

Output Hysteresis and Optimal Monetary Policy *

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Abstract

We analyze the implications for monetary policy when deficient aggregate demand can cause a permanent loss in potential output, a phenomenon termed as *output hysteresis*. We incorporate Schumpeterian endogenous growth into a business cycle model with nominal rigidities. In the model, incomplete stabilization of a temporary shortfall in demand reduces the return to innovation, thus reducing R&D and producing a permanent loss in output. Output hysteresis arises under a standard Taylor rule, but not under a strict inflation targeting rule when the nominal interest rate is away from the zero lower bound (ZLB). At the ZLB, a central bank unable to commit to future policy actions suffers from *hysteresis bias*: it does not offset past losses in potential output. A new policy rule that targets zero output hysteresis approximates the optimal policy by keeping output at the first-best level. Estimated structural impulse response functions for key variables align with predictions of the model. A quantitative model provides evidence of significant output hysteresis resulting from endogenous growth over the Great Recession.

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1 Introduction

”... a portion of the relatively weak productivity growth ... may be the result of the recession itself. ... In particular, investment in research and development has been relatively weak... Federal Reserve actions to strengthen the recovery may not only help bring our economy back to its productive potential, but it may also support the growth of productivity and living standards over the longer run.”

Janet L. Yellen, Chair of the Board of Governors of the Federal Reserve System

In the aftermath of the Great Recession, the US economy has experienced its slowest post-recession recovery since World War II. Eight years in, real GDP is still approximately 15% below the pre-recession trend (Figure 1). Similar trajectories have been observed in other OECD countries as well (Figure 2). One of the primary drivers of this output shortfall has been slow productivity growth (Hall 2016, Stock and Watson 2016), the source of which has been a subject of extensive debate. Fernald (2014a) and Cette, Fernald and Mojon (2016) show that total factor productivity (TFP) growth started slowing in 2004, three years before the recession started. Thus, they say slowed growth following the recession may not have been due to the recession itself. On the other hand, Decker, Haltiwanger, Jarmin and Miranda (2014) show that the recession accelerated the slowdown in startup entry, which is a significant channel for productivity growth. Similarly, investment in research and development (R&D), considered to be another important contributor to TFP growth, fell considerably during the recent recession (Figure 3).¹ These facts underscore Chairwoman Yellen’s concerns as cited above.

The standard theoretical treatment of monetary policy is largely silent on the interaction of monetary policy with the productive potential of the economy.² In this paper, we construct a model in which there is such an interaction. We embed a model of Schumpeterian Growth along the lines of Aghion and Howitt (1992) and Grossman and Helpman (1991) in a New Keynesian (NK) setting. A contraction in aggregate demand reduces the incentives for firms to invest in R&D, resulting in lower innovation. This leads to an endogenous slowdown in TFP growth, which results in a persistent output gap. Traditional NK models do not incorporate endogenous productivity and thus incorrectly predict a recovery to the pre-recession trend. Our model formalizes the possibility that following a recession, unemployment returns to its natural rate while output remains below the trend, what Barro (2016) refers to as the *job-filled non-recovery*. Using this framework, we ask whether monetary policy can affect the long-run potential output of the economy, and, if this is so, whether it is optimal for monetary policy to offset the permanent effects of temporary demand shocks on the level of GDP.

¹ Barlevy (2007) shows that Research and Development (R&D) expenditure is pro-cyclical. There is a considerable literature on innovation and patents, starting with Griliches (1957), which argues that innovation expenditures significantly determine the TFP growth rate. See Hall, Mairesse and Mohnen (2010) for a detailed survey.

²More recently, Yellen (2016) remarked, “Are there circumstances in which changes in aggregate demand can have an appreciable, persistent effect on aggregate supply? Prior to the Great Recession, most economists would probably have answered this question with a qualified “no.” ... This conclusion deserves to be reconsidered in light of the failure of the level of economic activity to return to its pre-recession trend in most advanced economies. This post-crisis experience suggests that changes in aggregate demand may have an appreciable, persistent effect on aggregate supply—that is, on potential output.” (October 14, 2016) There is a recent literature that explores these interactions, including Anzoategui et al. (2017), Bianchi, Kung and Morales (2017) and Benigno and Fornaro (2017). Ours is the first paper to analyze the interaction of optimal monetary policy at the ZLB, aggregate demand and endogenous growth. We discuss this later in related literature.

We use our framework to study an economy hit with temporary demand shortfalls. In this paper, we focus on liquidity demand shocks and monetary policy shocks. There are three sets of results: positive, normative and quantitative.

Our first main result is that output hysteresis is contingent on the monetary policy specification of the central bank. A central bank following a standard [Taylor \(1993\)](#) rule, targeting inflation and unemployment, admits permanent output gaps. When demand is low, the Taylor rule does not completely stabilize the economy. As a result, there is an endogenous decline in productivity-enhancing investment which slows down TFP growth. As the shock abates, Taylor rule prescribes setting a path for interest rates that does not offset past shortfalls in TFP growth. Hence, output is permanently below the initial deterministic trend. We define these (deterministic) trend-based output gaps as *output hystereses*. Our model thus formalizes what Lawrence Summers refers to as *Inverse Say's law*: a lack of demand distorts supply over time.³

If the central bank strictly targets inflation and the nominal interest rate is away from the zero lower bound (ZLB), there is no output hysteresis. However, such a policy is unable to stabilize aggregate demand perfectly when the ZLB is binding. As a result, a strict inflation targeting or a Taylor rule admit output hysteresis after a ZLB episode. On the other hand, there exist policy rules which, if credibly communicated to the public, could prevent output hysteresis following recessions induced by shortage of aggregate demand, whether or not the ZLB is binding. A new rule where the central bank targets zero output hysteresis emerges in the endogenous growth framework. The central bank commits to keeping interest rates lower until output is back at the initial trend. This rule signals ex-ante commitment by the central bank to running a *high-pressure economy* in the future when there is no slack in employment.

Our second set of results speak to the desirability of running a *high-pressure economy* using a hysteresis targeting rule. An optimizing policy-maker with ability to commit to future policy actions (optimal commitment policy) sets interest rates to offset the permanent output gap. When the economy is away from the ZLB, a central bank pursuing the objective of price-level-stability can achieve the first-best outcome of maintaining output at the pre-recession trend. However, in the event that the central bank is unable to do so, due to constraints on policy such as the ZLB, these shocks can lead to output hysteresis. At the ZLB, the optimal policy response is to credibly commit to keeping future interest rates lower in order to incentivize recovery close to the pre-recession trend. A zero output-hysteresis targeting policy rule can eliminate all the persistent effects resulting from the constrained monetary policy, while also replicating the welfare gains achieved under optimal policy. This rule has the relative advantage in ease of communicating the central bank's policy stance to the public, unlike optimal policy rules studied in the literature.

Importantly, we uncover a new dynamic inconsistency problem. A policy-maker unable to commit to future policy actions (discretionary policy) does not find in its interest to undo permanent output gaps, following a ZLB period. We label this as the *hysteresis bias* of a policymaker. It complements our first finding that hysteresis is a consequence of a central bank's policy constraints and not of inept or irrational

³Speech titled "Fiscal Policy and Full Employment" at the Center on Budget and Policy Priorities, April 2014.

behavior on part of the central bankers. More importantly, it implies that it is sub-optimal *ex post* for policy to be redesigned in order to offset the existing output hysteresis.

Our third main result is that both the liquidity demand shock and the central bank's policy rule can have a quantitatively large effect in a medium-scale version of our benchmark model. A one-time shock to the liquidity demand, calibrated to match the increase in premium associated with liquid assets during the crisis, can explain a third of the drop in output observed in the data and one percentage point drop in inflation rate. Under a Taylor rule, the nominal interest rate hits the zero lower bound and the recovery is sluggish. When policy is set using a hysteresis-augmented Taylor rule, the output drop is 90% lower (relative to Taylor rule) and the nominal interest rate is positive. The permanent effects on long-run output depend crucially on the calibration of innovation intensity elasticity. For calibration on two extreme values considered in the innovation literature, the output hysteresis is 1.24% and 0.08% respectively.

Further, using the local-projections instrumental-variables approach, we estimate structural impulse response functions (IRFs) for R&D investment, firm entry indicators and utilization-adjusted TFP to monetary policy shocks. A contractionary monetary policy shock temporarily reduces R&D investment and firm entry, and has a persistent negative effect on the level of TFP. Standard DSGE models cannot generate any effect on TFP following a monetary policy shock. The quantitative model with endogenous growth replicates the empirical impulse responses to monetary policy shocks, particularly the persistent effects on TFP.

Our paper is closely related to the recent work of [Anzoategui, Comin, Gertler and Martinez \(2017\)](#), [Benigno and Fornaro \(2017\)](#), [Bianchi, Kung and Morales \(2017\)](#), [Garcia-Macia \(2015\)](#), [Guerron-Quintana and Jinnai \(2016\)](#), and [Moran and Queraltó \(2017\)](#) who integrate endogenous growth into a business cycle framework. Among these papers, our framework is similar to the one used by [Benigno and Fornaro \(2017\)](#), who identify the possibility of an economy entering a phase of persistent liquidity trap and low growth due to pessimistic expectations. We complement their analysis by studying monetary policy in response to shocks to fundamentals such as liquidity demand shock, TFP shock and markup shock. We derive a closed-form expression for the linear-quadratic approximation of the representative agent's utility function. This expression generalizes the approximation derived by [Benigno and Woodford \(2004\)](#) to the endogenous growth environment and nests the exogenous growth as a special case.

The fully quadratic approximation allows us to analytically show the *hysteresis bias* of the policymaker at the ZLB. On the technical front, the hysteresis bias result may be surprising to scholars of business cycles given that we operate in an environment with an endogenous state variable (level of productivity) influenced by policy levers. Because of a standard assumption of linearity in the production function in endogenous growth models, past losses in productivity do not affect the allocation of resources between investment and consumption, and are *bygones* from the policy maker's perspective. Lack of credibility tools with the central bank results in permanent output shortfalls. This long-run consequence of policy constraints is, we argue, a reason for the policy-maker to pursue more aggressive stabilization policy through implementable rules during times of severe demand shortfalls.

In terms of our contribution to optimal monetary policy literature, we are closely related to [Galí \(2016\)](#) and [Erceg and Levin \(2014\)](#). [Galí \(2016\)](#) solves for optimal policy in an insider-outsider model of labor markets ([Blanchard and Summers, 1986](#)). [Erceg and Levin \(2014\)](#) evaluate monetary policy rules in an environment where workers may exit the labor force to reconcile the lower labor force participation rates in the economy. The focus of these analyses, like ours, is on an environment where the *Inverse Say's law* holds. We complement these analyses by allowing contractions in demand to negatively affect long-term supply via endogenous productivity growth. Because of the linearity assumption in the production function of endogenous growth model, the environment is tractable enough to yield closed form solutions. [Annicchiarico and Pelloni \(2016\)](#) study Ramsey policy and [Ikeda and Kurozumi \(2014\)](#) study the use of simple operational rules in an endogenous TFP growth setting away from the ZLB. To our knowledge, ours is the first paper to analyze the issue of permanent output gaps in the presence of severe demand shocks, particularly relevant once the ZLB is binding. The analytical result on *hysteresis bias* is new to the literature and has important implications for central banks' policy.

In terms of implications for stabilization policy, our paper is related to [DeLong and Summers \(2012\)](#), and [Fatás and Summers \(2015\)](#) who argue that these permanent deviations can be avoided using appropriate policy tools. The former two papers focus on fiscal policy as the appropriate mechanism to counteract the permanent negative effects, our analysis carves out a role for monetary policy, as suggested by [Yellen \(2016\)](#) recently. Further, [Reifschneider, Wascher and Wilcox \(2015\)](#) estimate that weaknesses in aggregate demand in the US had significant adverse effects on the supply side, and can account for 7% drop in the level of potential output. Exploiting exposure of publicly-traded firms to the 2007-09 financial crisis, [Ridder \(2017\)](#) finds evidence of persistence effect of the crisis on firm-level sales. Consistent with these results, our numerical examples indicate that the slowdown in productivity growth due to collapse of aggregate demand was quantitatively large during the Great Recession. Our theoretical analysis uncovers an implementable policy rule for the central bank that approximates welfare gains achieved under optimal policy.⁴

Our paper adds to the [Hansen/Summers/Gordon](#) secular stagnation literature. While our model can not generate permanent recessions (as in [Eggertsson and Mehrotra 2015](#), [Guerrieri and Lorenzoni 2017](#)) due to the representative agent setup, it formalizes how demand-side and supply-side secular stagnation ideas are related. These papers instead employ a permanent shock to the borrowing limit, and hence to the natural interest rate $r\text{-star}$. As a result, output is permanently depressed. In our setting, a temporary shock to $r\text{-star}$ propagates through a slowdown in TFP growth to generate a permanent effect on the level of output. We leave the quantitative investigation of this channel in a heterogeneous agent setting to future work.

While we model changes in R&D investment as the mechanism for persistent effects of transitory shocks in the presence of monetary policy constraints, we view our model as an organizing framework consistent with

⁴On fiscal policy, we show in Appendix [F](#) that investment tax credits are expansionary and in related work, it has been shown that debt-financed fiscal policy can be self-financing in hysteresis-prone environments (see [Eggertsson, Mehrotra, Singh and Summers \(2016\)](#)). However, our focus in the paper is on monetary policy. There has been an extensive literature on persistent effects of short-run fluctuations - [Stadler \(1990\)](#), [Fatas \(2000\)](#), and [Barlevy \(2004\)](#) study the effect of fluctuations on the long-run path of the economy.

other channels such as decline in firm entry (Gourio, Messer and Siemer 2016, Mehrotra and Sergeyev 2015), on-the-job training (Kiyotaki and Zhang 2017) or other mechanisms through which demand contractions can affect the path of potential output. Our empirical analysis finds that a contractionary monetary policy negatively affects firm entry. This extensive margin of monetary policy was uncovered by Bergin and Corsetti (2008). The *creative destruction* aspect of our setting can be interpreted to understand the persistent effects of decline in firm entry on the level of output.

The paper is organized as follows with some key results highlighted. Section 2 proposes a production economy with nominal wage rigidities, augmented with endogenous growth. We show that the model exhibits the *divine coincidence* property under liquidity demand shocks and monetary policy shocks. This property implies that monetary policy can completely undo these shocks and maintain the economy at the *first-best level*. An implication of this property is that the pre-recession trend of output is also first-best. Hence, these shocks allow us to tractably study monetary policy with endogenous growth. This new observation, we think, provides a useful pedagogical device to generalize the study of monetary policy to an endogenous productivity setting.

Sections 3 and 4 discuss the main theoretical results under liquidity demand and monetary policy shocks. Section 3 derives the positive results and Section 4 solves for optimal policy. A purely quadratic approximation of household's utility function allows us to decompose objectives of the policy-maker into key market distortions/wedges: the policymaker attempts to stabilize the labor wedge and the productivity growth rate at the efficient level. This latter consideration provides an additional rationale for stabilizing short-run fluctuations. We provide conditions under which the relative welfare weight on stabilizing fluctuations in the growth rate exceeds the corresponding weight on stabilizing the labor wedge and argue that these conditions are likely to hold in general. This has the implication that the optimal commitment policy equilibrium at the ZLB is more inflationary than the equivalent equilibrium in the exogenous productivity growth environment.

Section 5 solves for optimal policy under TFP shocks and wage markup shocks. We show that the optimal policy admits permanent output gaps in response to these shocks. This is because supply shocks reduce the amount of resources available for consumption and investment. As a result, it is indeed optimal to cut investment in R&D. A policy of hysteresis targeting is sub-optimal to deal with supply shocks relative to a standard Taylor rule. Our model makes a case for monetary policy to play an active role in committing to low interest rates to neutralize the supply side effects of adverse demand shocks. However as argued earlier, the entire productivity slowdown may not be attributed to the contraction in demand and thus having a policy rule that targets to recover back to the pre-recession trend may be highly inflationary and undesirable (also for reasons such as financial stability, not modeled in the paper). This final finding suggests the need for identifying the source of business cycle fluctuations in determining optimal stabilization policy, given that temporary shocks may have long run-effects on output. In Section 6, we assess the empirical relevance of output hysteresis and potency of monetary policy offsetting output hysteresis. Section 7 concludes and discusses extensions for future work.

2 A New Keynesian Model with Endogenous Growth

We integrate a textbook model of endogenous growth into a New Keynesian (NK) environment. Households set nominal wages in staggered contracts following [Calvo \(1983\)](#). On the production side, we use a discrete time version of the Schumpeterian growth model of [Aghion and Howitt \(1992\)](#), following [Aghion and Howitt \(2008, Ch. 4\)](#). There is a continuum of intermediate goods, each of which is produced by a sector-specific monopolist. Growth results from innovations that raise the productivity in the economy by improving the quality of products. These innovations are undertaken by profit-maximizing entrepreneurs in every sector, who spend final output in research. We assume that the central task of the monetary policy is to mitigate the effects of nominal rigidities, while fiscal policy is responsible for offsetting distortions associated with imperfect competition.

There are six main actors in our model - households, wage unions, firms, entrepreneurs, fiscal authority and the central bank - described below.

2.1 Households & Wage Setting

2.1.1 Households

There is a continuum of monopolistically competitive households (indexed on the unit interval), each of which supplies a differentiated labor service to the production sector. As is standard, we assume perfect risk sharing within the household. Household derives utility from consuming a final consumption good, disutility from supplying labor and utility from holding a risk-free bond.

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^j \left[\log(C_{t+s}) - \frac{\omega}{1+\nu} \int_0^1 L_{t+s}(j)^{1+\nu} dj + \xi_t \frac{B_{t+1}}{P_t} \right]$$

where $\nu > 0$ is the inverse Frisch elasticity of labor supply, $\omega > 0$ is a parameter that pins down the steady-state level of hours and the discount factor β satisfies $0 < \beta < 1$.

We use this particular specification of the utility function augmented with taste for holding risk-free bonds in order to introduce the liquidity demand shock ξ_t . [Fisher \(2015\)](#) models this shock as a micro-foundation for the risk-premia shock considered by [Smets and Wouters \(2007\)](#). The primary reason for our preference for this shock, as will become more clear in Section 3.2, is that ξ_t allows us to maintain *divine coincidence* ([Blanchard and Galí, 2007](#)).⁵ That is, a monetary policy authority following optimal policy

⁵[Anzoategui et al. \(2017\)](#) also use the same specification for the demand shock because this shock induces a co-movement in investment and consumption. This is also a relevant feature for our setting. This shock also has a standard interpretation of a shock to the money in the utility function if the central bank paid interest on reserves. In [Appendix G](#), we show the results for standard preference shocks to the household's utility as employed in [Eggertsson \(2008\)](#). Alternately, we could have introduced these shocks through the budget constraint of the household. [Amano and Shukayev \(2012\)](#) show that such shocks are important ingredients for building models with binding ZLB. We prefer introducing them as shocks to the “wealth in the utility” function. Intrinsic desirability for wealth is not unconventional. See for instance [Michaillat and Saez \(2014\)](#) for detailed references. One insightful reference is to [Keynes \(1919, Chapter II\)](#): Keynes observed that among the capitalist class of Europe before the First World War, “the duty of saving became nine-tenths of virtue and the growth of the cake the object of true religion”, where the “cake” refers to the stock of capital in the economy. Keynes adds that while saving is a sign of virtue and thus a source of utility, consumption is viewed as a sin: “There grew round the non-consumption of the cake all those instincts

rule does not face a trade-off in completely stabilizing fluctuations in output and inflation arising from such shocks. This shock is an example of *purely intertemporal* shocks considered by [Eggertsson \(2008\)](#) and we think is a desirable ingredient in order to generalize the study of monetary policy to an environment with endogenous TFP growth.

Labor income $W_t L_t$ is subsidized at a fixed rate τ^w . Households own an equal share of all firms, and thus receive Γ_t dividends from profits, and pay taxes τ^b on their incomes from riskfree bonds. Finally, each household receives a lump-sum government transfer T_t . Household's budget constraint in period t states that consumption expenditure plus asset accumulation must equal disposable income.

$$P_t C_t + B_{t+1} = (1 - \tau^b) B_t (1 + i_t) + (1 + \tau^w) W_t L_t + \Gamma_t + T_t \quad (1)$$

Utility maximization delivers the first order condition linking the inter-temporal consumption smoothing to the marginal utility of holding the riskfree bond

$$1 = \beta \mathbb{E}_t \left[\frac{C_{t+1}^{-1}}{C_t^{-1}} (1 + i_t) \frac{P_t}{P_{t+1}} (1 - \tau^b) \right] + \xi_t C_t \quad (2)$$

The stochastic discount factor by which financial markets discount nominal income in period $t + 1$ is given by:

$$Q_{t,t+1} = \beta \frac{C_{t+1}^{-1}}{C_t^{-1}} \frac{P_t}{P_{t+1}}$$

The household does not choose hours directly. Rather each type of worker is represented by a wage union who sets wages on a staggered basis. Consequently the household supplies labor at the posted wages as demanded by firms.

