

THE INCENTIVE CHANNEL OF CAPITAL MARKET INTERVENTIONS*

PRELIMINARY AND INCOMPLETE

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Abstract

We develop a tractable dynamic model of collateralized lending in which the degree of adverse selection evolves endogenously due to moral hazard. We use this model to study how government interventions designed to boost liquidity in frozen markets affect private incentives to maintain high-quality assets. We show that small interventions can lead to “intervention traps” – expectations concerning future interventions destroy private incentives to improve the quality of collateral, which stunts recovery and warrants continued market intervention – even when they restore market liquidity. Bigger interventions may lead to faster recoveries, and it may be efficient to continue to intervene even after market liquidity is restored. This runs counter to previous findings in static environments where it is optimal to keep interventions as small as possible, and to intervene only when markets are illiquid.

*The views expressed in this paper are those of the authors and do not necessarily reflect the position of the Federal Reserve Bank of New York or the Federal Reserve System.

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1 Introduction

Monetary authorities increasingly rely on asset purchase programs to stimulate economic activity and to restore liquidity in financial markets during times of stress. These programs, which initially focused on purchases of government debt, have recently been expanded to include a broader set of privately-produced financial assets, such as corporate bonds, mortgage-backed securities, or bank-originated asset-backed securities. For example, the European Central Bank began exchanging investment grade corporate bonds for government debt under the Eurosystem's Corporate Sector Purchase Programme starting in 2016. This raises the concern that the opportunity to sell relatively low-quality assets to monetary authorities may affect the incentives of private agents to produce high-quality assets, harming the long-run health of financial markets.

We explore this incentive channel of capital market interventions in a parsimonious dynamic model of collateralized lending under asymmetric information. As in standard models of adverse selection, markets are liquid in a given period if and only if the fraction of good asset is sufficiently high. A large negative shock to asset quality may therefore result in the breakdown of lending markets due to adverse selection. This provides the scope for the government to improve welfare through interventions that restore market liquidity.

The novel aspect of our analysis is that asset quality is endogenously determined by costly unobservable effort undertaken by borrowers. This means that markets remain liquid only if sufficiently many borrowers find it privately optimal to exert effort. Private incentives in turn depend on the relative value of owning a good or bad asset. Since valuations depend on market conditions, this gives rise to a two-way feedback between market liquidity and incentives. We use this framework to show that interventions which targets liquidity but ignore the feedback to incentives may lead to long-run declines in average asset quality. As a result, such interventions may be strictly dominated by bigger interventions that target both market liquidity and incentives.

In the model, good assets and bad assets differ in both the cash flows that can be pledged to lenders (the collateral value) and the cash flows that cannot be pledged conditional on investment (the divertable return). In order to obtain financing, thus, borrowers must post a fraction of their asset's pledgeable cash flows as collateral. Lenders are un-

informed about the quality of the underlying collateral. Hence they charge an adverse selection premium that is decreasing in the fraction of good assets. (Going forward, we refer to the fraction of good assets as *average asset quality*.) This force leads to an effort complementarity: owning a good asset is more valuable when the adverse selection premium is low, and so it is beneficial to exert effort when many other borrower do so as well.

Markets break down due to adverse selection if average asset quality is too low. If this is the case, then borrowers cannot invest. Hence they consume the cash flows they would otherwise have pledged as collateral. If the difference in collateral values between good and bad assets is sufficiently large, a local substitutability in effort arises at the liquidity threshold: since borrowers must pledge all of their collateral when markets are just liquid, the relative value of good assets drops as soon as markets become liquid. Consequently, it is privately optimal to shirk when average asset quality is expected to be such that markets are just liquid. Both channels combine to ensure that effort incentives are determined by expectations of future market liquidity.

In the absence of regulation, we show that this structure gives rise to at most three steady states for asset quality, and that there exist equilibrium paths for asset quality that asymptote to exactly one of these steady states. In a *full recovery equilibrium*, all borrowers exert effort in every period and the fraction of good assets asymptotes to 1. In a *no recovery equilibrium*, all borrowers shirk in every period and the fraction of good assets asymptotes to zero, leading to large inefficiencies. In a *partial recovery equilibrium*, a fraction of borrowers exerts effort in every period, average asset quality is equal to the liquidity cutoff, the liquidity cutoff, and only a fraction of borrowers are able to obtain funding. This fraction is chosen such that borrowers are indifferent between effort and shirking. Since it is efficient to fund all project at the liquidity threshold, this equilibrium features both inefficiently low liquidity and inefficient effort.

Against this background, we study the dynamic effects of market interventions designed to alleviate market breakdowns. Period-by-period, the efficiency gain from such interventions stems from the notion in Holmstöm and Tirole (1998) that the government can use its taxation power to render cash flows pledgeable that could otherwise not be pledged. In static settings with fixed asset quality, Tirole (2012) and Philippon and Skreta (2012) show that interventions aimed at providing relief to illiquid markets should keep kept as small as

possible. That is, conditional on asset quality, the regulator should intervene just enough to ensure that private markets are liquid.

Whether or not regulators can improve upon such minimal interventions in our dynamic setting depends on their ability to commit to future interventions. If regulators cannot commit, then the optimal policy problem is equivalent to a sequence of static intervention problems with fixed asset quality. Hence the insights from Tirole (2012) and Philippon and Skreta (2012) apply and regulators always intervene minimally. In the dynamic setting considered here, this may have deleterious long-run consequences, however. The reason is that minimal interventions may entirely eliminate private effort incentives precisely because they focus only on restoring liquidity. Hence asset quality declines and the regulator is forced to intervene time and again due to an “intervention trap” – it is optimal to intervene because markets are illiquid, but interventions further harm asset quality.

We show that regulators endowed with the ability to commit to policies one period ahead can do strictly better. Under commitment, regulators can credibly promise the terms of an intervention prior to borrowers’ effort decisions. This allows regulators to design interventions that explicitly account for their impact on both liquidity and incentives. Specifically, we find that it may be optimal to commit to per-unit subsidies that are larger than the minimal intervention. The reason is that “large” interventions boost private incentives by lowering the expected adverse selection premium. Committing to larger interventions may have the added benefit of strictly lowering the ex-post *aggregate* subsidy: by increasing the fraction of good assets, the cost of the intervention may be lower even if the per-unit subsidy is larger.

Regulators may also want to commit to intervene in states of the world where markets would be liquid even in the absence of intervention. The result contrasts with Tirole (2012) and Philippon and Skreta (2012), who find that it is optimal to intervene only if markets are illiquid. The intuition for this result stems from the approximate steady state at the liquidity threshold under *laissez-faire*. In this region, effort substitutability outweighs complementarities conditional on liquidity. As such, asset quality is high enough to prevent markets from deteriorating, but not high enough for all borrowers to consistently exert effort. Absent intervention, individual borrowers prefer to free-ride on aggregate quality improvements whenever a sufficiently large fraction of other borrowers exerts effort, hindering full

recovery. To overcome this dynamic coordination problem, a regulator can promise a small subsidy that absorbs enough of the lower adverse selection cost that restores full effort incentives until quality rises above to a level at which it self-sustains. Lower discount factor increases the scope for intervention to improve efficiency by resolving dynamic coordination problem.

Finally, regulators with a binding budget constraint may find it optimal to delay intervention. If market quality starts out below the approximate steady state around the liquidity threshold, then average asset quality in the laissez-faire economy asymptotes to the liquidity threshold. Intervening early efficiently restores market liquidity but does not improve upon private effort incentives (since borrowers would have exerted effort anyway). Intervening only once we reach the approximate steady foregoes the efficiency gains from early market liquidity, but allows the economy to transition into the self-sustaining effort region. Hence waiting may be optimal as long as the regulator is sufficiently patient and budget constraints prevent the regulator from intervening in every period. This result differs from Fuchs and Skrzypacz (2015), who study a setting with fixed asset quality but dynamic trading and show that it is optimal to intervene as soon as possible. Chiu and Koepl (2016) show that waiting to intervene be optimal in settings with fixed asset quality and search frictions. Their result is based on the notion that future interventions increase selling pressure in frictional markets today, alleviating adverse selection and giving rise to an announcement effect that can be exploited by regulators. In our setting instead regulators may prefer to wait because asset quality may partially recover even in the absence of interventions.

Related Literature. Our paper is most closely related to Tirole (2012) and Philippon and Skreta (2012), who study how public interventions can jump-start markets frozen due to adverse selection, and how to best design such interventions. Their focus lies on studying how participation constraints for the government program depend on the endogenous response of the competitive allocation. Fuchs and Skrzypacz (2015) consider a similar setting in which private trading occurs dynamically, and show an initial subsidy and subsequent tax on trade can improve allocations, fixing the distribution of asset quality. Our approach is to analyze how the expectation of future interventions affects private incentives to produce high-quality collateral, and to study conditions under which interventions may either

lead to a sustained lack of high-quality collateral, or complement private effort incentives. Camargo, Kim, and Lester (2016) and Bond and Goldstein (2015) argue that government interventions may have a detrimental effect on private information acquisition that must be traded off against the benefits of unfreezing asset markets. We share their focus on studying potential downsides of market interventions, but do so by studying the dynamics of asset quality itself.

A key feature of our model is that all lending must be collateralized as in Gorton and Ordoñez (2014) or Fostel and Geanakoplos (2015). This structure implies a divergence between pledgeable cash flows and those that are inalienable from the asset holder. When markets are liquid, collateral is pledged at a pooling price, thereby diluting the private value of holding a good collateral asset and reducing effort incentives. Hence it is precisely when there is dispersion in collateral values that expected market liquidity hampers effort incentives. Choi, Santos, and Yorulmazer (2016) also study a model of collateralized lending and ask which types of collateral central banks should lend against to boost asset market liquidity. They show that when the central bank can distinguish between good and bad collateral, policies demanding good collateral in exchange for liquidity provisions impose negative externalities on private markets. In our setting, we assume that the central bank is as uninformed as financial market participants when assessing asset value, and study the endogenous dynamics of asset quality under interventions.

Layout. Section 2 lays out the model environment. Section 3 analyzes the baseline model absent government intervention. We introduce government interventions in Section 4 and outline its impact on incentives. Section 5 concludes. All proofs can be found in Appendix A.

2 Model

Time is discrete and infinite, $t = 1, \dots, \infty$. There is a unit measure of borrowers (which we refer to as farmers), unit measure of investors, and a regulator. All agents are risk neutral. Farmers and the regulator are infinitely lived, while investors live for one period and are then replaced by a new generation. Hence there are infinitely many generations of

investors. Each investor generation is competitive with deep pockets, and have access to an outside storage technology with one-period return $1 + r_f$. Long-lived agents discount the future using the discount factor $\beta \in [0, 1)$.

2.1 Production technology and endowments.

Each farmer owns and operates a long-lived investment technology called an *asset*. Assets can only be operated by farmers. Assets are of quality $\theta \in \{g, b\}$. Hence they are either *good* or *bad*. In each period, an asset of quality θ generates a risk-free return of L_θ with or without investment, where $L_g \geq 1 + r_f > L_b$. Each farmer can also invest 1 unit of capital in his asset. This generates an additional return of R_θ with probability p_θ , and 0 otherwise. We assume $p_g R_g \geq \max\{p_b R_b, 1 + r_f\}$. Good assets thus offer greater safe returns and greater expected risky returns than bad assets.

At the beginning of each period, each farmer can exert unobservable effort at private non-pecuniary cost $\bar{c}$ to improve or maintain the quality of his asset. The realized quality of his asset conditional on effort is stochastic. Let $e \in \{0, 1\}$ denote the effort decision, where $e = 1$ if the farmer exerts effort. Given initial asset quality θ , the probability distribution over new asset quality θ' is:

$$Prob [\theta' = g | \theta, e] = \begin{cases} 1 & \text{if } \theta_1 = g \text{ and } e = 1 \\ 1 - \pi & \text{if } \theta_1 = g \text{ and } e = 0 \\ \pi & \text{if } \theta_1 = b \text{ and } e = 1 \\ 0 & \text{if } \theta_1 = b \text{ and } e = 0. \end{cases} \quad (1)$$

Here π captures the incremental value of effort for maintaining or producing a good asset. The effort outcome is i.i.d across agents. The realized fraction of good assets (conditional on effort outcomes) in period t is λ_t . Asset quality is persistent over time, in that the economy enters period t with asset quality $t - 1$. The initial condition is λ_0 . We permit mixed strategies. Without loss of generality with respect to aggregate outcomes, we restrict attention to effort strategies that are symmetric by type. We denote the probability with which farmers of type θ exert effort in period t by $\mu_{t,\theta}$, which $\mu_t = \{\mu_{t,b}, \mu_{t,g}\}$. By the law

of large numbers, the law of motion for λ_t thus is

$$\lambda_t = \Gamma(\lambda_{t-1}, \mu_t) \equiv \underbrace{\mu_g \lambda_{t-1}}_{\{\theta, e\}=\{g, 1\}} + (1 - \pi) \underbrace{(1 - \mu_{t,g}) \lambda_{t-1}}_{\{\theta, e\}=\{g, 0\}} + \pi \underbrace{\mu_{t,b} (1 - \lambda_{t-1})}_{\{\theta, e\}=\{b, 1\}}. \quad (\text{LoM})$$

2.2 Information Structure and Capital Markets

The fundamental source of inefficiency is adverse selection due to asymmetric information. Specifically, each farmer is privately informed about the quality of his asset. For simplicity, the fraction of good assets is common knowledge.

