

Technology Network, Innovation and Growth

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This paper develops a multi-sector endogenous growth model which embeds a technology network that captures heterogeneous intersectoral knowledge spillovers. Each sector serves both as a distributor and a consumer of knowledge: the former depends on a sector's position in the network; the latter depends on its efficiency in utilizing knowledge. The interaction of these forces influences long-run economic growth, sectoral shares and the firm size distribution. The sparsity of the network imposes an upper bound on the impact of knowledge spillovers. In this model, sectors converge to the same growth rate if they belong to the same *irreducible* network. However, their contributions to economic growth differ substantially, depending on their positions in the technology network and their efficiency in conducting innovation. Consequently, the model has implications for the allocation of innovation subsidies. The gain in economic growth derived from promoting innovation in the sector that utilises knowledge most efficiently is over 10,000 times larger than gain derived from promoting innovation in the least efficient sector.

Key words: Technology network, Firm innovation, Knowledge spillovers, Growth, Citation.

JEL Codes: O31 O33 O41 D85

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1. INTRODUCTION

The seminal work by [Klette and Kortum \(2004\)](#) provides a parsimonious model that is able to explain several stylized facts about firm’s innovation behavior. In their model, they treat firms as collections of knowledge and assume that past knowledge accumulation contributes to future innovation equally, regardless of which sector it comes from and which sector it contributes to. This simplifying assumption renders their model and the subsequent studies that have adopted the same framework silent on questions like how innovations in different sectors are connected with each other; what is the relationship between sectoral growth and the aggregate economic growth; does industry-oriented policy have impacts beyond its targeted sector? These questions are of contemporary relevance as many governments, especially those of developing countries, have implemented ad hoc industrial policies without overall evaluations.¹ Unfortunately, little research has been done on these issues. In this paper, we aim to bridge the gap by building a framework incorporating a technology network that captures heterogeneous intersectoral technology dependence among sectors.

Figure 1 showcases an example of such a network constructed from patent citations.² In the graph, the economy is represented by the network. Each node is a sector in the economy and each edge implies the existence of a technology connection between sectors.³ The thickness of an edge captures the strength of knowledge flows from one sector to another. The size of each node represents the strength of aggregate outward knowledge flows, measured by the cross-sector backward citation ratio.

Three features of Figure 1 are noteworthy. First, the strength of knowledge spillovers differs across sector pairs, as shown by the different thickness of the edges. Second, for a given sector pair, the knowledge spillover from one sector to the other is not the same as the spillover in the other direction, which implies that the network is asymmetric. Finally, knowledge spillovers do not exist among some sector pairs, which is reflected in the sparsity of the network. The above features all point to sector-pair-specific heterogeneity, one dimension of heterogeneity which is of interest in this paper.

¹For example, the Chinese government proposed to invest more than 161 billion dollars over 10 years to develop the semiconductor industry in 2015.

²The patent citation data is from US Patent and Trademark Office and will be discussed in detail shortly. The figure is drawn using all citation data available in the dataset between 1975 and 2006 .

³The definition of sectors here follows [Hall et al. \(2001\)](#), who assign all patents to 37 sectors.

heterogeneous efficiency in utilizing knowledge from other sectors

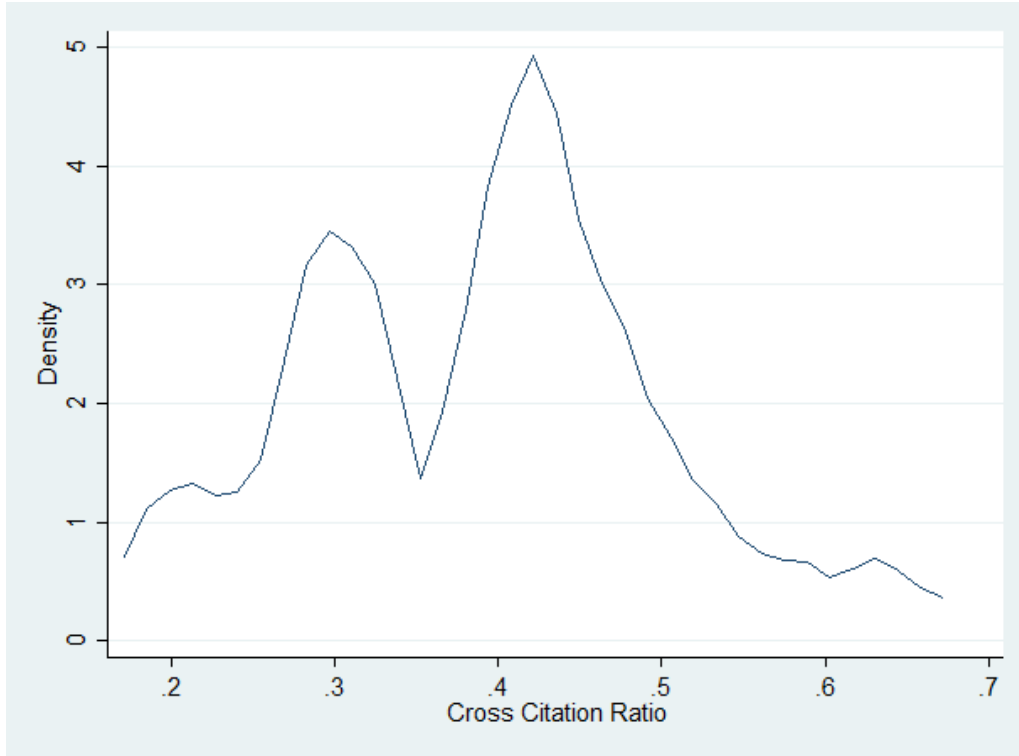


FIGURE 2
CROSS SECTOR CITATION RATIO

These empirical observations offer guidance with respect to the building of the theoretical model. To capture the notion of technology spillovers, individual firms must be allowed to accumulate knowledge in multiple sectors. Each sector differs with respect to its technological dependence on other sectors. Some sectors are technically closer to one-another than others. For instance, the Computer sector and the Communication sector share a strong technological connection while the Chemistry sector and the Metal sector do not. Such heterogeneity is assumed to be determined by the deep nature of the sector-specific knowledge itself and is treated as exogenous.

We study two types of networks in this paper: an irreducible network and a reducible network. The former represents an economy where each sector has knowledge spillovers to all the other sectors either directly or indirectly. In contrast, an economy denoted by a reducible network can be thought of as a collection of multiple technology clusters. In this case, knowledge spillovers only exist among sectors within each technology cluster but not across clusters. The difference between the reducible and irreducible networks is determined by the sparseness of the network. When the network is sufficiently sparse, it becomes a reducible network.

There are several theoretical findings in the benchmark case of the irreducible network. First, we show that the interaction of the technology network and the sectoral innovation intensities together

determines the long-run economic growth rate as well as individual sector growth rate. Specifically, the magnitude of the long-run growth rate is the dominant eigenvalue of a matrix, Φ , that reflects the interaction of these two forces. In addition, all sectors converge to the same growth rate as the aggregate economy.

Second, the change of the network structure itself has important implications for economic growth. On one hand, stronger intersectoral technological linkages always lead to a higher long-run growth rate. On the other hand, sectors are different in their relative importance for overall economic growth, which depends on whether a sector is a knowledge sender or a knowledge receiver. Increasing the strength of knowledge spillovers *from* a larger sector will benefit economic growth more than in the case of a smaller sector. Increasing the strength of knowledge spillovers *to* a sector with high innovation intensity results in a larger effect on economic growth than in the case of a low innovation intensity.

Third, the sectoral shares are equal to the corresponding generalized eigenvector centrality associated with a matrix, Ψ , which determines the dynamics of the sectoral share evolution. Eigenvector centrality is a measure of the importance of a node in a network. A higher score is assigned to nodes with connections to/from high score nodes. Generalized eigenvector centrality used in this paper takes into account the strength of each connection in addition to the number of connections. In the economy, a sector that is located in a more central position of the network economy occupies a larger share. The intuition is that a more central sector plays a more important role in diffusing knowledge to other sectors. Therefore, the relative size of a sector measures the influence of a sector in terms of spreading knowledge to the rest of the economy.

Lastly, the interaction of the technology network and the sectoral innovation intensities also plays a role in shaping the firm size distribution. The firm size distribution is shown to be a Pareto distribution, the right tail of which is determined by the ratio of the population growth rate and the long-run economic growth rate.

The previous theoretical results are established under the assumption that each sector displays global spillover effects. This assumption is relaxed when the network structure is assumed to be reducible. For a reducible network, the sectoral growth rate is driven by the local network of technology clusters that each sector belongs to and the converging behaviour of sectoral growth is limited only within each cluster. As a result, different technology clusters demonstrate heterogeneous growth and aggregate economic growth is determined by the cluster that grows fastest. Given that sectors belong to different technology clusters may grow at different rates, their shares diverge in the long run. Moreover, the firm size distribution is also technology cluster specific. For firms that belong to

different technology clusters, their size difference reflects the difference of those technology clusters' network structures.

We apply the framework to analyse industry-oriented policies. Specifically, We introduce a one-period temporary policy shock to a sector such that the innovation intensity of that sector increases by 1%. We then simulate the impact of the shock on output. This process is applied to every sector. We find that there is substantial heterogeneity across sectors in terms of the impacts of industry policies. The most effective industry policy leads to over 0.01% increase of output level, while the least effective policy has almost no effects. Similar results are found when a permanent policy shock is implemented so that the innovation intensity of the affected sector increases by 1% forever. The most effective policy can result in a percentage increase of economic growth that is 10000 times larger than in the case of the least effective policy.

The remainder of this paper is organised as follows. Section 2 reviews the past literature. Section 3 presents the empirical motivation of this paper. The formal model is developed in Section 4, while policy analyses are provided in Section 5. Section 6 concludes.

2. Related Literature

This paper contributes to several strands of literature. Among others, it is closely related to the literature that studies R&D via endogenous innovation.⁶ In particular, the model is an extension of [Klette and Kortum \(2004\)](#). In their paper, firms engage in a Schumpeterian-style innovation process and expand their products via a Poisson birth and death process. New products arise at a rate that depends on the knowledge accumulation embodied in past products, while some products of a firm are lost as a result of competition from rivals making them obsolete. A large literature has subsequently adapted their approach and applied it to different environments.⁷ A common feature of these models is that innovation is undirected: that is, knowledge accumulation in one sector will benefit innovation in other sectors equally. In contrast, we construct a multi-sector model that focuses on the interdependency of sectors and allows for heterogeneous innovation rates. These new characteristics make it possible to think about the linkage between individual sectors and the aggregate economy and to evaluate the aggregate effects of industrial policies.

There is a large literature trying to model various networks existing in the economy. Among them, [Jackson and Rogers \(2007\)](#) are the first to present a general model of network formation in an

⁶See, for example, [Romer \(1986\)](#), [Aghion and Howitt \(1992\)](#), [Aghion et al. \(1997\)](#), [Grossman and Helpman \(1991\)](#) and [Kortum \(1997\)](#).

⁷See, for example, [Lentz and Mortensen \(2005, 2008\)](#), [Acemoglu et al. \(2013\)](#) and [Akcigit and Kerr \(2016\)](#)

attempt to explain the salient features of various networks formed in the real world. They argue that nodes in a network form links with each other either randomly or via local search using the existing network, the so-called meeting friends of friends. The technology network in this paper shares the fundamental feature of the social network in [Jackson and Rogers \(2007\)](#). Firms enter sectors where their existing patent portfolios have strong knowledge spillovers and innovate intensively in those sectors. This is similar to the local search in the social network.

