

Reserve Accumulation, Foreign Direct Investment, and Economic Growth

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Abstract

This paper develops a quantitative small-open-economy model to assess the optimal pace of foreign reserve accumulation by developing countries. The model features endogenous growth with foreign direct investment (FDI) entry and sudden stops of capital inflows to incorporate benefits of reserve accumulation. Reserve accumulation depreciates the real exchange rate and attracts FDI, which endogenously promotes productivity growth. When a sudden stop happens, the government uses accumulated reserves to prevent a severe economic downturn. The calibrated model shows that two factors are the key determinants of the optimal pace of reserve accumulation: the elasticity of the foreign borrowing spread with respect to debt, and the entry cost for FDI. The model suggests that these two factors can explain a substantial amount of the cross-country variation in the observed pace of reserve accumulation.

Keywords: Sudden stop, Endogenous growth, Gross capital flows

JEL code: F23, F31, F32, F41, F43

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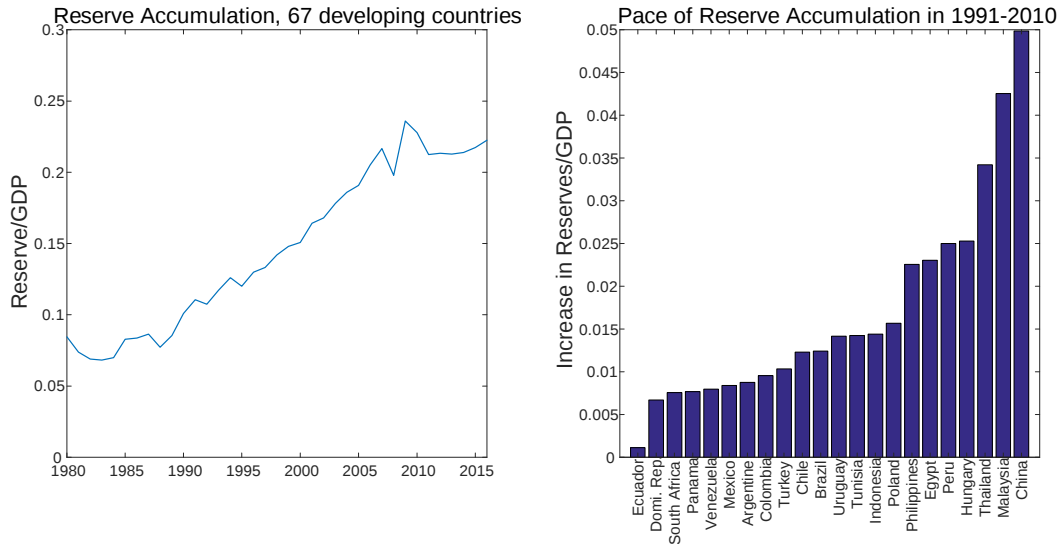
1 Introduction

The active accumulation of foreign reserves by developing countries, especially those in East and Southeast Asia, is one of the most prominent developments in the international financial system in the past 25 years. The left panel in Figure 1 shows that the average reserve-to-GDP ratio across 67 developing countries has increased from less than 10% before 1990 to almost 25% by 2010. While many developing countries have built up reserve holdings in this period, there is a wide cross-country variation in how quickly these countries have accumulated reserves. The right panel in Figure 1 shows the average increase in reserve holdings in terms of the ratio to GDP across developing countries in 1991-2010. It can be observed that Asian countries such as China, Malaysia, and Thailand have been accumulating reserves equivalent to 3.5-5% of GDP on average, while many Latin American countries are accumulating reserves less than 1% of GDP. Although the optimal reserve policy has been an active research area and a central policy question throughout the past decade, we still know little about what the optimal pace of reserve accumulation is, and why different countries accumulate reserves at different rates.

This paper develops a quantitative small-open-economy model to study the optimal pace of reserve accumulation by developing countries. The main novelty of this paper is twofold. First, the model incorporates the key benefits and costs of reserve accumulation into a quantitative framework. On the benefit side, the existing literature has identified two benefits of reserve accumulation: a growth-promoting effect and a precautionary effect. The growth-promoting effect goes through depreciating the real exchange rate and attracting foreign direct investment (FDI). The precautionary effect is that reserve holdings help to stabilize the economy in the face of volatile capital flows. Most existing theoretical papers studying optimal reserve policy incorporate only one of these effects. On the cost side, reserve accumulation crowds out domestic investment.¹ The second novel contribution is that this paper addresses why different countries accumulate reserves at different paces, instead of trying to identify the unique optimal pace of reserve accumulation for a representative developing country.

¹Crowding-out of investment resulting from reserve accumulation is documented by Reinhart, Reinhart, and Tashiro (2016) at the macro level, and empirically shown by Cook and Yetman (2012) at the micro level.

Figure 1: Reserve Accumulation by Developing Countries



The model is a small open economy with tradable and non-tradable sectors. The economy starts with scarce capital, accumulates capital by borrowing from abroad, and grows rapidly. The focus of the model is how reserve policy should be conducted during this transition period. To capture the two benefits of reserve accumulation mentioned above, I introduce two features into the model: endogenous growth with FDI entry and sudden stops of capital inflows. Endogenous growth is introduced to study the growth-promoting effect of reserve accumulation. The model framework is a version of the Schumpeterian growth model in which intermediate goods-producing firms endogenously innovate and increase aggregate productivity. I introduce FDI into the framework in order to capture the idea that reserve accumulation promotes growth in part by attracting FDI as well.² Sudden stops are introduced to capture the precautionary effect of reserve holdings. Sudden stops are modeled as an occasionally binding borrowing constraint on private debt and working capital financing.

The government reserve policy consists of two interventions. First, in normal times when the borrowing constraint is not binding, the government collects taxes at a fixed rate to accumulate reserves. Reserve accumulation causes real depreciation, which in turn shifts more labor to the tradable sector and reduces the real wage. This brings higher profits for intermediate firms, which induces more innovations and attracts FDI. Second, when the borrowing

²Dooley et al. (2007, 2014) argue that Asian countries' growth strategy is to repress the real wage by foreign exchange rate intervention and to attract FDI.

constraint binds, the government provides accumulated reserves to mitigate the shock to output, consumption, and investment. I call this intervention a bailout by the government. Since investment in innovation and FDI entry are forward-looking decisions, anticipation of future bailouts also induces investment in innovation and attracts FDI. Through these two interventions, the reserve policy achieves high and stable growth of the economy. The optimal pace of reserve accumulation is determined by the fixed tax rate that maximizes the expected utility of households.

The reason why the reserve policy may improve welfare is because private agents do not internalize that their actions affect FDI entry decisions, and the reserve policy corrects this externality. The main benefit of receiving FDI is that foreign firms invest more in innovation than domestic firms and therefore contribute to productivity growth.³ In order to attract FDI, the country can increase the growth rate by investing more in capital and shift more labor to the tradable sector, which increases profits for foreign firms. Avoiding sharp drops in foreign firms' profits during sudden stops also helps to attract FDI. The reserve policy corrects the externality by bringing about more investment and a labor shift to the tradable sector in normal times, and by preventing a severe economic downturn during sudden stops.

On the other hand, reserve accumulation involves costs. As government collects tax revenue to accumulate reserves, private agents borrow more from abroad to compensate for the loss of resources. In the model, the interest rate on foreign borrowing is debt-elastic, and the larger debt-to-GDP ratio increases the interest rate spread. The debt-elastic spread causes two costs of reserve accumulation. First, it prevents a full offset by private agents, thus lowering consumption in the short run. The optimal reserve policy therefore balances the costs of short-run austerity with the benefits of higher long-run consumption. Second, the higher spread discourages investment in capital and innovation, a form of crowding out. This reduces the growth-promoting effect of reserve accumulation and worsens the trade-off between current austerity and future consumption.

In the quantitative analysis, I calibrate the model to a sample of eight developing countries. I target the productivity gain from FDI entry, the innovation rate of foreign firms

³Arnold and Javorcik (2009) and Guadalupe, Kuzmina, and Thomas (2012) show that foreign firms invest more in innovation than domestic firms in Indonesia and Spain respectively.

relative to domestic firms, and the value added share of foreign firms in the tradable sector, all from empirical papers. The frequency and duration of sudden stops are derived from sudden stop episodes for a sample of 33 countries over the period of 1980-2009. I solve the model globally using a version of the policy function iteration algorithm to deal with the occasionally binding borrowing constraint.

The first important result using this model is that the optimal pace of reserve accumulation and its welfare impact crucially depend on two characteristics of each country: the debt-elasticity of the foreign borrowing spread, and the FDI entry cost. In countries with a higher debt-elasticity of this spread, reserve accumulation causes severe crowding-out of investment, which reduces the growth-promoting effect. In countries with a larger FDI entry cost, reserve accumulation is not as effective in attracting FDI and the growth-promoting effect is therefore limited. In these cases, the optimal pace of reserve accumulation is slower, and the welfare gain is limited. The decomposition analysis shows that 72% of the growth-promoting effect of the reserve policy comes from real depreciation in normal times, and 28% comes from anticipation of future bailouts during sudden stops. The model also shows that reserve accumulation without bailouts cannot improve welfare, because without bailouts private agents do not increase foreign borrowing to compensate for the loss of resources as much, and thus reduce short-run consumption substantially.

Given these results, I evaluate each developing country's pace of reserve accumulation accounting for these two factors. I empirically estimate the debt-elasticity of the spread using panel regression for a sample of 22 developing countries. I find that the default history is significantly associated with high elasticity of the spread with respect to debt. Accordingly I divide the sample 22 countries into 5 groups according to the number of past defaults, and estimate the elasticity for each group by interacting the debt-to-GDP ratio with dummy variables representing the default history. I also adjust the parameters for the FDI entry cost to match the FDI inflow-to-GDP ratio for each country. Using this model, I derive the optimal pace of reserve accumulation for each country, and compare it with the actual pace of reserve accumulation. The second important result is that many developing countries are roughly in line with the optimal pace suggested by the model, suggesting that these two factors can explain the observed cross-country variation in the pace of reserve accumulation.

A few countries including China, however, seem to be accumulating reserves too quickly. But this result may suggest that there is something else happening through reserve accumulation in China that is not captured by the model in this paper.

The remainder of the paper is organized as follows. Section 2 reviews the related literature. Section 3 introduces the model. Section 4 discusses the mechanisms of how the reserve policy works in the model. Section 5 presents the calibration of the model and the quantitative analysis. Section 6 studies the key determinants of the optimal pace of reserve accumulation. Section 7 evaluates the actual pace of reserve accumulation by developing countries. Section 8 concludes.

2 Related Literature

Foreign reserve accumulation by developing countries has been an active research area in the last decade. One strand of literature focuses on the growth-promoting effect. As an empirical motivation, Aguiar and Amador (2011) find that there is a positive correlation between the government net foreign asset growth and the GDP growth across developing countries, which is in a stark contrast to the prediction of neo-classical growth models. Gourinchas and Jeanne (2013) show that this correlation is driven mainly by reserve accumulation. Alfaro, Kalemli-Ozcan, and Volosovych (2014) find a similar correlation between reserve accumulation and growth, and further show that private capital inflows such as portfolio investment and FDI are in contrast positively correlated with productivity growth. Aizenman and Lee (2010) and Korinek and Servén (2016) develop models with a learning-by-doing externality in the tradable sector and study the optimal reserve policy. Attracting FDI is another proposed channel through which reserve accumulation promotes productivity growth. Dooley et al. (2007, 2014) argue that Asian countries' growth strategy is to repress the real wage by foreign exchange rate intervention in order to attract FDI from developed countries. Consistent with this view, Aizenman and Lee (2010) find that FDI inflows from Japan and Korea to China have increased along with China's reserve holdings since 2000.

Another strand of literature studies the precautionary benefits of reserve accumulation. Jeanne and Rancière (2011) model reserve accumulation as an insurance contract that pays

off in a sudden stop, and quantify the optimal amount of reserve holdings. Bianchi, Hatchondo, and Martinez (2016) build a sovereign default model in which the government holds reserve assets to insure against future defaults and loss of access to international financial market.

All of these theoretical papers focus on either of the growth-promoting effect or the precautionary effect of reserve accumulation, but not both. The model in this paper incorporates both effects into a unified framework and studies the interaction between the growth-promoting effect and the precautionary effect. In this sense, reserve policy in this paper is similar to that in Benigno and Fornaro (2012). The key difference between my model and theirs is the growth process and how reserve policy promotes growth. Benigno and Fornaro (2012) assume that productivity in the tradable sector increases as more imported inputs are used for production, which is the externality in the model. Reserve policy promotes growth and improves welfare by inducing private agents to use more imported inputs. In my model, on the other hand, productivity in the tradable sector improves through endogenous domestic and FDI entry and incumbent firms' innovation. These entry and innovation are forward-looking decisions, and reserve policy induces more innovations and attracts FDI by increasing expected future profits for domestic and foreign firms. In particular, bailouts during sudden stops prevent sharp drops in firms' profits, and thus anticipation of future bailouts induces more innovations and attracts FDI, while in Benigno and Fornaro (2012) interventions during sudden stops help private agents to import more inputs and promote growth. In addition, reserve accumulation in my model crowds out domestic investment through higher foreign borrowing spread, while in Benigno and Fornaro (2012) there is no crowding out by reserve accumulation.

There are a few papers that study cross-country differences in the amount or the pace of reserve accumulation. Obstfeld, Shambaugh, and Taylor (2010) consider the risk of double drain of capital and measure the optimal amount of reserve holdings relative to the size of the banking system. They show that even China does not appear to be an extreme outlier. Aguiar and Amador (2011) develop a neo-classical growth model with political frictions and show that differences in the degree of these frictions can explain cross-country differences in the speed of net public debt reduction. This paper focuses instead on two other factors that

can explain cross-country variation in the pace of reserve accumulation, the debt-elasticity of the spread on foreign borrowing and the FDI entry cost.

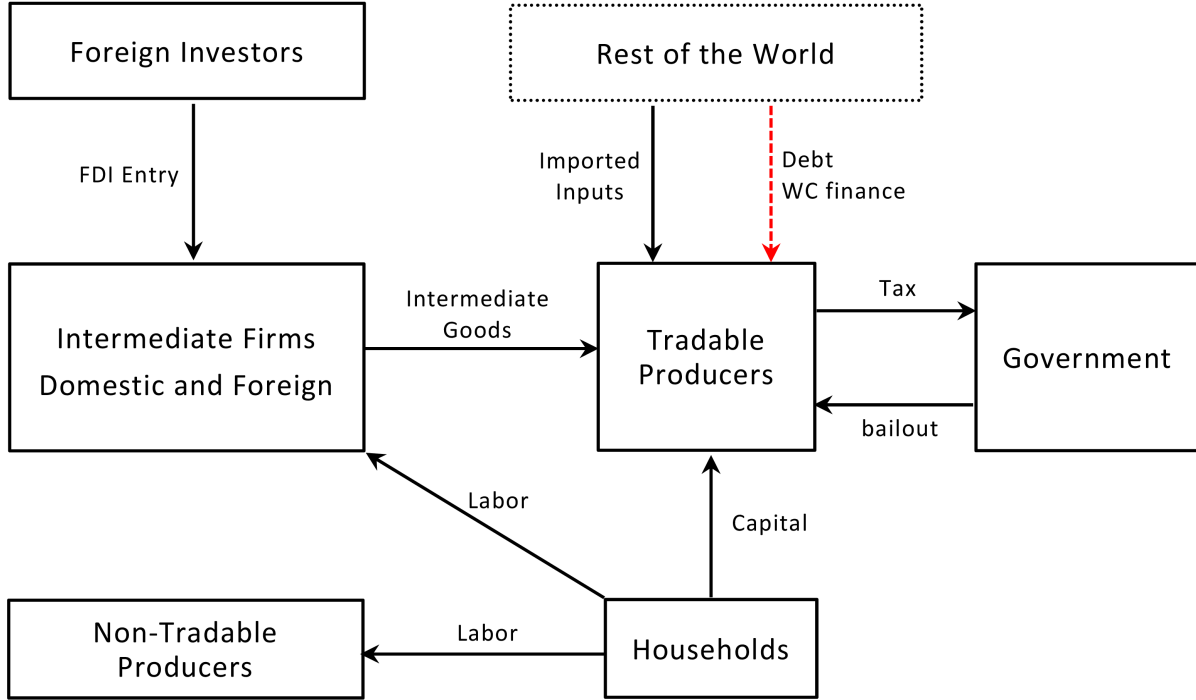
The model structure of this paper rests on two strands of literature. First, the endogenous growth framework is based on a version of the Schumpeterian growth model developed by Ates and Saffie (2016). They incorporate the growth model developed by Klette and Kortum (2004) into a DSGE framework, and also introduce heterogeneous innovations. I extend Ates and Saffie (2016) by introducing FDI and innovation by foreign firms. Second, sudden stops are modeled as an occasionally binding constraint on foreign borrowing. The borrowing constraint in this model is similar to Bianchi (2016), and different from Mendoza (2010) in that the fraction of capital used as collateral is stochastic, and the collateral value of capital is set at book value rather than market value.

Lastly, this paper shares with several recent papers the feature that crises have a permanent negative impact on productivity. Queralto (2015) builds a model based on the Comin and Gertler (2006) version of the product-variety expansion model and shows that the model can explain the permanent negative effect of the 1998 sudden stop on productivity in Korea. Gornemann (2015) also adopts the product-variety expansion model and develops a model that captures a very persistent negative effect of sovereign default on productivity. Ates and Saffie (2016) introduce heterogeneous innovations and financial selection into the Schumpeterian growth model and show that new firms created during sudden stops are fewer but better.

3 Model

The model framework is based on a standard infinite-horizon small open economy with tradable and non-tradable sectors. The overview of the model environment is as follows. The tradable goods producers use capital, a variety of differentiated intermediate goods, and imported inputs for production. Capital and differentiated intermediate goods are the two drivers of economic growth. The economy starts with scarce capital and grows quickly by accumulating capital. A variety of intermediate goods are produced by domestic and foreign firms. They are modeled as a version of the Schumpeterian growth model developed by

Figure 2: Model Economy



Klette and Kortum (2004) and Ates and Saffie (2016), in which new entry and incumbent firms' innovations increase aggregate productivity endogenously. I extend their framework to incorporate FDI entry and foreign firms' innovations. The country is also subject to a sudden stop in the form of an occasionally binding borrowing constraint on foreign debt and working capital financing. In particular, the borrowing limit is set as a fraction of capital holdings of the country, and the fraction occasionally tightens with an exogenous probability. Unlike the business cycle models such as Mendoza (2010), the possibility of sudden stops exists only in the transition periods of capital accumulation. In the transition where capital is scarce, there is a large need for foreign borrowing to accumulate capital, but the borrowing limit is tight. In this case, a negative shock on the borrowing limit causes a binding constraint and generates drops in output, consumption, investment, and FDI inflows. In the long run when capital has reached a steady state condition, the borrowing limit is large enough that the borrowing constraint never binds, and the economy follows a smooth balanced growth path. The focus of the analysis in this paper is therefore how the reserve policy should be conducted during the transition period of capital accumulation.

Figure 2 presents a diagram for the model economy. There are six agents in the model. First, tradable goods producers produce goods by using capital, a variety of intermediate goods, and imported inputs. They also borrow from abroad using non-contingent one-period debt and within-period working capital financing. Because households in this model do not have direct access to the international financial market, tradable goods producers borrow from abroad to smooth consumption and accumulate capital on behalf of households.⁴ Borrowing from abroad is subject to the stochastic borrowing limit as explained above. Second, intermediate goods producing firms produce a unit mass of differentiated intermediate goods using labor and sell output to tradable goods producers. There are domestic and foreign intermediate firms, and each firm produces one or more product line(s) of differentiated goods. There are endogenous entry and innovations by domestic firms and foreign firms, and through these activities aggregate productivity of intermediate goods increases over time. The detailed exposition of the firm dynamics and the growth process will be laid out below. Third, there are an infinite number of foreign investors who consider acquiring product lines from domestic firms and entering this country using FDI. Fourth, non-tradable goods producers produce goods using labor and sell output to households. Fifth, households consume tradable and non-tradable goods, accumulate and rent capital, and supply labor. They also invest in innovation to create new intermediate goods producing firms.

Given this model environment, the government engages in a reserve policy to improve household's welfare. As explained above, a reserve policy consists of two interventions. In normal times when the borrowing constraint is not binding, the government collects lump-sum taxes from tradable goods producers and accumulates reserves. When the borrowing constraint binds, the government provides accumulated reserves to tradable goods producers to help finance the working capital payment and prevent an economic downturn. I call this intervention a bailout by the government. I next turn to the characterization of each agent.

⁴Even if households directly borrow from abroad, it would be an equivalent model. The assumption here is just to simplify the model by avoiding two agents facing the same borrowing constraint.

