Puzzling Exchange Rate Dynamics and Delayed Portfolio Adjustment[1]

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Abstract

The objective of this paper is to show that the proposal by Froot and Thaler (1990) of delayed portfolio adjustment can account for a broad set of puzzles about the relationship between interest rates and exchange rates. The puzzles include: i) the delayed overshooting puzzle; ii) the forward discount puzzle (or Fama puzzle); iii) the predictability reversal puzzle; iv) the Engel puzzle (high interest rate currencies are stronger than implied by UIP); v) the forward guidance exchange rate puzzle; vi) the absence of a forward discount puzzle with long-term bonds. These results are derived analytically in a simple two-country model with portfolio adjustment costs. Quantitatively, this approach can match all the moments related to these puzzles.

1 Introduction

Richard Thaler won the 2017 Nobel Prize in Economics for incorporating "psychologically realistic assumptions into analyses of economic decision-making," according to the press release by the Nobel committee. One area in which Thaler has noticed behavior inconsistent with what he refers to as "rational efficient markets," is the foreign exchange market. Focusing on the forward discount puzzle that high interest rate currencies tend to appreciate, Froot and Thaler (1990) argue that "a rational efficient markets paradigm provides no satisfactory explanation for the observed results". They suggest that gradual portfolio adjustment could solve this puzzle. Their hypothesis is that "...at least some investors are slow in responding to changes in the interest differential." They argue that "It may be that these investors need some time to think about trades before executing them, or that they simply cannot respond quickly to recent information." In Bacchetta and van Wincoop (2010) we took this proposal seriously and showed that it can indeed account for the forward discount puzzle.

In this paper our objective is to show that gradual portfolio adjustment can in fact account for a much broader set of puzzles about the relationship between interest rates and exchange rates that have been identified more recently in the literature. The six puzzles that we will address are:

1. Delayed overshooting puzzle: a monetary contraction that raises the interest rate leads to a period of appreciation, followed by gradual depreciation.

2. Forward discount puzzle (or Fama puzzle): high interest rate currencies have higher expected returns over the near future.

3. Predictability reversal puzzle: high interest rate currencies have lower expected returns after some period of time.

4. Engel puzzle: high interest rate currencies are stronger than implied by uncovered interest parity.

5. Forward guidance exchange rate puzzle: the exchange rate is more strongly affected by expected interest rates in the near future than the distant future.

6. LSV puzzle: long-term bond return differentials across countries are not predictable by current interest differentials.

The delayed overshooting puzzle was first documented by Eichenbaum and Evans (1995) for the US and Grilli and Roubini (1996) for other countries. It should be pointed out that that some of the subsequent studies have shown that the evidence depends on identification strategies.[1] The second puzzle, the forward discount puzzle, is the best known on this list and continues to be a well established empirical fact.[2] The predictability reversal puzzle, first documented by Bacchetta and van Wincoop (2010), is related to the forward discount puzzle. They show that while the excess return over the next quarter is positive for higher interest rate currencies (forward discount puzzle), after about 8 quarters the quarterly excess return is negative for currencies whose current interest rate is relatively high. In other words, there is a reversal in the sign of expected excess returns. Engel (2016) confirms that this is a robust puzzle.

The fourth puzzle is documented in Engel (2016). The exchange rate can be written as the sum of all expected future interest differentials (the UIP exchange rate that applies under uncovered interest rate parity) plus the sum of all future expected excess returns. The Engel puzzle says that the sum of all expected future excess returns is negative for high interest rate currencies. In other words, the predictability reversal will ultimately dominate and investors demand a lower sum of future excess returns on currencies whose interest rate is currently high. Such currencies are therefore strong relative to what they would be under UIP. Engel (2016) finds that standard foreign exchange risk premium models cannot account for this.

The forward guidance exchange rate puzzle is developed by Galí (2018). As stated above, under UIP the exchange rate is equal to the unweighted sum of all future expected interest rate differentials. This implies that changes in expected interest rates in the near future have the same effect on the exchange rate today as changes in the expected interest differential in the more distant future. However, in the data Galí (2018) finds that expectations of interest differentials in the distant future have a much smaller effect on the current exchange rate than expectations of interest differentials in the near future.

The LSV puzzle stands for the puzzle developed by Lustig, Stathopoulos and

[1]See for example Cushman and Zha (1997), Faust and Rogers (2003) and Scholl and Uhlig (2008).

[2]Notice, however, that the puzzle does not seem to hold when we include post-2008 data. See Bussière et al. (2008).

Verdelhan (2018) (henceforth LSV). It says that the forward discount puzzle has no analogy in long-term bonds. While the international excess return on short-term bonds tends to be positive for currencies with a relatively high interest rate (forward discount puzzle), LSV find that this is not the case for long-term bonds. They show that the local excess return of long-term bonds over short-term bonds tends to be lower for high interest rate currencies and that this offsets the positive expected excess return for short-term bonds.[3] LSV find that no-arbitrage models in international finance cannot account for this.

Our objective is to show that a single friction, associated with portfolio adjustment costs, is able to account for each of these puzzles. An additional objective is to do so in an analytically tractable way, which significantly facilitates the analysis and makes the results more transparent. With the exception of the LSV puzzle, the key results are summarized through a set of propositions that follow directly from the closed form analytical solution of the model. We obtain analytic tractability by assuming that agents can adjust their portfolio each period, but face a simple quadratic portfolio adjustment cost.[4] This also allows us to abstract from investors' heterogeneity. By contrast, the existing literature on gradual portfolio adjustment has mostly assumed overlapping generations that make new portfolio decisions every T periods,[5] an assumption that has the disadvantage of both requiring a numerical solution and leading to a wobbly impulse response of asset prices to shocks.[6]

The role of a portfolio adjustment friction in accounting for these puzzles has received limited attention in the literature so far. An exception is Bacchetta and van Wincoop (2010), who address the first three puzzles in a model where agents

[3]The excess return of long-term bonds can be written as the sum of the excess return for short-term bonds plus the difference in local excess returns of long-term over short-term bonds.

[4]Vayanos and Woolley (2012) and Gârleanu and Pedersen (2016) also introduce a cost of changing portfolios to model portfolio inertia, but in different contexts.

[5]For examples, see Bacchetta and van Wincoop (2010), Bogousslavky (2016), Duffie (2010), Henderschott et al. (2013) and Greenwood et al. (2015).

[6]The wobbly impulse response results from the fact that investors anticipate that agents who changed their portfolio at the time of a shock will change their portfolio again T periods later. Apart from a portfolio adjustment cost as we assume here, the impulse response can also be smoothed by assuming that agents change their portfolio with a given probability each period, as recently proposed in Bacchetta and van Wincoop (2017). But this approach, which is analogous to Calvo price setting, is even less analytically tractable and requires a non-trivial numerical solution technique.

make a new portfolio decision every T periods. Apart from addressing all six puzzles, the approach in this paper has several key advantages. As Engel (2016) points out, the Bacchetta and van Wincoop (2010) model "is complex and requires numerical solution." In addition, the model fits the data best when a second friction is introduced, limited information processing, to avoid the wobbly impulse response. Some of the literature has speculated more informally about the role of gradual portfolio adjustment. This ranges from the positive views expressed by Froot and Thaler (1990), to an agnostic view by LSV regarding the LSV puzzle, to a negative view by Engel (2016) regarding the predictability reversal puzzle and Engel puzzle. As we will see, the latter is due to a misunderstanding of the implications of gradual portfolio adjustment.

While there is a vast literature on the forward discount puzzle, which we will not review here, the other puzzles have received much more limited attention. A few papers focus on delayed overshooting, e.g., Gourinchas and Tornell (2004) and Kim (2005). Recently, several papers analyze the predictability reversal puzzle. Engel (2016), Itshkoki and Mukhin (2017), and Vlachev (2017) propose explanations based on liquidity shocks. Chernov and Creal (2018) and Dahlquist and Penasse (2017) focus on the role of long-term real exchange rate adjustment. The forward guidance exchange rate puzzle and LSV puzzle have only been recently documented and no solution has been proposed yet.

It should finally be pointed out that a gradual portfolio adjustment friction is well motivated by evidence of delayed adjustment in investors' portfolios. Ameriks and Zeldes (2004) document that investors make changes to their TIAA-CREF allocations very infrequently. Using data from the Panel Study of Income Dynamics (PSID) and the Survey of Consumer Finances (SCF), Bilias et al. (2010) find widepread inertia of portfolios in response to stock market fluctuations. Brunnermeier and Nagel (2008) use PSID data to conclude that "...one of the major drivers of household portfolio allocation seems to be inertia: households rebalance only very slowly following inflows and outflows or capital gains and losses." Mitchell et al. (2006) find that 401(k) plan participants are characterized by "profound inertia". Duffie (2010) reviews a broad range of evidence motivating models of gradual portfolio adjustment.

The remainder of the paper is organized as follows. In Section 2 we discuss a two-country model with short-term bonds and gradual portfolio adjustment. In Section 3 we provide formal propositions related to the first five puzzles as well as a numerical illustration. Section 4 extends the model to incorporate long-term bonds to address the last puzzle. Section 5 concludes.

2 Model with Gradual Portfolio Adjustment and Short-Term Bonds

The six puzzles can be written both in terms of real interest rates and exchange rates and in terms of nominal interest rates and exchange rates. As Engel (2016) points out, the forward discount rate puzzle applies equally when using real variables. He proceeds to develop the puzzle that high interest rate currencies are stronger than expected based on UIP by using real interest rates and exchange rates. Galí (2018) also uses real interest rates and exchange rates to develop the forward guidance exchange rate puzzle. An advantage of stating the puzzles in terms of real variables is that the real exchange rate is stationary, while the nominal exchange rate is generally not stationary. We therefore use real variables, although we should stress that the equations can easily be written in nominal terms as well.

In this section we first describe the model and then the solution for the equilibrium real exchange rate. We also derive excess return predictability coefficients. There are two countries, Home and Foreign, with agents who invest in one-period bonds of both countries. We adopt a simplifying overlapping generations framework as in Bacchetta and van Wincoop (2007, 2010). But the model differs in that we adopt a cost of changing the portfolio share instead of a fixed interval of changing portfolios. Moreover, we focus on real variables and we consider investors from both countries rather than just the Home country.

