

Too Much Skin-in-the-Game?

The Effect of Mortgage Market Concentration on Credit and House Prices

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Abstract

During the housing boom, mortgage markets became increasingly concentrated with the government-sponsored enterprises (GSEs) being exposed to over 40 percent of U.S. mortgages in the 2000s. Research on the causes of the pre-crisis rise in risky lending has largely overlooked this trend. I develop a theory where this concentration in mortgage holdings can explain key features of the housing boom and bust. In the model, large lenders with many outstanding mortgages have incentives to extend risky credit to prop up house prices. An increase in concentration can lead to a credit boom with worsening credit quality and a subsequent bust with widespread defaults. The model can generate a negative correlation between credit and income growth across areas (such as ZIP codes) while maintaining a positive correlation between them across borrowers reconciling empirical evidence that has previously seemed contradictory.

Keywords: Concentration, GSEs, housing booms and busts, mortgage credit.

JEL Classifications: G01, G21, L11, L25, R21, R31.

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In the 2000s, there was an unprecedented surge in risky lending to borrowers with low FICO scores at high debt-to-income and loan-to-value ratios. These mortgages were associated with high default rates and ex-post did not seem to be profitable for credit suppliers.¹ What motivated this high-risk, seemingly unprofitable lending? A common explanation is that dispersion in mortgage holdings driven by securitization caused moral hazard problems in mortgage origination. More specifically, since credit providers could sell off mortgages, they no longer had skin-in-the-game in the mortgages they originated and therefore had a reduced incentive to monitor and originate quality mortgages.² This explanation has gained traction in macro-prudential policy following the crisis, with the Dodd-Frank act requiring a minimum level of risk retention by mortgage lenders.

However, during the housing boom, mortgage markets had a historically high level of concentration if we consider broader exposure to the mortgage market, such as mortgage holdings rather than just originations. In particular, the GSEs and a few banks had rising exposure to the mortgage market through the 1990s and amassed a large concentration of mortgage risk in the 2000s. The agencies share increased from about 7% of the U.S. mortgage market in the 1980s to over 40% in the 2000s.³ In this paper, I develop a theory of how this increase in concentration of mortgage risk can explain the surge in high-risk lending and other important characteristics of the housing boom and bust.

The model can explain key empirical features of the recent housing crisis. In particular, as mortgage markets become more concentrated, the model predicts a boom in credit characterized by increasing house prices and debt-to-income (DTI) ratios. Credit quality worsens over the life of the boom. A fundamental shock to a concentrated market can lead to a collapse in real estate prices accompanied by large-scale defaults. For a short period after the bust, lenders in concentrated markets continue to make high-risk loans. Importantly, the model can explain the timing of high-risk lending that started in the 2000s after the credit boom had already begun and the continuation of high-risk activity by the GSEs in 2007 once mortgage markets began to slow down (Bhutta and Keys (2017)).

The key idea of the model is that if credit affects house prices and house prices in turn affect the severity of default, large mortgage lenders internalize their effect on house prices

¹Following the crisis of 2007, many private securitizers went out of business, banks such as Lehman Brothers and Bear Sterns collapsed or had to be bailed out due to their exposure to the subprime market, and Fannie Mae and Freddie Mac were placed into government conservatorship.

²For theoretical models of this mechanism see Parlour and Plantin (2008) and Vanasco (2017).

³The GSEs' exposure to mortgages came in the form of portfolio holdings of their own loans (about half of which they held on to) and insurance guarantees on the securitized mortgages that they sold. Additionally, the agencies were the single largest investors in the private securitization market purchasing about 30% of the total dollar volume of private-label MBSes between 2003-2007 (Acharya, Richardson, Nieuwerburgh, and White (2011)) and Adelino, Frame, and Gerardi (2016).

and consequently on default probabilities and losses when making lending decisions.⁴ More specifically, prevailing house prices affect the profitability of previously issued mortgages since borrowers are less likely to default when house prices are high and upon default their house, which is collateral for lenders, is worth more. Lenders with a large amount of mortgages on their books therefore have an incentive to keep house prices high when they are due mortgage repayments. If lenders can influence house prices through increasing their supply of credit, they may find it optimal to extend credit to low-quality, high-risk borrowers not because of the return they expect to make on the loan itself, but because of the boost in house prices that comes from credit provision. Lenders trade off the loss they make on the issuance of mortgages to these borrowers with the profits they make by keeping house prices high on mortgages that are due repayment.

Concentration impacts both the quantity and quality of mortgage credit. In most models of industrial organization, as concentration increases, agents behave less like price-takers and the aggregate quantity supplied of the good in question decreases.⁵ While this “Cournot” effect is present in the model, there is a second effect of changes in concentration that is new, the “propping-up” effect. As concentration increases, individual lenders acquire larger market shares which creates an incentive to extend more credit to prop up house prices. If the propping-up effect dominates the Cournot effect, the aggregate supply of credit increases as mortgage markets become more concentrated. Furthermore, credit in more concentrated markets is generally riskier than credit in less concentrated markets. In the model, I show that it is possible for two areas with different levels of concentration to have the same level of credit provision. However, the area with higher concentration will have lower quality credit with higher default rates. This is because large banks are willing to compromise on the return they earn from the expected loan repayment because of the benefit they get from the resulting increase in house prices.

A calibration of the stylized model matches key moments of the U.S. housing market during the 1991-2009 credit cycle and demonstrates that changing concentration can produce significant differences in the likelihood of a credit boom and bust, and the quantity and quality of credit expanded during the credit cycle. Specifically, when concentration is set to approximately match the GSE market share, the model is able to explain about half of

⁴There is a large amount of empirical support for these assumptions. Many papers have found a connection between house prices and default. See Foote, Gerardi, and Willen (2008), Haughwout, Peach, and Tracy (2008), Palmer (2013), Ferreira and Gyourko (2015). Further, many papers also provide evidence that the availability of credit affects house prices. See Himmelberg, Mayer, and Sinai (2005), Khandani, Lo, and Merton (2009), Hubbard and Mayer (2009), Mayer (2011), Griffin and Maturana (2015), Landvoigt, Piazzesi, and Schneider (2015), An and Yao (2016) and Favilukis, Ludvigson, and Nieuwerburgh (2017).

⁵See Tirole (1988).

the boom and bust in house prices and over 90% of lending to sub-prime borrowers during the housing boom and bust.⁶ The model is also able to generate the GSEs increase in market share. In a counterfactual analysis of the calibrated model, I show that decreasing concentration by doubling the number of competing lenders in the mortgage market would have reduced the fraction of sub-prime lending in the housing boom and bust to 0. It would have also resulted in 30% lower growth in house prices during the boom and 80% smaller decline in house prices during the bust.

This paper also contributes to an important debate on the cause of the housing crisis that has centered around two narratives. The first is that distortions in the supply of credit led to lax lending standards. A key finding in support of this view is by [Mian and Sufi \(2009\)](#) who find that income growth decoupled from the growth in mortgage credit in the U.S. at the ZIP code-level. They point to innovations in the provision of credit to low-quality borrowers as an explanation for their findings. The second narrative is that the expectation of high future house prices led lenders and borrowers to over-estimate the profitability of mortgage loans. Recent evidence in support of this view is presented by [Adelino, Schoar, and Severino \(2016\)](#) who find that at a borrower-level, income growth did not decouple from the growth in mortgage credit.

This paper can provide a theoretical foundation for the seemingly contradictory empirical findings by [Mian and Sufi \(2009\)](#) and [Adelino et al. \(2016\)](#). Following an increase in mortgage market concentration, the model can generate a negative correlation between mortgage credit and income growth when looking across areas (such as ZIP codes), while at the same time maintaining a positive correlation between them at a borrower-level. In the model, lenders have relatively more market power in affecting housing prices in areas with low income growth since in such areas without the availability of credit there is little else to drive the demand for housing and keep house prices high. Therefore, for each additional mortgage loan, the percentage increase in house prices and consequently the return to propping up house prices is high. An increase in concentration can therefore lead to a credit supply shock in areas where income growth is low, leading to a decoupling of income growth from the growth in mortgage credit. However, banks' incentives to lend more to higher-quality borrowers do

⁶In this paper, I focus on the private mandate of the GSEs to maximize profits for shareholders to explain high-risk lending. Although the GSEs had private shareholders, they also had a public mandate to achieve goals to support housing amongst low- and moderate-income households and in underserved areas. This private/public nature of the agencies may mean that their motivations were not purely profit-maximizing. [Acharya et al. \(2011\)](#) argue that it is hard to explain GSE high-risk activity because of their public mandate alone. They report that GSE adherence to their housing targets seemed to be voluntary - the GSEs missed their housing targets on several occasions without any severe sanctions by regulators. Furthermore, the largest housing target increases for the GSEs took place in 1996 and 2001, yet the increase in GSE high-risk activity did not take place till later. See [Elenev, Landvoigt, and Van Nieuwerburgh \(2016\)](#) for a theory of the quasi-government nature of the GSEs.

not fundamentally change. All else equal, a bank would always prefer to make a loan to a high-quality borrower, if possible, as such a loan would also serve to increase house prices. Therefore when looking at borrower-level data, the growth in income and mortgage credit remains positively correlated.

The model is robust to concentration in the mortgage market at an originator level or at a secondary market level. At an originator level, Countrywide Financial was increasing its share of the U.S. mortgage market during the boom and accounted for about 15% of all mortgage origination in 2005. In the secondary market, the GSEs were the largest participants in the U.S. mortgage market but did not originate mortgages themselves. Rather their exposure to the mortgage market was through insurance guarantees on mortgage-backed securities (MBS) they sold to investors, through portfolio holdings of their own loans, and through the purchase of private-labeled MBS. Additionally, about 50% of all holdings of AAA rated non-GSE MBS were concentrated amongst a few large complex financial institutions (LCFIs). The key mechanism in the model simply requires concentration in mortgage markets. The basic model setup abstracts away from the secondary market. However, I provide an equivalent version of the model in which concentration is present in the secondary market rather than the primary originator market. The key mechanism works as long as there is concentration in mortgage holdings at some level and agents with exposure to mortgage payments have some market power. If secondary market players own a large share of the mortgage market, they benefit from high house prices. If they have market power, they can offer attractive prices on the secondary market for sub-prime mortgages that will incentivize mortgage originators to issue mortgages to risky borrowers. Holders of these mortgages will suffer losses on these purchases but the increase in house prices will be profitable for their outstanding mortgage exposure.

This paper also contributes to macro-prudential policy discussion in the aftermath of the crises. From a policy perspective, it is crucial to understand the different forces that drove the housing boom and bust. While steps have been taken to address the issue of securitization leading to a lack of skin-in-the game, with the Dodd-Frank act requiring a minimum level of risk retention by lenders, concentration in the mortgage market has not been discussed much by regulators and has increased since the crisis. *The Economist* recently reported that the GSEs and Federal Housing Association are funding about 65-80% of new mortgages. At least some of these mortgages appear to be highly risky and of questionable quality, with the report stating that 20% of all loans since 2012 have LTV ratios of over 95%. Further, the new regulations faced by banks have made them move out of mortgage lending. As a result, mortgage origination has become highly concentrated with new, independent firms Quicken

Loans and Freedom Mortgage originating roughly half of all new mortgages.⁷

This paper puts forward a theory that can explain the deterioration in lending standards and its link to the growth of the secondary market because there was concentration in the holdings of securitized loans. Papers by [Ben-David \(2011\)](#), [Carrillo \(2013\)](#), [Garmaise \(2015\)](#) and [Piskorski, Seru, and Witkin \(2015\)](#) have shown that mortgage originators were lowering underwriting standards, becoming more lax in loan screening and not monitoring loans carefully in the years leading up to the 2008 crisis. [Keys, Mukherjee, Seru, and Vig \(2011\)](#) connect this phenomenon to the development of the secondary market for mortgages.⁸ While securitization did create a new security with potential information frictions and moral hazard concerns, it also caused a large increase in the concentration of mortgage market exposure. In particular, the rise of securitization occurred after Salomon Brothers created a mortgage trading operation and found investors for MBS. Investor interest in MBS allowed the GSEs to grow their share of the mortgage market by becoming the key players in MBS issuance. This second effect of securitization has been largely overlooked by research into the housing crises.

Many recent papers provide support for the theory that large lenders were driving risky lending. In a paper testing my theory, [Elul, Gupta, and Musto \(2017\)](#) find that in 2007 as small private securitizers were withdrawing from the risky lending, the GSEs increased high-LTV mortgage purchases in MSAs in which they had high outstanding mortgage exposure. Additionally, [Favara and Giannetti \(2017\)](#) find that mortgage lenders in more concentrated markets internalize house price drops coming from foreclosure externalities and are less likely to foreclose on delinquent households. [Dell’Ariccia, Igan, and Laeven \(2012\)](#) find that the decline in lending standards was driven by large lenders and that loan denial rates were lower in areas that had a smaller number of competing lenders. [Adelino et al. \(2016\)](#) find that when private securitizers designed MBS pools for the agencies, loans in GSE pools were riskier based on observable risk characteristics than loans in non-GSE pools. [Nadauld and Sherlund \(2013\)](#) find that securitization of sub-prime mortgages increased 200% between 2003 and 2005 and was driven primarily by the five largest broker/dealer banks resulting in a lowering of lending standards in the primary market.⁹

The rest of this paper is arranged as follows. Section 1 provides a review of the literature related to this paper. Section 2 describes the main model setup. Section 3 illustrates

⁷Briefing: Housing in America. 2016. “Comradely Capitalism.” *The Economist*.

⁸[Keys et al. \(2011\)](#) find that loan performance was significantly worse for borrowers with a FICO score of just above 620 which conformed to a rule-of-thumb that made loans with a FICO score of 620 and above easier to securitize, than those just below. Also see [Elul \(2011\)](#) and [Griffin and Maturana \(2016\)](#).

⁹Also see [Jiang, Nelson, and Vytlačil \(2014\)](#).

the key mechanism of how concentration can affect credit in a simple three-period model. Section 4 discusses the main infinite horizon model and explains how the model generates housing booms and busts. Section 5 provides an equivalent model to the baseline model with concentration in the secondary mortgage market. Section 6 discusses an extension of the model with lender heterogeneity. It also provides details of the model calibration. The last section concludes. All proofs are in the appendix.

1. Related Literature

Although the effect of concentration in markets on resulting prices and quantities is widely studied in economics, research on the effect of concentration in mortgage markets on credit and house prices jointly is relatively sparse. [Scharfstein and Sunderam \(2014\)](#), [Fuster, Lo, and Willen \(2016\)](#) and [Agarwal, Amromin, Chomsisengphet, Landvoigt, Piskorski, Seru, and Yao \(2017\)](#) study how competition in the mortgage market affects mortgage interest rates, but take house prices as exogenous. [Poterba \(1984\)](#) and [Himmelberg et al. \(2005\)](#) study how mortgage interest rates affect house prices, but assume perfectly competitive mortgage markets. This paper combines these ideas and studies credit and house prices when lenders internalize the impact their credit provision has on house prices.

This paper is related to the literature on how size can affect incentives to take on risk. The main theory in this area of research is too-big-to-fail: large institutions take on excessive risks because they expect to be bailed out by the government ([Stern and Feldman \(2004\)](#)). In my paper, the key variable that causes institutions to take on mortgage risk is the size of their mortgage exposure rather than the size of the institution. This yields cross-sectional predictions, holding a lender fixed, and is consistent with empirical evidence. In a similar vein, [Bond and Leitner \(2015\)](#) develop a theory in which buyers with large inventories of assets, can make further asset purchases at loss-making prices because other market participants use prices to infer information about the underlying asset value. In their model, the buyer incurs a cost when the market value of his inventories falls too low and would therefore like to keep market prices high. In my setting, there is no asymmetric information and lenders with large outstanding mortgage make loans that are low-quality based on *observable* risk. This can therefore help explain the rise of sub-prime lending, which had observably higher LTV and DTI ratios and higher default rates than prime mortgages. [Milbradt \(2012\)](#) models how mark-to-market accounting can lead financial institutions to suspend trading. [Kumar and Seppi \(1992\)](#) show that uniformed investors have incentives to manipulate the spot price used to compute the cash settlement at delivery when they hold futures positions. My model focuses instead on how outstanding exposure can increase incentives to extend credit rather than cause a suspension of trade.

This paper also contributes to the recent debate on whether the housing boom and collapse was driven by a credit supply shock or by high house price expectations. The majority of this debate has been empirical with [Mian and Sufi \(2009\)](#), [Favara and Imbs \(2015\)](#), [Griffin and Maturana \(2015\)](#), [Landvoigt et al. \(2015\)](#) providing evidence supporting a credit supply shock and with [Glaeser, Gottlieb, and Gyourko \(2013\)](#) and [Adelino et al. \(2016\)](#) arguing that an expectations based explanation fits the data better. The theoretical literature reconciling observations from the crisis with either view is relatively sparse, and typically requires either irrationality or misinformation to justify the housing boom. The expectations-based view often requires that buyers and lenders in housing markets hold over-optimistic views about future housing prices.¹⁰ In the case of a credit supply shock, since borrowers, securitizers and the MBS buyers faced large losses in the crisis, it is hard to explain why the credit supply shock happened without an overoptimism or misinformation about the benefits of new ways to supply credit. This paper adds to this literature by providing a theoretical framework that can reconcile many of the empirical findings driving the current debate.

2. The Model

The model is an infinite horizon, discrete time model with overlapping generations. A number, N , of infinitely lived banks each with access to an equal share of borrowers make mortgage loans to households. Each period t a new generation is born that lives for two periods and consists of a continuum $[0, 1]$ of households. Households from generation t derive utility from consuming housing, $k_t \in \{0, 1\}$, when they are young, and a consumption good when they are old. Their life-time utility is given by,

$$u(k_t, c_{t+1}) = \gamma k_t + \beta c_{t+1}.$$

The extent to which households value housing consumption is captured by the preference parameter, γ , and $\beta < 1$ is a discount factor.¹¹ Households have access to a storage technology which yields a return of 1.

There are two types of households: a proportion α^{nb} of households (“non-borrowers”) receive their endowment when they are young and the remaining households (“borrowers”) receive their endowment when they are old. “Non-borrowers” from generation t are born with

¹⁰Arguments in favor of this have been made by [Cheng, Raina, and Xiong \(2014\)](#), [Shiller \(2014\)](#) and [Glaeser and Nathanson \(2015\)](#).

