

The Efficiency of Surplus Sharing in Sequential Labor and Goods Markets

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February 2018*

PRELIMINARY AND INCOMPLETE

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Abstract

What is the optimal sharing of value added between consumers, producers, and labor? We study the constrained efficiency properties of a model with search frictional labor and goods markets. The social planner's allocation seeks to minimize turnover costs in all markets. The decentralized allocation is constrained efficient if Hosios conditions hold in each of the labor and goods markets. Deviations from Hosios in the labor market can lead to either over or under hiring, and either excess tightness or slack in the goods market. Deviations from Hosios in the goods market lead to either excess or insufficient tightness in the goods market, and always lead to lower labor market tightness. A calibration of the model to the US economy indicates restoring efficiency to the goods market would lower long unemployment and markups in the goods market.

JEL Classification: E24, E32, J63, J64.

Keywords: Search and matching models of unemployment, search and matching models of goods markets, constrained efficiency.

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1 Introduction [To be completed]

In competitive markets products are sold at marginal cost, the consumption surplus is driven to zero, labor is paid its marginal product and factors are fully utilized. Modern economies experience persistent unemployment, production does not take place at capacity, wages are below marginal products and firms price goods at a markup. We study the optimal sharing of value added between consumers, producers, and labor when markets are frictional and participants have market power. The trading frictions take the form of search in labor and goods market, which lead to the existence of positive surplus to transactions. Market power in bargaining over the rents from trade determine whether a good is sold near its marginal cost or closer to its marginal utility, and whether labor is paid its marginal product or nearer its reservation wage.

Production in the model requires a worker, and once a consumer is found the product is sold until either the consumer changes tastes or the producer loses its worker. In the labor market, workers and firms search and bargain over wages. In the goods market, consumers decide how actively to search for products to add to their consumption basket and bargain with the seller over the price of the good. An increase in firm goods market power has two, opposing, effects on the equilibrium in the labor market. First, firms obtain a higher price. This increases the marginal revenue from hiring labor. Second, the higher price dissuades consumers from actively searching in the goods market and increases the difficulty for all firms to find new customers. This reduces the expected revenue from hiring labor and reduces the incentives to create jobs. There thus exists a level of market power in the goods market that maximizes job creation and tightness in the labor market.

The constrained efficient allocation can be achieved by placing restrictions on the agents' market power in goods and labor markets. Entry in markets cause congestion in the meeting rates which agents do not consider in their private decisions. In the efficient allocation the planner takes into account the constraints of the search trading frictions. The constrained efficient allocation can be achieved by adjusting the share of the surplus in each market to compensate for this externality. However, since the labor and goods market interact in general equilibrium, the bargaining power that achieves efficiency in one market is not independent of the market power and the equilibrium in the other market. In the labor market, constrained efficiency is achieved if the share of the surplus going to labor is equal to the elasticity of matching with respect to unemployment, and, importantly, if constrained efficiency in the goods market holds. In the goods market, the share of the surplus to the consumer to ensure constrained efficiency depends both on the elasticity of goods market matching to consumer search and the division of rents in the labor market as part of the cost of producing - the wage - is passed on to the consumer. A double Hosios constrained efficiency condition in both goods and labor markets arises.

Away from the Hosios conditions in the goods and labor markets four regimes emerge characterized by markets being either too slack or too tight. However, the mapping to prices and wages is not direct. That is, for instance, equilibria with prices in the goods market above those desired by a social planner can be associated with either too much or too little job creation in the labor market depending on the share of the surplus accruing to labor in wage setting. Similarly, equilib-

ria with excessive job creation can exist along side a goods market in which prices are either too high or too low depending of the degree of consumer market power in price setting.

Taking the model to the US data, we find that labor's share in bargaining is too low, and firms have too much market power. For comparison, moving market power from their calibrated values to the Hosios condition move the rate of unemployment from 6% to 4%. [INCOMPLETE]

A recent strand of research has revived the original ideas of the search literature where consumers needed to search in the goods market in order to consume (e.g., [Diamond, 1971, 1982](#)). [Petrosky-Nadeau and Wasmer \(2015\)](#) study the business cycle implication of extended the search model of equilibrium unemployment to a search frictional product market. Their main finding is an added factor of amplification and persistence of shocks stemming from the feedback from product to labor markets. [Bai et al. \(2011\)](#), in a direct search environment explore the implications for the measurement of aggregate productivity and the identification of the sources of business cycles of capacity utilization varying endogenously in response to demand shocks. Goods market frictions lead to under utilized capacity and, as the show, a time varying wedge between aggregate productivity and Solow residual. Similir finds arise in [Michaillat and Saez \(2014\)](#) who argue that changes in goods market frictions must account for a large share of fluctuations in the labor market.

An early attempt to integrate goods and labor frictions with money is in [Shi \(1998\)](#). Trading frictions in product markets studied in new monetarist models (see Rocheteau for a review), establish a similar mechanism through which product market frictions affect the marginal revenue of firm and the equilibrium in the labor market (see [Branch et al. \(2014\)](#) for instance). In [Berentsen et al. \(2011\)](#) it is inflation, reducing the real income of consumers, that affects the real demand faced by firms, their mark ups and demand for labor.

The rest of the paper is organized as follows. Section 2 sets out the environment, the trading frictions agents face in the labor and goods markets. Section 3 for the decentralized equilibrium, and section 4 solves for the constrained efficient allocation. The existence of constrained efficiency conditions in the labor and goods market is shown in section 4.2. The same section also takes the model to the data while section ?? extends to sequentially bargained wages.

2 Environment

2.1 Matching and separation in the labor market

Workers are either unemployed or employed. Let 1 be the total labor force, and $\mathcal{U}$ is the number of unemployed workers. Firms post a number $\mathcal{V}$ of vacancies. These two inputs combine through an increasing and constant returns to scale function $M_L(\mathcal{V}, \mathcal{U})$ into a number of matches in the labor market. Workers separate from these employment relations at constant rate s^L . The law of motion of unemployment is therefore:

$$\dot{\mathcal{U}} = s^L (1 - \mathcal{U}) - M_L(\mathcal{U}, \mathcal{V}) \quad (1)$$

2.2 Matching and separation in the goods market

Consumers supply D_U search units on the goods market. The number of firms searching in the goods market, $\mathcal{N}_g$, is predetermined by past transitions. These two inputs combine into a number of matches in the goods market through an increasing and constant returns to scale function $M_G(D_U, \mathcal{N}_g)$. This flow adds to the number of matched consumers $\mathcal{D}_M$ and the number of matched firms $\mathcal{N}_\pi$. A match with a seller continues until one of two events occurs. A consumer can quit a particular consumption relationship. This arrives at a constant rate s^G . Finally, consumer-firm relationships are also terminated when the producer, i.e., the firm, loses its worker. This occurs at rate s^L . A convenient notation used in subsequent sections is $s = s^G + s^L$. Therefore, the laws of motions in the goods market are:

$$\dot{\mathcal{D}}_M = M_G(D_U, \mathcal{N}_g) - s\mathcal{D}_M \quad (2)$$

$$\dot{\mathcal{N}}_\pi = M_G(D_U, \mathcal{N}_g) - s\mathcal{N}_\pi \quad (3)$$

$$\dot{\mathcal{N}}_g = M_L(\mathcal{U}, \mathcal{V}) + s^G\mathcal{N}_\pi - M_G(D_U, \mathcal{N}_g) - s^L\mathcal{N}_g \quad (4)$$

The inflows into the mass of firms searching in the goods market, in equation (4), stem from two sources. First there is the inflow of new hires in the labor market. Second, the inflow from firms that have lost a consumer, $s^G\mathcal{N}_\pi$. Likewise, there are two sources of outflows: new matches with consumers in the goods market, and separation from workers ($s^L\mathcal{N}_g$).

2.3 Identities in goods and labor markets

The total number of filled jobs corresponds to the total number of employed workers, that is:

$$1 - \mathcal{U} = \mathcal{N}_g + \mathcal{N}_\pi \quad (5)$$

The total number of consumers matched with a producer of the search good corresponds to the number of firms supplying to consumers, namely that:

$$\mathcal{D}_M = \mathcal{N}_\pi \quad (6)$$

The combination of (5) and (6) delivers a constraint that the number of unemployment workers, matched consumers and employed workers at firms without a goods market match sum to one:

$$1 = \mathcal{U} + \mathcal{D}_M + \mathcal{N}_g \quad (7)$$

2.4 Tightness and transitions in labor and goods markets

Under constant returns to scale in matching in the labor market, one can define tightness in the labor market θ , the job hiring rate $f(\theta)$, the vacancy filling rate $q(\theta)$, and their elasticity $\eta_L(\theta)$ as:

$$\begin{aligned}\theta &= \mathcal{V}/\mathcal{U} \\ f(\theta) &= M_L/\mathcal{U} = \theta q(\theta) \text{ with } f' > 0 \\ q(\theta) &= M_L/\mathcal{V} \text{ with } q' < 0 \\ \eta_L(\theta) &= -\theta q'(\theta)/q(\theta) \in (0, 1)\end{aligned}$$

Similarly, under constant returns to scale in matching in the goods market, one can define tightness in the goods market ξ , the consumer meeting rate $\check{\lambda}(\xi)$, the firm's meeting rate $\lambda(\xi)$, and their elasticity $\eta_G(\xi)$ as:

$$\begin{aligned}\xi &= D_U/\mathcal{N}_g \\ \check{\lambda}(\xi) &= M_G/D_U = \lambda(\xi)/\xi \text{ with } \check{\lambda}' < 0 \\ \lambda(\xi) &= M_G/\mathcal{N}_g \text{ with } \lambda' > 0 \\ \eta_G(\xi) &= \xi \lambda'(\xi)/\lambda(\xi) \in (0, 1)\end{aligned}$$

A higher value of ξ reflects a greater amount of consumers search for a good relative to number of firms searching for consumers. Hence it is easier for firms to match in the goods market, and more difficult for consumers.¹

3 Decentralized equilibrium

Each job produces 1 units of the search good sold at unit price $\mathcal{P}$, and valued by the consumer at the margin by $\Phi > 0$. There is a flow cost to searching in the goods market $\sigma > 0$. The non-employed have flow utility $z > 0$, and there is a flow cost to job vacancies $\gamma > 0$.

3.1 Consumer's surplus and seller's surplus in the goods market

Consumers set the privately optimal level of search units D_U in order to exhaust all consumption opportunities given a cost of search in the goods market σ . The consumer Bellman equations in a

¹In previous work (Petrosky-Nadeau and Wasmer, 2017, Chapter 7) we had studied the case where both the number of consumers and of searching firms is predetermined. The number of consumers was assumed to be equal to the number of employees (the unemployed could not consume the search good) and therefore the tightness in the goods market was determined by the identities above and the stock flow equations, independently of the optimal behavior of consumers and firms. Here, we study the case where one of the two inputs in M_G is endogenous to workers decisions, namely the consumer side. The symmetric assumption (D_U predetermined and $\mathcal{N}_g$ as a control variable of the firm) could also be studied, as well as both variables as controls variables of the agents.

steady-state are

$$rW_{Du} = -\sigma + \check{\lambda} (W_{D_M} - W_{Du}) \quad (8)$$

$$rW_{D_M} = (\Phi - \mathcal{P}) + s^G (W_{Du} - W_{D_M}) + s^L (W_{Du} - W_{D_M}) \quad (9)$$

where W_{D_M} and W_{Du} denote the value functions of matched and searching consumers, respectively. Consumer find goods at rate $\check{\lambda}$ and enjoy the consumption surplus $(W_{D_M} - W_{Du})$ in equation (8). The stream of net flow utility from consuming the search good $(\Phi - \mathcal{P})$ ends following either a change in tastes, which arrives at rate s^G , or following a labor turnover event at the firm selling the good. Indeed, the last term $s^L (W_{Du} - W_{D_M})$ captures the fact that if the firm from which the good is purchased loses its worker there is an interruption the flow of surplus as the good is no longer produced.

Implicitly, any resource not spent on the search good is used as a numeraire and spent into search activity.² We do not report the income of the consumer as it scales up both value functions in the same way, dropping out of the consumption surplus.