We treat interest rate shocks as exogenous to the model and consider the impact on the exchange rate. This is analogous to Bacchetta and van Wincoop (2010). One can for example think of the interest rate shocks as monetary policy shocks, although we do not explicitly model monetary policy. In order to derive all the results we do not explicitly model other shocks. Itskhoki and Muhkin (2017) argue that financial shocks, in the form of portfolio shocks, are the main drivers of exchange rates and can account for the exchange rate disconnect puzzle. Other shocks only matter to our results to the extent that they affect exchange rate volatility, which affects the portfolio response to expected returns. In numerical illustrations we simply use the observed exchange rate volatility.[7]

2.1 Model Description

There are overlapping generations of agents who live two periods and are born with a wealth of 1 in real terms. Agents in the Home country born at time t maximize

$$E_t \frac{C_{t+1}^{1-\gamma}}{1-\gamma} - 0.5\psi \left(z_t - z_{t-1}\right)^2 \tag{1}$$

where the second term is an adjustment cost. It captures a utility cost of choosing a different portfolio share z_t invested in Foreign bonds than that of "parents" one period ago. This reduced-form adjustment is more ad hoc than the alternative ways to generate gradual portfolio adjustment, but this comes at a significant gain of analytical tractability that delivers key insights. Consumption is equal to the portfolio return:

$$C_{t+1} = R_{t+1}^p = \left[z_t \frac{S_{t+1}}{S_t} e^{i_t^*} e^{-\tau} + (1 - z_t) e^{i_t} \right] \frac{P_t}{P_{t+1}} + T_{t+1} \tag{2}$$

i_t and i_t^* are the nominal interest rate on Home and Foreign bonds. S_t is the level of the nominal exchange rate, measured in terms of the Home currency per unit of the Foreign currency. P_t is the price level. We will assume that inflation over the next period is known, so that P_{t+1} is known. This captures the fact that there is much more uncertainty about exchange rates than inflation over the near future. We also allow for a cost τ of investing in Foreign bonds, which is an international financial friction. The aggregate of this cost across agents is reimbursed through T_{t+1}. R_{t+1}^p is therefore the same in equilibrium as it would be when $\tau = 0$, but agents take T_{t+1} as given, not under their control through portfolio choice.[8]

Define the gross real interest rates as $R_t = e^{i_t} P_t / P_{t+1}$ and $R_t^* = e^{i_t^*} P_t^* / P_{t+1}^*$ where P_t^* is the Foreign price level. The real exchange rate is defined as $Q_t = S_t P_t^* / P_t$. The first-order condition is then

$$E_t e^{-\gamma r_{t+1}^p + q_{t+1} - q_t + r_t^* - \tau} - E_t e^{-\gamma r_{t+1}^p + r_t} - \psi(z_t - z_{t-1}) = 0 \tag{3}$$

[7]We implicitly assume that other shocks that are important drivers of the exchange rate do not affect interest rates themselves. In other words, shocks affect exchange rates (mostly) either through interest rates or other channels (e.g. portfolio shift independent of interest rates).

[8]See Bacchetta and van Wincoop (2017) and Davis and van Wincoop (2017) for the same approach.

where lower case letters denote logs.

The first-order approximation of the log portfolio return is $r^p_{t+1} = z_t er_{t+1} + r_t$, where the excess return is

$$er_{t+1} = q_{t+1} - q_t + r^*_t - r_t \tag{4}$$

Substituting the first-order approximation of the log portfolio return into the first-order condition, assuming log-normality, and using the approximation $e^x = 1 + x$, we have

$$E_t er_{t+1} - \tau + 0.5var(er_{t+1}) - \gamma z_t var(er_{t+1}) - \psi(z_t - z_{t-1}) = 0 \tag{5}$$

Define the steady state fraction invested in Foreign bonds as

$$\bar{z} = \frac{0.5}{\gamma} - \frac{\tau}{\gamma var(er_{t+1})} \tag{6}$$

We take $\bar{z}$, which is related to τ, as a parameter that is given. The frictionless optimal portfolio in the absence of adjustment costs is

$$z^f_t = \bar{z} + \frac{E_t er_{t+1}}{\gamma\sigma^2}$$

where $\sigma^2 = var(er_{t+1})$. The optimal portfolio with adjustment costs can then be written as

$$z_t = \frac{\psi}{\psi + \gamma\sigma^2} z_{t-1} + \frac{\gamma\sigma^2}{\psi + \gamma\sigma^2} z^f_t \tag{7}$$

Expression (7) shows that the portfolio share z_t is a weighted average of the previous period's portfolio share and the frictionless optimal portfolio share. The portfolio share therefore gradually adjusts to the frictionless optimal portfolio share. However, in equilibrium z^f_t is not a fixed target as the expected excess return is endogenous and changes over time.

There is an analogous solution for the Foreign country giving the optimal fraction invested in Foreign bonds by Foreign investors z^*_t. By symmetry the steady state fraction invested in the Foreign bond for Foreign investors is $\bar{z}^* = 1 - \bar{z}$. We will focus on the average portfolio share invested in the Foreign bond $z^A_t = 0.5(z_t + z^*_t)$.

The real supply of bonds is assumed fixed at 1 in terms of the purchasing power of the respective countries. As a result of Walras' Law it is sufficient to focus on the Foreign bond market equilibrium. Taking the perspective of the Home country, the Foreign bond market equilibrium condition in real terms is

$$z_t + z_t^* Q_t = Q_t \tag{8}$$

It is useful to introduce a home bias parameter h. Home bias is usually defined as one minus the ratio of the share invested abroad and the share of the foreign asset in the world asset supply. The home bias parameter in steady state in our model is therefore $h = 1 - 2\bar{z}$. Using this, and linearizing the market clearing condition around the log of the real exchange rate $q = 0$, and portfolio shares $\bar{z}$ and $\bar{z}^*$, gives

$$z_t^A = 0.5 + bq_t \tag{9}$$

where $b = (1 - h)/4$ is a parameter between 0 and 0.25. Substituting this into the average of (7) and its foreign counterpart and using expression (4) for the excess return, we have

$$E_t q_{t+1} - \theta q_t + b\psi q_{t-1} + r_t^D = 0 \tag{10}$$

where $r_t^D = r_t^* - r_t$ is the Foreign minus Home real interest rate differential and $\theta = 1 + \psi b + \gamma\sigma^2 b$, with $\theta > 1$.

Equation (10) is a second-order difference equation in q_t. The presence of q_{t-1} comes from the adjustment cost and vanishes when $\psi = 0$. Notice that we can rewrite (10) as:

$$E_t er_{t+1} = \gamma\sigma^2 bq_t + \psi b(q_t - q_{t-1)} \tag{11}$$

The expected excess return depends on two terms. The first is a standard risk premium $\gamma\sigma^2 bq_t$ and will play a marginal role in the analysis. It is the second term, $\psi b(q_t - q_{t-1})$, generated by the adjustment cost, that is playing the key role.[9]

2.2 Solution Real Exchange Rate

Using (10) we can solve for the equilibrium q_t as a function of the lagged real exchange rate and expected future interest rate differentials. Using standard solution

[9]Chernov and Creal (2018) and Dahlquist and Penasse (2017) argue that there is a missing risk premium that should be related to the real exchange rate. Their proposed specification is however different from (11).

techniques for second-order stochastic difference equations, we have

$$q_t = \alpha q_{t-1} + E_t \sum_{i=0}^{\infty} \frac{1}{D^{i+1}} r_{t+i}^D \tag{12}$$

where α and D are the roots of the charcteristic equation of (10):

$$\alpha = \frac{\theta - \sqrt{\theta^2 - 4\psi b}}{2} \tag{13}$$

$$D = \frac{\theta + \sqrt{\theta^2 - 4\psi b}}{2} \tag{14}$$

It is easily verified that $0 \leq \alpha < 1$ and $D > 1$. The equilibrium real exchange rate therefore depends on the lagged real exchange rate and a present discounted value of expected future real interest rate differentials. Since $D > 1$, it is immediate that expected real interest rates in the more distant future have a smaller effect on the equilibrium real exchange rate than expected real interest rates in the near future. This addresses the forward guidance exchange rate puzzle. We will explore this more, and develop the intuition behind it, in the next section.

A couple of comments about the parameters α and D are in order as they are key to the solution. Appendix B derives the following Lemma:

Lemma 1. *The following properties describe the relationship between α, D and the portfolio adjustment cost parameter ψ:*

- *As ψ rises from 0 to ∞, α rises monotonically from 0 to 1.*
- *As ψ rises from 0 to ∞, D rises monotonically from $1 + \gamma\sigma^2$ to ∞.*

Higher portfolio adjustment costs imply that the real exchange rate depends to a greater extent on the value of the real exchange rate during the last period and future expected real interest rates are discounted more heavily in the equilibrium real exchange rate.

We will focus on the case where the real interest differential follows a simple AR(1) process:

$$r_t^D = \rho r_{t-1}^D + \varepsilon_t \tag{15}$$

In that case (12) gives us

$$q_t = \alpha q_{t-1} + \frac{1}{D - \rho} r_t^D \tag{16}$$

We can also write this as a function of current and past real interest rate shocks:

$$q_t = \frac{1}{D-\rho}\sum_{i=0}^{\infty}\nu_i\varepsilon_{t-i} \tag{17}$$

where

$$\nu_i = \begin{cases} \dfrac{\alpha^{i+1}-\rho^{i+1}}{\alpha-\rho} & \text{if } \alpha \neq \rho \\ (i+1)\rho^i & \text{if } \alpha = \rho \end{cases} \tag{18}$$

2.3 Excess Return Predictability Coefficients

Consider the following regression:

$$er_{t+k} = \alpha + \beta_k r_t^D + \varepsilon_{t+k}^{er} \tag{19}$$

Several of the puzzles are related to the excess return predictability coefficients β_k. The coefficient β_k tells us the effect of the current real interest differential on the expected excess return k periods from now. The forward discount puzzle focuses on $k = 1$, with one period usually being a month or a quarter. For the predictability reversal puzzle and the Engel puzzle we are also interested in β_k for $k > 1$, which relates to the effect of the current interest differential on the excess return further into the future.

In the model, the value of β_k is equal to

$$\beta_k = \frac{cov(er_{t+k}, r_t^D)}{var(r_t^D)} \tag{20}$$

Using the solution for the real exchange rate under the assumed AR(1) process for the real interest differential, Appendix C shows that this can be written as[10]

$$\beta_k = \begin{cases} \lambda_1\rho^{k-1} + \lambda_2\alpha^{k-1} & \text{if } \alpha \neq \rho \\ \dfrac{\rho^{k-1}}{D-\rho}\left(D - \dfrac{1}{1+\rho} - (1-\rho)(k-1)\right) & \text{if } \alpha = \rho \end{cases} \tag{21}$$

where

$$\lambda_1 = \frac{1}{D-\rho}\left(D - \rho\frac{\alpha-1}{\alpha-\rho}\right) \tag{22}$$

$$\lambda_2 = \frac{\alpha-1}{D-\rho}\left[\frac{\rho}{\alpha-\rho} + \frac{1}{1-\alpha\rho}\right] \tag{23}$$

[10] β_k is a continuous function of α (and therefore of ψ), but λ_1 and λ_2 are not defined at $\alpha = \rho$, which is why the expression for β_k at $\alpha = \rho$ is reported separately.

Lemma 2 in Appendix F characterizes the signs of λ_1 and λ_2. λ_1 and λ_2 are positive for low values of ψ, but turn negative as ψ increases.

3 Explaining Five Puzzles

We now use the simple model discussed above to address the first five puzzles. We do so by discussing a series of propositions and a numerical illustration. When describing the intuition behind the results, we will always consider an increase in the relative *Foreign* interest rate (rise in r_t^D), which leads to an appreciation of the *Foreign* currency (rise in q_t). We will always refer to the Foreign currency, so a depreciation refers to a Foreign depreciation or drop in q_t.

3.1 Delayed Overshooting Puzzle

First define

$$\bar{t} = \begin{cases} \dfrac{ln(1-\rho) - ln(1-\alpha)}{ln(\alpha) - ln(\rho)} & \text{if } \alpha \neq \rho \\ \dfrac{\rho}{1-\rho} & \text{if } \alpha = \rho \end{cases} \tag{24}$$

Using (17), Appendix D proves the following proposition:

Proposition 1. *Consider the impulse response of the real exchange rate to a positive shock to the relative Foreign interest rate r_t^D.*

- *if $\alpha < 1-\rho$: the real exchange rate appreciates at the time of the shock and subsequently gradually depreciates back to the steady state.*
- *if $\alpha > 1-\rho$: there is delayed overshooting. The real exchange rate appreciates at the time of the shock and keeps appreciating until time $t > \bar{t} > 1$. Then it gradually depreciates back to the steady state.*

Since Lemma 1 tells us that α rises from 0 to 1 as we raise the gradual portfolio adjustment parameter ψ, Proposition 1 implies that for sufficiently large ψ, and assuming $\rho > 0$, there is delayed overshooting of the type reported by Eichenbaum and Evans (1995) and others. They show that after monetary policy tightening, the currency continues to appreciate for another 25-39 months before it starts to depreciate. With less gradual adjustment, such that $\alpha < 1-\rho$, there is no delayed overshooting.

