Monetary Policy and Macroeconomic Stability Revisited*

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Abstract

A large literature has established that the Fed's change from a passive to an active policy response to inflation led to U.S. macroeconomic stability after the Great Inflation of the 1970s. This paper revisits the literature's view by estimating a generalized New Keynesian model using a full-information Bayesian method that allows for equilibrium indeterminacy and adopts a sequential Monte Carlo algorithm. The estimated model shows an active policy response to inflation even during the Great Inflation. Moreover, a more active policy response to inflation alone does not suffice for explaining the macroeconomic stability, unless it is accompanied by a change in either trend inflation or policy responses to the output gap and output growth. Our model empirically outperforms its canonical counterpart that is similar to models used in the literature, thus giving strong support to our view.

Keywords. Monetary policy, Great Inflation, Equilibrium indeterminacy, Generalized New Keynesian model, Sequential Monte Carlo algorithm

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1 Introduction

What led to macroeconomic stability in the U.S. after the Great Inflation of the 1970s? A large literature has regarded the Great Inflation as a consequence of self-fulfilling expectations in indeterminate equilibrium, which lasted until determinacy was restored by changes in the Fed's policy under the chairmanship of Paul Volcker and his successors.[1] In particular, this literature has established the view that the U.S. economy's shift from indeterminacy to determinacy was achieved by the Fed's change from a passive to an active policy response to inflation.[2] Clarida, Galí, and Gertler (2000) demonstrate this view by estimating a monetary policy rule of the sort proposed by Taylor (1993) during two periods, before and after Volcker's appointment as Fed Chairman, and combining the estimated rule with a calibrated New Keynesian (henceforth NK) model to analyze determinacy.[3] Lubik and Schorfheide (2004) confirm the view by estimating a Taylor-type rule and an NK model jointly during similar periods with a full-information Bayesian approach that allows for indeterminacy and sunspot fluctuations.[4]

This paper revisits the literature's view by estimating a generalized NK (henceforth GNK) model jointly with a Taylor-type rule.[5] This model differs from canonical NK (henceforth CNK) models used in the literature mainly in that, following micro evidence, each period some prices remain unchanged even under non-zero trend inflation. As a consequence, a generalized NK Phillips curve takes the place of a canonical one, and causes the GNK

[1] Following the literature, this paper explains the U.S. macroeconomic stability from the perspective of monetary policy. Other explanations emphasize a decline in the volatility of shocks to the U.S. economy (e.g., Sims and Zha, 2006; Justiniano and Primiceri, 2008) or the development of inventory management (e.g., Kahn, McConnell, and Perez-Quirós, 2002).

[2] A policy response to inflation is called *active* if it satisfies the Taylor principle, which claims that the (nominal) interest rate should be raised by more than the increase in inflation. Otherwise, it is called *passive*.

[3] Mavroeidis (2010) points to a weak-identification issue in the GMM estimation of the Taylor-type rule by Clarida, Galí, and Gertler (2000), and emphasizes the need to make use of identifying assumptions that can be derived from the full structure of their model.

[4] See also Boivin and Giannoni (2006), Kimura and Kurozumi (2010), and Lubik and Matthes (2016) among others for the monetary-policy explanation of U.S. macroeconomic stability after the Great Inflation.

[5] For a literature review on GNK models, see, e.g., Ascari and Sbordone (2014).

model to be more susceptible to indeterminacy than CNK models, as indicated by Ascari and Ropele (2009), Hornstein and Wolman (2005), and Kiley (2007).[6] Indeed, even an active policy response to inflation that generates determinacy in CNK models can induce indeterminacy in the GNK model.

Our estimation is performed using a full-information Bayesian approach based on Lubik and Schorfheide (2004).[7] A difficulty in their approach is that when a model is estimated over both determinacy and indeterminacy regions of the model's parameter space, its likelihood function is possibly discontinuous at the boundary of each region. As a consequence, the Random-Walk Metropolis-Hastings (henceforth RWMH) algorithm—which has been the most widely used in Bayesian estimation—can get stuck near a local mode and fail to find the entire posterior distribution for the model's parameters. To deal with this difficulty, our paper adopts the sequential Monte Carlo (henceforth SMC) algorithm developed by Herbst and Schorfheide (2014, 2015). As they illustrate, the SMC algorithm can produce more reliable estimates of model parameters than the RWMH algorithm when the parameters' posterior distribution is multimodal. This is particularly the case when the likelihood function of a model to be estimated exhibits discontinuity as in our paper.

Our empirical analysis makes three main contributions to the literature. First of all, the GNK model empirically outperforms its CNK counterpart in both periods before and after the Volcker disinflation of 1979–1982.[8] This finding indicates that the former model is more suitable than the latter for the analysis of what led to U.S. macroeconomic stability after the Great Inflation, which has been examined with CNK models in the literature. In other words, the feature of the GNK model that some prices remain unchanged in each time period

[6] See also Coibion and Gorodnichenko (2011), Kobayashi and Muto (2013), Kurozumi (2014, 2016), and Kurozumi and Van Zandweghe (2016, 2017).

[7] The full-information Bayesian approach of Lubik and Schorfheide (2004) has been used in previous studies, such as Benati and Surico (2009), Bhattarai, Lee, and Park (2012, 2016), Doko Tchatoka et al. (2017), and Hirose (2007, 2008, 2013, 2014).

[8] The CNK counterpart can be derived by altering the GNK model so that prices which are kept unchanged in the model are updated by indexing to trend inflation as in Yun (1996). Therefore, these two models coincide only when trend inflation is zero. This implies that the GNK model does not necessarily literally generalize the CNK counterpart.

is not only more consistent with micro evidence on price adjustment but also improves the model's fit to U.S. macroeconomic time series.

Second, the U.S. economy was likely in the indeterminacy region of the (GNK) model's parameter space before 1979, while it was likely in the determinacy region after 1982, in line with the result obtained in the literature. However, even during the pre-1979 period, the estimated response to inflation was active in the Taylor-type rule, which adjusts the interest rate for current values of inflation, the output gap, and output growth in the presence of interest rate smoothing.[9] This finding contrasts sharply with the literature's view that the policy response to inflation was passive during the Great Inflation and that the subsequent change to an active response led to the shift from indeterminacy to determinacy.[10]

Last but not least, the increase in the policy response to inflation from the pre-1979 to the post-1982 estimate alone does not suffice for explaining the U.S. economy's shift, unless it is accompanied by either the estimated decline in trend inflation or the estimated change in the policy responses to the output gap and output growth. This finding reveals that a lower rate of trend inflation (or equivalently a lower inflation target in the model), a more dampened response to the output gap, and a more aggressive response to output growth play a key role in accounting for the shift to determinacy, along with a more active response to inflation. Therefore, the finding extends the literature by emphasizing the importance of the changes in other aspects of monetary policy than its response to inflation.

This paper is an extension of Lubik and Schorfheide (2004) and a complementary study to Coibion and Gorodnichenko (2011).[11] Our paper strengthens the analysis of Lubik and Schorfheide by adopting the SMC algorithm in their full-information Bayesian approach and

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[9] Orphanides (2004) obtains active responses to *expected future* inflation in both periods before and after Volcker's appointment as Fed Chairman by estimating a Taylor-type rule with real-time data on the Federal Reserve Board's Greenbook forecast. See also Coibion and Gorodnichenko (2011).

[10] The CNK counterpart confirms the literature's view; that is, the policy response to inflation was passive and the U.S. economy was likely in the indeterminacy region before 1979, while the response was active and the economy was likely in the determinacy region after 1982.

