Off to a Bad Start? The Role of Leverage for Start-Up Productivity and Survival over the Business Cycle*

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Abstract

We propose a new method - the Regression-Augmented Dynamic Olley-Pakes Decomposition (RA-DOPD) - that integrates regression analysis into the productivity decomposition of Melitz and Polanec (2015). The approach uses estimated regression coefficients to split individual decomposition terms into several components, thus capturing distinct channels and offering deeper insights into the micro-drivers of aggregate productivity dynamics. We apply this novel framework to micro data on all Dutch start-ups established during 2003-2008. We track these cohorts over time and use our RA-DOPD approach to identify and quantify the channels through which leverage affects the survival rate and productivity growth of start-ups. Our results show that...

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1 Introduction

Contribution to the literature on:

Start-ups and leverage

Productivity slowdowns and in particular cleaning/sullying role recessions

Methodological contribution: RA-DOPD

2 Empirical approach

2.1 The RA-DOPD: An introduction

Decompositions of aggregate productivity have gained widespread popularity as a tool to understand the micro-drivers of productivity at the macro level.[1] Dynamic versions decompose the growth of productivity rather than the level. Such dynamic decompositions typically take the following form:

$$APG = WI + BE + EX. \tag{1}$$

That is, aggregate productivity growth (APG) is decomposed into three components: a term reflecting within-firm productivity growth (WI); a term capturing changes in the distribution of resources between firms of different sizes (BE) (Olley and Pakes, 1996); and a term capturing the effect of firm exit (EX).[2] Our methodological contribution is to integrate regression analysis into the Dynamic Olley-Pakes Decomposition of Melitz and Polanec (2015). This method, which we label the Regression-Augmented Dynamic Olley-Pakes Decomposition (RA-DOPD), allows for a precise reconstruction of the individual DOPD terms on the basis of estimated regression coefficients. Using the RA-DOPD, each term can be split into several components that capture distinct channels. This allows for

[1] Well-known examples include Baily et al. (1992); Olley and Pakes (1996); Foster et al. (2001); Petrin and Levinsohn (2012); and most recently Melitz and Polanec (2015).

[2] Many applications contain an additional term for firm entry. As we consider firm cohorts, entry plays no role and we therefore omit this term in what follows. In Appendix B, we provide further analysis for those who wish to use our approach to analyze data with firm entry.

deeper insights into the underlying drivers of aggregate productivity changes.

First, we show how, using a simple regression, the Olley-Pakes between term can be split as:

$$BE = AL + DI, \tag{2}$$

where AL reflects changes in the allocation of resources while DI reflects changes in the dispersion of productivity across firms. In the extant literature, the between term and the dispersion of productivity are typically viewed as alternative ways to assess the extent of misallocation, or changes therein. For example, Bartelsman et al. (2013) compare the two measures across a variety of countries and favor the Olley-Pakes between term over dispersion. Other authors, such as Hsieh and Klenow (2017), instead advocate the use of productivity dispersion as a measure of misallocation. Our analysis integrates the two measures into a single decomposition framework and makes clear that the Olley-Pakes between term captures both changes in allocation and in dispersion. Importantly, it also allows for a quantification of the relative importance of both mechanisms.

Second, we show how the exit term can be decomposed when analyzing the difference in productivity growth between two groups of firms, such as two cohorts. Specifically, the difference in the exit term can be decomposed as:[3]

$$dEX = dEXDR + dEXDS \tag{3}$$

In the above equation, $dEXDR$ captures changes in the death rate of firms, whereas $dEXDS$ captures differences in the selection of exiting firms, i.e., the extent to which these firms are on average less productive than continuing firms. The selection term thus allows for an empirical confrontation of theories on the role of selection over the business cycle. For example, Caballero and Hammour (1994) emphasize the “cleansing” effects of recessions – which should increase aggregate productivity – whereas Barlevy (2002) proposes a model in which recessions trigger “sullying” selection effects, reducing

[3] The same decomposition can be used to compare different groups of firms in the same time period.

aggregate productivity. Our method not only helps to distinguish between such theories qualitatively but also delivers a quantification of the impact on aggregate productivity (and the relative roles of the entry and exit margins in this respect).

Third, we show how to assess differences in productivity growth across different cohorts and/or firm sub-populations, while controlling for differences in (other) observable characteristics. For example, we can decompose the difference in aggregate productivity growth between a “bust” and a “boom” cohort as:

$$dAPG = dAPG^{bust} + dAPG^{controls}, \tag{4}$$

where the first term captures the pure effect of the bust while the second term reflects differences due to control variables (such as industry composition and geographical factors). We can perform this split not only for aggregate productivity growth but also separately for the within, between, and exit terms.

The need to control for other factors is particularly acute in our application, in which we compare different cohorts of firms, each of which may differ in terms of composition. However, controlling for multiple observable confounding factors is important in any application in which compositional differences may play a role. Examples are manifold and include the comparison of aggregate productivity growth across different stages of the business cycle or across different groups of firms (like high- and low leverage firms in our application in Section 4). Large micro data sets with many possible control variables are increasingly becoming available and our RA-DOPD method is designed to exploit all this additional information to refine productivity decompositions.

Lastly, we can also exploit the regressions to assess the statistical significance of the various channels through which productivity can differ across groups or across points in time. The use of regression analysis links our work to papers that have estimated regressions to understand variations in misallocation across different stages of the business cycle (Foster et al., 2014) or across countries (Bartelsman et al., 2017). These authors, however, focus on reallocation only and do not link their regressions to productivity decompositions.[4]

After this introduction, we now present our RA-DOPD approach in more technical detail. We start by a short exposition of the standard DOPD approach as proposed by Melitz and Polanec (2015).

2.2 The Dynamic Olley-Pakes Decomposition (DOPD)

Our object of interest is aggregate productivity. In accordance with the literature, we define aggregate productivity at time t as:

$$\Phi_t \equiv \sum_i s_{i,t}\varphi_{i,t}, \tag{5}$$

where $\varphi_{i,t}$ is the logged level of productivity of firm i and $s_{i,t}$ denotes its market share. Thus, aggregate productivity is the average of firm-level productivity, weighted by market share. The measurement of firm-level productivity will be discussed in detail at a later stage. The market share will be measured as the share of employment of the firm, i.e., $s_{i,t} \equiv l_{i,t}/\sum_i l_{i,t}$, where $l_{i,t}$ denotes the firm's employment level. Our purpose is to understand how aggregate productivity changes between two periods $t = 0$ and $t = 1$.[5] That is, we are interested in understanding $\triangle\Phi \equiv \Phi_1 - \Phi_0$. Since productivity at the firm level is measured in logarithms, we can interpret $\triangle\Phi$ as aggregate productivity growth. Melitz and Polanec (2015) show that aggregate productivity growth can be decomposed as in Equation (1), where $APG = \triangle\Phi$ and

$$WI = \triangle \underset{i\in C}{\overline{\varphi}_i}, \qquad BE = \triangle \underset{i\in C}{cov(\widetilde{l}_i, \varphi_i)}, \qquad EX = s_{X,0}\left(\Phi_{C,0} - \Phi_{X,0}\right).$$

Here, upper bars denote averages, X indicates the set of firms that exit between the two periods, and C is the complementary set of continuers. Thus, the within term WI is

[4] Foster, Grim and Haltiwanger (2014) compute counterfactual paths for firm-level market shares, using the regression to isolate the effect of business cycles, and assess the effect on aggregate productivity. A limitation of their approach is that firm-level productivity is held constant between any two periods, resulting in a discrepancy with the data even when market shares are unaffected by the business cycle. Moreover, their exercise requires a simulation for each individual firm in the data.

[5] In our application, we analyze cohorts. $t = 0$ corresponds to age zero (the year of startup) whereas $t = 1$ corresponds to the cohort at age four.

simply the change in the unweighted average of (logged) productivity across continuing firms. For the between term, BE, we define the *relative* employment level of a continuer as $\widetilde{l}_{i,t} \equiv l_{i,t}/\bar{l}_{C,t}$, where $\bar{l}_{C,t}$ is the average employment level across continuers. The between term equals the change in the covariance between (relative) firm size and productivity, as proposed by Olley and Pakes (1996).[6] Finally, in the expression for EX, $s_{X,t} \equiv \sum_{i \in X} s_{i,t}$ is the total market share of exiters, whereas $\Phi_{C,t} \equiv \sum_{i \in C} \frac{s_{i,t}}{s_{X,t}} \varphi_{i,t}$ and $\Phi_{X,t} \equiv \sum_{i \in X} \frac{s_{i,t}}{s_{C,t}} \varphi_{i,t}$ are, respectively, the aggregate productivities among continuers and exiters, where s_C is defined analogously to s_X. Intuitively, the exit term is positive if continuers are on average, weighted by market share, more productive than exiters.

2.3 The Regression Augmented Decomposition (RA-DOPD)

2.3.1 Simple RA-DOPD

We now show how the DOPD described above can be further refined using regressions and discuss what insights this delivers. The idea underlying the method is that we can reconstruct each of the terms in the DOPD using the regression equations given below. We start by estimating these regressions for a single group (a cohort in our application) but will subsequently extend these regressions to allow for multiple cohorts, in order to compare the drivers of aggregate productivity growth between the boom and bust cohort in our application. Also note that at this point none of the regressions include control variables beyond those needed to reconstruct the DOPD. We will add control variables in Section 2.3.3.

Within	$\triangle \varphi_i = \alpha_0 + \varepsilon_i$	$i \in C$
Between	$\widetilde{l}_{i,t} = \beta_{0,t} + \beta_{1,t} \varphi_{i,t} + \varepsilon_{i,t}$	$i \in C,\ t = 0,1$
	$\widetilde{\varphi}^2_{i,t} = \gamma_{0,t} + \varepsilon_{i,t}$	$i \in C,\ t = 0,1$
Exit	$x_i = \delta_0 + \varepsilon_i$	all i
	$\widetilde{l}_{i,0} \varphi_{i,0} = \lambda_0 + \lambda_1 x_i + \varepsilon_i$	all i

[6] Melitz and Polanec (2015) alternatively express the between term as $BE = \triangle N_C \underset{i \in C}{cov}(s_i, \varphi_i)$, where – in their notation – the market share s_i is defined among continuers only and N_C denotes the number of continuers (which in their formulas is omitted for notational simplicity). It can be shown that their expression is equivalent to ours, since it holds that $\widetilde{l}_{i,t} = N_C s_{i,t}$, for $i \in C$.