Another strand of literature studies the size distribution of firms and cities. [Luttmer \(2007, 2011\)](#) describes a balanced growth model that features a Pareto distribution of firm size, which is consistent with the observed size distribution of U.S. firms' employment. In a different context, [Gabaix \(1999\)](#) employs a model, underlying which is a geometric Brownian motion with a reflecting barrier, to explain the city size distribution in the U.S.. The way in which these studies generate size differences is assuming that entities experience random growth and have different sizes because of the realization of different shocks.⁸ This paper provides an alternative way to think about the firm size difference. In an economy represented by a sparse network, firms that belong to different technology clusters have different growth potentials, which are reflected by their sizes.

This paper is not the first attempt to incorporate networks into macro analysis. [Oberfield \(2012\)](#) develops a model of a business network through which firms form production chains and studies the endogenously emerging network. [Cai and Li \(2012\)](#) study the impact of intersectoral knowledge linkages on firms' innovation intensity and the sequence of entry into different industries. My paper treats the technology network as exogenous and explores how different structures of the network can affect economic growth, sectoral shares and the firm size distribution.

Finally, this paper contributes to the literature on knowledge spillovers. [Jaffe \(1986\)](#) constructs an empirical measure of technology spillovers to study the impact of the research of neighbouring firms on the success of a firm's R&D. He finds that high R&D firms tend to reap the benefit of knowledge spillovers while firms with low R&D are worse off. [Bloom et al. \(2013\)](#) study two types of spillovers: a positive technology spillover effect from other research firms and a negative business stealing effect from product market rivals. They employ and extend the measure of technology spillovers from [Jaffe \(1986\)](#) and conclude that the positive knowledge spillover effect dominates the negative business stealing effect.⁹ [Acemoglu et al. \(2016\)](#) map the upstream technology network and sectoral patent

⁸[Gabaix \(2009\)](#) surveys the theory and application of the power law in economics and finance.

⁹There is a large amount of research in the IO literature that studies the external knowledge spillover between firms and their competition in the product market. A short list includes: [D'Aspremont and Jacquemin \(1988\)](#), [Suzumura \(1992\)](#), [Amir and Wooders \(1999\)](#), [Anbarci et al. \(2002\)](#) and [Erkal and Piccinin \(2010\)](#). My paper abstracts away from product market competition and external knowledge spillovers, and instead focuses on the internal knowledge spillovers. This feature makes my model tractable and renders the relationship between the network structure and the economic

growth to predict future innovation after 1995 and find strong predictive power at the sector level. My paper serves as the complement to the previous studies by providing novel empirical evidence of intersectoral knowledge spillovers at the firm level and new theoretical insights on the importance of intersectoral technological linkages.

3. EMPIRICAL EVIDENCE OF TECHNOLOGY NETWORK

In this section, we present evidence on the spillover effects of firms' past knowledge accumulation on future innovation. We ask three questions here: (i) Does a firm's existing patent portfolio affect its entry to a new sector in the future? (ii) Does the past knowledge accumulation contribute to the innovation of new patents across sectors? and (iii) If so, how does this relationship vary across sectors? The first two questions present two margins of spillover effects: the former is what we term the pro-entry effect, which is intended to capture the extensive margin of knowledge spillover effects in expanding firms' patent portfolios. The latter is the pro-innovation effect. The pro-innovation effect captures whether past knowledge accumulation in one sector helps to promote innovation in technologically related sectors, representing an intensive margin of knowledge spillover effects. The third question deals with the heterogeneity of different sectors' efficiencies in absorbing and applying intersectoral knowledge.

The backbone for our analysis here is the NBER Patent Database from the United States Patent and Trademark Office (USPTO) from 1975 to 2006. The dataset contains patent records that provide information on the unique assignee number of the inventor of each patent, the country the assignees belong to, the date each patent is applied for and granted as well as the technology sector each patent belongs to, etc. We use the field classification proposed by [Hall et al. \(2001\)](#), who assign all patents into 37 technology sectors. Details of how each sector is defined can be found in [Hall et al. \(2001\)](#). Here, for convenience, we label each technology sector by a number from 1 to 37. TABLE A.1 in the Appendix shows the correspondence between the numeric label and the original field. We only use patents granted to U.S. companies and drop observations for which the assignees are missing. The patent database contains an associated dataset upon the citing-cited relationship of patents. The citation data will be critical for us to reveal the technology network of a firm's patent portfolio. Estimations in this paper will be based on the data between 1990 and 2001. The 2001 end date is chosen to allow for a 5-year window for patent reviews. The 1990 start date allows enough pre-sample data to implement the empirical strategy that will be discussed in detail later. Estimations extending the range of data deliver qualitatively same results.

growth transparent.

To address the first question, some idea about how close two technologies are, or more precisely, how easily certain technology can be applied to others is required. Therefore, we introduce a measure of technology applicability based on the patent citation data. A citation may serve as an indicator of knowledge spillovers. For instance, if patent A cites patent B, then the implication is that knowledge flows from B to A. My measure of technology applicability from sector y to x , $g(y, x)$, is constructed using the following formula:¹⁰

$$g(y, x) = \frac{N(\text{citation from } x \text{ to } y)}{N(\text{total citation to } y)}. \quad (1)$$

This measure is sector-pair specific and can be interpreted as the average spillover effect from sector y to sector x . Using this measure, we can continue to construct the average proximity between a firm's current patent portfolio and any targeted technology sector. The measure of average proximity is needed because firms can operate in multiple sectors, and our first measure cannot account for this. We use the variable $Mproximity_{f,t}$ to denote the average proximity, and it is calculated as

$$\sum_{y \in \{y \neq x\}} \frac{n_{f,t}^y}{n_{f,t}} g(y, x),$$

where $n_{f,t}^y$ is the patent stock for firm f at time t in sector y and $n_{f,t}$ is total patent stock at t .¹¹ Note that the targeted sector x is excluded in the summation because we want to analyze the intersectoral spillover effects. With the above measure, we run a Probit regression over a firm's probability of entering a given sector on its average proximity to the targeted sector:

$$Pr(y_{f,t}^x) = \alpha_1 Mproximity_{f,t-1}^x + \alpha_2 Controls_{f,t-1}^x + \mu_f^x + \vartheta_t + \zeta^x + \varepsilon_{f,t}^x, \quad (2)$$

where the dependent variable $y_{f,t}^x$ is a dummy variable that takes the value of 1 if the corresponding firm f produces patents in technology sector x at time t and zero otherwise. We control for whether firm f has previously operated in the same sector, $y_{f,t-1}^x$, as well as the total number of technology sectors firm f operates in at $t - 1$, $N_{f,t-1}$. To deal with unobserved heterogeneity, it is assumed that the error term is composed of μ_f^x , a firm-sector fixed effect, a full set of year dummies, ϑ_t , and sector dummies, ζ^x , as well as an idiosyncratic component, $\varepsilon_{f,t}^x$. We follow Wooldridge (2002) and instrument the unobserved firm-sector fixed effect μ_f^x with the time mean of all exogenous variables and the initial value of the dependent variable y_0^x . The key parameter that we are interested in is α_1 , which captures the pro-entry effect. A positive α_1 implies that a patent portfolio that is more technically related to sector x contributes to the firm's entry into sector x .

¹⁰This measure has the property that $\sum_x g(y, x) = 1$, which will become useful in the next section.

¹¹ $n_{f,t}^y$ can be zero if firm f does not hold any patent in sector y at time t .

Estimation results are presented in TABLE 1. As shown in column (1), a firm is more likely to enter a technology sector x when its patent portfolio contains technologies closer to that sector. The estimates remain significant after controlling for various factors (column (2) to (4)). The additional control variables behave as we would expect. In particular, a firm that currently owns patents in sector x is more likely to continue innovation in the future (column(2)), and there is a higher probability that a firm with a broad set of patents will expand into another sector (column(3)). The magnitude and significance of the above variables are largely unaffected by adding time and sector dummies to the regression as shown in column (4). The regression results taking account of unobserved firm-sector fixed effects are provided in column (5). After controlling for the unobserved firm-sector fixed effects, the magnitude of the coefficient for all controls decline significantly. However, the coefficient for “Mproximity” increases compared to column (4), demonstrating its robustness.

TABLE 1
PROBIT REGRESSIONS

	(1)	(2)	(3)	(4)	(5)
$Mproximity_{f,t-1}^x$	14.75*** (0.17)	13.55*** (0.12)	14.16*** (0.13)	11.89*** (0.14)	14.57*** (0.24)
$y_{f,t-1}^x$		2.17*** (0.01)	1.98*** (0.01)	1.90*** (0.01)	1.62*** (0.01)
$N_{f,t-1}$			0.06*** (0.00)	0.06*** (0.00)	0.00 (0.00)
<i>YearDummy</i>	No	No	No	Yes	Yes
<i>SectorDummy</i>	No	No	No	Yes	Yes
<i>Firm-sector FE</i>	No	No	No	No	Yes
<i>Constant</i>	-1.66*** (0.01)	-2.08*** (0.01)	-2.25*** (0.00)	-2.65*** (0.02)	-2.80*** (0.03)
Observations	2,042,178	2,042,178	2,042,178	2,042,178	2,042,178

A ***/*** next to the coefficient indicates significance at the 10/5/1% level. The variable $Mproximity_{y,t-1}$ is constructed using the data at time $t - 1$. Firm-sector fixed effect is instrumented using the method proposed by Wooldridge (2002). All standard errors are clustered at firm level.

One concern with the above approach is that the positive relationship between technology proximity and the probability of entering new sectors may be driven by a few sectors that demonstrate strong

positive effects instead of a universal phenomenon across all sectors. To address such a concern, TABLE A.2 in the Appendix reports the results of Probit regressions for each technology sector. The conclusion drawn is that the pro-entry effect is indeed observed in all sectors, though there is some heterogeneity regarding how strong the effect is across sectors. In general, the probability of entering a technology sector in the future is higher if a firm currently owns a patent portfolio that is technically closer the sector of interest.

We now turn to evaluate how much past knowledge accumulation contributes to the building of new knowledge. We construct a new measure, $Wpatent_{f,t}^x$, to capture the weighted patent stock with the weights equal to $g(y, x)$. $Wpatent_{f,t}^x$ for firm f at time t with respect to sector x is defines as

$$\sum_{y \in \{y \neq x\}} I(y_{f,t} = 1)g(y, x)n_{f,t}^y,$$

where $n_{f,t}^y$ is the patent stock for firm f in the technology class y at time t and $I(y_{f,t} = 1)$ is a dummy variable that takes the value of 1 if firm f does not hold a patent in sector y , and 0 otherwise. This measure captures the intersectoral spillover effect of the current patent portfolio with respect to sector x , since we do not include the patent stock in x . We follow Hall et al. (2005) and construct the patent stock using a perpetual inventory method with a 15% depreciation rate.¹² That is, $n_{f,t}^y = \Delta n_{f,t}^y + (1 - \delta)n_{f,t-1}^y$, where $\Delta n_{f,t}^y$ is the number of new patents firm f produces in sector y at time t and $\delta = 0.15$. We then run the following Negative Binomial model:

$$PatentCounts_{f,t}^x = \beta_1 Wpatent_{f,t-1}^x + \beta_2 Controls_{f,t-1}^x + \tilde{\mu}_f^x + \tilde{\vartheta}_t + \tilde{\zeta}^x + \tilde{\varepsilon}_{f,t}^x \quad (3)$$

where $PatentCounts_{f,t}^x$ represents the number of new patents produced by firm f in technology sector x from time $t - 1$ to time t and $\tilde{\mu}_f^x$ is the firm-sector fixed effect. $\tilde{\vartheta}_t$ and $\tilde{\zeta}^x$ are sets of time dummies and sector dummies respectively. Due to the nonlinearity of the Negative Binomial model, we follow Blundell et al. (1999) and use the pre-sample mean scaling method to control for fixed effects. The idea is to use pre-sample data on patenting behavior to instrument for unobserved heterogeneity. The long panel of patent data available from USPTO allows me to construct the pre-sample average between 1970 and 1989, which is ideal for the practice. The controls used here are the same as those in (2) except that we include a dummy to indicate whether a firm previously innovates in the targeted sector and the lag value of patent counts of the firm in the targeted sector.