[11] Arias et al. (2017) extend the analysis of Coibion and Gorodnichenko (2011) by employing a medium-scale GNK model based on Christiano, Eichenbaum, and Evans (2005), which is estimated during a post-1984 period within the determinacy region of the model's parameter space.

estimating the GNK model (jointly with the Taylor-type rule) as well as its CNK counterpart, which is similar to their model. Coibion and Gorodnichenko revisit the literature's view by using a calibrated GNK model in an approach analogous to Clarida, Galí, and Gertler (2000), and offer the alternative view that the U.S. economy's shift to determinacy after the Great Inflation is due to their estimated change in a Taylor-type rule and their calibrated fall in trend inflation.[12] An advantage of ours over these studies is that our GNK model empirically outperforms its CNK counterpart that is similar to models employed in the literature, giving strong support to our view on the economy's shift rather than the literature's view.

The remainder of the paper proceeds as follows. Section 2 presents a GNK model with a Taylor-type rule. Section 3 explains the estimation strategy and data. Section 4 shows the results of the empirical analysis. Section 5 concludes.

2 Generalized New Keynesian Model

This paper investigates the source of the U.S. economy's shift from indeterminacy of equilibrium to determinacy after the Great Inflation by estimating a GNK model jointly with a Taylor-type rule. This model differs from CNK models used in previous literature mainly in that, following micro evidence, each period a fraction of prices remains unchanged even under non-zero trend inflation. As a consequence, a generalized NK Phillips curve takes the place of a canonical one.

In the model there are a representative household, a representative final-good firm, a continuum of intermediate-good firms, and a central bank. The model introduces (external) habit formation in the household's consumption preferences, backward-looking price-setters among intermediate-good firms (as in Galí and Gertler, 1999), and interest-rate smoothing in the Taylor-type rule so that output, inflation, and the interest rate could exhibit persistence. This is because our estimation is conducted with a full-information Bayesian approach based on Lubik and Schorfheide (2004), which may have a bias toward indeterminacy unless the model can generate sufficient persistence, as argued by Beyer and Farmer (2007).

[12] In the estimation of the Taylor-type rule by Coibion and Gorodnichenko (2011), its constant term contains not only trend inflation but also other factors. They thus calibrate the level of trend inflation.

2.1 Households

The representative household consumes final goods $\tilde{C}_t$, supplies labor $\{l_t(i)\}$ specific to each intermediate-good firm $i \in [0,1]$, and purchases one-period riskless bonds B_t to maximize the utility function $E_0 \sum_{t=0}^{\infty} \beta^t \exp(z_{u,t}) \left\{ \log(\tilde{C}_t - hC_{t-1}) - [1/(1+1/\eta)] \int_0^1 (l_t(i))^{1+1/\eta} di \right\}$ subject to the budget constraint $P_t\tilde{C}_t + B_t = \int_0^1 P_t W_t(i) l_t(i) di + r_{t-1}B_{t-1} + T_t$, where E_t is the rational expectations operator conditional on information available in period t, $\beta \in (0,1)$ is the subjective discount factor, $h \in [0,1]$ is the degree of habit persistence in consumption preferences, $\eta \geq 0$ is the elasticity of labor supply, P_t is the price of final goods, $W_t(i)$ is the real wage paid by intermediate-good firm i, r_t is the (gross) interest rate on bonds and is assumed to equal the monetary policy rate, T_t consists of lump-sum taxes and transfers and firm profits received, and $z_{u,t}$ is a shock to current preferences.

The first-order conditions for utility maximization with respect to consumption, labor supply, and bond holdings become

$$\Xi_t = \frac{\exp(z_{u,t})}{C_t - hC_{t-1}}, \tag{1}$$

$$W_t(i) = \frac{(l_t(i))^{1/\eta} \exp(z_{u,t})}{\Xi_t}, \tag{2}$$

$$1 = E_t \frac{\beta\, \Xi_{t+1}}{\Xi_t} \frac{r_t}{\pi_{t+1}}, \tag{3}$$

where Ξ_t is the marginal utility of consumption, C_t is aggregate consumption, and $\pi_t = P_t/P_{t-1}$ is the (gross) inflation rate of the final-good price.

2.2 Firms

The representative final-good firm produces homogeneous goods Y_t by choosing a combination of intermediate inputs $\{Y_t(i)\}$ to maximize profit $P_tY_t - \int_0^1 P_t(i)Y_t(i)\, di$ subject to the CES production technology $Y_t = \left[\int_0^1 (Y_t(i))^{(\theta-1)/\theta} di\right]^{\theta/(\theta-1)}$, where $P_t(i)$ is the price of intermediate good i and $\theta > 1$ is the elasticity of substitution between intermediate goods.

The first-order condition for profit maximization yields the final-good firm's demand curve for intermediate good i

$$Y_t(i) = Y_t \left(\frac{P_t(i)}{P_t}\right)^{-\theta}, \tag{4}$$

and thus the CES production technology leads to

$$P_t = \left[\int_0^1 (P_t(i))^{1-\theta} \, di \right]^{\frac{1}{1-\theta}}. \tag{5}$$

The final-good market clearing condition is given by

$$Y_t = C_t. \tag{6}$$

Each intermediate-good firm i produces one kind of differentiated good $Y_t(i)$ under monopolistic competition using the production technology

$$Y_t(i) = A_t l_t(i), \tag{7}$$

where A_t denotes the technology level and follows the stochastic process

$$\log A_t = \log a + \log A_{t-1} + z_{a,t}, \tag{8}$$

where $\log a$ is the steady-state rate of technological change, which turns out to be equal to the steady-state rate of balanced growth, and $z_{a,t}$ is a (non-stationary) technology shock.

The first-order condition for cost minimization yields firm i's real marginal cost

$$mc_t(i) = \frac{W_t(i)}{A_t}. \tag{9}$$

Prices of intermediate goods are set on a staggered basis as in Calvo (1983). In each period, a fraction $\lambda \in (0, 1)$ of firms keeps prices unchanged, while the remaining fraction $1-\lambda$ sets prices in the following two ways. As in Galí and Gertler (1999), a fraction $\omega \in (0, 1)$ of price-setting firms uses a backward-looking rule of thumb, while the remaining fraction $1 - \omega$ optimizes prices.

The price set by the backward-looking rule of thumb is given by

$$P_t^r = P_{t-1}^a \pi_{t-1} \quad \text{or} \quad p_t^r = \frac{P_t^r}{P_t} = \frac{(P_{t-1}^a / P_{t-1}) \, \pi_{t-1}}{P_t / P_{t-1}} = \frac{p_{t-1}^a \pi_{t-1}}{\pi_t}, \tag{10}$$

where

$$P_t^a = (P_t^r)^{\omega} (P_t^o)^{1-\omega} \quad \text{or} \quad p_t^a = \frac{P_t^a}{P_t} = \left(\frac{P_t^r}{P_t} \right)^{\omega} \left(\frac{P_t^o}{P_t} \right)^{1-\omega} = (p_t^r)^{\omega} (p_t^o)^{1-\omega}, \tag{11}$$

and P_t^o is the price set by optimizing firms in period t. The price P_t^o maximizes the relevant profit function $E_t \sum_{j=0}^{\infty} \lambda^j Q_{t,t+j} \left(P_t(i)/P_{t+j} - mc_{t+j}(i) \right) Y_{t+j} (P_t(i)/P_{t+j})^{-\theta}$, where $Q_{t,t+j}$ is the stochastic discount factor between period t and period $t + j$.

