Asset prices and climate policy*

Larry Karp[†] Armon Rezai[‡]

February 11, 2018

Abstract

People might reduce carbon emissions to protect themselves, their wealth, or future generations from climate damage. An overlapping generations climate model with endogenous asset price and investment levels disentangles these incentives. Asset markets capitalize the future effects of policy, regardless of people's concern for future generations. These markets can lead self-interested agents to undertake significant abatement. A small climate policy that raises the price of capital increases old agents' welfare and also increases welfare of young agents with a high intertemporal elasticity of substitution. Climate policy can also have subtle distributional effects across the currently living generations.

Keywords: Climate externality, overlapping generations, climate policy, generational conflict, dynamic bargaining, Markov perfection, adjustment costs.
JEL, classification numbers: E24, H23, Q20, Q52, Q54

---

*Larry Karp thanks the Ragnar Frisch Centre for Economic Research for financial support. Armon Rezai benefited from the WU Visiting Fellowship and the Schrödinger fellowship of Austrian Science Fund (FWF): J 3633 and thanks the Department of Agricultural and Resource Economics at UC Berkeley for its hospitality. This research was made possible by a generous collaborative research grant under the Berkeley-Austria program of the Austrian Marshall Plan Foundation. We thank Terry Iverson, Rick van der Ploeg, Leo Simon, Steve Salant, and Christian Traeger for comments. The usual disclaimer holds.

[†]Department of Agricultural and Resource Economics, University of California, Berkeley, email: karp@berkeley.edu

[‡]Department of Socio-Economics, Vienna University of Economics and Business, email: arezai@wu.ac.at. Also affiliated with the International Institute for Applied Systems Analysis, IIASA.

1 Introduction

The standard climate narrative emphasizes that climate policy requires current generations to sacrifice to protect the climate for future generations. This narrative usually ignores asset prices or discusses them only in the context of "stranded assets", whose value is reduced by climate policy. Fossil fuel companies, the common example of stranded assets, constitute a large asset class, but a modest fraction of total world financial assets.[1] Society at large, and people with a well-diversified portfolio, should be more concerned about the effect of climate policy on asset prices writ large. Would recognizing this link affect selfish agents' incentives to reduce greenhouse gas emissions?

Climate policy can increase future capital productivity, and asset markets transfer some of those future benefits to currently living asset owners by changing asset prices. We find that recognizing the link between climate policy and asset prices can significantly increase incentives of currently living people to undertake climate policy, *even if they have no concern for their successors.*[2] (Intergenerational altruism of course increases equilibrium climate policy.) We also examine the distributional effects, across asset sellers and buyers, of policy changes arising from incentives created by asset markets.

Markets may give asset owners a stake in environmental protection, even when the owners' have no direct interest in the environment (Oates, 1972). Empirical evidence shows that environmental outcomes can affect asset prices, e.g. in real estate (Chay and Greenstone, 2005; Bushnell, Chong, and Mansur, 2011) and market portfolios (Bansal and Ochoa, 2011, Bansal, Kiku and Ochoa, 2015 and 2016).[3]

We use a model with selfish agents and a non-trivial asset market to assess whether asset markets significantly influence incentives to protect the climate. Two features of Integrated Assessment Models (IAMs), the standard tool for assessing climate policy (Stern, 2006; Nordhaus, 2008), make them unsuitable for this research question. First, their use of an Infinitely

[1] Fossil fuel companies are worth about \$5 trn (trillion) (Bullard, 2014). The world stock market capitalization exceeds \$69 trn, and the value of total financial assets exceeds \$284 trn (Witkowski, 2015).

[2] Our analysis is normative, insofar as it states how agents with particular preferences and facing particular constraints "should" behave. As we emphasize below, we do not regard the equilibrium abatement levels that these agents chose as optimal for society.

[3] Dietz et al. (2016) use DICE to estimate the "climate value at risk" at 1.8% of global financial assets, with much higher tail risks. Balvers et al. (2012) estimate the cost of past warming, as captured in asset prices, at 4.18% of wealth.

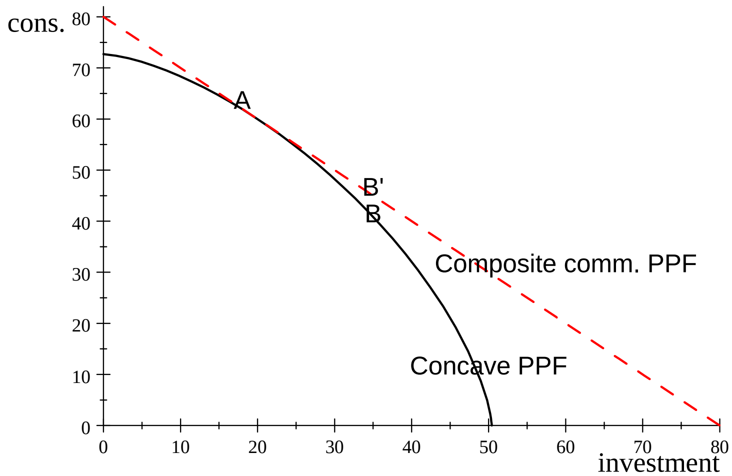

Figure 1: The solid graph shows a concave Production Possibility Frontier and the dashed graph shows this frontier under the composite commodity assumption. Both have the same value of output, 80, at the tangency, $(I, C) = (20, 60)$, where the relative price of investment is 1 and the value of output is 80. If the production point shifts south east, toward B or B', the relative price of investment changes for the concave PPF but remains constant in the composite commodity scenario

Lived Representative Agent (ILRA) means that they cannot represent the case where people currently alive have no concern for their successors. Second, IAMs use a composite commodity framework (the consumption and investment goods), rendering asset markets trivial.[4]

We replace the ILRA – composite commodity framework with a Diamond (1965) overlapping generations (OLG) model and strictly concave production possibility frontier (PPF). Figure 1 shows that changes to the production point alter the equilibrium price of investment if the PPF is concave, but not in the composite commodity framework. The dashed line in Figure 1 shows the PPF corresponding to the composite commodity framework. As the production point moves from A to B, investment increases, but the equilibrium price of investment (relative to the consumption numeraire) remains

[4] A composite commodity is a set of commodities having fixed relative prices. By choice of units, the fixed relative price(s) can be set to unity.

constant at 1. The solid curve shows a concave PPF. Here, a movement of the production point from A to B' alters the equilibrium price of investment.

In common with most IAMs, we use a model with a single stock of capital, and assume that the new capital produced by current investment and the undepreciated old capital remaining at the end of a period are homogenous. Arbitrage requires that the price of this undepreciated capital equals the price of investment. In the composite commodity framework, this price is fixed at 1, but with a concave PPF the price is endogenous.

In the OLG setting, old agents finance their consumption by renting their capital and then selling undepreciated capital. In both the composite commodity and concave PPF settings, current policy and the climate typically affect current factor prices, including the rental price of capital and the wage. In this respect, capital and labor are symmetric. With a concave PPF (but not in the composite commodity framework), current policy and the climate potentially also affect the end-of-period price of capital. This asset price effect might influence capital owners' attitude toward climate policy. Young agents use labor income to finance current consumption and to purchase capital, their means of saving. These agents will be alive in the next period, when they might benefit from current climate policy. Thus, the young and old agents in the OLG setting have different selfish reasons for caring about current climate policy.

It is difficult to directly measure the policy incentives arising from asset markets, but the econometric evidence cited above establishes a link between environmental outcomes and asset prices. An OLG model with a strictly concave PPF enables us to assess the importance of asset markets on incentives to protect the climate, and also to examine policy's distributional effects across young and old generations, the buyers and sellers of capital.

We conduct two experiments in which selfish young and old agents choose current greenhouse gas abatement to maximize a weighted sum of their own lifetime welfare, without concern for future generations. These agents play a dynamic game with their successors, taking as given successors' Markovian policy rules. In the first experiment, agents take the asset price as given, so asset markets can provide no incentive for climate policy. Of course, the debate about stranded assets arises precisely because owners of *some* assets recognize that the prices of their assets are endogenous. The second experiment, where agents understand that asset prices are endogenous, represents the situation where agents with a broad portfolio of assets are as sophisticated as actual owners of stranded assets. We also compare the outcomes with a benchmark where a discounted utilitarian chooses climate policy, just as in the standard Ramsey model. The equilibrium level of abatement when agents recognize the link between climate policy and the asset price is about twice the level chosen when agents take the asset price as fixed, and half the level chosen by the discounted utilitarian. Here, recognizing the link between climate policy and asset prices increases the equilibrium level of abatement that selfish agents choose, reaching a substantial fraction of the level a discounted utilitarian chooses.

This assessment might alter the climate debate by creating another constituency. Currently, people concerned about the long-run health of our economy, or even our species, and those concerned about the short run costs of reducing greenhouse gas emissions, are the primary parties of this debate. Asset owners with well-diversified portfolios have a (previously unidentified) stake in this debate, apart from their allegiance to the other factions. Self-interest is a powerful motivator only if it is recognized.

We also show analytically that a small increase in current abatement increases the old agent's welfare if and only if it increases the asset price. With a higher price, the young agent's welfare increases if and only if the elasticity of intertemporal substitution is finite and exceeds 1. Here, both agents benefit from a small level of abatement that increases the price of capital, even if they have no concern for their successors' welfare.

Additional literature and other considerations The discounted utilitarian benchmark is a reference point for the outcomes in the two experiments. It is *not designed to recommend optimal policy.* The practical importance of our results is to help reframe the debate about climate policy, particularly as it relates to stranded assets.[5] Many OLG models have been used to examine environmental policy (Howarth and Norgaard, 1992; John and Pecchenino, 1994; Gerlagh and Keyzer, 2001; Schneider et al. 2012,

[5]Karp and Rezai (2014) use a Lucas tree model to examine how asset markets can create incentives to conserve a generic resource (e.g. a fishery). There, all adjustment occurs via prices, possibly exaggerating the role of asset markets. Our model here, where investment is endogenous, includes the standard Ramsey and the Lucas tree models as limiting cases, neither assuming away nor giving undue prominence to the role of asset markets. The previous paper assumes that agents have infinite elasticity of intertemporal substitution. Our results are sensitive to this parameter, which here takes any positive value. The structure in Karp and Rezai (2014) makes the model unrelated to climate change. Here we adapt and then calibrate to DICE (2013) in order to study climate change.

Williams et al., 2015; and Iverson and Karp, 2017, Karp 2017). Bovenberg and Heijdra (1998) show that the issuance of public debt can support Pareto-improving environmental policy by transferring the cost of policy, across time, to those who benefit from it. We exclude public debt, social security, and other means of intergenerational transfers from the future to the present, precisely because our research question concerns the incentives to abate greenhouse gasses created by asset markets, not the design of optimal climate policy.[6]

We want to use the simplest model capable of assessing the magnitude of these incentives, not the most descriptive model across all dimensions. We therefore ignore several important issues in climate economics. First, motivated by its tractability, we use a deterministic model, thus excluding questions arising from uncertainty about damages and growth (Lemoine and Traeger 2014, Jensen and Traeger 2014, Kelly and Tan 2015, Lontzek et al 2015), and specifically about the "climate beta" (Traeger 2014, Giglio et al. 2015, Lemoine 2017, Dietz et al. 2018). Second, in keeping with the tradition of IAMs, we use a single stock of capital. In fact, there are many asset classes, which will be affected differently depending on their sectoral and geographical location. This consideration is important, but a single-stock model provides the obvious place to begin and also serves as a proxy for a broad index of assets. Third, again in keeping with most IAMs, we ignore the cross-country free-riding problem. Asset markets potentially attenuate intergenerational free riding, but do nothing to reduce cross-country free riding.

2 The Model

This section describes the young agent's savings decision and then discusses the economy-wide production function. A model of adjustment costs with congestion generates a tractable model with a strictly concave production possibility frontier (PPF). Without strict concavity, the asset price is fixed at the price of the numeraire. To emphasize the role of asset markets, we rule out other types of transfers across time, such as social security and public debt. The final subsection discusses the decentralized equilibrium.

[6]Intergenerational transfer schemes may face commitment constraints. Rangel (2003) shows that such transfers can be sustained in a subgame perfect equilibrium with overlapping generations by means of a trigger strategy. We consider Markov Perfect Equilibria, excluding trigger strategies.

2.1 The savings decision

In each period, a cohort of constant size, $l \equiv 1$ is born. Agents live two periods and maximize their lifetime welfare, Ω. Lifetime welfare is the discounted sum of utility, $U(\cdot)$, derived from consumption while young, c^y, and old, c^o: $\Omega_t^y = U(c_t^y) + \rho\, U(c_{t+1}^o)$ with ρ the constant utility discount factor and superscripts y and o denoting the young and old generation. Utility, $U(\cdot)$, satisfies the Inada conditions except in the case of linear utility (Section 3.2). The young agent receives labor income, w_t, but no inheritance, and spends c_t^y on consumption. With a depreciation rate δ, the amount of capital remaining at the end of period t is $(1-\delta)\, k_t$. Newly produced capital, i_t, and undepreciated old capital are equally productive; therefore, in equilibrium they have the same price, p_t. The young agent buys s_t shares of the old capital stock and i_t units of new capital at the cost $p_t\,[s_t(1-\delta)k_t + i_t]$. The rental rate on capital is r_t. When old in period $t+1$, the agent earns the factor payment $r_{t+1}(s_t(1-\delta)k_t + i_t)$, and obtains revenue from selling the end-of-period stock, $p_{t+1}(1-\delta)(s_t(1-\delta)k_t + i_t)$. Agents are selfish, so the old agent consumes all her income.

Agents take prices w_t and p_t as given and have rational point expectations of r_{t+1} and p_{t+1}. The young agent's maximization problem is

$$\max_{i_t,\ s_t,\ c_t^y,\ c_{t+1}^o} \quad U(c_t^y) + \rho\, U(c_{t+1}^o) \text{ subject to}$$

$$\begin{aligned} c_t^y &\le w_t - p_t\,[s_t(1-\delta)k_t + i_t] \\ c_{t+1}^o &\le (r_{t+1} + p_{t+1}(1-\delta))\,(s_t(1-\delta)k_t + i_t)\,. \end{aligned} \tag{1}$$

The optimal decision to buy shares of existing capital, s_t, satisfies

$$\psi_t \equiv \frac{U'(c_t^y)}{\rho\, U'(c_{t+1}^o)} = \frac{(r_{t+1} + p_{t+1}(1-\delta))}{p_t}, \tag{2}$$

which states that in equilibrium the marginal rate of intertemporal substitution equals the marginal rate of transformation. The right side in equation (2) gives the number of consumption units a young agent obtains in the next period by reducing consumption by 1 unit today and investing in capital instead. This ratio equals the marginal rate of intertemporal substitution, ψ_t, which equals 1 plus the endogenous interest rate between period t and $t+1$. In equilibrium, $s_t \equiv 1\ \forall t$, because the old generation has inelastic supply of undepreciated capital. Provided that $i_t > 0$ (as we hereafter assume), the optimality condition for i_t is identical to equation (2).

We rearrange the optimality condition (2) to obtain the asset price equation:

$$p_t = \frac{r_{t+1} + p_{t+1}(1-\delta)}{\psi_t} \text{ for } t < H, \tag{3}$$

where $H < \infty$ is the last period. The young generation in H only lives one period, so it does not accumulate capital, implying the asset price is zero, $p_H = 0$. (Section 4 further discusses this finite horizon assumption.)

