

A Model of Bank Credit Cycles*

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Abstract

This paper develops a model of financial intermediation in which the dynamic interaction between regulator supervision and banks' loophole innovation generates credit cycles. In the model, banks' leverages are constrained due to a risk-shifting problem. The regulator supervises the banks to ease this moral hazard problem, and its expertise in supervision improves gradually through learning-by-doing. At the same time, banks can engage in loophole innovation to circumvent supervision, which acts as an endogenous opposing force diminishing the value of the regulator's accumulated expertise. In equilibrium, banks' leverage and loophole innovation move together with the regulator's supervision ability. Our model generates pro-cyclical bank leverage and asymmetric credit cycles. We show that a crisis is more likely to occur and the consequences are more severe after a longer boom. In addition, we investigate the welfare implications of a maximum leverage ratio in the environment of loophole innovation.

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The more effective regulation is, the greater the incentive to find ways around it. With time and considerable money at stake, those within the regulatory boundary will find ways around any new regulation. The obvious danger is that the resultant dialectic between the regulator and the regulated will lead to increasing complexity, as the regulated find loopholes which the regulators then move (slowly) to close.

– Goodhart and Lastra (2010)

1 Introduction

Banks and other financial intermediaries play a prominent role in the economy by channeling funds from savers to borrowers. In the wake of the recent financial crisis, there is a surge in research aimed at understanding the relationship between financial intermediaries, financial instability and macroeconomic fluctuations. In this paper, we build a dynamic model of financial intermediation that emphasizes the interaction between the regulator and banks. We show that banks' moral hazard can endogenously lead to financial instability, and generate boom-bust credit cycles. In particular, the longer the boom, the more likely there will be a crisis and the more severe the consequences will be, which corresponds to Minsky (1986)'s hypothesis that good times sow the seeds for the next financial crisis. Moreover, the model's predictions reconcile well with some empirical facts related to credit cycles. For instance, our model predicts that banks' leverage is pro-cyclical, consistent with the findings in Adrian and Shin (2010, 2011). The model also generates asymmetric credit cycles, i.e., long periods of credit booms followed by sudden and sharp busts, while recovery is slow and gradual, as documented in Reinhart and Reinhart (2010).

The key element of our paper is banks' risk-shifting problem. It is now widely accepted that excessive risk-taking by banks contributed to the financial crisis of 2007-2009. While the causes of excessive risk-taking remain subject to debate, many observers and policymakers believe that bank supervision failure is one of the key contributing factors (Acharya and Richardson (2009), Acharya et al. (2011), Freixas et al. (2015)).¹ Indeed, several countries have made great efforts to improve their supervision of banks in the aftermath of the crisis.

¹Other mentioned factors include, for instance, shortcomings in financial institutions' incentive structures and risk management practices, misplaced reliance on credit rating agencies, etc.

What could explain the failure of bank supervision? Among various factors, financial innovation is mentioned as one key factor that can undermine the effectiveness of the regulator’s supervision (see, e.g., Silber (1983), Miller (1986), Kane (1988), Tufano (2003)). Undoubtedly, good financial innovations provide numerous benefits to the economy.² However, there are also bad financial innovations that create new ways for financial institutions to get around current supervision and take excessive risks, which we refer to as loophole innovations in this paper. For instance, Stein (2013) argues that second-generation securitization, like subprime CDOs, is a bad financial innovation that evolved in response to flaws in prevailing models and incentive schemes. Another related example is Credit Default Swap (CDS). CDS was widely used to free up regulatory capital in banks’ balance sheets prior to the crisis. However, when the risky assets of banks and the insurer are correlated, banks can use CDS to engage in regulatory arbitrage and take excessive risks under the Basel regulatory framework (Yorulmazer (2013)). As is illustrated in the recent financial crisis, the regulator was slow to understand the danger of loophole innovations in some instances, facilitating banks’ excessive risk-taking. In this paper, we put a central emphasis on how banks’ loophole innovation affects the effectiveness of regulator supervision, and investigate its macroeconomic implications over the credit cycles. To the best of our knowledge, this paper is the first to explicitly model the dynamic interaction between regulator supervision and banks’ loophole innovation.

In the model, banks borrow from depositors in the form of debt to finance their investment opportunities. The investment opportunities could be safe or risky projects. Due to limited liability, banks are subject to a risk-shifting problem. Therefore, banks have incentives to take on inefficient risky projects, in which they enjoy the upside of payoff if projects succeed but depositors bear the loss if projects fail. One solution to this moral hazard problem is market discipline: depositors impose a leverage constraint on banks. If banks have enough “skin in the game”, they will behave properly. However, market discipline is costly in the sense that it limits a bank’s investment capacity.

Another complementary solution to the moral hazard problem is supervision by the regulator. In this paper, we formally distinguish regulation from supervision, in terms of the verifiability of bank information and actions, following Eisenbach et al. (2016).³ Through

²For instance, financial innovations help improve risk sharing, complete the market, reduce trade costs, see Beck et al. (2016) for an excellent survey of the debate on the “bright” and “dark” sides of financial innovation.

³Regulation is written into law and enforced through the courts, so it can only be contingent on verifiable

actively monitoring banks' activities, the regulator can promote banks' safety and soundness. The leverage constraint and supervision from the regulator work together to address banks' risk-shifting problem. The better the regulator's supervision ability, the more banks can relax their borrowing constraint. As a result, the size of the banks depends on depositors' beliefs regarding the regulator's supervision ability. When depositors' confidence in the regulator's competence is high, they will permit banks to take high leverage without worrying about the risk-shifting problem. If the regulator's ability is perceived to be low, depositors have to tighten the leverage constraint to make sure banks behave properly.

Even though regulator supervision helps banks to increase their leverage from an *ex ante* perspective, banks always have incentives to find loopholes to circumvent regulator supervision *ex post*. In our paper, we model loophole innovation as discovering a new type of risky project which is not currently supervised by the regulator, thereby providing banks with opportunities to take on risky projects without being monitored by the regulator. This acts as an endogenous opposing force diminishing the regulator's expertise in supervision. When the loophole innovation eventually succeeds, the regulator's supervision becomes less effective. However, the regulator and depositors are not aware of the new loopholes immediately, so banks take on inefficient risky projects, thereby leading to massive defaults and a severe decline in output. After a bust, depositors realize that the regulator's expertise has become obsolete and they lose confidence in the financial system. In response, they constrain banks from taking high leverage to prevent their risk-taking activities, which implies a sharp contraction of the banking sector.

We incorporate regulator supervision and banks' loophole innovation into a dynamic model. We assume that the regulator's expertise in supervising banks regarding previous loopholes gradually improves through a learning-by-doing process. This assumption is supported by some recent studies on how prudential supervision works in practice (see, e.g., Dahlgren (2011), Dudley (2014), Eisenbach et al. (2016)).⁴ As the regulator's expertise information. In contrast, supervision involves the assessment of the safety and soundness of banks through monitoring by the regulator, and corrective actions in response to the assessment. Supervision can be contingent on non-verifiable information.

⁴For instance, according to Eisenbach et al. (2016), "The current structure and organization of FRBNY FISG supervisory staff dates from a significant reorganization that took place in 2011. That reorganization drew on lessons learned during the financial crisis to reshape the internal structure of the group and the way that staff interacts with one another to enhance communication and facilitate identification of emerging risks

grows, it has two effects on banks' moral hazard problem. On the one hand, it eases banks' risk-shifting problem related to previous loopholes, which allows banks to take a higher leverage. Therefore, banks have a larger investment size, and the total output goes up. In this way, the economy experiences a boom accompanied with rising leverage in the banking sector. On the other hand, however, banks will also engage in loophole innovation more actively. When supervision is more effective and banks' leverage is higher, the gain from finding a new loophole is larger. Banks' efforts to conduct loophole innovation increase, and thus banks are more likely to discover a new loophole. If the loophole innovation is successful, it provides banks with a new type of risky project which is not supervised by the regulator. There is a crisis in the economy.

Our main result of the dynamic model is that the interaction between regulator supervision and banks' attempts to circumvent supervision can lead to regime changes in banks' moral hazard problem, and generates macroeconomic fluctuations. In our economy, the sources of the economic downturns endogenously come from banks' loophole innovations. Moreover, the longer the boom, the more likely there will be a crisis and the more severe the consequences will be. This is because as banks' leverage rises in boom periods, they have stronger incentives to find new loopholes. Furthermore, the business cycles are asymmetric in our economy: periods of gradual expansions in banks' leverage, investment, and aggregate output are followed by sudden and sharp contractions, and then the economy starts the gradual growth again. This result arises from the asymmetric nature of loophole innovation. Although the regulator takes time to gradually improve its supervision ability through a learning-by-doing process, its expertise can be severely undermined the moment that new loopholes are discovered.

The 2007-2009 financial crisis is a good example to illustrate our mechanism. Before the crisis, banks discovered vulnerabilities in the rules of regulation and supervision, and by exploiting these loopholes, they took excessive risks. When the massive failures occurred and the crisis unfolded, regulators and investors realized that there had been so many cracks in the financial system. As Timothy F. Geithner (Geithner, 2010) recognized, "Our regulatory framework was built in a different era for a long extinct form of finance. It long ago fell through a greater emphasis on cross-firm perspectives. The reorganization was designed to foster enhanced and more frequent engagement between senior supervisory staff and senior managers and members of the board of directors at supervised firms".

behind the curve of market developments. Parts of the system were crawling with regulators but parts of the system were without any meaningful oversight. This permitted and even encouraged arbitrage and evasion on an appalling scale.” In response to the vulnerabilities in the financial system, investors cut their lending to the banks and there was a sharp deleveraging process in the financial sector.

We also investigate the regulation implications of this model. We consider the regulation with a maximum leverage ratio. The regulator’s supervision ability can be seen as the state of the economy. Banks’ loophole innovation effort determines the evolution rules for the regulator’s supervision ability, which characterizes the stationary distribution of the economy in the long run. We find that under certain conditions the regulator would set a maximum leverage ratio to restrict the upper-bound leverage for the banks. This regulation has two effects. First, it reduces banks’ leverage and can potentially decrease output in boom periods. Second, it decreases success probability of loophole innovation. A lower loophole innovation success probability shifts the stationary distribution of the economy towards more favorable states, which improves the average output in the long run. The regulator will trade off these two effects to set the optimal maximum leverage ratio.

The model’s empirical implications are broadly consistent with the stylized facts found in many empirical studies. First, Schularick and Taylor (2012) study 14 developed countries over 140 years, concluding that a long period of credit growth is the best single predictor of financial crises. Second, Reinhart and Reinhart (2010) find that credit cycles are asymmetric: long periods of credit expansion are followed by sudden stops, and then gradual recovery. Third, Adrian and Shin (2010, 2011) find that financial intermediaries’ leverage is pro-cyclical over the business cycles. Fourth, Dell’Ariccia et al. (2014) find that during a boom, financial intermediaries’ lending standards decrease and loan default rates increase, which is accompanied by massive failures in the financial sector. Our model’s results reflect these facts within a unified framework.

This paper contributes to the existing literature in several ways. First, unlike most regulation and supervision literature which focuses on static models, this paper studies regulator supervision in a dynamic framework. Second, complementary to a small but growing literature on endogenous business cycles, which focus on non-financial firms, this paper provides a novel mechanism to generate endogenous credit cycles originated from the financial sector. By analyzing the dynamics of banks’ moral hazard problem, this paper is able to rationalize

some of the key features of the credit cycles that are not explained by the existing literature. Third, this paper provides a new rationale for the maximum leverage ratio when there is an interaction between regulator supervision and banks' loophole innovation. We show that tightening banks' leverage ratio involves a systemic risk and output trade-off, and the regulator can lower the likelihood of systemic crises at the cost of decreasing output in boom periods.

The paper's structure is as follows. Section 2 discusses the related literature. Section 3 presents the static model for bank risk-shifting, supervision, and loophole innovation. Section 4 nests the static model in a dynamic model, analyzing the macroeconomic implications of the interaction between banks' loophole innovation and regulator supervision. Section 5 investigates the welfare implications of the maximum leverage ratio. Section 6 adds learning about unknown loopholes and the regulator's investigation choice in the model. Section 7 discusses several setups in the model. Section 8 is the conclusion.

2 Literature

This paper is linked to different strands of the literature on banks' risk-taking, financial innovation, financial crises, and credit cycles.

