Price rigidities and the relative PPP*

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Abstract

We measure the proportion of real exchange rate movements accounted for by cross-country movements in relative reset prices (prices that changed since the previous period) using CPI microdata for the UK, Austria and Mexico. Relative reset prices account for almost all of the real exchange rate movements in the data. This is at odds with the predictions of Sticky Price Open Economy models with complete markets, which generate volatile and persistent real exchange rates but not through movements in relative reset prices. We show that incomplete markets models featuring UIP deviations are much closer to replicating the empirical decomposition at low frequencies.

Keywords: PPP puzzle, real exchange rates, price stickiness

JEL Codes: F31, F41, G31

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1 Introduction

Movements in real exchange rates –nominal exchange rates adjusted for relative inflation– are large, persistent, and closely track movements in nominal rates, while cross-country differences in inflation rates are small and stable. A classic explanation for these observations is that prices are sticky and thus respond sluggishly to shocks affecting the nominal exchange rate. Since Chari et al. (2002), an extensive literature has evaluated this explanation quantitatively using Sticky Price Open Economy (SPOE) models. Under some extensions, these models have been successful in replicating the empirical properties of the real exchange rate.[1]

This paper evaluates this theory using a new real exchange rate decomposition. In particular, we define the (log) real exchange rate between countries i and n as $rer_{in,t} \equiv p_{i,t} + e_{in,t} - p_{n,t}$, and decompose it into:

$$rer_{in,t} = \underbrace{\bar{p}_{i,t} + e_{in,t} - \bar{p}_{n,t}}_{\equiv \overline{rer}_{in,t}} + \underbrace{[p_{i,t} - \bar{p}_{i,t}] - [p_{n,t} - \bar{p}_{n,t}]}_{\equiv rer_{in,t} - \overline{rer}_{in,t}}. \quad (1)$$

Here $p_{\cdot,t}$ and $e_{in,t}$ are the aggregate price levels and the nominal exchange rate, and $\bar{p}_{\cdot,t}$ are the price levels of the subsets of goods that adjusted prices between $t-1$ and t. The first term in this decomposition is what we label as the *'reset exchange rate'*, $\overline{rer}_{in,t}$. It measures the cross-country relative price, in a common currency, of the subset of goods that did adjust prices between $t-1$ and t. The second term captures cross-country differences in reset prices relative to aggregate price levels. The magnitude of these two terms informs whether sticky prices are an important factor dampening the adjustment of price levels to shocks affecting the nominal exchange rate. If they are not, the first term should account for a large fraction of real exchange rate movements. Intuitively, in this case the distinction between reset and non-reset prices would be irrelevant, and the term $rer_{in,t} - \overline{rer}_{in,t}$ would be small. In contrast, the second term should account for a larger fraction of real exchange rate movements the more important price stickiness is.

We evaluate this central prediction of SPOE models using the microdata that underlies the construction of the CPIs in the UK, Austria and Mexico. These data contain monthly price quotes for uniquely defined products, sampled in specific outlets, which can be

[1] For example, Steinsson (2008) generates volatile and persistent real exchange rates in a SPOE model with segmented labor markets and real shocks, while Carvalho and Nechio (2011) generate these properties in a SPOE model with multiple sectors and nominal shocks. The baseline SPOE model in Chari et al. (2002) can reproduce the observed volatility but not the persistence of real exchange rates. Bergin and Feenstra (2001), Benigno (2004), and Bouakez (2005) show that the persistence of real exchange rate movements in this model increases in extensions that allow for sticky wages or strategic complementarities in price setting.

traced through time. The combined dataset contains over 30 million monthly price quotes for the 2006-2015 period. To our knowledge, this is the first study combining CPI microdata from multiple countries to study the behavior of real exchange rates. An important feature of the data is that it covers almost the entire universe of consumption goods and services. This is important for the study of price rigidities, as the frequency of price changes varies significantly across different types of consumption goods.[2]

We show that in the data, real exchange rate movements are driven almost exclusively by movements in the reset exchange rate. We decompose the fraction of the variance of real exchange rate movements that can be attributed to each of the two terms in our decomposition, considering horizons from one month to up to one year.[3] Changes in the reset exchange rate account for about 90 percent of the variance of real exchange rate changes for most horizons.

We then apply this decomposition in a benchmark two country symmetric SPOE model with complete markets where all relative prices can be solved for analytically (to a first order). The model features Calvo-style nominal rigidities based on local currency pricing, real rigidities, and wage rigidities, and nests many of the models used in the literature. We show that in this model, the stochastic processes of the real and reset exchange rates depends on parameters governing: (i) the frequency of price changes, (ii) the persistence of the nominal exchange rate, (iii) the degree of real rigidities in price setting and (iv) the degree of nominal wage rigidities. While the model can generate volatile and persistent real exchange rates under different combinations of these parameters, in all these combinations the reset exchange rate accounts for only a fraction of the real exchange rate movements. Intuitively, this follows because even with strong real and wage rigidities, in these models reset prices react to shocks affecting the nominal exchange rate, driving a wedge between in $rer_{in,t}$ and $\overline{rer}_{in,t}$. We show that this central prediction of the SPOE models is robust to quantitative extensions that incorporate multiple sectors as in Carvalho and Nechio (2011), and a Taylor rule and real shocks as in Steinsson (2008). We conclude that SPOE models with complete markets are inconsistent with the observed movements in the reset exchange rate.

A well know limitation of open economy models with complete markets is that they impose uncovered interest rate parity, which is violated in the data. Recently, Mukhin and Itskhoki (2016) have shown that incorporating shocks to international asset demand into dynamic general equilibrium model with incomplete markets can help resolve many

[2]See Nakamura and Steinsson (2010) and Carvalho and Nechio (2011).

[3]We also decompose the fraction of the MSE of the change in the real exchange rate that can be attributed to each of the two terms following Engel (1999), and obtain very similar results.

of the puzzles associated with the exchange rate disconnect. We show that such models are closer to replicating the empirical decomposition of the real exchange rate at low frequencies than models featuring complete markets. Intuitively, complete market models impose a strong relation between nominal costs and nominal exchange rates which does not need to hold in models with exchange rate disconnect.

Our paper is related to a large literature that studies large and persistent deviations from relative PPP. Mussa (1986) argued that the observed volatility of the real exchange rate favored models with nominal shocks and price rigidities, a view echoed by Rogoff (1996). Engel (1999) showed that short run fluctuations in the real exchange rate are due to changes in the cross-country relative price of tradables, rather than to changes in the relative price of non-tradables. Betts and Devereux (2000) rationalized this finding in a general equilibrium model where firms price to market and prices are sticky in local currency. A more recent literature starting with Chari et al. (2002) quantitatively evaluates whether SPOE models can generate volatile and persistent real exchange rates.[4] Our main contribution to this literature is to evaluate the mechanisms in these models through a new decomposition of the real exchange rate applied to a new dataset of consumer prices across countries. In doing so, we provide support to incomplete markets models with shocks affecting international asset demand, as Gabaix and Maggiori (2015).

Our paper is also related to the recent literature that uses microdata to study exchange rate pass-through and deviations from the law of one price across countries. Gopinath et al. (2011) and Burstein and Jaimovich (2012) use scanner data from a supermarket chain to study retail price differences across the US and Canada, but don't distinguish between reset and non-reset prices. Gopinath et al. (2010) study pass-through into US import prices conditional on a price adjustment. Building on their seminal work, we separately focus on reset prices to disentangle the mechanisms through which SPOE models generate movements in real exchange rates. Also related to us are Cavallo et al. (2014), who study deviations of the law of one price for online prices of identical goods sold by four large global retailers. They show that absolute PPP holds well within currency unions, and that deviations from absolute PPP outside currency unions arise at the moment that new products are introduced. In contrast, our focus is on understanding deviations from relative PPP and the relation between real and nominal exchange rate movements, using new microdata for the universe of consumer prices.[5]

The rest of the paper is organized as follows. Section 2 presents a simplified SPOE

[4]See for example Bergin and Feenstra (2001),Benigno (2004), Bouakez (2005), Steinsson (2008), Carvalho and Nechio (2011) and Crucini et al. (2014), among many others.

[5]Our paper is also related to Bils et al. (2009) and Kryvtsov and Carvalho (2016), who compute different measures of 'reset inflation'.

model to derive intuition on the relationship between the real and the reset exchange rates. Section 3 presents the data and our empirical methodology. Section 4 presents our empirical results. Finally, Section 5 compares the data to quantitative SPOE models with complete and incomplete markets, and Section 6 concludes.

2 A simple SPOE model

Before presenting our data, we describe a simplified sticky price open economy model to build intuition on the relation between the real and the reset exchange rates.

Setup: Consider a one-period world economy consisting of two ex-ante symmetric countries indexed by i and n. The world economy can experience one of infinitely many shocks or states, s. In each country, there is a final good that can be used for consumption or as an intermediate input in production. This final good is produced by aggregating a continuum of intermediate goods. Using lower case variables to denote logs, we the write price of the final good in country i as:

$$p_i(s) \equiv \mu p_{ii}(s) + [1-\mu] \, p_{ni}(s),$$

where $p_{ni}(s)$ is the average price across all the intermediate goods produced in country n and sold in country i, expressed in the currency of country i. The parameter μ governs the share of domestic goods in the price index.

Intermediate goods are produced by a continuum of monopolistically competitive producers. We introduce price stickiness by assuming that a only fraction $1-\theta$ of these producers can observe the realization of the state before setting their prices. These producers set a price that denominated in the buyer's currency equals:

$$\bar{p}_{in}(s) = \bar{\rho} + mc_i(s) + e_{in}(s),$$

where $\bar{\rho}$ is a markup and $e_{in}(s)$ is the nominal exchange rate expressed in units of currency n per-unit of currency i. The remaining producers must set prices in the buyer's currency before observing the realization of the state. The price set by these producers is

$$p^e = \bar{\rho} + mc^e,$$

where mc^e denotes the expected marginal cost.[6]

[6]We omit country subscripts since countries are symmetric before the state is realized (and, for the same

All producers operate the same constant returns to scale technology that combines labor with intermediate inputs. The marginal costs of production is:

$$mc_i(s) = [1-\bar{\alpha}]\, w_i(s) + \bar{\alpha} p_i(s),$$

where $\bar{\alpha}$ is the share of intermediate inputs in production. Note that the parameter $\bar{\alpha}$ controls the degree of real rigidities in the model: the costs of the firms that get to observe the state depends on the prices set by the firms that do not observe the state. Reset prices will depend less on the realization of the state the larger is $\bar{\alpha}$.

Relative prices: We can now write the reset and the real exchange rates in this economy as a function of relative wages and the nominal exchange rate. The reset exchange rate is defined as:

$$\overline{rer}_{in}(s) \equiv \bar{p}_i(s) + e_{in}(s) - \bar{p}_n(s),$$

with $\bar{p}_i(s) \equiv \mu \bar{p}_{ii}(s) + [1-\mu]\, \bar{p}_{ni}(s)$. Substituting we obtain,

$$\overline{rer}_{in}(s) = \alpha rer_{in}(s) + [2\mu - 1 - \alpha]\, rer^w_{in}(s), \tag{2}$$

where $\alpha \equiv [2\mu-1]\,\bar{\alpha}$ and $rer^w_{in}(s) \equiv w_i(s) + e_{in}(s) - w_n(s)$ denotes relative wages expressed in a common currency. Finally, noting that the average price set by firms from country i selling in country n is $p_{in}(s) = [1-\theta]\,\bar{p}_{in}(s) + \theta p^e$, we can write the real exchange rate as:

$$rer_{in}(s) = \frac{\theta}{1-[1-\theta]\,\alpha} e_{in}(s) + \frac{[1-\theta]\,[2\mu-1-\alpha]}{1-[1-\theta]\,\alpha} rer^w_{in}(s), \tag{3}$$

where the equality follows from the definitions of rer_{in}, p_i, $\bar{p}_i$ and from equation (2).

Discussion: Equation (3) shows the determinants of the real exchange rate in this simple sticky price model. First, the real exchange rate mechanically tracks the nominal exchange rate if some prices are set before the realization of the state, $\theta > 0$. This is the primary mechanism driving real exchange rates in the SPOE models described in the introduction. Note that the partial elasticity of the real exchange rate with respect to the nominal exchange rate increases with the degree of real rigidities, α. Second, the real exchange rate also moves with the nominal exchange rate if relative wages expressed in a common

reason, set the expected log-exchange rate equal to zero).

currency move with the nominal exchange.[7] For large values of θ, the effect of relative wages on the real exchange rate is small.

Equation (2) shows the relation between the real and the reset exchange rates. First, note that the reset exchange rate is more sensitive to movements in relative wages rer^{w}_{in} than the real exchange rate, implying that $\overline{rer}_{in}$ is a better measure of relative marginal costs than the real exchange rate. Equation (2) also shows that if there are no real rigidities and relative wages are constant, $\alpha = rer^{w}_{in} = 0$, then the reset exchange rate is constant. More generally, if relative wages are constant, the reset exchange rate is only a fraction $\alpha < 1$ of the real exchange rate.[8] In the next section we'll show that in the data rer_{in} and $\overline{rer}_{in}$ move one to one with each other. This implies that models that generate real exchange rate movements through price stickiness, θ, with little movements in relative wages, rer^{w}_{in}, will fail to match the empirical decomposition between real and reset exchange rate.

3 Data

This section describes the CPI microdata from the UK, Austria and Mexico. It then describes how the datasets are harmonized and used to compute the empirical counterparts of the aggregate and reset price indices defined in the previous section.

3.1 Data sources and description

Our analysis combines the microdata that underly the construction of the Consumer Price Indexes in three countries: the United Kingdom, Austria, and Mexico. We describe the sources of the data below.

United Kingdom: The microdata used to compute the CPI in the UK is collected by the United Kingdom's Office for National Statistics (ONS). The product-level price quotes and item-level price indices used for the construction of the CPI were made publicly available in September of 2012. We use the portion of these data that spans the 2006-2015 period, containing over 14 million price quotes. For most item categories, the ONS collects price quotes of individual products by sampling outlets around 150 locations in

[7] Such movements in relative wages could potentially arise if there are nominal wage rigidities, or if wages are flexible but nominal exchange rate movements are disconnected from fundamentals, as in Mukhin and Itskhoki (2016).

[8] As we will show in Section 5, the fraction α overstates the relation between the reset and the real exchange rate relative to a dynamic version of this model, since in the dynamic version firms that reset prices will take into account the mean reversion in the real exchange rate.

the UK. Each elementary price quote collected through this method represents a unique product, sampled in a particular outlet. These items account for about 64 percent of consumer expenditures in the UK in the average month. Prices for the remaining CPI items are collected centrally by the ONS with no field work. Such items include shelter, university tuition fees, rail fares, and other services. Unfortunately, the ONS only provides item-level price indices for these items.[9] Since observing individual price trajectories is central for the study of reset prices, we exclude these items from our analysis.

