

Bank Runs, Monetary Policy, and (In)efficiency

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I study monetary policy and efficiency in a monetary model in which agents are subject to liquidity risk (i.e., preference shocks) and banks are subject to fundamental-based runs. I characterize the optimal deposit contract and study the conditions under which the possibility of runs generates a flight to liquidity (i.e., an increase in depositors' money holdings and a reduction in deposits). Temporary monetary injections are, in general, non-neutral: nominal prices increase less than one-for-one, money velocity drops, deposits at banks might increase or decrease, and welfare increases. Quantitatively, the non-neutrality of temporary injections depends on depositors' elasticity of money demand and elasticity of intertemporal substitution. The equilibrium is generically constrained-inefficient.

1 Introduction

The bankruptcy of Lehman Brothers in September 2008 was followed by a flight to safe and liquid assets as well as runs on several financial institutions. For instance, [Duygan-Bump et al. \(2013\)](#) and [Schmidt, Timmermann, and Wermers \(2016\)](#) document a \$400 billion run on money market mutual funds. In response to these events, the Federal Reserve implemented massive monetary interventions. Flight to liquidity, runs, and monetary interventions characterized the Great Depression as well, although the response of the Federal Reserve was more muted at the time, and the US economy experienced a large deflation ([Friedman and Schwartz, 1963](#)).

Despite the interactions between bank runs, flight to liquidity, and monetary policy interventions, very few models analyze the interconnections among these phenomena. Most of the literature on banking crises assumes that banks operate in environments with only one real good, without fiat money. While this approach is useful for many purposes, in practice banks take and repay deposits using money, giving rise to non-negligible interactions with monetary policy choices.¹

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¹A few other papers deal with this observation. I review this literature in the next section.

To fill this gap, I present a general equilibrium model of fundamentals-based bank runs with money. I make some stark assumptions to keep the model simple and tractable, and to convey the intuition behind the results in the simplest possible way. In particular, output is exogenous, there are no aggregate shocks, and prices are fully flexible. In this way, my results can easily be compared with those obtained in classical monetary models such as [Lucas and Stokey \(1987\)](#).

There are two main sets of results in the paper. First, I characterize the effects of monetary injections on prices, allocations, and welfare. In some cases, monetary injections are non-neutral and affect real allocations (i.e., consumption) and financial allocations (i.e., deposits and money holdings), even though prices are fully flexible. Second, I study the efficiency of the decentralized equilibrium for a given monetary policy stance. Despite the fact that banks compete to offer the optimal contract, the equilibrium is generically constrained-inefficient because of a pecuniary externality related to the role of money.

I address these questions in a three-period economy ($t = 0, 1, 2$) in which households are subject to preference shocks, in the spirit of [Diamond and Dybvig \(1983\)](#) and [Atkeson and Lucas \(1992\)](#), and need money to purchase consumption goods in a centralized market, similar to [Lucas and Stokey \(1987\)](#). In the language of [Lucas and Stokey \(1987\)](#), households are subject to a cash-in-advance constraint, which can be motivated by frictions that prevent the use of credit or of interest-bearing assets as means of payment.

I focus on three scenarios: an economy with strong fundamentals in which banks are well-functioning and there are no runs; an economy with weak fundamentals and runs on some banks; and a bankless economy in which banks do not operate at all. The bankless economy delivers similar results to those of the economy with runs, but the analysis is much simpler and thus the logic of such results can be conveyed in a more straightforward way.

The primary novel findings about monetary interventions are related to injections that are temporary (i.e., that are reverted when the crisis is over). The effects of temporary monetary injections depend on the condition of the banking system. If the banking sector is well-functioning and there are no runs, temporary monetary injections are neutral, similar to what would happen in a standard monetary model. In contrast, if banks are subject to runs or if the economy is bankless, temporary monetary injections are non-neutral: nominal prices move less than one-for-one with the monetary injections, money velocity drops, welfare increases, and, if banks are active, deposits at banks might either increase or decrease. I also study permanent injections (i.e., one-time permanent increases of the money supply), but these interventions are always neutral.

The second main result is related to the (in)efficiency of the decentralized equilibrium. If banks are well-functioning and there are no runs, the equilibrium achieves the first best. If instead there are runs or if the economy is bankless, the equilibrium is generically constrained-inefficient because of a pecuniary externality. The equilibrium is characterized by a cross-sectional hetero-

geneity in the Lagrange multipliers of the cash-in-advance constraints of ex-ante identical agents. Since the price level enters such a constraint, an economy-wide planner can reduce the heterogeneity by forcing households and banks to take actions that reduce the price level, which in turn allows cash-constrained households to increase their consumption. This inefficiency arises even if banks offer the “optimal” contract. This is because each bank is small and takes prices as given, but the externality acts through prices. This result implies that a crisis in my model is not “optimal” even if runs are fundamental-based, in contrast to the results of the non-monetary model of [Allen and Gale \(1998\)](#).

To convey intuitions in the simplest way, I begin by deriving most of the results in a bankless economy. All households are ex-ante identical and thus enter $t = 1$ with the same amount of money. However, because of preference shocks, some of them are impatient and have high marginal utility of consumption, while others are patient and have low marginal utility of consumption. As a result, impatient households would like to spend more money than they have, and their cash-in-advance constraints are binding. Patient households, on the other hand, spend only a fraction of their money, and their constraints are not binding. The fact that the cash-in-advance constraint binds only for a subset of agents produces a wedge that distorts the allocation of consumption, and prevents the equalization of the marginal utility across ex-ante identical agents.² In addition, the standard predictions of the quantity theory that arise in classic monetary models break down. Some households spend only a fraction of the money they have and store the rest, which produces a drop in nominal prices and velocity with respect to an economy with a well-functioning banking system, and is responsible for the non-neutral effects of temporary monetary injections.

The additional money injected with a temporary intervention allows cash-constrained households to increase their consumption. The behavior of unconstrained households, however, is different. If prices were fixed, unconstrained households would simply store the additional money. In equilibrium, however, prices increase but less than one-for-one with the monetary injection, and thus unconstrained households spend some of the additional money and store the rest.

Households’ money-demand elasticity governs the quantitative impact of a temporary injection on prices and velocity. More precisely, I am referring here to the elasticity of real money demand with respect to its opportunity cost (i.e., with respect to the nominal interest rate). As nominal prices move less than one-for-one, a monetary injection increases the supply of money in real terms, and the opportunity cost of money might need to adjust to clear the money market. The extent to which the opportunity cost adjusts depends on households’ money demand elasticity, and it can be characterized analytically in three special cases.

²Throughout the paper, I assume that the central bank does not run the Friedman rule. If instead the central bank runs the Friedman rule by implementing zero nominal interest rates, the equilibrium allocations always corresponds to the first best.

First, if money demand is very elastic, households are willing to absorb any increase in real money holdings even if there is no change in the opportunity cost. This case arises if the elasticity goes to infinity (i.e., money demand is flat), which in turn requires households' intertemporal elasticity of substitution to be infinite (i.e., linear preferences). A temporary injection does not change nominal prices, and the increase in real money supply due to the injection is maximal. In addition, velocity drops one-for-one with the monetary injection because the constant prices do not require households to increase consumption expenditure, so that the extra money is simply stored. Another way to understand this result is to note that if nominal prices change and trigger any movement in the opportunity cost of holding money, the amount of money demanded by the households will change dramatically because of the high elasticity, and the money market will not clear.

Second, in the opposite scenario in which money demand is very inelastic (i.e., money demand is almost vertical), the price level moves one-for-one with the monetary injection. This result implies that the real money supply is unchanged despite the increase in nominal money, and thus the money market clears. The change in the price level significantly alters the opportunity cost of holding money, but this is consistent with clearing in the money market. Because of the low elasticity, even large changes in the opportunity cost do not substantially alter the amount of money demanded by households. In the limit in which money demand is completely inelastic, which arises if households' intertemporal elasticity of substitution converges to zero, a temporary monetary injection is neutral.

Third, I characterize the results for the intermediate value of the elasticity of money demand that arises with log utility. Not surprisingly, the results are an intermediate case between those of infinite and zero elasticity.

The pecuniary externality arises because the price of consumption goods enters the binding cash-in-advance constraint of impatient households. If an economy-wide planner forced patient households to marginally reduce their consumption expenditure, the price level would drop and impatient households would be able to consume more. Welfare would increase because impatient households would face a first-order gain, whereas patient households would suffer only a second-order loss. A key condition to obtain the pecuniary externality is that households purchase goods in a competitive market, and thus the price level enters the cash-in-advance constraint. If instead households met bilaterally and bargained over the terms of trade, as in [Lagos and Wright \(2005\)](#), forcing cash-unconstrained households to cut consumption would not produce any benefits for cash-constrained households. Another important assumption is that output is exogenous. In a richer model, an additional trade-off might arise if forcing households to cut consumption expenditure reduced output. Nonetheless, the force that gives rise to the pecuniary externality is likely to be unaltered even in richer frameworks. By using a deliberately simple model, I am able to abstract

from other results and convey in a stark and simple way the logic of the pecuniary externality.

I then introduce banks that are subject to idiosyncratic shocks to their assets. Some banks are hit by bad shocks that destroy their investments and are thus subject to runs at $t = 1$. In line with [Allen and Gale \(1998\)](#) and [Keister and Narasiman \(2015\)](#), I refer to this scenario as a fundamental-based run. Households with deposits at banks subject to runs are essentially in the same situation as in the bankless economy; that is, they all have the same amount of money, even though some of them are impatient and thus have higher need to consume. By contrast, banks not hit by bad shocks can implement a standard deposit contract; that is, they distribute more money to impatient households at $t = 1$ more resources to patient households at $t = 2$. The possibility of runs increases households' money demand and reduces deposits at banks if the risk aversion of households is sufficiently high. When that is the case, I refer to this outcome as a “flight to liquidity.”

Temporary injections in the economy with bank runs produces the same results as in the bankless economy, but they also affect money held by households and deposits. To explain these results, it is useful to first study the benchmark case in which there are no runs. In this case, temporary injections reduce the opportunity cost of holding money, but their impact is neutral: prices, deposits, and money held by households move one-for-one with the injection. With no runs, households minimize their money holdings to economize on the opportunity cost of holding money, and a change in the opportunity cost does not alter households' decisions to hold the minimum amount of money in their wallets. If instead there are runs, the results depend on how close the equilibrium is to the economy with no runs in terms of money spent. If a large fraction of households spend all their money, deposits increase too. If instead only a few households spend all their money, deposits drop. To convey more intuition for the second case, consider the effect of injecting the extra money while keeping prices fixed. Most households have a slack cash-in-advance constraint, and thus are able to maintain the same consumption by holding more money—which is feasible because of the monetary injection—and cutting deposits. Allowing prices to change does not alter the results.

Finally, the logic of the pecuniary externality in the economy with banks is identical to that of the bankless economy. However, households differ not only in their preference shocks but also in whether or not they face a run on their own bank, and these multiple layers of heterogeneity create a more complicated cross-sectional dispersion in the Lagrange multipliers of the cash-in-advance constraints.

1.1 Additional comparisons with the literature

A few other papers analyze monetary injections in the context of bank runs. However, these papers differ from mine in important ways.

A first set of papers analyzes monetary injections in the context of bank runs driven by fundamentals. [Allen and Gale \(1998\)](#), [Allen, Carletti, and Gale \(2013\)](#), and [Diamond and Rajan \(2006\)](#) study how monetary policy should respond to aggregate shocks when deposit contracts are nominal and not contingent on the price level. However, crises in these models do not produce flight to liquidity in anticipation of runs or deflation, and small monetary injections may actually generate inflation. As a result, the main focus of these papers is on other aspects of banking crises.³ [Antinolfi, Huybens, and Keister \(2001\)](#) and [Rochet and Vives \(2004\)](#) study central bank lending in response to aggregate shocks in models that likewise do not produce any flight to liquidity.

A second set of papers present models in which monetary injections can eliminate bank runs driven by panics, in the sense of multiple equilibria. [Carapella \(2012\)](#), [Cooper and Corbae \(2002\)](#), and [Robatto \(2018\)](#) use general equilibrium models, whereas [Martin \(2006\)](#) analyzes a Diamond-Dybvig partial-equilibrium economy with money. I comment further on the two closest papers, [Cooper and Corbae \(2002\)](#), and [Robatto \(2018\)](#). In [Cooper and Corbae \(2002\)](#), depositors choose to hold some money in their wallets during crises, as in my model. However, they focus solely on steady states in which banks are either perpetually well-functioning or malfunctioning, and thus they consider only permanent injections. By contrast, my simpler model allows me to study a scenario in which crises eventually end, and to distinguish between temporary and permanent injections. In the infinite-horizon model of [Robatto \(2018\)](#), monetary injections might also reduce deposits; however, the richness of that model—required to study multiple equilibria in an infinite-horizon economy—imposes limitations on the analysis.⁴

The drop in money velocity that I obtain is also related to [Alvarez, Atkeson, and Edmond \(2009\)](#), who show that monetary injections reduce velocity in a monetary model with segmented-asset markets, abstracting from financial crises. However, the drop in velocity in my model is related to the temporary nature of a monetary injection, whereas [Alvarez, Atkeson, and Edmond \(2009\)](#) show that segmented asset markets are responsible for a drop in velocity when a monetary injection is permanent.