3.1 Tradable Goods Producer

Tradable goods are the numeraire of this model economy, and their price is normalized at one. The representative tradable goods producer uses capital K_t^D , unit-mass variety of intermediate goods $\{y_t(i)\}_{i=0}^1$, and imported inputs M_t to produce output Y_t^T following the Cobb-Douglas production function:

$$Y_t^T = (K_t^D)^\alpha (I_t^M)^\theta (M_t)^{1-\alpha-\theta} \quad (1)$$

where I_t^M is the composite of intermediate goods:

$$I_t^M = \exp \left[\int_0^1 \ln y_t(i) di \right] \quad (2)$$

Before production materializes, a fixed fraction ϕ of the cost of intermediate goods and imported inputs need to be paid. This payment is financed by within-period borrowing from abroad with no interest cost. In addition, the tradable goods producer borrows from abroad using one-period non-contingent debt B_t . Foreign borrowing is subject to an occasionally binding borrowing constraint. Specifically, the borrowing limit is given by $\kappa_t K_{t-1}$, where κ_t is a collateral shock and takes either of two values, κ_H or κ_L , following a two-state Markov process; and K_{t-1} is the capital stock of this country at the beginning of period t . κ_H is the value in normal times, and it is large enough that the borrowing constraint never binds. With exogenous probability P_{HL} , κ_t switches from κ_H to κ_L , which is small enough that the borrowing constraint may bind, depending on the state of the economy. In particular, when capital is scarce, the borrowing limit is low and at the same time there is a large incentive to borrow from abroad to accumulate capital. In this case, a collateral shock κ_L is likely to cause a binding borrowing constraint. As seen later, the binding borrowing constraint endogenously generates drops in output, consumption, investment in capital and innovation, and FDI inflows. When this happens, the government provides V_t units of reserves to help finance working capital payments and mitigate the negative impacts from a sudden stop. A negative collateral shock κ_L ends with probability P_{LH} , in which case κ_t switches back to the normal value κ_H .

Given these settings, the maximization problem by the representative tradable goods producer is as follows:

$$\max_{\{K_t^D, \{y_t(i)\}_{i=0}^1, M_t, B_t\}_{t=0}^\infty} E_o \sum_{t=0}^\infty \beta^t \lambda_t \Pi_t^T$$

subject to the production function (1) and

$$\Pi_t^T = Y_t^T - r_t K_t^D - \int_0^1 p_t(i) y_t(i) di - P^M M_t - B_t + R_{t-1} B_{t-1} - T_t + V_t \quad (3)$$

$$- B_t + \phi \left[\int_0^1 p_t(i) y_t(i) di + P^M M_t \right] - V_t \leq \kappa_t K_{t-1} \quad (4)$$

where λ_t is the marginal utility of tradable goods consumption by households, $p_t(i)$ is the price of intermediate goods i , P^M is the price of imported inputs, and R_{t-1} is the gross interest rate on foreign debt repaid at period t . T_t is a lump-sum tax that the government collects to accumulate reserves. Each period, the tradable goods producer chooses capital demand K_t^D , intermediate goods demand $\{y_t(i)\}_{i=0}^1$, imported inputs M_t , and foreign debt B_t to maximize the expected profit discounted by household's discount rate adjusted by the marginal utility λ_t . Let μ_t denote the Lagrange multiplier on the borrowing constraint (4). The first-order conditions with respect to the choice variables are as follows:

$$K_t^D : r_t = \alpha \frac{Y_t^T}{K_t^D} \quad (5)$$

$$y_t(i) : p_t(i) \left(1 + \phi \frac{\mu_t}{\lambda_t} \right) = \theta \frac{Y_t^T}{y_t(i)} \quad (6)$$

$$M_t : P^M \left(1 + \phi \frac{\mu_t}{\lambda_t} \right) = (1 - \alpha - \theta) \frac{Y_t^T}{M_t} \quad (7)$$

$$B_t : \lambda_t - \mu_t = \beta R_t E_t(\lambda_{t+1}) \quad (8)$$

$$\mu_t : \mu_t \left(B_t + \phi \left[\int_0^1 p_t(i) y_t(i) di + P^M M_t \right] - V_t - \kappa_t K_{t-1} \right) = 0, \mu_t \geq 0 \quad (9)$$

The first three equations are the demand functions for capital, intermediate goods, and imported inputs. When the borrowing constraint is slack, $\mu_t = 0$ and the demand functions for intermediate goods (6) and imported inputs (7) are the standard ones equating the price and the marginal products. When the borrowing constraint binds, strictly positive μ_t appears

as the external financing premium on working capital payments, which increases the effective cost of inputs. Equation (8) is the Euler equation with respect to foreign debt. Given that λ_t is the marginal utility of tradable goods consumption by households, it is the standard Euler equation except when the Lagrange multiplier on the borrowing constraint μ_t appears. This term captures the external financing premium on foreign debt when the borrowing constraint binds, which increases the effective real interest rate on foreign debt, as explained in Mendoza (2010). The last equation (9) is the complementary slackness condition for the borrowing constraint.

The gross interest rate on foreign borrowing R_t is endogenously determined. Following Schmitt-Grohé and Uribe (2003), R_t is a function of the aggregate debt-to-GDP ratio:

$$R_t = \bar{R} + \psi_b \left(\exp \left(\frac{B_t}{GDP_t} - \bar{b} \right) - 1 \right) \quad (10)$$

where GDP is given by $Y_t^T - P^M M_t + P_t^N Y_t^N$ with $P_t^N Y_t^N$ being the non-tradable goods price times output. As shown in Schmitt-Grohé and Uribe (2003), this formulation guarantees that the debt-to-GDP ratio will converge to the given value $\bar{b}$ and R_t will be $\bar{R}$ in the long run, so that the balanced growth path is uniquely pinned down. In this model, however, this debt-elastic spread plays an important role along the transition path. As the government collects tax revenue, the tradable goods producer borrows more from abroad to compensate for loss of resources. This causes a higher interest spread through equation (10), which triggers two key consequences of reserve accumulation. First, because foreign borrowing becomes more costly, the tradable goods producer does not fully offset the collected tax by borrowing the same amount. This in turn leads to lower tradable goods consumption by households and causes real depreciation. This is the mechanism of how reserve accumulation causes real depreciation in the model. Real depreciation in turn shifts more labor to the tradable sector and brings higher profits for intermediate firms, which induces innovation and promotes growth. Second, higher interest rate implies a higher cost for investment, and thus crowds out investment both in capital and innovation. Section 4.2 discusses these key mechanisms and their implications for the optimal policy in more detail.

3.2 Intermediate Goods Producing Firms

There is a unit-mass variety of differentiated intermediate goods in the tradable sector, indexed by i . Following the versions of the Schumpeterian growth model developed by Klette and Kortum (2004) and Ates and Saffie (2016), a firm is defined as a collection of one or more product line(s) among these differentiated goods.⁵ Each firm produces the product line(s) it owns using labor, and innovates over other product lines. The production technology is given by:

$$y_t(i) = a_t(i)\ell_t(i) \tag{11}$$

where $a_t(i)$ is the labor productivity and $\ell_t(i)$ is labor input. Labor productivity $a_t(i)$ is heterogeneous across i , and improves over time by entry and innovations by domestic and foreign firms. The entry and innovation processes will be laid out in the next section. I first show that only the productivity leader produces goods for each product line, and explain how profit is determined by the size of a productivity lead over rival firms. As shown in equation (6), demand for each product line from the tradable goods producer is a unit-elastic demand function. This implies that the solution to the profit-maximization problem for a monopolist is to set the price infinitely high. In the Schumpeterian growth model, however, there are rival firms that can produce the same type of good with lower productivity, who could steal the market by setting a price slightly below the monopoly price. Therefore, through Bertrand competition, the profit-maximizing behavior by the productivity leader is to set the price equal to the marginal cost of the closest rival firm and monopolize the demand.

Next I explain how a productivity lead over the closest rival firm is determined. There are three different cases for a productivity increase. First, when domestic innovation happens on a product line, either by new entry or an incumbent's innovation, it improves productivity of the product line by a factor of $(1 + \sigma_t^D)$. This implies that the productivity leader of a domestic-owned product line has $(1 + \sigma_t^D)$ times higher productivity than that of the previous leader, which is the closest rival. Second, when a domestic-owned product line is

⁵The number of firms is endogenously determined by entry and innovation, as in Klette and Kortum (2004) and Ates and Saffie (2016). Although the model structure allows me to study the firm age and size distribution, I focus on how reserve policy affects FDI and aggregate growth, and do not conduct firm-level analysis in this paper.

acquired by foreign investors, which I call FDI entry, productivity increases by a factor of $(1 + \sigma_t^F)/(1 + \sigma_t^D)$ with $\sigma_t^F > \sigma_t^D$. Since the productivity leader of a domestic-owned product line has $(1 + \sigma_t^D)$ times higher productivity than the closest rival, a productivity increase by a factor of $(1 + \sigma_t^F)/(1 + \sigma_t^D)$ upon FDI entry means that a product line acquired by foreign investors has $(1 + \sigma_t^F)$ times higher productivity than the closest rival. Third, when a foreign incumbent firm innovates on a product line, productivity increases by a factor of $(1 + \sigma_t^F)$. The productivity improvement process can be summarized as follows:

$$a_t(i) = \begin{cases} (1 + \sigma_t^D)a_{t-1}(i) & \text{if domestic entry or innovation} \\ (1 + \sigma_t^F)/(1 + \sigma_t^D)a_{t-1}(i) & \text{if FDI entry} \\ (1 + \sigma_t^F)a_{t-1}(i) & \text{if foreign innovation} \\ a_{t-1}(i) & \text{none of the above} \end{cases}$$

Under this process, the productivity lead over the closest rival is simply determined by whether the productivity leader is a domestic firm or a foreign firm. In the former case the productivity lead is by a factor of $(1 + \sigma_t^D)$, and in the latter case it is by a factor of $(1 + \sigma_t^F)$.

I assume that the productivity step sizes σ_t^D and σ_t^F are increasing in capital scarcity, defined as k_{ss}/k_{t-1} , where $k_{ss} = K_{t-1}/A_t$ is the capital stock K_{t-1} normalized by the aggregate productivity A_t along the balanced growth path, and $k_{t-1} = K_{t-1}/A_t$ is the same variable in the transition, which trends upward over time. Specifically I assume the following functional forms:

$$\sigma_t^D = \sigma^D \left(\frac{k_{ss}}{k_{t-1}} \right)^\rho \quad (12)$$

$$\sigma_t^F = \sigma^F \left(\frac{k_{ss}}{k_{t-1}} \right)^\rho \quad (13)$$

with $\rho > 0$ and $\sigma^F > \sigma^D$. In the transition in which capital is scarce, $k_{ss}/k_{t-1} > 1$ and the step sizes are large. As capital accumulates and k_{t-1} gets close to k_{ss} , k_{ss}/k_{t-1} declines toward 1 and the step sizes converge to σ^D and σ^F along the balanced growth path. The idea behind this assumption is that capital scarcity indicates the technological distance from the world frontier, and when there is a large distance, the economy can grow faster through the catching-up effect.

Now I show how profit for each intermediate firm is determined. As explained above, demand from the tradable goods producer (6) is unit-elastic, and thus the profit-maximizing behavior by the productivity leader is to set the price equal to the marginal cost of the closest rival. Given the productivity leader's productivity $a_t(i)$ for a product line i at period t , productivity of the closest rival is given by $a_t(i)/(1 + \sigma_t^s)$, where $s = D, F$ indicates whether the productivity leader is a domestic firm or a foreign firm. The optimal price $p_t(i)$ and corresponding profit $\pi_t(i)$ are given as follows:

$$p_t(i) = \frac{W_t(1 + \sigma_t^s)}{a_t(i)} \quad (14)$$

$$\pi_t(i) = \frac{\theta Y_t^T}{1 + \phi \mu_t / \lambda_t} - W_t \ell_t(i) = \frac{\sigma_t^s}{1 + \sigma_t^s} \frac{\theta Y_t^T}{1 + \phi \mu_t / \lambda_t} \quad (15)$$

where W_t is the real wage. The middle term of equation (15) implies that from the individual firm's viewpoint, larger demand from the tradable goods producer and a cheaper real wage bring higher profits. Using the expression in (14), the profit can be written as the last term of equation (15). This expression shows that the profit for each product line depends only on the owner type $s = D, F$, and is independent of the product-line specific productivity $a_t(i)$. It follows from the middle term of (15) that the labor input also depends only on the owner type. Hereafter I use π_t^D and π_t^F to denote the profits for a domestic-owned and a foreign-owned product line respectively, and ℓ_t^D and ℓ_t^F to denote the corresponding labor input. From equation (15) labor input is given as follows:

$$\ell_t^s = \frac{\pi_t^s}{\sigma_t^s W_t} \quad (16)$$

There are two more things to note from equation (15). First, the assumption $\sigma^F > \sigma^D$ implies $\pi_t^F > \pi_t^D$, i.e. foreign-owned product lines yield higher profits than domestic-owned product lines. Second, profits are affected by the borrowing constraint on the tradable goods producer through the Lagrange multiplier μ_t . When the borrowing constraint binds, the tradable goods producer faces a higher effective cost of buying intermediate goods, which thus reduces demand for them. Equation (15) shows that the smaller demand directly translates into a lower profit for the intermediate goods producing firms.

3.3 Innovation and Firm Dynamics

In the previous subsection I showed that only the productivity leader produces each product line and that its profit depends only on the owner type. I now turn to firm dynamics with a focus on how productivity leaders change through innovation. The firm dynamics in this economy are characterized by four different types of innovations: domestic entry, FDI entry, domestic incumbent innovation, and foreign incumbent innovation. Figure 3 illustrates an example of the evolution of firms from one period to the next. The left panel shows 6 product lines with productivity a_1 to a_6 . The first three lines are owned and produced by domestic firm 1, and the other three lines by foreign firm 1. In the next period, depicted in the right panel, foreign investors acquire product line 1 from a domestic firm 1 via FDI entry. FDI entry improves productivity of product line 1 by the factor of $(1 + \sigma_t^F)/(1 + \sigma_t^D)$. For product line 3, a foreign incumbent firm 1 succeeds in innovation and improves its productivity by the factor of $(1 + \sigma_t^F)$. Thus domestic firm 1 loses two product lines and shrinks its business. For product line 6, there is domestic entry and a new domestic firm obtains the product line from foreign firm 1. Through entry and innovation, firms compete with each other and endogenously enter, exit, expand and shrink, and increase the overall productivity of the country.

This framework captures the several features of FDI entry and foreign firms documented by empirical studies: (1) Most FDI entry is through the acquisition of domestic firms rather than greenfield investment.⁶ (2) Some domestic firms are forced to exit through competition with foreign firms.⁷ In later sections I will also show: (3) Foreign firms innovate more often than domestic firms.⁸ (4) In a crisis, foreign firms invest more in innovation than domestic firms and survive better.⁹ I now move on to the characterization of entry and innovation. I start from innovation by foreign incumbent firms.

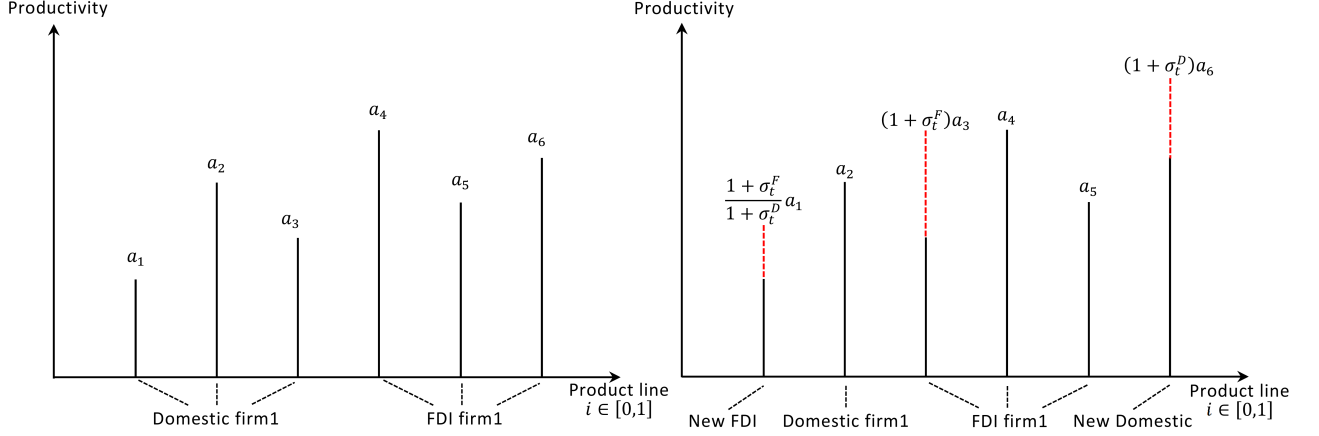
⁶Navaretti and Venables (2004) shows that 90% of FDI is in the form of acquisitions.

⁷See Aitken and Harrison (1999).

⁸See Arnold and Javorcik (2009) and Guadalupe et al. (2012).

⁹See Alfaro et al. (2012).

Figure 3: Firm Dynamics



3.3.1 Innovation by Foreign Incumbent Firms

Let θ_{t-1} denote the fraction of product lines owned by foreign firms at the beginning of period t . Consider a foreign firm that owns n product lines. As seen before, operating profit depends only on the owner type and is independent of the individual firms's productivity. Therefore the total operating profit of this firm is $n\pi_t^F$. I assume that a firm with n product lines has n opportunities to innovate. The idea behind this assumption is that an incumbent firm's innovation is based on and spins off from the existing technology in practice. Following Akcigit and Kerr (2015), the success probability of innovation i_t^F for each innovation opportunity is a concave function of tradable goods Z_t^F that a firm invests in innovation:

$$i_t^F = \eta^F \left(\frac{Z_t^F}{A_t} \right)^{1-\rho^F} \quad (17)$$

where $\eta^F > 0$ is the productivity coefficient and $0 < \rho^F < 1$ is the parameter that governs the concavity. As is common in Schumpeterian growth models, innovation is undirected in the sense that innovation is equally likely to apply to any product line. This feature is preserved in this model by the structure that the operating profit is independent of productivity. Undirected innovations by many firms imply that each product line faces the same replacement probability. Let d_t denote this probability, and define as $P(i, n, p)$ the probability of having

i successes in n trials for a binomial process with success probability p . Namely,

$$P(i, n, p) = \binom{n}{i} p^i (1-p)^{n-i}$$

The value of a foreign firm with n product lines can be written in a recursive form as follows:

$$V_t^F(n) = \max_{Z_t^F} \left\{ n\pi_t^F - nZ_t^F + \frac{1}{R^F} \left[\sum_{i=0}^n P(i, n, i_t^F) \left(\sum_{j=0}^n P(j, n, d_t) E_t(V_{t+1}^F(n+i-j)) \right) \right] \right\}$$

The first two terms are the operating profit minus the innovation investment cost. R^F is the world interest rate, and foreign investors who own foreign firms discount future profits by this rate. The bracketed term is the expected value of this firm next period. The first summation adds up the expected value over the $n+1$ cases for the number of successful innovations, from 0 to n . The second summation adds up over the $n+1$ cases for the number of replacements, again from 0 to n . Thus for a foreign firm with n product lines, there are $(n+1)^2$ different possible combinations of the number of successful innovations and replacements. Note, however, that the expected value of the firm in each case, $E_t(V_{t+1}^F(n+i-j))$, depends only on the number of the product lines, $n+i-j$, and not on the specific combination of the number of innovations and replacements. For example, 3 innovations and 2 replacements will give the same expected value as 4 innovations and 3 replacements, namely $E_t(V_{t+1}^F(n+1))$.

Following Ates and Saffie (2016), I use a guess-and-verify method to show that the value of a foreign firm with n product lines is equal to n times the value of a foreign firm with a single product line:

$$V_t^F(n) = nV_t^F(1)$$

The formal proof is left to the Appendix. This linear relation enables us to aggregate the firm dynamics in a tractable way and study how the firm dynamics affect the endogenous growth of the entire economy, without keeping track of the firm size distribution. The value of a foreign firm with a single product line is given by:

$$V_t^F(1) = \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} (1 + i_t^F - d_t) E_t(V_{t+1}^F(1)) \right\} \quad (18)$$

Taking into account equation (17), the first-order condition with respect to Z_t^F gives the optimal investment condition:

$$\eta^F (1 - \rho^F) \left(\frac{Z_t^F}{A_t} \right)^{-\rho^F} \frac{1}{A_t} \frac{1}{R^F} E_t(V_{t+1}^F(1)) = 1 \quad (19)$$

Since I assume $0 < \rho^F < 1$, investment Z_t^F and the probability of successful innovation i_t^F are increasing in the expected value of a product line next period.

3.3.2 FDI Entry

FDI entry takes the form of an acquisition of a domestic-owned product line by foreign investors. There are infinitely many foreign investors who can consider acquiring a product line and entering this country via FDI. There are three types of cost for FDI entry. First, foreign investors need to pay a fixed fraction $0 < \lambda < 1$ of the discounted expected value of a product line $(1/R^F)E_t(V_{t+1}^F(1))$ to the domestic firm that owns the product line. λ can be interpreted as the negotiation power of the domestic owner firm against foreign investors.¹⁰ Second, there is a fixed entry cost $A_t C^F$. This is in line with Helpman, Melitz, and Yeaple (2004) in that FDI entry is characterized by a large fixed entry cost. Third, there is a congestion cost of entry, which is linearly increasing in the aggregate number of product lines acquired by FDI in each period. Since there is an infinite number of potential foreign investors, FDI entry continues until the congestion cost pushes down the net expected profit of entry to zero. Therefore FDI entry in each period, denoted by e_t^F , is determined by the following zero-profit condition:

$$A_t \chi^F \left(\frac{e_t^F}{1 - \theta_{t-1}} \right) = (1 - \lambda) \frac{1}{R^F} E_t(V_{t+1}^F(1)) - A_t C^F \quad (20)$$

where χ^F is a coefficient on the congestion cost. The denominator $1 - \theta_{t-1}$ is the fraction of domestic-owned product lines at the beginning of period t . This is introduced to capture the

¹⁰For the domestic owner firm to be willing to sell a product line to foreign investors, the incentive compatibility condition must be satisfied. This condition is given by $\lambda(1/R^F)E_t(V_{t+1}^F(1)) \geq E_t(\Lambda_{t,t+1} V_{t+1}^D(1))$ where the right-hand side is the expected value of a domestic-owned product line discounted by the households' stochastic discount factor. Because this condition is always satisfied under the calibrated parameter values, I do not consider it explicitly.

idea that it is more costly to find a good product line to acquire as the number of domestic-owned product lines falls. Both the congestion cost and the fixed entry cost increase over time along with the aggregate productivity of the economy A_t , so that FDI entry e_t^F will be constant in the long run. Similarly to innovation by foreign incumbent firms, FDI entry e_t^F is an increasing function of the expected value of a product line.