We now show how to reconstruct the DOPD terms by estimating the equations in the box above. For the within term, we simply regress productivity growth, $\triangle\varphi_i$, on a constant. We immediately note that $WI = \widehat{\alpha}_0$, where $\widehat{\alpha}_0$ is the estimated constant. At this point, the regression is trivial and delivers no additional insight. This, however, will change once we start comparing different cohorts and/or add control variables.

In a subsequent step, we can deconstruct the between term using the next two regressions. Consider first the regression equation for $\widetilde{l}_{i,t}$, as given in the box above. We estimate this equation twice, once observing firms at $t = 0$ and once at $t = 1$.[7] Note that the estimated coefficient $\widehat{\beta}_{1,t}$ is the (semi-)elasticity of size with respect to productivity. This coefficient captures the extent to which more productive firms tend to be larger. Having estimated the equation, we can express the covariance between relative size and productivity as $cov\left(\widetilde{l}_{i,t}, \varphi_{i,t}\right) = \widehat{\beta}_{1,t} var\left(\varphi_{i,t}\right),\ i \in C$. These results allow us to decompose the between term according to Equation (3), where:[8]

$$AL = \underset{i\in C}{var}(\varphi_{i,1})\triangle\widehat{\beta}_1, \qquad DI = \widehat{\beta}_{1,0}\triangle\underset{i\in C}{var}(\varphi_i).$$

We interpret AL as a term capturing the change in allocation, since it is proportional to the change in the size-productivity elasticity $\triangle\widehat{\beta}_1$. This change is multiplied by the variance of productivity, since the aggregate effect of stronger allocation depends on how much productivity dispersion there is. In the extreme case in which there is no productivity dispersion, i.e., if $\underset{i\in C}{var}(\varphi_{i,1}) = 0$, there are no possible aggregate productivity gains from reallocating resources. The term DI is interpreted as the dispersion term, since it is proportional to the change in the cross-sectional variance of productivity between $t =$

[7] Alternatively, we could estimate one pooled regression for both periods and use an interaction term. We opt for separate regressions here for ease of exposition.

[8] Here, we use that the change in the product of two variables x and y, between two periods 0 and 1, can be expressed as $\triangle xy = x_1\triangle y + y_0\triangle x$. An alternative decomposition would be to use $\triangle xy = x_0\triangle y + y_0\triangle x + \triangle x\triangle y$. Accordingly, one would then instead set the allocation term to $AL = \underset{i\in C}{var}(\varphi_{i,0})\triangle\widehat{\beta}_1$ and add an interaction term $\triangle\widehat{\beta}_1\triangle\underset{i\in C}{var}(\varphi_i)$. While this decomposition would be somewhat more symmetric, it introduces an additional term. Since we found the interaction term to be quantitatively negligible in our application, we do not opt for this approach here.

0 and $t = 1$. Note that the impact of an increase in dispersion on aggregate productivity can be either negative or positive, depending on the sign of the level of the elasticity $\widehat{\beta}_{1,0}$.

The change in the variance in the dispersion term can be reconstructed using the regression for $\widetilde{\varphi}^2_{i,t}$, see the above table where we defined $\widetilde{\varphi}_{i,t} \equiv \varphi_{i,t} - \overline{\varphi}_{C,t}$. Intuitively, $\widetilde{\varphi}^2_{i,t}$ measures the absolute deviation of a continuer's productivity level from the average (up to a quadratic transformation). Based on this regression, we can express the change in the variance as $\triangle var(\varphi_i) = \widehat{\gamma}_{1,1} - \widehat{\gamma}_{1,0}$. That is, the change in the variance is the change in the regression constant. Again, the regression is trivial, but this will no longer be the case once we compare different cohorts and/or add control variables. Similarly, one could use the regression to express the variance term in the allocation equation as $\underset{i \in C}{var}(\varphi_{i,1}) = \widehat{\gamma}_{1,1}$.

Finally, we reconstruct the exit term using the two regression equations at the bottom of the above table. Here x_i is an indicator variable that equals one if the firm exits between $t = 0$ and $t = 1$. Moreover, $\widetilde{l}_{i,t}\varphi_{i,t}$ is the productivity of a firm, scaled by its relative size.[9] This variable captures the contribution of a firm to aggregate productivity. This contribution is larger if a firm is both relatively large and relatively productive. Based on these regressions, we can express the exit term as (see Appendix B for derivations):

$$EX = DR \cdot DS, \tag{6}$$

where

$$DR = \widehat{\delta}_0 = \overline{x}, \tag{7}$$

and

$$DS = \widehat{\lambda}_0 \frac{\overline{s}_{X,0} - \overline{s}_{C,0}}{\overline{s}_{C,0}} - \widehat{\lambda}_1, \tag{8}$$

where $\overline{s}_{X,0}$ ($\overline{s}_{C,0}$) is the average market share of exiters (continuers). The exit term is thus the product of two terms. The first, DR, is simply the exit rate, whereas the second, DS, captures selection. Specifically, the coefficient $\widehat{\lambda}_1$ captures the extent to which, on average, exiters contribute more to aggregate productivity than continuers. If $\widehat{\lambda}_1$ is

[9] Relative employment is defined as $\widetilde{l}_{i,t} \equiv l_{i,t}/\overline{l}_t$, i.e., employment relative to the average across all firms (not just continuers).

negative, the average exiter is less productive than the average continuer. That is, there is positive selection, $DS > 0$, which increases aggregate productivity. The magnitude of the selection effect on aggregate productivity, however, depends on the death rate DR. Finally, the term $\widehat{\lambda}_0 \frac{\bar{s}_{X,0}-\bar{s}_{C,0}}{\bar{s}_{C,0}}$ is a correction which arises when the average exiter has a different market share than the average continuer. Intuitively, if exiters are very small compared to continuers, then selection does not matter much for aggregate productivity.[10]

2.3.2 Comparing productivity growth across different groups

So far, we have discussed how to decompose productivity growth in one particular time period. In many applications, however, the aim is to understand how productivity growth varies across different samples such as different cohorts of start-ups, different types of firms, or different countries. For example, the goal may be to understand why aggregate productivity growth varies across phases of the business cycle.

Our method can be readily applied to such settings. To fix ideas, suppose we compare productivity growth of start-ups during an economic bust to productivity growth of start-ups during a boom. Let $dAPG \equiv APG^{bust} - APG^{boom}$ denote the difference in aggregate productivity growth across the two time groups. From Equation (1) it directly follows that $dAPG = dWI + dBE + dEX$. Moreover, we can decompose the difference in the between term, based on Equation (2), which implies that $dBE = dAL + dDI$. Finally, we can exploit the result that $EX = DR \cdot DS$ to derive the decomposition for dEX given by Equation (3), where[11]

$$dEXDR = DS^{bust} \cdot dDR \quad , \qquad dEXDS = DR^{boom} \cdot dDS$$

The first term, $dEXDR$ is driven by the difference in the death rate between the bust

[10] If continuers are on average larger than exiters at $t = 0$, then $\bar{s}_{X,0} > \bar{s}_{C,0}$ so that $\widehat{\lambda}_0 \frac{\bar{s}_{X,0}-\bar{s}_{C,0}}{\bar{s}_{C,0}} > 0$. That is, the exit term becomes smaller relative to the within and between terms. If, on the other hand, firms in both groups are on average equally large, then $\widehat{\lambda}_0 \frac{\bar{s}_{X,0}-\bar{s}_{C,0}}{\bar{s}_{C,0}}$ and the selection term collapses to $DS = \widehat{\lambda}_1$.

[11] Again, we could alternatively decompose the difference as $dEX = DS^{boom} \cdot dDR + DR^{boom} \cdot dDS + dDS \cdot dDR$, but since the interaction term is again very small, we do not use this decomposition for the sake of parsimony.

and the boom period, whereas the second term, $dEXDS$, is driven by the change in the selection of exiters.

2.3.3 Adding control variables

It is tempting to interpret the difference between a boom and a bust cohort as a pure business-cycle effect. However, differences may also be driven by other factors that are not directly related to the cyclical state of the economy. In particular, the composition of the firm population - in terms of characteristics such as industry, urbanization, and so on - may change over time and hence affect the productivity decomposition.

We therefore now extend the RA-DOPD to compare time periods while controlling for confounding factors. To this end, we extend the regression with control variables. The box below presents the extended regression equations. For simplicity, we assume that the sample spans only two start-up cohorts: a boom and a bust cohort. It is, however, straightforward to extend the setup to compare multiple groups of firms. In the box below, bu_i is a dummy variable which equals one if i is observed in the bust period and zero if it is observed in the boom period. Moreover, z_i is a vector of control variables, which may vary across the different regression equations.

Within	$\triangle\varphi_i = \alpha_0 + \alpha_1 bu_i + \alpha_z z_i + \varepsilon_i$	$i \in C$
Between: Allocation	$\widetilde{l}_{i,t} = \beta_{0,t} + \beta_{1,t}\varphi_{i,t} + \beta_{2,t} bu_i + \beta_{3,t}\varphi_{i,t} bu_i + \beta_{z,t} z_{i,t} + \varepsilon_{i,t}$	$i \in C,\ t = 0,1$
Between: Dispersion	$\widetilde{\varphi}^2_{i,t} = \gamma_{0,t} + \gamma_{1,t} bu_i + \gamma_{z,t} z_{i,t} + \varepsilon_{i,t}$	$i \in C,\ t = 0,1$
Exit: Death rate	$x_i = \delta_0 + \delta_1 bu_i + \delta_z z_i + \varepsilon_i$	all i
Exit: Selection	$\widetilde{l}_{i,0}\varphi_{i,0} = \lambda_0 + \lambda_1 x_i + \lambda_2 bu_i + \lambda_3 x_i bu_i + \lambda_z z_i + \varepsilon_i$	all i

We use these regressions to decompose the difference in aggregate productivity growth across the two periods as in Equation (4), i.e., in a component reflecting a "pure" bust effect and a component reflecting differences in the control variables. Specifically, from the regressions in the box above, we can reconstruct each of the components of $dAPG$, similarly to the previous subsections. We then split the reconstructed components into a part that is due to controls and a remaining part due to the bust itself. We do so by isolating those parts of the reconstructed terms which pertain to the coefficients on the control variables in the regression.