To understand the intuition, consider an increase in the Foreign interest rate. There will be an immediate appreciation of the Foreign currency as investors shift to Foreign bonds. Subsequent to the shock, there are two opposing forces at work. On the one hand, the Foreign interest rate starts to gradually decline again, which leads to a shift away from Foreign bonds and therefore a gradual depreciation. On the other hand, to the extent that portfolios are slow to adjust, there will be a continued flow towards Foreign bonds, which leads to a continued appreciation. When ψ is sufficiently large, the second force dominates and there will be delayed overshooting.

Expression (24) tells us how long the real appreciation will last in the case of delayed overshooting. Appendix D shows that the derivative of $\bar{t}$ with respect to α is positive. A larger gradual portfolio adjustment parameter ψ, which raises α (Lemma 1), will then lead to a longer duration of the delayed overshooting. In the extreme where α approaches 1, $\bar{t}$ approaches infinity.

Figure 1 provides a numerical illustration. The chart on the left shows the impulse response of the real exchange rate when $\psi = 15$ and $\gamma = 50$.[11] We refer to this as the benchmark case. The chart on the right shows the time to maximum overshooting for ψ varying from 0 to 20 and γ taking on the values 10, 50 and 100. Other parameters are calibrated to interest rates and exchange rates of the 6 G-7 countries relative to the United States (as in Engel, 2016). The real interest rate is computed as the monthly nominal interest rate minus the expected monthly inflation rate (estimated from annual inflation). We find $\rho = 0.9415$.[12] The standard deviation σ of the monthly excess return is computed as the average standard deviation of the monthly change in the real exchange rate, which is 0.0271. We set the home bias parameter h equal to 0.66, which is the average for the countries during Q2, 2017.[13] Further details on the data for the calibration can be found in Appendix A.

[11] A risk premium of 50 may seem very large, but analogous to the equity premium, which requires a very high rate of risk aversion to explain, currency premia are very small for low rates of risk aversion. One could alternatively introduce other features to introduce large premia, such as disaster risk, but that distracts from the topic of the paper and the analytic transparency.

[12] The average standard deviation of the relative real interest rate innovation is 0.000342. This is only used in the impulse response of the real exchange rate to a one standard deviation interest rate shock in Figure 1. It does not affect any of the other results.

[13] We combine BIS data on debt securities outstanding with external assets and liabilities for debt securities from the IMF International Investment Position Statistics.

Figure 1: IMPULSE RESPONSE q_t AND DELAYED OVERSHOOTING

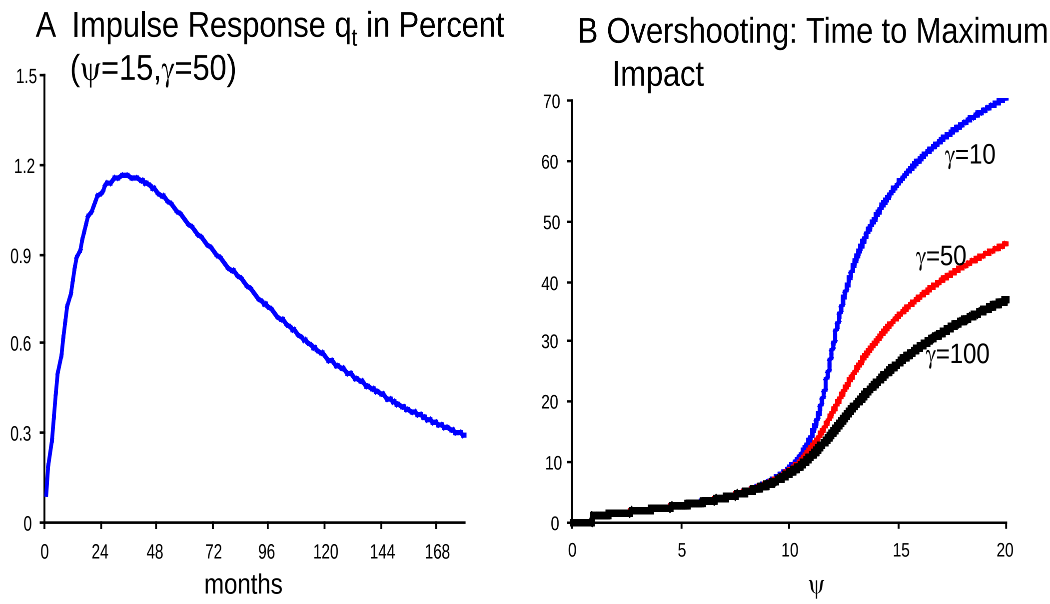


Chart A of Figure 1 shows that the real exchange rate overshoots, reaching a maximum after 35 months. This is consistent with the results in Eichenbaum and Evans (1995). Chart B shows that except for very small values of ψ, the model implies delayed overshooting. Consistent with Proposition 1, the time to maximum impact rises significantly with ψ. It is also larger the lower the rate of risk-aversion. A higher rate of risk aversion implies that portfolios are less responsive to expected returns. This weakens the appreciation subsequent to the shock and shortens the delayed overshooting.

3.2 Forward Discount Puzzle

While UIP implies that the Fama coefficient β_1 is zero, empirical evidence typically finds a positive number. Proposition 2 characterizes the sign of β_1 in the model:

Proposition 2. *The Fama predictability coefficient β_1 is positive, and larger when there is gradual portfolio adjustment ($\psi > 0$).*

The proof is given in Appendix E. Since $\beta_1 > 0$, a positive excess return is expected on the high interest rate currency, consistent with the forward discount puzzle. Moreover, Proposition 2 says that β_1 is larger when we introduce a cost of adjusting portfolios ($\psi > 0$). Even without this cost, there is some excess return predictability in the model through a risk premium channel.[14]

Proposition 1 on delayed overshooting is a useful starting point to understand the role of gradual portfolio adjustment in accounting for the forward discount puzzle. When $\alpha > 1-\rho$, so that there is delayed overshooting, the Foreign currency is expected to appreciate for at least one more period after the initial appreciation. The Foreign currency will then have a positive expected excess return both due to the higher interest rate and the expected appreciation. Therefore the portfolio adjustment parameter ψ, which causes a gradual portfolio shift to the Foreign currency that leads to continued appreciation, increases the Fama predictability coefficient β_1.[15]

In the benchmark case of Figure 1 ($\psi = 15$ and $\gamma = 50$), the excess return predictability coefficient β_1 is equal to 3.26. Figure 2 shows how β_1 varies with ψ and γ. It rises until ψ is about 12 and then gradually declines. When ψ is very small, there is not much delayed overshooting, weakening the excess return predictability. If instead ψ is very large, the weak portfolio response causes a very slow response of the exchange rate, which also diminishes excess return predictability. β_1 is therefore largest for an intermediate value of ψ. Figure 2 also shows that the excess return predictability coefficient β_1 is larger when risk aversion γ is smaller. A very large γ again leads to a weak portfolio response, diminishing excess return predictability associated with the Foreign currency appreciation.

3.3 Predictability Reversal Puzzle

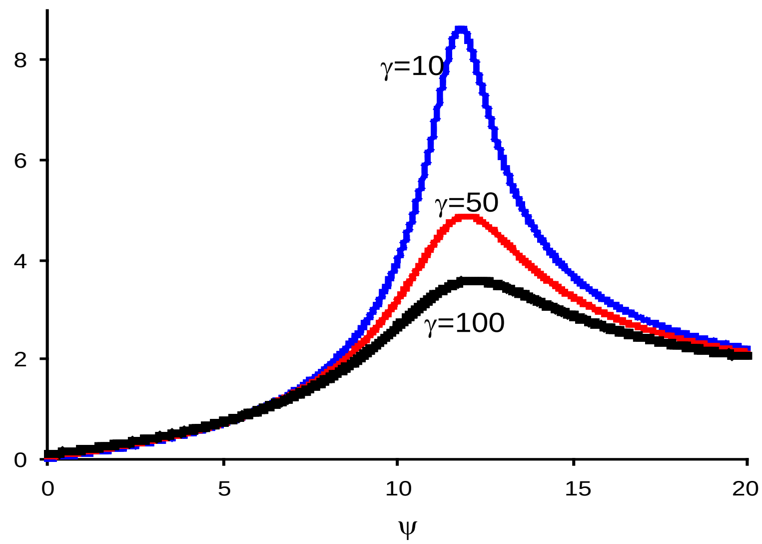
Define $\bar{\psi} = \rho\gamma\sigma^2/(1-\rho)$. Excess return predictability at longer horizons, measured by β_k, is described in the following proposition:

Proposition 3. *The following holds for β_k:*

[14]Specifically, a higher Foreign real interest rate leads to a real appreciation of the Foreign currency, which increases the relative value of the Foreign bond supply. To invest a larger portfolio share in Foreign bonds, investors demand a positive expected excess return on the Foreign bond.

[15]Even when $\alpha < 1-\rho$, so that there is no delayed overshooting, gradual portfolio adjustment leads to a higher Fama coefficient β_1 because the rate of depreciation subsequent to the shock is smaller due to gradual portfolio adjustment. The weaker subsequent depreciation implies a higher expected excess return on the Foreign currency and therefore a larger Fama coefficient β_1.

Figure 2: Forward Discount Puzzle: Predictability Coefficient β_1



- *if $\psi \leq \bar{\psi}$: β_k is positive for all k and drops monitonically to zero as $k \to \infty$*
- *if $\psi > \bar{\psi}$: there is a $\bar{k} > 1$ such that β_k is positive for $k < \bar{k}$ and negative for $k \geq \bar{k}$. It converges to zero as $k \to \infty$.*

The proof is given in Appendix F. Proposition 3 implies that when the gradual adjustment parameter is low, the Foreign currency continues to have positive expected excess returns in all future periods, although the predictability β_k vanishes to zero over time. But when the gradual adjustment parameter is sufficiently high ($\psi > \bar{\psi}$), there will be a predictability reversal. While initially, after the increase in the Foreign interest rate, the Foreign currency is expected to have a positive expected excess return, after a certain period of time it is expected to have a negative expected excess return. Bacchetta and van Wincoop (2010) first documented this reversal in the sign of predictability for nominal interest rates and exchange rates. They find that a high interest rate currency has a positive expected excess return for about 5-10 quarters, after which it has a negative expected excess return. Engel (2016) reports similar findings for real interest rates and exchange rates.