Assumption 1 (Information Structure). *At the beginning of each period, the quality of each individual asset is the private information of the farmer who owns the asset. The aggregate fraction of good trees λ_t is common knowledge.*

We assume that farmers cannot save. In order to invest, farmers must therefore borrow 1 unit of capital from investors. Investors competitively offer lending contracts to farmers, subject to outside option $1 + r_f$. The key financial friction is that farmers are subject to limited pledgability as in Tirole (2012). Specifically, we assume that L_θ is ex-post verifiable and thus pledgeable, while R_θ is not verifiable and thus fully divertible.¹

Assumption 2 (Limited Pledgeability). *Only L_θ is pledgeable to investors.*

Given that only L_θ is pledgeable and investors are short-lived, we take as given that investors competitively offer one-period collateralized debt contracts characterized by (i) face value B , and (ii) the right to seize up to B of farmers' risk-free return L_θ if the farmer defaults the promised payment B . This contract is optimal in that it minimizes the adverse selection discount faced by good farmers. Since assets are long-lived but investors are short-lived, it is inefficient to liquidate assets in order to enforce a payment less than or equal to L_θ . We therefore focus on renegotiation-proof contracts. This leads us to the tie-breaking rule that investors do not seize assets if farmers repay at least L_θ .² It follows that no farmer

¹The mechanisms are unchanged if L_θ is only partially pledgeable, or if R_θ is only partly divertible.

²If assets were to be liquidated after a default, we would have to specify the quality of new entrants. This added complication does not add much to the main mechanisms discussed here.

can credibly pledge more than L_g . This market structure implies that all contracting is static and time-invariant. This allows us to focus on how the endogenous dynamics of asset quality determine aggregate outcomes. To ensure equilibrium existence, we allow investors to play mixed strategies. A strategy for a investor thus consists of $\{B, \mu_l\}$, where B is face value of the offered contract, ϕ is the probability with which they offer the contract.

2.3 Timing.

The timing in each period is summarized in Figure 1. At the beginning of the period, each farmer makes an effort decision. Conditional on the effort decision, the new quality of each asset is realized according to the effort technology specified in Equation 1.

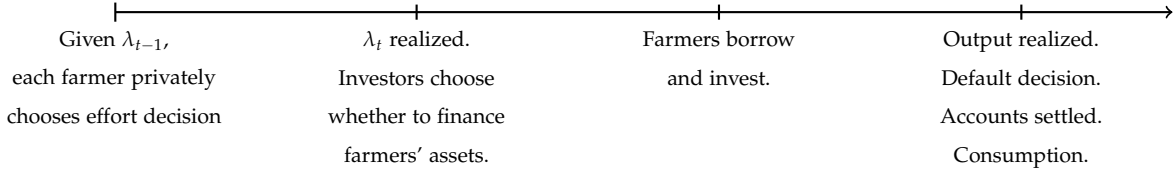


Figure 1: Timing of Events in Period t

After observing the aggregate quality λ_t , investors decide whether to finance farmers. If they choose to do so, farmers borrow and invest. If they do not, there is no investment. Lastly, output is realized, farmers decide whether to default, and accounts are settled.

3 Laissez-faire Economy

We now characterize the competitive equilibrium in the absence of government interventions. The first step is to define the equilibrium concept.

3.1 Equilibrium Concept

Each farmer of type θ chooses an infinite sequence $\{\mu_{\theta,t}\}_{t=0}^{\infty}$ of effort probabilities and an infinite sequence $\{\alpha_{\theta,t}\}_{t=0}^{\infty}$ mapping offered contract terms to acceptance probabilities.

Each investor generation t chooses a debt contract B_t and the probability with which this contract is offered ϕ_t , giving rise to infinite sequences $\{B_t\}_{t=0}^{\infty}$ and $\{\phi_t\}_{t=0}^{\infty}$.

Definition 1 (Perfect Bayesian Equilibrium). *A Perfect Bayesian Equilibrium given initial condition λ_0 is given by $\{B_t, \phi_t\}$ for each investor in generation t , sequences $\{\mu_{\theta,t}\}_{t=0}^{\infty}$ and $\{\alpha_{\theta,t}\}_{t=0}^{\infty}$ for each farmer, and an aggregate law of motion for asset quality Γ^* such that*

1. *Effort decisions and acceptance probabilities are individually optimal for each farmer at each t given $\{B_t\}_{t=0}^{\infty}$, $\{\phi_t\}_{t=0}^{\infty}$, and Γ^* .*
2. *B_t and ϕ_t are individually optimal for each investor of generation t given λ_0 , Γ^* and investor beliefs.*
3. *Investor beliefs are consistent with Bayes' Rule wherever possible.*
4. *Γ^* is consistent with $\{\mu_{\theta,t}\}_{t=0}^{\infty}$ given λ_0 .*

In line with this definition, our approach to constructing equilibrium is to conjecture paths for farmer effort, compute the implied law of motion for asset quality, and then verify whether the conjectured effort path is indeed consistent with individual optimality. In doing so, it will be convenient to summarize outcomes using the following taxonomy.

Definition 2 (Liquidity and Recovery). *For any t , market liquidity and the recovery from illiquidity are defined as follows.*

1. *Markets are **partially liquid** in period t if a strictly positive fraction of farmers obtains funding in that period. Markets are **liquid** in period t if all farmers obtain funding in that period. Markets are **illiquid** if they are not partially liquid.*
2. *Assume that markets are illiquid in some period t_0 . Then markets have **partially recovered** in period $t > t_0$ if markets are partially liquid in period t and in some period $\tau > t$. Markets have **fully recovered** in period t if markets are liquid for all $\tau \geq t$.*

3.2 Equilibrium of the Lending Game

The first step in our analysis is to characterize market liquidity in a given period. Since investors are short-lived, we study the lending decision given fixed λ_t . We call this stage the *lending game*.

Given λ_t , investors must choose (i) the probability with which they offer a contract to farmers ϕ , and (ii) the terms of the offered contract B . By renegotiation-proofness, a farmer of type θ will default if and only if $B > L_\theta$. Hence we can restrict attention to $B \leq L_g$ without loss of generality. Given λ and the outside option $1 + r_f$, the lender participation constraint is

$$\lambda \min\{B, L_g\} + (1 - \lambda) \min\{B, L_b\} \geq 1 + r_f. \quad (\text{PC})$$

Since the maximum pledgeable cash flow obtains when $B = L_g$ and $L_b < 1 + r_f$, it follows that there exists a threshold $\bar{\lambda}$ such that markets are liquid if and only if $\lambda \geq \bar{\lambda}$. This threshold is

$$\bar{\lambda} \equiv \frac{1 + r_f - L_b}{\Delta L} \quad \text{where} \quad \Delta L = L_g - L_b. \quad (2)$$

Since investors have deep pockets and are competitive, (PC) holds with equality whenever $\lambda \geq \bar{\lambda}$. The face value of debt that is consistent with zero profits is thus given by

$$B^*(\lambda) = \frac{1 + r_f - (1 - \lambda)L_b}{\lambda}, \quad (3)$$

and is decreasing in λ . Increases in average asset quality thus lower funding costs by reducing the adverse selection discount. Assumption 3 ensures that farmers strictly prefer to obtain funding even if they have to pledge their entire collateral. We maintain this assumption throughout. Proposition 1 characterizes the resulting equilibrium.

Assumption 3. $p_\theta R_\theta > L_\theta$ for all θ .

Proposition 1. Fix λ_t . Then the equilibrium face value of debt is $B^*(\lambda_t)$, and all farmers accept any contract with $B \leq L_g$. If $\lambda_t < \bar{\lambda}$, then $\phi_t^* = 0$ (no farmer receives funding). If $\lambda_t > \bar{\lambda}$, then $\phi_t^* = 1$ (all farmers receive funding). If $\lambda_t = \bar{\lambda}$, then $\phi_t^* \in [0, 1]$ (any degree of funding is consistent with investor optimality).

The argument is intuitive. If $\lambda_t < \bar{\lambda}$, then no investor is willing to lend. If $\lambda_t > \bar{\lambda}$, then all farmers must be funded, since else there would be profitable deviations for investors who fund unfunded farmers at some face value slightly higher than B^* but below L_g . If $\lambda_t = \bar{\lambda}$, then any degree of funding is consistent with optimality, since the expected value of a loan to an unfunded farmer is zero at the maximum face value L_g . Hence there are no profitable deviations.

It is useful to note that it is socially efficient to fund all farmers if $\lambda_t = \bar{\lambda}$. Adverse selection thus leads to inefficient production if $\phi^* < 1$. Nevertheless, the unique equilibrium of the dynamic model may feature $\phi^* < 1$. The reason is that asset quality is endogenous, and effort choices may be consistent with the equilibrium law of motion only if there is rationing on the equilibrium path.

3.3 Optimal Effort

We now turn to farmer's incentives to exert effort. In each period t , farmers make their effort decision based on their beliefs about the current and average asset quality $\{\lambda_\tau\}_{\tau \geq t}$. Since the aggregate law of motion for asset quality is in turn determined by individual effort decision, an equilibrium consists of a fixed point such that individual effort decisions give rise to a sequence $\{\lambda_t\}$ that is consistent with individual incentives given initial condition λ_0 . We start by defining the one-period payoff of an asset of type θ given λ as

$$v_\theta(\lambda) \equiv \begin{cases} p_\theta R_\theta + L_\theta - \min\{B(\lambda), L_\theta\} & \text{if } \lambda \geq \bar{\lambda} \\ L_\theta & \text{if } \lambda < \bar{\lambda} \end{cases} \quad (4)$$

and let $\Delta v(\lambda) = v_g(\lambda) - v_b(\lambda)$ denote the difference in one-period payoffs given λ . An important feature of the model is that effort is complementary conditional on market liquidity. The reason is that borrowing costs decrease in the fraction of good borrowers. This makes it more valuable to own a good asset.

Lemma 1 (Complementarities). *$\Delta v(\lambda)$ is strictly increasing in λ on the interval $[\bar{\lambda}, 1]$.*

To fully characterize a farmer's effort decision, we must take into account the entire dynamic of λ_t . We can characterize the value function $V_\theta(\lambda)$ of an individual farmer with

asset of type θ as follows:

$$V_\theta(\lambda) = \max_{e \in \{0,1\}} (1 - \pi)(v_\theta(\lambda) + \beta E[V_\theta(\lambda')]) \quad (5)$$

$$+ \pi [(1 - e)(v_b(\lambda) + \beta E[V_b(\lambda')]) + e(v_g(\lambda) + \beta E[V_g(\lambda')])] - e \cdot \bar{c}, \quad (6)$$

where λ' follows the law of motion (LoM). Let $\Delta V(\lambda) \equiv V_g(\lambda) - V_b(\lambda)$. This reveals the condition under which a farmer of type θ exerts effort:

Lemma 2. *Given a sequence $\{\lambda_t\}$ for $t = 0, 1, 2, \dots, \infty$, a farmer of type θ exerts effort in period τ if and only if:*

$$\underbrace{\Delta v(\lambda_\tau)}_{\text{current value of effort}} + \underbrace{\beta E[\Delta V(\lambda_{\tau+1})]}_{\text{future expected value of effort}} \geq c \equiv \frac{\bar{c}}{\pi} \quad (\text{IC})$$

Importantly, this incentive-compatibility condition is independent of the farmer's current type. Hence we can restrict attention to symmetric effort decisions without loss of generality with respect to aggregate outcomes.

3.4 Equilibrium

We now study the equilibrium evolution of asset quality in the absence of interventions. We are particularly interested in whether markets endogenously recover from periods of illiquidity. Complementarities typically give rise to equilibrium multiplicity. The same is true here, in that farmers may find it optimal to exert effort when all other farmers do, but prefer to shirk if all other farmers shirk. This type of multiplicity is not the focus of this paper. Hence we assume that farmers always coordinate on the equilibrium with maximum effort. This allows us to characterize conditions such that markets fail to recover even when there are no coordination failures.

As a first step, show that there exists a steady state in which all assets are good so long as the cost of effort is not too high.

Lemma 3. *Suppose that $c < \frac{\Delta v(1)}{1-\beta}$. If $\lambda_0 = 1$, then $\lambda_t = 1$ for $\forall t \geq 1$.*

That is, so long as $c < \frac{\Delta v(1)}{1-\beta}$, all farmers find it privately optimal to exert effort if $\lambda = 1$, and aggregate asset quality never declines. An economy with this property serves as a useful benchmark for understanding whether markets may recover from negative shocks to λ . We therefore maintain the following (stricter) assumption for the remainder of the paper.