2.2 Gross world product

We follow DICE and other IAMs in modeling gross world product (GWP) as a function of capital, k_t, labor, l, (normalized to 1), the stock of atmospheric carbon in excess of pre-industrial levels, e_t, and the fraction of emissions abated (the abatement rate) $\mu_t \in [0,1]$. The endogenously changing stocks, k and e, are the state variables, and the abatement rate is the policy variable. Absent abatement and climate change, GWP equals $F(k_t, l)$, an increasing, concave, constant returns to scale function. CO_2 in excess of pre-industrial levels creates climate change, causing output to equal $D(e)\,F(k_t, l)$, with $D(0) = 1$, $D' < 0$ for $e_t > 0$. Business as Usual (BAU) means that there is no climate regulation, $\mu = 0$. Under BAU firms choose emissions to minimize their production costs, leading to emissions $\zeta F(k, l)$; ζ is the constant BAU carbon intensity of output. Emissions raise future levels of e_t, lowering future productivity of both capital and labor.

Environmental policy obliges firms to abate the fraction $\mu_t \in [0,1]$ of emissions $\zeta F(k_t, l)$ in period t. Abatement reduces GWP by the factor $\Lambda(\mu_t)$. By definition, zero abatement minimizes costs, so $\Lambda(0) = \Lambda'(0) = 0$, with $\Lambda' > 0$ and $\Lambda'' > 0$ for $\mu_t > 0$. Output, y_t, and emissions, z_t, equal

$$y_t = (1 - \Lambda(\mu_t))D(e_t)\,F(k_t, l) \quad \text{and} \quad z_t = (1 - \mu_t)\,\zeta F(k_t, l)\,.$$

All markets are competitive and the representative firm hires labor and capital to equate a factor's wage or rental rate and its marginal product:

$$w_t = \frac{\partial y_t}{\partial l} \quad \text{and} \quad r_t = \frac{\partial y_t}{\partial k_t}. \tag{4}$$

With constant decay rates δ for capital and ϵ for atmospheric carbon, the transition equations for the stock of atmospheric carbon and capital are

$$e_{t+1} = (1-\epsilon)e_t + z_t \text{ and } k_{t+1} = (1-\delta)k_t + i_t, \text{ with } e_0 \text{ and } k_0 \text{ given.} \quad (5)$$

2.3 The production possibility frontier

Adjustment costs can explain firms' sluggish response to exogenous changes or to deviations from long run equilibria. These costs produce a simple model with a state-dependent price of capital in an OLG economy with productive assets (Huberman, 1984; Huffman, 1985, 1986; Labadie, 1986). Reduced form estimates (Shapiro 1986 and Hall 2001, 2004) and structural methods (Mumtaz and Zanetti 2015) establish the empirical importance of adjustment costs. Bond and van Reenen (2007) survey the literature.

Gross output, y_t, can be directly consumed and it can be transformed into the investment good. Investment i requires $A(i)\,i$ units of the composite commodity, so adjustment costs equal $(A(i)-1)\,i$. For $A(i) \equiv 1$, adjustment costs are 0 and the PPF between the consumption and the investment good is a line with slope -1, as in standard IAMs. For $A'(i) > 0$, adjustment costs are positive and the PPF is strictly concave (Figure 1).[7]

The consumption good is the numeraire, so nominal factor prices (equation 4) equal real returns. Moreover, just as in standard IAMs, the *relative* factor price, w/r, does not depend on the carbon stock or abatement. This independence makes it easier to identify the effect of asset prices on incentives to abate.[8]

We assume that firms producing (or transforming the composite good into) capital create congestion in their sector. Each firm takes its marginal production cost as constant at $A_t \equiv A(i_t)$ but this cost increases with aggregate i, unless adjustment costs are 0. The price of new capital equals average instead of marginal aggregate production costs ($A(i_t)$, not $A(i_t) + A'(i_t)\,i_t$),

[7] To further motivate our use of adjustment costs to produce a model with a concave PPF, Referees' appendix B.1 considers two alternatives.

[8] If the current wage and rental rates respond differently to climate policy agents have two incentives, not present in the standard IAMs, to use climate policy: to manipulate both the asset price and the relative factor price. When the relative factor price is independent of climate policy we obtain a model that differs from standard IAMs only by its inclusion of a non-trivial asset market.

creating a static market failure.[9] Old and new capital are equally productive, so their prices are equal when investment is positive:

$$i_t > 0 \Rightarrow p_t = A(i_t). \tag{6}$$

2.4 Equilibrium

Section 4 presents the political economy setting that determines climate policy, the sequence of emissions standards, $\left\{\mu_{t+h}\right\}_{h=0}^{H-t}$. For the time being, we take this policy sequence as given, and define the conditional equilibrium under the assumption of positive investment in every period:

Definition 1 *A competitive equilibrium at t, with initial condition k_t and e_t, conditional on $\left\{\mu_{t+h}\right\}_{h=0}^{H-t}$, is a sequence of the carbon and capital stocks and asset price, $\{e_{t+h}, k_{t+h}, p_{t+h}\}_{h=0}^{H-t}$, satisfying the asset market equilibrium (3) implied by the young agents' savings decision, the factor market conditions (4), the transition equations (5), and the no-arbitrage condition (6).*

We assume that preferences are constant elasticity, with $U(c) = \frac{c^{1-\eta}-1}{1-\eta}$ and $\eta \geq 0$ (so $U(c) = \ln c$ for $\eta = 1$); η is the inverse of the elasticity of intertemporal substitution (EIS).

Lemma 1 *For a given climate policy and $\eta \neq 1$, equilibrium lifetime welfare in period t is Ω_t^y for the young and Ω_t^o for the old agent, with*

$$\Omega_t^y \equiv U(c_t^y) + \rho\, U(c_{t+1}^o) = \frac{(c_t^y)^{-\eta}}{1-\eta} w_t - \frac{1}{1-\eta}(1-\rho) \tag{7}$$

and

$$\Omega_t^o \equiv U(c_t^o) = \frac{\left[(r_t + (1-\delta)p_t)k_t\right]^{1-\eta} - 1}{1-\eta}. \tag{8}$$

The Appendix contains proofs; Section 3.1 discusses the case $\eta = 1$.

[9] An adjustment cost model in which the price of the investment good equals marginal costs requires a sector-specific factor in the investment sector, and accompanying rent. The additional factor of production and its asset price make the model analytically intractable. The congestion assumption, implying that price equals average cost, is widespread in economics; for example, it is the basis for much of the literature on endogenous growth.

3 Climate Policy, Asset Price and Welfare

Climate policy diverts resources from consumption or savings to mitigation, imposing a cost on those who implement the policy. In an ILRA economy, today's climate investment benefits agents in the future. In an OLG economy with linear PPF and no bequest motive, the current old generation has no reason to abate. The current young generation benefits from lower climate-related damages when they are old. With concave PPF, the endogenous asset price has the potential to transfer some future benefits to the current period, creating an incentive for climate policy.

This section provides intuition for the interaction between climate policy, asset prices, and equilibrium welfare for a small level of current abatement ($\mu > 0$, $\mu \approx 0$). We approximate the welfare effect in the usual manner, using the first-order term of the Taylor expansion of welfare around $\mu = 0$.[10] Section 4 endogenizes the abatement decision, recognizing that future policy responds to current policy via changes in the state variable; here we treat future abatement levels as fixed. Because abatement costs are minimized at zero abatement ($\Lambda'(0) = 0$), the first unit of abatement has a zero first-order effect on current output and factor returns:

$$\left.\frac{\partial y_t}{d\mu_t}\right|_{\mu=0} = \left.\frac{\partial w_t}{d\mu_t}\right|_{\mu=0} = \left.\frac{\partial r_t}{d\mu_t}\right|_{\mu=0} = 0.$$

The second-order effect of abatement on output and factor returns is negative.

The old generation's consumption equals its income from renting capital and from selling undepreciated capital. Although the policy perturbation has zero first-order effect on rental income, it can change the old generation's welfare by altering the asset price:

Proposition 1 *For $\delta < 1$ and predetermined stocks of atmospheric carbon and capital and fixed future abatement levels, a small level of current abatement ($\mu > 0$, $\mu \approx 0$) increases the old generation's welfare if and only if this policy raises the asset price:*

$$\left.\frac{d\Omega_t^o}{d\mu}\right|_{\mu=0} > 0 \Leftrightarrow \left.\frac{dp_t}{d\mu}\right|_{\mu=0} > 0.$$

[10] Our results do not change if, instead of considering the perturbation of only current policy, we consider the perturbation of a sequence of abatement levels. In that case, we begin with $\bar{\boldsymbol{\mu}} \in \mathbb{R}^{H-t}$, $\bar{\mu}_h \geq 0$ and denote the sequence of climate policy as $\varepsilon\bar{\boldsymbol{\mu}}$. The comparative statics is then with respect to ε, evaluated at $\varepsilon = 0$.

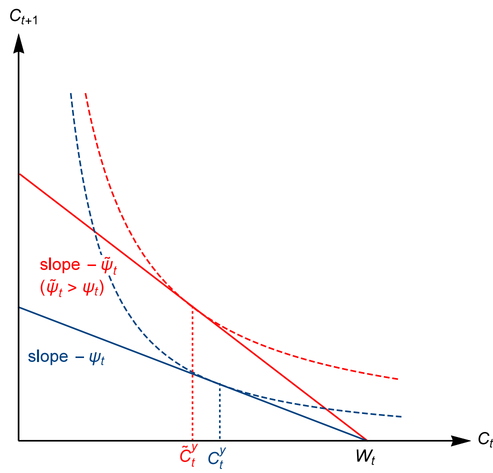

Figure 2: A higher asset price that also increases the young agent's welfare must rotate this agent's budget constraint clockwise. Market clearing implies that this change must reduce c_t^y. A fall in c_t^y following a steeper (outwardly rotated) budget constraint occurs if and only if the substitution effect is greater than the income effect, which (for constant EIS) requires $\eta < 1$.

For $\delta = 1$, where the old generation has no remaining capital at the end of the period, and therefore sells nothing, climate policy has zero first-order effect on the old generation's welfare.

The next proposition states that a policy that increases the asset price benefits the young agent if and only if $\eta \in (0, 1)$. To provide intuition for this result, suppose that a policy increases p_t and also increases the young agent's welfare. The young's current consumption is $c_t^y = w_t - p_t\ k_{t+1}$ and next period consumption is $c_{t+1}^o = \psi_t\ p_t\ k_{t+1}$, so the budget constraint is $c_{t+1}^o = \psi_t\,(w_t - c_t^y)$, shown as solid lines in figure 2. The hypothesis that the young benefit from the policy-induced increase in p_t implies that the budget constraint rotates clockwise, increasing ψ_t to $\widetilde{\psi}_t$.[11]

The increase to $\widetilde{\psi}_t$ creates an income effect that encourages higher consumption when young, because lifetime wealth increases, and a substitution effect that encourages lower consumption when young, because the return

[11]The policy has only a second-order effect on w_t. If the policy causes ψ_t to fall, the budget constraint pivots counter-clockwise around the c_t^y intercept (equal to w_t), necessarily lowering the young agent's welfare, contrary to our hypothesis.

on savings increases. However, the small abatement policy creates only a second order reduction in aggregate output at t, while it leads to a first-order increase in the old agent's consumption (by Proposition 1), and also increases equilibrium investment. Market clearing requires that the policy lowers the young agent's consumption (from c_t^y to $\tilde{c}_t^y$). Consequently, for welfare to rise, the substitution effect must dominate the income effect. For iso-elastic utility, the substitution effect dominates the income effect if and only if $\eta < 1$.

Proposition 2 *Assume that investment is positive and that the consumption-investment PPF is strictly concave.*

(i) If $\eta \in (0,1)$, a small level of abatement increases welfare of the current young if and only if the policy causes the asset price to rise:

$$\left.\frac{d\Omega_t^y}{d\mu}\right|_{\mu=0} > 0 \Leftrightarrow \left.\frac{dp_t}{d\mu}\right|_{\mu=0} > 0.$$

(ii) If $\eta > 1$, a small level of abatement increases welfare of the current young if and only if the policy causes the asset price to fall:

$$\left.\frac{d\Omega_t^y}{d\mu}\right|_{\mu=0} > 0 \Leftrightarrow \left.\frac{dp_t}{d\mu}\right|_{\mu=0} < 0.$$

(iii) Climate policy has the same qualitative effect on welfare of the two generations alive in the first period if $\eta \in (0,1)$; climate policy has opposite effects on their welfare if $\eta > 1$.

Today's equilibrium asset price depends on future prices, making it difficult to sign $\frac{dp_t}{d\mu}$. However, for logarithmic utility ($\eta = 1$) and for a linear model with $\eta = 0$, a small increase in period-t abatement strictly increases the welfare of one of the two agents alive in that period, has a 0 first-order effect on the other agent's welfare, and strictly increases welfare for the agent born in the next period. Here, the asset market gives agents an incentive to impose climate policy, despite their lack of altruism.

3.1 Logarithmic utility

If utility is logarithmic ($\eta = 1$), income and substitution effects cancel: Equilibrium saving is a constant fraction of income and the asset price is an increasing function of the wage.

Lemma 2 *With $\eta = 1$,*

$$(i) \;\; p_t k_{t+1} = \frac{\rho}{1+\rho} w_t \text{ and } (ii) \;\; p_t = f(w_t) \text{ with } f' > 0.$$

Lemma 2.i is a standard result: with logarithmic utility, an agent saves a constant fraction of her income. An increase in the wage shifts out demand for savings, increasing the price of capital due to equation 6.

Because climate policy has a zero first-order effect on the current wage, Lemma 2.ii implies that policy has zero first-order effect on the current asset price, and a zero first-order effect on the old agent's welfare. The policy has a zero first-order effect on the young agent's saving, but it creates a first-order reduction in next-period pollution stock, raising the next-period wage and asset price.[12] The higher asset price in $t = 1$ increases consumption of the old agent in period 1, thus increasing lifetime welfare of the agent who is young at $t = 0$. The higher wage in $t = 1$ also increases consumption and welfare of the young agent in $t = 1$, because of the constant saving rule. The rule also implies higher saving for the richer young agent at $t = 1$.

Proposition 3 *With $\eta = 1$, a small level of abatement: (i) has a zero first-order effect on the welfare of the old agent in period $t = 0$, (ii) increases welfare of the agent born in $t = 0$, and (iii) increases welfare of the agent born in the next period, $t = 1$.*

Agents born in subsequent periods inherit different stocks of both capital and pollution (relative to the zero-abatement equilibrium). Without imposing further structure, we cannot determine the changes in their welfare.

3.2 Linear utility

If utility is linear ($\eta = 0$), the interest rate equals the pure rate of time preference ($\psi_t - 1 = \frac{1}{\rho} - 1$). The young generation's lifetime welfare is

$$\Omega_t^y|_{\eta=0} = w_t - 1 + \rho, \tag{9}$$

by equation (7). Here, the young's welfare does not depend on the asset price. A small level of abatement has only a second-order effect on the same-period

[12] The statement that there is a first-order change in p_1 and only a second-order change in p_0 is consistent with equation 3. The change in ψ_0 offsets the change in p_1.

wage, and therefore creates a zero first-order effect on welfare for the young agent. (A non-negligible level of abatement lowers the wage, lowering the young agent's welfare. Section 4.4 returns to this issue.) If policy increases the asset price, it raises the old agent's welfare (by Proposition 1). With a higher asset price and associated increase in investment, the young agent born at $t = 1$ inherits a larger stock of capital and a smaller carbon stock. Both of these changes increase the wage at $t = 1$, benefitting the agent born in that period. The higher k_{t+1} increases emissions in $t + 1$, so the effect of the perturbation on agents' welfare at $t \geq 2$ is ambiguous.