The expressions for the bust and control components are presented in the box below. In Appendix A we derive these expressions and show that they add up to the total difference. Note that the coefficients pertaining to the control variables, indicated by subscript z, do not enter the expressions for the bust effect. Moreover, if these coefficients are set to zero, the corresponding controls component collapses to zero. Finally, we note that an advantage of the RA-DOPD is that one can use the estimated regressions to assess the statistical significance of the various channels.

Bust effects	
dWI	$\widehat{\alpha}_1$
dAL	$\left(\widehat{\beta}_{3,1}-\widehat{\beta}_{3,0}\right) var_{bu=1}(\varphi_{i,1})+\left(\widehat{\beta}_{1,1}-\widehat{\beta}_{1,0}\right)(var_{bu=1}(\varphi_{i,1})-var_{bu=0}(\varphi_{i,1}))$
dDI	$(\widehat{\gamma}_{0,1}+\widehat{\gamma}_{1,1}-\widehat{\gamma}_{0,0}-\widehat{\gamma}_{1,0})\left(\widehat{\beta}_{1,0}+\widehat{\beta}_{3,0}\right)-(\widehat{\gamma}_{0,1}-\widehat{\gamma}_{0,0})\widehat{\beta}_{1,0}$
dDR	$DS^{bust}\cdot\widehat{\delta}_1$
dDS	$DR^{boom}\left(\left(\widehat{\lambda}_0+\widehat{\lambda}_2\right)\frac{\overline{s}_{X,bu=1}-\overline{s}_{C,bu=1}}{\overline{s}_{C,bu=1}}-\widehat{\lambda}_3-\widehat{\lambda}_0\frac{\overline{s}_{X,bu=0}-\overline{s}_{C,bu=0}}{\overline{s}_{C,bu=0}}\right)$
Control effects	
dWI	$\widehat{\alpha}_z\left(\overline{z}_{i,bu=1}-\overline{z}_{i,bu=0}\right)$
dAL	$\triangle\left(\widehat{\beta}_z cov_{bu=1}(z_i,\varphi_i)-\widehat{\beta}_z cov_{bu=0}(z_i,\varphi_i)\right)$
dDI	$\left(\widehat{\beta}_{1,0}+\widehat{\beta}_{3,0}\right)(\widehat{\gamma}_{z,1}\overline{z}_{1,bu=1}-\widehat{\gamma}_{z,0}\overline{z}_{0,bu=1})-\widehat{\beta}_{1,0}(\widehat{\gamma}_{z,1}\overline{z}_{1,bu=0}-\widehat{\gamma}_{z,0}\overline{z}_{0,bu=0})$
dDR	$DS^{bust}\cdot\widehat{\delta}_z\left(\overline{z}_{i,bu=1}-\overline{z}_{i,bu=0}\right)$
dDS	$DR^{boom}\widehat{\lambda}_z\left(\overline{z}_{i,bu=1}-\overline{z}_{i,bu=0}\right)$

3 Interpreting the RA-DODP with a model

In this section we study a model with heterogeneous firms and micro-level distortions in order to interpret the regression equations underlying the RA-DOPD. The model setup builds upon Restuccia and Rogerson (2008), Hsieh and Klenow (2009), among others. Following these authors, we do not take a particular stance on the sources of the distortions, but rather model them as generic wedges which enter a firm's profit function as tax rates. We then show that the model maps into reduced-form equations which align with the regression equations of the RA-DOPD. This enables us tot gauge how the various distortions and model parameters map into the reduced-form regression coefficients, allowing for a more structural interpretations of the regressions.

The model economy is static and consists of a number of heterogeneous firms indexed by i. The variable production technology of firm i is given by $y_i = A_i l_i^{\gamma} k_i^{\alpha}$, $\alpha, \gamma > 0$, where l_i denotes labor, k_i is capital, and A_i is TFP. Operation also requires a fixed cost, denoted by c_f. A firm can avoid the fixed cost by exiting before production takes place. In that case, the firm does not produce anything and makes zero profits. Firms act as price takers and a market wage w and a rental rate of capital r as given.

We introduce three firm-specific distortions. The first of these, $\tau_{y,i}$, is a revenue distortion which acts as a tax on sales, see Restuccia and Rogerson (2008). The second distortion, $\tau_{k,i}$, acts as a tax on capital, see Hsieh and Klenow (2009). Finally, we introduce a distortion which acts as a tax on the fixed cost. A firm maximizes, π_i, taking into account the distortions:

$$\pi_i = \max_{x_i} \left\{ \max_{l_i, k_i} \left(1 - \tau_{y,i}\right) A_i l_i^{\gamma} k_i^{\alpha} - w l_i - \left(1 + \tau_{k,i}\right) r k_i - \left(1 + \tau_{f,i}\right) c_f, 0 \right\}. \tag{9}$$

Here, $x_i \in \{0, 1\}$ denotes the exit decision of the firm, i.e. it equals one if the firm exits and zero if it produces. While $\tau_{y,i}$, $\tau_{k,i}$ and $\tau_{f,i}$ enter the model as distortions, we not take a stance on their origin. Hence, they are not necessarily inefficient from a welfare perspective. For example, they may arise due to adjustment costs that are part of the firm's technology. Precisely because we do not take a stance on the origin of the distortions, it is important to allow for correlation between them, as well as with TFP. Such correlations may arise for many reasons. For example, suppose that$\tau_{y,i}$ captures adjustment costs. Startups with high TFP generally find it optimal to grow more, but as a result are also more affected by adjustment cost. In what follows, we assume that $\ln A_i$, $\ln\left(1 - \tau_{y,i}\right)$, $\ln\left(1 + \tau_{k,i}\right)$, and $\ln\left(1 + \tau_{f,i}\right)$ are drawn from a joint normal distribution.

Interpretation of the allocation regression

We first show how the structural model described above relates to the allocation regression. It turns out that the model can be mapped into the following reduced-form equation:

$$\ln l_i = \beta_0 + \beta_1 \varphi_i + \varepsilon_i, \tag{10}$$

where, as in the RA-DOPD allocation regression, φ_i is either TFP or labor productivity and ε_i has mean zero and is orthogonal to φ_i.[12] The reduced-form coefficients, β_0 and β_1 can be expressed as functions of the parameters, and ε_i as a function of the parameters and stochastic draws. These can be derived considering either TFP or labor productivity as the choice for φ_i. The details of the derivations are provided in the Appendix.

Let us start with logged TFP as the measure for productivity, i.e. $\varphi_i = \ln A_i$. Profit maximization leads to the following decisions for, respectively, labor and capital:

$$l_i = \left(\frac{\gamma}{w}\left(1-\tau_{y,i}\right) A_i k_i^{\alpha}\right)^{1/(1-\gamma)}, \quad k_i = \left(\frac{\alpha}{r}\frac{1-\tau_{y,i}}{1+\tau_{k,i}} A_i l_i^{\gamma}\right)^{1/(1-\alpha)}. \tag{11}$$

Given normality of the joint distribution, we can express the relation between the revenue friction and TFP as $\ln\left(1-\tau_{y,i}\right) = \xi_y + \phi_y \ln A_i + u_{y,i}$, where $u_{y,i}$ is a stochastic component which has zero mean and is orthogonal to$\ln A_i$. Similarly, the relation between the capital distortion and TFP can be expressed as $\ln\left(1+\tau_{k,i}\right) = \xi_k + \phi_k \ln A_i + u_{k,i}$, where $u_{k,i}$ has mean zero and is orthogonal to $\ln A_i$. Here, ϕ_y and ϕ_k capture the relation between TFP and, respectively, the revenue and capital distortion.

The reduced-form coefficient on productivity can now be expressed as $\beta_1 = \frac{1+\phi_y-\alpha\phi_k}{1-\alpha-\gamma}$, see the Appendix for details. To understand what this expression tells us about the regression, consider the first a simple case in which there is no correlation between the distortions and TFP, i.e. $\phi_y = \phi_k = 0$. The coefficient on productivity is then given by $\beta_1 = \frac{1}{1-\alpha-\gamma}$. That is, the coefficient equals the inverse of the degree of returns to scale

[12] Strictly speaking, in the RA-DOPD we put on the left-hand side relative size, $\widetilde{l}_i \equiv l_i/\bar{l}$. But note that a first-order approximation of logged labor gives $\ln l_i \simeq \ln \bar{l} + \frac{l_i - \bar{l}}{\bar{l}} = 1 + \ln \bar{l} + \widetilde{l}_i$, so up to a first-order approximation and the constant, $\ln l_i$ and, $\widetilde{l}_i$ are the same.

in production, $1-\alpha-\gamma$. Given that the degree of returns to scale is time invariant, we should find the same estimate for β_1 no matter the year or subset of firms we use in the estimation.

Now consider the general case in which there is correlation between the wedges and TFP. The coefficient β_1 now might change over time due to time variation in ϕ_y and ϕ_k, which capture changes in the extent to which allocation frictions depend on TFP. Alternatively, we might interpret n ϕ_y and ϕ_k as the extent to which distortions are size dependent.[13] A decrease in β_1 means that more productive (larger) firms face heavier distortions. The distinction between distortions that are correlated and uncorrelated with productivity (or size) is also made by Restuccia and Rogerson (2008). They consider two versions of their model: one in which distortions are uncorrelated with size and one in which larger firms face higher distortions. They show that uncorrelated distortions lead to only small declines in aggregate productivity, whereas correlated distortions can lead to much larger losses. In our RA-DOPD, the allocation term isolates the effects of the correlated distortions.

Next, consider labor productivity as the right-hand side variable, i.e. $\varphi_i = \ln(y_i/l_i)$. Under this choice the regression coefficient becomes $\beta_1 = -\frac{1+\phi_A(1-\alpha\phi_k)}{1-\alpha-\gamma}$, where $\phi_A = \phi_y \frac{var(\ln(1-\tau_{y,i}))}{var\ln(A_i)}$. In the absence of correlated distortions the reduced-form coefficient on labor productivity equals $\beta_1 = -\frac{1}{1-\alpha-\gamma}$, i.e. it is exactly the negative of the reduced-form coefficient under $\varphi_i = \ln A_i$. Moreover, β_1 is again a function of only returns to scale and should thus be constant. With correlated distortions, however, changes in β_1 can arise from changes in the the relation between the distortions and TFP, as captured by ϕ_A and ϕ_k.