The excess return on the Foreign currency is driven both by the higher Foreign interest rate and the change in the value of the Foreign currency. Under delayed overshooting the Foreign currency will at first appreciate for a while and therefore have a positive excess return. But after time $\bar{t}$ it will start to depreciate, which contributes to a negative excess return. If $\bar{t}$ is large, by the time the Foreign currency starts to depreciate, the interest differential will be small. The excess return is then mainly driven by the Foreign currency depreciation and is therefore negative.[16]

Engel (2016) claims that models with gradual portfolio adjustment cannot account for the predictability reversal. To show this, Engel (2016) does not develop a model of gradual portfolio adjustment, but instead considers a reduced form exchange rate equation to capture the idea that under gradual portfolio adjustment the real exchange rate gradually approaches the level that would apply under UIP. This equation involves an AR(1) process for $q_t - q_t^{IP}$, where q_t^{IP} is the exchange rate under UIP where the expected excess return is zero: $E_t(q_{t+1}^{IP} - q_t^{IP} + r_t^* - r_t) = 0$. The exchange rate under interest rate parity is

$$q_t^{IP} = \sum_{i=0}^{\infty} E_t r_{t+i}^D \tag{25}$$

This conjecture about the process of the real exchange rate under gradual portfolio adjustment is not correct though. First, as can be seen from (10), the real exchange rate is driven by an AR(2) process, not an AR(1) process. The same is the case for $q_t - q_t^{IP}$, which is described by the process

$$E_t\left(q_{t+1} - q_{t+1}^{IP}\right) - \theta\left(q_t - q_t^{IP}\right) + b\psi\left(q_{t-1} - q_{t-1}^{IP}\right) + \frac{1-\theta}{1-\rho} r_t^D + \frac{b\psi}{1-\rho} r_{t-1}^D = 0 \tag{26}$$

Second, as illustrated in Figure 3A for the benchmark parameterization, instead of the real exchange rate gradually approaching the interest parity exchange rate, the gap between q_t and q_t^{IP} grows significantly in months 6 to 40 after the shock. While the real exchange rate keeps appreciating as a result of delayed overshooting, the interest parity exchange rate gradually depreciates immediately following the shock.

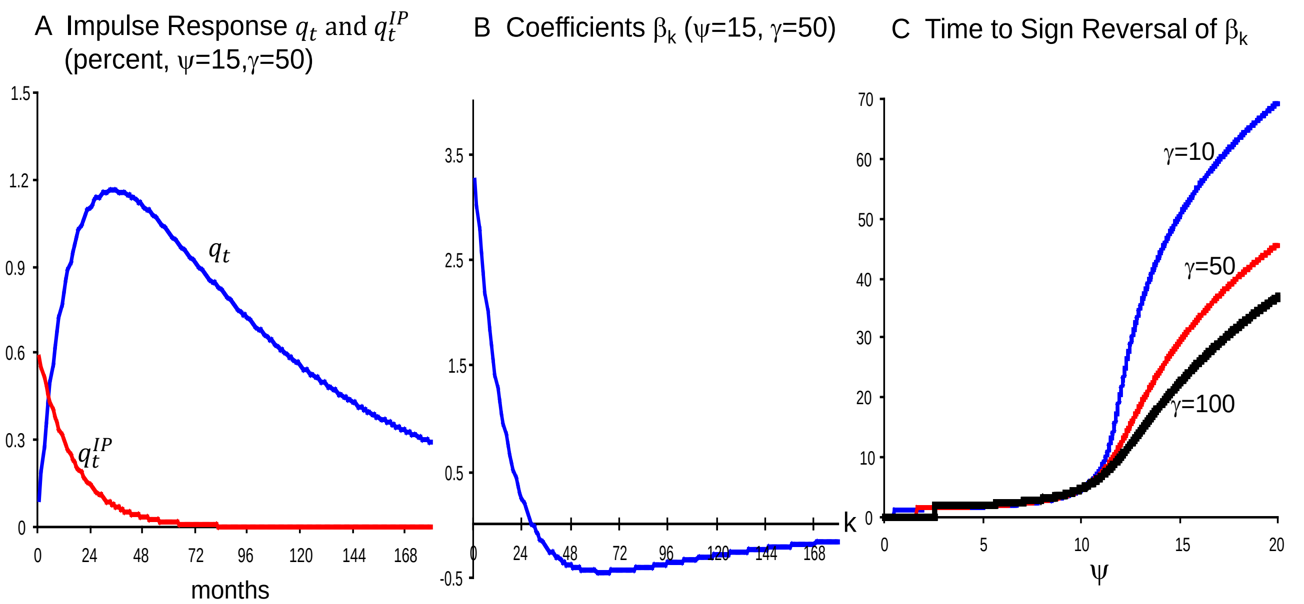
Chart B of Figure 3 reports β_k for k from 1 to 180 for the benchmark case where $\psi = 15$ and $\gamma = 50$. The reversal of the predictability coefficient from positive to negative occurs after 30 months. This is not too far from the reversal after 5-10 quarters reported in Bacchetta and van Wincoop (2010). It is also consistent with results reported in Engel (2016).[17] Chart C considers the impact of ψ and γ on

[16]Delayed overshooting is not a necessary condition for predictability reversal. Dependent on parameters, predictability reversal can also happen when $\alpha + \rho < 1$.

[17]Engel (2016) reports results of regressions of both the ex-post and ex-ante excess return on the interest differential. The ex-ante excess return relies on a VAR to compute expected returns and delivers a somewhat shorter time to reversal of about 12 months on average.

the time k where β_k reverses sign from positive to negative. This rises with ψ and falls with γ, consistent with the longer time to maximum impact of the exchange rate for higher ψ and lower γ shown in Chart 1B.

Figure 3: Sign Reversal of Predictability Coefficient β_k



3.4 Engel Puzzle

The Engel puzzle says that high interest rate currencies tend to be strong relative to the interest parity exchange rate. More formally:

$$cov(q_t - q_t^{IP}, r_t^D) > 0 \tag{27}$$

Engel (2016) provides evidence that this condition holds in the data for 6 currencies. We will refer to it as the Engel condition. Using that $\sum_{k=0}^{\infty}(q_{t+k+1} - q_{t+k} + r_{t+k}^D) = q_\infty - q_t + \sum_{k=0}^{\infty} r_{t+k}^D$, from (20) we have

$$var(r_t^D)\sum_{k=1}^{\infty}\beta_k = cov\left(q_\infty - q_t + \sum_{k=0}^{\infty} r_{t+k}^D, r_t^D\right) = cov(q_t^{IP} - q_t, r_t^D) \tag{28}$$

The last equality uses that q_∞ and $\sum_{k=0}^{\infty}(r^D_{t+k} - E_t r^D_{t+k})$ are unaffected by shocks that affect r^D_t. It follows that the Engel condition can be written as

$$\sum_{k=1}^{\infty} \beta_k < 0 \tag{29}$$

which is an equivalence used by Engel (2016) as well. Predictability reversal is a necessary condition for this to hold, so that $\psi > \bar{\psi}$ is a necessary, but not a sufficient condition. Negative expected excess returns on the Foreign currency for $k \geq \bar{k}$ must more than offset the positive expected excess returns when $k < \bar{k}$.

Define (ψ^E_1, ψ^E_2) as positive values of ψ, with $\psi^E_1 < \psi^E_2$, where $cov(q_t - q^{IP}_t, r^D_t) = 0$. Appendix G describes these values and proves the following proposition:

Proposition 4. *Necessary and sufficient conditions for the Engel condition to hold are*

1. $\psi^E_1 < \psi < \psi^E_2$.
2. $\gamma\sigma^2 b < \frac{1-\rho}{\rho}\left(1 - \sqrt{1-\rho}\right)^2$.

Proposition 4 imposes several restrictions on parameters for the Engel condition to be satisfied. First, the gradual portfolio adjustment parameter ψ cannot be too large or too small. Second, risk or risk aversion cannot be too large. Third, the Engel condition is only satisfied for intermediate values of the interest rate persistence ρ. It will not hold when ρ is very close to either 0 or 1. While these restrictions may appear restrictive, we will see below that they are satisfied for a broad range of values of γ and ψ.

In order to understand the intuition behind the proposition, and the role of the various parameters, we can write the Engel condition as

$$cov(q_t, r^D_t) > cov(q^{IP}_t, r^D_t) = \frac{1}{(1-\rho)^2(1+\rho)} var(\varepsilon_t) \tag{30}$$

First consider the role of the gradual portfolio adjustment parameter ψ. We already know from Proposition 3 that it cannot be too small as $\psi > \bar{\psi}$ is needed to obtain predictability reversal. At the same time, when ψ gets too large, the predictability reversal is not strong enough to satisfy the Engel condition. The gradual portfolio adjustment parameter only affects the left hand side of (30). When ψ is very large, portfolios are very slow to respond, so that q_t rises little at the time of the shock and in the initial periods after the shock when the interest differential is large. (30) will then not be satisfied.

Next consider the role of risk or risk aversion. If $\gamma\sigma^2$ is extremely large, portfolios are not very responsive to expected returns and therefore the Foreign currency appreciates very little. The left hand side of (30) is again too small for the Engel condition to be satisfied.

Finally, consider the persistence ρ of the real interest rate. The right hand side of (30) goes to infinity when ρ approaches 1, so that the Engel condition will not be satisfied. If the interest differential is very persistent, the Foreign currency will continue to experience high interest rates for a very long time, which by itself causes positive excess returns for a long time. Proposition 4 tells us that the Engel condition is also not satisfied when ρ is very close to zero. Such lack of persistence implies a weak appreciation of the real exchange rate at the time of the shock. The left hand side of (30) will then be small.

Figure 4 shows that the Engel result holds quite generally in the model as long as ψ is not too close to zero. Consistent with Proposition 4, the Engel result is stronger the lower the rate of risk-aversion γ and peaks for an intermediate value of ψ. For the benchmark parameterization where $\psi = 15$ and $\gamma = 50$, the Engel coefficient $\sum_{k=1}^{\infty} \beta_k$ is equal to -25. This is similar to the estimate in Engel (2016), who finds a -31 coefficient for the G6 average exchange rate against the dollar and an average of -21 for the individual G6 currencies against the dollar.[18]

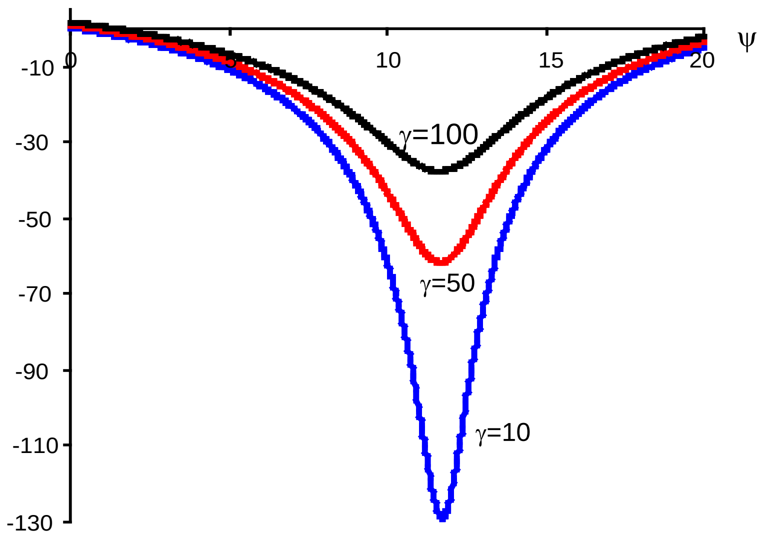
3.5 Forward Guidance Exchange Rate Puzzle

The following proposition addresses the forward guidance puzzle posed by Galí (2018):

Proposition 5. *The real exchange rate q_t gives less weight to expected interest differentials in the distant future than the near future. The higher the gradual portfolio adjustment parameter ψ, the more future expected interest differentials are discounted in the equilibrium real exchange rate.*

[18] Engel (2016) also reports a regression of the level of the real exchange rate q_t on r_t^D. The model implies a coefficient of 42 for the benchmark parameterization, which represents the fact that a high interest rate currency tends to be strong. Engel (2016) reports a coefficient of 43.7 when using the G6 average exchange rate against the dollar.