2.1.2 Wage Setting

Wage Setting follows the modeling of [Erceg, Henderson and Levin \(2000\)](#). Perfectly competitive labor agencies combine j type labor services into a homogeneous labor composite L_t according to a Dixit-Stiglitz aggregation:

$$L_t = \left[\int_0^1 L_t(j)^{\frac{1}{1+\lambda_{w,t}}} dj \right]^{1+\lambda_{w,t}}$$

where $\lambda_{w,t} > 0$ is the (time-varying) nominal wage markup. Labor unions representing workers of type j set wages (with indexation) on a staggered basis following [Calvo \(1983\)](#), taking given the demand for their

of puritanism which in other ages has withdrawn itself from the world and has neglected the arts of production as well as those of enjoyment". And while in principle saving could be used for consumption during retirement or consumption by offsprings, Keynes argues that this did not happen: "Saving was for old age or for your children; but this was only in theory the virtue of the cake was that it was never to be consumed, neither by you nor by your children after you." [Keynes \(1931, Chapter V\)](#) also comes back to this thesis.

specific labor input:

$$L_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\frac{1+\lambda_{w,t}}{\lambda_{w,t}}} L_t, \quad \text{where } W_t = \left[\int_0^1 W_t(j)^{\frac{-1}{\lambda_{w,t}}} dj \right]^{-\lambda_{w,t}}$$

In particular, with probability $1 - \theta_w$, the type- j union is allowed to re-optimize its wage contract and it chooses W_t^* to minimize the dis-utility of working for laborer of type j , taking into account the probability that it will not get to reset wage in the future. If a union is not allowed to optimize its wage rate, it indexes wage to steady state wage inflation $\bar{\Pi}^w$. Workers supply whatever labor is demanded at the posted wage. The first order condition for this problem is given by:

$$\mathbb{E}_t \sum_{s=t}^{\infty} (\beta \theta_w)^{s-t} C_s^{-1} \left[\frac{W_t^*(j)(\bar{\Pi}^w)^{s-t}}{P_s} - (1 + \lambda_{w,t}) \omega L_s^\nu(j) C_s \right] L_s(j) = 0 \quad (3)$$

By the law of large numbers, the probability of the nominal wage resetting corresponds to the fraction of types who actually change their wage. Consequently, the nominal wage evolves according to:

$$W_t^{\frac{-1}{\lambda_{w,t}}} = (1 - \theta_w) W_t^{* \frac{-1}{\lambda_{w,t}}} + \theta_w (W_{t-1} \bar{\Pi}^w)^{\frac{-1}{\lambda_{w,t}}} \quad (4)$$

2.2 Firms

2.2.1 Final Good producer

Households consume the *final* good, which is produced by perfectly competitive firms. These firms use identical production technology employing a homogeneous labor composite supplied by the wage union and a CES composite of intermediate goods weighted by their productivity:⁶

$$Y_t^G = M_t^{1-\alpha} L_t^{1-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^\alpha di, \quad (5)$$

where each x_{it} is the flow of intermediate product i used at time t , the productivity parameter, A_{it} reflects the quality of that product and M_t is the stationary (aggregate) productivity shock.

The firms choose L_t and $\{x_{it}\}_{i \in [0,1]}$ to maximize profits, taking as given both the wage index W_t and the prices of the intermediate goods $\{p_{it}\}_{i \in [0,1]}$. The first-order conditions for profit maximization give inverse demands for labor composite and intermediate good i .

$$\begin{aligned} \frac{W_t}{P_t} &= (1 - \alpha) M_t^{1-\alpha} L_t^{-\alpha} \int_0^1 A_{it}^{1-\alpha} x_{it}^\alpha di \\ \frac{p_{it}}{P_t} &= \alpha M_t^{1-\alpha} L_t^{1-\alpha} A_{it}^{1-\alpha} x_{it}^{\alpha-1} \end{aligned} \quad (6)$$

⁶We denote gross output by Y_t^G , to keep it distinct from Y_t (defined shortly after), which we refer to as the GDP analog of our model.

2.2.2 Intermediate goods producer

There is a continuum of intermediate products indexed by $i \in [0, 1]$, each of which is produced by a monopolist. The monopolist uses one unit of final good to produce one unit of his own good. As a result, every monopolist faces a marginal cost of P_t . Each intermediate monopolist chooses prices flexibly in every period to maximize his profits, taking as given the final sector's demand for its product. In particular, she solves

$$\max_{p_{it}} (1 - \tau^p) p_{it} x_{it} - P_t x_{it} \quad s.t. \quad \text{inverse demand in eq 6} \quad (7)$$

where τ^p is a sales tax/subsidy imposed on the monopoly price. Further, we assume that there is a competitive fringe in every sector who can produce the intermediate good with quality $\frac{A_{it}}{\gamma}$, where $\gamma > 1$ is the step-size of innovation and captures the quality distance between the frontier and laggard firms within a sector. As a result, the intermediate monopolist cannot charge a price higher than $p_{it} = \gamma^{1-\alpha} P_t$. In equilibrium, the monopolist charges a price given by:

$$p_{it} = \zeta P_t \equiv \min \left(\gamma^{1-\alpha}, \frac{1}{(1 - \tau^p)\alpha} \right) P_t$$

The linearity in the use of rival goods in the final goods' production function implies makes an intermediate firm's profits linear in the labor demanded in the final good production and own productivity.⁷ Higher own productivity enables the firm to capture a larger share of the demand for the final good. Profits are given by

$$\Gamma_t(A_{it}) = \chi^m P_t M_t L_t A_{it} \quad (8)$$

where $\chi^m = (\zeta - 1) \left(\frac{\alpha}{\zeta} \right)^{\frac{1}{1-\alpha}}$

2.2.3 Entrepreneurs

There is a single entrepreneur in each sector who invests $RD(z_{it})A_{it}$ of final good in research and development in period t , where $RD' > 0$, $RD'' > 0$.⁸ The dependence on productivity A_{it} is assumed for stationarity. With probability z_{it} , she is successful in making a process improvement. The productivity in sector i goes up by a factor of $\gamma > 1$ (step size of innovation) and she gets the monopoly rights (*patent*) over production of the intermediate good in the *following* period. If she fails to innovate, the incumbent monopolist continues

⁷This linear dependence on productivity is central to endogenous growth models. Jones (2005, Sec 6.2) formalizes this argument as “any model of sustained exponential growth requires that a particular differential equation is linear in some sense”.

⁸We follow Aghion, Akcigit and Howitt (2014) in this discrete time analog of their classic Schumpeterian model, but extend it to allow for a more general innovation production function that allows decreasing returns to R&D. Benigno and Fornaro (2017) use a similar model but with $RD'' = 0$. Assuming $RD'' > 0$ introduces decreasing returns to innovation, which is a feature stressed regularly in the innovation literature. As we will argue in Section 6, RD'' is a crucial parameter to quantify the output hysteresis.

to produce with productivity A_{it} until replaced by a successful entrant. Hence,

$$A_{it+1} = \begin{cases} \gamma A_{it} & \text{with probability } z_{it} \\ A_{it} & \text{otherwise} \end{cases} \quad (9)$$

The cost of research is increasing in the innovation intensity chosen by the entrepreneur and the existing level of technology in the intermediate goods' sector in which the entrepreneur operates. Specifically, we assume that $RD(z_{it}) = \delta z_{it}^\varrho$, where $\delta > 0$ and $\varrho > 1$ is the inverse elasticity of innovation intensity to R&D expenses. τ^r is a research subsidy provided by the government to the entrepreneur. The entrepreneur in every sector chooses z_{it} to maximize her expected discounted profits (from the patent):

$$\max_{z_{it} \in [0,1]} \{z_{it} \mathbb{E}_t Q_{t,t+1} V_{t+1} (\gamma A_{it}) - (1 - \tau^r) P_t RD(z_{it}) A_{it}\} \quad (10)$$

where value of the patent is given by:

$$V_t = \Gamma_t + (1 - z_{it}) \mathbb{E}_t Q_{t,t+1} V_{t+1} \quad (11)$$

The value function is linear in productivity due to the linearity in the production function (see Appendix A). Writing the normalized value function as $\tilde{V}_{it} \equiv \frac{V_{it}}{P_t A_{it}}$ and focusing on the symmetric equilibrium, we solve for interior solution (where $z_t > 0$):

$$\varrho z_t^{\varrho-1} = \beta \mathbb{E}_t \left(\frac{C_{t+1}}{C_t} \right)^{-1} \frac{\gamma \tilde{V}_{t+1}}{(1 - \tau_t^r) \delta} \quad (12)$$

According to equation (12), the entrepreneur chooses innovation intensity so that the discounted marginal revenue of an additional unit of innovation intensity is equal to the marginal cost of this unit. Increase in demand for final good increases the value of obtaining the patent. This is because of the *market size effect* - for a given cross-sectional distribution of productivities, increase in demand for final good requires higher quantities of intermediate goods to fulfill that demand. Since a monopolist's profits are increasing in the quality of its product, she can capture higher share of the increased market with a successful innovation.

2.3 Aggregation & Market clearing

The aggregate behavior of the economy depends on the aggregate (which also corresponds to the average in this case) productivity index defined as:

$$A_t = \int_0^1 A_{it} di \quad (13)$$

Because of the linearity assumption in the production function, we can aggregate the firm-level variables to form aggregate composites. Specifically $RD_t = \int RD_{it} di$ is the total R&D expenditure and $X_t = \int X_{it} di$

is the aggregate intermediate good produced in the economy. We can rewrite the aggregate output and nominal wage purely in the form of aggregates:

$$Y_t^G = \left(\frac{\alpha}{\zeta}\right)^{\frac{\alpha}{1-\alpha}} M_t L_t A_t \quad (14)$$

$$W_t = (1 - \alpha) \left(\frac{\alpha}{\zeta}\right)^{\frac{\alpha}{1-\alpha}} M_t A_t P_t \quad (15)$$

The growth rate of output in the economy is equal to the growth rate of aggregate productivity:

$$g_{t+1} = \frac{A_{t+1} - A_t}{A_t} \quad (16)$$

In any period, innovations occur in z_t sectors and $1 - z_t$ sectors use previous period's production technology. Aggregating across all the sectors, we get the following equation governing the dynamics of aggregate productivity:

$$A_{t+1} = \int_0^1 [z_t \gamma A_{it} + (1 - z_t) A_{it}] di = A_t + z_t (\gamma - 1) A_t \quad (17)$$

This means that the growth rate of the economy in period $t + 1$ is determined in period t and equals the number of innovating sectors times the step-size of innovation:

$$g_{t+1} = z_t (\gamma - 1) \quad (18)$$

The number of innovating sectors z_t may be interpreted as new entrants since incumbents do not undertake R&D investment in our model.

The final output produced in the economy is used for consumption, research and production of intermediate goods:

$$Y_t^G = C_t + RD_t + X_t \quad (19)$$

Henceforth, we define $Y_t^G - X_t = (1 - \frac{\alpha}{\zeta}) Y_t^G \equiv Y_t$ as GDP. To close the model, we assume net zero supply of risk-free bonds:

$$B_t = 0$$

2.4 Fiscal & Monetary policy

The government's budget is balanced every period, so total lump-sum transfers are equal to intermediate-good, wage and research subsidies.

$$P_t T_t = \tau^p \int_0^1 p_{it} x_{it} di + \tau^r P_t RD_t + \tau^w \int_0^1 W_t(h) L_t(h) dh \quad (20)$$

An independent central bank follows a Taylor rule in setting the nominal interest rate in the economy:

$$1 + i_t = \max \left(1, (1 + i_{ss}) \left(\frac{\Pi_{W,t}}{\bar{\Pi}_W} \right)^{\phi_\pi} \left(\frac{L_t}{\bar{L}} \right)^{\phi_y} \varepsilon_t^i \right); \quad \phi_\pi > 1, \phi_y \geq 0 \quad (21)$$

The nominal interest rate is set in order to target deviations of wage inflation and employment at respective steady state targets, as long as the implied nominal interest rate is non-negative. ε_t^i is assumed monetary policy shock. In conjunction with equation 14, we emphasize that this rule is analogous to the Taylor rule used to represent monetary policy response in an exogenous growth model.

2.5 Equilibrium

We formally define the competitive equilibrium of the economy in Appendix A. In order to get a stationary system of equations, we normalize the equilibrium equations by dividing the non-stationary variables such as consumption, output, real wage by the level of productivity. We define $c_t = \frac{C_t}{A_t}$ as the normalized (productivity adjusted) consumption and so forth. This allows us to solve for the steady state.

We find the steady state by imposing restrictions on the parameters such that the steady state satisfies a) $z \in (0, 1)$, b) consumption is non-negative and c) nominal interest rates are non-negative. In Appendix B, we analytically characterize the steady state by imposing additional assumptions on nature of wage rigidity and length of patent duration granted to the monopolist.

Steady State Efficiency

Because of the presence of research externalities and monopoly distortions, the private sector equilibrium is inefficient. We define the *efficient* steady state as the one in which the welfare of the representative household is maximized subject to the production technology of consumption good (eq 14), the law of motion of aggregate productivity (eq 17), and economy's resource constraint (eq 19) for a given level of initial productivity. The complete system of equations is provided in Appendix D.

Proposition 1 states that the steady state of the competitive equilibrium allocation is inefficient. This is due to the presence of three *static* distortions in our setup: (i) monopoly power in each intermediate goods sector, (ii) monopolistic competition in the labor market and (iii) inter-temporal research externalities. Whereas the first two distortions are common in the business cycle literature, the third distortion is specific to the endogenous growth literature. The entrepreneur is unable to reap all the benefits of her technology advancement because she gets replaced with positive probability by a new entrant (*surplus appropriability effect*). This makes her under invest in R&D. On the other hand, an entrant replaces the incumbent to profit from the full step size of innovation γ rather than the incremental gain in knowledge $\gamma - 1$. This *business stealing effect* (Aghion and Howitt, 1992) incentivizes the entrepreneur to over-invest in R&D. As a result of these two opposing forces, private investment in research can be higher or lower than the first-best.

We assume that the fiscal authority has access to lump-sum taxes, and so the first best allocation in the

steady state can be implemented by a set of constant taxes elaborated in the following proposition:

Proposition 1 (Steady State Efficiency). *Assuming the policy maker has access to non-distortionary lump-sum taxes, the steady state of the competitive equilibrium can be made efficient using the following three fiscal tools:*

- a) sales subsidy $\tau^p = 1 - \frac{1}{\alpha}$
- b) wage tax cut $\tau^w = \frac{1-\lambda_w}{\lambda_w}$, and
- c) research tax /subsidy $\tau^r = 1 - \left[\left(\frac{\gamma l^* (1-\alpha) \alpha^{\frac{1-\alpha}{1-\beta(1-z^*)}}}{1-\beta(1-z^*)} \right) \left(\frac{1-\beta}{(\gamma-1)c^*} \right) \right]$, where terms with * denote the efficient steady state values.

Proof. See Appendix E □

It is commonly argued in the endogenous growth literature that the private sector under invests in R&D (Jones and Williams 1998), and therefore growth rate is higher in the efficient steady state. These distortions would imply that in the absence of relevant fiscal instruments, monetary policy could affect the growth rate of output in the long-run. We follow the monetary economics literature and suppose that the average productivity growth rate is optimal and independent of monetary policy. As shown by Woodford (2003) and Benigno and Woodford (2004), the linear-quadratic approximation to the social welfare function around the non-stochastic efficient variables is justified if there are no distortions under price stability. In the parlance of the literature, there are no permanent differences between the efficient and natural rate of interest. The idea is to disassociate the welfare losses from fluctuations in growth rate from those arising from suboptimal growth solely due to monopoly distortions and research externalities. We make the following assumption:

Assumption 1. *The fiscal authority provides the set of constant subsidies described in Proposition 1 such that the competitive equilibrium is efficient in the steady state.*

The crucial difference to note from the earlier monetary economics literature is that monetary policy in our setting has a bearing on the long-run level of output even though we do not allow monetary policy to influence the steady state distortions. The monetary policy authority in our economy is assigned the single objective of offsetting short-run nominal distortions.

We log-linearize the competitive equilibrium around the efficient-steady state and define the following approximate equilibrium:⁹

Definition 2.1 (Approximate Equilibrium). The approximate competitive equilibrium in this economy with endogenous growth is defined as a sequence of variables $\{\hat{\pi}_t^w, \hat{c}_t, \hat{y}_t, \hat{g}_{t+1}, \hat{l}_t, \hat{L}_t, \hat{w}_t, \hat{\pi}_t, \hat{V}_t\}$ which satisfy the

⁹In the Appendix, we show the necessary condition for local determinacy for the model with perfect wage rigidity and single period patent granted to the monopolist. This derivation generalizes the local determinacy condition derived by Benigno and Fornaro (2017) to include diminishing returns to R&D investment. However, a closed form representation of determinacy is not possible in our benchmark setup with Calvo wage rigidities and stochastic patent duration. With this caveat in mind, we proceed by assuming local determinacy to characterize theoretical implications for policy. In all numerical simulations, we verify the local determinacy of the efficient steady state.

following equations, for a given sequence of exogenous shocks $\{\hat{\xi}_t, \hat{M}_t, \hat{\epsilon}_t^i, \hat{\lambda}_{wt}\}$.¹⁰

Aggregate Demand:

$$-(\mathbb{E}_t \hat{c}_{t+1} - \hat{c}_t + \hat{g}_{t+1}) + \hat{i}_t - \mathbb{E}_t \hat{\pi}_{t+1} + \hat{\xi}_t = 0 \quad (22)$$

Endogenous Growth equations:

$$(\varrho - 1)\eta_g \hat{g}_{t+1} = -(\mathbb{E}_t \hat{c}_{t+1} - \hat{c}_t + \hat{g}_{t+1}) + \mathbb{E}_t \hat{V}_{t+1} \quad (23)$$

$$\hat{V}_t = \eta_y \hat{y}_t - \eta_z \hat{g}_{t+1} - \eta_q (\mathbb{E}_t \hat{c}_{t+1} - \hat{c}_t + \hat{g}_{t+1}) + \eta_q \mathbb{E}_t \hat{V}_{t+1} \quad (24)$$

where $\eta_g = \frac{1+g}{g} > 1$, $\eta_y = 1 - \frac{(1-z)\beta}{1+g} > 0$, $\eta_z = \frac{\beta}{\gamma-1} > 0$, $\eta_q = \frac{(1-z)\beta}{1+g} > 0$

Market clearing:

$$\frac{c}{y} \hat{c}_t + \frac{\mathbb{R}}{Y} \varrho \eta_g \hat{g}_{t+1} = \hat{y}_t \quad (25)$$

$$\hat{y}_t = \hat{M}_t + \hat{L}_t \quad (26)$$

Wage setting:

$$\hat{\pi}_t^w = \tilde{\beta} \mathbb{E}_t \hat{\pi}_{t+1}^w + \kappa_w [\hat{c}_t + \nu \hat{L}_t - \hat{w}_t] + \kappa_w \hat{\lambda}_{wt} \quad (27)$$

$$\hat{w}_t = \hat{M}_t \quad (28)$$

$$\hat{\pi}_t^w = \hat{w}_t - \hat{w}_{t-1} + \hat{\pi}_t + \hat{g}_t \quad (29)$$

where $\kappa_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\nu(1+\frac{1}{\lambda_w}))} > 0$

Monetary policy rule:

$$\hat{i}_t = \max \left(-\frac{\bar{i}}{1+\bar{i}}, \phi_\pi \hat{\pi}_t^w + \phi_y \hat{L}_t + \hat{\epsilon}_t^i \right) \quad (30)$$

The Aggregate Demand, Market Clearing and Wage Phillips Curve are familiar to scholars of the new Keynesian business cycle literature. The new ingredient is the endogenous growth block (eqns 23-24). It is a log-linear transformation of profit-maximization condition of the entrepreneur.