¹¹[Green and White \(1997\)](#), [Sekkat and Szafarz \(2011\)](#) and [Sodini, Nieuwerburgh, Vestman, and Lilienfeld-toal \(2016\)](#) provide estimates of the benefits of home-ownership.

an endowment ω_t^{nb} at t . They receive a positive endowment, $\omega_t^{nb} = e^{nb}$, with probability ϕ_s^{nb} and 0 otherwise where s is a generation-specific income shock. “Borrowers” from generation t receive an endowment ω_t^b at $t + 1$. These households therefore need a mortgage to be able to buy a house at t . There are two types of borrowers: proportion α^{bh} of households are high-quality borrowers and the remaining are low-quality borrowers, with the former having a greater expected endowment. High and low-quality borrowers receive a positive endowment $\omega_t^b = e^b$ with probability ϕ_s^{bh} and $\phi_s^{bl} (< \phi_s^{bh})$ respectively and 0 otherwise.

Each generation t has a generation-specific shock, $s_t \in \{R, P\}$, and can be born rich or poor with q being the probability of a rich generation being born. In a rich generation, all agents have a higher expected endowment than in a poor generation: $\phi_P^{nb} < \phi_R^{nb}$, $\phi_P^{bh} < \phi_R^{bh}$ and $\phi_P^{bl} < \phi_R^{bl}$. At each time t , once a generation is born, the expected endowments of its borrowers and non-borrowers are common knowledge. There is therefore no adverse selection due to information frictions in the credit market.

2.1. Housing Market

The housing stock, h_t , depreciates at rate δ per period where $0 < \delta < 1$. Each period, competitive construction firms can also produce new housing, n_t , to add to the existing stock of housing. Firms have a quadratic cost of producing houses, $\frac{c}{2}h_t^2$, which depends on both the existing stock of housing and new houses produced. This particular cost function delivers tractable solutions and captures the idea that land availability is an important factor in the cost of housing construction.¹² The total supply of housing at time t is therefore given by:

$$h_t = (1 - \delta)h_{t-1} + n_t.$$

The demand for housing is given by the number of mortgage loans borrowers get from banks, h_t^b , and the number of houses purchased by non-borrowers, h_t^{nb} . I will make parameter restrictions (outlined at the end of this section) to ensure that there is some new construction every period. The price of housing, P_t , is then set to clear the housing market and is given by a linear function:

$$P_t = ch_t.^{13}$$

¹²The main results of the model also hold for a more general supply function in which construction costs are affected differentially by new construction and by the existing stock of housing.

¹³To obtain this we can solve the representative construction firm’s problem,

$$\max_{n_t} P_t n_t - \frac{c}{2} ((1 - \delta)h_{t-1} + n_t)^2$$

2.2. Mortgage Loans

At time t , a household i borrows $k_t^i P_t$ at an interest rate, $r_t^i(s_{t+1})$, that can be contingent on the future states of the world. At time $t + 1$, if a household pays back its loan, it keeps its house which it can sell to use the proceeds for consumption. If the household defaults on its loan, the bank forecloses on the house and is entitled to the household's endowment. In the model, mortgage loans are therefore similar to adjustable rate mortgages with recourse.¹⁴

2.3. The Household's Problem

Each period t , borrowers and non-borrowers from generation t decide whether to purchase a house. Households also have access to a storage technology which gives a rate of return of 1 at time $t + 1$. When deciding whether to purchase a house, non-borrowers account for both the utility they get from housing consumption and the future price at which they expect to sell their home (the proceeds of which are spent on the consumption good). At time t , a non-borrower with endowment $\omega_t^{nb} \geq P_t$ will buy 1 unit of housing if:

$$\gamma + \beta(1 - \delta)E[P_{t+1}] \geq \beta P_t.$$

Borrower households from generation t receive their endowment in the future and must borrow from banks at time t to buy housing. At time $t + 1$, a borrower who has successfully obtained a mortgage will either successfully repay their mortgage and can then sell their house, or default and lose their endowment and house. If a borrower's bank charges him a state-contingent interest rate of $r_t(s_{t+1})$, then he will buy 1 unit of housing if:

$$\gamma + \beta(1 - \delta)E[P_{t+1}] \geq \beta E[\min\{P_t(1 + r_t(s_{t+1})), \omega_t^b + (1 - \delta)P_{t+1}\}].$$

The LHS is the utility the household gains from living in the house in period t and the proceeds the household gets from selling the house at $t + 1$. The RHS represents the net cost of purchasing the house to the household. If the household does not have enough funds to repay its mortgage, $\omega_t^b + (1 - \delta)P_{t+1} < P_t(1 + r_t(s_{t+1}))$, then it defaults and loses its endowment and house.

The first order condition yields, $P_t = c((1 - \delta)h_{t-1} + n_t) = ch_t$.

¹⁴In a model with recourse, at time t , a household with a mortgage loan from generation $t - 1$, repays its mortgage if its net worth is larger than the repayment amount

$$\omega_{t-1}^b + (1 - \delta)P_t \geq P_{t-1}r_{t-1}^i(s_t).$$

If the household defaults, the bank gets the maximum amount the household can repay, i.e., $\omega_{t-1}^b + (1 - \delta)P_t$.

2.4. The Bank's Problem

There are N infinitely lived banks that can make mortgage loans to households. Each period t , banks observe the income shock of the current generation and decide how many loans to issue and at what interest rate. Each bank has access to an equal share, $\frac{1}{N}$, of the mortgage market. The mortgage market is thus segmented implying that households borrow from their local bank and do not shop around for mortgage rates. Therefore, each bank has access to a group of borrowers without having to compete with other banks on interest rates.¹⁵ Although banks do not compete directly on interest rates, they interact strategically with each other due to the collective effect of their actions on house prices. This gives rise to strategic substitution effects that are similar to those in models of Cournot competition.

I solve the model in both the case when a bank cannot commit to future lending and in the case when the bank can commit to future lending. Let $V(s_t, m_{t-1}^h, r_{t-1}^h, m_{t-1}^l, r_{t-1}^l, P_{t-1}, s_{t-1})$ be the value function of a bank at time t where $s_t = \{h, l\}$ represents the income shock of the generation born at time t , m_{t-1}^j represent the number of mortgage loans that the bank has made at time $t-1$ to borrowers of type $j = \{h, l\}$ at interest rate r_{t-1}^j , and P_{t-1} is the price of housing at time $t-1$ (and a function of m_{t-1}^h and m_{t-1}^l). Then at time t , a bank solves the following problem:

$$\begin{aligned}
V(s_{t-1}, s_t, m_{t-1}^h, m_{t-1}^l, r_{t-1}^h, r_{t-1}^l, P_{t-1}) &= \max_{m_t^h \geq 0, m_t^l \geq 0, r_t^h, r_t^l} \\
&\sum_{j=\{h,l\}} \underbrace{m_{t-1}^j \left(\phi_{s_t}^{bj} \min\{P_{t-1}(1 + r_{t-1}^j), e^b + (1 - \delta)P_t\} + (1 - \phi_{s_t}^{bj}) \min\{P_{t-1}(1 + r_{t-1}^j), (1 - \delta)P_t\} \right)}_{\text{Repayment}} \\
&- \sum_{j=\{h,l\}} \underbrace{m_t^j P_t}_{\text{New Lending}} + \underbrace{\beta E[V(s_t, s_{t+1}, m_t^h, m_t^l, r_t^h, r_t^l, P_t)]}_{\text{Continuation Value}} \\
\text{s.t. } \gamma + \beta(1 - \delta)E[P_{t+1}] &\geq \beta E[\min\{P_t(1 + r_t(s_{t+1})), \omega_t^b + (1 - \delta)P_{t+1}\}] \\
m_t^h &\leq \frac{1}{N} \alpha^{bh} \\
m_t^l &\leq \frac{1}{N} (1 - \alpha^{bh} - \alpha^{nb}).
\end{aligned}$$

¹⁵Lacko and Pappalardo (2007) and Amel, Kennickell, and Moore (2008) provide empirical evidence that supports this assumption. They find that consumers tend to bank locally and do not shop around for mortgage rates.

The first term in the bank's payoff is the amount the bank earns on loans made to borrowers from generation $t - 1$ which are due for repayment at time t . House prices at time t affect the bank's payoff from outstanding loans in two ways: they affect borrower net-worth which determines whether the borrower will repay or not; they also affect the bank's payoff in case of default. The second term is the cost of new lending and the final term is the bank's expected continuation value. The bank faces a borrower purchasing constraint - that given the repayment schedule chosen by the bank, the borrower wants to get a mortgage. The second and third constraints are the market share constraints of the bank.¹⁶

2.5. Parametric Restrictions

Given the $[0, 1]$ continuum of households born every period, the maximum housing price is c . To help understand the following parameter restrictions, it is useful to note that given these restrictions, the price of housing in the economy will never fall below $c\phi_P^{nb}\alpha^{nb}$. To close out the model, I make the following parametric restrictions.

1. The private benefit of housing is large enough, i.e., $\gamma \geq \beta(c - c(1 - \delta)\phi_P^{nb}\alpha^{nb})$, to guarantee that non-borrowers always demand housing and there is a positive interest rate at which borrowers demand housing.
2. Non-borrower endowment is large enough, i.e., $e^{nb} > c$, to guarantee that a non-borrower who receives a positive endowment can always afford to buy a house. Since non-borrowers in the model are proxying for outside housing demand, this assumption guarantees that credit is never the sole driver of house prices.¹⁷
3. In the theoretical results, depreciation is not too low, i.e. $\phi_P^{nb}\alpha^{nb} > 1 - \delta$, to guarantee that there is at least some new construction every period and that the bank's problem is thus continuous in house prices. In the calibrated version of the model, I do not restrict the parameters to satisfy this assumption.
4. Low-quality borrower endowment is small enough, i.e.,

$$\beta\phi_R^{bl}e^b + \beta(1 - \delta)c \leq c\phi_P^{nb}\alpha^{nb},$$

¹⁶Note that banks are taking into account the current and future lending decisions of all other banks when making their own decision about how many loans to make. In a slight abuse of notation, the problem as it is currently written does not make this explicit. Lending by other banks is embedded in the bank's decision when it accounts for current and future house prices.

¹⁷This also helps simplify the model solution as house prices will always increase with more credit. Banks do not crowd non-borrowers out of the market by making house prices too expensive.

to guarantee that it is never profitable for banks to lend to low-quality borrowers. The assumption on new construction every period guarantees that price never falls below $c\phi_P^{nb}\alpha^{nb}$. This restriction helps to clarify the key mechanism of the model since for any possible sequence of house prices and in any state, any mortgage loan made to low-quality borrowers is NPV negative. Therefore, there is no reason a bank would ever make loans to low-quality borrowers unless the return from propping up prices is high enough.

Model Robustness: There are two key requirements for the results. First, house prices affect a household's ability or incentive to repay a mortgage such that higher housing prices reduce the probability of default and/or the loss due to default. Second, credit provision has an effect on house prices. The model is robust to modeling mortgage loans without recourse and as fixed rate mortgages. The model is also robust to other market structures as long as banks are able to make profits in one period and offset them with losses from another. The model can also allow entry and exit so that banks lifetime profits are zero as long as they can make profits or losses period-by-period.

3. Three-Period Model

To demonstrate the key mechanisms of the model I start by discussing the equilibrium in a simplified three-period setting. This highlights how concentration affects both the quantity and quality of credit. It also explains how, in concentrated markets, mortgage growth can be negatively correlated with income growth across areas and positively correlated with income growth across borrowers. Uncertainty in future lending opportunities and intra-period borrower heterogeneity are not necessary to obtain the key results of the model, and therefore I abstract away from both in this simplified model. The full model keeps the intuition of the three-period model and is additionally able to produce boom and bust cycles with features that characterized the recent housing crisis.

In the first period the economy is in a rich-state with only non-borrowers and high-quality borrowers, and in the second period a poor-state hits with certainty in which there are only non-borrowers and low-quality borrowers. In the final period, no new generation is born and therefore I assume the price of housing falls to an endogenously specified liquidation value, $c\phi_P^{nb}\alpha^{nb} \geq \kappa \geq 0$. Since no high-quality borrowers are born in the second period, any $t = 2$ lending will only be to low-quality borrowers. Since by assumption low-quality loans are negative NPV, banks only lend a positive amount at $t = 2$ if they find it profitable to prop up house prices. This setup thus clearly demonstrates when a bank is incentivized to sacrifice loan quality for the return to keeping house prices high.

I characterize the results of the model both when banks cannot commit to a level of $t = 2$ lending when making loans at $t = 1$ and when banks can commit to future lending. As I will discuss, in both cases the results are qualitatively similar but the economic intuition for why banks want to prop up prices is different. In practice, there are reasons to think that the GSEs were able to commit, at least in part, to future lending. [Hurst, Keys, Seru, and Vavra \(2016\)](#) provide evidence that the GSEs faced political pressure that did not allow them to make substantial changes to interest rates. These constraints could credibly allow the GSEs to commit to future activity.

The three-period model can be solved by backwards induction. Since no new generation is born in the third period, banks do not lend at $t = 3$. In the second period, lending by any given bank, m_2 is stated in the following lemma, where M_2^{-i} is lending by all other banks at $t = 2$.

Lemma 1.A *In the three-period model, without commitment to future lending, a bank's period-2 lending, m_2 , is given by the following two cases.*

Case 1: If $\phi_R^{bh} e^b \leq \frac{\gamma}{\beta}$,

$$\max \left\{ 0, \frac{m_1(1-\delta)}{2} - \frac{\phi_P^{nb} \alpha + M_2^{-i}}{2} + \beta \frac{\phi_P^{bl} e^b + (1-\delta)\kappa}{2c} \right\}.$$

Case 2: If $\phi_R^{bh} e^b > \frac{\gamma}{\beta}$,

$$\max \left\{ 0, \frac{(1-\phi_R^{bh})m_1(1-\delta)}{2} - \frac{\phi_P^{nb} \alpha + M_2^{-i}}{2} + \beta \frac{\phi_P^{bl} e^b + (1-\delta)\kappa}{2c} \right\}.$$

In the three-period model, with commitment to future lending, a bank's period-2 lending, m_2 , is given by:

$$\max \left\{ 0, \frac{m_1(1-\delta)}{2} - \frac{\phi_P^{nb} \alpha + M_2^{-i}}{2} + \beta \frac{\phi_P^{bl} e^b + (1-\delta)\kappa}{2c} \right\}.$$

The loans a bank makes to low-quality borrowers, m_2 , is always increasing in outstanding loans, m_1 . When $m_1 = 0$ and the bank has no outstanding loans on its balance sheet, it will never make any loans at $t = 2$ to low-quality borrowers and $m_2 = 0$.¹⁸ As the amount of outstanding loans increases, m_2 can become positive. If at $t = 1$, a bank is unable to commit to a level of future lending, m_2 , then it props up house prices to improve its return on loans that are delinquent - when the borrower is unable to return the full face-value of the

¹⁸Since low-quality loans are assumed to be negative NPV, $-\phi_P^{nb} \alpha^{nb} - M_2^{-i} + \beta \frac{\phi_P^{bl} e^b + (1-\delta)\kappa}{c} < 0$.

loan. By increasing house prices through credit expansion, a bank is able to earn a higher return on defaulting loans since it has a claim on the house. If a bank is able to commit to future lending, it props up prices to improve its return on delinquent loans and additionally to increase the face-value it can charge on non-delinquent loans. With commitment, a bank therefore has greater incentives to prop up prices.¹⁹

Loans to low-quality borrowers, m_2 , is also increasing in the future expected income of low-quality borrowers, $\phi_P^{bl}e^b$. It is decreasing in the housing demand coming from non-borrowers and other banks, $\phi_P^{nb}\alpha^{nb} + M_2^{-i}$. A lower $\phi_P^{nb}\alpha^{nb} + M_2^{-i}$ implies that an individual bank effectively has larger market power in influencing house prices since outside sources of demand are lower. In other words, a lower $\phi_P^{nb}\alpha^{nb} + M_2^{-i}$ implies a larger *elasticity of house prices to credit*.²⁰ This increases the net benefit that credit expansion by the bank has on house prices.

At $t = 1$, a bank takes into account its lending at time $t = 2$ when determining how many loans to make. In period 1, a bank's lending is stated in the following lemma, where M_1^{-i} is lending by all other banks at $t = 1$.

Lemma 1.B *In the three-period model, without commitment to future lending, a bank's period-1 lending, m_1 , is given by the following two cases.*

Case 1: If $\phi_R^{bh}e^b \leq \frac{\gamma}{\beta}$,

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\frac{\beta}{c} (\phi_R^{bh}e^b + (1 - \delta)P_2) - \phi_R^{nb}\alpha^{nb} - M_1^{-i}}{2}, \frac{(1 - \alpha^{nb})}{N} \right\} \right\}.$$

Case 2: If $\phi_R^{bh}e^b > \frac{\gamma}{\beta}$,

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\frac{\beta}{c} \left(\frac{\gamma}{\beta} + (1 - \delta)P_2 \right) - \phi_R^{nb}\alpha^{nb} - M_1^{-i}}{2 - \left(\frac{\beta}{2}(1 - \phi_R^{bh})\phi_R^{bh}(1 - \delta)^2 \right) \cdot \mathbb{1}_{\{m_2 > 0\}}}, \frac{(1 - \alpha^{nb})}{N} \right\} \right\}.$$

In the three-period model, with commitment to future lending, a bank's period-1 lending, m_1 , is given by:

¹⁹When $\phi_R^{bh}e^b \leq \frac{\gamma}{\beta}$, bank lending at $t = 2$ is identical with and without commitment. In this case, $t = 1$ borrowers are willing to repay the bank $\phi_R^{bh}e^b + (1 - \delta)P_2$ and by setting a face-value of the loan slightly above this, banks can credibly raise house prices to improve their return on all outstanding loans at $t = 2$ by propping up prices. For more detail on this, see the appendix.