Figure 4: Engel Puzzle $\sum_{k=1}^{\infty} \beta_k$



Proposition 5 follows directly from (12) and Lemma 1. Future expected interest differentials are discounted at the rate D, which is larger than 1 and rises with ψ.

We have seen that under UIP the real exchange rate is equal to the interest parity real exchange rate (25), where there is no discounting. Even when $\psi = 0$, the discount rate is larger than 1 when we allow for exchange rate risk, which leads to a deviation from UIP. Specifically, we have $D = 1 + \gamma\sigma^2 b$. But as we shall see, the discount rate D is very close to 1 when $\psi = 0$.

To see the role of ψ, assume that we are currently at time t and consider an expected one-period increase in the interest rate differential at $t + k$. The only reason the real exchange rate appreciates prior to $t + k$ is an expectation of subsequent appreciation. The response of q_{t+k-1} to a given higher q_{t+k} is reduced as a result of a positive ψ as portfolios are less sensitive to expected returns. For the same reason the response of q_{t+k-2} to a given expected higher q_{t+k-1} is reduced as a result of the positive ψ. When going back all the way to time t, the response of q_t can be very small when k is large. There are multiple rounds of discounting as each period the real exchange rate response to an expected higher real exchange rate next period is reduced by the positive portfolio adjustment parameter ψ.[19]

The quarterly discount rate D in our benchmark numerical example with

[19]While this broadly captures the intuition, the actual response of the real exchange rate is somewhat complicated by the fact that the real exchange rate not only responds to the expected real exchange rate next period, but also to the lagged real exchange rate.

$\psi = 15$ and $\gamma = 50$ is 1.29. Future expected interest rates are therefore heavily discounted. This is consistent with results reported by Galí (2018), which imply that expected interest rates more than two years into the future have an effect on the current real exchange rate that is very small compared to the impact of expected interest rates over the next two years.[20] For comparison, when $\psi = 0$ (holding all other parameters the same), the discount rate is $D = 1.0031$. In that case expected interest rates two years into the future have an effect on the current exchange rate that is only 7 percent less than the effect of the current interest rate.

4 Lack of Predictability with Long Term Bonds

LSV show that excess return predictability vanishes when considering monthly returns of long-term bonds. In order to address this last puzzle we need to extend the model by introducing long-term bonds. In that case we need to solve not only for the equilibrium real exchange rate, but also long-term bond prices in both countries. In this extension an analytical solution is no longer feasible. We will describe the extended model, leaving all algebraic details to a separate Online Appendix.

There are now four assets: one-period bonds and long-term bonds in both countries. Agents in the Home country maximize

$$E_t \frac{C_{t+1}^{1-\gamma}}{1-\gamma} - \frac{1}{4}\psi \sum_{i=1}^{4} \left(z_{it} - z_{i,t-1}\right)^2 \tag{31}$$

z_{1t} is the fraction of wealth invested in Foreign short term bonds. z_{2t} is the fraction invested in Foreign long term bonds and z_{3t} is the fraction invested in Home long term bonds. The remaining fraction $z_{4t} = 1 - z_{1t} - z_{2t} - z_{3t}$ is invested in Home short term bonds. The adjustment cost term in (31) is the same as in (1) when we set the long term bond portfolio shares equal to 0. In that case $z_{1t} = z_t$ and $z_{4t} = 1 - z_t$.

[20] Galí (2018) regresses q_t on $\sum_{i=0}^{23} E_t r^D_{t+i}$ and $\sum_{i=24}^{\infty} E_t r^D_{t+i}$. We cannot do so in our model as both are proportional to r^D_t and therefore collinear. They would no longer be collinear if we adopted an AR(2) process. More generally, the precise coefficients that we would obtain for a Gali type regression depend on what we assume about the information about future expected interest differentials, which is auxiliary to the gradual portfolio adjustment aspect of the model.

Let R^L_{t+1} be the real return on Home long-term bonds from the perspective of Home agents and $R^{L,*}_{t+1}$ the real return on Foreign long-term bonds from the perspective of Foreign agents. The gross real interest rates on one-period bonds will continue to be denoted as R_t and R^*_t. Consumption of Home agents is equal to the portfolio return:

$$C_{t+1} = R_t + z_{1t}\left(\frac{Q_{t+1}}{Q_t}R^*_t e^{-\tau} - R_t\right) + z_{2t}\left(\frac{Q_{t+1}}{Q_t}R^{L,*}_{t+1}e^{-\tau_L} - R_t\right) + \\ z_{3t}\left(R^L_{t+1} - R_t\right) + T_{t+1} \tag{32}$$

As before, the cost of investing abroad is τ for short-term bonds. It is τ_L for long-term bonds. The aggregate of these costs is reimbursed through T_{t+1}.

Long-term bonds in both countries earn real coupons of κ, $(1-\delta)\kappa$, $(1-\delta)^2\kappa$, and so on. A smaller δ implies a longer maturity of debt. The real returns on Home and Foreign long-term bonds, from the perspective of respectively Home and Foreign agents, are then

$$R^L_{t+1} = \frac{(1-\delta)P^L_{t+1} + \kappa}{P^L_t} \tag{33}$$

$$R^{L,*}_{t+1} = \frac{(1-\delta)P^{L,*}_{t+1} + \kappa}{P^{L,*}_t} \tag{34}$$

Here P^L_t and $P^{L,*}_t$ are the prices of newly issued bonds at time t, measured in real terms from the perspective of Home and Foreign agents.

As before, denote logs with lower case letters. Log excess returns are defined as log real asset returns from the perspective of Home agents minus the real interest rate r_t of the Home country. The vector of excess returns on the first three assets, not including the cost of investing abroad, is

$$\mathbf{er_{t+1}} = \begin{pmatrix} er_{1,t+1} \\ er_{2,t+1} \\ er_{3,t+1} \end{pmatrix} = \begin{pmatrix} q_{t+1} - q_t + r^*_t - r_t \\ q_{t+1} - q_t + r^{L,*}_{t+1} - r_t \\ r^L_{t+1} - r_t \end{pmatrix} \tag{35}$$

Define Σ as the variance of $\mathbf{er_{t+1}}$. Using log normality of consumption and returns, the Online Appendix shows that the first-order conditions of Home agents can be written as

$$E_t\mathbf{er}_{t+1} - \begin{pmatrix} \tau \\ \tau_L \\ 0 \end{pmatrix} + 0.5diag(\Sigma) - \gamma\Sigma\mathbf{z}_t = \frac{\psi}{2R}(\hat{\mathbf{z}}_t - \hat{\mathbf{z}}_{t-1}) \tag{36}$$

where $\mathbf{z}_t = (z_{1t}, z_{2t}, z_{3t})'$ and $\hat{\mathbf{z}}_t$ subtracts z_{4t} from each element of $\mathbf{z_t}$. R is the steady state gross real interest rate. The analogous first-order conditions for Foreign agents are

$$E_t \mathbf{er}_{t+1} + \begin{pmatrix} \tau \\ \tau \\ \tau - \tau_L \end{pmatrix} - (1-\gamma)\Sigma_1 + 0.5 diag(\Sigma) - \gamma \Sigma \mathbf{z}_t^* = \frac{\psi}{2R}(\hat{\mathbf{z}}_t^* - \hat{\mathbf{z}}_{t-1}^*) \quad (37)$$

where $\mathbf{z}_t^* = (z_{1t}^*, z_{2t}^*, z_{3t}^*)'$ is the vector of portfolio shares of Foreign agents and $\hat{\mathbf{z}}_t^*$ subtracts z_{4t}^* from each element of $\mathbf{z}_t^*$. Σ_1 is the first column of Σ.

The asset market equilibrium conditions can be written

$$z_{1t} + Q_t z_{1t}^* = Q_t b^S \quad (38)$$

$$z_{2t} + Q_t z_{2t}^* = Q_t P_t^{L,*} b_t \quad (39)$$

$$z_{3t} + Q_t z_{3t}^* = P_t^L b_t \quad (40)$$

$$z_{4t} + Q_t z_{4t}^* = b^S \quad (41)$$

Here b^S is the real supply of short-term bonds in terms of the purchasing power of each country and b_t is the quantity of ong-term bonds. Define $b^L = \bar{P}^L \bar{b}$ as the steady state real value (in local purchasing power) of long-term bonds. We assume that $b^S + b^L = 1$ and in deviation from steady state

$$b_t = -p_t^{L,A} \quad (42)$$

where $p_t^{L,A} = 0.5(p_t^L + p_t^{L,*})$ is the average log long-term bond price. This assures that when we add up all (log-linearized) market clearing conditions we get an identity, which must be the case due to Walras' Law (the last market clearing condition is reduntant). (42) is not important in what follows as excess returns depend on relative log bond prices, not average log bond prices.[21]

Assuming (42), linearizing the first three market clearing conditions, we have

$$\mathbf{z}_t^A = 0.5 \begin{pmatrix} b^S \\ b^L \\ b^L \end{pmatrix} + 0.5 \begin{pmatrix} b^S \\ b^L \\ 0 \end{pmatrix} q_t - 0.5 \bar{\mathbf{z}}^* q_t + 0.25 b^L p_t^{L,D} \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \quad (43)$$

[21]The reason we assume (42) is a bit technical. One can think of the bonds as issued by Home and Foreign governments. Since there is no investment in the model, it must be the case that world saving (private plus government) is zero in equilibrium. Since there is no endogenous mechanism in the model to equate world saving to zero, we assume that world government saving adjusts to make world saving equal to zero. This happens when (42) is satisfied, which implies that the average world real bond supply remains constant.

where $\mathbf{z}_t^A$ is the average of $\mathbf{z_t}$ and $\mathbf{z_t^*}$, $\bar{\mathbf{z}}^*$ is the steady state of $\mathbf{z_t^*}$ and $p_t^{L,D} = p_t^L - p_t^{L,*}$ is the relative log long term bond price.

By symmetry $\bar{z}_1^* + \bar{z}_4^* = b^S$ and $\bar{z}_2^* + \bar{z}_3^* = b^L$. We can choose τ and τ_L to set $\bar{z}_4^*$ and $\bar{z}_3^*$ at any value. We will assume that these values are such that they generate the same home bias h for short and long-term bonds. This happens when

$$\bar{z}_4^* = 0.5(1-h)b^S \tag{44}$$

$$\bar{z}_3^* = 0.5(1-h)b^L \tag{45}$$

We then also have $\bar{z}_2^* = 0.5(1+h)b^L$ and $\bar{z}_1^* = 0.5(1+h)b^S$.

After substituting the market clearing conditions (43) into the average of the Home and Foreign first-order conditions (36)-(37), we obtain a dynamic system of three equations in q_t, $p_t^{L,D}$ and $p_t^{L,A}$ in deviation from steady states. We assume that both Home and Foreign interest rates r_t and r_t^* follow AR(1) processes with AR coefficients ρ. This is therefore also the case for the average interest rate $r_t^A = 0.5(r_t + r_t^*)$ and the interest differential $r_t^D = r_t^* - r_t$. The Online Appendix shows that one of the three equations of the dynamic system can be used to solve for the average long term bond price:

$$p_t^{L,A} = -\frac{1}{1-\lambda\rho} r_t^A \tag{46}$$

with $\lambda = (1-\delta)/R$. A higher average world real interest rate reduces the average long term bond price.