Our paper follows the literature on banks' risk-taking and financial stability (see, e.g., Keeley (1990), Suarez (1994), Matutes and Vives (1996), Boyd and Nicoló (2005) and Martinez-Miera and Repullo (2017)). Unlike most literature, which assumes an exogenous capital structure, in our paper, bank's leverage is endogenously chosen by the bank as a commitment device to reduce moral hazard. In this respect, our paper is mostly related to a recent paper by Dell'Ariccia et al. (2014), in which it is shown how interest rate affects a bank's risk-taking when the bank can choose its leverage optimally. However, none of these papers consider the role of regulator supervision in alleviating banks' moral hazard.

Our work is related to the literature on regulator supervision (see, e.g., Dewatripont et al. (1994), Bhattacharya et al. (2002), Prescott (2004), Marshall and Prescott (2006), Rochet (2008)). More recently, Eisenbach et al. (2015, 2016) formally distinguish bank supervision and regulation and develop a static framework to explain the relationship between supervisory efforts and bank characteristics observed in the data. We depart from this literature by focusing on the connection between the regulator's competence and credit cycles.

In this respect, our paper is closely related to Morrison and White (2005, 2013). They show that crises will only occur when public confidence in the regulator’s ability to detect bad banks through screening is low. While the regulator’s ability is constant in the static model in Morrison and White (2005, 2013), we study the dynamic interaction between regulator supervision and banks’ loophole innovation. In this regard, we consider our model a first attempt to formalize Kane (1988)’s influential idea of “regulatory dialectic”.

Our interest in endogenous business cycle relates to Suarez and Sussman (1997), Martin (2008), Favara (2012), Myerson (2012), and Gu et al. (2013). Among these papers, our paper is mostly related to Myerson (2012), who shows how boom-bust credit cycles can be sustained in economies with moral hazard in financial intermediation. Unlike Myerson (2012), our model focuses on the role of regulator supervision in curbing moral hazard in financial intermediation, and more importantly, our paper generates richer macroeconomic implications consistent with stylized facts found in the empirical literature.

Our work is also linked to the literature on asymmetric business cycles. Some papers, including Veldkamp (2005), Ordoñez (2013), and Kurlat (2015), study the asymmetric nature of the credit cycles from the perspective of the asymmetric information flow over the cycles. A recent paper by Asriyan and Vanasco (2014) studies the role of financial intermediaries’ learning in generating and amplifying the informational cycles. Our paper also features a regulator whose expertise grows through learning-by-doing. The key difference is that, in our paper, the shock to the fundamental is endogenously generated by the banking sector itself rather than exogenously. And our paper also stresses the role of banks’ leverage over the cycle, which is absent in their paper.

There is an emerging literature studying the close relationship between boom and bust in the business cycles. In Gorton and Ordoñez (2014, 2016), booms are associated with loss of information while crises happen when the economy transits from information-insensitive states to information-sensitive states. Boz and Mendoza (2014) and Biais et al. (2015) emphasize the role of investors’ belief regarding the strength of a financial innovation in generating boom and bust. Good belief builds up in boom periods, but adverse realization of the fundamental decreases belief dramatically and leads to a bust. Boissay et al. (2016) build a model featuring an interbank market with moral hazard and adverse selection problems. Increased savings during expansions drive down the return on loans, and when the fundamental becomes weak, the interbank market freezes due to an agency problem, which leads to a bank

crisis. Unlike these papers, we build a model focusing on the interaction between regulator supervision and banks' loophole innovation.

3 Static Model

Consider an economy with a mass-one continuum of banks, a large mass of households, and a regulator. All parties are risk-neutral. Each bank is endowed with w . Banks can use their own money ω and raise deposit (or more generally issue debt liabilities) from households to make investments. A household can invest in a storage technology with a fixed return of r_0 , or invest in the banks as a depositor. We assume that the deposit market is competitive, and there is no deposit insurance, so households are willing to invest in the banks as long as they break even relative to the return on storage technology. If a bank borrows x from depositors, the bank's investment size would be $\omega + x$.⁵ We denote a bank's leverage as $L \equiv \frac{\omega+x}{\omega}$. Banks are protected by limited liability and repay depositors only in case of success.

Banks can invest in a safe project or in a risky project. The safe project's payoff is R/η^s with probability η^s and zero with probability $1 - \eta^s$, so the expected return of the safe project is R . The risky project is more likely to fail than the safe project but will pay more if it succeeds. More specifically, the success probability of the risky project is $\eta < \eta^s$, and the payoff conditional on project success is $\bar{\lambda}R/\eta$, so the expected payoff of the risky project is $\bar{\lambda}R$. Banks can choose the success probability of the risky project, η , within the interval $[\underline{\eta}, \bar{\eta}]$, with $\bar{\eta} < \eta^s$ and $\bar{\lambda}/\bar{\eta} > 1/\eta^s$. As η is lower, the risky project is less likely to succeed, but, conditional on success, the payoff is higher. Therefore, η is also a measure of the riskiness of the risky project. The lower is η , the more risky is the project. We assume that $R > r_0 > \bar{\lambda}R$. Thus, the safe project has the highest expected return, and the risky project has a lower expected return than the storage technology. Banks' project choices are not observed by depositors and are not contractable.

There is a benevolent regulator who can supervise the banks. To model regulator supervision, we assume that the regulator can prevent banks from choosing high riskiness when

⁵In this paper, a bank's capital structure is endogenously determined, rather than exogenously given. This treatment is supported by two observations under existing bank regulations. First, a bank's true leverage may be higher than the regulatory limit because banks can overstate capital by not recognizing losses. Second, banks can save on capital by engaging in regulatory arbitrage of capital requirements.

taking the risky project.⁶ More specifically, when the regulator's supervision ability is η^* , banks can only choose the risky project success probability within the interval $[\eta^*, \bar{\eta}]$. The setup regarding supervision is similar to Eisenbach et al. (2016), where the regulator can take corrective actions to reduce the variance of bank' return.

However, the regulator's supervision is not perfect. Sometimes banks may discover a new type of risky project, which is not immediately known by the regulator and households. We call this discovery a successful loophole innovation. If a loophole innovation succeeds, banks are able to take the new risky project with any riskiness levels, since the regulator does not realize that it exists. Borrowing the setup from technology innovation literature, such as Aghion and Howitt (2009) and Laeven et al. (2015), we assume that only one bank is capable to conduct loophole innovation.⁷ We call this bank the capable bank. It is costly for the capable bank to conduct loophole innovation. When the capable bank's effort is $e \in [0, 1]$, the loophole innovation succeeds with probability e . The cost for the capable bank is $\frac{1}{2}ce^2 \cdot (\omega + x)$, where c is the coefficient governing the cost of innovation. If a loophole innovation succeeds, all banks learn about the new loophole, and a risky project immune from supervision is available to them.

The timing of the static model is as follows: at the beginning of the period, each bank offers a deposit menu to households, which specifies the leverage of the bank and deposit rate. Households decide whether or not to make deposits in the banks. After that, one of the banks knows it is the capable one and exerts loophole innovation effort. If the loophole innovation is successful, a new type of risky project emerges, and all other banks learn about it. Banks make project choices and choose riskiness levels under the regulator's supervision if they invest in the risky project. At the end of the period, banks' projects pay off. Banks pay back the depositors if their projects succeed. Otherwise, they default and go bankrupt.

To characterize the equilibrium, we take the following steps. First, we describe banks' menu choice problem, in which banks choose the deposit menus to maximize their expected profits. Second, we solve the capable bank's loophole innovation problem, in which the capable bank chooses the effort to conduct loophole innovation, given its leverage and interest rate. Third, we impose the equilibrium condition that the expected loophole innovation

⁶High riskiness would correspond to low success probability in our model.

⁷We can generalize this assumption for N banks, as long as N is finite. Otherwise, innovations succeed every period.

success probability is consistent with the capable bank's innovation effort, and solve the equilibrium.

First, we describe banks' menu choice problem. A bank's expected profit given leverage and interest rate is⁸

$$(1-p) \max\{RL - \eta^s r(L-1), \bar{\lambda}RL - \eta^* r(L-1)\} + p \max\{RL - \eta^s r(L-1), \bar{\lambda}RL - \underline{\eta}r(L-1)\} \quad (1)$$

where p is the probability that a loophole innovation succeeds in equilibrium, which both banks and households take as given. The first term is bank's expected profit if the loophole innovation fails. When the loophole innovation fails, banks are monitored by the regulator, thus the highest riskiness available to them is η^* . Due to limited liability, banks would like to choose the highest riskiness η^* if they invest in the risky project. Banks optimally decide between the safe project and the risky project with riskiness η^* . The second term is bank's expected profit if the loophole innovation is successful. If the loophole innovation succeeds, it provides banks with a new type of risky project to circumvent the regulator's supervision. In this case, if banks invest in the new risky project, they can choose the riskiness $\underline{\eta}$. Banks decide between the safe project and the risky project with riskiness $\underline{\eta}$ to maximize their expected profits.

In this paper, we focus on the case that banks' leverage is always constrained by the risk-shifting problem, so that their leverage is finite. The following assumption is the sufficient condition that guarantees it.

Assumption 1. $\frac{(1-\bar{\lambda})R}{(\eta^s - \underline{\eta}^*)r_0} < 1$.

This assumption implies two things. First, banks with sufficiently high leverage will choose the risky project. Second, the maximum supervision ability is not high enough to fully eliminate banks' risk-shifting problem.

To analyze banks' menu choice problem, we can divide the possible menus into three areas according to equation (1). First, if the deposit menu $\{L, r\}$ satisfies $RL - \eta^s r(L-1) \geq \bar{\lambda}RL - \underline{\eta}r(L-1)$, banks will never choose the risky project since the safe project yields a higher expected profit. Second, if $\{L, r\}$ satisfies $\bar{\lambda}RL - \underline{\eta}r(L-1) > RL - \eta^s r(L-1) \geq \bar{\lambda}RL - \eta^* r(L-1)$, which project banks will choose depends on whether there is a

⁸Since there is a continuum of banks and only one of them is capable, each bank expects itself to be the capable one with a probability of measure zero. Thus, banks do not consider the cost of loophole innovation when they choose the leverage at the beginning of the period.

successful loophole innovation or not. If the loophole innovation fails, banks will be under the monitoring of the regulator with supervision ability η^* . Therefore, they choose the safe project. Otherwise, banks will take advantage of the loophole to circumvent the supervision, and choose the new risky project with the highest riskiness. Third, if $\{L, r\}$ is in the area such that $RL - \eta^s r(L - 1) < \bar{\lambda}RL - \eta^* r(L - 1)$, banks will always choose the risky project, even if the loophole innovation fails. Banks choosing menus in this area have to offer households very high interest rates to attract deposits, which yields a negative expected profit for banks. Thus, menus in this area are never optimal for banks. In other words, the feasible menus have to provide banks with incentives to invest in the safe project if there is no successful loophole innovation. We can write this incentive compatibility constraint as

$$RL - \eta^s r(L - 1) \geq \bar{\lambda}RL - \eta^* r(L - 1) \quad (2)$$

The left side of the constraint is a bank's expected profit from taking the safe project. The right side is a bank's expected profit from taking the risky project if the loophole innovation fails, in which case the regulator's supervision ability is η^* .

Given the leverage, banks would like to offer the lowest possible interest rate to attract deposit from households. Since households are rational, they would conjecture banks' project choices given the leverage, and demand a deposit rate that leaves them indifferent between depositing in the bank and investing in the storage technology. Therefore, the interest rate in the deposit menu is related to the leverage. We define $L_0 \equiv 1/(1 - (1 - \bar{\lambda})R\eta^s/((\eta^s - \underline{\eta})r_0))$ and $L^* \equiv 1/(1 - (1 - \bar{\lambda})R((1 - p)\eta^s + p\underline{\eta})/((\eta^s - \eta^*)r_0))$. It is easy to see that for a small loophole innovation success probability p , L^* is larger than L_0 .

