Measuring Uncertainty

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Abstract

This paper distinguishes three concepts that are usually commonly referred as uncertainty: i) real uncertainty or the increase in dispersion of productivity, ii) informational uncertainty or the decrease in the precision of a common signal, and iii) disagreement or the decrease in the precision of idiosyncratic signals of firms. A simple dynamic labor search model is used to analytically show that these concepts have different implications for aggregate outcomes and for the distributio of firm-level outcomes. These differences are used in a DSGE labor-search model to identify and quantify each form of uncertainty in the United States over the last 30 years.

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1 Introduction

Uncertainty has been a key concept for the last ten years in the economic literature. Nevertheless, it is still unclear, at least in macroeconomics, what we mean by this term. Do we mean that firms are more heterogeneous in their underlying productivity? Or do we mean by uncertainty the fact that the forecast of every firm in the economy about aggregate productivity is less precise? Or is it perhaps that private information of every firm is more disperse increasing the disagreement in their forecast? The literature seems to assume that all of the former are in fact part of what we understand as uncertainty, even more, that the distinction of these components is futile as they might all trigger the same aggregate effects.

This paper shows that it is crucial to distinguish what we mean by uncertainty as the former interpretations generate different aggregate implications. We also provide a model based path for distinguishing three distinct concepts: i) Real uncertainty or the increase in dispersion of productivity, ii) informational uncertainty or the decrease in the precision of the common signal, and iii) disagreement or the decrease in the precision of the idiosyncratic signals of firms.

We use a simple two period labor search model to illustrate the difference of the three components and highlight the path for identification. Analytic results show that increases in real uncertainty can trigger cuts in aggregate employment but increase the cross-sectional dispersion of firm-level employments. Informational uncertainty, by contrast, generates a labor expansion while shrinking the dispersion of firms' employment. Larger disagreement about aggregate productivity among firms is expansionary. However, its prediction on employment dispersion is ambiguous. These differential responses provide a path for a model based identification of all three components.

The quantitative section of the paper develops a DSGE model with labor search, real and informational uncertainty, as well as disagreement. Labor dynamic data from the Census is used to estimate the model and disentangle the role that each form of uncertainty played during the last 30 years.

2 Literature Review

The effects of unceratinty on macro aggregates in the context of capital and labor adjustment frictions dates back at least to Bernanke (1983) and Pindyck (1991) and more recently by Bloom

(2009). Bloom et al. (2014) shows in a DSGE model with non convex adjustment costs that higher uncertainty (real uncertainty) triggers a recession as it raises the option value of waiting instead of adjusting inputs. However, the real-option effect has to be strong enough to offset an expansionary effects due to the convexity in the profit function, the Oi-Hartman-Abel effect (Oi, 1962; Hartman, 1972; Abel, 1983). Bachmann and Bayer (2014) highlights a trade-off between having a sizable contraction due to real uncertainty and generating pro-cyclical dispersion of investment rates across firms in a model with counter-cyclical uncertainty shocks. Jia (2016) distinguishes between real and informational uncertainty to conciliate both effects. His calibrated model is able to match the pro-cyclicality of cross-sectional investment rate dispersion, and at the same time, a sluggish recovery in the aggregate investment imostly due to information uncertainty. In a DSGE with labor search, Schaal (2017) finds that though real uncertainty can generate sharp decline in aggregate employment for the 2007 - 2009 recession, it fails to account for the slow recovery in the post-crisis labor market. Kozeniauskas et al. (2016) is the first paper that highlights the potential differences of real uncertainty and disagreement associated with higher order beliefs, they also suggest that this avenue should be promising to reconcile several macro co-movements. They do not explore the potentially different impacts of second moment shocks over business cycles. We contribute to this literature with a clear distinction between three key uncertainty concepts and a model based accounting exercise of their role during last 30 years.

3 Simple Model

In this section we present a two period model where firms differ only in their information about period 2 productivity. Firms have to post vacancies in period 1 in order to hire and produce in period 2.