³[Allen and Gale \(1998\)](#) and [Allen, Carletti, and Gale \(2013\)](#) emphasize that nominal deposit contracts allow the economy to achieve the first best in response to aggregate shocks under an appropriate monetary intervention. In contrast, in my model, there are no aggregate shocks and the denomination of deposits does not play any role. [Diamond and Rajan \(2006\)](#) emphasize the comparison between deposits denominated in foreign versus domestic currency. They also sketch an extension of their model in which runs are associated with deflation; however, they do not analyze monetary injections in the extended model with deflation.

⁴In addition, the focus of [Robatto \(2018\)](#) is on the monetary policy stance that eliminates multiple equilibria, similar to the main research question in [Carapella \(2012\)](#) and [Cooper and Corbae \(2002\)](#).

2 Model: the core environment

This section presents the core environment without banks, and [Section 3](#) derives the equilibrium. I then study the main effects of monetary injections on prices and allocations in [Sections 3.3-3.5](#) and the efficiency of the equilibrium in [Section 3.6](#). Finally, I extend the model to introduce banks and study monetary injections and efficiency with banks in [Section 4](#).

Time is discrete with three periods indexed by $t \in \{0, 1, 2\}$. The economy is populated by a double continuum of households indexed by $h \in \mathbb{H} = [0, 1] \times [0, 1]$; the double continuum is required when introducing banks in [Section 4](#).

The core environment combines preference shocks at $t = 1$, in the spirit of [Diamond and Dybvig \(1983\)](#), with a Lucas-tree cash-in-advance economy, similar to [Lucas and Stokey \(1987\)](#). Money is required to finance consumption expenditure at $t = 1$, after agents are hit by preference shocks. As a result, a precautionary demand for money arises at $t = 0$, so that households can finance consumption induced by preference shocks at $t = 1$.⁵ More formally, the monetary theory literature has identified some key frictions that give rise to the need of money, such as limited commitment and anonymity, which I am implicitly assuming ([Lagos et al., 2017](#)). In addition, other financial instruments, such as claims on the Lucas tree, cannot be used for transactions, which in turn has been motivated by other frictions such as the inability to recognize a counterfeited claim ([Lagos, 2010](#); [Lester et al., 2012](#)). In terms of the trading protocol, consumption goods are purchased in a centralized market in which pricing is competitive, similar to [Rocheteau and Wright \(2005\)](#).

To provide a terminal value for money in this three-period model, I introduce a fiscal authority that imposes taxes and uses the proceeds to repurchase the outstanding money supply at the end of $t = 2$. One way to understand this intervention is to assume that taxes must be paid with money. Following the literature on the fiscal theory of the price level ([Cochrane, 2005](#); [Woodford, 1995](#)), this approach pins down a unique value for the outstanding government liabilities, which in this model are represented by the money supply. [Diamond and Rajan \(2006\)](#) use the same approach to provide a terminal value of money in their three-period model of banking.

2.1 Preferences

Let C_1^h and C_2^h denote consumption of household h at $t = 1$ and $t = 2$, respectively. Households' utility depends on the preference shock ε^h that is realized at the beginning of $t = 1$:

$$\mathbb{E}_\varepsilon \left\{ \varepsilon_1^h u(C_1^h) + C_2^h \right\} \quad (1)$$

⁵The core environment builds on the model in [Robatto \(2018\)](#).

where

$$\varepsilon_1^h = \begin{cases} \varepsilon^H & \text{(impatient household) with probability } \kappa. \\ \varepsilon^L & \text{(patient household) with probability } 1 - \kappa \end{cases}$$

and $\varepsilon^H > 1 > \varepsilon^L > 0$ to capture the patience of the second group of households. I impose the normalization:

$$\mathbb{E}(\varepsilon_1^h) = 1.$$

The preference shock is i.i.d. across households, and I assume that the law of large numbers holds, so that the fraction of impatient agents in the economy equals κ . Moreover, I assume that the law of large numbers also holds for each subset of $\mathbb{H}$ with a continuum of households.⁶ The preference shock is private information of household h .

The utility of consumption at $t = 1$ is

$$u(C) = \frac{C^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}}, \quad (2)$$

where $\sigma \in (0, \infty)$ parametrizes the intertemporal elasticity of substitution.

The intertemporal elasticity σ plays a key role. In most of the analysis, I focus on three limiting cases: $\sigma \rightarrow \{0, 1, \infty\}$. The cases $\sigma \rightarrow \infty$ and $\sigma \rightarrow 0$ represent the extreme cases in which temporary monetary injections have only real or only nominal effects, respectively. The case $\sigma \rightarrow 1$ delivers—not surprisingly—intermediate results, but is useful because it is the only intermediate case in which the equilibrium can be easily characterized in closed form.

To clarify the role of σ , consider first the extreme case in which the elasticity of substitution converges to infinity (i.e., $\sigma \rightarrow \infty$). In this case, households' utility converges to

$$\varepsilon_1^h u(C_1) + C_2 \rightarrow \varepsilon_1^h C_1^h + C_2^h$$

and thus households have linear utility.⁷

At the other extreme case, in which the elasticity of substitution converges to zero (i.e., $\sigma \rightarrow 0$), households' utility converges to

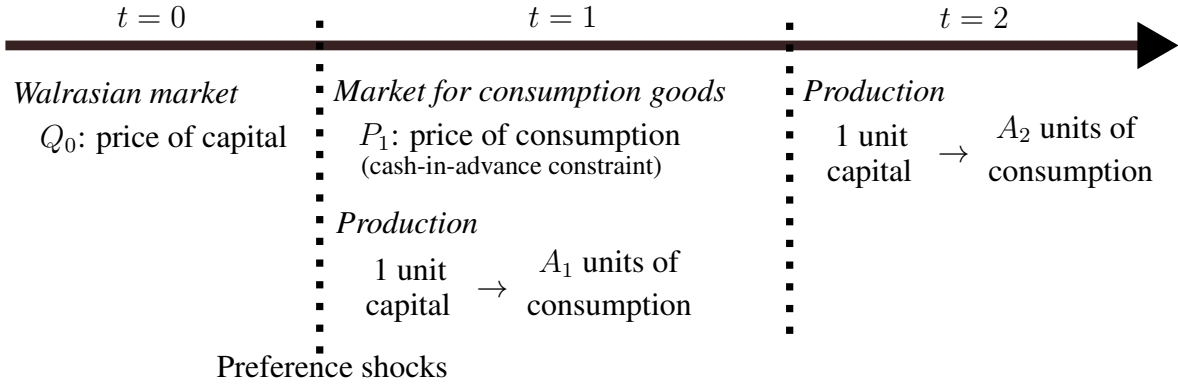
$$\varepsilon_1^h u(C_1) + C_2 \rightarrow \begin{cases} -\infty & \text{if } C_1 < 1 \\ C_2 & \text{if } C_1 \geq 1. \end{cases}$$

I will refer to this case as a *quasi-Leontief* utility function because, in this case, one can show that

⁶This is consistent with the results of [Al-Najjar \(2004\)](#) about the law of large numbers in large economies.

⁷More precisely, the utility function converges to $\varepsilon_1^h C_1 + C_2 - \varepsilon_1^h$ but the term $-\varepsilon_1^h$ can be ignored because it does not affect households' choices.

Figure 1: Timing of production and markets



households' indifference curves are L-shaped and thus similar to those of a Leontief function. With a zero elasticity of substitution, households would be willing to borrow or save at any interest rate to achieve $C_1 = 1$ and to leave any further resources for time-2 consumption. Note that the quasi-Leontief utility is independent of the preference shock ε_1^h . Thus, the role of preference shocks vanishes in the limit as $\sigma \rightarrow 0$.

2.2 Assets, endowments, and markets

There are two assets with exogenous supply: money and capital. At $t = 0$, households are equally endowed with $\overline{M}$ units of fiat money and $\overline{K}$ units of capital. Throughout the paper, money is the numeraire, and contracts are expressed in terms of money, without loss of generality.⁸ The central bank can change the supply of money at $t = 0, 1, 2$, as described in [Section 2.4](#), and the supply of money is denoted by M_t^S . The supply of capital is instead fixed at $\overline{K}$ for all $t = 0, 1, 2$.⁹ Capital produces output twice, that is, both at $t = 1$ and $t = 2$.

Next, I describe trading and production. The timing is represented in [Figure 1](#). At $t = 0$, there is a Walrasian market in which capital and money can be traded. The price of capital is denoted by Q_0 . At $t = 1$, after preference shocks are realized, each unit of capital produces A_1 units of consumption goods. There is a centralized market in which these consumption goods can be sold at price P_1 . Consumption expenditure must be financed with money, and thus households are subject to a cash-in-advance constraint; as in [Lucas and Stokey \(1987\)](#), households cannot consume goods produced by their own stock of capital. In addition, and without loss of generality, capital cannot

⁸In contrast to [Allen, Carletti, and Gale \(2013\)](#), the denomination of contracts is irrelevant in my model because there are no aggregate shocks; that is, the results are unchanged if deposits were contingent on prices.

⁹The results would remain unchanged if I were to endow households with goods rather than a fixed supply of capital. In this case, the entire endowment would be invested anyway because there is no consumption at $t = 0$. That is, I follow the approach of monetary models similar to [Lucas and Stokey \(1987\)](#), in which portfolio decisions involve holding money or physical capital, whereas perishable goods cannot be stored.

be traded at $t = 1$.¹⁰ I normalize output produced at $t = 1$ to

$$A_1 \bar{K} = 1. \quad (3)$$

Finally, at $t = 2$, each unit of capital produces A_2 units of consumption goods.

2.3 First-best allocation

This section derives the first best, which is then used as a benchmark against the decentralized equilibrium. The planner has the ability to observe the realization of households' preference shocks and can distribute consumption goods without being subject to the frictions that give rise to a role for money.

The planner's problem is to choose consumption of each agent at $t = 1$ and $t = 2$ subject to the feasibility and non-negativity constraints. At the optimum, the consumption at $t = 1$ is equalized among impatient households (denoted by C_1^H), and similarly for patient households (denoted by C_1^L). Thus, the planner problem can be written:

$$\max_{C_1^H, C_1^L, \{C_2^h\}_{h \in \mathbb{H}}} \kappa \varepsilon^H u(C_1^H) + (1 - \kappa) \varepsilon^L u(C_1^L) + \int C_2^h dh$$

subject to

$$\kappa C_1^H + (1 - \kappa) C_1^L \leq 1 \quad (4)$$

$$\int C_2^h dh \leq A_2 \bar{K}. \quad (5)$$

At $t = 1$, the resources available to the planner are the goods $A_1 \bar{K}$ produced by the fixed supply of capital, and $A_1 \bar{K} = 1$ because of the normalization in [Equation \(3\)](#). At $t = 2$, the resources include the goods produced both by the capital and the money. The first-order condition for the optimal allocation of time-1 consumption goods is:

$$\varepsilon^H u'(C_1^H) = \varepsilon^L u'(C_1^L). \quad (6)$$

That is, the planner equalizes the marginal utility of impatient households to that of patient households. Combining the first-order condition (6) to the resource constraint (4), I can solve for the first-best allocation at $t = 1$. At $t = 2$, any allocation $\{C_2^h\}_{h \in \mathbb{H}}$ that uses all the resources available (i.e., such that (5) holds with equality) is optimal.

¹⁰If capital could be traded at $t = 1$, the results would be unchanged provided that the proceeds from selling capital at $t = 1$ cannot be used to finance consumption expenditure at $t = 1$ and are available at $t = 2$. If instead proceeds from selling capital could be used to finance consumption at $t = 1$, there would be no role for banks, similar to [Jacklin \(1987\)](#).

The next proposition formalizes the first-best allocation at $t = 1$ for the three cases $\sigma \rightarrow \{0, 1, \infty\}$.

Proposition 2.1. *The first-best allocation at $t = 1$ converges to*

$$\{(C_1^H)^*, (C_1^L)^*\} \rightarrow \begin{cases} \{1, 1\} & \text{as } \sigma \rightarrow 0 \\ \{\varepsilon^H, \varepsilon^L\} & \text{as } \sigma \rightarrow 1 \\ \{\frac{1}{\kappa}, 0\} & \text{as } \sigma \rightarrow +\infty. \end{cases}$$

The intertemporal elasticity σ affects the dispersion of first-best consumption at $t = 1$ between impatient and patient households. That is, the ratio $(C_1^H)^* / (C_1^L)^*$ is maximal as $\sigma \rightarrow \infty$ and is minimized at one as $\sigma \rightarrow 0$. This result arises because σ interacts with the effects of the preference shocks. While for all $\sigma \in (0, \infty]$ the planner wants to allocate more time-1 consumption to impatient households rather than impatient households, the difference between $C_1^*(\varepsilon^H)$ and $C_1^*(\varepsilon^L)$ disappears in the limit as $\sigma \rightarrow 0$. In that case, the planner wants to allocate the same amount of consumption to all households, independently of the preference shocks. In other words, the effects of the preference shocks vanishes as $\sigma \rightarrow 0$. At this limit, it is as if such shocks were shut down. This point is relevant because the analysis of the decentralized equilibrium will show that, as $\sigma \rightarrow 0$, all the results converge to those of standard models with no preference shocks.

