Heterogenous Information Choice in General Equilibrium*

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Abstract

We study the incentives of heterogeneous households to acquire information about the state of the economy. In the standard Krusell and Smith (1998) environment, where households differ in their labor productivity and asset holdings, we find that: i) when agents use current productivity and aggregate capital to condition expectations (the Krusell and Smith (1998) benchmark), expected utility losses from forming expectations equal to unconditional means correspond to less than 0.05 percent of lifetime consumption. In other words, the Krusell and Smith (1998) information structure is an equilibrium only if information is essentially free. ii) when all agents decide to not acquire information beyond the unconditional mean of the capital distribution, the capital stock is significantly more volatile. This increases the loss of not using the current capital stock to forecast future variables to between 0.5 and 3.5 percent of lifetime consumption, depending on the level of capital. iii) The individual benefits of information acquisition under ii) strongly depend on an individual's position in the wealth distribution and the aggregate capital stock.

Keywords: Heterogenous information, rational expectations, bounded rationality

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1 Introduction

Rational expectations equilibria, the benchmark of modern macroeconomics, require agents to correctly perceive the stochastic transition laws that govern the dynamics of an economy. With many agents and endogenous heterogeneity, this requires substantial information about the distribution of individual state variables and individual decision rules. The resulting curse of dimensionality that inhibits solving for such an equilibrium has often led researchers to specify perceived equilibrium dynamics in heterogeneous agent economies in terms of a finite number of moments (typically simply the mean) of the cross-sectional distribution of individual state variables. The motivation for this is that higher moments of the distribution in any case do not help to predict variables of interest much in a statistical sense. So the inaccuracy of such "boundedly rational" expectations that neglect higher moments should imply only small costs.

We quantify the utility loss implied by different degrees of "bounded rationality", or the use of different information sets, in the standard Krusell and Smith (1998) (henceforth KS) environment. We find that the benefits of acquiring information strongly depend on the information that other households use to form expectations. When all agents use a simple productivity-dependent autoregression to forecast future wages and returns (the Krusell and Smith (1998) benchmark), an individual household may neglect all moments but the unconditional mean of the capital distribution at low utility cost (equivalent to less than 0.05 percent of lifetime consumption). When households only use the unconditional mean of the capital distribution to forecast future wages and returns, the aggregate capital stock becomes substantially more persistent, and thus fluctuates more strongly in equilibrium. This makes the utility loss for an individual household of ignoring the current capital stock in her forecast of future variables sizeable (up to 3.5 percent of lifetime consumption). The benefits of conditioning forecasts on the current state of aggregate productivity, or the variance of the capital distribution, are always small, independently of the law of motion that households use in their forecasts in equilibrium.

These results have two main implications. First, informational equilibria in economies with heterogeneous agents might be sensitive to the cost of information acquisition. Only when information comes at negligible costs is the KS benchmark an equilibrium. When such costs are substantial, in contrast, agents may prefer to form expectations equivalent to the unconditional mean of wages and returns. At intermediate costs, no symmetric equilibrium may exist. Second, information choice is likely to be heterogeneous. This is, first, because the costs of information acquisition may differ across households, and, second, because we find that the utility loss of using less information to form expectations differs strongly between households with different labor market status and wealth.

1.1 Relation to the literature

[To be added.]

2 The model

2.1 The general environment

The economic environment is a version of the KS economy with unemployment benefits, as studied in Den Haan et al. (2010) (and other articles in the same issue). Specifically, the economy consists of a continuum of ex ante identical households of unit mass. Households experience labor market shocks ϵ that make them transit from employment ($\epsilon = 1$) to unemployment ($\epsilon = 0$). When employed, a household earns wage w_t, when unemployed she receives unemployment benefits μw_t. The only asset in the economy is physical capital, whose net return equals $r_t - \delta$, the rental rate net of depreciation, and is equal for all households. Financial markets are thus incomplete and households can smooth consumption only trough their choice of capital k^i_{t+1} by saving and dissaving, subject to a no-borrowing limit ($k^i_t > 0$). The household thus solves the problem