3.1 Producers

There is a continuum of heterogeneous firms indexed by i. Firms use labor as their only input $N_{i,t}$. Aggregate productivity is common for every firm and denoted by z_t. The production function is given by:

$$Y_{i,t} = e^{z_t} N_{i,t}^{\xi}, \tag{1}$$

where $\xi \in (0,1)$. Aggregate productivity follows a stationary AR(1) process given by:

$$z_2 = \rho_z z_1 + \eta_2, \tag{2}$$

with $\rho_z \in (0,1)$, and $\eta_2 \sim \mathbf{N}(-\frac{1}{2}\frac{\sigma_\eta^2}{1+\rho_z}, \sigma_\eta^2)$. σ_η scales the conditional volatility of aggregate productivity.[4]

The economy starts at the end of period 1, when production has been realized. Firms post vacancy $V_{i,1}$ in order to build the target labor stock $N_{i,2}$ for period 2. For simplicity, we assume that all labor relations have been destroyed at the moment f posting vacancies. Therefore, if a firm does not post any vacancy, it cannot produce in period 2. At period 2, the labor force of firm i is:

$$N_{i,2} = q_1 V_{i,1} \tag{3}$$

where q_1 refers to the job vacancy's filling rate in the end of period 1.

3.2 Information Structure

Firms cannot directly observe aggregate productivity z_t but they have access to a public signal s_t subject to firm specific noise. In particular,

$$s_t = z_t + u_t \tag{4}$$

$$s_{i,t} = s_t + \epsilon_{i,t}, \tag{5}$$

where $\epsilon_{i,t} \sim \mathbf{N}(0, \sigma_\epsilon^2)$ and $u_t \sim \mathbf{N}(0, \sigma_u^2)$. For exposition purposes, we do not allow firms to perfectly learn z_1 by inverting the production function. In the full DSGE model this inversion is not feasible because firms cannot separate idiosyncratic from aggregate productivity.

[4] We set the unconditional expectation of log normal aggregate productivity e^{z_t} to 1 such that $\mathbb{E}e^{z_t} = e^{\mu_z + \frac{1}{2}\sigma_z^2} = 1$.

3.3 Disentangling Uncertainty

In the context of this simple example we can illustrate the three usually confounded meanings of uncertainty:

1. *Real Uncertainty:* σ_η^2 scales the conditional volatility of real aggregate productivity. This form of uncertainty is real in the sense that levels of productivity in the tails are now more likely. So the economy is actually more likely to produce with higher or lower productivity levels. In the full DSGE model, an increase in σ_η^2 is a real uncertainty shock.

2. *Information Uncertainty:* The inverse of σ_u^2 is the precision of the pubic signal. When σ_u^2 increases, tail realizations of productivity are not more likely, but every firm in the economy is less likely to predict aggregate productivity. In the full DSGE model, an increase in σ_u^2 is an informational uncertainty shock.

3. *Disagreement:* σ_ϵ^2 governs the dispersion of firm level noise. Idiosyncratic noise affect firm level forecasting. Therefore, the individual aggregate forecast of firms is more disperse. In the full DSGE model, an increase in σ_ϵ^2 is a disagreement shock.

3.4 Vacancy Filling

There is a search and matching process at the end of period 1. The matching function is given by

$$M_t = U_t^\phi V_t^{1-\phi}, \tag{6}$$

where U_t is the mass of unemployed workers and V_t is the mass of unfilled vacancies. $\phi \in (0,1)$ indexes the elasticity of successful matches with respect to the measure of unemployed workers. At the end of first period, the mass of unemployed workers is normalized to 1, then:

$$1 = U_1. \tag{7}$$

The job filling is given by:

$$q_1 = \frac{M_1}{V_1} = V_1^{-\phi} \tag{8}$$

3.5 Firm's Problem

Firm's hire $N_{i,2}$ workers in period 2 by optimally posting $V_{i,1}$ vacancies that solve:

$$\Omega = \max_{v_i} -c \cdot V_{i,1} - \bar{w} \cdot N_{i,1} + \alpha \mathbb{E}[e^{z_2}(V_1^{-\phi} V_{i,1})^{\xi}], \tag{9}$$

subject to Equation (3) and Equation (8), where c denotes the cost of posting a vacancy. Wage in period 2 is determined by Nash-bargaining parametrized by $\alpha \in (0, 1)$. FOC with respect to V_i is given by

$$c \cdot V_{i,1}^{1-\xi} = \alpha \xi \mathbb{E}[e^{z_2} V_1^{-\xi\phi}] \tag{10}$$

Firm i's posted vacancy increases in the expected values of TFP weighted by the probability of having a successful vacancy fill $V_1^{-\phi}$, an aggregate state variable.

3.6 Analytic Results

Appendix A.1, proves the following corollary:

Corollary 1 *In the economy of the simple model, firm's vacancy posting is log linear in aggregate productivity, aggregate signal noise , and idiosyncratic noise such that* $v_{i,1} = \log V_{i,1} = \zeta_0 + \zeta_1(e_1 + u_1 + \epsilon_{i,1})$. *Aggregate vacancy posted is log linear in aggregate productivity and aggregate noise such that* $v_1 = \int v_{i,1} \mathrm{d}i = \zeta_0 + \zeta_1(e_1 + u_1 + \epsilon_{i,1})$ *with* $\zeta_0 > 0$ *and* $\zeta_1 > 0$.