²⁰The elasticity of house prices is simply defined here as the percentage change in house prices for the marginal mortgage loan

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\frac{\beta}{c} \left(\min\{\phi_R^{bh} e^b, \frac{\gamma}{\beta}\} + (1 - \delta)P_2 \right) - \phi_R^{nb} \alpha^{nb} + M_1^{-i}}{2}, \frac{(1 - \alpha^{nb})}{N} \right\} \right\}.$$

In an equilibrium in which a bank props up house prices, if a bank lends more at $t = 1$, it also increases its $t = 2$ lending. This pushes up housing prices at $t = 2$ (P_2) in turn increasing the amount of loans a bank makes at $t = 1$. There is thus a feedback loop between $t = 1$ and $t = 2$ lending. Bank lending is also affected by the aggregate lending of other banks. The number of loans a bank makes at $t = 1$ is decreasing in the number of loans made by other banks, M_1^{-i} , but increasing in the number of loans made by banks in the future, M_2^{-i} . The more loans other banks make at $t = 1$, the higher is the price of housing at $t = 1$, making it more expensive for a bank to make mortgage loans. This causes a bank to decrease the amount it lends. The more loans other banks make at $t = 2$, the higher is the price of housing at $t = 2$, allowing banks to charge a larger interest rate on loans made at $t = 1$ and increasing their incentive to lend at $t = 1$. There is thus strategic substitution in bank lending within period but strategic complementarities in bank lending across periods. The full characterization of the equilibrium is discussed in the following subsection.

Numerical Example: To help understand the mechanism, I run through a numerical example with $N = 1$. I choose the following parameters: $\alpha^{nb} = .3$, $\delta = .3$, $e^b = \$100,000$, $\phi_R^{bh} = 1$, $\phi_P^{bl} = .35$, $\kappa = \$75,000$, $c = \$300,000$. For simplicity, I assume no discounting, i.e. $\beta = 1$, and also have no non-borrower income shocks, i.e. $\phi_R^{nb} = \phi_P^{nb} = 1$. I also assume $\gamma \geq \phi_R^{bh} e^b$, so that bank lending with and without commitment are equivalent.

Imagine a bank does not take into account the effect of house prices on the profitability of its outstanding share of loans. Then in the second period, a bank will not prop-up prices. It therefore makes no loans at $t = 2$ since all loans to low-quality borrowers are negative NPV. Only non-borrowers will buy housing at $t = 2$. Therefore, housing demand in the second period is $h_2^d = .3$, and resulting house prices are $ch_2^d = \$90,000$. We can check that loans to low-quality borrowers are negative NPV. House prices at $t = 3$ are given by the liquidation value $\kappa = \$75,000$ and low-quality borrowers' expected endowment is $\phi_P^{bl} e^b = \$35,000$. If a bank was to lend to low-quality borrowers, it would have to pay $\$90,000$ at $t = 2$ and receives an expected repayment of $(1 - \delta)\kappa + \phi_P^{bl} e^b = \$87,500$ at $t = 3$.

If a bank is not propping up house prices, it will make no loans in period 2. In the first period, the bank will make $m_1 = .12$ loans.²¹ Resulting house prices at $t = 1$, will be $ch_1^d = \$126,500$. The cost of making $t = 1$ loans to a bank is $\$126,500$ and the expected

²¹Loan amounts can be calculated using Lemma 1.

repayment from these loans is $(1 - \delta)P_2 + \phi_R^{bh}e^b = \$163,000$. The total profits earned by the bank are $.12 * (163,000 - 126,500) = \4441 .

Now, let's consider what happens if the bank takes into account its outstanding share of loans in the second period and wants to deviate to making $t = 2$ loans. Then if the bank's outstanding share is $m_1 = .12$, a bank will find it optimal to make $m_2 = .04$ loans. This will increase $t = 2$ price to $\$101,525$. The bank earns an increased return of $\$982$ on its outstanding loans while making a loss of $-\$539$ on new lending at $t = 2$. Banks are able to make this gain in profits at the expense of young non-borrowers. They are harmed by this increase in price and suffer an aggregate loss of $\alpha^{nb} * (\$101,525 - \$90,000) = \$3458$. This loss of young non-borrowers is transferred to banks, old non-borrowers, and construction firms.

The increase in house prices at $t = 2$, allows banks to make a greater return per loan they make at $t = 1$. Banks are now able to get an expected repayment of $\$171,068$ instead of $\$163,000$. This makes banks want to lend more at $t = 1$. This will in turn make the bank want to lend more at $t = 2$ and so on and so forth. Eventually, the bank will increase $t = 1$ lending to $.14$ and $t = 2$ lending to $.04$. House prices at $t = 2$ will be $\$102,937$ and at $t = 1$ will be $\$130,534$. The total profits earned by the banks make from $t = 1$ loans is $\$5,610$ (an increase from $\$4,441$). The bank earns losses on $t = 2$ lending totaling $-\$667$ which offsets some of these profits. Young non-borrowers at $t = 2$ account for the rest of the transfer to banks.

3.1. Concentration and Credit

When concentration in mortgage holdings is low and each bank holds a small share of the market, the return to propping up prices for any individual bank is low. Banks therefore do not issue any loans to low-quality borrowers. As concentration increases, banks have access to a larger share of high-quality borrowers at $t = 1$. In this case, they will issue loans to risky borrowers to increase house prices and consequently the rents that they get from high-quality borrowers. Formally, we can establish the following proposition,

Proposition 1 *The three-period model has a unique equilibrium. There exists a cutoff, $\bar{N}$, such that if $N \geq \bar{N}$, banks do not prop up houses prices and make no negative NPV loans. If $N < \bar{N}$, banks engage in risky lending to prop up house prices and supply a positive amount of negative NPV loans.*

When house prices at $t = 2$ are high, high-quality borrowers (who get a mortgage at $t = 1$) make larger mortgage repayments to banks. This allows banks to earn greater rents from them. As the market share of banks increases, they lend to more high-quality borrowers at $t = 1$. This increases the effect of $t = 2$ house prices on their profitability. As concentration

increases, banks begin to make low-quality loans at $t = 2$ since credit expansion keeps house prices high. As concentration decreases, banks begin to act more like price-takers in the mortgage market and no longer make loans to prop up house prices.

Despite strategic complementarities in bank lending across time, the equilibrium is unique.²² The uniqueness arises due to intra-temporal strategic substitution in bank lending. If other banks pull back on lending at $t = 2$, an individual bank is incentivized to increase its own lending at $t = 2$ and not cut back on its $t = 1$ lending enough to give rise to multiplicity. There is therefore a unique equilibrium of the model.

As concentration increases aggregate credit can increase or decrease. There are two competing effects. The first is a contemporaneous price effect. Large lenders internalize their effect on house prices more than small lenders. The marginal increase in price when making an additional loan affects large lenders' cost of total lending more than that of small lenders. Lenders in a concentrated market will therefore cut back on credit more than lenders in a market with many small lenders. This effect is similar to a typical mechanism in Cournot competition in which as concentration increases, the quantity of goods supplied on the market decreases as suppliers internalize price effects more. As the number of banks decreases, this "Cournot" effect leads to a decrease in credit supply. However, since concentration also creates incentives to prop up prices, there is a second effect of change in concentration on credit, the "propping-up" effect. Concentration increases banks' incentives to increase $t = 2$ prices through credit expansion and if this effect is large enough, it can cause overall lending to increase.

The following corollary summarizes the effect of concentration on mortgage lending:

Corollary 1 *In the unique equilibrium of the three-period model, as N decreases,*

1. *credit extended by any given bank to both high- and low-quality borrowers increases,*
2. *if $N \geq \bar{N}$ and banks are not propping up prices, aggregate credit decreases,*
3. *if $N < \bar{N}$ and banks are propping up prices, aggregate credit can increase.*

When $N \geq \bar{N}$ and banks are not propping up housing prices, aggregate credit is always decreasing with concentration because of the Cournot effect. As is typical in most models

²²Typically the presence of strategic complementarities gives rise to multiple equilibria. The typical reason for multiplicity is as follows: when banks expect aggregate lending to be high at $t = 2$, they lend more at $t = 1$, and the high $t = 1$ lending would lead to the high $t = 2$ lending that banks anticipated. Conversely, when banks expect lending at $t = 2$ to be low, they lend less at $t = 1$ which in turn leads to low $t = 2$ lending as banks anticipated.

of competition, as the number of banks decreases, banks behave more like price-takers and are willing to issue more loans. As discussed above, when $N < \bar{N}$, there is a second effect of concentration on credit, the propping-up effect. As banks acquire larger market shares, they issue more loans per bank at $t = 1$. This increases the incentive for banks to prop up prices and make negative NPV loans at $t = 2$. Higher $t = 2$ price further increase the incentive to issue $t = 1$ loans and so on and so forth. As concentration increases, this feedback loop can cause aggregate lending to increase.

Figure 1 illustrates the effect of concentration on house prices. As the market becomes more concentrated and the number of banks decreases, banks begin to prop up house prices. In this parametrization, credit increases with concentration in the region in which banks prop up prices, as the propping-up effect dominates the Cournot effect. As concentration decreases and $N > \bar{N}$, banks stop propping up prices and the amount of credit increases as competition in the market causes banks to behave more and more like price-takers.

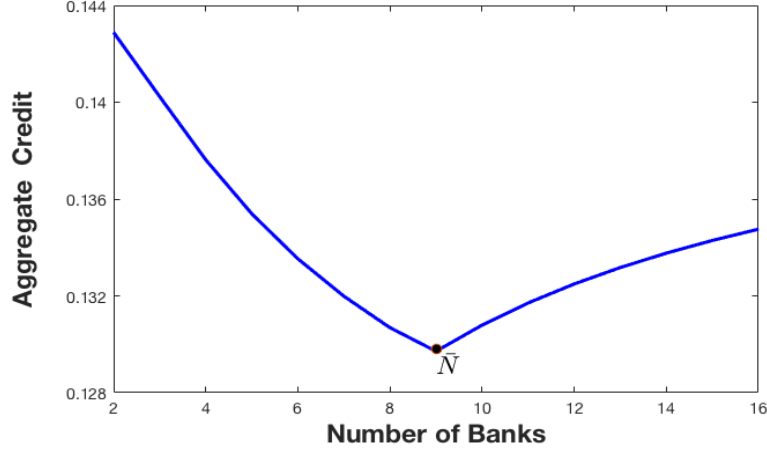


Figure 1: The figure above plots total credit, measured by the number of households who get a mortgage, against the level of concentration in the mortgage market. As we move along the x-axis, N increases and concentration decreases. The parametrization is as follows: $\delta = .01$, $\alpha^{nb} = .2$, $\phi_P^{bl} = .2$, $\phi_R^{bh} = 1$, $\phi_P^{nb} = .7$, $\phi_R^{nb} = 1$, $e_b = 2$, $\kappa = .45$, $\gamma = 4$, $c = 9.8$.

Looking at Figure 1, we can see that it is possible for two areas with different levels of concentration to have the same amount of aggregate credit. However, the composition of this credit is different. In particular, the credit in the area with larger concentration is riskier - a larger fraction of lending is to high-risk borrowers. Figure 2 overlays the first graph with different credit risk characteristics - debt-to-income ratios and default rates.

As Figure 2 illustrates, although two areas with differing concentration can have the same aggregate credit, the credit in the area with higher concentration is riskier. When banks are propping up prices, they extend credit to riskier households with high default rates and make

negative NPV mortgage loans. If there is an economic cost to high mortgage default rates, this result suggests that a safer way to expand homeownership would be through increased competition rather than through creating agencies that concentrate mortgage risk. This may however come at the cost of lower income (and negative NPV) households not getting credit.

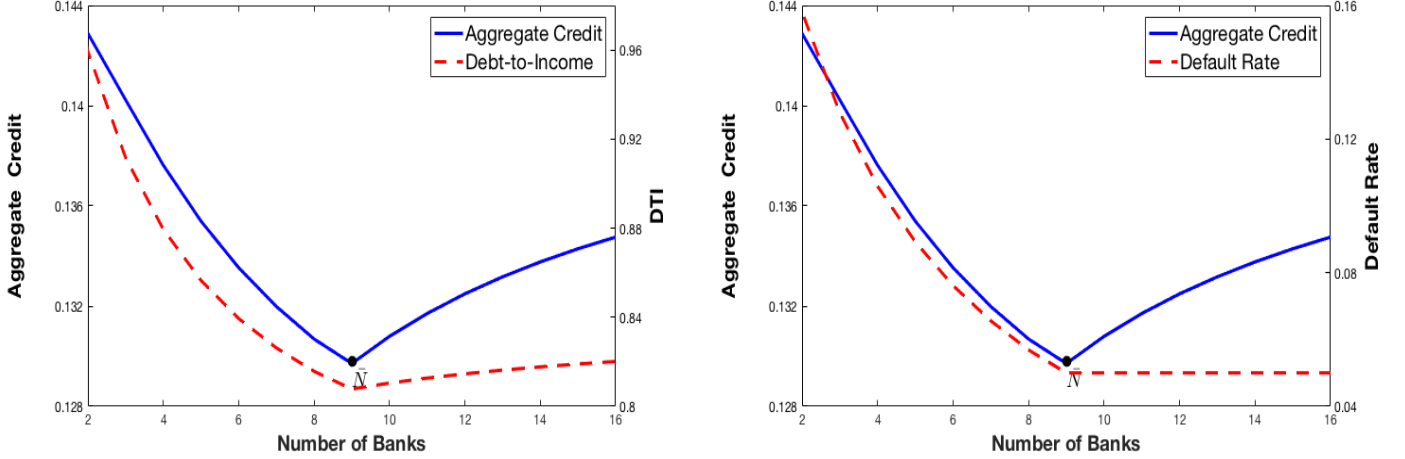


Figure 2: The figures above plot the debt-to-income ratio and default rates on the right y-axis against concentration. As we move along the x-axis, N increases and concentration decreases. The parametrization is as follows: $\delta = .01$, $\alpha^{nb} = .2$, $\phi_P^{bl} = .2$, $\phi_R^{bh} = 1$, $\phi_P^{nb} = .7$, $\phi_R^{nb} = 1$, $e_b = 2$, $\kappa = .45$, $\gamma = 4$, $c = 9.8$.

3.1.1. The Effect of Various Model Primitives

A number of factors affect banks' incentives to prop up house prices. When the expected income of low-quality borrowers, $\phi_P^{bl}e^b$, is high it is relatively more profitable to lend to low-quality borrowers and banks have to take a smaller loss on these loans in their effort to prop up prices. Therefore this increases the incentive to prop up house prices. When non-borrower income growth in the poor state, ϕ_P^{nb} , is low banks have relatively more market power when it comes to affecting house prices, increasing the incentive to prop up prices. Finally, when δ is low, houses are worth more in future periods increasing how much banks and households value the future asset value of a house. As banks are incentivized to prop up prices more when these primitives change, the threshold level of concentration necessary for banks to make high-risk loans decreases. The following corollary formalizes how $\bar{N}$ changes with the various primitives of the model.

Corollary 2 *In the three-period model, $\bar{N}$ increases as the expected income of low-quality borrowers increases, as the depreciation rate decreases, and as non-borrower growth in the poor state decreases. Formally,*

$$\frac{\partial \bar{N}}{\partial \phi_P^{bl}e^b} > 0, \quad \frac{\partial \bar{N}}{\partial \delta} < 0 \quad \text{and} \quad \frac{\partial \bar{N}}{\partial \phi_P^{nb}} < 0.$$

3.1.2. Asset Value of Housing

The key property of housing that gives rise to this mechanism is that housing is a durable asset with future value. This is distinct from other goods that only serve a consumption purpose. When $\delta = 1$ and housing depreciates completely, banks and households do not care about future house prices. There are no incentives to prop-up prices, and only the Cournot effect remains. As δ decreases, the asset value of the house increases, causing banks and households to value future house prices. This creates an incentive to prop up prices when banks have a large enough exposure to the housing market. At low values of δ , the incentives to prop up prices is stronger since housing is worth more upon repayment leading to a higher value of $\bar{N}$. Lower values of δ also increases household net worth upon repayment and allow banks to charge larger repayments from households when making mortgage loans increasing the total amount of credit banks are willing to supply. Figure 3 shows aggregate lending for different values of δ .

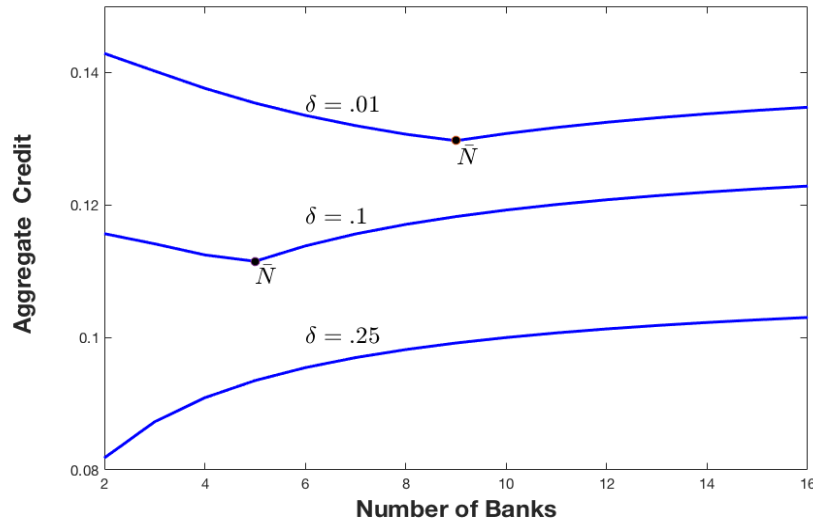


Figure 3: The figure above plots total credit, measured by the number of households who get a mortgage, against the level of concentration in the mortgage market for different δ s. As we move along the x-axis, N increases and concentration decreases. The parametrization is as follows: $\alpha^{nb} = .2$, $\phi_P^{bl} = .2$, $\phi_R^{bh} = 1$, $\phi_P^{nb} = .7$, $\phi_R^{nb} = 1$, $e_b = 2$, $\kappa = .45$, $\gamma = 4$, $c = 9.8$.