3.3.3 Innovation by Domestic Incumbent Firms

Characterization of domestic incumbent firms is similar to foreign incumbent firms, but different in one key aspect: there is the possibility that product lines are acquired by foreign investors via FDI. Consider a domestic firm with n product lines. This firm has n innovation opportunities as assumed for foreign firms. For each opportunity, the firm invests Z_t^D units of tradable goods, and the probability of successful innovation i_t^D is given by the following equation:

$$i_t^D = \eta^D \left(\frac{Z_t^D}{A_t} \right)^{1-\rho^D} \quad (21)$$

with $0 < \rho^D < 1$. Let Q_t denote the price that foreign investors pay to the domestic owner firm to acquire a product line via FDI. From the last subsection this is given by:

$$Q_t = \lambda \frac{1}{R^F} E_t(V_{t+1}^F(1)) \quad (22)$$

Using Q_t , the replacement probability d_t , and the probability for a binomial process $P(i, n, p)$, the value of a domestic firm with n product lines can be recursively written as follows:

$$\begin{aligned} V_t^D(n) = & \max_{Z_t^D} \left\{ n\pi_t^D - nZ_t^D \right. \\ & + \left[\sum_{i=0}^n P(i, n, i_t^D) \left\{ \sum_{j=0}^n P(j, n, d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t \left[\Lambda_{t,t+1} V_{t+1}^D(n+i-j-k) \right] \right) \right\} \right] \\ & \left. + \left[\sum_{j=0}^n P(j, n, d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) kQ_t \right) \right] \right\} \end{aligned}$$

Compared to the value of a foreign firm, the additional terms are the third summation in the second line and the third line. The third summation in the second line adds up the

expected value over the $n - j + 1$ cases for the number of product lines acquired via FDI by foreign investors, from 0 to $n - j$, given that j product lines are replaced. $e_t^F/(1 - \theta_{t-1})$ is the probability that each product line is acquired via FDI by foreign investors. Note that the expected value of the firm next period is discounted by the households' stochastic discount factor $\Lambda_{t,t+1}$. Note also that the number of product lines the firm owns next period is given by $n + i - j - k$. The third line adds up the acquisition price of FDI entry over the same $n - j + 1$ cases given j replacements. Using the same guess-and-verify method, it can be shown that a linear relation holds for the value of a domestic firm:

$$V_t^D(n) = nV_t^D(1)$$

and the value of a domestic firm with a single product line is given by:

$$V_t^D(1) = \max_{Z_t^D} \left\{ \pi_t^D - Z_t^D + \left[i_t^D + (1 - d_t) \left(1 - \frac{e_t^F}{1 - \theta_{t-1}} \right) \right] E_t(\Lambda_{t,t+1} V_{t+1}^D(1)) + (1 - d_t) \frac{e_t^F}{1 - \theta_{t-1}} Q_t \right\} \quad (23)$$

The first-order condition with respect to Z_t^D gives the optimal condition for domestic innovation investment:

$$\eta^D (1 - \rho^D) \left(\frac{Z_t^D}{A_t} \right)^{-\rho^D} \frac{1}{A_t} E_t(\Lambda_{t,t+1} V_{t+1}^D(1)) = 1 \quad (24)$$

3.3.4 Domestic Entry

Finally, entry of new domestic firms comes from innovation by households and poaches a product line from incumbent firms. Households invest Z_t^E units of tradable goods to create new firms. The number of firms created from Z_t^E units of investment is given by:

$$e_t^D = \eta^E \left(\frac{Z_t^E}{A_t} \right)^{1 - \rho^E} \quad (25)$$

The optimal investment Z_t^E satisfies that the marginal benefit of investment is equal to the marginal cost, therefore:

$$\eta^E (1 - \rho^E) \left(\frac{Z_t^E}{A_t} \right)^{-\rho^E} \frac{1}{A_t} E_t(\Lambda_{t,t+1} V_{t+1}^D(1)) = 1 \quad (26)$$

3.4 Aggregation and Productivity Growth

I now characterize how firm dynamics translate into macroeconomic dynamics, specifically the transition of the share of product lines owned by foreign firms, and productivity growth.

First, replacement of a product line happens through three different reasons: domestic incumbent innovations, foreign incumbent innovations, and domestic entry. Thus the replacement rate d_t is the sum of these three probabilities:

$$d_t = (1 - \theta_{t-1})i_t^D + \theta_{t-1}i_t^F + e_t^D \quad (27)$$

Note that the successful innovation probabilities by incumbents, i_t^D and i_t^F , are multiplied by the share of domestic-owned and foreign-owned product lines respectively. Next I derive the transition equation of θ_t , the share of product lines owned by foreign firms. θ_t increases for two reasons: foreign incumbent innovation over domestic-owned product lines, and FDI entry. θ_t decreases for two reasons: domestic incumbent innovation and domestic entry over foreign-owned product lines. The transition of θ_t is thus given by the following law of motion:

$$\begin{aligned} \theta_t &= \theta_{t-1} + \theta_{t-1}(1 - \theta_{t-1})i_t^F + e_t^F - \theta_{t-1}(1 - \theta_{t-1})i_t^D - \theta_{t-1}e_t^D \\ &= \theta_{t-1} + e_t^F - \theta_{t-1}e_t^D + (i_t^F - i_t^D)\theta_{t-1}(1 - \theta_{t-1}) \end{aligned} \quad (28)$$

Next I derive the expressions for aggregate productivity and its growth. I first combine (15) and (16) to obtain the ratio between labor input by domestic-owned product lines ℓ_t^D and foreign-owned product lines ℓ_t^F :

$$\frac{\ell_t^D}{\ell_t^F} = \frac{1 + \sigma^F}{1 + \sigma^D}$$

Combining this with total labor in the tradable sector $L_t^T = (1 - \theta_{t-1})\ell_t^D + \theta_{t-1}\ell_t^F$, I obtain the following expressions for labor input by domestic-owned and foreign-owned product lines:

$$\begin{aligned} \ell_t^D &= \frac{1 + \sigma^F}{(1 - \theta_{t-1})\sigma^F + \theta_{t-1}\sigma^D} \\ \ell_t^F &= \frac{1 + \sigma^D}{(1 - \theta_{t-1})\sigma^F + \theta_{t-1}\sigma^D} \end{aligned}$$

Plugging these equations and the production function for intermediate goods (11) into (2), the composite of intermediate goods I_t^M can be written as follows:

$$I_t^M = A_t L_t^T \frac{(1 + \sigma_D)^{\theta_{t-1}} (1 + \sigma_F)^{1-\theta_{t-1}}}{\theta_{t-1}(1 + \sigma_D) + (1 - \theta_{t-1})(1 + \sigma_F)} \quad (29)$$

where aggregate productivity A_t is given by:

$$A_t = \exp \left[\int_0^1 \ln a_t(i) di \right]$$

and L_t^T is total labor hired by intermediate goods producing firms. As is clear from the expression, aggregate productivity A_t grows as productivity of each product line $a_t(i)$ improves. Using the four different innovation rates and the innovation sizes, the growth rate of A_t is characterized as follows:

$$\frac{A_{t+1}}{A_t} = 1 + g_t = \left(\frac{1 + \sigma_F}{1 + \sigma_D} \right)^{e_t^F} (1 + \sigma_D)^{e_t^D} (1 + \sigma_D)^{(1-\theta_{t-1})i_t^D} (1 + \sigma_F)^{\theta_{t-1}i_t^F} \quad (30)$$

The four terms in the right-hand side correspond respectively to FDI entry, domestic entry, domestic incumbent innovation, and foreign incumbent innovation. This completes the characterization of firm dynamics and its effect on aggregate productivity growth.

3.5 Non-Tradable Goods Producer

The non-tradable goods producer hires labor from households and produces non-tradable goods. The production function is given as follows:

$$Y_t^N = A_t (L_t^N)^{1-\alpha^N} \quad (31)$$

where $0 < 1 - \alpha^N < 1$ is the labor share in non-tradable goods production. I assume that total factor productivity in non-tradable goods production increases at the same rate as aggregate productivity in the tradable sector. This assumption comes from the empirical fact that productivity spillovers from multinational firms to domestic firms happens through

worker mobility.¹¹ Since this spillover to non-tradable goods production is not internalized by innovation investment decisions, it works as an externality that may cause too little innovation. In the appendix, I study a version of the model with slow productivity spillovers to the non-tradable sector, and found that this alteration does not affect the optimal reserve policy or its welfare impact substantially. This spillover guarantees that production of tradable goods and non-tradable goods will grow at the same rate in the long run. Let P_t^N denote the non-tradable goods price. Since the law of one price holds for tradable goods between this country and the rest of the world, the non-tradable goods price P_t^N determines the real exchange rate of this country. Thus I call P_t^N the real exchange rate, and an increase in P_t^N is real appreciation. The first-order condition of the non-tradable goods producer is simply given by:

$$W_t = P_t^N A_t (1 - \alpha^N) (L_t^N)^{-\alpha^N} \quad (32)$$

Since labor is mobile between the tradable and non-tradable sectors, the real wage W_t is common in both sectors. Using P_t^N and W_t the profit for non-tradable goods producer is given by:

$$\Pi_t^N = P_t^N Y_t^N - W_t L_t^N \quad (33)$$

which is paid to households.

3.6 Household

The representative household consumes tradable goods C_t^T and non-tradable goods C_t^N , supplies labor L_t elastically, accumulates and rents capital K_t to the tradable goods producer, and invests Z_t^E units of tradable goods in domestic entry. They receive the wage income $W_t L_t$, capital income $r_t K_{t-1}$, and profits from tradable goods producers Π_t^T , non-tradable goods producers Π_t^N , and domestic intermediate goods producing firms $(1 - \theta_{t-1})(\pi_t^D - Z_t^D)$. They also receive FDI inflow $e_t^F Q_t^F$, which is revenue from the sales of domestic-owned product lines to foreign investors. The representative household's optimization problem is

¹¹Dasgupta (2012) reviews the relevant empirical literature.

then given as follows:

$$\max_{\{C_t^T, C_t^N, L_t, K_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} [\ln C_t - \psi(L_t)^\omega]$$

$$C_t = \left[(\gamma)^{1/\varepsilon} (C_t^T)^{\frac{\varepsilon-1}{\varepsilon}} + (1-\gamma)^{1/\varepsilon} (C_t^N)^{\frac{\varepsilon-1}{\varepsilon}} \right]^{\frac{\varepsilon}{\varepsilon-1}} \quad (34)$$

subject to

$$C_t^T + P_t C_t^N + I_t + Z_t^E = W_t L_t + r_t K_{t-1} + \Pi_t^T + \Pi_t^N + (1 - \theta_{t-1})(\pi_t^D - Z_t^D) + e_t^F Q_t^F \quad (35)$$

$$I_t = K_t - (1 - \delta)K_{t-1} + \frac{\psi_k}{2} K_{t-1} \left(\frac{K_t}{K_{t-1}} - (1 + \bar{g}) \right)^2 \quad (36)$$

where γ is a parameter to determine the weight of tradable goods in composite consumption C_t , ε is the constant elasticity of substitution between tradable and non-tradable consumption, and δ is the depreciation rate of capital. Capital accumulation is subject to a capital adjustment cost that slows down the transition process, and ψ_k is the parameter that governs the size of this cost. The functional form of the capital adjustment cost is taken from Neumeyer and Perri (2005) to be consistent with long-run growth of the economy.

Optimal investment in domestic entry Z_t^E is determined by equation (25). The first-order conditions for the rest of the choice variables can be summarized as follows:

$$\frac{C_t^T}{C_t^N} = \frac{\gamma}{1-\gamma} (P_t^N)^\varepsilon \quad (37)$$

$$\psi \omega (L_t)^{\omega-1} = \frac{W_t}{C_t} \left(\gamma \frac{C_t}{C_t^T} \right)^{1/\varepsilon} \quad (38)$$

$$\lambda_t \left[1 + \psi_k \left(\frac{K_t}{K_{t-1}} - (1 + \bar{g}) \right) \right] = \beta E_t \left[\lambda_{t+1} \left\{ r_{t+1} + 1 - \delta - \frac{\psi_k}{2} \left((1 + \bar{g})^2 - \left(\frac{K_{t+1}}{K_t} \right)^2 \right) \right\} \right] \quad (39)$$

where λ_t is the marginal utility of tradable consumption given by:

$$\lambda_t = \frac{1}{C_t} \left(\gamma \frac{C_t}{C_t^T} \right)^{1/\varepsilon} \quad (40)$$

The stochastic discount factor $\Lambda_{t,t+1}$ is then given by $\Lambda_{t,t+1} = \beta\lambda_{t+1}/\lambda_t$. Equation (37) relates the optimal ratio of tradable and non-tradable goods consumption to the real exchange rate. Equation (38) gives the optimal labor supply L_t , and equation (39) is the Euler equation with respect to capital.

3.7 Government

The government in this model engages in a reserve policy to improve household's welfare. The reserve policy consists of two types of interventions. First, when $\kappa_t = \kappa_H$ and the borrowing constraint is loose, the government collects T_t units of tradable goods through a lump-sum tax and accumulates reserves. In general T_t can be any function of the state of the economy, but in this paper I consider a simple tax rule that the government collects a fraction τ of tradable goods output Y_t^T each period. Second, when $\kappa_t = \kappa_L$ and the borrowing constraint binds, the government provides accumulated reserves to the tradable goods producer to help finance working capital payments, which I call a bailout.

The government keeps accumulating τY_t^T units of reserves each period until it becomes suboptimal to do so. There are two reasons why accumulating reserves becomes suboptimal at some point in the transition. First, the benefit from attracting FDI becomes smaller as capital accumulates and the step size $\sigma_t^F(k_t)$ becomes smaller. Second, as capital accumulates, the collateral value becomes large enough at some point that the borrowing constraint never binds and there is no need for bailouts. For these reasons it is optimal for the government to stop accumulating reserves once capital is sufficiently accumulated.¹²

Bailouts in sudden stops are modeled as follows. The borrowing constraint being binding implies that the tradable goods producer cannot borrow as much as they would if the borrowing constraint was loose. Let $Y_t^{T,loose}$ denote tradable goods output when the constraint is loose. The shortage of foreign borrowing, denoted by S_t , can be written as follows:

$$S_t = \max \left\{ -B_t + \phi(1 - \alpha) Y_t^{T,loose} - \kappa_L K_{t-1}, 0 \right\}$$

¹²It is also optimal for the government not to rebate reserves after reserve accumulation stops, because rebating reserves would reduce the growth rate in the model. The redundant reserves are lost from the economy, but the welfare loss is limited (smaller than 0.02% of permanent consumption) because the value of goods received after 40 periods or more in the future is heavily discounted.

The first two terms are the borrowing amount when the constraint is loose, and $\kappa_L K_{t-1}$ is the borrowing limit, so that the gap is the shortage of foreign borrowing. A negative gap implies that the tight borrowing limit $\kappa_L K_{t-1}$ is still large enough to cover the necessary amount of borrowing, and in this case a max operator sets $S_t = 0$. When S_t is positive, i.e. the borrowing constraint binds without a bailout, the government gives reserves to cover the shortage up to the amount of reserves at hand. The size of a bailout, denoted by V_t , is given as:

$$V_t = \min \{S_t, R^F F_{t-1}\}$$

where $R^F F_{t-1}$ is the amount of reserves at the beginning of period t .

Note that the size of a bailout is dependent on the amount of private debt, and thus the anticipation of bailouts induces private agents to borrow more. I investigate if this is the optimal bailout policy by trying two different types of bailout policies. First, I study a bailout policy in which the government lends reserves to help finance working capital payments, and tradable producers repay these loans after production. I found that this type of intervention has a similar effect on productivity growth, but the welfare gain is limited compared to the benchmark policy described here. Second, in the appendix I also study a bailout policy in which there is a ceiling on the size of bailouts so that the bailout size is independent of private debt, and I found the benchmark bailout policy described here achieves higher welfare.

Given the tax rule in normal times and bailouts in sudden stops, the amount of reserves F_t follows the transition equation given as follows:

$$F_t = \begin{cases} R^F F_{t-1} + T_t & \text{when } \kappa_t = \kappa_H \\ R^F F_{t-1} - V_t & \text{when } \kappa_t = \kappa_L \end{cases} \quad (41)$$

3.8 Market Clearing Conditions

To close the model, this subsection lists the market clearing conditions. The capital market, labor market, and non-tradable goods market clearing conditions are given as follows:

$$K_{t-1} = K_t^D \quad (42)$$

$$L_t = L_t^T + L_t^N \quad (43)$$

$$Y_t^N = C_t^N \quad (44)$$

and labor in the tradable sector satisfies:

$$L_t^T = (1 - \theta_{t-1})\ell_t^D + \theta_{t-1}\ell_t^F \quad (45)$$

This completes the exposition of the model economy. The appendix formally defines the equilibrium of the model economy and the stationarized equilibrium conditions that I use to solve the model numerically.

4 Discussion on Reserve Policy

This section elaborates on the key mechanism of the model, namely how the reserve policy attracts FDI, promotes growth, and improves welfare. The section starts by describing the main externality in the model and explains how the reserve policy corrects it. The section then discusses the cost of the reserve policy, and explains the key trade-off that the reserve policy faces, i.e. lower consumption in the short run and higher consumption in the long run.

4.1 Benefit of Reserve Policy

The key externality in the model is that private agents do not internalize that their actions affect FDI entry decisions by foreign investors. The main benefit of receiving FDI is that foreign firms innovate more often than domestic firms, and thus contribute to higher productivity growth. To attract more FDI, households should invest more in capital and innovation to increase the growth rate of the economy, and make more labor available for foreign firms. Higher growth and more labor will bring higher profits for foreign firms and attract more FDI entry.

The reserve policy is intended to correct this externality through two channels. The first one is the real depreciation channel. In normal times when the borrowing constraint

is loose, the government collects taxes from private agents to accumulate reserves. As some resources are taken away by the government and a profit for the tradable producer reduces, households reduce tradable goods consumption. One thing to note here is that private agents have an incentive to offset reserve accumulation by borrowing more from abroad. There are two reasons why they have an incentive to borrow more from abroad. First, they have an incentive to compensate for the loss of resources. Second, anticipation of future bailouts induces them to borrow more. As shown in Jeanne (2012), if reserve accumulation is completely offset by private foreign borrowing, it would have no effect on the consumption path, and thus it would not cause real depreciation. The key factor that prevents full offset in this model is the debt-elastic foreign spread. As private agents borrow more from abroad, the interest spread rises through equation (10) and makes foreign borrowing more costly. Therefore offset is only partial, and households reduce tradable goods consumption.

Next I provide a partial equilibrium intuition for how this reduction in tradable goods consumption leads to real depreciation and a labor shift to the tradable sector. First, I combine equations (1), (7), (15), (29) and (32) to obtain the wage equality condition across sectors:

$$W_t = A_t F(K_{t-1}, \theta_{t-1}) (L_t^T)^{-\frac{\alpha}{\alpha+\theta}} = P_t^N A_t (1 - \alpha^N) (L_t - L_t^T)^{-\alpha^N} \quad (46)$$

where $F(K_{t-1}, \theta_{t-1})$ is a function of the state variables. Since production is concave in labor in both the tradable sector and the non-tradable sector, the marginal product of labor is decreasing in labor in both sectors. Solving the second equation for P_t^N and plugging it into the optimality condition between tradable and non-tradable goods consumption (37), along with non-tradable goods production (31),

$$\frac{C_t^T}{A_t (L_t - L_t^T)^{1-\alpha^N}} = \frac{\gamma}{1-\gamma} \left(\frac{A_t F(K_{t-1}, \theta_{t-1}) (L_t^T)^{-\frac{\alpha}{\alpha+\theta}}}{A_t (1 - \alpha^N) (L_t - L_t^T)^{-\alpha^N}} \right)^\varepsilon$$

Note that labor in the tradable sector L_t^T is the only endogenous variable in this equation except C_t^T and total labor supply L_t , given the state variables. Therefore, this equation indicates that the reduction in tradable goods consumption C_t^T caused by reserve accumulation

requires a labor shift across sectors. Specifically, a labor shift to the tradable sector, i.e. higher L_t^T , decreases the right-hand side by decreasing the marginal product of labor in the tradable sector (numerator) and increasing the marginal product of labor in the non-tradable sector (denominator), and recovers the equality. This labor shift also reduces non-tradable goods production in the denominator of the left-hand side, but since the right-hand side is lower, the reduction in non-tradable goods production is smaller than the reduction in tradable goods consumption.¹³ Note also that the fraction inside the parenthesis in the right-hand side is the real exchange rate P_t^N . Therefore, reserve accumulation causes real depreciation and a labor shift to the tradable sector. In addition, it is clear from equation (46) that this labor shift also reduces the real wage W_t . This mechanism is in line with the empirical findings by Rodrik (2008) that real depreciation promotes productivity growth by shifting more production resources to the tradable sector.

To see the effect of this labor shift on FDI entry and innovation, I derive the expression for tradable output Y_t^T in terms of labor by combining (1), (7) and (29):

$$Y_t^T = A_t G(K_{t-1}, \theta_{t-1}) (L_t^T)^{\frac{\alpha}{\alpha+\theta}}$$

where $G(K_{t-1}, \theta_{t-1})$ is another function of the state variables. Thus tradable output is an increasing function of labor in the tradable sector. Now recall the expression for firms' profit when the constraint is not binding from equation (15):

$$\pi_t^s = \theta Y_t^T - W_t \ell_t^s = \frac{\sigma_s}{1 + \sigma_s} \theta Y_t^T$$

It follows that a labor shift to the tradable sector increases profits for intermediate firms. From the viewpoint of individual firms, they enjoy higher profits due to larger demand and a cheaper real wage.