The above chain of reasoning assumes that the perturbation increases the current asset price. To illustrate a situation where this assumption is correct, we adopt the following linear structure:

Assumption 1 *(Linearity) (i) Utility, the production technology, and average adjustment costs are linear:* $U(c_t) = c_t$, $F(k_t, l) = \alpha\, k_t + \varpi\, l$, *and* $A(i) = \phi i/2$ *with* $\alpha, \varpi, \theta, \phi > 0$. *Output is* $(1 - \Lambda(\mu_t))\,((\alpha - \xi e_t)\, k_t + (\varpi - \vartheta e_t)\, l)$. *(ii) Future policy is constant (*$\mu_t = \bar{\mu}$*, for* $t \geq 1$*). (iii) The stocks of capital and atmospheric carbon remain finite as* $H \to \infty$.

Assumption 1.i allows us to express the equilibrium asset price as a linear function of the state variables, where μ_0 enters parametrically. Assumption 1.ii (with $H = \infty$) makes the solution stationary, and Assumption 1.iii provides a transversality condition as $H \to \infty$.

Proposition 4 *Under Assumption 1, a small level of abatement: (i) has a zero first-order effect on the current (*$t = 0$*) young agent's welfare, (ii) increases the current asset price and (for* $\delta < 1$*) increases the current old agent's welfare, and (iii) increases the next-period young agent's welfare.*

4 Political Economy Equilibria

To move beyond the perturbation analysis we treat abatement as endogenous, using two benchmarks and two experiments. There is zero abatement in the BAU benchmark. In the second benchmark, a discounted utilitarian chooses a sequence of abatement levels to maximize the present discounted stream of utility from aggregate consumption. In both of the two experiments, abatement is the outcome of a simple political economy model imbedded in a dynamic game. By comparing abatement across the two experiments we can assess the importance of *recognizing* the effect of abatement on asset prices. By comparing abatement in the second experiment and under the discounted utilitarian we can assess the extent to which selfish agents who recognize the link between asset prices and climate policy act as if they care about future generations.

A probabilistic voting model underpins the political economy component (Lindbeck and Weibull, 1987; Persson and Tabellini, 2000). There are no transfers across generations. Voters care about their consumption-related welfare and about ideology. Political parties propose climate policies to maximize the probability of their election. The parties can attract swing voters (those more interested in consumption than ideology) by proposing an abatement level that increases those voter's consumption-related welfare. In equilibrium, parties support the same climate policy, the abatement level that maximizes a convex combination of young and old agents' consumption-related welfare, $\xi\Omega_t^y + (1-\xi)\Omega_t^o$, with $0 \leq \xi \leq 1$. The weight on each generation increases with the number of swing voters in that generation. Hereafter, we refer to the agent who maximizes the joint welfare function, thus implementing the political economy equilibrium, as the *planner* (as distinct from the *discounted utilitarian*).

In choosing current abatement, currently living agents in the two experiments are influenced by their expectations of future policy. Young and old generations in each period are part of a sequence of pairs of generations in a dynamic game; policy is a Markov perfect equilibrium (MPE) to this game. Hassler et al. (2003) and Conde-Ruiz and Galaso (2005) use this type of political economy model to determine intergenerational redistribution and the provision of a public good. In our model, the public good is Earth's capacity to absorb carbon emissions.

To identify the role of asset markets as clearly as possible, we ignore growth in technology or population, and we use a single climate state variable. Due to these simplifications, the equilibrium abatement levels are not prescriptive even for the discounted utilitarian. We are primarily interested in the differences, across settings, of abatement levels, rather than their level in a particular setting. To obtain a stationary solution while avoiding the multiplicity arising from an incomplete transversality condition, we study the limit equilibrium (as $H \to \infty$).[13]

[13] The use of a limit equilibrium does not guarantee uniqueness; for example there might be multiple solutions to the dynamic programming equations that determine the equilib-

4.1 The political economy setting

Selfish agents might abate in order to reduce climate damage that the current young suffer when they become old, and/or to alter the asset price. Absent both these incentives, price-taking selfish agents do not abate (the BAU benchmark). The two experiments isolate the selfish agents' two possible incentives to abate. The first experiment assumes "unsophisticated" agents, who take the asset price as given. However, they recognize that current abatement lowers current factor returns and increases next-period factor returns by reducing climate-related damages. The second experiment assumes "sophisticated" agents who additionally understand that current climate policy alters the current asset price via the change in the future returns to capital induced by the change in the carbon stock. The asset market exists in both scenarios, but only the sophisticated agents recognize it. By comparing equilibrium abatement across scenarios, we assess the importance (on the incentives to reduce emissions) of agents' *recognition* of the link between climate policy and asset prices.

We assume that in choosing climate policy, the political parties (and the planner who implements their platform) take the level of investment as given; climate policy is not used to influence investment:

Assumption 2 *(Nash) Planners take the current investment decision as given in choosing current abatement. Agents take prices and the current abatement policy as given in choosing investment.*

The planner in the probabilistic voting model maximizes the convex combination of currently living agents' welfare. Current and future abatement alter the trajectory of atmospheric carbon, thus affecting the current asset price and current welfare. Current and future planners play a sequential game; we consider a stationary Markov Perfect Equilibrium (MPE) to this game. The payoff-relevant state variable is the pair (k_t, e_t). The investment and abatement decisions jointly determine current consumption, utility levels, and the next period state variable, (k_{t+1}, e_{t+1}). A MPE is a mapping from the state variable in period t to the planner's abatement policy and young agents' saving policy in that period. Denote the abatement policy function as $\mu_t = M(k_t, e_t)$. The Markov-perfect condition to the game across periods requires that given agents' belief that subsequent generations will follow the

rium. No evidence of multiplicity arises in our numerical analysis.

equilibrium decision rule, $\mu_{t+j} = M(k_{t+j}, e_{t+j})$ for $j > 0$, the political economy equilibrium that determines the current abatement is $\mu_t = M(k_t, e_t)$.

Given a policy function $\mu_t = M(k_t, e_t)$, the current wage and rental rates are functions of the state variables, $W(k_t, e_t)$ and $R(k_t, e_t)$. Asset owners receive rent and revenue from asset sales. Via the dependence of factor prices on $M(\cdot)$, the equilibrium asset price, $p_t = \Psi(k_t, e_t)$, is a functional in $M(\cdot)$ satisfying equation (3). The political economy equilibrium in period t maximizes the convex combination of the lifetime welfare of agents at t:

$$\begin{gathered}\max_{\mu_t} \xi\ \Omega_t^y + (1-\xi)\ \Omega_t^o = \\ \max_{\mu_t} \frac{\xi\left([w_t - p_t k_{t+1}]^{1-\eta} + \rho[(r_{t+1} + (1-\delta)p_{t+1})k_{t+1}]^{1-\eta}\right) + (1-\xi)[(r_t + (1-\delta)p_t)k_t]^{1-\eta}}{1-\eta}\end{gathered} \tag{10}$$

subject to equations (5) and (1). For $\eta > 0$ and $0 < \xi < 1$, the choice of climate policy that maximizes $\xi\ \Omega_t^y + (1-\xi)\ \Omega_t^o$ involves a "redistribution motive" (Section 4.4).

Unsophisticated planners take asset prices p_t and p_{t+1}, as given, but understand that abatement affects factor returns w_t, r_t, r_{t+1}. Climate policy reduces current factor returns but increases the next-period rental rate. Sophisticated planners additionally understand that the policy-induced change in e_{t+1} alters the next-period asset price, $p_{t+1} = \Psi(k_{t+1}, e_{t+1})$, thereby changing the current price via equation (3). Both types of planner take the level of investment as given (the Nash Assumption 2). We close the model using the no-arbitrage condition, equation (6), so the investment rule, for $i > 0$, is

$$i_t = A^{-1}(\Psi(k_t, e_t)). \tag{11}$$

Substituting the primitives of the model, $F(k, l)$, $D(e)$, and $A(i)$, and the equilibrium factor prices, $W(k_t, e_t)$ and $R(k_t, e_t)$, into equation (10) produces the planner's problem. We obtain numerical solutions to the equilibrium problems in both experiments using the collocation method and Chebyshev polynomials (Judd, 1998; Miranda and Fackler, 2002).

Example: linear utility The case of linear utility shows how equilibrium climate policy changes with a change in agents' political influence (ξ) or their sophistication. The left side of equation 12 decomposes national income

+ wealth into the old and young agents' welfare (their lifetime consumption).

$$\begin{aligned}&\underbrace{(r_t+(1-\delta)p_t)k_t}_{\text{old agent's welfare}}+\underbrace{w_t-(p_t-\rho\,(r_{t+1}+(1-\delta)p_{t+1}))\,k_{t+1}}_{\text{young agent's welfare}}\\&\quad=\underbrace{r_t+w_t}_{\text{income}}+\underbrace{(1-\delta)p_t)k_t}_{\text{wealth}}\end{aligned}\tag{12}$$

An old agent who chooses policy ($\xi = 0$) maximizes the first under-bracketed expression. If abatement increases the asset price, the sophisticated old agent chooses a positive level of abatement. The unsophisticated old agent (who takes the asset price as fixed), chooses zero abatement, because climate policy lowers the current return to capital.

A young agent who chooses policy ($\xi = 1$) maximizes the second under-bracketed term. The sophisticated young agent understands that the asset price equation 3 (and the fact that here $\psi_t = \rho^{-1}$) implies that the term multiplying k_{t+1} equals zero. She wants to maximize the current wage and therefore chooses zero climate policy. The unsophisticated young agent takes asset prices as fixed, and wants to maximize $w_t + \rho r_{t+1} k_{t+1}$. She chooses a positive level of abatement to protect next period's return on capital.

Intuition suggests that the young agent, who experiences climate change late in life, is more willing than the old to protect the climate. However, the sophisticated young agent is not willing to reduce her wage because she understands that the current asset price changes to incorporate any increase in next-period returns. The unsophisticated young agent might want to use climate policy with the expectation of higher future returns in the next period. However (in our model) that benefit to the young agent is illusory; the old agent captures it via a change in the asset price.

4.2 The discounted utilitarian

The discounted utilitarian (DU) chooses investment and abatement to maximize the discounted sum of the welfare of aggregate consumption, using agents' pure rate of time preference: $\sum_{s=0}^{\infty} \rho^s U\left(c_t^y + c_t^o\right)$, with $c_t^y + c_t^0 = y_t - A\left(i_t\right) i_t$.[14] The dynamic programming equation is

[14] The consumption constraint, $c_t^y + c_t^0 = y_t - A\left(i_t\right) i_t$, means that the asset market does not affect the DU's optimization problem. However, we use the asset market equation (3) and the expressions for factor returns and welfare, equations (4), (7) and (8), to calculate

$$J(k_t, e_t) = \max_{i_t,\mu_t}\left[U\left(c_t^y + c_t^0\right) + \rho J(k_{t+1}, e_{t+1})\right] \tag{13}$$

subject to transition equations (5). Just as in the standard ILRA IAM, the resulting decision rules, $\mu_t = M(k_t, e_t)$ and $i_t = I(k_t, e_t)$, are first-best. The only difference here is that the PPF is concave.

4.3 Functional forms and calibration

Assumption 3 *(Functional forms) Production and abatement functions are iso-elastic:* $F(k,l) = \alpha l^\beta k^{1-\beta}$, $\Lambda(\mu) = \theta\,\mu^\nu$, *with* $0 < \beta < 1$, $\theta > 0$, $\nu > 1$. *Aggregate investment costs are* $A(i)i = i + \frac{\phi}{2}i^2$ *with* $\phi > 0$, *and climate damages are* $D(e_t) = (1 + \iota\; e_t^2)^{-1}$ *with* $\iota > 0$.

Labor obtains the constant output share β; θ is the fractional reduction in output under zero emissions ($\mu = 1$); ν is the elasticity of abatement costs.

$\rho = 0.7$ discount factor	$\eta = 2$ inverse EIS	$\beta = 0.6$ labor share	$\delta = 0.88$ capital depreciation
$\zeta = 0.062$ carbon intensity	$\epsilon = 0.126$ carbon decay rate	$\iota = 4 \times 10^{-7}$ damage parameter	$\phi = 0.0003$ adjustment cost parameter
$\alpha = 264$ TFP	$\xi = 0.5$ weight on young welfare	$\theta = 0.056$, abatement cost share	$\nu = 2.8$ abatement cost elasticity

Table 2: Parameter names and baseline values

We use DICE-07 (Nordhaus, 2008) for calibration, making the model comparable to familiar IAMs. Our baseline uses moderate rates of capital depreciation, adjustment costs, and damages, somewhat expensive (relative to DICE-07) abatement, and it places equal weight on currently living generations' welfare. Table 2 collects parameter names and baseline values. Agents

the generations' equilibrium welfare in Table 3 below. We experimented with an alternative specification for the DU's objective, replacing the utility of aggregate consumption with the sum of agents' utility. That alternative introduces a "redistribution motive" not commonly found in IAMS. It does not qualitatively change our results.

live for 70 years, so one period lasts 35 years. Our baseline elasticity of intertemporal substitution is 0.5, so $\eta = 2$, a conventional choice for IAMs. Agents discount future utility at $1\%/yr$, implying $\rho = 0.7$.

We scale nominal units by 10^9 2010 USD ($T). Year 2010 capital stock, K_0, is roughly 200 $\$T$ (Rezai et al., 2012). Yearly world output is roughly 63 $\$T$, so output during the first 35-year period is $y_0 = 35 \times 63\ \$T \cong 2200\ \$T$ (CIA, 2010). Given the initial endowments of capital and labor (normalized to 1), y_0, and $\beta = 0.6$, total factor productivity is calibrated to $\alpha = 264$. We set capital depreciation to $6\%/yr$, above the mean of $4\%/yr$ for 2010 of the Penn World Table and below the $10\%/yr$ used by Nordhaus (2008); this implies $\delta = 0.88$. The sensitivity analysis uses $4\%/yr$ depreciation.

We measure the carbon stock, e, in parts per million by volume (ppmv). With the pre-industrial level of 280 and the current level about 400, the initial value is $e_0 = 120$. Annual 2010 emissions were 8.6 $Gt\ C$ (BP Statistical Review of World Energy, 2017), corresponding to an annual increase in atmospheric CO_2 of 3.92 $ppmv$. With yearly world output of 63 $\$T$, this implies a carbon dioxide emission intensity $\zeta = \frac{3.92}{63} \approx 0.062\ \frac{ppmv}{\$T}$. The actual increase in atmospheric CO2 concentration in 2010 was 2.42 $ppmv$ (NOAA, 2017), implying dissipation was 1.5 $ppmv$. The corresponding depreciation factor equals $\frac{1.5}{390} = 0.0038\ \%/yr$, implying $\epsilon = 1 - (1 - 0.0038)^{35} = 0.126$. This number is close to the mean of the (0.0025%/yr, 0.0055%/yr) range of the implied dissipation rates of carbon in DICE-07 (Rezai, 2010).

The DICE-07 abatement cost elasticity is $\nu = 2.8$. The parameter θ measures the share of GDP necessary to abate all emissions ($\Lambda(1) = \theta\ 1^{\nu} = \theta$). In DICE-07, it costs 5.4% to abate all emissions today, 0.9% in 30 decades and 0.4% in 60 decades. We set $\theta = 0.054$, a constant, to obtain a stationary model. Consequently, future abatement is more expensive in our model than in DICE, tending to raise current abatement and reduce future abatement (flattening the "policy ramp").