The coefficient of interest is β_1 . A positive sign of this coefficient implies the existence of a pro-innovation effect. The estimates are shown in TABLE 2. From column (1), it is clear that the past patent portfolio does contribute to the production of new patents. Adding variables that control

¹²Hall et al. (2005) use R&D to calculate knowledge capital while we use patents here.

for the number of patents firm f owns a patent and how many patents firm f owns decreases the magnitude of the coefficient (column 2). Nonetheless, the inclusion of further controls does not affect it significantly (column (3) to (4)). Finally, in column (5), we demonstrate the results using the pre-sample mean scaling method to control for the firm-sector fixed effect, which changes neither the magnitude nor the significance of the estimate for the weighted patent stock variable. As shown in column (5), past knowledge accumulation has a substantial effect on the innovation of new patents. A 1% increase of the weighted patent stock leads to a 0.4% increase in the production of new patents.

TABLE 2
NEGATIVE BINOMIAL REGRESSIONS

	(1)	(2)	(3)	(4)	(5)
$Wpatent_{f,t-1}^x$	0.76*** (0.01)	0.40*** (0.01)	0.43*** (0.01)	0.39*** (0.01)	0.39*** (0.01)
$I(x_{f,t-1} = 1)$		2.87*** (0.04)	2.88*** (0.04)	2.82*** (0.04)	2.66*** (0.04)
$PatentCounts_{f,t-1}^x$		0.06*** (0.01)	0.06*** (0.01)	0.06*** (0.01)	0.04*** (0.01)
$N_{f,t-1}$			-0.02*** (0.00)	-0.01** (0.00)	-0.00 (0.00)
<i>YearDummy</i>	No	No	No	Yes	Yes
<i>SectorDummy</i>	No	No	No	Yes	Yes
<i>Firm-sector FE</i>	No	No	No	No	Yes
<i>Constant</i>	-0.47*** (0.02)	-2.22*** (0.02)	-2.09*** (0.02)	-2.80*** (0.07)	-2.73*** (0.09)
$\ln\alpha$	2.20*** (0.03)	0.73*** (0.02)	0.72*** (0.03)	0.72*** (0.03)	0.55*** (0.03)
Observations	2,029,470	2,029,396	2,029,396	2,029,396	1,050,316

A */**/***/ next to the coefficient indicates significance at the 10/5/1% level. $I(x_{f,t-1} = 1)$ is the indicator function that takes the value of 1 if firm f operates in sector x at time $t - 1$ and 0 otherwise. $Wpatent_{f,t-1}^x$ is in log form to make the interpretation more convenient. The firm-sector fixed effect is instrumented using the method proposed by [Blundell et al. \(1999\)](#) and is reported in column (5). All standard errors are clustered at the firm level.

The above analysis studies the average effect of intersectoral knowledge spillovers. In the rest

of this section, the focus will be on the heterogeneity of intersectoral knowledge spillovers. To this end, we run the same regression as (3) but restrict the sample to each sector instead of pooling all sectors together. The estimate β_1 for each sector can be interpreted as the average efficiency of firms in absorbing knowledge from other sectors to their own sectors.

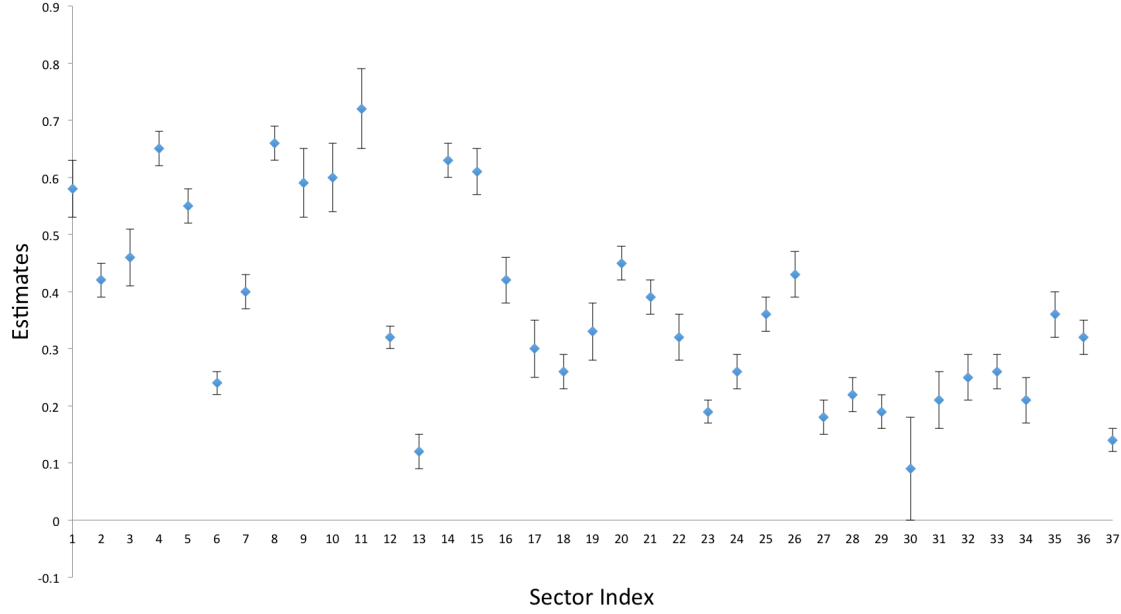


FIGURE 3
ESTIMATES OF SECTOR INNOVATION INTENSITIES

Each sector's estimate with one standard error is displayed in FIGURE 3.¹³ There is significant heterogeneity across sectors with the parameter estimates, ranging from 0.09 to 0.72 with a mean of approximately 0.3. The results reveal that intersectoral knowledge spillovers contribute significantly to knowledge building for most sectors but that the ability to absorb and use knowledge to produce research output differs across sectors.

To sum up, this section presents empirical evidence about the importance of technology network on the innovation at the micro level. In the next section, we will build a model that embeds the features shown here. In particular, the model will include two layers of heterogeneity: the intersectoral knowledge spillovers and the sector-specific innovation efficiency. The latter endogenously arises as the heterogeneous technology network renders the option value of research differs across sectors.

¹³Details of the regression results are shown in TABLE A.3 in the Appendix.

4. THEORETICAL MODEL

4.1. Multi-sector Innovation Model with an Irreducible Technology Network

There are a finite number of technology sectors in the economy. Each sector is a collection of technologies that share similar features. Sectors differ in their technology spillovers to other sectors. The heterogeneous sector-pair technology spillovers (edges) along with all sectors (nodes) form a technology network. Formally, the technology network is modeled as a $M \times M$ weighted directed adjacency matrix G , where each entry $g_{ij} \geq 0$ denotes the knowledge flow from sector i to sector j , and M is the number of sectors. There are intersectoral technology spillovers from i to j if $g_{ij} > 0$. The implications of different network structures will be explored here. As a benchmark case, we will study an irreducible technology network, whose definition is formalized as follows:

Definition 1. A network is irreducible if $\forall i, j$, there exists a sequence of indices $l_1, l_2, \dots, l_n$ such that $g_{il_1} g_{l_1 l_2} \dots g_{l_n j} \neq 0$.

Loosely speaking, an irreducible technology network represents an economy where all sectors have potential impacts on each other either directly or indirectly.

Entrepreneurs develop their knowledge portfolios based on their past knowledge accumulation. The innovation activity is modeled as a Poisson process. A novel feature is that old knowledge not only contributes to the building of new knowledge in the same sector but also in other sectors. In other words, innovation in one sector demonstrates both intrasectoral and intersectoral knowledge spillover effects. The strength of the intersectoral knowledge spillover effects is captured by the off-diagonal elements of the adjacency matrix G .

Incumbents use patents to store new knowledge. Each patent possesses some value to the owner. We will assume that the value of patents is randomly distributed with a sector-specific mean, $\bar{\pi}_i$. In general, the average value of a patent can be different across sectors, meaning that $\bar{\pi}_i \neq \bar{\pi}_j$ for $i \neq j$. There are several ways to interpret the value of a patent. A firm can use its patent to design a new product, and thus enjoy the monopoly profit of the product. Alternatively, a new patent may add value to a firm's current product and improve its quality. In an environment where quality matters to consumers, the two can be equivalent. Here, we take the first interpretation. Now assume that a firm at time t has n_i patents in the technology sector i , and the associated values of these patents are denoted by a vector $\tilde{\pi}_i = (\pi_{i1}, \pi_{i2}, \dots, \pi_{in_i})$. In addition, for a firm with a total of n patents, the value of those patents is represented by a vector of vectors $\tilde{\pi}^n = (\{\tilde{\pi}_i\}_{i \in \mathcal{T}})$, where $n = \sum_{i \in \mathcal{T}} n_i$

and $\mathcal{T}$ is the set of all sectors. Note that if a firm does not have any patent in a certain sector s , then $n_s = 0$.

Each firm can be regarded as a collection of research teams, each specializing in a certain technology sector. Each research team pools resources of other teams together and devotes effort in order to produce new patents. However, resources from other areas will not contribute symmetrically to innovation. In particular, it is assumed that 1 unit of knowledge stock (1 patent) in sector j will serve as g_{ji} units of effective knowledge stock when used for innovation in sector i . Therefore, the total effective knowledge stock in sector i for a firm with the patent portfolio specified above is $\sum_{j \in \mathcal{T}} n_j g_{ji}$. This term can be interpreted as total knowledge spillovers to sector i .

All firms have access to a common production technology that allows them to innovate at the rate λ_i per patent in sector i . The total cost is assumed to be a function of the total innovation rate and the effective knowledge stock and takes the following form:

$$C\left(\lambda_i \sum_{j \in \mathcal{T}} n_j g_{ji}, \sum_{j \in \mathcal{T}} n_j g_{ji}\right).$$

$C(.,.)$ is assumed to be homogeneous of degree one and increases with both arguments. As a result, the total cost function can be written as:

$$C = \sum_{j \in \mathcal{T}} n_j g_{ji} c(\lambda_i),$$

where the variable cost function is specified as follows:

$$c(\lambda_i) = \lambda_i^\epsilon \theta^{1-\epsilon}, \quad \epsilon > 1. \quad (4)$$

θ is a technology parameter which is constant across sectors. The assumption $\epsilon > 1$ ensures that the cost function is a convex function of the flow innovation rate. In addition, $c(.)$ is assumed to be twice differentiable.

Given the previous setup, a firm takes as given its current portfolio of patents and decides the optimal innovation rate by solving the following value function:

$$\begin{aligned} rV(\tilde{\pi}^n) - \dot{V}(\tilde{\pi}^n) = \max_{\{\lambda_i\}_{i \in \mathcal{T}}} & \left\{ \sum_{i \in \mathcal{T}} \sum_{s=1}^{n_i} \pi_{is} - \sum_{i \in \mathcal{T}} c(\lambda_i) \sum_{j \in \mathcal{T}} n_j g_{ji} \right. \\ & \left. + \sum_{i \in \mathcal{T}} \lambda_i \sum_{j \in \mathcal{T}} n_j g_{ji} (E[V(\tilde{\pi}^{n+1})|i] - V(\tilde{\pi}^n)) \right\}, \end{aligned} \quad (5)$$

where $E[V(\tilde{\pi}^{n+1})|i] = E[V(\tilde{\pi}^n \cup \pi_{i(n_i+1)})]$ and r is the interest rate.