The second channel through which the reserve policy attracts FDI and corrects the externality is the precautionary channel. When the borrowing constraint binds, bailouts help

¹³It can be easily seen that an opposite labor shift, i.e. lower L_t^T , is not consistent with a reduction in C_t^T . Lower L_t^T would increase the right-hand side, and at the same time decrease the left-hand side even more by increasing non-tradable goods production.

the tradable goods producer finance working capital payments, and prevent sharp drops in profits for intermediate firms. Since FDI entry and innovation investment decisions are forward-looking, anticipation of future bailouts increases the expected value of firms and induces more FDI entry and innovation today.

In summary, the reserve policy consists of reserve accumulation in normal times and bailouts in sudden stops. Anticipation of these policy interventions in the future increases the expected profits for intermediate firms today, and attracts more FDI entry and induces more innovation today. There is also a feedback loop that growth induces further growth. Harrison and Rodríguez-Clare (2010) review the empirical literature on FDI entry and summarize the extensive evidence that FDI is attracted to less risky and growing markets. The main mechanism here is consistent with this empirical fact, and in this model the reserve policy achieves high and stable growth. However, whether the reserve policy can improve welfare depends on its cost. The next section discusses this point.

4.2 Cost of Reserve Policy

There are two types of costs associated with the reserve policy in the model. First, as explained above, private agents do not fully offset reserve accumulation by foreign borrowing, so that consumption of tradable goods becomes lower in the short run. Consumption of non-tradable goods also becomes lower because reserve accumulation shifts labor to the tradable sector. As is clear from the discussion in the previous subsection, lower consumption and a labor shift to the tradable sector are the essential parts of the mechanism of how reserve policy works. In this sense they are the unavoidable cost of the reserve policy. At the cost of short-run lower consumption, the reserve policy promotes productivity growth and households enjoy higher consumption in the long run. Therefore, for the reserve policy to improve welfare, the long-run gain of higher consumption must exceed the short-run loss of lower consumption. This is the key trade-off that the reserve policy faces.

The second cost is a crowding-out effect of reserve accumulation. Reinhart, Reinhart, and Tashiro (2016) show that there is a strong negative correlation between the reserve-to-GDP ratio and the investment-to-GDP ratio in Asian countries after 2000, suggesting that active reserve accumulation crowds out investment. Cook and Yetman (2012) use micro-

level data to empirically show that reserve accumulation reduces bank lending in emerging Asian countries. In my model the crowding-out effect results from the debt-elastic spread on foreign borrowing. As the government accumulates reserves by collecting taxes, private agents borrow more from abroad to compensate for the loss of resources, at least partially. Higher borrowing then increases the interest rate on foreign debt through equation (10). To see how higher interest rate on foreign debt crowds out investment in capital, recall that a profit for the tradable producers are evaluated in terms of the marginal utility of tradable consumption by households λ_t . The Euler equation with respect to foreign debt therefore takes the standard form except the Lagrange multiplier on the borrowing constraint μ_t :

$$\lambda_t - \mu_t = \beta R_t E_t(\lambda_{t+1})$$

Comparing this equation with the Euler equation with respect to capital investment (39), it can be seen that higher interest rate on foreign debt requires higher capital return, and thus crowds out capital investment. Higher interest rate on foreign debt also crowds out investment in domestic firm entry and innovation. To see this, I arrange the Euler equation with respect to foreign debt to obtain the expression for the stochastic discount factor as follows:

$$E_t(\Lambda_{t,t+1}) = \frac{\beta E_t(\lambda_{t+1})}{\lambda_t} = \left(1 - \frac{\mu_t}{\lambda_t}\right) \frac{1}{R_t} \quad (47)$$

This equation indicates that higher interest rate on foreign debt reduces the stochastic discount factor, implying that households evaluate future tradable consumption less compared to tradable consumption today. Since investment in domestic firm entry and innovation are forward-looking, the lower stochastic discount factor leads to lower investment in domestic entry and innovation.

As seen from equation (10), the parameter ψ_b is crucial for the size of the crowding-out effect. If ψ_b is large, reserve accumulation is likely to cause severe crowding-out and slow down growth. In this case, the long-run gain from higher consumption is likely to be limited, and the welfare impact of reserve policy is smaller or can be even negative.

In summary, the reserve policy brings higher consumption in the long run at the cost of lower consumption in the short run. The optimal reserve policy is determined to hit the

balance between the marginal gain and the marginal loss. The policy analysis section shows how the value of ψ_b and the FDI entry cost affect the optimal reserve policy and its welfare gain.

5 Quantitative Analysis

This section calibrates the model parameters, demonstrates the baseline simulation, and discusses the model features. I solve the model numerically by two steps: First I divide the equilibrium conditions by productivity level A_t to stationarize the equations. In the second step I solve the stationarized model globally using a version of the policy function iteration algorithm to deal with the occasionally binding borrowing constraint. The stationarized equilibrium conditions and the details of the solution procedure are left to the appendix.

5.1 Calibration

One period in the model is meant to be annual. There are 30 parameters to be determined in the model, except the debt-elasticity of spread ψ_b which is estimated from the data below. I use conventional values in the literature if available, and calibrate the other parameters to target the data for a sample of eight developing countries from 1990-2010.¹⁴ Table 1 presents 19 externally-determined parameter values. Six parameters regarding preferences are set to the conventional values in the literature. The discount factor $\beta = 0.96$ and gross return on the safe asset $R^F = 1.02$ are standard values for annual models. The weight on tradable goods in consumption $\gamma = 0.34$ is set following Mendoza (2005) and Durdu, Mendoza, and Terrones (2009). The elasticity of substitution between tradable and non-tradable goods in consumption, $\varepsilon = 0.6$, is in the middle of the range discussed in Mendoza (2005). The coefficient on labor disutility $\psi = 0.525$ is set so that labor supply in the long run is equal to 1. The parameter for the labor supply elasticity $\omega = 1.455$ is set following Mendoza (1991). Regarding the production parameters, capital's share in tradable production $\alpha = 0.3$ and the capital depreciation rate $\delta = 0.1$ are set to the conventional values. The imported input

¹⁴The countries are Chile, Colombia, Malaysia, Mexico, Poland, Thailand, Tunisia, and Vietnam. These countries are chosen based on the availability of data for the calibration, such as the manufacturing share of FDI inflows and JP Morgan EMBI Global spread data.

price P^M is set to be 1, and labor's share in non-tradable production $1 - \alpha^N = 0.75$ is taken from Schmitt-Grohé and Uribe (2016). The share of intermediate goods θ is set so that the imported inputs-to-GDP ratio matches the data at 14%. The fraction of the input cost subject to the working capital requirement ϕ is determined by the method adopted in Mendoza (2010), in which the ratio of the domestic credit to private firms to GDP is used as a proxy for working capital. This ratio is 47% on average for the sample countries, and this results in $\phi = 1.18$. This parameter is set to 1 in Neumeyer and Perri (2005) and 1.25 in Uribe and Yue (2006). The long-run debt-to-GDP ratio $\bar{b}$ is set to the average of the sample countries in recent years at 36%. The long-run interest rate on foreign borrowing $\bar{R}$ is set to be consistent with the long-run growth rate satisfying $\beta\bar{R} = 1 + \bar{g}$, where the long-run growth rate $\bar{g}$ is determined below.

Table 1: Externally-determined parameters

Variable	Value	Source and Target
β Discount factor	0.96	Standard
R^F Return on reserve asset	1.02	Standard
γ Tradable share in consumption	0.34	Mendoza (2005)
ε CES between T and NT	0.6	Middle value in literature
ψ Labor disutility	0.525	Unit labor supply
ω Frisch elasticity $1/(\omega - 1)$	1.455	Mendoza (1991)
α Capital share	0.3	Standard
θ Intermediate input share	0.56	Imported input/GDP 14%
P^M Imported input price	1	Normalized value
$1 - \alpha^N$ Labor share in non-tradable	0.75	Schmitt-Grohe Uribe (2016)
δ Capital depreciation	0.1	Standard
ϕ Share of input subject to WC	1.18	Private credit/GDP 47.0%
$\bar{b}$ Long-run debt/GDP	-0.36	Data
$\bar{R}$ Long-run interest rate	1.0635	Consistent with BGP growth
P_{HL}, P_{LH} Probability matrix of κ_t	0.080, 0.851	Frequency and duration of SS
ρ^D, ρ^E, ρ^F Concavity of innovation investment	0.5	Akcigit and Kerr (2015)

The only uncertainty in the model is stochastic borrowing limit coefficient κ_t . κ_t follows a two-state Markov process with a 2×2 transition matrix. I follow Jeanne and Rancière (2011) and derive the average frequency and duration of sudden stop episodes in the following way:

a given country in a given year is in a sudden stop if the capital inflow-to-GDP ratio drops more than 5% from the previous year. Using the same sample of 33 countries as in Jeanne and Rancière (2011) over the period of 1980-2009, the unconditional probability of sudden stops is 8.6%, and each sudden stop episode continues for two years with probability 14.9%. Accordingly I set $P_{HL} = 0.080$ and $P_{LH} = 0.851$.

For the parameters related to innovation and FDI entry, the concavity parameters governing investment, ρ^D, ρ^E, ρ^F , are set to 0.5 following Akcigit and Kerr (2015) and their literature review. The remaining eight parameters, $\eta^D, \eta^E, \eta^F, \chi^F, \lambda, C^F, \sigma^D, \sigma^F$, are jointly determined to match eight moments in the data to those in the model in the long run. Each of the following moments are tightly related to the above eight parameters in the same order. (1) The ratio of R&D expenditure in the manufacturing sector to GDP is closely related to η^D , which governs the scale of domestic innovation. This ratio is in general small in developing countries and high in developed countries. I set the long-run ratio in the model to match the average of developed countries in the recent data, which is 2.4%. (2) The domestic entry rate is closely related to η^E . The data is taken from The World Development Indicators. The average of sample countries in 2007 is 8.56%. (3) The innovation rate of foreign firms relative to domestic firms identifies η^F , which governs foreign incumbents' innovation. Guadalupe, Kuzmina, and Thomas (2012) document that foreign firms in Spain conduct product innovations 1.387 times more often than domestic firms. I target this value. (4) The value added share of foreign firms in the manufacturing sector identifies χ^F , the congestion cost of FDI entry. This target is meant to pin down the economic presence of foreign firms in the tradable sector in the model. Ramstetter (2009) reports this value for Malaysia, Thailand, and Vietnam, and Ramondo (2009) reports this for Chile. The average of these 4 countries is 32.25%, hence I set this as a target. (5) The ratio of FDI inflows to the manufacturing sector to GDP helps to pin down the cost of acquisition for FDI. The average FDI inflow-to-GDP ratio of the sample countries over 1990-2009 is 3.8%. Data for FDI inflows by sector is available at the International Trade Centre's website. The data are available only for five of the eight sample countries. The average of these countries in 2012 is 50%, thus I set 1.9% as a target. (6) As there is no data or reliable estimation for the fixed cost of FDI C^F , I follow Fillat and Garetto (2015) and set the fixed entry cost so that

it is 72% of the operational profit of foreign-owned product lines. (7) The long-run growth rate helps to pin down σ^D , the productivity gain of domestic innovation. It is set to 2.1%, which is the average growth rate of developed countries in recent years. (8) The productivity gain from FDI entry identifies σ^F . Arnold and Javorcik (2009) and Guadalupe, Kuzmina, and Thomas (2012) estimate the productivity gain from FDI entry in Indonesia and Spain using the propensity score matching method to control for firm characteristics and mitigate the cherry-picking effect of FDI entry choice. These papers show that in the year of entry, firm productivity increases by 11%. Hence I set $(1 + \sigma^F)/(1 + \sigma^D) = 1.11$. When I evaluate each country's reserve policy below, I allow the FDI congestion cost χ^F and the fixed entry cost C^F to vary across countries to match variation in the relevant data moments within the sample.

Table 2: Jointly-determined parameters

Variable		Value	Target	Model
η^D	Domestic innovation coeff.	0.24	Manu. R&D/GDP 2.4%	2.40%
η^E	Domestic entry coeff.	0.74	Domestic entry rate 8.56%	8.56%
η^F	Foreign innovation coeff.	0.21	Relative innovation rate 1.387	1.387
χ^F	Coeff. of FDI congestion cost	0.15	FDI value-added in manu. 32.25%	32.25%
λ	Share of FDI firm value paid	0.93	Manu. FDI inflow/GDP 1.9%	1.86%
C^F	Fixed entry cost	0.037	Fixed entry cost/profit 72%	72%
σ^D	Domestic innovation size	0.21	Long-run growth rate 2.1%	2.1%
σ^F	Foreign innovation size	0.34	11% productivity gain upon FDI entry	

Table 3: Parameters related to transitional dynamics

Variable		Value	Target
κ_L	Borrowing limit coefficient	0.89	SS dynamics
ψ_k	Capital adjustment cost	15	Drop in investment in SS
ρ	Exponent on catch-up term	0.25	Avg. growth 3% in transition

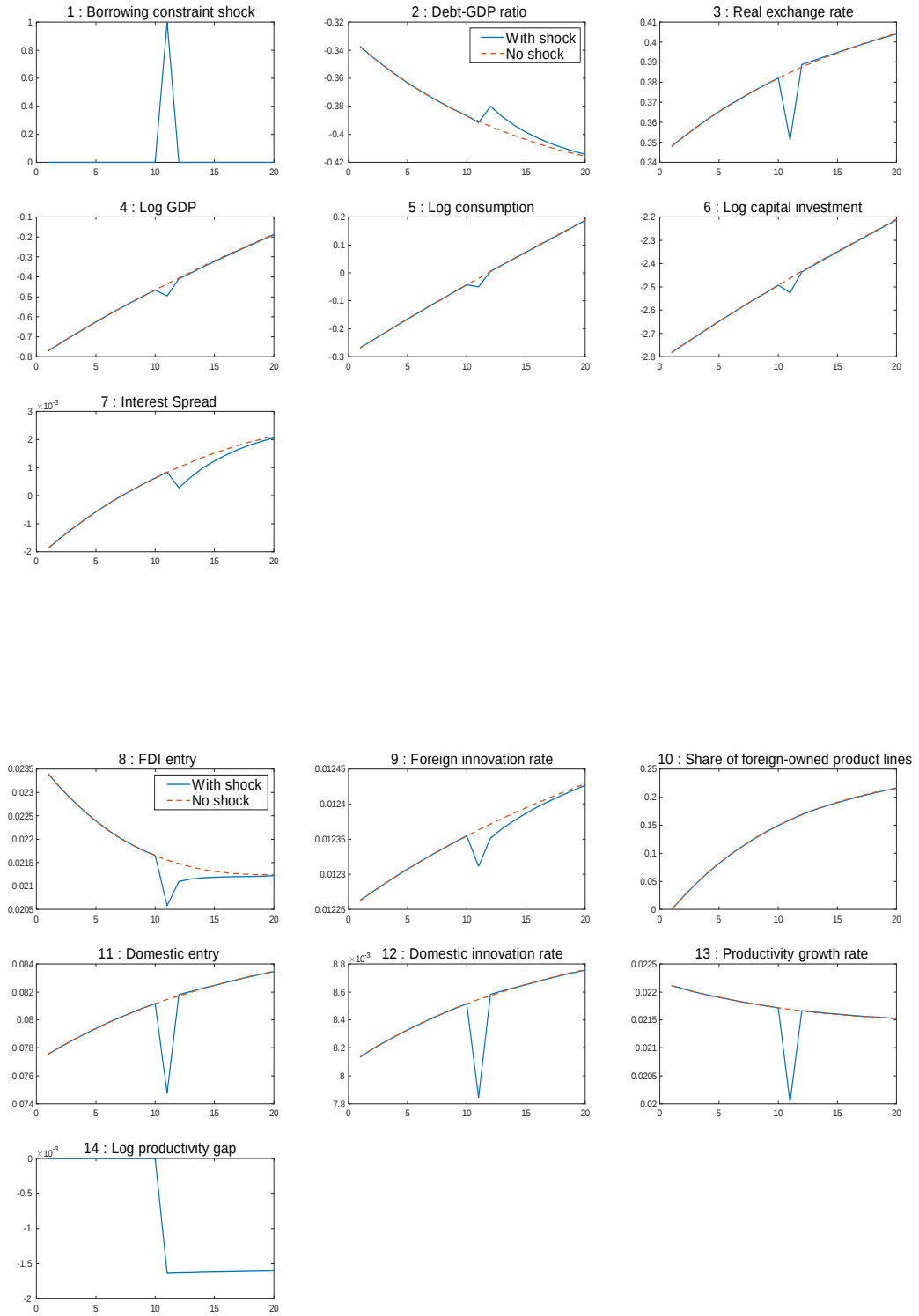
Finally, the borrowing limit coefficient κ_L , the capital adjustment cost parameter ψ_k , and the exponent on the catch-up term ρ are determined to target the model behavior in the transition, because these parameters are irrelevant along the balanced growth path. κ_L and ψ_k are set to match the sudden stop dynamics of the model with the data shown in the next subsection. ρ governs the growth rate of the economy in the transition, and thus is set to match the average growth rate of the model economy in the first 30 periods with the average

growth rate of the sample countries from 1980 to 2010. The average growth rate in the data is about 3%. With $\rho = 0.25$, along with the initial capital holdings $k_{-1} = 0.5k_{ss}$, the average growth rate of the first 30 periods is 2.9%. In determining these parameter values, I do not target the pace of capital accumulation. According to the Penn World Table, the capital holdings of the sample countries in 1980 is 28% of that in 2010. In the model, the initial capital K_{-1} is about 30% of the capital holdings at period 30, which is close to the data.

5.2 Quantitative Performance of the Model

This section documents the quantitative performance of the model and demonstrates the role of the reserve policy. I start by showing a sample dynamic simulation path to give an idea of what the transition dynamics and sudden stops look like. Figure 4 presents a sample dynamic path without policy intervention. The debt-elasticity of spread is set at $\psi_b = 0.0542$ as a benchmark; below, I estimate this parameter using a group of developing countries. Initial capital is 50% of its long-run level as measured by the productivity-adjusted value, initial debt is 33% of GDP to match with the data in 1980, and the initial share of foreign-owned product lines is 0. There is a one-time shock to the borrowing constraint at period 11. The solid lines are the paths with this shock, and the dashed lines are the smooth paths without shocks as a reference. Panel 2 shows the debt-to-GDP ratio, which increases gradually as capital accumulates and the borrowing limit expands. When a sudden stop happens the ratio jumps up, consistent with the stylized facts of sudden stops. The real exchange rate in Panel 3 drops sharply in a sudden stop. Panels 4-6 show that GDP, consumption, and investment all fall in a sudden stop, and investment drops the most, which is also in line with the empirical regularities of sudden stop events illustrated by Mendoza (2010). The interest spread in Panel 7 increases over time along with the debt-to-GDP ratio, and it declines in a sudden stop. This decline in the spread in the sudden stop may look odd, but the effective interest rate actually increases according to the external financing premium captured by the Lagrange multiplier on the borrowing constraint. Turning to innovation and FDI entry, Panels 8, 9, 11, and 12 show that FDI entry, foreign innovations, domestic entry, and domestic innovations all drop in a sudden stop. They also show that the size of the drop is much larger for domestic innovations compared to foreign innovations.

Figure 4: Model Simulation without Reserve Policy

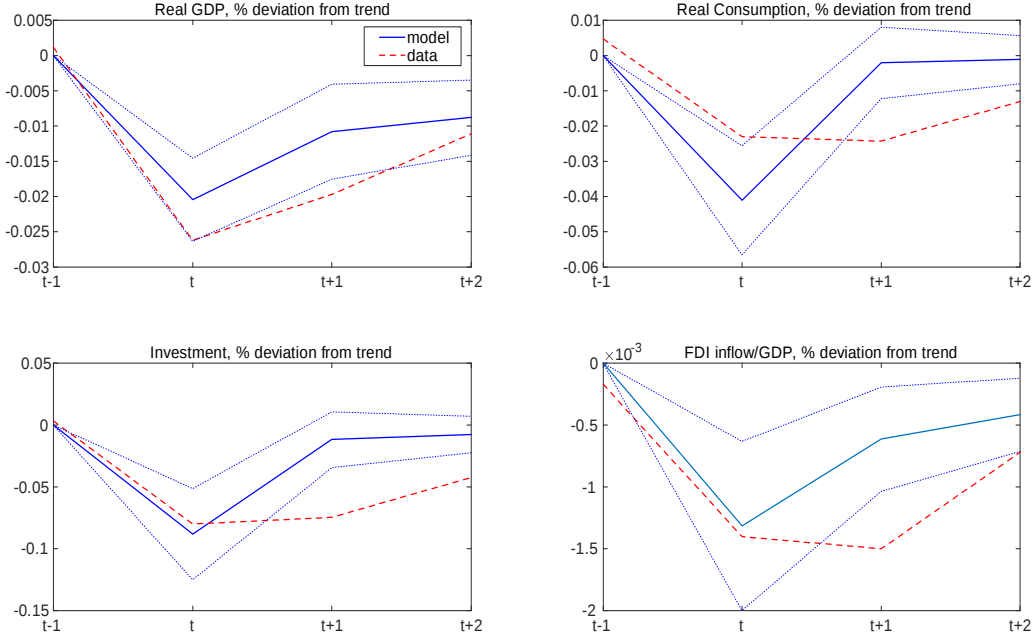


This difference comes from that the stochastic discount factor by domestic households drops due to the binding borrowing constraint, as shown in equation (47), and thus domestic firms reduce their investment substantially, while foreign firms discount future profits by the fixed rate $1/R^F$. These different responses by domestic and foreign firms are consistent with the empirical facts documented by Alfaro and Chen (2012) that domestic firms reduce investment to a larger extent than foreign-owned firms in crisis. The last panel shows that a sudden stop has a permanent level effect on productivity. Cerra and Saxena (2008) provide empirical evidence that the negative effect of a crisis on productivity is very persistent or almost permanent. This makes a clear contrast to previous business-cycle models of sudden stops such as Mendoza (2010). Permanent negative effects on productivity suggest that the cost of sudden stops may be underestimated in business-cycle models and point to the importance of the endogenous growth framework, especially when the focus is a normative analysis of policies trying to fight sudden stops.