2.4 Government policy: central bank and price-level determination

The government is composed of a central bank and a fiscal authority that act separately. The role of the fiscal authority is simply that of providing a terminal value of money in this three-period model. That is, the fiscal authority is committed to purchasing the entire money supply M_2^S at the end of period $t = 2$, and it finances this operation by imposing lump-sum taxes on households. This assumption implies that the nominal price level at $t = 2$ is determined by fiscal considerations, as in the fiscal theory of the price level; see [Cochrane \(2005\)](#) and [Woodford \(1995\)](#). Throughout the analysis, I keep fixed that fiscal policy that determines the price level, and I focus on policies in which the central bank changes the money supply M_t^S at $t = 0, 1, 2$.

2.4.1 Central bank and money supply

If there is no central bank intervention, the money supply is constant at $M_t^S = \bar{M}$ for all $t = 0, 1, 2$. If there is a policy intervention, the central bank changes the money supply by varying M_t^S . Interventions are announced at $t = 0$, before the Walrasian market opens, and the central bank fully commits to the policy announcement. Changes to the money supply are implemented with lump-sum taxes and transfers. However, in this bankless economy, the results are unchanged if the

central bank implements the monetary injections using asset purchases (i.e., purchasing physical capital) or providing loans to households to be repaid at $t = 2$, but the notation required to describe these policies would be more complicated.

If the money supply at $t = 0$ is $M_0^S > \bar{M}$, the central bank is injecting $M_0^S - \bar{M}$ units of money at $t = 0$ because the initial endowment of money is $\bar{M}$. Denoting T_0^{CB} to be the lump-sum transfer from the central bank to households at $t = 0$, we have:

$$T_0^{CB} = M_0^S - \bar{M}. \quad (7)$$

At $t = 1$, I restrict attention to the case in which the money supply does not change. Thus, $M_1^S = M_0^S$.

At $t = 2$, the central bank can again change the money supply. Nominal transfers T_2^{CB} to households (or taxes, if $T_2^{CB} < 0$) are:

$$T_2^{CB} = M_2^S - M_0^S. \quad (8)$$

Depending on the timing of the monetary interventions, there are three relevant types of monetary injections that I consider. The first type is a temporary injection (i.e., $M_0^S > \bar{M}$ and $M_2^S = \bar{M}$), and thus the central bank taxes households at $t = 2$ to reduce the money supply to the initial level $\bar{M}$. The second type is a permanent injection (i.e., $M_0^S = M_2^S > \bar{M}$). The third type is a change of the money supply at $t = 2$ that is fully anticipated at $t = 0$ (i.e., $M_0^S = \bar{M}$ and $M_2^S \neq \bar{M}$; recall that the central bank announces the path of money at $t = 0$ and commits to it).

2.4.2 Fiscal authority and price level determination

The fiscal authority collects taxes on the output produced by capital at $t = 2$. Recall that each unit of capital produces A_2 units of consumption at $t = 2$. The government imposes a proportional tax τ_2 on each unit of output, and uses these resources to purchase the stock of money M_2^S in circulation at the end of $t = 2$. Another way to understand this process is to assume that the taxes imposed at $t = 2$ must be paid with money, which in turn provides a value for the money in circulation at $t = 2$. Note that even though the tax τ_2 is proportional, it is not distortionary in this environment because the output produced at $t = 2$ is exogenous.

Let P_2 be the nominal price level at $t = 2$. The price level P_2 adjusts in equilibrium so that the real value of money, M_2^S/P_2 , is equal to the real taxes collected by the government, either because

the government repurchases money or because money must be used to pay such taxes:¹¹

$$\frac{M_2^S}{P_2} = \tau_2 A_2 \bar{K}. \quad (9)$$

To simplify the analysis even further, I set $\tau_2 = 1 / (A_2 \bar{K})$ and maintain this assumption unchanged throughout the rest of the paper. Thus, Equation (9) implies

$$P_2 = M_2^S. \quad (10)$$

That is, the price level at $t = 2$ is proportional to the quantity of money in circulation. The result in Equation (10) could alternatively be derived in an infinite-horizon economy, such as Lucas and Stokey (1987) or Lagos and Wright (2005), in which money has value because it can be carried to the next period.¹² Moreover, Equation (10) is consistent with a quantity-theory equation, in which the price level is proportional to the stock of money.

3 Bankless economy

I now analyze the equilibrium of this baseline model. I refer to this environment as the bankless economy because households are the only set of private agents and there are no banks.

Households choose money and capital (i.e., M_0^h and K_0^h), consumption at $t = 1$ if impatient or patient (i.e., C_1^H and C_1^L), and consumption at $t = 2$ (i.e., C_2^h) by solving

$$\begin{aligned} \max_{M_0^h, K_0^h, C_1^H, C_1^L} \kappa & \left\{ \varepsilon^H u(C_1^H) + \underbrace{\frac{M_0^h - P_1 C_1^H}{P_2} + \frac{Q_0 K_0^h (1 + r_2^K)}{P_2}}_{=C_2^h \text{ if } h \text{ is impatient}} \right\} \\ & + (1 - \kappa) \left\{ \varepsilon^L u(C_1^L) + \underbrace{\frac{M_0^h - P_1 C_1^L}{P_2} + \frac{Q_0 K_0^h (1 + r_2^K)}{P_2}}_{=C_2^h \text{ if } h \text{ is patient}} \right\} \end{aligned} \quad (11)$$

where $1 + r_2^K$ is the nominal gross return of holding capital, defined by

$$1 + r_2^K = \frac{P_1 A_1 + P_2 A_2 (1 - \tau_2)}{Q_0}. \quad (12)$$

¹¹Cochrane (2005) refers to Equation (9) as a “valuation equation,” as opposed to a government budget constraint. This is because, in the fiscal theory of the price level, there is no constraint that must be satisfied by the government. Rather, given τ_2 , the price level P_2 adjusts so that (9) holds in equilibrium, providing a value for money endogenously.

¹²In a related work (Robatto, 2018), I formalize the notion that a three-period model similar to the one in this paper can be understood as a snapshot of an infinite-horizon model, and the result in Equation (10) arises without the need to introduce a fiscal-based determination of the price level.

The maximization in (11) is subject to the budget and cash-in-advance constraints:

$$M_0^h + K_0^h Q_0 \leq \underbrace{\overline{M} + \overline{K} Q_0}_{\text{value of endowments}} + \underbrace{T_0}_{\text{monetary injection}}, \quad (13)$$

$$P_1 C_1^H \leq M_0^h, \quad P_1 C_1^L \leq M_0^h. \quad (14)$$

At $t = 0$, the household has access to the Walrasian market where it chooses its portfolio of money and capital, subject to the budget constraint (13); M_0^h and K_0^h denote the amount of money and capital that the household has after trading. At $t = 1$, consumption is subject to the cash-in-advance constraint (14). At $t = 2$, consumption is financed with unspent money ($M_0^h - P_1 C_1^H$ if the household is impatient, and $M_0^h - P_1 C_1^L$ if it is patient) and with the return $1 + r_2^K$ earned by investing $Q_0 K_0^h$ in capital.

To solve problem (11), I conjecture that the cash-in-advance constraint at $t = 1$ is binding for impatient households, and not binding for patient households. This conjecture is then verified because the opportunity cost of holding money, represented by the return on capital, is positive in equilibrium.

The first-order conditions for the choice of money and capital at $t = 0$ imply

$$\mathbb{E}_\varepsilon \{ \varepsilon_1^h u' (C_1^h) \} \frac{1}{P_1} = \frac{1 + r_2^K}{P_2}. \quad (15)$$

Households are indifferent between investing an extra dollar in money or in capital at $t = 0$. Investing in money allows households to increase consumption at $t = 1$, which gives a marginal utility $\mathbb{E}_\varepsilon \{ \varepsilon_1^h u' (C_1^h) \} / P_1$ per dollar. Investing in capital provides a marginal utility that depends on the return on capital R_2^K , because of the linear utility at $t = 2$.

For patient households, the first order condition for the choice of C_1^L at $t = 1$ is

$$\varepsilon^L u' (C_1^L) \frac{1}{P_1} = \frac{1}{P_2} \quad (16)$$

because their cash-in-advance constraint is not binding.

Time-1 consumption of impatient households, C_1^H , is determined by the binding cash-in-advance constraint. The fact that the cash-in-advance constraint is binding only for a subset of agents imposes a wedge that distorts the allocation of consumption at $t = 1$ away from the first best, as discussed later.

3.1 Equilibrium in the bankless economy

An equilibrium of this economy is a collection of prices Q_0 and P_1 and households' choices M_0^h , K_0^h , C_1^H , and C_1^L such that: (i) M_0^h , K_0^h , C_1^H , and C_1^L solve the problem (11); (ii) the money and capital markets clear at $t = 0$, $\int M_0^h dh = M_0^S$ and $\int K_0^h dh = \bar{K}$; and (iii) the goods market at $t = 1$ clears, $\int C_1^h dh = A_1 \bar{K}$.

I analyze the equilibrium of the three special cases $\sigma \rightarrow \{0, 1, \infty\}$. At $t = 0$, households' choices of money and capital holdings are independent of σ . Since all households are alike, they hold the same amount of money and capital, and market clearing implies $M_0^h = M_0^S$ and $K_0^h = \bar{K}$ for all h . At $t = 1$, the consumption of impatient households is determined by the binding cash-in-advance constraint (14), whereas the consumption of patient households is residually determined by the market clearing condition. However, prices and consumption allocation crucially depend on σ .

Consider first the the case of linear utility at $t = 1$, that is, $\sigma \rightarrow \infty$. Consumptions and prices are

$$C_1^H = \frac{M_0^S (1 - \kappa)}{M_2^S (1 - \kappa \varepsilon^H)}, \quad C_1^L = \frac{\varepsilon^L M_2^S - \kappa M_0^S}{M_2^S (1 - \kappa \varepsilon^H)},$$

$$P_1 = M_2^S \left[\frac{1 - \kappa \varepsilon^H}{1 - \kappa} \right], \quad Q_0 = M_2^S \left[A_2 \varepsilon^L - A_1 \frac{(\varepsilon^H - 1) \kappa \varepsilon^L}{1 - \kappa} \right], \quad r_2^K = \frac{\kappa (\varepsilon^H - 1)}{1 - \kappa \varepsilon^H}$$

provided that $M_2^S > (\kappa/\varepsilon^L) M_0^S$.¹³ Note that the price level at $t = 1$ depends on *future* money supply M_2^S , and is instead independent of the amount of money held by households at $t = 1$, which is given by M_0^S . This is because P_1 is pinned down by the first-order condition with respect to C_1^L of impatient households, that is, Equation (16). This equation depends on M_2^S through P_2 , but is independent of M_0^S . Although I analyze monetary injections in more details later, it is worth emphasizing a crucial implication of the equilibrium. Monetary interventions that change only the money supply M_0^S at $t = 0$ —such as a monetary injection implemented at $t = 0$ and reverted at $t = 2$ —do not produce changes to the price level P_1 , whereas they change the time-1 consumption allocation.

Consider now the other extreme case in which households' elasticity of substitution is $\sigma \rightarrow 0$. The equilibrium converges to:

$$C_1^H = 1, \quad C_1^L = 1,$$

$$P_1 = M_0^S, \quad Q_0 = M_0^S \left[\frac{M_0^S}{M_2^S} A_1 + (A_2 - A_1) \right], \quad r_2^K = \frac{M_2^S}{M_0^S} - 1,$$

provided that $M_2^S > M_0^S$.¹⁴ When the intertemporal elasticity of substitution converges to zero,

¹³If this parameter restriction does not hold, the equilibrium replicates the first best (i.e., $C_1^H = 1/\kappa$ and $C_1^L = 0$).

¹⁴If this restriction on money supply is not satisfied, the nominal interest rate is zero and $Q_0 = M_0^S A_1 +$

the equilibrium consumption converges to the first-best allocation (i.e., C_1^H and C_1^L are equalized). Patient and impatient households spend the same amount of money and, thus, the entire money supply M_0^S is spent.

In the intermediate case in which agents have log utility (i.e., $\sigma \rightarrow 1$):

$$C_1^H = \frac{M_0^S}{\kappa M_0^S + M_2^S (1 - \kappa \varepsilon^H)}, \quad C_1^L = \varepsilon^L \frac{M_2^S}{\kappa M_0^S + M_2^S (1 - \kappa \varepsilon^H)}$$

$$P_1 = \kappa M_0^S + M_2^S (1 - \kappa \varepsilon^H), \quad Q_0 = \frac{M_0^S [A_1 \kappa (M_0^S - \varepsilon^H M_2^S) + M_2^S A_2]}{[M_0^S (1 - \kappa) + M_2^S \kappa \varepsilon^H]}, \quad r_2^K = \kappa \left(\frac{M_2^S \varepsilon^H}{M_0^S} - 1 \right),$$

provided that $M_2^S > (1/\varepsilon^H) M_0^S$.¹⁵

Finally, I comment on welfare. In this bankless economy, the consumption allocation is in general not at the first best for all $\sigma > 0$. This is because the cash-in-advance constraint is binding only for a subset of agents, implying a wedge that distorts the marginal utility of consumption away from the first best. Impatient households, which face a binding cash-in-advance constraint, consume too little in comparison to the first best, and patient households consume too much.

