Math Matters: Education Choices and Wage Inequality*

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Abstract

Standard SBTC is a powerful mechanism in explaining the increasing wage gap between educated and uneducated individuals. However, SBTC cannot explain within-group wage inequality in the US. This paper provides an explanation for the observed intra-college group inequality by showing that the top decile earners' significant wage growth is underpinned by the link between *ex ante* ability, math-heavy college majors and highly quantitative occupations. We develop a general equilibrium model with multiple education outcomes, where wages are driven by individuals' *ex ante* abilities and acquired math skills. A large portion of within-group and general wage inequality is explained by math-biased technical change (MBTC).

JEL classification: E20, E24, E25, I20, I24, J24, J31

Keywords: wage inequality, SBTC, college majors, occupations, mathematics abilities

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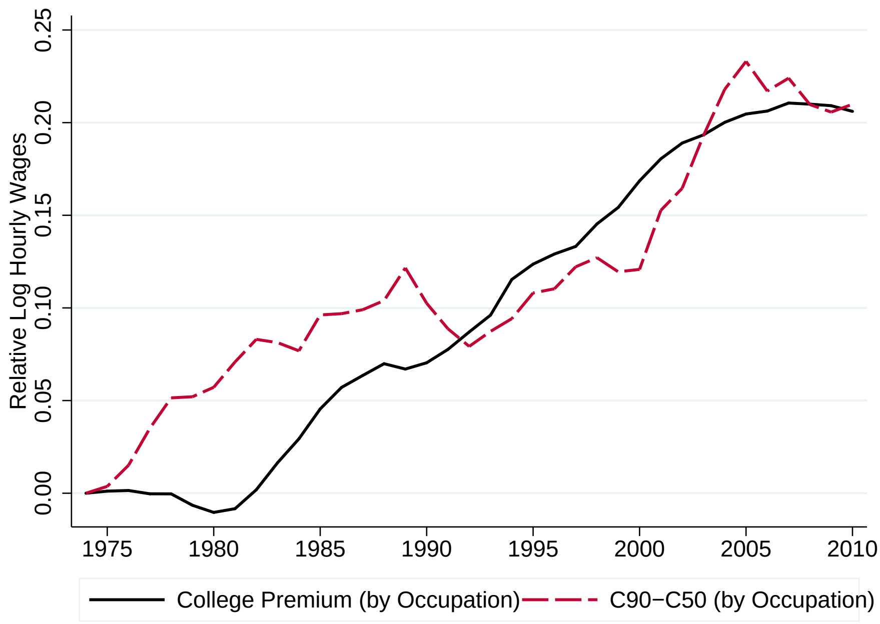


Source: IPUMS-CPS (see King et al., 2010). Log wages are average log residual wages by occupation. Where residual wages are computed from a regression of hourly log wages of full-time (at least 35 hours of work and 40 weeks per year) male workers aged 25 to 59 on age, age squared, dummies for education, race, state, and marital status.

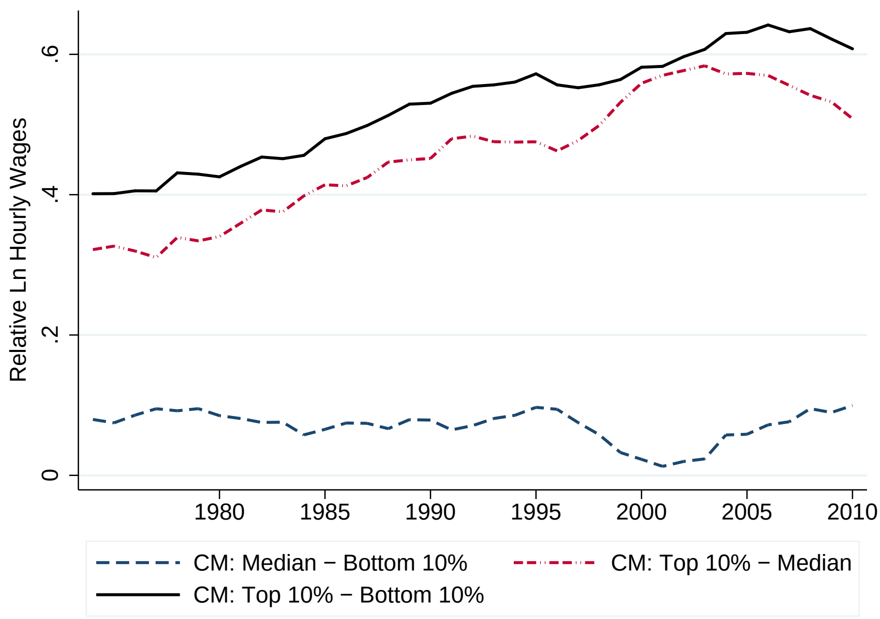
Figure 1: US Wage Inequality

1 Introduction

> Math is a language of logic. It is a disciplined, organized way of thinking. There is a right answer; there are rules that must be followed. More than any other subject, math is rigor distilled. Mastering the language of logic helps to embed higher-order habits in kids' minds: the ability to reason, for example, to detect patterns and to make informed guesses. Those kinds of skills [have] rising value in a world in which information [is] cheap and messy.[1]

The US has seen a large rise in wage inequality since the mid-1970s (see Figure 1). While standard skill-biased technical change (SBTC) can match the average wage trend as

[1] Amanda Ripley, *The Smartest Kids in the World: And How They Got That Way*



Source: IPUMS-CPS (see King et al., 2010). Log wages are defined as in Figure 1. Percentiles are determined by sorting occupations according to their numerical aptitude content in 1980.

Figure 2: US Wage Inequality by Occupation Math Content

measured by the college wage premium (i.e., college/non-college), the large and increasing within-group wage inequality is ignored, e.g., increases in inequality between the 90th percentile of college graduates and the 50th percentile (see Figure 1, line "C90-C50"). [2]

In this paper we argue that this rising within college wage inequality can be explained by the increasing importance of math skills in the labor market. Figure 2 decomposes within college wage inequality by comparing residual wages between occupations ranked according to their numerical requirements.[3] Time trends show that college occupations with a median math requirement have not gained compared to occupations with the lowest (bottom 10% of math). In contrast, occupations with very high numerical contents (top

[2]Figure 1 graphs wage residuals average by occupations effectively removing all individual idiosyncratic wage variation (or luck).

[3]Numerical requirements are taken from the Dictionary of Occupational Titles (DOT 1977, 1991) and O*net information.

90%) have gained substantially since 1975 compared to both the median and low math occupations. In magnitudes the increase between the top math occupation and the median is similar to the within college increase documented in Figure 1 of about 20 percentage points. Therefore, we conjecture that labor augmenting technical change biased toward math skills can account for a large part of the observed within-group inequality for college graduates.

While Figure 2 points to substantial differences between college occupations with and without high numerical content over time, there could be possible other correlated factors explaining the overall increase of within college inequality. Other theories such as the emergence of routine biased technical change (RBTC) within the task biased literature (see for example Autor et al., 2003; Autor & Dorn, 2013) have been successful in explaining large increases in wage inequality across both the US and European countries.[4]

To provide further evidence in favor of our theory when explaining within college wage inequality, we provide some suggestive data evidence in two ways. First, we borrow the methodology from the RBTC literature and directly compare a potential RBTC hypothesis with our math biased technical change (MBTC) hypothesis for college graduates. The results on this comparison show three important pattern for college graduates, (1) employment has mostly increased for occupation in the lowest skill percentiles (sorted by wage rates in 1980); (2) wages have increases most for occupations in the highest skill percentiles, with a slight overall U-shape pattern; and (3) occupations with the highest employment increase have higher routine task requirements, while occupations with the highest wage increase have the highest abstract skill requirements. Second, we follow Firpo et al. (2009) and provide a partial equilibrium wage decomposition over time by different skill requirements. While this is a simple accounting exercise and partial equilibrium by nature it provides further insight into the unique importance of numerical requirements

[4]RBTC focuses on how technology has replaced mostly routine tasks, usually associated with middle-wage occupations, through computers and left employment opportunities only in highly abstract (high-wage) or manual (low-wage) occupations.

when explaining within college, but not necessarily overall wage inequality.

Given the supportive data evidence on the rising importance of math skills over time, but lack of good time-series data to study general equilibrium effects (both demand and supply) we develop a model that links innate abilities and acquired math/general skills with rising wage inequality.[5] While the main driving force over time is a mechanism of MBTC, the micro-foundations of the model are based on three facts, (1) highly quantitative occupations have exhibited increasing relative wages since the mid-1970s as depicted in Figure 2; (2) the numerical skill content of occupations and the math content of college majors are highly correlated; and (3) students attempt to study majors with the highest math content, but are constrained by their initial abilities. Therefore, the model starts with an education choice of attending college or directly enter the labor market at age 20. Individuals attempting college earn new math and further general skills if successful. That is, we directly assign individuals math credits subject to ability constraints, with some individuals dropping out of college. Individuals supply both their *ex post* ability and any acquired math skills to the labor market. Only college graduates can supply the math skills after graduation. Firms hire college and non-college labor. Wage inequality is driven by both generic SBTC and specific MBTC.