To take a closer look at the quantitative performance of the model in capturing sudden stop episodes, I compare the average dynamics of sudden stop events in the model and in the data. I simulate the model with the same initial conditions as above 1,000 times, and compute deviations of the key variables from the smooth paths without sudden stops. Then I take the average of these deviations from the smooth paths in all sudden stop events. For the data, I derive deviations of the key variables from the smooth trend using the Hodrick-Prescott filter, using data for the same 33 developing countries in 1980-2009 that I used to determine the Markov transition matrix for sudden stops. The data for GDP, consumption, and investment are taken from the World Development Indicators, and the FDI inflow-to-GDP ratio is computed using the data from Broner, Didier, Erce, and Schmukler (2013). Then I take the average of these deviations for all the sudden stop events. Figure 5 shows the results for real GDP, real consumption, investment, and the FDI-to-GDP ratio for the model and the data. The dotted lines are the one standard deviation band for the model dynamics. It is clear that the model captures the quantitative dynamics of average sudden stop episodes in the data quite well.

Next I introduce reserve policy and demonstrate how it affects the transitional dynamics of the economy. I set a fixed tax rate τ as 3%, which is the optimal policy for this economy

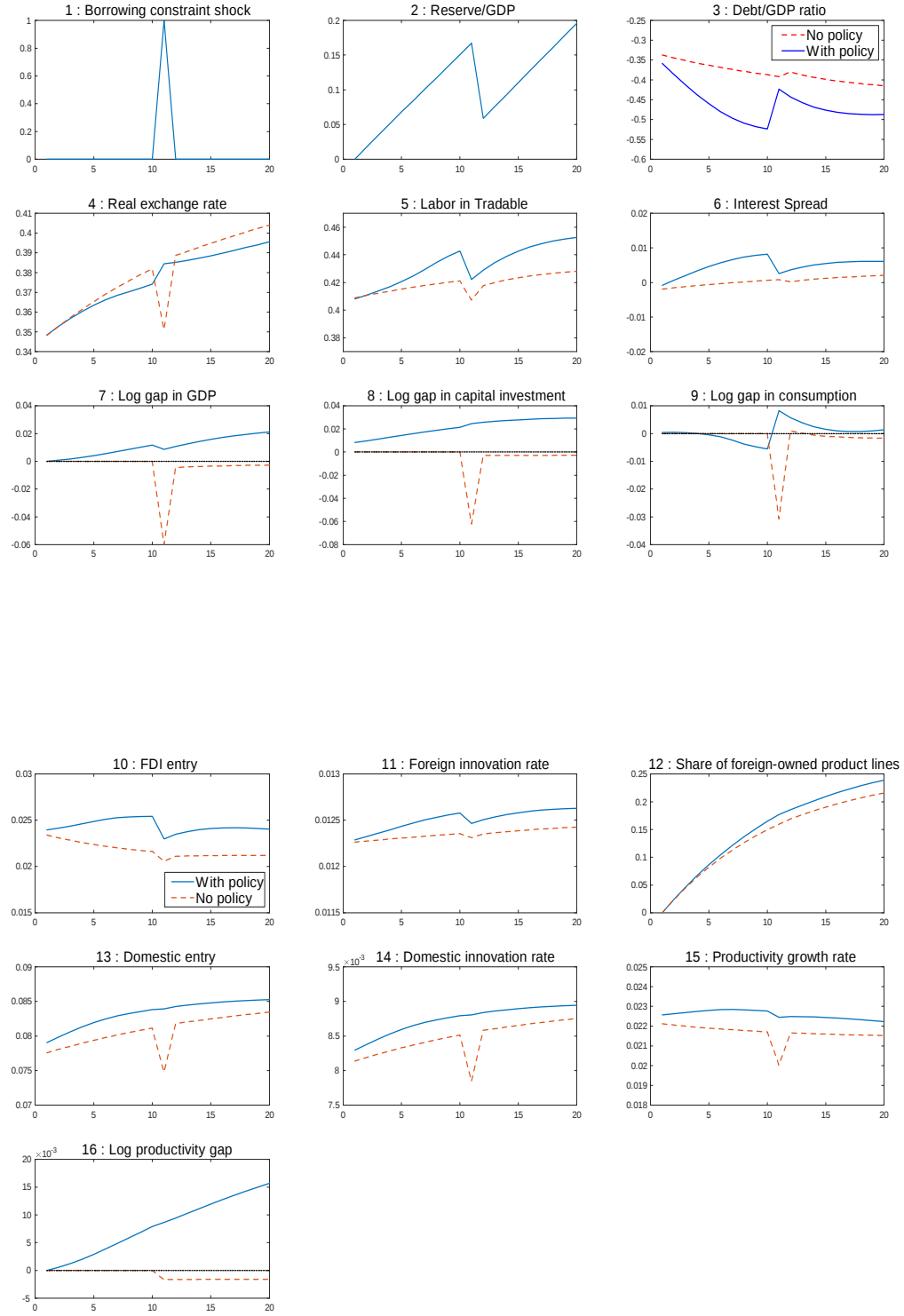
Figure 5: Sudden Stop Dynamics, Model and Data



as shown in the next section. The government stops accumulating reserves when capital holdings adjusted by the productivity level reach 95% of the balanced growth path level as is true in the optimal policy.¹⁵ Figure 6 presents the simulation results for the same one-time shock as above. The solid lines are the paths with the reserve policy and the dashed lines are the paths without policy. Panel 7,8,9, and 16 show the log gaps from the path without shocks, with and without policy. As the tax will be zero in the long run, both paths will converge to the same balanced growth path measured by productivity-adjusted values, but they are different in productivity levels. Panel 2 shows that reserves are accumulated in normal times, and provided to private agents when a sudden stop happens. Responding to reserve accumulation, private agents borrow more to compensate for the collected tax as shown in Panel 3. As the debt-to-GDP ratio increases, the foreign spread rises as shown in Panel 6, which makes the offset only partial and reduces tradable consumption. This in turn causes a real depreciation and a labor shift to the tradable sector as shown in panel 4 and 5. As shown

¹⁵To be precise, the tax rate is declining linearly from 3% to 0 as k_{t-1}/k_{ss} increases from 85% to 95%. This enables me to avoid an abrupt change in the decision rules and makes numerical solution easier and more accurate.

Figure 6: Model Simulation with Reserve Policy



in Panel 7 and 8, GDP and capital investment are always larger with reserve accumulation, but consumption is lower in the short run because the offset is partial. It may look puzzling that investment is larger with reserve accumulation, because that higher interest spread is likely to crowd out investment. The reason is that the increased labor in the tradable sector and the higher growth rate of productivity under reserve accumulation increases the marginal product of capital, and this positive effect dominates the negative effect from the higher spread. When a sudden stop happens, the government gives accumulated reserves to the private agents, which prevents sharp drops in GDP, investment, consumption, and real exchange rate. The latter 7 panels are related to firm dynamics and productivity growth. With reserve accumulation all types of entry and innovation are larger. These panels also show that intervention in a sudden stop prevents a drop in domestic entry and innovation, and achieves high and stable growth. The productivity level is higher by 1.7% after 20 periods, and the gap goes to almost 2.5% in the long run.

6 Optimal Reserve Policy

This section studies how the optimal reserve policy is determined, and shows the first main result of the paper: the debt-elasticity of the interest rate spread and the FDI entry cost are key determinants of the optimal pace of reserve accumulation. As discussed in the model section, I consider a simple policy rule that the government collects a fixed fraction τ of tradable output every period to accumulate reserves, and eliminates the tax once the economy has accumulated enough capital. For the analyses below with different parameters and tax rate τ , I found that it is optimal to stop collecting taxes when capital holdings adjusted by the productivity level reach 95% of the balanced growth level. Thus this rule is fixed for any of the analyses hereafter.¹⁶

¹⁶Again, to be precise, I assume the tax rate is declining linearly from τ to 0 as capital accumulates from 85% to 95%. See footnote 15.

6.1 Role of Debt-Elasticity of Spread

In this subsection I show how the debt-elasticity of spread ψ_b affects the optimal reserve policy. To have a quantitatively reasonable value for ψ_b , I estimate it using data for developing countries. The detailed estimation method is described in the next section, in which each country's reserve policy is evaluated quantitatively. The result is that the mean debt-elasticity of the spread over 22 developing countries is 0.0542, which implies that if the debt-to-GDP ratio increases by 10%, the spread increases by 54.2 basis points or 0.542%. I also estimate ψ_b for five subgroups of countries, according to the number of past defaults for each country. The estimated values for ψ_b are 0.0205, 0.0382, 0.0559, 0.0736, and 0.0915 from the fewest defaults to the most defaults. To clarify how different ψ_b affects the optimal reserve policy, I use 0.0205, 0.0559, and 0.0913 for the following analysis.

The optimal reserve policy is derived in the following way: Given each value for ψ_b , I first numerically solve the model without reserve policy, simulate the model with stochastic shocks for 300 periods 100,000 times, and compute the expected utility. Next I solve the model with reserve policy $\tau = 0.01$ to 0.06 and compute the expected utility in the same way. The welfare gain/loss for each reserve policy is evaluated in terms of the permanent consumption gain/loss in percentage as is common in the literature. The result is summarized in Figure 7. It is clear that the welfare gain from the reserve policy is larger with a smaller debt-elasticity of spread ψ_b . It can also be observed that the optimal pace of reserve accumulation τ is faster with smaller ψ_b .

To understand the role of ψ_b , Figure 8 plots the dynamics of key variables from simulations with $\psi_b = 0.0205$ and $\psi_b = 0.0913$. Log gaps refer to logged variables for $\psi_b = 0.0205$ minus the logged variables for $\psi_b = 0.0913$. The tax rate τ is set to 0.04 for both simulations, and shocks are shut down to see the difference in a clean setting. The first panel shows that the debt-to-GDP ratio is always smaller with a more elastic spread, implying that private agents offset less with a more elastic spread. The smaller offset makes reserve accumulation more effective, thus for the first several periods the economy grows faster with a more elastic spread. However, as the debt-to-GDP ratio becomes larger, the interest spread becomes higher with more elastic spread as shown in the second panel. This higher spread

Figure 7: Welfare Impact with Different ψ_b

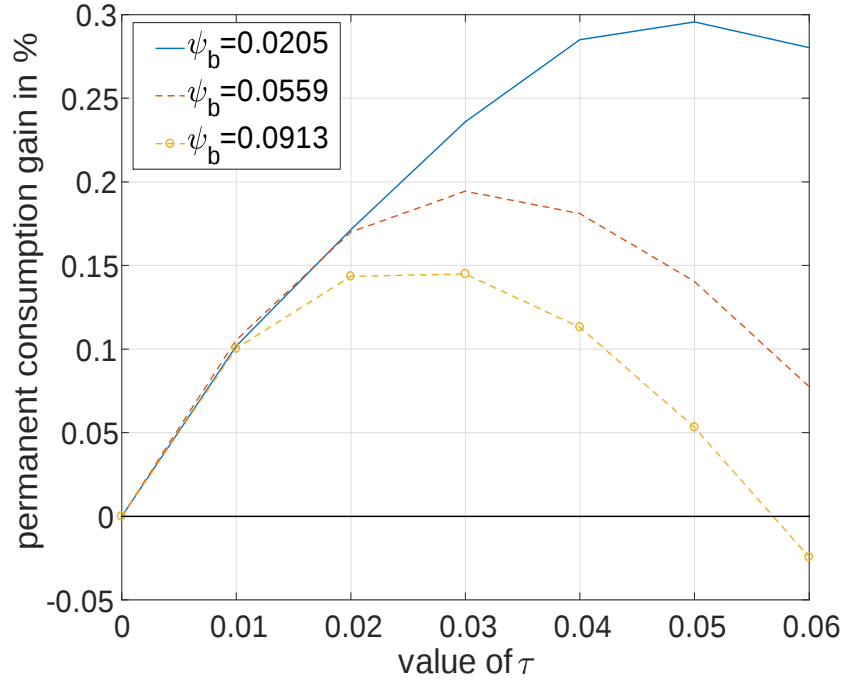
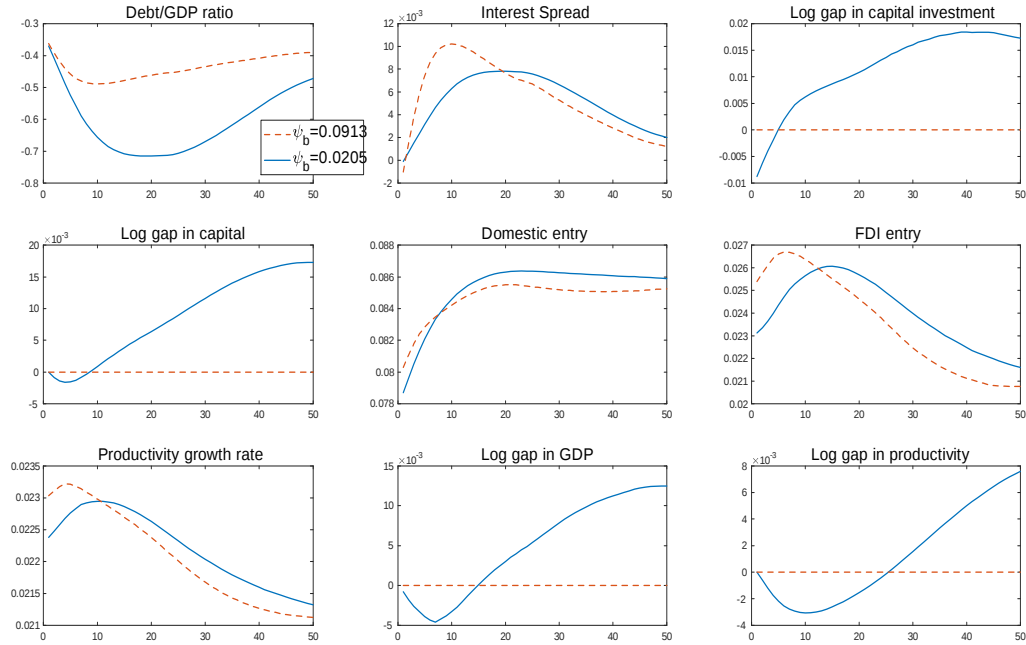


Figure 8: Role of ψ_b



discourages capital investment and slows down capital accumulation as shown in the third and fourth panels. Lower capital in the tradable sector leads to smaller profits for intermediate goods producing firms, which thus discourages domestic and foreign entry, and slows down productivity growth. As a result, GDP and the productivity level will eventually be lower with more elastic spread.

This mechanism has an important implication for the optimal pace of reserve accumulation. As discussed in the previous section, the key trade-off that the reserve policy faces is lower short-run consumption against higher long-run consumption. The debt-elasticity of the foreign spread determines the extent of crowding-out due to reserve accumulation, and thus is the key determinant of the long-run gain from the reserve policy. If the spread is more elastic to private debt, reserve accumulation causes more severe crowding-out and reduces the growth-promoting effect of reserve accumulation. In this case, the optimal pace of reserve accumulation is slower, and the welfare gain is limited.

6.2 Role of the FDI Entry Cost

Another key determinant of the optimal reserve policy in the model is the FDI entry cost. There is a vast literature on the determinants of FDI inflows, and many factors have been identified as significant determinants, such as the host country's institutions, relative labor endowments, and so on.¹⁷ FDI entry costs in the model can be interpreted as these implicit factors that affect FDI inflow and govern the size of FDI inflows to the country.¹⁸

To show why the FDI entry cost is important for the reserve policy, I change the FDI entry cost parameters from the baseline calibration, and create an economy with larger FDI entry cost. In particular, I target the ratio of manufacturing FDI inflows to GDP to 1.08% rather than the benchmark 1.9%. I use this lower value in the next section to evaluate the reserve policy in some countries. This target requires me to increase the coefficient on the FDI entry congestion cost χ^F . I also adjust the fixed entry cost C^F to keep the fixed entry cost-to-profit ratio at 72%, and adjust the innovation step sizes σ^D and σ^F

¹⁷Blonigen (2005) and Blonigen and Piger (2011) review the literature on the determinants of FDI.

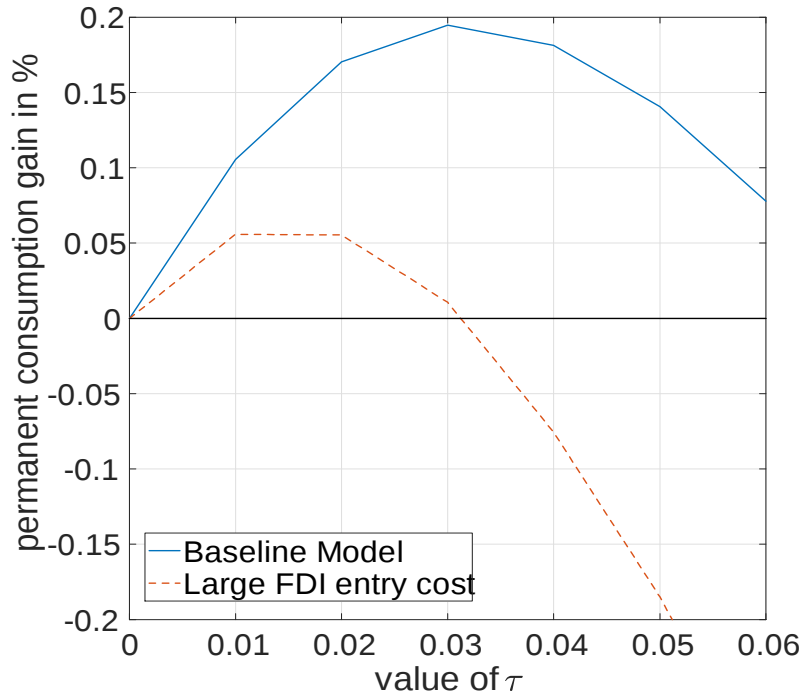
¹⁸Alternatively, in the appendix I estimate the FDI entry cost across countries using the index for the cost to start up new businesses.

to have the same long-run growth rate as in the baseline model, keeping the relative size $(1 + \sigma^F)/(1 + \sigma^D) = 1.11$ unchanged. The other parameters are left unchanged. New parameter values are summarized in Table 4.

Table 4: Parameters, Large FDI Entry Cost

Variable		Value	Target	Model
χ^F	Coeff. of FDI congestion cost	0.32	Manu. FDI inflow/GDP 1.08%	1.08%
C^F	Fixed entry cost	0.040	Fixed entry cost/profit 72%	72%
σ^D	Domestic innovation size	0.24	Long-run growth rate 2.1%	2.1%
σ^F	Foreign innovation size	0.38	11% productivity gain upon FDI entry	

Figure 9: Welfare Impact with Different FDI Entry Cost



Given these new parameter values, I compute the welfare impact of reserve policy for different rates of accumulation τ . Figure 9 presents the results, with the results from the baseline model for comparison. Both models are solved assuming $\psi_b = 0.0542$. It is clear that the welfare gain is substantially smaller for the case with larger FDI entry cost. The figure also shows that the optimal pace of reserve accumulation is slower, and as the pace becomes faster the welfare impact quickly turns negative. This result suggests that attracting FDI is

an important channel through which the reserve policy improves welfare. If some factors of the country impede FDI inflow and the reserve policy is not effective in attracting FDI, the optimal pace of reserve accumulation is slower, and the welfare impact is likely to be limited.

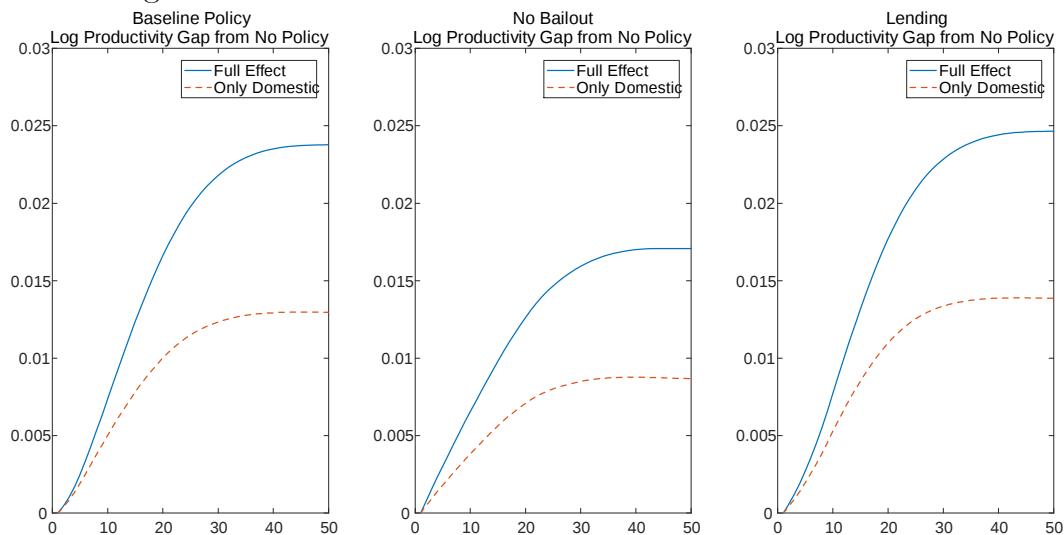
6.3 Decomposition of Policy Effect

So far I study the effect of the reserve policy as a whole, but the reserve policy consists of two types of interventions, reserve accumulation in normal times and bailouts in sudden stops. This subsection conducts a decomposition analysis of the policy effect. Specifically, in order to highlight the effect of bailouts on growth and welfare, I compare two different types of bailout schemes with the policy in the baseline model. The first scheme is that the government accumulates reserves but never uses them for bailouts. I call this a "no-bailout" scheme. The second scheme is that the government provides accumulated reserves to private agents to help finance working capital payments, but private agents need to repay these reserves to the government after production. I call this a "lending" scheme. The model parameters are the same as the baseline model, and the debt-elasticity of spread is set at its benchmark value $\psi_b = 0.0542$. Borrowing constraint shocks are shut down to highlight the effect of bailout policies through anticipation without actual bailouts.