Damages depend on the single climate state, atmospheric carbon stock, e, with $D(e_t) = (1 + \iota\ e_t^2)^{-1}$; emissions in one period cause damages in the next period.[15] As in DICE, we assume that doubling the carbon stock relative to

[15] Ricke and Caldeira (2014) estimate that most of the warming effect of current emissions, and thus most of the temperature-related damage, occurs within a decade. Allen et al. (2009) estimate that the maximum warming effect occurs after many decades. Economic IAMs also disagree about the lag between emissions and damage response. In Nordhaus' (2008) DICE, the maximum temperature response occurs after about 60 years. In Golosov et al (2014), the maximum damage response occurs within a decade. With a 35

preindustrial levels reduces national income by 3%, implying $\iota = 4 \times 10^{-7}$.[16] We also consider a high damage scenario, where doubling the carbon stock lowers national income by 6% (implying $\iota = 1 \times 10^{-6}$). This change increases damages and its sensitivity to the stock.

Aggregate investment cost, inclusive of adjustment costs, in our framework is $A(i)i = i + \frac{\phi}{2}i^2$; $\phi \geq 0$ determines the concavity of the production possibility frontier between the consumption and the investment goods. With a 20% investment rate, abatement cost as a percent of output equals $2\phi y = 4400\phi$ ($\$T$) for $y = 2200$ $\$T$. Hall's (2004) estimates imply negligible adjustment costs. Shapiro's (1986) estimates of adjustment costs are about 3% of output. Mumtaz and Zanetti's (2015) estimate adjustment costs at slightly over 3% of output (Appendix B.4). If adjustment costs are zero, the PPF is linear, and asset markets create no incentives to protect the climate. An increase in adjustment costs increases the concavity of the PPF, making it more likely that asset markets are important to incentives. Given the uncertainty about the actual magnitude of adjustment costs, we conservatively, set $\phi = 0.0003$, so that when investment equals 20% of output, adjustment costs equal 1.3% of output (about half way between Hall's estimates and Shapiro's low estimates). Our robustness check doubles our baseline value of ϕ, bringing our estimate of adjustment costs (as a percent of income) close to Shapiro's low estimate.

4.4 Application

Agents in the baseline political economy equilibria reduce emissions, despite their indifference to future generations' welfare. The sophisticated planner abates at approximately twice the level of the unsophisticated planner, and half the level of the DU. This result suggests that the recognition that asset prices are endogenous can significantly increase abatement.

The unsophisticated planner, who takes into account only current factor returns and next-period return to capital, initially reduces emissions by

---

year time step, and current emissions affecting next period damages, our model represents a middle ground.

[16] We can compare the damage functions in our model and in DICE-07, using the equilibrium relation between temperature, τ, and stocks, e: $\tau = 3 \ \log[e/280]/\log[2]$. Our damage function is lower than in DICE for temperatures below $\tau < 2°C$ but rises more steeply beyond this point. The two damage functions differ by less than one percent for temperatures below $3.3°C$.

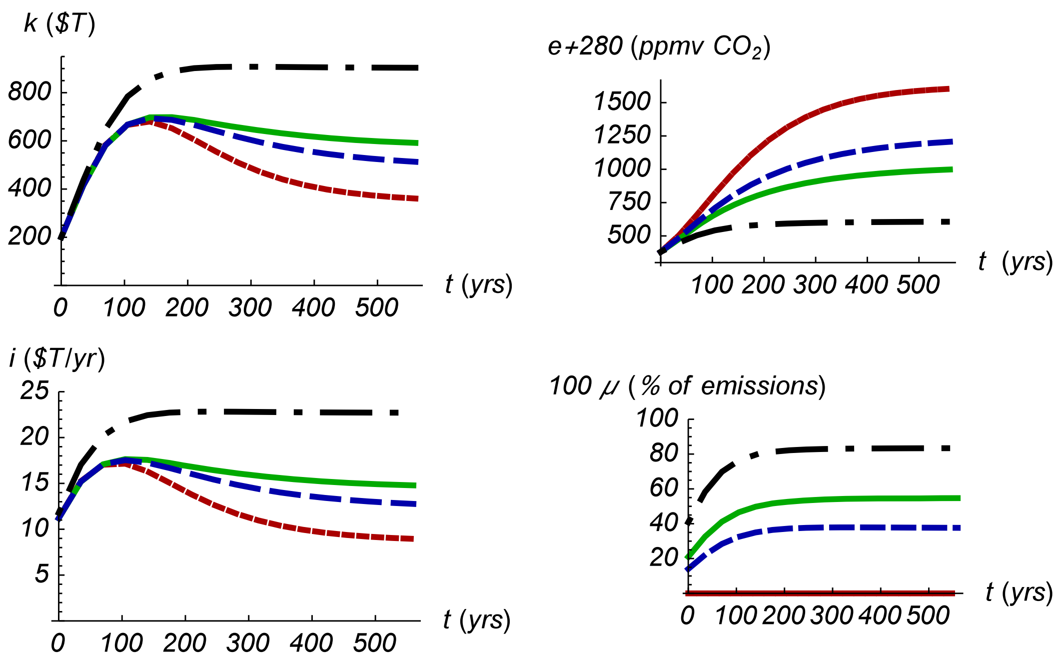

Figure 3: Equilibrium trajectories under: BAU (red dotted); unsophisticated political economy (blue dashed); sophisticated political economy (green solid); discounted utilitarian (black dot-dash). Abatement increases as planners internalize damages occurring during their lifetime (blue dashed), recognize asset price effect (green solid), or take into account future generations' welfare (black dot-dash).

14%, eventually increasing abatement to 40%. The sophisticated planner, who additionally takes into account the endogenous asset price, initially reduces emissions by 21%, increasing to nearly 55% after centuries. The DU initially reduces emissions by 40%, increasing to 85%. Reduced emissions lead eventually to lower trajectories of atmospheric carbon, lower damages, and higher levels of capital stock and investment. Figure 3 shows the trajectories of capital stock, atmospheric carbon, investment and abatement for the unsophisticated and sophisticated political equilibrium, the DU, and under BAU (zero abatement).

Although investment levels diverge across the scenarios after a century, they are nearly the same early in the program. The asset price and investment level are linearly related, so in the short run the equilibrium asset price is nearly the same across scenarios. However, the comparison between the unsophisticated and the sophisticated political economies shows that the incentive to alter the asset price increases abatement even in the short run. The fact that the incentive to alter asset prices changes abatement without changing short run asset prices might seem paradoxical. The explanation is that future generations also abate emissions. In the short run, the lower factor returns resulting from the sophisticated planner's higher abatement approximately offset the increase in factor returns due to the (slightly) lower climate stock. (In a tug of war, increasing the prize money causes both sides to work harder, but may not affect the outcome.) This result may be important for empirical work: the lack of a statistically significant relation between climate variables and the price of a broad portfolio of assets does not imply that asset markets are unlinked from climate policy.

The capital stock initially increases in all scenarios. Under the unsophisticated planner (blue, dashed), the low abatement leads to high carbon stocks and high climate damage, reducing both income and the return to capital, lowering equilibrium investment. Eventually the stock of capital, and thus emissions, fall, slowing the growth in the carbon stock. Carbon levels stabilize at 1245 ppmv, leading to an equilibrium temperature increase of $6.4°C$ and damages of 27% of output. The eventual fall in capital stocks and rise in carbon stocks is more extreme under zero-abatement BAU (red dotted).

As noted, in the short run the trajectories of capital and investment are similar in the three scenarios with abatement, but they later diverge, lagging the divergence in the carbon trajectories. Lower levels of emissions and atmospheric carbon support a higher equilibrium capital stock and output. Under the DU, carbon levels stabilize at 610 ppmv, creating an equilibrium temperature increase of $3.4°C$ and a 4% loss of output. This temperature increase and loss in output is significantly higher than in DICE, because our model has no exogenous decrease in abatement costs or carbon intensity. Our higher abatement costs and emissions imply higher steady state carbon stocks and damages.

The DU and the unsophisticated political economy trajectories sandwich the trajectories under the sophisticated political economy (green, solid). Equilibrium abatement there is eventually about 50% greater than in the unsophisticated scenario, and 65% of the DU level. Carbon concentration in the sophisticated equilibrium stabilizes at 1025 ppmv and temperature increases to $5.6°C$, resulting in climate damages of about 18% of output.

Table 3 presents the current and future percent welfare changes under the sophisticated equilibrium and the DU, relative to the unsophisticated equilibrium. In the political economy setting, where generations care only for their own welfare, the recognition of asset markets increases abatement (relative to the unsophisticated scenario) but initially has negligible welfare effect.[17] The DU imposes tighter emission standards, leading to the highest welfare after a century, but causing welfare losses for generations alive today and the generation born in the next period. Only generations born in 2115 or later are best off under the DU, because of the higher capital stock and lower carbon stock there.

	sophisticated political economy	discounted utilitarian planner
$\Delta\Omega_t^O$	0.%	- 0.4%
$\Delta\Omega_t^y$	0.%	- 1.7%
$\Delta\Omega_{t+1}^y$	0.1%	- 1.5%
$\Delta\Omega_{t+2}^y$	0.7%	0.6%
$\Delta\Omega_{t+3}^y$	1.8%	4.1%

Table 3. Percent change in equilibrium life-time welfare ($\Delta\Omega$) of current generations and future young generations, relative to their welfare in the unsophisticated equilibrium.

Robustness and distribution Table 4 reports initial abatement levels for different parameters; results in bold correspond to our baseline. The first set of columns corresponds to abatement for $\eta = 2$ and $\eta = 0$ and three values of ξ, the welfare weight on the young in the political economy setting. This experiment sheds light on the inter-generational distributional effects of policy and on the role of η. The next column shows abatement for $\xi = 0.5$, but higher adjustment costs and lower capital depreciation; the final column doubles the assumed damages under a doubling of carbon stocks.

For $\eta = 2$, current abatement tends to decrease the asset price. For our baseline calibration, the first-period young have higher marginal utility of income than the first-period old. Therefore, reducing the asset price increases the aggregate payoff for $\xi = 0.5$. This redistributive effect accounts

[17] In the baseline sophisticated equilibrium, the generations alive at t have a small welfare loss, rounded to zero. The move from the unsophisticated to the sophisticated scenario changes future as well as current actions, and the future actions affect current welfare via the asset price.

for the increase in abatement when moving from the unsophisticated to the sophisticated equilibrium. The experiments with $\xi = 1$ and $\xi = 0$ reinforce this conclusion. The young agent wants to increase abatement, while the old agent prefers zero abatement. For $\eta = 2$, abatement leads to a small reduction in the asset price, lowering the old agent's welfare.

For $\eta = 0$, current abatement increases the asset price (as in Proposition 4) and both agents have the same constant marginal utility. The discussions below equations 9 and 12 note that for $\eta = 0$, a non-negligible level of abatement harms the young, whose utility equals their wage. Therefore, the young agents who choose policy and recognize the endogeneity of asset prices prefer zero abatement. If instead they treat the asset price as fixed, they prefer substantial abatement, in order to increase the next-period return on capital. These incentives are reversed for the old agent. Taking the asset price as fixed, this agent thinks that abatement only lowers current returns, and therefore prefers zero abatement. The old agent who understands that asset prices are endogenous wants modest abatement, to raise the asset price.

		baseline			high adj. & low dep.	high dam.
		$\xi = 0.5$	$\xi = 1$	$\xi = 0$	$\xi = 0.5$	$\xi = 0.5$
$\eta = 2$	soph	**21%**	37%	0%	20%	51%
	unsoph	**14%**	18%	0%	14%	21%
	DU	**41%**	N/A	N/A	41%	53%
		$\xi = 0.5$	$\xi = 1$	$\xi = 0$	$\xi = 0.5$	$\xi = 0.5$
$\eta = 0$	soph	3%	0%	5%	5%	4%
	unsoph	24%	32%	0%	24%	34%
	DU	64%	N/A	N/A	63%	78%

Table 4: Robustness. Percentage first period abatement for "soph." and "unsoph." (the sophisticated and the unsophisticated political economy), and the DU (discounted utilitarian) with: (i) equal welfare weights ($\xi = 0.5$); (ii) the young generation chooses policy ($\xi = 1$); and (iii) the old generation chooses policy ($\xi = 0$). "N/A" means "not applicable". The baseline uses values from Table 3. The "high adjustment cost and low depreciation" scenario use baseline values except for $\phi = 0.0006$, $\delta = 0.76$. The "high damage" scenario uses baseline values except for $\iota = 1 \times 10^{-6}$.

The move from $\eta = 2$ to $\eta = 0$ leads to a large increase in abatement for the DU, a familiar result from the Ramsey formula for the social discount rate. With positive consumption growth, the consumption discount rate increases with η. For low η, the DU is willing to incur higher contemporaneous sacrifices to increase future consumption. The same force operates in the unsophisticated political equilibrium. With high elasticity of intertemporal substitution, and taking the asset price as exogenous, the young agent is willing to make current sacrifices to obtain a cleaner environment in the future. Therefore, if $\xi = 0.5$ or $\xi = 1$, equilibrium abatement is higher in the political economy (with unsophisticated agents) under $\eta = 0$, compared to $\eta = 2$. As above, the old agent who takes the asset price as exogenous has no interest in a cleaner future environment, so abatement remains at zero if $\xi = 0$. Moving to the sophisticated equilibrium reverses these incentives, because abatement increases the asset price. The old agent therefore favors abatement and the young agent opposes it.

Table 4 shows that the recognition of endogenous asset prices can significantly change the incentives to abate, altering equilibrium abatement. The direction of these incentives is sensitive to the intertemporal elasticity of substitution and to the distributional weights on the two generations. However, a change in parameters or in sophistication that leads to a large change in abatement causes negligible change in short run price of assets, for reasons explained above.

The third column of Table 4 reports results for $\xi = 0.5$ with higher adjustment costs ($\phi = 0.0006$ instead of 0.0003) and a 4% annual depreciation rate for capital (instead of the baseline of 6%). The higher abatement cost increases the concavity of the PPF, making the asset price more sensitive to the level of investment. The slower depreciation rate means that the old agent has more capital at the end of a period, increasing the distributional importance of the asset price. Thus, both of these changes move our baseline further from the standard IAM with linear PPF. The equilibrium levels of abatement are similar to the baseline levels, suggesting that the results are robust to our choice of parameter values.

Increasing the level and slope of damages (the final column) leads to a much higher level of abatement. For $\eta = 2$, the higher damages cause equilibrium abatement by sophisticated agents (with $\xi = 0.5$) to approach the DU's abatement level, and to more than double relative to the unsophisticated level. For $\eta = 0$, higher damages also increase the difference between abatement levels chosen by sophisticated and unsophisticated agents.

5 Conclusion

Discussions of climate policy usually ignore the link between policy and asset prices. The recognition is selective when it occurs, lamenting that meaningful climate policy will lower the price of "stranded assets". Society as a whole, and anyone with a well-diversified portfolio of assets, is more concerned about asset value writ large. Prominent IAMs use a single stock of capital, which represent assets writ large. However, IAMs assume a composite commodity and infinitely lived agents, and thereby circumvent any role of asset markets in affecting the choice of climate policy. We stick with a single stock of capital, but assume a strictly concave production possibility frontier and overlapping generations, thereby making it possible for asset markets to affect incentives to protect the climate.