Denote the wage paid to labor when the firm is searching in the goods market by w_g , and by w_π when the firm is selling its product. The firm's Bellman equations are:

$$rJ_g = -w_g + \lambda (J_\pi - J_g) + s^L (J_v - J_g) \quad (10)$$

$$rJ_\pi = \mathcal{P} - w_\pi + s^G (J_g - J_\pi) + s^L (J_v - J_\pi) \quad (11)$$

The firm matches with a consumer at rate λ , and gains the goods market surplus $(J_\pi - J_g)$. However, it faces the risk of a labor turnover shock in both stages g and π , and a loss $(J_v - J_g)$ and $(J_v - J_\pi)$, respectively. The profit flow $\mathcal{P} - w_\pi$ from selling the good may also be interrupted following a loss of the consumer, at rate s^G . In this event the worker remains with the firm and the capital loss is $(J_g - J_\pi)$.

The above Bellman equations lead to expressions for the consumer and firm private surpluses in a goods market match:

$$W_{D_M} - W_{Du} = \frac{(\Phi - \mathcal{P}) - rW_{Du}}{r + s} \quad (12)$$

$$J_\pi - J_g = \frac{\mathcal{P} - w_\pi - (r + s^L) J_g + s^L J_v}{r + s} \quad (13)$$

3.2 Entry and price determination in the market for goods

Free entry of consumer search in the goods market leads to $W_{Du} = 0$ in equilibrium. Applying this to equations (8) and (9) we have a goods market entry condition:

$$\frac{\sigma}{\check{\lambda}(\xi)} = \frac{(\Phi - \mathcal{P})}{r + s} \quad (14)$$

²Alternatively, it can be spend on a perfectly supplied numeraire good with no incidence on the results.

under which entry occurs until the average cost of finding a search good, $\sigma/\check{\lambda}$, equals the consumer's consumption surplus $W_{D_M} - W_{D_U} = (\Phi - \mathcal{P}) / (r + s)$, which is simply the discounted present value of the net utility flow from consumption $(\Phi - \mathcal{P})$.

The price in the goods market $\mathcal{P}$ divides the total consumption surplus, $(W_{D_M} - W_{D_U}) + (J_\pi - J_g)$, into shares α_G and its complement $(1 - \alpha_G)$, where $\alpha_G \in (0, 1)$ is the bargaining strength of a consumer. Such a price solves the Nash problem given by

$$\mathcal{P} = \operatorname{argmax} (W_{D_M} - W_{D_U})^{\alpha_G} (J_\pi - J_g)^{1-\alpha_G} \quad (15)$$

The slope of J_π in equation (11) and the slope of W_{D_M} in equation (9) are opposite but identical in absolute value. Therefore the price must satisfy the sharing rule

$$(1 - \alpha_G) (W_{D_M} - W_{D_U}) = \alpha_G (J_\pi - J_g)$$

which leads to a simple rule for determining prices:

$$\mathcal{P} = (1 - \alpha_G) (\Phi + \xi\sigma) + \alpha_G (w_\pi - w_g) \quad (16)$$

The detailed derivations are available in Appendix B.1. The price increases in the bargaining strength of the firm, $1 - \alpha_G$, the marginal utility of consumers Φ and their relative threat points, captured by $\xi\sigma$. The latter term has the interpretation that higher search costs and a tighter search market for goods push the negotiated price up as the seller has a relatively better outside option. The last element of the price rule is the wage differential between the search and selling stages in the goods market, $(w_\pi - w_g)$. When the difference is positive, that is, when the firm must pay labor a higher wage when selling, part of this increase in cost is past on to the consumer as a higher price.

3.3 Surpluses in the labor market

The value of a vacancy to a firm, which is the sum of a outflow cost γ and a capital gain $J_g - J_v$ following a match with a worker at rate $q(\theta)$, can be written as:

$$rJ_v = -\gamma + q(\theta) (J_g - J_v) \quad (17)$$

The Bellman equations for workers distinguish employment at a firm that is matched with a consumer and selling its goods, and one that is not. The respective asset values $W_{e\pi}$ and W_{eg} are:

$$rW_{e\pi} = w_\pi + s^L (W_u - W_{e\pi}) + s^G (W_{eg} - W_{e\pi}) \quad (18)$$

$$rW_{eg} = w_g + s^L (W_u - W_{eg}) + \lambda (W_{e\pi} - W_{eg}) \quad (19)$$

$$rW_u = z + f(\theta) (W_{eg} - W_u) \quad (20)$$

while W_u is the asset value of an unemployed worker with flow utility z and job finding rate

$f(\theta)$. A worker's and firm's surplus generated from forming a match in the labor market are, respectively:

$$\begin{aligned} W_{eg} - W_u &= \frac{w_g + \lambda (W_{e\pi} - W_{eg}) - rW_u}{r + s^L} \\ J_g - J_v &= \frac{-w_g + \lambda (J_\pi - J_g)}{r + s^L} \end{aligned}$$

The worker's surplus is the present discounted value of the wage w_g , plus the change in match surplus ($W_{e\pi} - W_{eg}$) when a consumer is found, net of the worker's outside option rW_u . The firm's surplus is the present discounted value of the difference between the wage paid to the worker and the expected gain once a customer has been found, $(J_\pi - J_g)$.

The match surpluses between a worker and firm, once a match in the goods market is formed and the firm moves the profit state, gain respectively:

$$W_{e\pi} - W_{eg} = \frac{w_\pi - w_g}{r + s + \lambda} \quad (21)$$

$$J_\pi - J_g = \frac{\mathcal{P}x - (w_\pi - w_g)}{r + s + \lambda} \quad (22)$$

A worker gains the present value of any change in the bargained wage. A firm gains the present value of the revenue from selling its good net of any change in the cost of labor.

3.4 Entry and wage determination in the market for labor

In keeping with the assumption in the goods market, the wage split the labor match surplus through bargaining into shares α_L and its complement $(1 - \alpha_L)$, where $\alpha_L \in (0, 1)$ is the bargaining strength of a worker. Several alternative assumptions can be made over which wages are bargained, when they are bargained, and on the nature of the outside option in bargaining. Our baseline assumption is a case in which wages are bargained in stages g and π separately. The alternative is a wage that is bargained to apply in both stages, that is, $w_\pi = w_g = w$, a case also considered below. We discuss how the second assumption affects the main results when differences arise, and provide complete details in appendix section B. Finally, firms are assumed to freely enter the labor market, resulting in an equilibrium with $J_v = 0$.

Sequential wage setting

The wage in each stage is negotiated independently of the other. The outside option in negotiating the wage in stage g is the dissolution of the labor match. That is,

$$w_g = \operatorname{argmax} (W_{eg} - W_u)^{\alpha_L} (J_g - J_v)^{1-\alpha_L} \quad (23)$$

The solution for the stage g wage must satisfy the sharing rule $(1 - \alpha_L) (W_{eg} - W_u) = \alpha_L (J_g - J_v)$, such that we have:

$$w_g = (1 - \alpha_L) z + \alpha_L \gamma \theta \quad (24)$$

The wage in stage g is increasing in the worker flow utility of non-employment z and labor market tightness θ .

There are two alternatives for the outside option when bargaining over the wage in stage π . In the first case the outside option is a loss of the consumer, but not the dissolution of the labor match. In the second case the outside option is the dissolution of the labor match. Both assumptions result in the same wage rule w_π .

(i) Loss of the consumer as outside option

When bargaining over the wage while matched with a consumer, in stage π , the worker and the firm consider the consequence of failing to reach an agreement to be the loss of the consumer and the preservation of the labor match. That is, the wage solves the bargaining problem

$$w_\pi = \operatorname{argmax} (W_{e\pi} - W_{eg})^{\alpha_L} (J_\pi - J_g)^{1-\alpha_L} \quad (25)$$

The private surpluses in this objective function have the same absolute slope (see equations (21) and (22)). As such, the wage must satisfy the sharing rule $(1 - \alpha_L) (W_{e\pi} - W_{eg}) = \alpha_L (J_\pi - J_g)$, which leads immediately to $w_\pi = \alpha_L \mathcal{P} + w_g$ – the increment in the wage is a share α_L of the revenue from the sale of the good – and

$$w_\pi = (1 - \alpha_L) z + \alpha_L (\gamma \theta + \mathcal{P}) \quad (26)$$

(ii) Dissolution of the labor match as outside option

When the outside option is the dissolution of the labor match, the wage bargained while the firm is in a match with a consumer solves:

$$w_\pi = \operatorname{argmax} (W_{\pi g} - W_u)^{\alpha_L} (J_\pi - J_v)^{1-\alpha_L} \quad (27)$$

The resulting wage must satisfy the sharing rule $(1 - \alpha_L) (W_{e\pi} - W_u) = \alpha_L (J_\pi - J_v)$. Since the wage bargained in stage g must still satisfy the sharing rule $(1 - \alpha_L) (W_{eg} - W_u) = \alpha_L (J_g - J_v)$, it remains the case that we have $(1 - \alpha_L) (W_{e\pi} - W_{eg}) = \alpha_L (J_\pi - J_g)$, as in the previous assumption on the outside option during stage π wage bargaining. The wage rule in π is thus once again:

$$w_\pi = (1 - \alpha_L) z + \alpha_L (\gamma \theta + \mathcal{P})$$

This wage increases with leisure and labor market tightness. It also reflects the outcome of the equilibrium in the goods market. The wage is increasing in the bargained price $\mathcal{P}$.

Constant wage profile

The alternative that the wage is to apply in both stages, that is, $w_\pi = w_g = w$, implies that the value of employment $rW_{eg} = rW_{e\pi} = rW_e = w + s^L (W_u - W_e)$. The wage solves the problem

$$w = \operatorname{argmax} (W_e - W_u)^{\alpha_L} (J_g - J_v)^{1-\alpha_L} \quad (28)$$

and must satisfy the surplus sharing rule $(1 - \alpha_L) (W_e - W_u) = \alpha_L (J_g - J_v)$. This results in a wage:

$$w = (1 - \alpha_L) z + \alpha_L \left[\gamma\theta + \frac{\lambda}{r + s + \lambda} \mathcal{P} \right] \quad (29)$$

This wage increases with non-employment utility z and labor market tightness. It also reflects the outcome of the equilibrium in the goods market along two dimensions. First, the wage is increasing in the bargained price $\mathcal{P}$. Second, it is increasing in the rate at which firms find consumers λ .

Job creation

Under free entry in the labor market, combining (10), (17), and (22), we have:

$$\frac{\gamma}{q} = \frac{\frac{\lambda}{r+s+\lambda} \mathcal{P} - \bar{w}}{r + s^L} \quad (30)$$

where $\bar{w} = \frac{\lambda w_\pi + (r+s)w_g}{r+s+\lambda}$ is an expected wage payment from the perspective of a firm entering the labor market. This expected wage payment is identical across all wage setting assumptions. Indeed, from the solution for the wages in stages g and π in the sequential wage bargaining case we have $\bar{w} = (1 - \alpha_L) z + \alpha_L \left[\gamma\theta + \frac{\lambda}{r+s+\lambda} \mathcal{P} \right]$, which corresponds to the wage in (29). We can rewrite the job creation condition under either set of assumptions on wage setting as:

$$(r + s^L) \frac{\gamma}{q} = (1 - \alpha_L) \left(\frac{\lambda}{r + s + \lambda} \mathcal{P} - z \right) - \alpha_L \gamma\theta \quad (31)$$

The factor $\frac{\lambda}{r+s+\lambda}$ discounts the revenue of the firm from selling by the necessary time it takes to form a relationship with a customer in the goods market.