Figure 10 shows the simulation results for the three different policy schemes: the baseline scheme, the no-bailout scheme, and the lending scheme. The solid curves show the productivity gaps relative to the case without policy. The dashed curves are created by fixing the FDI entry rate and foreign innovation rate to their values in the case without policy. Therefore they show the policy effect on growth through only the domestic entry and innovation channels, and the gaps between the solid curves and the dashed curves show the policy effect on growth through the FDI entry and foreign innovation channels.

There are three important observations from this figure. First, note that the no-bailout scheme promotes growth only through the channel of reserve accumulation causing real depreciation. Therefore the gap between the impact of the baseline policy and the no-bailout scheme is the effect of policy on growth through bailout anticipation. Comparing the baseline policy and the no-bailout scheme tells us that 72% of the effect of the reserve policy on growth comes from the real depreciation channel, and 28% comes from the bailout

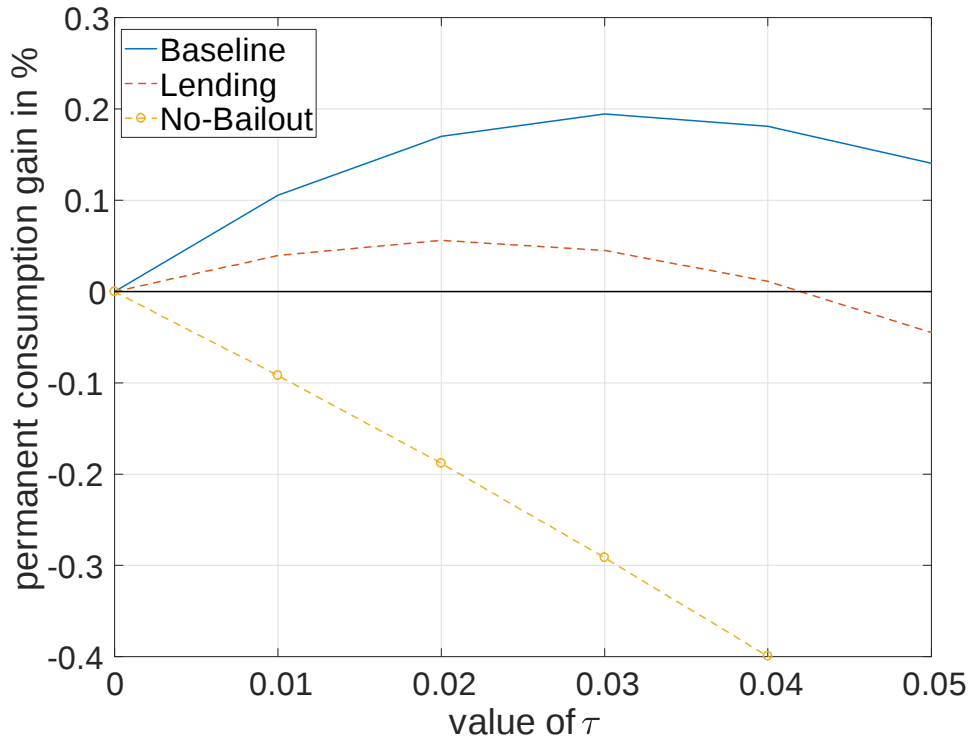
Figure 10: Effect of Different Bailout Schemes on Growth



anticipation channel. Second, the effect of the lending scheme on growth is almost the same as the effect of the baseline policy. This reveals that the bailout anticipation channel is actually anticipation for bailouts to help working capital finance, and not anticipation for rebating reserves. Third, in all three schemes the effect on promoting growth comes from domestic and foreign factors roughly equally. In each case, 55%, 51%, and 56% of the growth-promoting effect comes from domestic entry and innovation, and the rest comes from FDI entry and foreign innovation.

Next I study the effect of these three policy schemes on welfare. I compute the welfare gain/loss from the three policy schemes using the same method as above, namely simulating the model with each policy 100,000 times with stochastic shocks, and measuring the welfare gain/loss in terms of the permanent consumption gain. Figure 11 shows the welfare gain/loss for each policy scheme for different rates of reserve accumulation. The first observation is that reserve accumulation alone without any type of bailout cannot improve welfare. There are two reasons for this result. First, the growth-promoting effect is limited without bailouts, and thus the long-run gain from higher consumption is smaller. Second, since there are no bailouts, private agents do not increase foreign borrowing to compensate for reserve accumulation as much. This reduces short-run consumption even more, and increases the short-run cost of reserve accumulation. The second observation is that the lending scheme

Figure 11: Welfare Impact of Different Bailout Schemes



substantially improves the welfare impact from the reserve policy compared to the no-bailout scheme. The welfare plot indicates that about 70% of the welfare improvement over the no-bailout scheme comes from lending, i.e. helping working capital financing. The rest of the welfare improvement comes from rebating reserves in bailouts. In the lending scheme, private agents actually do not receive any rebate of accumulated reserves, and all reserves are lost from the economy. In contrast, in the baseline policy some of the reserves are rebated to private agents depending on the number of sudden stops, and thus the loss of resources is smaller. Since the productivity gain is almost the same between the baseline policy and the lending scheme, the welfare gap between the baseline policy and the lending scheme comes solely from rebating reserves.

7 Evaluation of Reserve Policy

This section conducts the second main analysis of the paper: evaluation of the actual reserve policies of developing countries. The last section shows that the optimal reserve policy

and its welfare impact are crucially dependent on the debt-elasticity of the spread and the FDI entry cost. In the first subsection I estimate the debt-elasticity of the spread for developing countries from the data. Then I proceed to evaluate whether observed reserve accumulation policies of developing countries are roughly optimal, and whether the model can quantitatively explain observed variation in the pace of reserve accumulation across countries.

7.1 Estimation of Debt-Elasticity of Spread

There is a large amount of literature on the determinants of the interest rate spread in developing countries.¹⁹ I first follow Dell’Erba, Hausmann, and Panizza (2013) and conduct a parsimonious panel regression to estimate the relationship between the spread and the debt-to-GDP ratio. The regression equation is given as follows:

$$S_{i,t} = \beta_0 + \beta_1 \text{debtGDP}_{i,t} + \alpha_i + \tau_t + \varepsilon_{i,t}$$

where $S_{i,t}$ is the interest rate spread on external borrowing in percent, α_i is a country-specific fixed effect, τ_t is a time-specific fixed effect, and $\varepsilon_{i,t}$ is an error term. The data for the spread is taken from JP Morgan’s EMBI Global, as is common in the literature. Since the available time period of this data is different across countries, this is an unbalanced panel regression with 22 countries with maximum time period 1994-2015. The debt-to-GDP ratio is computed using the data from Lane and Milesi-Ferretti (2017). The data is annual and the total number of observations is 379. The result is that β_1 is estimated to be 0.0542 with a standard error 0.0073 and a t-value of 7.45. This means that as the debt-to-GDP ratio increases by 10%, the spread increases by 54.2 basis points or 0.542%. This is similar to the results of other papers that include more controls, such as 0.0447 in Dell’Erba, Hausmann, and Panizza (2013) and 0.0567 in Kennedy and Palerm (2014).

Next I introduce dummy variables into the regression to differentiate countries into several groups with different debt-elasticities of the spread. I found that the number of past defaults

¹⁹One of the main interests in the literature is whether the developing countries’ spread is determined by the global factors or the countries’ fundamentals. See for example Kennedy and Palerm (2014) and their literature review.

is significantly associated with high elasticity of the spread, which is consistent with the finding by Reinhart and Rogoff (2009). According to the data in the Chartbook for their book, Reinhart (2010), the number of defaults before the sample period for the sample 22 countries varies from 0 to 9. Accordingly, I divide the sample countries into five groups with the number of defaults 0 or 1, 2 or 3, 4 or 5, 6 or 7, and 8 or 9, and assign dummy variables from 0 to 4 for each group. Then I estimate the following regression:

$$S_{i,t} = \beta_0 + \beta_1 \text{debtGDP}_{i,t} + \beta_2 (\text{debtGDP}_{i,t} \times \text{Default}_i) + \alpha_i + \tau_t + \varepsilon_{i,t}$$

The result is presented in Table 5. This result implies that the debt-elasticity of the spread for countries with 0 or 1 default is 0.0205, and the elasticity increases by 0.0177 as the number of defaults increases by two: 0.0382 for 2 or 3 defaults, 0.0559 for 4 or 5 defaults, 0.0736 for 6 or 7 defaults, and 0.0913 for 8 or 9 defaults.

Table 5: Estimation of debt-elasticity of spread

Explanatory Variables	Coefficient (S.E.)	t-value
β_1 : Debt-GDP ratio	0.0205* (0.0107)	1.92
β_2 : Debt-GDP ratio $\times$ Default	0.0177*** (0.0042)	4.23

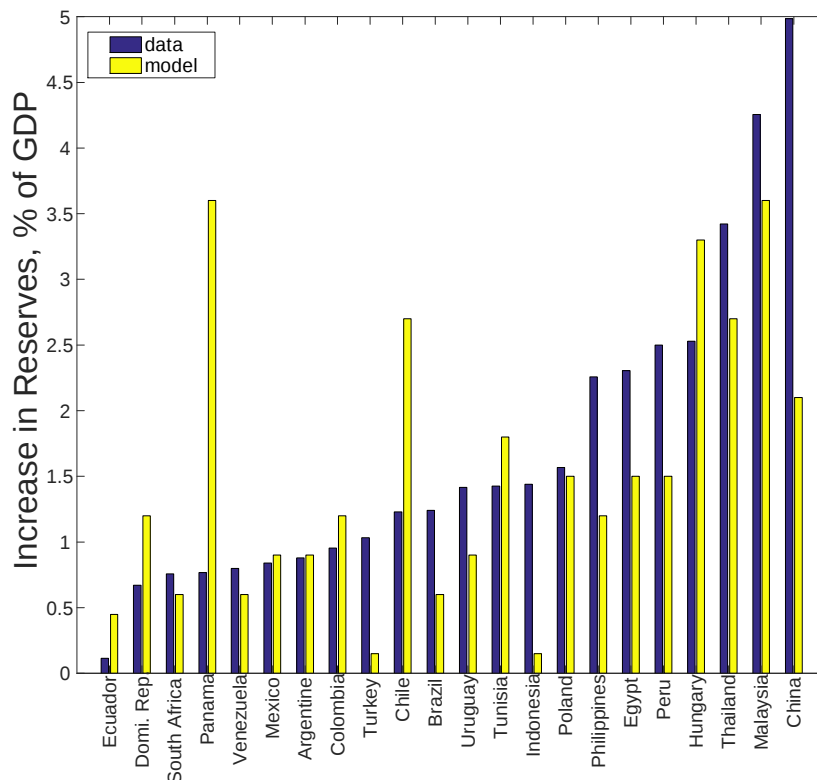
7.2 Evaluation of Each Country's Reserve Policy

Given the estimated debt-elasticity of the spread, this subsection evaluates each country's reserve policy. I proceed with the following steps: (1) I adjust the FDI entry cost parameters and innovation step sizes to match the FDI inflow-to-GDP ratio in the model to the data.²⁰²¹ (2) Given the new parameters and given the estimated ψ_b , I solve the model and find the optimal reserve accumulation pace τ that maximizes household's expected utility. (3) I compute the average pace of reserve accumulation for each country from the data. In particular, I divide increases in reserve holdings by GDP, both expressed in terms of current

²⁰As I did for the baseline model, I assume that 50% of FDI inflow goes into the manufacturing sector. Accordingly the target is 50% of the FDI-to-GDP ratio for each country.

²¹It may be too much to let a single FDI entry cost parameter χ^F explain all the cross-country differences in the FDI inflow-to-GDP ratio. In the appendix, I alternatively estimate the FDI entry cost using the Starting a Business Index from the World Bank's Doing Business Surveys.

Figure 12: Reserve Accumulation Pace, Data and Model



US dollars, for every year from 1991-2010, and take the average across years. The data is from the World Development Indicators. (4) I solve the model assuming that each country's reserve accumulation pace τ corresponds to the data, and compute the expected utility.²² The results are presented in Figure 12 and Tables 6.

Figure 12 lines up the sample countries in the order of the observed pace of reserve accumulation from the slowest to the fastest, along with the optimal pace suggested by the model. Overall, most developing countries are roughly in line with the optimal pace suggested by the model. It shows that Panama and Chile may have more room for welfare gains by accumulating reserves more quickly. On the other hand, Turkey, Indonesia, and China seem to be accumulating reserves too fast.

Table 6 presents more detailed results including the welfare gain/loss by the actual and optimal pace of reserve accumulation. It can be observed that the welfare gain from the actual policy is close to the optimal level for many countries. This is because the welfare

²²In the model a 1% tax on tradable output corresponds to 0.6% of GDP on average in the first 30 periods.

gain is not very sensitive to the pace of reserve accumulation around the optimal pace, and thus a small deviation from the optimal pace does not reduce the welfare gain. Therefore, many countries are in fact accumulating reserves at the optimal pace or very close to the optimal in terms of welfare. Looking at each country in detail, most Latin American countries have high elasticity of the spread because of their default history. This reduces the optimal pace of reserve accumulation, but most Latin American countries are actually in line with the optimal pace. Only three countries, China, Indonesia, and Turkey, incur a welfare loss due to the accumulation pace substantially faster than the optimal pace.

Table 6: Evaluation of Reserve Accumulation Pace

Country	Accum. Pace (%)		Welfare (%)		Elasticity of Spread	FDI Inflow / GDP (%)
	Actual	Optimal	Actual	Optimal		
Argentina	0.9	0.9	0.05	0.05	0.0736	2.16
Brazil	1.2	0.6	0.01	0.04	0.0913	2.12
Chile	1.2	2.7	0.20	0.21	0.0913	5.57
China	5.0	2.1	-0.13	0.18	0.0382	3.24
Colombia	1.0	1.2	0.08	0.08	0.0736	2.60
Dominican Rep.	0.7	1.2	0.09	0.12	0.0736	3.11
Ecuador	0.1	0.5	0.01	0.02	0.0913	1.67
Egypt	2.3	1.5	0.10	0.13	0.0382	2.62
Hungary	2.5	3.3	0.35	0.37	0.0382	5.05
Indonesia	1.4	0.2	-0.11	0.01	0.0382	0.76
Malaysia	4.3	3.6	0.39	0.39	0.0205	4.69
Mexico	0.8	0.9	0.06	0.06	0.0913	2.48
Panama	0.8	3.6	0.16	0.43	0.0382	6.05
Peru	2.5	1.5	0.07	0.13	0.0913	3.47
Philippines	2.3	1.2	0.01	0.06	0.0205	1.51
Poland	1.6	1.5	0.09	0.13	0.0382	2.71
South Africa	0.8	0.6	0.02	0.02	0.0382	1.22
Thailand	3.4	2.7	0.18	0.21	0.0205	2.96
Tunisia	1.4	1.8	0.16	0.16	0.0559	3.32
Turkey	1.0	0.2	-0.05	0.01	0.0559	1.04
Uruguay	1.4	0.9	0.03	0.06	0.0913	2.44
Venezuela	0.8	0.6	0.04	0.04	0.0913	2.06

This overall result suggests that the debt-elasticity of the foreign borrowing spread and the FDI entry cost can explain why different countries accumulate reserves at different rates. In countries with high debt-elasticity of the spread and/or the large FDI entry cost, the optimal pace of reserve accumulation is slow. If these countries accumulate reserves at a pace similar to Asian countries such as Malaysia or Thailand, it may cause a large welfare loss through severe crowding-out of investment and/or little gain from FDI. Such over-accumulation could occur in principle for countries with serial default history and/or small FDI inflows, typically Latin American countries. However, these countries are not actually accumulating reserves quickly, and thus are not subject to welfare losses from over-accumulation in practice.

8 Conclusion

In the past decade, foreign reserve accumulation by developing countries has been both an active research area and a central area of policy debate. However, our understanding regarding the optimal reserve policy and its benefits and costs is still limited. We also know little about the reason for the wide variation in the amount and the pace of reserve accumulation across developing countries.

This paper contributes to our understanding on these issues by developing a quantitative framework to assess the optimal reserve policy, incorporating the key benefits and costs of reserve accumulation. On the benefit side, I combine elements in two strands of literature, endogenous growth and sudden stops, to incorporate the growth-promoting effect and the precautionary effect of reserve accumulation. I also introduce FDI into the endogenous growth framework, which constitutes an important channel through which reserve accumulation promotes growth. On the cost side, I introduce crowding out of investment resulting from reserve accumulation.

Using the model, I identify two factors that are important determinants of the optimal pace of reserve accumulation: the debt-elasticity of the interest rate spread, and the FDI entry cost. In countries with a high debt-elasticity of the spread, active reserve accumulation severely crowds out investment, which reduces the growth-promoting effect. In countries with large FDI entry costs, reserve policies are not effective in attracting FDI, and the growth-

promoting effect is limited. In these cases, the optimal pace of reserve accumulation is slower, and the welfare gain is limited.

I show that both real depreciation by reserve accumulation in normal times and bailouts during sudden stops are important in terms of both the productivity gain and the welfare gain. Quantitative analysis shows that 72% of the growth-promoting effect comes from real depreciation by reserve accumulation in normal times, and 28% is from the anticipation of future bailouts in sudden stops. I also show that real depreciation alone cannot improve welfare without bailouts, and 70% of the welfare gain by bailouts comes from helping to finance working capital payment.

Accounting for differences in the debt-elasticity of the spread and the FDI entry cost across countries, most developing countries are roughly in line with the optimal pace of reserve accumulation suggested by the model. This result implies that these two factors can explain a substantial amount of the cross-country variation in the observed pace of reserve accumulation.

In addition, the model developed in this paper provides a useful framework for broader research areas. One possible line is to introduce a pecuniary externality into the borrowing constraint, as in Mendoza (2010) and Bianchi and Mendoza (2013), and to study optimal macroprudential policies. The key difference from their models is that the model in this paper has endogenous growth, thus enabling the study of the interaction between macroprudential policies and growth. It is important to incorporate growth into the framework because countries subject to a sudden stop of capital inflows are developing countries, which grow faster than the rest of the world during tranquil times. The model here also incorporates innovations by heterogeneous firms in the endogenous growth framework, as originally developed in Ates and Saffie (2016). Thus the model enables the study of the effects of policy on various types of firms.

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Appendix

A Equilibrium and Stationarized Equilibrium

This section defines the equilibrium of the model economy and the stationarized equilibrium.