For a small subset of items and regions, the ONS does not report outlet identifiers to comply with confidentiality guidelines. In such cases, there could be multiple price quotes with the same product-outlet identifier in a given month in the dataset. In most of these cases, there is no variation in prices that share an identifier in a given month.[10] For the few cases in which we do observe different prices with the same identifier, we use the algorithm described in Appendix A.1 to recover: i) a unique price-trajectory associated to a product-outlet pair, and ii) a weight for each price trajectory reflecting the frequency of each price under a particular identifier.

Austria: The microdata used for the construction of the Austrian CPI was obtained from Statistics Austria. The dataset spans the 2006-2015 period, and contains about 50,000 elementary price quotes in the average month. In total, there are roughly six million price quotes covering about 95 percent of Austrian consumption expenditures in the dataset. The remaining expenditures correspond to the shelter portion of the Austrian CPI, for which Statistics Austria only reports price indices. Each elementary price quote corresponds to a unique product sampled in a specific outlet, and contains information on the date, the outlet, and the units in which prices are quoted. For confidentiality reasons the raw dataset has been anonymized with respect to product brands and outlet identifiers. The majority of the price quotes are sampled from individual outlets across 20 major Austrian cities. For about 40 percent of the product categories, price quotes are collected centrally.

Mexico: The microdata used to compute the Mexican CPI are collected by the National Institute of Statistics and Geography (INEGI). Since January 1994, these data are published monthly in the Diario Oficial de la Federacion (DOF), the official bulletin of the

[9] For some of these items, such as housing rent, price indices are obtained from alternative surveys, such as the household survey. For most items, including rail fares and tuition fees, price indices are computed by aggregating price-quotes that are centrally collected the ONS. Unfortunately, the ONS does not disseminate these quotes.

[10] See Blanco (2017) for a detailed description.

Mexican government. Each elementary price quote in the DOF represents a unique product-outlet combination that can be traced through time. As with our other datasets, for each elementary product we observe its monthly price, the city in which it is sold and the units in which prices are quoted. The data for the 2006-2015 period are sampled from 46 Mexican cities and contain about 85,000 monthly price quotes, representing about 84 percent of consumption expenditures in the average month. The remaining expenditures correspond to shelter and car insurance, which we exclude from our analysis since for these items the INEGI only reports aggregated price indices. For the same reasons, we also exclude computers for the 2011-2016 sub-period. The DOF also publishes a list of products that are added to the CPI basket due to product substitutions or changes in the set of outlets sampled by the INEGI. Earlier versions of this dataset has been used previously by Ahlin and Shintani (2007), Gagnon (2009) and Cravino and Levchenko (Forthcoming).

3.2 Harmonization and cleaning

As described above, statistical agencies in the UK, Austria and Mexico share similar methodologies for collecting consumer prices. The datasets do differ in terms of the number and the composition of the products sampled. In addition, in each country we had to exclude different product categories in cases where only aggregated price indices were available (the most important category, which we exclude in all three datasets, is shelter). Table 1 and Appendix Tables A1a and A1b summarize these differences in our final sample. The datasets also differ in other aspects, such as the treatments of temporary sales, product substitutions, and updates to the basket of goods sampled, which we describe below.

Table 1: Summary Statistics, CPI Microdata

	UK	Austria	Mexico
Number of price quotes			
Total	14,172,863	5,908,223	10,099,642
Average month	118,107	49,235	84,164
Number of item categories	581	779	300
% of CPI expenditures	65	95	83
% of non-shelter CPI	69	100	98

Note: This table summarizes our final sample of the microdata used for the construction of the CPI in the UK, Austria and Mexico. Each price quote corresponds to a specific product-outlet combination. 'Item categories' are the most dissagregated categories for which the statistical agencies construct price indices and provide CPI weights.

Temporary sales: Temporary sales are flagged in the UK and the Austrian data, but not in the Mexican data. To maintain consistency across datasets, we create our own indicator for temporary sales by defining them as a price change that is reverted after one or two months. In our baseline calculations, we do not consider temporary sales as price changes (that is, we replace sale prices for regular prices). Table A1b reports frequencies of price changes by product category both inclusive and exclusive of final sales. As expected, sales are concentrated in Food and Beverages, Clothing and Furnishing.

Forced substitutions: In a given month, price inspectors may not be able to sample some elementary-products in the basket due to unanticipated reasons. Such reasons include changes in product characteristics, products that get discontinued, and in rare cases outlets that shut down. In such cases, statistical agencies typically sample another product which they consider to be a close substitute to the original product. Forced substitutions are flagged in all three datasets. Most of these substitutions are flagged as 'comparable', indicating that the new product is sufficiently close to the one being substituted. Statistical agencies include changes in prices arising from these substitutions in the CPI. Some substitutions are flagged as 'non-comparable', and changes in prices arising from these type of substitutions are excluded from the CPI. To keep our results comparable to the official inflation numbers, we only exclude the non-comparable substitutions from our dataset.

Planned changes to the CPI basket: Statistical agencies periodically update the basket of goods that comprise the CPI to reflect changes in expenditure patterns or the introduction of new products. In the UK, updates to the CPI basket are introduced every February. In Austria, the CPI basket was updated in January 2005 and then in every since January 2011. Because some new products are introduced to the CPI basket with each update, the number of price changes that we can compute in the month of an update is slightly smaller than in the typically month. Appendix Table A2 summarizes the share of new products that are introduced after each update. In the UK, in the average update month 13 percent of the price quotes correspond to a new products (relative to 3 percent of new products in the average month). In Austria, in the average update month only 3 percent of the price quotes correspond to new products. In practice, the only important update to the Austrian basket is on January 2011, where about 12 percent of the products are new. In our main calculations, we treat update months in the Austria and the UK as regular months (the caveat is that we cannot compute price changes for the new products introduced in the update, so the effective sample in these months is slightly smaller).

In Mexico, the CPI basket was updated in September 2002 and January 2011. In the case of Mexico, all the product-store identifiers were changed between December 2010 and January 2011. This means that we cannot compute inflation between these months with our microdata. In our baseline results, we use the official Dec 2010 - Jan 2011 inflation rate to chain the aggregate and reset price index between these two months months.

Weighting and aggregation: The statistical agencies in the UK, Austria and Mexico divide products into Item categories, similarly to how the BLS divides products into 'Entry Level Items' (ELIs) for the construction of the US CPI. These are the most dissagregated categories for which the agencies construct price indexes and provide CPI weights. From now on we refer to these detailed categories as simply as 'Items'. The categorization of products into Items varies across the three countries in our sample. For instance, Mexico is the only country that assigns an Item for 'Tortillas', while Austria is the only country that assigns an item for 'Hazelnut Cuts'. In contrast, all three countries assign an Item to 'Beer'. All Items can be grouped into broader categories following the international standards in the 'Classification of Individual Consumption According to Purpose' (COICOP). In the example above, 'Tortillas' and 'Hazelnut Cuts' are both part of the COICOP category 'Bread and cereals', which in turn is part of the broader COICOP category 'Food'. The number of items groups in each country is listed in Table 1. In what follows, we use the country-specific Item weights to aggregate the price quotes from the microdata into measures of aggregate inflation.

3.3 Computing aggregate and reset price indexes

We now describe how we use the microdata to compute real and reset exchange rates. To be consistent with the theory in Section 2, we compute inflation between two consecutive months as the change in the average log price for the set of goods that continue in the sample for the two months. A limitation of focusing on changes in average prices is that they may reflect both changes in prices and changes in the composition of the basket of goods used to construct the CPI.[11] This issue is problematic when measuring changes in reset prices because, even if the broad the basket goods is fixed, the set of goods for which we observe a price change can differ across months.[12]

With this in mind, we first normalize the price trajectories in our data to construct elementary-product level price indexes. In particular, for each product ω, we construct a price index defined as:

$$p_{i,t}^{j}(\omega) \equiv \begin{cases} log\left[P_{i,t}^{j}(\omega)/P_{i,t_0}^{j}(\omega)\right] & if\ t_0 = \text{Jan 2006} \\ log\left[P_{i,t}^{j}(\omega)/P_{i,t_0}^{j}(\omega)\right] + p_{i,t_0}^{j,off} & if\ t_0 \neq \text{Jan 2006} \end{cases}. \tag{4}$$

Here t_0 is the first month in which product ω appears in our data. $P_{i,t}^{j}(\omega)$ and $P_{i,t_0}^{j}(\omega)$ are the raw prices in country i of product ω belonging to Item category j, at dates t and t_0. $p_{i,t_0}^{j,off}$ is the log of the official price index for item category i, normalized to take a value of zero on January 2006. Equation (4) defines log price indexes that take a value of zero in the first month of our sample. For a product that is introduced to the CPI basket in the middle of the sample, the log price index is normalized so that it takes the value of the Item-level index in the month that the product is introduced. Note that the normalization in (4) only adds a product-specific constant to the log of the price of the product, and hence does not affect product-level inflation nor aggregate inflation for continuing goods. Yet the normalization does ensure that compositional changes in the basket of products that adjust prices do not mechanically affect our measure of inflation for reset prices.[13]

We then construct Item-level price indexes as simple averages of the product-level price indexes in each Item category. That is, for each Item category j we compute a price

[11]Alternatively, one could define inflation as the average price change across two periods. The two approaches are equivalent when the basket of goods sampled is fixed.

[12]Note that compositional changes do not affect the price indices in the theory in Section 2, since in that theory products are homogeneous.

[13]Kryvtsov and Carvalho (2016) use an alternative normalization for their measure of reset inflation. In particular, instead of normalizing by the initial prices of the elementary product, they normalize all prices relative to an item-strata level mean.

index given by:

$$p_{i,t}^{j}=\frac{1}{\left|\Omega_{i,t}^{j}\right|}\sum_{\omega\in\Omega_{i,t}^{j}}p_{i,t}^{j}\left(\omega\right),$$

where $\Omega_{i,t}^{j}$ and $\left|\Omega_{i,t}^{j}\right|$ are the set and the number of products in item category j that are in the sample at dates t and $t-1$. Similarly, we compute a reset price index for each category as

$$\bar{p}_{i,t}^{j}=\frac{1}{\left|\bar{\Omega}_{i,t}^{j}\right|}\sum_{\omega\in\bar{\Omega}_{i,t}^{j}}p_{i,t}^{j}\left(\omega\right),$$

where $\bar{\Omega}_{i,t}^{j}$ and $\left|\bar{\Omega}_{i,t}^{j}\right|$ are the set and the number of products in item i for which $p_{i,t}^{j}\left(\omega\right)\neq p_{i,t-1}^{j}\left(\omega\right)$.

Finally, we construct aggregate and reset price indexes as weighted averages of the item level indexes in equations (4), using the official CPI weights.[14] That is, our aggregate price indices are defined as:

$$p_{i,t}=\sum_{i}s_{i,t}^{j}p_{i,t}^{j} \tag{5}$$

and

$$\bar{p}_{i,t}=\sum_{i}s_{i,t}^{j}\bar{p}_{i,t}^{j}, \tag{6}$$

where $s_{i,t}^{j}$ is the official weight of Item category j in the CPI of the country i.

To compute real exchange rates, we obtain bilateral nominal exchange rates for Austria-UK and Mexico-UK from Eurostat. We construct a log-index for the bilateral nominal exchange rate as

$$e_{iUK,t}\equiv log\left[E_{iUK,t}/E_{iUK,\text{Jan06}}\right], \tag{7}$$

where $E_{iUK,t}$ is the bilateral nominal exchange rate, expressed in terms of pounds per domestic currency. We then combine the price indices constructed according to equations (5), (6) and (7) to construct indices for real exchange rates and for relative reset prices

[14]We obtain these weights from the same Statistical agencies that provide the microdata. Aggregate weights at the 1-digit COICOP level are listed in Appendix Table A1a.

according to equation (1). Note that by construction, these indices are zero in January 2006, and reflect cumulative changes relative to January 2006 at other dates.

Comparison with official statistics Appendix Figure 5 compares aggregate inflation computed from the microdata to the official inflation series in each of the three countries in our sample. The inflation from the microdata mimics the official inflation extremely well in all three countries. Appendix Figure 6 compares the UK-Austria and the UK-Mexico real exchange rates based on official inflation to real exchange rates based on the inflation rates we obtain from the microdata. The figure shows that, although the microdata does not represent 100 percent of consumption expenditures, the two real exchange rate series are extremely close to each other. We conclude that despite differences in methodologies and coverage, the inflation and real exchange rate measures we compute from the microdata are representative of the measures computed from official data.

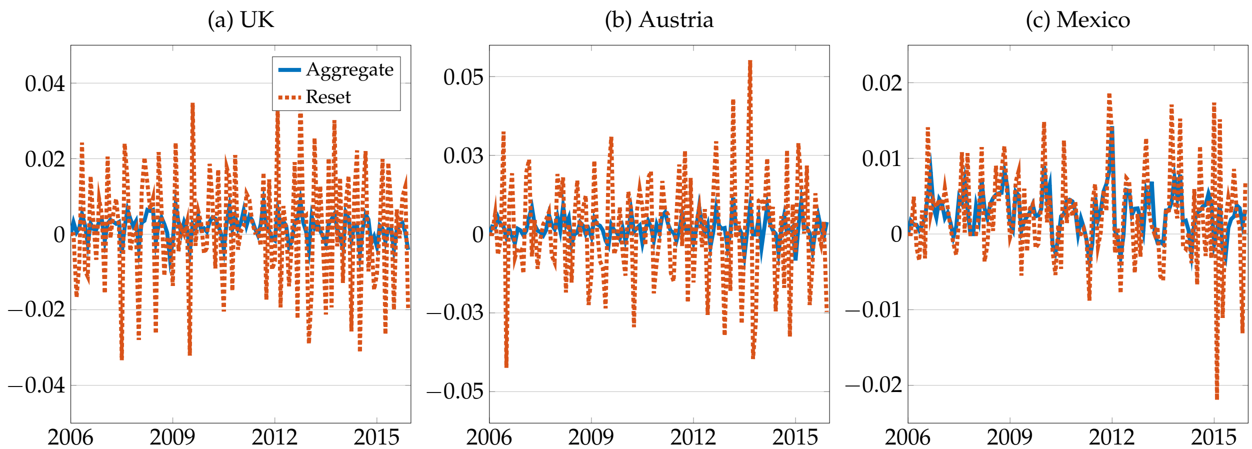
Changes in aggregate and reset prices We conclude this section by comparing our measures of inflation for aggregate and reset prices. Figure 1 plots monthly changes and levels of the aggregate and reset and price indexes defined in equations (5) and (6), and summary statistics for monthly changes are presented in Table 2. As expected, reset inflation is substantially more volatile than aggregate inflation. In the UK and in Austria, the standard deviation of reset inflation is 5 times the standard deviation of aggregate inflation. In Mexico the standard deviation of reset inflation is 'only' 2.2 times larger than the standard deviation of aggregate inflation. Note also that the reset price index is much closer to the aggregate price index in Mexico than in the other two countries. This is not surprising since, as summarized in Table 1, prices adjust more frequently in Mexico than in Austria and UK.