3.5 Equilibrium

The decentralized equilibrium is a pair (ξ, θ) that solve a goods market and a labor market entry condition:

$$(r + s) \frac{\sigma}{\check{\lambda}(\xi)} = \frac{\alpha_G (1 - \alpha_L)}{1 - \alpha_G \alpha_L} \Phi - \frac{(1 - \alpha_G)}{1 - \alpha_G \alpha_L} \xi \sigma \quad (32)$$

$$(r + s^L) \frac{\gamma}{q(\theta)} = (1 - \alpha_L) \left[\Phi - z - \frac{(r + s) \sigma}{\check{\lambda}(\xi)} \left(\frac{1 - \alpha_G \alpha_L}{\alpha_G (1 - \alpha_L)} \right) \right] - \alpha_L \gamma\theta \quad (33)$$

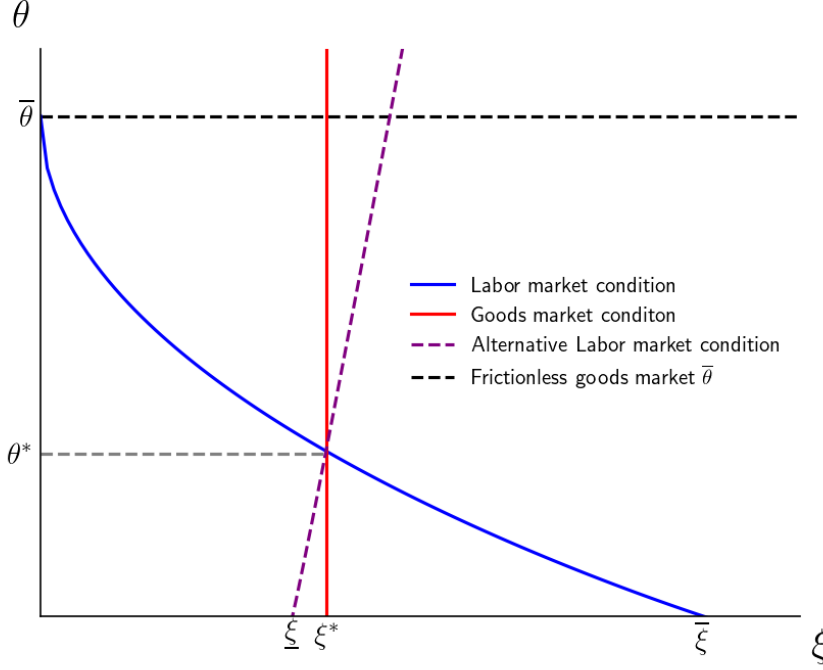


Figure 1: Decentralized equilibrium goods and labor market tightness ξ^* and θ^*

The goods market condition (32) is obtained from the consumer entry condition (14) along with the price rule (16) and the wage rules (24) and (26). The labor market condition (33) is obtained by expressing the term $\frac{\lambda P}{r+s+\lambda}$ as $\Phi\chi - \frac{(r+s)\sigma}{\tilde{\lambda}(\xi)} \left(\frac{1-\alpha_G\alpha_L}{\alpha_G(1-\alpha_L)} \right)$ in equation (31).

The equilibrium conditions are plotted in (ξ, θ) space in Figure 1. The equilibrium level of tightness in the goods market ξ^* is uniquely determined by equation (32), and depicted as a vertical line. Although it is independent of equilibrium labor market tightness, it depends on the worker's share of the labor market surplus in wage setting α_L . The equilibrium condition (33) describes a decreasing relation between labor and goods market tightness. This is represented by decreasing curve in (ξ, θ) space of Figure (1). Goods market frictions unambiguously reduce labor market tightness relative to the limit when $\sigma/\tilde{\lambda}$ tends toward zero. In this limit case labor market tightness converges to $\bar{\theta}$, which is plotted a horizontal dashed line in Figure 1. Forming a match in the goods market is costly and the rents in the goods market must be shared with consumers, reducing the net surplus to creating employment.

The labor market entry condition (33) may also be expressed as:

$$(r + s^L) \frac{\gamma}{q(\theta)} = (1 - \alpha_L) \left(\frac{1 - \alpha_G}{(1 - \alpha_L) \alpha_G} \sigma \xi - z \right) - \alpha_L \gamma \theta \quad (34)$$

In this new expression labor market tightness is increasing in goods market tightness, and it is depicted by the increasing dashed line in Figure 1. Equations (33) and (34) intersect at the equilibrium level of goods market tightness ξ^* . Equation (34) places a lower bound $\underline{\xi}$ on goods market tightness for an equilibrium with positive labor market tightness, while equation (33) places an up-

per bound $\bar{\xi}$. Under parameter restrictions ensuring $\underline{\xi} < \bar{\xi}$, detailed next, an equilibrium with positive tightness in the goods and labor market exists and is unique. Indeed, equation (34) requires that $\sigma \xi \left(\frac{1-\alpha_G}{(1-\alpha_L)\alpha_G} \right) > z$, placing a lower bound for goods market tightness of $\underline{\xi} = \frac{z}{\sigma} \left(\frac{1-\alpha_G}{(1-\alpha_L)\alpha_G} \right)$ for an equilibrium with positive θ . The existence of an equilibrium with positive labor market tightness in condition (33) requires that $\frac{\Phi-z}{r+s} > \frac{\sigma}{\lambda(\xi)} \left(\frac{1-\alpha_G\alpha_L}{\alpha_G(1-\alpha_L)} \right)$, placing the upper bound on goods market tightness $\bar{\xi}$.

Equilibrium effects of changes in market power

It is informative to describe the decentralized equilibrium as defined by the triplet $(\xi, \theta, \mathcal{P})$ that solves (i) the consumer search effort condition (35), (ii) the goods market price rule (36), and (iii) the labor market equilibrium condition (37):

$$\frac{\sigma}{\lambda(\xi)} = \frac{(\Phi - \mathcal{P})}{r + s} \quad (35)$$

$$\mathcal{P} = (1 - \alpha_G) \frac{(\Phi + \xi\sigma)}{(1 - \alpha_G\alpha_L)} \quad (36)$$

$$(r + s^L) \frac{\gamma}{q(\theta)} = (1 - \alpha_L) \left(\frac{\lambda(\xi)}{r + s + \lambda(\xi)} \mathcal{P} - z \right) - \alpha_L \gamma \theta \quad (37)$$

The equilibrium in the goods market is represented in $(\mathcal{P}, \xi)$ space in Figure 2a. The consumer search effort condition (35) describes a decreasing relation between the price and the tightness in the goods market. The lower the price, the more consumers expend search effort D_U , leading to a tighter market. The price rule (36) is an increasing function of goods market tightness. As long as consumer market power α_G is strictly positive, and the worker's share in wage bargaining $\alpha_L < 1$, there is a unique equilibrium with positive goods market tightness.

Turning to the equilibrium in the labor market, represented in $(\mathcal{P}, \theta)$ space in Figure 2b, the job creation condition (37) describes a strictly increasing relation between the price in the goods market and tightness of the labor market. The equilibrium price in the goods market is independent of labor market tightness and, as long as the expected revenue for the firm $\frac{\lambda}{r+s+\lambda} \mathcal{P}$ is greater than the flow value of non-employment z , there exists a unique equilibrium with positive labor market tightness.

We consider first the equilibrium effects of changes in α_G .³ An increase in the goods market power of consumers, α_G , tilts and shifts the price curve outwards (Figure 2a). The decentralized equilibrium shifts down the entry curve to a higher level of tightness ξ^* and lower price $\mathcal{P}^*$. Consumers receive more surplus in the goods market, through a lower price, and are induced to greater entry. In the labor market, the equilibrium price curve shifts down, in itself pushing towards lower labor market tightness along the original job creation curve. However, the increased goods market tightness shifts the job creation curve outward as firms match with consumers more quickly. This increase in the incentives to enter the labor market and create jobs can overturn the

³Additional comparative statics are discussed in the appendix.

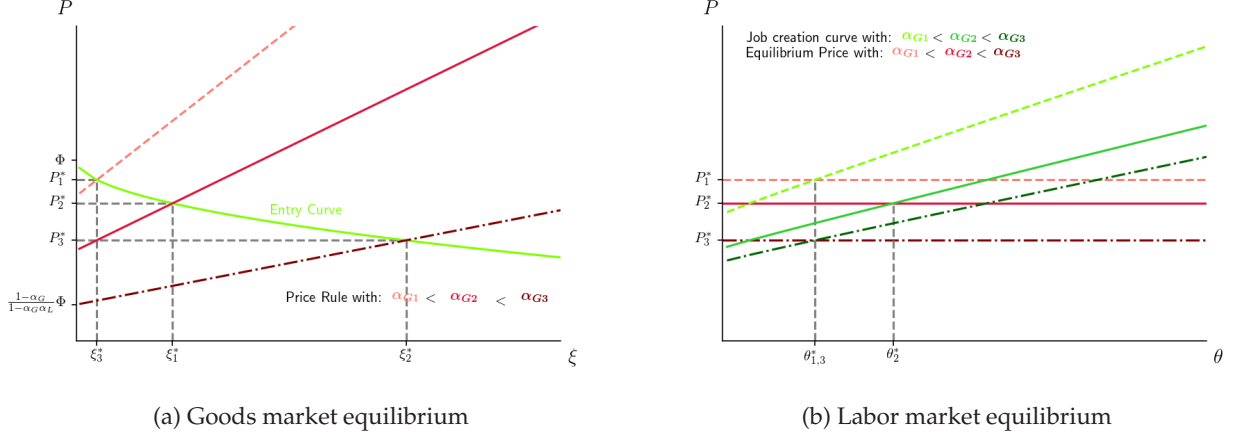


Figure 2: Equilibrium effects of increases in goods market bargaining strength α_G

decline in labor market tightness due to the drop in price.

The effect of goods market bargaining power on tightness in the labor market is non-monotonic. While goods market tightness ξ^* is strictly increasing in α_G , in the labor market the equilibrium effect of changes in goods market power α_G depends on whether the price or the trading effect dominates. An increase in α_G always reduces the price, which reduces the incentives for firms to enter the labor market. However, the increase in goods market tightness accelerates firm's meeting rate λ , which increases the expected payoff from hiring labor. In the region $(0, \eta_G / [1 - \alpha_L (1 - \eta_G)])$ greater bargaining power for consumers leads to greater equilibrium labor market tightness. The trading effect dominates the (declining) price effect. Beyond this point, when the seller's market power $(1 - \alpha_G)$ is less than the elasticity of the goods market matching function η_G (adjusting for labor's bargaining strength α_L), further increases in market power reduce equilibrium labor market tightness. In terms of Figure 2b, the downwards shift in the price curve exceeds the outward shift in the job creation curve.

This is summarize in the following proposition:⁴

Proposition 1 - The decentralized equilibrium goods market tightness ξ^* is strictly increasing in the consumer bargaining power α_G . The decentralised equilibrium tightness of the labor market θ^* is an increasing an concave function of α_G , maximized when the consumer's share of the goods market surplus equals $\alpha_G = \eta_G / [1 - \alpha_L (1 - \eta_G)]$.

Next we consider the equilibrium effects of changes in α_L . The goods market effect of an increase in α_L is an increase in the price $\mathcal{P}$ and decrease in tightness ξ . Part of the greater cost of labor is passed on to the consumer through higher prices, and consumers have less incentive to

⁴Under the assumption of an identical wage in both stages g and π , equilibrium tightness in the goods market is independent of rent sharing in labor market, and labor market tightness is maximized when the consumer's bargaining share α_G equals the elasticity η_G .

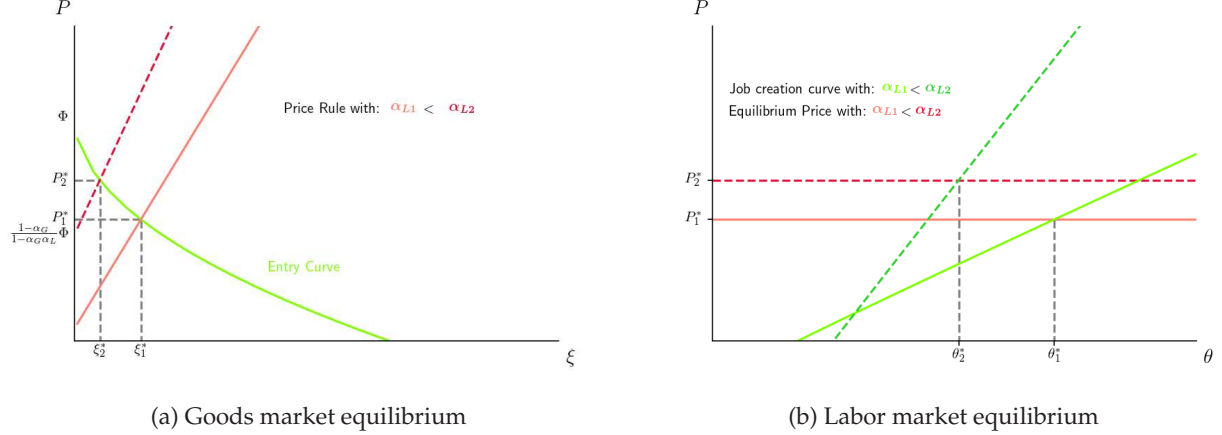


Figure 3: Equilibrium effects of increases in labor market bargaining strength α_L

search for products. In the labor market, the decline in meeting rate for firms from lower goods market tightness always dominates any effect on job creation incentives from the increase in the price.