3.2 Optimal monetary policy

The central bank can implement the first best with a monetary policy that drives to zero the nominal interest rate (i.e., the return on capital r_2^K), that is, the Friedman rule. With a zero nominal interest rate, the opportunity cost of holding money is zero, and thus households at $t = 0$ hold enough money to be able to finance any desired amount of consumption at $t = 1$. As a result, the cash-in-advance constraint at $t = 1$ is never binding, not even for impatient households, and the wedge that precludes the first best is eliminated.

In the rest of paper, I consider a monetary policy that does not implement the Friedman rule. The literature has provided various reasons why the Friedman rule might not be optimal or not even feasible (Andolfatto, 2013; Berentsen et al., 2007). To keep the model as simple as possible, I do not include in the model any of these considerations, and I simply study how monetary injections affect the equilibrium away from the Friedman rule.

3.3 Monetary injections: some preliminary remarks

As a preliminary step in the analysis of monetary injections, it is useful to make two sets of remarks. The first set of remarks concerns households' money demand. The second set of remarks concerns

$M_2^S (A_2 - A_1)$; however, the consumption allocation is unchanged.

¹⁵If this restriction does not hold, the nominal interest rate is zero (i.e., the central bank implements the Friedman rule) and the equilibrium allocation is the same as in the first best.

the effects of monetary injections that arise mechanically on velocity and the price level.

Households' money demand elasticity. Households' real money demand elasticity with respect to its opportunity cost—namely, the return on capital r_2^K —is a crucial element that governs the equilibrium response to monetary injections. Real money demand is given M_0^h/P_1 , and its elasticity $\eta\left(\frac{M_0^h}{P_1}, r_2^K\right)$ can be derived by combining Equations (15) and (16) and the binding cash-in-advance constraint for impatient households, and totally differentiating with respect to r_2^K :

$$\eta\left(\frac{M_0^h}{P_1}, r_2^K\right) \equiv \frac{d(M_0^h/P_1)}{dr_2^K} \frac{r_2^K}{(M_0^h/P_1)} = -\sigma \left[\frac{r_2^K}{\kappa \varepsilon^H u'(C_1^H) (P_2/P_1)} \right]. \quad (17)$$

The key results that I want to emphasize are how the elasticity behaves in the limiting cases $\sigma \rightarrow \infty$ and $\sigma \rightarrow 0$.

When $\sigma \rightarrow \infty$ and thus households have linear preferences, the elasticity of money demand converges to $\eta\left(\frac{M_0^h}{P_1}, r_2^K\right) \rightarrow -\infty$. This means that any small change to return on capital r_2^K produces an enormous change in the money holdings M_0^h of households. For instance, if a policy intervention produces an arbitrary small increase of r_2^K , optimal households' money holding drops to $M_0^h = 0$, no matter what the money holding was without policy intervention. If instead the intervention produces an arbitrary small decrease of r_2^K , money demand increases to the point that a household wants to invest all its time-0 wealth in money and nothing in capital (i.e., using the budget constraint (13), $M_0^h = Q_0 \bar{K} + \bar{M}$). While such dramatic swings in money demand are possible in partial equilibrium, they might not be consistent with market clearing in general equilibrium. As a result, and anticipating some of the general equilibrium results for the case $\sigma \rightarrow \infty$, markets clear only if the opportunity cost r_2^K does not change in response to a monetary injection.

When $\sigma \rightarrow 0$, households' money demand elasticity depends on the value of C_1^H at which it is evaluated. I focus only on the relevant case $C_1^H = 1$, which corresponds to the equilibrium of the bankless economy. In this case, we have $\eta\left(\frac{M_0^h}{P_1}, r_2^K\right) \rightarrow 0$, and thus money demand is completely inelastic to changes in the interest rate. That is, unlike the $\sigma \rightarrow \infty$, households' choices are not affected by r_2^K . Thus, even in general equilibrium, the requirement that markets must clear does not prevent r_2^K from changing in response to a policy intervention.

The equation of exchange and the effects of monetary injections on velocity and the price level. A second remark is related to the elasticity of money velocity and the price level. Since $M_1^S = M_2^S$, money velocity v at $t = 1$ is defined by the equation of exchange:

$$M_0^S v = P_1 (A_1 \bar{K}). \quad (18)$$

To understand how a change in M_0^S affects v and P_1 , I totally differentiate (18) with respect to M_0^S and use the fact that output $A_1 \bar{K}$ is exogenous to obtain:

$$1 = \eta(P_1, M_0^S) - \eta(v, M_0^S) \quad (19)$$

where

$$\eta(P_1, M_0^S) \equiv \frac{dP_1}{dM_0^S} \frac{M_0^S}{P_1}, \quad \eta(v, M_0^S) \equiv \frac{dv}{dM_0^S} \frac{M_0^S}{v} \quad (20)$$

are the elasticities of the equilibrium price level and money velocity with respect to M_0^S , respectively.

A change in M_0^S is associated with a change in the price level P_1 or a change in money velocity v , or both. Moreover, these changes are such that the sum of the absolute values of the respective elasticities must be equal to one, as stated in (19). This result follows from the identity in (18) and thus is purely mechanical. Nonetheless, it sheds some light on the results of the policy analysis. If a monetary intervention that changes M_0^S is neutral (i.e., it affects only nominal prices), then the elasticity of the price level is $\eta(P_1, M_0^S) = 1$ and the elasticity of velocity is $\eta(v, M_0^S) = 0$. In the opposite scenario in which an intervention has no nominal effect—that is, the elasticity of the price level is $\eta(P_1, M_0^S) = 0$ —the intervention must change velocity with an elasticity $\eta(v, M_0^S) = -1$; that is, velocity reduces one-for-one with M_0^S . For intermediate scenarios, both the price level P_1 and money velocity v are affected.

Crucially, any change in velocity is associated with a real effect or, more precisely, with a change in the allocation of consumption at $t = 1$ between patient and impatient households.

3.4 Monetary injections in the bankless economy

This section provides the key results of the paper about the effects of monetary injections on prices, velocity, and consumption allocations. Even though the model of this section does not include either banks or runs, it contains the key elements that are necessary for the analysis. The results are qualitatively unchanged in the economy with banks and runs. In a sense, the bankless economy studied in this section can be viewed as an economy experiencing a financial crisis that is so severe that no bank operates. Section 4.5 extends the analysis of monetary injections to an economy with banks to study the effects of monetary injections on deposits.

The effects of a temporary monetary injection on equilibrium prices and allocations depend on the value of σ . As $\sigma \rightarrow \infty$ (i.e., linear utility), a temporary monetary injection does not affect nominal prices but only affects the equilibrium allocation. At the other extreme $\sigma \rightarrow 0$ (i.e., quasi-Leontief), the opposite result arises; that is, a temporary monetary injection affects only nominal prices.

Temporary monetary injections. With a temporary monetary injection, the central bank marginal increases the money supply M_0^S at $t = 0$, and then reverts the monetary injection at $t = 2$, so that the money supply M_2^S at $t = 2$ is unchanged. This exercise should be understood as a comparative static between two equilibria: the equilibrium that arises if money supply is M_0^S , and the equilibrium that arises if the money supply is marginally greater than M_0^S . In other words, the results of this section are a comparative static analysis with respect to policy announced before time-0 Walrasian market opens. That is, the results refer to how the equilibrium varies as a function of monetary policy.

I begin the analysis by characterizing the effects of temporary injections in the limit as the preference shocks disappear, that is, $\{\varepsilon^H, \varepsilon^L\} \rightarrow \{1, 1\}$. This result provides a benchmark because it shows the effects of injections in a standard cash-in-advance economy. I express all the results in terms of elasticities. For instance, the elasticity of P_1 and v with respect to M_0^S is defined in (20), and the elasticities of the return on capital r_2^K , time-1 consumption of impatient households C_1^H , and time-1 consumption of patient households C_1^L are defined similarly and denoted by $\eta(r_2^K, M_0^S)$, $\eta(C_1^H, M_0^S)$, and $\eta(C_1^L, M_0^S)$, respectively.

Proposition 3.1. *If $\{\varepsilon^H, \varepsilon^L\} \rightarrow \{1, 1\}$ and the central bank is not implementing the Friedman rule:*

$$\begin{aligned} \eta(P_1, M_0^S) &\rightarrow 1, & \eta(v, M_0^S) &\rightarrow 0, & \eta(r_2^K, M_0^S) &\rightarrow -1 \\ \eta(C_1^H, M_0^S) &\rightarrow 0, & \eta(C_1^L, M_0^S) &\rightarrow 0. \end{aligned}$$

Without preference shocks, a temporary monetary injection is neutral: velocity is constant, real allocations are not affected by monetary policy, and the price level P_1 and the nominal interest r_2^K move one-for-one with the monetary injection.

Next, I turn to the full model with preference shocks, and I start by focusing on the case $\sigma \rightarrow 0$, that is, the quasi-Leontief utility. In this case, the money supply M_0^S at $t = 0$ is also neutral.

Proposition 3.2. *(Temporary monetary injection, quasi-Leontief utility) If $\sigma \rightarrow 0$ and the central bank is not implementing the Friedman rule:*

$$\begin{aligned} \eta(P_1, M_0^S) &\rightarrow 1, & \eta(v, M_0^S) &\rightarrow 0, & \eta(r_2^K, M_0^S) &\rightarrow -1 \\ \eta(C_1^H, M_0^S) &\rightarrow 0, & \eta(C_1^L, M_0^S) &\rightarrow 0. \end{aligned}$$

As noted in Sections 2.1 and 2.3, the consequence of taking the limit $\sigma \rightarrow 0$ is to eliminate any effects of the preference shocks on households' utility. As a result, the responses to a temporary monetary injection are identical to that of Proposition 3.1, that is, the economy with no preference shocks.

Next, I consider the other extreme case, $\sigma \rightarrow \infty$, in which households have linear utility. In this case, the effects of temporary monetary injections are completely reversed. A marginal increase in M_0^S has no effects on nominal prices, but it alters allocations and welfare. In particular, the higher money supply at $t = 0$ relaxes the cash-in-advance constraint of impatient households and allows them to increase their consumption. By market clearing, the consumption of patient households reduces. In addition, because of [Equation \(19\)](#) and the fact that the monetary injection does not change P_1 , money velocity decreases one-for-one with the monetary injection. The next proposition summarizes these results.

Proposition 3.3. *(Temporary monetary injection, linear utility) Assume that $\sigma \rightarrow \infty$ and the central bank is not implementing the Friedman rule. Then:*

$$\begin{aligned} \eta(P_1, M_0^S) &\rightarrow 0, & \eta(v, M_0^S) &\rightarrow -1, & \eta(r_2^K, M_0^S) &\rightarrow 0 \\ \eta(C_1^H, M_0^S) &\rightarrow 1, & \eta(C_1^L, M_0^S) &\rightarrow -\frac{\kappa}{1-\kappa} \frac{C_1^H}{C_1^L} < 0. \end{aligned}$$

To understand the effects of a monetary injection when $\sigma \rightarrow 0$ ([Proposition 3.2](#)) and $\sigma \rightarrow \infty$ ([Proposition 3.3](#)), recall how households' real money demand elasticity behaves in these limiting cases. As $\sigma \rightarrow 0$ and $\sigma \rightarrow \infty$, this elasticity converges to zero and infinity, respectively.

With $\sigma \rightarrow 0$ and thus zero elasticity of real money demand, households' demand for real money is essentially constant (i.e., vertical) at $M_0^h/P_1 = 1$, for any r_2^K . To guarantee clearing in the money market, the supply of real money must also be equal to one. Thus, any increase of the money supply at $t = 0$ must be compensated with a one-for-one increase in P_1 . This higher price reduces the interest rate r_2^K . This effect, however, does not preclude clearing in the money market. Because of the inelastic money demand, the lower r_2^K does not change the amount of money demand by the household.

With $\sigma \rightarrow \infty$ and thus infinite elasticity of real money demand, the logic is reversed. Households' money demand is flat (i.e., horizontal), pinning down the nominal interest rate r_2^K at which households are willing to hold any amount of real money. Thus, a temporary monetary injection does not change P_1 —a necessary condition to keep r_2^K constant since the price in the last period, P_2 , is unchanged as well—and real money supply expands one-for-one with the monetary injection. Households fully absorb the monetary injection and store the additional money injected by the central bank.

Finally, if $\sigma \rightarrow 1$ (i.e., if households have log utility), the results are intermediate between the economies with $\sigma \rightarrow \infty$ and $\sigma \rightarrow 0$. That is, a temporary monetary injection that increases M_0^S affects both nominal prices (i.e., increases the price level P_1 and reduces the nominal interest rate $1 + r_2^K$) and consumption allocations (i.e., it relaxes the cash-in-advance constraint of impatient

households and thus increases C_1^H , and reduces C_1^L by market clearing).