Table 2: Empirical properties of aggregate inflation and reset inflation

	Relative St. dev.	Correlation
UK	6.71	0.36
Austria	5.84	0.58
Mexico	2.24	0.56

Note: 'Relative St. dev.' is the ratio of the standard deviation of monthly reset inflation to the standard deviation of monthly inflation, given by $\sigma_{\Delta\bar{p}_i}/\sigma_{\Delta p_i}$. 'Correlation' is the coefficient of correlation between monthly aggregate inflation and reset monthly inflation.

Figure 1: Empirical inflation and reset inflation (monthly)



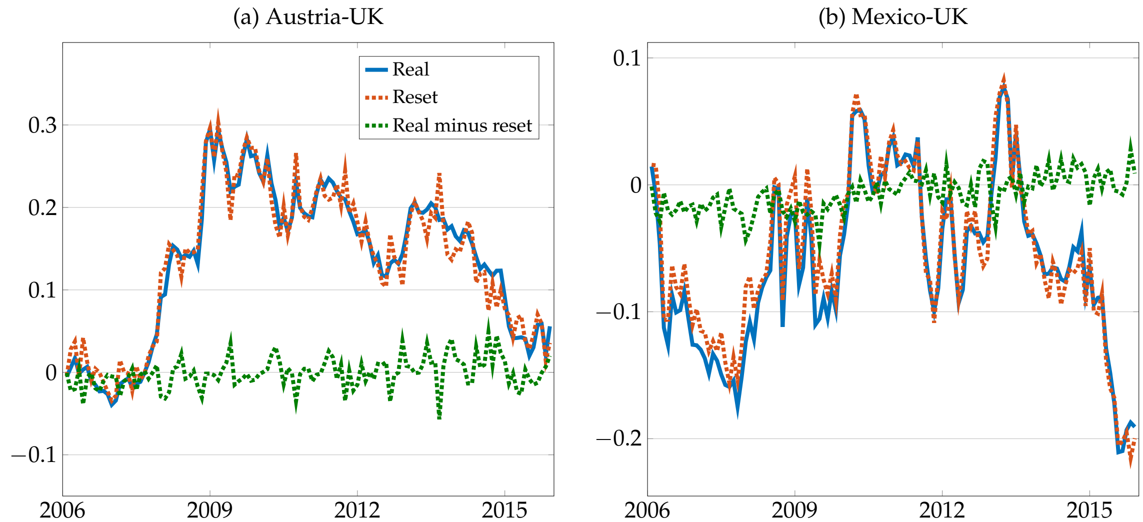
Note: The figures report monthly changes in inflation calculated as $p_{i,t} - p_{i,t-1}$ and $\bar{p}_{i,t} - \bar{p}_{i,t-1}$ from equations (5) and (6).

Discussion and relation to Bils et al. (2009) The reset price index in equation (5) measures the cumulative inflation, relative to January 2006, for the subset of goods that adjusted prices in period t. In a model of Calvo pricing such as the ones presented in the next section, changes in this index would be a model consistent measure of the changes in the reset price. Bils et al. (2009) computed an alternative measure of 'reset inflation', which imputes the reset price changes exhibited by price changers to all items, those changing and those not. While this measure is also consistent with the reset price in a Calvo model, it can differ from our measure in small samples.

4 Empirical results

This section conducts an empirical decomposition of the Austria-UK and the Mexico-UK real exchange rates according to equation (1). Figure 2 plots cumulative changes in real exchange rates, reset exchange rates, and in the difference between these two. The figure shows that real exchange rate movements are driven almost exclusively by movements in the reset exchange rate. In particular, movements in the reset exchange rate are as large and are strongly correlated to those in real exchange rates. In contrast, the difference between these two is much less volatile and almost uncorrelated to the real exchange rate. In fact, the cumulative change in this difference is centered around zero, while sharp movements in the real exchange rate are persistent.

Figure 2: Empirical decomposition of the real exchange rate

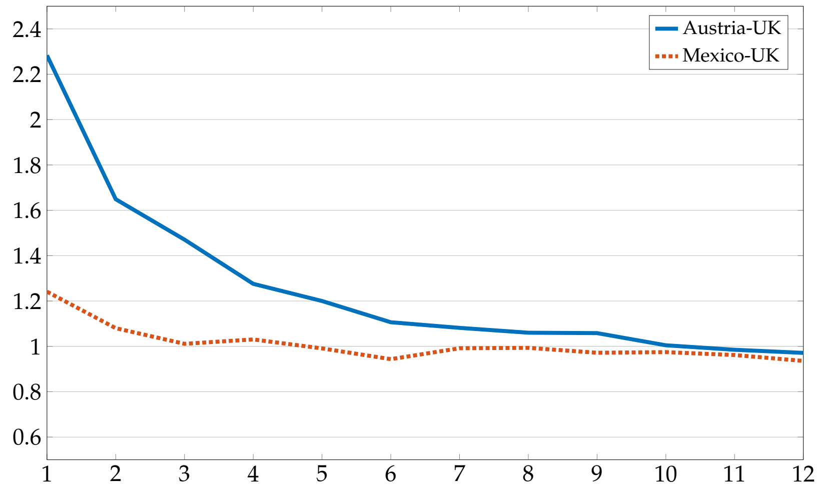


Note: 'Real' and 'Reset' refer to the real and the reset exchange rates reset, defined in equation (1). The figure plots cumulative changes relative to January 2006.

Variance decomposition We evaluate our decomposition more formally by measuring how much of the variance of the change in the real exchange rate is attributable to changes in relative reset prices. In particular, we follow Engel (1999) and measure the fraction of the variance of $\Delta_k rer_{in,t}$ accounted for by the variance of $\Delta_k \overline{rer}_{in,t}$ for k monthly differences up to a horizon of one year. Figure 3 plots the fraction $var\left(\Delta_k \overline{rer}_{in,t}\right) / var\left(\Delta_k rer_{in,t}\right)$. The figure shows that both for the Austria-UK and the Mexico-UK real exchange rates, the variance of the reset exchange rate is larger than that of the real exchange rate for high frequency movements, and that for lower frequency movements (horizons greater than two months) the two variables have about the same variance.

We conclude that in the data, movements in relative reset prices account for almost the totality of the movements in the real exchange rate. This evidence is at odds with the canonical SPOE model presented in Section 2. The following section derives a fully fledged quantitative model that allows for multiple sectors and wage rigidities to show that the predictions of this bigger model are still at odds with the data.

Figure 3: Relative Variances: Reset vs Real



Note: The figures plots the ratio of $\mathbb{V}\left[\overline{rer}_{t+k} - \overline{rer}_t\right]$ to $\mathbb{V}\left[rer_{t+k} - rer_t\right]$ for different values of k.

Understanding reset exchange rate movements We conclude this section by evaluating the sources of comovement between the reset and real exchange rates. As documented by Mussa, real exchange rates track nominal exchange rates because movements in relative inflation rates are small. Table 3 shows that in our data, relative inflation rates are far less volatile than nominal exchange rates. The standard deviation of relative price changes is about 0.16-0.15 for of the standard deviation of nominal exchange rate changes at a monthly frequency, and 0.08-0.15 at a yearly frequency.

Why does the reset exchange rate also track the nominal rate? Table 3 shows that the volatility of relative reset inflation is sizable, even relative to the observed changes in the nominal exchange rate. For the Austria-UK pair, the standard deviation of the monthly relative reset inflation is 1.2 times the standard deviation of the monthly nominal exchange rate changes. Why does the reset exchange rate comoves with the nominal rate? The answer can be gleaned from the second panel of Table 3, which shows that changes in relative reset prices are uncorrelated to nominal exchange rate change at the monthly frequency. That is, while relative reset prices move a lot, they don't do so in a way that offsets the changes in the nominal exchange rate. A similar pattern emerges for the Mexico-UK country pair. The correlation between relative reset price changes and nominal exchange rates is still low at -0.2 (-0.33) at a yearly horizon. At this horizon, however, changes in relative reset prices are substantially less volatile than nominal exchange rate changes.

Table 3: Relative inflation and relative reset inflation

Relative to $\Delta e\downarrow$	Austria-UK $\Delta(p_i - p_n)$	$\Delta(\bar{p}_i - \bar{p}_n)$	$\Delta Wedge$	Mexico-UK $\Delta(p_i - p_n)$	$\Delta(\bar{p}_i - \bar{p}_n)$	$\Delta Wedge$
Std. Dev.						
Monthly	0.16	1.20	2.29	0.15	0.59	1.69
Quarterly	0.12	0.75	1.08	0.17	0.42	1.30
Yearly	0.08	0.28	0.38	0.15	0.26	0.55
Correlation						
Monthly	0.18	0.03	0.11	0.16	-0.03	0.01
Quarterly	0.08	0.00	0.01	0.10	-0.11	-0.10
Yearly	-0.15	-0.20	-0.03	-0.15	-0.33	0.01

Note: 'Relative St. dev.' is the ratio of the standard deviation of monthly reset inflation to the standard deviation of monthly inflation, given by $\sigma_{\Delta\bar{p}_i}/\sigma_{\Delta p_i}$. 'Correlation' is the coefficient of correlation between monthly aggregate inflation and reset monthly inflation.

5 Quantitative SPOE models

This section describes different quantitative SPOE models that the literature has used to study the joint dynamics of the nominal and the real exchange rates. It then evaluates the implications of these models for the terms in the real exchange rate decomposition presented in Section 3.

5.1 SPOE models with complete markets

We consider a world economy consisting of two countries indexed by i and n. Each country is inhabited by a continuum of households indexed by $h \in [0,1]$, a producer of final goods, a continuum of monopolistic intermediate producers indexed by $\omega \in [0,2]$, a government, and a monetary authority. Below we describe the preferences, technologies, and monetary rules in each country.

Households: Each household in country i has preferences given by

$$U_i = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta_t \left[\varphi_{i,t} logC_{i,t} - N_{i,t}(h)\right], \tag{8}$$

where $C_{i,t}$ and $N_{i,t}$ denote consumption and labor, $\varphi_{i,t}$ is an exogenous shock to the utility of consumption, and β_t is a stochastic discount factor which is the same for the two countries.

Households supply differentiated labor services, have monopoly power over their wage, and face Calvo-type constraints on their ability to adjust wages, as in Kollmann (2001). The probability that a household can change its wage in a given period is given by $1-\theta_w$.

Financial markets are complete. The date 0 inter-temporal budget constraint is:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \Theta_{i,t} \left[P_{i,t}C_{i,t} - W_{i,t}(h) N_{i,t}(h) - \Upsilon_{i,t} - T_{i,t}\right] = 0. \tag{9}$$

Here $P_{i,t}$, $W_{i,t}(h)$, $\Upsilon_{i,t}$, and $T_{i,t}$ respectively denote the price of consumption, nominal wages, firm's profits, and government transfers, all denominated in the currency of country i.[15] $\Theta_{i,t}$ is the date $t=0$ local-currency price of an arrow security that pays one unit of the local currency in at time t. By definition, the nominal exchange rate satisfies: $E_{in,t} = E_{in,0}\frac{\Theta_{i,t}}{\Theta_{n,t}}$.

Final goods producers: The final good in country i, $Y_{i,t}$, is produced with the Kimball (1995) aggregator:

$$1 = \mu\Omega\left(\frac{Y_{ii,t}}{\mu Y_{i,t}}\right) + [1-\mu]\Omega\left(\frac{Y_{ni,t}}{[1-\mu]\, Y_{i,t}}\right), \tag{10}$$

$$1 = \int \Upsilon_p\left(\frac{Y_{ni,t}(\omega)}{Y_{ni,t}}\right) d\omega. \tag{11}$$

Here $Y_{ni,t}(\omega)$ denotes the quantity of intermediate good ω produced in country n and consumed in country i, and μ is the share of domestic goods in absorption in the symmetric steady state. The function Ω satisfies $\Omega(1) = 1$, $\Omega'(.) > 0$, and $\Omega''(.) < 0$. In what follows, we adopt the specification of Klenow and Willis (2007) for $\Upsilon_p(.)$ that yields $\Upsilon_p'^{-1}(x) = \left[1-\phi_p x\right]^{\frac{\gamma_p}{\phi_p}}$. Under this specification, the elasticity of demand is $\varepsilon \equiv -\frac{\partial log \Upsilon_p'^{-1}(x)}{\partial log x} = \frac{\gamma_p}{1-\phi_p log(x)}$, which is constant and equal to γ_p when $\phi_p = 0$, and increasing in x if $\phi_p > 0$.

Intermediate good producers: Intermediate producers behave as monopolistic competitors and set prices as in Calvo (1983). Importantly, producers set prices in the cur-

[15] Note that while the wage rigidities make labor income idiosyncratic, the expected discounted value of the labor income is common across all households.

rency of the country where they sell. The probability that a producer can change its price in any period is given by $1-\theta_p$. The production function for intermediate goods is

$$Y_{ni,t}(\omega) = \bar{a} N_{ni,t}(\omega)^{1-\bar{\alpha}} X_{ni,t}(\omega)^{\bar{\alpha}}, \tag{12}$$

$$1 = \int_0^1 \Upsilon_w \left(\frac{n_{ni,t}(\omega,h)}{N_{ni,t}(\omega)} \right) dh, \tag{13}$$

with $\bar{a} \equiv \bar{\alpha}^{\bar{\alpha}} [1-\bar{\alpha}]^{1-\bar{\alpha}}$ and $\Upsilon_w'^{-1}(x) = [1-\phi_w x]^{\frac{\gamma_w}{\phi_w}}$. Here $N_{ni,t}(\omega)$ and $X_{ni,t}(\omega)$ denote the quantities of labor and intermediate inputs used to produce good ω in country n to serve market i. The profit maximizing price for an intermediate producer that gets to adjust prices satisfies:

$$\bar{P}_{ni,t} = \arg\max \left\{ \mathbb{E}_t \sum_{j=0}^{\infty} \frac{\Theta_{i,t+j}}{\Theta_{i,t}} \theta_p^j \left[\frac{\Theta_{n,t+j}}{\Theta_{i,t+j}} \bar{P}_{in,t} - W_{i,t+j}^{1-\alpha} P_{i,t+j}^{\alpha} \right] \bar{Y}_{in,t+j}(\bar{P}_{ni,t}) \right\}, \tag{14}$$

where $\bar{Y}_{in,t}(\bar{P}_{ni,t})$ is the demand function associated with the aggregator (11).