The two remaining equations of the dynamic system can be written as

$$A_1 E_t \begin{pmatrix} q_{t+1} \\ p_{t+1}^{L,D} \end{pmatrix} + A_2 \begin{pmatrix} q_t \\ p_t^{L,D} \end{pmatrix} + A_3 \begin{pmatrix} q_{t-1} \\ p_{t-1}^{L,D} \end{pmatrix} + A_4 r_t^D = 0 \tag{47}$$

The matrices A_1 through A_4 are described in the Online Appendix. They have coefficients that depend on model parameters as well as the variance Σ of excess returns. This dynamic system can be used to solve for $(q_t, p_t^{L,D})'$:

$$\begin{pmatrix} q_t \\ p_t^{L,D} \end{pmatrix} = \sum_{k=0}^{\infty} M_1^k M_2 r_{t-k}^D \tag{48}$$

where M_1 and M_2 are two-by-two matrices and M_1^0 is the identity matrix.

For numerical analysis, we need to make assumptions about the parameters h, γ, ψ, ρ, δ, R and the variance Σ of excess returns. As in the benchmark parameterization of Section 3, we set $h = 0.66$, $\rho = 0.9415$, $\gamma = 50$ and $\psi = 15$. We set $R = 1.0033$ for monthly data, corresponding to a 4 percent anual interest rate. Lustig, Stathopoulos and Verdelhan (2018) consider the returns on 10-year coupon bonds. A 10-year bond with face value of 1 and coupons of $R-1 = 0.0033$ has a Macauley duration of 99.3 months or 8.3 years. We set $\delta = 0.0071$, which yields a Macauley duration of 99.3 months. We use data on real exchange rates and long term bond returns to compute Σ (see Appendix A for data details).[22] Using the symmetry of the model we can write

$$\Sigma = \begin{pmatrix} \sigma_1^2 & \sigma_1^2 - \sigma_{13} & \sigma_{13} \\ \sigma_1^2 - \sigma_{13} & \sigma_1^2 + \sigma_3^2 - 2\sigma_{13} & \sigma_{23} \\ \sigma_{13} & \sigma_{23} & \sigma_3^2 \end{pmatrix} \tag{49}$$

where $\sigma_i^2 = var(er_{i,t+1})$ and $\sigma_{ij} = cov(er_{i,t+1}, er_{j,t+1})$. We therefore compute four moments: the variance of the real exchange rate σ_1^2, the covariance σ_{13} between the real exchange rate and Home real long-term bond return, the variance σ_3^2 of real long-term bond returns and the covariance σ_{23} between the Home and Foreign real long-term bond returns. We use the same standard deviation 0.0271 of the real exchange rate as in the benchmark parameterization of Section 3. For the other three moments we find $\sigma_3 = 0.0206$, $\sigma_{13} = 0.0000538$ and $\sigma_{23} = 0.000267$.

Using this parameterization, we can regress excess returns on the interest differential $r_t^* - r_t$ as in LSV. Table 1 compares the results of three regressions from LSV based on the data to the theoretical regression coefficients from our model. The first column shows the predictability coefficient for the one-month excess FX return $er_{1,t+1} = q_{t+1} - q_t + r_t^* - r_t$. The coefficient of 1.97 is very close to LSV. The second column shows the regression of the one-month excess return on the Foreign long-term bond minus the Home long-term bond $er_{2,t+1} - er_{3,t+1} = q_{t+1} - q_t + r_{t+1}^{L,*} - r_t^L$. The coefficient of 0.34 is slightly lower than LSV. But their coefficient of 0.65 is statistically insignificant. The last column of Table 1 considers the regression coefficient for the monthly excess return of long-term bonds over short-term bonds in the Foreign country minus that in the Home country, $(r_{t+1}^{L,*} - r_t^*) - (r_{t+1}^L - r_t)$. The coefficient is -1.64, again similar in magnitude to LSV.[23] The model with gradual

[22]As mentioned at the start of Section 2, while we focus on interest rate shocks, asset returns are also driven by other shocks that we do not explicitly model. These other shocks only affect the response to interest rate shocks to the extent that they affect Σ.

[23]These three coefficients are consistent with each other as we can write $q_{t+1} - q_t + r_{t+1}^{L,*} - r_t^L =$

portfolio adjustment therefore delivers results consistent with the puzzle uncovered by LSV.

Table 1: PREDICTABILITY WITH LONG TERM BONDS

Regressions on r_t^D			
	Currency excess return $q_{t+1} - q_t + r_t^* - r_t$	Bond excess return $q_{t+1} - q_t + r_{t+1}^{L,*} - r_t^L$	Bond local currency return diff. $(r_{t+1}^{L,*} - r_t^*) - (r_{t+1}^L - r_t)$
Benchmark model	1.97	0.34	-1.64
LSV panel estimate	1.98	0.65	-1.34

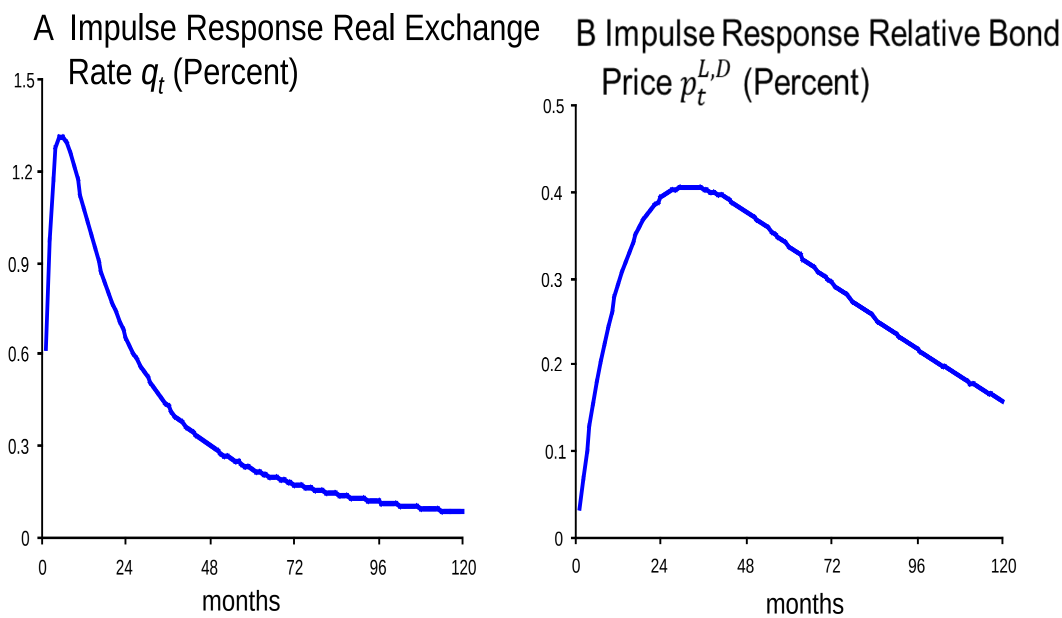
Note : The table shows the slope coefficient of a regression of the dependent variable on the interest differential r^D. The benchmark model is described in the text and the LSV panel estimates are from Lustig, Stathopoulos and Verdelhan (2018), Table 1.

Figures 5 and 6 show some impulse response functions that help shed light on these results. They show the response to a one standard deviation increase in r_t^D. We observe the same delayed overshooting for the real exchange rate that we saw in Section 3. What is new is the response of the relative long-term bond price, shown in chart B of Figure 5. The relative Home bond price $p_t^{L,D}$ rises in response to the shock and then continues to rise for 32 months before it starts to fall. This delayed overshooting for the relative bond price is critical to understanding the results reported above. The higher Foreign interest rates causes especially Foreign investors to reallocate their portfolio from Foreign long-term bonds to Foreign short-term bonds. This lowers the price of Foreign long-term bonds, explaining the increase in the relative price $p_t^{L,D}$ of Home long-term bonds in Figure 5B. However, the process of reallocating from Foreign long-term bonds to Foreign short-term bonds continues over time as a result of gradual portfolio adjustment, leading to a continued decline in the relative price of Foreign bonds. This generates a negative Foreign minus Home excess return of long-term bonds over short-term bonds, as can be seen also in Figure 6B. Even though the Foreign currency is appreciating over time, this is offset by the negative Foreign local excess return of long-term bonds over short-term bonds. The latter dominates from month 5 to 50, as can be

$(q_{t+1} - q_t + r_t^* - r_t) + \left([r_{t+1}^{L,*} - r_t^*] - [r_{t+1}^L - r_t]\right)$

seen from Figure 6A.

Figure 5: IMPULSE RESPONSE REAL EXCHANGE RATE AND RELATIVE BOND PRICE

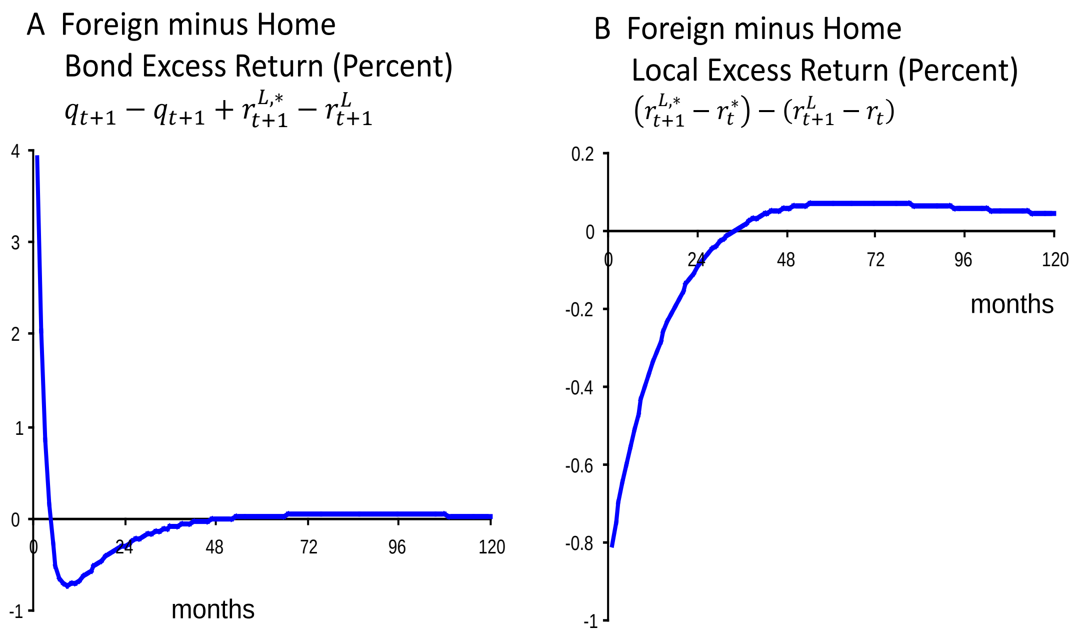


A couple of other aspects of this result are worth emphasizing. First, the real exchange rate overshoots earlier than in the benchmark model of Section 3. This is because the continued negative long-term Foreign bond return weakens the shift towards Foreign assets that is caused by the higher Foreign short-term rate. This also explains the somewhat lower one-month FX excess return predictability coefficient in the model with long-term bonds (1.98 versus 3.26). Second, the Engel coefficient in this parameterization is -34, which again closely matches the results reported in Engel (2016).

5 Conclusion

We have explored the implications of delayed portfolio adjustment for exchange rate dynamics. We have shown that when adjustment is sufficiently gradual it can solve the forward premium puzzle, as suggested by Froot and Thaler (1990). Moreover, it can explain five other puzzles related to the relationship between exchange rates and interest rates. Not surprisingly, gradual adjustment is consistent with

Figure 6: Impulse Response Excess Returns



delayed overshooting. More strikingly, it can explain excess return predictability reversal and the fact that high interest rate currencies are stronger than implied by UIP. This is contrary to the claim of Engel (2016). Gradual portfolio adjustment can also explain why there is no forward premium puzzle for long-term bond returns, as documented by Lustig et al. (2018). Finally, it implies that interest rates in the far future have a smaller impact on the exchange rate than interest rates in a near future, thereby giving an explanation to the forward guidance exchange rate puzzle raised by Galí (2018).