The first line on the right-hand side captures net flow profits generated by the firm's current patent portfolio over the total cost incurred for innovation. The second line represents the expected change

of the firm's value from the arrival of new patents. Note that the expected value change is the sum of all sectoral changes. In particular, $\lambda_i \sum_{j \in \mathcal{T}} n_j g_{ji}$ is the sector-specific aggregate innovation rate, while $E[V(\tilde{\pi}^{n+1})|i] - V(\tilde{\pi}_n)$ is the expected increase in a firm's value due to the arrival of a new patent in sector i .

The next proposition characterizes the solution to the value function and the associated optimal innovation choices.

Proposition 1. *The value function for the firm's problem has a solution as follows:*

$$V(\tilde{\pi}^n) = \sum_{i \in \mathcal{T}} \sum_{s=1}^{n_i} \frac{\pi_{is}}{r} + \sum_{j \in \mathcal{T}} n_j R_j, \quad (6)$$

where $(R_1 \cdots R_M)$ are the solutions to a system of M nonlinear equations:

$$rR_j = \sum_i \theta \left(\frac{\bar{\pi}_i + R_i}{\epsilon} \right)^{\frac{1}{\epsilon-1}} g_{ji} \left[\left(1 - \frac{1}{\epsilon} \right) \left(\frac{\bar{\pi}_i}{r} + R_i \right) \right] \quad \forall j \in \mathcal{T}, \quad (7)$$

and the optimal innovation choices are given by:

$$\lambda_i = \theta \left(\frac{\bar{\pi}_i + R_i}{\epsilon} \right)^{\frac{1}{\epsilon-1}}. \quad (8)$$

Proof. See Appendix. □

From the proposition, we can see that the value function of a firm with a patent portfolio $\tilde{\pi}^n$ is equal to the discounted additive sum of random profits generated from the whole portfolio plus the aggregation of the sector-specific option value of research, $n_i R_i$. It is interesting to note that the per patent option value of research in each sector is itself a function of option values of all the other sectors. This is an intrinsic feature of the model because each piece of knowledge created in one sector will be useful for innovation in other sectors. The ultimate research value of each patent is determined fundamentally by the technology network, G . The option value of research will be homogeneous across sectors if the technology network, G , is a diagonal matrix, namely, there is no intersectoral technology spillovers in the economy.

The innovation rate in each sector depends on both the average profit and the research value of patents that belong to the corresponding sector. A higher average profit or a higher option value of research induces firms to increase their effort and thus results in a higher innovation rate.

To complete our analysis, it is necessary to specify the entry of new firms. Suppose that the population is growing at a constant rate η in the economy. There is a fixed proportion of each generation that has the potential to become entrepreneurs. The proportion of potential entrepreneurs is constant over time. As a result, the number of firms grows at the same rate as the population. Further assume

that for a firm to obtain an entry rate of 1, it must pay a fixed cost f . We specify the following free entry condition:

$$E[V(\pi)] = f,$$

where $E[V(\pi)]$ is the expected value of an entrant. Upon entry, firms draw from a common probability distribution function that determines which sector they will enter. Specifically, let p_i denote the probability of entering sector i . The expected value of a patent for an entrant is therefore:

$$E[V(\pi)] = \sum_i p_i V(\pi_i).$$

The probability distribution function of entry is assumed to be exogenous. This assumption is innocuous for the main results in this paper.

4.2. Dynamics of Firms' Innovation

As shown in the previous section, for a firm with patent portfolio $\{n_{i,t}\}_{i \in \mathcal{T}}$, the increase of the patent stock in sector i for a small time interval Δt is:

$$n_{i,t+\Delta t} - n_{i,t} = \lambda_i \Delta t \sum_{j \in \mathcal{T}} n_{j,t} g_{ji} \quad \forall i \in \mathcal{T} \quad (9)$$

where $n_{i,t}^i - n_{i,t+\Delta t}^i$ is the accumulation of new patents in technology sector i for the time interval Δt . The above equation reveals that the evolution of a firm's patent portfolio is a nested function of the technology network and sectoral innovation rates. To see this, rewrite equation (9) in matrix form:

$$\begin{bmatrix} n_{1,t+\Delta t} - n_{1,t} \\ n_{2,t+\Delta t} - n_{2,t} \\ \vdots \\ n_{M,t+\Delta t} - n_{M,t} \end{bmatrix} = \begin{bmatrix} \lambda_1 g_{11} & \lambda_1 g_{21} & \cdots & \lambda_1 g_{M1} \\ \lambda_2 g_{12} & \lambda_2 g_{22} & \cdots & \lambda_2 g_{M2} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_M g_{1M} & \lambda_M g_{2M} & \cdots & \lambda_M g_{MM} \end{bmatrix} \begin{bmatrix} n_{1,t} \\ n_{2,t} \\ \vdots \\ n_{M,t} \end{bmatrix} \Delta t$$

or more compactly in a continuous form:

$$\dot{n}_t = \Phi n_t$$

where

$$\dot{n}_t = \begin{bmatrix} \dot{n}_{1,t} \\ \vdots \\ \dot{n}_{M,t} \end{bmatrix} \quad \Phi = \begin{bmatrix} \lambda_1 g_{11} & \lambda_1 g_{21} & \cdots & \lambda_1 g_{M1} \\ \lambda_2 g_{12} & \lambda_2 g_{22} & \cdots & \lambda_2 g_{M2} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_M g_{1M} & \lambda_M g_{2M} & \cdots & \lambda_M g_{MM} \end{bmatrix}$$

Matrix Φ captures a version of intersectoral knowledge spillovers which is adjusted by the sectoral innovation intensity. A firm's patent accumulation in any sector depends indirectly on the innovation intensities of other sectors. This is because innovation in one sector provides useful knowledge that

can be used in other sectors. A higher innovation intensity in the i_{th} sector leads to faster accumulation of knowledge in sector i and also contributes to the rate of innovation in sector j since $n_{i,t}g_{ij}$ enters the expression for $n_{j,t+\Delta t} - n_{j,t}$ given that $g_{ij} \neq 0$. Even if $g_{ij} = 0$, as long as the network is irreducible, then there exists a path $(i, l_1, l_2, \dots, l_n, j)$ so that $n_{i,t}$ impacts $n_{j,t+\Delta t} - n_{j,t}$ indirectly. The existence of an irreducible network guarantees that each sector *communicates* with each other.

It is interesting to compare the technology network here and the social development network in [Jackson and Rogers \(2007\)](#). In their paper, the main way for a person to make new friends is by meeting friends of friends. In particular, if a person has a well-connected friend, it will be much easier for the person to make new friends through the local search of her well-connected friend's social network. In our case, new firms enter a certain sector of the economy and accumulate knowledge in that sector. They are more likely to subsequently enter new sectors where they can apply their existing knowledge more efficiently. This is analog to local search in the social networks.

Due to the nature of random innovation, the evolution of a firm's patent portfolio is history dependent. In order to proceed with the analytical analysis, the rest of the paper adopts the mean-field approximation, popularized by [Jackson and Rogers \(2007\)](#). The mean-field approximation assumes that all innovations happen deterministically at the expected rate. Under the mean-field approximation, aggregating individual firms' patent accumulation and taking into account entrants' innovation, we obtain the sectoral patent accumulation as follows:

$$N_{i,t+\Delta t} - N_{i,t} = p_i L_t \eta \Delta t + \lambda_i \Delta t \sum_{j \in \mathcal{T}} N_{j,t} g_{ji} \quad \forall i \in \mathcal{T}, \quad (10)$$

where $N_{i,t}$ is the total number of patents in sector i , L_t is the population of firms at time t and p_i is the probability that a new entrant will enter sector i . The increase of patents in sector i for a short time interval Δt is the result of both entrants' and incumbents' innovations. We are free to aggregate individual firms' knowledge stocks, $n_{i,t}$, to an aggregate knowledge stock, $N_{i,t}$ because of the constant return to scale of patent production. The next proposition characterizes growth of the economy at both the aggregate and sectoral level, where growth is defined as the increase in the stock of knowledge.

Proposition 2. *In the long run, the growth rates of different sectors are the same and are equal to the aggregate growth rate of the whole economy:*

$$\begin{aligned} \dot{N}_t / N_t = \dot{N}_{i,t} / N_{i,t} &= \tau^* \text{ if } \eta < \tau^* \\ \dot{N}_t / N_t = \dot{N}_{i,t} / N_{i,t} &= \eta \text{ if } \eta > \tau^* \end{aligned}$$

where τ^* is the dominant eigenvalue of the matrix Φ .

Proof. See Appendix. □

In an economy where each sector is connected with each other, it is no surprise that all sectors eventually grow at the same rate, as does the whole economy. There are two sources of growth in the economy: both entrants and incumbents contribute to the accumulation of knowledge. On one hand, the dominant eigenvalue of Φ determines how fast knowledge in every sector grows due to incumbents' innovation. On the other hand, entrants in every period bring new ideas to the economy at the rate of population growth. Ultimately, the magnitude of economic growth depends on whether the former dominates the latter or not. The rest of the paper will focus on the theoretical interesting case of $\tau^* > \eta$, namely, the incumbents' contribution to innovation outweighs the entrants.

Proposition 2 sheds light on our understanding of economic growth. It highlights the role of the innovation-adjusted technology network, Φ , in determining economic growth. For an economy without intersectoral knowledge spillovers, economic growth is simply determined by the fastest growing sector. Here every sector contributes to the knowledge accumulation in the economy in two ways. On one hand, each sector applies both intrasectoral and intersectoral knowledge to conduct innovation. Their relative contribution to knowledge accumulation is captured by sector-specific innovation intensities. On the other hand, each sector produces knowledge spillovers to other sectors, and their importance is determined by the technology network structure. It is the interplay of these two forces that determines the growth rate of the economy.

Given the *equal* growth rate of all sectors in our economy, it naturally gives rise to a stationary patent distribution over sectors. To characterize this feature, we first define a vector $Q = (Q_1, Q_2, \dots, Q_M)$, where $Q_i = \lim_{t \rightarrow \infty} \frac{N_{i,t}}{N_t}$ is sector i 's share of total patent stock in the long run. Let $\Delta t = 1$, and rewrite equation (10) as follows:

$$N_{i,t+1} = p_i L_t \eta + N_{i,t} + \lambda_i \sum_{j \in \mathcal{T}} N_{j,t} g_{ji} \quad \forall i \in \mathcal{T} \quad (11)$$

Define

$$\Psi = \begin{bmatrix} 1 + \lambda_1 g_{11} & \lambda_1 g_{21} & \cdots & \lambda_1 g_{M1} \\ \lambda_2 g_{12} & 1 + \lambda_2 g_{22} & \cdots & \lambda_2 g_{M2} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_M g_{1M} & \cdots & \cdots & 1 + \lambda_M g_{MM} \end{bmatrix}$$

Now equation (11) can be written in matrix form:

$$\tilde{N}_{t+1} = L_t \eta \tilde{p} + \Psi \tilde{N}_t \quad (12)$$

where $\tilde{p} = (p_1, p_2, \dots, p_M)$ and $\tilde{N}_t = (N_{1,t}, N_{2,t}, \dots, N_{M,t})'$. Matrix Ψ determines the evolution of the sector size as well as the sector shares. The next proposition summarizes the main findings.