The endogenous growth condition (eq 23) states that the entrepreneur makes her R&D investment decision based on the expected present discounted value of the future profits. Thus there are two forces governing her decision: the rate at which she discounts the future, and the expected value of future profits. In our model, the households retain ownership over all firms. Therefore, the rate at which a firm discounts the future is given by the stochastic discount factor of the household. A higher stochastic discount factor increases the entrepreneur's incentive to innovate (*discounting effect*) because of lower discounting of future profits. Second, higher expected future output increases her incentive to invest in innovation because of the *market size effect*, discussed above. Furthermore, a percentage change in innovation investment translates into $\frac{1}{\varrho\eta_g}$ p.p. change in productivity (gross) growth rate, where $\frac{1}{\varrho}$ is the elasticity of innovation intensity, and ϱ is assumed to be greater than 1 following the innovation literature (see Acemoglu and Akgigit 2012). This

¹⁰For any variable x , $\hat{x}_t = \log \left(\frac{x_t}{\bar{x}} \right)$, where $\bar{x}$ is the efficient/non-distortionary steady state. With few exceptions: $\hat{g}_{t+1}$ is the deviation of gross growth rate from the steady state value that is $\hat{g}_{t+1} \equiv \log \left(\frac{1+g_{t+1}}{1+g} \right)$. For liquidity demand shock $\hat{\xi}_t \equiv cA_t \xi_t$ since the steady state value of the shock is 0.

implies decreasing returns to investment in innovation- a higher value of ϱ signifies lower responsiveness of innovation success (and productivity growth rate as a result) to innovation investment.

Equilibrium Concepts and Policy instruments

We now provide a brief discussion of the natural rate allocation, the first-best allocation and the pre-recession trend allocation under the endogenous growth setting. Under liquidity demand and monetary policy shocks, the natural-rate and the first-best allocations coincide. Hence, the distinction between the three equilibrium concepts is immaterial. Specifically, it is the case that these shocks do not affect the flexible-wage equilibrium output, consumption and investment in R&D. The economy stays on the initial balanced growth path (BGP), also referred to as the *pre-recession trend*. Thus, any change in output in the sticky-wage economy emerges solely because of the nominal rigidities. This helps to isolate the role of monetary policy. Hence, in sections 3 and 4 we focus only on these two shocks. Such shocks are sometimes referred to as *purely intertemporal* shocks in the literature (Eggertsson, 2008) since these only affect the aggregate demand block.

The distinction between the natural-rate, the first-best and the pre-recession trend allocations will become crucial in section 5, because under supply shocks there can be divergence among these concepts depending on the type of shocks – productivity or mark-up shock. In order to provide a data counterpart for hysteresis, we define hysteresis as the deviation of output under a competitive equilibrium allocation from the pre-recession trend.

Before proceeding further, we clarify the distinction between *natural rate/flexible wage* and *first-best* allocations. We assume that the (normalized) economy is in the efficient steady state at beginning of time $t = 0$.

The *first-best allocation* is the competitive equilibrium allocation under flexible wages such that the fiscal authority utilizes (non-distortionary) time-varying taxes in order to maximize the representative agent's welfare. The *natural-rate* allocation (or interchangeably flexible-wage allocation) is the competitive equilibrium allocation under flexible wages such that the fiscal authority provides (non-distortionary) constant tax instruments outlined in Proposition 1.

There are two concepts of *flexibility* in the presence of a pre-determined state variable. One is the Neiss and Nelson (2003) definition of flexible wages, under which wages have been set flexibly since time 0 and remain flexible indefinitely. Wages set under this concept are called time-0 flexible wages. Second concept of flexibility is the Woodford (2003)'s definition where wages are set flexibly in the current and future periods taking as given the evolution of state variable. Wages set under this concept are called time-t flexible wages. Based on two concepts of flexible wages, there are *time-0 first best*, *time-0 natural rate*, *time-t first best* and *time-t natural rate* allocations.

We emphasize that the distinction between two natural rate concepts defined here is different from that imposed in exogenous growth environments with capital investment. Here, the natural rate of interest is always same under the two concepts. Only the levels of productivity and output differ. Importantly, this

difference in levels may be permanent depending on the central bank’s policy rule. In contrast, an exogenous growth environment renders a temporary difference in levels of capital, output as well as the interest rates. In the long-run, these concepts yield an identical allocation in that framework.

To avoid clutter of notation, henceforth we will use *first best* allocation for time-0 first best allocation and *natural rate* for time-0 natural rate allocation whenever possible without ambiguity. What is usually referred to as the potential output in the literature coincides with the time- t first-best allocation in our setting. For the ease of exposition, we refer to time-0 flexible wages as *flexible wages*.

Sticky wage allocation is the equilibrium allocation under staggered (nominal) wages such that the fiscal authority provides (non-distortionary) constant tax instruments outlined in Proposition 1. We refer the reader to Appendices D.9.1, D.9.2 and D.9.3 for a formal definition of these equilibria concepts.

Proposition 2. *The (time-0) natural rate allocation coincides with the (time-0) first-best allocation under liquidity demand and monetary policy shocks.*

Proof. See Appendix E □

Proposition 2 implies that the representative agent’s welfare is maximized if the policy maker could replicate the natural rate allocation. This outcome is always possible if the policymaker has access to time-varying tax instruments (see for example Correia, Nicolini and Teles 2008, and Correia, Farhi, Nicolini and Teles 2013). In Appendix D.6, we illustrate how the first-best can be implemented by appropriate state-contingent fiscal instruments even at the ZLB. Henceforth, we assume that the policy maker does not have access to these time varying fiscal instruments: fiscal authority satisfies Assumption 1, and adjusts lump-sum taxes every period to balance the budget. The Central Bank sets the nominal interest rate i_t on the risk-free (nominal) bond B_t subject to the ZLB constraint:

$$i_t \geq 0 \quad \forall t. \tag{31}$$

This is the bank’s only policy instrument.

Calibration and Impulse Responses

Our approximate equilibrium is linearized around a locally determinate steady state. We can analytically solve for the impulse responses under the assumption of AR(1) process for shocks. This is illustrated in the Appendix C for the case of liquidity demand shocks. However in order to illustrate the dynamics for the benchmark model, we calibrate the model with parameters reported in Table 1. Time is quarterly. There are eight parameters- we calibrate five of these using values standard in the New Keynesian literature. The discount factor β equals 0.99. Labor share $1 - \alpha$ is set to 0.67. Preferences are logarithmic in consumption and the inverse Frisch elasticity ν is set at 2. The wage adjustment probability is set such that wages are reset once every 4 quarters and the steady state wage markup is 10%. Monetary policy is assumed to follow a standard Taylor rule (eq 30) with $\phi_\pi = 1.5$ and $\phi_y = 0.5$.

We choose remaining three innovation parameters - step size of innovation γ , the (inverse of the) innovation elasticity ϱ , and cost parameter in R&D investment δ such that the model replicates annual steady state growth rate of 2%, annual firm entry rate of 10%, and R&D to GDP ratio of 25%. ¹¹ In the data, the private R&D to GDP ratio is 2% (NIPA 1953-2007). We do not have a data counterpart of this ratio under an efficient steady state. As shown in Proposition 1, the steady state in the private sector equilibrium may feature over/under-investment in R&D because of the research externalities. Jones and Williams (1998) estimated that the social return on R&D investment is at least four times the private return on R&D. In a more general setup than ours, they find that the forces such as business stealing effect are quantitatively dominated by loss in profits to innovating firms due to technological spillovers to other firms. Because agents cannot appropriate the full extent of profits from innovation, they underinvest. We choose a higher value of R&D to GDP ratio for ease of illustrating our results. We show robustness of our results to choosing a grid of R&D - GDP ratio in the range of 10% - 30% later in Section 4. In Section 6, we show a quantitatively realistic calibration of model, away from an efficient steady state, that can replicate key variable moments in the data.

Rows 1 and 2 in Figure 4 plot the impulse responses for normalized output, wage inflation, real interest rate and productivity growth rate for a positive shock to liquidity demand ξ_t and a contractionary monetary policy shock ε_t , each following an AR(1) process with persistence 0.90 and 0.92 respectively. A positive liquidity demand shock corresponds to a fall in annualized *natural interest rate* of one percentage point. It increases the desire for saving in the risk-free bond and thus diverts the resources away from consumption. Lower anticipated aggregate demand reduces investment in R&D by entrepreneurs, exerting a drag on productivity growth. Furthermore, a positive liquidity demand shock reduces household's stochastic discount factor, for a given nominal interest rate. This is equivalent to an increase in the “borrowing cost” for investment in innovation for the entrepreneur. These two forces act in the same direction to reduce investment in innovation. Hence, the productivity growth rate is lower following a contraction in demand induced by the liquidity demand shock.

Similarly, a surprise contractionary monetary policy shock (annualized 68 basis points) implies a 10 basis points increase in the real interest rate and tends to lower the nominal wage. Due to the stickiness of nominal wages, aggregate demand adjusts downwards. The equilibrium increase in the real interest rate combined with expectations of a lower future aggregate demand leads to a reduction in investment in R&D and, therefore, in TFP growth.

3 Demand Shocks and Output Hysteresis: Positive Implications

Standard New Keynesian models do not make a distinction between output Y_t and normalized output y_t because productivity is modeled as an exogenous process. Because of endogeneity of productivity, the relevant

¹¹Innovation success probability is interpreted as firm entry rate, consistent with the “creative destruction” literature.

variables of interest in the endogenous growth environment are the levels of output Y_t and consumption C_t . While the normalized output and the normalized consumption regain the pre-shock values, it is not obvious whether output and consumption recover back to the pre-shock trajectory. Is it the case that the level of output and consumption return to the counterfactual level that would have prevailed in the absence of demand shocks? Do shocks always lead to permanent displacement of these variables from their trajectory?

Here we examine the three positive implications of our model: (i) under a Taylor rule (eq 30), negative demand shocks displace the economy to a permanently lower output path, (ii) strict inflation targeting mitigates these permanent effects. But, (iii) even under a policy of strict inflation targeting these permanent gaps can be sizable when the ZLB is binding. However an inertial policy rule can aid the economy in recovery to the pre-shock trend. This section thus formalizes the argument that the *potential* output can contract following adverse demand shocks, but appropriately designed monetary policy rules can avoid such phenomena. We defer the discussion on optimal policy to the following section.

Let superscript e denote the *counterfactual path* of productivity and output in the absence of nominal wage rigidities. As discussed in previous section, we assume *time-0* flexible wages. Under the Taylor rule, a permanent gap emerges in the level of productivity and output relative to this counterfactual path. This follows intuitively from the observation that transitory shocks affect investment spending in research, which in turn determines productivity growth. If the policy rule is not a strict targeting (i.e. $\phi_\pi \not\rightarrow \infty \cup \phi_y \not\rightarrow \infty$) rule, presence of a positive real interest rate gap causes output and investment in innovation to decline. Since the policy rule has no inertia, the nominal interest rate recovers to steady state as the shock abates, leaving the economy on a parallel but lower trajectory relative to the counterfactual path.

Given local determinacy, we can derive the deviations in level of productivity and output from the respective levels under flexible wages as:

$$\log A_t - \log A_t^e = \sum_{s=0}^{t-1} \psi_g^i \epsilon_s^i; \quad \log Y_t - \log Y_t^e = \hat{y}_t + \sum_{s=0}^{t-1} \psi_g^i \epsilon_s^i$$

where $\psi_g^i > 0$ (detailed expression in the appendix C) and ϵ_t^i is the liquidity demand shock or the monetary policy shock at time t . We refer to the permanent deviation in output from the flexible wage benchmark as the *output hysteresis* (or alternately as *permanent gap*). Proposition 3 summarizes the first implication as follows:

Proposition 3 (Output hysteresis). *Given the monetary policy rule (eq 30) and in the absence of a zero lower bound constraint on the nominal interest rate, transitory (modeled as AR(1) process) liquidity demand shocks or monetary policy shocks induce a permanent gap in the time series of output from the counterfactual (flexible wage-) level of output if and only if monetary policy is not a strict targeting rule i.e.*

$$Y_T \neq Y_T^e \iff \{\phi_\pi, \phi_y > 0 : \phi_\pi \not\rightarrow \infty \cup \phi_y \not\rightarrow \infty\}$$

where $1 < T < \infty$ such that $y_T = y$ (steady state value) and $y_T \equiv \frac{Y_T}{A_T}$ is the normalized (or stochastically detrended) output.

Proof. See Appendix E □

Intuitively, as long as there is incomplete stabilization of normalized output i.e. $\hat{y}_t \neq 0 \forall t$, permanent gaps emerge in this economy. This is a consequence of a standard monetary policy specification assumed in eq 30. Normalized output (and the growth rate of productivity) exhibits a monotonic response to the shocks which approaches zero as the shocks die out. Thus, the sum of the productivity growth rate deviations from the steady state cumulate to the output hysteresis denoted henceforth by $h_t \equiv \sum_{s=1}^t \hat{g}_s = \hat{g}_t + h_{t-1}$.

Since entrepreneurs are forward looking, expectations of low future demand depresses investment in innovation. This causes a slowdown in productivity growth, which is not offset by the monetary policy rule. Hence, the potential output is permanently lower relative to the flexible wage economy (dashed line). As inflation and employment approach the steady state, output tends to this permanently lower level of potential output. Had the monetary policy followed a strict inflation targeting rule, these permanent effects would not have emerged. Note that under the considered demand shocks, the property of *divine coincidence* (Blanchard and Galí, 2007) holds. This implies that the central bank faces no trade-off in stabilizing output and inflation. Setting nominal interest rate so as to track the natural interest rate leads to perfect stabilization of the economy, and therefore there are no long-lasting supply effects from the demand shocks. Inability of the central bank to track the natural interest rate perfectly gives rise to permanent supply side deviations following demand shocks. This is the second key implication of our framework and formalizes the concept of *Inverse Say's law* recently put forward by Lawrence Summers.

An alternative to the strict inflation targeting policy is an inertial policy rule. Instead of strict inflation (or output) targeting, the central bank can target the history of deviations of productivity growth rate due to current and past shocks, which we refer to as *zero output hysteresis targeting* rule. Specifically if the central bank followed a hysteresis-augmented Taylor rule of the form:

$$\hat{i}_t = \max \left(-\frac{\bar{i}}{1 + \bar{i}}, \phi_\pi \hat{\pi}_t^w + \phi_y \hat{L}_t + \phi_h h_{t+1} + \hat{\varepsilon}_t^i \right), \quad (32)$$

which incorporates an additional target of cumulative sum of all deviations in productivity growth rate h_{t+1} resulting from history of shocks until time t , it could avoid the permanent gaps by committing to maintaining a path of interest rates until output is restored to the counterfactual path of output. Under this policy, the central bank credibly signals the willingness to tolerate excess wage inflation, in turn allowing real interest rate to fall so as to close the gap from the natural interest rate. A policy rule is defined as a *strict output hysteresis targeting* rule if $\phi_h \rightarrow \infty$ in equation 32. Such a policy may be especially desirable under circumstances such as the binding ZLB constraint.¹² We turn to this scenario next.

¹² Note that our use of “targeting” is distinct from that in the delegation literature. See Vestin (2006) for examples and references. That literature specifies a target for monetary authority in that the monetary authority chooses instrument in order

Binding Zero Lower Bound Constraint

In our discussion above, we show that a non-inertial policy of incomplete-stabilization (the Taylor rule in equation 30) causes permanent negative effects on the economy's potential output. A strict targeting rule, while non-inertial, could mitigate the permanent effects by tracking the natural interest rate. However, at the ZLB such a policy that tracks natural interest rate perfectly may not be implementable. This is because the nominal interest rate consistent with complete stabilization of (normalized) output is negative. Thus monetary policy may be constrained by a lower bound on the nominal interest rate resulting in episodes of output hystereses.

To illustrate, we follow Eggertsson and Woodford (2003) in setting up a two-state Markov Chain for the natural interest rate $\hat{r}_t^n$ in the endogenous growth economy.¹³ Structurally, a negative shock to the natural interest rate is an increase in the demand for risk-free bonds representing the *flight to safety* aspects of the financial crisis of 2007-09 (Krishnamurthy and Vissing-Jorgensen, 2012). We assume that the economy hits the ZLB unexpectedly in period 1, that is the nominal interest rate consistent with the stable inflation target breaches a policy lower bound constraint $r_t^n < i^{LB}$ (assume $i^{LB} = 0$).

$$\text{A1a} \quad \hat{r}_t^n = \hat{r}_S < 0 \quad \forall 1 \leq t < T^e \quad (33)$$

With probability μ , it continues to stay in the low state and with complementary probability, the shock returns to the steady state. We assume that the economy is back at the no-deflation steady state after a stochastic but finite time $T^e < \infty$.

$$\text{A1b} \quad \hat{r}_t^n = (1 - \beta) > 0 \quad \forall t \geq T^e \quad (34)$$

Further, we assume restrictions on parameters such that the equilibrium is locally determinate around the no-deflation steady state (Assumption A2).¹⁴ We calibrate the expected duration of ZLB at 4.6 quarters (14 months approx.) and the natural interest rate is set to -3% (annual). This calibration implies a drop of 5% in (normalized) output and 1% in nominal wage inflation relative to the target. The Central Bank is assumed to follow the Taylor rule (eq 30), targeting inflation and employment deviations from steady state.

Proposition 4 (Output Hysteresis at the ZLB). *Given the monetary policy rule (eq 30), a positive shock to liquidity demand such that the zero lower bound is binding for finite time T^e results in a permanent gap*

to maximize a welfare-objective with a quadratic term for the target. In most cases, this welfare objective is different from the societal welfare objective function. Instead, we simply augment the policy rule of central bank with an additional objective following Chung, Herbst and Kiley (2015). Strict targets may be implemented without an explicit instrument rule as in Chung, Herbst and Kiley (2015). For example, they implement a nominal GDP target with an equation that sums price level and output gap (from flexible level) to zero. We leave the extension of our framework to delegating problems as in Vestin (2006) to future work.

¹³In the notation of our framework, $\hat{r}_t^n = -\xi_t + (1 - \beta)$. $\xi > 1 - \beta$ makes the ZLB binding.

¹⁴In the appendix, we show the determinacy condition in analytically tractable setting with exogenous wages and one-period lived patents.

in output from the flexible wage counterfactual.

Proof. See Appendix E □

This result follows from the fact that a) when the ZLB ($t < T^e$) is binding, there is wage deflation and low output along equilibrium path, and b) after time $t \geq T^e$ (when the ZLB is non-binding), monetary authority raises the nominal interest rate to the level consistent with wage inflation target and full employment. While the economic indicators of employment and wage inflation return back to full capacity levels, the productive potential of the economy is permanently lower relative to the counterfactual path, in which the ZLB is not binding. Such losses in potential output can be sizable for reasonable durations of binding ZLB constraint. While, we leave a detailed quantitative analysis to Section 6, we illustrate the extent of hysteresis in our model at the efficient steady state.

In Figure 5, we plot output (solid line) when ZLB is binding for 28 quarters. Output falls on impact by 5% and in the subsequent periods productivity continues to grow at a rate slower than its (annual) steady state growth rate of 2% because investment in R&D is reduced during the recessionary period.

The output hysteresis is a key implication of assuming a textbook specification for the monetary policy rule, which prescribes raising interest rates as soon as deflationary pressures subside and employment is back to full capacity. Thus the long-lasting supply effects of demand shocks in our framework suggest a role for policy based on an inertial rule. Reifschneider and Williams (2000), Eggertsson and Woodford (2003) and others have shown that optimal policy at the ZLB involves some form of history dependence. The key new result is that an inertial rule is needed in order to offset negative supply side effects at the ZLB. The dashed line in figure 5 tracks the level of output under the hysteresis-augmented Taylor rule. This is an inertial policy which signals commitment by the central bank to maintain a path of nominal interest rates consistent with reversing past policy constraints/mistakes. A positive liquidity demand shock results in a drop in (normalized) output and wage inflation. However, since the central bank is committed to undoing any permanent gaps in output, it is willing to tolerate excess wage inflation (Figure 5, panel B). This reduces the real interest rate gap, which results in lower growth rate deviations on impact, and allows subsequent growth rate overshooting to undo past constraints on policy. Thus the hysteresis targeting policy embeds a *forward guidance* mechanism, credibly signaling the intention to tolerate excess inflation.

This channel of forward guidance offers a relatively clear description of the central bank's intentions to the public. So long as the potential output is lower than the counterfactual trend, the central bank maintains a low interest rate. It initiates gradual tightening of interest rates when actual output has sufficiently overshoot the trend so as to make up for the losses in economy's productive potential. Hence this policy is equivalent to targeting what is referred to in the literature as the *unconditional* output gap. Adolfson et al. (2011), following Neiss and Nelson (2003), define the unconditional potential output as the level of output that would prevail in the economy had prices and wages been flexible since the beginning of time. Consequently, the difference of output from this unconditional potential output is the unconditional

output gap. Previously, the new Keynesian literature has explored implications of unconditional output gap targeting in models with exogenous productivity, where the potential level of output is exogenous. As a result the economy always finds its way back to the (unconditional) potential output. In an endogenous productivity setting, potential output is responsive to fluctuations in demand because productivity growth results from pro-cyclical investment decisions of entrepreneurs. When the ZLB is binding, due to severe deficiency in aggregate demand, it is possible to have long-lasting losses in economy's potential relative to the flexible wage counterpart.