Thus changes in market power in labor market lead to equilibria that feature a declining labor market tightness along with a rising price. The effect on the equilibrium wage however depends on whether the price or trading channels dominate in the goods market. Beyond a threshold share of the labor market surplus accruing to workers, further increases in α_L reduce the wage.

4 Constrained efficiency in goods and labor markets

4.1 Social planner's problem

The social planner seeks to maximize the value of consumption of the search good and non-employment utility, net of labor and goods market search costs:

$$\Omega = \int_0^\infty e^{-rt} [D_M \Phi + z\mathcal{U} - \gamma\mathcal{V} - \sigma D_U] dt$$

Equations (1) to (4) are dynamic frictional constraints. They apply to both the decentralized equilibrium and the social planner's program. Equations (5) and (6) are also common to the decentralized equilibrium and the social planner's program. Using these constraints, and in particular constraint (7) to replace $\mathcal{N}_g$ by a function of $\mathcal{U}$ and D_M , we have:

$$\begin{aligned} \Omega = \max_{D_U, \mathcal{V}, \mathcal{U}, D_M} & \quad e^{-rt} [D_M \Phi + z\mathcal{U} - \gamma\mathcal{V} - \sigma D_U] \\ & + \Psi_U [M_L(\mathcal{U}, \mathcal{V}) - s^L(1 - \mathcal{U})] \\ & + \Psi_{D_M} \left[(s^G + s^L) D_M - M_G(D_U, 1 - \mathcal{U} - D_M) \right] \end{aligned}$$

where the planner chooses consumer search units D_U and job vacancies $\mathcal{V}$, subject to the laws of motion of the co-states of unemployment $\mathcal{U}$ and matched consumers D_M . The associated multipliers are Ψ_U and Ψ_{D_M} , respectively.

The social planner's allocation is a pair of efficient goods and labor market tightness $(\xi^{opt}, \theta^{opt})$ that solve the pair of conditions :

$$(r + s) \frac{\sigma}{\check{\lambda}(\xi)} = \eta_G \Phi - (1 - \eta_G) \sigma \xi \quad (38)$$

$$(r + s^L) \frac{\gamma}{q(\theta)} = (1 - \eta_L) \left[\Phi - z - \frac{\sigma(r + s)}{\check{\lambda}(\xi)} \frac{1}{\eta_G} \right] - \eta_L \gamma \theta \quad (39)$$

See appendix A for details of the derivations. The constrained efficient level of tightness in the goods market ξ^{opt} is uniquely determined by equation (38). It is independent of equilibrium labor market tightness. The equilibrium condition (39) describes a decreasing relation between labor and goods market tightness. Goods market frictions unambiguously reduce labor market tightness relative to what the social planner could achieve if $\sigma/\check{\lambda}$ were to tend toward zero.⁵

4.2 Second best efficiency and the double Hosios rule

Comparing the equilibrium conditions (32) in the decentralized economy to the social planner's allocation in (38), tightness in the goods market is constrained efficient if $\alpha_G = \frac{\eta_G}{1 - \alpha_L(1 - \eta_G)}$. Turning to the labor market, from equations (33) and (39) the decentralized equilibrium in the labor market corresponds to the social planner's allocation if $\alpha_L = \eta_L$ and $\alpha_G = \frac{\eta_G}{(1 - \alpha_L(1 - \eta_G))}$. At this point, the search externalities in the goods and labor markets are internalized and the decentralized agents reach an optimal allocation. This leads to a double Hosios condition:

Proposition 2 - When wages are set separately at firms searching in the goods market and matched in the goods market, the decentralized allocation in an economy with search and bargaining in goods and labor markets is constrained efficient if and only if $\alpha_L = \eta_L$ and $\alpha_G = \frac{\eta_G}{1 - \alpha_L(1 - \eta_G)}$.

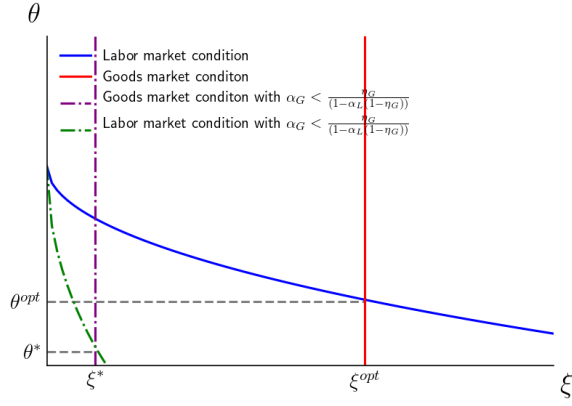
We thus obtain a double Hosios condition, similar to that obtained in the labor and credit markets in Petrosky-Nadeau and Wasmer (2013).

The equilibrium effects of deviations from the Hosios conditions can be illustrated by representing the social planner's and decentralized economy' equilibrium condition in (ξ, θ) space, as in Figure 4. The equilibrium curves overlap when $\alpha_L = \eta_L$ and $\alpha_G = \frac{\eta_G}{1 - \alpha_L(1 - \eta_G)}$, at which point $\xi^* = \xi^{opt}$ and $\theta^* = \theta^{opt}$.

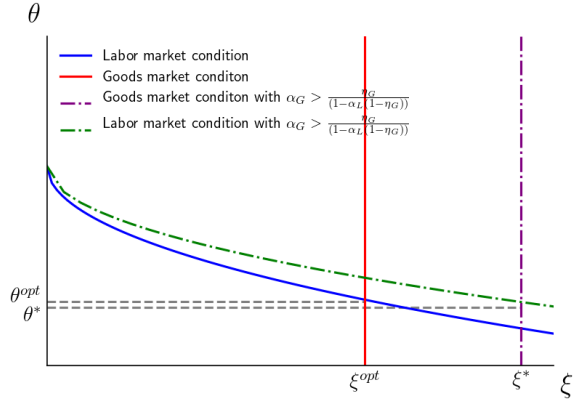
Reducing α_G in Figure 4a shifts the vertical goods market equilibrium condition to the left to

⁵As in the decentralized equilibrium, the labor market condition (39) may be re-expressed as:

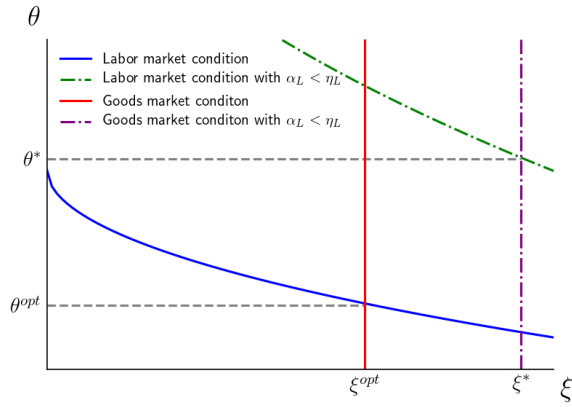
$$(r + s^L) \frac{\gamma}{q(\theta)} = (1 - \eta_L) \left[\sigma \xi \left(\frac{1 - \eta_G}{\eta_G} \right) - z \right] - \eta_L \gamma \theta \quad (40)$$



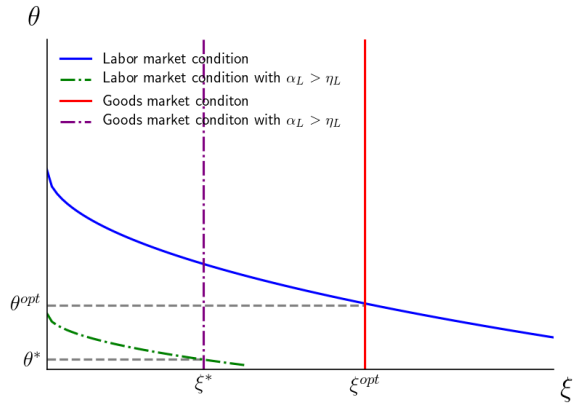
(a) Deviation $\alpha_G < \frac{\eta_G}{(1-\alpha_L)(1-\eta_G)}$: $\xi^* < \xi^{opt}$ and $\theta^* < \theta^{opt}$



(b) Deviation $\alpha_G > \frac{\eta_G}{(1-\alpha_L)(1-\eta_G)}$: $\xi^* > \xi^{opt}$ and $\theta^* < \theta^{opt}$



(c) Deviation $\alpha_L < \eta_L$: $\xi^* > \xi^{opt}$ and $\theta^* > \theta^{opt}$



(d) Deviation $\alpha_L > \eta_L$: $\xi^* < \xi^{opt}$ and $\theta^* < \theta^{opt}$

Figure 4: Deviations from the constrained efficient goods and labor market tightness ξ^{opt} and θ^{opt}

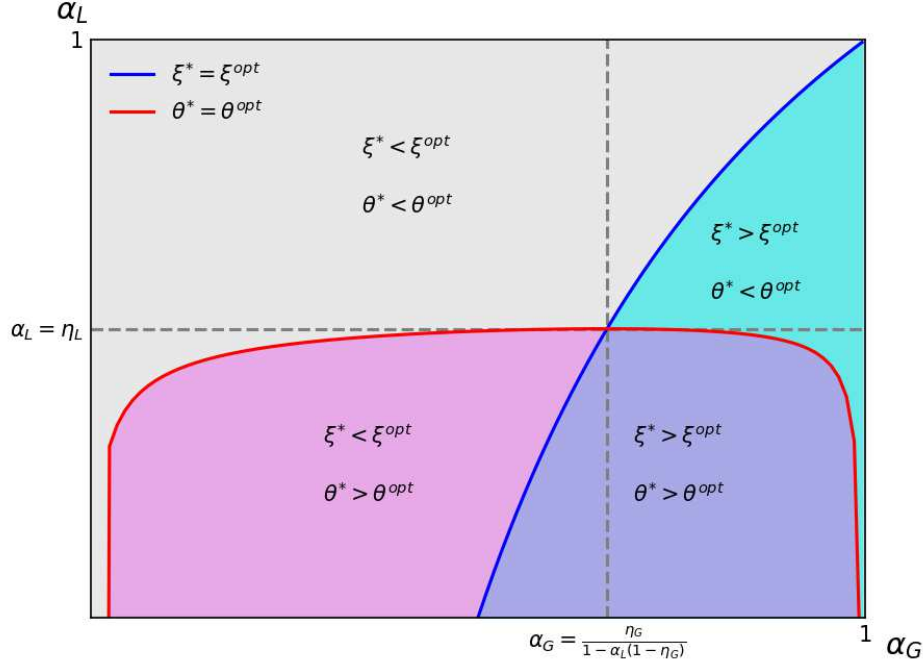


Figure 5: Efficiency of tightness in the goods and labor markets

a lower tightness of the goods market ξ^* . Along the original (solid) job creation curve, the higher price for the good would lead to an increase in firm entry in the labor market and a tighter labor market. However, the decline in α_G tilts the labor market equilibrium curve inwards as firms face greater difficulty in finding a consumer to sell to. This second effect dominates and the equilibrium labor market tightness is too low.

The opposite comparative static, an increase in α_G , is shown in Figure 4b. In the goods market, consumers face a lower price, obtaining a larger share of the surplus and goods market tightness is inefficiently high. In itself this leads to a decline in labor market tightness along the original labor market condition from a lower price. However there is an upward tilt in the job creation from the shortening in the duration of search for sellers in the goods market, mitigating some of the decline in θ . Both deviations, an increase and decrease in α_G , result in a decline in labor market tightness relative to the constrained efficient allocation, although for different underlying reasons in the goods market.

Turing market power in the labor market, a decrease in the worker's share in the labor match surplus α_L leads to a decentralized equilibrium in which the goods market is too tight and the labor market are too tight. As seen in Figure 4c, a lower α_L shifts the job creation curve out as firms respond to receiving a greater share of the labor surplus by creating jobs. The increase in θ^* is mitigated by the increase in ξ^* above ξ^{opt} . An increase in α_L has the exact opposite effect, as shown in Figure 4d.