3.2. Mortgage Growth and Income Growth

There has been an active debate on whether the housing boom and bust was driven by a supply shock or by expectations of high future house prices. In support of the first hypothesis, [Mian and Sufi \(2009\)](#) find that income growth decoupled from the growth in mortgage credit in the U.S. at the ZIP code level. They stress that such a result is in line with the supply shock hypothesis since lending seemed to have increased disproportionately to

borrowers at the lower end of the income distribution. In a recent paper, looking at borrower-level data [Adelino et al. \(2016\)](#) find that income and mortgage credit growth remained positively correlated at the borrower-level. They argue that this is more in line with high house price expectations since credit increased across the income distribution and was not disproportionately extended to high-risk low-quality borrowers.²³

My model incorporates elements of both hypotheses in a rational framework with homogeneous beliefs across all agents and simultaneously produces the results of [Mian and Sufi \(2009\)](#) and [Adelino et al. \(2016\)](#). In the model, an exogenous increase in concentration can cause a credit-supply shock that propagates through the expectation of higher future house prices as large lenders have an incentive to prop up house prices. This can generate a negative correlation between the growth in mortgage credit and income growth across areas (eg. across ZIP codes) while still maintaining a positive correlation between the two at a borrower-level. Specifically, consider a change in concentration when the number of banks decreases from $N_1 > \bar{N}$ to $N_2 < \bar{N}$ such that banks move from not propping up prices at $t = 2$ to propping up prices. Define ΔM as the change in mortgage credit following this increase in concentration. Then, we can establish the following proposition,

Proposition 2 *Following an increase in concentration when the number of banks decreases from $N_1 \geq \bar{N}$ to $N_2 < \bar{N}$ and banks begin to prop up house prices, mortgage credit growth is negatively correlated with non-borrower income growth in the poor state and positively correlated with borrower income growth in the poor state i.e.,*

$$\frac{\partial \Delta M}{\phi_P^{nb}} < 0, \quad \text{and} \quad \frac{\partial \Delta M}{\phi_P^{bl}} > 0.$$

As concentration increases the magnitude of the credit supply shock is largest in areas that have the lowest growth in non-borrower income.²⁴ All else equal, when non-borrower income growth is low at $t = 2$, the intra-period strategic substitution effect leads banks to extend more credit. In the absence of other sources of demand to drive up housing prices, the effective market power of banks in the housing market is higher. As a result, for every additional mortgage loan that banks extend to low-quality borrowers, the percentage

²³It is worth noting that both the credit-supply hypothesis and the expectations hypothesis typically require irrationality, misinformation or differences in beliefs amongst agents to justify the housing boom. In the case of a credit supply shock due to securitization and other innovative ways of channeling loans to low-quality borrowers, since both borrowers and the buyers of MBSes faced losses in the crisis, it is hard to explain why the credit supply shock happened without an overoptimism about the benefits of securitization and other forms of credit expansion. On the other hand, the expectations hypothesis is silent about what drives expectations and often requires irrational or heterogeneous beliefs concerning future house prices.

²⁴Note that since $\bar{N}$ depends on ϕ_P^{bl} and ϕ_P^{nb} , this proposition only compares areas given an N_1 and N_2 s.t. $\bar{N}$ for each area falls within $(N_1, N_2]$.

increase in housing prices is greater when non-borrower income growth is low. The returns to propping up prices are therefore highest in areas with low-income growth. Looking at borrower-level income growth, the return from making mortgage loans to borrowers is higher when borrowers have larger expected income growth. All else equal, a bank would always prefer to make a loan to a high-quality borrower, if possible, as such a loan would also serve to increase house prices. The growth in mortgage credit is therefore positively correlated to the growth in borrower income, ϕ^b .

Following an exogenous increase in concentration, this negative correlation between non-borrower income and mortgage credit growth can cause areas with low-income growth to experience larger credit-supply shocks than areas with higher income growth. Figure 4 illustrates such a case by plotting aggregate credit across an area with high versus low income growth. In the region in which banks do not prop up prices, mortgage credit growth is positively correlated to income growth. Banks only consider the return they make on the mortgage loan itself, which increases when income growth is higher. When the mortgage market is concentrated and banks are propping up prices, it is possible for credit to be higher in areas with relatively low-income growth as the return to propping up prices is higher in areas with low non-borrower income growth. In the example in Figure 4, imagine an increase in concentration which causes the number of banks to go from 8 to 6. In this case, the area with high-income growth will experience a decline in credit due to decreased competition amongst banks. At the same time, the area with low-income growth will experience a positive growth in credit due to an increase in the market power of banks.

This mechanism gives a theoretical foundation to the finding in [Mian and Sufi \(2009\)](#) that mortgage growth decoupled from income growth in the period leading up to the crisis at a ZIP code-level. The housing boom coincided with mortgage holdings in the U.S. becoming increasingly concentrated as Fannie Mae and Freddie Mac grew to hold a very large share of the mortgage market. The GSEs business model depended heavily on house prices staying high and the mechanism in this model suggests that they may have successively extended worse and worse credit to maintain high house prices.

At the borrower-level, the growth in mortgage credit and income growth can remain positively correlated. In the simplified three-period model, I provide the intuition for this result but do not show it explicitly as there is no inter-period heterogeneity amongst borrowers. Appendix C outlines an example with inter-period borrower heterogeneity that shows how the model can simultaneously produce a negative correlation between credit growth and income growth amongst areas (such as ZIP codes) while maintaining a positive correlation at the borrower-level. The model can thus reconcile the seemingly contradictory findings in the empirical literature.

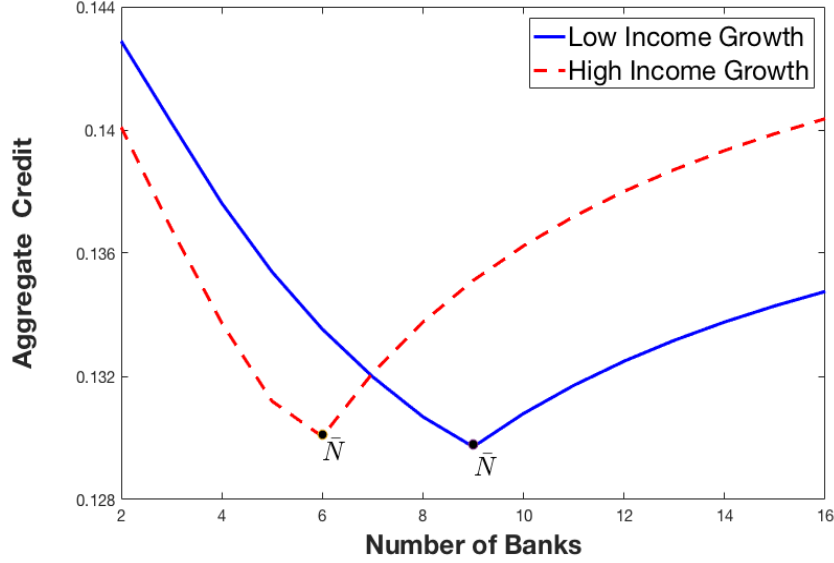


Figure 4: The figure above plots total credit, measured by the number of households who get a mortgage, against the level of concentration in the mortgage market for different income growths between $t = 1$ and $t = 2$. As we move along the x-axis, N increases and concentration decreases. The parametrization is as follows: $\delta = .01$, $\alpha^{nb} = .2$, $\phi_P^{bl} = .2$, $\phi_R^{bh} = 1$, $\phi_R^{nb} = 1$, $e_b = 2$, $\kappa = .45$, $\gamma = 4$, $c = 9.8$. ϕ_P^{nb} is varied to get changes in income growth across the two plots - it is equal to .73 in the high-income growth area and .7 in the low-income growth area.

In the three-period model, by construction in the last period house prices fall to κ demonstrating that banks may lend to low-quality borrowers even if a crash in house prices is inevitable. This helps illustrate the incentives banks have to prop up prices in the simplest possible setup. In the infinite horizon model in Section 4, we get more realistic price processes for housing prices and credit and can assess how concentration can lead to endogenous booms and busts in house prices.

4. Infinite Horizon Model

I now solve the infinite-horizon model which gives rise to boom and bust cycles that can explain various empirical facts that defined the housing crisis. In the full model, there is intra-period heterogeneity: each period has both high- and low-quality borrowers. Additionally, there is uncertainty about the state of the world: with probability q a rich generation is born and with probability $1 - q$ a poor generation is born in which all households have a lower expected income.

The path dependency of this problem can make it complicated to solve since in every state banks have to decide how much to lend taking into account outstanding loans and future lending. Furthermore, they also have to account for how the lending decisions of

other banks will affect both current and future house prices. Given the model setup, it is possible to simplify a bank's maximization in a similar way as the three-period model to get a tractable problem.²⁵ In the infinite horizon model, we can show that similar to the model with three periods, once markets become concentrated banks have incentives to prop up house prices. Furthermore, because of intra-period strategic substitution amongst banks, the economy has a unique equilibrium despite strategic complementarities in lending across periods. Initialize initial loans to 0. Then we can establish the following proposition analogous to the three-period case,

Proposition 3 *The infinite-horizon model has a unique equilibrium. There exists a cutoff, $\bar{N}$, such that if $N \geq \bar{N}$, for any possible sequence of shocks, banks do not make any loans to high-risk, low-quality borrowers to prop up prices. If $N < \bar{N}$, there are a strictly positive number of sequences of shocks in which banks will extend credit to high-risk, low-quality borrowers to prop up house prices.*

The key intuition for this proposition is identical to that in the three-period case. When N is large, each individual bank has a small amount of loans on its books. Therefore banks do not benefit from making loans that are unprofitable to push up house prices as the return from propping up prices is low. As N increases and individual banks acquire larger market shares, increasing house prices allows them to profit from a greater number of loans. This increases the return from keeping house prices high. Therefore, as concentration increases, the equilibrium begins to feature loans that are made to high-risk borrowers even if the loan itself is negative NPV.

An important feature of the equilibrium is that conditional on the state of the economy, lending per bank is increasing in its outstanding loans. Formally,

Corollary 3 *Conditional on the state of the economy, aggregate lending, M_t , is increasing in aggregate outstanding loans, M_{t-1} . Lending per bank, m_t , is similarly increasing in outstanding loans, m_{t-1} , conditional on the state of the economy.*

This feature of the equilibrium naturally generates housing boom and bust cycles and is discussed in the remainder of this section.

4.1. Boom and Bust Cycles

An exogenous increase in concentration can lead to credit booms and busts with features that match key empirical facts about the recent housing crisis.

²⁵Details are provided in the appendix.

Credit Boom, Rising House Prices and Rising DTI Ratios: In the equilibrium, the amount of loans a bank makes is increasing in the its outstanding loans, conditional on the state of the economy. This feature of the equilibrium naturally gives rise to an increasing time-series of credit and house prices following a series of positive income shocks.²⁶ This increase in credit happens even though the “fundamentals” of the shock, the underlying income of households, stays the same. The increase in credit is rather driven by the fact that lenders find it profitable to increase credit expansion to prop up house prices as the size of their outstanding exposure to the mortgage market increases. This effect is most pronounced for lenders with access to a large number of potential borrowers since they make more loans per bank, increasing the incentive to prop up prices in subsequent periods. Furthermore, they are able to make a larger number of mortgages before being limited by the market share constraint. They are therefore able to acquire larger and larger mortgage exposures, increasing their incentive to prop up prices. High concentration can therefore cause credit booms to be much larger than when concentration is low. Since the fundamentals of the economy in terms of expected borrower income remain the same, the only thing that is changing over time are house prices driven by greater credit expansion. The credit boom is therefore accompanied by an increase in house prices and rising debt to income ratios.

Figure 5 shows how a credit boom and bust differ in high concentration versus low concentration areas in an economy that has a series of consecutive high shocks followed by low shocks. In this example, the area with a higher concentration has approximately 25% more credit expanded at the height of the boom than the area with low concentration.

Rise in Risky Lending: The recent housing crisis had an unprecedented increase in risky lending with mortgages being extended to borrowers with low FICO scores at high DTI and LTV ratios. Figure 6 shows the composition of credit during the boom and bust cycle. The right y-axis plots the percentage of credit extended to low-quality borrowers in the credit boom. Not only do areas with high concentration have larger credit booms, the composition of that credit is riskier. Also note that there is a boom in credit to high-quality borrowers during rich-shocks in both high- and low- concentration areas. This is by virtue of fundamentals being better during high shocks and is in line with findings by [Adelino et al. \(2016\)](#) that credit expansion increased across the income distribution during the credit boom. However, the increase in outstanding loans in high concentration areas also creates an incentive to prop up prices, leading to loans additionally being made to low-quality borrowers. The overall composition of credit is therefore riskier in high concentration

²⁶For example, imagine that all banks start with 0 loans on their books and at $t = 1$ a high shock occurs and banks make $m_1 > 0$ loans. Then at $t = 2$ banks have $m_1 > 0$ loans outstanding on their books. If at $t = 2$ another high shock occurs, given that m_t is increasing in m_{t-1} conditional on the state of the economy, $m_2 > m_1$. And so on and so forth for any consecutive high shocks.

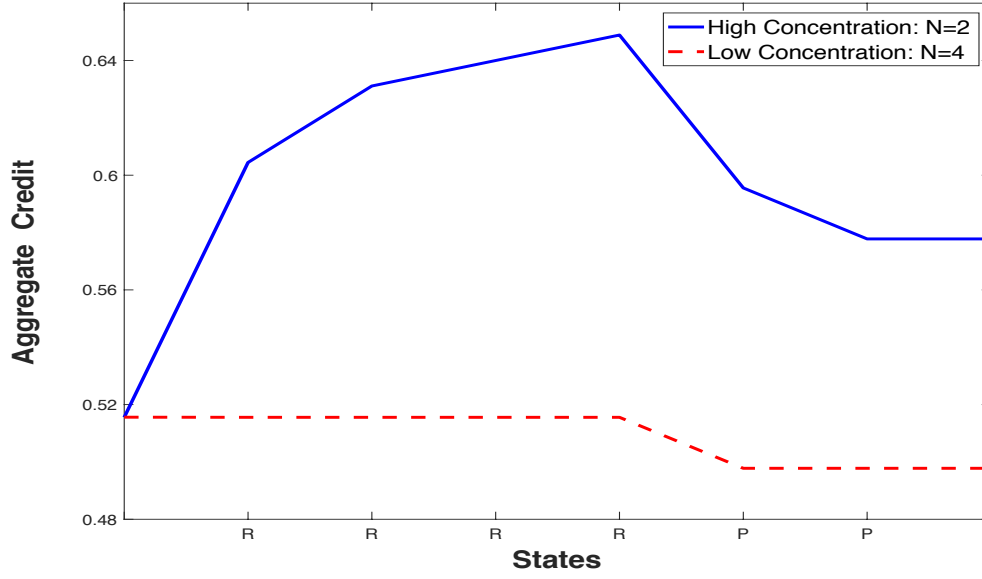


Figure 5: The figure above plots total credit, measured by the number of households who get a mortgage, across two areas with different concentration for a series of income-shocks on the x-axis. Details of the parametrization are in Appendix B.

areas during credit booms.

Timing of Risky Lending during Boom: The rise of risky lending happened in the early- to mid-2000s even though house prices had been rising since the 1990s. Most theories on the rise of risky lending do not account for the timing of the rise. For example, the securitization of mortgages was common since the 1990s but the rise in risky lending was not observed until later. As Figure 6 illustrates, the amount of risky lending increases over the life of the credit boom and does not start immediately on entering a rich state. Since low-quality loans are negative NPV, lending to risky borrowers is profitable only when the return from propping up house prices is high. A lender's portfolio needs to be large enough for there to be an incentive to make risky loans. Since the number of outstanding loans increases over the life of the boom, high-risk lending will begin after the start of the boom once the outstanding mortgage exposure is large enough. The exact timing of the start of risky lending depends on the amount of concentration and the profitability of high- and low-quality loans. As banks are incentivized to prop up prices, they will first saturate the high-quality market and only then move on to riskier lending.

Continuation of Risky Lending after Boom: Lastly, Figure 6 illustrates that once a low-shock hits the economy, lenders in the highly concentrated area continue to make risky loans for a short time. Even though fundamentals in a poor state worsen, since large lenders have a lot of outstanding loans, they have an incentive to keep lending to keep house prices high. Essentially, large lenders react less to fundamental shocks than small lenders because

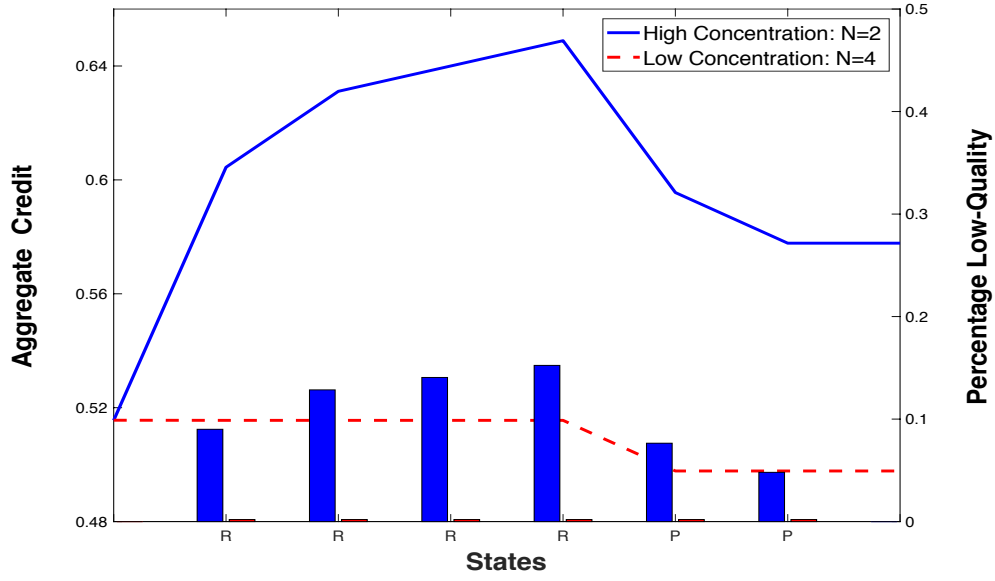


Figure 6: The bars in the figure above plot the percentage of loans (right y-axis) made to high-risk, low-quality borrowers across two areas with different concentration for a series of income-shocks on the x-axis. The lines plot total credit (left y-axis), measured by the number of households who get a mortgage. Details of the parametrization are in Appendix B.

of the size of their balance sheet. In recent work, Elul, Gupta and Musto (2017) find that the GSEs increased their risky activity in 2007 when markets began to slow down and when private securitizers started withdrawing from the market. Additionally, Bhutta and Keys (2017) find that private mortgage insurance issuance which allowed the GSEs to securitize loans with riskier fundamentals also increased in 2007 as the housing market was beginning its downturn. Both papers also find that this seemed to be happening in areas where other players were pulling out of markets. As house prices are likely to fall when other sources of housing demand slow down, this is in line with my model predictions.