A.1 Equilibrium

Definition: *The equilibrium of the model economy is defined by the initial states $A_0, R_{-1}B_{-1}, K_{-1}, \theta_{-1}, F_{-1}, \kappa_{-1}$, the stochastic process $\{\kappa_t\}_{t=0}^\infty$, the government policy rules $\{T_t, V_t\}_{t=0}^\infty$ and the following:*

1. *Tradable goods producer: Given prices $\{r_t, W_t, R_t\}_{t=0}^\infty$ and the government policy rules $\{T_t, V_t\}_{t=0}^\infty, \{K_t^D, M_t, B_t, I_t^M, Y_t^T, \Pi_t^T, \mu_t\}_{t=0}^\infty$ satisfy (1), (3), (5), (7), (8), (9), (29).*
2. *Foreign intermediate goods producing firms: Given prices $\{W_t\}_{t=0}^\infty$ and tradable goods output $\{Y_t^T\}_{t=0}^\infty, \{\ell_t^F, Z_t^F, \pi_t^F, i_t^F, V_t^F, \sigma_t^F\}_{t=0}^\infty$ satisfy (13), (15), (16), (17), (18), (19).*
3. *Domestic intermediate goods producing firms: Given prices $\{W_t\}_{t=0}^\infty$ and tradable goods output $\{Y_t^T\}_{t=0}^\infty, \{\ell_t^D, Z_t^D, \pi_t^D, i_t^D, V_t^D, \sigma_t^D\}_{t=0}^\infty$ satisfy (12), (15), (16), (21), (23), (24).*
4. *Foreign investors: $\{e_t^F, Q_t^F\}_{t=0}^\infty$ satisfy (20) (22).*
5. *Non-tradable goods producer: Given prices $\{W_t, P_t^N\}_{t=0}^\infty, \{Y_t^N, L_t^N, \Pi_t^N\}_{t=0}^\infty$ satisfy (31), (32), (33).*
6. *Households: Given prices $\{r_t, W_t, P_t^N\}_{t=0}^\infty, \{C_t, C_t^T, C_t^N, L_t, K_t, Z_t^E, I_t, e_t^E, \lambda_t\}_{t=0}^\infty$ satisfy (25), (26), (34), (35), (36), (37), (38), (39), (40).*
7. *Foreign reserve: $\{F_t\}_{t=0}^\infty$ follows the transition equation given by (41).*
8. *Aggregate variables $\{A_t, \theta_t, d_t\}_{t=0}^\infty$ satisfy (27), (28), (30).*
9. *Prices $\{r_t, W_t, P_t^N, R_t\}_{t=0}^\infty$ and labor in tradable sector $\{L_t^T\}_{t=0}^\infty$ satisfy (10), (42), (43), (44), (45).*

A.2 Stationarized Equilibrium

To stationarize the model, I divide the equilibrium conditions by aggregate productivity A_t . I denote stationarized variables by the lower-case letters, and use g_t to denote the productivity growth rate A_{t+1}/A_t . I also make some arrangements and reduce the number of equations. The following is the complete list of equations to characterize the stationarized equilibrium of the model:

Tradable goods producers

$$\begin{aligned}
y_t^T &= \left(\frac{k_{t-1}}{1 + g_{t-1}} \right)^\alpha (i_t^M)^\theta (m_t)^{1-\alpha-\theta} \\
i_t^M &= L_t^T \frac{(1 + \sigma_D)^{\theta_{t-1}} (1 + \sigma_F)^{1-\theta_{t-1}}}{\theta_{t-1}(1 + \sigma_D) + (1 - \theta_{t-1})(1 + \sigma_F)} \\
w_t &= \frac{\theta_{t-1}(1 + \sigma_D) + (1 - \theta_{t-1})(1 + \sigma_F)}{(1 + \sigma_D)(1 + \sigma_F)} \frac{\theta y_t^T}{L_t^T} \frac{1}{1 + \phi \mu_t / \lambda_t} \\
r_t^k &= \alpha \frac{y_t^T}{k_{t-1} / (1 + g_{t-1})} \\
(1 - \alpha - \theta) \frac{y_t^T}{m_t} &= P^M \left(1 + \phi \frac{\mu_t}{\lambda_t} \right) \\
1 - \frac{\mu_t}{\lambda_t} &= \beta R_t E_t \left(\frac{\lambda_{t+1}}{\lambda_t (1 + g_t)} \right) \\
R_t &= \bar{R} + \psi_b \left(\exp \left(\frac{b_t}{g d p_t} - \bar{b} \right) - 1 \right) \\
\mu_t \left[-b_t + \frac{\phi(1 - \alpha) y_t^T}{1 + \phi \mu_t / \lambda_t} - \kappa_t \right] &= 0
\end{aligned}$$

Foreign intermediate goods producing firms

$$\begin{aligned}
\sigma_t^F &= \sigma^F \left(\frac{k_{ss}}{k_{t-1}} \right)^\rho \\
\pi_t^F &= \frac{\sigma_t^F}{1 + \sigma_t^F} \theta y_t^T \frac{1}{1 + \phi \mu_t / \lambda_t} \\
v_t^F &= \pi_t^F - z_t^F + [i_t^F + (1 - d_t)] (1 + g_t) \frac{1}{R^F} E_t(v_{t+1}^F)
\end{aligned}$$

$$i_t^F = \eta^F (z_t^F)^{1-\rho^F}$$

$$\eta^F (1 - \rho^F) (z_t^F)^{-\rho^F} (1 + g_t) \frac{1}{R^F} E_t(v_{t+1}^D) = 1$$

Domestic intermediate goods producing firms

$$\sigma_t^D = \sigma^D \left(\frac{k_{ss}}{k_{t-1}} \right)^\rho$$

$$\pi_t^D = \frac{\sigma_t^D}{1 + \sigma_t^D} \theta y_t^T \frac{1}{1 + \phi \mu_t / \lambda_t}$$

$$v_t^D = \pi_t^D - z_t^D + \left[i_t^D + (1 - d_t) \left(1 - \frac{e_t^F}{1 - \theta_{t-1}} \right) \right] (1 + g_t) E_t(\Lambda_{t,t+1} v_{t+1}^D) + (1 - d_t) \frac{e_t^F}{1 - \theta_{t-1}} q_t^F$$

$$i_t^D = \eta^D (z_t^D)^{1-\rho^D}$$

$$\eta^D (1 - \rho^D) (z_t^D)^{-\rho^D} (1 + g_t) E_t(\Lambda_{t,t+1} v_{t+1}^D) = 1$$

FDI entry

$$\frac{e_t^F}{1 - \theta_{t-1}} = \chi^F \left[(1 - \lambda)(1 + g_t) \frac{1}{R^F} E_t(v_{t+1}^F) - c^F \right]$$

$$q_t^F = \lambda(1 + g_t) \frac{1}{R^F} E_t(v_{t+1}^F)$$

Aggregate variables

$$d_t = e_t^D + (1 - \theta_{t-1}) i_t^D + \theta_{t-1} i_t^F$$

$$\theta_t = \theta_{t-1} + e_t^F - \theta_{t-1} e_t^D + (i_t^F - i_t^D) \theta_{t-1} (1 - \theta_{t-1})$$

$$1 + g_t = \left(\frac{1 + \sigma_t^F}{1 + \sigma_t^D} \right)^{e_t^F} (1 + \sigma_t^D)^{e_t^D} (1 + \sigma_t^F)^{(1-\theta_{t-1})i_t^D} (1 + \sigma_t^F)^{\theta_{t-1}i_t^F}$$

Non-tradable goods producer

$$y_t^N = (L_t - L_t^T)^{1-\alpha^N}$$

$$w_t = P_t^N (1 - \alpha^N) (L_t - L_t^T)^{-\alpha^N}$$

Households

$$c_t^T + b_t + k_t + z_t^E = y_t^T - P^M m_t - \theta_{t-1} \pi_t^F - (1 - \theta_{t-1}) z_t^D + (1 - \delta) \frac{k_{t-1}}{1 + g_{t-1}} + \frac{R_{t-1} b_{t-1}}{1 + g_{t-1}} + e_t^F q_t^F - \tau_t - \psi_t^K$$

$$\frac{c_t^T}{y_t^N} = \frac{\gamma}{1 - \gamma} (P_t^N)^\varepsilon$$

$$\psi \omega (L_t)^{\omega-1} = \frac{w_t}{c_t} \left(\gamma \frac{c_t}{c_t^T} \right)^{\frac{1}{\varepsilon}}$$

$$1 + \psi_k \left(\frac{k_t(1 + g_{t-1})}{k_{t-1}} - (1 + \bar{g}) \right) = \beta E_t \left[\frac{\lambda_{t+1}}{\lambda_t(1 + g_t)} \left\{ r_{t+1} + 1 - \delta - \frac{\psi_k}{2} \left((1 + \bar{g})^2 - \left(\frac{k_{t+1}(1 + g_t)}{k_t} \right)^2 \right) \right\} \right]$$

$$\lambda_t = \frac{1}{c_t} \left(\gamma \frac{c_t}{c_t^T} \right)^{1/\varepsilon}$$

Domestic firm entry

$$e_t^D = \eta^E (z_t^E)^{1-\rho^E}$$

$$\eta^E (1 - \rho^E) (z_t^E)^{-\rho^E} (1 + g_t) E_t (\Lambda_{t,t+1} v_{t+1}^D) = 1$$

Foreign reserve transition

$$f_t = R^F \frac{f_{t-1}}{1 + g_{t-1}} + \tau_t - v_t$$

The stationarized equilibrium is characterized by 33 variables $\{y_t^T, k_t, g_t, i_t^M, m_t, L_t^T, \theta_t, w_t, \mu_t, \lambda_t, r_t^k, \sigma_t^F, \pi_t^F, v_t^F, z_t^F, i_t^F, \sigma_t^D, \pi_t^D, v_t^D, z_t^D, i_t^D, e_t^F, q_t^F, d_t, R_t, y_t^N, L_t, P_t^N, c_t^T, b_t, z_t^E, e_t^D, f_t\}_{t=0}^\infty$ and the above 33 equations, given the initial state $R_{-1}b_{-1}/(1+g_{-1}), k_{-1}/(1+g_{-1}), \theta_{-1}, f_{-1}/(1+g_{-1}), \kappa_{-1}$, the government policy $\{\tau_t, v_t\}_{t=0}^\infty$, and the stochastic process $\{\kappa_t\}_{t=0}^\infty$.

B Numerical Solution

In this section I sketch the numerical solution method and present the accuracy of the solution.

B.1 Solution Method

The solution method is a version of the policy function iteration, modified to deal with the occasionally binding constraint. Below is the procedure to obtain the numerical solution.

1. I set the equally-spaced grid points for the endogenous state variables, foreign debt $R_{t-1}b_{t-1}/(1 + g_{t-1})$, capital $k_{t-1}/(1 + g_{t-1})$, share of product lines owned by foreign firms θ_{t-1} , and foreign reserve holdings $f_{t-1}/(1 + g_{t-1})$. I set 31 grid points for debt, capital, and reserves. I set 5 grid points for the share of foreign product lines, since the decision rules are close to linear over this state variable. There are also 2 states for the borrowing limit κ_t .
2. For each grid point, I set the initial guess for 5 variables: b_t , z_t^D , z_t^F , L_t^T , and the right-hand side of the Euler equation with respect to capital (RHSEE).
3. For each grid point, I do the following:
 - (a) I leave the 5 variables I have made guess for as unknown variables, and express all the other endogenous variables in terms of the state variables and 5 unknowns. In this process I first assume that the borrowing constraint is not binding and proceed. Later I check if the constraint is satisfied. If it is not satisfied, I recalculate all the variables using the binding borrowing constraint. The other endogenous variables, which include next-period state variables, are now functions of the 5 variables.
 - (b) Using multi-dimensional linear interpolation over the next-period state variables and the guess for the 5 variables (b_t , z_t^D , z_t^F , L_t^T , RHSEE), I compute all the endogenous variables next period. I then calculate all the forward-looking expectation terms, such as the right-hand side of the Euler equations and the value functions.
 - (c) All the equilibrium conditions are now the functions of the initial 5 unknowns. There are 4 equations I did not use in step (a), and the explicit expression for RHSEE, thus 5 equations in total. I solve for the 5 unknowns using non-linear solver.

4. I check the gap between the guess and the newly-obtained values for the 5 variables. If they are close enough, I stop. If not, I update the guess by the newly-obtained values, and go back to step 3. Repeat this process until the gap becomes sufficiently small.

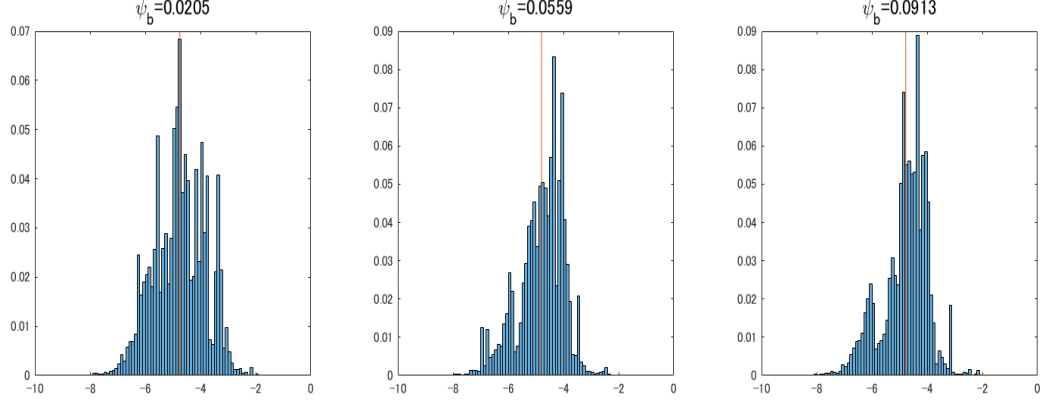
B.2 Accuracy of the Solution

Next I present the accuracy of the numerical solution obtained by the above method. Following Aruoba, Fernández-Villaverde, and Rubio-Ramírez (2006), I compute the Euler equation error of the solution. I use the Euler equation with respect to foreign borrowing, because it is subject to the occasionally binding borrowing constraint, and thus likely to cause a larger error. For each value of ψ_b and τ , I simulate the model for 50 periods with the initial states used in the main analysis and stochastic shocks to the borrowing constraint. The reason why I stop simulation at period 50 is because the economy after period 50 follows a smooth path with no borrowing constraint binding, and thus errors are very small. I repeat this simulation 10,000 times. For each period t in each simulation i , I compute the Euler equation error defined as follows:

$$error_{t,i} = \log_{10} \left[1 - \frac{c_{t,i}^{T,EE}}{c_{t,i}^T} \right]$$

where $c_{t,i}^T$ is tradable consumption computed directly from the decision rules, and $c_{t,i}^{T,EE}$ is computed by using the Euler equation with respect to foreign borrowing. Figure 13 plots the distribution of the Euler equation errors obtained by this method. As a reference, I plot the distributions for the models with three different ψ_b that are used to study the effect of the debt-elasticity of the spread on the optimal policy, with the corresponding optimal τ . For each case, the average error is smaller than -4 and the maximum error is smaller than -2, which are reasonably small compared to the literature. For other models with different values of ψ_b and τ , the distributions of errors are similar.

Figure 13: Euler Equation errors



C Alternative Policy and Model

This section studies a bailout policy with an upper bound for the bailout size, and also the model with slow productivity spillover to the non-tradable sector as robustness checks.

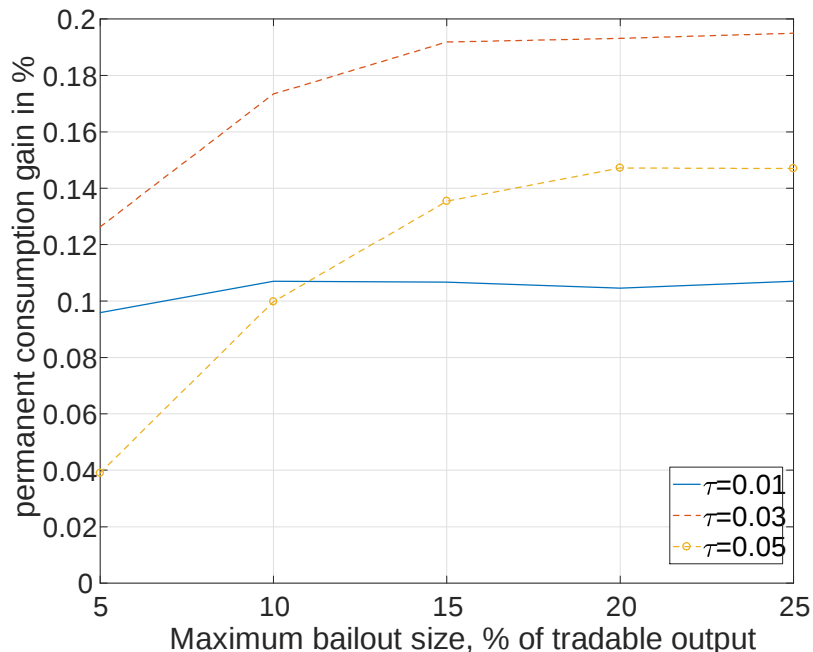
C.1 Bailout Policy with Upper Bound

For the bailout policy studied in the main text, the size of a bailout is dependent on the amount of private debt, thus there is a moral-hazard borrowing by private agents. This potentially makes reserve accumulation less effective in depreciating the real exchange rate and promoting growth. This section considers a bailout policy in which the size of a bailout is independent of the amount of private debt. Specifically, I introduce an upper bound for the size of a bailout in terms of a fixed fraction χ of tradable output Y_t^T . The size of a bailout $\tilde{V}_t$ is then given as follows:

$$\tilde{V}_t = \min\{V_t, \chi Y_t^T\}$$

where V_t is the size of a bailout for the baseline policy discussed in the main text. This bailout policy implies that as the private debt becomes larger, the size of a bailout hits the upper bound χY_t^T and is independent of the amount of the private debt. To compare with the baseline policy, $\tilde{V}_t$ units of reserves are given to private agents upon bailouts. I try different values for χ with different tax rate τ to see how the optimal τ is affected by the upper bound

Figure 14: Bailout Policy with Upper Bound



χ . Figure 14 presents the result. It shows that the higher upper bound monotonically gives higher welfare. It also shows that as χ becomes larger, the welfare impact is not affected by χ and becomes flat. This is because the size of a bailout is not likely to be larger than the upper bound, thus the upper bound never binds. This is essentially the baseline bailout policy in which there is no upper bound for the size of a bailout. This analysis therefore shows that the baseline bailout policy with full rebating is better than the bailout policy with an upper bound.

C.2 Slow Productivity Spillover to the Non-Tradable Sector

This subsection considers the model with slow productivity spillover from the tradable sector to the non-tradable sector. This might potentially have an important impact on the result, because slower productivity growth in the non-tradable sector compared to the tradable sector would cause real appreciation through the Balassa-Samuelson effect and might mitigate the policy effectiveness in promoting growth and attracting FDI. One possible way

to introduce slow spillover is to model the non-tradable productivity A_t^N as follows:

$$A_t^N = (A_{t-1}^N)^\iota (A_t)^{1-\iota}$$

where ι is the parameter that governs the speed of productivity spillover. In terms of the stationarized model, the relative productivity $a_t^N = A_t^N/A_t$ is given by the following equation:

$$a_t^N = \left(\frac{A_{t-1}^N}{A_t} \right)^\iota = \left(\frac{a_{t-1}^N}{1 + g_{t-1}} \right)^\iota$$

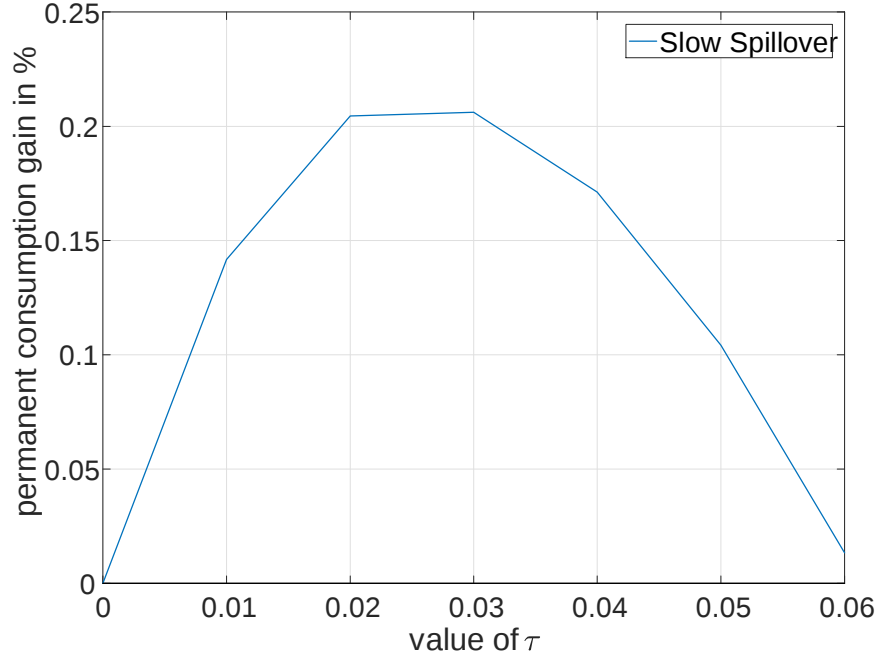
If $a_{-1}^N = 1$ and $\iota = 0.9$ for example, the relative productivity declines over time from 1 to 0.83 at the balanced growth path.

This specification, however, would require another state variable in the model and make the numerical solution substantially difficult, given that there are 5 state variables in the model. Therefore I consider a reduced form spillover function as follows:

$$a_t^N = \left(\frac{k_{-1}}{k_{t-1}} \right)^{\iota'}$$

With this functional form, the relative productivity declines over time from 1 to some value determined by the parameter ι' . I set the value for ι' to match the long-run relative productivity equal to 0.83, consistent with the case with $\iota = 0.9$ for the original specification. This gives $\iota' = 0.27$, and I adjust the other parameter values so that the long-run growth rate $\bar{g}$ is the same as the baseline model. I then try different values for τ and find the value that maximizes the expected utility of households. The result is presented in Figure 15: The optimal pace of reserve accumulation is still $\tau = 0.03$, and the size of welfare gain is very close to the baseline model. Therefore I conclude that slow productivity spillover to the non-tradable sector does not change the main result of the paper.

Figure 15: Slow Spillover to Non-Tradable Sector



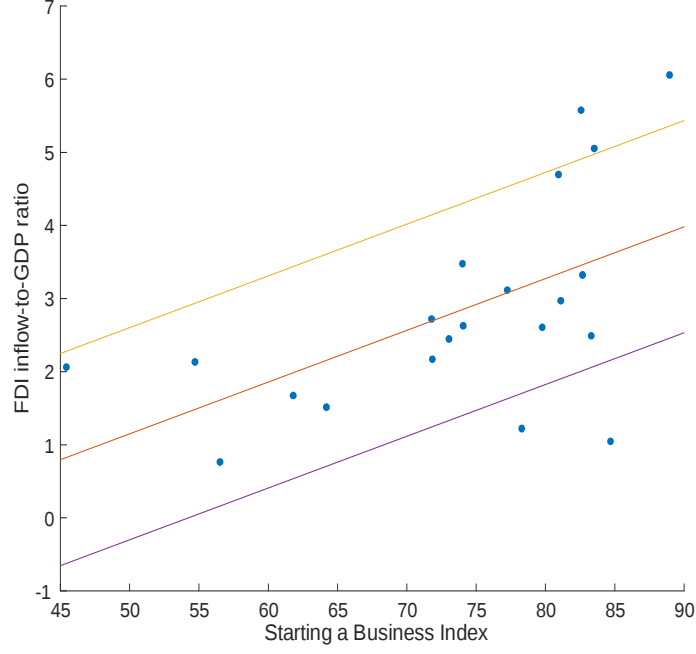
D Policy Evaluation with Estimated FDI Entry Cost

This section presents an alternative analysis of reserve policy evaluation. In the main text, I adjust the FDI entry cost parameters to target the FDI inflow-to-GDP ratio for each country and evaluate the reserve policy. In this section, I estimate the FDI entry cost for each country using Starting a Business Index from the World Bank's Doing Business Surveys, and evaluate each country's pace of reserve accumulation.