First, we see that firm-level vacancy posting increases in aggregate productivity, and in aggregate and idiosyncratic information noises. By Corollary 1 and Equation (3), log aggregate employment in period 2 can be expressed as

$$n_2 = (1 - \phi)v_1 = (1 - \phi)(\zeta_0 + \zeta_1 z_1 + \zeta_1 u_1) \tag{11}$$

Hence, the aggregate employment n_2 and the aggregate vacancy posting v_1 are both pro-cyclical for $\zeta_1 > 0$.

Second, we show that the magnitude of responses of aggregate vacancy and employment to aggregate productivity changes $\zeta_1 = \frac{\rho_z \kappa_s}{1-\xi+\xi\phi\kappa_s}$ increases in the Kalman Gain κ_s, which measures the information weight assigned to firm's signal $s_{i,1}$ regarding aggregate productivity realization z_1. The weight $\kappa_s = \frac{\tau_x}{\tau_x+\tau_\eta}$ combines the real uncertainty, information uncertainty, and disagreement where $\tau_x = \frac{1}{\sigma_\epsilon^2+\sigma_u^2}$ and $\tau_\eta = \frac{1}{\sigma_\eta^2}$. Note that, $\frac{\partial\kappa_s}{\partial\sigma_\eta^2} > 0$, $\frac{\partial\kappa_s}{\partial\sigma_\epsilon^2} < 0$, and $\frac{\partial\kappa_s}{\partial\sigma_u^2} < 0$. Intuitively, more volatile real productivity σ_η^2 means less precise prior beliefs that can be used for forecasting z_1. By contrast, more imprecise signals driven by higher σ_u^2 or greater disagreement among firms σ_ϵ^2 reduce the Kalman gain. Hence, the results suggest differential amplification: greater real uncertainty amplifies aggregate employment responses to shocks to aggregate productivity whereas greater informational uncertainty and larger disagreement dampen these responses.

Third, the constant term $(1-\phi)\zeta_0$ is also a function of the three second moments. Exploiting the impacts of second moments on ζ_0, then we have the following proposition regarding aggregate employment responses to changes in real uncertainty, information uncertainty, and disagreement.[5]

Proposition 1 (Aggregate Employment and Second Moment Measures) *Under certain regularity conditions, higher real uncertainty σ_η^2 reduces aggregate employment. Higher informational uncertainty σ_u^2 and higher disagreement σ_ϵ^2 increase employment. Summarizing, $\frac{\partial n_2}{\partial\sigma_\eta^2} < 0$, $\frac{\partial n_2}{\partial\sigma_\epsilon^2} > 0$, and $\frac{\partial n_2}{\partial\sigma_u^2} > 0$.*

Note that, because the model has only two periods, we do not introduce an option value of waiting, thus abstracting from the traditional "wait-and-see" of uncertainty. In our setting, the information frictions alone imply that greater real uncertainty triggers a recession through a hiring freeze. Conversely, re-weighting of prior beliefs relative to new signals, driven by greater information uncertainty and disagreement, can generate expansions.

Finally, Proposition 2 show the differential effects of each component on the dispersion of firm level employment.

Proposition 2 (Dispersion of Firm-level Employment and Second Moment Measures) *Greater real uncertainty expands the dispersion of firm-level employments. Larger information uncertainty shrinks it. The change of employment dispersion across firms is ambiguous given by higher disagreement among firms about aggregate productivity.*

[5] proof in Appendix A.2.

The proof is provided in Appendix A.3. The key mechanism purely arises from the information frictions. Precisely, greater real uncertainty (higher σ_η^2), by increasing the Kalman gain κ_s, makes firms weight more their idiosyncratic noise $\epsilon_{i,1}$ when forecasting z_1. As a result, firms' employments become more dispersed. However, when information uncertainty is larger (higher σ_u^2), firms underweigh newly received signals by weighing more on prior beliefs of $\mu_{z^\cdot}$, and thus, firms see the future in a similar way. Interestingly, two forces are at work when disagreement increases. First, greater disagreement (higher σ_ϵ^2) makes idiosyncratic signal more imprecise, decreasing κ_s and making firms more similar in their forecasts. Nevertheless, greater disagreement also increase the dispersion of signals and therefore increase the heterogeneity of hiring. The overall effect depends on the quantitative magnitude of these forces.