The first-order condition for the optimized price P_t^o becomes

$$E_t \sum_{j=0}^{\infty} (\beta\lambda)^j \frac{\Xi_{t+j}}{\Xi_t} \frac{Y_{t+j}}{Y_t} \prod_{k=1}^{j} \pi_{t+k}^{\theta} \left(p_t^o \prod_{k=1}^{j} \frac{1}{\pi_{t+k}} - \frac{\theta}{\theta-1} mc_{t+j}^o \right) = 0, \tag{12}$$

where the equilibrium condition $Q_{t,t+j} = \beta^j \Xi_{t+j}/\Xi_t$ is used and mc_{t+j}^o denotes period-$t+j$ real marginal cost of firms that optimize prices in period t. From (1), (2), (4), (6), (7), and (9), it follows that the marginal cost is given by

$$mc_{t+j}^o = \left(p_t^o \prod_{k=1}^{j} \frac{1}{\pi_{t+k}} \right)^{-\frac{\theta}{\eta}} \left(\frac{Y_{t+j}}{A_{t+j}} \right)^{\frac{1}{\eta}} \left(\frac{Y_{t+j}}{A_{t+j}} - h \frac{Y_{t+j-1}}{A_{t+j}} \right). \tag{13}$$

Under the staggered price-setting, the final-good price equation (5) can be rewritten as

$$1 = (1-\lambda) \left[(1-\omega)(p_t^o)^{1-\theta} + \omega \left(p_t^r\right)^{1-\theta} \right] + \lambda \pi_t^{\theta-1}. \tag{14}$$

For the steady state to be well defined, the following condition is assumed.

$$\lambda \pi^{\theta(1+1/\eta)} < \min(1, \pi^{1+\theta/\eta}), \tag{15}$$

where π is the steady-state value of π_t and represents the (gross) rate of trend inflation.

2.3 Central bank

The central bank conducts monetary policy according to a Taylor-type rule. This rule adjusts the policy rate r_t in response to inflation π_t, the output gap x_t, and output growth Y_t/Y_{t-1} in the presence of policy rate smoothing:

$$\log r_t = \phi_r \log r_{t-1} + (1-\phi_r) \left[\log r + \phi_\pi (\log \pi_t - \log \pi) + \phi_x \log x_t + \phi_{\Delta y} \left(\log \frac{Y_t}{Y_{t-1}} - \log a \right) \right] + z_{r,t}, \tag{16}$$

where the output gap is defined as

$$x_t = \frac{Y_t}{Y_t^n}, \tag{17}$$

Y_t^n is the natural rate of output, $z_{r,t}$ is a monetary policy shock, $r \geq 1$ is the steady-state (gross) policy rate, $\phi_r \in [0,1)$ is the degree of policy rate smoothing, and ϕ_π, ϕ_x, $\phi_{\Delta y}$ are the degrees of policy responses to inflation, the output gap, and output growth.

By considering flexible prices (i.e., $\lambda = \omega = 0$) in the intermediate-good price equation (12) and the final-good price equation (14), and combining the resulting two equations with the marginal cost equation (13), the law of motion for the natural rate of output is given by

$$\left(\frac{Y_t^n}{A_t}\right)^{1+\frac{1}{\eta}} = \frac{\theta-1}{\theta} + h\left(\frac{Y_t^n}{A_t}\right)^{\frac{1}{\eta}} \frac{Y_{t-1}^n}{A_t}. \tag{18}$$

2.4 Log-linearized equilibrium conditions

The equilibrium conditions consist of (1), (3), (6), (8), (10)–(17), and (18). Combining these conditions, rewriting the resulting equations in terms of the detrended variables $y_t = Y_t/A_t, y_t^n = Y_t^n/A_t$, and log-linearizing them under the assumption (15) yields

$$\hat{\pi}_t = \gamma_b \hat{\pi}_{t-1} + \gamma_f E_t \hat{\pi}_{t+1} + \kappa\left[\hat{y}_t + \frac{h}{(a-h)(1+1/\eta)}(\hat{y}_t - \hat{y}_{t-1} + z_{a,t})\right] + \psi_t, \tag{19}$$

$$\psi_t = \beta\lambda\pi^{\theta-1}E_t\psi_{t+1} + \kappa_f(E_t\hat{y}_{t+1} - \hat{y}_t + \theta E_t\hat{\pi}_{t+1} - \hat{r}_t)\,, \tag{20}$$

$$\hat{y}_t = \frac{h}{a+h}(\hat{y}_{t-1} - z_{a,t}) + \frac{a}{a+h}(E_t\hat{y}_{t+1} + E_t z_{a,t+1}) - \frac{a-h}{a+h}(\hat{r}_t - E_t\hat{\pi}_{t+1} + E_t z_{u,t+1} - z_{u,t})\,, \tag{21}$$

$$\hat{r}_t = \phi_r \hat{r}_{t-1} + (1-\phi_r)\left[\phi_\pi \hat{\pi}_t + \phi_x \hat{x}_t + \phi_{\Delta y}(\hat{y}_t - \hat{y}_{t-1} + z_{a,t})\right] + z_{r,t}, \tag{22}$$

$$\hat{x}_t = \hat{y}_t - \hat{y}_t^n, \tag{23}$$

$$\hat{y}_t^n = \frac{h\eta}{a(1+\eta)-h}\left(\hat{y}_{t-1}^n - z_{a,t}\right)\,, \tag{24}$$

where hatted variables denote log-deviations from steady-state values, ψ_t is an auxiliary variable, and the coefficients in (19) and (20) are given by $\gamma_b = \omega/\varphi$, $\gamma_f = \beta\lambda\pi^{\theta(1+1/\eta)}/\varphi$, $\kappa = (1-\lambda\pi^{\theta-1})(1-\beta\lambda\pi^{\theta(1+1/\eta)})(1+1/\eta)(1-\omega)/[\varphi(1+\theta/\eta)]$, $\kappa_f = \beta\lambda\pi^{\theta-1}(\pi^{1+\theta/\eta}-1)(1-\lambda\pi^{\theta-1})(1-\omega)/[\varphi(1+\theta/\eta)]$, and $\varphi = \lambda\pi^{\theta-1} + \omega(1-\lambda\pi^{\theta-1} + \beta\lambda\pi^{\theta(1+1/\eta)})$. Equation (19) is the generalized NK Phillips curve.

Each of the three shocks $z_{j,t}$, $j \in \{u, a, r\}$ is assumed to follow the stationary first-order autoregressive process

$$z_{j,t} = \rho_j z_{j,t-1} + \varepsilon_{j,t}, \tag{25}$$

where $\rho_j \in [0, 1)$ is the autoregressive parameter and $\varepsilon_{j,t} \sim$ i.i.d. $N(0, \sigma_j^2)$ is the innovation to each shock.

2.5 Canonical New Keynesian counterpart

In addition to the GNK model presented above we consider its CNK counterpart, to confirm the literature's view on the U.S. economy's shift from indeterminacy to determinacy after the Great Inflation and compare that view with ours. The CNK counterpart can be derived by altering the model so that firms that keep prices unchanged in the aforementioned setting update prices using indexation to trend inflation as in Yun (1996). The resulting system of log-linearized equilibrium conditions consists of (21)–(25) and the NK Phillips curve

$$\hat{\pi}_t = \gamma_{b,1}\hat{\pi}_{t-1} + \gamma_{f,1}E_t\hat{\pi}_{t+1} + \kappa_1\left[\hat{y}_t + \frac{h}{(a-h)(1+1/\eta)}(\hat{y}_t - \hat{y}_{t-1} + z_{a,t})\right], \tag{26}$$

where the coefficients $\gamma_{b,1}$, $\gamma_{f,1}$, κ_1 correspond to γ_b, γ_f, κ with zero trend inflation (i.e., $\pi = 1$). Hence, the GNK model differs from the CNK counterpart only in that the generalized NK Phillips curve (19) takes the place of the NK Phillips curve (26) along with the auxiliary-variable equation (20). Moreover, the two models coincide only when trend inflation is zero.

3 Estimation Strategy and Data

This section describes the strategy and data for estimating the GNK model and its CNK counterpart presented in the preceding section. To investigate the source of the U.S. economy's shift from indeterminacy of equilibrium to determinacy after the Great Inflation, both models are estimated with a full-information Bayesian approach based on Lubik and Schorfheide (2004). Specifically, each model's likelihood function is constructed not only for the determinacy region of the model's parameter space but also for the indeterminacy region.[13] The likelihood function can then exhibit discontinuity at the boundary of each region.[14] As a consequence, the posterior distribution for parameters in the model is possibly multimodal and thus the extensively used RWMH algorithm can get stuck near a local

[13] The full-information Bayesian approach of Lubik and Schorfheide (2004) allows for indeterminate equilibrium by including a sunspot shock and its related arbitrary coefficient matrix in solutions to linear rational expectations models. By estimating the coefficient matrix with a fairly loose prior for it, a set of particular solutions that are the most consistent with data can be selected from a full set of solutions.