Proposition 3.4. *(Temporary monetary injection, log utility) Assume that $\sigma \rightarrow 1$ and the central bank is not implementing the Friedman rule. Then:*

$$\eta(P_1, M_0^S) \rightarrow \frac{\kappa M_0^S}{\kappa M_0^S + M_2^S(1 - \kappa \varepsilon^H)}, \quad \eta(v, M_0^S) \rightarrow -\frac{M_2^S(1 - \kappa \varepsilon^H)}{\kappa M_0^S + M_2^S(1 - \kappa \varepsilon^H)},$$

$$\eta(r_2^K, M_0^S) \rightarrow -\frac{\varepsilon^H M_2^S}{\varepsilon^H M_2^S - M_0}$$

$$\eta(C_1^H, M_0^S) \rightarrow \frac{M_2^S(1 - \kappa \varepsilon^H)}{\kappa M_0^S + M_2^S(1 - \kappa \varepsilon^H)}, \quad \eta(C_1^L, M_0^S) \rightarrow -\frac{\kappa M_0^S}{\kappa M_0^S + M_2^S(1 - \kappa \varepsilon^H)},$$

and thus $\eta(P_1, M_0^S), \eta(C_1^H, M_0^S) \in (0, 1)$, and $\eta(v, M_0^S), \eta(r_2^K, M_0^S), \eta(C_1^L, M_0^S) \in (-1, 0)$.

Permanent monetary injections. To complete this analysis, I characterize the response to permanent injections. A permanent injection at $t = 0$ implies that money supply increases at $t = 0$ and stays constant afterward, $M_1^S = M_0^S$ and $M_2^S = M_0^S$. Given the similarity with a classic monetary framework, permanent monetary injections are neutral.

Proposition 3.5. *(Permanent monetary injection at $t = 0$) Let $M_0^S = M_1^S = M_2^S = \widehat{M}$, so that any change to the money supply implemented at $t = 0$ is permanent. If the central bank is not implementing the Friedman rule, then¹⁶*

$$\eta(P_1, \widehat{M}) \rightarrow 1, \quad \eta(v, \widehat{M}) \rightarrow 0, \quad \eta(r_2^K, \widehat{M}) \rightarrow 0$$

$$\eta(C_1^H, \widehat{M}) \rightarrow 0, \quad \eta(C_1^L, \widehat{M}) \rightarrow 0.$$

3.5 The welfare effect of temporary monetary injections

As a last step in the analysis of monetary injections in the bankless economy, I analyze the welfare implications of small temporary monetary injections. A temporary injection relaxes the cash-in-advance constraints of impatient households, closing the gap with the first-best allocation. Thus, temporary monetary injections are welfare improving, as stated in the next proposition.

Proposition 3.6. *If $\sigma > 0$ and the central bank does not implement the Friedman rule:*

$$\frac{d \mathbb{E}_\varepsilon \{ \varepsilon_1^h u(C_1^h) + C_2^h \}}{d M_0^S} > 0.$$

¹⁶The elasticity of the price level P_1 is defined as $\eta_{P_1, \widehat{M}} = (dP_1/d\widehat{M}) \times (\widehat{M}/P_1)$, and the other elasticities are defined similarly.

3.6 Inefficiency and pecuniary externality

I now ask whether the equilibrium is constrained-efficient, taking as given the monetary and fiscal policy. The answer is negative because of a pecuniary externality related to the binding cash-in-advance constraints.

The pecuniary externality is related to a cross-sectional heterogeneity of the Lagrange multiplier of the cash-in-advance constraint. Recall that impatient households face a binding cash-in-advance constraint, whereas patient households face a non-binding constraint. In addition, the price level P_1 enters the constraint. If a planner could force patient households to cut spending at $t = 1$, the price level P_1 would drop, relaxing the binding cash-in-advance constraint of patient households. This process would increase welfare because the welfare gain for impatient households would be first-order, since their cash-in-advance is binding, and the welfare loss for patient households would be only second order, since their cash-in-advance constraint is not binding in the decentralized equilibrium.

To begin the analysis, I describe the planner problem. The planner has limited planning abilities, similar to the approach used, for instance, by [Bianchi \(2011\)](#). The planner chooses households' portfolio at $t = 0$ and consumption expenditure of each household at $t = 1$, but households purchase the consumption goods in a centralized market at $t = 1$, taking as given the price level P_1 . Crucially, the planner internalizes the effects of each household's consumption expenditure on the price level P_1 , unlike households that take P_1 as given in the decentralized equilibrium. The objective function of the planner is utilitarian, with the same weights on all households at $t = 0$.

At $t = 0$, the planner allocates money and capital equally among households, as in the decentralized equilibrium. At $t = 1$, the planner dictates that impatient households spend all their money, and thus their consumption, C_1^H , is determined by the binding cash-in-advance constraint, $P_1 C_1^H = M_0^h$. However, the planner's first-order condition for consumption of impatient households, C_1^L , implies

$$u'(C_1^L) \frac{1}{P_1} = \frac{1}{P_2} + \frac{dP_1}{dC_1^L} \frac{C_1^L}{P_1} \frac{\kappa C_1^H}{(1 - \kappa) C_1^L} (\mu_1^H) \quad (21)$$

where μ_1^H is the Lagrange multiplier of the cash-in-advance constraint of impatient households (i.e., households with preference shock ε^H) and, similar to the decentralized equilibrium, the Lagrange multiplier of patient households is zero.

The key difference with the decentralized equilibrium is that the marginal private cost of increasing patient households' expenditure by an extra dollar (i.e., right-hand side of [Equation \(16\)](#)) is different from the marginal social cost (i.e., right-hand side of [Equation \(21\)](#)). The marginal private cost includes only the value of carrying the dollar to $t = 2$. In contrast, the marginal social cost includes an extra term that derives from the fact that the planner internalizes the effects of patient households' consumption expenditure on prices. This extra term depends on three elements.

The first element is the elasticity of the price level, P_1 , with respect to the consumption choice, C_1^L ; the higher this elasticity is, the bigger the impact on prices of spending an extra dollar, and the higher the social costs. The second is the ratio of total consumption of impatient, κC_1^H , to that of impatient, $(1 - \kappa) C_1^L$; if the fraction of impatient κ is small, only few households suffer from the binding cash-in-advance constraint, and thus the marginal social costs of a higher price level are limited. The third element is the Lagrange multiplier of the cash-in-advance constraint of impatient households, μ_1^H ; if this multiplier is very large, a further increase in the price level creates high welfare losses for impatient households.

One way to reduce the cross-sectional heterogeneity would be to redistribute resources from patient to impatient households at $t = 1$. However, this is not possible if the planner faces the same frictions that gives rise to the need of money, such as limited commitment and lack of pledgeability of capital. Nonetheless, the planner can alter the consumption expenditure of impatient households to create some redistribution through a general equilibrium effect.

The result that the decentralized equilibrium is constrained-inefficient follows trivially from comparing the planner's first-order condition (21) with that of the decentralized equilibrium, [Equation \(16\)](#), and noting that the Lagrange multiplier of the cash-in-advance constraint of impatient households, μ_1^H , is strictly positive.

4 Model with banks

The results of the bankless economy open up a role for banks to redistribute money between households after the realization of the preference shocks. This section studies the optimal arrangement that is offered by competitive banks to insure against preference shocks, and the implications for monetary policy and welfare. I show that the effects of monetary injections on prices and allocations in the economy with banks are qualitatively similar to those of the bankless economy, and I also characterize the effects of monetary injections on households' decisions to hold money and deposits.

I consider an environment that is very similar to that of the bankless economy, with the addition of some shocks to capital. The role of these shocks is to generate fundamentals-based runs, similar to [Allen and Gale \(1998\)](#), thereby preventing banks from implementing the first best. All the results of the bankless economy are not affected if one solves for the bankless equilibrium in the economy with shocks to capital.

I derive the optimal contract while preserving the general-equilibrium approach in which decision makers are small and thus take prices as given. To do so, I partition the set of households into many subsets, which I will refer to as "islands." Each island is small with respect to the whole economy but big enough so that the law of large numbers about preference shocks hold within the

subset. I then study the problem of island-level planners, which I will later reinterpret as banks. Each island-level planner pools the endowment of the households on the respective island at $t = 0$, and sets up an optimal mechanism to redistribute resources at $t = 1$ and $t = 2$, taking prices as given.¹⁷ As a result, the mechanism chosen by each island-level planner resembles that of [Allen and Gale \(1998\)](#). However, prices are then determined in general equilibrium. Despite the mechanism chosen by each island-level planner is optimal given prices, a pecuniary externality arises and thus the equilibrium constrained-inefficient, similar to the bankless economy.

As a next step, I show how to implement the island-level planner mechanism using banking contracts. I assume that each island can host at most one bank, although for each island, multiple banks can compete to offer the best contract (and thus banks make zero profits). Households have access only to one bank—the bank on their own island—and thus the bank plays a role that is similar, although not necessarily identical, to that of the island-level planner. At $t = 0$, households can deposit at the bank and trade in the centralized market, and banks can trade in the centralized market as well. At $t = 1$, households can withdraw money from their bank and purchase consumption goods in a centralized market subject to a cash-in-advance constraint. At $t = 2$, households can contact their own bank again.

As a final step, I study the effects of monetary injections. I focus, in particular, on the effects of monetary injections on households' money-holding decisions and on deposits.

4.1 Extended environment

I consider an environment very similar to that of the bankless economy, with the addition of idiosyncratic shocks that hit physical capital at $t = 1$. To clarify the exposition, it is useful to imagine that the households are located on a continuum of spatially separated islands. Recall that there is a double continuum of households, and thus each island hosts a continuum of households. Let me stress that the notion of households located on separated islands is made just for expositional simplicity—introducing islands simply helps to clarify the modeling of the idiosyncratic shocks to capital and banks. Even though households are on separate islands, they have access to the centralized Walrasian market at $t = 0$, and to a centralized market where they can purchase consumption goods using money at $t = 1$.

The idiosyncratic shocks to capital that hit the economy at the beginning of $t = 1$ are denoted by $\psi \in \{\psi^H, \psi^L\}$. These shocks resemble the capital quality shocks in [Gertler and Kiyotaki \(2010\)](#), although they are idiosyncratic rather than aggregate. The shocks are island-specific, that is, the realization of the shock is the same for all agents within an island. The effect of these shocks is to reallocate capital among islands, leaving the aggregate stock of capital unchanged at

¹⁷This approach has similarities to that described by [Atkeson and Lucas \(1992\)](#).

$\bar{K}$.¹⁸ The shocks hit at the beginning of $t = 1$, and thus they reallocate capital after trades in the time-0 Walrasian market have taken place. For a fraction $1 - \alpha$ of islands, $\alpha \in (0, 1)$, the stock of capital increases by a factor $\psi^H > 1$. For the remaining fraction α of islands, the stock of capital reduces by a factor ψ^L , with $\psi^L = 0$; that is, the stock of capital on islands hit by ψ^L is completely wiped out. Since the shocks are idiosyncratic, we have

$$\alpha (1 + \psi^L) + (1 - \alpha) (1 + \psi^H) = 1. \quad (22)$$

I assume that markets are exogenously incomplete with respect to the shocks to capital. That is, it is not possible to write contracts that insure against the realization of the bad shock to capital ψ^L .

At the level of each island, introducing shocks to capital produces results that resemble those in [Allen and Gale \(1998\)](#). If an island is hit by the bad shock ψ^L , the island-level planner does not have any capital to distribute to households at $t = 2$, and thus is forced to pay money at $t = 1$ to all households—including those that are patient—to elicit truthful revelation of the preference shocks. This case corresponds to a run. If instead an island is hit by a good shock ψ^H , the island-level planner can distribute more money to impatient households at $t = 1$ and compensate patient households by giving them more goods at $t = 2$.

For future reference, let $1 + r_2^K(\psi)$ be the nominal return on capital at $t = 2$ for an island that is hit by the idiosyncratic shock to capital ψ . This return is defined by

$$1 + r_2^K(\psi) = (1 + \psi) \frac{A_2 P_2 + A_1 P_1}{Q_0}. \quad (23)$$

4.2 Island-level planner

The first step of the analysis is to study the problem of an island-level planner that pools the resources of households on a given island and sets up the optimal mechanism to provide insurance against preference shocks. The island-level planner collects endowments at $t = 0$, distributes resources to each household at $t = 1$ and $t = 2$, and takes prices as given. In [Section 4.3](#), I describe how the results of this section can be implemented by profit-maximizing banks that compete to offer the best contract.

The approach of this section is similar to standard mechanism design problems used to compute the optimal contract in banking models, such as [Allen and Gale \(1998\)](#), but with some crucial differences. First, the island-level planner distributes money at $t = 1$, rather than goods. Second, while the island-level planner chooses the optimal mechanism by taking the price of capital Q_0 and that of consumption goods P_1 as given, such prices are then determined in general equilibrium to

¹⁸Alternatively, these disturbances could be modeled as idiosyncratic shocks that affect the productivity of capital at $t = 1$ and $t = 2$.

clear the market for capital and money at $t = 0$ and for consumption goods at $t = 1$.