Since the focus of this paper is to assess the contribution of MBTC, coupled with innate ability constraints, to the increasing wage inequality in the US from 1975 to 2010, we first calibrate the model to 1975 and then increase skill-biased and math-biased productivity growth rates to match the rise in college attainment and the college/math wage premium. Given the same innate distribution of skill in 1975 and 2010, we can measure the contribution of MBTC and innate constraints to the rise in wage inequality over time. The calibrated model does well in generating the main trends over time, such as college attendance, college students' abilities and drop out rates. The counterfactuals highlight the strength of MBTC as a determinant of intra-college graduate income inequality. With

[5] Note, we use the terms "initial-" and "*ex ante*" ability interchangeably. Initial or *ex ante* ability refers to the ability an individual has upon completing high school.

SBTC alone, the college wage premium is matched through all college graduate wages growing over time, which ignores the observed intra-group trends. In contrast, a combination of SBTC and MBTC matches both the aggregate and intra-group trends observed in the data: the stagnation of wages at the bottom of the college graduate group and the substantial increase in wages for the top college graduates. While high ability individuals respond by studying more math, the bottom of the college graduates are unable to respond because of their limited skill set. Nonetheless, given the rising college premium, more individuals attend college, decreasing average ability (and increasing the variance) of college students. This phenomena feeds back into an even larger math gap between the top and bottom students, increasing wage inequality even further. That is, MBTC is a type of SBTC to which some people cannot respond because of their ability constraint. Given empirical evidence, we argue that math is a valid proxy/measure for these skills.

Section 2 summarizes the related literature. Section 3 provides (1) a comparison with the RBTC mechanism focusing on college graduates; (2) evidence of MBTC focusing on a partial equilibrium wage decomposition; and (3) the key facts underlying the microfoundations of the general equilibrium model. The general equilibrium model is outlined in Section 4, Section 5 explains the calibration procedure, and Section 6 provides analytical results. Section 7 concludes.

2 Literature

The theory of MBTC put forth in this paper is complimentary to the task-biased technical change (TBTC) literature first developed by Autor et al. (2003). In order to study a rise in wage inequality in the US, the authors decompose occupation requirements into three task types: manual (hand and finger dexterity), routine (repetitive) and abstract (analytical). Generally, the low, middle and high portions of the income distribution are linked to manual, routine and abstract tasks, respectively. Therefore, a rise in abstract tasks and a fall in routine tasks can generate the wage polarization observed in the US.

The abstract measure used in the TBTC literature is composed of two parts, with one capturing managerial/interpersonal tasks and the other measuring mathematical skills. MBTC focuses on the second measure, as math skills can be measured at an individual level and cleanly linked to educational choices. Since math skills can be acquired, the supply of these skills can most likely be influenced through education systems and individual choices. In contrast, beyond significant measurement issues, linking interpersonal skills to acquired knowledge is difficult. Moreover, unlike the TBTC literature, we focus on explaining intra-education group inequality linked to education choices, rather than overall wage polarization in the US.

Intra-education group income inequality motivate Altonji et al. (2012), who find that male electrical engineers earn 51.6 percent more than male education majors, which is comparable to the college wage premium of 57.7 percent. More specifically, a pronounced wage gap between low- (e.g., communication, psychology, music and drama) and high-math credit majors (e.g., mathematics, engineering, economics, finance) is documented in Table 1 of Altonji et al. (2012). Given this heterogeneity, low- and high-wage college majors cannot be split along a routine/abstract task dimension. Due to the empirical focus of their research, the authors only estimate disaggregated cross-sectional returns to college majors with associated math and verbal SAT scores. The authors note that this area of research is relatively unexplored, but is important for understanding the structural mechanisms underpinning the *ex post* outcomes of higher education. A comprehensive review of the existing empirical studies on the returns to college major can be found in Table 2 of Altonji et al. (2012). Specifically, the authors note that there is lack of research explaining why individuals choose different education types, and how this translates into occupational choices. A crucial difference between our research and Altonji et al. (2012) is, by using the information on mathematic skill requirements within occupations from the Dictionary of Occupational Titles[6] (DOT), we show that mathematics-focused majors are

[6]Dictionary of Occupational Titles, 1977 and 1991

highly correlated with *ex post* wage outcomes through the occupational choices available to these majors. We hypothesize that wage inequality is driven, to a large extent, by individuals' initial abilities, which limit education options and, consequently, occupational choices.

Addressing similar wage discrepancies as Altonji et al. (2012), Silos & Smith (2012) look at the trade-off between acquiring specific and targeted human capital. They concentrate on individuals' choices between education paths leading to specific occupations versus education paths that have broader applicability, and thus more occupational choice. The authors show that policies directed at occupation-specific human capital accumulation lead to lower income growth and lower inequality. MBTC sits within the broader educational transition described above. We emphasize the importance of the skill types accumulated, with particular attention given to mathematics as either a specific necessary ability or as a strong indicator of associated abilities. Those who major in math-intensive areas may initially sort into high-wage occupations, with little incentive of switching (to alternative) occupations.

Carneiro et al. (2011) and Eisenhauer et al. (2013) explore another dimension of intra-education group income inequality and find that the returns to college enrollment are approximately zero for low ability individuals, and possibly negative. MBTC is a potential mechanism that explains this observation, as math-light college majors, without exception, occupy the bottom of the college group wage distribution. Furthermore, there is a significant mass of college graduates with zero and three college math credits.

The idea of SBTC found in Acemoglu (2002), which builds upon the empirical work of Bartel & Lichtenberg (1987) and Autor et al. (1998), formalizes a model in which the labor augmenting technical change is divided along the education dimension (college/non-college). This model has become a workhorse for analyzing and explaining the persistent increase in the relative wages of college graduates. Our work builds on this existing framework by focusing on the distributional wage changes between college graduates. We add a separate mechanism that approximates the specific skills driving wage inequality intra-college graduates.

The college attendance mechanism in this paper is loosely based on Hendricks & Schoellman (2014). In that paper the authors look at the discrete education choices of individuals (i.e., high school, some college, and college), focusing on *ex ante* abilities as measured by IQ scores. Their results show that one-third of the college wage premium and one-fourth of its growth is driven by ability ("ability premium"). While looking at ability as a driver of wage outcomes, Hendricks & Schoellman (2014) define broad education categories that mask the sub-group mainly driving wage inequality: the top earning college graduates, who exhibit strong mathematical abilities.

3 Data

This section has three parts, we first draw a comparison with the RBTC hypothesis but focusing exclusively on college graduates. Second, we preform a partial equilibrium accounting exercise to assess the importance of different skills in explaining the observed rise in wage inequality from the late 1970s to 2010. Lastly, we provide some evidence for the micro-foundations of the general equilibrium model developed in this paper.

3.1 RBTC for College Graduates

Using the skill classification as in Autor & Dorn (2013) we sort college occupations by their mean 1980 wage.[7] Using the same smoothing technique, we compute changes in employment shares for college men from 1980 to 2008 (see Figure 3a) and wage growth by skill percentile (see Figure 3b).

College graduate wage growth exhibits a slight U-shape with most of the rise concentrated at the top of the skill distribution. In contrast, increases in employment occurred mostly at the bottom of the skill distribution suggesting that evermore college graduates

[7]Given that we only focus on college graduates, the resulting skill ranking is not identical.

(a) Employment Change

(b) Wage Growth

Source: 1980 Census and 2008 ACS. Data from Acemoglu & Autor (2011).

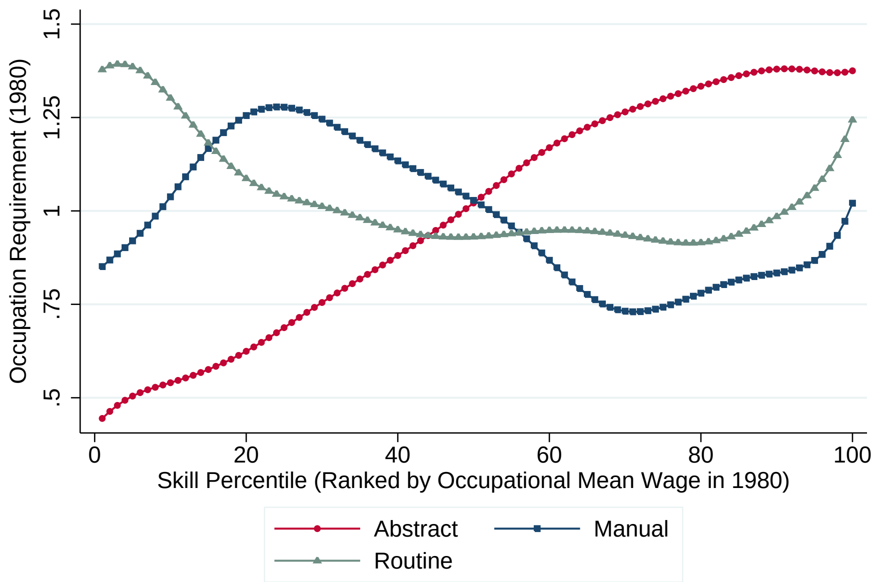
Figure 3: RBTC for College Graduates

are bunching at occupations that are falling behind in terms of wage growth. Figure 4 shows skill sets in 1980 (using the Autor & Dorn (2013) classifications) by skill percentile. Each skill measure is normalized by average skill used by all college graduates. Not consistent with an RBTC story for rising within college wage inequality, low skill occupation have high routine (repetitive) task requirements, while high wage growth occupations have high abstract (analytical and managerial) skill requirements. In conclusion, while the rise in college wage inequality is related to abstract skills, it does not seem to support a routine-biased hypothesis.