All possible deviations of allocations away from the constrained efficient can be represented in (α_G, α_L) space as in Figure 5. The condition for constrained efficiency in the goods market,

$\alpha_G = \frac{\eta_G}{(1-\alpha_L(1-\eta_G))}$, is plotted as the upward slope curve in the rang $\alpha_G \in [\eta_G, 1]$. The curve represents all the pairs of bargaining weights in the goods and labor market that deliver $\zeta^* = \zeta^{opt}$. To the right of the curve the goods market is too tight. To the left of the curve the goods market is too slack. In particular, it states that for greater worker bargaining strength α_L consumers need to receive a greater share of the surplus in the goods market to ensure the efficient tightness of the goods market. If not, if consumer market power is too weak, there is an insufficient entry of consumer in the goods market relative to goods as the decentralized price $\mathcal{P}$ is too high. At the limit, as the worker share α_L tends to 1, the consumer needs to receive a share of the goods market surplus α_G that also tends to 1. At the other end of rent sharing, when the worker receives its reservation wage ($\alpha_L = 0$), efficient goods market tightness is achieved for $\alpha_G = \eta_G$. Below $\alpha_G = \eta_G$ it is not possible to restore constrained efficiency in the goods market by varying market power in the labor market.⁶

A second important curve separating the bargaining weight space is defined by the combinations of bargaining weights α_L and α_G which deliver a decentralized labor market tightness θ^* exactly equal to the constrained efficient θ^{opt} . Indeed, combining the price rule and the job creation equation (35) yields an expression that implicitly defines α_L as function of α_G for a given level of labor market tightness.⁷ This function is plotted as the increasing and concave curve in Figure 5. All location below the curve correspond to a decentralized labor market allocation with $\theta^* > \theta^{opt}$. All locations above the curve correspond to decentralized allocations with $\theta^* < \theta^{opt}$. The intersection of both curves, which corresponds to of $\alpha_L = \eta_L$ and $\alpha_G = \eta_G / [1 - \alpha_L (1 - \eta_G)]$, is the constrained efficient allocation in which $\zeta^* = \zeta^{opt}$ and $\theta^* = \theta^{opt}$.

We thus have four regimes depending on which market is either too tight or too slack relative to the socially efficient in a model with fully flexible prices.

Special case of a constant wage profile

Efficiency of the decentralized equilibrium in the case of a constant wage profile is discussed in appendix B.5, and summarized as:

Proposition 3 - The decentralized allocation in an economy with search and bargaining in goods and labor markets is constrained efficient if and only if $\alpha_G = \eta_G$ and $\alpha_L = \eta_L$.

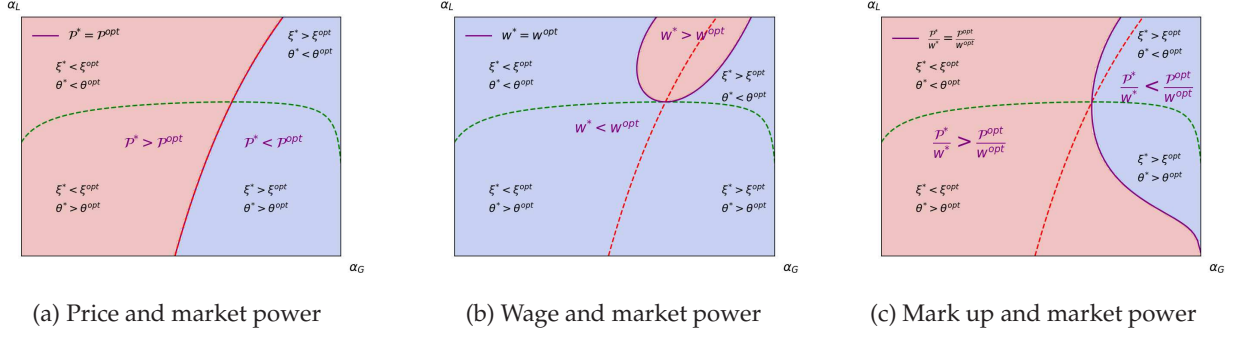


Figure 6: Efficiency of prices, wages, and markups market goods and labor market power space. Regions in which the decentralized equilibrium features an inefficiently high outcome of interest are in red. Inefficiently low decentralized values are in blue. The locii of bargaining strengths for which $\xi^* = \xi^{opt}$ and $\theta^* = \theta^{opt}$ are represented by the dashed lines.

4.3 Efficiency of prices, wages and markups

We can now address whether prices, wages, and their ratio are inefficiently high or low, and how this maps to market power in goods and labor market. Figures in 6 superimposes on Figure 5 regions where an outcome of interest is above (red) or below (blue) the same outcome in the social planner's allocation implementing the constrained efficient allocation for a given pair of market power α_G and α_L .

All decentralized allocations that result in a goods market tightness $\xi^* < \xi^{opt}$ have a price $\mathcal{P}^*$ that is inefficiently high, and conversely when $\xi^* > \xi^{opt}$ the decentralized allocation price is too low (see Figure 6a). The labor market, however, can be either too tight or too slack when the price in the goods market deviates from $\mathcal{P}^{opt}$, depending on the share of the surplus accruing to the worker in wage setting. For a given market power in the goods market $\alpha_G < \frac{\eta_G}{1-\alpha_L(1-\eta_G)}$, we have a decentralized equilibrium with prices that are too high along with too little job creation when α_L is above a value ensuring $\theta^* = \theta^{opt}$.

In all four tightness regimes identified earlier, the price in the goods market is either too high or too low but never both. This is not the case for wages, as seen in Figure 6b. Allocation with insufficient job creation, $\theta^* < \theta^{opt}$, can arise when the wage is either too high or too low. This only occurs in the standard model of equilibrium unemployment when the wage is too high. In the presence of product market frictions this can occur even with a low wage the further the equilibrium departs from the constrained efficient allocation in the goods market. This is because the expected revenue $\lambda \mathcal{P}$ is at a maximum at $\xi^* = \xi^{opt}$. As we move to the left of the dashed curve for which $\xi^* = \xi^{opt}$, the price is too high, too few consumers enter the goods market and the expected revenue falls because it is hard for firms to find consumers. To the right the opposite

⁶In addition, the implied α_L is always strictly less than α_G when $\eta_G \geq 0.5$. For any $0 < \eta_G < 0.5$, there a crossing at $\tilde{\alpha}_G$ with $\alpha_L < \alpha_G$ when $\alpha_G < \tilde{\alpha}_G$, and $\alpha_L > \alpha_G$ when $\alpha_G > \tilde{\alpha}_G$. See Figure ?? of the appendix. In the constant wage profile case the goods market Hosios condition with $\alpha_G = \eta_G$ is a vertical line. See Figure ?? of the appendix.

⁷The curved is defined by the function $(r + s^L) \frac{\gamma}{q(\theta^{opt})} = (1 - \alpha_L) \left[\frac{\lambda(\xi^*)}{r + s + \lambda(\xi^*)} \left(\frac{1 - \alpha_G}{1 - \alpha_L \alpha_G} \right) (\Phi + \xi^* \sigma) - z \right] - \alpha_L \gamma \theta^{opt}$ in which θ^{opt} is taken as given, and ξ^* is the decentralized goods market tightness that solves (32).

occurs: the price is too low, and even though it is easier to find costumers as tightness in the goods market is higher, the expected revenue from entering the labor market falls. Allocation with excessive job creation, $\theta^* > \theta^{opt}$, only arise with wages that are too low.

The last panel, Figure 6c, presents the complementary information of the mark up of price of wage. The tight labor market with excessive seller market power always have a markup that is inefficiently high. The mirror regime of a slack labor market with too much market power for consumers and a tight goods market presents a markup that is inefficiently low.

5 Evaluating costs of frictions in the goods and labor markets

In what follows we calibrate the model to the US economy in order to recover the key parameters and evaluate the costs of frictions in labor and goods markets.

5.1 Calibrating to the US economy

The basic unit of time is a month. The rate of time preference is set to match an average return on 3-month Treasury bill of 2.5 percent at an annualized rate. The calibration of labor market and goods market parameters are described next, and summarized in Table 1.

We begin with the set of parameters in the labor market s^L , η_L , χ_L , α_L , γ , and z . The rate of job separation s^L is set to 0.032 corresponding to estimates from the Job Openings and Labor Turnover Survey (JOLTS) over the period January 2001 to December 2016. The matching function in the labor market is assumed to be a Cobb-Douglas $M_L = \chi_L \mathcal{U}^{\eta_L} \mathcal{V}^{\eta_L}$. Petrongolo and Pissarides (2001) provide an extensive survey of empirical estimates of the labor market matching function. The most frequent finding is an elasticity of the matching function $\eta_L = 0.5$, which we adopt here. The average vacancy rate in the US over the period spanning January 2001 to December 2016 was 4 percent, which pins down the value of the level parameter in the matching function χ_L . The average unemployment rate over the same period, 6 percent, serves as calibration target for the worker's share α_L of the labor surplus. We then obtain the flow cost of vacancy posting γ by targeting average recruiting costs relative to starting wage estimated in Silva and Toledo (2009) to be approximately 40% of a month's wage ($\gamma/(q \times w) = 0.4$). The flow value of non-employment $z = 0.71$ as per Hall and Milgrom (2008).

Next we address the set of parameters in the goods market s^G , η_G , χ_G , α_G , σ , and Φ . We use estimates of product exit rates from household consumption baskets provided in Broda and Weinstein (2010) to set $s^G = 0.001$, as in Petrosky-Nadeau and Wasmer (2015). The matching function in the goods market is assumed to be a Cobb-Douglas $M_G = \chi_G \mathcal{D}_U^{\eta_G} \mathcal{N}_S^{1-\eta_G}$, where η_G is the elasticity with respect to searching consumers $\mathcal{D}_U$ and χ_G a level parameter. The model implies a price elasticity of demand⁸

$$d\mathcal{D}_M/d\mathcal{P} \times \mathcal{P}/\mathcal{D}_M = -\frac{\eta_G}{1-\eta_G} \frac{\mathcal{P}}{\mathcal{P}-\Phi}$$

⁸See appendix B.6 for the derivation.

We use empirical estimates of this elasticity to calibration the value of the goods market matching function elasticity η_G . We target an elasticity of -7 based on the work of Hottman et al (2016) and present results for a range of value further below. The rate of capacity utilization in the US manufacturing sector of the period spanning January 2001 to December 2016 averaged 75 percent. This rate in the model is defined as $\mu = \mathcal{N}_\pi / (\mathcal{N}_g + \mathcal{N}_\pi)$ and serves as a calibration target for the level parameter of the matching function χ_G . Broda and Weinstein (2010) additionally provide estimates of product entry rates that imply target a value of λ for consumers of 0.22 at an annualized rate to determine the value of the marginal utility Φ . As in Petrosky-Nadeau and Wasmer (2015), we use estimates from the American Time Use Survey of the time spent shopping for goods and services to calibration the search cost parameter σ . That is, σ is such that the value of consumer search σD_U equals 5 percent of labor earnings. Finally, we target a markup over the wage $\mathcal{P}/w$, where $w = \mu w_\pi + (1 - \mu) w_g$, to calibrate the value of the goods market bargaining power α_G . There is a range of estimates of the markup, and our baseline results target a 15% markup over the wage. We present the results for a range of values further below.

The calibration results in a worker's wage bargaining weight α_L of 0.X, a consumers share of the surplus in the goods market α_G of 0.Y, and an elasticity of the goods market matching function is found to have an elasticity of $\eta_G = 0.27$.⁹

Table 1: A calibration: Targets and Parameter Values

	Parameter	Value	Target or reference:
Technology and discounting:			
time discount rate	r	$e^{(-2.5/1200)} - 1$	3-month Treasury bill
Labor market:			
job-separation rate	s^L	0.032	JOLTS
elasticity of matching function	η_L	0.50	Petrongolo and Pissarides (2001)
level of matching function	χ_L	0.63	Job vacancy rate
worker bargaining weight	α_L	0.32	Unemployment rate
vacancy cost	γ	0.24	Recruiting costs $\gamma / (q \times w)$
non-employment value	z	0.71	Hall and Milgrom (2008)
Goods market:			
goods exit rate	s^G	0.001	Product exit rate
elasticity of matching function	η_G	0.27	Price elasticity of demand
level of matching function	χ_G	0.07	Rate of capacity utilization
consumer bargaining weight	α_G	0.24	Price markup over wage
cost of search	σ	0.04	American Time Use Survey
marginal utility of search good	Φ	1.10	Product entry rate

⁹The Bai et al using a different approach to infer this elasticity find a value of 0.29. Gourio and Rudanko (2013), on the other hand find a calibrated value of 0.11 in a model where firms acquire customers as capital through search.