The model thus explains many facts about the recent housing crisis. As discussed earlier, when Salomon Brothers got investors interested in MBS as an investment vehicle, Fannie Mae and Freddie Mac grew their market share tremendously having a monopoly on prime securitization and later became the biggest investors in private label MBS. This event can be viewed as an exogenous increase to the GSEs market power, giving them access to large share of potential borrowers. The model can help us understand the credit boom and bust following this increase in GSE market power.

5. Secondary Market Equivalency

During the housing boom, there was a large concentration in mortgage holdings at the GSE level and amongst a few large banks who purchased many MBS. In the model described

so far, mortgage originators are assumed to be the final holders of these mortgages. The baseline model can be reframed as an equivalent problem in which mortgage holders purchase mortgages from a secondary market and do not originate any loans themselves. The key mechanism works as long as there is concentration in mortgage holdings at some level and agents with exposure to mortgage payments have some market power. If secondary market players own a large share of the mortgage market, they want to keep house prices up. If they have market power, they can offer attractive prices on the secondary market for sub-prime mortgages that will incentivize mortgage originators to issue mortgages to risky borrowers. Holders of these mortgages will suffer losses on these risky purchases but the increase in house prices will be profitable for their outstanding mortgage exposure.

The equivalent model is as follows. A continuum $[0, 1 - \alpha^{nb}]$ of banks competitively originate mortgages and sell them to a secondary market. Each bank has access to one borrower in the continuum of borrowers. Final mortgage holders purchase mortgages on the secondary market from originators. Each holder has access to a fraction $\frac{1}{N}$ of originators and thereby to a fraction $\frac{1}{N}$ of borrowers. Assuming mortgage originators follow an originate-to-distribute model and do not hold mortgages is equivalent to assuming that their main source of funding comes from secondary markets. The originate-to-distribute model was common amongst mortgage originators during the housing boom ([Purnanandam \(2011\)](#)). [Loutskina and Strahan \(2009\)](#) and [An and Yao \(2016\)](#) provide evidence that the GSEs were a key source of liquidity provision for non-jumbo loans issued by banks.

A bank that originates mortgages is offered a secondary market price, $Y_t(r_t^j, \omega_t^{bj})$, for a mortgage originated at time t to a household j with expected endowment ω_t^{bj} at interest rate r_t^j . An originator will make a mortgage loan to a borrower if:

$$Y_t(r_t^j, \omega_t^{bj}) - P_t \geq 0.$$

Each period holders of mortgages will choose to post secondary market prices taking into account their effect on originator decisions and how that influences housing prices. I assume that secondary market holders can purchase mortgages at different rates from different mortgage originators thereby allowing them to control how many mortgages of a type they wish to purchase. Therefore, secondary market mortgage holders maximize the following, where m_{t-1}^j is the number of mortgages of type $j \in \{h, l\}$ they purchased in the previous period,

$$\begin{aligned}
V(s_t, s_{t-1} m_{t-1}^h, m_{t-1}^l, r_{t-1}^h, r_{t-1}^l, P_{t-1}) &= \max_{m_t^h \geq 0, m_t^l \geq 0, Y_t^j, Y_t^l, r_t^h, r_t^l} \\
&\sum_{j=\{h,l\}} \underbrace{m_{t-1}^j \left(\phi_{s_t}^{bj} \min\{P_{t-1}(1 + r_{t-1}^j), e^b + (1 - \delta)P_t\} + (1 - \phi_{s_t}^{bj}) \min\{P_{t-1}(1 + r_{t-1}^j), (1 - \delta)P_t\} \right)}_{\text{Repayment}} \\
&- \sum_{j=\{h,l\}} \underbrace{m_t^j Y_t^j}_{\text{New Lending}} + \underbrace{\beta E[V(s_{t+1}, m_t^h, m_t^l, r_t^h, r_t^l, P_t, s_t)]}_{\text{Continuation Value}} \\
\text{s.t. } &Y_t(r_t^j, \omega_t^{bj}) - P_t \geq 0 \\
&\gamma + \beta(1 - \delta)E[P_{t+1}] \geq \beta E[\min\{P_t(1 + r_t(s_{t+1})), \omega_t^b + (1 - \delta)P_{t+1}\}] \\
&m_t^h \leq \frac{1}{N} \alpha^{bh} \\
&m_t^l \leq \frac{1}{N} (1 - \alpha^{bh} - \alpha^{nb}).
\end{aligned}$$

The mortgage holder will pick Y_t so that the originator is just willing to lend, implying that $Y_t = P_t$. Furthermore, they will choose an r_t so that borrowers repay the maximum they are willing to pay. This is equivalent to the problem faced by banks that hold onto the mortgages they originate. The same logic can be applied if mortgages are resold on the secondary market. The key requirement for the mechanism to work is that the final holder of mortgages has some market power.

6. Extension - Lender Heterogeneity: Large and Small Lenders

During the housing boom and bust, while the GSEs and some large LCFIs acquired a large share of the mortgage market, there were also smaller mortgage market participants. The model can be extended to evaluate the case of some large lenders and many small, dispersed lenders who behave like price-takers in the mortgage market. Specifically, I modify the model to allow small, dispersed lenders to have access to a share s of households while large lenders have access to the remaining ones.

The previous results and intuition all apply here. Furthermore, in a boom-bust cycle, following a series of h shocks the *effective market share* of a large lender as measured by the percentage share of all mortgages held by that lender over the total number of mortgages outstanding, is increasing over time even though access to borrowers stays the same. This is

because as a large lender acquires more outstanding mortgages over the boom, the incentive to prop up prices increases, leading the lender to increase credit. This effect is not present for small lenders who are price-takers. The exposure of the large lender to the total mortgage market therefore increases over time. This extension of the model matches the empirically observed increase in Fannie Mae and Freddie Mac's market share over the housing boom.

I also calibrate this extended model to U.S. data. The calibration is discussed in section 6.1. As concentration increases, the size of the credit boom and extension of risky credit also increase. Importantly, now the market share of large lenders increases over the boom. Figure 7 shows a credit boom and busts in areas with differing concentration. It also shows that the gain in effective market share of a large lender is lower when there are more competing lenders.²⁷

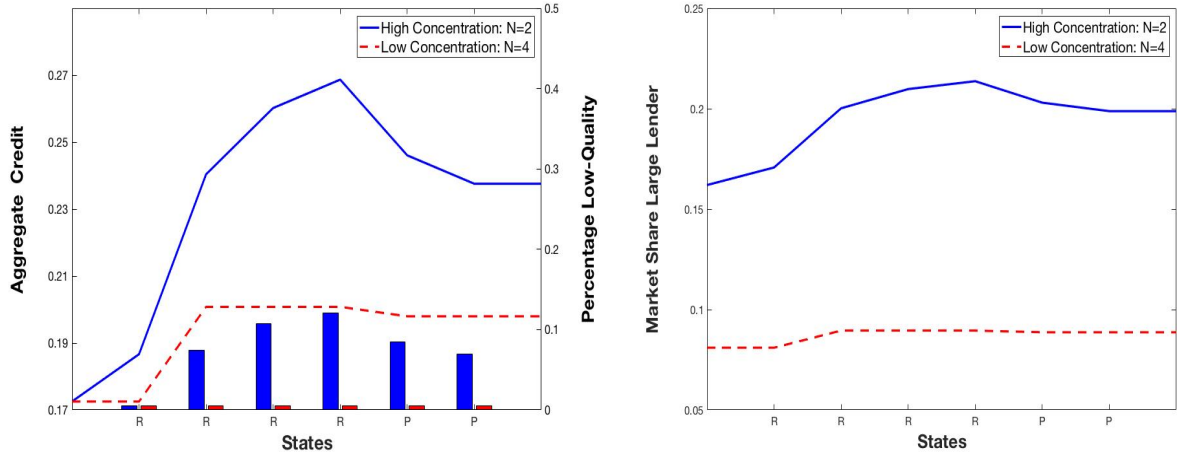


Figure 7: The panel on the right above plots the total amount of credit extended, measured by the number of households who get a mortgage loan on the y-axis and the percentage of loans made to high-risk, low-quality borrowers, on the right y-axis, across two areas with different concentration for a series of income-shocks on the x-axis. The panel on the left shows the effective market share of large lenders across two areas with different concentration for a series of income-shocks on the x-axis. Details of the parametrization are in Appendix B.

6.1. Calibration Exercise

I now calibrate the stylized model to the U.S. economy. The main purpose of this model is to clearly illustrate the theory of how concentration in the mortgage market can create an incentive to extend risky credit to keep house prices high and how that can produce credit cycles with dynamics similar to that of the recent housing crisis. In doing so, the model

²⁷This version of the calibration has the market share of large lenders decreasing upon the bust. The effective market share may also increase if non-borrower demand and the provision of mortgages by small price-taking lenders slows down.

abstracts away from various aspects of mortgage markets, housing decisions by households, details of mortgage contracts, etc. A deeper examination of the quantitative implications of concentration on mortgage credit requires a more detailed quantitative model. However, this calibration of the stylized model can help us address two questions. First, does the model produce quantitatively significant credit and house price dynamics that match the U.S. experience in the recent housing crisis? Second, does simply changing the level of concentration produce significant variation in the path of credit and house prices? If the answers to these questions are positive, then it provides support for this paper's theory that the housing boom and bust was driven by a change in concentration.

The model is calibrated to the 1991-2009 U.S. housing market with the boom quantities aggregated across 1991-2006 and the bust quantities aggregated across 2007-2008. For this exercise, I assume that the economy experienced a sequence of rich shocks in the boom years followed by a sequence of poor shocks in the bust years. The rich and poor shocks are mapped to Federal Reserve Economic Data on the growth rate of personal income. From 1991-2006 real median personal income growth at an annual rate of 1.5% while from 2007-2009 it grew at an annual rate of -1.6%. Appendix B provides a detailed description of the data.²⁸

Table 4 in Appendix B summarizes the benchmark configuration of the model parameters. I choose $s=.6$ and $N = 2$ to roughly match the GSEs' eventual market share at the height of the boom. The income shocks of high- and low-quality borrowers in the rich- and poor-states of the economy are chosen to match the default rates on prime and sub-prime loans during the boom and bust. The fraction of high-quality borrowers is chosen to match the fraction of prime versus sub-prime lending while the fraction of non-borrowers is chosen to match the fraction of cash-only house purchases.

Table 5 in Appendix B compares the model-generated quantities for the calibration to those in the data. A one-time change in concentration is able to match the house price increase and decrease well, explaining approximately 45% of the boom in house prices. A fundamental shock to the concentrated market explains about 30% of the bust in house prices. It also does a good job of matching the fraction of sub-prime borrowers during the boom explaining over 90% of the lending to sub-prime borrowers in the boom but overestimates the lending to sub-prime borrowers slightly during the bust. This may be partially driven by the fact that during 2007, the GSEs kept lending to high-risk borrowers but private securitizers started pulling out of the high-risk market.

Table 6 in Appendix B does a counterfactual analysis. I decrease concentration by doubling N from 2 to 4 while keeping the other parameters of the benchmark calibration the

²⁸Appendix B tables 1-3 also provide results for calibration of the benchmark model without lender heterogeneity.

same. This change in concentration decreases the fraction of sub-prime lending in the boom to 0 in both the boom and bust. The boom also has 30% lower growth in house prices while the bust has about 80% smaller decline in house prices.

The model can therefore produce quantitatively meaningful magnitudes and changes in credit and prices when looking at the housing boom and bust. Moreover, changing concentration can result in significant changes to the model-implied credit boom and bust. This suggests that concentration can be an important channel that contributes to credit cycles and a more comprehensive quantitative exploration of this channel is interesting for future research.

Conclusion

This paper provides a novel theory of how concentration in mortgage markets can affect both the quantity and the quality of mortgage credit. Lenders with a large outstanding mortgage exposure have incentives to extend risky credit to prop-up house prices. An increase in concentration in mortgage markets can generate housing booms and busts with features that match the recent U.S. housing crisis.

In the aftermath of the housing crisis, policy makers have wanted to design policy to curb high-risk lending. However, the role that concentration can play in creating incentives to extend risky mortgage credit has been largely overlooked in this process. The Economist recently reported that concentration in mortgage markets has increased since the crisis as a *side-effect* of new regulations faced by banks which have made them move out of mortgage lending. Somewhat ironically, many of these regulations are intended to reduce risky lending. The GSEs and the FHA are currently funding between 65-80% of new mortgages, many of which appear to be highly risky. A fifth of all loans since 2012 have LTV ratios of over 95%. This is comparable to the fraction of sub-prime lending in the years before the housing collapse. As this paper demonstrates, such high concentration can have a significant impact on both the quantity and quality of credit. It is therefore crucial to comprehensively understand the different forces that incentivize high-risk lending when designing policy.

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Appendix A - Proofs of Key Propositions

Proof of Lemma 1. The three-period model can be solved by backwards induction. Since no new generation is born at $t = 3$, banks do not lend at $t = 3$. Additionally, the price of housing at $t = 3$ is given by the liquidation value κ . At $t = 2$, a bank solves,

$$\begin{aligned} \max_{m_2 \geq 0} \quad & \underbrace{m_1 \left(\phi_R^{bh} \min\{P_1(1+r_1), e^b + (1-\delta)P_2\} + (1-\phi_R^{bh}) \min\{P_1(1+r_1), (1-\delta)P_2\} \right)}_{\text{Repayment Outstanding Loans}} - \underbrace{m_2 P_2}_{\text{New Lending}} \\ & + \underbrace{\beta m_2 \left(\phi_P^{bl} \min\{P_2(1+r_2), e^b + (1-\delta)\kappa\} + (1-\phi_P^{bl}) \min\{P_2(1+r_2), (1-\delta)\kappa\} \right)}_{\text{Repayment of New Loans}} \end{aligned}$$

$$\text{s.t. } \gamma + \beta(1-\delta)\kappa \geq \beta E[\min\{P_2(1+r_2), \omega_2^b + (1-\delta)\kappa\}]$$

$$\cdot \quad 0 \leq m_2 \leq \frac{1}{N}(1 - \alpha^{nb}).$$

In the following analysis, I refer to the first constraint faced by the bank as the borrower purchasing constraint. The above problem can be simplified by focusing on the bank's choice of interest rate. Since banks have monopoly power over their borrowers when setting interest rates and loans have full recourse, a bank will charge the maximum interest rate that borrowers are willing to pay.

I start by considering the bank's choice of interest rate at $t = 1$. If a bank can not commit to future lending, there are two cases depending on the relative values of ϕ_R^{bh} and γ .

Case 1: If $\phi_R^{bh} e^b \leq \frac{\gamma}{\beta}$, at $t = 1$ a bank will charge borrowers,

$$P_1(1+r_1) > e^b + (1-\delta)P_2.$$

In this case, a proportion ϕ_R^{bh} of bank borrowers receive a positive endowment, have net worth equal to $e^b + (1-\delta)P_2$ and repay the bank $e^b + (1-\delta)P_2$ while the remaining do not get an endowment, default and the bank gets $(1-\delta)P_2$. In expectation, borrowers repay the bank $\phi_R^{bh} e^b + (1-\delta)P_2$ which satisfies their purchasing constraint. In the model without commitment to $t = 2$ lending when banks make $t = 1$ loans, banks can only credibly prop up house prices to improve their return on loans when the borrower cannot repay the full face-value of the loan. The bank will therefore choose a face-value that is strictly higher than $e^b + (1-\delta)P_2$.

Case 2: If $\phi_R^{bh} e^b > \frac{\gamma}{\beta}$, a bank will charge,

$$P_1(1 + r_1) = \frac{\gamma}{\beta\phi_R^{bh}} + (1 - \delta)P_2.$$

In this case, a proportion ϕ_R^{bh} of bank borrowers receive a positive endowment, have net worth equal to $e^b + (1 - \delta)P_2$ and repay the bank in full $\frac{\gamma}{\beta\phi_R^{bh}} + (1 - \delta)P_2$ while the remaining do not get an endowment, default and the bank gets $(1 - \delta)P_2$. Borrowers repay the bank $\frac{\gamma}{\beta} + (1 - \delta)P_2$ in expectation which just satisfies their purchasing constraint. Note that in this case, a bank cannot charge a higher face-value because borrowers with a positive endowment would be able to repay the higher face-value and this would violate their purchasing constraint. A bank will therefore only be able to improve its return on all defaulting loans from a proportion $(1 - \phi_R^{bh})$ of borrowers.

By similar reasoning, the interest rate a bank will charge borrowers at $t = 2$ is s.t. $P_2(1 + r_2) \geq e^b + (1 - \delta)\kappa$. The assumption on low-quality loans being unprofitable implies that $\phi_P^{bl}e^b < \frac{\gamma}{\beta}$.

The equilibrium solution in both cases is similar. In Case 1, I can write an equivalent maximization problem for the bank at time 2 only incorporating the portion of its profits from outstanding loans that will be affected by its $t = 2$ lending,

$$\begin{aligned} \max_{m_2 \geq 0} \quad & m_1(1 - \delta)P_2 - m_2P_2 + \beta m_2 (\phi_P^{bl}e^b + (1 - \delta)\kappa) \\ \text{s.t.} \quad & 0 \leq m_2 \leq \frac{1}{N}(1 - \alpha^{nb}). \end{aligned}$$

Define M_2^{-i} as the total lending by all other banks at $t = 2$. Then, taking the FOC, the optimal number of loans issued by the bank at $t = 2$, m_2 , is given by,

$$m_2 = \max \left\{ 0, \frac{m_1(1 - \delta) - \phi_P^{nb}\alpha^{nb} - M_2^{-i} + \beta \frac{\phi_P^{bl}e^b + (1 - \delta)\kappa}{c}}{2} \right\}.$$

Note that m_2 is always less than $\frac{1}{N}(1 - \alpha^{nb})$ so the maximum constraint on m_2 never binds. This is because the assumption on low-quality loans always being negative NPV gives implies that $-\phi_P^{nb}\alpha^{nb} - M_2^{-i} + \beta \frac{\phi_P^{bl}e^b + (1 - \delta)\kappa}{c} < 0$ and m_1 always have to be less than $\frac{1}{N}(1 - \alpha^{nb})$ due to bank's $t = 1$ lending constraint.