To raise money from depositors, the interest rate needs to be sufficiently high to compensate for the bank's risk. The interest rate that leaves depositors indifferent between depositing in the bank and investing in the storage technology is

$$r = \begin{cases} \frac{r_0}{\eta^s}, & \text{if } L \leq L_0 \\ \frac{r_0}{(1-p)\eta^s + p\underline{\eta}}, & \text{if } L_0 < L \leq L^* \\ \frac{r_0}{\eta^*}, & \text{if } L > L^* \end{cases} \quad (3)$$

First, if a menu has a leverage lower than or equal to L_0 and an interest rate r_0/η^s , the bank's expected profit from taking the safe project is always higher than the risky project. Therefore, the bank will never invest in the risky project, even if there is a successful loophole innovation. Since the safe project succeeds with probability η^s , the interest rate for depositors

to break even is r_0/η^s . Second, for a menu with a leverage between L_0 and L^* and an interest rate $r_0/((1-p)\eta^s + p\underline{\eta})$, the bank's expected profit from taking the safe project is higher than the risky project when the loophole innovation fails and lower than the risky project when the loophole innovation succeeds. With probability p , the loophole innovation is successful, and banks will choose the risky project with the highest riskiness $\underline{\eta}$. With probability $1-p$, the loophole innovation fails, and banks will choose the safe project. From an ex-ante perspective, the bank succeeds with probability $(1-p)\eta^s + p\underline{\eta}$, thus depositors demand an interest rate of $r_0/((1-p)\eta^s + p\underline{\eta})$. Third, if a bank's leverage is higher than L^* , it will always take the risky project even if the loophole innovation fails, so the interest rate needs to be as high as r_0/η^* to compensate for the risk. It is easy to see that this leads to a negative profit for banks. Therefore, banks will never choose a leverage higher than L^* .

In the case that p is small, we can show that a bank's expected profit with menu $\{L^*, r_0/((1-p)\eta^s + p\underline{\eta})\}$ is higher than that with menu $\{L_0, r_0\}$, so all banks will choose a leverage of L^* . From now on, we will focus on this case.

Next, let us solve the loophole innovation effort problem of the capable bank. After all banks raise deposits, one bank knows that it is the capable one, and it can exert effort to conduct loophole innovation. Given the leverage level and deposit rate, the innovation effort problem of the capable bank is

$$\max_e (1-e)[RL - \eta^s r(L-1)] + e[\bar{\lambda}RL - \underline{\eta}r(L-1)] - \frac{1}{2}ce^2L \quad (4)$$

With probability $1-e$, the loophole innovation fails, so the capable bank chooses the safe project. With probability e , the loophole innovation is successful, so the capable bank chooses the risky project with riskiness $\underline{\eta}$.

The first-order condition can be written as

$$- [R - \eta^s r(1 - 1/L)] + [\bar{\lambda}R - \underline{\eta}r(1 - 1/L)] = ce \quad (5)$$

First, we can see that given the leverage and interest rate, a higher loophole innovation cost coefficient c reduces the capable bank's innovation effort. Second, other things equal, a higher leverage L induces the capable bank to choose a higher loophole innovation effort. This is because when the bank's leverage is higher, the gain from finding a new loophole is larger. Third, a higher interest rate r results in a higher loophole innovation effort, since a new loophole provides the capable bank with an opportunity to avoid paying interest.

The definition of equilibrium in the static model is as follows.

Definition 1. *An equilibrium in the static model consists of the success probability of loophole innovation and decision rules $\{L(\eta), r(\eta), e(\eta)\}$ such that (i) the deposit menu $\{L(\eta), r(\eta)\}$ solves the banks' problem (1) given (3); (ii) $e(\eta)$ solves the capable bank's problem (4); (iii) the success probability of loophole innovation is consistent with the capable bank's innovation effort, i.e., $p = e$.*

In equilibrium, the ex ante probability that the loophole innovation succeeds must be equal to the innovation effort chosen by the capable bank, i.e., $p = e$. The break-even condition for depositors implies that the interest rate is

$$r_0 = [(1 - e)\eta^s + e\underline{\eta}]r \quad (6)$$

Here with probability $1 - e$, the loophole innovation fails. Banks take on the safe project, which has a success probability η^s . With probability e , the loophole innovation succeeds, and banks take on the risky project with success probability $\underline{\eta}$. From an ex-ante view, the bank succeeds with probability $(1 - e)\eta^s + e\underline{\eta}$. The interest rate r_0 compensates for the bank's default risk.

As we mentioned before, we need the loophole innovation success probability p to be small, so that banks will choose a leverage of L^* . The following lemma shows that a large innovation cost c will guarantee this.

Lemma 1. *Under Assumption 1 and a large innovation cost coefficient c , the incentive compatibility constraint equation (2) is always binding for each bank.*

With Lemma (1), we can solve the bank's problem in an explicit form. From equations (2), (6), and (5), we can solve the innovation effort, deposit rate, and leverage in equilibrium given the supervision ability η^* ,

$$e = \frac{\eta^* - \underline{\eta}}{\eta^s - \eta^*} \frac{(1 - \bar{\lambda})R}{c} \quad (7)$$

$$r = \frac{r_0}{(1 - e)\eta^s + e\underline{\eta}} \quad (8)$$

$$L = \frac{1}{1 - \frac{(1 - \bar{\lambda})R}{(\eta^s - \eta^*)r}} \quad (9)$$

From the above equations, we have the following proposition.

Proposition 1. *Under Assumption 1 and a large innovation cost coefficient c , as the regulator's supervision ability increases,*

- (I) the capable bank's loophole innovation effort increases;
- (II) the banks' deposit rate increases;
- (III) the banks' leverage increases.

The total output depends on whether the loophole innovation succeeds or not. If the loophole innovation fails, all banks take the safe project, and the total output is $(R - r_0)\omega L$. If the loophole innovation is successful, all banks take the risky project, and the total output is $(\bar{\lambda}R - r_0)\omega L$. Thus, the expected output at the beginning of the period is $\omega \cdot [(1 - e)(R - r_0) + e(\bar{\lambda}R - r_0)]L$. We plot these results in Figure 1.

Next we study the comparative statics. We focus on how the cost coefficient of loophole innovation (c), the expected payoff of the safe project (R), and the relative payoff of the risky project ($\bar{\lambda}$) will affect banks' deposit rate (r), leverage (L), and capable bank's innovation effort (e).

Lemma 2. *Under Assumption 1 and a large innovation cost coefficient c ,*

- (I) the capable bank's loophole innovation effort e decreases in c , increases in R , and decreases in $\bar{\lambda}$;
- (II) the banks' deposit rate r decreases in c , increases in R , and decreases in $\bar{\lambda}$;
- (III) the banks' leverage L is increases in c , increases in R , and decreases in $\bar{\lambda}$;

It is easy to see that with a larger innovation cost coefficient c , the capable bank will exert less effort to conduct loophole innovation, thus the success probability of the loophole innovation decreases. A lower loophole innovation success probability will reduce the deposit rate demanded by households, since banks are less likely to take the risky project. And a lower deposit rate relaxes banks' incentive compatibility constraint, so banks can have a higher leverage.

The effects of increasing the expected payoff of the safe project, R , are more complicated. On the one hand, a larger R makes the safe project more attractive, which directly dampens the incentive of loophole innovation. On the other hand, a larger R also increases the leverage of banks, which indirectly gives the capable bank stronger incentive to innovate. The latter effect dominates the former one, so the capable bank's innovation effort increases. Following a similar logic, a larger $\bar{\lambda}$, increases the attractiveness of the risky project, but the low leverage

associated with it decreases the capable bank's incentive to innovate. Overall, the capable bank's innovation effort is lower with a larger $\bar{\lambda}$.

4 Dynamic Model

4.1 Setup

In this section, we extend the static model into a dynamic model. Each bank lives for one period. Each bank is endowed with ω at the beginning of each period. To raise deposit, banks offer deposit menus to households. In each period, only one bank has a capable idea, and it chooses the effort it will make to conduct loophole innovation. The loophole innovation, once successful in one period, spreads in two dimensions. First, all banks in that period learn about it and are able to exploit the loophole, as in the static model. Second, all banks in the periods following the successful loophole innovation will learn about it.

One key element in the dynamic model is the evolution of the regulator's supervision ability. There are two countervailing forces that affect the regulator's supervision ability. On the one hand, a successful loophole innovation discovers a new type of risky project that is off the radar of the regulator's accumulated monitoring skills, which undermines the regulator's expertise. On the other hand, after a successful loophole innovation, the regulator recognizes the existence of a new loophole and starts to investigate it.⁹ Over time, the regulator learns more and more about the new loophole, and improves its monitoring skills each period through learning-by-doing. As mentioned in the introduction, the assumption that the regulator engages in learning-by-doing is supported by some recent empirical papers (see, e.g., Dahlgren (2011), Dudley (2014), Eisenbach et al. (2016)). These papers find that, in reality, regulators have drawn on lessons learned during the financial crisis and make effort to improve their supervision abilities.

In this paper, we capture the regulator's learning-by-doing in a reduced form. More specifically, the evolution law for the regulator's supervision ability for a new loophole, i.e., the regulator's supervision ability η_t in the period t since the last loophole innovation that

⁹Since there is an infinite number of banks in the economy, the public can infer the occurrence of a successful loophole innovation from the share of bank failures at the end of the period.

succeeded in the period $\hat{t}$ is

$$\eta_t = \begin{cases} \eta_k^*, & \text{if } t - \hat{t} = k < K \\ \eta_K^*, & \text{if } t - \hat{t} = k \geq K \end{cases} \quad (10)$$

Here k is the period following the last successful loophole innovation. If a loophole innovation is successful in one period, regulator supervision becomes ineffective for this new loophole. From the next period on, the regulator's supervision ability starts to evolve gradually according to the evolution law. We assume that η_k^* increases with k , so regulator's supervision ability regarding the new loophole increases for each period. After K periods, it will stay constant unless another new loophole innovation succeeds. This guarantees that there is an upper-bound for the regulator's supervision ability, so banks' risk-shifting problem always exists. For each loophole, we denote the regulator's supervision ability space as $\{\eta_1^*, \eta_2^*, \dots, \eta_k^*, \dots, \eta_K^*\}$.

Regarding banks' project choices in each period, we need to consider two possible cases. First, if a loophole innovation is successful, all banks can take the new risky project without being detected by the regulator. Second, if a loophole innovation fails, it is easy to see that if banks want to take risky projects, they would only take the risky project discovered in the latest loophole innovation. This is because the regulator's supervision ability is lowest for the risky project discovered in the latest loophole innovation, so banks can choose the highest riskiness when taking the new risky project.

When banks borrow from depositors at the beginning of each period, whether the loophole innovation will succeed or fail is not yet known. Thus the regulator's supervision ability for the latest discovered loophole determines the deposit contracts between banks and depositors, and the capable bank's loophole innovation effort. Therefore, the regulator's supervision ability related to the latest loophole is sufficient to describe the state for the economy, which implies that regulator's supervision ability space for the latest discovered loophole, $\{\eta_1^*, \eta_2^*, \dots, \eta_k^*, \dots, \eta_K^*\}$, is also the state space for the economy.

The timing of the dynamic model is as follows: at the beginning of each period, the regulator's supervision ability is updated according to the evolution law, which is common knowledge. Banks offer deposit menus to households. Households decide whether or not to make deposits in the banks. After banks raise deposits, one of the banks knows it is the capable bank, and it chooses to make loophole innovation effort. If the loophole innovation is successful, all other banks can learn from it. Banks make project choices and choose riskiness

if they invest in risky projects. At the end of each period, projects pay off. Banks pay back the depositors if their projects succeed. Otherwise, they default. In the next period, the regulator improves its supervision ability on the loopholes according to the evolution law.

4.2 Dynamics

Within each period, the problem is the same as the static model. As shown in the static model, in normal times without successful loophole innovation, all banks choose the safe project. However, if loophole innovation is successful, all banks choose the risky project. Given the regulator's supervision ability η_t in the period t , we have the following results

$$e_t = \frac{\eta_t - \underline{\eta}}{\eta^s - \eta_t} \frac{(1 - \bar{\lambda})R}{c} \quad (11)$$

$$r_t = \frac{r_0}{(1 - e_t)\eta^s + e_t\underline{\eta}} \quad (12)$$

$$L_t = \frac{1}{1 - \frac{(1 - \bar{\lambda})R}{(\eta^s - \eta_t)r_t}} \quad (13)$$

As the regulator's supervision ability improves, banks have a higher leverage ratio. If the loophole innovation fails in the period t , all banks choose the safe project. A fraction $1 - \eta^s$ of banks fail at the end of the period, and the output is $y_t^n = \omega \cdot (R - r_0)L_t$. Thus the output in the economy increases as banks' leverage rises. We say that the economy is in boom. However, if the loophole innovation succeeds in the period t , all banks choose the risky project. A fraction $1 - \underline{\eta}$ of banks default, and the output is $y_t^i = \omega \cdot (\bar{\lambda}R - r_0)L_t$. Due to the widespread defaults and declining output, we say that there is a crisis in the economy when a loophole innovation is successful.