Our framework formalizes the hypothesis that the productive potential of the economy is endogenous to the monetary policy rule implemented by the central bank. We find that sizable permanent output gaps emerge in response to transitory shocks under standard Taylor rules for monetary policy. While appropriately designed monetary policy rules can offset these permanent gaps, the question arises - is it optimal for the policy maker to undo these long-run effects of transitory shocks? To answer this question, we next study the normative implications for the conduct of monetary policy in our endogenous growth framework.

4 Normative Implications for Conduct of Monetary Policy

In order to understand the *normative* implications for the conduct of monetary policy, we derive a quadratic approximation of the welfare function of the household. We use this to analyze optimal policy in response to liquidity demand and monetary policy shocks. We highlight three results. One, we show that the equilibrium under optimal policy does not involve permanent shifts in output. Away from the ZLB, this policy is equivalent to a strict inflation targeting rule discussed in the previous section. Two, at the ZLB, optimal policy commits to keeping interest rates lower in the future. Such a policy keeps the economy close to the pre-shock trend. Three, we show that a discretionary policy at the ZLB involves excessive output hysteresis relative to commitment policy. We label this as the *hysteresis bias* of discretionary policy. Numerically, we show that the strict hysteresis targeting policy implies significant welfare gains over a standard monetary policy rule. These welfare gains approximate the welfare gains achieved under optimal policy for a range of calibration of a key parameter ϱ , that determines the innovation sensitivity.

Quadratic approximation of Welfare

Another contribution of our paper is that we find a closed form solution for the quadratic approximation for the representative household under endogenous growth. This expression will help us solve for optimal policy, in a tractable manner. It generalizes the quadratic objective derived by [Benigno and Woodford \(2004\)](#) to an endogenous growth setting.

Proposition 5. *Assume that the economy is at the efficient steady state at time $t = 0$, with given productivity level A_0 . Under the sticky wage allocation, quadratic approximation of representative agent's lifetime utility function $\mathbb{W}_0$ around the non-stochastic efficient steady state is given by*

$$\frac{\mathbb{W}_0 - \mathbb{W}_0^*}{U_{c_{ss}} y_{ss}} = -\frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[\underbrace{\lambda_y \left(\hat{y}_t - \frac{\beta}{1-\beta} \frac{1}{\nu + \frac{y}{c}} \hat{g}_{t+1} \right)^2}_{(i)} + \underbrace{\lambda_g \hat{g}_{t+1}^2}_{(ii)} + \underbrace{\lambda_{\pi} (\hat{\pi}_t^w)^2}_{(iii)} \right] + \mathcal{O}(\|\hat{\xi}_t, \hat{\varepsilon}_t^i\|^3) + t.i.p. \quad (35)$$

(i) : labor efficiency gap, (ii): productivity growth rate gap, and (iii): wage inflation gap

where $\lambda_y = (\nu + \frac{y}{c}) > 0$, $\lambda_g = \frac{c}{y} \frac{\beta}{1-\beta} \left[\frac{\nu}{\nu + \frac{y}{c}} \frac{\beta}{1-\beta} + [(\varrho - 1)\eta_g + 1] \right] > 0$, $\lambda_{\pi} = \frac{1+\lambda_w}{\lambda_w} \frac{1}{\kappa_w} > 0$, $\kappa_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\nu(1+\frac{1}{\lambda_w}))} > 0$, $\eta_g = \frac{1+g}{g} > 1$ and *t.i.p.* stands for “terms independent of policy”. $\mathbb{W}^*$ denotes welfare under the (time-0) first-best allocation. The approximation is scaled by the constant $U_{c_{ss}} y_{ss} = \frac{y_{ss}}{c_{ss}}$ (evaluated at the efficient steady state).

Proof. See Appendix E □

This approximation is composed of three gaps/wedges - (i) labor efficiency gap, (ii) productivity growth rate gap, and (iii) wage inflation gap. These are the three stabilization goals for a planner maximizing social welfare.

Labor efficiency gap is the difference between the marginal product of labor and the marginal rate of substitution between consumption and leisure for the representative household.

$$(i) = mrs_t - mpn_t$$

where these terms denote deviations from the respective steady state values. Since we do not model price setting frictions in this simple benchmark model, and do not consider price-markup shocks, mpn_t corresponds to the (productivity-adjusted) real wage. Thus the labor efficiency gap captures the time-varying wedge in the disutility of the household from supplying labor at a pre-set nominal wage.

The third term (wage inflation gap) describes the loss in efficiency resulting from dispersion in wages across the members of the household. Wage dispersion, similar to price dispersion in standard New Keynesian models, is costly because firms hire different number of hours from various members of the household, causing marginal disutility of labor to vary within the household. Under flexible wages, both labor inefficiency gap and the wage inflation gap go to zero.

The new component - *productivity growth rate gap* - is a key ingredient of the endogenous growth model. Investment in R&D in a given period contributes to increase in productivity which persists into the indefinite future. These inter-temporal spillovers of R&D investment may not be internalized by the private agents and may result in too high or low responsiveness of investment relative to the first-best. Starting from a productivity level A_0 , the growth rate gap in eq. (35) captures the sub-optimality of deviations from the first-best level of productivity given by $A_t^* = A_0(1 + g_{ss})^t$ at all times $t > 0$. Under nominal rigidities, as discussed in last section, demand shocks may induce this permanent gap, thus leaving the agent permanently

worse off. This gap disappears under the exogenous growth assumption and the quadratic approximation simplifies to an exogenous growth setting discussed in Galí (2015).

In Corollary 1 we show the conditions under which the welfare loss resulting from these productivity growth rate deviations impose larger welfare loss than the changes in the labor efficiency gap. We provide a sufficient condition for the growth rate gap to be of higher importance for stabilization than the labor efficiency wedge. We argue below that this condition is likely to be satisfied even for extreme values of parameters, considered in the literature.

Corollary 1 (Importance of Growth Stabilization). *The relative weight on growth rate gap is higher than the relative weight on labor efficiency wedge if*

$$\frac{\beta}{1-\beta} > \frac{y}{c} \left(\nu + \frac{y}{c} \right) \quad (36)$$

Proof. See Appendix E □

Common calibration values of discount rate β at quarterly frequency lie in the range of $[0.98, 1)$. This implies a lower bound on the left hand side of the condition (36) at 49. We bound the right hand side as follows: Consumption Output ratio in the US has fluctuated between 0.54 and 0.66 from 1960 -2014 (BEA). Estimates of Frisch elasticity of labor $1/\eta$ in the micro literature lie between 0.1 and 0.5 (Chetty et al. 2016) while the macro literature uses the estimates in the range of (2,4). Using value of 0.1 for η^{-1} and 0.54 for c/y ratio, this implies an upper bound on the Right Hand Side at 22. Hence for a wide range of parameter calibration and estimates used in the macroeconomics literature, the welfare loss from a given growth rate deviations is higher than the welfare loss from a similar change in labor efficiency gap. This may not be surprising in light of the results from Section 3. A given deviation in growth rate from steady state has long run, potentially permanent effects. On the other hand, fluctuations in the labor efficiency pertain to welfare losses only in the period these are encountered. This highlights the importance of stabilizing the productivity growth rate around the first-best allocation.

Optimal Policy away from ZLB

We now turn to investigating the implications for the conduct of monetary policy in our model and show the main results outlined at the beginning of this section. First, we show that optimal policy involves setting the nominal interest rate in order to perfectly stabilize output and productivity along the first-best allocation.

Proposition 6 (Optimal Policy away from ZLB). *Given a process for liquidity demand and monetary policy shocks, optimal policy under sticky wage allocation tracks the natural rate of interest when the Zero Lower Bound constraint is slack.*

Proof. See Appendix E □

From Proposition 2, we know that the flexible wage allocation coincides with the first-best allocation. Under a sticky wage allocation, setting the nominal interest rate to track the natural interest rate implements the flexible wage allocation, thereby replicating the first-best allocation. This implies that the output follows a trend stationary process since the normalized output and productivity growth rate are always at the steady state. Hence, the following corollary follows;

Corollary 2. *When the ZLB is slack, the time series of output under optimal policy is a trend stationary process (integrated of order zero), that is,*

$$\log Y_t = a + b * t$$

where $a = \log Y_0$ is the initial level of output, and $b = \log(1 + g_{ss})$ is the steady state productivity growth rate.

Proof. See Appendix E □

We established that permanent output gaps are an undesirable feature of the endogenous growth economy in response to temporary demand shocks. The optimal policy does not allow for these hysteresis effects. But, as discussed in Section 3, it may not be possible to implement the optimal policy due to a binding ZLB constraint. As a result, under standard monetary policy rule, temporary contractions in aggregate demand result in permanent downward shifts in output. Should monetary policy offset these hysteresis effects at the ZLB? We take up this question next. Our analysis at the ZLB retains the assumptions (A1 and A2) regarding the exogenous dynamics of natural interest rate and local determinacy we made in Section 3.

Optimal Policy at the ZLB

We now turn to solving the optimal commitment policy, when the central bank can credibly commit to future state-contingent policy actions. At the ZLB, the economy is characterized by deflation and drop in output. By committing to pursuing accommodative policy in the future, the central bank manages expectations of private agents regarding the future path of inflation. Commitment policy achieves two objectives - (i) it reduces the severity of economic contraction during the ZLB, and (ii) it allows aggregate demand to overshoot the steady state level after the the ZLB stops binding. While the first (*forward guidance*) channel reduces the drops in output from the trend through reduced contraction in demand, the second channel tends to reverse past drops in output that occurred during the ZLB. The key takeaway from this analysis is that the optimal policy returns the economy close to the pre-recession trend. In the baseline calibration, the Taylor rule admits a permanent output gap of 0.88 percent. On the other hand, the optimal policy involves a permanent gap of only 0.085 percent.

The policy maker maximizes the lifetime utility of the household subject to assumption 1 and the competitive equilibrium conditions: (i) Euler Equation (eq 22), (ii) Wage Setting Block (eqns 27-29), (iii) Endogenous

growth block (eqns 23-24), (iv) resource constraints and market clearing conditions (eqns 25-26), and (v) the lower bound on the nominal interest rate (eq 31).

Since the first order conditions involve a complementary slackness condition, the solution to the optimal policy problem does not have a closed form. We solve it numerically for each state contingent realization of the shock. We provide the first order conditions in the appendix. The solution method is a version of shooting algorithm defined in Eggertsson and Woodford (2003).

Figure 6 shows the equilibrium output, inflation and nominal interest rate under a realization of the shock binding for 28 quarters. A central bank with the ability to credibly commit offsets the permanent output gap by promising to keep interest rates lower after the ZLB stops binding. Under optimal policy, the central bank minimizes total losses in welfare by trading welfare losses during the ZLB against the welfare losses from policy that arise after the ZLB stops binding. By committing to keeping interest rates lower upon exit from the ZLB, the central bank creates anticipation of a boom, which lowers the real interest rate during the ZLB. This has the effect of reducing the impact of the shock relative to a discretionary policy. On impact, the drop in wage inflation and output are only 0.04% and 1.23% respectively.

Upon exit from the ZLB, the central bank keeps interest rate lower for two additional quarters to follow through with its promise and thus creates a boom in output and inflation. Because of procyclicality of investment in innovation, the boom in output allows for growth rate to overshoot its target. Hence the permanent output gap is reduced substantially on account of two reasons a) *the forward guidance* channel of optimal policy and (b) the accommodation of excess wage inflation upon exit from the ZLB. In the steady state, output is only 0.085 percent below the (time -0) efficient path of output (solid line). In our numerical example, we have a two-state Markov chain for the shock process with an expected duration of ZLB of 4.6 quarters. On average, agents expect the central bank to keep interest rates lower for two quarters beyond 4.6 quarters implied by the shock. While we illustrate one realization of ZLB binding for 28 quarters, we want to emphasize that the expansionary effects of commitment do not arise because agents at time 0 expect the central bank to keep interest rates lower after 28 quarters. Such a long duration of expected ZLB generally violates local determinacy of the ZLB steady state.

Note that this is the optimal policy subject to the binding ZLB constraint. If the policymaker had access to time-varying proportional tax instruments such that it could replicate the flexible wage allocation, then the first-best allocation can be implemented (as shown in Appendix D.6). However, the optimal policy trades off welfare losses during the ZLB episode against welfare losses in the future in the absence of appropriate time-varying tax instruments. It is possible to avoid the permanent output gap altogether by a commitment to accommodating even higher inflation post the ZLB. Such a policy would be optimal had the social planner put higher weight on growth rate stabilization relative to the “true” welfare weight in eq (35) (as shown in row 3 of Table 2). However, under the “true” welfare weights, the policy maker allows some permanent output gap because perfectly neutralizing the permanent output gap comes at the expense of higher wage dispersion inefficiency upon exit from ZLB. Thus the ZLB introduces a short-run versus long-run tradeoff

for the central bank even when we have assumed away initial steady state distortions (by Assumption 1).

How does this optimal policy compared to the policy when the central bank does not internalize that it can influence productivity growth rate? That is, a policy-maker solves the optimal policy problem as before except she does not choose productivity growth rate. The optimal policy under this non-internalizing scenario does not allow the central bank to accommodate as high inflation after a ZLB episode as the optimal policy considered above would. Consequently, the permanent output gap is somewhat larger. Figure 7 shows the optimal policy under this “misspecified” setting and compares it to the optimal policy when the central bank internalizes the consequence of its actions on TFP growth rate. Quantitatively, this difference in the optimal policies is negligible (permanent output loss is 0.09% under misspecified problem compared to 0.085% under fully optimal rule). This is because the key problem in this economy is deficiency of aggregate demand. Since the R&D investment is pro-cyclical under liquidity demand shocks, stabilizing inflation stabilizes aggregate output and hence R&D investment. The main implication of this analysis is that while optimal commitment policy prescriptions are not quantitatively different under the two environments, the cost of not adhering to optimal commitment rules is elevated because of permanent output gaps. The key insight that we illustrate next is that we do not need a Taylor rule to generate output gaps. A minimum departure from fully optimal policy by introducing non-credibility is sufficient to generate permanent output gaps.

Markov-Perfect Policy at the ZLB

We analyze the optimal policy when the policy maker is *unable* to commit to policy actions announced in the future. Such a policy is referred to as the *discretionary* policy and the resulting equilibrium as the *Markov Perfect equilibrium* (MPE, formally defined in Maskin and Tirole 2001). The key result here is that the discretionary policy is characterized by a new dynamic inconsistency (Kydland and Prescott 1977) problem that we label as the *hysteresis bias*: once the ZLB stops binding, the nominal interest rate is set without any intent to offset the long-run effects of past contractions in aggregate demand. Hence, a policy of committing to lower future interest rates is not time-consistent because the central bank would increase the interest rates as soon as employment recovers back to full employment. The discretionary policy-maker treats past productivity losses as *bygones*.

The policy maker sets the current short-term nominal interest rate in order to maximize the quadratic approximation of the welfare function (eq 35) subject to assumption 1 and the constraints: (i) Euler Equation (eq 22), (ii) Wage Setting Block (eqns 27-29), (iii) Endogenous growth block (eqns 23-24), (iv) resource constraints and market clearing conditions (eqns 25-26), and (v) the lower bound on the nominal interest rate (eq 31). The problem is similar to the optimal commitment problem, except the policy maker cannot commit to future policy actions.

Proposition 7 (Optimal Discretionary Policy at the ZLB). *If Assumptions A1 and A2 hold and for a given level of productivity at time 0, A_0 , the Markov equilibrium is characterized by:*

$$\log A_1 = \log A_0 + \log(1 + g_{ss})$$

for $0 < t < T^e$

$$\hat{y}_t = \psi_y r_S^n < 0; \hat{\pi}_t^w = \psi_p r_S^n < 0; \hat{g}_t = \psi_g r_S^n < 0$$

$$\log A_{t+1} = \log A_t + \psi_g r_S^n$$

and when $t \geq T^e$

$$\hat{y}_t = \hat{\pi}_t^w = \hat{g}_t = 0$$

$$\log A_{t+1} = \log A_{t+1}^* + (T^e - 1)\psi_g r_S^n < \log A_{t+1}^*$$

where $\psi_y = \frac{(1-\beta\mu)\eta_C^{-1}}{(1-\beta\mu)(1-\mu)-\kappa_w(\nu+\eta_C)\mu\eta_C^{-1}} > 0$, $\psi_p = \frac{\kappa_w(\nu+\eta_C)}{1-\mu\beta}\psi_y > 0$, and $\psi_g = \frac{1-\frac{\varepsilon}{\beta}\eta_C}{\frac{\varepsilon}{\beta}\eta_g}\psi_y > 0$. A_{t+1}^* is the (time-0) first-best output at time $t+1$.

Proof. See Appendix E.1.2 □

Since the policymaker is unable to commit to future actions, optimal policy involves setting interest rates such that the economy returns to the (normalized) steady state as soon as the shock abates. This leads to excessive deflation during the ZLB relative to the commitment policy that involves $\hat{\pi}_{T^e}^w > 0$. This dynamic inconsistency problem identified as the *deflation bias* by Eggertsson (2006) is present in our setup. The new feature is that when the ZLB stops binding at stochastic time T^e , the discretionary policy maker does not offset the difference in level of productivity from the first-best. MPE thus admits a unit root in the time-series of productivity and hence output. This is the *hysteresis bias* we identify. Absence of credibility is sufficient to generate a permanent output shortfall.

Under discretionary policy, the policymaker re-optimizes every period, hence past deviations in growth rate from the steady state are no longer under the influence of a policy-maker at time T^e onwards. In order to bring the output back to the first-best output, the policy maker needs to incentivize excess investment in R&D after the economy has recovered back to full employment. Such an allocation is not desirable from the perspective of policymaker from time T^e onwards. This can be seen by directly looking at the first-order conditions of discretionary equilibrium. Once the shock to the natural interest rate is over, the policy-maker sets interest rate equal to the natural interest rate implying zero slack in the economy. Intuitively this happens because of the following reason: even though the level of productivity is an endogenous state variable, it only affects the absolute level of the stochastically-trending variables. The efficiency of resource allocation in the normalized economy is independent of the level of productivity. As soon as the central bank is able to set the normalized variables to their steady state values, it would do so. Past deviations of growth rate enter the welfare-loss as additive inefficiencies that cannot be influenced by policymaker optimizing at time $t \geq T^e$. In other words, what is relevant for the stabilization at time t is the gap from the time- t first-best allocation. Once the ZLB has stopped binding, setting interest rates such that employment is back

to the efficient steady state implements the time- t first-best allocation.

Figure 6 plots the path of output under MPE. There is an unanticipated shock at time $t = 1$. The output falls by 5% and continues to grow at a slower pace. When the shock stops binding in period $T^e = 28$, the economy is permanently at a lower output trajectory. This also corresponds to the policy under a standard monetary policy specification we discussed in Section 3. The output in the new steady state is permanently lower by 0.88 percent. Compare the equilibrium evolution of variables under discretionary policy to that under optimal commitment policy. The discretionary policy leads to excessive deflation and slack in the economy during the ZLB. Since the discretionary policy does not offset output hysteresis, it also leads to a larger permanent output gap.

This hysteresis bias of discretionary policy thus strengthens the result from Section 3 that output hysteresis is an artifact of policy-constraints faced by the central bank and does not arise because of irrational or inept behavior on part of the central bank. An implication of the hysteresis bias, we emphasize, is that it is sub-optimal for the central bank to redesign policy ex-post in order to offset past output hystereses. Hence, if the central bank could *credibly commit to being irresponsible* as suggested by Krugman (1998), it could not only reduce the deflation during the ZLB but also minimize the permanent output gaps. This raises the stakes for optimal commitment policy that the central bank must credibly communicate to the public ex-ante.

Alternative Policy Rules at the ZLB

Eggertsson and Woodford (2003) have underscored the complex nature of the optimal commitment policy in that it may not be feasible to properly communicate the policy stance to the public even if full credibility can be achieved. On the other hand, we showed that the discretionary policy-maker suffers from *hysteresis bias* and does not offset past inefficiencies. In this regard, alternate simple policy rules that have built-in commitment to reverse past policy mistakes assume importance. Such policy rules are presumably easier to communicate to the public such as commitment to keep interest rates low until the permanent output gap is filled. We illustrated the potency of this strict output hysteresis targeting rule in Section 3, given by:

$$h_{t+1} \equiv \sum_{s=1}^{t+1} \hat{g}_s = 0$$

where growth rate g_{t+1} is determined at time t .