To characterize the optimal mechanism, I now state the problem of the island-level planner and then describe each element in detail. The island-level planner holds money M_0 and capital K_0 at $t = 0$, and distributes money $M(\varepsilon_1^h, \psi)$ at $t = 1$ and consumption goods $Y_2(\varepsilon_1^h, \psi)$ at $t = 2$ to a household that reports its type to be ε_1^h , where ψ denotes the realization of the idiosyncratic shock to capital in the island of household h . The island-level planner maximizes:

$$\max_{\{M_0 K_0 \{M(\varepsilon_1^h, \psi), Y_2(\varepsilon_1^h, \psi)\}_{\varepsilon_1^h \in \{\varepsilon^H, \varepsilon^L\}, \psi \in \{\psi^H, \psi^L\}\}}} \mathbb{E}_{\varepsilon, \psi} \{ \varepsilon_1^h u(C_1(\varepsilon_1^h, \psi)) + C_2(\varepsilon_1^h, \psi) \} \quad (24)$$

subject to:

$$M_0 + Q_0 K_0 \leq \bar{M} + Q_0 \bar{K} + T_0 \quad (25)$$

$$\int M(\varepsilon_1^h, \psi) dh \leq M_0 \quad (26)$$

$$C_1(\varepsilon_1^h, \psi) = \arg \max_{c \leq M(\varepsilon_1^h, \psi)/P_1} \left\{ \varepsilon_1^h u(c) + \frac{M(\varepsilon_1^h, \psi) - P_1 c}{P_2} \right\} \quad (27)$$

$$P_2 \int Y_2(\varepsilon_1^h, \psi) dh \leq Q_0 K_0 [1 + r_2^K(\psi)] + \left[M_0 - \int M(\varepsilon_1^h, \psi) dh \right] \quad (28)$$

$$\left\{ \max_{c \leq M(\varepsilon^L, \psi)/P_1} \varepsilon^L u(c) + \frac{M(\varepsilon^L, \psi) - P_1 c}{P_2} + Y_2(\varepsilon^L, \psi) \right\} \geq \left\{ \max_{c \leq M(\varepsilon^H, \psi)/P_1} \varepsilon^H u(c) + \frac{M(\varepsilon^H, \psi) - P_1 c}{P_2} + Y_2(\varepsilon^H, \psi) \right\} \quad (29)$$

and where

$$C_2(\varepsilon_1^h, \psi) = Y_2(\varepsilon_1^h, \psi) + \frac{M(\varepsilon_1^h, \psi) - P_1 C_1(\varepsilon_1^h, \psi)}{P_2}. \quad (30)$$

In addition, non-negativity constraints on $M(\varepsilon_1^h, \psi)$ and $Y_2(\varepsilon_1^h, \psi)$ must hold for all ε_1^h and all ψ .

Equation (25) is the budget constraint at $t = 0$. **Equation (26)** requires that the distribution of money at $t = 1$ is feasible; that is, money distributed to households at $t = 1$ cannot exceed money holdings M_0 at $t = 0$. **Equation (27)** refers to the optimal consumption choice made by households, given the money distribution rule $M(\varepsilon_1^h, \psi)$. Unlike classic banking models where the planner distributes goods at $t = 1$, the island-level planner here distributes money, and then households choose how to allocate such money between consumption expenditure (subject to the cash-in-advance constraint) and savings between $t = 1$ and $t = 2$. **Equation (28)** requires that the nominal value of the time-2 consumption goods distributed to households at $t = 2$ (i.e., $P_2 \int Y_2(\varepsilon_1^h, \psi) dh$) does not ex-

ceed the the gross return of the resources invested in physical capital and the money not distributed at $t = 1$, if any, given by $M_0 - \int M(\varepsilon_1^h, \psi) dh$.¹⁹ Equation (29) is the incentive-compatibility (IC) constraint that must be satisfied so that households truthfully report the realization of their own preference shock ε_1^h to the island-level planner. Finally, Equation (30) states that time-2 consumption is the sum of goods received from the island-level planner, $Y_2(\varepsilon_1^h, \psi)$, plus the real value of unspent money, $[M(\varepsilon_1^h, \psi) - P_1 C_1(\varepsilon_1^h, \psi)] / P_2$.

To solve the island-level planner problem, I proceed as follows. First, I note that for an island hit by the low shock $\psi^L = -1$, the stock of capital is destroyed, and thus no time-2 goods produced by capital are available; that is, Equation (28) and the non-negativity constraints imply $Y_2(\varepsilon^H, \psi^L) = 0$ and $Y_2(\varepsilon^L, \psi^L) = 0$. Thus, the IC constraint (29) simplifies to

$$M(\varepsilon^L, \psi^L) \geq M(\varepsilon^H, \psi^L)$$

and holds with equality at the planner's optimal choice. Thus, using (26), we have

$$M(\varepsilon^H, \psi^L) = M(\varepsilon^L, \psi^L) = M_0. \quad (31)$$

In addition, I conjecture (and later verify) that the cash-in-advance constraint for an impatient household on a island hit by the low shock ψ^L is binding and that the cash-in-advance constraint is not binding if the household is patient. Thus, (27) implies that consumption of impatient and patient households is determined by

$$P_1 C_1(\varepsilon^H, \psi^L) = M_0, \quad (32)$$

$$\varepsilon^L u'(C_1(\varepsilon^L, \psi^L)) \frac{1}{P_1} = \frac{1}{P_2}, \quad (33)$$

respectively. To analyze the implications of the binding cash-in-advance constraint for impatient households, let $P_1 \mu_1(\varepsilon_1^h, \psi) \geq 0$ be the Lagrange multiplier of the cash-in-advance constraint $P_1 c \leq M(\varepsilon_1^h, \psi)$. The planner's first-order conditions for the allocation of money in the ψ^L -island imply:

$$\varepsilon^H u'(C_1(\varepsilon^H, \psi^L)) \frac{1}{P_1} - \mu_1(\varepsilon^H, \psi^L) = \varepsilon^L u'(C_1(\varepsilon^L, \psi^L)) \frac{1}{P_1}. \quad (34)$$

Because of the binding IC constraint, the island-level planner cannot redistribute money toward impatient households, and thus there is a misallocation of time-1 consumption that manifests through a cash-in-advance constraint that is binding only for impatient households. This result is similar to

¹⁹Without loss of generality, the analysis of this section can be restricted to mechanisms in which (26) holds with equality and thus the planner distributes all the money at $t = 1$. However, allowing for mechanisms in which (26) is slack and thus some money is distributed at $t = 2$ is crucial when studying efficiency.

that of the bankless economy, even though it holds here only for islands hit by ψ^L .

For an island hit by the good shock ψ^H , I show next that the island-level planner can successfully reallocate money toward impatient households, thereby providing insurance against preference shocks. For such an island, I conjecture (and later verify) that the IC constraint [Equation \(29\)](#) is not binding. The optimal distribution of money at $t = 1$ is such that the marginal utility of consumption is equalized across all agents, that is,

$$\varepsilon^H u' (C_1 (\varepsilon^H, \psi^H)) = \varepsilon^L u' (C_1 (\varepsilon^L, \psi^H)). \quad (35)$$

Because of the linearity of households' consumption at $t = 2$, and so long as the IC constraint is slack, there are possibly many allocations of time-2 consumption that are optimal. Note that the optimality in [\(35\)](#) holds independently whether the cash-in-advance constraint of households binds or not. Letting $P_1 \mu_1 (\varepsilon_1^h, \psi) \geq 0$ be the Lagrange multiplier of the cash-in-advance constraint $P_1 c \leq M (\varepsilon_1^h, \psi)$, and using [\(27\)](#), the household's optimal consumption expenditure at $t = 1$ solves

$$\varepsilon_1^h u' (C_1 (\varepsilon_1^h, \psi^H)) \frac{1}{P_1} = \frac{1}{P_2} + \mu_1 (\varepsilon_1^h, \psi^H) \quad (36)$$

and thus [\(35\)](#) implies that the value of the Lagrange multiplier of the budget constraint is equalized across households of a ψ^H -island:

$$\mu_1 (\varepsilon^H, \psi^H) = \mu_1 (\varepsilon^L, \psi^H). \quad (37)$$

Thus, so long as the island-level planner faces a non-binding IC constraint, such a planner is able to eliminate a key distortion that arises in the economy with no banks, namely, that the value of the Lagrange multiplier is not equalized among ex-ante identical households.

The last step of the island-level planner problem is to solve for the time-0 portfolio allocation among money and capital. The optimal choice implies

$$\mathbb{E}_{\varepsilon, \psi} \{ \varepsilon_1^h u' (C_1 (\varepsilon_1^h, \psi)) \} \frac{1}{P_1} = \frac{1}{P_2} \mathbb{E}_{\psi} \{ 1 + r_2^K (\psi) \}. \quad (38)$$

That is, on the margin, the island-level planner must be indifferent between investing an extra dollar in money, which allows to purchase $1/P_1$ units of consumption goods at $t = 1$ which in turn gives marginal utility $\mathbb{E}_{\varepsilon, \psi} \{ \varepsilon_1^h u' (C_1 (\varepsilon_1^h, \psi)) \}$ per unit, or in physical capital that provides an expected return $\mathbb{E}_{\psi} \{ 1 + r_2^K (\psi) \}$.

The equilibrium definition is similar to that of the bankless economy, and the equilibrium variables are determined by [Equations \(26\)](#) evaluated at $\psi = \psi^H$, [\(28\)-\(30\)](#), [\(31\)-\(34\)](#), [\(36\)](#) evaluated at $\varepsilon_1^h \in \{ \varepsilon^H, \varepsilon^L \}$, [\(36\)-\(38\)](#), and the market clearing for money and capital at $t = 0$, and for

consumption goods at $t = 1$.

At $t = 0$, all islands are alike and, thus, market clearing implies $M_0 = M_0^S$ and $K_0 = \bar{K}$ for all island-level planners. At $t = 1$, the equilibrium allocation depends on the value of the intertemporal elasticity σ .

If $\sigma \in (0, +\infty)$, consumption for households on an island hit by the good shock to capital, ψ^H , is

$$C_1(\varepsilon^H, \psi^H) > (C_1^H)^*, \quad C_1(\varepsilon^L, \psi^H) > (C_1^L)^*$$

and consumption for households on a island hit by the bad shock to capital, ψ^L , is

$$C_1(\varepsilon^H, \psi^L) < (C_1^H)^*, \quad C_1(\varepsilon^L, \psi^L) > (C_1^L)^*,$$

where $(C_1^H)^*$ and $(C_1^L)^*$ denote the first-best level of consumption, which are the same as those in [Section 2.3](#). The consumption allocation in a ψ^L -island is qualitatively similar to that of the bankless economy. On islands hit by ψ^L , the island-level planner does not have the ability to redistribute money because the IC constraint is binding, and thus a wedge between the marginal utility of patient and impatient households arises. Different, on an island hit by the good shock to capital ψ^H , the consumption allocation is efficient within the island. Nonetheless, agents on such an island consume too much in comparison to the first best because of a general equilibrium effect. Some of the households on the islands hit by ψ^L are patient, and yet they receive from the planner at $t = 1$ more money than they need to truthfully report their own type. These patient households consume only a fraction of the money they receive, and store the rest until $t = 2$. The lower consumption expenditure of these households depresses the price of consumption goods and allows households on islands hit by ψ^H to consume more than in the first best.

If instead $\sigma \rightarrow 0$, the consumption allocation is the same as in the first best because all households want to consume the same amount, independently of their own type, similar to the bankless economy. As a result, the planner distributes the same amount of money to all households, irrespective of the realization of the idiosyncratic shocks to capital.

The result for the price level is similar to the bankless economy:

$$P_1 \rightarrow \begin{cases} M_2^S \frac{\varepsilon^L}{\varepsilon^H} < P_2 & \text{if } \sigma \rightarrow \infty \\ M_0^S [1 - \alpha(1 - \kappa)] + M_2^S \alpha(1 - \kappa \varepsilon^H) & \text{if } \sigma \rightarrow 1 \\ M_0^S & \text{if } \sigma \rightarrow 0. \end{cases}$$

If $\sigma \rightarrow \infty$, the price level P_1 depends only on money supply at $t = 2$, M_2^S , and does not depend on the money supply at $t = 0$. If $\sigma \rightarrow 0$, it depends only on M_0^S . And in the intermediate case,

$\sigma \rightarrow 1$, P_1 depends on both M_0^S and M_2^S . Qualitatively, the results about P_1 are identical to those of the bankless economy.

Finally, I study whether the cash-in-advance constraint is binding for households on a ψ^H -islands. This will be important for the analysis of monetary injections because the results will depend on whether the cash-in-advance constraint binds or not for ψ^H -islands. Whether or not the constraint is binding in equilibrium depends on the value of the intertemporal elasticity σ . I first state the results and then provide some comments.