D.1 Estimation of FDI Entry Cost

Starting a Business Index measures the effective cost of starting a new business in each country by taking into account the minimum capital requirement, number of procedures, and time and cost to start up a new business. The Index is focused on 100% domestically-owned firms, but I use this Index as a proxy for cost to start a new business by foreign investors. To validate that this Index can be used as a proxy for FDI entry cost, I first show the correlation between the Index and the FDI inflow-to-GDP ratio across developing countries. Since the Index is not available for China, I remove China from the sample in the

Figure 16: Correlation between FDI inflow and Starting a Business Index



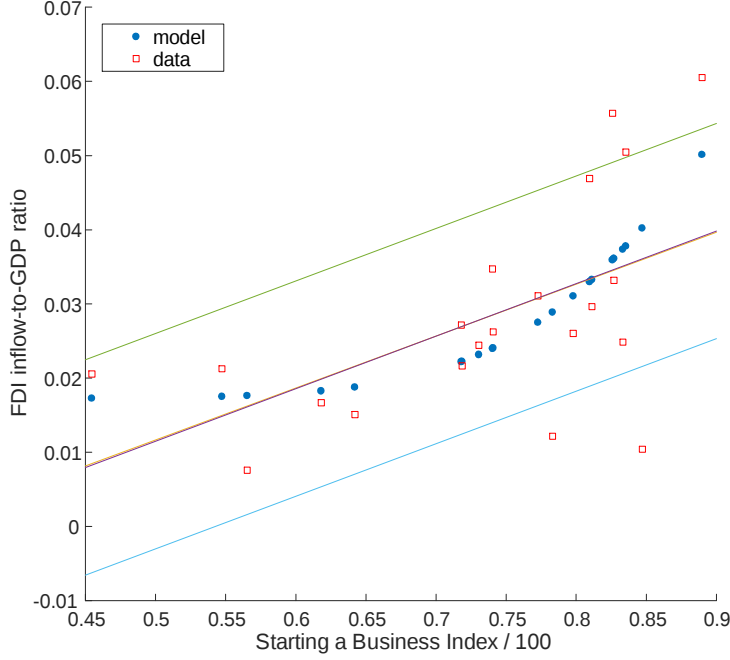
main text and use a sample of 21 developing countries. The Index takes a value between 0 and 100, and a higher value implies smaller cost to start a new business. The regression result is presented in Figure 16, with a band for one standard deviation. It is clear that there is a positive correlation between the Index and FDI inflows. The slope is 0.071 with a standard deviation 0.024 and a t-value 2.90. This result validates that Starting a Business Index can be used as a proxy for FDI entry cost.

Given this result, I estimate the FDI entry cost parameter in the model using Starting a Business Index. To do this, I assume that the congestion cost coefficient for FDI entry is a function of the Index:

$$1/\chi_i^F = \beta_0 + \beta_1(\text{Index}_i)^{\beta_2}$$

where χ_i^F is the congestion cost coefficient for country i , and Index_i is the average of Starting a Business Index for country i in 2004-2017. The reason for taking the inverse of χ_i^F is because higher Index implies smaller cost. I then choose β_0 , β_1 , β_2 to minimize the sum of squared gaps in the FDI inflow-to-GDP ratios between the model and the data across countries.

Figure 17: FDI inflows based on estimated FDI entry cost



Namely,

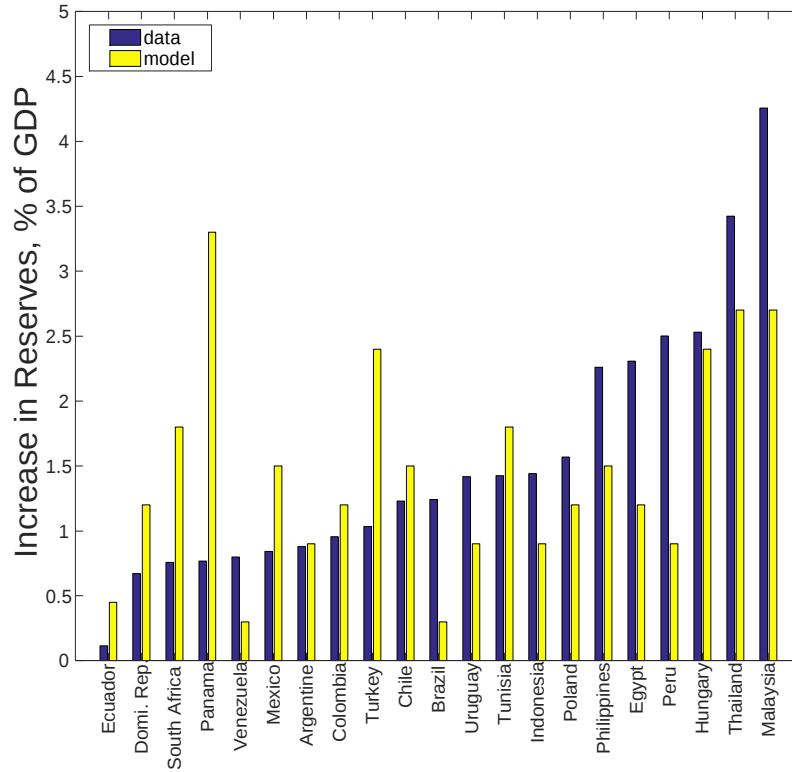
$$\min_{\beta_0, \beta_1, \beta_2} \sum_{i=1}^{21} \left[\left(\frac{\text{FDI}}{\text{GDP}} \right)_i^{\text{data}} - \left(\frac{\text{FDI}}{\text{GDP}} \right)_i^{\text{model}} \right]^2$$

In doing this, I adjust the fixed entry cost C^F to keep the fixed entry cost-to-profit ratio at 72%, and the step sizes σ^D and σ^F to have the same long-run growth rate as in the baseline model for each country. As a result, I obtain $\beta_0 = 2.36$, $\beta_1 = 33.1$, and $\beta_2 = 10.9$. Figure 17 plots the FDI inflow-to-GDP ratios using the estimated FDI entry cost χ_i^F along with the ratios in the data. The model captures the variation in the FDI inflow-to-GDP ratios across countries relatively well, although there are some large gaps for countries with high Index. The regression lines for the data and the model perfectly coincide.

D.2 Evaluation of Each Country's Reserve Policy

Now I evaluate each country's reserve policy using the estimated FDI entry cost. I follow the same steps as in the main text, namely, I derive the optimal pace of reserve accumulation for each country, and compare the welfare gain/loss between the actual pace and the optimal

Figure 18: Reserve Accumulation Pace using Estimated FDI Entry Cost



pace. The results are presented in Figure 18 and Table 7. It seems from Figure 18 that there are more gaps between the actual pace and the optimal pace, compared to the analysis in the main text. It suggests that countries such as Ecuador, South Africa, Panama, and Turkey, may have room for welfare improvement by accumulating reserves more quickly. But Table 7 shows that welfare gains for most countries are again close to the optimal level, suggesting that most countries are still roughly in line with the optimal pace. The largest discrepancy from the result in the main text is Turkey, which has a -0.05% welfare loss due to over-accumulation in the main analysis, but now has a significant positive welfare gain and still some room for welfare improvement by accumulating more quickly. This is because Turkey has a small FDI inflow-to-GDP ratio at 1.04% in the data, while Starting a Business Index is high at 84.7, and thus the estimated FDI inflow-to-GDP ratio is 4.02%, which is four times as large as the actual ratio.

Table 7: Evaluation of Reserve Accumulation Pace

Country	Accum. Pace (%)		Welfare (%)		Elasticity of Spread	Estimated FDI/GDP
	Actual	Optimal	Actual	Optimal		
Argentina	0.9	0.9	0.05	0.05	0.0736	2.22
Brazil	1.2	0.3	-0.03	0.02	0.0913	1.75
Chile	1.2	1.5	0.14	0.14	0.0913	3.59
Colombia	1.0	1.2	0.11	0.12	0.0736	3.10
Dominican Rep.	0.7	1.2	0.08	0.09	0.0736	2.75
Ecuador	0.1	0.5	0.01	0.02	0.0913	1.82
Egypt	2.3	1.2	0.06	0.10	0.0382	2.40
Hungary	2.5	2.4	0.24	0.24	0.0382	3.78
Indonesia	1.4	0.9	0.03	0.05	0.0382	1.76
Malaysia	4.3	2.7	0.17	0.25	0.0205	3.30
Mexico	0.8	1.5	0.13	0.15	0.0913	3.73
Panama	0.8	3.3	0.15	0.37	0.0382	5.01
Peru	2.5	0.9	-0.13	0.05	0.0913	2.40
Philippines	2.3	1.5	0.06	0.09	0.0205	1.88
Poland	1.6	1.2	0.08	0.09	0.0382	2.22
South Africa	0.8	1.8	0.10	0.15	0.0382	2.89
Thailand	3.4	2.7	0.23	0.25	0.0205	3.32
Tunisia	1.4	1.8	0.18	0.19	0.0559	3.61
Turkey	1.0	2.4	0.17	0.22	0.0559	4.02
Uruguay	1.4	0.9	0.02	0.05	0.0913	2.31
Venezuela	0.8	0.3	0.00	0.02	0.0913	1.73

E Proof of Linear Relations in Value Functions

This section shows the detailed procedure of the guess-and-verify method to prove the linear relation in value functions for intermediate producing firms.

E.1 Foreign Firms

I guess the linear relation $V_t^F(n) = nV_t^F(1)$. I first work on the value of a foreign firm with a single product line:

$$V_t^F(1) = \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} \left[\sum_{i=0}^1 P(i, 1, i_t^F) \left(\sum_{j=0}^1 P(j, 1, d_t) E_t(V_{t+1}^F(1 + i - j)) \right) \right] \right\}$$

There are 4 cases next period, depending on whether innovation is successful or not, and replacement happens or not. Writing out all 4 cases and noting $V_{t+1}^F(0) = 0$,

$$\begin{aligned} V_t^F(1) &= \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} [P(0, 1, i_t^F)P(0, 1, d_t)E_t(V_{t+1}^F(1))] \right. \\ &\quad + P(1, 1, i_t^F)P(1, 1, d_t)E_t(V_{t+1}^F(1)) \\ &\quad \left. + P(1, 1, i_t^F)P(0, 1, d_t)E_t(V_{t+1}^F(2)) \right\} \\ &= \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} [(1 - i_t^F)(1 - d_t)E_t(V_{t+1}^F(1))] + i_t^F d_t E_t(V_{t+1}^F(1)) + i_t^F(1 - d_t)E_t(V_{t+1}^F(2)) \right\} \end{aligned}$$

Using the linear relation $V_{t+1}^F(2) = 2V_{t+1}^F(1)$,

$$\begin{aligned} V_t^F(1) &= \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} [(1 - i_t^F)(1 - d_t)E_t(V_{t+1}^F(1))] + i_t^F d_t E_t(V_{t+1}^F(1)) + 2i_t^F(1 - d_t)E_t(V_{t+1}^F(1)) \right\} \\ &\quad \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} [(1 + i_t^F - d_t)E_t(V_{t+1}^F(1))] \right\} \end{aligned}$$

which is equation (18) in the main text. Next I work on the value of a foreign firm with n product lines:

$$V_t^F(n) = \max_{Z_t^F} \left\{ n\pi_t^F - nZ_t^F + \frac{1}{R^F} \left[\sum_{i=0}^n P(i, n, i_t^F) \left(\sum_{j=0}^n P(j, n, d_t) E_t(V_{t+1}^F(n + i - j)) \right) \right] \right\}$$

Using the linear relation $V_{t+1}^F(n+i-j) = (n+i-j)V_{t+1}^F(1)$,

$$\begin{aligned} V_t^F(n) &= \max_{Z_t^F} \left\{ n\pi_t^F - nZ_t^F + \frac{1}{R^F} \left[\sum_{i=0}^n P(i, n, i_t^F) \left(\sum_{j=0}^n P(j, n, d_t)(n+i-j) E_t(V_{t+1}^F(1)) \right) \right] \right\} \\ &= \max_{Z_t^F} \left\{ n\pi_t^F - nZ_t^F + \frac{1}{R^F} E_t(V_{t+1}^F(1)) \left[\sum_{i=0}^n P(i, n, i_t^F) \sum_{j=0}^n P(j, n, d_t)(n+i-j) \right] \right\} \end{aligned}$$

Inside of bracket can be written as follows:

$$\sum_{i=0}^n P(i, n, i_t^F) \sum_{j=0}^n P(j, n, d_t)(n+i-j) = n + \sum_{i=0}^n P(i, n, i_t^F)i - \sum_{j=0}^n P(j, n, d_t)j = n + ni_t^F - nd_t$$

Note that the last two terms are just the expected number of successes for each binomial process. Thus $V_t^F(n)$ can be written as follows:

$$\begin{aligned} V_t^F(n) &= \max_{Z_t^F} \left\{ n\pi_t^F - nZ_t^F + \frac{1}{R^F} n(1 + i_t^F - d_t) E_t(V_{t+1}^F(1)) \right\} \\ &= n \max_{Z_t^F} \left\{ \pi_t^F - Z_t^F + \frac{1}{R^F} (1 + i_t^F - d_t) E_t(V_{t+1}^F(1)) \right\} \\ &= nV_t^F(1) \end{aligned}$$

This verifies that my initial guess $V_t^F(n) = nV_t^F(1)$ is correct.

E.2 Domestic Firms

I guess the linear relation $V_t^D(n) = nV_t^D(1)$. Again I first work on the value of a domestic firm with a single product line, this time taking into account acquisition by foreign investors:

$$\begin{aligned} V_t^D(1) &= \max_{Z_t^D} \left\{ \pi_t^D - Z_t^D \right. \\ &\quad + \left[\sum_{i=0}^1 P(i, 1, i_t^D) \left\{ \sum_{j=0}^1 P(j, 1, d_t) \left(\sum_{k=0}^{1-j} P\left(k, 1-j, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t[\Lambda_{t,t+1} V_{t+1}^D(1+i-j-k)] \right) \right\} \right] \\ &\quad \left. + \left[\sum_{j=0}^1 P(j, 1, d_t) \left(\sum_{k=0}^{1-j} P\left(k, 1-j, \frac{e_t^F}{1-\theta_{t-1}}\right) kQ_t \right) \right] \right\} \end{aligned}$$

There are now 6 cases next period: Whether innovation is successful or not, and whether the product line is replaced, acquired, or survives. Writing out the second line,

$$\begin{aligned}
& \sum_{i=0}^1 P(i, 1, i_t^D) \left\{ \sum_{j=0}^1 P(j, 1, d_t) \left(\sum_{k=0}^{1-j} P\left(k, 1-j, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t [\Lambda_{t,t+1} V_{t+1}^D(1+i-j-k)] \right) \right\} \\
= & P(0, 1, i_t^D) P(0, 1, d_t) P\left(0, 1, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t (\Lambda_{t,t+1} V_{t+1}^D(1)) \\
& + P(1, 1, i_t^D) P(0, 1, d_t) P\left(0, 1, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t (\Lambda_{t,t+1} V_{t+1}^D(2)) \\
& + P(1, 1, i_t^D) P(0, 1, d_t) P\left(1, 1, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t (\Lambda_{t,t+1} V_{t+1}^D(1)) \\
& + P(1, 1, i_t^D) P(1, 1, d_t) E_t (\Lambda_{t,t+1} V_{t+1}^D(2))
\end{aligned}$$

Using the linear relation $V_{t+1}^D(2) = 2V_{t+1}^D(1)$,

$$\begin{aligned}
& = \left[P(0, 1, i_t^D) P(0, 1, d_t) P\left(0, 1, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t (\Lambda_{t,t+1} V_{t+1}^D(1)) \right] \\
& + \left[P(1, 1, i_t^D) P(0, 1, d_t) P\left(0, 1, \frac{e_t^F}{1-\theta_{t-1}}\right) 2E_t (\Lambda_{t,t+1} V_{t+1}^D(1)) \right] \\
& + \left[P(1, 1, i_t^D) P(0, 1, d_t) P\left(1, 1, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t (\Lambda_{t,t+1} V_{t+1}^D(1)) \right] \\
& + [P(1, 1, i_t^D) P(1, 1, d_t) E_t (\Lambda_{t,t+1} V_{t+1}^D(1))] \\
= & (1 - i_t^D)(1 - d_t) \left(1 - \frac{e_t^F}{1-\theta_{t-1}}\right) E_t (\Lambda_{t,t+1} V_{t+1}^D(1)) \\
& + 2i_t^D(1 - d_t) \left(1 - \frac{e_t^F}{1-\theta_{t-1}}\right) E_t (\Lambda_{t,t+1} V_{t+1}^D(1)) \\
& + i_t^D(1 - d_t) \frac{e_t^F}{1-\theta_{t-1}} E_t (\Lambda_{t,t+1} V_{t+1}^D(1)) \\
& + i_t^D d_t E_t (\Lambda_{t,t+1} V_{t+1}^D(1)) \\
= & i_t^D + (1 - d_t) \left(1 - \frac{e_t^F}{1-\theta_{t-1}}\right) E_t (\Lambda_{t,t+1} V_{t+1}^D(1))
\end{aligned}$$

Next, writing out the third line,

$$\sum_{j=0}^1 P(j, 1, d_t) \left(\sum_{k=0}^{1-j} P\left(k, 1-j, \frac{e_t^F}{1-\theta_{t-1}}\right) k Q_t \right) = (1 - d_t) \frac{e_t^F}{1-\theta_{t-1}} Q_t$$

Therefore $V_t^D(1)$ can be written as follows:

$$V_t^D(1) = \max_{Z_t^D} \left\{ \pi_t^D - Z_t^D + \left[i_t^D + (1 - d_t) \left(1 - \frac{e_t^F}{1-\theta_{t-1}}\right) \right] E_t (\Lambda_{t,t+1} V_{t+1}^D(1)) + (1 - d_t) \frac{e_t^F}{1-\theta_{t-1}} Q_t \right\}$$

which is equation (23) in the main text. Next I work on the value of a domestic firm with n product lines:

$$\begin{aligned}
V_t^D(n) &= \max_{Z_t^D} \left\{ n\pi_t^D - nZ_t^D \right. \\
&\quad + \left[\sum_{i=0}^n P(i, n, i_t^D) \left\{ \sum_{j=0}^n P(j, n, d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) E_t [\Lambda_{t,t+1} V_{t+1}^D(n+i-j-k)] \right) \right\} \right] \\
&\quad + \left[\sum_{j=0}^n P(j, n, d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) kQ_t \right) \right] \left. \right\}
\end{aligned}$$

Using the linear relation $V_{t+1}^D(n+i-j-k) = (n+i-j-k)V_{t+1}^D(1)$,

$$\begin{aligned}
V_t^D(n) &= \max_{Z_t^D} \left\{ n\pi_t^D - nZ_t^D \right. \\
&\quad + \left[\sum_{i=0}^n P(i, n, i_t^D) \left\{ \sum_{j=0}^n P(j, n, d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) (n+i-j-k) E_t [\Lambda_{t,t+1} V_{t+1}^D(1)] \right) \right\} \right] \\
&\quad + \left[\sum_{j=0}^n P(j, n, d_t) \left(\sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) kQ_t \right) \right] \left. \right\} \\
&= \max_{Z_t^D} \left\{ n\pi_t^D - nZ_t^D \right. \\
&\quad + E_t [\Lambda_{t,t+1} V_{t+1}^D(1)] \left[\sum_{i=0}^n P(i, n, i_t^D) \sum_{j=0}^n P(j, n, d_t) \sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) (n+i-j-k) \right] \\
&\quad + Q_t \left[\sum_{j=0}^n P(j, n, d_t) \sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) k \right] \left. \right\}
\end{aligned}$$

The bracketed term in the second line is:

$$\begin{aligned}
&\sum_{i=0}^n P(i, n, i_t^D) \sum_{j=0}^n P(j, n, d_t) \sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) (n+i-j-k) \\
&= n + ni_t^D - nd_t - \sum_{j=0}^n P(j, n, d_t) \sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) k \\
&= n + ni_t^D - nd_t - \sum_{j=0}^n P(j, n, d_t) \left[(n-j) \frac{e_t^F}{1-\theta_{t-1}} \right] \\
&= n + ni_t^D - nd_t - n \frac{e_t^F}{1-\theta_{t-1}} + nd_t \frac{e_t^F}{1-\theta_{t-1}} \\
&= n \left[i_t^D + (1-d_t) \left(1 - \frac{e_t^F}{1-\theta_{t-1}} \right) \right]
\end{aligned}$$

The bracketed term in the last line is:

$$\sum_{j=0}^n P(j, n, d_t) \sum_{k=0}^{n-j} P\left(k, n-j, \frac{e_t^F}{1-\theta_{t-1}}\right) k = n \left[(1-d_t) \frac{e_t^F}{1-\theta_{t-1}} \right]$$

Therefore $V_t^D(n)$ can be written as follows:

$$\begin{aligned} V_t^D(n) &= \max_{Z_t^D} \left\{ n\pi_t^D - nZ_t^D \right. \\ &\quad + n \left[i_t^D + (1-d_t) \left(1 - \frac{e_t^F}{1-\theta_{t-1}} \right) \right] E_t [\Lambda_{t,t+1} V_{t+1}^D(1)] \\ &\quad \left. + n \left[(1-d_t) \frac{e_t^F}{1-\theta_{t-1}} \right] Q_t \right\} \\ &= nV_t^D(1) \end{aligned}$$

This verifies that my initial guess $V_t^D(n) = nV_t^D(1)$ is correct.