Table 2: Estimates of the costs of goods and labor market frictions in the calibrated economy

		Baseline calibration	Constrained efficient allocation	Allocation with $\alpha_L = \eta_L$ and $\alpha_G \neq \eta_G$	Allocation with $\alpha_L \neq \eta_L$ and $\alpha_G = \frac{\eta_G}{1-\alpha_L(1-\eta_G)}$
Goods market tightness	ξ	5.27	333.45	5.27	333.45
Price	$\mathcal{P}$	1.08	1.04	1.08	1.04
Markup	$\mathcal{P}/w$	1.30	1.06	1.37	1.06
Capacity utilization rate	μ	75%	96%	75%	96%
Labor market tightness	θ	0.67	1.30	0.36	2.39
Wage	w	0.79	0.98	0.79	0.98
Unemployment rate	$\mathcal{U}$	6.00%	4.37%	7.96%	3.26%

Notes: In the efficient allocation case the bargaining parameters are set to the Hosios condition and all other parameters are kept constant. In the efficient labor market allocation only the workers weight is set to the Hosios condition. In the efficient goods market case only the consumer's bargaining weight is set to the Hosios condition.

5.2 Estimating the costs of market frictions [TO BE REWRITTEN]

The calibration of the model to moments from the US economy yielded bargaining shares in goods and labor market that deviate from the Hosios conditions. In the labor market $\alpha_L < \eta_L$, which, where the Hosios condition in the goods market to hold, would imply inefficiently high θ with excessive entry of firms. However, $\alpha_G < \eta_G$ by a margin sufficient that the decentralized equilibrium in a region with inefficiently low labor market tightness due to the low entry of consumers in the goods market. In order to evaluate the economic magnitude of these departures, we calculate the allocation restoring the Hosios conditions in both markets, keeping all other parameters constant. That is, we set $\alpha_G = \eta_G$ and $\alpha_L = \eta_L$. The first column of results in Table 2 presents the value of central and target variables in the baseline calibration. The second column presents their values at the constrained efficient allocation.

The unemployment rate in the constrained efficient allocation, at 4.37 percent, is 1.6 percentage points lower than the calibration target for the decentralized allocation. The rate of capacity utilization would be 96 percent, up from 75 percent, and the markup 6 percent, down from 37 percent.

In the last two columns of Table 2 we report the effects of imposing a Hosios condition in one of the markets alone.

6 Conclusion

[TO BE WRITTEN]

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A Social planner's problem

This section provides the detailed derivations for section 4. Recall the social planner's program:

$$\begin{aligned}
H = \max_{D_U, \mathcal{V}, \mathcal{U}, D_M} & e^{-rt} [D_M \Phi x + z\mathcal{U} - \gamma\mathcal{V} - \sigma D_U] \\
& + \Psi_{D_M} \left[(s^G + s^L) D_M - M_G(D_U, 1 - \mathcal{U} - D_M) \right] \\
& + \Psi_U \left[M_L(\mathcal{U}, \mathcal{V}) - s^L(1 - \mathcal{U}) \right]
\end{aligned}$$

using $\frac{\partial M_G}{\partial \mathcal{U}} = \frac{\partial M_G}{\partial \mathcal{N}_g} \frac{\partial \mathcal{N}_g}{\partial \mathcal{U}}$, $\frac{\partial M_G}{\partial D_M} = \frac{\partial M_G}{\partial \mathcal{N}_g} \frac{\partial \mathcal{N}_g}{\partial D_M}$, and the follow properties of the matching functions: $\frac{\partial M_L}{\partial \mathcal{U}} = \eta_L f(\theta)$; $\frac{\partial M_G}{\partial D_U} = \eta_G \check{\lambda}$; $\frac{\partial M_L}{\partial \mathcal{V}} = (1 - \eta_L) q(\theta)$; $\frac{\partial M_G}{\partial \mathcal{N}_g} =$; $(1 - \eta_G) \lambda(\theta)$, the first order conditions are:

$$\frac{\partial H}{\partial D_U} = 0 \rightarrow e^{-rt}(-\sigma) - \Psi_{D_M} \frac{\partial M_G}{\partial D_U} = 0 \quad (\text{A.1})$$

$$\frac{\partial H}{\partial \mathcal{V}} = 0 \rightarrow e^{-rt}(-\gamma) + \Psi_U (1 - \eta_L) q = 0 \quad (\text{A.2})$$

$$\frac{\partial H}{\partial D_M} = -\dot{\Psi}_{D_M} \rightarrow e^{-rt}(\Phi x) + \Psi_{D_M} \left[(s^G + s^L) + (1 - \eta_G) \lambda(\theta) \right] = -\dot{\Psi}_{D_M} \quad (\text{A.3})$$

$$\frac{\partial H}{\partial \mathcal{U}} = -\dot{\Psi}_U \rightarrow e^{-rt}z + \Psi_U \left(\eta_L f + s^L \right) + \Psi_{D_M} (1 - \eta_G) \lambda(\theta) = -\dot{\Psi}_U \quad (\text{A.4})$$

A.1 Constrained efficient goods market tightness

To obtain the efficient goods market tightness, begin with (A.1):

$$\Psi_{D_M} = -\frac{e^{-rt}\sigma}{\eta_G \check{\lambda}} \quad \text{and} \quad \dot{\Psi}_{D_M} = \frac{re^{-rt}\sigma}{\eta_G \check{\lambda}}$$

Combining with (A.3) we have

$$\begin{aligned}
e^{-rt}(\Phi x) + \Psi_{D_M} \left[(s^G + s^L) + (1 - \eta_G) \lambda(\theta) \right] &= -\dot{\Psi}_{D_M} \\
e^{-rt}\Phi x - \frac{e^{-rt}\sigma}{\eta_G \check{\lambda}} \left[(s^G + s^L) + (1 - \eta_G) \lambda \right] &= \frac{re^{-rt}\sigma}{\eta_G \check{\lambda}} \\
\eta_G \Phi x - \sigma \check{\zeta} (1 - \eta_G) &= \frac{\sigma}{\check{\lambda}} (r + s)
\end{aligned}$$

A.2 Constrained efficient labor market tightness

Solving for Ψ_U in (A.2) we obtain:

$$\Psi_U = \frac{e^{-rt}\gamma}{q(1 - \eta_L)} \quad \text{and} \quad \dot{\Psi}_U = \frac{-re^{-rt}\gamma}{q(1 - \eta_L)}$$

and in combination with (A.4), we have:

$$\begin{aligned}
-\dot{\Psi}_U &= e^{-rt}z + \Psi_U (\eta_L f + s^L) + \Psi_{D_M} (1 - \eta_G) \lambda \\
\frac{-re^{-rt}\gamma}{q(1-\eta_L)} &= e^{-rt}z + \frac{e^{-rt}\gamma}{q(1-\eta_L)} (\eta_L f + s^L) - \frac{e^{-rt}\sigma}{\eta_G \check{\lambda}} (1 - \eta_G) \lambda \\
\frac{-r\gamma}{q(1-\eta_L)} &= z + \frac{\gamma}{q(1-\eta_L)} (\eta_L f + s^L) - \frac{\sigma}{\eta_G \check{\lambda}} (1 - \eta_G) \lambda \\
\frac{(r+s^L)\gamma}{q(1-\eta_L)} &= \frac{\sigma}{\eta_G \check{\lambda}} (1 - \eta_G) \lambda - z - \frac{\eta_L \gamma f}{q(1-\eta_L)} \\
\frac{(r+s^L)\gamma}{q} &= (1-\eta_L) \left[\frac{\sigma}{\eta_G \check{\lambda}} (1 - \eta_G) \lambda - z \right] - \eta_L \gamma \theta \\
\frac{(r+s^L)\gamma}{q} &= (1-\eta_L) \left[\sigma \check{\xi} \left(\frac{1-\eta_G}{\eta_G} \right) - z \right] - \eta_L \gamma \theta
\end{aligned}$$

Note that from the equilibrium condition for goods market tightness we have that $\sigma \check{\xi} \left(\frac{1-\eta_G}{\eta_G} \right) = \Phi x - \frac{\sigma}{\eta_G \check{\lambda}(\check{\xi})} (r+s)$. This delivers the social planner's job creation condition (39).

B Decentralized equilibrium

The consumer, worker, and firm Bellman equations are reproduced here for convenience:

$$rW_{D_U} = -\sigma + \check{\lambda} (W_{D_M} - W_{D_U}) \quad (\text{A.5})$$

$$rW_{D_M} = (\Phi - \mathcal{P})x + s^G (W_{D_U} - W_{D_M}) + s^L (W_{D_U} - W_{D_M}) \quad (\text{A.6})$$

$$rW_{e\pi} = w_\pi + s^L (W_u - W_{e\pi}) + s^G (W_{e\pi} - W_{e\pi}) \quad (\text{A.7})$$

$$rW_{eg} = w_g + s^L (W_u - W_{eg}) + \lambda (W_{e\pi} - W_{eg}) \quad (\text{A.8})$$

$$rW_u = z + f(\theta) (W_{eg} - W_u) \quad (\text{A.9})$$

$$rJ_v = -\gamma + q(\theta) (J_g - J_v) \quad (\text{A.10})$$

$$rJ_g = -w_g + \lambda (J_\pi - J_g) + s^L (J_v - J_g) \quad (\text{A.11})$$

$$rJ_\pi = \mathcal{P}x - w_\pi + s^G (J_g - J_\pi) + s^L (J_v - J_\pi) \quad (\text{A.12})$$

B.1 Price determination

Starting from the sharing rule

$$(1 - \alpha_G) (W_{D_M} - W_{D_U}) = \alpha_G (J_\pi - J_g) \quad (\text{A.13})$$

the left hand side can be replaced with equation (12), and the right hand side with (13):

$$(1 - \alpha_G) [(\Phi - \mathcal{P})x] = \alpha_G [\mathcal{P}x - w_\pi + s^L (J_v - J_g)]$$

Re-arranging, we obtain an expression for the price:

$$\mathcal{P}x = (1 - \alpha_G) \Phi x + \alpha_G \left[w_\pi + s^L (J_g - J_v) \right]$$

We further have, using (10),

$$\begin{aligned} s^L (J_g - J_v) &= -w_g + \lambda (J_\pi - J_g) = -w_g + \lambda \frac{1 - \alpha_G}{\alpha_G} (W_{D_M} - W_{D_U}) \\ &= -w_g + \lambda \frac{1 - \alpha_G}{\alpha_G} \frac{\sigma}{\check{\lambda}(\xi)} = -w_g + \frac{1 - \alpha_G}{\alpha_G} \sigma \xi \end{aligned}$$

such that one obtains:

$$\mathcal{P}x = (1 - \alpha_G) [\Phi x + \sigma \xi] + \alpha_G (w_\pi - w_g)$$

Inserting this price rule into the free-entry condition for consumer search units, one has:

$$\frac{\sigma}{\check{\lambda}(\xi)} = \frac{\alpha_G (\Phi x - w_\pi + w_g) - (1 - \alpha_G) \sigma \xi}{r + s}$$

B.2 Useful equivalences

Use $\frac{\sigma}{\check{\lambda}} = W_{D_M} - W_{D_U}$; $W_{D_M} - W_{D_U} = \frac{\alpha_G}{1 - \alpha_G} (J_\pi - J_g)$; $(J_\pi - J_g) = \frac{\mathcal{P}x - (w_\pi - w_g)}{r + s + \lambda}$; $\frac{\sigma}{\check{\lambda}} = \frac{(\Phi - \mathcal{P})x}{r + s} w_\pi = \alpha_L \mathcal{P}x + w_g$; $\mathcal{P}x = (1 - \alpha_G) (\Phi x + \sigma \xi) + \alpha_G (w_\pi - w_g) = (1 - \alpha_G) (\Phi x + \sigma \xi) + \alpha_G \alpha_L \mathcal{P}x$.