Assumption 4. $c < v(1)$.

Given this benchmark, we now study equilibrium starting from some arbitrary $\lambda_0 < 1$. This initial condition need not be a steady state. Hence one might think of our exercise as studying the market response to a negative shock to asset quality at some steady state. Given an initial λ_0 , an equilibrium consists of a sequence $\{\lambda_t^*\}_{t \geq 0}$, farmers' equilibrium effort decisions consistent with the law of motion specified by Equation (LoM), and investor optimality. We define three types of equilibria, and note that any equilibrium belongs to one of these types.

Definition 3. *Given an initial condition λ_0 , we call an equilibrium a **full recovery equilibrium** if full recovery occurs at some $\tau > 0$, a **partial recovery equilibrium** if partial recovery occurs at some $\tau > 0$, and a **no recovery equilibrium** if there exists a τ such that markets remain illiquid for all $t \geq \tau$.*

We next show that there exist at most three approximate steady states for average asset quality, and that each is associated with one of the three types of equilibrium defined above.

Proposition 2. *There exist at most three equilibrium steady states for average asset quality, $\lambda^* = 0$, $\lambda^* = 1$ and $\lambda^* = \bar{\lambda}$. In any full recovery equilibrium, $\lim_{t \rightarrow \infty} \lambda_t = 1$. In any no recovery equilibrium, $\lim_{t \rightarrow \infty} \lambda_t = 0$. In any partial recovery equilibrium, $\lim_{t \rightarrow \infty} \lambda_t = \bar{\lambda}$.*

Given Proposition 2, we can focus on characterizing the set of equilibria with long-run steady state $\lambda^* \in \{1, 0, \bar{\lambda}\}$. The first step is to identify the set of permissible λ_0 for which there exists an equilibrium path to full recovery. To do so, we identify conditions under which farmers' effort is compatible with a path of $\{\lambda_t\}$ consistent with full recovery. Since future payoffs can be recursively expressed as a sum of one-period returns, we can build on Lemma 1 to infer properties of a feasible equilibrium sequence $\{\lambda_t\}$. In particular, noticing that $\Delta v(\lambda)$ is monotonically increasing in λ for $\lambda \geq \bar{\lambda}$ leads to the following cutoff argument.

Lemma 4. *There exists a minimum cutoff $\hat{\lambda} \in [\bar{\lambda}, 1)$ such that, if $\lambda_{t+1} \geq \hat{\lambda}$, then it is privately optimal for each farmer to exert effort in all periods $\tau \geq t$ conditional on all other farmers exerting effort in all periods $\tau \geq t$.*

The logic behind this result is the following. Suppose that average asset quality tomorrow is expected to be such that the value difference exceeds the cost of effort, and that the expected asset quality will be realized if all farmers exert effort today. Then it is indeed optimal for all farmers to exert effort today, and asset quality increases. Since incentives are strictly increasing in λ , moreover, this argument also holds from tomorrow onwards. For the same reason, it is possible to find the smallest λ in $[\bar{\lambda}, 1)$ such that effort is optimal from today onward. This defines the minimum cutoff $\hat{\lambda}$. Noting that the law of motion for asset quality is $\lambda_{t+1} = \lambda_t + \pi(1 - \lambda_t)$ if all farmers exert effort gives the next result.

Corollary 1. *For any $\lambda_0 \geq \frac{\hat{\lambda} - \pi}{1 - \pi}$, there exists a full recovery equilibrium.*

The previous results hold for any c so long as Assumption 4 is satisfied. However, they rely on λ_0 being sufficiently high. We now extend the set of initial values that permit full recovery by considering all λ_0 such that there exists an equilibrium path to some $\lambda_{t'}$ that satisfies Lemma 4.

Proposition 3. *Suppose that $\bar{\lambda} + (1 - \bar{\lambda})\pi \geq \hat{\lambda}$. Then:*

- (i) *If $c < \frac{\Delta L}{1 - \beta}$, a full recovery equilibrium exists for all $\lambda_0 \in [0, 1]$;*
- (ii) *If $c > \frac{\Delta L}{1 - \beta}$, a full recovery equilibrium exists for all $\lambda_0 \in [\frac{\hat{\lambda} - \pi}{1 - \pi}, 1]$, and a no recovery equilibrium exists for $\lambda_0 \in [0, \frac{\hat{\lambda} - \pi}{1 - \pi})$ for some cutoff $\check{\lambda} \in [0, \bar{\lambda}]$.*

The opposite argument can be used to identify the set of $\lambda_{t'}$ that preclude full recovery. Note that no effort is a strictly dominant strategy for any $\lambda_{t'} \in (\bar{\lambda}, \hat{\lambda})$. If $\hat{\lambda} > \bar{\lambda} + (1 - \bar{\lambda})\pi$, then there does not exist any $\lambda_{t'-1} \leq \bar{\lambda}$ that permits a transition into some $\lambda_t \geq \hat{\lambda}$. Together this implies the following:

Proposition 4. *If $\bar{\lambda} + (1 - \bar{\lambda})\pi < \hat{\lambda}$, then full recovery does not exist for $\lambda_0 \leq \frac{\hat{\lambda} - \pi}{1 - \pi}$.*

Nonexistence of a full recovery equilibrium means that markets do not recover from a sufficiently bad initial condition (or, equivalently, from a sufficiently negative shock). In such cases, the long-run λ^* either takes a value $\bar{\lambda}$ or 0. We now show that recovery occurs only if $c < \frac{\Delta L}{1-\beta}$, but that average asset quality deteriorates to 0 otherwise.

Proposition 5. *Suppose that $\bar{\lambda} + (1 - \bar{\lambda})\pi < \dot{\lambda}$. Then:*

- (i) *if $c < \frac{\Delta L}{1-\beta}$, then a full recovery equilibrium only arises for $\lambda_0 \in [\frac{\dot{\lambda}-\pi}{1-\pi}, 1]$, and a partial recovery equilibrium arises for $\lambda_0 \in [0, \frac{\dot{\lambda}-\pi}{1-\pi})$;*
- (ii) *if $c > \frac{\Delta L}{1-\beta}$, then a full recovery equilibrium only arises for $\lambda_0 \in [\frac{\dot{\lambda}-\pi}{1-\pi}, 1]$, and a no recovery equilibrium arises for $\lambda_0 \in [0, \frac{\dot{\lambda}-\pi}{1-\pi})$.*

3.5 Discussion

The laissez-faire equilibrium demonstrates how markets cope with adverse selection in the absence of regulation. We showed that farmers can collectively improve asset quality and quickly restore market liquidity if initial asset quality is not too poor. After large negative shocks, instead, markets either deteriorate to the point where no good assets are left, or they partially recover, with asset quality hovering around the liquidity threshold. In either case, markets are no longer liquid with certainty, and there may be inefficient breakdowns in trade. This provides scope for government interventions. We discuss this next.

4 Model with Capital Market Interventions

Given the benchmark laissez-faire economy developed in the previous section, we now turn to assessing the impact of government interventions designed to restore liquidity. We focus particularly on the interaction of interventions and private effort incentives.

In many real-world situations, a constraint on pledgeability (e.g. shortage of good collateral) is thought to be at the heart of market breakdowns due to adverse selection. In this context, a natural policy to consider is one in which the regulator injects liquidity into the

economy in order to relax borrowing constraints. Recent examples include the Fed’s Term Securities Lending Facility (stopped in 2010) and the ECB’s Securities Lending Programme (continued), which explicitly aimed to upgrade the collateral available to private borrowers by providing downside insurance to lenders or by engaging in swaps for low-quality collateral prone to adverse selection. With this in mind, we extend the model by introducing a regulator that can implement such policies with a view towards affecting market liquidity. We assume that the regulator is long-lived and risk-neutral, and has the same discount factor $\beta \in [0, 1)$ as farmers.

Policy instrument. The main policy instrument we consider is government insurance for investors against farmer defaults. At the beginning of every period t , and *before* farmers make their effort decisions, the regulator announces a policy $\{q, \mathcal{I}\}$. This policy consists of a payment q to an investor in the event that the farmer he has lent to defaults (by failing to pay the full amount B), and specifies the set of values of $\lambda \in \mathcal{I} \subseteq [0, 1]$ conditional on which the policy is implemented. Hence, the policy is a partial state-contingent guarantee for investors, and thus serves to relax participation constraints. After λ_t is realized, the regulator implements the intervention. In Section 4.4, we show an equivalence between this policy program and a broader set of intervention tools, such as direct intervention or interest rate policy. This is reminiscent of Philippon and Skreta (2012), who argue that the precise implementation of policies is not important, and that a sufficient statistic is the borrowing rate in the private market. The same is true here: all policies that deliver the same equilibrium face value of debt B conditional on market liquidity are equivalent.

Policy Regimes. We focus on two policy regimes, which differ with respect to commitment. The first is *no commitment*. Under no commitment, the regulator is not committed to implementing the policy he announced at the beginning of the period (prior to effort decision). Hence it is as if he is choosing the intervention policy $\{q, \mathcal{I}\}$ *after* observing λ_t . The second is *commitment*. Under commitment, the regulator credibly commits to implementing the policy $\{q, \mathcal{I}\}$ he announced prior to the effort decision. In either regime, we assume that the regulator cannot commit to a time path of future interventions. Hence we permit the regulator to condition on the relevant aggregate state variable λ , but we do not allow for policies that are contingent on calendar time.³ The timing of events with market

³Given the investors are short-lived, we will however show that intra-period commitment is sufficient to

intervention is summarized in Figure 2.

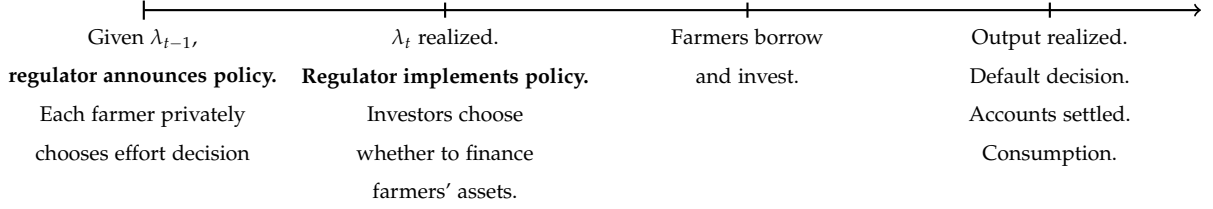


Figure 2: Timing of Events in Period t

Regulator's objective. We use $S(\lambda, \{q, \mathcal{I}\})$ to denote the per-period cost of intervention for a policy $\{q, \mathcal{I}\}$ for λ . Since only bad farmers default, it is given by

$$S(\lambda, \{q, \mathcal{I}\}) = \mathbb{1}_{\{\lambda \in \mathcal{I}\}}(1 - \lambda)q. \quad (7)$$

We start by assuming that the regulator is unconstrained in terms of resources, and his expected payoff is given by the discounted sum of total expected farmer utility net of the cost of intervention.

$$W = \sum_{t=1}^{\infty} \beta^{t-1} E_t [\lambda_t \cdot v_g(\lambda_t) + (1 - \lambda_t)v_b(\lambda_t) - S(\lambda_t, \{q, \mathcal{I}\}_t)] \quad (8)$$

This is an appropriate welfare criterion here given that investors are competitive and have deep pockets, and thus and always earn $1 + r_f$.

4.1 Market Intervention and Impact on Incentives

We now turn to characterizing the equilibrium under market interventions. We begin by incorporating intervention into investors' lending decisions. For a given intervention

obtain first-best.

$\{q, \mathcal{I}\}$, investors' participation constraint is

$$\begin{cases} \lambda L_g + (1 - \lambda)(L_b + q) \geq 1 + r_f & \text{if } \lambda \in \mathcal{I} \\ \lambda L_g + (1 - \lambda)L_b \geq 1 + r_f & \text{if } \lambda \notin \mathcal{I} \end{cases} \quad (9)$$

That is, if the intervention is implemented, then investors expect to receive an additional q units of capital from the government when a farmer defaults. Conditional on investors' participation condition being satisfied, the competitive face value of debt $B(\lambda, \{q, \mathcal{I}\})$ as a function of intervention is thus given by

$$B(\lambda, \{q, \mathcal{I}\}) = \begin{cases} \frac{1+r_f-(1-\lambda)(L_b+q)}{\lambda} & \text{if } \lambda \in \mathcal{I} \\ \frac{1+r_f-(1-\lambda)L_b}{\lambda} & \text{if } \lambda \notin \mathcal{I} \end{cases} \quad (10)$$

As expected, the borrowing cost drops with respect to the subsidy q provided by the government to investors.