Proposition 3. *The sector shares approach to Q in the long run, which satisfy the following system of equations:*

$$\psi^* Q = \Psi Q$$

where $\psi^* = \tau^* + 1$.

Proof. See Appendix. □

The above proposition provides new insights on the relative size of each sector in equilibrium. Q is the eigenvector associated with the dominant eigenvalue ψ^* of the matrix Ψ . The relative size of each sector is equal to the corresponding entry of the eigenvector. In the network literature, elements in Q are called the *generalized eigenvector centrality*. This is a measure of the importance of nodes in the network. It assigns scores to each node. A higher score implies a more central position of a node. A node gets higher scores if it is connected with other high score nodes. In the context of this paper, the relative size of a sector in the long run is determined by its position in the network, Ψ , which is a slightly modified version of Φ .

Note that the relative sector size here is very different from that in a model without intersectoral knowledge spillovers. For the latter, the largest sector is always the one with the highest innovation rate. In contrast, with intersectoral knowledge spillovers, that may not be the case. A sector that innovates at a slow rate may turn out to have a large size if it enjoys strong knowledge spillovers from other sectors. The following example illustrates this point. Suppose there are 4 sectors in the economy, and the network structure of the economy is represented by a adjacency matrix as follows:

$$G' = \begin{bmatrix} 1 & 0.3 & 0.3 & 0.3 \\ 0.1 & 1 & 0.1 & 0.1 \\ 0.1 & 0.1 & 1 & 0.1 \\ 0.1 & 0.1 & 0.1 & 1 \end{bmatrix}$$

The network structure in the above matrix is such that sector 1 receives the strongest knowledge spillovers in the economy. Now let's assume that the innovation rates for each sector are:

$$\lambda_1 = 0.01, \lambda_2 = 0.011, \lambda_3 = 0.012, \lambda_4 = 0.013,$$

where sector 1 has the lowest innovation rate. Given this information, the stationary sectoral distribution can be easily calculated according to Proposition 3:

$$Q_1 = 0.3236, Q_2 = 0.1727, Q_3 = 0.2193, Q_4 = 0.2844.$$

From the above results, sector 1 is the largest sector in this example. This simple numerical analysis showcases a possibility that a sector can become the dominant sector even if it does not innovate as fast as the other sectors do. Sectors that are capable of using resources from other sectors most efficiently can grow large.

We have provided a simple formula to calculate the long-run economic growth rate. It is natural to ask what factors affect the value of the growth rate. In particular, given our interest in the technology network, it is interesting to explore how changes in the technology network can impact on long-run economic growth. The exercise here is to consider if there were some exogenous changes to the intersectoral technological linkages over the network, how would it affect the long-run economic growth rate? To focus on the role of the technology network, we assume that sectoral innovation rates are constant. The major findings are provided in the following proposition.

Proposition 4. *Given fixed sectoral innovation rates and $(1 + \varepsilon_{Q_{jt}g_{ji}}) > 0 \forall t, i, j$, higher intersectoral knowledge spillovers lead to higher economic growth.*

$$\frac{\partial \tau^*}{\partial g_{ji}} > 0, \quad \forall i \neq j$$

where $\varepsilon_{Q_{jt}g_{ji}}$ is the elasticity of Q_{jt} with respect to g_{ji} .

For any two sectors j and j' where $Q_{j'} > Q_j$, if

$$\lim_{t \rightarrow \infty} Q_{j't}(1 + \varepsilon_{Q_{j't}g_{j'i}}) > \lim_{t \rightarrow \infty} Q_{jt}(1 + \varepsilon_{Q_{jt}g_{ji}})$$

then

$$\frac{\partial \tau^*}{\partial g_{j'i}} > \frac{\partial \tau^*}{\partial g_{ji}}$$

For any two sectors i and i' where $\lambda_{i'} > \lambda_i$, if

$$\lim_{t \rightarrow \infty} \lambda_{i'}(1 + \varepsilon_{Q_{jt}g_{j'i'}}) > \lim_{t \rightarrow \infty} \lambda_i(1 + \varepsilon_{Q_{jt}g_{ji}})$$

then

$$\frac{\partial \tau^*}{\partial g_{ji'}} > \frac{\partial \tau^*}{\partial g_{ji}}$$

Proof. See Appendix. □

Let me provide some intuition for the proposition. The proposition contains three parts. The first part states that increasing the connectedness of the network increases economic growth. The second part deals with the marginal value of strengthening the strength of knowledge spillovers from different sectors. In particular, higher knowledge spillovers from more central sectors have a greater impact on economic growth than those from less central sectors, given that the relative size ranking

is still preserved after the structure change. The last part of the proposition is similar to the second part but it discusses the marginal benefit of higher knowledge spillovers *to* different sectors. The last two arguments highlight that the marginal benefit of increasing knowledge spillovers depends on whether a sector is a knowledge spreader or a knowledge utilizer. As a knowledge spreader, a large sector, which is equivalent to a central sector according to Proposition 3, has more and stronger technological connections with other central sectors, and plays a more important role in the diffusion of knowledge. As a knowledge user, sectors that are more efficient in absorbing and applying cross-sector knowledge are more important.

To have a better understanding of the proposition, we provide a simple stylised example to show how changing the network structure can influence economic growth. Specifically, we construct a 4×4 matrix that represents an economy of 4 sectors, indexed by 1, 2, 3, 4 respectively. We normalize the diagonal elements of the matrix to be 1, implying that the intrasectoral knowledge spillovers are the same over sectors and serve as the reference for intersectoral knowledge spillovers. We randomly assign values to the off-diagonal entries but ensure that they are all smaller than 1 such that the intersectoral spillovers are weaker than intrasectoral spillovers. The resulting matrix is as follows:

$$G^T = \begin{bmatrix} 1 & 0.3485 & 0.5276 & 0.5274 \\ 0.8253 & 1 & 0.8791 & 0.4422 \\ 0.1097 & 0.2406 & 1 & 0.6913 \\ 0.7999 & 0.4789 & 0.8500 & 1 \end{bmatrix}$$

Next, we assume the following innovation rates for each sector

$$\lambda_1 = 0.02, \lambda_2 = 0.04, \lambda_3 = 0.06, \lambda_4 = 0.08.$$

The innovation rate increases monotonically with the index.¹⁴ It transpires that the sector with the highest innovation rate is also the largest sector in equilibrium. Given this setup, we change each off-diagonal element of the network matrix one-at-a-time by adding 0.1 to the original value, then calculate the corresponding economic growth rate. The results are shown in the table below.

The value shown in each entry of the table is the economic growth rate, τ , calculated for each new technology network. For example, 0.1449 in the first row, second column corresponds to the economic growth rate for the technology network which is identical to the benchmark network except that the knowledge spillover from sector 2 to sector 1 is changed from 0.3485 to 0.4485. As a reference, the economic growth rate of the original network is 0.1446.

The table shows results consistent with the proposition. First, note that values in the table are all larger than 0.1446, which implies that an increase of intersectoral knowledge spillovers always

¹⁴The specific values of innovation rates do not matter, our results hold qualitatively as long as the ranking is preserved.

TABLE 3
ECONOMIC GROWTH WITH CHANGING TECHNOLOGY NETWORK

	1	2	3	4
1		0.1449	0.1450	0.1454
2	0.1448		0.1453	0.1463
3	0.1452	0.1460		0.1479
4	0.1453	0.1463	0.1469	

contributes to higher growth. Second, a horizontal examination of the table shows that stronger knowledge spillovers from larger sectors lead to higher economic growth. Last, look at the table vertically. Stronger knowledge spillovers to higher indexed sectors result in higher economic growth. This result reflects the fact that fast-growing sectors are more efficient in absorbing knowledge, so a technology network that places more weight on such sectors is expected to have higher growth.

We have shown how the technology network together with heterogeneous sectoral innovation intensities affects growth and sector share distribution. Next, we will show that the same forces also affect the size distribution of firms.

The size of a firm is measured by the number of patents it holds. Aggregating equation (9) across all sectors gives the dynamics of firm size:

$$n_{t+\Delta t} - n_t = \sum_{i \in \mathcal{T}} \lambda_i \Delta t \sum_{j \in \mathcal{T}} n_{j,t} g_{ji} \quad (13)$$

In the economy, the number of firms grows at the rate η . At time t , the number of firms in the age cohort a is proportional to $e^{\eta(t-a)}$. As a result, the age distribution of firms is an exponential distribution. As time passes, large firms tend to accumulate patents at the same rate, which is equal to the growth rate of the economy. Therefore the size of a firm is a deterministic function of age, which gives rise to a Pareto distribution. The proposition below summarizes the details.

Proposition 5. *The right tail of the accumulative firm size distribution $F(n)$ is given by:*

$$F(n) = 1 - \alpha n^{-\frac{\eta}{\tau^*}}$$

where α is a constant.

Proof. See Appendix. □

The firm size distribution displays a heavy tail. The thickness of the right tail depends on the shape parameter, η/τ^* . A higher growth rate of the firm population, η , leads to smaller firm size

differences across time and thus a thinner right tail. On the other hand, a higher long-run innovation rate, τ^* , results in a higher proportion of large firms and therefore a thicker right tail.

The firm size distribution found in this paper is consistent with the literature ([Gabaix \(2009\)](#)). Firms grow over time at the same rate. The growing population of firms gives rise to different age cohorts, which, combined with the growth of firms, generates different size cohorts of firms. An economic growth rate, τ^* , that is close to the population growth rate, η , will deliver Zipf's distribution. The fundamental linkage between economic growth and the firm size distribution makes the latter subject to the impacts of the same forces that influence the former.

4.3. Reducible Network

We have so far established an innovation model under an irreducible network and explored several impacts of the technology network on the economy. One implication of the irreducible network is that sectors benefit from knowledge spillovers among each other, thus growing at the same rate in the long run. This implication is at odds with heterogeneous growth rates of different sectors observed in the real world. To break this stringent implication, in this section, the assumption that the technology network is irreducible will be relaxed. The key difference between an irreducible network and a reducible network is that, for the latter, technologies in one area need not communicate with technologies in all other areas. In terms of the network structure, this means that the network is more sparse than it was before. Knowledge spillovers in such a network display local effects instead of global effects. The definition below formally defines a reducible network.

Definition 2. A network is reducible if it can be partitioned exclusively into different subnetworks that are irreducible.

A reducible network represents an economy formed of multiple technology clusters. Each technology cluster is comprised of several sectors that are technically related to each other. Denote the i_{th} technology cluster by $\tilde{G}^i$ as follows:

$$\tilde{G}^i = \begin{bmatrix} \tilde{g}_{11}^i & \cdots & \tilde{g}_{1M_i}^i \\ \vdots & \ddots & \vdots \\ \tilde{g}_{M_i1}^i & \cdots & \tilde{g}_{M_iM_i}^i \end{bmatrix}$$

Every technology cluster is an irreducible subnetwork. Let the number of clusters be $\mathcal{N}$, then $\mathcal{N}M_i = M$, where M is the number of sectors. The whole reducible network can be expressed as a matrix:

$$\tilde{G} = \begin{bmatrix} \tilde{G}^1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \tilde{G}^2 & \cdots & \vdots \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \cdots & \tilde{G}^{\mathcal{N}} \end{bmatrix}$$

where $\mathbf{0}$ is a zero matrix. Also define the sectoral innovation-adjusted cluster network as:

$$\tilde{\Phi}^i = \begin{bmatrix} \lambda_1^i \tilde{g}_{11}^i & \cdots & \lambda_1^i \tilde{g}_{1M_i}^i \\ \vdots & \ddots & \vdots \\ \lambda_{M_i}^i \tilde{g}_{M_i1}^i & \cdots & \lambda_{M_i}^i \tilde{g}_{M_iM_i}^i \end{bmatrix}$$

Within each technology cluster, most of the intuition developed previously in the case of an irreducible networks is retained. In particular, economic growth is determined by the technology network of each cluster coupled with sectoral innovation rates within each cluster. In the aggregate, however, different clusters may experience different growth rates and economic growth is driven by the cluster that grows fastest. These findings are summarized by the following proposition.