Asset markets exist regardless of whether discussions about climate policy take them into account. We asked whether recognizing the link between climate policy and asset prices would affect selfish agents' incentives to reduce greenhouse gas emissions. This question is relevant precisely because discussions of climate policy ignore the link or invoke it selectively.

In our political economy setting, currently living selfish generations choose climate policy to maximize a convex combination of their lifetime welfare. In our baseline calibration, with the commonly used value for the intertemporal elasticity of substitution of 0.5 ($\eta = 2$), there is an incentive to abate in order to benefit the current young late in their life. When the two generations have equal political influence, the *recognition* that the asset price is endogenous increases near-term abatement by 50%, reaching over half the level a (standard) discounted utilitarian chooses.

Climate policy in our baseline lowers short run asset prices, but by a negligible amount. Higher abatement levels and the resulting lower return on capital in the future approximately offset the increased return on capital due to the induced reduction in carbon stocks. This result suggests that it may be difficult to detect econometrically a link between climate policy and the price of a broad portfolio of assets. However, the increased incentive to abate arising from the recognition of endogenous asset prices significantly increases investment and asset prices in the long run. Climate policy increases agents' aggregate welfare, benefiting the young agent but in our baseline harming the old agent.

An increase in the young agent's relative political influence increases equilibrium abatement. This conclusion flips if the elasticity of intertemporal substitution is infinite; here, abatement tends to increase the asset price. We showed analytically that a marginal level of abatement that increases the asset price always benefits the old agent, and also benefits the young agent if and only if the elasticity of intertemporal substitution exceeds 1 (the substitution effect exceeds the income effect).

In order to focus on incentives created by asset markets, we excluded intergenerational transfers. We recognize (in line with earlier studies) that such transfers can be important as a means of inducing currently living people to undertake investments that benefit their successors. Our results do not alter this conclusion. However, our results suggest that even if asset markets significantly change incentives to undertake climate policy, they result in small inter-generational redistribution across agents in the first period.

References

Allen, M., D. Frame, C. Huntingford, C. Jones, J. Lowe, M. Meinshausen, and N. Meinshausen (2009): "Warming caused by cumulative carbon emissions towards the trillionth tonne," *Nature*, 458(7242), 1163–1166.

Balvers, R., D. Du, and X. Zhao (2012): "The Adverse Impact of Gradual Temperature Change on Capital Investment," 2012 Annual Meeting, August 12-14, 2012, Seattle, Washington 124676, Agricultural and Applied Economics Association.

Bansal, R., D. Kiku, and M. Ochoa (2015): "Climate Change and Growth Risks," Working paper.

——— (2016): "Price of long-run temperature shifts in capital markets," NBER working paper 22529.

Bansal, R., and M. Ochoa (2011): "Welfare Costs of Long-Run Temperature Shifts," NBER Working Papers 17574, National Bureau of Economic Research, Inc.

Bond, S., and J. van Reenen (2007): "Microeconometric Models of Investment and Employment," in *Handbook of Econometrics*, ed. by J. Heckman, and E. Leamer, vol. 6A, chap. 65. Elsevier, 1 edn.

Bovenberg, A. L., and B. J. Heijdra (1998): "Environmental tax policy and intergenerational distribution," *Journal of Public Economics*, 67(1), 1–24.

BP (2011): "Statistical Review of World Energy," .

Bullard, N. (2014): "Fossil fuel divestment: a 5 trillion dollar challenge," Bloomberg New Energy Finance White Paper.

Bushnell, J., H. Chong, and E. Mansur (2013): "Profiting from regulation: evidence from the European Carbon market," *American Economic Journal: Economic Policy*, 5(4), 78–106.

Chay, K., and M. Greenstone (2005): "Does air quality matter? Evidence from the housing market," *Journal of Political Economy*, 113(2), 376–424.

CIA (2010): *The world factbook*. https://www.cia.gov/library/publications/the-world-factbook/.

Conde-Ruiz, J., and V. Galasso (2005): "Positive arithmetic of the welfare state," *Journal of Public Economics*, pp. 933 – 955.

Diamond, P. A. (1965): "National Debt in a Neoclassical Growth Model," *The American Economic Review*, 55(5), 1126–1150.

Dietz, S., A. Bowen, C. Dixon, and P. Gradwell (2016): "'Climate value at risk' of global financial assetsNational Debt in a Neoclassical Growth Model," *Nature Climate Change Letters*, DOI: 10.1038/NCLIMATE2972.

Dietz, S., C. Gollier, and L. Kessler (2018): "The Climate Beta," *Journal of Environmental Economics and Management*, 87, 258 – 274.

Feldman, L., M. Nisbet, A. Leiserwitz, and E. Maibach (2010): "The Climate Change Generation," American University School of Communication.

Foley, D. (2009): "The economic fundamentals of global warming," in *Twenty-first century macroeconomics: Responding to the Climate Challenge*, ed. by J. Harris, and N. Goodwin. Edward Elgar Publishing, Cheltenham UK.

Gerlagh, R., and M. Keyzer (2001): "Sustainability and intergenerational distribution of natural resource entitlements.," *Journal of Public Economics*, 79, 315–341.

Giglio, S., M. Maggiori, J. Stroebel, and A. Weber (2015): "Climate Change and Long Run Discount Rates: Evidence from Real Estate," NBER Working Paper 21767.

Golosov, M., J. Hassler, P. Krusell, and A. Tsyvinski (2014): "Optimal taxes on fossil fuels in general equilibrium," *Econometrica*, 82, 41–88.

Hall, R. E. (2001): "The Stock Market and Capital Accumulation," *American Economic Review*, 91(5), 1185–1202.

——— (2004): "Measuring Factor Adjustment Costs," *The Quarterly Journal of Economics*, 119(3), 899–927.

HASSLER, J., J. V. RODRIGUEZ MORA, K. STORELETTEN, AND F. ZILIBOTTI (2003): "The survival of the welfare state," *American Economic Review*, 93(1), 87 – 112.

HOWARTH, R. B., AND R. B. NORGAARD (1992): "Environmental Valuation under Sustainable Development," *American Economic Review*, 82(2), 473–77.

HUBERMAN, G. (1984): "Capital asset pricing in an overlapping generations model," *Journal of Economic Theory*, 33(2), 232–248.

HUFFMAN, G. W. (1985): "Adjustment Costs and Capital Asset Pricing," *Journal of Finance*, 40(3), 691–705.

——— (1986): "Asset Pricing with Capital Accumulation," *International Economic Review*, 27(3), 565–82.

IVERSON, T., AND L. KARP (2017): "Carbon taxes and climate commitment with non-constant time preference," Working Paper.

JENSEN, S., AND C. TRAEGER (2014): "Optimal Climate Change Mitigation under Long-Term Growth Uncertainty: Stochastic Integrated Assessment and Analytic Findings," *European Economic Review*, 69, 104– 125.

JOHN, A., AND R. PECCHENINO (1994): "An Overlapping Generations Model of Growth and the Environment," *Economic Journal*, 104(427), 1393–1410.

JONES, R., D. COX, AND J. NAVARO-RIVERA (2014): "Believers, Sympathizers and Skeptics," Public Religion Research Institute.

JUDD, K. (1998): *Numerical Methods in Economics*. MIT Press, Cambridge, Massachusetts.

KARP, L. (2017): "Provision of a public good with multiple dynasties," *Economic Journal*, 127, 2641–2664.

KARP, L., AND A. REZAI (2014): "The political economy of environmental policy with overlapping generations," *International Economic Review*, 55, 711 – 733.

Kelly, D. L., and Z. Tan (2015): "Learning and Climate Feedbacks: Optimal Climate Insurance and Fat Tails," *Journal of Environmental Economics and Management*, 72, 98–122.

Labadie, P. (1986): "Comparative Dynamics and Risk Premia in an Overlapping Generations Model," *Review of Economic Studies*, 53(1), 139–52.

Lemoine, D. (2017): "The Climate Risk Premiiuim: How Uncertainty Affects the Social Cost of Carbon," University of Arizona Working Paper 15 - 01.

Lemoine, D., and C. Traeger (2014): "Watch your step: optimal policy in a tipping climate," *American Economic Journal: Economic Policy*, 6, 1–31.

Lindbeck, A., and J. Weibull (1987): "Balanced budget redistibution as political equilibrium," *Public Choice*, 52, 273–297.

Lontzek, T., Y. Cai, K. Judd, and T. Lentonl (2015): "Stochastic integrated assessment of climate tipping points indicates the need for strict climate policy," *Nature Climate Change*, pp. 441–444.

Lucas, Robert E, J. (1978): "Asset Prices in an Exchange Economy," *Econometrica*, 46(6), 1429–45.

Miranda, M. J., and P. L. Fackler (2002): *Applied Computational Economics and Finance*. MIT Press, Cambridge, Massachusetts.

Mumtaz, H., and F. Zanetti (2015): "Factor adjustment costs: A structural investigation," *Journal of Economic Dynamics and Control*, 51, 341 – 355.

NOAA (2017): *Full Mauna Loa CO2 record.* https://www.esrl.noaa.gov/gmd/ccgg/trends/full.html.

Nordhaus, W. D. (2008): *A Question of Balance: Economic Modeling of Global Warming*. Yale University Press, New Haven.

Oates, W. (1972): *Fiscal Federalism*. Harcourt Brace Javonovich, New York.

Persson, T., and G. Tabellin (2000): *Political Economics - Explaining Economic Policy*. MIT Press, Cambridge.

Powell, A. A., and F. H. G. Gruen (1968): "The Constant Elasticity of Transformation Production Frontier and Linear Supply System," *International Economic Review*, 9(3), 315–328.

R. C. Williams III, H. Gordon, D. Burtraw, J. C. Carbone, and R. D. Morgenstern (2015): "The Initial Incidence of a Carbon Tax Across Income Groups," *National Tax Journal*, 68(1), 195–214.

Ramsey, F. P. (1928): "A Mathematical Theory of Saving," *The Economic Journal*, 38(152), 543–559.

Rangel, A. (2003): "Forward and Backward Intergenerational Goods: Why is Social Security Good for the Environment?," *American Economic Reviewl*, 93, 813 – 834.

Rezai, A. (2010): "Recast the DICE and its policy recommendations," *Macroeconomic Dynamics*, 14, 275–289.

Rezai, A., D. Foley, and L. Taylor (2012): "Global warming and economic externalities," *Economic Theory*, 49, 329–351.

Ricke, K., and K. Caldeira (2014): "Maximum warming occurs about one decade after a carbon dioxide emission," *Environmental Research Letters*, 9, http://dx.doi.org/10.1088/1748-9326/9/12/124002.

Schneider, M., C. Traeger, and R. Winkler (2012): "Trading off generations: equity, efficiency and climate change," *European Economic Review*, 56, 1621–1644.

Shapiro, M. (1986): "The Dynamic Demand for Capital and Labor," *The Quarterly Journal of Economics*, 101(3), 513–542.

Stern, N. (2006): *The Economics of Climate Change*. Cambridge University Press.

Traeger, C. (2014): "Why uncertainty matters - Discounting under Intertemporal risk aversion and ambiguity," *Economic Theory*, 56, 627–664.

WITKOWSKI, W. (2015): "Global stock market cap has doubled since QE's start," http://www.marketwatch.com.

A Appendix: Proofs

Proof. (Lemma 1) The old generation consumes all of its income, so $c_t^o = (r_t + (1-\delta)\, p_t)\, k_t$, which implies equation 8. To obtain equation 7 we begin with the definition of the young agent's lifetime welfare

$$\Omega_t^y \equiv U(c_t^y) + \rho\, U(c_{t+1}^o).$$

With isoelastic utility, $\psi_t = \frac{1}{\rho}\left(\frac{c_{t+1}^o}{c_t^y}\right)^{\eta}$. Using this result in equation 3 gives

$$p_t = \rho \left(\frac{c_{t+1}^o}{c_t^y}\right)^{-\eta} (r_{t+1} + (1-\delta)p_{t+1})\frac{k_{t+1}}{k_{t+1}} = \rho \frac{\left(c_{t+1}^o\right)^{1-\eta}}{\left(c_t^y\right)^{-\eta} k_{t+1}},$$

where the second equality follows from the second constraint in equation 1. Rearranging this equation gives

$$\left(c_{t+1}^o\right)^{1-\eta} = p_t \frac{\left(c_t^y\right)^{-\eta} k_{t+1}}{\rho}.$$

Using this expression for the utility of the agent who is old in period $t+1$, we write the lifetime welfare for the agent who is young in period t as

$$\begin{aligned}
\Omega_t^y &= \frac{1}{1-\eta}\left(\left(c_t^y\right)^{1-\eta} - 1 + \rho\left(\left(p_t \frac{\left(c_t^y\right)^{-\eta} k_{t+1}}{\rho} - 1\right)\right)\right) \\
&= \frac{\left(c_t^y\right)^{-\eta}}{1-\eta}\left(c_t^y + p_t k_{t+1}\right) - \frac{1}{1-\eta}(1-\rho) \\
&= \frac{\left(c_t^y\right)^{-\eta}}{1-\eta} w_t - \frac{1}{1-\eta}(1-\rho),
\end{aligned}$$

where the last equality follows from the first constraint in equation 1. ■

Proof. (Proposition 1) Both claims follow from inspection of equation 8, together with the facts that k_t is predetermined and the policy has only a second order effect on the current return to capital. For $\delta = 1$, the old agent has nothing to sell, so the price of capital does not affect her welfare. ■

Proof. (Proposition 2) We hold future policy levels fixed, and consider the effect of a small current policy, μ_0, at the initial time, $t = 0$. Using equation 7 (and ignoring the constant) we write the young agent's lifetime welfare as

$$\begin{aligned}
\Omega_0^y(\mu_0) &= \frac{\left(c_0^y\right)^{-\eta}}{1-\eta} w_0 \Rightarrow \\
\frac{d\Omega_0^y(\mu_0)}{d\mu_0} &= \frac{1}{1-\eta}\left(\frac{\partial w_0}{\partial \mu_0}\left(c_0^y\right)^{-\eta} - \eta w_0 \left(c_0^y\right)^{-\eta-1}\frac{dc_0^y}{d\mu}\right). \qquad (14)
\end{aligned}$$

The young agent's budget constraint is

$$\begin{aligned}
c_0^y &= w_0 - p_0\left((1-\delta)k_0 + i_0(p_0)\right) \Rightarrow \\
\frac{dc_0^y}{d\mu} &= \frac{\partial w_0}{\partial \mu} - \left(p_0\frac{\partial i_0}{\partial p_0} + ((1-\delta)k_0 + i_0)\right)\frac{dp_0}{d\mu}. \qquad (15)
\end{aligned}$$

A small climate policy ($\mu_0 \approx 0$) has zero first-order effect on output and on factor prices (because $\Lambda'(0) = 0$). This fact and equations 14 and 15 imply

$$\left.\frac{d\Omega_0^y(\mu_0)}{d\mu_0}\right|_{\mu_0=0} = \frac{1}{1-\eta}\eta w_0 \left(c_0^y\right)^{-\eta-1}\left(p_0\frac{\partial i_0}{\partial p_0} + (1-\delta)k_0 + i_0\right)\frac{dp_0}{d\mu}. \qquad (16)$$