Proposition 2. *Under Assumption 1 and a large innovation cost coefficient c , the longer the boom,*

- (I) *the higher the bank's leverage;*
- (II) *the more likely a crisis is to occur;*
- (III) *conditional on a crisis occurring, the larger the decline in output.*

Since the regulator improves its supervision ability each period through learning-by-doing, the regulator's supervision ability is higher when the boom is longer. From Proposition 2,

we know that banks' leverage and the capable bank's loophole innovation effort increase with supervision ability. Therefore, banks' leverage is higher for a longer boom, and at the same time capable banks' innovation effort is higher, which implies that crises are more likely to happen. Conditional on loophole innovation being carried out, the output is $y_t^i = \omega \cdot (\bar{\lambda}R - r_0)L_t$. Since $\bar{\lambda}R < r_0$, the greater the leverage, the larger the drop in output.

To illustrate Proposition 2, we simulate a certain path of loophole innovation in the economy. The results are in Figure 2. Two successful loophole innovations take place in the period 15 and 25, so there are crises in these two periods. The boom period before the first crisis is longer than the one before the second. As we can see, both leverage and output increase in boom periods. The longer the boom, the higher the leverage and output. At the same time, the capable bank's innovation effort also increases, which means there is a higher probability that a crisis is to occur. When the loophole innovation eventually succeeds, banks choose the risky project. As is shown in Figure 2, conditional on a crisis occurring, the drop in output is larger in the first crisis.

4.3 Long-run Distribution Properties

Next, we investigate the long-run distribution for the economy. As we have shown before, the regulator's supervision ability regarding the latest discovered loophole characterizes the states of the dynamic economy. Given the regulator's supervision ability η_i^* , all banks offer the same contracts to households, which determines the leverage, deposit contract, and capable bank's loophole innovation effort. At the same time, the evolution of the supervision ability state depends on whether loophole innovation succeeds or not. To make a more general case, we let the regulator's supervision ability for known risky projects grow with probability q , and stay at the same level with probability $1 - q$. Whether or not the regulator's supervision ability grows is public knowledge. Note that when $q = 1$, we go back to the previous case where supervision ability grows in each period with certainty. If the current supervision ability is η_i^* , i.e., $\eta_t = \eta_i^*$, we can write down the general rule for supervision ability evolution. For the case $i < K$,

$$\eta_{t+1} = \begin{cases} \eta_{i+1}^*, & \text{with prob. } q \text{ in case of no successful loophole innovation;} \\ \eta_i^*, & \text{with prob. } 1 - q \text{ in case of no successful loophole innovation;} \\ \eta_1^*, & \text{in case of successful loophole innovation.} \end{cases} \quad (14)$$

For the case $i = K$,

$$\eta_{t+1} = \begin{cases} \eta_K^*, & \text{in case of no successful loophole innovation;} \\ \eta_1^*, & \text{in case of successful loophole innovation.} \end{cases} \quad (15)$$

For supervision ability η_i^* , with probability $1 - e_i$, loophole innovation fails in the current period. In this case, with probability q , the regulator's supervision ability will evolve to η_{i+1}^* if $i < K$, or stay at η_K^* if $i = K$ in the next period. With probability $1 - q$, the regulator will stay at the same level of supervision ability η_i^* . With probability e_i , loophole innovation succeeds in the current period. In this case, the regulator's supervision ability resets to η_1^* in the next period. Thus, the regulator's supervision ability follows a Markov process. We can write the transition matrix for the Markov process as

$$P = \begin{bmatrix} e_1 + (1 - q)(1 - e_1) & q(1 - e_1) & 0 & \dots & 0 \\ e_2 & (1 - q)(1 - e_2) & q(1 - e_2) & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ e_{K-1} & 0 & 0 & \dots & q(1 - e_{K-1}) \\ e_K & 0 & 0 & \dots & 1 - e_K \end{bmatrix} \quad (16)$$

The element P_{ij} denotes the probability that the economy evolves from state i in the current period, to state j in the next period. If the current state is i , the regulator's supervision ability is η_i^* , and the capable bank's innovation effort is e_i . For states $1 \leq i < K$, with probability e_i , the loophole innovation succeeds, and the economy will evolve to state 1 in the next period. With probability $1 - e_i$, the loophole innovation fails, and the economy will evolve to the next state $i + 1$ with probability q and stay at the same state i with probability $1 - q$ in the next period. For state K , the difference is that the economy will stay the same state in the next period if there is no successful loophole innovation in the current period.

Since there is only a finite number of recurrent states which follow a Markov process, we can deduce the following lemma.

Lemma 3. *Under Assumption 1 and a large innovation cost coefficient c , there is a stationary distribution π for the supervision ability Markov process, i.e., $\pi = \pi P$.*

The stationary distribution π is a $1 \times K$ row vector, where the i th element π_i is the probability of the economy with supervision ability η_i^* . Since the first state occurs only after a successful loophole innovation, the first element π_1 equals the probability of crises in the long run.

As is shown in Lemma 2, when the values of parameters such as c , R , and $\bar{\lambda}$ change, the success probability of loophole innovation changes. This leads to changes in the transition matrix and the stationary distribution. We can deduce the following lemma.

Lemma 4. *Under Assumption 1 and a large innovation cost coefficient c , if c increases, R decreases, or $\bar{\lambda}$ increases, the probability of the lowest supervision ability state decreases, and the probability of the highest supervision state increases.*

The intuition is that if c increases, R decreases, or $\bar{\lambda}$ increases, the success probability of loophole innovation in each state decreases. On the one hand, this implies that there are fewer crises, and the economy is less likely to return to the lowest supervision ability state. On the other hand, the economy is more likely to evolve into the state with higher supervision ability, thus the probability of the highest supervision ability state increases.

We can further characterize the property for the whole distribution in the following proposition.

Proposition 3. *Under Assumption 1 and a large innovation cost coefficient c , if c increases, R decreases, or $\bar{\lambda}$ increases, the new stationary distribution will first-order stochastically dominate the original one.*

First-order stochastic dominance means that the cumulative density function of the new stationary distribution is lower than that of the original one, so the whole distribution shifts to the higher supervision states on average. In other words, the probability that the regulator has a high supervision ability is higher in the long run. In Figure 3, we plot the innovation probability and stationary distribution with different innovation cost coefficients. As shown in the figure, we can see that with a small c , the stationary distribution has a higher probability for low supervision ability, i.e., the economy is more likely to stay in the low supervision ability states in the long run.

5 Regulation: Maximum Leverage Ratio

In this section, we discuss the policy implications of banks' loophole innovation. When a capable bank engages in loophole innovation, it will not internalize the negative externalities for other banks. The negative externalities of loophole innovation have two dimensions. First, successful loophole innovation will reduce the output in the current period by allowing all

banks to invest in inefficient risky projects. Second, after a new loophole innovation, the regulator has to learn about it and improve its supervision ability gradually from the start. This leads to a low leverage for the banks in the following periods. These externalities provide the regulator with justification for setting the maximum leverage ratio to curb loophole innovation probability. As shown before, when the regulator has a high supervision ability, the market allows the banks to have a high leverage. But at the same time, the market-determined leverage results in a high probability of innovation. To curb the high probability of innovation, the regulator can set a maximum leverage ratio for the banks.

Under the regulator's supervision ability η_i^* , let us denote the market-determined leverage as L_i^m . Here L_i^m is the bank's privately optimal leverage where there is no regulation, as in the benchmark model. Now suppose that the regulator sets the maximum leverage ratio as $\bar{L}$. If $L_i^m \leq \bar{L}$, banks can choose the market-determined leverage without violating the regulation. In this case, the maximum leverage ratio will not affect the bank's decision. However, if $L_i^m > \bar{L}$, regulation constrains banks' leverage choices. Banks cannot choose the privately optimal leverage L_i^m due to the regulation, instead they can only take a leverage of $\bar{L}$. From Proposition 1, we know that L^m increases with the regulator's supervision ability, so regulation is more likely to be effective when supervision ability is high. We refer to the states that the maximum leverage ratio constrains market-determined leverage as the affected states.

When the leverage regulation is effective, banks optimally choose the regulated maximum leverage, and the incentive compatibility constraint becomes slack. The first order condition for the capable bank's innovation effort is

$$- [R - \eta^s r(1 - 1/\bar{L})] + [\bar{\lambda}R - \underline{\eta}r(1 - 1/\bar{L})] = ce \quad (17)$$

and the interest rate in the equilibrium is

$$r = \frac{r_0}{(1 - e)\eta^s + e\underline{\eta}} \quad (18)$$

By solving the above two equations, we can get the innovation probability $\bar{e}$ when banks' leverages are restricted by the regulation. Thus, the innovation probability under regulation is

$$e_i^r = \begin{cases} e_i^m, & \text{if } L_i^m \leq \bar{L} \\ \bar{e}, & \text{if } L_i^m > \bar{L} \end{cases} \quad (19)$$

Since banks' leverage is constrained with a maximum leverage ratio, the innovation probability under regulation is always smaller than or equal to that without regulation, i.e., $e_i^r \leq e_i^m$. With the above innovation probability, we can write the transition matrix under regulation as

$$P^r = \begin{bmatrix} e_1^r + (1-q)(1-e_1^r) & q(1-e_1^r) & 0 & \dots & 0 \\ e_2^r & (1-q)(1-e_2^r) & q(1-e_2^r) & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ e_{K-1}^r & 0 & 0 & \dots & q(1-e_{K-1}^r) \\ e_K^r & 0 & 0 & \dots & 1-e_K^r \end{bmatrix} \quad (20)$$

where the element $P^r(i, j)$ is the probability of moving from state i to state j in the next period. With this transition matrix, we can get the stationary distribution under regulation, π^r . Since we know that certain states' innovation probabilities are smaller with regulation if there are some $L_i^m > \bar{L}$, we can compare the two stationary distributions with and without regulation in the following proposition.

Proposition 4. *Under Assumption 1 and a large innovation cost coefficient c , if the regulator sets a maximum leverage lower than the highest one determined by the market, the stationary distribution under regulation will first-order stochastically dominate the one without regulation.*

The results are shown in Figure 4. The figure includes three cases: without regulation, lenient regulation (high maximum leverage), and strict regulation (low maximum leverage). In fact, we can consider the case without regulation as a special case of regulation, when the maximum leverage ratio is sufficiently high for banks' leverage choice to never be restricted. As we can see in the figure, as regulation becomes stricter, leverage and innovation probability under more states deviate from the case with only market discipline. Also, the leverage and innovation probability in those affected states are lower under stricter regulation. The changes in innovation probability affect the transition matrix and also stationary distribution. As we see in the graph, the stationary distribution shifts more to the high states under stricter regulation.

We assume that the regulator sets the maximum leverage ratio to maximize average output in the long run. The expected output in state i is

$$y_i^r = \omega \cdot [(1-e_i^r)(R-r_0) + e_i^r(\bar{\lambda}R-r_0)]L_i^r \quad (21)$$

and the average output in the stationary distribution is

$$EY = \sum_{i=1}^K \pi_i^r y_i^r \quad (22)$$

The maximum leverage ratio can affect average output in two ways. First, it can directly affect expected output y_i^r in certain states through its effects on leverage and loophole innovation success probability. Effective regulation decreases banks' leverage in the affected states, which has a negative effect on output in the affected states given the expected output per unit investment. But at the same time, regulation reduces loophole innovation success probability, which increases the expected output per unit investment. The overall effect of regulation on expected output in the affected states depends on which effect dominates. Usually when supervision ability is high, the former effect dominates, so expected output in the affected states will decrease with strict regulation. Second, it can affect the stationary distribution π_i^r through its effect on loophole innovation success probability. Strict regulation will shift the distribution towards high states, which usually have a higher output. If regulation decreases expected output in the affected states, there is a trade-off for the regulator between expected output in affected states and the probability of staying in high states in the stationary distribution.