3.2 Decomposing Rising Wage Inequality by Skill Types

Abstract skills are defined as a combination of numerical and managerial skills. Given data availability in the DOT 1977, DOT 1991 to O*net surveys, we focus on verbal and numerical aptitudes plus the non-cognitive skills used in Autor et al. (2003).[8] We proceed in doing a wage decomposition following Firpo et al. (2009) using a rercentered influence function (RIF) regression for both variances and quantiles. Essentially, an alternative regression

[8]In Appendix **??** we also include a general intelligence measure available in the DOT 1977/1991 surveys.



Source: 1980 Census and 2008 ACS. Data from Acemoglu & Autor (2011).

Figure 4: 1980 Task Requirements for College Graduates

approach where the estimated coefficient can be interpreted as the effect of increasing the mean value of X on the unconditional quantile Q_r, similar to a standard Oaxaca-Blinder decomposition. Given the presence of general equilibrium effects that cannot be captured here, this exercise should only be viewed as a simple accounting exercise providing suggestive evidence for MBTC. Decompositions are provided for both college and non-college graduates to highlight the importance of numerical skills only within college. Moreover, to avoid confounding results from the oil crisis and the financial crisis, we compare the early 1980s (1980-1985) with the early 2000s (2000-2005) periods. Residual wages are as in Figure 1, that is we only focus on wage inequality by occupation. We compute comparable measure of verbal/numerical aptitudes and non-cognitive skills from 1980 to 2005 using the DOT 1977, 1991 and O*net 2002-2005 surveys.[9]. Table 1 provides the results for the

[9]See Appendix **??** for details

Table 1: Decomposition (1980s-2000s)

	Inequality Measures (C+)			Education Comparison	
	90-10	90-50	50-10	Var (C+)	Var (LTC)
Unadjusted change	.2298	.1627	.0671	.0213	.0070
Composition effects attributable to					
DCP	-.0054	- .0019	-.0035	-.0016	.0057
Verbal	.0031	.0021	.0010	.0005	-.0054
Math	.0611	.0410	.0201	.0210	0
Wage structure effects attributable to					
DCP	.0448	.0050	.0398	-.0101	-.0135
Verbal	-.3206	-.0161	-.3046	-.1181	.0299
Math	.0760	.0480	.0281	.0209	.0011

Notes: CPS data see notes in Figure 1, using mean wage

following two counterfactual exercise given the wage decomposition

$$\Delta_{2000s-1980s} = \Delta_X + \Delta_p$$

1. Composition effect: Everything, but the distribution of X is the same for the two time periods (1980s vs 2000s)

2. Wage structure: Everything, but the returns to X are the same for the two time periods

Within college graduates we record a considerable larger increase in log wage inequality for the 90th-10th percentile and the 90th-50th percentile than the 50th-10th percentile as evidence in Figure 1 and 2. Moreover, math composition and wage structure effects contribute roughly equally to the rise in wage inequality for the 90th-10th and 90th-50th percentile. Verbal aptitudes have none or a negative effect, while non-cognitive skills (DCP) have also contributed to a rise in inequality but to a smaller degree than math aptitudes. In magnitude, numerical aptitudes can account for over 50 percent of the rise in inequality by log percentile. The contribution of math skills (both composition and wage effects) is even larger for the variance decomposition. In contrast, for non-college graduates (last column labeled “LTC”) only returns to verbal skills produce a sizable increase in inequality over time.

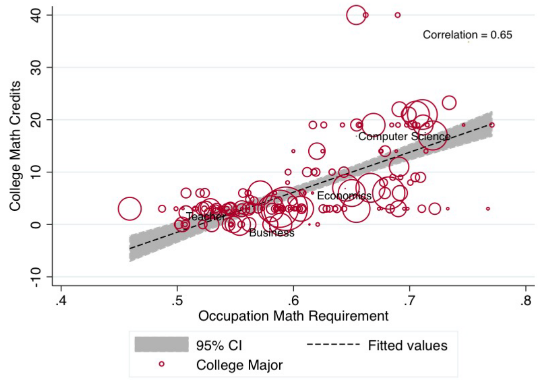
3.3 Skills, Majors and Occupations

The general equilibrium model developed in Section 4 relies on three facts listed in the introduction and expanded upon in this section. Together, these three points present a coherent story of *ex ante* mathematical ability dictating college major options, from which occupations and, ultimately, wages are determined. Those with higher mathematics abilities pursue math-heavy majors and occupations. These particular occupations also enjoy the highest wages. Furthermore, the math intensive majors that lead to higher wage occupations are increasingly shunned by each subsequent generation of college degree holders. This shift away from math-heavy majors further exacerbates wage inequality.

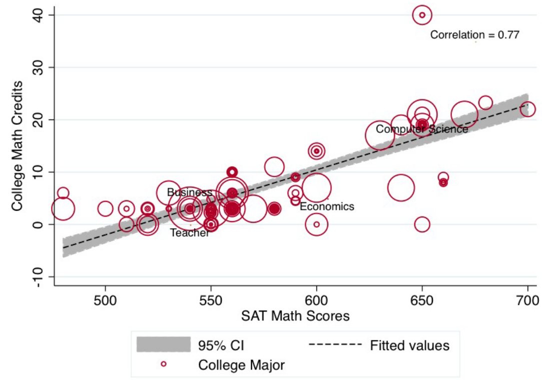
Figure 5a depicts the relationship between college majors’ math credits and the numerical skill requirements of occupations in 2010 for individuals aged 23 and 62. The figure uses the individual-level observations with college major and occupation information from the American Community Survey (ACS), combined with the DOT numerical job requirements.[10] All individuals are first grouped by their college major and the average occupation math requirement is computed, as there are multiple occupation outcomes within each college major. This figure shows that occupation-specific math skills are highly correlated with college-level math credits by college major, with a 0.65 correlation coefficient.

Looking at the initial characteristics that lead to college major sorting, Figure 5b merges college-level math credits and mean SAT Math scores by college major from the National Center for Education Statistics (NCES) to individuals in the ACS. The results illustrate that *ex ante* abilities are correlated with college-level math credits with a correlation coefficient of 0.77. Thus, those with high-math abilities prior to college, as measured by the average SAT Math scores of those graduating within a specific college major, are

[10] The ACS 2010 is used throughout. The ACS 2009 is the first year in which college major is included. Note that the trends observed in the ACS 2010 are virtually identical to the ACS 2009.



(a) Occupation Math Requirements and College-Level Math Credits by Major



(b) SAT Math Scores and College-Level Math Credits by Major

Source: NCES and 2010 ACS.

Figure 5: Skills, Majors and Occupations

more likely to graduate from math intensive college majors, as measured by college-level math credits.

The correlation between SAT I scores (math and verbal combined) and math credits is 0.65. By construction, SAT I scores (general ability) are highly correlated with SAT Math (math ability) at 0.94.

4 Model

As this research is focused on the evolution of wage inequality over time, a general equilibrium model is required to provide the time dynamics that would invariably be ignored by estimating a regression model on the cross sections of available data. The overlapping generations model used here features a unit mass of finitely lived agents and a single representative firm.

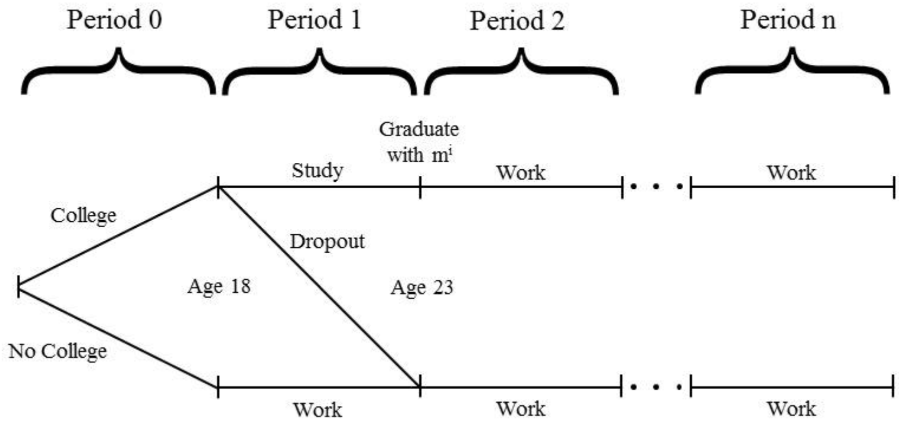


Figure 6: Timeline for Individuals

4.1 Individuals

The model is loosely based on Hendricks & Schoellman (2014). However, the schooling decision is modified to incorporate a choice between "college majors" and the fact that individuals face constraints based on their ability. Figure 6 provides the basic model timeline for individuals. For each generation, there are two primary periods with all decisions taking place in period 0 and the realization of the decisions occurring in period 1. During period 0, individuals choose to either enter or forgo college (college or non-college). At the beginning of period 1, individuals are about 18 years old. If they attempt college there are two outcomes: (1) drop out/fail or (2) graduate with a specific degree characterized by specific acquired math skills, m^i. Both college graduates and dropouts enter the labor force in period 2, with individuals who drop out/fail losing one period of income as an opportunity cost of attempting college. Each subsequent period, individuals supply their general ability and acquired math skills to firms in exchange for wages.