Sequential wage setting

$$\begin{aligned} \frac{\sigma}{\check{\lambda}} &= \frac{\alpha_G}{1 - \alpha_G} (J_\pi - J_g) \\ \frac{\sigma}{\check{\lambda}} \xi &= \frac{(1 - \alpha_L) \alpha_G}{1 - \alpha_G} \frac{\mathcal{P}x}{r + s + \lambda} \\ \frac{\lambda \mathcal{P}x}{r + s + \lambda} &= \frac{1 - \alpha_G}{(1 - \alpha_L) \alpha_G} \sigma \xi \end{aligned}$$

and

$$\begin{aligned} \frac{\sigma}{\check{\lambda}} &= \frac{\alpha_G (1 - \alpha_L) \Phi x - (1 - \alpha_G) \sigma \xi}{(1 - \alpha_L \alpha_G) (r + s)} \\ \frac{\sigma (r + s)}{\check{\lambda}} \left(\frac{1 - \alpha_L \alpha_G}{\alpha_G (1 - \alpha_L)} \right) &= \Phi x - \frac{1 - \alpha_G}{(1 - \alpha_L) \alpha_G} \sigma \xi \\ \frac{1 - \alpha_G}{(1 - \alpha_L) \alpha_G} \sigma \xi &= \Phi x - \frac{\sigma (r + s)}{\check{\lambda}} \left(\frac{1 - \alpha_L \alpha_G}{\alpha_G (1 - \alpha_L)} \right) \end{aligned}$$

Constant wage profile

$$\begin{aligned}\frac{\sigma}{\check{\lambda}} &= \frac{\alpha_G}{1 - \alpha_G} (J_\pi - J_g) \\ \frac{\sigma}{\check{\lambda}} \check{\xi} &= \frac{\alpha_G}{1 - \alpha_G} \frac{\mathcal{P}x}{r + s + \lambda} \\ \frac{\lambda \mathcal{P}x}{r + s + \lambda} &= \frac{1 - \alpha_G}{\alpha_G} \sigma \check{\xi}\end{aligned}$$

and

$$\begin{aligned}\frac{\sigma}{\check{\lambda}} &= \frac{\alpha_G (\Phi x) - (1 - \alpha_G) \sigma \check{\xi}}{r + s} \\ (r + s) \frac{\sigma}{\alpha_G \check{\lambda}} &= \Phi x - \frac{1 - \alpha_G}{\alpha_G} \sigma \check{\xi} \\ \frac{1 - \alpha_G}{\alpha_G} \sigma \check{\xi} &= \Phi x - \frac{\sigma (r + s)}{\check{\lambda}} \frac{1}{\alpha_G}\end{aligned}$$

B.3 Wage determination

Sequential wage bargaining

In both extensions the wage in stage g solves

$$w_g = \operatorname{argmax} (W_{eg} - W_u)^{\alpha_L} (J_g - J_v)^{1 - \alpha_L}$$

The solution for the stage g wage must satisfy the sharing rule $(1 - \alpha_L) (W_{eg} - W_u) = \alpha_L (J_g - J_v)$, such that we have:

$$\begin{aligned}(1 - \alpha_L) (w_g - rW_u + \lambda(W_{e\pi} - W_{eg})) &= \alpha_L (-w_g + \lambda (J_\pi - J_g)) \\ w_g &= (1 - \alpha_L) rW_u \\ w_g &= (1 - \alpha_L) z + \alpha_L \gamma \theta\end{aligned}$$

(i) Loss of consumer as outside option

$$w_\pi = \operatorname{argmax} (W_{e\pi} - W_{eg})^{\alpha_L} (J_\pi - J_g)^{1 - \alpha_L}$$

for which the sharing rule is

$$\begin{aligned}
(1 - \alpha_L) (W_{e\pi} - W_{eg}) &= \alpha_L (J_\pi - J_g) \\
(1 - \alpha_L) \left(\frac{w_\pi - w_g}{r + s + \lambda} \right) &= \alpha_L \left(\frac{\mathcal{P}x - (w_\pi - w_g)}{r + s + \lambda} \right) \\
(1 - \alpha_L) (w_\pi - w_g) &= \alpha_L (\mathcal{P}x - (w_\pi - w_g)) \\
w_\pi &= \alpha_L \mathcal{P}x + w_g \\
w_\pi &= (1 - \alpha_L) z + \alpha_L (\gamma\theta + \mathcal{P}x)
\end{aligned}$$

(ii) Dissolution of labor match as outside option

$$w_\pi = \operatorname{argmax} (W_{\pi g} - W_u)^{\alpha_L} (J_\pi - J_v)^{1-\alpha_L}$$

for which the sharing rule is

$$\begin{aligned}
(1 - \alpha_L) (W_{e\pi} - W_u) &= \alpha_L (J_\pi - J_v) \\
(1 - \alpha_L) (w_\pi - rW_u + s^G (W_{eg} - W_{e\pi})) &= \alpha_L (\mathcal{P}x - w_\pi + s^G (J_g - J_\pi)) \\
w_\pi &= \alpha_L \mathcal{P}x + (1 - \alpha_L) rW_u \\
w_\pi &= (1 - \alpha_L) z + \alpha_L (\gamma\theta + \mathcal{P}x)
\end{aligned}$$

Constant wage profile

Under the baseline assumption that the wage in both stages g and π are identical, that is $w = w_g = w_\pi$, we have that $W_{e\pi} = W_{eg} = W_e$. In addition, $J_\pi - J_g$ does not depend on the wage. Hence, we have

$$\frac{\partial (W_e - W_u)}{\partial w} = \frac{1}{r + s^L} = -\frac{\partial (J_g - J_v)}{\partial w_g}$$

and the simple Nash-bargaining rule applies:

$$(1 - \alpha_L) (W_e - W_u) = \alpha_L (J_g - J_v) \quad (\text{A.14})$$

The wage is therefore the solution to

$$\begin{aligned}
(1 - \alpha_L) [w - rW_u + \lambda (W_{e\pi} - W_{eg})] &= \alpha_L [-w + \lambda (J_\pi - J_g)] \\
w &= (1 - \alpha_L) rW_u + \alpha_L \lambda (J_\pi - J_g)
\end{aligned}$$

using $W_{e\pi} - W_{eg} = 0$. From the wage bargaining sharing rule and the firm's free-entry we have $rW_u = z + f(\theta) (W_e - W_u) = z + \gamma\theta\alpha_L / (1 - \alpha_L)$. Moreover, the firm's surplus can be expressed

as $J_\pi - J_g = \mathcal{P}x / (r + s + \lambda)$, such that we have the wage rule:

$$w = (1 - \alpha_L) z + \alpha_L \left[\gamma\theta + \frac{\lambda \mathcal{P}x}{r + s + \lambda} \right]$$

B.4 Labor market equilibrium condition

Sequential wage bargaining

The labor market entry condition:

$$\begin{aligned} (r + s^L) \frac{\gamma}{q} &= -w_g + \lambda (J_\pi - J_g) \\ (r + s^L) \frac{\gamma}{q} &= -w_g + \lambda \left(\frac{\mathcal{P}x - (w_\pi - w_g)}{r + s + \lambda} \right) \\ (r + s^L) \frac{\gamma}{q} &= \frac{\lambda \mathcal{P}x - \lambda (w_\pi - w_g) - (r + s + \lambda) w_g}{r + s + \lambda} \\ (r + s^L) \frac{\gamma}{q} &= \frac{\lambda (\mathcal{P}x - w_\pi) - (r + s) w_g}{r + s + \lambda} \end{aligned}$$

by inserting the bargained wages becomes:

$$\begin{aligned} (r + s^L) \frac{\gamma}{q} &= -(1 - \alpha_L) z - \alpha_L \gamma\theta + \lambda \left(\frac{\mathcal{P}x - (\alpha_L \mathcal{P}x)}{r + s + \lambda} \right) \\ (r + s^L) \frac{\gamma}{q} &= -(1 - \alpha_L) z - \alpha_L \gamma\theta + \lambda \left(\frac{\mathcal{P}x - (\alpha_L \mathcal{P}x)}{r + s + \lambda} \right) \\ (r + s^L) \frac{\gamma}{q} &= (1 - \alpha_L) \left(\frac{\lambda \mathcal{P}x}{r + s + \lambda} - z \right) - \alpha_L \gamma\theta \end{aligned}$$

Constant wage profile

Inserting the rule wage into the free-entry condition of firms leads to the labor market equilibrium presented in the text:

$$\begin{aligned} \frac{\gamma}{q(\theta)} &= \frac{-w + \lambda (J_\pi - J_g)}{r + s^L} \\ (r + s^L) \frac{\gamma}{q(\theta)} &= -(1 - \alpha_L) z - \alpha_L \left[\gamma\theta + \frac{\lambda \mathcal{P}x}{r + s + \lambda} \right] + \frac{\lambda \mathcal{P}x}{r + s + \lambda} \\ (r + s^L) \frac{\gamma}{q(\theta)} &= (1 - \alpha_L) \left[\frac{\lambda \mathcal{P}x}{r + s + \lambda} - z \right] - \alpha_L \gamma\theta \end{aligned}$$

B.5 Constant wage profile equilibrium conditions

In the constant wage profile case the equilibrium conditions are:

$$\frac{\sigma}{\check{\lambda}(\xi)} = \frac{\alpha_G \Phi x - (1 - \alpha_G) \xi \sigma}{r + s} \quad (\text{A.15})$$

$$(r + s^L) \frac{\gamma}{q(\theta)} = (1 - \alpha_L) \left[\Phi x - z - \frac{\sigma}{\alpha_G \check{\lambda}(\xi)} (r + s) \right] - \alpha_L \gamma \theta \quad (\text{A.16})$$

Note that the job creation condition (A.16) expression can be rewritten by replacing the revenue term $\lambda \mathcal{P} x / (r + s + \lambda)$ with $(1 - \alpha_G) \sigma \xi / \alpha_G$ as:

$$(r + s^L) \frac{\gamma}{q(\theta)} = (1 - \alpha_L) \left(\frac{1 - \alpha_G}{\alpha_G} \sigma \xi - z \right) - \alpha_L \gamma \theta \quad (\text{A.17})$$

B.6 Properties of the decentralized allocation

Price elasticity of demand

We are interested in the price elasticity of demand keeping supply constant to calibrate to the equivalent empirical concept estimated in the data. Using

$$M_G(\mathcal{D}_U, \mathcal{N}_g) = s \mathcal{D}_M \quad (\text{A.18})$$

and a Cobb-Douglas matching function, $M_G = \chi_G \mathcal{D}_U^{\eta_G} \mathcal{N}_g^{1-\eta_G}$, we have log differences:

$$\eta_G \frac{d\mathcal{D}_U}{\mathcal{D}_U} = \frac{d\mathcal{D}_M}{\mathcal{D}_M}$$

and

$$\frac{d\mathcal{D}_M}{d\mathcal{D}_U} \frac{\mathcal{D}_U}{\mathcal{D}_M} = \eta_G$$

From the equilibrium condition (14), the goods market entry condition:

$$\frac{\sigma}{\check{\lambda}(\xi)} = \frac{(\Phi - \mathcal{P}) x}{r + s}$$

we have:

$$\begin{aligned} \frac{\sigma}{\chi_G} \mathcal{D}_U^{1-\eta_G} \mathcal{N}_g^{\eta_G-1} &= \frac{(\Phi - \mathcal{P}) x}{r + s} \\ (1 - \eta_G) \frac{d\mathcal{D}_U}{\mathcal{D}_U} \frac{(\Phi - \mathcal{P}) x}{r + s} &= -\frac{x}{r + s} d\mathcal{P} \\ \frac{d\mathcal{D}_U}{d\mathcal{P}} \frac{\mathcal{P}}{\mathcal{D}_U} &= \frac{-1}{(1 - \eta_G)} \frac{\mathcal{P}}{\Phi - \mathcal{P}} \end{aligned}$$

and finally

$$\begin{aligned}\frac{d\mathcal{D}_M}{d\mathcal{P}} \frac{\mathcal{P}}{\mathcal{D}_M} &= \frac{d\mathcal{D}_M}{d\mathcal{D}_U} \frac{\mathcal{D}_U}{\mathcal{D}_M} \frac{d\mathcal{D}_U}{d\mathcal{P}} \frac{\mathcal{P}}{\mathcal{D}_U} \\ &= -\frac{\eta_G}{(1-\eta_G)} \frac{\mathcal{P}}{\Phi - \mathcal{P}}\end{aligned}$$

B.7 Constrained efficiency

Compare

$$(r+s) \frac{\sigma}{\check{\lambda}(\xi)} = \frac{\alpha_G (1-\alpha_L)}{1-\alpha_G \alpha_L} \Phi x - \frac{(1-\alpha_G)}{1-\alpha_G \alpha_L} \xi \sigma \quad (\text{A.19})$$

$$(r+s^L) \frac{\gamma}{q(\theta)} = (1-\alpha_L) \left[\Phi x - z - \frac{(r+s) \sigma}{\check{\lambda}(\xi)} \left(\frac{1-\alpha_G \alpha_L}{\alpha_G (1-\alpha_L)} \right) \right] - \alpha_L \gamma \theta \quad (\text{A.20})$$

with

$$(r+s) \frac{\sigma}{\check{\lambda}(\xi)} = \eta_G \Phi x - (1-\eta_G) \sigma \xi \quad (\text{A.21})$$

$$(r+s^L) \frac{\gamma}{q(\theta)} = (1-\eta_L) \left[\Phi x - z - \frac{(r+s) \sigma}{\check{\lambda}(\xi)} \frac{1}{\eta_G} \right] - \eta_L \gamma \theta \quad (\text{A.22})$$

In the goods market

Suppose

$$\frac{\alpha_G (1-\alpha_L)}{1-\alpha_G \alpha_L} = \eta_G$$

Then

$$\begin{aligned}1-\eta_G &= 1 - \frac{\alpha_G (1-\alpha_L)}{1-\alpha_G \alpha_L} \\ &= \frac{1-\alpha_G \alpha_L - \alpha_G + \alpha_G \alpha_L}{1-\alpha_G \alpha_L} \\ &= \frac{1-\alpha_G}{1-\alpha_G \alpha_L}\end{aligned}$$

Therefor, if

$$\begin{aligned}\alpha_G (1-\alpha_L) &= \eta_G - \eta_G \alpha_G \alpha_L \\ \alpha_G (1-\alpha_L + \eta_G \alpha_L) &= \eta_G \\ \alpha_G (1-(1-\eta_G) \alpha_L) &= \eta_G \\ \alpha_G &= \frac{\eta_G}{(1-(1-\eta_G) \alpha_L)}\end{aligned} \quad (\text{A.23})$$

from equations (A.19) and (A.21) $\xi^* = \xi^{opt}$.