Lemma 5. *For $\lambda \in \mathcal{I}$, $B(\lambda, \{q, \mathcal{I}\})$ strictly decreases in q .*

This implies that interventions affect the market value of good and bad assets by reducing the adverse selection discount. Given some λ and policy $\{q, \mathcal{I}\}$, the per-period value of an asset of type θ is given by

$$v_\theta(\lambda, \{q, \mathcal{I}\}) = \begin{cases} p_\theta R_\theta + L_\theta - \min\{B(\lambda, \{q, \mathcal{I}\}), L_\theta\} & \text{if } \lambda \in \mathcal{I} \text{ and } \lambda L_g + (1 - \lambda)(L_b + q) \geq 1 + r_f \\ p_\theta R_\theta + L_\theta - \min\{B(\lambda, \{q, \mathcal{I}\}), L_\theta\} & \text{if } \lambda \notin \mathcal{I} \text{ and } \lambda \geq \bar{\lambda} \\ L_\theta & \text{otherwise.} \end{cases} \quad (11)$$

Similar to the laissez-faire economy, a wedge arises conditional on market liquidity. As long as market quality is sufficiently high (i.e. $\lambda \geq \bar{\lambda}$), farmers are able to access capital and invest even if $\lambda \notin \mathcal{I}$. However, the intervention boosts asset values by lowering B for all $\lambda \in \mathcal{I}$. In this way, the policy may enable farmers to invest even when λ is such that adverse selection would otherwise lead to market breakdowns.

Optimality of full recovery. It is clear that market interventions can restore liquidity. In principle, interventions may also induce a full recovery equilibrium by solving a dynamic coordination problem between borrowers who would otherwise find it privately optimal shirk (as in the laissez-faire equilibrium for sufficiently low λ_0 and high c). Before studying whether this is the case, it is useful to first understand the conditions under which full recovery is, in fact, efficient relative to laissez-faire.

To highlight this, suppose for now that there exists some policy such that, for some initial value $\lambda_0 \leq \bar{\lambda}$, an equilibrium path $\{\lambda_t\}$ leading to full recovery exists under this policy. Let $F(\lambda, t) = \pi \sum_{i=0}^{t-1} (1 - \pi)^i + (1 - \pi)^t \lambda$ for $t \geq 1$ and $F(\lambda, 0) = \lambda_0$ define the law of motion under full effort by all farmers in every period. We can then express the discounted net output under such this proposed policy recursively:

$$\sum_{t=1}^{\infty} \beta^{t-1} [F(\lambda_0, t)(p_g R_g + L_g) + (1 - F(\lambda_0, t))(p_b R_b + L_b) - (1 + r_f)] - \sum_{t=1}^{\infty} \beta^{t-1} \bar{c} \quad (12)$$

Four insights are worth noting:

1. since any subsidy is transfer to investors, optimality solely depends on the sum of discounted payoffs net of the effort cost ($\bar{c}$) and opportunity cost ($1 + r_f$);
2. Relative to a full recovery laissez-faire equilibrium, if the expected payoff from the stochastic return $F(\lambda_0, 1)(p_g R_g + L_g) + (1 - F(\lambda_0, 1))(p_b R_b + L_b) - (1 + r_f) \geq 0$, then full recovery induced by the intervention generates (weakly) greater output. Hence, there exists a cutoff $\underline{\lambda}$ for which $\lambda_0 > \underline{\lambda}$ implies that intervention improves efficiency relative to a full recovery laissez-faire equilibrium;
3. As β increases, the value of resolving the dynamic coordination problem increases. To see this, consider the stark case when β approaches 1 and $\lambda_0 \in (\underline{\lambda}, \frac{\bar{\lambda} - \pi}{1 - \pi})$. It is straightforward to see that the long-run one-period net output under intervention $p_g R_g + L_g - (1 + r_f) - \bar{c}$ is strictly greater than the long-run one-period net output under laissez-faire equilibria with $\lambda^* < 1$. This implies that for any $\lambda_0 \in (\underline{\lambda}, \frac{\bar{\lambda} - \pi}{1 - \pi})$, there exists a sufficiently large β such that intervention improves efficiency for λ_0 . Noting that first term in Equation 12 monotonically increases in λ_0 , let $\lambda_{\phi}(\beta)$ denote

the minimum λ_0 for a given β for which intervention obtains (weakly) greater output relative to laissez-faire.

Given that our primarily goal is understanding how market intervention interacts with incentives, we focus on the case in which interventions that induce full recovery and provide liquidity improve efficiency relative to laissez-faire for $\lambda_0 \geq \underline{\lambda}$:

Assumption 5. β satisfies $\lambda_{\varnothing}(\beta) \leq \underline{\lambda}$.

4.2 Minimal Intervention and Intervention Traps

Under no commitment, the intervention is essentially chosen after observing λ_t . Hence we must characterize the optimal $\{q, \mathcal{I}\}$ given λ . Since future λ' is independent of current policy, it is optimal to restore liquidity in illiquid markets so long as the expected output of the average asset is above a threshold. Moreover, the regulator chooses the smallest q that is consistent with market liquidity, and intervenes only if markets are illiquid. This leads to the following result, which is closely related to Tirole (2012).

Proposition 6. *Under no commitment, a regulator chooses $\{q^{min}, \mathcal{I}^{min}\}$ where q^{min} is set such that $B(\lambda, \{q^{min}, \mathcal{I}^{min}\}) = L_g$ and $\mathcal{I}^{min} = [\underline{\lambda}, \bar{\lambda}]$ where cutoff $\underline{\lambda} = \frac{1+r_f-p_b R_b}{\Delta R}$.*

Under no commitment, the regulator's policy strategy involved intervening only when necessary to provide relief to an otherwise illiquid market, and offering the minimal subsidization q required to restore market liquidity. In effect, the regulator implements a policy of “**minimal intervention**”, in that he intervenes if and only if markets are ex-post illiquid, i.e. $\lambda < \bar{\lambda}$, and provides the minimum effective intervention.⁴

While a policy of minimal intervention aims to provides liquidity only when necessary, farmers' ex-ante incentives are affected by the expectation of intervention. In particular, because minimal intervention involves injecting just enough liquidity such that markets are liquid, farmers effectively anticipate market conditions akin to $\lambda = \bar{\lambda}$, as described below:

⁴Other motives to implement a minimal intervention includes political economy costs associated with bailing out financial institutions and markets. For example, see Dam and Koetter (2012) and Behn, Haselmann, Kick, and Vig (2015).

Lemma 6. Suppose that $\lambda \in [\underline{\lambda}, \bar{\lambda}]$. Under minimal intervention, one-period payoff of projects is given by $v_\theta(\lambda, \{q^{\min}, \mathcal{I}^{\min}\}) = p_\theta R_\theta$.

Putting this together, we can characterize the equilibrium under minimal intervention following a large adverse shock that results in $\lambda_0 < \bar{\lambda}$:

Proposition 7. Suppose that $\lambda_0 \in [\underline{\lambda}, \bar{\lambda}]$. Under minimal intervention we obtain a full recovery equilibrium if (1) $c < \frac{\Delta R}{1-\beta}$ or (2) $\bar{\lambda} + (1 - \bar{\lambda})\pi \geq \dot{\lambda}$ and $\lambda_0 \geq \ddot{\lambda}$ for some threshold $\ddot{\lambda} \leq \bar{\lambda}$. Otherwise, we obtain a no recovery equilibrium.

A key insight is that even though the intent of minimal intervention is to intervene only when necessary, illiquidity necessitates intervention. That is, just as the regulator cannot commit to providing subsidies when markets are liquid, the regulator cannot commit to *not* providing subsidies when markets are illiquid. As a result, no commitment, while increasing net output in the current period, potentially *destroys* incentives. When this is the case, λ_t decreases, which necessitates intervention to continue indefinitely.

Proposition 8 (Intervention Trap). Suppose that $\lambda_0 \in [\underline{\lambda}, \bar{\lambda}]$. Under minimal intervention:

1. if (1) $c < \frac{\Delta R}{1-\beta}$ or (2) $\bar{\lambda} + (1 - \bar{\lambda})\pi \geq \dot{\lambda}$ and $\lambda_0 \geq \ddot{\lambda}$, intervention occurs for finite periods to a steady state $\lambda^* = 1$;
2. Otherwise, if $c > \frac{\Delta L}{1-\beta}$, intervention occurs for finite periods, until market quality drops below $\underline{\lambda}$ to a steady state $\lambda^* = 0$; if $c < \frac{\Delta L}{1-\beta}$, intervention occurs indefinitely to approximate steady state $\lambda^* = \underline{\lambda}$.

Note, even if asset quality deteriorates under intervention, intervention directly increases output by enabling investment. As a result, whether welfare improves or declines relative to laissez-faire largely depends on the intertemporal trade-off (i.e. long-lived agents' discount rate β). Intervention does, however, have unambiguous effects on incentives relative to laissez-faire:

Proposition 9. Suppose that $\lambda_0 \in [\underline{\lambda}, \bar{\lambda}]$. Under minimal intervention:

1. if (1) $c < \frac{\Delta R}{1-\beta}$ or (2) $\bar{\lambda} + (1 - \bar{\lambda})\pi \geq \dot{\lambda}$ and $\lambda_0 \geq \ddot{\lambda}$, output and asset quality is weakly greater relative to laissez-faire;
2. otherwise, asset quality is weakly lower relative to laissez-faire.

4.3 Jump-starting Liquidity

Implicitly, no commitment enables the regulator to react to market conditions. At the cost of this flexibility, the regulator cannot influence private incentives of market participants, who correctly anticipate intervention under illiquid conditions. In this section, we demonstrate that, under commitment, promising “over-stimulus” could expedite recovery.

Under commitment, the regulator’s intervention is announced prior to farmers’ effort decisions. As a result, $\{q, \mathcal{I}\}$ takes into its account its impact on farmers’ incentives. Our main result is that the regulator optimally commits to a *bigger* subsidization to investors than under the minimal intervention in order to ensure effort incentives.

Proposition 10. *Suppose that $\lambda_0 \in [\underline{\lambda}, \bar{\lambda})$. Under commitment, the regulator equilibrium policy is given by $\{q^c, \mathcal{I}^c\}$ is given by , where:*

1. $\{q^c, \mathcal{I}^c\} = \{q^{\min}, \mathcal{I}^{\min}\}$ if (1) $c < \frac{\Delta R}{1-\beta}$ or (2) $\bar{\lambda} + (1 - \bar{\lambda})\pi \geq \dot{\lambda}$ and $\lambda_0 \geq \ddot{\lambda}$
2. Otherwise, $\{q^c, \mathcal{I}^c\}$ is such that q^c sets $B(\lambda, \{q, \mathcal{I}\}) = c$ and $\mathcal{I}^c = [\underline{\lambda}, \dot{\lambda})$;

and for which we obtain a full recovery equilibrium.

Under commitment, the regulator fully internalizes the impact of regulation on incentives. The equilibrium intervention in the commitment regime, referred to henceforth as “**c-intervention**” reveals two important qualities. First, the regulator offers (weakly) larger subsidies q relative to minimal intervention. When incentives are taken into account, the regulator chooses a larger subsidy q , and by doing so lowers the cost of capital to farmers. As conditional on holding a good asset, farmers are able to retain a larger fraction of their certainty return L_g , intervention actually improves the incentive problem. Second, the regulator activates intervention even when $\lambda > \bar{\lambda}$. In a commitment regime, because intervention can not only relieve illiquidity but ultimately maximize total output, intervention occurs as long as farmers are unable to collectively improve market quality. Together, c-intervention always leads to a full recovery equilibrium.

Importantly, because interventions support farmers’ incentives until markets recover to $\lambda = \dot{\lambda}$, markets are able to transition into a long-run $\lambda^* = 1$ within finite periods of time. This yields the following result:

Proposition 11 (Bigger Interventions). *Suppose that $\lambda_0 \in [\underline{\lambda}, \bar{\lambda})$. Under c-intervention, intervention occurs for finite periods to a steady state $\lambda^* = 1$.*

As with minimal intervention, c-intervention immediately restores liquidity, thereby creating short-term gains relative to laissez-faire. Additionally, c-intervention induces effort by all farmers, thereby (weakly) improving the speed of asset quality:

Proposition 12. *Suppose that $\lambda_0 \in [\underline{\lambda}, \bar{\lambda})$. Under c-intervention, output and asset quality is weakly greater relative to laissez-faire and minimal intervention.*

4.4 Policy Equivalence

In this section, we show that insights on the main intervention policy instrument applies to a broader set of policies. In this section, we show that our main policy intervention can be implemented with a (1) program that directly subsidizes farmers and (2) interest rate policy with respect to r_f .