Interpretation of the dispersion regression

In the RA-DOPD, aggregate TFP is also affected by changes in the dispersion of productivity across firms, for which we run separate regressions. In the literature, dispersion in *labor productivity* has been highlighted by Hsieh and Klenow (2009) and others as a key

[13]Note that a firm's optimal size is a function of productivity only, so we can reinterpret correlation between the distortions and TFP as correlation between distortions and (optimal) size.

measure of misallocation. In the model outlined above, the firm's optimal labor decision implies that $\ln(y_i/l_i) = \ln w - \ln\gamma - \ln(1-\tau_{y,i})$. Thus, cross-sectional variation in labor productivity varies across firms is driven entirely by the revenue distortion. If there were no dispersion in the distortion, the marginal product of labor would be equalized across firms, reflecting an efficient allocation of labor. This, however, does not imply that the dispersion in the distortion is all that matters for misallocation. Given that there is dispersion, the degree of correlation with TFP also matters, as shown above and as emphasized by Restuccia and Rogerson (2008). Thus, by distinguishing between the allocation and dispersion term, we can asses how the importance of both channels changes over time.

Dispersion in TFP is not directly a source of misallocation, since TFP is a fundamental (as opposed to labor productivity, which is an endogenous outcome). Thus, one should be careful in interpreting an increase in TFP dispersion as a sign of misallocation. However, changes in dispersion do affect the Olley-Pakes (OP) covariance term, which implies that changes in the OP do not necessarily reflect misallocation. The RA-DOPD separates the dispersion channel from the allocation channel, and hence helps to avoid potential misinterpretations.[14]

Interpretation of the selection regression

Finally, we consider the selection regression. For simplicity, let us consider first a model without revenue and capital distortions. In this case, a firm will exit if and only if profits are negative, i.e. if

$$(1+\tau_{c,i})\,c_f > A_i l_i^{\gamma} k_i^{\alpha} - wl_i - rk_i = y_i\,(1-\alpha-\gamma)\,. \tag{12}$$

[14] That said, it is possible that dispersion in TFP indirectly creates misallocation, since TFP can correlate with the distortion. Within the model, we do not take a stance on any causality behind these correlations, but one might think of reasons why higher TFP dispersion may lead to higher dispersion in the distortions, which in turn creates misallocation. Thus, a change in TFP dispersion term might partly indirectly reflect misallocation through such channels. The extent of such misallocation, however, can be assessed more directly from the dispersion in labor productivity.

This is essentially a Roy (1951) model, which can be mapped into the following reduced-form equation:

$$\ln A_i = \lambda_0 + \lambda_1 x_i + \varepsilon_i, \tag{13}$$

where ε_i has mean zero and is orthogonal to the exit variable x_i. Note that λ_0 is the mean of logged TFP among continuing firms, whereas $\lambda_0 +$ λ_1 is the mean among exiters. The difference, λ_1, captures the selection.

Let σ_A^2 denote the variance of $\ln A_i$ and σ_{Af} the covariance between $\ln A_i$ and $\ln(1+\tau_{c,i})$, i.e. the covariance between TFP and the entry friction. In the Appendix, we show that the selection coefficient can be written as

$$\lambda_1 = -\frac{\sigma_A^2 - (1-\alpha-\gamma)\,\sigma_{Af}}{\sigma}\omega, \tag{14}$$

where $\sigma \equiv std\,(\ln A_i - (1-\alpha-\gamma)\ln(1+\tau_{f,i})) > 0$ and $\omega > 0$ is a constant which is defined in the Appendix and which does not depend on the joint distribution of TFP and the entry cost friction.

The above equation makes clear that, if there is no correlation between the entry friction and TFP, i.e. if $\sigma_{Az} = 0$, then $\lambda_1 < 0$, i.e. there is positive selection on TFP. This selection effect is stronger the higher the dispersion in TFP. By contrast, a positive correlation between the entry friction and TFP, i.e.$\sigma_{Az} > 0$, pushes up λ_1, thus reducing the strength of selection. Intuitively, if there is such a positive correlation, then more productive firms tends to have higher effective costs of production, triggering more exit of relatively productive firms and thus weakening selection. The above equation thus makes clear that weaker selection, as reflected by a reduction in λ_1, can arise both from an increase in TFP dispersion, and from a reduction in the correlation between TFP and entry distortions.

4 Data

4.1 Start-ups and other firms

Our data set contains the complete population of Dutch firms that had to declare corporate income tax during the period 2000-2013 (i.e., all firms with a legal personality). These confidential micro data are provided by Statistics Netherlands (CBS) and are based on a merger of the Dutch general business register (ABR) and data on corporate tax declarations from the Ministry of Finance. The matched data include annual balance sheets as well as profit and loss statements. Importantly, this detailed firm information is also available for all new firms that were established during this period (start-ups).

We restrict our sample to private firms in non-financial sectors. For multi-establishment firms we take the 2-digit sector code (NACE Rev.1 2-digit) of the firm with the largest number of employees. The resulting sample contains a total of 367,453 firms, which we observe on average for 4 consecutive years.

We categorize all firms as either micro, small, medium-sized, or large.[15] Moreover, we define start-ups as firms that newly enter our database in a given year, were not previously registered in the business register, and are not a subsidiary or establishment of another company.[16] We exclude new firms with more than twenty employees or total assets in excess of EUR 10 million in the start-up year as these are typically spin-offs of large existing companies. Moreover, we disregard new firms with zero employees in the start-up year and/or less than two employees in each year that we observe the firm. These observations typically concern freelancers, independent contractors and professionals rather than regular start-ups. Based on this definition, our final data set contains 31,526 start-ups. In the remainder of this paper, we focus on the survival rate and productivity growth of these newly established enterprises.

[15] Micro (small) enterprises have less than 10 (50) employees and an annual balance sheet total of below EUR 2 (10) million. Medium-sized (large) enterprises have less (more) than 250 employees and an annual balance sheet total of no more (less) than EUR 43 million. A firm's number of employees includes the firm owners. This means that sole proprietors are classified as firms with one employee.

[16] We keep start-up firms in our sample that appear in the business register one year prior to their first tax-filing. The reason is that some entrepreneurs register their firm in the business register but are only legally obliged to file for taxes when the firm is fully operational, which can take several months.

4.2 The global financial crisis and start-up cohorts

The Netherlands, a typical small open economy, was impacted strongly by the global financial crisis. After average annual economic growth of 2.7 per cent during the period 2003-H1 2008, growth turned sharply negative in the 3$^{\text{th}}$ quarter of 2008. The 1$^{\text{st}}$ quarter of 2009 marked the largest negative post-war quarter-on-quarter negative growth recorded. After a short-lived growth rebound in 2011, the Dutch economy fell back into a recession following the eurozone sovereign debt crisis and only regained momentum in 2013.

In order to exploit the sharp and unexpected business cycle downturn as of the end of 2008, we contrast two start-up cohorts: those that were established *and* matured during the pre-crisis period 2003-05 (the boom cohort) and those established during 2006-08, just before the boom abruptly ended (the bust cohort). Importantly, both cohorts were established *before* the global financial crisis. The crucial difference is that while start-ups in the first cohort enjoyed a beneficial economic environment during the first years of their existence, the second cohort grew up during a financial crisis and the associated recession.

We provide summary statistics for these two cohorts in Table 1. We observe each cohort both at age 0 and age 4.[17] The boom cohort includes 10,490 start-ups whereas the bust cohort comprises 21,036 new firms. The average four-year exit rates in these two start-up cohorts are 34.6 and 39.0 per cent, respectively. That is, firms that soon after start-up were confronted with the global financial crisis had a 4.4 percentage point higher probability of dying within four years' time as compared with firms that matured in a more benign economic environment.

The average start-up in the boom cohort begins operations with 6.4 employees. This number is slightly lower for those starting just before the crisis: 6.0 employees. During the first four years of their life, start-ups expand the number of employees on average by 31.1 per cent (boom cohort) and 20.4 per cent (bust cohort). That is, firms that soon

[17] Table A1 in the appendix provides similar statistics for the start-up sample for which we are able to calculate TFP.

after establishment enter a crisis not only have a lower survival probability, they also expand less quickly (conditional on survival) as compared with firms that mature during a boom period.

For each firm, we measure leverage at start up as the sum of long-term debt ($>$ 1 year remaining maturity), short-term debt ($<$1 year remaining maturity) and trade credit, divided by total assets. The average start-up has an initial leverage level of 79 per cent (boom cohort) and 74 per cent (bust cohort). During the first years of operation, firms tend to slightly reduce leverage by about 11 per cent (boom cohort) and 12 per cent (bust cohort) as they generate earnings and pay off debt. We divide the total population of start-ups (i.e., those in both cohorts) according to whether they had above or below-median leverage at the time of their establishment relative to all other firms in the same two-digit industry. The percentage high-leverage start-ups was 54 (48) per cent in the boom (bust) cohort. We find that the reduction in debt is concentrated among the high-leverage start-ups: these firms reduce debt by 16 per cent (boom cohort) and 10 per cent (bust cohort) during the first four years of their existence. In contrast, firms that at start-up had relatively modest leverage levels, either slightly increase leverage by 12 per cent (boom cohort) or keep leverage constant (bust cohort).

Table 1 also shows a clear difference in both exit rates and employee growth among initially high-leverage versus low-leverage firms. The former group increases its average number of employees by 37.4 and 26.7 per cent in the boom and bust cohorts, respectively. The speed of expansion is significantly lower among low-leverage firms at 25.4 and 17.3 per cent, respectively. At the extensive margin, the exit rate among high-leverage firms goes up from 40.7 to 46.5 when we compare the boom and the bust cohorts. Among low-leverage firms, the risk of exit increases as well but from a much lower base: 27.3 per cent in the boom and 32.2 per cent in the bust cohort. That is, in both cohorts the default probability is about 50 per cent higher among high-leveraged firms than among low-leveraged firms.