[14] With a univariate model, Lubik and Schorfheide (2004) illustrate discontinuity of the model's likelihood function that is constructed for both determinacy and indeterminacy regions of its parameter space.

mode and fail to find the entire posterior distribution for the parameters. To deal with this problem, the SMC algorithm developed by Herbst and Schorfheide (2014, 2015) is adopted to generate the posterior distribution.[15] The SMC algorithm can overcome the problem inherent in multimodality by building a particle approximation to the posterior distribution gradually through tempering the likelihood function.

This section begins by explaining the method for solving linear rational expectations (henceforth LRE) models under indeterminacy. It then accounts for how Bayesian inferences over both determinacy and indeterminacy regions of the parameter space are made with the SMC algorithm. Moreover, it presents the data and prior distributions used in estimation.

3.1 Rational expectations solutions under indeterminacy

Lubik and Schorfheide (2003) derive a full set of solutions to LRE models by extending the solution algorithm developed by Sims (2002).[16] Any LRE model can be written in the canonical form

$$\Gamma_0(\vartheta)s_t = \Gamma_1(\vartheta)s_{t-1} + \Psi(\vartheta)\varepsilon_t + \Pi(\vartheta)\xi_t, \tag{27}$$

where $\Gamma_0(\vartheta)$, $\Gamma_1(\vartheta)$, $\Psi(\vartheta)$, and $\Pi(\vartheta)$ are coefficient matrices that depend on model parameters ϑ, s_t is a vector of endogenous variables including those expected at time t, ε_t is a vector of fundamental shocks, and ξ_t is a vector of forecast errors. Specifically, in the GNK model, these vectors are given by

$$\begin{aligned}
s_t &= [\hat{y}_t, \hat{\pi}_t, \hat{r}_t, \hat{y}_t^n, \hat{x}_t, \psi_t, z_{u,t}, z_{a,t}, z_{r,t}, E_t\hat{y}_{t+1}, E_t\hat{\pi}_{t+1}, E_t\psi_{t+1}]', \\
\varepsilon_t &= [\varepsilon_{u,t}, \varepsilon_{a,t}, \varepsilon_{r,t}]', \\
\xi_t &= [(\hat{y}_t - E_{t-1}\hat{y}_t), (\hat{\pi}_t - E_{t-1}\hat{\pi}_t), (\psi_t - E_{t-1}\psi_t)]'.
\end{aligned}$$

According to Lubik and Schorfheide (2003), a full set of solutions to the LRE model (27) is of the form

$$s_t = \Phi_x(\vartheta)s_{t-1} + \Phi_\varepsilon(\vartheta, \tilde{M})\varepsilon_t + \Phi_\zeta(\vartheta)\zeta_t, \tag{28}$$

[15] Creal (2007) is the first paper that uses an SMC algorithm in Bayesian estimation of a dynamic stochastic general equilibrium model.

[16] Sims (2002) generalizes the solution algorithm of Blanchard and Kahn (1980) and characterizes one particular solution in the case of indeterminacy. In this solution, the contribution to forecast errors of fundamental shocks and that of sunspot shocks are orthogonal.

where $\Phi_x(\vartheta)$, $\Phi_\varepsilon(\vartheta, \tilde{M})$, and $\Phi_\zeta(\vartheta)$ are coefficient matrices, $\tilde{M}$ is an arbitrary matrix, and ζ_t is a reduced-form sunspot shock, which is a non-fundamental disturbance.[17] For estimation, it is assumed that $\zeta_t \sim$ i.i.d. $N(0, \sigma_\zeta^2)$. In the case of determinacy, the solution (28) is reduced to

$$s_t = \Phi_x^D(\vartheta)\, s_{t-1} + \Phi_\varepsilon^D(\vartheta)\, \varepsilon_t. \tag{29}$$

The solution (28) shows two key features under indeterminacy. First, the dynamics of the LRE model is driven not only by the fundamental shocks ε_t but also by the sunspot shock ζ_t. Second, the solution cannot be unique due to the presence of the arbitrary matrix $\tilde{M}$, that is, the LRE model induces indeterminate solutions. Thus, to specify the law of motion of the endogenous variables s_t, the matrix $\tilde{M}$ must be pinned down.

The arbitrary matrix $\tilde{M}$ is inferred from the data used in estimation, following Lubik and Schorfheide (2004). The prior distribution for $\tilde{M}$ is set so that it is centered around the matrix $M^*(\vartheta)$ given in a particular solution. That is, $\tilde{M}$ is replaced with $M^*(\vartheta) + M$, and M is estimated with prior mean zero. The matrix $M^*(\vartheta)$ is selected so that the contemporaneous impulse responses of endogenous variables to fundamental shocks (i.e., $\partial s_t / \partial \varepsilon_t$) are continuous at the boundary between determinacy and indeterminacy regions of the parameter space. More specifically, for each set of ϑ, the procedure searches for a vector ϑ^* that lies on the boundary of the determinacy region, and selects $M^*(\vartheta)$ that minimizes the discrepancy between $\partial s_t / \partial \varepsilon_t(\vartheta, M^*(\vartheta))$ and $\partial s_t / \partial \varepsilon_t(\vartheta^*)$ using a least-squares criterion. In the search for ϑ^*, the procedure numerically finds ϑ^* by perturbing the parameter ϕ_π in the monetary policy rule (22), given the other parameters in ϑ.

3.2 Bayesian inference with a sequential Monte Carlo algorithm

The LRE model is estimated using a full-information Bayesian approach that extends the model's likelihood function to the indeterminacy region of the parameter space. Following

[17] Lubik and Schorfheide (2003) originally express the last term in (28) as $\Phi_\zeta(\theta, M_\zeta)\zeta_t$, where M_ζ is an arbitrary matrix and ζ_t is a vector of sunspot shocks. For identification, Lubik and Schorfheide (2004) impose the normalization $M_\zeta = 1$ with the dimension of the sunspot shock vector being unity. Such a normalized shock is referred to as a "reduced-form sunspot shock" in that it contains beliefs associated with all the expectational variables.

Lubik and Schorfheide (2004), the likelihood function for a sample of observations $X^T = [X_1, ..., X_T]'$ is given by

$$p(X^T|\vartheta, M) = 1\{\vartheta \in \Theta^D\}\, p^D(X^T|\vartheta) + 1\{\vartheta \in \Theta^I\}\, p^I(X^T|\vartheta, M),$$

where Θ^D and Θ^I are the determinacy and indeterminacy regions of the parameter space; $1\{\vartheta \in \Theta^i\}$, $i \in \{D, I\}$ is the indicator function that equals one if $\vartheta \in \Theta^i$ and zero otherwise; and $p^D(X^T|\vartheta)$ and $p^I(X^T|\vartheta, M)$ are the likelihood functions of the state-space models that consist of observation equations and either the determinacy solution (29) or the indeterminacy solution (28). Then, by Bayes' theorem, updating a prior distribution $p(\vartheta, M)$ with the sample X^T leads to the posterior distribution

$$p(\vartheta, M|X^T) = \frac{p(X^T|\vartheta, M)p(\vartheta, M)}{p(X^T)} = \frac{p(X^T|\vartheta, M)p(\vartheta, M)}{\int p(X^T|\vartheta, M)p(\vartheta, M)d\vartheta \cdot dM}.$$

To approximate the posterior distribution, this paper exploits the generic SMC algorithm with likelihood tempering described in Herbst and Schorfheide (2014, 2015). In the algorithm, a sequence of tempered posteriors are defined as

$$\varpi_n(\vartheta) = \frac{[p(X^T|\vartheta, M)]^{\tau_n} p(\vartheta, M)}{\int [p(X^T|\vartheta, M)]^{\tau_n} p(\vartheta, M)d\vartheta \cdot dM}, \quad n = 0, ..., N_\tau.$$