Individuals' decisions within the model are straightforward. Individuals decide to pursue college education or enter the labor market in period 0. This is the only choice available to individuals in this model. Dropping out is determined by a single ability cutoff. The college major "choice" is simplified to a direct mapping from human capital, h_m, to acquired math skills, m, without an explicit choice. Arcidiacono et al. (2012) and Zafar (2013) find strong evidence that men optimize their education choices in order to earn the highest wage return. Also exploring men's education choices, Paglin & Rufolo (1990) find empirical evidence that men choose college majors based on their math ability. This result is similar to that of Stinebrickner & Stinebrickner (2014), who show that many people attempt math-heavy college majors, but learn about their abilities through failure. These people move into college majors with lighter math loads, with the failure and "drop" process repeating until the student's ability is matched to the math content of the college major or they drop out. Thus, for men, combining these results points to a direct mapping from initial ability to the highest possible college math outcome.

The education choice is determined by weighing the financial benefits against the utility and opportunity costs (lost wages) of studying. Markets are complete, such that income maximization and consumption maximization yield the same results, with discounting of $\beta = \frac{1}{1+r}$. Thus, the individuals' objective function is:

$$\max_{s^i} \quad \Bigg\{ \sum_{t=2}^{N} \left(\frac{1}{1+r}\right)^t E\big[p(\theta)\omega_{et}(\theta, \theta_m) + (1 - p(\theta))\omega_{ut}(\theta)\big] - \zeta^i, \\ \sum_{t=1}^{N} \left(\frac{1}{1+r}\right)^t E(\omega_{ut}(\theta)) \Bigg\}. \tag{1}$$

Individuals make a schooling choice, $s^i \in \{e, u\}$, between attending college or not. Individual wages are given by ω^i_{jt}, with $j = \{e, u\}$ denoting the college graduate (educated) or non-college (uneducated) outcomes. Wages are a function of individuals' initial general ability, θ^i. College-graduate wages are, in addition, also a function of individuals' initial math ability, θ^i_m. The probability of graduating from college is represented by p and depends on an individuals' initial general ability, θ^i. Individuals are heterogeneous across initial general ability, θ^i, initial math ability, θ^i_m, and taste for college, ζ^i.

While initial ability is defined in terms of θ^i and θ^i_m, agents are aware that ability translates into human capital, h^i, through a noisy process, that affects their performance both at school and at work. This process is defined as $h^i = \exp(\theta^i + \epsilon^i)$ and $h^i_m = \exp(\theta^i_m + \epsilon^i)$ for general and math human capital, respectively. Individuals' *ex ante* estimate of their general and math human capital is given by $\hat{h}$ and $\hat{h}_m$. Students generally overestimate their human capital when making schooling choices, which we call overconfidence (for an empirical motivation see Bordalo et al., 2014, and references therein). This overconfidence is seen in individuals' *ex ante* general and math human capital estimates: $\hat{h} = \exp(\theta + \hat{\epsilon})$ and $\hat{h}_m = \exp(\theta_m + \hat{\epsilon})$, where $E(\hat{\epsilon}) > E(\epsilon)$.

College is not reversible and dropping out occurs only if an individual does not meet the minimum graduation requirement set by $\overline{h}$. I.e., $p(\theta^i) = 1$ if $h^i(\theta) > \overline{h}$ otherwise $p = 0$ and the individual drops out of college.

Math human capital has no value in the labor market unless an individual studies math in college. We will approximate this college major matching process by allocating math credits directly to individuals based on their ability. Acquired math skill in college, $m^i(h_m)$, is an increasing function of math human capital, $\frac{\partial m^i(h_m)}{\partial h_m} > 0$.

The wages for college educated individuals are determined by,

$$\omega^i_{et} = w_{et}\left(w_{het}h^i + w_{met}m^i(h_m)\right)\exp(\eta^i_t), \tag{2}$$

where w_{et} are general wage returns to a college degree for general human capital (indexed h) and acquired math skill (indexed m). A transitory luck component, η, is drawn each period.

The wages for uneducated individuals are,

$$\omega^i_{ut} = w_{hut}h^i\exp(\eta^i_t), \tag{3}$$

as math human capital has no value unless refined in college. Uneducated individuals face the same transitory luck component, η, as educated individuals.

4.2 Firms

A representative firm hires college and non-college labor to produce a final good (Y_t). The production function is a CES between non-college and college labor. College labor is a nested-CES between general human capital and math. Non-college labor, by definition, only supplies general human capital.

$$Y_t = \Big[\alpha L_{hut}^{\nu} + (1-\alpha)\big[\lambda\,(A_t L_{het})^{\rho} + (1-\lambda)\,(A_t M_t L_{met})^{\rho}\,\big]^{\nu/\rho}\Big]^{1/\nu} \tag{4}$$

The elasticity of substitution between education types is $\frac{1}{1-\nu}$. The elasticity of substitution between general human capital and acquired math skills is $\frac{1}{1-\rho}$. Labor shares are comprised of two components for educated individuals, general human capital and acquired math skills, and general human capital alone for uneducated individuals. Formally, labor shares are defined as,

$$L_{hjt} = \int_i (\mathbf{1}_{(s^i=j)} h^i) di,\ \forall j = e, u \quad \text{and} \quad L_{met} = \int_i (\mathbf{1}_{(s^i=e)} m^i) di.$$

A_t is skill-biased technical change (SBTC) and M_t is math-biased technical change (MBTC) over time, with growth rates γ_{at} and γ_{mt} for SBTC and MBTC, respectively. Thus, SBTC is $A_t = (1+\gamma_{at})A_{t-1}$ and MBTC is $M_t = (1+\gamma_{mt})M_{t-1}$.

The model solutions follow from the firm's cost minimization problem. If we define college labor output as,

$$Y_{et} = \big[\lambda\,(L_{het})^{\rho} + (1-\lambda)\,(M_t L_{met})^{\rho}\,\big]^{1/\rho},$$

with a price of $p_{et} = w_{et}$, given perfect competition, then the relative demand for college labor output from the firm's minimization is,

$$\left(\frac{Y_{et}}{L_{hut}}\right)_{demand} = \left(\frac{A_t^{\nu}(1-\alpha)}{\alpha}\frac{w_{hut}}{w_{et}}\right)^{\frac{1}{1-\nu}}. \tag{5}$$

Firms demand relatively more college labor when the wage rate decreases or college labor productivity (A_t) increases. The relative demand for acquired math skills ("college math") from the firm's solution is,

$$\left(\frac{L_{met}}{L_{het}}\right)_{demand} = \left(\frac{M_t^{\rho}(1-\lambda)}{\lambda}\frac{w_{het}}{w_{met}}\right)^{\frac{1}{1-\rho}}. \tag{6}$$

Firms demand relatively more college math when the wage rate decreases or the productivity of acquired math skills (M_t) increases.

4.3 Equilibrium

The general equilibrium conditions are dependent on the individuals' and the firm's optimization problems.

An equilibrium, given wage rates $\{w_{hut}, w_{et}, w_{het}, w_{met}\}$, is defined by:

1. The education choice, $s^i = \{e, u\}$, that maximizes the individual problem, subject to the graduation constraint $h^i \geq \overline{h}$;
2. The demand for labor $\{L_{hut}, L_{het}, L_{met}\}$, that minimizes the firm's production cost; and
3. Labor markets clear, both for general human capital, $(L_{hjt})_{demand} = (L_{hjt})_{supply}$ for $j = \{e, u\}$, and for college math, $(L_{met})_{demand} = (L_{met})_{supply}$.