Finally, for each start-up firm we know the concentration of human activity, proxied by the density of addresses per km^2, in the locality where the firm is located. We create a

City dummy variable that is '1' if the location is characterized by very high urbanity ($\geq$ 2,500 addresses per km^2) or high urbanity ($\geq$1,500 but $<$2,500 addressed per km^2) and '0' otherwise (i.e., $<$1,500 addresses per km^2). Around 42 per cent of all Dutch start-ups are located in cities. This holds for both start-up cohorts as well as for high- and low-leverage firms. We will use *City* as one of the control variables in our empirical analysis.

4.3 Measuring firm productivity

Throughout the paper we analyze the role of initial leverage for both start-up labor productivity and total factor productivity (TFP). We define labor productivity as total turnover minus raw material costs and all other business costs (except for salaries, depreciation and interest payments) divided by the number of employees.[18] Table 1 shows that labor productivity is higher among low-leverage firms. However, within the first four years of a start-up's life, high-leverage firms partially catch up as their labor-productivity growth outstrips that of low-leverage firms. In the boom cohort, labor-productivity growth is 30 per cent among high-leverage and only 21.8 per cent among low-leverage firms. For the bust cohort, we find that high-leverage firms manage to increase productivity by 6.6 per cent whereas the low-leverage firms actually experience a decline in productivity of about percentage point.

While labor productivity can be readily computed from our raw data, estimating firm-level TFP requires a more complex estimation technique. The object of interest is A_{it}, in the following equation:

$$RVA = A_{it} K_{it}^{\alpha} L_{it}^{(1-\alpha)} \tag{15}$$

[18]To create real firm-level variables, we apply sector-specific (NACE Rev.2, 2-digit level) deflators from Eurostat. Turnover and intermediate inputs are deflated by the GDP deflator while capital is deflated by the deflator for gross fixed capital formation in the private non-household sector. We drop the top and bottom 1 per cent of observations for each variable, either in levels or in terms of growth rate. Several financial indicators are ratios that should theoretically lie between 0 and 1. They are dropped if they do not satisfy this condition. The following additional cleaning rules are applied: all observations with negative or zero labor, labor costs or capital are replaced by missing values; all observations with negative turnover, total assets, liquid assets, long-term debt, trade creditors/debtors, interest payments, dividends, current assets and depreciation are replaced by missing values; and all yearly observations of 2-digit sectors that contain less than 10 firms are dropped.

where *RVA* is real value added, K is the real book value of net capital and L is total employment. Building on the approach developed by, among others, Olley and Pakes (1996); Levinsohn and Petrin (2003); Wooldridge (2009), we estimate A_{it} by applying a logarithmic transformation to the above equation. Firm-level TFP is then estimated following the methodology proposed by Galuscak and Lizal (2012): an implementation of the Olley and Pakes (1996); Levinsohn and Petrin (2003) methodology in a GMM framework, controlling for capital measurement error. This estimation is performed on all firms in our data set (start-ups and existing firms) on a 2-digit industry level.[19] Since labor and TFP are simultaneously determined, while capital takes time to build, labor is instrumented by its first lag. The regression also contains several higher order and interaction terms between capital and materials to control for non-linearities. Moreover, a full set of year dummies is included to control for sector-specific trends.

Based on the estimation of this GMM framework, firm-level TFP is then retrieved on the basis of real value added and the fitted values for real capital and labor:[20]

$$TFP = RVA_{it} / \left(K_{it}^{\hat{\beta}_1} * L_{it}^{\hat{\varpi}} \right) \tag{16}$$

The dynamics in TFP broadly mimic those in labor productivity (see Appendix Table A1). Initial TFP tends to be higher among low-leveraged firms as compared with high-leveraged ones. However, during the first four years after establishment, high-leveraged firms manage to boost TFP by 31.1 and 9.9 per cent in the boom and bust cohort, respectively. These numbers are again much lower among low-leveraged firms: 10.9 and -8.6 per cent, respectively.

Figure 1 provides a further year-by-year breakdown of productivity-growth of start-up firms. For each year we plot cumulative growth in labor productivity (Figure 1a) and

[19] To obtain consistent estimates with sufficient degrees of freedom, a minimum of 25 observations per sector and year is required. We replace the TFP estimates of sectors that do not meet this minimum by a TFP estimate at the macro-sector level.

[20] Extreme outliers in the TFP distribution - observations above (below) the 99.5^{th} (0.5^{th}) percentile - are excluded. In unreported robustness tests we also use the Hsieh and Klenow (2009)TFP measure which uses firm-level marginal products of capital and labor weighted with their respective estimated production function coefficients. The results are qualitatively and quantitatively very similar to these we report here based on the regular TFP measure.

TFP (Figure 1b) during the first four years after establishment. This graphical evidence confirms that productivity growth was considerable higher for firms established in 2003-05 (the boom cohort) than for those established in 2006-08 (the bust cohort). In the latter cohort, labor productivity even turns negative while TFP growth is basically zero.

The charts also show clearly that high-leverage firms experience significantly higher productivity growth and this holds especially true for the bust cohort. In fact, while among low-leverage start-ups both labor-productivity growth and TFP growth turn negative for the bust cohort (especially so for firms started in 2008, just before the outbreak of the crisis), this is not the case for high-leverage firms. For instance, while low-leverage firms established in 2008 saw a decline in cumulative TFP of 13 percent over the period 2008-2011, high-leverage firms established in 2008 experienced an *increase* in TFP of 19 per cent. For labor productivity, these numbers are -17 and 10 per cent, respectively. The very sharp drop in productivity growth among firms that soon after establishment had to weather a deep financial crisis is almost exclusively driven by firms with low leverage at start up (relative to other firms in the same two-digit industry).

In sum, our data show that the development of high-leverage firms during the first years of their existence differs from that of low-leverage firms in the following key ways: (1) they reduce leverage more; (2) they increase employment more; (3) they increase labor-productivity and TFP more; and (4) they are more likely to go bankrupt. Taken together, this suggests that high-leverage firms are characterized more by up-or-out dynamics than low-leverage firms. Section 5 will shed more light on the precise mechanisms that drive these different dynamics.

5 Results

5.1 The productivity growth of start-ups: Boom versus bust

We start our analysis of the micro-drivers of start-up productivity growth by presenting a dynamic Olley-Pakes decomposition in the vein of Melitz and Polanec (2015). We explain how the individual productivity components can be derived from a series of straightforward firm-level regressions.

Figure 2 depicts decompositions of labor-productivity growth (Figure 2a) and TFP growth (Figure 2b) for the two start-up cohorts. The first four columns in Table 2 provide the related numerical decomposition. In line with Figure 1, we observe a sharp drop in aggregate productivity growth among firms that soon after establishment faced the global financial crisis. Cumulative labor-productivity growth during the first four years of operation, indicated by the red dots, declined from 47.2 per cent for the boom cohort to -2.4 per cent for the bust cohort. The equivalent numbers for TFP growth are 35.6 and 2.6 per cent, respectively. The steeper decline in labor-productivity growth likely reflects the sharp drop in firm investment during the crisis. At the same time, a decline in aggregate demand combined with labor hoarding may have played a role.[21]

We can decompose aggregate productivity growth into three underlying micro-drivers: within-firm productivity growth (the blue component), between-firm reallocation (green), and firm exit (yellow). First of all, we observe a very sharp decline in within-firm productivity growth. While this was by far the main driver of total productivity growth for the boom cohort, its contribution to both labor-productivity growth and TFP growth turns negative for the bust cohort. Column 4 in Table 2 shows that this sharp reversal in within-firm productivity growth can explain 78.8 (94.0) per cent of the total decline in labor-productivity growth (TFP growth) between the two cohorts.

As explained in Section 2.3, our RA-DOPD framework uses firm-level regressions to determine the exact size of the various components in Figure 2. The within-term can be derived directly from Table 3. The dependent variable in this table is firm-level productivity growth among start-ups that do not exit the sample during the first four years after establishment. We pool both cohorts. The constant in the first column (0.370) then captures average productivity growth among continuers in the boom cohort (the blue component on the left-hand side of Figure 2a) while the interaction term (-0.391) shows the change in the contribution of within-firm growth across both cohorts. The sum of these coefficients (-0.021) equals within-firm productivity growth in the bust cohort (the

[21] See Campello et al. (2010) and Duchin et al. (2010) for international and U.S. evidence on the negative impact of the global financial crisis on firm investment, respectively.

blue component on the right-hand side of Figure 2a).

A second source of productivity dynamics relates to changes in productivity *between* firms (the green component). This component is positive if more productive firms manage to expand faster (in terms of number of employees) than less productive firms. Figure 2 shows that the role of between-firm reallocation is virtually absent in the boom cohort: firm growth during the first four years after establishment was largely unrelated to firm productivity. However, in the bust cohort we appear to observe an increasingly distorted economic allocation, whereby *less* productive start-ups capture additional market share. This effect is most pronounced for labor-productivity growth, where it explains 15.5 per cent of the total decline in productivity between the two cohorts. The absence of a meaningful between term in the TFP decomposition suggests that during the financial crisis there was limited reallocation of labor towards firms with a high capital intensity. As discussed in Section 2, further analysis is needed to assess whether the negative between effect indeed reflects increased misallocation or (also) a change in the dispersion of start-up productivity (combined with a constant market-share elasticity vis-à-vis productivity). We return to this important issue in Section 5.2 when we derive the between term, and its two sub-components, from our RA-DOPD regressions.

The third source of productivity dynamics relates to firm exit. If relatively unproductive start-ups die whereas more productive start-ups continue to live, this selection at the extensive margin will contribute positively to aggregate productivity growth. Indeed, the yellow components in Figure 2 show that selection-at-exit was an important driver of productivity growth during the boom period (explaining about a quarter of aggregate labor-productivity and TFP growth). The role of exit selection becomes even much more important for the bust cohort as it is now the only (labor-productivity) or by far the main (TFP) source of productivity growth. In broad terms, the size of the exit-at-selection contribution to aggregate productivity growth depends on the number of firms that exit as well as on the productivity difference between these firms and those that continue to operate. In Section 5.3, we use the RA-DOPD framework to quantify these two sub-components.

5.2 Decomposing the between term: Allocation versus dispersion

Figure 2 showed a large, negative contribution of the between term to labor-productivity growth for the bust cohort. This term as a whole equals the change (between age 0 and age 4) in the covariance between (relative) firm size and productivity. Does the negative term therefore indicate an increasingly distorted economic allocation during the crisis, whereby less productive firms grow more in terms of number of employees? To answer this question, we need to drill down further into the data. To do so, Figure 3 splits the between term into two components: an allocation term and a dispersion term (everything else remains the same as in Figure 2). We again provide the underlying numbers in the first four columns of Table 2.