$$\max_{\{c^i_t, k^i_{t+1}\}^\infty_{t=0}} E_{\Omega_{i,t}} \sum_{t=0}^{\infty} \beta^t \frac{(c^i_t)^{1-\gamma} - 1}{1-\gamma} \tag{1}$$

$$s.t.\ \ c^i_t + k^i_{t+1} = r_t k^i_t + (1-\tau_t)\bar{l}\epsilon^i_t + \mu(1-\epsilon^1_t) w_t + (1-\delta) k^i_t \tag{2}$$

$$k^i_{t+1} \geq 0 \tag{3}$$

where $E_{\Omega_{i,t}}$ is the mathematical expectation conditional on information set $\Omega_{i,t}$ of household i in period t, c^i_t is household i's consumption, $\bar{l}$ her time endowment and τ a proportional tax on labor income.

Competitive firms rent capital and hire labor to produce the single output good in the economy using a standard Cobb-Douglas production technology

$$Y_t = z_t K_t^\alpha (\bar{l} L_t)^{1-\alpha} \tag{4}$$

Markets for labor, capital, and consumption goods are competitive, so factor prices are given by

$$w_t = z_t(1-\alpha)\left(\frac{K_t}{\bar{l}L_t}\right)^\alpha \tag{5}$$

$$r_t = z_t \alpha \left(\frac{K_t}{\bar{l}L_t}\right)^{\alpha-1} \tag{6}$$

where z_t is an exogenous process for aggregate productivity that takes on two values $1 - \Delta_z, 1 + \Delta_z$.

The only role of the government in this economy is to run a balanced-budget unemployment insurance scheme, such that

$$\tau_t = \frac{\mu u_t}{\bar{l}L_t} \tag{7}$$

where L_t and $u_t = 1 - L_t$ are, respectively, the employment and unemployment rates in the economy.

There are two exogenous sources of uncertainty in the economy: aggregate productivity z_t and individual employment status ϵ_t^i. Both follow a joint markov process, chosen such that the unemployment rate is a function only of z_t, and thus only takes on two values u_b, u_g with $u_b > u_g$.

2.2 Information-conditional General Equilibrium

In this section, we define a recursive competitive general equilibrium conditional on household information sets. Household decision rules and value function take as arguments their individual state variables k_t^i, ϵ_t^i, and the current state of the aggregate economy. Note that household decisions and cash-on-hand depend on aggregate capital through the wage rate and the return on investment, but are not directly affected by the distribution of capital. Given the assumption of incomplete markets, however, decision rules on individual capital holdings do not aggregate to a transition law for aggregate capital. In order to forecast future wages and returns accurately, households thus have to take into account the joint distribution of capital and employment status in the economy, which we denote as Γ. To form expectations further into the future, moreover, households use a law of motion $\Gamma' = H(\Gamma, z, z')$, where H allows for dependence of Γ' on the realization of productivity z' through wages and returns.

The dynamic program of a typical household i is therefore

$$V(k, \epsilon; \Gamma, z) = \max_{c,k'} E_{\Omega} \frac{c^{1-\gamma} - 1}{1-\gamma} \beta V(k', \epsilon'; \Gamma', z') \tag{8}$$

$$s.t. \quad c + k' = r_t k + (1-\tau)\bar{l}\epsilon + \mu(1-\epsilon^1)w + (1-\delta)k \tag{9}$$

$$k' \geq 0 \tag{10}$$

The joint distribution Γ is a high-dimensional object. To avoid the curse of dimensionality in their solution based on numerical dynamic programming, KS assume households to be *boundedly rational*, and perceive current and future prices to only depend on the first I moments of the joint distribution. They then use an iterative procedure to derive an approximate equilibrium. We assume a similar form of bounded rationality by conditioning the expectation operator in (8) on an arbitrary set of variables Ω that describes the state of the economy, and by assuming that households perceive Ω to follow a law of motion G. For now, we assume Ω to be the same across households and constant through time. Later, we will allow it to differ across households and time. Trivially, whenever Ω includes the current aggregate productivity state Ω and the first I moments of the distribution of capital, our approach is identical to that in KS.