4.4 Dynamics

To compute actual wage rates, the final good price is normalized to one, $p_t = 1$. Using the unit cost of producing one unit of output, it is straight forward to derive all wages from Equations (4)-(6). The wage rates of college labor input are,

$$w_{het} = w_{et}\lambda^{1/\rho}\left(1 + \frac{M_t^{\rho}(1-\lambda)}{\lambda}\left(\frac{L_{met}}{L_{het}}\right)^{\rho}\right)^{(1-\rho)/\rho} \tag{7}$$

and

$$w_{met} = w_{het}\frac{M_t^{\rho}(1-\lambda)}{\lambda}\left(\frac{L_{het}}{L_{met}}\right)^{1-\rho}, \tag{8}$$

where

$$w_{et} = A_t(1-\alpha)^{1/\nu}\left(1+\frac{\alpha}{A_t^{\nu}(1-\alpha)}\left(\frac{L_{hut}}{Y_{et}}\right)^{\nu}\right)^{(1-\nu)/\nu}. \tag{9}$$

SBTC (A_t) increases the returns to all college labor. There are two channels that increase the returns to math college, (1) a direct technical change effect, and (2) an indirect supply effect. First, MBTC (M_t) increases the returns to college math directly. Second, a relatively faster increase in general human capital compared to the supply of college math, $\frac{L_{het}}{L_{met}}$, given an elasticity parameter of $\nu < 1$, also increases the wage rate on college math, w_{met}. Therefore, consistent with our main hypothesis, if individuals are constrained in learning math, or new college entrants are unable to learn/study math, it is possible that the returns to math increase faster than the returns to college, leading to a larger spread between the top and bottom percentile wages within the college educated group. As a consequence, the larger the absolute number of college entrants, given that every new marginal entrant will have a lower ability level, the larger the post-education wage inequality. For completeness the wage rate of uneducated workers is determined by,

$$w_{hut} = w_{et}\frac{\alpha}{A_t^{\nu}(1-\alpha)}\left(\frac{Y_{et}}{L_{hut}}\right)^{1-\nu}, \tag{10}$$

where the non-college relative wage decreases with SBTC, which is consistent with the SBTC literature.

5 Calibration

We calibrate the model to 1980. The model parameters can be grouped into four categories: (1) standard parameter values, $\{\beta, N\}$; (2) individual-specific parameters, $\{\mu_\theta, \sigma_\theta, \sigma_\zeta, \hat{\mu}_\epsilon, \sigma_\epsilon, \sigma_\eta\}$; (3) college-specific parameters, $\{\delta_0, \delta_1, \overline{h}, \overline{m}\}$; and (4) firm-specific parameters $\{\alpha, \lambda, \nu, \rho\}$. Each category of parameters is discussed in separate subsections below. The calibration procedure estimates all parameter values jointly. To generate the time trends, the growth rates of productivity $\{\gamma_{jt}\}$ (SBTC and MBTC) are calibrated to match the rise in college attainment and the college wage premium.

5.1 General Parameters

The model has five-year time periods and is simulated for seven periods, from 1980 to 2010. We start the model economy at 1980 for two reasons: (1) the 1960 cohort of the NLSY79 is the first reasonable target available, i.e., individuals making their college decisions in 1980; and (2) the Vietnam War distorted the college decision of cohorts born before 1960, as the draft could be avoided by college enrollment (see Lemieux & Card, 2001). In addition, the decompositions in Section 3 show that income divergence began around 1980, with a relatively flat trend between 1975 and 1980. The model uses a standard discount factor of $\beta = 0.9$ per period, which implies a discount rate of approximately two percent per year.

We set $N = 9$, meaning that individuals live for nine periods after the college/no-college decision. Thus, each period contains nine generations and the modeled working life-time of an individual covers the equivalent of 45-years. As the period 0 decision is assumed to take place around the age of 18, the model covers the age range of 18 to 63. The simulation accounts for the baby boom/bust that generates different cohort sizes. However, the results are not sensitive to these cohort size differences.

5.2 Individual and Education Specific Parameters

The individual and education parameters interact directly. Therefore, this subsection discusses these two parameter groups together. Given that the NLSY79 was administered to individuals aged 14 to 22 in 1979, and model simulations start in 1980, we drop the youngest individuals for NLSY79 targets described below. That is, we match the 1960 cohort definition, only including individuals born before 1963.

Initial ability and human capital. The two types of initial ability, general (θ) and math (θ_m), are distributed normally. Formally, $\theta \sim N(0, \sigma_\theta^2)$ and $\theta_m \sim N(0, \sigma_\theta^2)$. The correlation ($\rho$) between initial general ability and initial math ability is set to $\phi = 0.9367$, matching the correlation between SAT I and SAT I Math scores in the ACS, $E(\theta_m) = \phi\theta$.

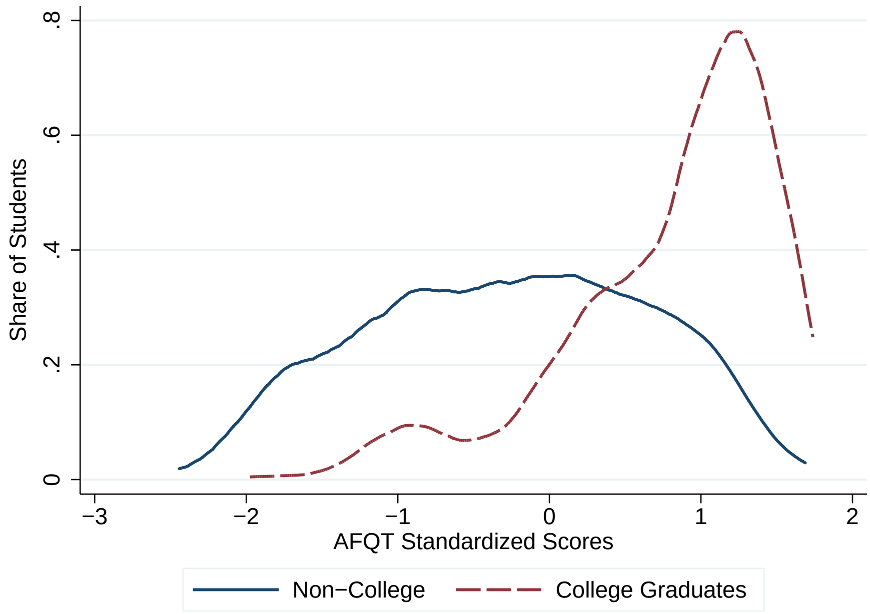
To generate the mapping between initial ability measures (θ and θ_m) and human capital (h and h_m), the noisy process ϵ is assumed $N(0, \sigma_\epsilon^2)$. The *ex ante* process $\hat{\epsilon}$ is also assumed $N(\hat{\mu}_\epsilon, \sigma_\epsilon^2)$.

Schooling choice. Preferences for studying impact the initial college/no college schooling choice. Study preferences are defined by ζ, which can be considered an individual's taste for college. While individuals sort into college based on their initial ability, Figure 7 shows that this sorting is not perfect. Thus, $\zeta \sim N(0, \sigma_\zeta^2)$, where a negative ζ is a cost and a positive ζ is "love" for studying.

Schooling outcome. There are two outcomes for those who attempt college education: (1) dropout/failure or (2) graduate in a college major characterized by math credits. In the model, the dropout rate is governed by a minimum human capital standard, $\overline{h}$, which is assumed constant over time. Those who do not drop out of college accumulate a representative measure of acquired math skill in college (m^i), with functional form,

$$m^i = \min\left\{\overline{m}, \max\left(0, \delta_0 + \delta_1 \exp(h_m)\right)\right\}. \tag{11}$$

Math acquired in college is subject to a cap, $\overline{m}$, which is set to match the share of individuals with 21 or more math credits in the 1960 cohort. The choice of 21 math credits is driven by returns to math credits in the ACS sample. A regression of math credits and math credits squared on wages, controlling for a number of characteristics, for full-time



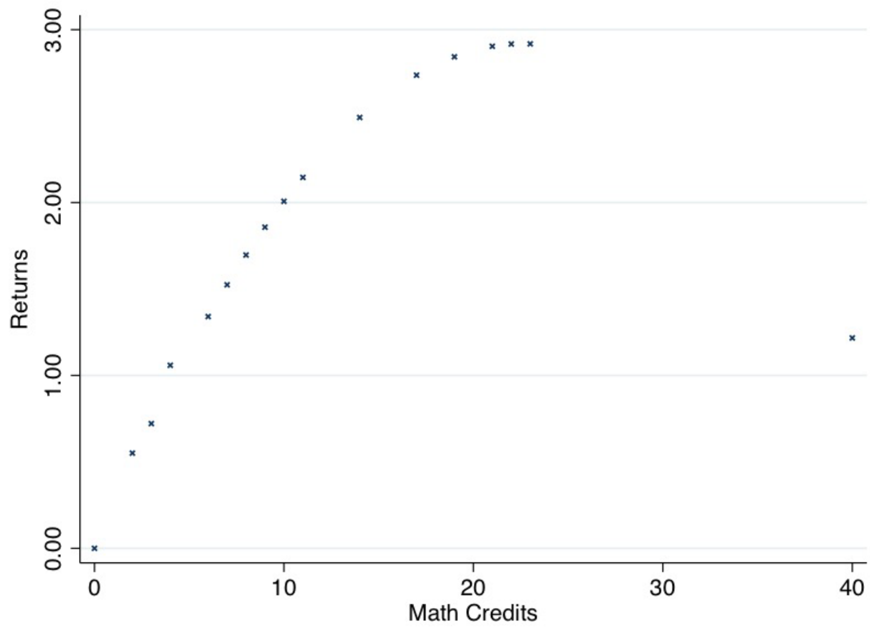
Source: NLSY79. Males born before 1963 (1960 cohort). Standardized test scores as computed by the method of Altonji et al. (2012).