Monetary and fiscal policy: As Carvalho and Nechio (2011) we leave the monetary policy implicit, and assume that aggregate nominal expenditures in each country are exogenous and given by $Z_{i,t} \equiv P_{i,t} C_{i,t}$.[16] Additionally, government expenditures G_t are exogenous. The government balances its budget every period, $G_t = T_t$.

Equilibrium: An equilibrium for this economy is a set of allocations for the households $\{C_{i,t}, W_{i,t}(h)\}_{\forall i,h,t}$, final good producers $\{Y_{i,t}, Y_{in,t}, \{Y_{in}(\omega)\}_{\omega}\}_{\forall n,t}$, and price policy functions for intermediate producers $\{\bar{P}_{in,t}\}_{\forall i,n,t}$, such that given prices: (i) households maximize (8) subject to (9); (ii) final good producers minimize cost according to equations (10) and (11); (iii) intermediate producers maximize profits according to equation (14); and (iv) labor and goods markets clear.

5.1.1 Analytic results

This section characterizes, to a first approximation, the stochastic processes of the real and the reset exchange rates in parameterizations of the model where the change in the nominal exchange rate follows an $AR(1)$. We highlight that the results in this section are valid irrespectively of whether the model is driven by shocks to the discount factor, preferences, government expenditures or to nominal demand, β_t, G_t, φ_t or Z_t.

[16] In the next section, we evaluate the case in which the monetary authority follows a Taylor rule.

Proposition 1. (Relative price processes) *If $\frac{\varphi_{i,t}}{\varphi_{i,t-1}}\frac{Z_{i,t-1}}{Z_{i,t}}$ follows an AR(1) with persistence ρ, then the law of motion for the nominal exchange rate satisfies $\mathbb{E}_t\left[\Delta e_{in,t+1}\right]=\rho\Delta e_{in,t}$. In addition, relative reset wages are:*

$$rer^{w}_{in,t}=A_{1w}rer^{w}_{in,t-1}+A_{2w}\Delta e_{in,t},$$

the real exchange rate is

$$rer_{in,t}=A_{0,p}rer^{w}_{in,t}+A_{1,p}rer_{in,t-1}+A_{2,p}\Delta e_{in,t},$$

and the reset exchange rate is

$$\overline{rer}_{in,t}=\frac{A_{0,p}}{1-\theta_p}rer^{w}_{in,t}+\frac{A_{1,p}-\theta_p}{1-\theta_p}rer_{in,t-1}+\frac{A_{2,p}-\theta_p}{1-\theta_p}\Delta e_{in,t},$$

where $A_{0p}\equiv\frac{\left[\theta_p^{-1}-1\right]\left[\theta_p^{-1}-\beta\right]\iota_p}{\lambda_p-\beta\left[A_{1w}+A_{1p}\right]}$ and for $x\in\{p,w\}$, $\lambda_x\equiv 1+\beta+\left[\theta_x^{-1}-1\right]\left[\theta_x^{-1}-\beta\right]\left[1-\alpha_x\right]$, $A_{1x}=\frac{\lambda_x-\sqrt{\lambda_x^2-4\beta}}{2\beta}$, $A_{2x}\equiv\frac{2-2\beta\rho\left[1-A_{0,p}A_{2w}\mathbb{I}(x=p)\right]}{\lambda_x+\sqrt{\lambda_x^2-4\beta}-2\beta\rho}$, $\alpha_p\equiv\frac{\Gamma_p+\bar{\alpha}[2\mu-1]}{1+\Gamma_p}$, $\iota_p=\frac{[1-\bar{\alpha}][2\mu-1]}{1+\Gamma_p}$, $\Gamma_p\equiv\frac{\phi_p}{\gamma_p-1}$, $\alpha_w\equiv\frac{\Gamma_w}{1+\Gamma_w}$, $\Gamma_p\equiv\frac{\phi_w}{\gamma_w-1}$.

Proposition 1 is useful for understanding the joint dynamics of the real and the reset exchange rate. A large literature has studied the persistence of the real exchange rate persistence in sticky price models, showing that it increases in: (i) the degree of nominal price rigidities, θ_p (Chari et al. 2002), (ii) the persistence of the exogenous shocks, ρ, (Benigno 2004, Carvalho and Nechio 2011 and Engel, 2016), (iii) the degree of real rigidities, $\bar{\alpha}$ and $\bar{\Gamma}$, (Bergin and Feenstra 2001, Bouakez 2005), and in the degree of nominal wage rigidities, θ^{w}, (Kollmann 2001). In what follows, we proceed in steps and evaluate alternative special parameterizations that isolate how each of these mechanisms affects the contribution of the reset exchange rate to real exchange rate movements. To this end, we study the cumulative impulse response of the real exchange rate at period $t+j$ to shocks in period t, and evaluate the fraction χ_j of this impulse response that is accounted for by the reset exchange rate.[17] When the reset exchange rate is a constant fraction of the real exchange rate, χ_j is related to the variance decomposition presented in Section 4 by $var\left[\Delta_j\overline{rer}_{in}\right]/var\left[\Delta_j rer\right]=\chi_j^2$.

[17]Formally, starting from a symmetric steady state in period $t-1$, we evaluate how different parameters affect $\mathcal{CIR}_{t+j}\left[rer_{in,t}\right]\equiv\mathbb{E}_t\left[rer_{in,t+j}\right]/\Delta e_{in,t}$, and $\chi_j\equiv\mathbb{E}_t\left[\overline{rer}_{in,t+j}\right]/\mathbb{E}_t\left[rer_{in,t+j}\right]$.

Case 1- baseline: Consider first a version of the model in which wages are flexible, $\theta_w = 0$, the nominal exchange rate follows a random walk, $\rho = 0$, and there are no real rigidities, $\alpha = 0$. In this case, the persistence of the real exchange rate as measured by the autocorrelation coefficient is given by θ_p, and the reset exchange rate is constant, $\overline{rer}_{in,t} = 0$,[18] so $\chi_j = 0$ as depicted in the solid blue lines in Figure 4. Intuitively, the persistence of the real exchange rate arises only from the infrequent price adjustment, but the relative prices for the firms that do adjust (i.e. the reset exchange rate) are constant. If we calibrate $\theta_p = 0.88$ to match the average frequency of price changes in the UK microdata, the model cannot generate enough persistence in the real exchange rate, as noted by Chari et al. (2002).

Carvalho and Nechio (2011) show that multisector models yield more persistent real exchange rates than a one-sector model calibrated to match the average frequency of price changes. Online Appendix C shows that the multi-sector model yields very similar results to a one-sector model in which θ_p is chosen to match the average duration of prices, which in our data corresponds to setting $\theta_p = 0.93$. While this increases the persistence of the real exchange rate, the reset exchange rate is still constant, $\chi_j = 0$.

Case 2- persistent shocks: Consider now increasing the persistence of the shocks without changing the remaining parameters. As illustrated in the red x-doted lines in Figure 4 for $\rho = 0.2$,[19] this makes the real exchange rate more persistent, but also makes the reset exchange rate negatively correlated to the real exchange rate, $\overline{rer}_{in,t} = \frac{-\theta\beta\rho}{1-\theta\beta\rho}\Delta e_{in,t}$. Intuitively, firms that reset prices may be unable to change prices again in the future, so they respond more than one to one to a persistent shock. This implies that increasing the real exchange rate persistence by increasing ρ moves the reset exchange rate in the opposite direction than in the data.

Case 3- real rigidities: We now set $\rho = 0$ again, but introduce real rigidities, $\alpha_p > 0$. In this case, the autocorrelation of the real exchange rate equals $A_1 = A_2$, and $\overline{rer}_{in,t} = \frac{1-\theta/A_1}{1-\theta} rer_{in,t}$, so $\chi_j = \frac{1-\theta/A_1}{1-\theta}$. Since A_1 increases with α, real rigidities simultaneously increase the persistence of the real exchange rate and the contribution of the reset exchange rate to the real exchange rate. We explore quantitatively if real rigidities alone can replicate our empirical decomposition by calibrating $\bar{\alpha}$, Γ_p and μ following the standard

[18] See Appendix B for a proof for all the statements in this section. All the results follow directly from Proposition 1.

[19] We use $\rho = 0.2$ to match the persistence of the growth rate of the UK-Austria nominal exchange rate.

practices in the literature.[20] This case is depicted in the green doted lines in Figure 4. The fraction of variance of the real exchange rate accounted for by the reset exchange rate in this parameterization is only $\chi_j^2 = 0.19$, well below what observed in the data.

Case 4- nominal wage rigidities: Finally, consider a case with no real rigidities $\alpha = 0$, but with nominal price and wage rigidities, $\theta_p, \theta_w > 0$. Introducing wage rigidities increases the persistence of the real exchange rate. In this case, we can show that $\chi_j = \frac{[2\mu-1][1-\beta\theta]}{1+[2\mu-1][1-\beta\theta][1+j]}$: the reset exchange rate accounts for a positive fraction of the impulse response of the real exchange rate. Note, however, that this fraction declines with j. Intuitively, as workers reset wages, the reset exchange rate converges to steady state level faster than the real exchange rate,[21] so χ_j declines. This case is depicted in the dashed purple lines in Figure 4 for $\theta_w = 0.9$.[22] The figure shows that in this case χ_j is still below 0.5 at every horizon.

5.1.2 Quantitative results

We now consider different quantitive versions of the model presented above. The parameters used for each of these models are reported in Table 4. First, we evaluate a model that satisfies the conditions in Proposition 1 and that combines all the mechanisms that were studied separately under Cases 1-4. The impulse responses of this calibration are plotted in the circled-black lines in Figure 4 to facilitate comparisons with our analytic results. Combining the different ingredients studied in the literature, the model generates persistent real exchange rates, with a half life that is close to 30 months, in line with the data.[23] However, the fraction of the impulse response of the real exchange rate accounted by the reset exchange rate χ_j is only around 0.5. Business-cycle statistics of this model are listed in the second column of Table 5, labeled 'Benchmark'. While the model is close to matching the autocorrelation of the real exchange rate and its correlation with the nominal exchange rate, it does not produce enough movements in the reset exchange rate. In addition, the residual term in the decomposition is perfectly correlated with the real exchange rate in this model, but is uncorrelated with the real exchange rate in the data.

Next, we deviate from the restrictions needed for Proposition 1 by introducing more

[20]In particular, we set $\bar{\alpha} = 0.5$ to match the share of intermediate inputs in production, $\Gamma_p = 0.66$ to match the elasticity of markups following Mukhin and Itskhoki (2016), and $\mu = 0.93$ to match set the share of home products in the CPI.

[21]The last statement follows because, on top of the wage rigidities, the price rigidities add persistence to the real exchange rate but not to the reset exchange rate, as explained in Case 1.

[22]We take this value from Negro et al. (2007).

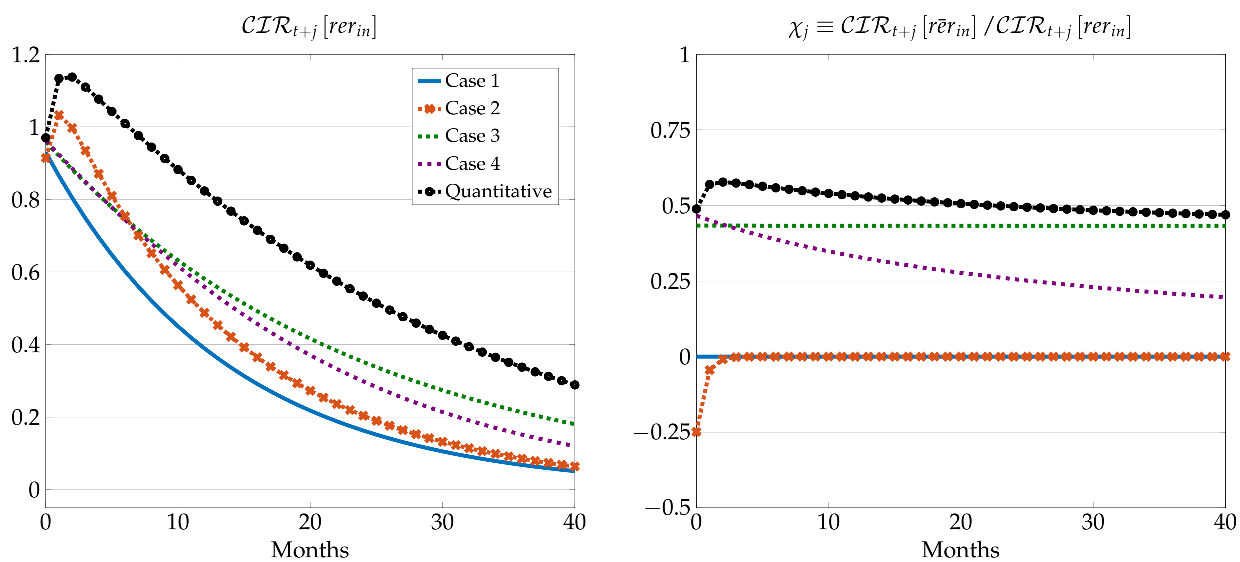
[23]See i.e. Rogoff (1996) and Burstein and Gopinath (2015)

general preferences and by incorporating money in the utility function, as in Chari et al. (2002).[24] The third Column of Table 5 shows that the results from this model are remarkably close to those in the Benchmark case. This implies that the additional restrictions imposed to derive Proposition 1 do not seem to be quantitatively important for the results of the models studied so far.

Finally, we allow for a more realistic monetary policy by assuming that the monetary authority follows a Taylor Rule as in Steinsson (2008). As noted by Steinsson (2008), this version of the model can generate persistent real exchanges rates, the fourth column in Table 5 shows that it produces an autocorrelation of the real exchange rate that close to the data. However, the model produces a reset exchange rate that is too volatile relative to the data.

We conclude that quantitative SPOE models with complete markets can generate persistent real exchange rates, but are hard to reconcile with the reset exchange rate data. The following section evaluates whether the behavior of the reset exchange rate can be approximated in models with incomplete markets.

Figure 4: Impulse Responses



Note: The left panel plots the impulse response of the real exchange rate and right panel plots the χ_j in equation defined in the main text.

[24] Appendix C presents a full description of this and the following model.