The central bank ex-ante announces to set interest rates in order to completely cut down permanent losses in output. Such a rule is fully optimal in the absence of the zero lower bound. At the ZLB, though not fully optimal this rule may have a relative advantage in ease of communication to the public. Figure 8 plots nominal interest rate, output and wage inflation under such a rule contrasting with the realized paths of these variables under optimal commitment rule. The central bank keeps the interest rates low for an additional quarter as in the optimal policy. The forward guidance element through anticipation of higher inflation leads to a reduction in the real interest rate, which implies a lower drop in inflation and normalized

output (on impact). In the calibrated experiment, output drops by 1.17% on impact. The commitment to this simple rule implies that the central bank accommodates excess wage inflation up to 0.25% before it starts to raise interest rates gradually. Such a policy is relatively more accommodative than the optimal policy. Rows 2 and 5 in table 2 show that the hysteresis targeting policy achieves most of the welfare gains under optimal policy relative to a strict inflation target (or a discretionary) policy, conditional on ZLB being binding in period 1. An optimal commitment policy with higher weight on output gap can also close the output gap (as shown in row 3 of Table 2), resulting in similar welfare losses as the strict hysteresis targeting rule.

Contrast this policy with the policy of nominal wage level targeting (analogue of a simple price level targeting rule), where the central bank ex-ante announces its intention to set interest rates in order to attain a particular level w^* for the normalized output y_t adjusted nominal wages w_t^n :

$$w_t^n + \lambda y_t = w^*; \quad \text{where } \lambda \equiv \frac{1 + \lambda_w}{\lambda_w}$$

Figure 8 shows the realized paths of output inflation and nominal interest rate under wage level targeting against those obtained under optimal commitment policy. This simple policy also approximates the welfare gains achieved under optimal commitment policy (as seen in row 6 of Table 2) relative to the discretion policy, but results in a permanent output gap of 0.4 percent given that it is not as accommodative as the optimal policy.

Compared with wage level targeting, the hysteresis targeting rule requires the central bank to be more tolerant of higher inflation upon exit from ZLB. But it may have an advantage in communication and operationalization over a policy of wage-level targeting. A central bank's commitment to keep interest rate lower until output has been restored to pre-shock trend is more readily observable and such a policy may presumably be easier to communicate, assuming that achieving credibility is not a constraint for the central bank. Such a policy of hysteresis targeting is equivalent to a real GDP targeting rule because:

$$\log Y_t - \log Y_t^e = h_t$$

where Y_t^e denotes the counterfactual path of output under time-0 flexible-wage allocation

A third simple targeting rule is the Nominal GDP (NGDP) targeting rule (see Woodford (2012) and references therein). Since our benchmark model features only nominal wage-frictions, a comparison with conventional NGDP targeting rule may not be a useful comparison. The analogue of NGDP targeting in this simple framework is the $W \times Y$ rule:

$$W_t \times Y_t = W_t^e \times Y_t^e$$

where W_t^e is the counterfactual path of nominal wages under time-0 flexible-wage allocation. The central

bank commits to adjusting interest rates in order to achieve this target relationship whenever possible. As shown in row 6 of table 2, this $W \times Y$ rule also implies significant welfare gains over the discretionary policy and resulting in permanent output gaps close to those under optimal commitment policy. This is expected given that the calibrated nominal wage rigidity parameter is 0.75, which implies considerable nominal wage rigidity. As nominal wages are made more flexible by increasing the probability of wage adjustment, the $W \times Y$ rule approximates the strict wage-level targeting rule.

Table 3 compares permanent output gaps and welfare losses in these three operational rules against the optimal commitment policy for a range of innovation elasticity parameters. *Ceteris paribus*, we vary ϱ (inverse of innovation intensity elasticity) and δ (R&D cost parameter) in order to hit 2% growth, 10% firm entry rate and R&D to GDP ratio in the range of 10% to 30%, while keeping all other parameters fixed at values described in Table 1. Hysteresis targeting policy closely approximates the welfare gains achieved under optimal policy for this range of parameters. This analysis primarily highlights that a new operational rule that approximates welfare gains achieved under optimal policy is available for implementation in our framework. Since the standard NK models feature exogenous productivity, this rule is not available to the policy-maker in those environments.

5 Supply Shocks and Potential Output

In order to isolate the implications of negative demand shocks, we have so far abstracted from supply shocks. We complete the positive and normative analysis by examining the implications for monetary policy in the presence of stationary productivity shocks $\hat{M}_t$ and wage markup shocks $\hat{\lambda}_{wt}$. Our analysis highlights the importance of identifying the source of business cycle fluctuations in the design of optimal monetary policy.

We underscore two main results : (i) Taylor rule admits a unit root in output under temporary aggregate productivity shocks and wage markup shocks, (ii) permanent output gaps also emerge under optimal monetary policy. The reason for optimality of permanent output gaps under the two shocks is different. Under productivity shocks, time-0 first best allocation is characterized by output following a unit root process (relative to the pre-recession trend). Because optimal policy closely approximates the first-best, permanent output gaps are optimal under productivity shocks. Under wage markup shock, in contrast, the first-best allocation does not entertain permanent gaps. But optimal policy still features permanent gaps. This is because the central bank trades off high inflation volatility by committing to a lower output growth in the future.

Supply shocks and Taylor rule

We first show that under a Taylor rule, output is permanently below its pre-shock trend following a negative productivity shock or an exogenous increase in wage markups. The rationale for permanent deviations is similar to that discussed in Section 3. Under a Taylor rule, the endogenous (normalized) variables return to their steady state values as soon as the shock subsides. Since the transition to the steady state in the

approximate equilibrium is monotonic, the cumulative effect of the shocks on the productivity growth rate constitutes the hysteresis.

Figure 9 plots the impulse response of output to a productivity shock and wage markup shock. Shocks are parameterized such that output falls by 1 percent on impact.¹⁵ A negative productivity shock reduces the resources available for consumption and investment. Because of contraction in the size of the pie, there is excess inflation and low output. Since less resources are available for investment in R&D, productivity growth rate shrinks leaving output permanently below the pre-shock trend. Similarly, an increase in wage markups reduces demand for labor and hence output contracts. Because of lack of inertia in the Taylor rule, the effect of shocks on the the level of output is permanent.

Supply shocks and first-best allocation

Now we show the result that the (time-0) first best allocation in the presence of productivity shocks allows a unit root in aggregate productivity and hence output. The rationale for this result is as follows: A negative (temporary) productivity shock M_t reduces the resources available for consumption and investment into R&D. As a result, the growth rate of productivity declines and output is on a lower trajectory.

Proposition 8. *Given i.i.d. aggregate productivity shocks, output follows a unit root under the (time-0) first best allocation.*

Proof. See Appendix E.2 □

Further, note that the (time-0) first-best allocation in the presence of wage markup shocks is a trend stationary process. This is because these shocks can be neutralized with time-varying taxes.

Figure 9 plots the first-best allocation in response to productivity and markup shocks. In our numerical example, output under Taylor rule is close to the first-best allocation following a productivity shock. However, under a markup shock the Taylor rule keeps the economy further from the efficient path. We next discuss why allowing for a permanent output gap is optimal in response to supply shocks.

Optimal Policy

Optimal commitment policy under supply shocks admits large permanent output gaps. If the first-best allocation admits permanent output gaps, as is the case for productivity shocks, the optimal allocation will admit hysteresis as well, because monetary policy targets the first-best. But in the case of wage markup shocks, the first-best allocation does not admit hysteresis, yet the optimal allocation does.

This is because in order to reduce volatility of short-run inflation, the central bank accommodates a drop in the long run level of output. Consider the flexible wage allocation. Under this allocation, there is a loss in output on impact as shown in Figure 9. Positive wage markup shock implies an increase in wages, which

¹⁵We assume persistence of 0.9 for each of the shocks. As stated in Section 3, and illustrated for the case of liquidity demand shocks in the appendix, an analytical solution for these impulse response functions is straightforward to derive. As such, these parameter choices are for illustration.

reduces the demand for labor and hence output falls. The central bank can reduce this excess wage inflation by creating deflationary expectations. If the central bank could “credibly” commit to maintaining a negative output gap in the future, wage setters do not raise the nominal wages by the full extent of the wage markup shock. Thus, a persistent negative output gap implies entrepreneurs invest lower resources into innovation, which lowers the TFP growth rate temporarily. Thus, in a bid to reduce current wage inflation, the central bank promises to keep output permanently below time-0 efficient trend. This discussion generalizes the textbook inflation-output tradeoff to an endogenous growth setting.

Under productivity shocks, the optimal policy closely follows the first best allocation (figure 9) and also results in a permanent output shortfall. Unlike the textbook New Keynesian models, optimal policy under productivity shocks does not coincide perfectly with the (time-0) efficient allocation. This is because of the presence of dynamic inefficiencies in this model. Nuño (2011) shows in a real business cycle economy version of Aghion-Howitt model that the natural rate allocation is inefficient because of myopic entrepreneurs. The entrepreneurs discount future at a higher rate relative to the social planner because of a threat of replacement by a future entrant in their sector. This time-varying externality, in a creative destruction setting, generates a tradeoff for the central bank in stabilizing output and inflation at the first-best levels. In our calibration, we found a quantitatively small difference between the natural rate allocation and the first-best allocation, which is why the optimal policy and the efficient allocation almost overlap in Figure 9. We leave the quantitative investigation of these endogenous tradeoffs for future work.¹⁶

Welfare Evaluation of Alternative Policy Rules

We now compare welfare under various policies using the quadratic approximation derived earlier. Specifically, we compare Taylor rule (eq 30), simple nominal wage level targeting, simple hysteresis targeting, strict inflation targeting and optimal commitment policy.¹⁷ Table 4 reports the consumption equivalent welfare loss under various policies and shocks. Optimal policy minimizes the welfare loss, but also leads to a significant permanent output gap under supply shocks. A simple Taylor rule based policy outperforms strict hysteresis targeting rule under productivity and markup shocks - demonstrating the non-optimality of closing the permanent gap. A nominal GDP targeting rule closely approximates the optimal policy.

In sections 3 and 4, we underscored the importance of closing the permanent output gap and found that the hysteresis targeting rule approximates welfare gains under optimal policy. The contrasting findings under supply shocks suggest the need for carefully identifying the source of business cycle fluctuations.

¹⁶In an ongoing work, we analytically derive that persistent productivity shocks as well as discount rate shocks introduce a time-varying wedge between time-0 first-best and time-0 natural rate allocation, which we refer to as *dynamic research externalities*. This finding that productivity shocks introduce inefficient “wedges” implies that the set of shocks that maintain “divine coincidence” is possibly narrower under endogenous growth and that monetary policy faces short-run tradeoffs beyond those introduced by exogenous markup shocks. We provide an illustration of this externality for the case of discount rate shocks in Appendix G.

¹⁷As noted earlier, we follow Chung, Herbst and Kiley (2015) in implementing a simple version of operational rules. Simple nominal wage level targeting takes the form: $\hat{w}_t + \hat{y}_t - \hat{y}_t^f = 0$, where $w_t = \frac{W_t}{A_t}$ is the normalized wage level, $y_t = \frac{Y_t}{A_t}$ is normalized output, hats refer to log deviations from steady state and superscript f denotes the corresponding variable under flexible wage allocation. Simple hysteresis targeting rule is $h_{t+1} + \hat{y}_t - \hat{y}_t^f = 0$, where h_{t+1} is the hysteresis term defined as a sum of productivity growth rate deviations (from steady state) because of the history of shocks realized until time t .

6 Quantitative Evaluation

So far, we advanced a channel for hysteresis by allowing monetary policy to have an effect on R&D investments and hence TFP growth. Second, we solved for optimal policy at ZLB assuming a liquidity demand shock. Our analysis raises two questions: (i) does monetary policy influence productivity enhancing investments and the level of TFP in the data, and (ii) can a realistically calibrated liquidity demand shock generate a sizable recession. We answer both questions in the affirmative. We show empirical evidence consistent with key model prediction regarding monetary policy shocks. Contractionary monetary policy tends to temporarily reduce R&D investment, firm entry, and has a persistent effect on TFP. Further, we conduct numerical exercises using a medium scale version of our model. A one time increase in liquidity demand, calibrated to match the increase in premium associated with very liquid assets during the financial crisis, can explain a third of the drop in output observed in the data during the Great Recession.

6.1 Empirical Evidence

We estimate dynamic causal impacts of monetary policy on R&D investment, firm-entry and aggregate TFP. We interpret firm entry as an indicator for productivity enhancing investment for two reasons. First, we observe R&D investment for large firms in the data. These firms may not be significant drivers of TFP growth. Second, [Decker et al. \(2014\)](#), among others, have shown that firm entry is a significant driver of TFP growth. Consistent with the creative destruction literature, we interpret the number of innovating sectors in our model as counterpart of net firm entry in the data. The estimated impulse responses lend support for key predictions of our model: a contractionary monetary policy shock has a transitory negative effect on R&D investment and firm entry, and a persistent negative effect on TFP.

Empirical Strategy

Our empirical strategy is based on the recent literature ([Jordà, Schularick and Taylor 2017](#), [Ramey and Zubairy 2017](#), and [Barnichon and Brownlees 2016](#)) that combines the instrumental variables with the local projections (LP-IV) approach to directly estimate the structural IRFs. The series of (narratively- and high frequency-) identified monetary surprises e_t^m are treated as proxy for the true shocks ϵ_t^m . In the first-stage, we instrument a policy indicator (fed funds rate) with the relevant proxy.¹⁸ In the second stage, we run a sequence of predictive regressions of the dependent variable on the instrumented policy indicator for different prediction horizons. The estimated sequence of regression coefficients of the instrumented policy indicator are then the impulse responses.

¹⁸The use of external instruments or proxy SVAR was developed by [Stock \(2008\)](#), and extended by [Stock and Watson \(2012\)](#) and [Mertens and Ravn \(2013\)](#). [Gertler and Karadi \(2015\)](#) combine high-frequency identification and proxy SVARs to estimate monetary policy impulse responses. [Stock and Watson \(2017\)](#) discuss connections between proxy SVAR and LP-IV approaches.

More specifically, we estimate the following second-stage LP specification for horizons $h \in 0, \dots, H$:

$$y_{t+h} = \alpha^h + \beta^h \hat{f}f r_t + \sum_p \theta^{ph} Z_{t-p} + \nu_{t+h} \quad (37)$$

$\hat{f}f r_t$ is the predicted policy instrument from the first-stage regression using identified monetary policy instruments e_t^m . The set Z_t includes lags of dependent variable, the policy indicator, the policy instrument, and the current and lagged conditioning variables that identify exogenous fluctuations in the monetary policy instrument and improve precision of standard errors (see [Stock and Watson 2017](#)). The conditioning variables are log real GDP and log GDP deflator. The dynamic coefficients of interest are, therefore, the estimates of β^h for $h = 0, 1, \dots, H$. We compute standard errors based on heteroskedasticity and autocorrelation robust covariance matrix (Newey-West) estimators. The impulse responses for R&D investment at the firm-level are estimated in a similar manner, by conditioning on time-invariant firm-fixed effects, an aggregate time trend as well as two lags of time-varying firm-level controls (assets, cash holdings, short-term debt, and annual employment). The standard errors, in this case, are clustered at the firm-level.

Data: Instruments and Variables of Interest

We obtain two sequences of monetary policy surprises identified in the empirical literature. One is narratively-identified series from [Romer and Romer \(2004\)](#) (RR). They decompose changes in the intended federal funds rate at the FOMC meetings into a *systematic* and a *residual shock* component. The residual shock is extracted from unexplained variation in a regression of target funds rate changes on changes in Greenbook forecasts of inflation, output growth and unemployment. The original monthly series from 1969-1996 has been recently extended by [Wieland and Yang \(2016\)](#) until 2007. The second set of surprises are measured using high-frequency data on the federal funds futures contracts. The rates on these contracts reflect market expectations of the average federal funds rate during that month. To identify the exogenous part of announced changes in monetary policy, [Gürkaynak, Sack and Swanson \(2005\)](#) (GSS) calculate changes in the traded rate in a narrow 30 minutes window around the FOMC press releases. We obtain this series for 1990-2007 by combining the data from GSS with that extended by [Gorodnichenko & Weber \(2016\)](#). We sum up both the series to get a quarterly series of surprises, as in [Ottonello and Winberry \(2017\)](#) and [Wong \(2015\)](#). Figure 10 plots series of obtained shocks against the effective federal funds rate. We use information on surprises until 2005Q4, with response variables measured up to three years later (2008Q4), before the financial crisis.¹⁹

As measures for R&D investment, we use two quarterly data series (denoting sample lengths used in parentheses): (i) log R&D investment deflated by GDP deflator available from NIPA (1969-2007), and (ii) firm-level R&D investment constructed from COMPUSTAT database (1990-2007). The construction of firm-level R&D investment data is described in the Appendix and follows the methodology common in the

¹⁹We exclude the rate cut of September 2001, to avoid the noise in the rates caused by the terrorist attacks.

literature (Brown, Fazzari, and Petersen 2009, Terry 2017).²⁰ As measures of firm entry, we obtain two aggregate data series: (1) log number of business incorporations, and (2) log number of (net) establishment births. The first series is aggregated to quarterly level from a monthly Survey of Current Business produced until 1994 run by the Bureau of Labor and Statistics (BLS). The second series comes from a quarterly National Private Sector Business Employment Dynamics Data of BLS available 1993 onwards. Finally, log utilization-adjusted TFP and non-adjusted TFP measures are constructed by cumulating the respective TFP growth rate series obtained from (Fernald, 2014b) over 1969-2007. For brevity, we leave the discussion on data-selection, sample end-points and other robustness checks to the appendix.

Results

Figures 11 and 12 report our main empirical results using the GSS and RR instruments over different sample lengths. We report deviations from a constant trend following a 100 bps increase in federal funds rate. The shaded areas represent the 95% confidence intervals. We report the F-statistics for respective IRFs in the figures to verify instrument relevance. In most cases, the F statistic is above 23, a threshold for ten percent level constructed by Montiel-Olea and Pflueger (2013). Because of the shorter sample length, the GSS instrument does suffer from the weak-instrument issue.

In figure 11, we plot the IRFs for utilization-adjusted TFP and raw TFP. Consistent with the dynamics of the model, the utilization-adjusted TFP declines gradually after a monetary policy shock. The IRFs for raw TFP decline by more than the fall in adjusted TFP because of higher fluctuations in factor utilizations induced by monetary policy shocks. The leveling off of the decline in raw TFP is consistent with the persistent decline in adjusted TFP. This decline in TFP reaches -0.4% after 32 quarters (estimated on data from 1969-1999). Recently, Moran and Queraltó (2017) identify monetary policy shocks using a Cholesky ordering and find that a shock which increases the federal funds rate by 70 basis points, permanently reduces adjusted-TFP by 0.25%. With a non-parametric estimation strategy, we reach similar results.²¹

In Figure 12, we plot the response of the number of new incorporations, establishment births, aggregate R&D and firm-level R&D investments. Contractionary monetary policy shocks are found to have a temporary effect on these indicators. There is a delayed negative effect on R&D investment, which is not statistically significant for aggregate R&D but is statistically significant at the firm-level. Our benchmark model does not feature adjustment costs or frictions in R&D investment. As a result, the benchmark model exhibited a linear response of R&D investment to monetary policy shocks. In the medium scale model, we introduce adjustment costs in order to generate the curvature in the R&D response. The empirical findings align with the key predictions of our model: monetary policy influences long-run level of TFP. We next use these

²⁰To provide a broad picture, the firm-level R&D sample data in year 2000 contained 3441 firms for which R&D investment information was available. These firms collectively employed 9.7% of total US Employment (Fred code: PAYEMS), spent 86% of total private R&D measured by NIPA and had sales worth 26% of US nominal GDP. Data construction discussed in Appendix H.

²¹In the appendix, we show the IRFs for log real GDP and the implicit price deflator. The responses are similar to those documented by Ramey (2016). We confirm that the real GDP traces the response of raw TFP and levels off after 32 quarters to match the permanent decline in TFP. The permanent effect is, however, statistically indistinguishable from zero.

empirical findings to assess the quantitative relevance of our model.

6.2 Medium Scale DSGE Model with Schumpeterian growth

We showed in Section 6.1 that monetary policy can influence variables relevant for endogenous growth. Through numerical examples, we show that the liquidity demand shocks were quantitatively relevant during the recent financial crisis. Further, these shocks had a significant permanent effect on the level of GDP. The magnitude of this long-run effect depends crucially on parametrization of ϱ (inverse innovation intensity elasticity), which we elaborate next

6.2.1 Model

For brevity, we sketch the additional features introduced into the benchmark model and leave the detailed model discussion to appendix I. Capital is introduced in the production of intermediate good, following Howitt and Aghion (1998). Households own and accumulate capital subject to investment adjustment costs and rent it out to the intermediate good monopolists. The specification for investment adjustment costs follows the new Keynesian literature (Christiano, Eichenbaum and Evans, 2005). We append price-rigidity by introducing a retail sector that sells the final good produced by the perfectly competitive producer. Monopolistically competitive retailers set prices on a staggered basis following Calvo (1983). Further, we allow for variable capital utilization, and (internal) habits in consumption. Relative to the existing new Keynesian literature, we introduce adjustment costs in R&D expenditure. A particular functional form we use is $S^{rd} = \frac{\varkappa}{2} \left(\frac{R_t}{(1+g_{ss})R_{t-1}} - 1 \right)^2$, which the entrepreneur takes as given while making her R&D investment decision. This feature helps the model match the curvature in R&D responses that is found in empirical IRFs (see Figure 12 discussed above, as well as Moran and Queraltó 2017).