4 Current work

Estimating a large model to use these results as a path for identification.

A Appendix

A.1 Proof of Corollary 1

Proof. We proceed through guess and verify. Assuming that aggregate vacancy posted V_1 is log linear in aggregate productivity z_1 and aggregate noise u_1 up to a constant such that:

$$\log V_1 = v_1 = \zeta_0 + \zeta_1 \cdot z_1 + \zeta_2 \cdot u_1 \tag{12}$$

Plugging Equation (12) into Equation (10), we have

$$\begin{aligned} c \cdot V_{i,1}^{1-\xi} &= \alpha\xi\mathbb{E}\{\exp[z_2 - \xi\phi(\zeta_0 + \zeta_1 \cdot z_1 + \zeta_2 \cdot u_1)]\} \\ &= \exp\{\log(\alpha\xi) - \xi\phi\zeta_0 + (\rho_z - \xi\phi\zeta_1) z_{i,1|1} + \hat{\Sigma}\} \\ &= \exp\{\log(\alpha\xi) - \xi\phi\zeta_0 + (\rho_z - \xi\phi\zeta_1)[\kappa\mu_z + (1-\kappa)s_{i,1}] + \hat{\Sigma}\} \end{aligned} \tag{13}$$

The second equality follows by exploiting the log-normality property of the productivity component and their autocorrelations. $z_{i,1|1}$ denotes firm i's posterior belief about the realized aggregate productivity z_1. $\hat{\Sigma}$ captures the posterior variance for forecasting the matching probability weighted marginal product of posting vacancy.

Applying the Bayes' rule, we have the posterior belief $z_{i,1|1}$ as weighted averages of prior belief $\mu_z = -\frac{1}{2}\frac{\sigma_\eta^2}{1-\rho_z^2}$ and the newly received information $s_{i,1}$. The information weight attached to the prior belief is given by $\kappa = \frac{\tau_\eta}{\tau_\eta + \tau_x}$ and $\tau_\eta = \frac{1}{\sigma_\eta^2}$ and $\tau_x = \frac{1}{\sigma_\epsilon^2 + \sigma_u^2}$. Thus the Kalman Gain, the weight attached to the signal is given by $\kappa_s = 1 - \kappa = \frac{\tau_x}{\tau_x + \tau_\eta}$. $\hat{\Sigma} = \frac{1}{2}(\rho_z - \xi\phi\zeta_1)^2[\tau_\eta + \tau_x]^{-1} + \frac{\rho_z}{2(1+\rho_z)\tau_\eta} + \frac{1}{2}[\xi\phi\zeta_2]^2\sigma_u^2$. Then we have

$$v_{i,1} = \log(V_{i,1}) = \gamma_0 + \gamma_1 s_{i,1} = \gamma_0 + \gamma_1 z_1 + \gamma_1 u_1 + \gamma_1 \epsilon_{i,1} \tag{14}$$

where $\gamma_0 = \frac{\log\frac{\alpha\xi}{c} - \xi\phi\zeta_0 + \hat{\Sigma} + \kappa\mu_z(\rho_z - \xi\phi\zeta_1)}{1-\xi}$ and $\gamma_1 = \frac{(\rho_z - \xi\phi\zeta_1)(1-\kappa)}{1-\xi}$.

Applying the law of large numbers on heterogeneous noises $\int \epsilon_{i,1} \mathrm{d}i = 0$, we have

$$v_1 = \int \log(V_{i,1})\mathrm{d}i = \gamma_0 + \gamma_1 e_1 + \gamma_1 u_t \tag{15}$$

Then we have verified that

$$\gamma_0 = \zeta_0 = \frac{\log \frac{\alpha\xi}{c} + \hat{\Sigma} + \kappa\theta\rho_z\mu_z}{1-\xi+\xi\phi} > 0 \tag{16}$$

$$\gamma_1 = \zeta_1 = \frac{\rho_z\kappa_s}{1-\xi+\xi\phi\kappa_s} > 0 \tag{17}$$

where $\theta = \frac{1-\xi}{1-\xi+\xi\phi(1-\kappa)} \in (0,1)$. Note that $\rho_z - \xi\phi\zeta_1 = \rho_z\theta < \rho_z$ ■

A.2 Proof of Proposition 1

Proof. We set first moments (aggregate productivity shocks and noise shocks) to their unconditional means to approximate for the no shocks scenario. Therefore, $z_1 = \mu_z = -\frac{1}{2}\frac{\sigma_\eta^2}{1-\rho_z^2} < 0$ and $u = 0$. Hence we have aggregate vacancy posted $v_1 = \zeta_0 + \zeta_1\mu_z$ and aggregate employment $n_2 = (1-\phi)v_1 = (1-\phi)(\zeta_0 + \zeta_1\mu_z)$. We first evaluate $\frac{\partial\zeta_0}{\partial\sigma_j^2}$ where $j = \eta, \epsilon, u$.