Proposition 6. *If an economy is represented by a reducible technology network, $\tilde{G}$, then the long-run growth rates of each technology cluster as well as the economy are given by:*

$$\dot{N}_t^i / N_t^i = \tau_i^*$$

$$\dot{N}_t / N_t = \tau^{max} = \max_i \{\tau_i^*\}$$

where τ_i^* is the dominant eigenvalue of $\tilde{\Phi}^i$ and $\tau^{max} = \max_i \{\tau_i^*\}$.

The reducible network structure allows sectors to grow at different rates across clusters but retain convergence within clusters. Note that it is possible for different clusters to grow at the same rate. Nonetheless, the observation that two clusters show the same growth rate does not necessarily imply that their underlying network structures are the same. It could be the case that one cluster has limited knowledge spillovers but higher sectoral innovation rates while the other cluster has strong knowledge spillovers but lower sectoral innovation rates. As an illustration, define two 4×4 matrices as follows:

$$G^1 = \begin{bmatrix} 1 & 0.01 & 0.01 & 0.01 \\ 0.01 & 1 & 0.01 & 0.01 \\ 0.01 & 0.01 & 1 & 0.01 \\ 0.01 & 0.01 & 0.01 & 1 \end{bmatrix} \quad G^2 = \begin{bmatrix} 1 & 0.5 & 0.5 & 0.5 \\ 0.5 & 1 & 0.5 & 0.5 \\ 0.5 & 0.5 & 1 & 0.5 \\ 0.5 & 0.5 & 0.5 & 1 \end{bmatrix}$$

As a result, the whole network can be expressed as:

$$G = \begin{bmatrix} 1 & 0.01 & 0.01 & 0.01 & 0 & 0 & 0 & 0 \\ 0.01 & 1 & 0.01 & 0.01 & 0 & 0 & 0 & 0 \\ 0.01 & 0.01 & 1 & 0.01 & 0 & 0 & 0 & 0 \\ 0.01 & 0.01 & 0.01 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.5 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 1 & 0.5 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 & 1 & 0.5 \\ 0 & 0 & 0 & 0 & 0.5 & 0.5 & 0.5 & 1 \end{bmatrix}$$

Let $\lambda^1 = 0.1$ be the innovation rate for all sectors in cluster 1, and $\lambda^2 = 0.0412$ be the innovation rate for all sectors in cluster 2. It is easy to show that the dominant eigenvalues for both clusters are the same (0.1030), which means that the long-run growth rate for both clusters is the same.

In this example, sectors in cluster 1 are technologically isolated while those in cluster 2 are well connected. However, sectors in both clusters grow at the same rate. They grow due to different reasons. Sectors in cluster 1 grow mainly because they have a high innovation rate, while sectors in cluster 2 grow because they benefit from intersectoral knowledge spillovers. This also has implications for individual firms' growth dynamics. Firms initially rooted in some sector of cluster 1 are more likely to specialize in one area because the benefits of intersectoral knowledge spillovers are limited and firms have less incentive to internalize these benefits by expanding their portfolios. This is not the case for firms entering the second cluster. In this case, there is a higher probability that a firm will develop a more diversified portfolio.

The fact that clusters grow at different rates has natural implications for the dynamics of sector shares. The discrepancy of cluster growth rates leads to the shrinking of the shares of all clusters except the fastest growing cluster.¹⁵ Therefore, there is no stationary sector shares. The shares of all sectors, other than those belong to the fastest growing cluster, vanish in the long run. However, the relative sector sizes within each cluster still approach constant in the long run.

To show the above results formally, we must first introduce some new notations. Define the vector of sector shares in cluster i at time t as $Q_t^i = (Q_{1,t}^i, Q_{2,t}^i, \dots, Q_{M_i,t}^i)$. The aggregate share of a cluster at time t is thus $\hat{Q}_t^i = \sum_h (Q_{h,t}^i)$, and the relative share of sector h within cluster i at time t is $\Pi_t(h|i) = \frac{Q_{h,t}^i}{\hat{Q}_t^i}$. Denote the vector that contains the conditional sector shares within cluster i as Π_t^i . The sector shares within cluster i converge to $\Pi^i = \lim_{t \rightarrow \infty} \Pi_t^i$ in the long run.

Proposition 7. *The conditional sector shares within cluster i , Π^i , satisfy the following system of equations:*

$$\psi_i^* \Pi^i = \Psi^i \Pi^i \quad \forall i \in (1, 2, \dots, \mathcal{N}),$$

¹⁵To simplify the analysis here, it is assumed that the fastest growing cluster is unique.

where ψ_i^* is the dominant eigenvalue of Ψ^i . Except the fastest growing cluster, the shares of all other clusters shrink and eventually vanish:

$$\lim_{t \rightarrow \infty} \hat{Q}_t^i \rightarrow 0, \quad \forall i \neq i_{max}.$$

Comparing Proposition 7 with Proposition 4, the predictions of sector shares differ substantially under different assumptions of the network structures. When the economy is represented by a reducible network, the model predicts divergent sector shares. This is consistent with the real world observation that some sectors are shrinking while others are expanding.

The features of the reducible network also play an important role in shaping the distribution of firm sizes. To see this, note that in a reducible technology network, firms innovate locally within a cluster. They stochastically enter a sector and accumulate knowledge in that sector. They then apply their knowledge to other sectors where they have a comparative advantage. However, they will not be able to expand their portfolios in an unlimited fashion, because knowledge in one technology cluster may not be useful in others. In addition, the network structure of a cluster imposes some restrictions on how valuable each piece of knowledge is on average, thus providing a limit upon how fast a firm can innovate.

Given the impact of different clusters on firm growth, the firm size distribution should be considered conditional on clusters. In particular, there are different firm size distributions over different clusters. Denote the accumulative firm size distribution in cluster i as $F_i(n)$. The following proposition characterizes the tail behavior of these distributions.

Proposition 8. *The right tail of the accumulative firm size distribution $F_i(n)$ for cluster i is given by:*

$$F_i(n) = 1 - \alpha_i n^{-\eta/\tau_i^*} \quad \forall i \in (1, 2, \dots, \mathcal{N})$$

where τ_i^* is the dominant eigenvalue of $\tilde{\Phi}^i$ and α_i is a cluster specific constant.

The firm size distribution stated above differs across clusters regarding the shape parameter $-\eta/\tau_i^*$. The cluster-specific growth rate in principle determines the thickness of the right tail of the firm size distribution. Proposition 8 provides a new angle to think about firm size differences compared to the traditional literature.¹⁶ In the traditional literature, the main mechanism to generate a Pareto-type distribution is the assumption that firms follow a random growth process. In that case, firms in the same age cohort have different sizes due to the realizations of innovation shocks. Firms with good luck may expand and grow, while firms with bad luck shrink over time. In contrast, in our model, firms in the same age cohort can have different sizes if the underlying technologies that

¹⁶See, among others, [Luttmer \(2007, 2011\)](#).

sustain their growth are different. Put differently, the fact that firms specialize in different technology clusters creates a fundamental difference in their growth potential, which, in turn, has an impact on their relative sizes.

This section demonstrates that different network structures have important implications on the long term behaviors of the economy. In next section, a simple numerical analysis is conducted to highlight these differences. The purpose of such an exercise is to offer a visual comparison of the irreducible and reducible network structures.

4.4. Irreducible v.s. Reducible Network: A Numerical Analysis

Two network structures are constructed, an irreducible network and a reducible network. The irreducible network is represented by a 8×8 matrix, whose off-diagonal entries are generated randomly from a uniform distribution with support $(0,0.2)$, while the diagonal entries are normalized to 1.

$$G = \begin{bmatrix} 1 & 0.0844 & 0.1357 & 0.0554 & 0.0877 & 0.1419 & 0.1919 & 0.1782 \\ 0.1930 & 1 & 0.1515 & 0.0092 & 0.0763 & 0.1509 & 0.0681 & 0.1919 \\ 0.0315 & 0.1584 & 1 & 0.0194 & 0.1531 & 0.0552 & 0.1171 & 0.1094 \\ 0.1941 & 0.1919 & 0.0784 & 1 & 0.1590 & 0.1359 & 0.0448 & 0.0277 \\ 0.1914 & 0.1311 & 0.1311 & 0.1390 & 1 & 0.1310 & 0.1503 & 0.0299 \\ 0.0971 & 0.0071 & 0.0342 & 0.0634 & 0.0980 & 1 & 0.0510 & 0.0515 \\ 0.1601 & 0.1698 & 0.1412 & 0.1900 & 0.0891 & 0.0238 & 1 & 0.1681 \\ 0.0284 & 0.1868 & 0.0064 & 0.0069 & 0.1293 & 0.0997 & 0.1398 & 1 \end{bmatrix}$$

The sector-specific innovation rates are drawn from a uniform distribution, and the realization is:

$$\lambda = (0.1629, 0.1812, 0.0254, 0.1827, 0.1265, 0.0195, 0.0557, 0.1094)$$

An initial population of 4000 firms, 500 for each sector, are simulated, following the process specified by equation (9), for a total of 300 periods to generate the dynamics of firm development. In addition, a 4% growth rate of the population is assumed, and every entrant enters a sector following a constant probability distribution function $\tilde{p}$. $\tilde{p}$ is constructed to be equal to the equilibrium sector size distribution, which, in our example, is:

$$\tilde{p} = (0.2695, 0.3127, 0.0140, 0.1436, 0.1059, 0.0119, 0.0350, 0.1075)$$

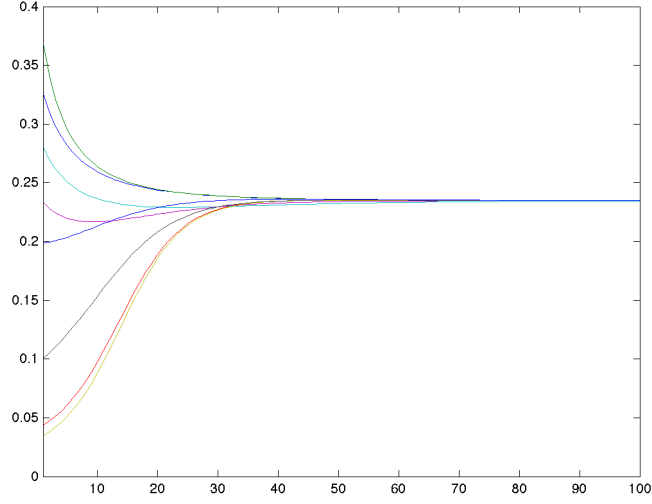
The results below are not dependent on this assumption. The simulation is applied to every generation of firms. The construction of an economy represented by a reducible network is similar. The economy now contains two 4×4 subnetworks in the diagonal blocks, each of which represents a technology cluster. The off-diagonal blocks are replaced with zero matrices. The intersectoral

knowledge spillovers within each cluster of the reducible network are assumed to be the same as those in the irreducible network.