Figure 7: Ability Distribution by Education Type

full-year male employees, suggests an increasing and concave relationship for the returns up-to 21 college math credits (see Figure 8). The returns to additional credits above 21 flattens and then drops sharply at 40 credits. These 40-credit college majors are primarily math majors. We argue that numerical skills and the tools learned in mathematics are valuable on the job market, but we do not, *per se*, think that esoteric math courses (e.g., chaos and dynamical systems) are the main driving force. However, the US share of college graduates with 40 credits is small.[11]

The number of college graduates studying zero and the maximum math credits can be seen in Figure 9. The individuals with zero math will benefit from SBTC, but not from MBTC. We define δ_0 such that the share of college graduates from the 1960 cohort with zero math credits is matched. Given the mapping function for m^i, δ_0 will be a negative

[11]Only 1.6 percent of college graduates within the entire 2009/2010 sample obtain 40 math credits.



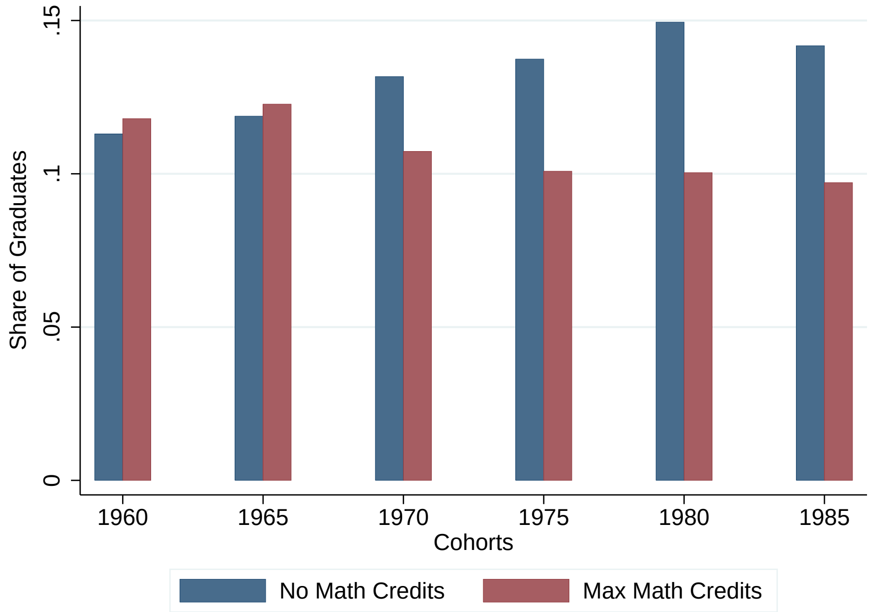
Source: ACS, NCES. Math credit returns are computed from a regression of hourly log wages of full-time (at least 35 hours of work and 40 weeks per year) male workers aged 25 to 59 on math credits, math credits squared, age, age squared, dummies for education, race, marital status, and year.

Figure 8: ACS Returns to College Math Credits

value.

Additional wage component. The model features a standard luck component (e.g., Storesletten et al., 2001) in wages over the life-cycle. For the precise process used in this paper, see Guvenen & Kuruscu (2010). This luck component is i.i.d. with mean zero and standard deviation σ_η.

Remaining moment conditions. The six structural parameters that do not have a one-to-one mapping to empirical moments are estimated by matching seven US data targets simultaneously. The seven moments that govern individual actions pertain to the 1960 cohort (i.e., individuals making education choices in 1980) or the year 1980. We group the



Source: ACS, NCES. College graduate males by birth cohort.

Figure 9: College Major Graduation Share by Cohorts

moments that we believe to be particularly informative of a given parameters.

- $\overline{h}$ is governed by the college dropout rate in 1980. Although there is a wide range of estimates for the dropout rate, we use estimates by Bound et al. (2010) for two reasons: (1) the authors provide estimates disaggregated by gender; and (2) their definition aligns with our interpretation of college dropouts. That is, the college dropout rate defined as the share of all individuals age 25 in 1980, who have some college but lack a four-year college degree. More importantly for our research, the results presented below are not sensitive to the precise value of the dropout rate. Note that the model generates an increasing college dropout rate over time, which is a characteristic that the literature agrees on (for example, Bound et al., 2010; Bailey & Dynarski, 2011, both show a rise in college dropout rates).

- $\hat{\mu}_\epsilon$, σ_ϵ, σ_ζ are determined by the average general ability of college graduates (NLSY79, Armed Forces Qualification Test (AFQT)), the correlation of high school GPA and freshman college GPA of 0.4 (Rothstein, 2004), and the average ability of non-college workers (NLSY79, AFQT). That is, $\overline{h}$ provides a clear dropout cutoff, with over-confidence, $\hat{\mu}_\epsilon$, contributing to the dropout rate, i.e., people attempt college who cannot graduate. Uncertainty over actual abilities is generated by σ_ϵ, with a large literature suggesting that SAT scores and high school performance are an imperfect measure of college performance, and significant information about own ability being revealed through studying at a college level.[12] Lastly, σ_ζ generates imperfect sorting, since individuals have idiosyncratic preferences for school, which translates into a larger variance in the ability of college graduates. This variance is used to explain differences in the average ability between college and non-college individuals.

- σ_θ and δ_1 are determined through three 1980 relative log wage targets: NC90-NC10, C90-C50, and C90-C10. These second moments cover the main intra-group inequality measures that we believe are important. Note that the model has an extra wage target. This additional second moment will provide information on the relevant parameters determining variance: σ_θ, σ_ϵ and σ_ζ. Thus, this extra wage target is important in matching both first and second moments when analyzing wage inequality in the model.

5.3 Firm-Specific Parameters

There are four parameters associated with the firm that must be pinned down in 1980, along with two time trends.

Time invariant firm parameters. Two parameters are set outside the estimation procedure. More precisely, the parameter ν is set within the range of standard estimate for college to non-college labor elasticities (see Autor et al., 1998, 2008), and the share

[12] Stinebrickner & Stinebrickner (2013) find that 45 percent of the college dropout rate at Berea College (a small liberal arts college in Kentucky) can be explained by students learning their academic performance in the first two years of college.

parameter on human capital is normalized, $\lambda = 0.5$.

The parameters α and ρ are pinned down by matching the share of college graduates (age 25 to 30) in 1980 and the college wage premium in 1980. The resulting elasticity parameter is $\rho = 0.707$.[13]

Time trends. Given the definition of SBTC and MBTC, we normalize A and M to one in 1980. By definition, a rise in A_t will affect both the returns to college ability and math equally, but a rise in M_t will only increase the returns to math. We restrict the growth rate of SBTC such that $\gamma_{a,t} \in [0.018, 0.028]$, which follows from the range of estimates found in Table 2 of Autor et al. (2008). The two growth rates are then calibrated to match the rise in the share of college graduates from 1980 to 2010, along with the rise in the college wage premium. This process yields a SBTC growth rate of $\gamma_a = 0.027$ per annum, which lies at the upper range of possible estimates. As relatively high SBTC decreases the effect of MBTC (Section 4.4), the SBTC growth rate estimate is conservative. The calibration suggests that MBTC is substantial during this time period, with $\gamma_m = 0.043$. We present a counterfactual is Section 6.1 to understand how important this precise value of γ_m is for the model.

5.4 Calibration Summary

Table 2 summarizes the estimated and calibrated parameters, with estimated parameters above the center line and calibrated parameters at the bottom.

The 1980 data targets used in pinning down the calibrated parameters are summarized in Table 3. The model does well in matching all targets. It only slightly overpredicts the average ability of college graduates and the C90-C50 wage differential of college graduates. For the time trends, the model is unable to match the full rise in the share of college

[13] Alternatively, using the ACS data, but instead computing elasticities across occupations rather than time, yields similar results as the calibration, with the interval of plus/minus one standard suggesting $\rho \in [0.33, 0.72]$. However, since this method is subject to various assumptions related to the computation of relative wage returns, efficiency units and grouping of occupations, our preferred estimate is using the calibrated elasticity.

Table 2: Calibration Summary

Parameter	Value	Source / Type
β	0.9	standard discounting
N	9	retirement at 63 (5 year periods)
σ_η	0.367	transitory wage luck (Guvenen & Kuruscu, 2010)
θ^i	$\sim N(0, \sigma_\theta^2)$	initial ability NLSY79
ν	0.597	elasticity parameter: college to non-college (Autor et al., 2008)
λ	0.5	ability share parameter - normalized
A_{0t}	1.0	SBTC 1980 - normalized
M_{0t}	1.0	MBTC 1980 - normalized
σ_θ	0.142	initial ability
$\hat{\mu}_\epsilon$	0.229	overconfidence
σ_ϵ	0.205	unknown ability component
σ_ζ	0.042	utility of studying
δ_0	-2.953	zero math outcome
δ_1	2.769	math skill slope
$\overline{h}$	0.109	minimum college requirement
$\overline{m}$	0.625	maximum math credits
α	0.411	college share parameter
ρ	0.707	elasticity parameter: ability to math
γ_a	0.027	SBTC growth rate
γ_m	0.043	MBTC growth rate

graduates by 2010. However, the model is able to match the share of new college graduates in 2010. This discrepancy can be explained by the draft during the Vietnam War generating above average college graduation rates (Lemieux & Card, 2001).