Proposition 4.1. *Assume $M_0^S = M_2^S = \overline{M}$. Then:*

- *If $\sigma \rightarrow 0$, the cash-in-advance constraint of households on a ψ^H -island is not binding;*
- *If $\sigma \rightarrow 1$, the cash-in-advance constraint of households on a ψ^H -island holds with equality but is not binding (i.e., its Lagrange multiplier is zero);*
- *If $\sigma \rightarrow \infty$, the cash-in-advance constraint of households on a ψ^H -island is binding.*

Under the assumption of a constant money supply, and as $\sigma \rightarrow 0$, the cash-in-advance constraint is not binding. Recall that, with $\sigma \rightarrow 0$, households' utility is quasi-Leontief and thus, given prices, households have enough money to finance the desired consumption expenditure at $t = 1$. By contrast, the cash-in-advance constraint is binding when $\sigma \rightarrow \infty$ (i.e., linear utility). In this case, because $P_1 < P_2$, impatient households would find it optimal to consume more if they had extra money, and thus the cash-in-advance constraint binds. In the intermediate case with $\sigma \rightarrow 1$, the cash-in-advance constraint holds with equality but is not binding, in the sense that its Lagrange multiplier is not zero. In addition, and focusing on the case in which the money supply is constant at $M_0^S = M_2^S = \overline{M}$, I can show numerically that a slack cash-in-advance constraint arises also for $\sigma \in (0, 1)$, and a binding cash-in-advance constraint arises also for $\sigma \in (1, \infty)$.

4.3 Implementing the island-planner equilibrium with banking contracts

I now turn to the implementation of the equilibrium of the island-level planner economy using banking contracts. I show that there are many banking contracts that implement the same equilibrium, but I focus on one implementation that allows me to match some stylized facts in the data and that is robust to a simple extension.

The simplest type of contract is one in which a household deposits all its endowment in a bank, and the bank distributes money $P_1 C_1(\varepsilon_1^h, \psi)$ at $t = 1$ and goods $C_2(\varepsilon_1^h, \psi)$ at $t = 2$. By choosing $C_1(\varepsilon_1^h, \psi)$ and $C_2(\varepsilon_1^h, \psi)$ equal to those that arise in [Section 4.2](#), the contract is trivially optimal.

However, this is not the only optimal contract. There are other optimal contracts that deliver the following result: households deposit only a fraction of their own endowment at the respective bank, and manage the remaining part by directly holding money and capital. Characterizing this second type of optimal contract is crucial to match the data. For instance, to study the Great

Depression, this second type of contract replicates the stylized fact that the private sector held currency directly before the Depression (i.e., before 1929), and it increased its holdings during the Depression (i.e., between 1929 and 1933).²⁰ In addition, and even in recent times, not all resources are intermediated through the banking system, and thus it is reasonable for a general equilibrium model that describes the whole economy to consider a limited role for banks' intermediation.

The multiplicity of banking contracts that implements the island-planner choices leaves open the question of the equilibrium value of deposits predicted by the model. Such a value varies depending on whether households deposit a large or small fraction of their own endowments. However, there is a simple extension of the model that delivers a unique equilibrium value of deposits, which I sketch briefly. If banking technology is costly and banks charge a proportional fee on deposits to cover such costs, then the fee represents a force that reduces the amount of deposits held in the banking system. In the limit in which the costs faced by banks is positive but arbitrarily small (i.e., it converges to zero), depositing the least amount of endowment required to insure against preference shocks is preferred, and the contract that implements this allocation is the one that arises in equilibrium.

Motivated by the above observations, I now characterize the optimal contract in which households deposit the minimum fraction of their endowment that is required to implement the island-level-planner outcome, and invest the remainder in money and capital directly. Let M_0^h , D_0^h , and K_0^h be the amount of money, deposits, and capital chosen by households at $t = 0$, which must satisfy the budget constraint

$$M_0^h + D_0^h + Q_0 K_0^h \leq \bar{M} + Q_0 \bar{K} + T_0.$$

Recall that deposits D_0^h are held only at the bank on the same island where the household is located, in line with the assumption that the household has access to only one island-level planner.

I search for the contract such that impatient households of an island hit by ψ^H withdraw all their deposits at $t = 1$ and are able to finance the consumption $C_1(\varepsilon^H, \psi^H)$ chosen by island-level planner of [Section 4.2](#):

$$\frac{M_0^h + D_0^h}{P_1} = C_1(\varepsilon^H, \psi^H). \quad (39)$$

and patient households on an island hit by ψ^H do not withdraw any money, so that their cash-in-advance constraint is $P_1 C_1(\varepsilon^L, \psi^H) \leq M_0^h$. The budget constraint of bank b is

$$M_0^b + Q_0 K_0^b \leq D_0^b \quad (40)$$

where M_0^b and K_0^b denote the bank's holdings of money and capital, and D_0^b denotes the sum of

²⁰See [Friedman and Schwartz \(1963\)](#).

deposits of households that are located on the same island as b :

$$D_0^b = \int_{\{h \mid h \text{ is on the same island as } b\}} D_0^h dh. \quad (41)$$

The objective of bank b is to choose M_0^b and K_0^b in order to (i) offer the optimal contract to depositors, and (ii) satisfy [Equation \(39\)](#).²¹ The next proposition characterizes the choice of the bank.

Proposition 4.2. *The unique banking contract that implements the island-level planner's solution and satisfies (39)-(41) is such that banks choose $M_0^b = \kappa D_0^b$.*

As a result of the previous proposition, banks invest a fraction κ of deposits in money (recall that κ is also the fraction of impatient households). Thus, the deposits contract is such that all the money held by bank at $t = 0$ is withdrawn at $t = 1$ by impatient households only, provided that the island is hit by the shock to capital ψ^H . If instead the island is hit by ψ^L , the bank distributes money equally to all of its depositors to elicit truthful revelation of depositors' types.

4.4 Flight to liquidity

As discussed in the Introduction, a feature of financial crises associated with bank runs is an increase in the private sector's holdings of liquid assets and a drop in the resources intermediated by banks and other financial intermediaries, that is, a flight to liquidity. This section shows that the model produces a flight to liquidity if and only if the cash-in-advance constraints of households on a ψ^H -island are slack. By combining this result with that of [Proposition 4.1](#), I conclude that a flight to liquidity arises if households are sufficiently risk-averse.

First, I formally define a flight to liquidity in the model by comparing the results of an economy in which $\alpha > 0$ (i.e., in which some islands are hit by the shock ψ^L) to an economy in which $\alpha = 0$ (i.e., in which there are no idiosyncratic shocks to capital and thus no runs).

Definition 4.3. *Given a money supply M_0^S , M_2^S , an equilibrium with $\alpha > 0$ displays a flight to liquidity if households hold more money and less deposits at $t = 0$ in comparison to an otherwise identical economy with no runs (i.e., with $\alpha = 0$); that is, if $M_0^h|_{\alpha>0} > M_0^h|_{\alpha=0}$ and $D_0^h|_{\alpha>0} < D_0^h|_{\alpha=0}$.*

There are two forces in the model that affect households' money-holdings decisions: the possibility of runs and the opportunity cost of holding money. The possibility of runs increases households' willingness to hold money to self-insure against the risk of facing a run (i.e., against the

²¹It is possible to reformulate the problem of banks as the choice over contracts that maximize profits. However, because of perfect competition in the banking sector, the optimal contract implies that banks earn zero profits and offer a contract that implements the same outcome of the island-level planner.

risk that the island where they live is hit by the shock ψ^L). However, households' decisions to hold money depends also on the opportunity cost, represented by the expected return on capital, $\mathbb{E}_{\psi} \{r_2(\psi)\}$.

I claim that a flight to liquidity arises when the risk aversion of households, $1/\sigma$, is sufficiently high. Intuitively, a high risk aversion implies that the possibility of runs is a main concern of households, so that households increase money holdings by reducing deposits, despite the opportunity cost of holding money.

The next proposition shows that a flight to liquidity arises if and only if the cash-in-advance constraints of households on ψ^H -island are slack. This proposition characterizes the existence of a flight to liquidity in terms of another endogenous object (i.e., whether or not some cash-in-advance constraints are binding), rather than in terms of the primitives of the model. However, we know from [Proposition 4.1](#) and the related discussion what the conditions are under which these cash-in-advance constraints are slack. That is, these constraints are slack when the intertemporal elasticity of substitution σ is low or, equivalently, the risk aversion $1/\sigma$ is high.

Proposition 4.4. *Let $\alpha > 0$. The equilibrium displays a flight to liquidity if and only if the cash-in-advance constraints of households on islands hit by ψ^H are slack (i.e., if these constraints hold with inequality).*

4.5 Monetary injections in the economy with banks

I now analyze temporary monetary injections in the economy with banks. The effects of a small temporary monetary injection on prices, consumption allocation, and welfare are similar to that of [Section 3.4](#), so long as some bank is subject to runs, and are similarly affected by the value of the intertemporal elasticity σ . Temporary injections have only real effects if $\sigma \rightarrow 0$, only nominal effects if $\sigma \rightarrow \infty$, or both if $\sigma \in (0, \infty)$. If instead no bank is subject to runs (i.e., $\alpha = 0$), monetary policy is neutral. However, there are some new results that are derived in this section and that are related to the response of money held by households and deposits. Money held by households always increases, whereas deposits may increase or decrease.

I begin by analyzing the benchmark case in which there are no runs, that is, the fraction α of islands hit by the shock ψ^L is zero. In this case, banks provide full insurance against preference shocks, similar to the good equilibrium of [Diamond and Dybvig \(1983\)](#), and temporary monetary injections that marginally change the money supply at $t = 0$ are neutral.

Proposition 4.5. *If $\alpha \rightarrow 0$, there exists an equilibrium that implements the first best. If, in addition, $M_2^S > M_0^S$, temporary monetary injections are neutral, that is,*

$$\eta(P_1, M_0^S) = 1, \quad \eta(v, M_0^S) = 0, \quad \eta(\mathbb{E}\{r_2^K(\psi)\}, M_0^S) < 0,$$

$$\eta(C_1(\varepsilon_1^h, \psi), M_0^S) = 0 \text{ for all } (\varepsilon_1^h, \psi), \quad \eta(M_0^h, M_0^S) = 1, \quad \eta(D_0^h, M_0^S) = 1.$$

In an economy with no runs, a temporary monetary injection increases nominal prices at $t = 0$, that is, Q_0 and P_1 , and reduces the opportunity cost of holding money, $\mathbb{E}\{r_2^K(\psi)\}$. However, households do not change the real amount of money and deposits held by households (i.e., the nominal quantity of money and deposits, M_0^h and D_0^h , increase one-for-one with the monetary injection to keep the real quantities, M_0^h/P_1 and D_0^h/P_1 , unchanged). This is because if there are no runs, households hold the minimum amount of money in their wallets—they hold the money necessary to finance consumption expenditure if they turn out to be patient at $t = 1$ —and a change in the opportunity cost of holding money does not alter households' portfolio decisions.²²

The results of [Proposition 4.5](#) are a starting point to understand also the effects of monetary injections in an economy with runs. If some bank is subject to runs, the effects on temporary injections on money M_0^h and deposits D_0^h crucially depends on whether households on islands hit by ψ^H spend all their money or not. If they spend all their money because their cash-in-advance constraints are binding, the results are similar to those of [Proposition 4.5](#). With binding cash-in-advance constraints, households in an island hit by ψ^H want to use the extra money injected by the central bank to increase their consumption expenditure at $t = 1$. The best way to do so is a mix of higher money holdings and higher deposits at $t = 0$. In a sense, when cash-in-advance constraints are binding for households on ψ^H -islands, most of the households in the economy spend all their money at $t = 1$, similar to what happens in an economy with no runs, and thus monetary injections increase both money and deposits.

The results are quite different, however, if households on islands hit by ψ^H do not spend all their money. In this case, the cash-in-advance constraints of these households are slack. To gain intuition, assume for a second that a temporary monetary injection does not change the price level P_1 . If that were the case, households that face a non-binding cash-in-advance constraint would not change their consumption expenditure. However, given the monetary injection at $t = 0$, it is now feasible for them to hold more money and fewer deposits and still be able to achieve the same consumption expenditure. Allowing the price level P_1 to adjust to the monetary injection does not change the result. Even abstracting from the special case with linear utility, most of the new money injected is unspent anyway, and thus the impact on the price level P_1 is limited.

I can now state these results formally.

Proposition 4.6. *Let $\alpha > 0$ and assume that the central bank implements a temporary monetary injection, that is, it marginally increases M_0^S . If the cash-in-advance constraint of households on islands hit by ψ^H is binding (i.e., $\mu_1(\varepsilon^H, \psi^H), \mu_1(\varepsilon^L, \psi^H) > 0$), both money held by households,*

²²The assumption $M_2^S > M_0^S$ is required to make sure that the nominal interest rate $\mathbb{E}\{r_2^K(\psi)\}$ is bounded away from zero, so that a small monetary injection can reduce it without hitting the zero lower bound.