In certain parameter space, the regulator optimally chooses a maximum leverage level at which the incentive compatibility constraint is not binding when the regulator has a high supervision ability. The results are shown in Figure 5. As we can see, the optimal regulation sets a maximum leverage ratio which is effective in some states. Expected output in those affected states becomes lower under regulation. However, the loophole innovation success probability is also reduced in those high supervision ability states, because the capable bank has less incentive to innovate under regulation. The change of loophole innovation success probability shifts the stationary distribution. Compared to the case of no regulation, the economy has a higher probability of staying in high supervision ability states, as shown in the fourth graph.

6 Learning about Loophole Innovation

In the previous sections, if a loophole innovation succeeds, the regulator and investors have full knowledge about the existence of the new loophole by the end of the period. Next we

study the dynamics when there is some uncertainty concerning whether or not there has been an unknown loophole in the economy.

We assume that there are N banks in the economy. Each bank lives for one period. A finite number of banks can prevent the revelation of the existence of a new loophole through the fraction of failed banks. In each period, one of the N banks is a capable one and can choose to make effort to conduct loophole innovation.

Regarding the uncertainty surrounding an unknown loophole, we assume that at the end of each period, the public only observes the number of bank failures. Therefore, the public needs to infer whether bank failures come from the safe projects or risky ones, and updates its belief about an unknown loophole using this information.¹⁰ Let us denote the public's belief about the probability of there being an unknown loophole as θ .

Unlike in previous sections, we give the regulator the additional role of investigator. Since there is uncertainty about the existence of loophole innovation, the regulator can pay a fixed cost χ/r_0 to investigate the banking sector at the beginning of each period, and the investigation result are publicly observed. We assume that the investigation cost comes from a lump-sum tax from households. If there is a loophole, the public knows about it, and the regulator's supervision ability for it starts to grow gradually from the lowest level. If there is no unknown loophole, it is revealed to the public, and the supervision ability evolves. Thus, investigation plays two roles in the model. First, it eliminates the uncertainty regarding an unknown loophole. Second, it is the starting point for the gradual growth of supervision ability for a certain type of risky project. Eisenbach et al. (2015) discusses that one of the supervisory jobs for the central bank is "discovery examination", which focuses on understanding a specific business activity and filling the knowledge gap. In our model, investigation from the regulator serves a similar role.

The timing is as follows: at the beginning of each period, the regulator decides whether or not to investigate. If it investigates and finds a loophole, its supervision ability resets to the lowest level. The investigation result is publicly observed, and the public updates its belief regarding an unknown loophole. Banks offer menus of leverage and deposit rate to households. Households decide whether or not to make deposits in the banks. After banks raise deposits, banks know whether there is a loophole that is unknown to the regulator, and

¹⁰On the contrary, if the public can observe banks' payoff, they know whether banks have invested in risky projects, and can clearly infer the existence of a loophole.

they learn about the loophole if there is one. One of the banks knows it is the capable bank, and it makes loophole innovation effort. If the innovation is successful, all other banks can learn about it. Then banks make project choices under the regulator's supervision. At the end of each period, projects pay off. Banks pay back the depositors if their projects succeed. Otherwise, they default. The public updates its belief about the existence of an unknown loophole in the economy. In the next period, the regulator's supervision ability on known risky projects evolves.

Consider the deposit menus banks offer to depositors. As in previous sections, we focus on the case where the innovation cost coefficient is large so that the success probability of loophole innovation is small. It is easy to see that banks will offer at most two types of contracts, one with low leverage and the other with high leverage. The first one is that banks offer a leverage and deposit rate menu $\{L_0, r_0/\eta^s\}$, where $L_0 \equiv 1/(1 - (1 - \bar{\lambda})R\eta^s/((\eta^s - \underline{\eta})r_0))$. Under this menu, the bank will never invest in any risky project even if there is an unknown loophole, so they only need to pay a low interest rate r_0/η^s to allow the depositors to break even. Also, if the capable bank offers this menu, it will have no incentive to conduct loophole innovation. Thus, the expected profit of the banks choosing this menu is

$$\pi_0 = RL_0 - r_0(L_0 - 1)$$

The second one is a menu with a leverage higher than L_0 , and a deposit rate that allows depositors to break even. For a large innovation cost coefficient, the incentive compatibility constraint is binding, i.e., $R - \eta^s r(1 - 1/L) = \bar{\lambda}R - \eta^* r(1 - 1/L)$. Since there is uncertainty about an unknown loophole, the belief about the probability that there is an unknown loophole, θ , plays a role in the deposit contract. Note that the capable bank will make loophole innovation efforts only if it chooses the high-leverage menu. Therefore, the number of banks choosing the high-leverage menu affects the success probability of loophole innovation, which also determines the expected profit of banks with high-leverage menus. Let us use n to denote the banks choosing a high-leverage menu. We have the following results related to

banks choosing a high-leverage menu

$$e = \frac{\eta^* - \underline{\eta}}{\eta^s - \eta^*} \frac{(1 - \bar{\lambda})R}{c} \quad (23)$$

$$r = \frac{r_0}{(1 - \theta)(1 - \frac{n}{N}e)\eta^s + [\theta + (1 - \theta)\frac{n}{N}e]\underline{\eta}} \quad (24)$$

$$L = \frac{1}{1 - \frac{(1 - \bar{\lambda})R}{(\eta^s - \eta^*)r}} \quad (25)$$

This menu is only feasible if the interest rate r is not higher than R/η^s . Since r depends on θ , this implies that the belief that there is an unknown loophole cannot be too large.

The expected profit for banks choosing a high-leverage menu is

$$\pi(n, \theta, \eta^*) = (1 - \theta) \left(1 - \frac{n}{N}e\right) [RL - \eta^s r(L - 1)] + \left[\theta + (1 - \theta)\frac{n}{N}e\right] [\bar{\lambda}RL - \underline{\eta}r(L - 1)] - \frac{1 - \theta}{N} \frac{1}{2} c e^2 L$$

With probability $(1 - \theta) \left(1 - \frac{n}{N}e\right)$, no unknown loophole existed before this period, and no new loophole innovation is carried out in this period, so banks with a high-leverage menu will choose the safe project. With probability $\theta + (1 - \theta)\frac{n}{N}e$, either there is unknown loophole, or a new loophole is discovered in this period, so banks with a high-leverage menu invest in the risky project evading the regulator's supervision. The probability that one high-leverage bank is a capable one is $1/N$, and it will exert innovation effort when there is no unknown loophole. We can see that $\pi^*(n, \theta, \eta^*)$ decreases in n , decreases in θ , and increases in η^* for a large innovation cost coefficient. The number of banks choosing high leverage, n , is endogenously determined in the equilibrium, where no bank has the incentive to switch to the low leverage menu. Let n^* denote the number of banks choosing a high-leverage menu in equilibrium, then

$$n^* = \begin{cases} 0, & \text{if } \pi(1, \theta, \eta^*) < \pi_0 \\ n, & \text{if } \pi(n, \theta, \eta^*) \geq \pi_0 > \pi(n + 1, \theta, \eta^*) \\ N, & \text{if } \pi(N, \theta, \eta^*) \geq \pi_0 \end{cases} \quad (26)$$

Firstly, if banks' expected profit with the low-leverage menu is higher than with the high-leverage menu, even if only one bank chooses the high-leverage menu, all banks will offer the low-leverage one. This case occurs when the belief is very pessimistic, i.e., θ is large. Secondly, if banks' expected profit with the high-leverage menu is higher than with the low-leverage menu, even if all banks choose low-leverage menu, all banks will offer the high-leverage one. This case occurs when the belief is very optimistic, i.e., θ is small. Thirdly, when θ is in

the medium range, some banks may choose the high-leverage menu while others choose the low-leverage one. The number of banks choosing the high-leverage menu is determined in such a way that banks' expected profit with high leverage is higher than or equal to the expected profit with low leverage, with one extra bank switching to the high-leverage menu making banks prefer the low-leverage menu. For a bank that chooses the high-leverage menu, let r^* , L^* , and π^* denote respectively the bank's interest rate, leverage, and expected profit in equilibrium. For a capable bank, let us use e^* to denote its loophole innovation effort in equilibrium.

Next we consider the belief updating problem. At the end of each period, the public can update its belief about the existence of an unknown loophole from the performance of banks in that period. For the banks choosing the low-leverage menu, there is no information about the existence of an unknown loophole since they never choose risky projects. Thus, all the information related to belief updates comes from those banks choosing the high-leverage menu. If the public observes m banks failing out of n^* banks choosing the high-leverage menu, the updated belief is

$$\tilde{\theta}(m, \theta, \eta^*) = \frac{[\theta + (1 - \theta)\frac{n^*}{N}e^*] \underline{\eta}^m (1 - \underline{\eta})^{n^* - m}}{(1 - \theta) \left(1 - \frac{n^*}{N}e^*\right) (\eta^s)^m (1 - \eta^s)^{n^* - m} + [\theta + (1 - \theta)\frac{n^*}{N}e^*] \underline{\eta}^m (1 - \underline{\eta})^{n^* - m}} \quad (27)$$

For a certain belief θ and supervision ability η^* , the updated belief after observing banks' performance can only have $n^* + 1$ possible values. Let us denote by $\mathcal{M}(\theta, \eta^*)$ the set for all possible updated belief,

$$\mathcal{M}(\theta, \eta^*) = \{\tilde{\theta}(m, \theta, \eta^*) | m = 0, 1, \dots, n^*\}$$

For a belief $\tilde{\theta}(m)$ in the set $\mathcal{M}(\theta, \eta^*)$, the probability that the public will have that belief after observing banks' performance is

$$\Gamma(\tilde{\theta}(m) | \theta, \eta^*) = \binom{n^*}{m} (1 - \theta) \left(1 - \frac{n^*}{N}e^*\right) (\eta^s)^m (1 - \eta^s)^{n^* - m} + \binom{n^*}{m} \left[\theta + (1 - \theta)\frac{n^*}{N}e^*\right] \underline{\eta}^m (1 - \underline{\eta})^{n^* - m}$$

At the beginning of each period, if the public belief is too pessimistic, all banks will offer low-leverage menu, and there is no belief updating. Let $\bar{\theta}(\eta^*)$ denote the threshold belief at which at least one bank will choose the high-leverage menu given the supervision ability η^* . It satisfies the following condition

$$\pi(1, \bar{\theta}(\eta^*), \eta^*) = \pi_0$$

Since $\pi(n, \theta, \eta^*)$ decreases in θ and increases in η^* for a large c , we get the following lemma

Lemma 5. *Under Assumption 1 and a large innovation cost coefficient c , there is an unique belief threshold, above which no bank will choose the high-leverage menu, and thus the belief about an unknown loophole is not updated in the period. The belief threshold increases in the regulator's supervision ability.*

When the belief θ is higher than $\bar{\theta}(\eta^*)$, no bank chooses the high-leverage menu, so there is no update about an unknown loophole from the banks' performance. The belief at the end of the period will be the same as θ . We call $\bar{\theta}(\eta^*)$ the belief-update threshold because there is updating of belief only if the belief is lower than $\bar{\theta}(\eta^*)$ for supervision ability η^* .

Next we discuss the effects of changes in belief and supervision ability on the economy. The analysis is complicated by the fact that the number of banks choosing the high-leverage menu also changes with these factors. We use n^*e^*/N , $[(N - n^*)r_0/\eta^s + n^*r^*]/N$, and $[(N - n^*)L_0 + n^*L^*]/N$ to denote expected innovation effort, average interest rate, and average leverage respectively. We have the following proposition

Proposition 5. *Under Assumption 1 and a large innovation cost coefficient c ,*

(I) *if θ increases, n^* stays the same or decreases.*

(i) *if n^* stays the same, expected innovation effort stays the same, average interest rate increases, average leverage decreases;*

(ii) *if n^* decreases, expected innovation effort decreases, interest rate may increase, decrease or stay the same, average leverage decreases.*

(II) *if supervision ability η^* increases, n^* stays the same or increases. Expected innovation effort increases, average interest rate increases, average leverage increases.*

For the first part of Proposition 5, the effects of belief θ mainly come from its effect on interest rate. When it is large, depositors worry about the unknown loophole, so banks choosing the high-leverage menu have to pay a high interest rate. A higher interest rate lowers the leverage through incentive compatibility constraint. Its effect on capable bank's loophole innovation effort comes from the extensive margin, i.e., banks switch to low-leverage menu. For the second part of Proposition 5, the effects of supervision ability could come from both the intensive and extensive margin. For the intensive margin, banks choosing the high-leverage menu can offer a higher leverage. This also leads to a higher loophole innovation

effort if the capable bank chooses the high-leverage menu. If more banks choose the high-leverage menu with increasing supervision ability, this increases the average leverage and expected innovation probability from the extensive margin. This shows that the results in Proposition 1 are robust even if we include learning in the model.