The tempering schedule $\{\tau_n\}_{n=0}^{N_\tau}$ is determined by $\tau_n = (n/N_\tau)^\chi$, where χ is a parameter that controls the shape of the tempering schedule. The SMC algorithm generates parameter draws and associated importance weights—which are called particles—from the sequence of posteriors $\{\varpi_n\}_{n=1}^{N_\tau}$; that is, at each stage, $\varpi_n(\vartheta)$ is represented by a swarm of particles $\{\vartheta_n^i, w_n^i\}_{i=1}^N$, where N denotes the number of particles. For $n = 0, ..., N_\tau$, the algorithm sequentially updates the swarm of particles $\{\vartheta_n^i, w_n^i\}_{i=1}^N$ through importance sampling.[18] Posterior inferences about parameters to be estimated are made based on the particles $\{\vartheta_{N_\tau}^i, w_{N_\tau}^i\}_{i=1}^N$ from the final importance sampling. The SMC-based approximation of the marginal data density is given by

$$p(X^T) = \prod_{n=1}^{N_\tau} \left(\frac{1}{N} \sum_{i=1}^{N} \tilde{w}_n^i w_{n-1}^i \right),$$

where $\tilde{w}_n^i$ is the incremental weight defined as $\tilde{w}_n^i = [p(X^T|\vartheta_{n-1}^i, M)]^{\tau_n - \tau_{n-1}}$.

[18] This process includes one step of a single-block RWMH algorithm.

In the subsequent empirical analysis, the SMC algorithm uses $N = 10,000$ particles and $N_\tau = 200$ stages. The parameter that controls the tempering schedule is set at $\chi = 2$ following Herbst and Schorfheide (2014, 2015).

3.3 Data

Our estimation is performed using three U.S. quarterly time series: the per-capita real GDP growth rate ($100\Delta \log Y_t$), the inflation rate of the GDP implicit price deflator ($100 \log \pi_t$), and the federal funds rate ($100 \log r_t$). The observation equations that relate the data to model variables are given by

$$\begin{bmatrix} 100\Delta \log Y_t \\ 100 \log \pi_t \\ 100 \log r_t \end{bmatrix} = \begin{bmatrix} \bar{a} \\ \bar{\pi} \\ \bar{r} \end{bmatrix} + \begin{bmatrix} \hat{y}_t - \hat{y}_{t-1} + z_{a,t} \\ \hat{\pi}_t \\ \hat{r}_t \end{bmatrix},$$

where $\bar{a} = 100(a-1)$, $\bar{\pi} = 100(\pi - 1)$, and $\bar{r} = 100(r-1)$.

To examine the shift from indeterminacy to determinacy after the Great Inflation of the 1970s, estimation is conducted during two periods: the pre-1979 period from 1966:I to 1979:II and the post-1982 period from 1982:IV to 2008:IV. Following Lubik and Schorfheide (2004), the Volcker disinflation period from 1979:III to 1982:III is excluded. The post-1982 period ends in 2008:IV, since the estimation strategy is not able to deal with the nonlinearity that arises from the zero lower bound on the (nominal) interest rate.

3.4 Fixed parameters and prior distributions

Before estimation, the elasticity of labor supply and the elasticity of substitution between intermediate goods are fixed at $\eta = 1$ and $\theta = 9.32$ to avoid an identification issue. The former value is a standard one in the macroeconomics literature, while the latter is the estimate of Ascari and Sbordone (2014). All the other parameters are estimated; their prior distributions are shown in Table 1.[19]

The prior distributions for the steady-state (quarterly) rates of output growth, inflation, and interest $\bar{a}, \bar{\pi}, \bar{r}$ are centered around their respective averages over the period from 1966:I

[19] For the subjective discount factor β, the steady-state condition $\beta = \pi a / r$ is used in estimation.

Table 1: Prior distributions for parameters of the GNK model and its CNK counterpart

Parameter	Distribution	Mean	St. dev.
$\bar{a}$	Normal	0.370	0.150
$\bar{\pi}$	Normal	0.985	0.750
$\bar{r}$	Gamma	1.597	0.250
h	Beta	0.700	0.100
ω	Beta	0.500	0.150
λ	Beta	0.500	0.050
ϕ_r	Beta	0.750	0.100
ϕ_π	Gamma	1.500/1.100	0.750
ϕ_x	Gamma	0.125	0.100
$\phi_{\Delta y}$	Gamma	0.125	0.100
ρ_u	Beta	0.500	0.200
ρ_a	Beta	0.500	0.200
ρ_r	Beta	0.500	0.200
σ_u	Inverse gamma	0.627	0.328
σ_a	Inverse gamma	0.627	0.328
σ_r	Inverse gamma	0.627	0.328
σ_ζ	Inverse gamma	0.627	0.328
M_u	Normal	0.000	1.000
M_a	Normal	0.000	1.000
M_r	Normal	0.000	1.000

Notes: The prior mean of the policy response to inflation ϕ_π is 1.5 for the GNK model and 1.1 for its CNK counterpart. The prior probability of equilibrium determinacy is 0.482 for the former and 0.485 for the latter. Inverse gamma distributions are of the form $p(\sigma|\nu, s) \propto \sigma^{-\nu-1}e^{-\nu s^2/2\sigma^2}$, where $\nu = 4$ and $s = 0.5$.

to 2008:IV. The priors for the structural and policy parameters—h (spending habit persistence), ω (fraction of backward-looking price-setters), λ (probability of no price change[20]), ϕ_r (policy rate smoothing), ϕ_π (policy response to inflation), ϕ_x (policy response to the output gap), $\phi_{\Delta y}$ (policy response to output growth)—are based on Smets and Wouters (2007). For the GNK model, these distributions generate the prior probability of equilibrium determinacy of 0.482, which is almost even, thus indicating that there is a priori no substantial bias toward determinacy or indeterminacy.[21] For the CNK counterpart, the prior mean of ϕ_π is set at 1.1 (as in Lubik and Schorfheide, 2004), so that the prior probability of determinacy

[20] In the CNK counterpart, λ represents the probability of trend inflation-indexed price-setting.

[21] The prior probability of equilibrium determinacy can be computed as the prior distributions' probability mass assigned to the determinacy region of the parameter space.

is 0.485.

Regarding the structural shocks, the prior distributions for the autoregressive parameters $\rho_i, i \in \{u, a, r\}$ are beta distributions with mean of 0.5 and standard deviation of 0.2, while those for the standard deviations of the shock innovations $\sigma_i, i \in \{u, a, r\}$ are inverse gamma distributions with mean of 0.63 and standard deviation of 0.33. As for the indeterminacy solution, the priors for the coefficients $M_i, i \in \{u, a, r\}$ are normal distributions with mean zero and standard deviation of unity, while that for the standard deviation of the sunspot shock σ_ζ is the same as those for the standard deviations of the structural shock innovations.

4 Results of Empirical Analysis

This section presents the results of the empirical analysis. First, the estimation results are explained. Then, the main question of what led to the U.S. economy's shift from indeterminacy of equilibrium to determinacy after the Great Inflation is addressed.

4.1 Estimation results

This subsection begins by comparing the empirical performance between the GNK model and its CNK counterpart. Table 2 presents the estimation results for both models in the pre-1979 and post-1982 periods. The second to last row of the table shows that the GNK model has a much larger value of the log marginal data density $\log p(X^T)$ than the CNK counterpart in both periods, i.e., $-127.10 > -133.24$ in the pre-1979 period and $-67.51 > -77.51$ in the post-1982 period. Therefore, the GNK model empirically outperforms its CNK counterpart during both periods before and after the Volcker disinflation. This result indicates that the former model is more suitable than the latter for the analysis of what led to the U.S. macroeconomic stability after the Great Inflation, which has been examined with CNK models in previous literature. In other words, the feature of the GNK model that some prices are kept unchanged in each time period is not only more consistent with micro evidence on price adjustment but also improves the model's fit to the U.S. macroeconomic time series.