At $t = 1$ a bank takes into account its lending at time $t = 2$ and solves,

$$\begin{aligned}
\max_{m_1 \geq 0} \quad & -m_1 P_1(m_1) + \beta m_1 (\phi_R^{bh} e^b + (1 - \delta) P_2(m_1)) - \beta m_2(m_1) P_2(m_1) \\
& + \beta^2 m_2(m_1) (\phi_P^{bl} e^b + (1 - \delta) \kappa) \\
\text{s.t.} \quad & m_1 \leq \frac{1}{N} (1 - \alpha^{nb}).
\end{aligned}$$

I now solve for m_1 when $m_2 > 0$ and when $m_2 = 0$. The first order condition for both, gives the following choice of m_1 for the bank,

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\frac{\beta}{c} (\phi_R^{bh} e^b + (1 - \delta) c (\phi_P^{nb} \alpha^{nb} + M_2^{-i} + m_2)) - \phi_R^{nb} \alpha^{nb} - M_1^{-i}}{2}, \frac{1 - \alpha^{nb}}{N} \right\} \right\}.$$

Note that the above solution is under the assumption that at $t = 2$, other banks take m_2 as given. An alternative approach involved other banks taking m_2 as a function of m_1 at $t = 2$, in which case at $t = 1$ when a bank chooses m_1 , it would also take into account its decision on M_2^{-i} . This approach assumes that deviations at $t = 1$ from equilibrium are not observable. The model solution if equilibrium deviations are observable is similar but less tractable.

In Case 2, I can similarly write an equivalent maximization problem for the bank at time 2 only incorporating the portion of its profits from outstanding loans that will be affected by its $t = 2$ lending,

$$\begin{aligned}
\max_{m_2 \geq 0} \quad & (1 - \phi_R^{bh}) m_1 (1 - \delta) P_2 - m_2 P_2 + \beta m_2 (\phi_P^{bl} e^b + (1 - \delta) \kappa) \\
\text{s.t.} \quad & 0 \leq m_2 \leq \frac{1}{N} (1 - \alpha^{nb}).
\end{aligned}$$

Taking the FOC, the optimal number of loans issued by the bank at $t = 2$, m_2 , is given by,

$$m_2 = \max \left\{ 0, \frac{(1 - \phi_R^{bh}) m_1 (1 - \delta) - \phi_P^{nb} \alpha^{nb} - M_2^{-i} + \beta \frac{\phi_P^{bl} e^b + (1 - \delta) \kappa}{c}}{2} \right\}.$$

The constraint $m_2 \leq \frac{1}{N} (1 - \alpha^{nb})$ will not bind for the same reason as in Case 1. At $t = 1$ a bank takes into account its lending at time $t = 2$ and solves,

$$\begin{aligned}
& \max_{m_1 \geq 0} -m_1 P_1(m_1) + \beta m_1 \left(\frac{\gamma}{\beta} + (1-\delta) P_2(m_1) \right) - \beta m_2(m_1) P_2(m_1) \\
& \quad + \beta^2 m_2(m_1) (\phi_P^{bl} e^b + (1-\delta) \kappa) \\
& \text{s.t. } m_1 \leq \frac{1}{N} (1 - \alpha^{nb}).
\end{aligned}$$

Taking the first order condition, the bank's choice of m_1 when $m_2 > 0$ is given by,

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\frac{\beta}{c} \left(\frac{\gamma}{\beta} + (1-\delta) c(\phi_P^{nb} \alpha^{nb} + M_2^{-i} + m_2) \right) - \phi_R^{nb} \alpha^{nb} - M_1^{-i}}{2 - \frac{\beta}{2} (1 - \phi_R^{bh}) \phi_R^{bh} (1-\delta)^2}, \frac{1 - \alpha^{nb}}{N} \right\} \right\}.$$

When $m_2 = 0$, the bank's choice of m_1 is given by,

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\frac{\beta}{c} \left(\frac{\gamma}{\beta} + (1-\delta) c(\phi_P^{nb} \alpha^{nb} + M_2^{-i} + m_2) \right) - \phi_R^{nb} \alpha^{nb} - M_1^{-i}}{2}, \frac{1 - \alpha^{nb}}{N} \right\} \right\}.$$

With commitment, the solution is similar. If $\phi_R^{bh} e^b \leq \frac{\gamma}{\beta}$, at $t = 1$ a bank will charge borrowers,

$$P_1(1 + r_1) \geq e^b + (1 - \delta) P_2.$$

The bank can set the interest rate equal to $e^b + (1 - \delta) P_2$. The bank now solves for both m_1 and m_2 at $t = 1$. It therefore solves,

$$\begin{aligned}
& \max_{m_1, m_2 \geq 0} -m_1 P_1(m_1) + \beta m_1 (\phi_R^{bh} e^b + (1-\delta) P_2(m_2)) - \beta m_2 P_2(m_2) \\
& \quad + \beta^2 m_2 (\phi_P^{bl} e^b + (1-\delta) \kappa) \\
& \text{s.t. } m_1, m_2 \leq \frac{1}{N} (1 - \alpha^{nb}).
\end{aligned}$$

The first order conditions give,

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\frac{\beta}{c} (\phi_R^{bh} e^b + (1-\delta) c(\phi_P^{nb} \alpha^{nb} + M_2^{-i} + m_2)) - \phi_R^{nb} \alpha^{nb} - M_1^{-i}}{2}, \frac{1 - \alpha^{nb}}{N} \right\} \right\}.$$

$$m_2 = \max \left\{ 0, \frac{m_1(1 - \delta) - \phi_P^{nb} \alpha^{nb} - M_2^{-i} + \beta \frac{\phi_P^{bl} e^b + (1 - \delta) \kappa}{c}}{2} \right\}.$$

If $\phi_R^{bh} e^b > \frac{\gamma}{\beta}$, at $t = 1$ a bank will charge borrowers,

$$P_1(1 + r_1) = \frac{\gamma}{\beta \phi_R^{bh}} + (1 - \delta) P_2.$$

The bank now solves for both m_1 and m_2 at $t = 1$. It therefore solves,

$$\begin{aligned} \max_{m_1, m_2 \geq 0} \quad & -m_1 P_1(m_1) + \beta m_1 \left(\frac{\gamma}{\beta} + (1 - \delta) P_2(m_2) \right) - \beta m_2 P_2(m_2) \\ & + \beta^2 m_2 (\phi_P^{bl} e^b + (1 - \delta) \kappa) \\ \text{s.t.} \quad & m_1, m_2 \leq \frac{1}{N} (1 - \alpha^{nb}). \end{aligned}$$

The first order conditions give,

$$\begin{aligned} m_1 = \max \left\{ 0, \min \left\{ \frac{\frac{\beta}{c} \left(\frac{\gamma}{\beta} + (1 - \delta) c (\phi_P^{nb} \alpha^{nb} + M_2^{-i} + m_2) \right) - \phi_R^{nb} \alpha^{nb} - M_1^{-i}}{2}, \frac{1 - \alpha^{nb}}{N} \right\} \right\}. \\ m_2 = \max \left\{ 0, \frac{m_1(1 - \delta) - \phi_P^{nb} \alpha^{nb} - M_2^{-i} + \beta \frac{\phi_P^{bl} e^b + (1 - \delta) \kappa}{c}}{2} \right\}. \end{aligned}$$

■

Proof of Proposition 1. I start the proof by showing that an equilibrium in symmetric strategies exists and then show that this equilibrium is unique. I work through Case 1 in which the bank can not commit to future lending. Case 2 and the equilibrium with commitment can be proved similarly.

I first consider equilibria in which all banks are propping up prices ($m_2 > 0$), and then equilibria in which no bank is propping up prices. I later show that these are the only two possible equilibria, as an equilibrium in which some banks prop up prices and some do not, does not exist.

First, I consider equilibria in which all banks prop up house prices ($m_2 > 0$). Then, using Lemma 1, at $t = 2$, a bank's choice of m_2 and m_1 are given by,

$$m_2 = \frac{m_1(1 - \delta) - \phi_P^{nb} \alpha^{nb} - M_2^{-i} + \beta \frac{\phi_P^{bl} e^b + (1 - \delta) \kappa}{c}}{2}.$$

$$m_1 = \min \left\{ \frac{\frac{\beta}{c} (\phi_R^{bh} e^b + (1 - \delta) c (\phi_P^{nb} \alpha^{nb} + M_2^{-i} + m_2)) - \phi_R^{nb} \alpha^{nb} - M_1^{-i}}{2}, \frac{1 - \alpha^{nb}}{N} \right\}.$$

If $m_1 = 0$, m_2 can never be greater than 0, therefore we can drop the minimum constraint from m_1 .

In a symmetric equilibrium, $M_2^{-i} = (N - 1)m_2$ and $M_1^{-i} = (N - 1)m_1$. Substituting these into the above expressions, m_1 and m_2 are given by,

$$m_2 = \frac{m_1(1 - \delta) - \phi_P^{nb} \alpha^{nb} + \beta \frac{\phi_P^{bl} e^b + (1 - \delta) \kappa}{c}}{N + 1}.$$

$$m_1 = \min \left\{ \frac{1}{N + 1 - \beta(1 - \delta)^2 \frac{N}{(N + 1)}} \left(\beta \frac{1 - \delta}{2} \left(\phi_P^{nb} \alpha^{nb} + \frac{(N - 1) \left(-\phi_P^{nb} \alpha^{nb} + \beta \frac{\phi_P^{bl} e^b + (1 - \delta) \kappa}{c} \right)}{N + 1} \right) \right. \right. \\ \left. \left. + \beta^2 \frac{1 - \delta}{2c} (\phi_P^{bl} e^b + (1 - \delta) \kappa) + \frac{\beta}{c} \phi_R^{bh} e^b - \phi_R^{nb} \alpha^{nb} \right), \frac{1 - \alpha^{nb}}{N} \right\}.$$

For $N \geq 1$ The denominator of the RHS is increasing in N , while the numerator is decreasing in N since by assumption $-\phi_P^{nb} \alpha^{nb} + \beta \frac{\phi_P^{bl} e^b + (1 - \delta) \kappa}{c} < 0$. Therefore m_1 is decreasing in N .

This is an equilibrium as long as banks want to make $m_2 > 0$ at $t = 2$. This is the case when,

$$m_1(1 - \delta) - \phi_P^{nb} \alpha^{nb} + \beta \frac{\phi_P^{bl} e^b + (1 - \delta) \kappa}{c} > 0.$$

Rearranging,

$$m_1 > \frac{\phi_P^{nb} \alpha^{nb} - \beta \frac{\phi_P^{bl} e^b + (1 - \delta) \kappa}{c}}{1 - \delta}.$$

Define $\bar{N}$ as the value of N at which $m_1 = \frac{\phi_P^{nb} \alpha^{nb} - \beta \frac{\phi_P^{bl} e^b + (1 - \delta) \kappa}{c}}{1 - \delta}$. This is given by,

$$\frac{\phi_P^{nb} \alpha^{nb} - \beta \frac{\phi_P^{bl} e^b + (1-\delta)\kappa}{c}}{1-\delta} = \min \left\{ \frac{\frac{\beta}{c} (\phi_R^{bh} e^b + (1-\delta)c\phi_P^{nb} \alpha^{nb}) - \phi_R^{nb} \alpha^{nb}}{\bar{N} + 1}, \frac{1 - \alpha^{nb}}{\bar{N}} \right\}.$$

This can be rearranged to give the following expression for $\bar{N}$,

$$\bar{N} = \min \left\{ \frac{\frac{\beta}{c} ((1-\delta)\phi_R^{bh} e^b + (1-\delta)^2 c\phi_P^{nb} \alpha^{nb} + \phi_P^{bl} e^b + (1-\delta)\kappa) - \alpha^{nb}((1-\delta)\phi_R^{nb} + \phi_P^{nb})}{\phi_P^{nb} \alpha^{nb} - \beta \frac{\phi_P^{bl} e^b + (1-\delta)\kappa}{c}}, \right. \\ \left. \frac{(1 - \alpha^{nb})(1 - \delta)}{\phi_P^{nb} \alpha^{nb} - \beta \frac{\phi_P^{bl} e^b + (1-\delta)\kappa}{c}} \right\}.$$

Since m_1 is decreasing in N , when $N < \bar{N}$, an equilibrium in symmetric strategies in which banks prop up prices exists.

Uniqueness: I now show that when all banks are propping up prices, the symmetric equilibrium is the unique equilibrium. We can write any given bank's $t = 1$ and $t = 2$ optimal lending in terms of prices,

$$m_2 = m_1(1 - \delta) - \frac{P_2}{c} + \frac{\beta}{c} (\phi_P^{bl} e^b + (1 - \delta)\kappa).$$

$$m_1 = \min \left\{ \frac{\beta}{c} \phi_P^{bl} e^b + \frac{\beta}{c} (1 - \delta) P_2 - \frac{P_1}{c}, \frac{1 - \alpha^{nb}}{N} \right\}.$$

Since all banks face the same P_1 and P_2 , if this equilibrium exists, all players must be behaving symmetrically. When banks are behaving symmetrically the equilibrium solution is unique (calculated above). Therefore, the symmetric equilibrium is the unique equilibrium when banks are propping up house prices.

I now consider equilibria in which banks do not prop up house prices. Similar to the case before, I first establish an equilibrium in symmetric strategies in which banks do not prop up prices and then show that this is the unique equilibrium.

When the bank is not propping up prices, $m_2 = 0$. In this case $P_2 = c\alpha^{nb}\phi_P^{nb}$. At $t = 1$, a bank solves,

$$\max_{m_1} -m_1 P_1(m_1) + \beta m_1 (\phi_R^{bh} e^b + (1 - \delta) c \alpha^{nb} \phi_P^{nb}).$$

The FOC is given by,

$$-cm_1 - c(\alpha^{nb}\phi_R^{nb} + m_1 + M_1^{-i}) + \beta\phi_R^{bh}e^b + \beta(1-\delta)c\alpha^{nb}\phi_P^{nb} = 0.$$

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\beta\frac{\phi_R^{bh}e^b}{c} + \beta(1-\delta)\alpha^{nb}\phi_P^{nb} - \alpha^{nb}\phi_R^{nb} - M_1^{-i}}{2}, \frac{1 - \alpha^{nb}}{N} \right\} \right\}.$$

In a symmetric equilibrium,

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\beta\frac{\phi_R^{bh}e^b}{c} + \beta(1-\delta)\alpha^{nb}\phi_P^{nb} - \alpha^{nb}\phi_R^{nb}}{N+1}, \frac{1 - \alpha^{nb}}{N} \right\} \right\}.$$

This is an equilibrium as long as banks do not want to make $m_2 > 0$ at $t = 2$. This is the case when,

$$m_1(1-\delta) - \phi_P^{nb}\alpha^{nb} + \beta\frac{\phi_P^{bl}e^b + (1-\delta)\kappa}{c} \leq 0.$$

This is satisfied whenever $N \geq \bar{N}$.

Uniqueness: Uniqueness follows as it did before when banks are not propping up prices. We can write any given bank's $t = 1$ and $t = 2$ optimal lending in terms of prices,

$$m_2 = 0.$$

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\beta}{c}\phi_P^{bl}e^b + \frac{\beta}{c}(1-\delta)P_2 - \frac{P_1}{c}, \frac{1 - \alpha^{nb}}{N} \right\} \right\}.$$

Since all banks face the same P_1 and P_2 , if this equilibrium exists, all players must be behaving symmetrically. When banks are behaving symmetrically the equilibrium solution is unique (calculated above). Therefore, the symmetric equilibrium is the unique equilibrium when banks are propping up house prices.

To complete the proof, I also need to rule out the case when some banks are propping up prices and some are not. I prove this by contradiction. Imagine bank i is propping up prices and bank j is not. Then at $t = 1$,

$$m_1^i = \min \left\{ \frac{\beta}{c}\phi_P^{bl}e^b + \frac{\beta}{c}(1-\delta)P_2 - \frac{P_1}{c}, \frac{1 - \alpha^{nb}}{N} \right\}.$$

$$m_1^j = \max \left\{ 0, \min \left\{ \frac{\beta}{c}\phi_P^{bl}e^b + \frac{\beta}{c}(1-\delta)P_2 - \frac{P_1}{c}, \frac{1 - \alpha^{nb}}{N} \right\} \right\}.$$

These expressions imply that $m_1^i = m_1^j$. However, if this is the case, $m_2^j = m_1^i$ which is a

contradiction. Therefore, in any equilibrium either all banks will be propping up prices, or no bank will be propping up prices.

For Case 2 without commitment, the proposition can be proven in a similar way. In this case, $\bar{N}$ is given by the following expression,

$$\bar{N} = \min \left\{ \frac{\frac{\beta}{c} ((1-\delta)(1-\phi_R^{bh})\phi_R^{bh}e^b + (1-\delta)^2(1-\phi_R^{bh})c\phi_P^{nb}\alpha^{nb} + \phi_P^{bl}e^b + (1-\delta)\kappa)}{\phi_P^{nb}\alpha^{nb} - \beta\frac{\phi_P^{bl}e^b + (1-\delta)\kappa}{c}}, \frac{(1-\alpha^{nb})(1-\phi_R^{bh})(1-\delta)}{\phi_P^{nb}\alpha^{nb} - \beta\frac{\phi_P^{bl}e^b + (1-\delta)\kappa}{c}} \right\}.$$

With commitment, $\bar{N}$ is given by,

$$\bar{N} = \min \left\{ \frac{\frac{\beta}{c} \left((1-\delta) \min \left\{ \phi_R^{bh}e^b, \frac{\gamma}{\beta} \right\} + (1-\delta)^2c\phi_P^{nb}\alpha^{nb} + \phi_P^{bl}e^b + (1-\delta)\kappa \right) - \alpha^{nb}((1-\delta)\phi_R^{nb} + \phi_P^{nb})}{\phi_P^{nb}\alpha^{nb} - \beta\frac{\phi_P^{bl}e^b + (1-\delta)\kappa}{c}}, \frac{(1-\alpha^{nb})(1-\delta)}{\phi_P^{nb}\alpha^{nb} - \beta\frac{\phi_P^{bl}e^b + (1-\delta)\kappa}{c}} \right\}.$$

■

Proof of Corollary 1. Consider Case 1 without commitment. If $N \geq \bar{N}$, banks do not prop up prices in equilibrium. The total credit extended at $t = 1$ by a single bank is given by,

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\beta\frac{\phi_R^{bh}e^b}{c} + \beta(1-\delta)\alpha^{nb}\phi_P^{nb} - \alpha^{nb}\phi_R^{nb}}{N+1}, \frac{(1-\alpha)^{nb}}{N} \right\} \right\}.$$

When $N \geq \bar{N}$, as N increases, m_1 decreases. $m_2 = 0$ for all $N \geq \bar{N}$. Therefore when $N \geq \bar{N}$, credit by any given bank increases as N decreases.