$$\frac{\partial\zeta_0}{\partial\sigma_j^2} = \frac{1}{1-\xi+\xi\phi}[\frac{\partial\hat{\Sigma}}{\partial\sigma_j^2} + (\rho_z - \xi\phi\gamma_1)\mu_z\frac{\partial\kappa}{\partial\sigma_j^2} - \xi\phi\kappa\mu_z\frac{\partial\zeta_1}{\partial\sigma_j^2}] \tag{18}$$

Clearly we see the second and third terms in the bracket associated with σ_η^2 added are negative. Those two terms associated with σ_ϵ^2 and σ_u^2 combined are positive. Under certain parametrizations, we have the following that

Corollary 2 *Posterior variance $\hat{\Sigma}$ increases in second moments σ_j^2 for $j = \eta, \epsilon, u$ and these second order effects do not dominate the first moment effects driven by information frictions $\frac{\partial\kappa}{\partial\sigma_j^2}$ and $\frac{\partial\zeta_1}{\partial\sigma_j^2}$.*

Then we check $\frac{\partial\zeta_1\mu_z}{\partial\sigma_j^2}$ where $j = \eta, \epsilon, u$. For $\mu_z < 0$, $\frac{\partial\zeta_1\mu_z}{\partial\sigma_\eta^2} = \frac{\partial\zeta_1}{\partial\sigma_\eta^2}\mu_z + \zeta_1\frac{\partial\mu_z}{\partial\sigma_\eta^2} < 0$. It also follows that $\frac{\partial\zeta_1\mu_z}{\partial\sigma_\epsilon^2} > 0$ and $\frac{\partial\zeta_1\mu_z}{\partial\sigma_u^2} > 0$. Combining results, the proposition is proved. ■

A.3 Proof of Proposition 2

Proof. We use the dispersion of firm-level vacancy postings to examine the cross-sectional heterogeneity in employments.

$$VAR[v_{i,1}] = \mathbb{E}(v_{i,1} - v_1)^2 = \mathbb{E}[\zeta_1 \cdot \epsilon_{i,1}]^2 = [\frac{\rho_z}{(1-\xi)\kappa_s^{-1} + \xi\phi}]^2 \sigma_\epsilon^2$$
$$= [\frac{\rho_z}{(1-\xi)(1+\frac{\sigma_\epsilon^2+\sigma_u^2}{\sigma_\eta^2}) + \xi\phi}]^2 \sigma_\epsilon^2 \tag{19}$$

By Equation (11), the dispersion of firm-level employments is given by

$$VAR[n_{i,2}] = VAR[v_{i,1}] \tag{20}$$

Real Uncertainty Changes. Considering changes in σ_η^2, the following prediction holds:

$$\frac{\partial VAR[n_{i,2}]}{\partial \sigma_\eta^2} > 0 \tag{21}$$

Information Uncertainty Changes. Taking partial derivatives with respect to σ_u^2, the informational uncertainty, we have following

$$\frac{\partial VAR[n_{i,2}]}{\partial \sigma_u^2} < 0 \tag{22}$$

Disagreement Changes. Taking partial derivatives with respect to σ_ϵ^2, the disagreement measure, we have the sign depending on relative size of information uncertainty σ_u^2, σ_ϵ^2 and σ_η^2.

$$\frac{\partial VAR[n_{i,2}]}{\partial \sigma_\epsilon^2} = [\frac{\rho_z}{(1-\xi)(1+\frac{\sigma_\epsilon^2+\sigma_u^2}{\sigma_\eta^2}) + \xi\phi}]^2 \frac{(1-\xi)(1+\frac{\sigma_u^2-\sigma_\epsilon^2}{\sigma_\eta^2}) + \xi\phi}{(1-\xi)(1+\frac{\sigma_u^2+\sigma_\epsilon^2}{\sigma_\eta^2}) + \xi\phi}$$
$$= \zeta_1^2 \frac{(1-\xi)(\sigma_\eta^2 + \sigma_u^2 - \sigma_\epsilon^2) + \sigma_\eta^2 \xi\phi}{(1-\xi)(\sigma_\eta^2 + \sigma_u^2 + \sigma_\epsilon^2) + \sigma_\eta^2 \xi\phi} \tag{23}$$

■

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