The model is stylized in order to derive basic insights and analytical results. Gradual adjustment has been modeled with an adjustment cost. Our conjecture is that using alternative modeling approaches, such as a constant probability of portfolio adjustment, would be more complex, but yield similar results. The analysis has focused on short-term excess returns. An interesting extension would be to consider returns over longer horizons. This would allow an analysis of longer term relationships, such as Chinn and Meredith (2004), as well as the link between the yield curve and exchange rates.

Appendix

A Data Description

To calibrate the model, we use monthly data for G7 countries over the interval December 1992 to December 2017 (the interval for which all data is available for all countries). Nominal exchange rates are end-of-period from FRED. Prices come from OECD CPI series. Nominal interest rates are end-of-period one-month Eurorates from Datastream. Long-term bond returns come from Benchmark 10Y Datastream Government Total Return Index. The monthly return is computed as $ln(TRI^i_t/TRI^i_{t-1})$ where TRI^i_t is the total return index for country i.

To compute real returns, we compute monthly inflation expectations using a regression of monthly inflation rate on lagged annual inflation. We compute short-term and long-term real return differentials and log real exchange rates for the six countries with respect to the US. We compute the moments of interest for each country pair and take the simple average of these moments.

B Proof of Lemma 1

It is immediate from the definitions of α and D that they are respectively equal to 0 and $1+\gamma\sigma^2$ when $\psi = 0$. To show that they both monotonically rise with ψ, we take their derivatives:

$$\frac{\partial \alpha}{\partial \psi} = \frac{0.5b}{\sqrt{\theta^2 - 4\psi b}}\left(\sqrt{\theta^2 - 4\psi b} - (\theta - 2)\right) \tag{B.1}$$

$$\frac{\partial D}{\partial \psi} = \frac{0.5b}{\sqrt{\theta^2 - 4\psi b}}\left(\sqrt{\theta^2 - 4\psi b} + (\theta - 2)\right) \tag{B.2}$$

It is easy to see that $\sqrt{\theta^2 - 4\psi b}$ is larger than both $\theta - 2$ and $2 - \theta$. This is automatic when these are negative. When they are positive, it follows because $\theta^2 - 4\psi b > (\theta - 2)^2$. The latter can be written as $-\psi b > -\theta + 1$, which holds when substituting $\theta = 1 + \psi b + \gamma\sigma^2$.

Next consider the limit of $\psi \to \infty$. We can write

$$\lim_{\psi\to\infty} \alpha = 0.5 \lim_{\psi\to\infty} \frac{1 - \sqrt{1 - \frac{4\psi b}{\theta^2}}}{1/\theta} \tag{B.3}$$

Since both the numerator and denominator approach 0 when $\psi \to \infty$, we can use $L'Hopital's$ rule:

$$\lim_{\psi\to\infty} \alpha = -0.5 \lim_{\psi\to\infty} \frac{0.5(1/\theta^2)(1-\frac{4\psi b}{\theta^2})^{-0.5}(-4b+8\psi b^2/\theta)}{-b/\theta^2} = 1 \tag{B.4}$$

It is immediate that $D \to \infty$ as $\theta \to \infty$ and $D/\theta \to 1$ when $\psi \to \infty$.

C Excess Return Predictability Coefficients

We will now derive the excess return predictability coefficients

$$\beta_k = \frac{cov(er_{t+k}, r_t^D)}{var(r_t^D)} \tag{C.1}$$

From $q_t = \alpha q_{t-1} + \frac{1}{D-\rho} r_t^D$, we have

$$q_t = \frac{1}{D-\rho}\left(r_t^D + \alpha r_{t-1}^D + \alpha^2 r_{t-2}^D + ...\right) \tag{C.2}$$

Therefore

$$\begin{aligned} &er_{t+k} = q_{t+k} - q_{t+k-1} + r_{t+k-1}^D = \\ &\frac{1}{D-\rho} r_{t+k}^D + \frac{1}{D-\rho}(\alpha-1)\left(r_{t+k-1}^D + \alpha r_{t+k-2}^D + \alpha^2 r_{t+k-2}^D + ...\right) + r_{t+k-1}^D \end{aligned} \tag{C.3}$$

Then

$$\begin{aligned} &cov(q_{t+k} - q_{t+k-1} + r_{t+k-1}^D, r_t^D) = \frac{1}{D-\rho}\rho^k var(r_t^D) + \rho^{k-1} var(r_t^D) + \\ &\frac{1}{D-\rho}(\alpha-1) var(r_t^D)\left(\rho^{k-1} + \alpha\rho^{k-2} + ... + \alpha^{k-2}\rho + \frac{\alpha^{k-1}}{1-\alpha\rho}\right) \end{aligned} \tag{C.4}$$

It follows that

$$\begin{aligned} &\beta_k = \frac{1}{D-\rho}\rho^k + \rho^{k-1} + \\ &\frac{1}{D-\rho}(\alpha-1)\left(\rho^{k-1} + \alpha\rho^{k-2} + ... + \alpha^{k-2}\rho + \frac{\alpha^{k-1}}{1-\alpha\rho}\right) \end{aligned} \tag{C.5}$$

Consider the last term, but not including the ratio at the end of the large bracketed term. We can rewrite this as

$$\frac{\alpha-1}{D-\rho}\alpha^{k-1}\left(\left(\frac{\rho}{\alpha}\right)^{k-1} + ... + \left(\frac{\rho}{\alpha}\right)\right) \tag{C.6}$$

When $\alpha \neq \rho$, we can write it as

$$\frac{\alpha-1}{D-\rho}\alpha^{k-1}\frac{\left(\frac{\rho}{\alpha}\right)-\left(\frac{\rho}{\alpha}\right)^k}{1-\left(\frac{\rho}{\alpha}\right)} \tag{C.7}$$

which can be written as

$$\frac{\alpha-1}{D-\rho}\frac{\rho}{\alpha-\rho}\alpha^{k-1}-\frac{\alpha-1}{D-\rho}\frac{1}{\alpha-\rho}\rho^k \tag{C.8}$$

Adding to this the remaining terms of (C.5), we obtain the expression (21) for β_k in the text when $\alpha \neq \rho$. When $\alpha = \rho$, (C.6) is equal to

$$\frac{\alpha-1}{D-\rho}(k-1)\alpha^{k-1} \tag{C.9}$$

Adding to this the remaining terms of (C.5), we obtain the expression (21) for β_k when $\alpha = \rho$.

D Proof of Proposition 1

Equation (17) shows the impulse response to an interest rate shock. First assume $\alpha \neq \rho$. If the interest rate shock starts at time $t = 0$, and we normalize the shock to $D - \rho > 0$ without loss of generality, it implies that in response to this shock

$$q_t - q_{t-1} = \frac{(1-\rho)\rho^t - (1-\alpha)\alpha^t}{\alpha-\rho} \tag{D.1}$$

This implies that $q_1 - q_0 = \alpha + \rho - 1$. More generally, $q_t < q_{t-1}$ when

$$t > \bar{t} = \frac{ln(1-\rho) - ln(1-\alpha)}{ln(\alpha) - ln(\rho)} \tag{D.2}$$

while $q_t > q_{t-1}$ when $t < \bar{t}$. Below we show that $\partial\bar{t}/\partial\alpha > 0$. Since $\bar{t} = 1$ when $\alpha = 1 - \rho$, it follows that $\bar{t} < 1$ when $\alpha + \rho < 1$. The condition (D.2) is therefore satisfied for all $t \geq 1$, so that $q_t < q_{t-1}$ for all $t \geq 1$. This proves the first part of Proposition 1. When $\alpha + \rho > 1$, $\partial\bar{t}/\partial\alpha > 0$ implies that $\bar{t} > 1$. Therefore the real exchange rate continues to appreciate for at least one additional period after the shock ($t = 1$), and will start to depreciate once $t > \bar{t} > 1$. Finally, when $\alpha = \rho$, we have $q_t - q_{t-1} = \rho^{t-1}(\rho - (1-\rho)t)$ and the same results as those above apply with $\bar{t} = \rho/(1-\rho)$. In this case $\alpha + \rho < 1$ corresponds to $\rho < 0.5$, where $\bar{t} < 1$, and $\alpha + \rho > 1$ implies $\rho > 0.5$, so that $\bar{t} > 1$.

It remains to show that $\partial\bar{t}/\partial\alpha > 0$ when $\alpha \neq \rho$. We have

$$\frac{\partial \bar{t}}{\partial \alpha} = \frac{1}{\alpha(1-\alpha)} \frac{\alpha(ln\alpha - ln\rho) + (1-\alpha)(ln(1-\alpha) - ln(1-\rho))}{[ln(\alpha/\rho)]^2} \tag{D.3}$$

The sign is determined by the numerator in the large fraction. Note that it is positive for $\alpha = 0$ and $\alpha = 1$. The derivative of the numerator with respect to α is $ln(\alpha/\rho) - ln(1-\alpha)/(1-\rho)$, which is positive when $\alpha > \rho$, zero when $\alpha = \rho$ and negative when $\alpha < \rho$. The numerator of the large expression in (D.3) is therefore smallest when $\alpha = \rho$, where it is zero. It is therefore positive for all $\alpha \neq \rho$.

E Proof of Proposition 2

We have

$$\beta_1 = \lambda_1 + \lambda_2 = \frac{1}{D-\rho}\left(D - \frac{1-\alpha}{1-\alpha\rho}\right) = \frac{1}{D-\rho}\frac{1}{1-\alpha\rho}\left(D - \alpha D\rho - 1 + \alpha\right) \tag{E.1}$$

Using that $\alpha D = \psi b$ and $\alpha + D = \theta$, we have

$$\beta_1 = \frac{1}{D-\rho}\frac{1}{1-\alpha\rho}\left(\theta - 1 - \rho\psi b\right) = \frac{1}{D-\rho}\frac{1}{1-\alpha\rho}\left((1-\rho)\psi + \gamma\sigma^2\right)b > 0 \tag{E.2}$$

Next consider the second part of Proposition 2. When $\psi = 0$, we have $\alpha = 0$, $\theta = 1 + \gamma\sigma^2 b$ and $D = \theta$. The second part of Proposition 2 then holds when

$$\frac{1}{D-\rho}\frac{1}{1-\alpha\rho}\left(\gamma\sigma^2 + (1-\rho)\psi\right) > \frac{1}{1+\gamma\sigma^2 b - \rho}\gamma\sigma^2 \tag{E.3}$$

This implies

$$(D-\rho)(1-\alpha\rho)\gamma\sigma^2 < (\gamma\sigma^2 + (1-\rho)\psi)(1 + \gamma\sigma^2 b - \rho) \tag{E.4}$$

Collecting terms multiplying $\gamma\sigma^2$ and using $D\alpha = \psi b$, we have

$$(D - \psi b\rho + \alpha\rho^2 - 1 - \gamma\sigma^2 b - (1-\rho)\psi b)\gamma\sigma^2 < (1-\rho)^2\psi \tag{E.5}$$

Using $D = \theta - \alpha = 1 + \psi b + \gamma\sigma^2 b - \alpha$, this becomes

$$-\alpha(1-\rho^2)\gamma\sigma^2 < (1-\rho)^2\psi \tag{E.6}$$

which clearly holds.