Table 4: Parameterization

Parameter	Description	Value	Target/source
β	Discount factor	$0.96^{1/12}$	Chari et al. (2002)
μ	Home bias parameter	0.93	Imports/abs. WIOD
ρ	Persistence of nominal exchange rate	0.20	Persistence of nominal exchange rate
$1-\theta_p$	Frequency of price changes	0.07	Average price duration across items, UK
$1-\theta_w$	Frequency of price changes	0.10	Negro et al. (2007)
$\frac{1}{\Gamma_p+1}$	Markup elasticity	0.40	Amiti et al. (2014)
$\bar{\alpha}$	Share of intermediates in production	0.5	VA/Output. WIOD

5.2 Models of incomplete market and UIP violations

A well know limitation of open economy models with complete markets is that they impose uncovered interest rate parity, which is violated in the data. Gabaix and Maggiori (2015), Jeanne and Rose (2002), and Kiyotaki et al. (2013) are some examples of incomplete market models in which UIP fails to hold due to shocks to international asset demand. More recently, Mukhin and Itskhoki (2016) show that incorporating these type of shocks into a dynamic general equilibrium model can help resolve many of the 'puzzles' associated with the exchange rate disconnect. In this section, we evaluate whether a version of the model that incorporates these features can rationalize the observed exchange rate decomposition presented in Section 4. We study this question in a version of the model that follows Mukhin and Itskhoki (2016): financial markets are incomplete and there is a variable wedge between the return on foreign bonds for the home households and the foreign interest rate. The monetary authority follows a Taylor rule that targets inflation. The full model and its calibration are presented in Appendix C.

The fifth column in Table 5 summarizes the dynamic properties of the real and the reset exchange rate in this model. As noted by Mukhin and Itskhoki (2016), the model produces very persistent real exchange rates that are almost perfectly correlated to the nominal exchange rates. It also does a much better job in replicating the observed real exchange rate decomposition at long horizons, despite the fact that reset exchange rate data is not used in the calibration. In particular, the model matches the correlation and the relative standard deviations between the two terms in the decomposition and the real exchange rate in levels, and does a good job in replicating the relative variance of the changes in the reset and real exchange rates for long run changes horizons. In contrast, the model fails to reproduce the relative variances compited for monthly growth rates, as it produces a residual term that is less volatile much more correlated with the real exchange rate than what is observed in the data.

Table 5: Business Cycle Moments: Models and Data

Moments	Data	Benchmark	CKM2002	STE2008	IM2017
Standard deviations rel. to real exchange rate (growth rates)					
Nominal	0.96	1.03	1.01	1.03	0.96
Reset	1.53	0.50	0.67	2.47	0.55
Difference	1.10	0.52	0.33	2.11	0.60
Correlations with real exchange rate (growth rates)					
Nominal	0.99	0.99	1.00	0.85	0.99
Reset	0.69	0.98	1.00	0.54	0.86
Difference	-0.05	0.98	1.00	-0.16	0.88
Standard deviations rel. to real exchange rate (levels)					
Reset	0.99	0.53	0.65	2.12	1.00
Difference	0.19	0.47	0.36	1.29	0.14
Correlations with real exchange rate (levels)					
Reset	0.98	1.00	0.99	0.91	0.99
Difference	0.13	1.00	0.98	-0.72	0.07
Autocorrelations (levels)					
Real	0.97	0.96	0.94	0.98	1.00
Reset	0.94	0.96	0.94	0.98	1.00
Difference	0.25	0.95	0.95	0.96	0.95
$\mathbb{V}\left[\overline{rer}_{in,t+k} - \overline{rer}_{in,t}\right] / \mathbb{V}\left[rer_{in,t+k} - rer_{in,t}\right]$					
1	2.28	0.25	0.45	6.12	0.30
3	1.47	0.30	0.45	5.56	0.53
6	1.11	0.31	0.45	5.10	0.69
9	1.06	0.30	0.45	4.87	0.78
12	0.97	0.30	0.45	4.75	0.83

Notes: The table presents business cycle moments of the data and the simulation series from the models. The time period in the data and in the model have the same lenght. All the moments in the model are the median over 5000 simulations.

6 Conclusion

Addressing the relative PPP puzzle is one of the central questions in international economics. This paper sheds light on the puzzle by showing that real exchange rates movements are primarily driven by movements in what we label as the 'reset exchange rate', i.e. relative reset prices across countries. This empirical finding is at odds with the predictions of Sticky Price Open Economy models, which can generate volatile and persistent real exchange rates but only through cross-country movements in the difference between reset and non-reset prices.

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Online Appendix

Price rigidities and the relative PPP puzzle

Andres Blanco and Javier Cravino

Appendix A Data

A.1 Algorithm used in the UK data

Let i denote an identifier at item-location-shop level and t denote time at monthly frequency. Let $P_{it} = \{p_{1ti}, p_{2ti}, ..., p_{nti}\}$ denote the set of different prices at i, t level, $n^{max}(i) = \max_t S_{it}$ the maximum number of repetitions for item–location--shop i, and $X_{it} = \{x_{1ti}, x_{2ti}, ..., x_{nti}\}$ the relative prices with respect to January of that year. Notice that by construction P_{it} has all different values. Next we describe the algorithm. After applying this filter, a good is a different item, shop and location for the price quotes $\{\hat{p}_{it}^k\}_{k=1}^{n(i)}$. At the time of applying the algorithm, it is important to keep track of the indicator box to filter the data with sales, comparable substitutions, etc. Additionally, for each price quote we generate a weight with the amount of repetitions across id. For example, if for $id = 1$ and $date = 1997m1$ there are three prices 1, 1, and 1.5, then the weight for 1 is 2/3 and the weight for 1.5 is 1/3.

i. Set $k = 1$, $P_{it}^k = P_{it}$ and $X_{it}^k = X_{it}$.

ii. Generate $\hat{p}_{ti}^k = p_{zti}$ and $\hat{x}_{ti}^k = x_{zti}$ for the first non-missing value over t. Set $\hat{p}_{t+1i}^k$ equal to whenever p_{zt+1i} and $\hat{x}_{t+1i}^k$ equal to whenever x_{zt+1i} if

$$\left(\hat{x}_{ti}^k I(month \neq Feb) + I(month = Feb)\right) \frac{p_{zt+1i}}{\hat{p}_{ti}^k} = \hat{x}_{zt+1i}^k$$

Generate $P_{it}^{k+1} = P_{it}^k | p_{zt+1i}$ and $X_{it}^{k+1} = X_{it}^k || x_{zt+1i}$. If this not possible, generate an empty-value. Iterate over t for a given k.

iii. Stop if P_{it}^{k+1} is empty for all t. If not, set $k = k + 1$ and go to step 2.

It is important to take into account missing pricing for less than one year but it is easy to modify step *ii* in the previous algorithm to include this feature in the data.

A.2 Algorithm used to define temporary sales

To describe the algorithm to filter sales let us define the following object: p_{ti} the log-price at time t and product i, $\Delta p_{ti} = p_{ti} - p_{t-1i}$ the price change between $t-1$ and t, $\mathcal{F}_k$ all the dates–products such that the sum of price changes between t and $t+k$ is close to zero

$$\mathcal{F}_k = \{(t, i) : \quad |\Delta p_{ti} + \cdots + \Delta p_{t+ki}| \leq \epsilon \quad , \quad \Delta p_{ti} \neq 0\} \, ,$$

$\mathcal{L}_k$ all the dates–products such that the sum of price changes between $t-k$ and t is close to zero

$$\mathcal{L}_k = \{(t, i) : \quad |\Delta p_{t-ki} + \cdots + \Delta p_{ti}| \leq \epsilon \quad , \quad \Delta p_{ti} \neq 0\} \, ,$$

Notice that for every price there has to be a negative of this price change. Therefore we drop all the elements in $(t, i) \in \mathcal{F}_k$ such that $(t+k, i) \notin \mathcal{L}_k$. After this, we drop all the price changes between (t^*, i) and $(t^* + k, i)$.

Table A1: Sectorial summary statistics

(a) CPI weights

	UK		Austria		Mexico	
	Official	Microdata	Official	Microdata	Official	Microdata
Food & non-alc. beverages	0.11	0.11	0.12	0.12	0.19	0.19
Alc. beverages & tobacco	0.04	0.04	0.03	0.03	0.03	0.03
Clothing & footwear	0.06	0.06	0.06	0.06	0.05	0.05
Housing & utilities	0.13	0.04	0.18	0.14	0.22	0.07
Furnishings	0.06	0.06	0.08	0.08	0.05	0.05
Health	0.02	0.02	0.05	0.05	0.04	0.04
Transport	0.15	0.05	0.15	0.15	0.14	0.13
Communication	0.03	0.01	0.03	0.03	0.04	0.04
Recreation & culture	0.15	0.06	0.12	0.12	0.05	0.05
Education	0.02	0.01	0.01	0.01	0.05	0.05
Restaurants & hotels	0.13	0.11	0.08	0.08	0.08	0.08
Miscellaneous	0.10	0.06	0.10	0.10	0.06	0.05
Aggregate	**1.00**	**0.65**	**1.00**	**0.96**	**1.00**	**0.84**

(b) Monthly frequency of price changes

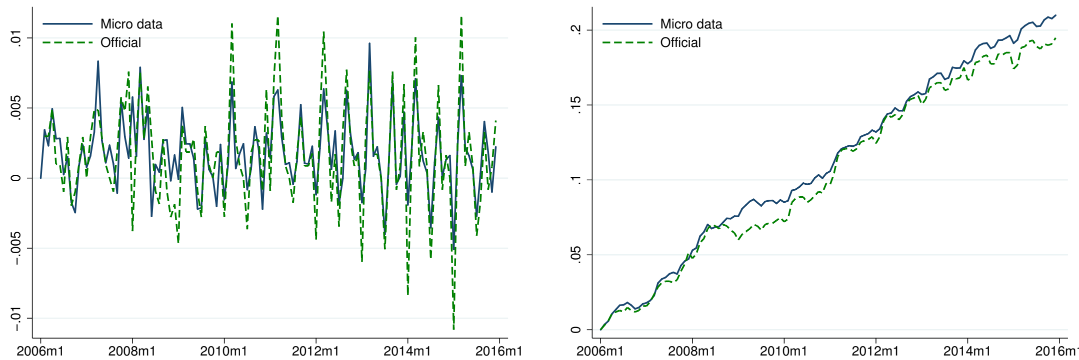
	UK		Austria		Mexico	
Treatment of Sales→	Included	Excluded	Included	Excluded	Included	Excluded
Food & non-alc. beverages	0.20	0.13	0.19	0.10	0.50	0.44
Alc. beverages & tobacco	0.31	0.23	0.12	0.08	0.28	0.24
Clothing & footwear	0.20	0.15	0.18	0.14	0.18	0.14
Housing & utilities	0.13	0.12	0.22	0.20	0.65	0.64
Furnishings	0.20	0.13	0.11	0.09	0.32	0.27
Health	0.10	0.09	0.27	0.27	0.17	0.15
Transport	0.12	0.11	0.16	0.15	0.38	0.37
Communication	0.21	0.20	0.14	0.12	0.38	0.37
Recreation & culture	0.19	0.14	0.26	0.24	0.24	0.22
Education	0.09	0.09	0.06	0.06	0.13	0.13
Restaurants & hotels	0.10	0.09	0.11	0.09	0.14	0.13
Miscellaneous	0.14	0.09	0.08	0.07	0.40	0.34
Aggregate	**0.16**	**0.12**	**0.17**	**0.15**	**0.35**	**0.32**

Table A2: Updates to the CPI basket

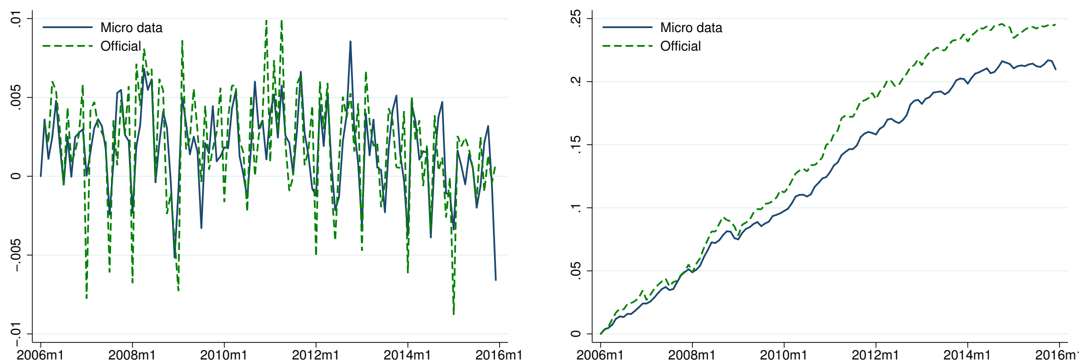
	UK			Austria		
	Fraction of new products	CPI weight of new items	Frequency	Fraction of new products	CPI weight of new items	Frequency
Months of basket update						
Average	0.13	0.03	0.17	0.03	0.01	0.26
Minimum	0.07	0.02	0.10	0.00	-	0.24
Maximum	0.22	0.10	0.25	0.12	0.05	0.29
Regular months						
Average	0.03	0.00	0.17	0.00	-	0.14
Minimum	-	-	0.10	-	-	0.09
Maximum	0.22	0.10	0.40	0.01	-	0.30

Note: ’Months of basket update’ refers to every February for the UK, and every January starting in January 2011 for Austria. ’Regular months’ refers to every other month. ’Fraction of new products’ is the ratio of the number of new products over the total number of products. ’CPI weight new of new items’ refers to the expenditure share of new item categories. ’Frequency’ is the is the share of price changes relative to the number of price quotes in the month.