M_0^h , and deposits, D_0^h , increase. If instead the cash-in-advance constraint of households on islands hit by ψ^H are not binding (i.e., $\mu_1(\varepsilon^H, \psi^H) = \mu_1(\varepsilon^L, \psi^H) = 0$), money held by households, M_0^h , increases and deposits, D_0^h , decreases.

As discussed at the end of [Section 4.2](#), cash-in-advance constraints are not binding if households are sufficiently risk averse. In particular, I can formally prove more precise results for the case of log utility (i.e., $\sigma \rightarrow 1$). When $\sigma \rightarrow 1$ and money supply is constant at $M_0^S = M_2^S = \bar{M}$, [Proposition 4.1](#) shows that the cash-in-advance constraints on ψ^H -islands hold with equality but are not binding, in the sense that their Lagrange multipliers are zero. This case is important because if I consider an economy with an arbitrary small difference in σ or in the money supply, the cash-in-advance constraints are either binding or slack. If risk aversion is marginally lower (i.e., $\sigma < 1$) or money at $t = 0$ marginally higher (i.e., the $M_0^S > \bar{M}$), the cash-in-advance constraints are not binding, and vice versa.²³

Proposition 4.7. Assume $\sigma = 1$, $\alpha > 0$, and $M_2^S = \bar{M}$. If $M_0^S = \bar{M}$,

$$\frac{d_- D_0^h}{dM_0^S} > 0, \quad \frac{d_- M_0^h}{dM_0^S} > 0, \quad \frac{d_+ D_0^h}{dM_0^S} < 0, \quad \frac{d_+ M_0^h}{dM_0^S} > 1$$

where d_- and d_+ denotes the left- and right-derivative, respectively. If $M_0^S > \bar{M}$:

$$\frac{dD_0^h}{dM_0^S} < 0, \quad \frac{dM_0^h}{dM_0^S} > 1,$$

and if $M_0^S < \bar{M}$

$$\frac{dD_0^h}{dM_0^S} > 0, \quad \frac{dM_0^h}{dM_0^S} > 0.$$

4.6 (In)efficiency: constrained first best and pecuniary externality

Similar to the bankless economy, the decentralized equilibrium with runs on some banks is constrained-inefficient because of a pecuniary externality. Since banks offer the optimal contract, one might be tempted to conclude that the equilibrium is efficient, as in most models in which runs are driven by fundamentals and banks offer the optimal contract, such as [Allen and Gale \(1998\)](#). This conclusion, however, is incorrect.

The analysis is similar to that of [Section 3.6](#), but is slightly more complicated by the presence of idiosyncratic shocks to capital and banks. In this section, the economy-wide planner with limiting planning abilities chooses the allocation of money and deposits at $t = 0$, and the truth-telling mechanisms to distribute money at $t = 1$ and goods at $t = 2$ on each island. This is slightly

²³The result about a marginally lower or higher σ can only be shown numerically.

different from [Section 3.6](#), in which the planner instructed households about the amount of money to be spent at $t = 1$. However, this approach clarifies that the mechanisms set-up by the island-level planner are socially suboptimal. Similar to [Section 3.6](#), households purchase consumption goods at $t = 1$ in a competitive market subject to a cash-in-advance constraint, and the objective function of the economy-wide planner's is utilitarian with the same weights on all households.

Given the above considerations, the objective function of the economy-planner is the same as that of island-level planners.²⁴ There are however, two differences in the constraints. First, the budget constraint (25) is replaced by two feasibility conditions: $M_0 \leq \bar{M}$, and $K_0 \leq \bar{K}$. Second, and more importantly, the constraints (26)-(30) are unchanged, but P_1 should be replaced with $P_1 \left(\{M(\varepsilon_1^h, \psi)\}_{\varepsilon_1^h \in \{\varepsilon^H, \varepsilon^L\}, \psi \in (\psi^H, \psi^L)} \right)$ to emphasize that the economy-wide planner internalizes the effects of the mechanism—and, in particular, the effects of the time-1 allocation of money $\{M(\varepsilon_1^h, \psi)\}$ —on the price level at $t = 1$. Nonetheless, to simplify the notation, I will omit the argument of P_1 in what follows.

Given these premises, the optimality conditions of the economy-wide planner imply

$$\varepsilon_1^h u' (C_1(\varepsilon_1^h, \psi^H)) \frac{1}{P_1} = \frac{1}{P_2} + \mu_1(\psi^H) + \frac{1}{(1-\alpha)Pr(\varepsilon_1^h)} \frac{dP_1}{dM(\varepsilon_1^h, \psi^H)} \int \left[\left(\varepsilon_1^{\tilde{h}} u' (C_1^{\tilde{h}}) \frac{1}{P_1} - \frac{1}{P_2} \right) C_1^{\tilde{h}} \right] d\tilde{h}$$

for $\varepsilon_1^h \in \{\varepsilon^H, \varepsilon^L\}$, and

$$\varepsilon^H u' (C_1(\varepsilon^H, \psi^L)) \frac{1}{P_1} = \frac{1}{P_2} + \mu_1(\psi^L) + \frac{1}{\alpha\kappa} \frac{dP_1}{dM(\varepsilon^H, \psi^L)} \int \left[\left(\varepsilon_1^{\tilde{h}} u' (C_1^{\tilde{h}}) \frac{1}{P_1} - \frac{1}{P_2} \right) C_1^{\tilde{h}} \right] d\tilde{h}.$$

Moreover, $M(\varepsilon^L, \psi^L) = M(\varepsilon^H, \psi^L)$ must hold to satisfy the incentive compatibility constraint of ψ^L -islands.

The first-order conditions of the economy-wide planner are conceptually similar to those of [Section 3.6](#), but their formulation is more complicated. Similar to [Section 3.6](#), the key difference with the decentralized equilibrium lies in the dissimilarities between private and social marginal costs. In this section, the social costs include the effect of distributing an extra unit of money on the price level P_1 and on the utility of constrained households. The latter element depends on the difference between the marginal utility of a constrained household $\tilde{h}$, $\varepsilon_1^{\tilde{h}} u' (C_1^{\tilde{h}})$, and the value that such marginal utility would take if the cash-in-advance constraint were not binding, $1/P_2$,

²⁴The choices M_0 and K_0 should be interpreted as the amount of money and capital per island, respectively, after imposing that all islands get the same money and capital at $t = 0$ because of the equal weights.

weighted by the amount $C_1^{\tilde{h}}$ consumed by this household.

Similar to the bankless economy, the decentralized equilibrium is characterized by a cross-sectional heterogeneity in the value of the households' Lagrange multipliers of the cash-in-advance constraint. In this economy with banks and runs, the cash-in-advance constraint is more binding for impatient households on islands hit by ψ^L (i.e., households that face runs), in comparison to households on islands hit by ψ^H (i.e., households that do not face runs) and patient households on islands hit by ψ^L . The planner can alter the mechanism set up by the island-level planner to redistribute resources from richer to poorer islands. In comparison to the decentralized equilibrium, the economy-wide planner reduces the money that is distributed at $t = 1$ to households on islands hit by ψ^H , thereby reducing their consumption. As a result, the amount of money spent at the economy-wide level decreases, and the price level P_1 (which is proportional to money spent) is lower. The lower P_1 allows impatient households on an island with runs to increase consumption, thereby relaxing their cash-in-advance constraint. The welfare gains of this process are positive because the gains of households with very binding cash-in-advance constraints are greater than the losses of households with a less binding or non-binding constraint.

5 Conclusion

I have presented a monetary, general equilibrium model to study monetary injections in an economy with preference shocks, banks, and, possibly, bank runs. If banks are well-functioning and there are no runs, all monetary injections are neutral. If instead banks are subject to runs or are not active, permanent injections are neutral, but temporary injections are, in general, non-neutral. The real effects of monetary injections depend on households' elasticity of real money demand, which is in turn affected by households' intertemporal elasticity of substitution. In addition, the equilibrium is generically constrained-inefficient because of a pecuniary externality related to binding cash-in-advance constraints.

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Appendix

A More on temporary injections

This appendix provides two alternative explanations to understand the effects of temporary injections when $\sigma \rightarrow \infty$ (i.e., under linear utility). First, I deconstruct a temporary injection at $t = 0$ into two separate interventions that I analyze separately: a permanent injection at $t = 0$ and a reduction of money supply implemented at $t = 2$ but announced at $t = 0$. Second, I appeal to the mathematical structure of the model and the equations that define the equilibrium.

A.1 Deconstructing a temporary injection into two separate interventions

I now deconstructs a temporary injection at $t = 0$ in the bankless economy into two separate interventions and analyzes them separately: a permanent injection implemented at $t = 0$ and a reduction of money supply at $t = 2$. Crucially, the reduction of money supply in the second intervention is

announced at $t = 0$ before the Walrasian market opens, and thus is fully anticipated. I focus on the case $\sigma \rightarrow \infty$ (i.e., linear utility), in which a temporary injection affects only allocations, but the logic can be extended to any $\sigma > 0$.

As shown in [Proposition 3.5](#), a permanent monetary injection does not affect money velocity, and thus prices respond one-for-one with the monetary injection. More precisely, the elasticity of nominal prices with respect to the monetary injection is one. Since a permanent injection has purely nominal effects, it does not affect consumption allocations or welfare.

Next, I turn to the analysis of an anticipated reduction of money at $t = 2$.²⁵

Proposition A.1. *(Monetary injection at $t = 2$, linear utility) If $\sigma \rightarrow \infty$ and the central bank is not implementing the Friedman rule, then:*²⁶

$$\begin{aligned} \eta(P_1, M_2^S) &\rightarrow 1, & \eta(Q_0, M_2^S) &\rightarrow 1, & \eta(v, M_2^S) &\rightarrow 1, & \eta(r_2^K, M_2^S) &\rightarrow 0 \\ \eta(C_1^H, M_2^S) &\rightarrow -1, & \eta(C_1^L, M_2^S) &\rightarrow \frac{\kappa}{1 - \kappa} \frac{C_1^H}{C_1^L} < 0. \end{aligned}$$

Using [Equation \(10\)](#), a reduction of money at $t = 2$ triggers a one-for-one reduction in P_2 . Crucially, I claim that P_1 and Q_0 must respond one-for-one with M_2^S as well, so that the interest rate $1 + r_2^K$ defined in [Equation \(12\)](#) does not change. To see this, recall from [Section 3.3](#) and [Equation \(17\)](#) that households' money demand with respect to its opportunity cost $1 + r_2^K$ is a crucial element to understand monetary injections. In the limiting case $\sigma \rightarrow \infty$, households' money demand elasticity converges to infinity. Thus, any variation in the opportunity cost $1 + r_2^K$ would produce a large swing in money demand, violating market clearing in the money market. Thus, the opportunity cost $1 + r_2^K$ must be unaffected by any policy intervention—including a reduction in the money supply M_2^S at $t = 2$. Because the monetary injection at $t = 2$ increase P_2 one-for-one, Q_0 and P_1 must respond one-for-one as well to keep $1 + r_2^K$ unchanged.

A.2 Understanding temporary injections using the equilibrium equations

The second explanation of [Proposition 3.3](#) is based on the mathematical structure of the model. Note that households' time-1 consumption C_1^H and C_1^L change in response to a temporary monetary injection despite prices remaining unchanged. This is because, given P_2 , the households' first-order

²⁵Note that [Proposition A.1](#) presents the elasticity of endogenous variables with respect to a marginal increase of money at $t = 2$. To understand the effects of a reduction of money at $t = 2$, the signs of the results in [Proposition A.1](#) must be flipped.

²⁶The elasticity of the price level P_1 is defined as $\eta_{P_1, M_2^S} = (dP_1/dM_2^S) \times (M_2^S/P_1)$, and the other elasticities are defined similarly.

conditions with respect to C_1^H and C_1^L in (15) and (16) become, as $\sigma \rightarrow \infty$,

$$\left[\kappa + (1 - \kappa) \frac{\varepsilon^L}{\varepsilon^H} \right] \frac{1}{P_1} = \frac{1}{Q_0} \left(\frac{A_1 P_1}{P_2} + A_2 \right)$$

$$\frac{1}{P_1} \frac{\varepsilon^L}{\varepsilon^H} = \frac{1}{P_2}.$$

Given P_2 , the two above equations depend only on prices Q_0 and P_1 , and are independent of the consumption allocation because households' utility is linear in consumption. Moreover, the first-order conditions are independent of the money supply M_0^S too, because M_0^S affects the money market clearing but does not enter directly into the households' utility maximization problem. As a result, the price of capital Q_0 and the price level P_1 that sustain the two first-order conditions do not depend on M_0^S , and thus a temporary monetary injection does not change Q_0 or P_1 .

This mathematical structure provides guidance for the intuition behind the results, not only for the case $\sigma \rightarrow 0$ but also for the more general case $\sigma > 0$. Temporary monetary injections that vary M_0^S affect the equilibrium allocation even though, in the limiting case $\sigma \rightarrow 0$, they do not change prices. Thus, the best and more general way to provide intuition for the result of a temporary injection is to either appeal to how endogenous variables should react to a monetary injection to clear the money market, as I do in the main text of the paper, or to deconstruct a such a policy into a permanent injection at $t = 1$ and a contraction of money at $t = 2$.