6.2.2 Calibration

We calibrate the model at a quarterly frequency. Table 5 reports the calibrated values of parameters, that we discuss next:

Steady State Parameters

Steady state labor supply is normalized to 1. Six parameters are set to match six steady state targets. Table 6 reports the steady state moments targeted by the model. We set β to 0.9990, to match an annualized real interest rate of 2.40%, along with (annualized) steady state output growth rate of 2%. Innovation step size γ is set to 1.55 to match the creative destruction rate of 3.6%. Howitt (2000) selects this value as it matches the empirical finding that a non-innovating U.S. company loses value at a 3.6-percent annual rate. Capital depreciation rate is set to an annual rate of 10% and steady state price markup is set to 15%. These are commonly used values in the business cycle literature. We calibrate α , δ , and ϱ such that model replicates

following (annual) steady state targets: Gross Private Domestic investment to GDP ratio of 17.2%, growth rate of 2%, R&D to GDP ratio of 2%, and Profits to GDP ratio of 6.2%. These are calculated from quarterly NIPA tables over 1947-2007.

We consider two variants of the model to vary the innovation sensitivity. Under first calibration, following [Benigno and Fornaro \(2017\)](#), we introduce an exogenous probability of patent loss $\mu = 11.4\%$. This implies that value of owning an intermediate goods' patent defined in equation 11 is modified to:

$$V_t = \Gamma_t + (1 - z_{it} - \mu)\mathbb{E}_t Q_{t,t+1} V_{t+1}$$

μ is chosen in order to match the (annual) R&D depreciation rate of 15%. An exogenous probability of patent loss reduces profitability from successful innovation, and in turn reduces R&D investment. Ceteris paribus, a higher exogenous patent loss probability requires higher returns from R&D investment, and thus lower ϱ . As a result, we find $\varrho = 1.07$. Schumpeterian growth literature following [Aghion and Howitt \(1992\)](#) has largely focused on the analytically tractable case of $\varrho = 1$ (cf. [Nuño 2011](#)). There is an extensive empirical literature that estimates this parameter (surveyed in [Hall, Mairesse and Mohnen 2010](#)) and finds a relatively wide range $\varrho \in (1.10, 5)$. Low ϱ implies higher sensitivity of innovation probability to R&D investment, which invariably allows the model to generate large growth rate fluctuations.²² Additionally, we recalibrate the model without the exogenous patent loss to get a calibration with higher $\varrho = 3.08$.

Parameters characterizing Endogenous Propagation

Remaining set of parameters are chosen from the standard business cycle literature, and we closely follow [Del Negro et al. \(2017\)](#) in calibrating these parameters. Inverse Frisch elasticity of labor supply is set to 1, wage markup is set to steady state markup of 15% to mirror the degree of monopolistic competition assumed in the product market ($\lambda_w = 0.15$). Nominal rigidities parameters are chosen, following the empirical evidence of [Nakamura and Steinsson \(2008\)](#) who find an average duration of price and wage contracts to be 4 quarters ($\theta_p = \theta_w = 0.75$). We calibrate habits parameter at $h = 0.5$. Varying these parameters to ranges considered in the literature does not significantly change our results. Investment adjustment cost parameter $S''(1)$ is set to 0.75, consistent with the estimates of price elasticity of investment (in the range of 1.22 – 1.36) in [Eberly \(1997\)](#) as well as [Christiano and Fisher \(1998\)](#).

As discussed above, we introduce curvature in R&D investment in order to replicate the curvature in the estimated impulse responses. [Brown, Fazzari and Petersen \(2009\)](#) estimate an Euler equation model for R&D investment at the firm level using Compustat data and find a baseline estimate for $\frac{\varkappa}{2} = 0.384$. Consequently, we set $\varkappa = 0.768$.²³

²²The marginal probability of success is decreasing in ϱ , keeping fixed the profitability upon successful innovation. Exogenous patent loss reduces the profitability of successful innovation, for a given probability of success.

²³They estimate the following equation for firm j , investing R&D $rd_{j,t}$ at time t :

$$rd_{j,t} = \beta_1 rd_{j,t-1} + \beta_2 rd_{j,t-1}^2 + \text{controls} + \text{fixed effects} + \text{error}_{j,t}$$

We interpret β_2 to be our model equivalent of $\frac{\varkappa}{2}$.

Policy Rule parameters and Exogenous shocks

We set the feedback coefficient on inflation and (normalized) output at 1.50 and 0.125 respectively (Taylor, 1993). Steady state government spending share $(1 - \frac{1}{\lambda_g})$ is set to 0.20. We discuss the persistence of shocks in the next exercises.

6.2.3 Quantitative Assessment

Impulse Response Functions

We shock the economy with a monetary policy shock that generates a 100 basis point (annualized) increase in nominal interest rate on impact. This is the same shock we used in the estimation in section 6.1, so that the results are comparable. We choose the persistence of monetary policy shock equal to 0.9, a commonly used estimate in the literature. We report the model IRFs in percent deviations from steady state at time 0. Figure 13 plots the IRFs for two calibrations of the model against the estimated IRFs for R&D investment, average firm entry, and utilization-adjusted as well as raw TFP.²⁴ While we do not explicitly model firm entry, we interpret probability of innovation z_t as the average firm entry in the following period consistent with the *creative destruction* aspect of our framework. The monetary policy shocks induce a negative transitory response for R&D investment, average firm entry and a permanent effect on TFP. Because of the presence of adjustment costs in R&D investment, R&D impulse response exhibits an U-shaped response, as seen in the estimated IRFs. R&D investment and firm entry are important sources of TFP growth in the model. While firm entry and R&D investment decline immediately, endogenous slow TFP growth results in a permanently lower level of TFP. Because of absence of technology adoption, TFP monotonically declines to a permanently lower level. As in the data, initial decline in raw TFP exceeds that of the adjusted TFP because of variability in factor utilizations. Overall, the model replicates the estimated dynamic impacts.

Importantly, the impulse response comparisons highlight a tradeoff in calibrating a value for ϱ . Lower ϱ implies higher sensitivity of R&D investment and hence a significant innovation gap emerges. The model, however, is unable to match the empirical response of R&D. Even for the extreme value of $\varrho = 3.08$, the model predicts a larger fall in R&D investment relative to that observed in the data. On the other hand, the model with low ϱ closely replicates the empirical impulse responses for TFP. Given the low responsiveness of R&D investment in the data, the model tends to fit the data under a firm entry interpretation. To the extent firm entry and other forms of investment are significant drivers of TFP growth, there is little reason to treat R&D expenditure in the model solely as the R&D expenditure incurred by publicly-traded firms. Another takeaway we emphasize is that there is lack of a clear disciplining device for calibrating the value for ϱ without making an assumption on the drivers of TFP growth. As a result, we will show results for both parameter calibrations.²⁵

²⁴In the model, we define raw TFP as sum of two terms (1) deviations in capital utilization from steady state, and (2) deviations in log TFP (pure) from its deterministic trend at time 0.

²⁵The range of values for ϱ considered is consistent with wide range of estimates found in aggregate and firm level studies (see

Simulating the Great Recession

We now simulate the model with a liquidity demand shock to study its ability to explain the Great Recession episode. In the model, the liquidity demand shock is characterized by the rise in premium for holding Treasuries - referred to as the *convenience yield* (Krishnamurthy and Vissing-Jorgensen, 2012). The size of the liquidity demand shock is calibrated to generate a rise in the liquidity premium of 180 basis points. This is the preferred parameter choice of Del Negro, Eggertsson, Ferrero and Kiyotaki (2017), who estimate the convenience yield using financial market data.²⁶ We chose the persistence of the shock to equal 0.938 and 0.95 in two calibrations of ϱ . These are chosen in order to generate a ZLB episode with expected duration of six quarters. This expected duration lies within the range of estimates found in financial market surveys during 2009-2010.

Figure 14 plots the evolution of output, inflation and nominal interest rate to the calibrated liquidity demand shock and compares it with the data, for sixteen quarters starting in 2008Q3. Column 1 shows the changes in the data relative to 2008Q3 (Lehman Brother's bankruptcy). We report percentage change in output from a linear trend estimated from 2000Q1 to 2007Q4, normalized to zero in 2008Q3. Output is constructed as the log sum of consumption, and investment from the NIPA tables. For inflation, we report the deviation of the annualized percentage change in the GDP deflator from 1.6% annual inflation rate. We chose this number to get the model to match annualized nominal interest rate of 4%. The nominal interest rate is the effective federal funds rate.

Given a relatively modest shock, the model can explain a significant component of the decline in output (-2.6% in the model versus -8.6% in the data). Furthermore, it implies a reduction in inflation of 0.9 percentage points following the shock, compared to an initial drop of 1% in the data. The nominal interest rate hits the zero lower bound, stays at zero for six quarters and sluggishly recovers back. We emphasize the close fit in the dynamics of the model with the data. The model implies no recovery to the 2000Q1-2007Q4 trend, as has been observed in the data. Calibrations of $\varrho = 1.07$ and 3.08 imply a 1.25% and 0.08% permanently lower output respectively, relative to pre-recession trend. In figure 14, we compare the evolution of consumption, investment and R&D investment with the data. The model replicates the broad empirical pattern of generating more decline in investment relative to consumption. Moreover, it generates a persistent decline in consumption relative to investment. The model with low ϱ (line with crosses) implies a more sluggish recovery in consumption relative to high ϱ (line with circles). Because of higher sensitivity of R&D investment, low ϱ generates a counterfactually large response of R&D investment. In the data,

Hall, Mairesse and Mohnen 2010). One of the commonly cited estimates come from Griliches (1990), who surveys the literature estimating relationship between R&D and patents (as an indicator of innovation output). Results differ on the estimation strategy: cross-sectional estimates of ϱ lie in range of 1 - 1.67, while within-firm time-series estimates are in the range of 1.5-3.3. Kortum (1993) reports estimates in the range of (1.3,10). More recently, Bloom et al. (2017) emphasize that research effort has gone up, with declining research productivity suggesting an increasing ϱ over time.

²⁶The results are qualitatively similar, but larger in magnitude, when we calibrated the shock to match rise in spread between AAA and 20 year Treasuries, or the spread between most recently used and older 10 year Treasury bonds of same maturity, called the *on-the-run/ off-the-run* spread.

R&D investment declined by 6%, while low ϱ implies a decline of 16%. In contrast, the model with high ϱ generates a 1.8% decline in R&D investment.²⁷

Hysteresis targeting during the Great Recession

How does a hysteresis targeting rule perform in a quantitative model? We assume that the central bank sets interest rate using the following hysteresis-augmented interest rate rule, with $\phi_h = 0.5$:

$$\hat{i}_t = \max \left(-\frac{\bar{i}}{1 + \bar{i}}, \phi_\pi \hat{\pi}_t^w + \phi_y (\hat{L}_t - \hat{L}_t^f) + \phi_h h_{t+1} + \hat{\varepsilon}_t^i \right) \quad (38)$$

where superscript f denotes the flexible-price-wage allocation, hysteresis $h_{t+1} = h_t + \hat{g}_{t+1}^f$, where g_{t+1} is determined by R&D investments in period t .

Figure 16 compares the evolution of output, inflation and interest rate under the above interest rate rule with $\phi_h = 0.5$ (Hysteresis targeting) to rule with $\phi_h = 0$ (Standard Taylor rule). We only plot the figures for the case of $\varrho = 1.07$. The results are similar in the case of $\varrho = 3.08$, although the permanent output shortfall is significantly smaller in that setting. Output falls by only 0.3% under hysteresis targeting compared to the 2.6% drop under Taylor rule. Inflation and federal funds rate are positive (in contrast to Taylor rule). An explicit commitment to targeting permanent output shortfalls creates inflationary expectations, which lowers the natural interest rate. Higher expected inflation provides more room for the central bank to offset declines in natural interest rates, as the central bank in this exercise has the power to reduce the impact of the shock by lowering the nominal interest rate. This example illustrates that the *hysteresis bias* embedded in a standard Taylor rule has quantitatively significant implications for the permanent level of output.

7 Conclusion and Discussion

This paper undertakes optimal monetary policy analysis in an environment where the long-run potential output of the economy is endogenous to short-run fluctuations in demand. An optimizing policy maker at the ZLB commits to keeping interest rates lower in order to offset the long-run effects of contraction in aggregate demand. However, a policymaker unable to commit to future interest rates does not offset permanent output gaps following a ZLB episode. This is the hysteresis bias of discretionary policy that we formalize.

There are however certain shortcomings in our analysis that we now highlight.

Our modeling assumption in the paper is that a new innovation gets adopted with certainty in the following period. This is clearly unrealistic. Comin and Hobijn (2010) and others have found that firms

²⁷Note that persistence of the simulated shock is calibrated such that the expected duration of ZLB is six quarters. Consequently, the recession is less severe. In the Appendix I.10, we show that a more persistent shock where the ZLB is expected to bind for twelve quarters performs better at replicating the drop in output, inflation, consumption and investment in the data. Moreover, the drop in consumption is more persistent and less severe than output and investment. Because of pro-cyclicality of R&D investment, a more severe recession, however, also implies a larger drop in R&D.

adopt new technology with average lags of up to 7 years. As long as contraction in demand results in lower investment in *knowledge creation*, the model of output hysteresis presented in this paper has insights for the conduct of monetary policy. The key elasticity determining the long-run effect of sub-optimal monetary policy is the elasticity of innovation to R&D expenditure. We have discussed robustness to calibrating various parameterizations of this elasticity. However we leave the investigation of optimal monetary policy in a richer model with implementation lags and technology diffusion (see for example [Anzoategui et al. 2017](#)) for future work.

While the empirical evidence on the interaction between monetary policy and long-term investment in research is still scant, there is a large literature emphasizing the potency of tax credits for spurring R&D growth ([Aghion, Hemous and Kharroubi 2014](#), [Dechezleprêtre, Einiö, Martin, Nguyen and Reenen 2016](#) among others). While time-varying fiscal instruments in the presence of non-distortionary lump-sum taxation can replicate the first-best outcome in our framework, we limited our focus in this paper to time-varying use of monetary policy instrument. We leave the analysis of optimal fiscal policy to future research.

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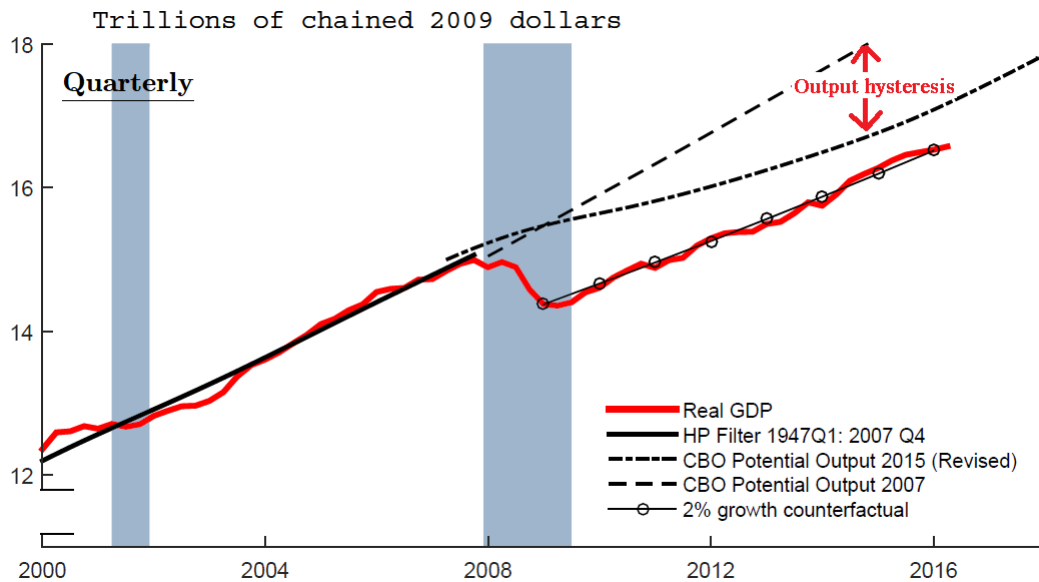
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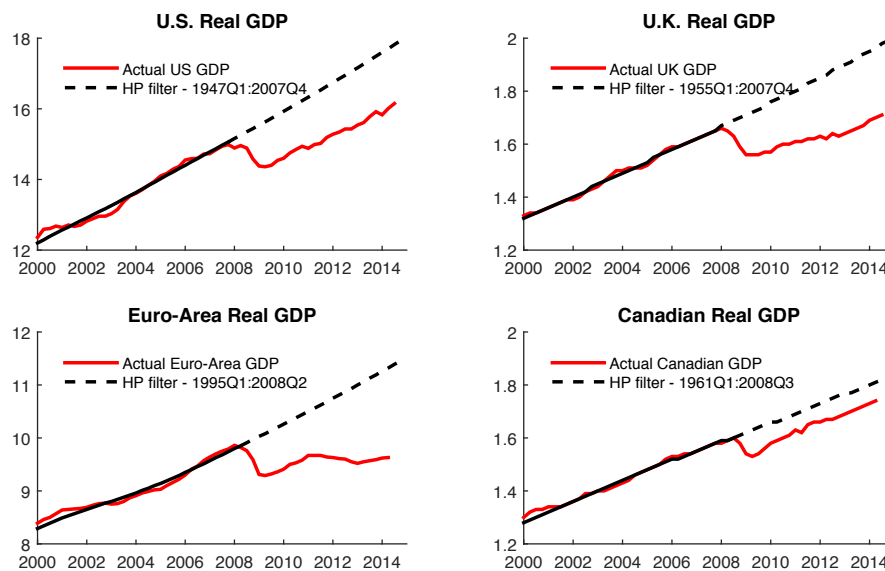
Figures

Figure 1: Real GDP



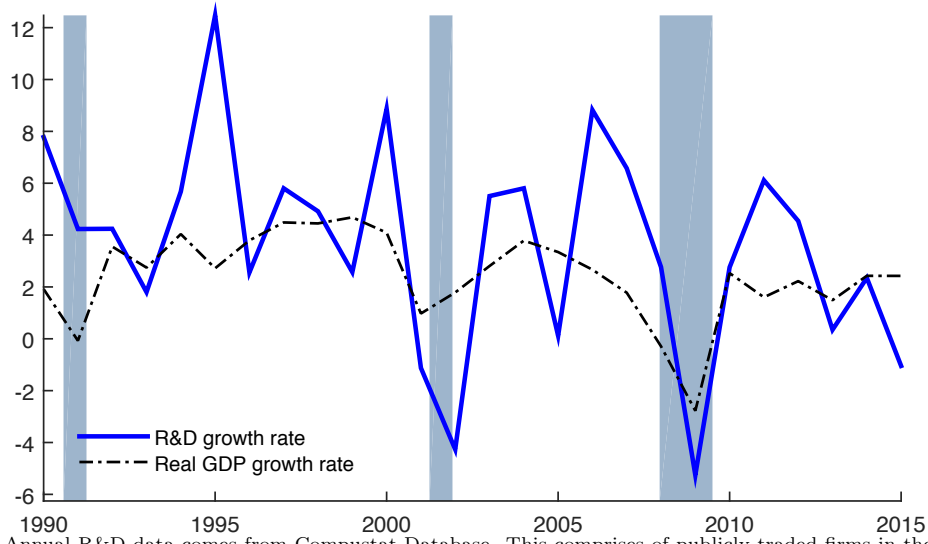
Note: Quarterly Real GDP data from St. Louis FRED database. CBO Potential Output 2015 and CBO Potential Output 2007 estimates are taken from the Congressional Budget Office February 2016 releases. The trend line until 2007Q4 is estimated on quarterly data from 1947 Q1: 2007 Q4 using Hodrick-Prescott filter with a smoothing parameter of 1600. The solid black line with circles is constructed using 2% annual growth rate starting from 2009. The shaded areas represent the recessions dated by NBER.