Unlike in previous sections, the regulator faces an investigation problem now, i.e., when to pay a fixed cost to investigate whether there is an unknown loophole. The regulator uses a lump-sum tax from households to fund the investigation cost. The regulator has a discount factor β , and its aim is to maximize the discounted expected output including the loss from the investigation cost. The expected output, given belief θ and supervision ability η^* , is

$$y(\theta, \eta^*) = n^* \left[(1 - \theta) \left(1 - \frac{n^*}{N} e^* \right) (R - r_0) + \left(\theta + (1 - \theta) \frac{n^*}{N} e^* \right) (\bar{\lambda} R - r_0) \right] L^* + (N - n^*) (R - r_0) L_0 \quad (28)$$

The regulator makes a decision concerning investigation based on the belief at the beginning of each period, which is the same as updated belief based on banks' performance in the last period. If the regulator does not investigate, the belief stays the same, and banks offer menus based on this belief. Otherwise, the belief will reset to zero after investigation, since the investigation eliminates the uncertainty about an unknown loophole in the economy. If the regulator finds a loophole through investigation, the regulator has to accumulate supervision ability from the beginning for the new type of risky project. If the investigation does not find a loophole, the regulator's supervision ability continues to evolve from the last period. Let $\tilde{\theta}$ be the belief before investigation in the current period, and $\tilde{\theta}'$ be the belief before investigation in the next period. We can write down the regulator's problem in the recursive form

$$V(\tilde{\theta}, \eta_i^*) = \max_{d \in \{0,1\}} \left\{ (1 - d) \left[y(\tilde{\theta}, \eta_i^*) + \beta \sum_{\tilde{\theta}' \in \mathcal{M}(\tilde{\theta}, \eta_i^*)} \Gamma(\tilde{\theta}' | \tilde{\theta}, \eta_i^*) (q * V(\tilde{\theta}', \eta_{i+1}^*) + (1 - q) * V(\tilde{\theta}', \eta_i^*)) \right] \right. \\ \left. + d \left\{ -\chi + \tilde{\theta} \left[y(0, \eta_1^*) + \beta \sum_{\tilde{\theta}' \in \mathcal{M}(0, \eta_1^*)} \Gamma(\tilde{\theta}' | 0, \eta_1^*) (q * V(\tilde{\theta}', \eta_2^*) + (1 - q) * V(\tilde{\theta}', \eta_1^*)) \right] \right. \right. \\ \left. \left. + (1 - \tilde{\theta}) \left[y(0, \eta_i^*) + \beta \sum_{\tilde{\theta}' \in \mathcal{M}(0, \eta_i^*)} \Gamma(\tilde{\theta}' | 0, \eta_i^*) (q * V(\tilde{\theta}', \eta_{i+1}^*) + (1 - q) * V(\tilde{\theta}', \eta_i^*)) \right] \right\} \right\} \quad (29)$$

If the regulator chooses not to investigate, i.e., $d = 0$, the expected output is $y(\tilde{\theta}, \eta_i^*)$, the belief in the next period $\tilde{\theta}'$ is updated from $\tilde{\theta}$ through the banks' performance, and the supervision ability evolves to η_{i+1}^* with probability q and stays the same with probability $1 - q$ in the next period. If the regulator chooses to investigate, i.e., $d = 1$, it needs to collect the tax from the household and pay the fixed cost at the beginning of the period, and the related loss in the output is χ . If the regulator finds a loophole through investigation, the regulator

has to accumulate its expertise for this new type of risky project from the beginning, and its supervision ability resets to the lowest level. The expected output is $y(0, \eta_1^*)$ in the current period, and the supervision ability and belief evolve following the rules. If the regulator does not find a loophole, the expected output is $y(0, \eta_i^*)$, and the supervision ability and belief evolve. In this economy, the belief $\tilde{\theta}$ and the supervision ability for a known loophole η_i^* are important states characterizing the evolution of the economy.

We can see that if the investigation cost is zero, the regulator will choose to investigate each period, because eliminating uncertainty can increase banks' leverage and reduce the risk related to an unknown loophole. Thus, there is no uncertainty about an unknown loophole when banks offer menus to households. The results will be the same to those in Section 4. If the investigation cost is too large, there are some absorbing states with positive probability, where the economy will stay forever once it enters. Since the supervision ability for an known loophole increases with positive probability, the absorbing states can only include the highest supervision ability η_K^* . If the belief at the beginning of the period is higher than the belief-update threshold $\bar{\theta}(\eta_K^*)$, banks will always choose the low-leverage menu if the regulator does not investigate. Given a sufficiently large investigation cost, the regulator would not choose to investigate. In this case, there is no belief update and no evolution for the supervision ability, and the economy stays where it is.

The case with a medium investigation cost is more interesting. We plot the belief thresholds in Figure 6 for a certain parameter space where the investigation cost is not too large or too small. The black dashed line denotes the belief-update threshold. If the belief θ is higher than this threshold, all banks will choose the low-leverage menu, and there will be no update regarding an unknown loophole. As is shown in Lemma 5, the threshold is higher for high supervision ability, and the black dashed line is higher on the right side. The blue solid line denotes the belief threshold for investigation. If the belief is higher than the threshold, the regulator will investigate whether there is an unknown loophole. We can see that the relationship between the investigation threshold and the supervision ability is not monotonic. Within the belief-update region, on the one hand, higher supervision ability leads to higher leverage, and the drop in the expected output will be larger if there is an unknown loophole. This force leads to the belief threshold decreasing with the supervision ability. In the extreme case where β is zero, it is easy to show that the belief threshold for investigation is a decreasing function for supervision ability. On the other hand, the investigation cost is irreversible,

so investigation is an option for the regulator. There could be a wait-and-see effect. The regulator may need more information before paying the fixed investigation cost. This force makes the regulator willing to delay investigation. Within the no-belief-update region, the regulator has an incentive to investigate to eliminate the uncertainty so that banks can have higher leverage. Also, since the belief-update threshold increases with supervision ability, the regulator may withhold investigation to allow the belief to fall below the threshold in the higher supervision ability state. Thus, the relationship between the belief threshold for investigation and supervision ability may not be monotonic.

7 Discussion

In this section, we discuss several setups in the model in relation to the loophole innovation and regulator’s learning-by-doing.

Regarding the loophole innovation, we assume that there is only one capable bank in each period. We can easily extend the model to the case that there is a finite number of capable banks. In this case, we can show that the choice of innovation effort of one capable bank depends on the choices of other banks. This extension makes the model more complicated, without adding any new insights. Second, we assume that all current and future banks can learn about the new loophole if the capable bank succeeds. There are two reasons that we make this assumption. Firstly, since a successful loophole innovation provides banks with the opportunities to take the risky projects, which the regulator tries to forbid, the capable bank cannot rely on any legal system to protect the successful loophole innovation. Secondly, there is no competition among banks in the model, so the capable bank has no incentive to prevent other banks from taking advantage of the new loophole.

In this paper, we model the regulator’s learning-by-doing process in a reduced form. We do this for two reasons. First, we treat the regulator’s supervision as passive in the model, so we can focus on the decisions of banks, especially the loophole innovation. Second, the passive evolution of the regulator’s supervision ability makes our model much more tractable. However, in Section 6, we add an investigation role for the regulator, which can be considered as a form of active learning-by-doing. In a companion paper on shadow banking that is still working in progress, we provide a complete micro-foundation for investors’ learning-by-doing, which we expect to incorporate into this model in the future.

8 Conclusion

In this paper we develop a model on the dynamic interaction between regulator supervision and banks' loophole innovation, and study its implications on banks' credit cycles. In the model, banks' leverages are constrained due to a risk-shifting problem. The regulator supervises the banks to ease this moral hazard problem, and its expertise in supervision improves gradually through learning-by-doing. At the same time, banks can engage in loophole innovation to circumvent supervision, which acts as an endogenous opposing force diminishing the value of the regulator's accumulated expertise. In equilibrium, banks' leverage and loophole innovation move together with the regulator's supervision ability. The model shows that long periods of gradual expansion in banks' leverage, investment, and aggregate output, are followed by sudden and sharp recessions. In our model, even in the absence of exogenous perturbations, banks themselves can become the sources of adverse shocks to the real economy. We show that the longer the boom, the more likely there is a crisis and the more severe the consequences, which corresponds to Minsky's hypothesis that good times sow the seeds for the next financial crisis. The model's empirical implications are broadly consistent with the stylized facts from empirical studies related to credit cycles.

Based on this model, we also discuss the welfare implications of a maximum leverage ratio in the environment of loophole innovation. We show that the regulator faces a trade-off between financial stability and output in boom periods. A higher maximum leverage ratio is associated with higher output in good times but more frequent crises, while a lower maximum leverage ratio is associated with lower output in good times but less frequent crises. Also, we extend the benchmark model by allowing households to have uncertainty regarding the regulator's supervision ability, and study how the economy evolves with both the regulator's supervision ability and households' beliefs regarding the regulator's supervision ability.

In the paper, the sources of credit cycles come from the interaction between regulator supervision and banks' loophole innovation. Without a doubt, there are other important sources for the credit cycles, which have been widely discussed in the literature. We consider our mechanism as a novel and complementary one to those in the previous literature. To highlight our mechanism, we have omitted other sources for the business cycles from this paper. We can potentially incorporate some common shocks in the business cycle literature into our model.

Although the present model is stylized, it would be interesting to test the implications of this model with data in future work. First, our model shows that longer boom periods predict higher probability of crises and more severe consequences. We can test the relationship between conditional frequency, as well as consequences of crises and the length of boom periods with cross-country data. Second, as more data on regulation and supervision emerges, we can study the linkage between bank regulation and business cycle patterns across countries.

A Proofs

Proof for Proposition 1

From equation (7), we can get

$$\frac{\partial e}{\partial \eta^*} = \frac{\eta^s - \underline{\eta}}{(\eta^s - \eta^*)^2} \frac{(1 - \bar{\lambda})R}{c} > 0 \quad (30)$$

Thus, innovation effort (probability) is increasing with supervision ability.

From equation (8), we can get

$$\frac{\partial r}{\partial \delta} = \frac{(\eta^s - \underline{\eta})r^2}{r_0} \cdot \frac{\partial e}{\partial \eta^*} > 0 \quad (31)$$

Thus, deposit rate is increasing with supervision ability.

From equation (9), we can get

$$\frac{\partial L}{\partial \eta^*} = \frac{(1 - \bar{\lambda})R}{(\eta^s - \eta^*)^3 c L^2 r_0} [\eta^s(\eta^s - \eta^*)c - (1 - \bar{\lambda})(\eta^s - \underline{\eta})(\eta^s + \eta^* - 2\underline{\eta})R] \quad (32)$$

From the above equation we can see that, as long as $c \geq \frac{(1 - \bar{\lambda})(\eta^s - \underline{\eta})(\eta^s + \bar{\eta}^* - 2\underline{\eta})R}{\eta^s(\eta^s - \bar{\eta}^*)}$ holds, leverage is always increasing with supervision ability. Q.E.D.