To calculate an equilibrium, we choose an iterative procedure similar to that in KS: i) Choose Ω. ii) Postulate a law of motion G. iii) solve the consumer's problem conditional on Ω and G. iv) Using the resulting decision rules, simulate a large number of households for a large number of periods. v) From this simulation, calculate time series for the conditioning variables Ω, and estimate a new law of motion G'. vi) Compare this to G used in ii). If the two are different, update the guess for G and start again; if the two are sufficiently similar, stop.

Our approach differs from that in KS only insofar Ω may or may not include the aggregate productivity state or the mean of the capital distribution (in other words, we also consider $I = 0$), and that we do not necessarily require the resulting law of motion G to be an

Table I: Benchmark parameters

	β	γ	α	δ	$\bar{l}$	μ	Δ_z
Values	0.99	1	0.36	0.025	1/0.9	0.15	0.01

Table II: Transition probabilities

	$1-\Delta_z$,0	$1-\Delta_z$,1	$1+\Delta_z$,0	$1+\Delta_z$,1
$1-\Delta_z$,0	0.525	0.35	0.03125	0.09375
$1-\Delta_z$,1	0.038889	0.836111	0.002083	0.122917
$1+\Delta_z$,0	0.09375	0.03125	0.291667	0.583333
$1+\Delta_z$,1	0.009115	0.115885	0.024306	0.850694

accurate description of the dynamics of Ω. Hence the label "information-conditional general equilibrium". After we have compared the properties of general equilibria conditional on Ω in the next section, we will consider several ways of determining equilibrium sets Ω chosen by households that may differ across individuals and time.

2.3 Parameter Choice

As our benchmark we use exactly the same parameters as in Den Haan et al. (2010), given in table I and II.

3 The utility benefit of information

We are interested in the individual incentives to acquire information in KS-type economies. For this, we first solve for an equilibrium where all households use an information set $\widetilde{\Omega}$. We then calculate the costs of "deviations", where an individual uses an information set Ω that differs from the information set $\widetilde{\Omega}$ used by all other households. To do this, we estimate laws of motion $G(\Omega)$ for the endogenous elements of this alternative information set Ω using the panel of individual variables generated in step iv) of the last iteration round of our algorithm. We then calculate decision rules for an individual who uses Ω to forecast the future. Finally, we calculate the expected utility from using those decision rules, evaluating expectations using a most comprehensive set Ω_{max} and its associated law of motion that are likely to allow accurate forecasts of the future. Finally, we calculate a relative utility loss of using Ω rather than the most comprehensive Ω_{max}. Evidently, the resulting cost measure for deviations is conditional on the equilibrium information set $\widetilde{\Omega}$, which may or may not coincide with Ω_{max}. In practice, we use the following formular:

$$CE_{\Omega}(k,\epsilon;\Omega_{max}) = (1-\beta)\left[(1-\gamma)\left(V_{\Omega}(k,\epsilon;\Omega_{max}) - V_{\Omega_{max}}(k,\epsilon;\Omega_{max})\right)+1\right]^{\frac{1}{1-\gamma}} \tag{11}$$

where $\Omega \subset \Omega_{max}$, $V_{\Omega}(k,\epsilon;\Omega_{max})$ equals the expected discounted utility households with capital k and labor market status ϵ expect when they use the information set Ω and the aggregate state of the economy is described by particular values of the elements in Ω_{max}. Note that all terms in (11) also depend on the equilibrium law of motion $\widetilde{\Omega}$, suppressed for simplicity. The utility cost of using only a subset of the potential information about the state of the economy Ω is thus the expected utility loss relative to a comprehensive set Ω_{max}, tansformed into units of permanent consumption. Note that the incentives to acquire information may depend on both the individual state variables k, ϵ and the aggregate state of the economy, as given by specific values of the state variables (or their summary measures) in the comprehensive information set Ω_{max}. Moreover, as we will see, incentives also differ between the information sets $\widetilde{\Omega}$ on which the equilibrium is conditioned.