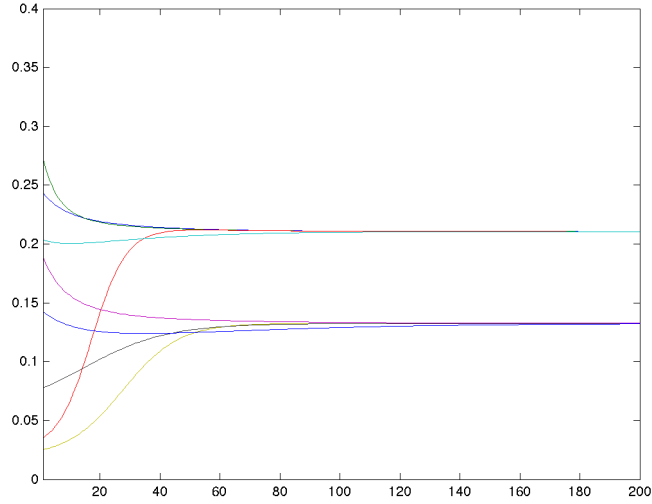
$$G' = \begin{bmatrix} 1 & 0.0844 & 0.1357 & 0.0554 & 0 & 0 & 0 & 0 \\ 0.1930 & 1 & 0.1515 & 0.0092 & 0 & 0 & 0 & 0 \\ 0.0315 & 0.1584 & 1 & 0.0194 & 0 & 0 & 0 & 0 \\ 0.1941 & 0.1919 & 0.0784 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.1310 & 0.1503 & 0.0299 \\ 0 & 0 & 0 & 0 & 0.0980 & 1 & 0.0510 & 0.0515 \\ 0 & 0 & 0 & 0 & 0.0891 & 0.0238 & 1 & 0.1681 \\ 0 & 0 & 0 & 0 & 0.1293 & 0.0997 & 0.1398 & 1 \end{bmatrix}$$

To facilitate a fair comparison, all other parameters are equal to those used before. The simulation is done with the new network structure to generate a panel of firms for the reducible network.

To build intuition, our first exercise here is to compare the long-run growth rate under these two different network structures. Using the simulated panel of firms, we are able to calculate the time series of sectoral growth rates. The results are shown in the figure below.



(a) IRREDUCIBLE



(b) REDUCIBLE

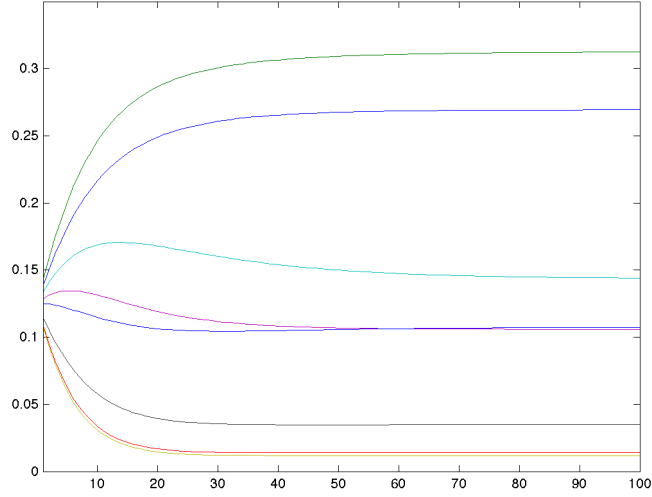
FIGURE 4

IRREDUCIBLE V.S. REDUCIBLE TECHNOLOGY NETWORK: SECTORAL GROWTH

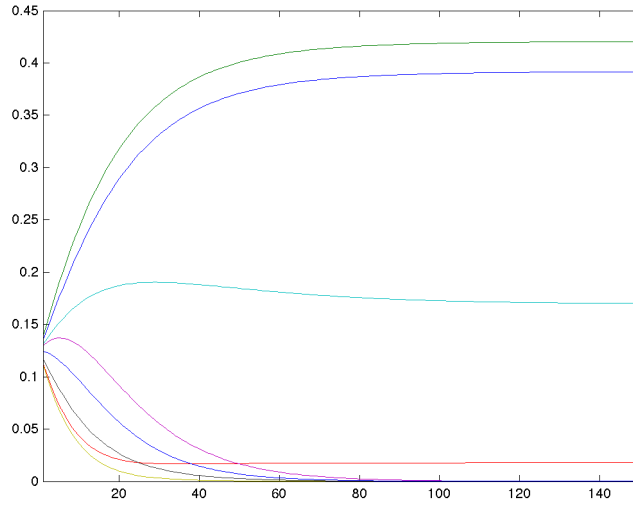
The top panel of the Figure 4 depicts the dynamics of sectoral growth in the irreducible network. All sectors grow at the same rate in the long run, as predicted by Proposition 2. In contrast, sectors in the reducible network are divided into two clusters, as shown in the bottom panel of Figure 4. Sectors converge to the same growth rate in the long run within each cluster but diverge across clusters. As explained in Proposition 6, the divergence of sectoral growth rate in different clusters is driven by the fact that different clusters are isolated from each other and have varying network structures.

Next, we analyse the change of sector share distributions over time in Figure 5. The comparison of the two panels in the figure reveals the significant difference of the sector share evolution. In

the irreducible network, the sector shares approach a fixed number and stay constant in the long run. Nonetheless, in the reducible network, the shares of some sectors asymptotically approach zero, while others remain positive. The discrepancy of the dynamics of the sector shares is a direct result of heterogeneous sectoral growth. Sectors in the slow-growing cluster shrink over time relative to those in the fast-growing cluster.



(a) IRREDUCIBLE



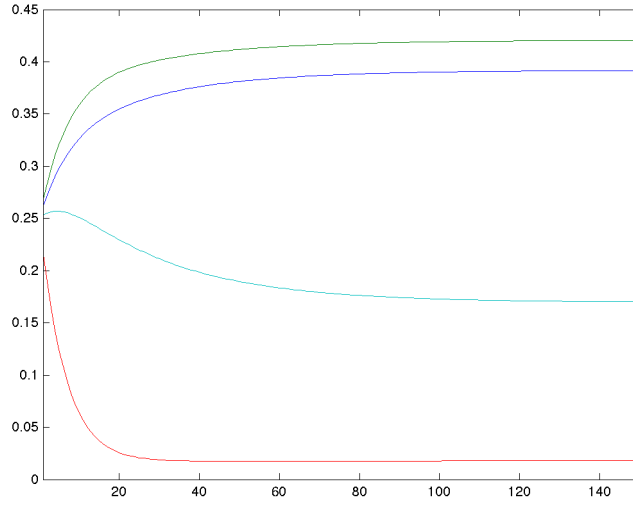
(b) REDUCIBLE

FIGURE 5

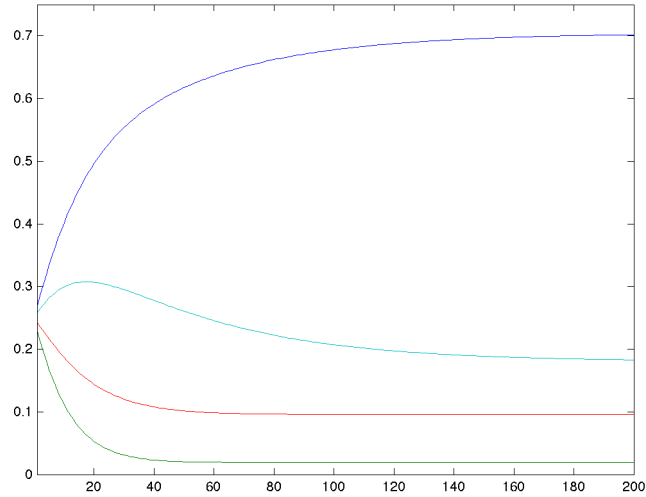
IRREDUCIBLE V.S. REDUCIBLE TECHNOLOGY NETWORK: SECTOR SHARE DISTRIBUTION

Highlighted by Proposition 7, the long-run within-cluster sector share distribution is constant in the reducible network. This feature is shown in FIGURE 6. Panel (a) and (b) in the figure corresponds to the top left and the bottom right blocks of the matrix G' respectively. The observation of the stable

sector share distribution within each cluster implies that sectors in the slow-growing cluster shrink at the same rate relative to sectors in the fast-growing cluster.



(a) CLUSTER 1



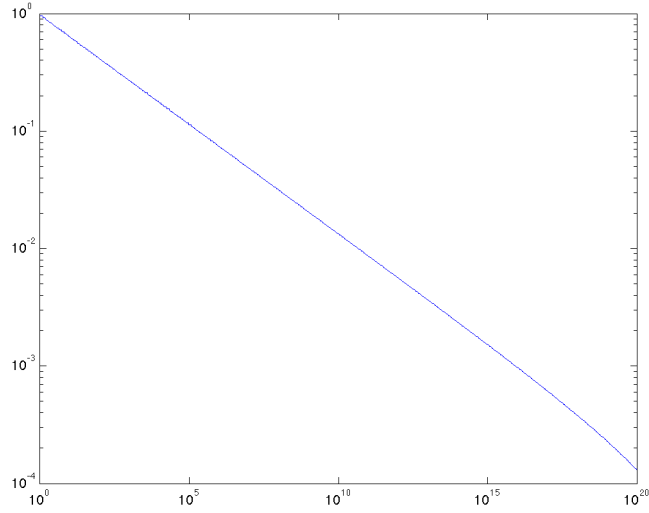
(b) CLUSTER 2

FIGURE 6

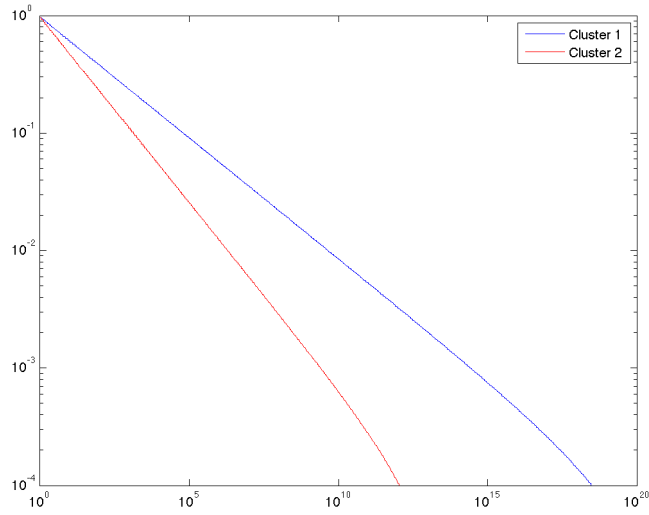
WITHIN CLUSTER SECTOR SHARE DISTRIBUTIONS

Lastly, we compare the firm size distributions in the irreducible and reducible networks in FIGURE 7. Note that the horizontal axis and the vertical axis are both in log scale. The firm size distributions in both cases are clearly log-linear, one notable feature of the Pareto distribution. For the reducible network, the firm size distribution in the two technology clusters differs in terms of the thickness of the right tails. As shown in Proposition 8, the thickness of the right tail is determined by the long-run growth rate of each technology cluster. Consistent with the theory, the firm size

distribution in fast-growing cluster demonstrates a thicker tail.