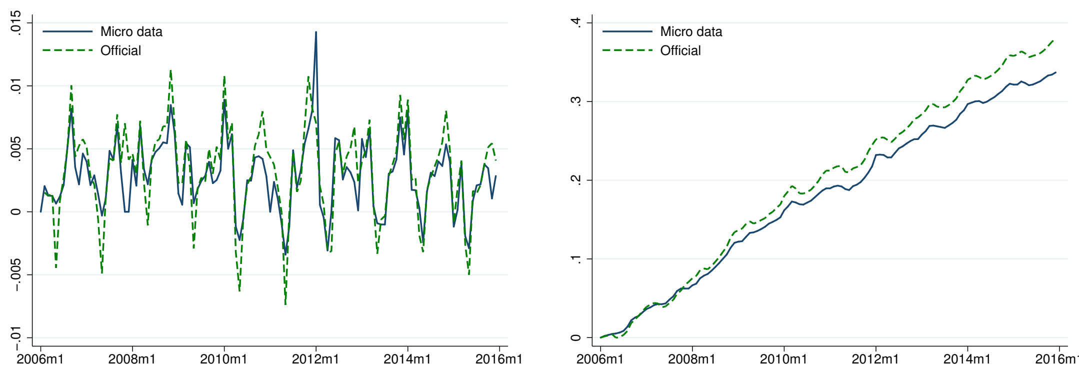
Figure 5: Inflation: Official sources vs. microdata



(a) Austria

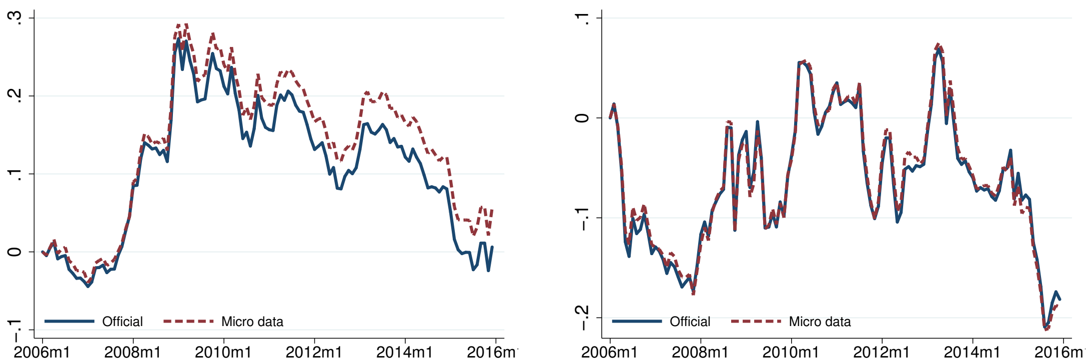


(b) UK



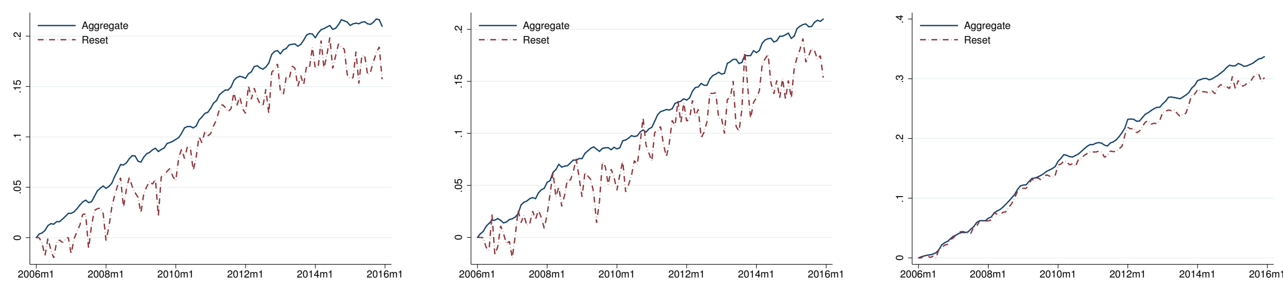
(c) Mexico

Figure 6: Real exchange rates: Official sources microdata



(a) UK-Austria (b) UK-Mexico

Figure 7: Empirical inflation and reset inflation (monthly)



(a) UK (b) Austria (c) Mexico

Note: The figures report cumulative inflation given by equations (5) and (6).

Appendix B Proofs and formulas

B.1 Proofs of Proposition 1

We start by deriving the optimality conditions for the household's problem. In each country, the marginal utility of consumption must satisfy:

$$\beta_t \frac{\partial u_j(C_{j,t}, X_{j,t})}{\partial C_{j,t}} = \lambda_i P_{i,t} \Theta_{i,t},$$

where λ_i is the Lagrange multiplier on country-i's budget constraint. Taking log-differences across countries and using the definition of the nominal exchange rate, assuming $E_{in,0} = 1$

we have that

$$rer_{in,t} = u^c_{i,t} - u^c_{n,t}.$$

Under log-separable preferences, and using $Z_{i,t} \equiv P_{i,t}C_{i,t}$ we have that

$$\beta_t \varphi_{i,t} = Z_{n,t}\lambda_n \Theta_{i,t}$$

with $\frac{u(X_{i,t})}{u(X_{n,t})}\frac{Z_{n,t}}{Z_{i,t}} = E_{in,t}$ and $\mathbb{E}_t\left[\Delta e_{in,t+1}\right] = \rho \Delta e_{in,t}$.

To a first order, the price indices are:

$$p_{i,t} = \mu p_{ii,t} + [1-\mu]\, p_{ni,t}.$$

Under calvo prices, the laws of motion for average prices under Calvo are :

$$p_{ni,t} = \left[1-\theta_p\right] \bar{p}_{ni,t} + \theta_p p_{ni,t-1},$$

and optimal reset prices satisfy:

$$\bar{p}_{ni,t} = \left[1-\beta\theta_p\right] \tilde{p}_{ni,t} + \beta\theta_p \mathbb{E}_t\left[\bar{p}_{ni,t+1}\right].$$

Here, $\tilde{p}_{ni,t}$ is the 'optimal spot price', which satisfies

$$\tilde{p}_{ni,t} = \frac{\Gamma_p}{1+\Gamma_p} p_{ni,t} + \frac{1}{1+\Gamma_p}\left[mc_{n,t} - e_{in,t}\right], \tag{B.1}$$

and $\Gamma_p \equiv \frac{\phi_p}{\gamma_p - 1}$ is the elasticity of markups implied by the aggregator in (13), and $mc_{i,t} = \bar{\alpha} w_{i,t} + [1-\bar{\alpha}]\, p_{i,t}$ are marginal costs for firms from country i. Combining these equations with the definition of the real exchange rate we obtain:

$$rer_{in,t} = \left[1-\theta_p\right] \overline{rer}_{in,t} + \theta_p \left[rer_{i,t-1} + \Delta e_{in,t}\right],$$

and

$$\overline{rer}_{in,t} = \left[1-\beta\theta_p\right] \widetilde{rer}_{in,t} + \beta\theta_p \mathbb{E}_t\left[\overline{rer}_{in,t+1} - \Delta e_{in,t}\right],$$

with $\widetilde{rer}_{in,t} \equiv \tilde{p}_{i,t} + e_{in,t} - \tilde{p}_{n,t}$. In combination with B.1 we obtain:

$$\widetilde{rer}_{in,t} = \alpha_p rer_{in,t} + \iota_p rer^w_{in,t},$$

with $\alpha_p \equiv \frac{\Gamma_p + \bar{\alpha}[2\mu-1]}{1+\Gamma_p}$ and $\iota_p \equiv \frac{[1-\bar{\alpha}][2\mu-1]}{1+\Gamma_p}$.

The Calvo assumption for wage setting implies that the law of motion for wages is

$$w_{i,t} = [1-\theta_w]\, \bar{w}_{i,t} + \theta_w w_{i,t-1},$$

and that reset wages satisfy

$$\bar{w}_{i,t} = [1-\beta\theta_w]\, \tilde{w}_{i,t} + \beta\theta_w \mathbb{E}_t\left[\bar{w}_{i,t+1}\right],$$

with

$$\tilde{w}_{i,t} = \frac{\Gamma_w}{1+\Gamma_w} w_{i,t} + \frac{1}{1+\Gamma_w} \left[p_{i,t} - u_{c,i,t}\right],$$

and $\Gamma_w \equiv \frac{\phi_w}{\gamma_w - 1}$. Relative wages are then:

$$rer^w_{in,t} = [1-\theta_w]\, \overline{rer}^w_{in,t} + \theta_w \left[rer^w_{i,t-1} + \Delta e_{in,t}\right],$$

and

$$\overline{rer}^w_{in,t} = \left[1-\beta\theta_p\right] \alpha_w rer^w_{in,t} + \beta\theta_w \mathbb{E}_t \left[\overline{rer}^w_{in,t+1} - \Delta e_{in,t}\right],$$

with $\alpha_w = \frac{\Gamma_p}{1+\Gamma_p}$.

Collecting these results, we list the set of equilibrium conditions that from which we will derive Proposition 1:

$$rer_{in,t} = \left[1-\theta_p\right] \overline{rer}_{in,t} + \theta_p \left[rer_{i,t-1} + \Delta e_{in,t}\right], \tag{B.2}$$

$$rer^w_{in,t} = [1-\theta_w]\, \overline{rer}^w_{in,t} + \theta_w \left[rer^w_{i,t-1} + \Delta e_{in,t}\right], \tag{B.3}$$

$$\overline{rer}_{in,t} = \left[1-\beta\theta_p\right] \left[\iota_p rer^w_{in,t} + \alpha_p rer_{in,t}\right] + \beta\theta_p \mathbb{E}_t \left[\overline{rer}_{in,t+1} - \Delta e_{in,t+1}\right], \tag{B.4}$$

$$\overline{rer}^w_{in,t} = [1-\beta\theta_w] \left[\alpha_w rer^w_{in,t}\right] + \beta\theta_w \mathbb{E}_t \left[\overline{rer}^w_{in,t+1} - \Delta e_{in,t+1}\right], \tag{B.5}$$

$$\mathbb{E}_t \left[\Delta e_{in,t+1}\right] = \rho \Delta e_{in,t}. \tag{B.6}$$

Combining equations B.3, B.5 and B.6 yields the following stochastic in difference equation:

$$rer^w_{in,t} = b^w_1 rer^w_{in,t-1} + b^w_2 \Delta e_{in,t} + b^w_3 \mathbb{E}_t [rer^w_{in,t+1}]$$

where $b^w_1 \equiv \frac{1}{\Upsilon_w}$, $b^w_2 \equiv \frac{1-\beta\rho}{\Upsilon_w}$, $b^w_3 \equiv \frac{\beta}{\Upsilon_w}$ and $\Upsilon_w \equiv 1 + \beta + \frac{[1-\theta_w][1-\beta\theta_w][1-\alpha_w]}{\theta_w}$. We can solve this equation by guessing and verifying the answer. In particular, guessing that $rer^w_{in,t} = Crer^w_{in,t-1} + D\Delta e_{in,t}$, we have that

$$Crer^w_{in,t-1} + D\Delta e_{in,t} = b^w_1 rer^w_{in,t-1} + b^w_2 \Delta e_{in,t} + b^w_3 C^2 rer^w_{in,t-1} + b^w_3 \left[C + \rho\right] D\Delta e_{in,t}.$$

Which implies that

$$C = \frac{\Upsilon_w \pm \sqrt{\Upsilon_w^2 - 4\beta}}{2\beta}.$$

Using the Viete's rule it is easy to see that the two roots over C, (C_1, C_2) satisfy

$$C_1 + C_2 = \frac{\Upsilon_w}{\beta} \quad , \quad C_1 C_2 = \frac{1}{\beta};$$

Now we show that for all $\beta \in (0,1)$, the solution satisfies $0 < C_1 < 1 < C_2$. To see this property notice that using the previous two equations, $f(C) = C + \frac{1/\beta}{C} = 1 + \frac{1}{\beta} + H = \Delta$ where $H = \left[\theta_w^{-1} - 1\right]\left[\theta_w^{-1} - \beta\right]\left[1 - \bar{\alpha}\right]/\beta$. $f(C)$ has an unique minimum at $\beta^{-1/2}$. Since $f(C)$ is decreasing between $(0, \beta^{-1/2})$ and $f(1) \leq \Delta$, there exist a $C_1 < 1$ s.t. $f(C_1) = \Delta$. Since the the function is increasing after the minimum, there exist other root with $C_2 > \beta^{-1/2} > 1$. Thus if we discard the explosive root, we have

$$C = \frac{\Upsilon_w - \sqrt{\Upsilon_w^2 - 4\beta}}{2\beta}$$

and

$$D = \frac{2\left[1 - \beta\rho\right]}{\Upsilon_w + \sqrt{\Upsilon_w^2 - 4\beta} - 2\beta\rho}.$$

Thus the solution for the real exchange rate is given by

$$rer^w_{in,t} = A_{1w} rer^w_{in,t-1} + A_{2w} \Delta e_{in,t}, \tag{B.7}$$

with $A_{1w} \equiv \frac{\Upsilon_w - \sqrt{\Upsilon_w^2 - 4\beta}}{2\beta}$ and $A_{2w} \equiv \frac{2[1-\beta\rho]}{\Upsilon_w + \sqrt{\Upsilon_w^2 - 4\beta} - 2\beta\rho}$.

We proceed in a similar way to characterize the dynamics of the real exchange rate. Combining equations (B.2), (B.4) and (B.6) we obtain the following difference equation:

$$rer_{in,t} = b_0^p rer^w_{in,t} + b_1^p rer_{in,t-1} + b_2^p \Delta e_{in,t} + b_3^p \mathbb{E}_t[rer_{in,t+1}]$$

where $b_0^w \equiv \left[\theta_p^{-1} - 1\right]\left[\theta_p^{-1} - \beta\right]\iota_p/\Upsilon_p$, $b_1^p \equiv \frac{1}{\Upsilon_p}$, $b_2^p \equiv \frac{1-\beta\rho}{\Upsilon_p}$, $b_3^p \equiv \frac{\beta}{\Upsilon_p}$ and $\Upsilon_p \equiv 1 + \beta + \frac{\left[1-\theta_p\right]\left[1-\beta\theta_p\right]\left[1-\alpha_p\right]}{\theta_p}$. Doing a guess and verify as before, we obtain

$$rer_{in,t} = A_{0p} rer^w_{in,t} + A_{1p} rer_{in,t-1} + A_{2p} \Delta e_{in,t}, \tag{B.8}$$

with $A_{0p} \equiv \frac{\left[\theta_p^{-1}-1\right]\left[\theta_p^{-1}-\beta\right]\iota_p}{\lambda_p - \beta\left[A_{1w}+A_{1p}\right]}$, $A_{1p}, \equiv \frac{\Upsilon_p - \sqrt{\Upsilon_p^2 - 4\beta}}{2\beta}$ and $A_{2p} \equiv \frac{2\left[1-\beta\rho(1-A_{0,p}A_{2w})\right]}{\Upsilon_p + \sqrt{\Upsilon_p^2 - 4\beta} - 2\beta\rho}$. Finally, substituting (B.8) into (B.2) we obtain:

Therefore the equilibrium dynamics of the exchange rates are given by

$$\overline{rer}_{in,t} = \frac{A_{0,p}}{1-\theta_p} rer^w_{in,t} + \frac{A_{1,p} - \theta_p}{1-\theta_p} rer_{in,t-1} + \frac{A_{2,p} - \theta_p}{1-\theta_p} \Delta e_{in,t}. \tag{B.9}$$

Equations (B.7), (B.8), and (B.9) give Proposition 1: in the text.