Proof for Lemma 2

From equation (7), we can get

$$\frac{\partial e}{\partial c} = -\frac{\eta^* - \underline{\eta}}{\eta^s - \eta^*} \frac{(1 - \bar{\lambda})R}{c^2} < 0 \quad (33)$$

$$\frac{\partial e}{\partial R} = \frac{\eta^* - \underline{\eta}}{\eta^s - \eta^*} \frac{1 - \bar{\lambda}}{c} > 0 \quad (34)$$

$$\frac{\partial e}{\partial \bar{\lambda}} = -\frac{\eta^* - \underline{\eta}}{\eta^s - \eta^*} \frac{R}{c} < 0 \quad (35)$$

From equation (8), we can get

$$\frac{\partial r}{\partial c} = \frac{(\eta^s - \underline{\eta})r^2}{r_0} \cdot \frac{\partial e}{\partial c} < 0 \quad (36)$$

$$\frac{\partial r}{\partial R} = \frac{(\eta^s - \underline{\eta})r^2}{r_0} \cdot \frac{\partial e}{\partial R} > 0 \quad (37)$$

$$\frac{\partial r}{\partial \bar{\lambda}} = \frac{(\eta^s - \underline{\eta})r^2}{r_0} \cdot \frac{\partial e}{\partial \bar{\lambda}} < 0 \quad (38)$$

From equation (9) and under large innovation cost coefficient c , we can get

$$\frac{\partial L}{\partial c} = -\frac{(1 - \bar{\lambda})R}{(\eta^s - \eta^*)L^2r^2} \cdot \frac{\partial r}{\partial c} > 0 \quad (39)$$

$$\frac{\partial L}{\partial R} = \frac{1 - \bar{\lambda}}{(\eta^s - \eta^*)^2 c L^2 r_0} [\eta^s(\eta^s - \eta^*)c - 2(1 - \bar{\lambda})(\eta^s - \underline{\eta})(\eta^* - \underline{\eta})R] \quad (40)$$

$$\frac{\partial L}{\partial \bar{\lambda}} = -\frac{R}{(\eta^s - \eta^*)^2 c L^2 r_0} [\eta^s(\eta^s - \eta^*)c - 2(1 - \bar{\lambda})(\eta^s - \underline{\eta})(\eta^* - \underline{\eta})R] \quad (41)$$

It is easy to see that for sufficiently large c , $\frac{\partial L}{\partial R} > 0$ and $\frac{\partial L}{\partial \bar{\lambda}} < 0$. In fact, as long as $\frac{\partial L}{\partial \eta^*} > 0$, $\frac{\partial L}{\partial R} > 0$ and $\frac{\partial L}{\partial \bar{\lambda}} < 0$. Q.E.D.

Proof for Lemma 4

From Lemma 2, we know that when c increases, R decreases, or $\bar{\lambda}$ increases, the innovation probabilities are lower for each state. If we can show that lower innovation probabilities lead to a lower probability in the lowest state and a higher probability in the highest state, we can prove this lemma. We use superscripts o to denote the old states and n to denote the new states.

From $\pi = \pi P$, we can get $\pi(I - P) = 0$. We can write down the relationship between the probabilities of two nearby states as follows

$$\pi_{j+1} = \begin{cases} \frac{q(1-e_j)}{1-(1-q)(1-e_{j+1})} \pi_j, & \text{if } 1 < j < K-1 \\ \frac{q(1-e_{K-1})}{e_K} \pi_{K-1}, & \text{if } j = K-1 \end{cases} \quad (42)$$

With this relationship, we can write down the relationship between the probability of the lowest state and that of others

$$\pi_j = \begin{cases} \prod_{i=1}^{j-1} \frac{q(1-e_i)}{1-(1-q)(1-e_{i+1})} \cdot \pi_1, & \text{if } 1 < j < K \\ \frac{q(1-e_{K-1})}{e_K} \prod_{i=1}^{K-1} \frac{q(1-e_i)}{1-(1-q)(1-e_{i+1})} \cdot \pi_1, & \text{if } j = K \end{cases} \quad (43)$$

We can define Δ_j as

$$\Delta_j = \begin{cases} \prod_{i=1}^{j-1} \frac{q(1-e_i)}{1-(1-q)(1-e_{i+1})}, & \text{if } 1 < j < K \\ \frac{q(1-e_{K-1})}{e_K} \prod_{i=1}^{K-1} \frac{q(1-e_i)}{1-(1-q)(1-e_{i+1})}, & \text{if } j = K \end{cases} \quad (44)$$

So $\pi_j = \Delta_j \cdot \pi_1$ for any $j \geq 2$. It is easy to see that as c increases, R decreases, or $\bar{\lambda}$ increases, all e_j 's decrease, so all Δ_j 's increase. Substitute π_j into $\sum_{j=1}^K \pi_j = 1$, we can get $\pi_1 = 1/(\sum_{j=1}^K \Delta_j)$, so π_1 decreases.

We will prove $\pi_K^n > \pi_K^o$ by contradiction. If $\pi_K^n \leq \pi_K^o$, since e_j becomes smaller, from equation (42), we can get $\pi_j^n < \pi_j^o$ for all $1 < j < K$. And from above, we know $\pi_1^n < \pi_1^o$. So $\sum_{j=1}^K \pi_j^n < \sum_{j=1}^K \pi_j^o = 1$, and there is contradiction. Thus, $\pi_K^n > \pi_K^o$. Q.E.D.

Proof for Proposition 3

To prove the new stationary distribution first-order stochastic dominates the original one, we just need to show that the cumulative probability $\sum_{j=1}^k \pi_j^n$ is smaller or equal to $\sum_{j=1}^k \pi_j^o$ for all k and with strict inequality for some k following the definition of first-order stochastic dominance.

If c increases, R decreases, or $\bar{\lambda}$ increases, $e_j^n < e_j^o$ for all j . From equation (42), it is easy to see that (1) if $\pi_j^n > \pi_j^o$ for some j , this inequality holds for all k larger than j ; (2) if $\pi_j^n < \pi_j^o$ for some j , this inequality holds for all k smaller than j . From Lemma 4, there must exist a $1 < k < K$, where $\pi_k^n \leq \pi_k^o$ and $\pi_{k+1}^n > \pi_{k+1}^o$. For $j < k$, $\pi_j^n < \pi_j^o$, so $\sum_{i=1}^j \pi_i^n < \sum_{i=1}^j \pi_i^o$. For $j > k$, $\pi_j^n > \pi_j^o$, so $\sum_{i=j}^N \pi_i^n > \sum_{i=j}^N \pi_i^o$. For $k < j < N$, $\sum_{i=1}^j \pi_i^n = 1 - \sum_{i=j}^N \pi_i^n < 1 - \sum_{i=j}^N \pi_i^o = \sum_{i=1}^j \pi_i^o$. Thus, we can show that $\sum_{i=1}^j \pi_i^n \leq \sum_{i=1}^j \pi_i^o$ for all j and with strict inequality for $j < K$. Q.E.D.

Proof for Proposition 4

With innovation probability under regulation, we can write down the relationship between the probabilities of two nearby states as follows

$$\pi_{j+1}^r = \begin{cases} \frac{q(1-e_j^r)}{1-(1-q)(1-e_{j+1}^r)} \pi_j^r, & \text{if } 1 < j < K-1 \\ \frac{q(1-e_{K-1}^r)}{e_K^r} \pi_{K-1}^r, & \text{if } j = K-1 \end{cases} \quad (45)$$

With this relationship, we can write down the relationship between the probability of the lowest state and that of others

$$\pi_j^r = \begin{cases} \prod_{i=1}^{j-1} \frac{q(1-e_i^r)}{1-(1-q)(1-e_{i+1}^r)} \cdot \pi_1^r, & \text{if } 1 < j < K \\ \frac{q(1-e_{K-1}^r)}{e_K^r} \prod_{i=1}^{K-1} \frac{q(1-e_i^r)}{1-(1-q)(1-e_{i+1}^r)} \cdot \pi_1^r, & \text{if } j = K \end{cases} \quad (46)$$

We can define Δ_j^r as

$$\Delta_j^r = \begin{cases} \prod_{i=1}^{j-1} \frac{q(1-e_i^r)}{1-(1-q)(1-e_{i+1}^r)}, & \text{if } 1 < j < K \\ \frac{q(1-e_{K-1}^r)}{e_K^r} \prod_{i=1}^{K-1} \frac{q(1-e_i^r)}{1-(1-q)(1-e_{i+1}^r)}, & \text{if } j = K \end{cases} \quad (47)$$

From $\sum_{j=1}^K \pi_j^r = 1$, we get $\pi_1^r = 1/(\sum_{j=1}^K \Delta_j^r)$. If the regulator sets a maximum leverage lower than the highest one determined by the market, there exists at least one e_j^r which is

smaller than that without regulation. We can get that all Δ_j^r 's are larger than or equal to the correspondent without regulation, Δ_j^m 's, and some are strictly larger. Then, $\sum_{j=1}^K \Delta_j^r$ is larger, so π_1^r is lower than that without regulation, π_1^m . It is easy to see that π_K^r is higher than the case without regulation, π_K^m .

Since the regulator sets a maximum leverage lower than the highest one determined by the market, there must exist a $1 \leq \bar{k} \leq K$, where all states lower than or equal to $\bar{k}$ are not affected by the regulation, while all states higher than $\bar{k}$ are affected by the regulation. For $j \leq \bar{k}$, $e_j^r = e_j^m$, and for $j > \bar{k}$, $e_j^r < e_j^m$. For equations (42) and (45), we can see that $\pi_j^r < \pi_j^m$ for $j \leq \bar{k}$. And if $\pi_j^r > \pi_j^m$ for some j , this inequality holds for all k larger than j . Since $\pi_K^r > \pi_K^m$, there must exist one $\bar{k} < \hat{k} \leq K$, where $\pi_j^r \leq \pi_j^m$ for $j < \hat{k}$ and $\pi_j^r > \pi_j^m$ for $j \geq \hat{k}$. For $j < \hat{k}$, $\pi_j^r \leq \pi_j^m$ with some strict inequality, so $\sum_{i=1}^j \pi_i^r < \sum_{i=1}^j \pi_i^m$. For $j \geq \hat{k}$, $\pi_j^r > \pi_j^m$, so $\sum_{i=j}^K \pi_i^r > \sum_{i=j}^K \pi_i^m$. For $\hat{k} \leq j < K$, $\sum_{i=1}^j \pi_i^r = 1 - \sum_{i=j}^K \pi_i^r < 1 - \sum_{i=j}^K \pi_i^m = \sum_{i=1}^j \pi_i^m$. Thus, we can show that $\sum_{i=1}^j \pi_i^r \leq \sum_{i=1}^j \pi_i^m$ for all j and with strict inequality for $j < K$. Q.E.D.

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Figure 1: Relationship with Supervision Ability

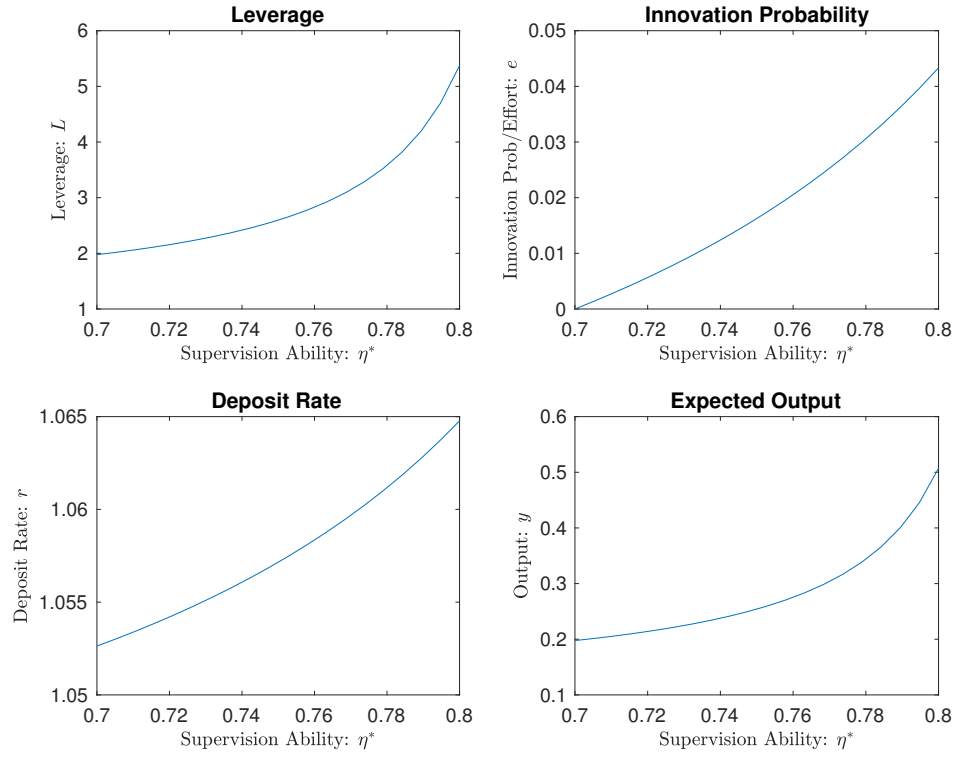
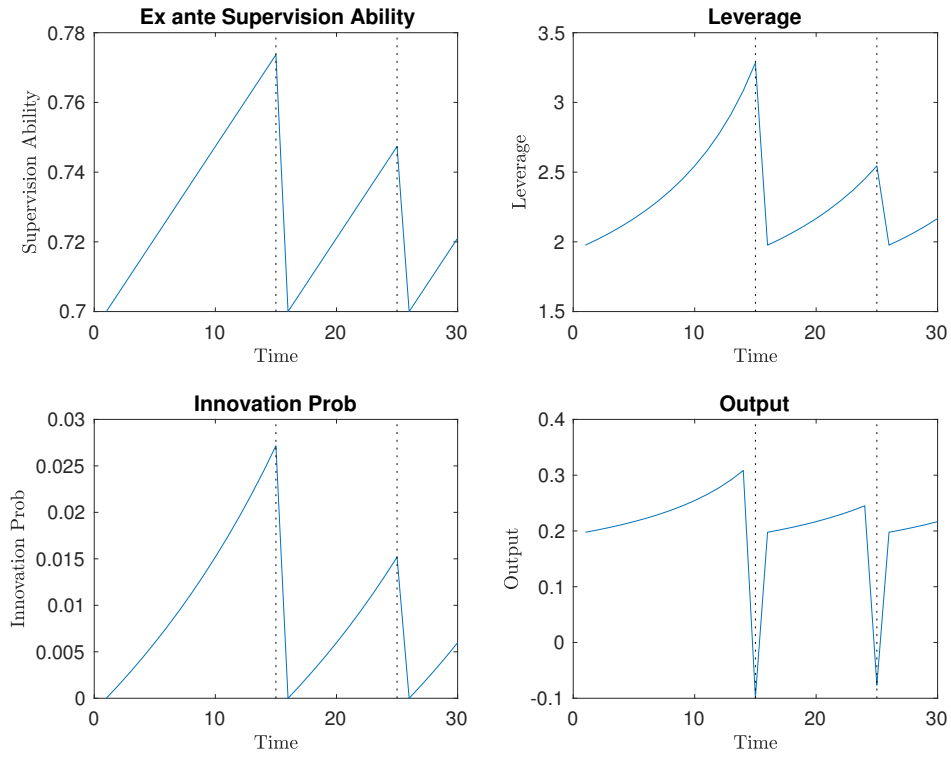