Figure 2: Real GDP in the OECD



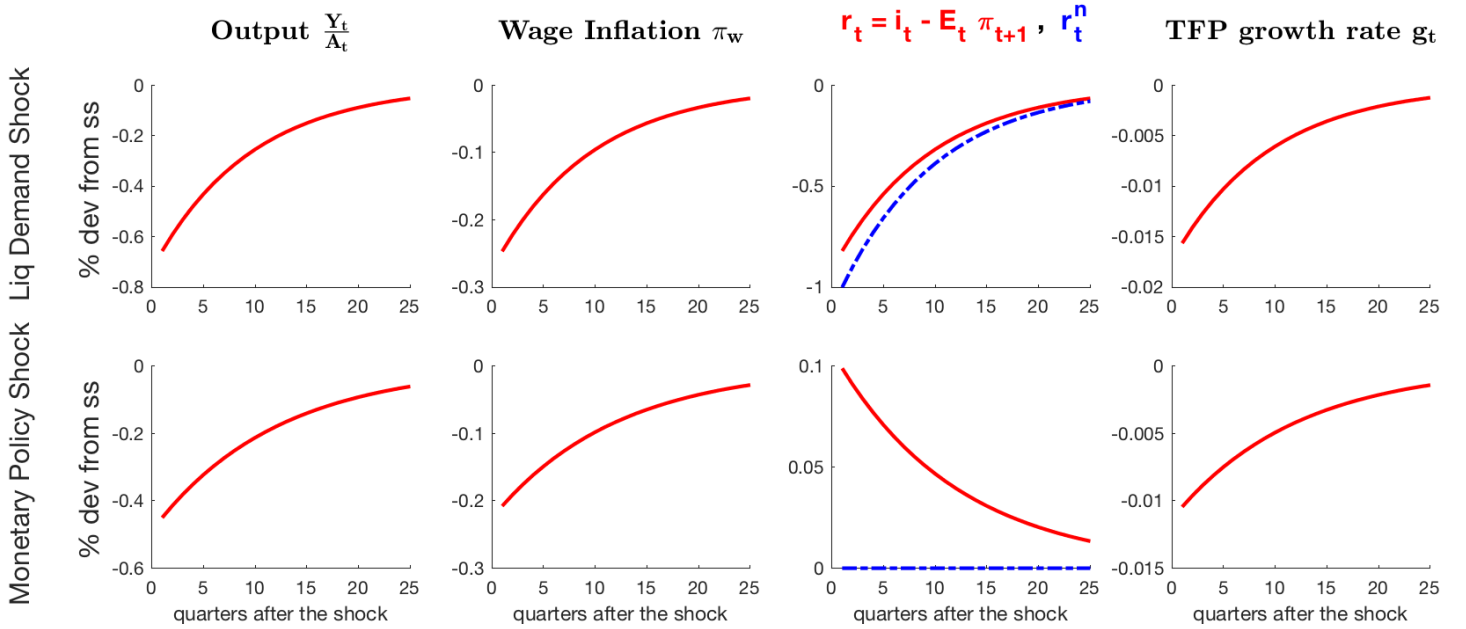
Note: Source: Martin, Munyan and Wilson (2015). The dashed black line is extension of HP-filtered GDP, using the 2007q4 growth rate.

Figure 3: Procyclicality of Private R&D



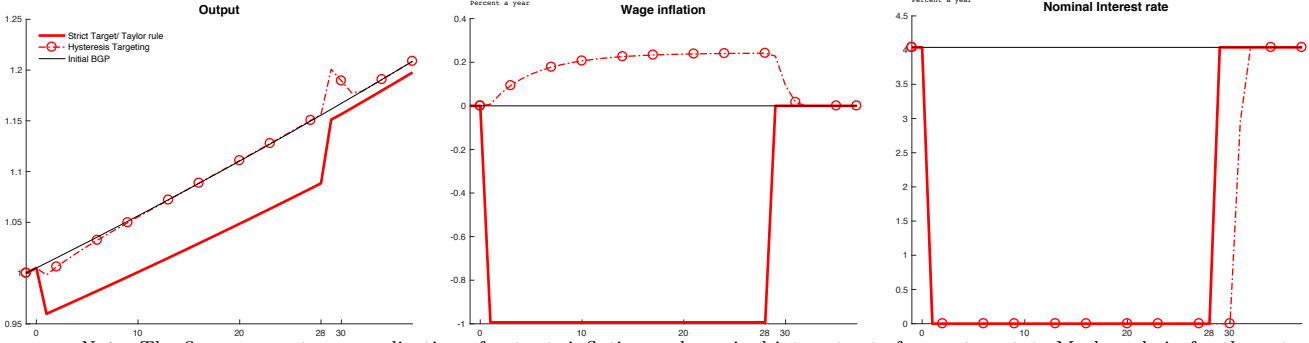
Note: Source: Annual R&D data comes from Compustat Database. This comprises of publicly traded firms in the US that undertake R&D in a given year. Annual GDP growth rate is taken from St. Louis FRED database. Original Graph by Barlevy (2007) used data until 2004. The correlation in the two series is 0.34. Y-axis plots growth rates in percentage points. The shaded areas represent the recessions dated by NBER.

Figure 4: Model based impulse response functions



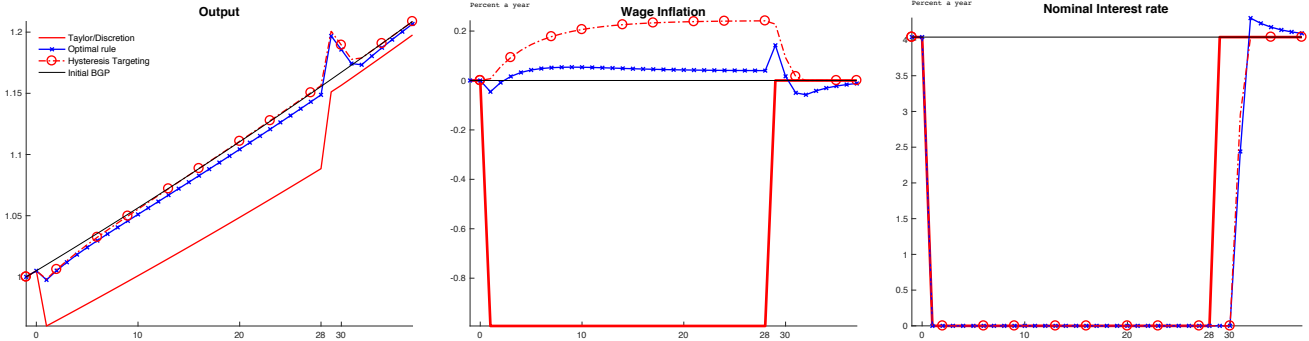
Note: The figures illustrate the impulse response functions from the benchmark model presented in Section 2. The IRFs are plotted in response to liquidity demand shock, and monetary policy shock, with persistence 0.9 and 0.92 respectively..

Figure 5: Strict Targeting Policy at ZLB



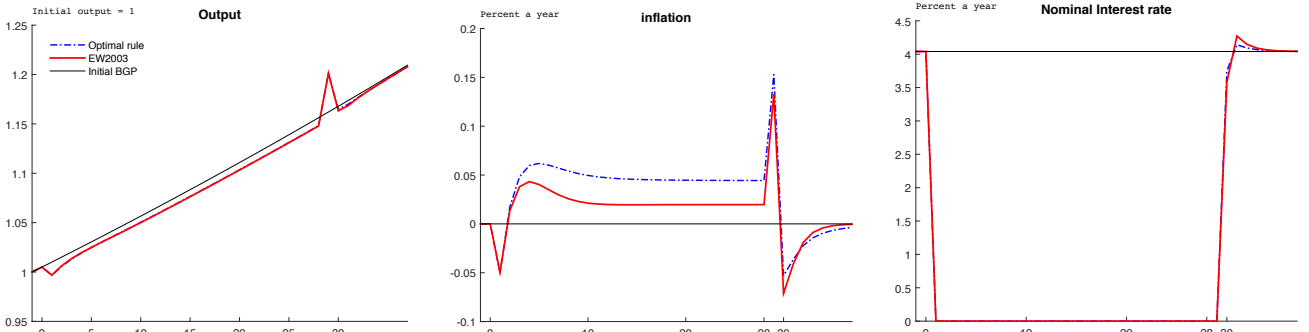
Note: The figures reports one realization of output, inflation and nominal interest rate from a two-state Markov chain for the natural interest rate under alternate policy equilibria. In period 1, the natural interest rate becomes negative and stays there for 28 quarters, and returns back to the full employment steady state. Realizations under a strict inflation targeting rule and under hysteresis targeting rule are shown. Wage inflation is plotted in deviation from steady state. Output in period -1 is normalized at 1. Black line in the output graph plots evolution of deterministic trend at an annual 2% steady state growth rate.

Figure 6: Optimal Policy at the ZLB



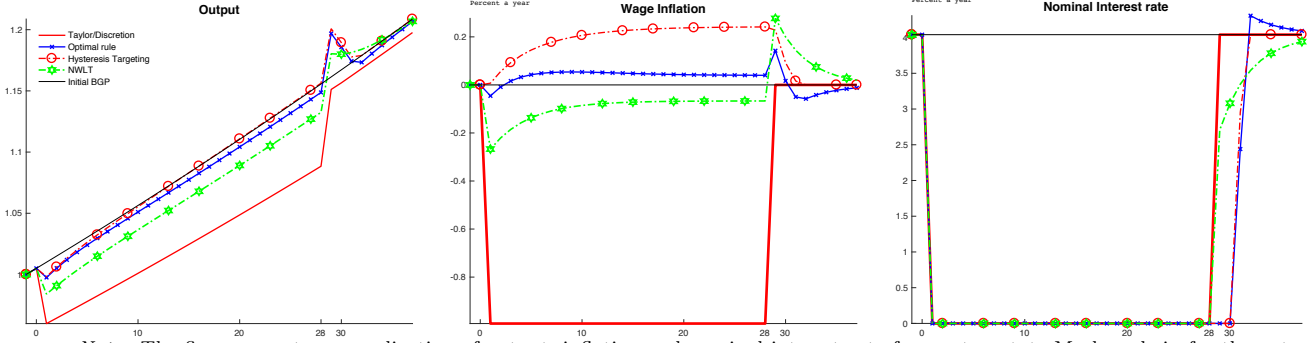
Note: The figure reports one realization of output, inflation and nominal interest rate from a two-state Markov chain for the natural interest rate under alternate policy equilibria. In period 1, the natural interest rate becomes negative and stays there for 28 quarters, and returns back to the full employment steady state. Realizations under a Taylor rule, Markov-Perfect Equilibrium (or discretionary) optimal policy, optimal commitment policy and hysteresis targeting are shown. Wage inflation is plotted in deviation from steady state. Output in period -1 is normalized at 1. Black line in the output graph plots evolution of deterministic trend at an annual 2% steady state growth rate.

Figure 7: Exogenous Productivity Comparison



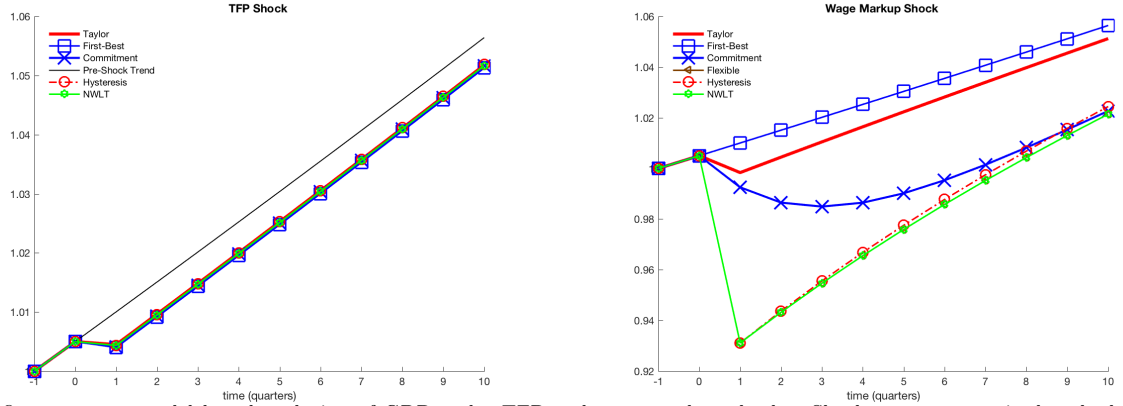
Note: The figure reports one realization of output, inflation and nominal interest rate from a two-state Markov chain for the natural interest rate under alternate policy equilibria. In period 1, the natural interest rate becomes negative and stays there for 28 quarters, and returns back to the full employment steady state. EW2003 denotes optimal “misspecified” policy when the central bank does not choose productivity growth rate. Optimal rule (dashed) denotes the optimal commitment equilibrium allocation. Wage inflation is plotted in deviation from steady state. Output in period -1 is normalized at 1. Black line in the output graph plots evolution of deterministic trend at an annual 2% steady state growth rate.

Figure 8: Alternate Rules at the ZLB



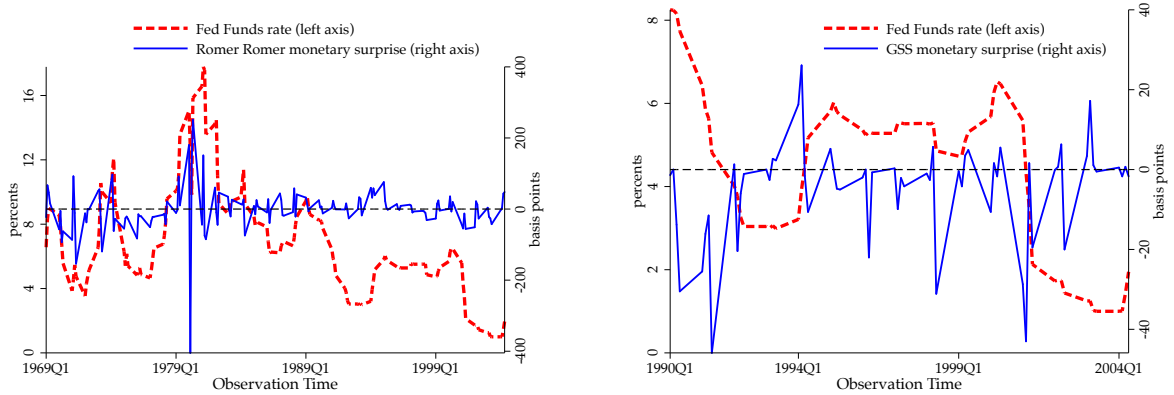
Note: The figure reports one realization of output, inflation and nominal interest rate from a two-state Markov chain for the natural interest rate under alternate policy equilibria. In period 1, the natural interest rate becomes negative and stays there for 28 quarters, and returns back to the full employment steady state. Realizations under a Taylor rule, Markov-Perfect Equilibrium (or discretionary) optimal policy, optimal commitment policy, hysteresis targeting and nominal wage level targeting rule are shown. Wage inflation is plotted in deviation from steady state. Output in period -1 is normalized at 1. Black line in the output graph plots evolution of deterministic trend at an annual 2% steady state growth rate.

Figure 9: Path of GDP under TFP and wage markup shocks



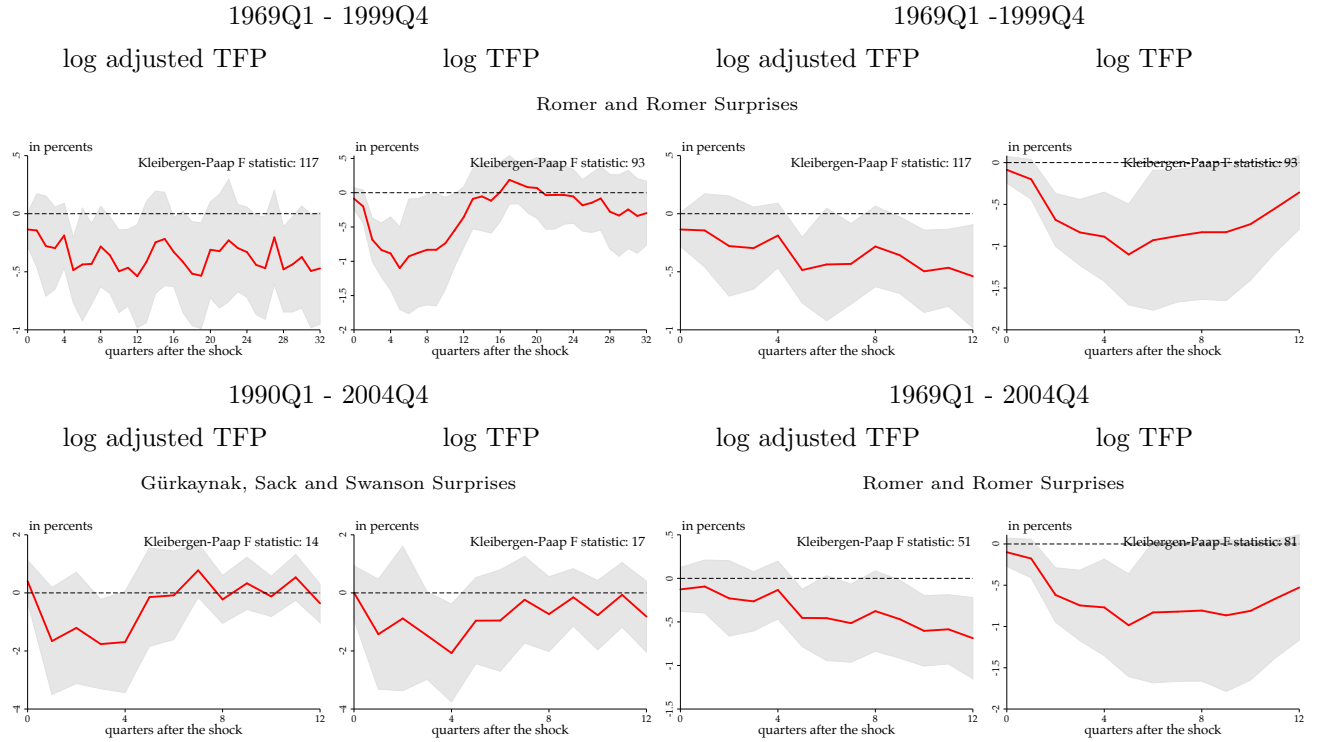
Note: The figure reports model based evolution of GDP under TFP and wage markup shocks. Shocks are parametrized such that output falls by 1 percent on impact. For illustration, persistence of shocks is chosen to equal 0.9. Output in period -1 is normalized at 1. Black line plots evolution of deterministic trend at an annual 2% steady state growth rate.