This augmented decomposition shows, first, that the absence of a between term for the boom cohort was the net outcome of two opposing forces of almost the same size: a positive disperion effect of 6.2 percentage points and a negative allocation effect of -6.8 percentage points. The positive dispersion term in the boom cohort reflects that during the first four years of this cohort's life, the productivity distribution tightened. For the bust cohort, the dispersion effect virtually disappears (that is, there is no longer a change in the productivity dispersion between age 0 and age 4). This is a first reason why the between term deteriorates in the bust cohort.

A second reason is that the allocation effect becomes more negative (from -6.8 to -8.4 percentage points). That is, there is increased misallocation between firms during the crisis. Yet, this effect is small compared with the decline in productivity dispersion as firms age. Our RA-DOPD allows us also to precisely quantify the relative importance of these two channels. Column 4 in Table 2 shows that changes in the between term across cohorts can explain 15.5 per cent of the overall decline in aggregate productivity growth. Of this, 12.4 percentage points is due to a smaller role for reduced dispersion when firms age, while 3.2 percentage points reflects a less efficient cross-firm re-allocation. In other words, the impact of dispersion is about four times as large as the role of misallocation.

The magnitude of these effects is again calculated on the basis of firm-level auxiliary regressions. We derive the allocation and dispersion effects in Tables 4 and 5, respectively. In Table 4, we pool both cohorts and use the relative number of employees of continuing firms as the dependent variable (i.e., the number of employees of the firm divided by the average number of firm employees). We run these regressions for age 0 (the start-up year) and age 4. The main explanatory variables are *Firm productivity*, a dummy for the *Bust cohort*, and the interaction of the two. The estimated coefficients in Table 4 for *Firm productivity* can then be interpretated as the semi-elasticity of firm size with respect to firm productivity, at age 0 or at age 4. We can now calculate the allocation term by multiplying the change in the semi-elasticity between age 4 and age 0 with the variance of firm productivity. For the boom cohort, the semi-elasticities are -0.226 (age 4) and -0.110 (age 0) as per columns 1 and 2 of Table 4. This shows that at the time of start-up more productive firms tend to be smaller and that when the cohort matures this negative relationship between productivity and firm size only worsens.

In Table 5, the dependent variable is the square of the difference between firm productivity and average firm productivity. The main explanatory variable is a dummy for the *Bust cohort*. We can now calculate the dispersion term by multiplying the semi-elasticity at age zero (taken from Table 4) with the change in the cross-sectional variance of productivity between age 0 and age 4. This latter term is defined as the change in the constants in Table 5. For the boom cohort these are 1.156 (age 0) and 0.591 (age 4). That is, firm-productivity dispersion in this cohort declines over time: $\triangle var(\varphi_i)$ is negative. This may be the case because lagging firms learn from more productive ones and hence catch up over time. Since we multiply this negative difference with a negative semi-elasticity ($\widehat{\beta}$), the total dispersion effect in the boom cohort is positive (see the right-hand side equation on page 6). The intuition is as follows: the negative semi-elasticity indicates that productive firms tend to be smaller. The negative aggregate impact of this inefficient allocation will be larger in case of a wider productivity distribution (if there would be no dispersion in firm productivity, one would not need to worry about a negative correlation between productivity and firm size). If dispersion is wide, the negative semi-elasticity has more 'bite' and leads to large aggregate effects. The fact that dispersion declines over time is welcome in our setting: it reduces the impact of the negative semi-elasticity.

In the bust cohort, the semi-elasticity only becomes more negative (the decline in the elasticity is -0.068 at age 0 and -0.100 at age 4, cf. columns 1 and 2 of Table 4). This leads to a further worsening if the allocation term (the green component in Figure 3a). At the same time, the dispersion effect disappears. This is because in the bust cohort the change in the cross-sectional variance of productivity between age 0 (1.156 - 0.583 = 0.573) and age 4 (0.591 - 0.022 = 0.569) is virtually zero. That is, during the crisis there is no longer a gradual reduction in productivity dispersion as start-up firms mature. Multiplying this zero change with the semi-elasticity yields a zero dispersion term.

The picture for TFP growth is slightly different (Figure 3b). Here we observe positive and increasing semi-elasticities (columns 7-12, Table 4). That is, at start-up more efficient firms tend to be larger and this positive relationship only strengthens when firms mature. However, as with labor productivity, we see that the cross-sectional variance of productivity between age 0 and age 4 declines (columns 7-12, Table 5). The product of these two developments - which determines the dispersion term - is therefore slightly negative.

5.3 Decomposing the exit term: Death rate versus selection

During the financial crisis, selection-at-exit became the main source of labor-productivity and TFP growth among Dutch start-ups. The exit term is the product of the exit rate and a selection effect. The latter reflects the productivity difference between exiters and continuers as well as their respective market shares (see Equations 6 to 8 on page 7).

We can use our RA-DOPD framework to quantify these sub-components and gauge their relative importance. To this end, Table 6 provides linear probability regressions where the dependent variable is a dummy that is '1' if a start-up firm dies within four years; '0' if it survives. The independent variable of interest is the *Bust cohort* dummy as well as the constant. Together these show that in the boom (bust) cohort, the exit probability is 0.346 and 0.390, respectively.

In Table 7, the dependent variable is the productivity of a firm scaled by its relative size. This variable therefore measures for each firm its contribution to aggregate productivity. On the right-hand side, we include the *Exit* dummy, the *Bust cohort* dummy and their interaction. The negative *Exit* coefficient implies a positive selection effect: start-ups that exit in the first four years after establishment tend to be less productive than those that survive.

Importantly, this productivity difference between exiters and continuers narrows in the bust cohort. This difference between both is -0.33 in the boom cohort and only -0.14 (-0.33 + 0.19) in the bust cohort. This indicates that selection at the extensive margin deteriorates during the crisis: there is more exit (Table 6) but each exit contributes less to aggregate productivity growth (Table 7).

Figures 4a and 4b provide a graphical breakdown of the change between cohorts in labor-productivity growth and TFP growth, respectively (see column 3 in Table 2 for the data). This figure confirms that the drop in firm productivity was mainly caused by a sharp reduction in within-firm productivity growth. It also shows that changes in selection-at-exit played a relatively small role. The small, positive orange bars reflect a small increase in the exit rate. This is more than counterbalanced, however, by a negative selection effect: each exit contributes less to productivity growth as was the case during the boom cohort.

Our RA-DOPD framework thus allows us to reach a relatively nuanced conclusion about the 'cleansing' (Caballero and Hammour, 1994) or 'sullying' (Barlevy 2002) role of firm exit during recessions. Overall, our evidence is mostly in line with a sullying effect: the positive effect of the increase in exit is outstripped by the negative impact of the much less discriminatory nature of firm exit. The absolute size of the cleansing exit effect is 1.8 percentage points (1.4 for TFP growth) of the total decline in aggregate productivity. This is far from sufficient to counterbalance the various other negative productivity components that all point contribute to a reduction in aggregate productivity. Indeed, the sullying effect due to the deterioration in the selection process alone contributes already about 7.4 and 14.9 percentage points to the decline in aggregate labor productivity and TFP, respectively.

5.4 Adding controls

One may worry that the stark differences between the bust and the boom cohort that we have documented so far, are not driven by the business cycle per se but rather by a changing composition of the start-up population across both cohorts. That is, boom and bust start-ups may for instance differ substantially in terms of type of industry or their geographical location (cities versus more rural areas). To assess whether such confounding factors are important in our context, we supplement the RA-DOPD regressions that we have used so far with a number of control variables (*City*, *Relative employment* and *Relative productivity*) as well as two-digit industry fixed effects. Standard errors are clustered at the two-digit industry level as well.

The results in Tables 3 to 8 show that all our results remain qualitatively and quantitatively the same. Visually this can be seen in Figure 5 where we parse out changing labor-productivity (Figure 5a) and TFP (Figure 5b) into a "clean" bust effect (the solid bars) and a component reflecting changing control variables (the shaded bars). A comparison of Figures 4 and 5 shows that not controlling for confounding factors would lead to a slight underestimation of the within component (blue) and the selection effect at exit (yellow). At the same time there is a slight overestimation of the two between components. The fact that the cleaned within-component is larger than the uncleaned one indicates that during the crisis there was an increase in start-ups in industries with somewhat higher average within-firm productivity growth.

5.5 Productivity growth among high-leverage versus low-leverage start-ups

A key question that we posed at the beginning of this paper was whether high-leverage firms that soon after their establishment had to weather the global financial crisis did less well - in terms of survival probability and productivity growth conditional on survival - as compared with low-leverage start-ups or with high-leverage start-ups during the boom.

We can now answer this question by repeating our RA-DOPD analysis on two distinct sub samples: start-ups with below median and start-ups with above median leverage (we benchmark a firm's start-up leverage to the median leverage level within that firm's two-digit industry). Tables 8 to 12 provide the required RA-DOPD background regressions and we summarize the productivity-growth components we derive from these regressions in graphical format in Figures 6 (boom versus bust) and 7 (bust minus boom) and in numerical format in columns 5 to 12 of Table 2.

It becomes immediately apparent that the decline in both labor-productivity and TFP between both cohorts is sharper among firms with relatively low leverage levels. It is striking that in the bust cohort high-leverage firms continue to increase their productivity during the first four years of their existence while productivity growth is negative for their less leveraged counterparts (-11.3 and -9.3 percentage points for labor-productivity and TFP growth, respetively). This indicates that high levels of leverage in fact helped to partially stem the decline in aggregate productivity growth during the crisis. Indeed, if productivity growth of high-leverage firms would have been affected by the crisis in the same way as low-leverage firms, aggregate productivity growth would have been 1.3 to 3.4 percentage points lower (depending on the control variables used).

Our decomposition allows us to say more about the underlying mechanisms, three of which play a particularly crucial role. First, the decline in the within component is sharper for low leveraged firms (see the blue bars in Figure 7). In fact, only for low-leveraged firms does within-firm productivity growth become negative for the bust cohort (Figure 6). Crisis-related drops in within-firm productivity may to some extent reflect a lack of demand and related excess capacity. A priori one may expect this to apply most to high-leverage firms which may have used external funding to boost capacity during the firm's start-up phase. Our results, however, seem to indicate that, if anything, excess capacity may have been more of a problem for low than for high-leverage start-ups. More generally, the smaller decline in the within-component for high-leverage firms is in line with theories that indicate that binding financial constraints may force firms to operate on a smaller scale and hence at higher productivity.