The last row of Table 2 reports the posterior probability of equilibrium determinacy

$\mathbb{P}\{\vartheta \in \Theta^D | X^T\}$.[22] For both the GNK model and the CNK counterpart, the probability of determinacy is (almost) zero in the pre-1979 period, whereas it is unity in the post-1982 period. Hence, both models share the estimation result that the U.S. economy was likely in the indeterminacy region of the parameter space before 1979, while the economy was likely in the determinacy region after 1982, in line with the result obtained in previous literature. However, there is an important difference between the estimation results of the two models. In the CNK counterpart, the policy response to inflation ϕ_π was passive (i.e., less than unity) during the pre-1979 period and then became active (i.e., greater than unity) during the post-1982 period. This result is consistent with that obtained in previous literature, and thus the CNK counterpart confirms the literature's view that ascribes the U.S. economy's shift from indeterminacy to determinacy after the Great Inflation to the Fed's change from a passive to an active policy response to inflation. The GNK model shows that the policy response to inflation was already active during the pre-1979 period, in sharp contrast with the literature's view.[23] Because the GNK model outperforms its CNK counterpart during both periods in terms of its fit to the data, our finding is more compelling than the literature's view.

[22]The posterior probability of equilibrium determinacy can be calculated as the posterior distribution's probability mass assigned to the determinacy region of the parameter space.

[23]For an estimated Taylor-type rule, Orphanides (2004) obtains an active response to expected future inflation during the pre-1979 period, thus claiming that self-fulfilling expectations cannot be the source of U.S. macroeconomic instability during the Great Inflation. This claim, however, does not necessarily hold for the GNK model, since an active policy response to inflation is not a sufficient condition for equilibrium determinacy, as also stressed by Coibion and Gorodnichenko (2011).

Table 2: Posterior distributions for parameters of the GNK model and its CNK counterpart

	Pre-1979 period				Post-1982 period			
	GNK model		CNK counterpart		GNK model		CNK counterpart	
Parameter	Mean	90% interval	Mean	90% interval	Mean	90% interval	Mean	90% interval
$\bar{a}$	0.353	[0.156, 0.572]	0.387	[0.172, 0.619]	0.399	[0.211, 0.584]	0.410	[0.223, 0.576]
$\bar{\pi}$	1.512	[1.189, 1.836]	1.349	[0.900, 1.794]	0.701	[0.537, 0.880]	0.679	[0.491, 0.873]
$\bar{r}$	1.663	[1.395, 1.941]	1.585	[1.270, 1.914]	1.442	[1.168, 1.741]	1.385	[1.119, 1.672]
h	0.550	[0.439, 0.653]	0.548	[0.426, 0.669]	0.625	[0.540, 0.713]	0.605	[0.523, 0.682]
ω	0.143	[0.050, 0.222]	0.110	[0.039, 0.180]	0.064	[0.024, 0.102]	0.069	[0.026, 0.110]
λ	0.521	[0.450, 0.594]	0.513	[0.428, 0.595]	0.458	[0.389, 0.534]	0.435	[0.365, 0.503]
ϕ_r	0.707	[0.591, 0.833]	0.692	[0.573, 0.814]	0.678	[0.602, 0.768]	0.617	[0.530, 0.701]
ϕ_π	1.028	[0.399, 1.640]	0.401	[0.083, 0.696]	2.730	[1.924, 3.574]	2.358	[1.795, 2.893]
ϕ_x	0.313	[0.095, 0.562]	0.163	[0.002, 0.320]	0.114	[0.001, 0.229]	0.085	[0.002, 0.168]
$\phi_{\Delta y}$	0.119	[0.003, 0.235]	0.125	[0.003, 0.243]	0.466	[0.269, 0.673]	0.409	[0.239, 0.565]
ρ_u	0.414	[0.125, 0.711]	0.459	[0.124, 0.762]	0.905	[0.868, 0.947]	0.914	[0.881, 0.949]
ρ_a	0.741	[0.492, 0.943]	0.428	[0.123, 0.731]	0.133	[0.014, 0.241]	0.095	[0.011, 0.167]
ρ_r	0.424	[0.200, 0.655]	0.439	[0.252, 0.626]	0.660	[0.579, 0.748]	0.672	[0.598, 0.753]
σ_u	0.968	[0.252, 2.194]	0.681	[0.252, 1.131]	2.032	[1.407, 2.712]	1.974	[1.405, 2.512]
σ_a	0.726	[0.351, 1.075]	1.737	[1.010, 2.487]	1.456	[1.139, 1.761]	1.410	[1.137, 1.685]
σ_r	0.282	[0.229, 0.333]	0.272	[0.226, 0.319]	0.233	[0.177, 0.286]	0.255	[0.193, 0.319]
σ_ζ	0.387	[0.286, 0.475]	0.471	[0.315, 0.610]	1.196	[0.267, 2.402]	0.689	[0.314, 1.039]
M_u	0.010	[−0.479, 0.477]	0.043	[−0.838, 0.784]	−0.608	[−2.243, 1.202]	−0.906	[−1.903, 0.100]
M_a	−0.263	[−0.786, 0.376]	−0.266	[−0.559, 0.009]	−0.751	[−2.202, 0.730]	−0.327	[−1.353, 0.655]
M_r	0.102	[−0.475, 0.708]	1.025	[0.118, 1.897]	0.055	[−1.596, 1.817]	−0.473	[−1.658, 0.705]
$\log p(X^T)$	−127.100		−133.240		−67.513		−77.511	
$\mathbb{P}\{\vartheta \in \Theta^D \vert X^T\}$	0.000		0.002		1.000		1.000	

Notes: This table shows the posterior mean and 90 percent highest posterior density intervals based on 10,000 particles from the final importance sampling in the SMC algorithm. In the table, $\log p(X^T)$ represents the SMC-based approximation of log marginal data density and $\mathbb{P}\{\vartheta \in \Theta^D|X^T\}$ denotes the posterior probability of equilibrium determinacy.

Before investigating the source of the U.S. economy's shift, the remainder of this subsection examines whether abstracting from some properties of the GNK model can improve its fit to the data even further. In light of the result of Cogley and Sbordone (2008) that there is no empirical support for intrinsic inertia of inflation in their generalized NK Phillips curve, our GNK model is estimated in the absence of backward-looking price-setters, i.e., $\omega = 0$. As presented in the second to last row of Table 3, the value of the log marginal data density of the model with $\omega = 0$ is -120.20 for the pre-1979 period and -55.59 for the post-1982 period. These values are substantially greater than those of the model without such a restriction shown in the second to last row of Table 2 (i.e., -127.10 for the pre-1979 period and -67.51 for the post-1982 period), which demonstrates that the data favors the model with $\omega = 0$, as is consistent with the result of Cogley and Sbordone.

In the model with no intrinsic inflation inertia (i.e., $\omega = 0$), Table 3 shows that four of the estimated parameters changed substantially between the pre-1979 and post-1982 periods. First, the policy response to inflation ϕ_π more than doubled from the pre-1979 to the post-1982 period. Second, trend inflation $\bar{\pi}$ fell by more than half. Third, the policy response to output growth $\phi_{\Delta y}$ increased by a factor of almost five. These three changes are significant in that the 90 percent highest posterior density intervals of the three parameters do not overlap between the two periods. Last but not least, the policy response to the output gap ϕ_x decreased considerably. To examine whether this decrease suggests virtually no response to the output gap, the model is further estimated with the additional restriction that the response is fixed at zero, i.e., $\phi_x = 0$. The second to last row of the table shows that the model with $\omega = \phi_x = 0$ fits the data better than the model with only $\omega = 0$ in the post-1982 period but not in the pre-1979 period. This result indicates that the policy response to the output gap diminished from the estimate of 0.37 during the pre-1979 period to zero during the post-1982 period. It suggests that the Fed during the post-1982 period was inclined to disregard the output gap—which involves great uncertainty about the measurement of unobservable potential output, as discussed by Orphanides (2001)—and put more emphasis on output growth as an indicator of real economic activity.