Direct subsidized lending program. Consider an intervention policy in which the regulator directly provides subsidized lending to farmers. Let the regulator offers to lend s bonds with zero coupon and face value 1, for which in return farmers are required to pay τ (or up to L_θ). The intervention offers up to K bonds to farmers at first-come first-serve basis, and is activated for a set of $\lambda \in \mathcal{I}$. We show that:

Proposition 13. *For any market intervention with policy $\{q, \mathcal{I}\}$, there exists a corresponding $\{s, \tau, \mathcal{I}, K\}$ that is outcome and cost equivalent.*

Risk-free interest rate policy. Let $r_{f,t}$ be used to denote the one-period risk-free return in period t that is accessible to investors. Consider an alternate policy where at every period t , the regulator chooses the short-term riskfree rate $r_{f,t}$. As long as $r_{f,t}$ is unbounded (e.g. no zero lower bound), there exists a mapping between any policy $\{q, \mathcal{I}\}$ can some r_f that achieves the same outcome:⁵

Proposition 14. *For any market intervention with policy $\{q, \mathcal{I}\}$, there exists an interest rate policy $\{r_f, \mathcal{I}\}$ that is outcome equivalent.*

⁵Assessing whether implementation through interest rate policy is cost equivalent is outside of the scope of the model.

4.5 Delaying Intervention

So far, we characterized intervention in a setting in which the regulator does not face a budget constraint. When unconstrained, the regulator always optimally intervenes early to capture the benefits of efficiently restoring liquidity. When the regulator is constrained, however, the timing of intervention matters because interventions in certain periods may offer more bang-for-the-buck than others.

Recall that when $c > \frac{\Delta L}{1-\beta}$, farmers had incentives to exert effort even when markets are illiquid. For a large shock that leads to $\lambda_0 < \bar{\lambda}$, delaying intervention is costly as markets are impaired. However, the regulator may not have enough resources to implement the unconstrained c – *intervention*. Delaying intervention until it is necessary to incentivize collective effort could economize on resources and ensure long-run liquidity.

We illustrate this point using a stark example. Let $c > \frac{\Delta L}{1-\beta}$ and $\bar{\lambda} + (1 - \bar{\lambda})\pi < \dot{\lambda}$, which corresponds to the recovery equilibrium in Proposition 5. Suppose that the regulator can only use at most $K = 1 + r_f - \bar{\lambda}(\Delta R + L_g - c) - (1 - \bar{\lambda})L_b$, which corresponds to the cost of a one-time c -intervention when $\lambda = \bar{\lambda}$. If the regulator were to intervene at any time before market quality recovered to $\bar{\lambda}$, K would be sufficient to finance a large enough intervention program that would induce incentives. As a result, intervention would achieve short-term market liquidity, coupled with a decline in market quality. By delaying intervention until λ_t reached $\bar{\lambda}$, the regulator is able to induce effort and ensure full recovery. This implies that as long as the regulator is patient (i.e. large β), delaying intervention strictly dominates.

5 Conclusion

In this paper, we develop a dynamic model of a collateralized asset market with asymmetric information. We show that minimal interventions markets can fall into “intervention traps” – expectations concerning future interventions eliminate private incentives to improve the quality of collateral, which stunts recovery and warrants continued market intervention. Committing to bigger interventions may lead to faster recoveries, and prolonging intervention even after market liquidity is restored may be optimal. A direct takeaway is that government interventions designed to boost liquidity in frozen markets must also take

into account its impact on private incentives to maintain high-quality assets. More generally, our paper provides a rationale for maintaining a program of market intervention even outside of extreme conditions, such as a financial crisis. Offering timely relief through interventions can be shown to be an economical solution to preserving market health, especially if more severe conditions will eventually necessitate intervention.

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A Proofs

A.1 Lemma 1

Proof. For $\lambda \in [\bar{\lambda}, 1]$, $\Delta v(\lambda) = \Delta R + L_g - B(\lambda)$. Since $B(\lambda) = \frac{1+r_f-(1-\lambda)L_b}{\lambda}$ decreases in λ , $\Delta v(\lambda)$ increases in λ . $\square$

A.2 Lemma 2

Proof. Follows directly from text. $\square$

A.3 Lemma 3

Proof. Suppose that $c < \frac{\Delta v(1)}{1-\beta}$. Then, $\sum_{t=0}^{\infty} \beta^t \Delta v(1) > c$. Hence, for a sequence $\lambda_t = 1$ for $t = 0, 1, 2, \dots$, Condition (IC) holds, i.e. individual farmers find it optimal to exert effort. Under the law of motion given by Equation (LoM), $\lambda_{t+1} = 1$ for $\lambda_t = 1$ if $e = 1$ for all farmers. Since effort is incentive compatible conditional on all other farmers exerting effort, under coordinated effort, $\lambda_t = 1$ for $\forall t$. $\square$

A.4 Proposition 2

Proof. First we show existence by example. Lemma 3 directly implies the existence of long-run value of 1. Next, suppose that $c > \frac{\Delta L}{1-\beta}$ and consider a candidate path where $\lambda_t = 0$ for $\forall t$. Since $\Delta v(0) + \beta V(0) = \frac{\Delta L}{1-\beta}$, there exists an equilibrium in which no farmers exert effort for any t , and λ_t trivially approaches 0. Finally, suppose that c is such that $\frac{\Delta L}{1-\beta} > c > \frac{\Delta R}{1-\beta}$ and assume conditions under which full recovery does not occur, in line with Proposition 4. Consider a sequence where $\lambda_0 = \bar{\lambda}$ and a candidate equilibrium path with $\lambda_t = \tilde{\lambda}$ for $\forall t > 0$, where

$$\tilde{\lambda} = \begin{cases} \bar{\lambda} & \text{with probability } \frac{\Delta L - (1-\beta)c}{\Delta L - \Delta R} \\ \bar{\lambda} - \epsilon & \text{with probability } \frac{(1-\beta)c - \Delta R}{\Delta L - \Delta R} \end{cases} \quad (13)$$

for some arbitrarily small $\epsilon > 0$.⁶ Note that conditional on $\lambda_0 = \bar{\lambda}$, conditional on full effort where $\lambda_1 = \bar{\lambda} + (1 - \bar{\lambda})\pi$, individual farmers do not choose $e = 1$. Conditional on $\lambda_t = \bar{\lambda}$ for $\forall t > 0$, we can express gains from effort:

$$\frac{(1 - \beta)c - \Delta R}{\Delta L - \Delta R} \Delta L + \frac{\Delta L - (1 - \beta)c}{\Delta L - \Delta R} = c \quad (14)$$

Hence, farmers are indifferent between $e = 0$ and $e = 1$. Symmetric mixed strategy of exerting effort with probability $\mu = \frac{1}{2}$ rationalizes the sequence $\bar{\lambda}$. Hence, an equilibrium exists with $\lambda^* \approx \bar{\lambda}$.

Second, We rule out all other long-run values of λ_t by contradiction. Consider an equilibrium in which λ_t approaches some value Y . Suppose that this is a no recovery equilibrium. This implies that there exists some $\lambda = Y \in (0, \bar{\lambda})$ such that $\lambda' = Y$ given the law of motion. Since $Y \neq Y + (1 - Y)\pi$ or $Y = (1 - \pi)Y$, this implies that farmers play mixed strategies such that $Y = \mu_g Y + (1 - \mu_g)\pi Y + \mu_b \pi(1 - Y)$. Since this requires that farmers are indifferent between $e = 0, 1$, we can infer that $\frac{\Delta v(Y)}{1 - \beta} = c$. However, since $\Delta v(Y) = \Delta L$, Condition (IC) is weakly satisfied. This implies that coordinated effort is possible, i.e. $\mu_g = \mu_b = 1$, which contradicts long-run value Y . Next, suppose that this is a full recovery equilibrium. Following a similar argument, this implies that there exists some $Y \in (\bar{\lambda}, 0)$ such that $Y = Y + (1 - Y)\pi$ or $Y = (1 - \pi)Y$ which is not true. Since mixing requires $\frac{\Delta v(Y)}{1 - \beta} = c$, in which case coordinated effort is possible, no such full recovery equilibrium exists. Lastly, suppose that this is a recovery equilibrium. Since only for $Y = \bar{\lambda}$ does there exist an arbitrarily small ϵ such that markets are illiquid for $Y - \epsilon$, there does not exist a $Y \neq \bar{\lambda}$ for a recovery equilibrium. $\square$

⁶Strategic substitutability at $\bar{\lambda}$ precludes an equilibrium. We allow for infinitesimally small perturbations to allow for a mixed-strategy equilibrium to exist. Alternatively, one could assume that investors play random strategy at $\bar{\lambda}$ since they are indifferent between investing and not investing.

A.5 Lemma 4

Proof. By Lemma 3, full recovery equilibrium exists for $\lambda_0 = 1$. For some small ϵ :

$$\begin{aligned} \frac{\Delta v(1)}{1-\beta} - \Delta v(1-\epsilon) - \beta \Delta V((1-\epsilon) + \epsilon\pi) &= \left[1 + r_f - \frac{1 + r_f - \epsilon L_b}{1-\epsilon} \right] \\ &+ \beta \left[1 + r_f - \frac{1 + r_f - \epsilon(1-\pi)L_b +}{1-\epsilon + \epsilon\pi} \right] + \dots \end{aligned}$$

which approaches 0 as $\epsilon \rightarrow 0$. Hence, there exists a $\dot{\lambda} = 1 - \bar{\epsilon}$ for some $\bar{\epsilon} > 0$ such that $\Delta v(\dot{\lambda}) + \beta \Delta V(\dot{\lambda} + (1 - \dot{\lambda})\pi) = c$. $\square$

A.6 Corollary 1

Proof. Since under coordinated effort, when $\lambda_0 = \frac{\dot{\lambda} - \pi}{1 - \pi}$, $\lambda_1 = \dot{\lambda}$, by Proposition 4, a full recovery equilibrium exists. $\square$

A.7 Proposition 3

Proof. Let $\bar{\lambda} + (1 - \bar{\lambda})\pi \geq \dot{\lambda}$ and consider the first case where $c > \frac{\Delta L}{1-\beta}$. We show that there exists a full recovery equilibrium for any $\lambda_0 \in [\frac{\ddot{\lambda} - \pi}{1 - \pi}, 1]$ for some threshold $\frac{\ddot{\lambda} - \pi}{1 - \pi}$ by characterizing a cutoff $\ddot{\lambda}$ for which individual farmers' effort decisions are consistent with a λ_t path with full effort. Let $\ddot{\lambda} = \dot{\lambda}$ if $\dot{\lambda} > \bar{\lambda}$. Otherwise, if $\dot{\lambda} = \bar{\lambda}$ then it implies that there exists some $\lambda < \bar{\lambda}$ where:

$$\Delta L + \beta \Delta V(\lambda + (1 - \lambda)\pi) \geq c \quad (15)$$

Let $\ddot{\lambda} \leq \bar{\lambda}$ be the cutoff at which:

$$\Delta L + \beta E[V(\ddot{\lambda} + (1 - \ddot{\lambda})\pi)] = c \quad (16)$$

Given the characterization of $\ddot{\lambda}$, effort is exerted by all farmers for any $\lambda_0 \geq \frac{\ddot{\lambda}-\pi}{1-\pi}$. For any $\lambda < \frac{\ddot{\lambda}-\pi}{1-\pi}$, gains from effort conditional on a path of full effort is such that:

$$\Delta L + \beta E[\Delta V(\lambda + (1 - \lambda)\pi)] < \Delta L + \beta E[\Delta V(\ddot{\lambda} + (1 - \ddot{\lambda})\pi)] \quad (17)$$

where the above holds because $c > \frac{\Delta L}{1-\beta}$ implies that $E[\Delta V(\ddot{\lambda} + (1 - \ddot{\lambda})\pi)] > c$, from which we can infer that $\Delta V(\lambda)$ (weakly) increases in λ . This implies that effort is not incentive compatible even if all other farmers exert effort for all periods. Hence, a full recovery equilibrium does not exist. Consider instead a candidate equilibrium path in which no farmers exert effort. Again, since $c > \frac{\Delta L}{1-\beta}$, individual farmers find it optimal to choose $e = 0$. Since the candidate path is consistent with no farmers exerting effort, there exists an equilibrium with no recovery for $\lambda_0 \in (0, \frac{\ddot{\lambda}-\pi}{1-\pi})$.