6 Results

The model accurately captures a variety of inequality dynamics, including many of the general wage trends and all of the intra-college group wage decomposition between 1980 and 2010. These results are driven by the introduction of MBTC into a standard SBTC framework, with the importance of MBTC highlighted through the counterfactual presented in Section 6.1. Table 4 compares the total US inequality trends and the modeled results. The base model explains nearly half of the rise in inequality of the aggregate US

Table 3: Targets Summary

Target	Data (1980)	Model
Fraction 0 Math Credits	0.113	0.113
Fraction 21 Math Credits	0.118	0.118
College Dropout Rate	0.550	0.548
$corr(\theta, h)$	0.400	0.401
$\theta_{college\ graduate}$	0.805	0.821
$\theta_{non-college\ worker}$	-0.282	-0.275
C90-C50	0.568	0.579
C90-C10	1.174	1.154
NC90-NC10	1.104	1.072
College Wage Premium	0.247	0.247
Young College Graduates	0.241	0.247
2010 College Graduates	0.301	0.281
2010 College Wage Premium	0.493	0.495

Table 4: US Wage Inequality & Model Results

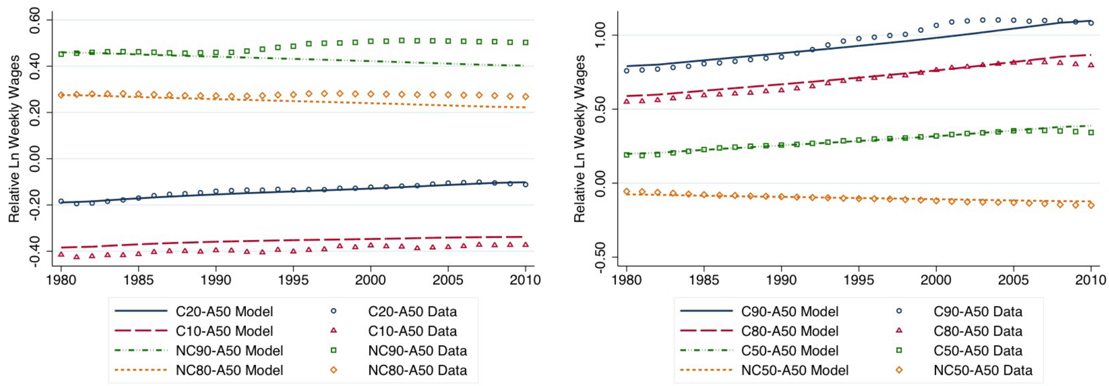
			Relative Log Wages	
	Year	90-10	90-50	50-10
All				
Data	1980	116	55	62
	2010	150	77	73
Model	1980	112	55	57
	2010	126	67	59
% Explained		42	52	23
College				
Data	1980	117*	57*	61
	2010	145	74	72
Model	1980	118*	59*	58
	2010	144	71	73
% Explained		92	68	131

Notes: * Moment targeted in the calibration.

male population.[14]

The model's strength lies in explaining intra-college income inequality, given that MBTC only impacts educated individuals. The model explains almost all the trends for

[14]The rise in the fraction of college graduates and the college wage premium are matched by construction.



(a) Bottom

(b) Top

Figure 10: Model Wage Trend Decomposition

college graduates, both at the top and bottom. At the other end of the income distribution, none of the aggregate non-college trends (NC90-NC10, NC-90-NC50 and NC50-NC10) are well explained. Figure 10 shows that the model matches the NC80-A50 and NC50-A50 wage evolution, but it predicts a slight fall in both the NC90-A50 and only a small fall in the NC20-A50 and NC10-A50 percentile (not pictured). In contrast, the data shows a mild increase in the NC90-A50 relative wage and a large decrease in the NC20-A50 and NC10-A50 relative wage. That is, the model is able to replicate the average uneducated worker's wage, but not the extremes. These unmatched trends are unsurprising, as the model ignores the human capital accumulation of uneducated individuals (e.g., unemployment, dropping out of high school or completing vocational 2-year college programs) and MBTC only affects college labor directly. Therefore, the model is more suited to predict income trends within the top half of the distribution, which includes all college graduates. Both Figures 10a and 10b show an almost perfect matching of the college distribution across the top (C90-A50, C80-A50), median (C50-A50) and bottom (C20-A50, C10-A50) deciles.

In addition to matching inequality trends, the model also preforms well in matching trends not directly targeted, such as ability distributions by school outcome, dropout rates, etc. Matching these trends is important when assessing the validity of the model and mechanism put forth in this paper. The following paragraphs discuss each trend in detail.

Initial ability. The NLSY79 and NLSY97 show that *ex ante* ability, as measured by the AFQT of college graduates, has fallen from 0.805 to 0.775. Since individuals were aged 12 to 16 in the NLSY97 sample, the natural comparison of college cohorts in the model is 2000-2005. The model slightly overpredicts the average initial ability in 1980 and predicts a fall to 0.769 by 2000 and 0.754 by 2005. While the fall is slightly larger than observed from the NLSY79 to NLSY97, the estimates are close to suggest that the model is not generating the above wage trends through an incorrect composition of college graduates.

Dropout rate. The model generates a rise in the college dropout rate from the matched target of 55 percent in 1980 to 59 percent by 2010. Bound et al. (2010) show that the non-completion rate for males aged 25 went from about 55 percent in 1980 to about 60 percent in 2000. By the year 2000 the model generates a dropout rate of 58 percent, just shy of the data estimates.

College math credits (0 and 21). The model shifts each subsequent cohort towards lower college math levels. In the 1960 cohort 11.3 percent of college graduates (in both the data and model) had zero math credits. By the 1980 cohort this number had risen to 14.9 percent in the data and 14.8 percent in the model. Looking at the fraction of individuals with 21 credits or more, the data falls from 11.8 percent for the 1960 cohort to 10.0 percent for the 1980 cohort. The model replicates just over one-third of this drop, generating a fall from 11.8 to 11.1 percent from 1980 to 2010 for new college graduates. The change over time in the model mimics the general pattern observed in the US (Figure 9).

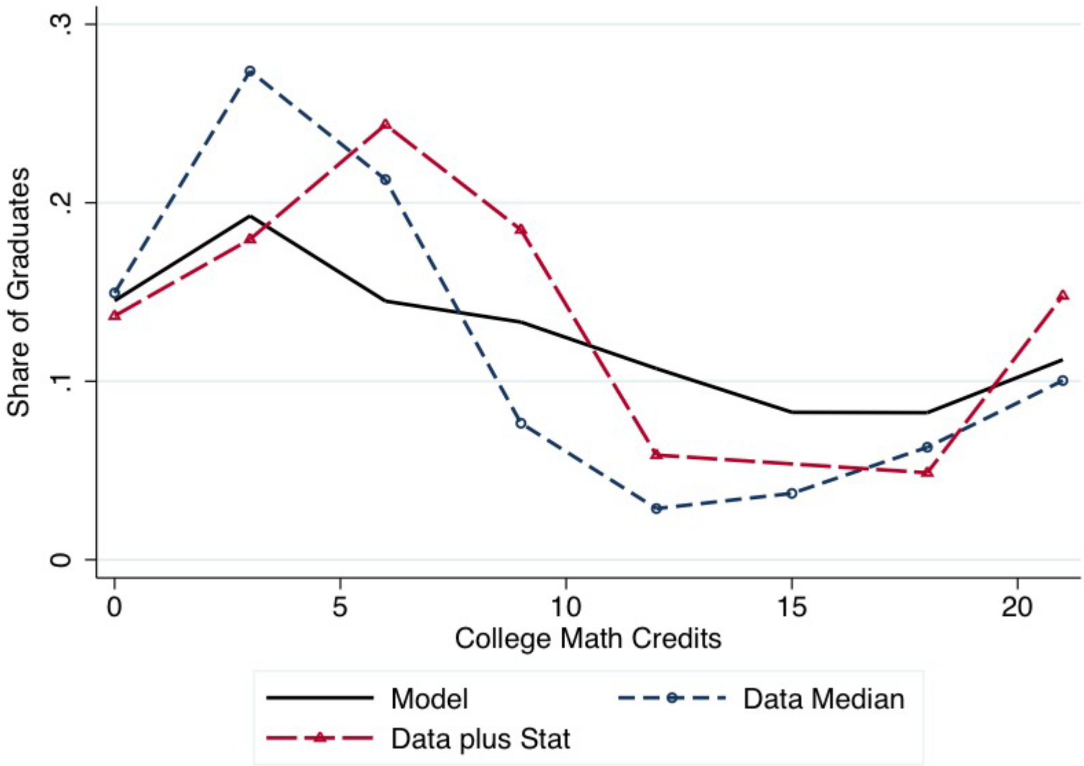


Figure 11: 1980 Cohort Data versus Model

College majors. Figure 11 graphs the share of college graduates over all math outcomes ("college majors") for the 1980 cohort of the model and data. As math is a continuous variable within the model, Figure 11 is computed by scaling all math outcomes such that the maximum math credits earned is 21, and then rounded up to the nearest three credit equivalent. The model, using a simple math technology, does well in matching the overall shape of college graduates over math outcomes. However, the continuous math outcome variable in the model leads to a smaller mass around three credits compared to the data. It should be noted that a large share of college majors might only require three math credits to graduate, suggesting a kink in the college math "production function." Additionally, reclassifying college credits in statistics as college math credits does not affect the share of individuals with zero math credits, but does generate a larger spread (away from three

Table 5: US Wage Inequality & Constant Skill Supply

	Relative Log Wages		
	90-10	90-50	50-10
College (2010)			
Data	145	74	72
Benchmark	144	71	73
Partial Equilibrium	140	69	71
% Explained Skill Supply	15	17	7

math credits). Given that the model estimates lie between the two types of math measures (with and without statistics) and the model studies aggregates at the top and bottom of the wage distribution, it is not of first order importance for the results to generate the mass at three credits.