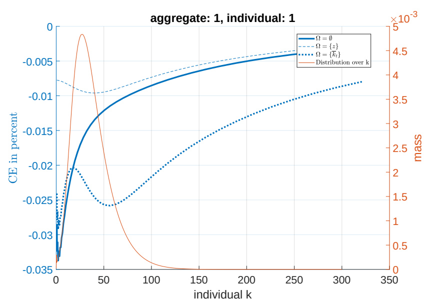
3.1 Incentives for information acquisition in the KS economy

This section presents results for the benchmark economy in KS. In other words, we consider $\widetilde{\Omega} = \{z, \overline{k}_t\}$, such that households' information sets contain aggregate productivity and the conditional mean of the capital distribution, and G takes the form of two simple autoregressions of $\overline{k'}$ on $\overline{k}$ conditional on z.

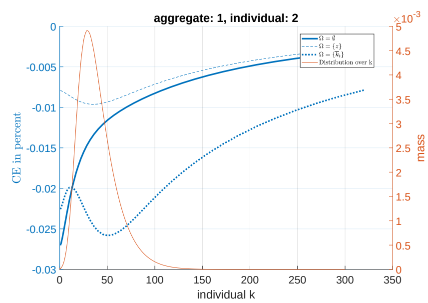
3.2 Incentives for foregoing information

We first look at the utility losses households suffer from having less information than contained in the KS $\{z, \overline{k}_t\}$. For this, we consider three different specifications of the alternative information set Ω: first, the empty set $\Omega = \emptyset$, implying that agents do not condition their expectations on anything, and therefore use only average transitions and the unconditional mean of capital in their forecasts for the future. Second, $\Omega = \{z_t\}$, implying that households can condition their expectations on the current aggregate productivity state, and use productivity-specific unconditional means of k' in their forecast for the aggregate capital stock. And finally, $\Omega = \{\overline{k}_t\}$, such that agents know the current mean of the capital distribution but not the realisation of productivity z. This implies that they forecast the mean of capital in the next period using an unconditional AR(1) process, and use transition probabilities that do not condition on z to form expectations.

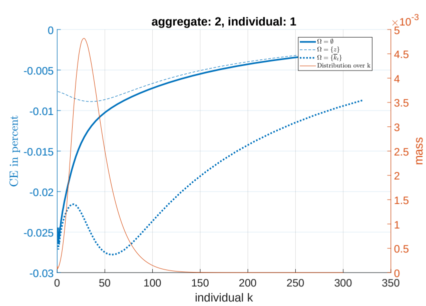
Figure 1 presents the utility cost of using less information in the KS economy, where $\widetilde{\Omega} = \Omega_{max} = \{z, \overline{k}\}$, for different levels of individual wealth along the bottom axis, and the four different combinations of the aggregate productivity and individual employment states in different panels, evaluated at the median of the distribution of aggregate capital. As a reference, the red lines indicate the average cross-sectional distributions of individual capital in periods where the aggregate capital stock is within 1 percentage point of the median, conditional on the aggregate productivity state.



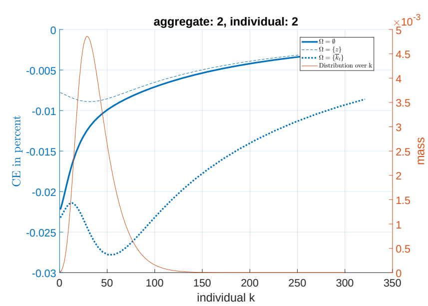
(a) $z = 1 - \Delta_z, \epsilon = 0$



(b) $z = 1 - \Delta_z, \epsilon = 1$



(c) $z = 1 + \Delta_z, \epsilon = 0$



(d) $z = 1 + \Delta_z, \epsilon = 1$

Figure 1: The figure presents $CE_{\Omega}(k, \epsilon; \Omega_{max})$, where $\Omega_{max} = \{z, k\}$ is the information set of KS, and Ω takes three specifications: first, the period-specific cross-sectional mean of k unconditional on z (implying $\overline{k}$ is forecast using an unconditional autoregression for k, solid lines), z only (implying forecasts equal to the mean of k across simulations, conditional on aggregate productivity, dashed lines), and the empty set (implying forecasts of aggregate capital equal its unconditional mean, dotted line). All losses are evaluated at the median of the distribution of aggregate capital conditional on productivity.