Using $\frac{\partial i_t}{\partial p_t} = (A^{-1})' > 0$, we conclude:

$$\begin{aligned}
\text{sign}\left(\left.\frac{d\Omega_0^y(\mu)}{d\mu}\right|_{\mu=0}\right) &= \text{sign}\left(\frac{dp_0}{d\mu_0}\right) \qquad \text{for } 0<\eta<1 \\
\text{sign}\left(\left.\frac{d\Omega_0^y(\mu)}{d\mu}\right|_{\mu=0}\right) &= -\text{sign}\left(\frac{dp_0}{d\mu_0}\right) \qquad \text{for } \eta>1.
\end{aligned}$$

Proposition 2.iii follows from the first two statements and from Proposition 1. ■

Proof. (Lemma 2) The budget constraint for the agent who is old in period $t+1$ requires $\frac{c_{t+1}^o}{k_{t+1}} = r_{t+1} + (1-\delta)p_{t+1}$. Using this relation and the fact that $\psi_t = \frac{c_{t+1}^o}{\rho c_t^y}$ under logarithmic utility, equation 3 becomes

$$p_t = \rho\frac{c_t^y}{c_{t+1}^o}\left(r_{t+1} + (1-\delta)p_{t+1}\right) = \rho\frac{c_t^y}{k_{t+1}} \Longrightarrow$$
$$c_t^y = \frac{p_t k_{t+1}}{\rho}$$

Using the last equation and the young agent's budget constraint at time t, we have

$$c_t^y = w_t - p_t k_{t+1} = \frac{p_t k_{t+1}}{\rho} \Rightarrow p_t k_{t+1} = \frac{\rho}{1+\rho}w_t \Rightarrow c_t^y = \frac{1}{1+\rho}w_t.$$

The last equality states that the agent spends a constant fraction $\frac{1}{1+\rho}$ of her wage on first period consumption and saves the rest. Using the fact that

$p_t = A(i_t)$, or $i_t = A^{-1}(p_t)$, we obtain an implicit function for p_t. This function, $p_t = f(w_t)$, solves

$$p_t k_{t+1} = p_t \left[(1-\delta)k_t + A^{-1}(p_t)\right] = \frac{\rho}{1+\rho} w_t. \tag{17}$$

Replacing p_t with $f(w_t)$, using the fact that A^{-1} is an increasing function, and totally differentiating the last equation in 17 implies that $f'(w) > 0$: an exogenous increase in the current wage causes an increase in the equilibrium price of capital. ■

Proof. (Proposition 3) (i) A small policy at $t = 0$ has zero first-order effect on the wage at $t = 0$. By Lemma 2.ii, this policy therefore has zero first-order effect on the asset price at $t = 0$. By Proposition 1, the policy has zero first-order effect on the old agent's welfare at $t = 0$.

(ii) By Lemma 2.i and the fact that the policy has a zero first-order effect on wage at $t = 0$, the policy has zero first-order effect on the $t = 0$ young agent's consumption, and thus on her savings, k_1. However, the policy creates a first-order reduction in the $t = 1$ pollution stock, leading to a first-order increase in both r_1 and w_1, and (by Lemma 2.ii) on the equilibrium value of p_1. The policy therefore leads to a first-order increase in her period $t = 1$ consumption, $c_1^o = (r_1 + (1-\delta)p_1)\, k_1$, and thus in her lifetime welfare.

(iii) The period-0 policy increases w_1 by reducing the period-1 pollution stock. By Lemma 2.i, the period-0 policy therefore increases the consumption, c_1^y, of the agent born at $t = 1$. This agent's consumption in the subsequent period equals $c_2^o = (r_2 + p_2(1-\delta))\, k_2$. Because the policy increases p_1, it also increases investment, i_1, and therefore increases k_2. The period-0 policy lowers e_1 without altering k_1, so the policy lowers the subsequent pollution stock, e_2. The reduction in e_2 and the increase in k_2 both increase the equilibrium wage, w_2. By Lemma 2.ii, the policy therefore increases p_2, and thus increases $p_2 k_2$. The equilibrium condition for the rental rate implies

$$r_2 k_2 = (1 - D(e_2))\, \Lambda(\mu_2)\, (1-\beta)\, \alpha k_2^{1-\beta}.$$

Because the policy has increased k_2 and reduced e_2, it increases $r_2 k_2$. Therefore, the policy also increases consumption, at $t = 2$, of the agent who was born at $t = 1$. Consequently, the policy increases this agent's welfare. ■

Proof. (Proposition 4). We invoke the turnpike theorem in letting $H \to \infty$.

(i) This result is an immediate consequence of equation 9, $w = (1 - \Lambda(\bar{\mu}))(\varpi - \vartheta e)$, and the fact that $\left.\frac{d\Lambda(\bar{\mu})}{d\bar{\mu}}\right|_{\bar{\mu}=0} = 0$.

(ii) (sketch; see Reviewers' Appendix B.2 for details) Using Proposition 1 we need only establish that $\left.\frac{dp_0}{d\mu_0}\right|_{\mu_0=0} > 0$. We confirm this inequality by examining the equilibrium conditions, a system of three linear difference equations in p, k, e. Price p is a forward looking variable, and the equilibrium price is linear in k, e. Using a linear trial solution and the assumption that the state variables, k, e, remain finite, yields $\left.\frac{dp_0}{d\mu_0}\right|_{\mu_0=0} > 0$.

(iii) This result follows from equation 9, $w = (1 - \Lambda(\mu))(\varpi - \vartheta e)$, $\Lambda'(0) = 0$, and $\frac{de_1}{d\mu_0} = -\zeta(\alpha k_0 + \varpi) < 0$. ■

B Reviewers' Appendix (not for publication)

This appendix provides the detailed proof of Proposition 4.ii, and it contains details of our numerical calibration and solution.

B.1 Alternative ways to obtain a concave PPF

To further motivate our choice of adjustment costs (+ congestion) as a means of introducing a concave PPF, we briefly consider alternatives:

- The Hecksher-Ohlin-Samuelson and Ricardo-Viner models also give rise to a concave PPF, but neither is analytically tractable. Furthermore, for both of these models, a change in climate policy or the carbon stock changes relative factor returns, distracting attention from our focus on asset markets. The Ricardo-Viner model also requires asset prices for the sector-specific factors.

- Under a constant elasticity of transformation between the consumption and investment goods, $c = c^y + c^o = y\left(b - a\,(i/y)^{(1+\sigma)/\sigma}\right)^{\sigma/(1+\sigma)}$, where σ, a and b are parameters (Powell and Gruen, 1968). This formulation retains the independence between relative factor returns and climate-related variables, but it makes the price of investment depend on i/y, instead of merely on i as in our adjustment cost model. This complication eliminates the analytic results in Section 3.

As $\sigma \to \infty$, the CES function approaches $c = b\left(y - \frac{a}{b}i\right)$. Using this fact, we can define adjustment costs in the CES setting as

$$A\left(\frac{i}{y};\sigma\right) \equiv y\left[\left(b - a\,(i/y)^{(1+\sigma)/\sigma}\right)^{\sigma/(1+\sigma)} - (b - a\,(i/y))\right].$$

B.2 Proof of Proposition 4.ii

Using Proposition 1 we need only establish that $\left.\frac{dp_0}{d\mu_o}\right|_{\mu_0=0} > 0$. Under Assumption 1, the dynamics of the competitive equilibrium, equations 3 and 5, from periods $t = 1$ onward, reduce to the following system of linear difference equations (with $\Delta = 1 - \delta$, $E = 1 - \epsilon$, and $\varphi = 2/\phi$ to simplify notation):

$$\begin{aligned} p_t &= \rho\ \left[(1 - \Lambda(\bar{\mu}))(\alpha - \xi e_{t+1}) + \Delta\ p_{t+1}\right] \\ k_{t+1} &= \Delta\ k_t + \varphi\ \ (p_t - 1) \\ e_{t+1} &= E\ e_t + \zeta\ (1 - \bar{\mu})\ \ (\alpha\ k_t + \varpi)\,. \end{aligned} \tag{18}$$

The initial conditions for this system are k_1 and e_1. These values depend on the state variables and the policy at time 0, k_0, e_0, and μ_0. In this linear model, where at t the states k_t and e_t are predetermined, and p_t is "forward looking", the equilibrium p_t is a linear function of k_t and e_t: $p_t = X + Y\ k_t + Z\ e_t$. The coefficients X, Y, Z depend on model parameters, including the constant $\bar{\mu}$. Substituting the trial solution, $p_t = X + Y\ k_t + Z\ e_t$, into system 18 we obtain

$$p_t = \begin{pmatrix} 1 & k_t & e_t \end{pmatrix} \cdot \begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \rho \begin{pmatrix} 1 & k_t & e_t \end{pmatrix} \begin{pmatrix} A_X \\ A_Y \\ A_Z \end{pmatrix} \quad \text{with}$$

$$\begin{pmatrix} A_X \\ A_Y \\ A_Z \end{pmatrix} = \begin{pmatrix} A_{x1} + A_{x2} \\ Y\ \Delta\ \ (\Delta + Y\ \varphi) + \alpha\ \zeta\ (1 - \bar{\mu})\ \ (Z\ \Delta - \xi\ (1 - \Lambda(\bar{\mu}))) \\ Z\ \Delta\ (E + Y\ \varphi) - E\ \xi\ (1 - \Lambda(\bar{\mu})) \end{pmatrix}$$

and

$$A_{x1} = \alpha\ (1 - \Lambda(\bar{\mu})) + X\ \Delta + Y\ \Delta\ \varphi\ (X - 1)$$

$$A_{x2} = \varpi\ \zeta\ (1 - \bar{\mu})\,(Z\ \Delta - \xi\ (1 - \Lambda(\bar{\mu})))\,.$$

Equating coefficients yields

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \rho \begin{pmatrix} A_X \\ A_Y \\ A_Z \end{pmatrix} \tag{19}$$

We show that there is a unique negative value of Z that satisfies this system: the asset price declines in the pollution stock. (Other roots of the system are either complex or positive.)

The last two equations in system 19 are independent of X. Solving the last equation for Z gives

$$Z(Y) = -\frac{E\xi\rho(1-\Lambda(\bar{\mu}))}{1-\Delta\rho(E+Y\varphi)}. \tag{20}$$

with the implication that $Z<0 \Leftrightarrow 1-\Delta\rho(E+Y\varphi)>0$. Substituting the expression for Z into the second equation of system 19, we obtain

$$0 = \Upsilon(Y) \equiv$$
$$Y - \rho\left[Y\ \Delta\ \ (\Delta + Y\ \varphi) - \alpha\ \zeta(1-\bar{\mu})\ \xi\ (1-\Lambda(\bar{\mu}))\left(1+\tfrac{\Delta\rho E}{1-\Delta\rho(E+Y\varphi)}\ \right)\right].$$

Define

$$\Theta = \Upsilon \times [1-\Delta\rho(E+Y\varphi)].$$

For

$$Y \neq Y_{crit} \equiv \frac{1-\Delta\ E\ \rho}{\Delta\ \rho\ \varphi} > 0,$$
$$\Upsilon(Y) = 0 \Leftrightarrow \Theta(Y) = 0.$$

Θ, is a cubic and thus has one or three real roots (as does Υ). The cubic Θ can be written

$$\begin{aligned}
\Theta &= B_3Y^3 + B_2Y^2 + B_1Y + B_0 \quad \text{with} \\
B_3 &\equiv \left(\Delta^2\rho^2\varphi^2\right) > 0 \\
B_2 &\equiv \Delta\rho\varphi\left(\Delta\rho(\Delta+E)-2\right) < 0 \\
B_1 &\equiv (1-\rho\Delta E)(1-\rho\Delta^2) - \alpha\zeta\xi\varphi\Delta\rho^2(1-\bar{\mu})(1-\Lambda) \\
B_0 &\equiv \alpha\zeta\xi\rho\,(1-\Lambda)\,(1-\bar{\mu}) > 0
\end{aligned}$$

The sign pattern of $\begin{pmatrix} B_3 & B_1 & B_1 & B_0 \end{pmatrix}$ is either $\begin{pmatrix} + & - & + & + \end{pmatrix}$ or $\begin{pmatrix} + & - & - & + \end{pmatrix}$. In either case, there are two sign differences between consecutive coefficients. By Descartes' rule of signs, there exist either two or zero positive roots. (In a knife-edge case, there is a single positive root.) To apply the corollary of Descartes' rule of signs, multiply the coefficients of odd powered terms by -1. This gives the sign pattern $\begin{pmatrix} - & - & - & + \end{pmatrix}$ or $\begin{pmatrix} - & - & + & + \end{pmatrix}$. In either case, there is one change in signs. The corollary states that the number of negative roots is either the number of sign changes (one), or fewer than that number by a multiple of 2. Because there cannot be a negative number of negative roots, we conclude that there is a unique negative root. We cannot rule out the existence of up to two roots with positive values for Y.

The system is explosive for $Y \geq Y_{crit}$; therefore, if Y is positive and the state variables remain finite (as assumed), $Y \in (0, Y_{crit})$. Substituting $p_t = X + Y\ k_t + Z\ e_t$ into the last two equations in system 18 gives

$$\begin{aligned} k_{t+1} &= \Delta\ k_t + \varphi\ \left(X + Y\ k_t + Z\ e_t - 1\right) \\ e_{t+1} &= E\ e_t + \zeta\ (1 - \overline{\mu})\ \left(\alpha\ k_t + \varpi\right). \end{aligned}$$

The Jacobian of this system (with respect to the state variables, k_t and e_t) is

$$J = \begin{pmatrix} \Delta + \varphi Y & \varphi Z \\ \alpha\zeta(1 - \overline{\mu}) & E \end{pmatrix}.$$

Stability requires that all eigenvalues of J lie within the unit circle. A weaker necessary condition requires the sum of the eigenvalues, or the trace of J, $Tr(J) = \Delta + E + \varphi Y$, to be less than 2. For the parameter restrictions $0 \leq \Delta \leq 1$ (capital decays) and $\rho > 0$ (positive discount factor) we have $Y \geq Y_{crit} \Longrightarrow Tr(J) > 2$:

$$Tr(J)|_{Y=Y_{crit}} = \Delta + E + \varphi \frac{1 - \Delta\ E\ \rho}{\Delta\ \rho\ \varphi} = \Delta + \frac{1}{\Delta\ \rho} > 2.$$

Because $Tr(J)$ is increasing in Y, we rule out all positive roots $Y^+ > Y_{crit}$. Therefore, any equilibrium root (which we have shown to exist) satisfies $Y < Y_{crit}$; consequently, $Z < 0$.