Figure 2: Dynamics



Note: The dotted vertical lines indicate the periods when loophole innovation occurs.

Figure 3: Long-run Stationary Distribution

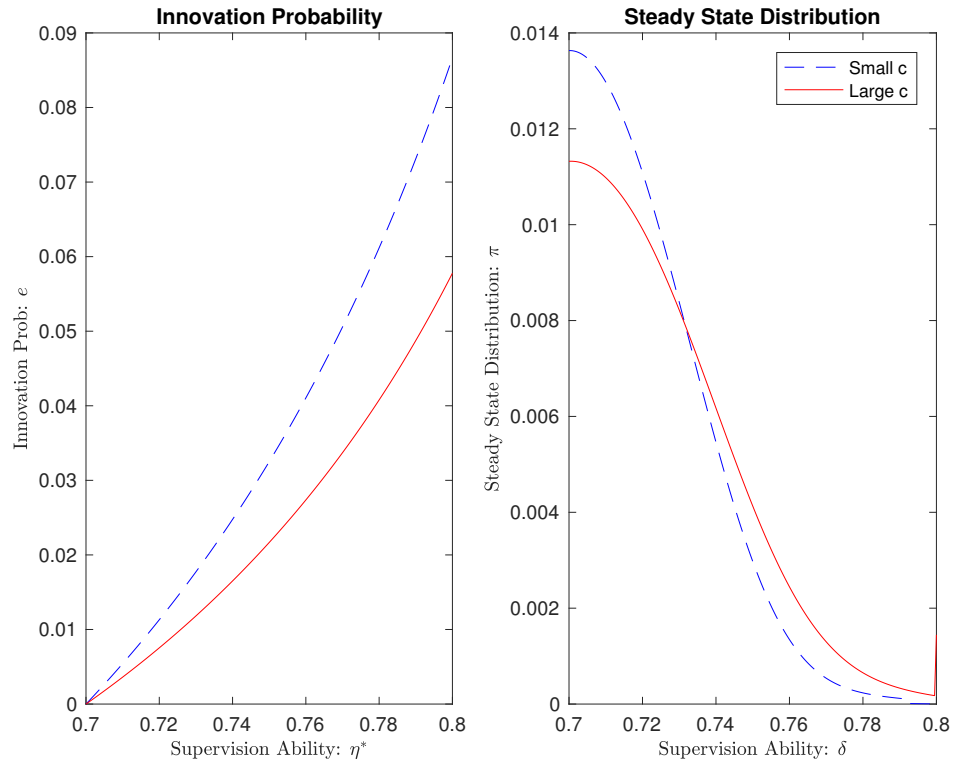
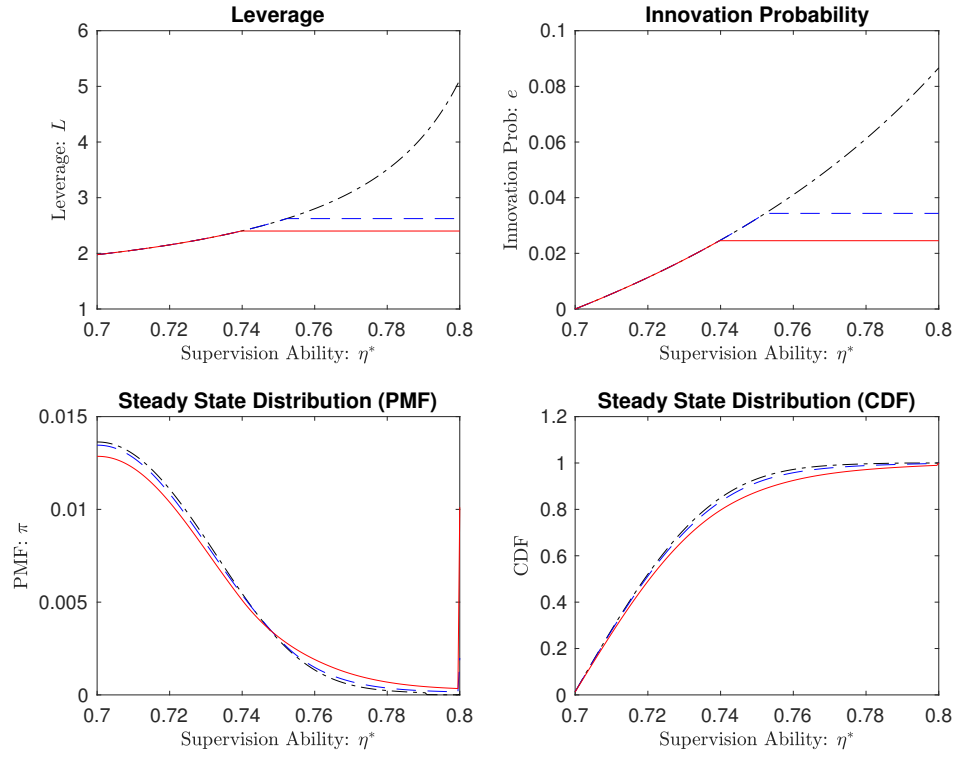
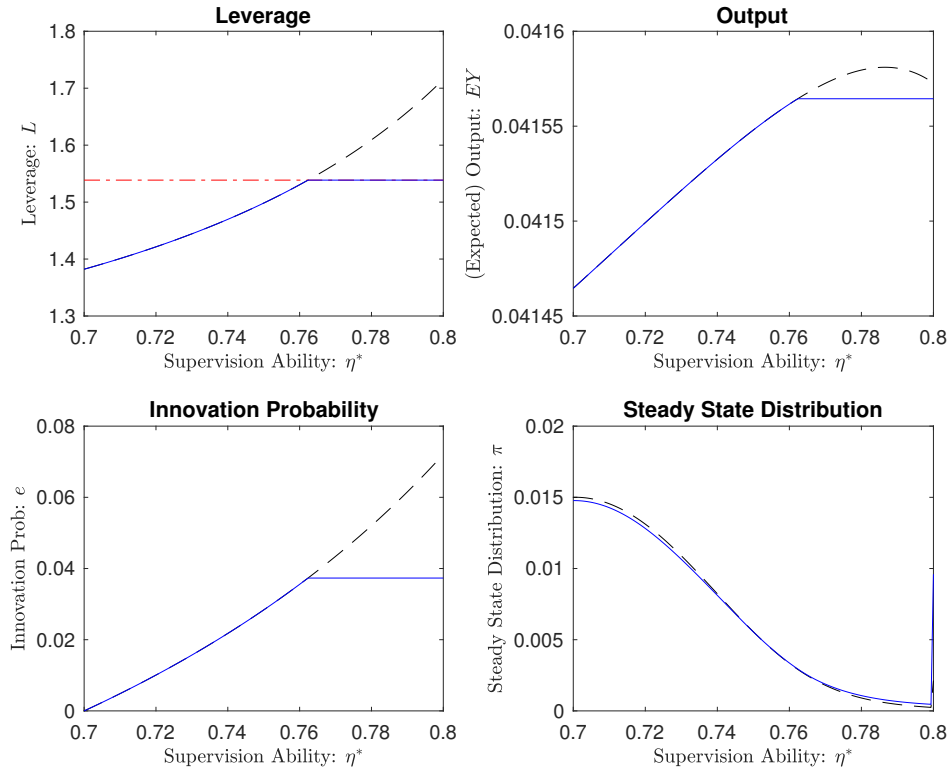


Figure 4: Maximum Leverage Ratio



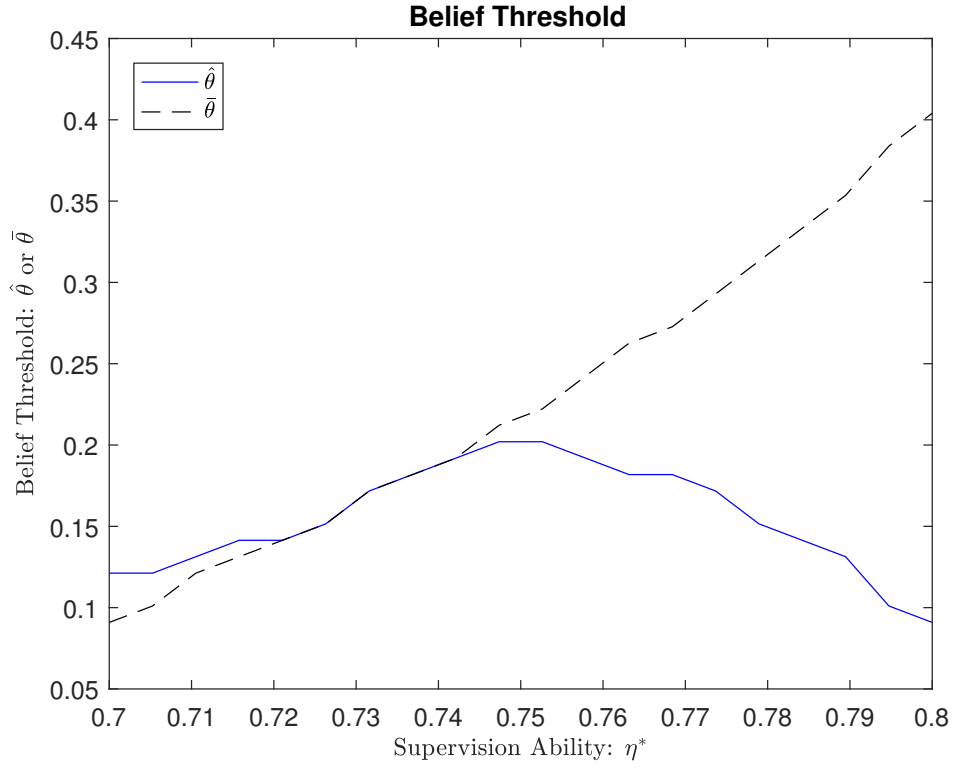
Note: Black dot-dashed line: without regulation; Blue dashed line: lenient regulation; Red solid line: strict regulation.

Figure 5: Optimal Regulation



Note: Black dashed line: without regulation; Blue solid line: optimal regulation.

Figure 6: Belief Threshold



Note: Black dashed line: belief threshold for no belief update; Blue solid line: belief threshold for investigation.