Figure 10: Policy Indicator and Monetary Policy Surprises



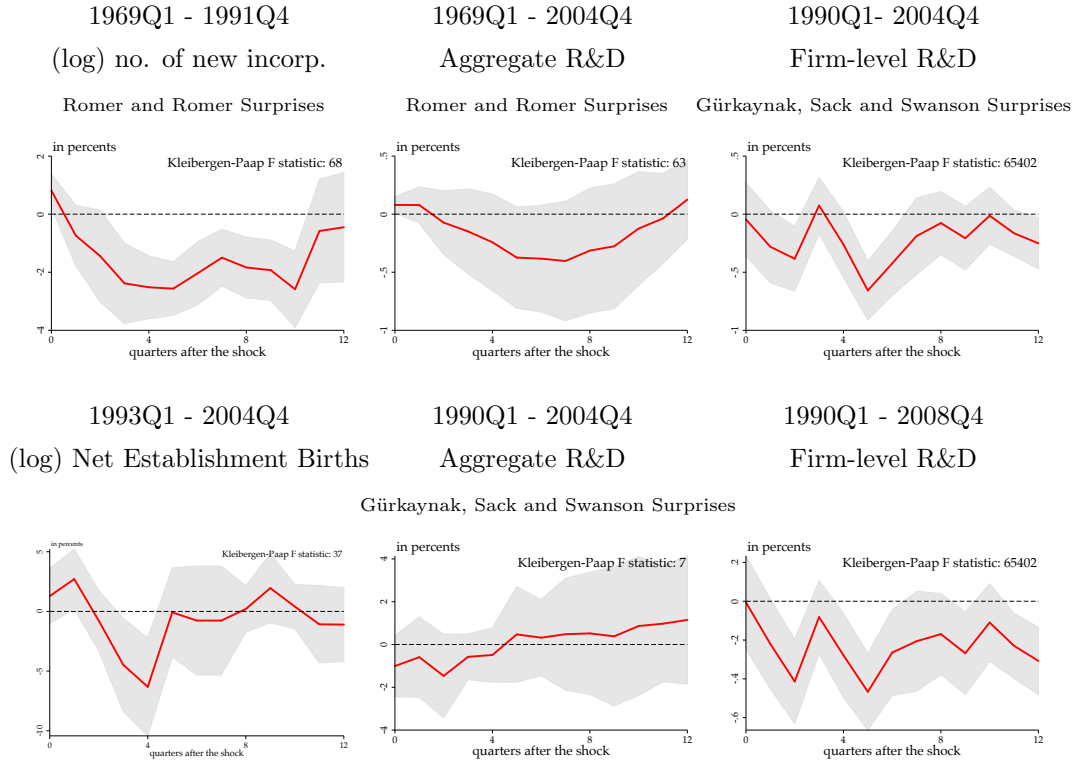
Note: The figure plots the Federal Funds rate against the monetary surprises. Two measures of monetary surprises are used in the main text. On the left, we plot the Romer & Romer (2004) narrative-identified monetary policy instruments. On the right, we plot the changes in current-month federal funds rate futures in a narrow 30 minute window around FOMC meeting announcements. These daily indicators are aggregated to the monthly frequency by adjusting for number of days left in the month. Monthly monetary surprises are summed to get the quarterly frequency aggregates. We take the FOMC days' announcement surprises from [Gürkaynak, Sack and Swanson \(2005\)](#)

Figure 11: Response of utilization adjusted TFP and TFP to 100 bps increase in Federal Funds Rate



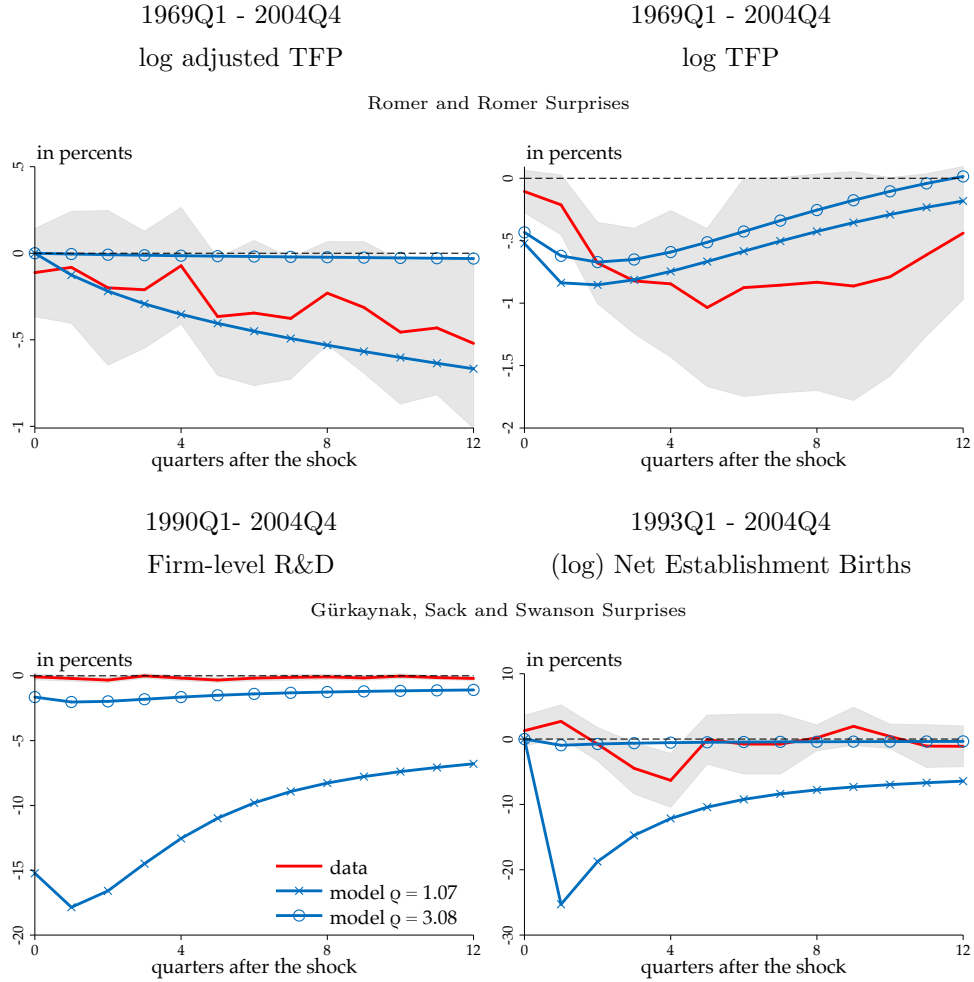
Notes: The figure plots the estimated impulse response functions for log utilization adjusted TFP and non-adjusted TFP. Time is in quarters. Sample length, and instrument used are denoted at the top of each row. IRFs are computed using a local-projections IV approach. Current and two past-lagged values of log real GDP and inflation rate are used as conditioning variables. Regressions also include past values of the proxy, the federal funds rate, and the dependent variable. Kleibergen-Paap F statistic for weak instruments are reported in the figures. The standard errors are calculated using HAR-Newey-West standard errors. The shaded areas denote 95% confidence intervals.

Figure 12: Response of Firm Entry, Aggregate R&D and Firm-level R&D to 100 bps increase in Federal Funds Rate



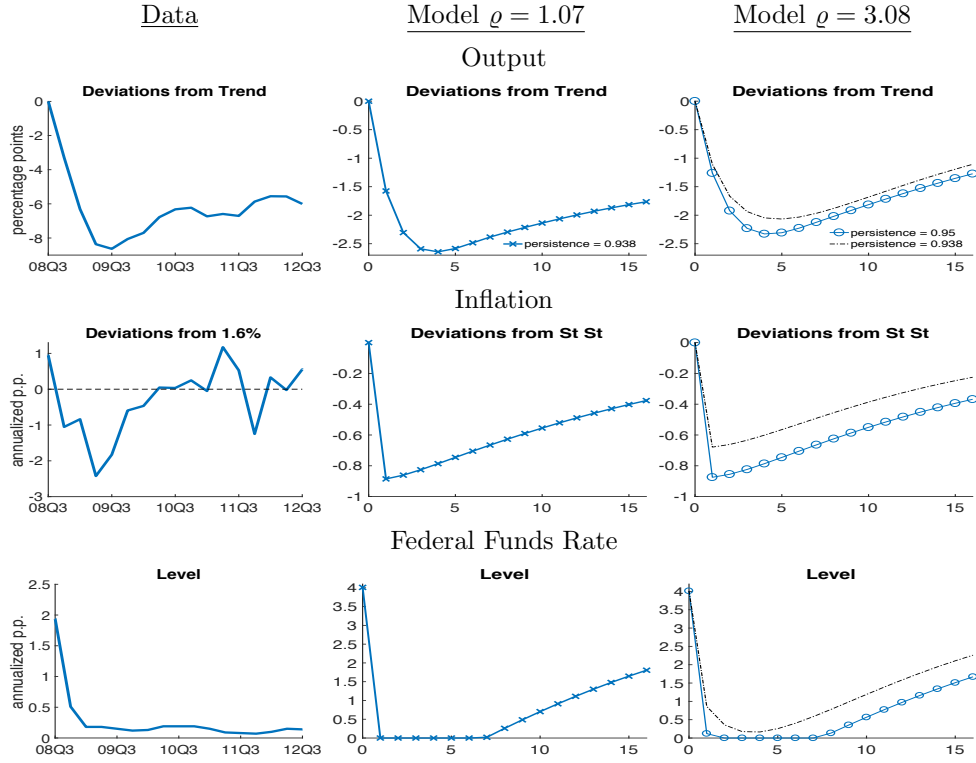
Notes: The figure plots the estimated impulse response functions for firm entry, aggregate R&D and firm-level R&D. Two indicators for firm-level R&D are used: (1) log number of new incorporations available over 1969-994, and (2) log number of net establishment births available since 1993. Time is in quarters. Sample length, and instrument used are denoted at the top of each row. IRFs are computed using a local-projections IV approach. Current and two past-lagged values of log real GDP and inflation rate are used as conditioning variables. Regressions also include past values of the proxy, the federal funds rate, and the dependent variable. Kleibergen-Paap F statistic for weak instruments are reported in the figures. The standard errors are calculated using HAR-Newey-West standard errors. The shaded areas denote 95% confidence intervals. Firm -level R&D regressions also include two lags of assets, short debt, cash, employment, and firm-fixed effects. The standard errors are robust clustered at the firm-level.

Figure 13: Response of Firm Entry, Aggregate R&D and Firm-level R&D to 100 bps increase in Federal Funds Rate



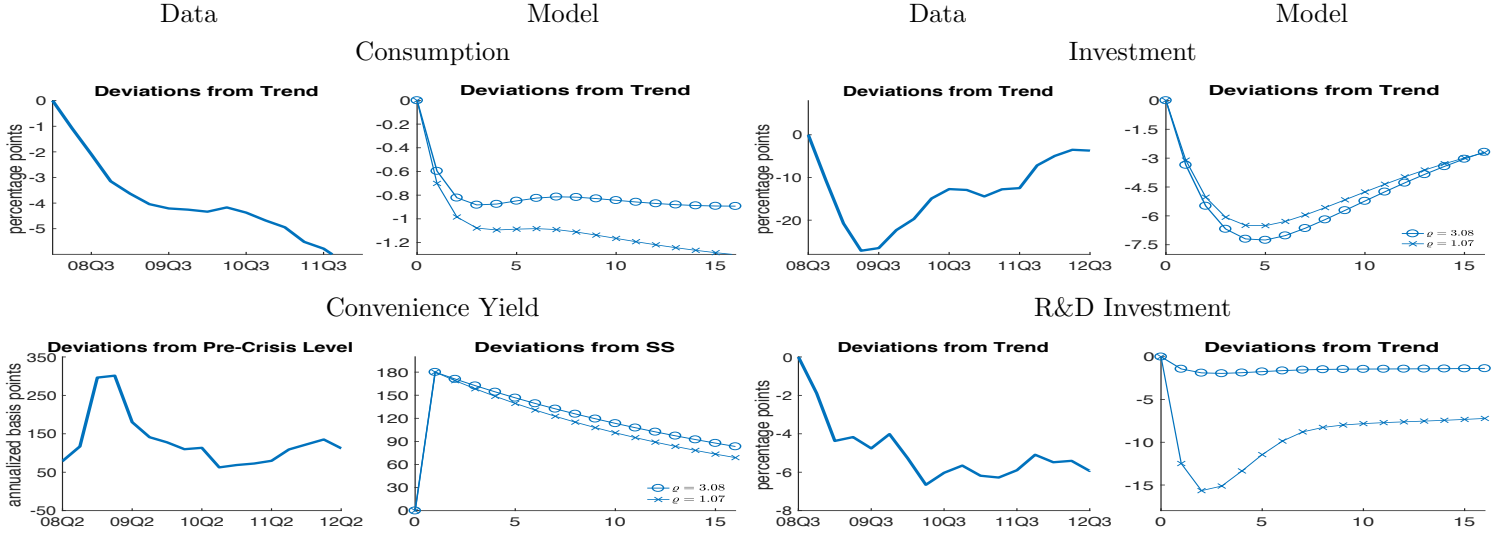
Notes: The figure compares model-implied IRFs to the estimated impulse response functions for utilization adjusted TFP, raw TFP, firm-level R&D and net establishment births. Time is in quarters. Sample length, and instrument used are denoted at the top of each row. IRFs are computed using a local-projections IV approach. Current and two past-lagged values of log real GDP and inflation rate are used as conditioning variables. Regressions also include past values of the proxy, the federal funds rate, and the dependent variable. Kleibergen-Paap F statistic for weak instruments are reported in the figures. The standard errors are calculated using HAR-Newey-West standard errors. The shaded areas denote 95% confidence intervals. Firm-level R&D regressions also include two lags of assets, short debt, cash, employment, and firm-fixed effects. The standard errors are robust clustered at the firm-level. The model impulse responses are extracted from two calibrations with $\rho = 1.07$ and $\rho = 3.08$. In the model, IRFs are traced following a one-time exogenous shock in the federal funds rate of 100 bps (annualized).

Figure 14: Response of Output, Inflation, and the Nominal Interest Rate to the Liquidity Shock



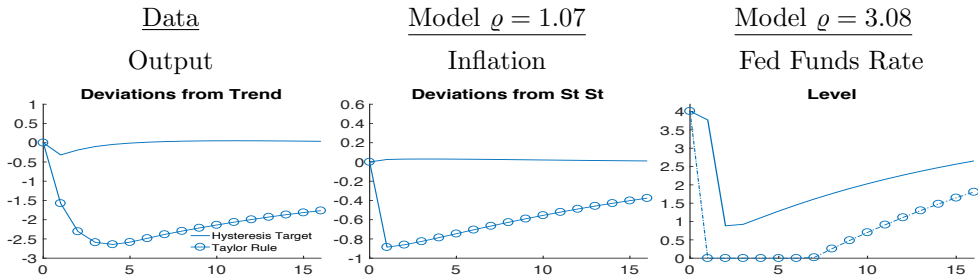
Notes: The figure compares the evolution of output, inflation, and the nominal interest rate in the data (left column) and in the two variants of the model in response to the calibrated liquidity shock (right columns). The data start in 2008Q3. Both data and model are plotted for 16 quarters. Output in the data (top-left) is the sum of consumption and investment, in percentage log-deviations from a linear trend estimated from 2000Q1 to 2007Q4, and is normalized to zero in 2008Q3. Inflation in the data (middle-left) is the annualized quarterly inflation rate of the GDP deflator minus 1.6%. Value of 1.6% is chosen for the model to hit a steady state nominal interest rate of 4%. The interest rate in the data (bottom-left) is the annualized effective Federal Funds Rate. Output in the model (top-right) is the log-deviation from steady state in percentage points. Inflation in the model (middle-right) is expressed in annualized percentage points. The interest rate in the model (bottom-right) is the annualized level of the nominal interest rate in percentage points (the horizontal line is its steady state value).

Figure 15: Response of Consumption, Investment, R&D Investment, and Convenience Yield to the Liquidity Shock



Note: The figure compares the evolution of consumption, investment, R&D investment, and convenience yield in the data (left column) and in the model in response to the calibrated liquidity shock (right column). The data start in 2008Q3. Both data and model are plotted for 16 quarters. Consumption in the data (top-left) is total consumption minus durable consumption. Investment in the data (top-middle-left) is investment plus durable consumption minus Intellectual Property Investment. R&D Investment in the data (bottom-middle-left) is the Intellectual Property Investment. These three variables are expressed in percentage log-deviations from a linear trend estimated from 2000Q1 to 2007Q4, and are normalized to zero in 2008Q3. The convenience yield in the data (bottom-left) is in annualized basis points (produced by (Del Negro et al., 2017)). Consumption (top-right), investment (top-middle-right), and R&D investment (bottom-middle-right) in the model are log-deviations from steady state in percentage points. The convenience yield in the model (bottom-right) is the annualized absolute deviation from steady state expressed in basis points.

Figure 16: Hysteresis Targeting: Response of Output, Inflation, and the Nominal Interest Rate to the Liquidity Shock



Notes: The figure compares the evolution of output, inflation, and the nominal interest rate under Hysteresis targeting rule and assumed Taylor rule in the model with $\rho = 1.07$ in response to the calibrated liquidity shock. All graphs are plotted for 16 quarters. Output in the model (top-right) is the log-deviation from steady state in percentage points. Inflation in the model (middle-right) is expressed in annualized percentage points, deviation from steady state value of 1.6%. The interest rate in the model (bottom-right) is the annualized level of the nominal interest rate in percentage points (the horizontal line is its steady state value). Hysteresis targeting rule is implemented by adding an additional term called the *hysteresis* with a coefficient of 0.5. Hysteresis is defined as sum of all endogenous growth rate deviations induced by history of shocks at time t .

Tables

Table 1: Parameters for Welfare Analysis in the Simple Benchmark Model

<i>Standard Parameters</i>	<i>Formula</i>	<i>Value</i>
Labor share	$1 - \alpha$	0.67
Discount rate	β	0.99
Steady State Wage Markup	λ_w	0.10
Calvo probability of wage adjustment	$(1 - \theta_w)$	$1 - 0.75$
Inverse Frisch Elasticity	ν	2
Innovation Step Size	γ	1.20
Inverse Innovation Elasticity	ϱ	1.47
Innovation Cost parameter	δ	22.6

Table 2: Policy Rules at the ZLB: Welfare Comparison

<i>Policy Rule</i>	<i>Welfare Loss</i>	<i>Permanent Output Gap</i>
Optimal rules		
Discretion (MPE)	100%	-0.88%
Commitment	0.043%	-0.085%
Commitment with higher wt on $\hat{g}_t$	0.11%	0
Simple rules		
Taylor rule eq 27	100%	-0.88%
Hysteresis Targeting	0.049%	0
Wage Level Targeting	0.053%	-0.30%
$W \times Y$ targeting	0.311%	-0.37%

Notes: Values report the conditional welfare loss starting from an efficient steady state. Loss is expressed in consumption equivalent units relative to discretionary rule. Computation details in the Appendix. The true relative weight on growth rate gap is 3.94. Under a weight of 165, the permanent output gap is 0.

Table 3: Policy Rules at the ZLB: Welfare Comparison for Range of ϱ

Innovation Intensity ϱ	1.02	1.09	1.20	Bench 1.47	1.50	1.71	2.78
Permanent Output Gap							
Discretion (MPE)	-1.74%	-1.58%	-1.25%	-0.88%	-0.867%	-0.695%	-0.346%
Commitment	0.0149%	-0.073%	-0.085%	-0.085%	-0.084%	-0.076%	-0.044%
Hysteresis Targeting	0	0	0	0	0	0	0
Wage Level Targeting	0.048%	-0.291%	-0.354%	-0.297%	-0.29%	-0.246%	-0.129%
$W \times Y$ targeting	0.026%	-0.11%	-0.136%	-0.145%	-0.143%	-0.132%	-0.08%
Welfare Loss							
Commitment	0.021%	1.17%	2.26%	4.33%	4.48%	5.48%	7.62%
Hysteresis Targeting	0.031%	1.27%	2.53%	4.87%	5.02%	6.09%	8.13%
Wage Level Targeting	0.073%	5.90%	13.83%	5.25%	23.41%	26.67%	32.63%
$W \times Y$ targeting	0.04%	1.36%	2.88%	6.43%	6.66%	8.56%	13.13%

Notes: Values report the conditional welfare loss starting from an efficient steady state. Loss is expressed in consumption equivalent units relative to discretionary rule. Only two parameters are adjusted. Innovation Intensity elasticity ($1/\varrho$) and research cost δ to target 2% annual growth rate.

Table 4: Policy Rules : Welfare Comparison

<i>Policy Rule</i>	<i>Markup shock</i>	<i>Productivity Shock</i>	<i>Liq Demand Shock</i>	<i>MP shock</i>
Optimal rules				
Commitment	0.18%	0.0001%	0	0
Discretion	0.753%	0.0002%	0	0
Simple rules				
Taylor rule eq 27	2.04%	0.001%	0.022%	0.019%
Hysteresis Targeting	17.68%	0.61%	0	0
Wage Level Targeting	0.2881%	0.0001%	0	0
$W \times Y$ targeting	4.6%	0.0017%	0	0

Notes: Values report the conditional welfare loss starting from an efficient steady state. Welfare losses are computed as an average over 10,000 simulations, each starting at the same efficient steady state. Loss is expressed in consumption equivalent units. Hysteresis target rule takes the following form $h_{t+1} + y_t - y_t^f = 0$ rule, where superscript f denotes flexible wage allocation, h_t is (log) hysteresis determined at time $t - 1$ and y_t is (log) stationarized output. Wage level targeting rule is implemented as $W_t + y_t - y_t^f = 0$, where W_t is the (log) nominal wage. $W \times Y$ targeting takes the form: $W_t + h_{t+1} + y_t - y_t^f = 0$.

Table 5: Parameters

Steady State Parameters					
	β	λ_p	δ_k	α	γ
	Discount factor	Price s.s. markup	Capital depreciation rate	Capital share	Innovation step size
	0.999	0.15	0.025	0.28	1.55
Calibrations		ϱ	δ	μ	
		Inverse innovation elasticity	Innovation cost parameter	Probability of patent loss	
	1. low ϱ	1.07	5.88	0.0285	
	2. high ϱ	3.08	7.47×10^4	0.0	
Parameters Characterizing the Dynamics					
ν	λ_w	θ_p	θ_w	h	$\frac{a''(1)}{a'(1)}$
Inverse Frisch elasticity	Wage s.s. markup	Price Calvo probability	Wage Calvo probability	(Internal) habit	Capital utilization cost
1.00	0.15	0.750	0.750	0.5	4
$\varkappa$	$S''(1)$	ϕ_π	ϕ_y	$1 - \frac{1}{\lambda_g}$	
R&D adjustment cost	Investment adjustment cost	Taylor rule inflation response	Taylor rule (normalized) output response	Government spending share	
0.768	0.75	1.50	0.125	0.20	

Notes: The table shows the parameter values of the model for the baseline calibration.

Table 6: Targets and Model-Implied Values in Calibration of Steady State Parameters

Targets	GDP growth rate	Creative Destruction rate	Real rate	Investment/GDP Ratio	R&D/GDP Ratio	Profits/GDP Ratio
Data	2	3.6	2.40	17.18	2	6.50
Model	2	3.6	2.40	17.18	2	6.59

Notes: The table shows the empirical targets and the model-implied values in the calibration of the six steady state parameters. The sample used to compute the data counterparts of the targets is 1948Q1-2007Q4.