$ . Reversing the argument, we recover the  $v$  that makes the observed  $c_1$  be in the stable arm of the model economy.<sup>9</sup>

### 5.3 Results

TO BE WRITTEN

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<sup>9</sup>In practice, we look for the  $v$  such that the capital stock at  $t = T + 1000$  equals the capital stock in BGP.

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