Table 3: Posterior distributions for parameters of the GNK model with no intrinsic inflation inertia ($\omega = 0$)

	Pre-1979 period				Post-1982 period			
	$\omega = 0$		$\omega = \phi_x = 0$		$\omega = 0$		$\omega = \phi_x = 0$	
Parameter	Mean	90% interval	Mean	90% interval	Mean	90% interval	Mean	90% interval
$\bar{a}$	0.379	[0.193, 0.555]	0.374	[0.141, 0.606]	0.392	[0.221, 0.560]	0.404	[0.231, 0.578]
$\bar{\pi}$	1.447	[1.116, 1.768]	1.431	[1.125, 1.766]	0.700	[0.560, 0.839]	0.699	[0.556, 0.841]
$\bar{r}$	1.641	[1.359, 1.920]	1.632	[1.329, 1.928]	1.446	[1.173, 1.722]	1.452	[1.173, 1.713]
h	0.568	[0.430, 0.700]	0.574	[0.458, 0.705]	0.598	[0.520, 0.682]	0.582	[0.500, 0.664]
ω	0	–	0	–	0	–	0	–
λ	0.530	[0.455, 0.601]	0.503	[0.415, 0.585]	0.458	[0.390, 0.522]	0.462	[0.398, 0.535]
ϕ_r	0.702	[0.583, 0.819]	0.651	[0.515, 0.790]	0.690	[0.607, 0.776]	0.678	[0.588, 0.762]
ϕ_π	1.179	[0.260, 2.065]	0.720	[0.143, 1.563]	2.989	[2.228, 3.792]	3.013	[2.143, 3.825]
ϕ_x	0.370	[0.106, 0.620]	0	–	0.125	[0.001, 0.252]	0	–
$\phi_{\Delta y}$	0.106	[0.003, 0.212]	0.214	[0.007, 0.406]	0.526	[0.322, 0.746]	0.525	[0.302, 0.725]
ρ_u	0.456	[0.178, 0.731]	0.521	[0.174, 0.866]	0.923	[0.887, 0.962]	0.918	[0.887, 0.954]
ρ_a	0.554	[0.207, 0.900]	0.597	[0.251, 0.893]	0.146	[0.035, 0.263]	0.154	[0.028, 0.268]
ρ_r	0.412	[0.195, 0.624]	0.410	[0.231, 0.591]	0.608	[0.521, 0.705]	0.591	[0.505, 0.680]
σ_u	1.859	[0.287, 3.333]	0.738	[0.252, 1.359]	2.323	[1.365, 3.318]	2.082	[1.417, 2.787]
σ_a	0.651	[0.291, 0.990]	1.207	[0.470, 2.053]	1.340	[1.066, 1.583]	1.301	[1.041, 1.539]
σ_r	0.282	[0.213, 0.343]	0.289	[0.219, 0.355]	0.240	[0.185, 0.297]	0.240	[0.185, 0.296]
σ_ζ	0.385	[0.284, 0.482]	0.449	[0.260, 0.589]	0.640	[0.263, 1.036]	0.655	[0.266, 1.033]
M_u	−0.013	[−0.370, 0.316]	0.044	[−0.702, 0.834]	0.146	[−1.436, 1.785]	−0.031	[−1.726, 1.573]
M_a	−0.102	[−0.711, 0.629]	0.092	[−0.734, 0.656]	−0.089	[−1.612, 1.554]	−0.003	[−1.675, 1.544]
M_r	0.093	[−0.546, 0.704]	−0.467	[−1.886, 0.702]	−0.065	[−1.698, 1.554]	−0.024	[−1.629, 1.636]
$\log p(X^T)$	−120.200		−124.040		−55.590		−54.214	
$\mathbb{P}\{\vartheta \in \Theta^D \| X^T\}$	0.001		0.168		1.000		1.000	

Notes: This table shows the posterior mean and 90 percent highest posterior density intervals based on 10,000 particles from the final importance sampling in the SMC algorithm. In the table, $\log p(X^T)$ represents the SMC-based approximation of log marginal data density and $\mathbb{P}\{\vartheta \in \Theta^D|X^T\}$ denotes the posterior probability of equilibrium determinacy.

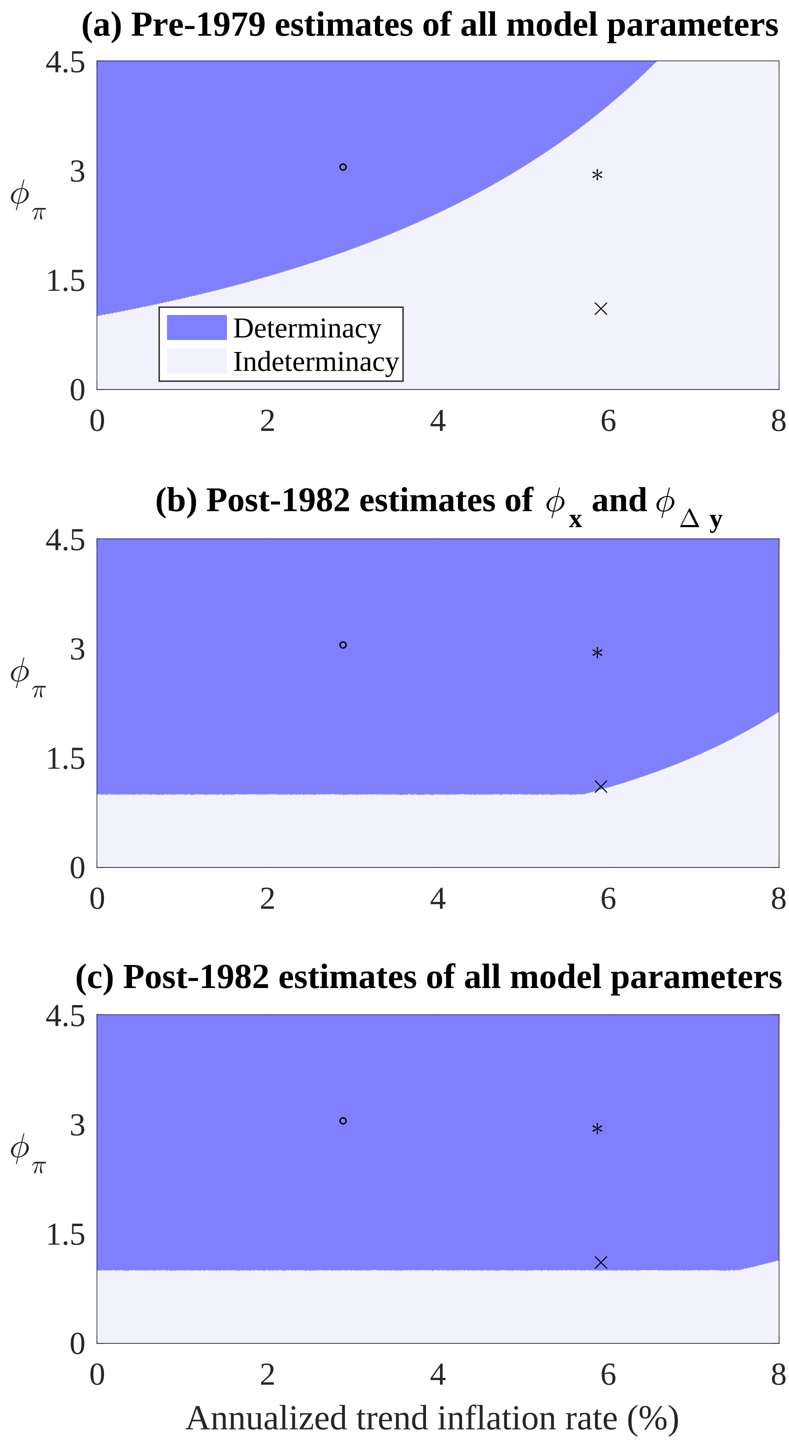
4.2 Source of the U.S. economy's shift from indeterminacy to determinacy

This subsection addresses the main question of what led to the U.S. economy's shift from indeterminacy to determinacy after the Great Inflation. In light of the estimation results in the preceding subsection, the present analysis examines the source of the shift by focusing on the changes in trend inflation and the policy responses to inflation, the output gap, and output growth from the pre-1979 estimates in the GNK model with $\omega = 0$ to the post-1982 estimates in the model with $\omega = \phi_x = 0$.