Figure 1 shows that there is substantial heterogeneity in the utility loss from using less information in forecasting the future, but that the magnitude of losses is small for all values of the aggregate and individual states, and all specifications of Ω.

[Results to be discussed further.]

3.3 Incentives for adding information

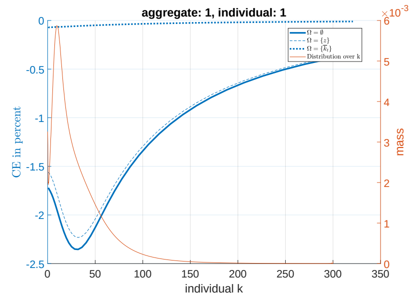
Do households have incentives to acquire more information in the KS benchmark economy? KS show that the gains from adding additional moments to their benchmark improves the in-sample fit of the predictive regression for the capital stock in the future by only a small amount. They also argue that the welfare gains are "vanishingly small" (p. 878). We confirm this result: The maximum welfare gains from increasing the information set to also contain the variance, i.e. $\Omega = \{z_t, \overline{k}_t, var(k)_t\}$, are less than one thousandth of a percent of permanent consumption (where the maximum is taken over all individual and aggregate states).

3.4 Incentives for information acquisition in a low-information version of the economy

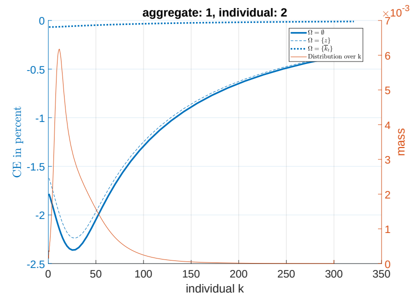
The last section showed that losses from suboptimal expectations are small in the KS economy. Does this mean agents should be expected to use simpler rules in equilibrium? Not necessarily. Figure 2 presents the same information as in 1 when the equilbrium information set is empty, so $\widetilde{\Omega} = \emptyset$ and agents only use unconditional expectations in their forecast of future variables, and when we evaluate losses at the lower 10th percentile of the distribution of aggregate capital. The losses from using simpler rules that do not contain the current mean of the capital distribution are two orders of magnitudes larger than those contained in Figure 1. Figure 3 shows the reason for this: the dynamics of the aggregate capital stock strongly depend on the information set that the equilibrium conditions on. Specifically, when agents only use unconditional means to forecast the future, the aggregate capital stock is substantially more persistent than in the KS benchmark. This makes the distribution of the capital stock substantially more dispersed around its mean, and thus increases the errors from using the unconditional mean as an approximation for tomorrow's capital stock.

3.5 Summary measures of utility losses

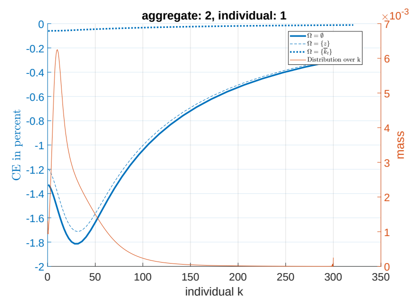
This section summarizes the utility losses from using less than the full information set Ω_{max} using two measures: average and maximum expected losses, where both the average and max-