Second, the reduction in the dispersion part of the between term is also significantly stronger for low-leverage firms (and its contribution to aggregate labor-productivity growth turns even negative).

Third, while for the high-leverage firms there is a positive contribution of the allocation part of the between term, this contribution is negative for low-leverage firms. That is, misallocation between firms is *reduced* among high-leverage firms but deteriorates further during the crisis for low-leverage firms. Among entrepreneurs who start their business just before the crisis and did so without leveraging up, we thus observe an increasingly negative correlation between initial productivity and market-share growth. If this relation (captured by the semi-elasticity of market share with regard to productivity) for high-leverage firms would have been affected by the crisis in the same way as for low-leverage firms, aggregate productivity growth would have been between 2.2 and 2.7 percentage points lower. This is clearly add odds with models of financial frictions and misallocation, based on which we would expect misallocation to be concentrated among relatively leveraged firms.

Finally, the data show that productivity growth among low-leverage firms not only suffers due to more misallocation among surviving start-ups, but also because selection at exit becomes less efficient (the productivity difference between exiters and continuers narrows). However, this holds for both high and low-leverage firms (the difference is statistically insignificant: p=0.11). That is, we find no evidence that financial frictions significantly change the impact of firm exit on aggregate productivity growth over the business cycle. If anything, among both high and low-leverage firms we observe an increase in the death rate in the bust cohort which contributes to productivity growth as the average exiter was still less productive than the average continuer (though less so than during the boom).

Overall, these results provide next to no evidence for meaninfull financial frictions related to leverage. Figure 7 shows that for almost each component high-leverage frms outperform low-leverage firms when we compare the bust with the boom cohort. Overall, high-leverage start-ups tend to perform better than otherwise similar competitors in the same industry with lower initial leverage levels.

6 Conclusions

- We start our analysis of the micro-drivers of start-up productivity growth by presenting a dynamic Olley-Pakes decomposition.

- We observe a sharp drop in aggregate productivity growth among firms that soon after establishment faced the global financial crisis. First of all, we observe a very sharp decline in within-firm productivity growth. While this was by far the main driver of total productivity growth for the boom cohort, its contribution to both labor-productivity growth and TFP growth turns negative for the bust cohort. When we adequately control for changing covariates, this decline in the within-term becomes even more pronounced.

- The role of between-firm reallocation is virtually absent in the boom cohort: firm growth during the first four years after establishment was largely unrelated to firm productivity. However, in the bust cohort we APPEAR TO observe an increasingly distorted economic allocation, whereby *less* productive start-ups capture additional market share. This effect is most pronounced for labor-productivity growth. When we further split up the between term, we find that the absence of a between term for the boom cohort was the net outcome of two opposing forces of almost the same size: a positive disperion effect of 6.2 percentage points and a negative allocation effect of -6.8 percentage points. For the bust cohort, the dispersion effect virtually disappears (that is, there is no longer a change in the productivity dispersion between age 0 and age 4). This is a first reason why the between term deteriorates in the bust cohort. A second reason is that the allocation effect becomes more negative. That is, there is some increased misallocation between firms during the crisis but this effect is small compared to the decline in productivity dispersion as firms age. Our RA-DOPD allows us also to precisely quantify the relative importance of these two channels. In other words, the impact of dispersion is about four times as large as the role of misallocation.

- The third source of productivity dynamics relates to firm exit. We find that selection-at-exit was an important driver of productivity growth during the boom period and that exit selection becomes even much more important - relative to other sources of productivity growth - for the bust cohort. The size of the exit-at-selection contribution to aggregate productivity growth depends on the number of firms that exit as well as on the productivity difference between these firms and those that continue to operate. We use the RA-DOPD framework to quantify these two sub-components. This indicates that selection at the extensive margin deteriorates during the crisis: there is more exit but each exit contributes less to aggregate productivity growth. Overall, our evidence is mostly in line with a sullying effect: the positive effect of the increase in exit is outstripped by the negative impact of the much less discriminatory nature of firm exit (this holds in particular for TFP growth and even more when we control for changing covariates). The absolute size of the positive ('cleansing') exit effect (higher death rate) is about four times (labor productivity) and eight times (TFP) smaller than countervailing ('sullying') effect of a less sharp selection process at exit.

- We also observe substantial difference between start-ups with low versus high initial levels of leverage. It becomes immediately apparent that the decline in both labor-productivity and TFP between both cohorts is sharper among firms with relatively low leverage levels. High levels of leverage in fact helped to partially stem the decline in aggregate productivity growth during the crisis. For almost each RA-DOPD component, high-leverage frms outperform low-leverage firms when we compare the bust with the boom cohort. In particular, the decline in the within component is less pronounced for high-leverage firms; the reduction in the dispersion part of the between term is weaker for high-leverage firms; and while for the high-leverage firms there is a positive contribution of the allocation part of the between term, this part is negative for low-leverage firms. Overall, these results provide next to no evidence for meaninfull financial frictions related to leverage. If anything, high-leverage firms tend to perform well during the crisis period in particular. This may reflect that banks manage do an effective job in screening borrowers in terms of their longer-term growth potential and resilience. While differences in performance at the extensive and intensive margin between high and low-leverage firms are not apparent during a boom period, when everyone benefits from the benign external environment, the latent differences between high and low-leverage firms only emergence once the economic tide turns.

XXX

Bovenstaande substantieel inkorten...

XXX

“Stronger up-or-out dynamics”

XXXX

Expliciet teruglinken naar vraag in titel: Where highly-leveraged start-up firms off to a bad start?

XXXX

Appendices

Appendix A - Interpreting the RA-DODP with a model

Consider the profit maximization problem. The first-order conditions for labor and capital are given, respectively, by:

$$w = \gamma \left(1 - \tau_{y,i}\right) A_i l_i^{\gamma - 1} k_i^{\alpha} \tag{17}$$

and

$$r = \alpha \frac{1 - \tau_{y,i}}{1 + \tau_{k,i}} A_i l_i^{\gamma} k_i^{\alpha - 1} \tag{18}$$

Rearranging these equations leads to Equation (XXX). Also note that

$$y_i = A_i \left(\frac{\gamma}{w} \left(1 - \tau_{y,i}\right) A_i k_i^{\alpha} \right)^{\gamma/(1-\gamma)} \left(\frac{\alpha}{r} \frac{1 - \tau_{y,i}}{1 + \tau_{k,i}} A_i l_i^{\gamma} \right)^{\alpha/(1-\alpha)} \tag{19}$$

Taking logs of (XXX) gives:

$$\ln l_i = \frac{1}{1-\gamma} \left(\ln \frac{\gamma}{w} + \ln \left(1 - \tau_{y,i}\right) + \ln A_i + \alpha \ln k_i \right) \tag{20}$$

$$\ln k_i = \frac{1}{1-\alpha} \left(\ln \frac{\alpha}{r} + \ln \left(\frac{1 - \tau_{y,i}}{1 - \tau_{k,i}} \right) + \ln A_i + \gamma \ln l_i \right) \tag{21}$$

Substituting out $\ln k_i$ in the first of these equations then leads to:

$$\ln l_i = \frac{1}{1-\gamma} \ln \frac{\gamma}{w} + \frac{1}{1-\gamma} \ln A_i + \frac{1}{1-\gamma} \ln \left(1 - \tau_{y,i}\right) + \frac{\alpha}{1-\gamma} \left(\frac{1}{1-\alpha} \ln \frac{\alpha}{r} + \frac{\gamma}{1-\alpha} \ln l_i + \frac{1}{1-\alpha} \ln A_i + \frac{1}{1-\alpha} \ln \left(\frac{1 - \tau_{y,i}}{1 + \tau_{k,i}} \right) \right) \tag{22}$$

$\Longleftrightarrow$

$$\ln l_i - \frac{\alpha}{1-\alpha} \frac{\gamma}{1-\gamma} \ln l_i = \frac{1}{1-\gamma} \ln \frac{\gamma}{w} + \frac{1}{1-\gamma} \frac{\alpha}{1-\alpha} \ln \frac{\alpha}{r} + \left(1 + \frac{\alpha}{1-\alpha}\right) \frac{1}{1-\gamma} \ln A_i + \frac{\alpha}{1-\alpha} \frac{1}{1-\gamma} \ln \left(\frac{1 -}{1 +} \right. \tag{23}$$

$\Longleftrightarrow$

$$\frac{1-\alpha-\gamma}{(1-\alpha)(1-\gamma)}\ln l_i=\frac{1}{1-\gamma}\ln\frac{\gamma}{w}+\frac{\alpha}{(1-\alpha)(1-\gamma)}\ln\frac{\alpha}{r}+\frac{1}{(1-\alpha)(1-\gamma)}\ln A_i+\frac{\alpha}{(1-\alpha)(1-\gamma)}\ln\left(\frac{1-}{1+}\right. \tag{24}$$

$\Longleftrightarrow$

$$\ln l_i=\frac{1-\alpha}{1-\alpha-\gamma}\ln\frac{\gamma}{w}+\frac{\alpha}{1-\alpha-\gamma}\ln\frac{\alpha}{r}+\frac{1}{1-\alpha-\gamma}\ln A_i-\frac{\alpha}{1-\alpha-\gamma}\ln\left(1+\tau_{k,i}\right)+\frac{1}{1-\alpha-\gamma}\ln\left(1-\tau_{y,i}\right) \tag{25}$$

Plugging in $\ln(1-\tau_{y,i})=\xi_y+\phi_y\ln A_i+u_{y,i}$ and $\ln(1+\tau_{k,i})=\xi_k+\phi_k\ln A_i+u_{k,i}$ now gives:

$$\ln l_i=\frac{1-\alpha}{1-\alpha-\gamma}\ln\frac{\gamma}{w}+\frac{\alpha}{1-\alpha-\gamma}\ln\frac{\alpha}{r}+\frac{1}{1-\alpha-\gamma}\ln A_i-\frac{\alpha}{1-\alpha-\gamma}\left(\xi_k+\phi_k\ln A_i+u_{k,i}\right)+\frac{1}{1-\alpha-\gamma}\left(\xi\right. \tag{26}$$

$\Longleftrightarrow$

$$\ln l_i=\frac{1-\alpha}{1-\alpha-\gamma}\ln\frac{\gamma}{w}+\frac{\alpha}{1-\alpha-\gamma}\ln\frac{\alpha}{r}+\frac{\xi_y-\alpha\xi_k}{1-\alpha-\gamma}+\frac{1+\phi_y-\alpha\phi_k}{1-\alpha-\gamma}\ln A_i+\frac{u_{y,i}-\alpha u_{k,i}}{1-\alpha-\gamma} \tag{27}$$

This equation corresponds precisely to the reduced-form equation (XXX) with $\varphi_i=\ln A_i$ and$\beta_0=\frac{1-\alpha}{1-\alpha-\gamma}\ln\frac{\gamma}{w}+\frac{\alpha}{1-\alpha-\gamma}\ln\frac{\alpha}{r}+\frac{\xi_y-\alpha\xi_k}{1-\alpha-\gamma}$, $\beta_1=\frac{1+\phi_y-\alpha\phi_k}{1-\alpha-\gamma}$, and $\varepsilon_i=\frac{u_{y,i}-\alpha u_{k,i}}{1-\alpha-\gamma}$.