Total credit at $t = 2$ is $M_2 = 0$. Total credit at $t = 1$ is given by,

$$M_1 = \max \left\{ 0, \min \left\{ N\frac{\beta\frac{\phi_R^{bh}e^b}{c} + \beta(1-\delta)\alpha^{nb}\phi_P^{nb} - \alpha^{nb}\phi_R^{nb}}{N+1}, (1-\alpha)^{nb} \right\} \right\}.$$

This is increasing in N .

If $N < \bar{N}$, the economy is in an equilibrium in which banks prop up house prices. The

total credit at $t = 1$ extended by a single bank is given by,

$$m_1 = \frac{1}{N+1 - \beta \frac{(1-\delta)^2}{2} - \beta(1-\delta)^2 \frac{N-1}{2(N+1)}} \left(\beta \frac{1-\delta}{2} \left(\phi_P^{nb} \alpha^{nb} + \frac{(N-1) \left(-\phi_P^{nb} \alpha^{nb} + \beta \frac{\phi_P^{bl} e^b + (1-\delta)\kappa}{c} \right)}{N+1} \right) \right. \\ \left. + \beta^2 \frac{1-\delta}{2c} (\phi_P^{bl} e^b + (1-\delta)\kappa) + \frac{\beta}{c} \phi_R^{bh} e^b - \phi_R^{nb} \alpha^{nb} \right).$$

m_1 increases as N decreases (from the proof of Proposition 1). The total credit at $t = 2$ extended by a single bank is given by,

$$m_2 = \frac{m_1(N)(1-\delta) - \phi_P^{nb} \alpha^{nb} + \beta \frac{\phi_P^{bl} e^b + (1-\delta)\kappa}{c}}{N+1}.$$

As N decreases, m_2 increases. Therefore, the total credit extended by a single bank increases as N decreases.

The total credit in the economy is given by,

$$M_1 + M_2 = \frac{N}{N+1 - \beta \frac{(1-\delta)^2}{2} - \beta(1-\delta)^2 \frac{N-1}{2(N+1)}} \left(1 + \frac{1-\delta}{N+1} \right) \left(\beta \frac{1-\delta}{2} (\phi_P^{nb} \alpha^{nb} + \right. \\ \left. \frac{(N-1) \left(-\phi_P^{nb} \alpha^{nb} + \beta \frac{\phi_P^{bl} e^b + (1-\delta)\kappa}{c} \right)}{N+1} \right) + \beta^2 \frac{1-\delta}{2c} (\phi_P^{bl} e^b + (1-\delta)\kappa) + \phi_R^{bh} e^b \\ \left. - \phi_R^{nb} \alpha^{nb} \right) + \frac{N}{N+1} \left(-\phi_P^{nb} \alpha^{nb} + \beta \frac{\phi_P^{bl} e^b + (1-\delta)\kappa}{c} \right).$$

Taking the derivative w.r.t N ,

$$- \frac{((1-\delta)^2 \beta + (1-\delta) - 1) N^2 - 2N - (1-\delta) - 1}{(N^2 + (2 - (1-\delta)^2 \beta) N + 1)^2} \left(\beta \frac{1-\delta}{2} \phi_P^{nb} \alpha^{nb} + \beta^2 \frac{1-\delta}{2c} (\phi_P^{bl} e^b + (1-\delta)\kappa) \right. \\ \left. + \frac{\beta}{c} \phi_R^{bh} e^b - \phi_R^{nb} \alpha^{nb} \right) - \frac{-\phi_P^{nb} \alpha^{nb} + \beta \frac{\phi_P^{bl} e^b + (1-\delta)\kappa}{c}}{(N+1)^2 (N^2 + (2 - (1-\delta)^2 \beta) N + 1)^2} (((1-\delta)^2 \beta + 1 - \delta - 4) N^4 \\ + (4(1-\delta)^2 \beta - 2(1-\delta) - 12) N^3 + (-(1-\delta)^4 \beta^2 + (2(1-\delta)^3 + 5(1-\delta)^2) \beta - 6(1-\delta) - 12) N^2 \\ + 2((1-\delta)^2 \beta - 2(1-\delta) - 4) N + 1 - \delta).$$

The second term in the above expression is always negative and this can cause the value of the total derivative to be negative.²⁹ Therefore aggregate credit can increase as N decreases. The proof in Case 2 and the case with commitment follow similarly. ■

Proof of Corollary 2. Consider Case 1 without commitment. From Proposition 1, $\bar{N}$ is given by,

$$\bar{N} = \min \left\{ \frac{\frac{\beta}{c} ((1-\delta)\phi_R^{bh}e^b + (1-\delta)^2c\phi_P^{nb}\alpha^{nb} + \phi_P^{bl}e^b + (1-\delta)\kappa) - \alpha^{nb}((1-\delta)\phi_R^{nb} + \phi_P^{nb})}{\phi_P^{nb}\alpha^{nb} - \beta\frac{\phi_P^{bl}e^b + (1-\delta)\kappa}{c}}, \right. \\ \left. \frac{(1-\alpha^{nb})(1-\delta)}{\phi_P^{nb}\alpha^{nb} - \beta\frac{\phi_P^{bl}e^b + (1-\delta)\kappa}{c}} \right\}.$$

It is straightforward from the above expression that,

$$\frac{\partial \bar{N}}{\partial (\phi_P^{bl}e^b)} > 0.$$

As δ increases, the denominator in the expression for $\bar{N}$ increases. At the same time the numerator of the above expression is decreasing. To see this clearly for the first term of the minimization, taking the derivative of the numerator of $\bar{N}$ w.r.t. δ , we get,

$$-\frac{\beta}{c} (\phi_R^{bh}e^b + 2(1-\delta)c\phi_P^{nb}\alpha^{nb} + \kappa) + \alpha^{nb}\phi_R^{nb} \quad (1)$$

Recall from Proposition 1, that when banks are not propping up prices,

$$m_1 = \max \left\{ 0, \min \left\{ \frac{\beta\frac{\phi_R^{bh}e^b}{c} + \beta(1-\delta)\alpha^{nb}\phi_P^{nb} - \alpha^{nb}\phi_R^{nb}}{N+1}, \frac{1-\alpha^{nb}}{N} \right\} \right\}.$$

Therefore, in the relevant range, $\frac{\beta}{c}\phi_R^{bh}e^b + \beta(1-\delta)\phi_P^{nb}\alpha^{nb} - \alpha^{nb}\phi_R^{nb} > 0$ since $m_1 \geq 0$. This implies that (1) is negative. Therefore $\bar{N}$ is decreasing in δ .

Similarly as ϕ_P^{nb} increases, the denominator of $\bar{N}$ is increasing. Additionally in the first term of the minimization, the numerator is decreasing: taking the derivative of the numerator of $\bar{N}$ w.r.t. ϕ_P^{nb} , we get,

²⁹For $N \geq 1$, the multiplier on $\phi_P^{nb}\alpha^{nb}$ and $\phi_R^{nb}\alpha^{nb}$ are always negative. The mathematics of when exactly this expression is negative is tedious and does not add anything much to understanding the main mechanism in the paper but can be made available on request. An example of this can be seen in the graphical illustration in the paper.

$$\alpha^{nb}(\beta(1-\delta)^2 - 1) < 0.$$

Therefore $\bar{N}$ is decreasing in ϕ_P^{nb} .

The proof in Case 2 and the case with commitment follow similarly. ■

Proof of Proposition 2. Consider Case 1. Since $\bar{N}$ depends on ϕ_P^{bl} and ϕ_P^{nb} , this proposition only compares areas given an N_1 and N_2 s.t. $\bar{N}$ for each area falls within $(N_1, N_2]$. Following an increase in concentration from $N_1 \geq \bar{N}$ to $N_2 < \bar{N}$, the change in total lending is given by,

$$\begin{aligned} \Delta M = & \left(N_2 + \frac{(1-\delta)N_2}{N_2+1} \right) \frac{\beta^{\frac{1-\delta}{2}} \left(\phi_P^{nb} \alpha^{nb} + \frac{(N_2-1) \left(-\phi_P^{nb} \alpha^{nb} + \beta \frac{\phi_P^{bl} e^b + (1-\delta)\kappa}{c} \right)}{N_2+1} \right) + \frac{\beta^2(1-\delta)}{2c} (\phi_P^{bl} e^b + (1-\delta)\kappa)}{N_2+1 - \beta \frac{(1-\delta)^2}{2} - \beta(1-\delta)^2 \frac{N_2-1}{2(N_2+1)}} \\ & + N_2 \left(1 + \frac{1-\delta}{N_2+1} \right) \frac{\frac{\beta}{c} \phi_R^{bh} e^b - \phi_R^{nb} \alpha^{nb}}{N_2+1 - \beta \frac{(1-\delta)^2}{2} - \beta(1-\delta)^2 \frac{N_2-1}{2(N_2+1)}} \\ & + \frac{N_2}{N_2+1} \left(-\phi_P^{nb} \alpha^{nb} + \beta \frac{\phi_P^{bl} e^b + (1-\delta)\kappa}{c} \right) - N_1 \frac{\beta \frac{\phi_R^{bh} e^b}{c} + \beta(1-\delta) \alpha^{nb} \phi_P^{nb} - \alpha^{nb} \phi_R^{nb}}{N_1+1}. \end{aligned} \quad (2)$$

Taking the derivative w.r.t ϕ_P^{nb} ,

$$\frac{\partial \Delta M}{\partial \phi_P^{nb}} = N_2 \left(1 + \frac{1-\delta}{N_2+1} \right) \frac{\beta^{\frac{1-\delta}{2}} \left(\alpha^{nb} - \frac{(N_2-1)\alpha^{nb}}{N_2+1} \right)}{N_2+1 - \beta(1-\delta)^2 \frac{N_2}{N_2+1}} - \frac{N_2 \alpha^{nb}}{N_2+1} - N_1 \beta \frac{(1-\delta) \alpha^{nb}}{N_1+1}.$$

If $\frac{\partial \Delta M}{\partial \phi_P^{nb}} < 0$, then income growth and the growth in mortgage credit can be negatively correlated. For this to be the case, we require that,

$$N_2 \left(1 + \frac{1-\delta}{N_2+1} \right) \frac{\beta(1-\delta) \frac{\alpha^{nb}}{N_2+1}}{N_2+1 - \beta(1-\delta)^2 \frac{N_2}{N_2+1}} - \frac{N_2 \alpha^{nb}}{N_2+1} - N_1 \beta \frac{(1-\delta) \alpha^{nb}}{N_1+1} < 0.$$

Simplifying,

$$(1-\delta) \frac{N_2+2-\delta}{N_2^2+1+2N_2-\beta(1-\delta)^2 N_2} \frac{N_2}{N_2+1} - \frac{1}{\beta} \frac{N_2}{N_2+1} - N_1 \frac{1-\delta}{N_1+1} < 0.$$

The denominator of the coefficient multiplying the first term is greater than the numerator. Therefore the first term is strictly less than the value of the second term and this expression is always less than 0. Therefore, $\frac{\partial \Delta M}{\partial \phi_P^{bb}} < 0$.

Looking at (2), we can see that all the terms multiplying ϕ_P^{bl} are always positive. Therefore, $\frac{\partial \Delta M}{\partial \phi_P^{bl}} > 0$.

The proof in Case 2 and the case with commitment follow similarly. ■

Proof of Proposition 3. I start the proof by simplifying the problem in a similar way to the three-period model by focusing on the bank's choice of interest rate. Given state-contingent repayments, state by state, a bank will charge the maximum interest rate such that borrowers are willing to pay. Consider the interest rate a bank will charge borrowers from generation t when the state of the world is $s_t \in \{R, P\}$. As in the three-period model, depending on the relative values of $\phi_{s_t}^{bh}$, $\phi_{s_t}^{bl}$ and γ , there are various possible cases. Here, I will work through the interest rate problem in the case in which $\phi_R^{bh}e^b < \frac{\gamma}{\beta}$ without commitment. The other cases are similar.

If $\phi_{s_t}^{bj}e^b < \frac{\gamma}{\beta}$, where $j = \{h, l\}$ represents borrower-type, a bank will charge:

$$P_{t-1}(1 + r_{t-1}(s_t)) > e^b + (1 - \delta)P_t(s_t).$$

In this case, a proportion $\phi_{s_t}^{bj}$ of bank borrowers get a positive endowment and pay the bank $e^b + (1 - \delta)P_t(s_t)$ while the remaining do not get an endowment and pay the bank $(1 - \delta)P_t(s_t)$. In expectation, borrowers repay the bank $\phi_{s_t}^{bj}e^b + (1 - \delta)E[P_t(s_t)]$ which satisfies their purchasing constraint. When $\phi_R^{bh}e^b < \frac{\gamma}{\beta}$, because of the assumptions on the income of high- versus low-quality borrowers and in rich versus poor states, $\phi_{s_t}^{bj}e^b < \frac{\gamma}{\beta}$, $\forall s_t, \forall j$. As in the three-period case, in the model without commitment, banks can only credibly prop up house prices to improve their return on loans when the borrower cannot repay the full facevalue of the loan. State-by-state the bank will therefore choose a facevalue that is strictly higher than $e^b + (1 - \delta)P_t(s_t)$.

In this case, define $m_t = m_t^h + m_t^l$. Then, we can write the bank's maximization problem at any time t as:

$$\begin{aligned} V(s_t, m_{t-1}) &= \max_{m_t^h \geq 0, m_t^l \geq 0} m_{t-1}(1 - \delta)P_t - \sum_{j=\{h,l\}} m_t^j P_t + \beta \sum_{j=\{h,l\}} m_t^j \phi_{s_t}^{bj} e^b + \beta E[V(s_{t+1}, m_t)] \\ \text{s.t. } m_t^h &\leq \frac{1}{N} \alpha^{bh}, \quad m_t^l \leq \frac{1}{N} (1 - \alpha^{bh} - \alpha^{nb}). \end{aligned}$$

The above problem is independent of interest rates chosen by the bank. Therefore each

period a bank only decides on the amount of loans they wish to issue to each type of borrower. Further, it is independent of the income of generation $t - 1$. The housing price only depends on the total number of loans outstanding to generation $t - 1$. In this case, the problem is also independent of the type of loans made to high- versus low-quality borrowers at $t - 1$ but in other cases, it is not.

Using the envelope theorem,

$$\frac{\partial E[V(s_{t+1}, m_t)]}{\partial m_t^h} = \frac{\partial E[V(s_{t+1}, m_t)]}{\partial m_t^l} = (1 - \delta)E[P_{t+1}]$$

This gives the following FOCs for a bank,

$$\frac{\partial V(s_t, m_{t-1})}{\partial m_t^h} = m_{t-1}(1 - \delta)c - P_t - m_t^h c + \beta \phi_{s_t}^{bh} e^b + \beta(1 - \delta)E[P_{t+1}].$$

$$\frac{\partial V(s_t, m_{t-1})}{\partial m_t^l} = m_{t-1}(1 - \delta)c - P_t - m_t^l c + \beta \phi_{s_t}^{bl} e^b + \beta(1 - \delta)E[P_{t+1}].$$

The equilibrium lending by a bank is given by,

$$m_t^h = \max \left\{ 0, \min \left\{ \frac{m_{t-1}(1 - \delta)c - P_t + \beta \phi_{s_t}^{bh} e^b + \beta(1 - \delta)E[P_{t+1}]}{c}, \frac{\alpha^{bh}}{N} \right\} \right\}.$$

$$m_t^l = \max \left\{ 0, \min \left\{ \frac{m_{t-1}(1 - \delta)c - P_t + \beta \phi_{s_t}^{bl} e^b + \beta(1 - \delta)E[P_{t+1}]}{c}, \frac{1 - \alpha^{bh} - \alpha^{nb}}{N} \right\} \right\}.$$

Given a choice of lending to high-quality borrowers over low-quality borrowers, it is always dominant for a bank to make a loan to a high-quality borrower. Therefore, if,

$$\frac{m_{t-1}(1 - \delta)c - P_t + \beta \phi_{s_t}^{bh} e^b + \beta(1 - \delta)E[P_{t+1}]}{c} \leq \frac{\alpha^{bh}}{N}$$

$$m_t^l = 0.$$

I now show an equilibrium is always symmetric if all banks start with the same level of initial loans m_0 . Consider a bank's lending at $t = 1$.

$$m_1^h = \max \left\{ 0, \min \left\{ \frac{m_0(1 - \delta)c - P_1 + \beta \phi_{s_1}^{bh} e^b + \beta(1 - \delta)E[P_2]}{c}, \frac{\alpha^{bh}}{N} \right\} \right\}.$$

$$m_1^l = \max \left\{ 0, \min \left\{ \frac{m_0(1-\delta)c - P_1 + \beta\phi_{s_t}^{bl}e^b + \beta(1-\delta)E[P_2]}{c}, \frac{1 - \alpha^{bh} - \alpha^{nb}}{N} \right\} \right\}.$$

If all banks have the same m_0 , then the above equations will be identical for all banks and they will choose the same m_1 . Similarly at $t = 2$, we can show that if all banks have the same m_1 , they will choose the same m_2 and so on and so forth. Given the symmetric equilibrium solution, I can rewrite equilibrium lending as,

$$\frac{M_t^h}{N} = \max \left\{ 0, \min \left\{ \frac{\frac{M_{t-1}}{N}(1-\delta)c - cM_t^h - cM_t^l - c\alpha^{nb}\phi_{s_t}^{nb} + \beta\phi_{s_t}^{bh}e^b + \beta(1-\delta)E[P_{t+1}]}{c}, \frac{\alpha^{bh}}{N} \right\} \right\}.$$

$$\frac{M_t^l}{N} = \max \left\{ 0, \min \left\{ \frac{\frac{M_{t-1}}{N}(1-\delta)c - cM_t^h - cM_t^l - c\alpha^{nb} + \beta\phi_{s_t}^{bh}e^b + \beta(1-\delta)E[P_{t+1}]}{c}, \frac{1 - \alpha^{bh} - \alpha^{nb}}{N} \right\} \right\}.$$