F Proof of Proposition 3

It first useful to characterize the signs of λ_1 and λ_2. The value of ψ where $\lambda_1 = 0$ is $\bar{\psi}$ defined in the text. Moreover, the value of ψ where $\alpha = \rho$, is $\bar{\psi} + \rho/b$. We can write the following Lemma:

Lemma 2. *There are three regions that determine the sign of λ_1 and λ_2:*

- $0 < \psi < \bar{\psi}$: $\lambda_1 > 0$ *and* $\lambda_2 > 0$
- $\bar{\psi}_1 < \psi < \bar{\psi} + \rho/b$: $\lambda_1 < 0$ *and* $\lambda_2 > 0$
- $\psi > \bar{\psi} + \rho/b$: $\lambda_1 > 0$ *and* $\lambda_2 < 0$

When $\psi = 0$, $\lambda_1 > 0$ and $\lambda_2 = 0$. When $\psi = \bar{\psi}$, $\lambda_1 = 0$ and $\lambda_2 > 0$.

Proof. First consider λ_1. Since $D - \rho > 0$, the sign is determined by

$$D - \rho \frac{\alpha - 1}{\alpha - \rho} \tag{F.1}$$

$\bar{\psi}$ is defined such that this term is equal to 0. To see this, setting (F.1) equal to zero and substituting the expressions (13) and (14) for α and D, we have

$$\frac{\theta + \sqrt{\theta^2 - 4\psi b}}{2} = \rho \frac{\theta - \sqrt{\theta^2 - 4\psi b} - 2}{\theta - \sqrt{\theta^2 - 4\psi b} - 2\rho} \tag{F.2}$$

Cross multiplying delivers

$$\psi b = \rho\theta - \rho \tag{F.3}$$

Substituting $\theta = 1 + \psi b + \gamma\sigma^2 b$ gives $\psi = (\rho/(1-\rho))\gamma\sigma^2 = \bar{\psi}$.

Now go back to (F.1). It is immediate that this term is positive when $\alpha > \rho$, so that $\lambda_1 > 0$. This happens when $\psi > \bar{\psi} + \rho/b$. So we need to consider $\psi < \bar{\psi} + \rho/b$, so that $\alpha < \rho$. Consider D and $\rho(\alpha - 1)/(\alpha - \rho)$ as functions of ψ. It follows from Lemma 1 that both rise monotonically with ψ. At $\psi = 0$, so that $\alpha = 0$, $D > \rho(\alpha - 1)/(\alpha - \rho)$. But $\rho(\alpha - 1)/(\alpha - \rho)$ rises to infinity as α approaches ρ from below, which happens when ψ approaches $\bar{\psi} + \rho/b$ from below. Therefore the schedule for $\rho(\alpha - 1)/(\alpha - \rho)$ must cross that for D between $\psi = 0$ and $\psi = \bar{\psi} + \rho/b$. This happens at $\psi = \bar{\psi}$. It follows that $\lambda_1 > 0$ when $\psi < \bar{\psi}$, $\lambda_1 = 0$ when $\psi = \bar{\psi}$ and $\lambda_1 < 0$ when $\bar{\psi} < \psi < \bar{\psi} + \rho/b$.

Next consider λ_2. It is immediate from (23) that $\lambda_2 < 0$ when $\alpha > \rho$, which happens when $\psi > \bar{\psi} + \rho/b$. So consider $\psi < \bar{\psi} + \rho/b$, so that $\alpha < \rho$. (23) then implies that $\lambda_2 > 0$ when $1/(1-\alpha\rho) < \rho/(\rho-\alpha)$. Cross multiplying, this gives $\alpha > \alpha\rho^2$. This holds as long as $\alpha > 0$ or $\psi > 0$. When $\psi = 0$, $\alpha = 0$ and $\lambda_2 = 0$. $\square$

The first part of Proposition 3 follows immediately from Lemma 2. When $\psi = 0$, we have $\beta_k = \lambda_1 \rho^{k-1}$, which is positive ($\lambda_1 > 0$) and monotonocally declines to zero as k rises. When $0 < \psi < \bar{\psi}$, Lemma 2 says that both λ_1 and λ_2 are positive. Since $0 < \alpha < 1$, it follows that $\beta_k = \lambda_1 \rho^{k-1} + \lambda_2 \alpha^{k-1}$ is positive and monotonocally declines to zero with an increase in k. Finally, when $\psi = \bar{\psi}$, Lemma 2 implies that $\beta_k = \lambda_2 \alpha^{k-1}$, with $\lambda_2 > 0$ and $0 < \alpha < 1$. It again follows that β_k is positive and declines monotonically to zero as k rises.

Next consider the second part of Proposition 3, where $\psi > \bar{\psi}$. It is immediate from (21) that $\lim_{k\to\infty} \beta_k = 0$. When $\psi \neq \bar{\psi} + \rho/b$, so that $\alpha \neq \rho$, we can write

$$\frac{\beta_k}{\alpha^{k-1}} = \lambda_1 \left(\frac{\rho}{\alpha}\right)^{k-1} + \lambda_2 \tag{F.4}$$

$$\frac{\beta_k}{\rho^{k-1}} = \lambda_2 \left(\frac{\alpha}{\rho}\right)^{k-1} + \lambda_1 \tag{F.5}$$

The sign of β_k corresponds to the sign of either of the two right hand side expressions. Assume first that $\bar{\psi} < \psi < \bar{\psi} + \rho/b$, so that $\alpha < \rho$, $\lambda_1 < 0$ and $\lambda_2 > 0$ (Lemma 2). Then (F.4) implies that $\beta_k > 0$ when $k < \bar{k}_1$ and $\beta_k < 0$ when $k > \bar{k}_1$ with

$$\bar{k}_1 = 1 + \frac{ln(-\lambda_2/\lambda_1)}{ln(\rho/\alpha)} \tag{F.6}$$

We know from Proposition 2 that $\beta_1 = \lambda_1 + \lambda_2 > 0$, so that $\lambda_2 > -\lambda_1$, which implies that $\bar{k}_1 > 1$. The $\bar{k}$ in Proposition 3 is the first whole number larger than $\bar{k}_1$.

A similar reasoning applies to the case where $\psi > \bar{\psi} + \rho/b$, so that $\alpha > \rho$, $\lambda_1 > 0$ and $\lambda_2 < 0$ (Lemma 2). Then (F.5) implies that $\beta_k > 0$ when $k < \bar{k}_2$ and $\beta_k < 0$ when $k > \bar{k}_2$ with

$$\bar{k}_2 = 1 + \frac{ln(-\lambda_1/\lambda_2)}{ln(\alpha/\rho)} \tag{F.7}$$

From Proposition 2, $\lambda_1 > -\lambda_2$, so that $\bar{k}_2 > 1$. Again the $\bar{k}$ in Proposition 3 is the first whole number larger than $\bar{k}_2$.

Finally consider the special case of $\psi = \bar{\psi} + \rho/b$, so that $\alpha = \rho$. In that case (21) implies that $\beta_k > 0$ when $k < \bar{k}_3$ and $\beta_k < 0$ when $k > \bar{k}_3$ with

$$\bar{k}_3 = 1 + \frac{D - (1/(1+\rho))}{1-\rho} > 1 \tag{F.8}$$

Again the $\bar{k}$ in Proposition 3 is the first whole number larger than $\bar{k}_3$.

G Proof of Proposition 4

The Engel condition is $\sum_{k=1}^{\infty} \beta_k < 0$. We will focus here on $\alpha \neq \rho$, which is sufficient as the β_k are continuous at $\alpha = \rho$. Then

$$\sum_{k=1}^{\infty} \beta_k = \lambda_1 \frac{1}{1-\rho} + \lambda_2 \frac{1}{1-\alpha} = \frac{1}{1-\rho} - \frac{1}{(D-\rho)(1-\alpha\rho)} \tag{G.1}$$

The Engel condition can therefore be written as $(D-\rho)(1-\alpha\rho) < 1-\rho$. Using that $D\alpha = \psi b$ and $D = \theta - \alpha$, we can also write it as

$$\alpha > \frac{\psi b}{1+\rho} + \frac{\phi}{1-\rho^2} \tag{G.2}$$

where $\phi = \gamma\sigma^2 b$. Using $\theta = 1 + \psi b + \phi$ and the definition of α, this becomes

$$\sqrt{(1+\psi b+\phi)^2 - 4\psi b} < 1 - \frac{(1+\rho^2)\phi}{1-\rho^2} - \frac{1-\rho}{1+\rho} b\psi \tag{G.3}$$

We can, for convenience, refer to the left and right hand sides of (G.3) as $f(\psi)$ and $g(\psi)$. $f(\psi)$ is a convex function, which is always positive and is symmetric around the axis $\psi = (1-\phi)/b$, where it reaches a minimum. $g(\psi)$ is a line with a negative slope. Moreover $f(0) > g(0)$. These properties imply that there are only two possibilities. Either $f(\psi)$ remains above $g(\psi)$ for all ψ and therefore the Engel condition is never satisfied, or $f(\psi)$ crosses $g(\psi)$ twice and the Engel condition is satisfied for an intermediate range of ψ that we will refer to as the interval (ψ_1^E, ψ_2^E), with the boundaries of the interval equal to the solutions to $f(\psi) = g(\psi)$.

To consider the solutions of $f(\psi) = g(\psi)$, we square both sides. We need to be careful doing so. If $f^2(\psi) = g^2(\psi)$ has two solutions, it is either the case that $f(\psi) = g(\psi)$ for both solutions or $f(\psi) = -g(\psi)$ for both solutions. We know that $f(\psi)$ is convex with an axis of symmetry $\psi = (1-\phi)/b$. If it crosses the symmetric

$f(\psi)$ twice, there will be two solutions that average to less that $(1-\phi)/b$ since $g(\psi)$ is a negatively sloping line.

We can write $f^2(\psi) = g^2(\psi)$ as

$$A\psi^2 + B\psi + C = 0 \tag{G.4}$$

where

$$A = \rho b^2 \tag{G.5}$$

$$B = b\rho(\phi - 1 - \rho) \tag{G.6}$$

$$C = \frac{\phi}{(1-\rho)^2}\left(1 - \rho^2 - \rho^2\phi\right) \tag{G.7}$$

In order for the Engel condition to be satisfied over some intermediate range (ψ_1^E, ψ_2^E) for ψ, two conditions need to hold. First, as discussed above, it must be the case that the average of these solutions is less than $(1-\phi)/b$, which implies $\phi < 1-\rho$. Second, it must the case that two solutions to $f^2(\psi) = g^2(\psi)$ exist, which requires $B^2 - 4AC > 0$, which can be written as

$$\rho\phi^2 - 2(2-\rho)(1-\rho)\phi + \rho(1-\rho)^2 > 0 \tag{G.8}$$

This is a quadratic that is positive when $\phi = 0$, then turns negative and then positive again. When $\phi = 1-\rho$, the quadratic is negative, so that both $\phi < 1-\rho$ and (G.8) will be satisfied when ϕ is between zero and the smaller of the two solutions to (G.8) as an equality. The latter is equal to

$$\bar{\phi} = \frac{1-\rho}{\rho}\left(1 - \sqrt{1-\rho}\right)^2 \tag{G.9}$$

To summarize, the Engel condition is satisfied if and only if $\phi < \bar{\phi}$ and $\psi_1^E < \psi < \psi_2^E)$, where $\bar{\psi}_1^E$ and ψ_2^E are the solutions to the quadratic (G.4).

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