Figure 1 illustrates how the determinacy region of the GNK model's parameter space for the annualized trend inflation rate $4\bar{\pi}$ and the policy response to inflation ϕ_π expands with changes in the other model parameters. In each panel of the figure, the mark "$\times$", "$*$", and "o" respectively represent the pair of $(4\bar{\pi}^{pre79}, \phi_\pi^{pre79})$, $(4\bar{\pi}^{pre79}, \phi_\pi^{post82})$, and $(4\bar{\pi}^{post82}, \phi_\pi^{post82})$, where $\bar{\pi}^{pre79}$ and ϕ_π^{pre79} denote the posterior mean estimates of the trend inflation rate and the policy response to inflation during the pre-1979 period presented in the second column of Table 3 and $\bar{\pi}^{post82}$ and ϕ_π^{post82} denote those during the post-1982 period presented in the eighth column of the table.

Panel (a) shows the case in which all the model parameters (except trend inflation and the policy response to inflation) are fixed at the pre-1979 estimates (presented in the second column of Table 3). In this panel, the pair of the pre-1979 estimates of trend inflation and the policy response to inflation $(4\bar{\pi}^{pre79}, \phi_\pi^{pre79})$—which is represented by "$\times$"—lies in the indeterminacy region of the parameter space, in line with the estimation result that the posterior probability of determinacy during the pre-1979 period is (almost) zero. The panel also demonstrates that the pair of the pre-1979 estimate of trend inflation and the post-1982 estimate of the policy response to inflation $(4\bar{\pi}^{pre79}, \phi_\pi^{post82})$—which is denoted by "$*$"—is located within the indeterminacy region. This indicates that the increase in the policy response to inflation from the pre-1979 estimate ϕ_π^{pre79} to the post-1982 estimate ϕ_π^{post82} alone does not suffice for explaining the shift from indeterminacy to determinacy. Moreover, the pair of the post-1982 estimates of trend inflation and the policy response to inflation $(4\bar{\pi}^{post82}, \phi_\pi^{post82})$—which is represented by "o"—lies inside the determinacy region. This finding suggests that the shift can be explained by the fall in trend inflation from the

Figure 1: Equilibrium-determinacy region of the GNK model's parameter space



Notes: For the annualized trend inflation rate $4\bar{\pi}$ and the policy response to inflation ϕ_π, the figure illustrates the equilibrium-determinacy region of the GNK model's parameter space. In each panel, the mark "×", "∗", and "∘" respectively represent the pair of $(4\bar{\pi}^{pre79}, \phi_\pi^{pre79})$, $(4\bar{\pi}^{pre79}, \phi_\pi^{post82})$, and $(4\bar{\pi}^{post82}, \phi_\pi^{post82})$, where $\bar{\pi}^{pre79}$ $(\bar{\pi}^{post82})$ and ϕ_π^{pre79} (ϕ_π^{post82}) denote the mean estimates of the trend inflation rate and the policy response to inflation in the pre-1979 (post-1982) period.

pre-1979 estimate $4\bar{\pi}^{pre79}$ to the post-1982 estimate $4\bar{\pi}^{post82}$ along with the increase in the policy response to inflation.

Panel (b) displays the case in which the policy responses to the output gap and output growth, ϕ_x and $\phi_{\Delta y}$, are set at the post-1982 estimates (presented in the eighth column of Table 3), keeping the other model parameters fixed at the pre-1979 estimates. As the difference between panels (a) and (b) shows, the change in the policy responses to the output gap and output growth from the pre-1979 to the post-1982 estimates significantly expands the determinacy region. As a consequence, in panel (b), the pair of the pre-1979 estimates of trend inflation and the policy response to inflation $(4\bar{\pi}^{pre79}, \phi_\pi^{pre79})$ is located near the boundary between the indeterminacy and determinacy regions, whereas the pair of the pre-1979 estimate of trend inflation and the post-1982 estimate of the policy response to inflation $(4\bar{\pi}^{pre79}, \phi_\pi^{post82})$ lies inside the determinacy region. This finding indicates that the decrease in the policy response to the output gap and the increase in the response to output growth, along with the rise in the response to inflation, can account for the shift from indeterminacy to determinacy, regardless of the fall in trend inflation.[24]

Panel (c) presents the case in which all the model parameters are set at the post-1982 estimates. In this panel, the pair of the post-1982 estimates of trend inflation and the policy response to inflation $(4\bar{\pi}^{post82}, \phi_\pi^{post82})$ is located inside the determinacy region, in line with the estimation result that the posterior probability of determinacy during the post-1982 period is unity. Panel (c) is not so different from panel (b), suggesting that the change from the pre-1979 to the post-1982 estimates of all the model parameters other than trend inflation and the policy responses to inflation, the output gap, and output growth plays a minor role in accounting for the shift from indeterminacy to determinacy.

These panels demonstrate that the increase in the policy response to inflation from the pre-1979 to the post-1982 estimate alone does not suffice for explaining the U.S. economy's shift from indeterminacy to determinacy after the Great Inflation, unless it is accompanied by either the estimated fall in trend inflation or the estimated change in the policy responses to the output gap and output growth. Taking into consideration that trend inflation is

[24] In a GNK model with a Taylor-type rule, the destabilizing role of the policy response to the output gap is indicated by Ascari and Ropele (2009), while the stabilizing role of the policy response to output growth is pointed out by Coibion and Gorodnichenko (2011).

equivalent to the central bank's inflation target in the model, this finding indicates that the changes in the Fed's implicit inflation target and policy responses to real economic activity have played a key role in the shift to determinacy, in addition to its more active response to inflation.

5 Conclusion

This paper has revisited a large literature's view that U.S. macroeconomic stability after the Great Inflation of the 1970s was achieved by the Fed's change from a passive to an active policy response to inflation. The paper has estimated a GNK model jointly with a Taylor-type rule during two periods, before and after the Volcker disinflation of 1979–1982, by adopting an SMC algorithm in a full-information Bayesian approach based on Lubik and Schorfheide (2004). It has shown that the GNK model empirically outperforms its CNK counterpart during both periods. This indicates that the former model is more suitable than the latter for analyzing the source of the U.S. macroeconomic stability, which has been examined with CNK models in the literature.

According to the estimated GNK model, the U.S. economy was likely in the equilibrium-indeterminacy region of the model's parameter space before 1979, while it was likely in the determinacy region after 1982, in line with the result obtained in the literature. However, the policy response to (current) inflation was active even during the pre-1979 period, in addition to the post-1982 period, which contrasts sharply with the literature's view that the response to inflation was passive during the Great Inflation and that the subsequent change to an active response led to the U.S. economy's shift from indeterminacy to determinacy. This paper has demonstrated that the increase in the policy response to inflation from the pre-1979 to the post-1982 estimate alone does not suffice for explaining the shift, unless it is accompanied by the change from the pre-1979 to the post-1982 estimates of either trend inflation or the policy responses to the output gap and output growth. This finding extends the literature on the role of monetary policy in achieving macroeconomic stability in the U.S. after the Great Inflation, by emphasizing the importance of the changes in the Fed's implicit inflation target and responses to real economic activity.

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