Next, consider when $c < \frac{\Delta L}{1-\beta}$. Suppose that conditional on a path with full effort, τ is the first period at which $\lambda_\tau \geq \dot{\lambda}$. We can express farmers' gain from effort at $t = 0$ as:

$$\Delta L + \beta \Delta L + \dots + \beta^{\tau-1} \Delta V(\lambda_\tau) > \frac{1 - \beta^{\tau-1}}{1 - \beta} \Delta L + \beta^{\tau-1} c \quad (18)$$

$$> c \quad (19)$$

This establishes that effort is incentive compatible for each farmer conditional on a full effort path. Hence there exists a full recovery equilibrium for any λ_0 . $\square$

A.8 Proposition 4

Proof. Recall, full recovery requires there to exist a incentive compatible transition path that reaches λ_t where $\lambda_\tau \geq \dot{\lambda}$ in some period τ . By construction, $\bar{\lambda} + (1 - \bar{\lambda})\pi < \dot{\lambda}$ implies that for set $[\bar{\lambda}, \dot{\lambda}]$ where for $\lambda_{t'} \in [\bar{\lambda}, \dot{\lambda}]$, individual farmers' effort decision is such that $e = 0$ is strictly dominating strategy at t' conditional on beliefs that all other farmers exert effort in the current and all future periods. From this, we can infer that there does not exist feasible transition path to full recovery: any candidate path with $\lambda_0 < \frac{\dot{\lambda}-\pi}{1-\pi}$, must have some t' at which $\lambda_{t'} \in [\bar{\lambda}, \dot{\lambda}]$, as there does not exist a $\lambda \leq \bar{\lambda}$ where $\lambda + (1 - \lambda)\pi \geq \dot{\lambda}$. However, effort is not incentive compatible given such $\lambda_{t'}$, which implies that full recovery does not

exist. □

A.9 Proposition 5

Proof. Let $\bar{\lambda} + (1 - \bar{\lambda})\pi < \dot{\lambda}$. Proposition 4 implies that for $\lambda_0 \leq \frac{\dot{\lambda} - \pi}{1 - \pi}$, no full recovery exists. We show that this implies that $c \leq \frac{\Delta R}{1 - \beta}$. To see this, note that $\frac{\Delta R}{1 - \beta} = \frac{\Delta v(\bar{\lambda})}{1 - \beta}$, which corresponds to the marginal value of effort conditional on a path of $\lambda_t = \bar{\lambda}$ for $t \geq 0$. However, since $\frac{\Delta v(\bar{\lambda})}{1 - \beta} < \Delta v(\bar{\lambda}) + \beta \cdot \frac{\bar{\lambda} + (1 - \bar{\lambda})\pi}{1 - \beta}$, if $c \leq \frac{\Delta R}{1 - \beta}$, then a full recovery equilibrium exists.

Rearranging $\bar{\lambda} + (1 - \bar{\lambda})\pi < \dot{\lambda}$ we get $\frac{\dot{\lambda} - \pi}{1 - \pi} < \bar{\lambda}$. Given this, suppose that $c > \frac{\Delta L}{1 - \beta}$. Since

$$\Delta L + \beta \cdot \frac{\max\{\Delta L, \Delta R\}}{1 - \beta} < c,$$

effort is not compatible for any $\lambda'_t < \frac{\dot{\lambda} - \pi}{1 - \pi}$ conditional on a recovery path with long-run $\lambda^* = \bar{\lambda}$. Hence, there does not exist any $\lambda_0 < \lambda'_t < \frac{\dot{\lambda} - \pi}{1 - \pi}$ that permits a transition to $\bar{\lambda}$, i.e. no equilibrium with recovery exists. Consider instead a candidate equilibrium where $\lambda_{t+1} = \lambda_t(1 - \pi)$ for all $t \geq 0$. Since $c > \frac{\Delta L}{1 - \beta} = \frac{\Delta v(\lambda_0)}{1 - \beta}$, the candidate path of λ_t is consistent with no effort exerted by any farmer. Hence, a no-recovery equilibrium exists.

Next, suppose that $c \leq \frac{\Delta L}{1 - \beta}$ and consider a farmer's effort decision at t' conditional on $\lambda_t = \tilde{\lambda}$ for $\forall t > t'$, where

$$\tilde{\lambda} = \begin{cases} \bar{\lambda} & \text{with probability } \frac{\Delta L - (1 - \beta)c}{\Delta L - \Delta R} \\ \bar{\lambda} - \epsilon & \text{with probability } \frac{(1 - \beta)c - \Delta R}{\Delta L - \Delta R}, \end{cases} \quad (20)$$

consistent with the characterization in Proposition 2. Note that when $\lambda_{t'} = \tilde{\lambda}$,

$$E[\Delta v(\lambda_{t'})] = \frac{(1 - \beta)c - \Delta R}{\Delta L - \Delta R} \Delta L + \frac{\Delta L - (1 - \beta)c}{\Delta L - \Delta R} \Delta R = c. \quad (21)$$

For any $\lambda_{t'-1} \in (\frac{\bar{\lambda} - \pi}{1 - \pi}, \frac{\dot{\lambda} - \pi}{1 - \pi})$, there exists a subgame equilibrium with recovery, in which all farmers exert effort with probability μ , where $\mu\lambda_{t'-1} + (1 - \mu)(1 - \pi)\lambda_{t'-1} + \mu\pi(1 -$

$\lambda_{t'-1}) = \bar{\lambda}$ and $\lambda_t \approx \bar{\lambda}$ for $t \geq t'$. Next, consider any $\lambda_0 < \frac{\bar{\lambda}-\pi}{1-\pi}$. Since:

$$\frac{\Delta L}{1-\beta} > \Delta L + \beta c = \underbrace{\Delta L - (1-\beta)c}_{>0} + c \geq c \quad (22)$$

effort is incentive compatible for any $\lambda_0 < \frac{\bar{\lambda}-\pi}{1-\pi}$ conditional on a sequence of λ_t that evolves according to $\lambda' = \lambda + (1-\lambda)\pi$ until it reaches $\bar{\lambda}$, after which it is followed by an infinite sequence of $\lambda_t \bar{\lambda}$. This establishes the existence of a recovery equilibrium for $\lambda_0 < \frac{\bar{\lambda}-\pi}{1-\pi}$. $\square$

A.10 Lemma 5

Proof. $\frac{\partial B(\lambda, \{q, \mathcal{I}\})}{\partial q} = -\frac{1-\lambda}{\lambda} < 0$. $\square$

A.11 Proposition 6

Proof. Consider the regulator's policy decision given λ . The regulator chooses some $\{q, \mathcal{I}\}$ to maximize:

$$W(\lambda, \{q, \mathcal{I}\}) = \lambda v_g(\lambda) + (1-\lambda)v_b(\lambda) - S(\lambda, \{q, \mathcal{I}\}) + \beta E[W(\lambda')] \quad (23)$$

First, suppose that $\lambda > \bar{\lambda}$. Taking the differential of the net output between a policy $\{q', \mathcal{I}'\}$ where $\lambda \in \mathcal{I}'$ and policy $\{q'', \mathcal{I}''\}$ with $\lambda \notin \mathcal{I}''$,

$$-\lambda B(\lambda, \{q', \mathcal{I}'\}) - (1-\lambda)q' + \lambda B(\lambda, \{q'', \mathcal{I}''\}) \quad (24)$$

$$= \lambda \left(\frac{1+r_f - (1-\lambda)L_b}{\lambda} - \frac{1+r_f - (1-\lambda)(L_b + q')}{\lambda} \right) - (1-\lambda)q' \quad (25)$$

$$= \lambda(1-\lambda)q' - (1-\lambda)q' < 0 \quad (26)$$

This implies that under no commitment, no $\lambda \notin \mathcal{I}$ for any $\lambda \geq \bar{\lambda}$. Next, consider $\lambda < \bar{\lambda}$. Following the same exercise, taking the differential of the net output between a policy $\{q', \mathcal{I}'\}$ where $\lambda \in \mathcal{I}'$ and q' sufficiently large such that $\lambda L_g + (1-\lambda)(L_b + q') \geq 1+r_f$, and

policy $\{q'', \mathcal{I}''\}$ with $\lambda \notin \mathcal{I}''$,

$$\underbrace{\lambda(p_g R_g + L_g - B(\lambda, \{q', \mathcal{I}'\})) + (1 - \lambda)(p_b R_b + L_b - \min\{B(\lambda, \{q', \mathcal{I}'\}), L_b\})}_{\text{farmers' one-period utility}} \quad (27)$$

$$+ \underbrace{\lambda(B(\lambda, \{q', \mathcal{I}'\}) - (1 + r_f)) + (1 - \lambda)(\min\{B(\lambda, \{q', \mathcal{I}'\}), L_b\} - (1 + r_f) + q')}_{\text{investors' net utility}} \quad (28)$$

$$- \underbrace{(1 - \lambda)q'}_{\text{intervention cost}} - \underbrace{[\lambda L_g + (1 - \lambda)L_b]}_{\text{output without intervention}} \quad (29)$$

$$= \lambda p_g R_g + (1 - \lambda)p_b R_b - (1 + r_f) \quad (30)$$

Let $\underline{\lambda} \equiv \frac{1+r_f-p_b R_b}{\Delta R}$. This implies that under no commitment, an intervention policy that restores market liquidity is desirable for $\lambda \in [\underline{\lambda}, \bar{\lambda})$. Finally, consider the optimal q conditional on $\mathcal{I} = [0, \bar{\lambda})$. Note that the minimum q such that $\lambda L_g + (1 - \lambda)(L_b + q') \geq 1 + r_f$ is given by

$$q^{\min} \equiv \frac{1 + r_f - \lambda L_g - (1 - \lambda)L_b}{1 - \lambda}. \quad (31)$$

For any $q' > q^{\min}$, the differential of the net output is given by

$$\begin{aligned} & -\lambda B(\lambda, \{q', \mathcal{I}\}) - (1 - \lambda)(q' - q^{\min}) + \lambda B(\lambda, \{q^{\min}, \mathcal{I}\}) \\ & = \lambda \underbrace{(B(\lambda, \{q^{\min}, \mathcal{I}\}) - B(\lambda, \{q', \mathcal{I}\}))}_{<0} - (1 - \lambda) \underbrace{(q' - q^{\min})}_{>0} \\ & < 0. \end{aligned}$$

Hence, the optimal intervention policy under no commitment is $\{q, \mathcal{I}\} = \{q^{\min}, [\underline{\lambda}, \bar{\lambda})\}$. $\square$

A.12 Lemma 6

Proof. Under minimal intervention, $q^{\min} = \frac{1+r_f-\lambda L_g-(1-\lambda)L_b}{1-\lambda}$. Hence, $B(\lambda, \{q^{\min}, \mathcal{I}\}) = \frac{1+r_f-(1-\lambda)(L_b+q^{\min})}{\lambda} = L_g$. This implies that $v_\theta(\lambda, \{q^{\min}, \mathcal{I}\}) = p_\theta R_\theta$. $\square$

A.13 Proposition 7

Proof. Consider the case when $\bar{\lambda} + (1 - \bar{\lambda})\pi > \dot{\lambda}$. Consider a candidate path $\{\lambda_t\}$ where $\lambda' = \lambda + (1 - \lambda)\pi$, i.e. full effort at every period. In period t' , a farmer's effort condition under minimal intervention is:

$$\Delta v_\theta(\max\{\lambda_{t'}, \bar{\lambda}\}) + \beta \Delta V_\theta(\max\{\lambda_{t'+1}\}) \geq c \quad (32)$$

From Proposition 3, we know that if $\bar{\lambda} + (1 - \bar{\lambda})\pi > \dot{\lambda}$, then the farmers' effort condition is satisfied for $\lambda_{t'+1} = \bar{\lambda}$, i.e.

$$\Delta v_\theta(\bar{\lambda}) + \beta \Delta V_\theta(\bar{\lambda} + (1 - \bar{\lambda})\pi) \geq c \quad (33)$$

This implies that if $c < \frac{\Delta R}{1-\beta}$, then for any $\kappa > 1$,

$$\sum_{t=1}^{\kappa} \beta^{t-1} \Delta v_\theta(\bar{\lambda}) + \beta^\kappa \Delta V_\theta(\bar{\lambda}) = \frac{1 - \beta^\kappa}{1 - \beta} \Delta R + \beta^\kappa \Delta V_\theta(\bar{\lambda}) \quad (34)$$

$$\geq \frac{1 - \beta^\kappa}{1 - \beta} \Delta R + \beta^\kappa c \quad (35)$$

$$\geq c. \quad (36)$$

which satisfies to Condition 32. Hence, there exists a full recovery equilibrium. More generally, we can write $\frac{\Delta R}{1-\beta}$ as:

$$\Delta R + \beta \Delta R + \dots \leq \sum_{t=1}^{\kappa} \beta^{t-1} \Delta v_\theta(\bar{\lambda}) + \beta^\kappa \Delta V_\theta(\bar{\lambda}) \quad (37)$$

Hence, as long as $c < \frac{\Delta R}{1-\beta}$, a full recovery exists.