6.1 Counterfactual

We present two counterfactuals: (1) a partial equilibrium exercise, where the graduating cohort attributes remain fixed at the 1960 cohort, and (2) eliminating MBTC by setting $\gamma_{m,t} = 0$.

Supply Effects. Maintaining the attributes of the 1960 cohort for all successive cohorts eliminates the supply effects from the results. Attributes here refer to the distribution of *ex ante* and *ex post* skills. That is, the counterfactual assesses the importance of the falling ability levels of college graduates in driving inequality. Table 5 shows how much of the rise in wage inequality in the benchmark is driven by the new "marginal" college graduate being of worse quality (in terms of math skills and general human capital) in 2010 compared to 1980. The aggregate effect is shown in the last row, stating the percentage contribution of changing skill supplies to the rising wage inequality from 1980 to 2010. Of the total wage inequality (90th to 10th percentile college graduate) generated in the benchmark model, 15 percent of the increase in inequality is explained by a deterioration of the skills at the

Table 6: US Wage Inequality & Counterfactual Results

	Year	Relative Log Wages 90-10	90-50	50-10
All				
Data	2010	150	77	73
SBTC + MBTC	2010	126	67	59
SBTC	2010	129	68	62
% SBTC Explained		51	55	42
College				
Data	2010	145	74	72
SBTC + MBTC	2010	144	71	73
SBTC	2010	118	60	58
% SBTC Explained		3	4	1

bottom relative to the top graduate from 1980 to 2010. For the 90th to 50th percentile the explanatory power is slightly larger at 17 percent.

MBTC. The removal of the MBTC mechanism clearly reveals the contribution of the returns to acquired math skills. The counterfactual model predicts more individuals attempt college, with a college dropout rate of 60.4 percent in 2010, and more individuals graduate from college, with the share of college graduates increasing to 28.6 percent in 2010 from 28.1 percent in the benchmark model. The share of zero-math credit graduates increases to 15.9 percent and the share of individuals with 21 credits decreases to 10.8 percent by 2010. Simultaneously, the average quality of a college graduate drops from 0.744 to 0.712 in terms of initial ability.

Table 6 compares the 2010 wage inequality levels between the data, the benchmark model and the counterfactual model results, omitting the 1980 values, as they are identical to Table 4 by construction. At an aggregate level, the counterfactual model is marginally better than the benchmark model in matching broad income inequality trends, particularly at the bottom of the wage distribution. However, these broad measurements hide the intra-college inequality trends that are ignored by the counterfactual model. The second part

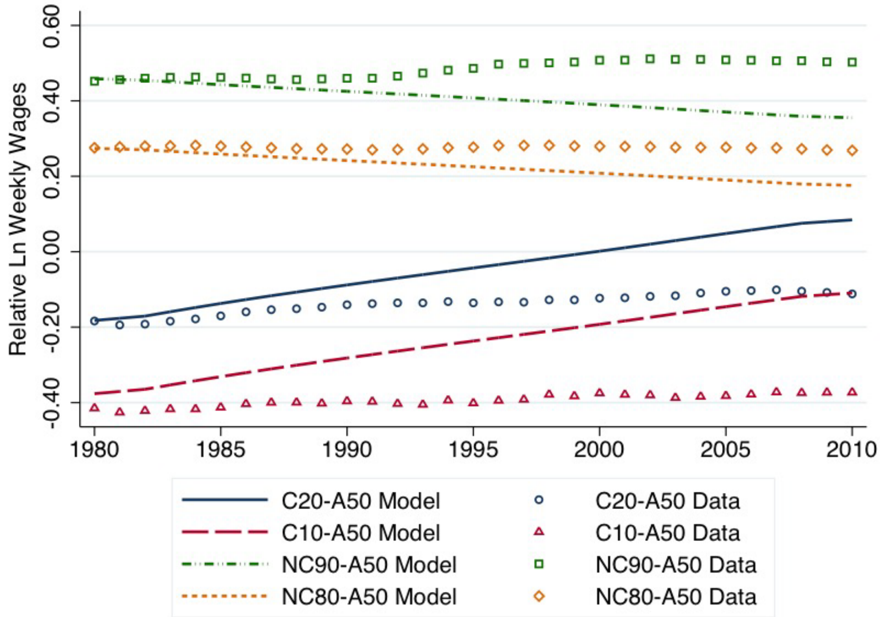


Figure 12: US Wage Inequality & Counterfactual Results

of Table 6 shows that the counterfactual model is unable to match any of the trends in wage inequality between college graduates, explaining only one to four percent of the rise in inequality from 1980 to 2010.

Figure 12 further highlights how the counterfactual model matches the aggregated income inequality trends for the wrong reasons. The counterfactual predicts a sharp increase in wages for the bottom college deciles and a sharp fall for wages of the top non-college deciles. This first point is driven by higher returns to the general human capital of college graduates (relative to math). The non-college results are driven by composition. That is, more low ability students enter and graduate from college, effectively decreasing the ability of the top non-college deciles.

The mechanisms of SBTC and MBTC work in opposite directions for the bottom of the college distribution. SBTC alone benefits all college graduates in wage terms. The bottom deciles, in particular, gain compared with the average individual in the economy. Further highlighting the broad power of the SBTC mechanism, the counterfactual pushes the income of the C20 group above the A50 wage, while the C10 wage approaches parity with the A50 wage. In contrast, MBTC is effective at only increase the top income deciles, leaving the bottom deciles with decreasing or stagnant wages. Thus, only the combination of these two different technical change concepts can generate the observed wage inequality trends of the US male college graduate population, along with part of the divergence in wages across the entire US male population.

7 Conclusion

This paper studies the role of math in determining both inter- and intra-education group wage inequality. The connection between *ex ante* math abilities, college math, and the labor demand for math skills provides a simple and powerful mechanism, explaining a large component of male wage inequality in the US. The estimated structural model highlights the importance of MBTC in explaining both aggregate wage inequality and the wage trends at the extremes of the college distribution. The model is also able to generate average trends for both college and non-college groups, closely matching the general trends achieved by SBTC alone.

Given the results, there are a number of interesting research extensions. As we extend the research on the determinants of wage inequality from a post-education perspective (Kambourov & Manovskii, 2009; Huggett et al., 2011) to a pre-college education perspective, we assume initial math ability is determined prior to college. Given the central role of math, studying the origins of initial math skills is of primary importance in determining college math outcomes and is a natural first extension of the MBTC mechanism presented here. A second research extension focuses on the gender dimension of education choices, where the specific characteristics of college majors favored by women are assessed. While men optimize their education choices to maximize pecuniary outcomes and choose high-math college majors given initial ability constraints, women exhibit more complex preferences with respect to non-pecuniary outcomes that seem to distort the education decision. In ongoing work, we study women's college decisions in a life-cycle model that accounts for atrophy and repair of skills due to career breaks.

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A Data Appendix: Aptitude Measures

This appendix lists the definition of the three DOT variables referenced in the paper. The three aptitude variables used are from the DOT 1977 and 1991 editions. We are unable to use the DOT successor, the Occupational Information Network (O*net), due to a measurement discontinuity in the recorded aptitudes.

In general, aptitudes measure the ability an individual must possess in order to perform a job successfully. More precisely, the measure is a function of the share in the population that meets this ability level. That is, there are five categories: (1) the bottom 10 percent of the population, (2) the bottom third excluding the bottom 10 percent, (3) the middle third, (4) the top third excluding the top 10 percent, and (5) the top 10 percent. We translate these measures to a scale ranging from zero to one, e.g., an aptitude above 0.66 would correspond to an individual in the top-third of the population.

Out of the 11 measures reported in the DOT 1977 and 1991, we use the three measures: general, numerical and verbal ability.

- General ability is the ability to understand instructions, understand principles and to make judgments. It encompasses a number of skills, e.g., using logic and scientific thinking, understanding procedures, establishing facts and drawing conclusions, etc. This measure is highly correlated with the ability to perform well in school.

- Numerical aptitude is the ability to perform arithmetic. The complexity and speed of operations is taken into account when assigning the category.

- Verbal aptitude is the ability to understand and use language effectively. Both oral and written skills, including the use of technical terminology, are taken into account when assigning categories for each occupation.