B.2 Impulse Responses

Iterating forward expression (B.7) for relative wages and using the definition of the cumulative impulse response we can write:

$$\begin{aligned}\mathcal{CIR}_{t+j}\left[rer_{in}^{w}\right] &\equiv \frac{\mathbb{E}_t\left[rer_{in,t+j}^{w}\right]}{\Delta e_{in,t}} = A_{2w}\sum_{i=0}^{j} A_{1w}^{i}\rho^{j-i} \\ &= A_{2w}\frac{A_{1w}^{j+1}-\rho^{j+1}}{A_{1w}-\rho}.\end{aligned}$$

Following the same steps using equation (B.8), for the real exchange rate we obtain

$$\begin{aligned}\mathcal{CIR}_{t+j}\left[rer_{in}\right] \equiv \frac{\mathbb{E}_t\left[rer_{in,t+j}\right]}{\Delta e_{in,t}} &= A_{0p}\sum_{i=k}^{j} A_{1p}^{j-k}\mathcal{CIR}_{t+k}\left[rer_{in}^{w}\right] + A_{2p}\sum_{i=0}^{j} A_{1p}^{i}\rho^{j-i} \\ &= A_{0p}\sum_{i=0}^{j} A_{1p}^{j-i}\mathcal{CIR}_{t+i}\left[rer_{in}^{w}\right] + A_{2p}\sum_{i=0}^{j} A_{1p}^{i}\rho^{j-i} \\ &= A_{2p}\frac{A_{1p}^{j+1}-\rho^{j+1}}{A_{1p}-\rho} + \frac{A_{0p}A_{2w}}{A_{1w}-\rho}\left[A_{1w}\frac{A_{1p}^{j+1}-A_{1w}^{j+1}}{A_{1p}-A_{1w}} - \rho\frac{A_{1p}^{j+1}-\rho^{j+1}}{A_{1p}-\rho}\right].\end{aligned}$$

Finally, using (B.9) we obtain:

$$\mathcal{CIR}_{t+j}\left[\overline{rer}_{in}\right] = \frac{A_{0,p}}{1-\theta_p}\mathcal{CIR}_{t+j}\left[rer_{in}^{w}\right] + \frac{A_{1,p}-\theta_p}{1-\theta_p}\mathcal{CIR}_{t+j}\left[rer_{in,t-1}\right] + \frac{A_{2,p}-\theta_p}{1-\theta_p}\rho^{j}$$

B.3 Special cases

Case 1- baseline: In this case, $\theta^w = 0$, then $A_{1w} = A_{2w} = 0$, and $rer_{in,t}^{w} = 0$. In addition, if $\alpha_p = 0$, and $\rho = 0$ then $A_1^p - A_2^p = \theta$. Substituting in (B.8) and (B.9) we get, $rer_{in,t} = \theta\left[rer_{in,t-1} + \Delta e_{in,t}\right]$, and $\overline{rer}_{in,t} = 0$. Note that if $\rho = 0$, then $\mathbb{E}\left[\Delta e_{in,t+1}\right] = 0$ and the persistence of the real exchange rate is θ.

Case 2- persistent shocks: In this case, $\theta^w = 0$, then $A_{1w} = A_{2w} = 0$, and $rer_{in,t}^{w} = 0$. In addition, if $\alpha_p = 0$, then $A_1^p = \theta$ and $A_2^p = \frac{-\theta\beta\rho}{1-\theta\beta\rho}$. The autocorrelation of the real exchange rate is $\mathbb{P} = \frac{\theta_p+\rho}{1+\rho\theta_p}$.

Case 3- real rigidities In this case, $\theta^w = 0$, then $A_{1w} = A_{2w} = 0$, and $rer_{in,t}^{w} = 0$. In addition, if $\rho_p = 0$, then $A_1^p = A_2^p > \theta_p$ and the autocorrelation of the real exchange rate is $\mathbb{P} = A_{1,p}$. Note that Υ_p is increasing in $1-\bar{\alpha}$, so both the persistence and χ_j increase with α.

Case 4- nominal wage rigidities In this case case: $\rho = 0$ and $\alpha_w = \alpha_p = 0$ and $\theta_p = \theta_w = \theta$. In this case it is easy to see that in this case $\theta = A_{1p} = A_{1w} = A_{2p} = A_{2w}$, and that:

$$\frac{\mathbb{V}\left[rer^{w}_{in,t}\right]}{\mathbb{V}\left[\Delta e_{in,t}\right]} = \frac{\theta^2}{1-\theta^2}$$

and that

$$\frac{\mathbb{E}[rer^{w}_{in,t}\Delta e_t]}{\mathbb{V}\left[\Delta e_{in,t}\right]} = \mathbb{P}\left[rer^{w}_{in,t}\right] = \theta.$$

We start by computing $\mathbb{E}\left[rer^{w}_{in,t}, rer_{in,t}\right]$. Note that

$$\begin{aligned}
\mathbb{E}\left[rer^{w}_{in,t}, rer_{in,t}\right] &= A_{0p}\mathbb{V}\left[rer^{w}_{in,t}\right] + A_{1p}A_{1w}\mathbb{E}\left[rer^{w}_{in,t}, rer_{in,t}\right] + A_{1p}A_{2w}\rho\mathbb{E}[rer^{w}_{in,t}\Delta e_t] + A_{2p}\mathbb{E}[rer^{w}_{in,t}\Delta e_t] \\
&= \frac{A_{0p}\mathbb{V}\left[rer^{w}_{in,t}\right] + A_2\mathbb{E}[rer^{w}_{in,t}\Delta e_t]}{1-A_1^2},
\end{aligned}$$

then:

$$\frac{\mathbb{E}\left[rer^{w}_{in,t}, rer_{in,t}\right]}{\mathbb{V}\left[\Delta e_{in,t}\right]} = \frac{A_{0p}\frac{\theta^2}{1-\theta^2} + \theta^2}{1-\theta^2} = \theta^2\frac{A_{0p}+1-\theta^2}{\left[1-\theta^2\right]^2}.$$

Similarly, we can write

$$\mathbb{E}\left[\Delta e_{in,t}, rer_{in,t}\right] = A_{0p}\mathbb{E}[rer^{w}_{in,t}\Delta e_{in,t}] + A_{1p}\rho\mathbb{E}[rer^{w}_{in,t}\Delta e_{in,t}] + A_{2p}\mathbb{V}\left[\Delta e_{in,t}\right],$$

so

$$\frac{\mathbb{E}\left[\Delta e_{in,t}, rer_{in,t}\right]}{\mathbb{V}\left[\Delta e_{in,t}\right]} = A_{0p}A_2 + A_2 = \theta\left[1 + A_{0p}\right].$$

With these results we have the variance and the persistence of the real exchange rate:

$$\begin{aligned}
\mathbb{V}\left[rer_{in,t}\right] &= A_{0p}^2\mathbb{V}\left[rer^{w}_{in,t}\right] + A_{1p}^2\mathbb{V}\left[rer_{in,t}\right] + A_{2p}^2\mathbb{V}\left[\Delta e_{in,t}\right] + ... \\
&\quad ... + 2A_{0p}A_{2p}\mathbb{E}\left[rer^{w}_{in,t}\Delta e_t\right] + 2A_{0p}A_{1p}\mathbb{E}\left[rer_{in,t-1}rer^{w}_{in,t}\right] \\
\frac{\mathbb{V}\left[rer_{in,t}\right]}{\mathbb{V}\left[\Delta e_{in,t}\right]} &= A_{0p}\frac{A_2^2}{\left[1-A_1^2\right]^2} + \frac{A_2^2}{\left[1-A_1^2\right]} + \frac{2A_{0p}A_2^2}{1-A_1^2} + 2A_{0p}A_1\left[A_1\mathbb{E}\left[rer_{in,t}rer^{w}_{in,t}\right] + \rho A_2\mathbb{E}\left[rer_{in,t}\Delta e_t\right]\right] \\
&= \frac{\theta^2}{\left[1-\theta^2\right]^2}\left[1-\theta^2 + A_{0p}\left[3 + 2\theta^2\left[A_{0p} - \theta^2\right]\right]\right],
\end{aligned}$$

and:

$$\begin{aligned}
\mathbb{P}\left[rer_{in,t}\right] &= A_{0p}A_{1w}\frac{\mathbb{E}\left[rer^{w}_{in,t}rer_{in,t}\right]}{\mathbb{V}\left[rer_{in,t}\right]} + A_{1p} + \left[A_{2p} + A_{0p}A_{2w}\right]\rho\frac{\mathbb{E}\left[\Delta e_{in,t}, rer_{in,t}\right]}{\mathbb{V}\left[rer_{in,t}\right]} \\
&= A_0 A_1 \frac{A_{0p} + 1 - A_1^2}{\left[1 - A_1^2 + A_{0p}\left[3 + 2A_1^2\left[A_{0p} - A_1^2\right]\right]\right]} + A_1 = \theta\left[1 + \frac{A_0}{1 + \frac{A_{0p}\left[2+2\theta^2\left[A_{0p}-\theta^2\right]\right]}{1-\theta^2+A_0}}\right]
\end{aligned}$$

Substituting $A_0 = \iota_p\left[1-\theta\right]\left[1-\beta\theta\right]$ we obtain:

$$\mathbb{P}\left[rer_{in,t}\right] \approx \theta\left[1 + \left[2\mu - 1\right]\left[1-\theta\right]\left[1-\beta\theta\right]\right].$$

The impulse response over wages is given by

$$\mathcal{CIR}_{t+j}\left[rer^{w}_{in}\right] \equiv \theta^{j+1},$$

and for the real exchange we have that

$$\begin{aligned}
\mathcal{CIR}_{t+j}\left[rer^{p}_{in}\right] &\equiv \theta^{j+1} + A_{0p}\frac{\theta}{1-\theta}\theta^{j+1}\lim_{x\to 1}\frac{1-x^{j+1}}{1-x} \\
&= \theta^{j+1}\left[1 + A_0\frac{\theta}{1-\theta}\left[j+1\right]\right]
\end{aligned}$$

Therefore, we obtain:

$$\begin{aligned}
\chi_j &= \frac{A_0}{1-\theta}\frac{\theta^{j+1}}{\theta^{j+1}\left[1 + A_0\frac{\theta}{1-\theta}\left[j+1\right]\right]} \\
&= \frac{A_0}{1-\theta+A_0\left[j+1\right]} = \frac{\left[2\mu-1\right]\left[1-\beta\theta\right]}{1+\left[2\mu-1\right]\left[1-\beta\theta\right]\left[j+1\right]},
\end{aligned}$$

which coincides with the expression in the text.

Appendix C Quantitative Models

This section describes and analyzes the quantitative models from Section 5.

C.1 Setup

Chari et al. (2002) The Chari et al. (2002) model is analogous to the model in Section 5, except that household preferences are given by

$$U_n(h) = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\sigma} \left[\kappa C_{n,t}(h)^{\frac{\xi-1}{\xi}} + [1-\kappa] \left[\frac{M_{n,t}(h)}{P_{n,t}} \right]^{\frac{\xi-1}{\xi}} \right]^{\frac{\xi[1-\sigma]}{\xi-1}} - \frac{N_{n,t}(h)^{1+\varphi}}{1+\varphi} \right], \tag{C.1}$$

where C_n, N_n, and $\frac{M_n}{P_n}$ denote consumption, labor and real money balances in the n-country. Note that we are now introducing money explicitly in the utility function. We follow CKM and assume that the only shocks in the model are shocks to the money supply in each country $M_{i,t}$, which we assume follow an$AR(1)$ process in growth rates.

Taylor rule with real shocks This extension uses a Taylor rule to describe the monetary policy as in Steinsson (2008). In this model, we assume that preferences are given by (C.1), but replace the assumption that money supply in each country follows an AR(1) for a Taylor rule. In particular, we follow Steinsson (2008) and assume use the Taylor rule

$$1 + i_t = \left[\frac{\bar{\Pi}}{\beta} \right]^{1-\rho_i} [1 + i_{t-1}]^{\rho_i} \left[\left[\frac{\Pi_t}{\bar{\Pi}} \right]^{\phi_\pi} \left[\frac{C_t}{\bar{C}} \right]^{\phi_c} \right]^{1-\rho_i}.$$

Additionally we change the technology to exhibit productivity shocks in the following form

$$y_{in,t}(\omega) = A_t \bar{a} n_{in,t}(\omega)^{1-\bar{\alpha}} x_{in,t}(\omega)^{\bar{\alpha}},$$

where A_t follows an $AR(1)$ with persistence ρ_A.

Incomplete markets and UIP shocks We now describe how to extend the model to allow for incomplete markets and deviation from UIP, as in Mukhin and Itskhoki (2016). In particular, we maintain the structure of the model described in Section 5, but assume financial markets are incomplete. Household supply differentiated labor services and have monopoly power over their wage. They face Calvo-type constraints on their ability to adjust wages, the probability that a household can adjust its wage in a given period is given by $1-\theta^w$, but they can insure with state contingent securities $A_{n,t}(h)$ at time t. In country n the flow budget constraint is:

$$P_{n,t}C_{n,t} + \frac{B_{n,t+1}}{R_{n,t}} + \frac{B_{ni,t+1}}{E_{in,t}e^{\psi_{n,t}}R_{i,t}} + M_{n,t+1} = B_{n,t} + \frac{B_{ni,t}}{E_{in,t}} + M_{n,t} + A_{n,t}(h) + W_{n,t}(h)L_{n,t}(h) + T_{n,t}. \tag{C.2}$$

The flow budget constraint in country i is:

$$P_{i,t}C_{i,t} + \frac{B_{i,t+1}}{R_{i,t}} + M_{i,t+1} = B_{i,t} + M_{i,t} + A_{i,t}(h) + W_{i,t}(h)L_{i,t}(h) + T_{i,t}. \qquad \text{(C.3)}$$

Following Mukhin and Itskhoki (2016), we assume that the monetary authority follows the Taylor rule given by

$$1 + i_t = \left[\frac{\bar{\Pi}}{\beta}\right]^{1-\rho_i} [1 + i_{t-1}]^{\rho_i} \left[\frac{\Pi_t}{\bar{\Pi}}\right]^{[1-\rho_i]\varphi_p}.$$

C.2 Calibration

We calibrate the quantitative models to monthly data for the UK. All the parameters are calibrated following the standard practice in the literature, with the exception of those that come from the UK microdata. Table A3 list the calibrated parameters. The discount factor, β and the intertemporal and intertemporal elasticities of substitution, σ, γ and ζ are taken from Chari et al. (2002). The share of home products in the CPI are calibrated to match their empirical counterparts for the UK, with we take from the WIOD. The innovation and persistence of the monetary shocks, ψ and ρ, are calibrated to match the stochastic process of the monetary base in UK , while the inflation target is chosen $\bar{\Pi}$ to match average inflation in our sample for these countries. The frequency of price changes, $1-\theta$, is chosen to match the average price duration observed in the microdata.We take all the parameters for the Taylor rule from Steinsson (2008) and Mukhin and Itskhoki (2016), respectively.

C.3 Impulse responses

This section shows the normalized impulse-response functions of the nominal and the real exchange rates. It also shows the faction of the impulse response of the real exchange rate that is accounted for by the impulse response of the reset exchange rate, χ_j. We normalize all the impulse responses by the for the initial change in the nominal exchange rate. Figure 8 shows the impulse response functions for the complete market models and the figure 9 shows the impulse response in the incomplete market environment.