The next step obtains the current ($t = 0$) asset price as a function of the next period pollution stock. Using equation 3 and $p_t = X + Y\ k_t + Z\ e_t$, we have

$$\begin{aligned} p_0 &= \rho\ \left[(1 - \Lambda(\mu_0))(\alpha - \xi e_1) + \Delta\ p_1\right] \\ &= \rho\left[(1 - \Lambda(\mu_0))(\alpha - \xi e_1) + \Delta\left(X + Y\ k_1 + Z\ e_1\right)\right] \\ &= \rho\left[(1 - \Lambda(\mu_0))\,\alpha + X\Delta\right] + \rho Y \Delta k_1 - \rho\left[(1 - \Lambda(\mu_0))\,\xi - Z\Delta\right] e_1. \end{aligned}$$

We now use the relation $k_{t+1} = \Delta\ k_t + \varphi\ (p_t - 1)$ to eliminate k_1 and write

$$\begin{aligned} p_0 = {} & \rho\left[(1 - \Lambda(\mu_0))\,\alpha + X\Delta\right] + \rho Y \Delta\left(\Delta\ k_0 + \varphi\ (p_0 - 1)\right) \\ & -\rho\left[(1 - \Lambda(\mu_0))\,\xi - Z\Delta\right] e_1. \end{aligned}$$

Solving for p_0 gives

$$p_0 = \tfrac{1}{(1-\rho Y\Delta\varphi)}\times$$
$$\left[\rho\left[\left(1-\Lambda\left(\mu_0\right)\right)\alpha + X\Delta\right] + \rho Y\Delta\left(\Delta k_0 - \varphi\right) - \rho\left[\left(1-\Lambda\left(\mu_0\right)\right)\xi - Z\Delta\right]e_1\right].$$

The assumption that abatement costs are minimized at zero abatement, $\Lambda'(0) = 0$, means that we now need only to consider the effect of μ_0 on p_0 via e_1. Using the transition equation of e_t in 18, we establish

$$\begin{aligned}
\left.\frac{dp_0}{d\mu_o}\right|_{\mu_0=0} &= B_4\frac{de_1}{d\mu_0} > 0 \quad \text{with} \\
B_4 &\equiv -\frac{\rho\left(\left(1-\Lambda\left(\bar{\mu}\right)\right)\xi - Z\Delta\right)}{\left(1-\rho Y\Delta\varphi\right)} = \frac{Z}{E} < 0 \quad \text{and} \\
\frac{de_1}{d\mu_0} &= -\zeta\left(\alpha k_0 + \varpi\right) < 0.
\end{aligned} \tag{21}$$

B.3 Definition of Political Economy Equilibria

The standard cases of policy-making are either business-as-usual (with zero abatement) or first-best (socially optimal policy) chosen by the DU. Our political economy equilibria are situated between these two extremes and, depending on the degree of sophistication, are closer to BAU or the DU. (Given the absence of bequest motives, the political economy equilibria will never reach the DU outcome.) Restating equation 10, we have the political preference function

$$\xi\ \Omega_t^y + (1-\xi)\ \Omega_t^o =$$
$$\tfrac{\xi\left\{\left[w_t - p_t k_{t+1}\right]^{1-\eta} + \rho\left[\left(r_{t+1}+(1-\delta)p_{t+1}\right)k_{t+1}\right]^{1-\eta}\right\} + (1-\xi)\left[\left(r_t+(1-\delta)p_t\right)k_t\right]^{1-\eta}}{1-\eta}.$$

Under BAU, agents do not abate. In our unsophisticated equilibrium, agents recognize that emissions affect current and next-period factor returns, but they take the asset price as given. Because the next period rental rate directly affects the currently young agents, and because the marginal cost of abating the first unit is zero, young agents want positive abatement. In the sophisticated equilibrium, agents understand the formation of asset prices following equation 3

$$p_t = \frac{r_{t+1} + p_{t+1}(1-\delta)}{\psi_t}.$$

Mitigation today reduces future levels of carbon stocks, affecting all future rental rates. These rates are capitalized in today's asset price, possibly creating an additional motive for climate policy. Mitigation also reduces investment levels, but agents take these as given by Assumption 2.

The example of linear utility, presented in Section 4.1, helps clarify the differences between the two equilibrium concepts. For linear utility ($\eta = 0$), we have the political preference function

$$\xi\,\Omega_t^y + (1-\xi)\,\Omega_t^o = (1-\xi)(r_t + (1-\delta)p_t)k_t + \xi\left[(w_t - p_t k_{t+1}) + \rho\left((r_{t+1} + (1-\delta)p_{t+1})\,k_{t+1}\right)\right]$$

Setting $\xi \in \{0, 0.5, 1\}$, we obtain the maximand for the unsophisticated planner in the political equilibria, shown in Table 1. We obtain the maximand for the sophisticated planner by using the asset price equation, $p_t = \rho\,(r_{t+1} + (1-\delta)p_{t+1})$, resulting other entries in Table 1.

B.4 Calibration of Adjustment Costs

For gross investment I, our estimate of marginal adjustment cost is ϕI. Shapiro (1986) estimates marginal adjustment costs of gross investment using quarterly data, and Hall (2004) estimates marginal adjustment costs of net investment using annual data. These studies use the functional forms for marginal costs:

$$\begin{aligned}
\text{Shapiro } I \text{ (1986)} &: \quad \phi_S^I\; Y\; I \\
\text{Shapiro } II \text{ (1986)} &: \quad \phi_S^{II}\; Y\; \left(\frac{I}{\Delta K}\right) \\
\text{Hall (2004)} &: \quad \phi_H\; \left(\frac{I^{net}}{K}\right)
\end{aligned}$$

where K is capital, Y equals after tax income, $\Delta = 1 - \delta$ equals the depreciation factor, and I^{net} is net investment. We use Shapiro's estimates to calibrate our adjustment cost parameter ϕ. His point estimates are $\phi_S^I = 0.0014$ and $\phi_S^{II} = 0.25$. Hall's median estimate is $\phi_H = 0.15$ and his high value for certain industries is $\phi_H = 1$.

We begin by reviewing Hall's derivation of the relation between the parameter estimates in his and in Shapiro's formulations (Hall 2001, appendix C, http://web.stanford.edu/~rehall/SMCA-AER-Dec-2001.pdf). With quarterly data, $\Delta \approx 1$ and $I \approx I^{net}$. Using these approximations, Hall ignores the distinction between net and gross investment, and the depreciation factor. Denote as $\phi_H^{[\text{Shapiro I}]}$ the parameter that, with Hall's functional form, gives the same marginal adjustment cost as Shapiro (I). That is, $\phi_H^{[\text{Shapiro I}]}\left(\frac{I}{K}\right) \equiv \phi_S^I\ Y\ I$ or

$$\phi_H^{[\text{Shapiro I}]} \equiv \phi_S^I\ YK.$$

Similarly, $\phi_H^{[\text{Shapiro II}]}$ is the parameter that, with Hall's functional form, gives the same marginal adjustment cost as Shapiro (II): $\phi_H^{[\text{Shapiro II}]}\left(\frac{I}{K}\right) \equiv \phi_S^{II}\ Y\ \left(\frac{I}{K}\right)$, or

$$\phi_H^{[\text{Shapiro II}]} \equiv \phi_S^{II}\ Y.$$

Shapiro uses $Y = 33\ bn$ and $K = 203\ bn$. Using these values in the above identities, we have

$$\phi_H^{[\text{Shapiro I}]} \equiv \phi_S^I\ Y\ K = 0.0014 * 33 * 203 = 9.24 \text{ and}$$
$$\phi_H^{[\text{Shapiro II}]} \equiv \phi_S^{II}\ Y = 0.25 * 33 = 8.25.$$

To compare these converted estimates to Hall's, we divide the former by 4 (because Shapiro uses quarterly and Hall uses annual data) to obtain 2. 3 and 2. 1. These values are over twice as large as Hall's largest estimate, and 15 times as large as Hall's median estimate of 0.15. Thus, Hall's estimate of adjustment cost are much smaller than Shapiro's.

The units of both adjustment costs and investment are $\left[\frac{\text{dollars}}{\text{time}}\right]$, so marginal adjustment cost is unit-free. With quarterly data, the units of $\frac{I}{K}$ are $\left[\frac{\frac{\$}{\text{quarter}}}{\$}\right] = \left[\frac{1}{\text{quarter}}\right]$. Because marginal adjustment costs are unit-free, the units of $\phi_H^{[\text{Shapiro I}]}$ and $\phi_H^{[\text{Shapiro II}]}$ are [quarters]. Our unit of time is $35 \times 4 = 140$ quarters, so we need to divide $\phi_H^{[\text{Shapiro i}]}$ by 140; we also need to divide by K to account for difference between the Hall/Shapiro formulations and ours. For example, if quarterly investment is I^{quarter}, investment over a 35 year period is $I^{35\text{ year}} = 140 I^{\text{quarter}}$. In order that our estimate of marginal adjustment costs is comparable to Shapiro's, we choose our adjustment cost parameter ϕ by setting marginal cost in our formulation equal to marginal cost in Hall's conversion of Shapiro's formulation:

$$\phi^{35\text{ years}} \times 140 \times I^{\text{quarter}} = \phi_H^{\text{Shapiro i}} \times I^{\text{quarter}} \times \frac{1}{K}$$
$$\Rightarrow \phi^{35\text{ years}} = \phi_H^{\text{Shapiro i}} \times \frac{1}{K \times 140}$$

Using our estimate $K_0 = K = 200$ (see Section 4.3) and the two values of $\phi_H^{\text{Shapiro i}}$ we have

$$\begin{array}{r} \phi^{35\ years} = \frac{1}{140}\frac{9.24}{200} = 0.00033 \text{ and} \\ \phi^{35\ years} = \frac{1}{140}\frac{8.24}{200} = 0.000295 \end{array} .$$

We set our benchmark to $\phi = 0.0003$.

A simple calculation shows that at plausible levels of investment, our baseline estimate of adjustment costs, expressed as a percent of income, is much less than Shapiro's estimates, but much larger than Hall's. If investment is 20% of output, and given a 35-year output of $\$T$ 2200, our baseline estimate of adjustment costs equals $0.0003\frac{(0.2\times 2200)^2}{2}$. Adjustment cost as a percent of *income* is $0.0003\frac{(0.2\times 2200)^2}{2\times 2200}100\% = 1.32\%$.

Shapiro's first estimate of adjustment cost is $\frac{\phi_S^I\ Y\ I^2}{2}$, so his estimate of adjustment cost as a percent of income is $\frac{\phi_S^I\ Y\ I^2}{2Y}100$. If investment is 20% of income ($= Y = 33$ for his data), his estimate of adjustment cost as a percent of income is $0.0014\frac{(.2\times 33)^2}{2}100 = 3.05$, over twice our baseline estimate. Shapiro's two estimates of adjustment costs are equal if $\phi_S^I\ Y\ I = \phi_S^{II}\ Y\ \left(\frac{I}{\Delta K}\right)$. Using his value for capital, $K = 203$, equality of his estimates of adjustment costs requires $0.0014 = 0.25\frac{1}{203\Delta}$, or $\Delta = 0.88$. A larger value of Δ implies that Shapiro's second estimate of adjustment cost, as a percent of income, is smaller than his first estimate. An annual depreciation rate of 6% implies that $\Delta = 0.9898$. Using this value, $\phi_S^{II} = 0.25$, $I = 0.2Y$, $Y = 33$, and $K = 203$, Shapiro's second estimate of adjustment cost as a percent of income, is

$$\phi_S^{II}\ \left(\frac{I^2}{2\Delta K}\right)100\% = 0.25\left(\frac{.04Y^2}{2(0.9898)K}\right)100\%$$
$$= 0.25\left(\frac{.04(33)^2}{2(0.9898)203}\right)100\% = 2.71\%,$$

over twice the percentage implied by our baseline. These estimates are similar to Mumtaz and Zanetti's (2015); using a structural model, they estimate total adjustment costs to equal 3.3% of output.

Hall's median estimate implies negligible adjustment costs. His largest estimate implies roughly the same magnitude of adjustment costs as our baseline, but only if we set depreciation to zero. Using the definition of net investment, $I^{net} + \delta K = I$, Hall's estimate of adjustment cost, as a percent of income, is

$$\frac{\phi_H \frac{\left(I^{net}\right)^2}{2K}}{Y} 100 = \frac{\phi_H \frac{(I-\delta K)^2}{2K}}{Y} 100.$$

Evaluated at $I = 0.2Y$ and using $\phi_H = 0.15$, adjustment cost as a percent of income equals

$$\frac{0.15 \frac{(0.2Y-\delta K)^2}{2K}}{Y} 100 = \frac{0.15 \frac{\left(0.2\frac{Y}{K}-\delta\right)^2 K^2}{2K}}{Y} 100 = \frac{0.15 \left(0.2\frac{Y}{K} - \delta\right)^2}{\frac{Y}{K}} 100.$$

At an annual depreciation rate of 6% ($\delta = 0.06$) and using the estimates $Y = 63$ and $K = 200$ (Section 4.3), we obtain the estimate

$$\frac{0.15 \left(0.2\frac{63}{200} - 0.06\right)^2}{\frac{63}{200}} 100\% = 4.286 \times 10^{-4}\% \approx 0.$$

Even for $\delta = 0$, the estimate is only 0.189%. Thus, Hall's median estimate implies that adjustment costs are negligible. Using his largest estimate (1) and a 6% annual depreciation rate, his estimate of adjustment cost as a percent of income is 2.9×10^{-3}, rising to 1.26 if we set depreciation to 0.

B.5 Numerical Approximation

Multiplicity: The possibility of multiple equilibria arises for two kinds of reasons. First, for a given candidate $M^c(k_{t+1}, e_{t+1})$, there might be multiple solutions to the political economy problem at time t. Because the function $M^c(k_{t+1}, e_{t+1})$ is a polynomial approximation, we cannot rely on curvature properties to guarantee uniqueness; instead, we depend on the numerical algorithm. Our numerical work finds no evidence of this kind of multiplicity. Second, the infinite horizon (required in our stationary setting) generically raises the possibility of non-uniqueness, a standard result in dynamic games where there is an "incomplete transversality condition". However, our algorithm works backwards, beginning with a scrap function to represent the last period; we iterate until convergence, so that the algorithm selects a solution that is close to the "limit equilibrium", as the horizon goes to infinity. We confirm that the converged equilibrium is insensitive to changes in the scrap function.

The political economy equilibria: The solution algorithm is standard. For both the unsophisticated and the sophisticated equilibria, we select a grid of points in state space (k, e) and choose Chebyshev polynomials to interpolate the endogenous functions investment function, $\mu_t = M(k_t, e_t)$, and price function, $p_t = \Psi(k_t, e_t)$, at points off this grid. The solution requires finding coefficients of these polynomials such that the approximations to the endogenous functions satisfy all equilibrium conditions at points on the grid. We begin with a guess of these coefficients to obtain candidate policy and investment functions, $M^c(k_t, e_t)$ and $\Psi^c(k_t, e_t)$ and the associated investment rule, $A^{-1}(\Psi^c(k_t, e_t))$. Using these candidates, we solve the optimization problem (10) (for both the unsophisticated and sophisticated planners) at every point on the grid, thus generating values of p_t and μ_t at every point on the grid. We update the Chebyshev coefficients so that the revised candidates satisfies the equilibrium conditions at points on the grid. We continue this iteration until the estimated coefficients, and thus the endogenous functions, approximately converge.

That is, we approximate $M(k_{t+1}, e_{t+1})$ and $\Psi(k_{t+1}, e_{t+1})$ as polynomials in k_{t+1} and e_{t+1}. We find coefficients of those polynomials so that, on the grid points, the recursion is satisfied:

$$p_t = \Psi(k_t, e_t) = \rho \frac{k_{t+1}^{-\eta}\left[R(k_{t+1}, e_{t+1}) + \Psi(k_{t+1}, e_{t+1})(1-\delta)\right]^{1-\eta}}{\left[W(k_t, e_t) - \Psi(k_t, e_t)\, k_{t+1}\right]^{-\eta}}. \tag{22}$$

In addition, the maximization of the joint welfare

$$\begin{aligned} &\max_{\mu_t} (1-\xi)\frac{\left[(r_t + (1-\delta)p_t)k_t\right]^{1-\eta} - 1}{1-\eta} + \\ \xi \quad &\left\{ \frac{\left[w_t - p_t k_{t+1}\right]^{1-\eta} - 1}{1-\eta} + \rho \frac{\left[r_{t+1} + (1-\delta)p_{t+1}k_{t+1}\right]^{1-\eta} - 1}{1-\eta} \right\} \end{aligned} \tag{23}$$

subject to

$$\begin{aligned} e_{t+1} &= (1-\epsilon)e_t + \zeta(1-\mu_t)F(k_t, l). \\ k_{t+1} &= (1-\delta)k_t + A^{-1}(p_t). \end{aligned} \tag{24}$$

with k_t, e_t given and $A^{-1}(.)$ the inverse function of $A(.)$, equals $M(k_t, e_t)$ on the grid points.