Given these first order conditions, I can write an equivalent maximization problem of a representative bank in this economy. This is given by,

$$\begin{aligned} V(s_t, M_{t-1}) &= \max_{M_t^h \geq 0, M_t^l \geq 0} \\ &\frac{M_{t-1}}{N}(1-\delta)P_t + \sum_{j=\{h,l\}} \left(-\frac{M_t^j}{N}P_t - \frac{N-1}{N}\frac{M_t^{j^2}}{2}c - \frac{(N-1)}{N}M_t^j\phi_{s_t}^{nb}\alpha^{nb}c + \beta M_t^j\phi_{s_t}^{bj}e^b \right) \\ &- \frac{(N-1)}{N}M_t^hM_t^lc + \beta E[V(s_{t+1}, M_t)] \\ s.t. \quad &M_t^h \leq \alpha^h \\ s.t. \quad &M_t^l \leq 1 - \alpha^h - \alpha^{nb}. \end{aligned}$$

The first order conditions for this representative bank give the same aggregate lending as those of the individual banks. I can show that the above maximization of the equivalent representative bank is a contraction mapping. Define

$$U(s_t, M_{t-1}, M_t^h, M_t^l) = \frac{M_{t-1}}{N}(1 - \delta)P_t +$$

$$\sum_{j=\{h,l\}} \left(-\frac{M_t^j}{N}P_t - \frac{N-1}{N} \frac{M_t^{j^2}}{2}c - \frac{(N-1)}{N} M_t^j \phi_{s_t}^{nb} \alpha^{nb} c + \beta M_t^j \phi_{s_t}^{bj} e^b \right) - \frac{(N-1)}{N} M_t^h M_t^l c$$

Since M_t^h , M_t^l and M_{t-1} are bounded, P_t is bounded and therefore U is bounded. Define an operator,

$$(TV)(s_t, M_{t-1}) = \max_{0 \leq M_t^h \leq \alpha^{bh}, 0 \leq M_t^l \leq 1 - \alpha^{bh} - \alpha^{nb}} \{U(s_t, M_{t-1}, M_t^h, M_t^l) + \beta E[V(s_{t+1}, M_t)]\}.$$

Take V to be bounded. Since U is bounded by assumption, then TV is also bounded. TV satisfies monotonicity. Suppose $V < W$. Let $g_h(M_{t-1}, s_t)$ and $g_l(M_{t-1}, s_t)$ be the optimal policy functions (not necessarily unique) corresponding to V for M_t^h and M_t^l respectively. Then for all $M_{t-1} \in [0, 1 - \alpha^{nb}]$,

$$\begin{aligned} TV(s_t, M_{t-1}) &= U(s_t, M_{t-1}, g_h(M_{t-1}, s_t), g_l(M_{t-1}, s_t)) + \beta E[V(s_{t+1} + g_h(M_{t-1}, s_t), g_l(M_{t-1}, s_t))] \\ &\leq U(s_t, M_{t-1}, g_h(M_{t-1}, s_t), g_l(M_{t-1}, s_t)) + \beta E[W(s_{t+1}, g_h(M_{t-1}, s_t) + g_h(M_{t-1}, s_t))] \\ &\leq \max_{0 \leq M_t^h \leq \alpha^{bh}, 0 \leq M_t^l \leq 1 - \alpha^{bh} - \alpha^{nb}} \{U(s_t, M_{t-1}, M_t^h, M_t^l) + \beta E[W(s_{t+1}, M_t^h + M_t^l)]\} \\ &= TW(s_t, M_{t-1}) \end{aligned}$$

TV also satisfies discounting. Let $a > 0$. Then,

$$\begin{aligned} T(V + a)(s_t, M_{t-1}) &= \max_{0 \leq M_t^h \leq \alpha^{bh}, 0 \leq M_t^l \leq 1 - \alpha^{bh} - \alpha^{nb}} \{U(s_t, M_{t-1}, M_t^h, M_t^l) + \beta E[V(s_{t+1}, M_t) + a]\} \\ &= \max_{0 \leq M_t^h \leq \alpha^{bh}, 0 \leq M_t^l \leq 1 - \alpha^{bh} - \alpha^{nb}} \{U(s_t, M_{t-1}, M_t^h, M_t^l) + \beta E[V(s_{t+1}, M_t)]\} + \beta a. \\ &= TV(s_t, M_{t-1}) + \beta a \end{aligned}$$

The model therefore satisfies Blackwell's conditions and is bounded and is therefore a contraction mapping with modulus β . Therefore, an equilibrium of this economy exists and

can be found through value function iteration.

The Hessian matrix of U is given by,

$$\begin{pmatrix} -\frac{N+1}{N}c & -\frac{N-1}{N}c \\ -\frac{N-1}{N}c & -\frac{N+1}{N}c \end{pmatrix}.$$

The determinant of the Hessian is given by,

$$\left(\frac{N+1}{N}c\right)^2 - \left(\frac{N-1}{N}c\right)^2 < 0.$$

The Hessian matrix is negative semi-definite since the determinant is less than 0 and $-\frac{N+1}{N}c < 0$. Therefore, for all M_t^h and M_t^l , U is a strictly concave function. Let S_{M_t} be the set of all possible values of M_t . Then since S_{M_t} is convex, the correspondence which gives the set of all feasible allocations given M_t is convex, and U is continuous and bounded, there is a unique policy function associated with the above problem and V^* is strictly concave. The equilibrium is therefore unique.

I now show that M_t is increasing in M_{t-1} . I first show that M_t^h is strictly increasing in M_{t-1}^h when $M_{t-1}^h \leq \frac{\alpha^{bh}}{N}$. Since in this range $M_t^l = 0$, it implies that $M_t = M_t^h$ and $M_t = M_{t-1}^h$ in this region. Let $M_{t-1}^{h'} > M_{t-1}^h$. If $M_t(M_{t-1}^{h'}, s_t) \leq M_t(M_{t-1}^h, s_t)$. Since conditional on s_t , U is increasing in M_{t-1} , it must be the case that

$$U(M_{t-1}^{h'}, s_t) - M_t(M_{t-1}^{h'}, s_t) > U(M_{t-1}^h, s_t) - M_t(M_{t-1}^h, s_t)$$

From the concavity of U , this implies that,

$$U'(M_{t-1}^{h'}, s_t) - M_t'(M_{t-1}^{h'}, s_t) < U'(M_{t-1}^h, s_t) - M_t'(M_{t-1}^h, s_t)$$

The FOC then implies that,

$$V^*(M_t^h(M_{t-1}^{h'}, s_t)) < V^*(M_t^h(M_{t-1}^h, s_t))$$

Since V^* is strictly concave, this is a contradiction. Therefore M_t is strictly increasing in M_{t-1} when $M_t \in \left[0, \frac{\alpha^{bh}}{N}\right)$ conditional on state. We can similarly show that M_t is strictly increasing in M_{t-1} when $M_t \in \left(\frac{\alpha^{bh}}{N}, \frac{1-\alpha^{bh}-\alpha^{nb}}{N}\right)$. When $M_{t-1} = \frac{\alpha^{bh}}{N}$ as mentioned above, $M_t = \frac{\alpha^{bh}}{N}$. M_t is therefore increasing in M_{t-1} .

Finally, from the first order conditions, it is straightforward that conditional on the state of the economy, M_t is increasing in M_{t-1} (and therefore m_t is increasing in m_{t-1} .) In the first order conditions, we can substitute in for P_{t+1} as a function of M_{t+1} . We can then write

M_{t+1} using as a function of M_t and P_{t+2} using the first order conditions. Then it is easy to rearrange the equation to see that any term multiplying M_{t-1} will always be linear.

An equilibrium in which banks do not make any loans to low-quality borrowers to prop-up prices is equivalent to an equilibrium in which $\forall t$ and $\forall s$,

$$\frac{m_{t-1}(1-\delta)c - P_t + \beta\phi_{s_t}^{bh}e^b + \beta(1-\delta)E[P_{t+1}]}{c} < 0$$

Since m_t is linearly increasing in m_{t-1} (and strictly increasing in the range $m_t \in \left[0, \frac{\alpha^{bh}}{N}\right]$), then $\exists T$ s.t. a series of T consecutive R shocks that will eventually give $m_T = \frac{\alpha^{bh}}{N}$ (as long as $V(m_0, R) > 0$) in such an equilibrium. Since the incentive to prop up prices is highest when outstanding loans are the highest, if at $T+1$, in either state, a bank does not want to prop up prices given that they expect no other banks to be propping up prices, then this is an equilibrium in which banks do not prop up prices. For this to be the case we require that,

$$\frac{\alpha^{bh}}{N}(1-\delta)c - P_t + \beta\phi_{s_t}^{bl}e^b + \beta(1-\delta)E[P_{t+1}] < 0.$$

Since banks will always make high-quality loans before low-quality loans, this implies that $m_t^h = \frac{\alpha^{bh}}{N}$. Furthermore, since in this range all banks act symmetrically, $M_t^h = \alpha^{bh}$. Substituting this in,

$$\frac{\alpha^{bh}}{N}(1-\delta)c - c(m_t^l + \alpha^{nb}\phi_{s_t}^{nb} + \alpha^{bh}) + \beta\phi_{s_t}^{bl}e^b + \beta(1-\delta)E\left[P_{t+1}\left(\frac{\alpha^{bh}}{N} + m_t^l\right)\right] < 0.$$

Since m_t is increasing in m_{t-1} , $E\left[P_{t+1}\left(\frac{\alpha^{bh}}{N} + m_t^l\right)\right]$ is decreasing in N (all other bank's lending decisions are taken as given and the bank only thinks about deviations to its own lending). The LHS is therefore decreasing in N . Therefore, $\exists \bar{N}$ s.t. when $N \geq \bar{N}$, an equilibrium in which banks do not prop up house prices exists. When $N < \bar{N}$, bank will make loans to low-quality borrowers and prop up prices. The other cases can be worked through similarly. ■

Proof of Corollary 3.

The proof for this is contained in the proof for Proposition 3. Please refer to that above. ■

Appendix B

Calibration of Main Model

Data Description: The fraction of sub-prime borrowers is from a research report by the Financial Inquiry Commission using data from Inside Mortgage Finance. U.S. house price changes are calculated from Federal Reserve Economic Data. The home-ownership rate is taken from the U.S. Census Bureau. The default rates on prime and sub-prime loans are taken from a research report by the U.S. Census Bureau. The fraction of cash-only house purchases are from RealtyTrac. The private benefit of home-ownership is hard to measure in the data and therefore I choose a value of γ so that it does not determine any equilibrium quantities.

Table 1: Configuration of Model Parameters

Description	Symbol	Value	Source/Target
Fraction of high-quality borrowers	α^{bh}	.5	Loans to prime borrowers
Fraction of non-borrowers	α^{nb}	.06	Cash only house purchases
Low-quality borrower shock rich state	ϕ_R^{bl}	.9	Default rate on sub-prime loans in boom
Low-quality borrower shock poor state	ϕ_P^{bl}	.75	Default rate on sub-prime loans in bust
High-quality borrower shock rich state	ϕ_R^{bh}	.98	Default rate on prime loans in boom
High-quality borrower shock poor state	ϕ_P^{bh}	.94	Default rate on prime loans in bust
Discount Factor	β	.99	Standard
Depreciation	δ	.02	Standard
Number of banks	N	2	Mortgage market concentration
Borrower endowment	e_b	2.4	House prices, sub-prime fraction
Construction cost	c	5.6	House prices, sub-prime fraction
Likelihood of rich state	q	.5	House prices, sub-prime fraction

To limit the number of free parameters, $\gamma \geq 2.6$, $\phi_R^{nb} = \phi_P^{nb} = 1$.

Table 2: Aggregate Moments

	Model	Data
Fraction of sub-prime borrowers in boom	.12	.12
Fraction of sub-prime borrowers in bust	.05	.05
Default rate sub-prime loans in boom	.10	.12
Default rate sub-prime loans in bust	.25	.25
Default rate prime loans in boom	.02	.02
Default rate on prime loans in bust	.06	.06
Fraction of cash only house purchases boom	.09	.13
Fraction of cash only house purchases bust	.10	.22
House price increase boom	.52	1.43
House price decrease bust	-.11	-.20
Change home-ownership rate boom	.07	.05
Change home-ownership rate bust	.07	.02

In the model without commitment, since the banks charge a face-value slightly higher than $e^b + (1 - \delta)P_2$, I consider a mortgage as delinquent when borrowers repay the banks less than $e^b + (1 - \delta)P_2$.

Table 3: Counterfactual Analysis

	N=2	N=4
Fraction of sub-prime borrowers in boom	.12	0
Fraction of sub-prime borrowers in bust	.05	0
House price increase boom	.52	.22
House price decrease bust	-.23	-.03

Calibration of Model with Dispersed Lenders

Data Description: The data is as before. The GSE market share is calculated from the Federal Reserve and Federal Housing Finance.

Table 4: Configuration of Model Parameters

Description	Symbol	Value	Source/Target
Fraction of high-quality borrowers	α^h	.5	Fraction of prime/sub-prime borrowers
Fraction of non-borrowers	α^{nb}	.06	Fraction of cash only house purchases
Low-quality borrower shock rich state	ϕ_R^{bl}	.9	Default rate on sub-prime loans in boom
Low-quality borrower shock poor state	ϕ_P^{bl}	.75	Default rate on sub-prime loans in bust
High-quality borrower shock rich state	ϕ_R^{bh}	.98	Default rate on prime loans in boom
Low-quality borrower shock poor state	ϕ_P^{bh}	.95	Default rate on prime loans in bust
Discount Factor	β	.99	Standard
Depreciation	δ	.02	Standard
borrower endowment	e_b	1.4	house price boom/bust
Elasticity	c	5.8	house price boom/bust
Number of banks	N	2	Mortgage market concentration
Borrower share of dispersed banks	s	.6	Mortgage market concentration
Likelihood of rich state	q	.5	

To limit the number of free parameters, $\gamma \geq 1.4$, $\phi_R^{nb} = \phi_P^{nb} = 1$.

Table 5: Aggregate Moments

	Model	Data
Fraction of sub-prime borrowers in boom	.11	.12
Fraction of sub-prime borrowers in bust	.07	.05
Default rate sub-prime loans in boom	.10	.12
Default rate sub-prime loans in bust	.25	.25
Default rate prime loans in boom	.02	.02
Default rate on prime loans in bust	.06	.06
Fraction of cash only house purchases boom	.18	.13
Fraction of cash only house purchases bust	.20	.22
House price increase boom	.63	1.43
House price decrease bust	-.06	-.20
Change home-ownership rate boom	.08	.05
Change home-ownership rate bust	.02	.02
GSE Market Share '07	.42	.42

In the model without commitment, since the banks charge a face-value slightly higher than $e^b + (1 - \delta)P_2$, I consider a mortgage as delinquent when borrowers repay the banks less than $e^b + (1 - \delta)P_2$.

Table 6: Counter factual Analysis

	N=2	N=4
Fraction of sub-prime borrowers in boom	.11	0
Fraction of sub-prime borrowers in bust	.07	0
House price increase boom	.63	.45
House price decrease bust	-.06	-.01

Appendix C

Income Growth and Mortgage Growth with Borrower Heterogeneity

In the three-period model in the main text it is clear that non-borrower income growth can cause there to be a negative correlation between credit and income-growth when looking at an area following an increase in concentration. However, to additionally be able to show that a positive correlation can still exist between credit and income growth at the borrower-level, we need to allow for borrower level heterogeneity. To illustrate the two results simultaneously, it is enough to allow borrower-level heterogeneity at $t = 2$ and look at $t = 2$ credit when concentration changes. At $t = 2$, let there be a proportion α^{bh} of high-quality borrowers who get an endowment with probability ϕ_R^{bh} . Then we get the following graphs for credit at a borrower-level versus an area-level as concentration changes.

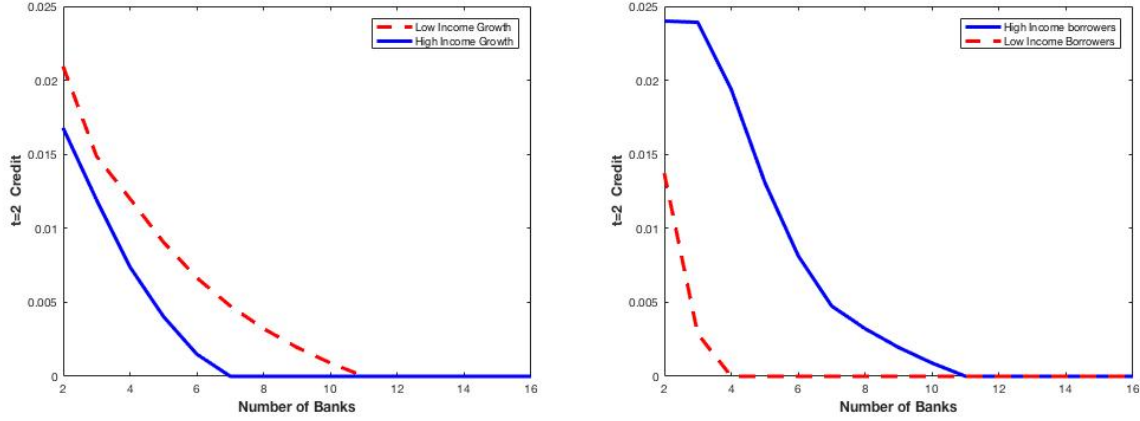


Figure 8: The figure on the left plots total credit, measured by the number of households who get a mortgage, against the level of concentration in the mortgage market for different income growths between $t = 1$ and $t = 2$. The figure on the right plots credit received by high-quality borrowers versus by low-quality borrowers inside the area with lower-income growth against the level of concentration. As we move along the x-axis, N increases and concentration decreases. The parametrization is as follows: $\delta = .01$, $\alpha^{nb} = .2$, $\phi_P^{bl} = .2$, $\phi_R^{bh} = 1$, $\phi_R^{nb} = 1$, $e_b = 2$, $\kappa = .45$, $\gamma = 4$, $b = 9.8$, $\alpha^{bh} = .01$, $\phi_P^{bh} = .41$. ϕ_P^{nb} is varied to get changes in income growth across the two plots - it is equal to .73 in the high-income growth area and .7 in the low-income growth area.