Next, relax our assumption that $c < \frac{\Delta R}{1-\beta}$. For any $\lambda \geq \frac{\dot{\lambda} - \pi}{1 - \pi}$ (note that by definition $\dot{\lambda} \geq \bar{\lambda}$), under intervention and full effort, farmers' value function conditional on effort is given by:

$$v_\theta(\dot{\lambda}) + \beta V_\theta(\dot{\lambda} + (1 - \dot{\lambda})\pi) - \bar{c} \quad (38)$$

Consider a farmer who deviates to no effort. The farmer obtains:

$$(1 - \pi) [v_\theta(\dot{\lambda}) + \beta V_\theta(\dot{\lambda} + (1 - \dot{\lambda})\pi)] + \pi [v_b(\dot{\lambda}) + \beta V_\theta(\dot{\lambda} + (1 - \dot{\lambda})\pi)] \quad (39)$$

Deviating is not profitable since:

$$c \leq \Delta v(\dot{\lambda}) + \Delta V(\dot{\lambda} + (1 - \dot{\lambda})\pi) \quad (40)$$

Hence, a full recovery equilibrium exists. Finally, consider when $\dot{\lambda} = \bar{\lambda}$. We show that there exists a full recovery equilibrium for $\lambda_0 \geq \ddot{\lambda}$ for some threshold $\ddot{\lambda}$. If $\dot{\lambda} = \bar{\lambda}$, then:

$$\Delta R + \beta \Delta V(\bar{\lambda} + (1 - \bar{\lambda})\pi) \geq c$$

Since ΔV increases in λ for $\lambda \geq \bar{\lambda}$, this implies that there exists some $\ddot{\lambda} \leq \bar{\lambda}$ which is given by:

$$\Delta R + \beta \Delta V(\ddot{\lambda} + (1 - \ddot{\lambda})\pi) = c.$$

This establishes the existence of a recovery equilibrium when $\lambda_0 \geq \ddot{\lambda}$.

Consider the remaining case when $\bar{\lambda} + (1 - \bar{\lambda})\pi < \dot{\lambda}$. This implies that even under full effort, an individual farmer does not find it optimal to exert effort:

$$\Delta R + \beta \Delta V(\bar{\lambda} + (1 - \bar{\lambda})\pi) < c \quad (41)$$

Recall that $\frac{\Delta R}{1-\beta} < c$ if $\bar{\lambda} + (1 - \bar{\lambda})\pi < \dot{\lambda}$. Since $\Delta V(\bar{\lambda}) > \Delta R$, this shows that an equilibrium with full recovery does not exist. Note also that no effort is a best response to any $\lambda \in [\underline{\lambda}, \bar{\lambda}]$. Hence, given initial condition $\lambda_0 \in [\underline{\lambda}, \bar{\lambda})$, it follows that $\lambda_t < \bar{\lambda}$ for all subsequent t . Hence, we obtain an equilibrium of no recovery. $\square$

A.14 Proposition 8

Proof. Following Proposition 7, we know that a full recovery exists if (1) $c < \frac{\Delta R}{1-\beta}$ or (2) $\bar{\lambda} + (1 - \bar{\lambda})\pi \geq \dot{\lambda}$ and $\lambda_0 \geq \ddot{\lambda}$. Since full recovery implies that intervention is no longer

activated at some period $\tau > 0$, intervention occurs for finite periods to a steady state $\lambda^* = 1$.

Consider the remaining cases. From Proposition 7, we know that in all other cases, we obtain no recovery in equilibrium. Suppose that $c > \frac{\Delta L}{1-\beta}$. Since:

$$\frac{\max\{\Delta L, \Delta R\}}{1-\beta} < c \quad (42)$$

for any path with no recovery, effort is not incentive compatible for any farmer. Hence, in equilibrium, $\lambda^* = 0$. In addition, since λ_t monotonically decreases, there exists some τ for which $\lambda_\tau \leq \underline{\lambda}$, which implies that for any $t > \tau$, $\lambda_t \notin \mathcal{I}^{min}$.

Finally, consider when $c < \frac{\Delta L}{1-\beta}$. We show by contradiction that the no recovery equilibrium does not have steady state $\lambda^* = 0$. Suppose that is the case, and $\lambda_{t'-1} = 0 < \underline{\lambda}$. Under a steady state, $\lambda_{t'} = 0$. However, conditional on $\lambda_t = 0$ for all $t > t' - 1$, a farmer chooses $e = 1$ if:

$$\Delta L + \beta \Delta V(0) \geq c \quad (43)$$

Since $V(0) = \frac{\Delta L}{1-\beta}$ for the proposed equilibrium path, and $c < \frac{\Delta L}{1-\beta}$, farmers find it optimal to exert effort. However, this implies that $\lambda_t > 0$. We conjecture and verify that instead there exists an approximate steady state $\lambda^* = \underline{\lambda}$. Let $\hat{\lambda} = \underline{\lambda}$ given by

$$\hat{\lambda} = \begin{cases} \underline{\lambda} & \text{with probability } \frac{\Delta L - (1-\beta)c}{\Delta L - \Delta R} \\ \underline{\lambda} - \epsilon & \text{with probability } \frac{(1-\beta)c - \Delta R}{\Delta L - \Delta R} \end{cases} \quad (44)$$

Consider when $\lambda = \hat{\lambda}$. The farmers' effort condition is given by

$$\frac{\Delta L - (1-\beta)c}{\Delta L - \Delta R} \Delta R + \frac{(1-\beta)c - \Delta R}{\Delta L - \Delta R} \Delta L = (1-\beta)c \quad (45)$$

This implies that

$$E[\Delta v(\hat{\lambda}) + \beta \Delta V(\hat{\lambda})] = c$$

Note, farmers are indifferent between $e = 1$ and $e = 0$. A symmetric mixing strategy $\mu = \frac{1}{2}$ is consistent with a equilibrium approximate steady state with $\lambda^* = \underline{\lambda}$. Following the argument in Proposition 2, it can be shown than $\underline{\lambda}$ is the unique approximate steady state. $\square$

A.15 Proposition 9

Proof. Under minimal intervention, full recovery implies that all farmers exerted effort for all $t > 0$. Hence, asset quality improves at least as fast as laissez-faire. In addition, for any $\lambda_0 < \frac{\bar{\lambda} - \pi}{1 - \pi}$, markets are liquid only under intervention, which implies that output is strictly greater (by $\lambda_1 p_g R_g + (1 - \lambda_1) p_b R_b$), while for any λ_t , weakly greater quality implies that output is at least as large as in the case of laissez-faire.

Recall, in the cases for which minimal intervention results in no recovery ($c > \frac{\Delta R}{1 - \beta}$ and $\lambda_0 < \ddot{\lambda}$), the corresponding laissez-faire equilibrium is given by:

- (i) if $\ddot{\lambda} < \ddot{\lambda}$ and $\bar{\lambda} + (1 - \bar{\lambda})\pi > \dot{\lambda}$, then for $\lambda_0 \in [\ddot{\lambda}, \ddot{\lambda}]$, a full recovery equilibrium;
- (ii) Otherwise, if $c < \frac{\Delta L}{1 - \beta}$, a partial recovery equilibrium with $\lambda^* = \bar{\lambda}$;
- (iii) or, if $c > \frac{\Delta L}{1 - \beta}$, no recovery equilibrium with $\lambda^* = 0$.

In case (i), asset quality is greater under laissez-faire since it undergoes full recovery. In case (ii), since $\bar{\lambda} > \underline{\lambda}$, asset quality is greater under laissez-faire. In case (iii), asset quality is identical. Together this confirms that asset quality is (weakly) greater under laissez-faire. $\square$

A.16 Proposition 10

Proof. Consider the regulator's policy decision given some initial value $\lambda_{t'-1}$. The regulator chooses some $\{q, \mathcal{I}\}$ to maximize the net output, taking into consideration the impact that intervention has on $\lambda_t(\lambda_{t'-1}, \{q, \mathcal{I}\})$, the quality after farmers' effort decision.

We conjecture and verify that the optimal policy is $\{q^c, \mathcal{I}^c\}$ is given by:

1. if (1) $c < \frac{\Delta R}{1 - \beta}$ or (2) $\bar{\lambda} + (1 - \bar{\lambda})\pi \geq \dot{\lambda}$ and $\lambda \geq \ddot{\lambda}$, then $\{q^c, \mathcal{I}^c\} = \{q^{min}, \mathcal{I}^{min}\}$

2. otherwise, $\{q^c, \mathcal{I}^c\}$ is such that q^c sets $B(\lambda, \{q^c, \mathcal{I}^c\}) = \Delta R + L_g - (1 - \beta)c$ and $\mathcal{I}^c = [\underline{\lambda}, \dot{\lambda})$.

First, note that setting when full recovery is possible and efficient given Assumption 5 under the minimal intervention, the regulator commits to minimal intervention, which is the lowest cost intervention that induces full recovery. Hence, it suffices to check that when conditions for case 1. are not met, the specified $\{q^c, \mathcal{I}^c\}$ is optimal.

Under case 2., minimal intervention, q^c sets $B(\lambda, \{q^c, \mathcal{I}^c\}) = \Delta R + L_g - (1 - \beta)c$ if $B(\lambda) > \Delta R + L_g - (1 - \beta)c$. We can explicitly express q^c when $q^c \neq q^{min}$ as

$$q^c = \frac{1 + r_f - (1 - \lambda)L_b - \lambda(\Delta R + L_g - (1 - \beta)c)}{1 - \lambda} \quad (46)$$

First, we show that for any policy, it is not optimal to have $\lambda \in \mathcal{I}$ for any $\lambda \in [\dot{\lambda}, 1]$. Consider a farmers' effort decision for some $\lambda_{t'-1}$. If $\lambda_{t'-1} \geq \frac{\dot{\lambda} - \pi}{1 - \pi}$, as a consequence of Lemma 4, all farmers exert effort even without intervention. From Lemma 1 we can infer that intervention weakly improves incentives in $\lambda_{t'}$ for $\lambda_{t'} > \bar{\lambda}$. Given a policy $\{q', \mathcal{I}'\}$ where $\lambda \in \mathcal{I}'$ and policy $\{q'', \mathcal{I}''\}$ with $\lambda \notin \mathcal{I}''$, this implies that $\lambda_{t'}(\lambda_{t'-1}, \{q', \mathcal{I}'\}) = \lambda_{t'}(\lambda_{t'-1}, \{q'', \mathcal{I}''\})$ since maximal effort is exerted by farmers with or without intervention. Hence, $\lambda \notin \mathcal{I}$ for any $\lambda > \dot{\lambda}$.

Next, we show that under $\{q^c, \mathcal{I}^c\}$, gains from effort conditional on a full recovery path (i.e. full effort by all farmers for all subsequent t) is bounded below by:

$$\frac{\Delta R + L_g - B(\lambda, \{q^c, \mathcal{I}^c\})}{1 - \beta} = c \quad (47)$$

Since effort is incentive compatible, $\{q^c, \mathcal{I}^c\}$ achieves a full recovery equilibrium for any λ_0 . Since q^c is set such that agents' incentive conditions are binding conditional on intervention, q^c is the minimum cost subsidy conditional on restoring liquidity *and* restoring incentives.

Given this, consider the net output for $\lambda_{t'-1} < \frac{\lambda - \pi}{1 - \pi}$ under $\{q^c, \mathcal{I}^c\}$.

$$\underbrace{\lambda_{t'}(p_g R_g + L_g - B(\lambda_{t'}, \{q^c, \mathcal{I}^c\})) + (1 - \lambda_{t'})(p_b R_b + L_b - \min\{B(\lambda_{t'}, \{q^c, \mathcal{I}^c\}), L_b\}) - \bar{c}}_{\text{farmers' one-period utility}} \quad (48)$$

$$+ \underbrace{\lambda_{t'}(B(\lambda_{t'}, \{q^c, \mathcal{I}^c\}) - (1 + r_f)) + (1 - \lambda_{t'})(\min\{B(\lambda_{t'}, \{q^c, \mathcal{I}^c\}), L_b\} - (1 + r_f) + q^c)}_{\text{investors' net utility}} \quad (49)$$

$$+ \beta \underbrace{E[\lambda_{t'+1} V_g(\lambda_{t'+1}, \{q^c, \mathcal{I}^c\}) + (1 - \lambda_{t'+1}) V_b(\lambda_{t'+1}, \{q^c, \mathcal{I}^c\}) - S(\lambda_{t'+1}, \{q^c, \mathcal{I}^c\})]}_{\text{future expected output} = W(\lambda_{t'+1}, \{q^c, \mathcal{I}^c\})} \quad (50)$$

$$- \underbrace{(1 - \lambda_{t'}) q^c}_{\text{intervention cost}} \quad (51)$$

$$= \lambda_{t'}(p_g R_g + L_g) + (1 - \lambda_{t'})(p_b R_b + L_b) - \bar{c} - (1 + r_f) + \beta W(\lambda_{t'+1}, \{q^c, \mathcal{I}^c\}) \quad (52)$$