In the labor market

Suppose now that $\alpha_L = \eta_L$. If α_G satisfies condition (A.20), then from equation from equations (A.19) and (A.22) $\theta^* = \theta^{opt}$.

C Data

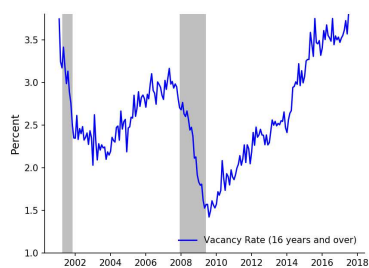
The unemployment and vacancy rates are monthly, seasonally adjusted series spanning from January 2001 to July 2017. The capacity utilization series is also monthly and seasonally adjusted, and spans from January 2001 to August 2017. The series for firm markup is quarterly, seasonally adjusted, and spans from January 2001 to July 2017. Finally, the series for corporate bond yields is monthly, not seasonally adjusted, and spans from January 2001 to July 2017. All data used are freely and publicly available, unless otherwise noted.

- We construct our U.S. unemployment rate series from January 2001 to July 2017 by drawing from data produced by the Bureau of Labor Statistics (BLS). Our complete, final unemployment rate series is plotted in Figure .
- We construct our U.S. vacancy rate series from January 2001 to July 2017 using the total number job openings from the Job Openings and Labor Turnover Survey (JOLTS) produced by the BLS divided by the total civilian labor force for those 16 years and over as calculated by the BLS. Figure plots our U.S. vacancy rate series.
- We construct our capacity utilization series from January 2001 to July 2017 by drawing from data produced by the Federal Reserve Board. We use the capacity utilization series for manufacturing. Figure plots the capacity utilization series for manufacturing.
- We construct our U.S. firm markup series from January 2001 to July 2017 by drawing from three sources from the BLS for the non-farm business sector: (i) compensation per hour, (ii) real output per hour of all persons, and (iii) an implicit price deflator. We calculate the markup charged by the firm as normalized real output per hour for all persons divided by compensation per hour. Figure plots our complete firm markup series.
- We construct our bond yield series from January 2001 to July 2017 by drawing from Moody's daily corporate AAA bond yield averages. This series is calculated based upon yields on bonds with maturities 20 years and above. Figure plots our U.S. bond yield series.

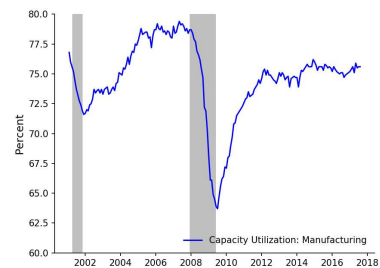
D Additional figures



(a) Unemployment rate

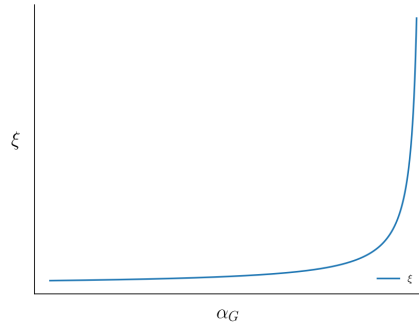


(b) Vacancy rate

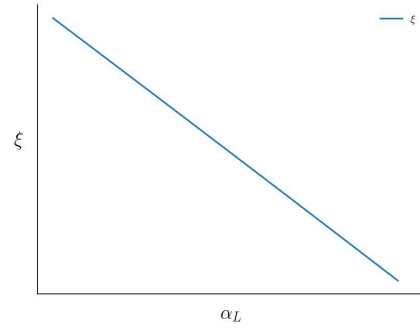


(c) Rate of capacity utilization

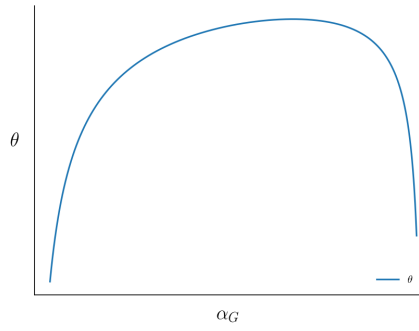
Figure 7: US time series, Jan. 2001 to July 2017



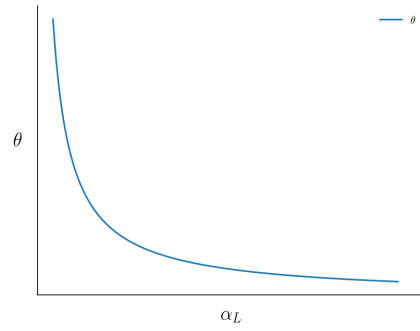
(a) Goods market tightness



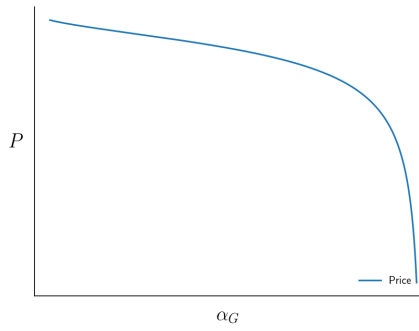
(b) Goods market tightness



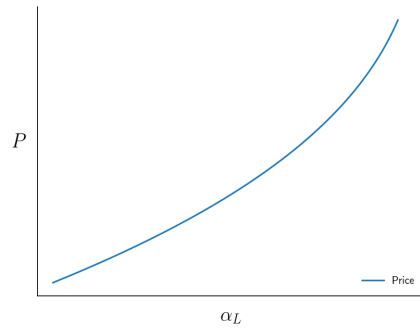
(c) Labor market tightness



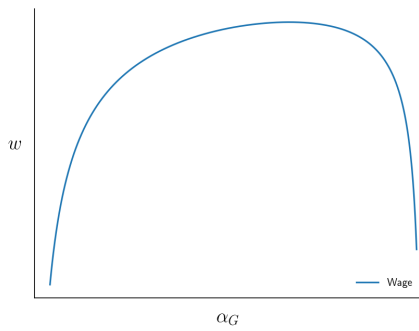
(d) Labor market tightness



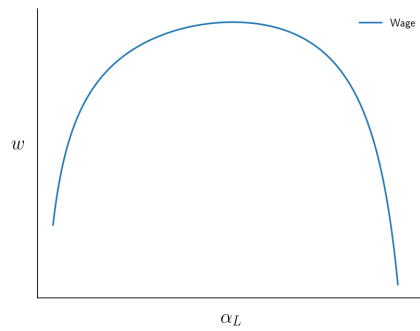
(e) Price



(f) Price

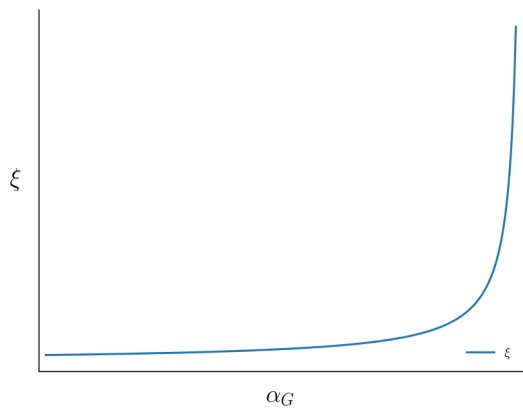


(g) Wage

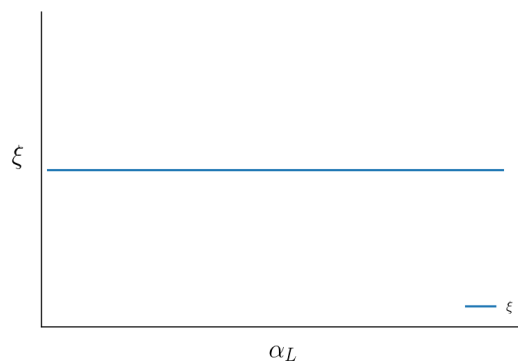


(h) Wage

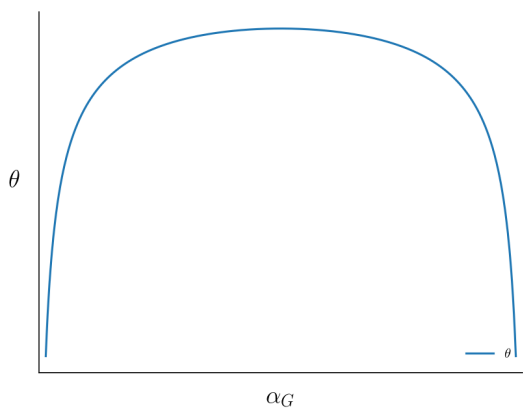
Figure 8: Comparative Statics on market power - Sequential wage setting



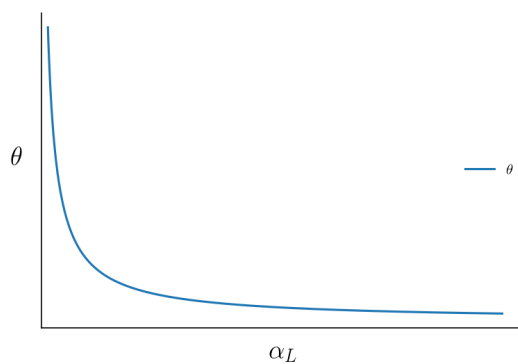
(a) Goods market tightness



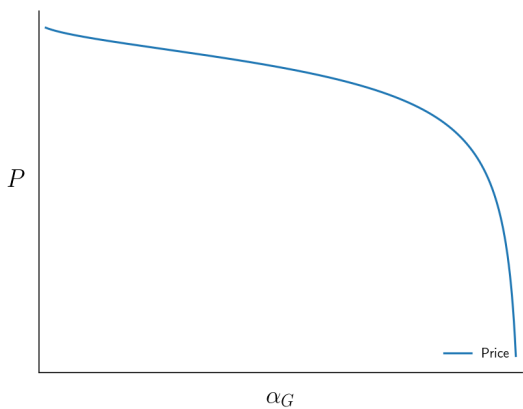
(b) Goods market tightness



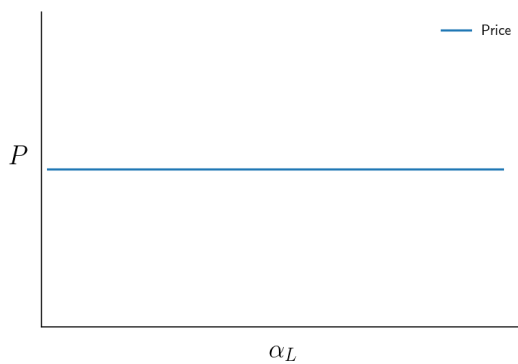
(c) Labor market tightness



(d) Labor market tightness



(e) Price



(f) Price