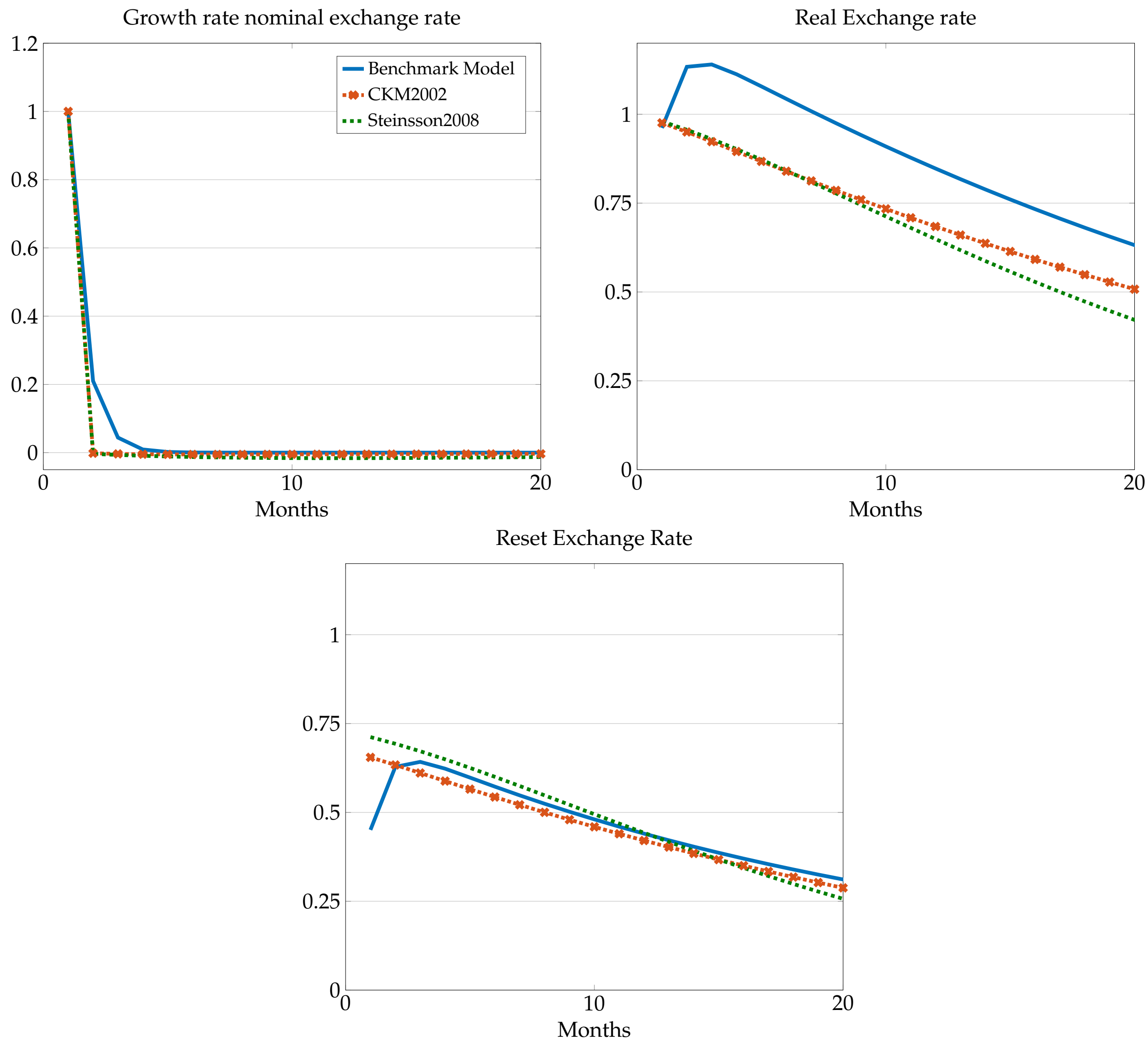
C.4 Relating the quantitative extensions to the benchmark model

This section shows that the business cycle dynamics in the Chari et al. (2002) and Carvalho and Nechio (2011) models can be described by our model under a suitable re-calibration of the paramters. In particular, if we calibrate the persistence of the shocks in our benchmark model to $\rho = 0$, then the model can generate business cycle dynamic for the nominal, real and reset exchange rates that are very close to those in Chari et al. (2002). Table C describes the variance decomposition and the business cycle moments of the nominal real and reset exchange rates in the Chari et al. (2002) and the our benchmark model cali-

Table A3: Parameter values

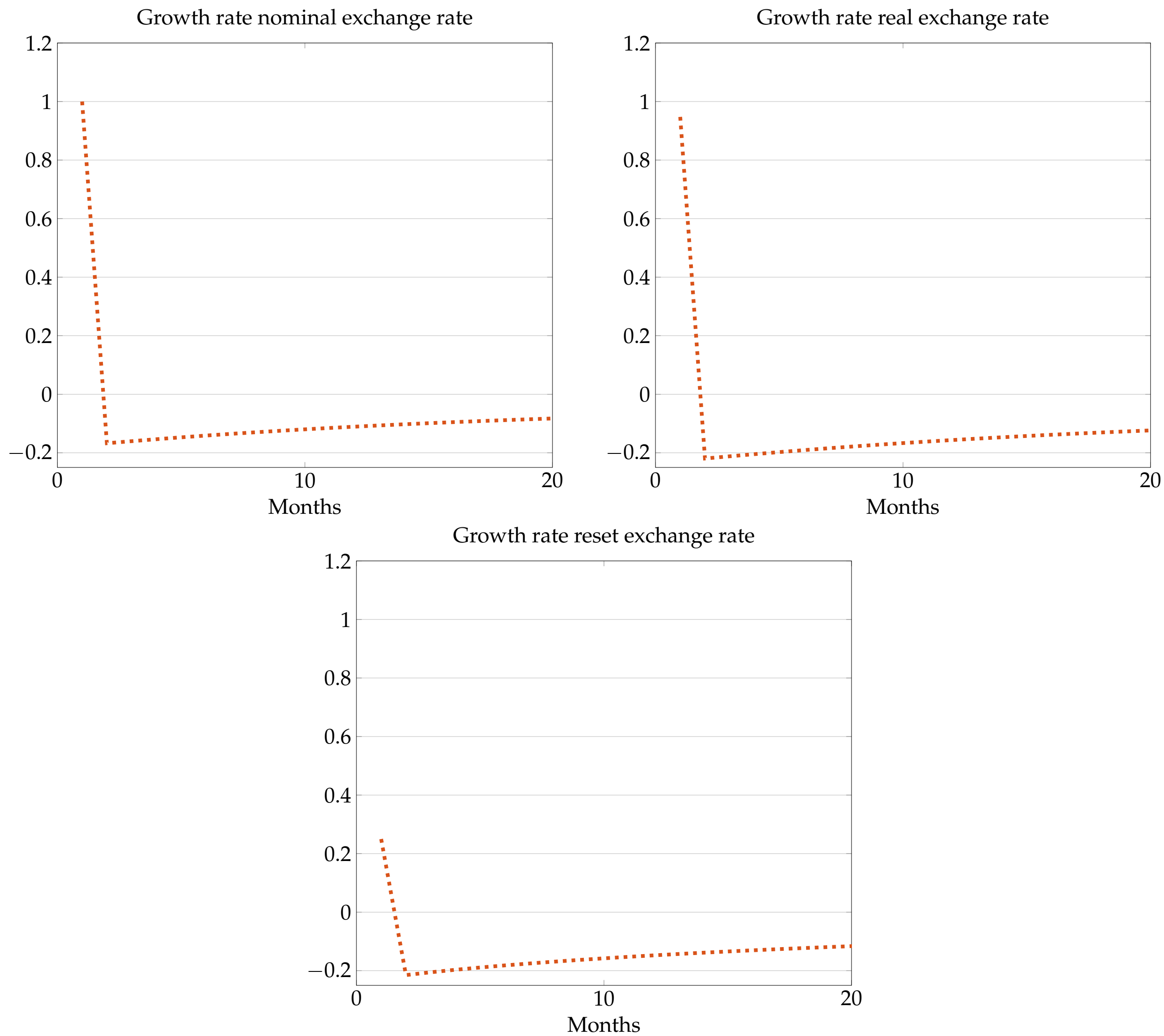
Parameter	Description	Value	Target/source
		Benchmark model	
β	Discount factor	$0.96^{1/12}$	Chari et al. (2002)
γ	Elasticity subst: varieties same country	10	Chari et al. (2002)
μ	Home bias parameter	93	Imports/abs. WIOD
ρ	Persistence of nominal exchange rate	0.54	Persistence M1 growth
$1-\theta_p$	Frequency of price changes	0.07	Average price duration, UK
$1-\theta_w$	Frequency of wage changes	0.90	Negro et al. (2007)
$\frac{1}{\Gamma_p+1}$	Weight in price marginal cost in static optimal price	0.10	Amiti et al. (2014)
$\bar{\alpha}$	Share of intermediates in production	0.5	VA/Output. WIOD
		Additional parameters for Chari et al. (2002)	
γ_p	Elasticity subst: varieties same country (prices)	10	Chari et al. (2002)
γ_w	Elasticity subst: varieties same country (wages)	7	Negro et al. (2007)
σ	Inter temporal elasticity of substitution	5	Chari et al. (2002)
κ	weight on consumption vs real balance	0.94	Chari et al. (2002)
ξ	Elasticity of subst.: consumption-real balances	0.34	Chari et al. (2002)
$\rho_{\Delta M}$	Persistence of monetary shocks	0.54	Persistence money growth, UK
φ	Elasticity of leisure	1	Frisch Elasticity
		Additional parameters for Steinsson (2008)	
ρ_i	Interest smoothing coefficient	0.95	Steinsson (2008)
ϕ_π	Inflation coefficient	2	Steinsson (2008)
ϕ_c	Consumption coefficient	0.5	Steinsson (2008)
$\frac{\sigma_{monetary}}{\sigma_{real}}$	Ratio between nominal and real shocks	300	Persistence nominal exchange rate, UK
$\rho_{nominal}$	Persistence innovation Taylor rule	0.94	Steinsson (2008)
ρ_{real}	Persistence of government expenditure shock	0.96	Steinsson (2008)
		Additional parameters for incomplete market model	
ρ_i	Interest smoothing coefficient	0.95	Mukhin and Itskhoki (2016)
ϕ_π	Inflation coefficient	2.15	Mukhin and Itskhoki (2016)
ρ_ψ	Persistence UIP shocks	0.99	Mukhin and Itskhoki (2016)

Figure 8: Impulse Responses in Quantitative Complete Markets Models



Note: The left panel plots the impulse response of the growth rate of the nominal exchange rate, the center panel plots the impulse response of the real exchange rate and the right panel plots the impulse response of the reset exchange rate.

Figure 9: Impulse Responses in Quantitative Incomplete Markets Model



Note: The left panel plots the impulse response of the growth rate of the nominal exchange rate, the center panel plots the impulse response of the growth rate of the real exchange rate and the right panel plots the impulse response of the growth rate of the reset exchange rate.

brated with $\rho = 0$ (Columns 2 and 3). As we can see, both models generate business cycle dynamics that are quantitative very similar.

We now briefly describe a version of the model that intends to capture the mechanisms in Carvalho and Nechio (2011). In particular, we extend the model to allow for multiple sectors that differ in the degree of price stickiness. We consider an economy where final goods producers aggregate the output of j different sectors according to

$$1 = \sum_j k_j g_1 \left[\frac{Y^j_{i,t}}{k_j Y_{i,t}} \right].$$

Here, $Y^j_{i,t}$ is a bundle of goods from sector j in country i, g_1 is a Kimball aggregator as in the main text, and the parameter k_j controls the share of sector j in total the final good. Sectorial output is produced by aggregating first across home and foreign goods and then by aggregating a continuum of differentiated products according to:

$$1 = \mu g \left[\frac{Y^j_{ii,t}}{\mu Y^j_{i,t}} \right] + (1-\mu) g \left[\frac{Y^j_{ni,t}}{(1-\mu) Y^j_{i,t}} \right],$$
$$1 = \int h_p \left(\frac{Y^j_{hi,t}(\omega)}{Y_{ji,t}} \right) d\omega.$$

Here $Y^j_{hn,t}(\omega)$ denotes the quantity of intermediate good ω in sector j produced in country j and consumed in country i, μ determines the share of imports in absorption in the symmetric steady state. The share of imports are symmetric across sector, as the Kimball's aggregators. The function g satisfies $g(1) = 1$, $g'(.) > 0$, and $g''(.) < 0$ with $\zeta = -\frac{\partial \log(g'^{-1}(x))}{\partial x}|_{x=1}$ as the demand elasticity between home and foreign goods.

We compare this model to a version of our benchmark model calibrated so that the average duration of prices to be the same in the two models. In particular, in our benchmark model, we the price stickiness parameter to $\theta^*_p = \left(\sum_j \theta^{-1}_{p,j} \right)^{-1}$, where θ^*_p is the frequency of price changes in the 1-sector model and $\{\theta_{p,j}\}^N_{j=1}$ are the sectorial frequency of price changes. Columns 4 and 5 in Table C show that the business cycles moments of the benchmark model and Carvalho and Nechio (2011) models are quantitatively almost identical.

Table A4: Business Cycle Moments and Variance Decomposition Across Quantitative Models

Moments	Benchmark	CKM2002	Benchmark	CN2011
Standard deviations rel. to real exchange rate (growth rates)				
Nominal	1.02	1.01	1.02	1.02
Reset	0.58	0.67	0.58	0.65
Difference	0.42	0.33	0.42	0.35
Correlations with real exchange rate (growth rates)				
Nominal	1.00	1.00	1.00	0.99
Reset	1.00	1.00	1.00	1.00
Difference	1.00	1.00	1.00	1.00
Standard deviations rel. to real exchange rate (levels)				
Reset	0.54	0.65	0.54	0.63
Difference	0.46	0.36	0.46	0.38
Correlations with real exchange rate (levels)				
Reset	1.00	0.99	1.00	0.99
Difference	1.00	0.98	1.00	0.98
Autocorrelations (levels)				
Real	0.94	0.94	0.94	0.93
Reset	0.93	0.94	0.93	0.93
Difference	0.95	0.95	0.95	0.94
Variance Decomposition (growth rates)				
1	0.58	0.67	0.58	0.65
3	0.58	0.67	0.58	0.65
6	0.57	0.67	0.57	0.65
9	0.57	0.67	0.57	0.65
12	0.56	0.67	0.56	0.65
Variance decomposition with respect to structural shocks				
To innovation in ϵ^e shocks				
real	100.00	96.32	100.00	97.37
reset	100.00	83.17	100.00	89.37
growth rate nominal	100.00	100.00	100.00	100.00
To innovation in ϵ^{mc} shocks				
real	100.00	96.32	100.00	97.37
reset	100.00	83.17	100.00	89.37
growth rate nominal	100.00	100.00	100.00	100.00
To innovation in ϵ^{uip} shocks				
real	100.00	96.32	100.00	97.37
reset	100.00	83.17	100.00	89.37
growth rate nominal	100.00	100.00	100.00	100.00

Notes: The table presents business cycle moments from the models, together with the variance decomposition of the nominal, real and reset exchange rates.