Consider an alternative policy $\{q', \mathcal{I}^c\}$. Since q^c is the minimum cost subsidy conditional on restoring liquidity *and* restoring incentives, it suffices to check for policies with $q' < q^c$. First, suppose that given $\lambda_{t'-1}$, $q^c > 0$. This implies that for any $q' < q^c$, farmers do not exert effort, i.e. $\lambda_{t'} = (1 - \pi)$. Let $\lambda'_{t'+1}$ be used for shorthand the quality in period $t' + 1$ under $\{q', \mathcal{I}^c\}$. Then net output if q' restores liquidity (but not incentives) is:

$$\underbrace{(1 - \pi) \lambda_{t'-1} (p_g R_g + L_g - B((1 - \pi) \lambda_{t'-1}, \{q', \mathcal{I}^c\})) + (1 - (1 - \pi) \lambda_{t'-1}) (p_b R_b + L_b - \min\{B((1 - \pi) \lambda_{t'-1}, \{q', \mathcal{I}^c\}), L_b\})}_{\text{farmers' one-period utility}} \quad (53)$$

$$+ \underbrace{(1 - \pi) \lambda_{t'-1} (B((1 - \pi) \lambda_{t'-1}, \{q', \mathcal{I}^c\}) - (1 + r_f)) + (1 - (1 - \pi) \lambda_{t'-1}) (\min\{B((1 - \pi) \lambda_{t'-1}, \{q', \mathcal{I}^c\}), L_b\} - (1 + r_f) + q')}_{\text{investors' net utility}} \quad (54)$$

$$+ \beta \underbrace{E[\lambda'_{t'+1} V_g(\lambda'_{t'+1}, \{q^c, \mathcal{I}^c\}) + (1 - \lambda'_{t'+1}) V_b(\lambda'_{t'+1}, \{q^c, \mathcal{I}^c\}) - S(\lambda'_{t'+1}, \{q^c, \mathcal{I}^c\})]}_{\text{future expected output} = W(\lambda'_{t'+1}, \{q', \mathcal{I}^c\})} \quad (55)$$

$$- \underbrace{(1 - (1 - \pi) \lambda_{t'-1}) q'}_{\text{intervention cost}} \quad (56)$$

$$= (1 - \pi) \lambda_{t'-1} (p_g R_g + L_g) + (1 - (1 - \pi) \lambda_{t'-1}) (p_b R_b + L_b) - (1 + r_f) + \beta W(\lambda'_{t'+1}, \{q', \mathcal{I}^c\}) \quad (57)$$

Taking the difference, we obtain:

$$\pi(\Delta R + \Delta L) - \bar{c} + \beta [W(\lambda_{t'+1}, \{q^c, \mathcal{I}^c\}) - W(\lambda'_{t'+1}, \{q', \mathcal{I}^c\})] \quad (58)$$

$$\geq \pi(\Delta R + \Delta L - c) + \underbrace{\beta [W(\lambda_{t'+1}, \{q^c, \mathcal{I}^c\}) - W(\lambda'_{t'+1}, \{q', \mathcal{I}^c\})]}_{>0} \quad (59)$$

which is strictly greater than zero under Assumption 4.

Finally, we show that conditional on a full recovery equilibrium being optimal given some initial value $\lambda_0 \geq \underline{\lambda}$, c-intervention, relative to laissez-faire is desirable.

Consider the net output under c-intervention:

$$\lambda_{t'}(p_g R_g + L_g) + (1 - \lambda_{t'})(p_b R_b + L_b) - \bar{c} - (1 + r_f) + \beta W(\lambda_{t'+1}, \{q^c, \mathcal{I}^c\}) \quad (60)$$

$$= \underbrace{\lambda_{t'}(p_g R_g - (1 + r_f)) + (1 - \lambda_{t'})(p_b R_g - (1 + r_f))}_{>0 \text{ for } \lambda_{t'} \geq \underline{\lambda}} + \beta W(\lambda_{t'+1}, \{q^c, \mathcal{I}^c\}) \quad (61)$$

$$+ \lambda_{t'} L_g + (1 - \lambda_{t'}) L_b - \bar{c} \quad (62)$$

Suppose that under laissez-faire, effort is exerted. This implies that under laissez-faire, we get:

$$\lambda_{t'} L_g + (1 - \lambda_{t'}) L_b - \bar{c} + \beta E[\lambda_{t'} V_g(\lambda_{t'+1}) + (1 - \lambda_{t'}) V_b(\lambda_{t'+1})] \quad (63)$$

Taking the difference:

$$\underbrace{\lambda_{t'}(p_g R_g - (1 + r_f)) + (1 - \lambda_{t'})(p_b R_g - (1 + r_f))}_{>0 \text{ for } \lambda_{t'} \geq \underline{\lambda}} + \underbrace{\beta [W(\lambda_{t'+1}, \{q^c, \mathcal{I}^c\}) - E[\lambda_{t'} V_g(\lambda_{t'+1}) + (1 - \lambda_{t'}) V_b(\lambda_{t'+1})]]}_{\geq 0} \geq 0$$

where the future expected difference is positive since the one-period net output difference (weakly) increases for consecutive periods. Assumption 5 directly implies that output under the c-intervention is greater than that under laissez-faire without full recovery for $\lambda \leq \underline{\lambda}$. $\square$

A.17 Proposition 11

Proof. Under c-intervention, all farmers exert effort for $t > 0$. Since there exists finite κ for which $F(\lambda_0, \kappa) \geq \dot{\lambda}$, c-intervention occurs for finite periods and $\lambda_t \rightarrow 1 = \lambda^*$. $\square$

A.18 Proposition 12

Proof. Follows from Proposition 10. $\square$

A.19 Proposition 13

Proof. We show that for any $\{q, \mathcal{I}\}$, there exists a policy $\{s, \tau, \mathcal{I}, K\}$ that is equivalent. Fix policy $\{q, \mathcal{I}\}$, and WLOG, suppose that q is such that markets are liquid whenever $\lambda \in \mathcal{I}$. Recall that $B(\lambda, \{q, \mathcal{I}\}) = \frac{1+r_f-(1-\lambda)(L_b+q)}{\lambda}$.

Consider $\{s', \tau', \mathcal{I}', K'\}$ where $s' = 1 + r_f$, $\tau' = \frac{1+r_f-(1-\lambda)(L_b+q)}{\lambda}$, $\mathcal{I}' = \mathcal{I}$, and K' be such that:

$$\frac{(1 + r_f) - (1 - \lambda) L_b - K'(1 + r_f - L_b)}{\lambda} = \frac{1 + r_f - (1 - \lambda)(L_b + q)}{\lambda} \quad (64)$$

i.e. $K' = \frac{(1-\lambda)q}{1+r_f-L_b}$. Conditional on K' bad farmers participating in this program (and the rest enter private markets), a bad farmer's one-period expected payoff conditional on participation is:

$$p_b R_b + s' - (1 + r_f) + \max\{L_b - \tau', 0\} = p_b R_b \quad (65)$$

Hence, bad farmers are indifferent between participation and non-participation under policy $\{s', \tau', \mathcal{I}', K'\}$. If K' bad farmers participate in the market, this implies that no good farmers opt into the program. Since a good farmer's expected payout conditional on participation is:

$$p_g R_g + s' - (1 + r_f) + \max\{L_g - \tau', 0\} = p_g R_g + L_g - B(\lambda, \{q, \mathcal{I}\}), \quad (66)$$

good farmers are indifferent between participation and non-participation under policy $\{s', \tau', \mathcal{I}', K'\}$. Policy $\{s', \tau', \mathcal{I}', K'\}$ replicates farmers' payoffs under policy $\{q, \mathcal{I}\}$. In addition, cost of policy $\{s', \tau', \mathcal{I}', K'\}$ is:

$$K'(s' - L_b) = \frac{(1 - \lambda)q}{1 + r_f - L_b}(s' - L_b) = (1 - \lambda)q$$

which is equivalent to the that of policy $\{q, \mathcal{I}\}$. □

A.20 Proposition 14

Proof. We show that for any $\{q, \mathcal{I}\}$, there exists a policy $\{r'_f, \mathcal{I}'\}$ that is equivalent. Fix policy $\{q, \mathcal{I}\}$, and WLOG, suppose that q is such that markets are liquid whenever $\lambda \in \mathcal{I}$. Recall that $B(\lambda, \{q, \mathcal{I}\}) = \frac{1 + r_f - (1 - \lambda)(L_b + q)}{\lambda}$.

Set $\mathcal{I}' = \mathcal{I}$ and consider a policy r'_f where:

$$\frac{1 + r'_f - (1 - \lambda)L_b}{\lambda} = \frac{1 + r_f - (1 - \lambda)(L_b + q)}{\lambda} \tag{67}$$

Rearranging, we get $r'_f = r_f - (1 - \lambda)q$. Since lowering the riskfree rate to r'_f leads to a cost of capital B that is equivalent to that induced by $\{q, \mathcal{I}\}$, policy $\{r'_f, \mathcal{I}'\}$ is outcome equivalent. □

B Supplementary Figures

All figures are simulations of the model with $\beta = 0$ and $L_b R_b \geq 1 + r_f + \bar{c}$, which implies that $\underline{\lambda} = 0$.

B.1 Minimal Intervention

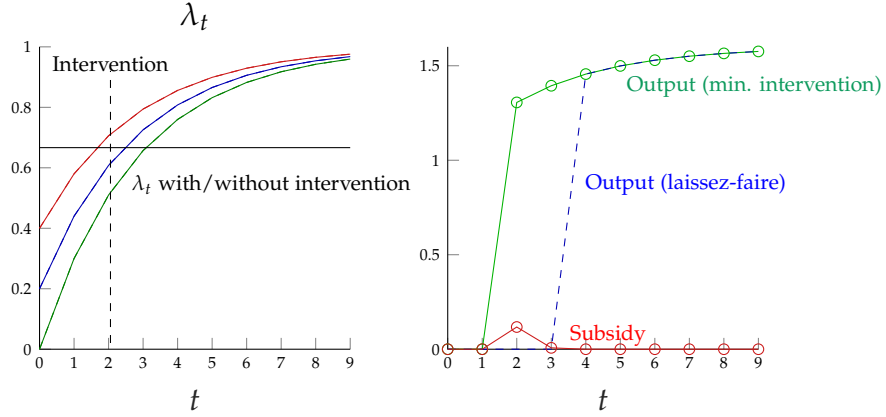


Figure 3: Market recovery under minimal intervention.

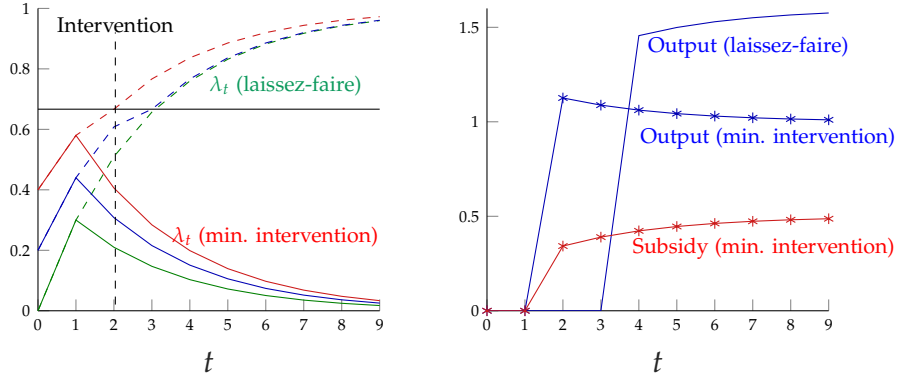


Figure 4: Market deterioration under minimal intervention.

B.2 c-Intervention

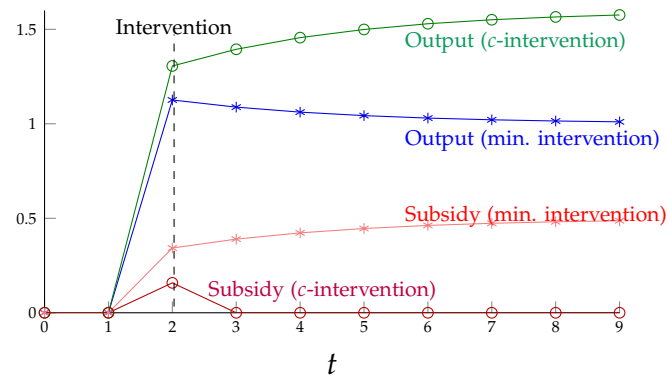


Figure 5: Output and subsidy comparison between minimal and c-intervention.

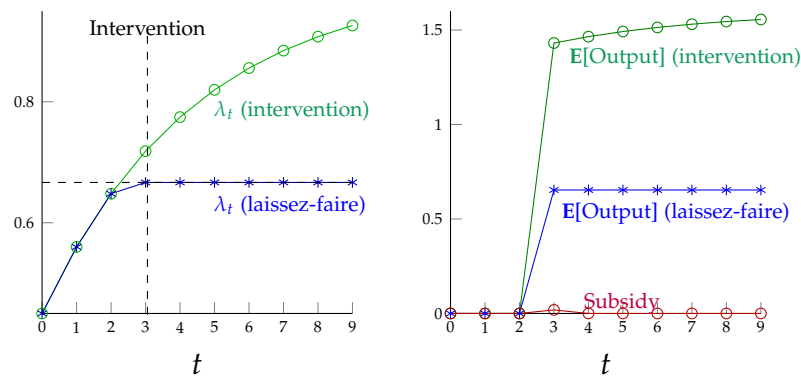


Figure 6: Delaying intervention until $t = 3$.