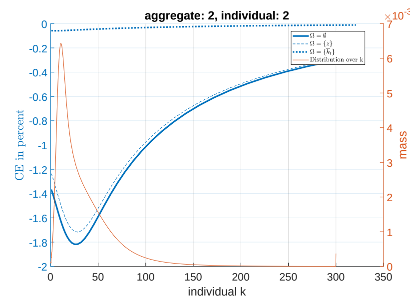
(a) $z = 1 - \Delta_z$,$\epsilon = 0$



(b) $z = 1 - \Delta_z$,$\epsilon = 1$



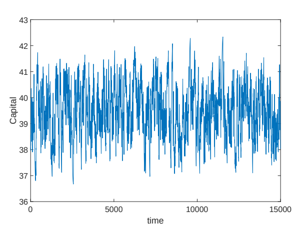
(c) $z = 1 + \Delta_z$,$\epsilon = 0$



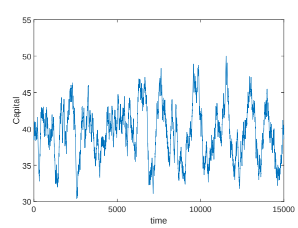
(d) $z = 1 + \Delta_z$,$\epsilon = 1$

Figure 2: The figure presents the measure of utility losses $CE_\Omega(k, \epsilon; \Omega_{max})$ in a 'low-information economy' where $\widetilde{\Omega} = \emptyset$ such that agents only use unconditional averages in forecasting the future. Ω takes three specifications: first, the period-specific cross-sectional mean of k unconditional on z (implying $\overline{k}$ is forecast using an unconditional autoregression for k, solid lines), z only (implying forecasts equal to the mean of k across simulations, conditional on aggregate productivity, dashed lines), and the empty set (implying forecasts of aggregate capital equal its unconditional mean, dotted line). All losses are evaluated at the 10th percentile of the distribution of aggregate capital conditional on productivity.

Figure 3: Aggregate time series of capital



(a) $\widetilde{\Omega} = \{z, \overline{k}_t\}$



(b) $\widetilde{\Omega} = \emptyset$

imum are taken over the whole ergodic distribution of individual and aggregate states.[1] Table III identifies the expected loss from using the three smaller information sets (corresponding

[1]In practice, since the maximum losses in some cases we consider increase strongly at low capital values, we restrict ourselves to the 2.5-97.5 percentile range.

Table III: Expected losses in percent CE

	$\widetilde{\Omega} = \{z, \overline{k}_t\}$	$\widetilde{\Omega} = \emptyset$	$\widetilde{\Omega} = \{z\}$	$\widetilde{\Omega} = \{\overline{k}_t\}$
$\Omega = \emptyset$	-0.0136	-0.7050	-0.7664	-0.0048
$\Omega = \{z\}$	-0.0096	-0.6841	-0.7451	-0.0017
$\Omega = \{\overline{k}_t\}$	-0.0270	-0.0370	-0.0378	-0.0121

Table IV: Maximum losses in percent CE

	$\widetilde{\Omega} = \{z, \overline{k}_t\}$	$\widetilde{\Omega} = \emptyset$	$\widetilde{\Omega} = \{z\}$	$\widetilde{\Omega} = \{\overline{k}_t\}$
$\Omega = \emptyset$	-0.0468	-3.4892	-3.1275	-0.0280
$\Omega = \{z\}$	-0.0170	-3.3257	-2.9770	-0.0024
$\Omega = \{\overline{k}_t\}$	-0.0466	-0.1092	-0.1419	-0.0269

to different rows) in four different economies that differ in the information that agents use in equilibrium (corresponding to different columns). For example, column 1 of table III shows the average losses in the KS economy ($\widetilde{\Omega} = \{z, \overline{k}_t\}$), when agents use only unconditional means ($\Omega = \emptyset$, first row), conditional means ($\Omega = \{z_t\}$, second row), or an unconditional autoregression for the mean capital stock ($\Omega = \{\overline{k}_t\}$, third row) to forecast future variables. As suggested by figure 1, the expected losses are less than 0.05 percent of permanent consumption equivalents. The same is true when agents ignore the level of current productivity in equilibrium ($\widetilde{\Omega} = \{\overline{k}_t\}$, column 4). Expected losses from using rules that ignore the current capital stock are two orders of magnitude larger, however, whenever the equilibrium information set does not include the current mean capital stock (columns two and three).

4 Equilibrium information choice

4.1 Symmetric information choice with ex ante criteria

[To be added.]

4.2 Heterogeneous time-varying information choice

[To be added.]

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