Equation 22 is an implicit expression in $\Psi(k_t, e_t)$. For the specific values of η used in our simulations, equation 22 can be reformulated to give explicit expressions of $\Psi(k_t, e_t)$. For $\eta = 0$, we have $p_t = \Psi(k_t, e_t) = \rho\ [R(k_{t+1}, e_{t+1}) + \Psi(k_{t+1}, e_{t+1})(1-\delta)]$ and, for $\eta = 2$,

$$\Psi(k_t, e_t) = \frac{W(k_t, e_t)}{k_{t+1}} + \frac{c^o_{t+1}}{2\ \rho} \pm \frac{\sqrt{c^o_{t+1}\left\{k_{t+1}c^o_{t+1} + 4\rho W(k_t, e_t)\right\}}}{2\ \rho\ \sqrt{k_{t+1}}}, \tag{25}$$

with $c^o_{t+1} = [R(k_{t+1}, e_{t+1}) + \Psi(k_{t+1}, e_{t+1})(1-\delta)]$. Of the two solutions for $\eta = 2$, only one is consistent with positive investment levels.

In the "unsophisticated" equilibrium, policy-makers take p_t and p_{t+1} as given. In the "sophisticated" equilibrium policy-makers understand the process under which asset prices are formed. We replace $\Psi(k_t, e_t)$ in objective (23) with its explicit expression (for $\eta = 2$ the right-hand side of (25)).

For both equilibria we use 36-degree Chebyshev polynomials evaluated at 6x6 Chebyshev nodes on the $[200, 800]$ interval for k and the $[200, 1000]$ interval for e. At each node the recursion defining $\Psi(k_t, e_t)$ is satisfied. For the political economy equilibria we have the optimality condition,

$$\frac{d}{d\mu_t}\left[\xi\frac{(W(k_t, e_t) - p_t k_{t+1})^{-\eta}}{1-\eta}W(k_t, e_t) + (1-\xi)\frac{[(R(k_t, e_t) + (1-\delta)p_t)k_t]^{1-\eta} - 1}{1-\eta}\right] = 0, \tag{26}$$

the Nash equilibrium condition, $\mu_t = M(k_t, e_t)$, and $p_t = \Psi(k_t, e_t)$ in the "unsophisticated" and p_t equal its "forward-looking" expression (for $\eta = 2$ the right-hand side of (25)) in the "sophisticated" equilibrium. In addition, system (24) must be satisfied. The first order condition (26) takes i_t (and consequently k_{t+1}) as given, implying that when political representatives choose abatement they take current investment as given. This procedure means that current generations do not use the abatement instrument to target investment.

Starting with an initial guess for the coefficients of the approximations of $\Psi(\cdot)$ and $M(\cdot)$, we evaluate the right side of equation 22 for at each node. Using these function values, we obtain new coefficient values for the approximation of $\Psi(\cdot)$. We then use the optimality condition 26 to find the values of μ at the nodes; we use those values to update the coefficients for the approximation of $\Psi(\cdot)$. We repeat this iteration until the coefficients' relative difference between iterations falls below 10^{-6}. See chapter 6 of Miranda and Fackler (2002) for details. Figures 4 and 5, graph the asset and abatement functions, the differences (the "residuals") between the right and left sides of equations 22 and 26, and the %-change in the coefficients of the approximated function between iterations. Residuals equal 0 at the nodes because we set both the degree of the polynomial and the number of nodes equal to n. We choose $n = 36$ to ensure that residuals are at least 4 orders of magnitudes below the solution values on the approximation interval. In certain simulations it proved numerically easier to iterate on the investment rather than the asset price function with the former a linear transformation of the latter: $p_t = A(i_t)$.

The DU: For the DU's problem, we approximate $J(k_{t+1}, e_{t+1})$, $I(k_{t+1}, e_{t+1})$ and $M(k_{t+1}, e_{t+1})$ as polynomials in k_{t+1} and e_{t+1}, and find coefficients of those polynomials so that the solution to the maximization of welfare

$$\max_{\iota_t, \mu_t} \frac{[y_t - A(i_t)i_t]^{1-\eta} - 1}{1-\eta} + \rho\ J(k_{t+1}, e_{t+1}) \tag{27}$$

subject to

$$\begin{aligned} e_{t+1} &= (1-\epsilon)e_t + \zeta(1-\mu_t)F(k_t, l). \\ k_{t+1} &= (1-\delta)k_t + i_t. \end{aligned} \tag{28}$$

with k_t, e_t given, approximately equals $I(k_t, e_t)$ and $M(k_t, e_t)$. At each node, the value function recursion is

$$J(k_t, e_t) = \frac{[y_t - A(i_t)i_t]^{1-\eta} - 1}{1-\eta} + \rho\ J(k_{t+1}, e_{t+1}). \tag{29}$$

the DU's optimality conditions are

$$\begin{aligned} \frac{d}{d\iota_t}\left[\frac{[y_t - A(i_t)i_t]^{1-\eta} - 1}{1-\eta} + \rho\ J(k_{t+1}, e_{t+1})\right] &= 0 \\ \frac{d}{d\mu_t}\left[\frac{[y_t - A(i_t)i_t]^{1-\eta} - 1}{1-\eta} + \rho\ J(k_{t+1}, e_{t+1})\right] &= 0. \end{aligned} \tag{30}$$

For the DU, we use 72-degree Chebyshev polynomials evaluated at 12x6 Chebyshev nodes on the $[200, 1000]$ interval for k and $[100, 1000]$ interval for e. Starting with an initial guess for the coefficients of the approximations of $J(\cdot)$, $I(\cdot)$ and $M(\cdot)$, we evaluate the right side of equation 29 for at each node. Using these function values, we obtain new coefficient values for the

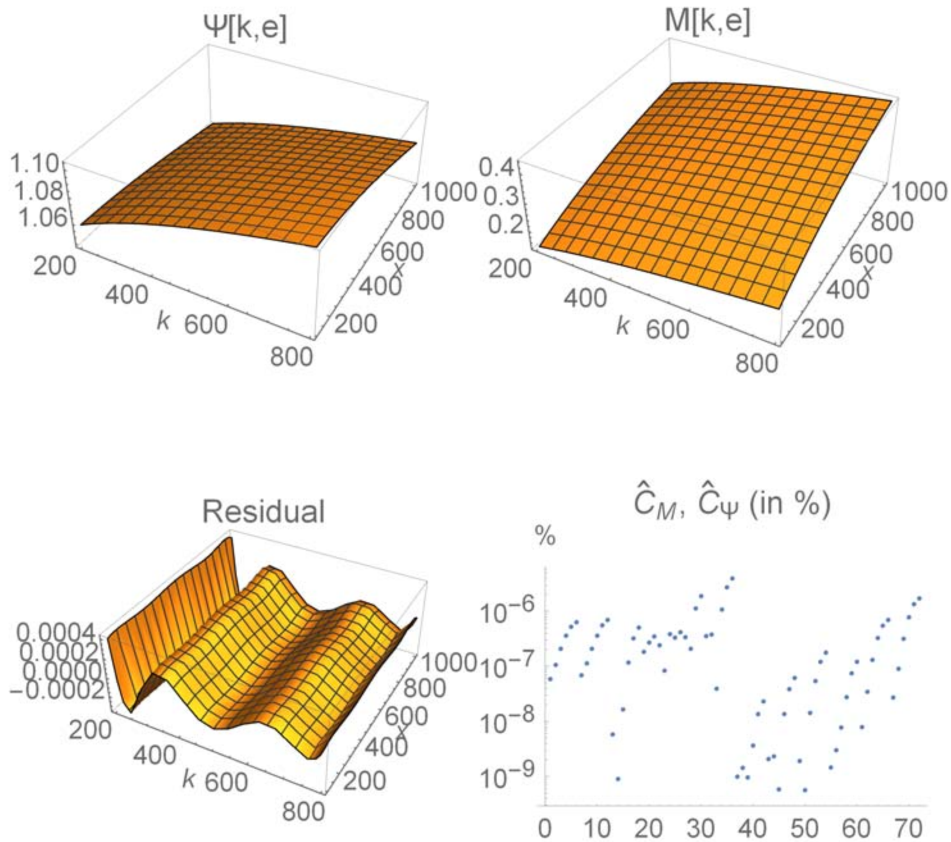

Figure 4: Benchmark "unsophisticated" equilibrium solution: *upper* Asset price, $\Psi(\cdot)$, and policy, $M(\cdot)$, function; *lower* deviation from true asset value outside of approximation nodes ("Residuals) and %-change of coefficients between iterations ("$\hat{C}$")

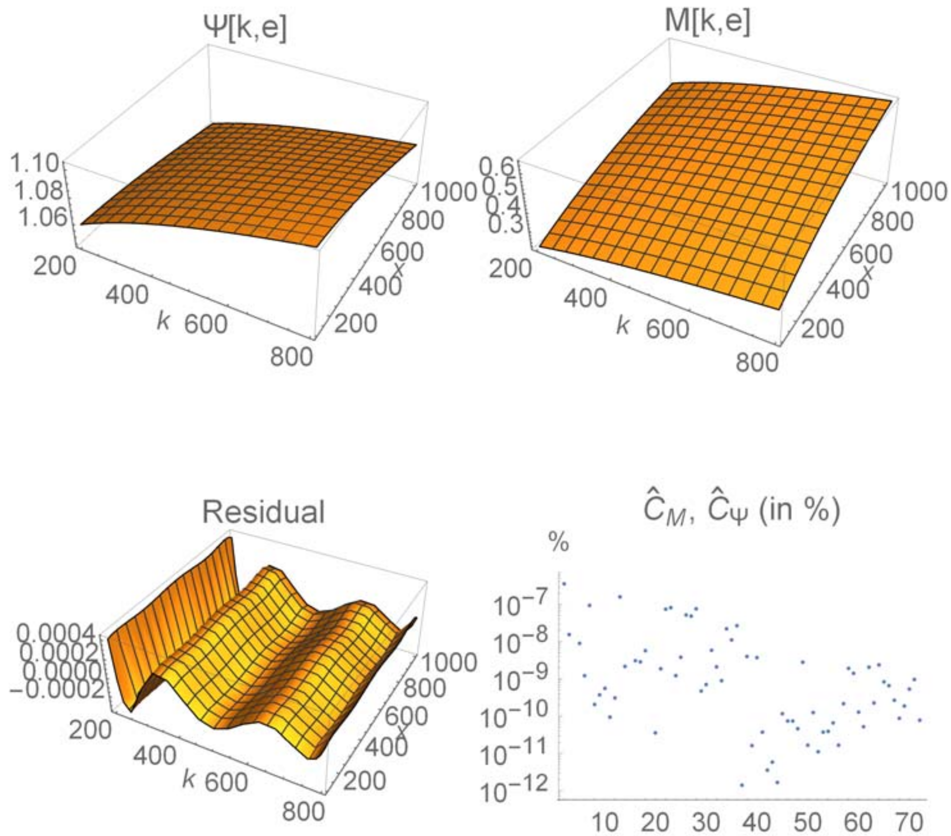

Figure 5: Benchmark "sophisticated" equilibrium solution: *upper* Asset price, $\Psi(\cdot)$, and policy, $M(\cdot)$, function; *lower* approximation from true asset value outside of approximation nodes ("Residuals") and %-change of coefficients between iterations ("$\hat{C}$")

approximation of $J(\cdot)$. We then use the optimality conditions 30 to find the values of i and μ at the nodes; we use those values to update the coefficients for the approximation of $J(\cdot)$. We repeat this iteration until the coefficients' relative difference between iterations falls below 10^{-6}. Figure 6 graphs the value, investment, and abatement functions, the differences (the "residuals") between the right and left sides of equations 22 and 26, and the %-change in the coefficients of the approximated function between iterations. Residuals equal 0 at the nodes because we set both the degree of the polynomial and the number of nodes equal to n. We choose $n = 72$ to ensure that residuals are at least 5 orders of magnitudes below the solution values on the approximation interval.

The approximation of the political economy equilibria requires a dynamic programming approach. The DU problem, however, can also be solved using optimal control. As a consistency check, we also solving the social planning problem using the optimization software GAMS and found that both approaches give the same results, accounting for the finite horizon of the numerical optimal control approach.

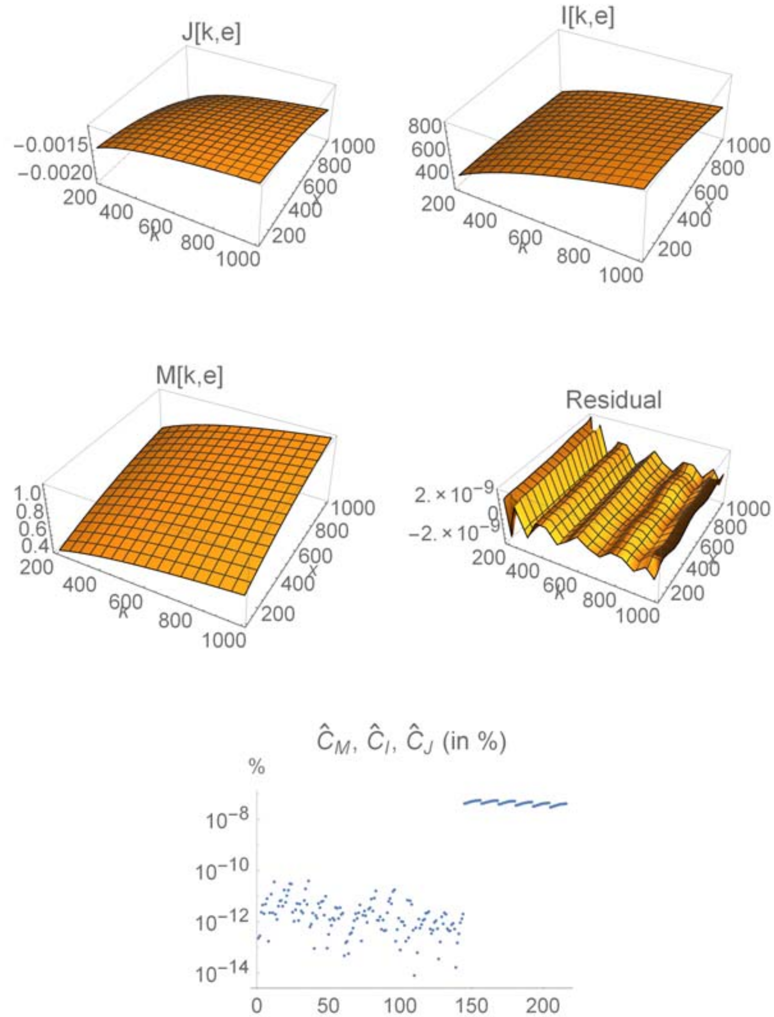

Figure 6: SP solution: *upper* Value, $J(\cdot)$, and investment function, $I(\cdot)$; *middle* mitigation policy function, $M(\cdot)$, and deviation from true value function value outside of approximation nodes ("Residuals"); *lower* %-change of coefficients between iterations ("$\hat{C}$")