We now derive the reduced-form coefficients for $\varphi_i=\ln(y_i/l_i)$. First note that the first-order condition for labor can be written as $w=\gamma(1-\tau_{y,i})\,y_i/l_i$. It directly follows that:

$$\ln\left(1-\tau_{y,i}\right)=\ln w-\ln\gamma-\ln\left(y_i/l_i\right) \tag{28}$$

The equation $\ln l_i$ derived above can be expressed as:

$$\ln l_i=\frac{1-\alpha}{1-\alpha-\gamma}\ln\frac{\gamma}{w}+\frac{\alpha}{1-\alpha-\gamma}\ln\frac{\alpha}{r}+\frac{1}{1-\alpha-\gamma}\ln A_i-\frac{\alpha}{1-\alpha-\gamma}\left(\xi_k+\phi_k\ln A_i+u_{k,i}\right)+\frac{1}{1-\alpha-\gamma}\ln \tag{29}$$

$$\iff$$

$$\ln l_i = \frac{1-\alpha-\alpha\xi_k}{1-\alpha-\gamma}\ln\frac{\gamma}{w}+\frac{\alpha}{1-\alpha-\gamma}\ln\frac{\alpha}{r}+\frac{1-\alpha\phi_k}{1-\alpha-\gamma}\ln A_i-\frac{\alpha}{1-\alpha-\gamma}u_{k,i}+\frac{1}{1-\alpha-\gamma}\ln\left(1-\tau_{y,i}\right) \tag{30}$$

The relation between TFP can alternatively be expressed as $\ln A_i = \xi_y + \phi_A \ln(1-\tau_{y,i}) + u_{A,i}$, where $u_{A,i}$ is orthogonal to $\ln(1-\tau_{y,i})$. Using this relation gives

$$\ln l_i = \frac{1-\alpha-\alpha\xi_k}{1-\alpha-\gamma}\ln\frac{\gamma}{w}+\frac{\alpha}{1-\alpha-\gamma}\ln\frac{\alpha}{r}+\frac{1-\alpha\phi_k}{1-\alpha-\gamma}\left(\xi_y+\phi_A\ln\left(1-\tau_{y,i}\right)+u_{A,i}\right)-\frac{\alpha}{1-\alpha-\gamma}u_{k,i}+ \tag{31}$$

$$\iff$$

$$\ln l_i = \frac{1-\alpha-\alpha\xi_k+\xi_y\left(1-\alpha\phi_k\right)}{1-\alpha-\gamma}\ln\frac{\gamma}{w}+\frac{\alpha}{1-\alpha-\gamma}\ln\frac{\alpha}{r}+\frac{1+\phi_A\left(1-\alpha\phi_k\right)}{1-\alpha-\gamma}\ln\left(1-\tau_{y,i}\right)+\frac{1-\alpha\phi_k}{1-\alpha-\gamma}u_{A,i}+ \tag{32}$$

Now use that $\ln(1-\tau_{y,i}) = \ln w - \ln\gamma - \ln(y_i/l_i)$:

$$\ln l_i = \frac{1-\alpha-\alpha\xi_k+\xi_y\left(1-\alpha\phi_k\right)+1+\phi_A\left(1-\alpha\phi_k\right)}{1-\alpha-\gamma}\ln\frac{\gamma}{w}+\frac{\alpha}{1-\alpha-\gamma}\ln\frac{\alpha}{r}-\frac{1+\phi_A\left(1-\alpha\phi_k\right)}{1-\alpha-\gamma}\ln\left(y_i/l_i\right) \tag{33}$$

Now consider the exit decision. First note that in the absence of revenue and capital distortions, the optimal level of labor satisfies

$$\ln l_i = \frac{1-\alpha}{1-\alpha-\gamma}\ln\frac{\gamma}{w}+\frac{\alpha}{1-\alpha-\gamma}\ln\frac{\alpha}{r}+\frac{1}{1-\alpha-\gamma}\ln A_i \tag{34}$$

Log output is given by

$$\ln y_i = \ln A_i + \gamma\ln l + \alpha\ln k_i \tag{35}$$

Substitute out log capital:

$$\ln y_i = \ln A_i + \gamma\ln l + \alpha\left(\frac{1}{1-\alpha}\left(\ln\frac{\alpha}{r}+\ln A_i+\gamma\ln l_i\right)\right) \tag{36}$$

$$\iff$$

$$\ln y_i = \frac{\alpha}{1-\alpha}\ln\frac{\alpha}{r}+\frac{1}{1-\alpha}\ln A_i+\frac{\gamma}{1-\alpha}\ln l \tag{37}$$

Substitute out log labor:

$$\ln y_i = \frac{\alpha}{1-\alpha}\ln\frac{\alpha}{r} + \frac{1}{1-\alpha}\ln A_i + \frac{\gamma}{1-\alpha}\left(\frac{1-\alpha}{1-\alpha-\gamma}\ln\frac{\gamma}{w} + \frac{\alpha}{1-\alpha-\gamma}\ln\frac{\alpha}{r} + \frac{1}{1-\alpha-\gamma}\ln A_i\right) \tag{38}$$

$\iff$

$$\ln y_i = \frac{\alpha}{1-\alpha-\gamma}\ln\frac{\alpha}{r} + \frac{\gamma}{1-\alpha-\gamma}\ln\frac{\gamma}{w} + \frac{1}{1-\alpha-\gamma}\ln A_i \tag{39}$$

Now consider the selection regression. Take logs of the exit condition:

$$\ln(1+\tau_{c,i}) + \ln c_f > \ln y_i + \ln(1-\alpha-\gamma) \tag{40}$$

Substitute out log output:

$$\ln(1+\tau_{c,i}) + \ln c_f > \ln(1-\alpha-\gamma) + \frac{\alpha}{1-\alpha-\gamma}\ln\frac{\alpha}{r} + \frac{\gamma}{1-\alpha-\gamma}\ln\frac{\gamma}{w} + \frac{1}{1-\alpha-\gamma}\ln A_i \tag{41}$$

so we write the condition as s

$$\ln(1+\tau_{c,i}) + \kappa > \frac{1}{1-\alpha-\gamma}\ln A_i. \tag{42}$$

where $\kappa \equiv \ln c_f - \ln(1-\alpha-\gamma) - \frac{\alpha}{1-\alpha-\gamma}\ln\frac{\alpha}{r} - \frac{\gamma}{1-\alpha-\gamma}\ln\frac{\gamma}{w}$.

To ease notation, we normalize the mean of $\ln A_i$ and $\ln(1+\tau_{f,i})$ to zero and define $z_i \equiv (1-\alpha-\gamma)\ln(1+\tau_{f,i})$. The variables $\ln A_i$ and z_i are jointly normally distributed:

$$\begin{bmatrix} \ln A_i \\ z_i \end{bmatrix} \sim N\left(\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}\right), \begin{pmatrix} \sigma_A^2 & \sigma_{Az} \\ \sigma_{Az} & \sigma_z^2 \end{pmatrix}\right). \tag{43}$$

A firm produces if $(x_i = 0)$ if $\ln A_i > z_i + (1-\alpha-\gamma)\kappa$. The mean productivity of a producer is given by

$$\lambda_0 = E\left[\ln A_i \mid \ln A_i - z_i > (1-\alpha-\gamma)\kappa\right] \tag{44}$$

Defining $\sigma^2 \equiv var\left(\ln A_i - z_i\right)$, this conditional expectation can be re-written as (see standard treatments of the Roy (1951) model, as in e.g. French and Taber (2011)):

$$\lambda_0 = \frac{\sigma_A^2 - \sigma_{Az}}{\sigma} \frac{\phi\left(\left(1-\alpha-\gamma\right)\kappa\right)}{1-\Phi\left(\left(1-\alpha-\gamma\right)\kappa\right)} \tag{45}$$

where ϕ and Φ are, respectively, the pdf and cdf of the standard normal distribution. Similarly, we can write the mean productivity of an exiter as

$$\lambda_0 + \lambda_1 = -\frac{\sigma_A^2 - \sigma_{Az}}{\sigma} \frac{\phi\left(\left(1-\alpha-\gamma\right)\kappa\right)}{\Phi\left(\left(1-\alpha-\gamma\right)\kappa\right)} \tag{46}$$

It now follows that

$$\lambda_1 = -\frac{\sigma_A^2 - \sigma_{Az}}{\sigma} \left(\frac{\phi\left(\left(1-\alpha-\gamma\right)\kappa\right)}{\Phi\left(\left(1-\alpha-\gamma\right)\kappa\right)} + \frac{\phi\left(\left(1-\alpha-\gamma\right)\kappa\right)}{1-\Phi\left(\left(1-\alpha-\gamma\right)\kappa\right)} \right) \tag{47}$$

The term between brackets is always positive. Finally, we define the function $\omega \equiv \frac{\phi((1-\alpha-\gamma)\kappa)}{\Phi((1-\alpha-\gamma)\kappa)} + \frac{\phi((1-\alpha-\gamma)\kappa)}{1-\Phi((1-\alpha-\gamma)\kappa)}$.

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