(a) IRREDUCIBLE



(b) REDUCIBLE

FIGURE 7

IRREDUCIBLE V.S. REDUCIBLE TECHNOLOGY NETWORK: FIRM SIZE DISTRIBUTION

5. POLICY ANALYSIS

The theoretical framework established above enables us to analyze industry-oriented policies and examine its impacts on the aggregate economy. From equation (8), the sectoral innovation rate is a function of a patent's average profit. We consider policies that change the average profitability of firms in a sector so that the innovation intensity of that sector increases by 1%. We consider both

temporary and permanent policy shocks. The former lasts for one period while the latter is permanent once implemented. Although the policy is industry-oriented, due to the existence of knowledge spillovers, a higher innovation intensity in one sector provides additional resources for innovation in other sectors. As a result, industry-oriented policies can generate impacts beyond their targeted sectors.

In order to implement the policy exercises, the parameters that determine the evolution of knowledge accumulation are needed. As a start point, we use the patent citation network as a proxy for the underlying technology network. The backward citation ratio is calculated to represent the strength of knowledge spillovers from the cited sector to the citing sector. We then proceed to calculate the sector-specific entry rate ηp_i . We first calculate the average annual growth rate of firm population from the NBER Patent Citation Data. The average growth rate is for the 11-year period between 1990 and 2000 where the data is most complete. The average probability of sector entry is derived for the same period. We exclude sector 33, Genetics, from the sample because this is a new sector that entered the dataset in 1977 and experienced volatile growth.

The last set of parameters required are the sectoral innovation intensities. They are constructed following equation (10). We calculate the increase in the number of patents per year due to the innovation of incumbents, and subtract that from the total annual increase of patent to get the number of patents from incumbents. Using the technology network, we infer the effective knowledge stocks for each sector which, combined with the previous results, gives rise to the sectoral innovation intensity. The detailed parameter values are shown in TABLE A.4 in the Appendix.

With these parameters, we simulate the dynamics of the economy, starting with 500 firms for each sector, for 800 periods. The temporary policy shock is introduced at period 400. The results are shown in FIGURE 8. The figure depicts the percentage increase in output relative to the status quo case for each sector shock. The horizontal axis represents the sector index. The figure shows significant heterogeneity across sectors. Policy shocks to most sectors have negligible impacts on aggregate output, with only a few exceptions.

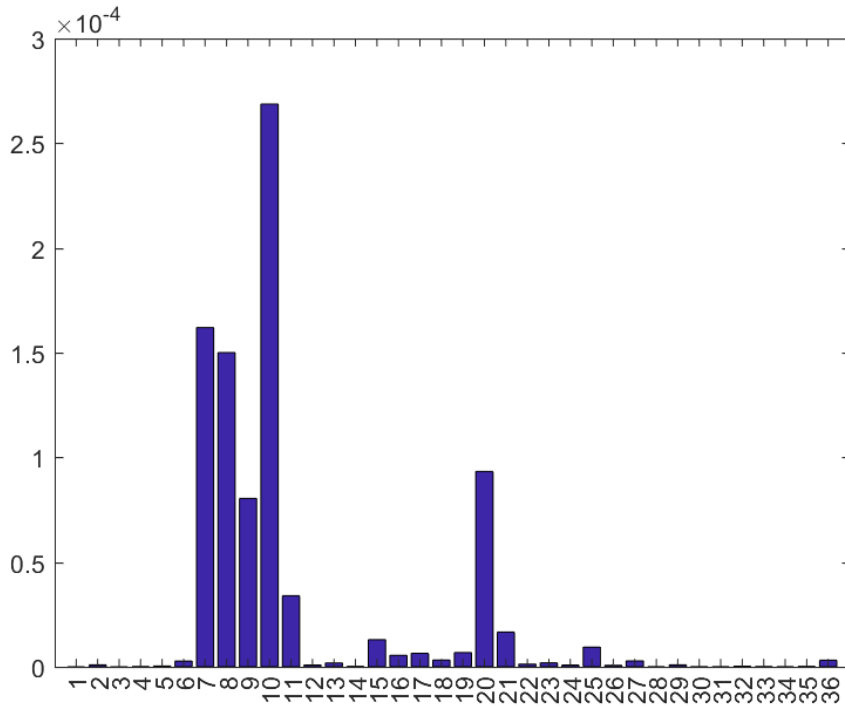


FIGURE 8

THE IMPACTS OF TEMPORARY POLICY SHOCKS AFFECTING EACH SECTOR

We list the top 4 and bottom 4 sectors in TABLE 4. The ranking is consistent with our expectation on which sector is important and which sector is not. For example, Information Storage is the most effective sector to target. A one-period 1% increase of the innovation intensity in this sector leads to an increase in output of 0.27%. By contrast, sectors such as Gas and Agriculture are in the bottom of the ranking. It is no surprise that innovations in the Agriculture sector have little impact on other sectors since there are limited knowledge spillovers originating from the sector.

TABLE 4

SECTOR RESPONSES TO TEMPORARY POLICY SHOCKS: TOP 4 V.S. BOTTOM 4

Top 4 Sectors		Bottom 4 Sectors	
Sector Name	Output Increase (%)	Sector Name	Output Increase (%)
Information Storage	0.027	Apparel & Textile	0.0000089
Communication	0.016	Earth Working & Wells	0.0000073
Computer Hardware & Software	0.015	Gas	0.0000039
Semiconductor Devices	0.0093	Agriculture, Food & Textiles	0.0000023

We now turn to the case of permanent policy shocks. FIGURE 9 shows the effects of permanent policy shocks on economic growth in the long run. The y axis represents the growth rate change due to policy shocks. The values shown in the figure are calculated when the economy converges to a

constant growth rate. Again, industry policies demonstrate considerable differences in their effects on economic growth. Depending on which sector is targeted, the impact of a 1% increase of sectoral innovation intensity on additional economic growth ranges from almost zero up to about 0.32%.

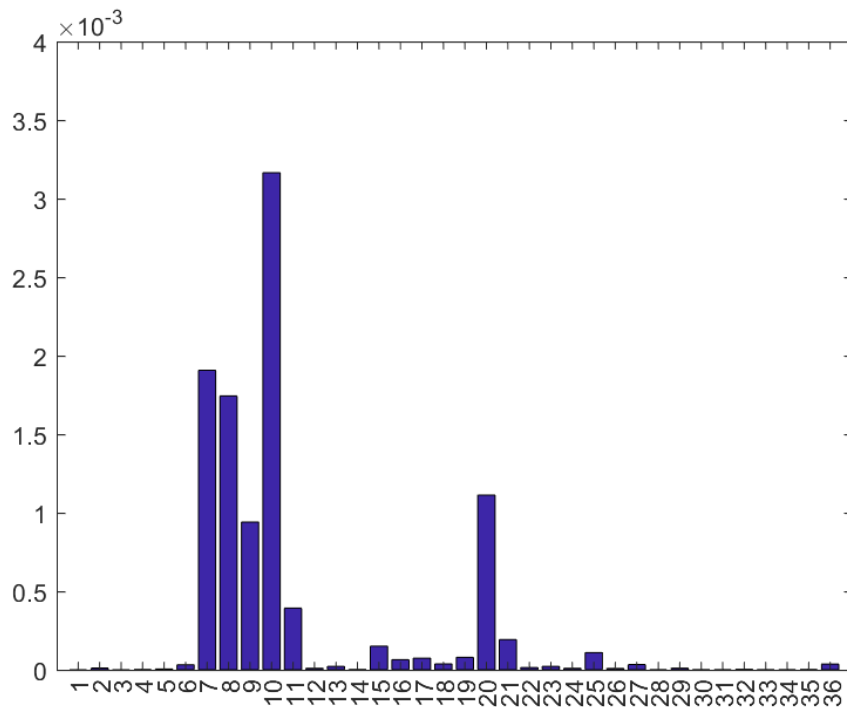


FIGURE 9

THE IMPACTS OF PERMANENT POLICY SHOCKS AFFECTING EACH SECTOR

TABLE 5 provides a counterpart of TABLE 4 for the permanent policy shocks. It is worthy to note that the ranking of top 4 sectors here are the same as the ranking in TABLE 5. This result suggests that to maximize the potential benefit of industry policies, the same set of sectors can be targeted no matter what type of policy is concerned.

TABLE 5

SECTOR RESPONSES TO PERMANENT POLICY SHOCKS: TOP 4 V.S. BOTTOM 4

Top 4 Sectors		Bottom 4 Sectors	
Sector Name	Growth Rate Change (%)	Sector Name	Growth Rate Change (%)
Information Storage	0.32	Apparel & Textile	0.00010
Communication	0.19	Earth Working & Wells	0.000085
Computer Hardware & Software	0.17	Gas	0.000045
Semiconductor Devices	0.11	Agriculture, Food & Textiles	0.000027

For the top 4 sectors in the table, permanent policy shocks have non-trivial effects on stimulating economic growth. For instance, the policy shock to Information Storage increases the economic

growth rate by 0.32%, followed by 0.19% for Communication. As a comparison, the policy shock to the least responsive sector, Agriculture, Food & Textiles, contributes a 0.000027% increase to the final growth rate. Thus, the most effective industry policy can generate an impact which is more than 10,000 times larger than the least effective industry policy.

6. CONCLUSION

The paper provides novel empirical evidence on the importance of intersectoral knowledge spillovers for innovation using the patent citation data from U.S. Patent and Trademark Office. In particular, the paper documents that cross-sector knowledge spillovers are important for an individual firm's innovation in both the intensive margin as well as the extensive margin. In addition, the empirical evidence reveals that sectors differ in their efficiencies in utilizing cross-sector knowledge, reflecting significant sectoral heterogeneity.

We construct a model of innovation on multiple technology sectors, which takes into account the features obtained in the empirical section. The economy as a whole is modeled as a technology network that captures both intrasectoral and intersectoral knowledge spillovers. Firms enter the economy by producing a new patent in a particular sector and then accumulate knowledge to expand their patent portfolios to other sectors. The paper makes theoretical contributions to the understanding of the relationship between the technology network and a range of important economic issues, including economic growth, the sector share distribution and the firm size distribution. Moreover, the paper demonstrates how changes in the network structure can affect the economic growth rate, and how the magnitudes of such impacts depend on the nature of the sectors.

The framework proposed in the paper offers a potential toolkit for policy analysis. We evaluate the effectiveness of industry-oriented policies and compare their impacts on economic growth. Sectors are ranked in terms of their response to policies. The policy exercises show that there are enormous differences across sectors in regard to their economic impacts. The most effective industrial policy can generate a 10,000 times larger impact than the least effective policy.

Appendix

Proposition 1. The value function for the firm's problem has a solution as follows:

$$V(\tilde{\pi}^n) = \sum_{i \in \mathcal{T}} \sum_{s=1}^{n_i} \frac{\pi_{is}}{r} + \sum_{j \in \mathcal{T}} n_j R_j$$

where $(R_1 \cdots R_M)$ are the solutions to a system of M nonlinear equations:

$$r R_j = \sum_i \theta \left(\frac{\bar{\pi}_i + R_i}{\epsilon} \right)^{\frac{1}{\epsilon-1}} g_{ji} \left[\left(1 - \frac{1}{\epsilon} \right) \left(\frac{\bar{\pi}_i}{r} + R_i \right) \right] \quad \forall j \in \mathcal{T}$$

And the optimal innovation choices are given by:

$$\lambda_i = \theta \left(\frac{\bar{\pi}_i + R_i}{\epsilon} \right)^{\frac{1}{\epsilon-1}} \quad \forall i \in \mathcal{T}$$

Proof. Guess the value function $V(\tilde{\pi}_n) = \sum_{i \in \mathcal{T}} \sum_{s=1}^{n_i} a \pi_{is} + \sum_{j \in \mathcal{T}} n_j R_j$. Substitute the guessing value function form into the Bellman equation, then we have:

$$\begin{aligned} r \left(\sum_{i \in \mathcal{T}} \sum_{s=1}^{n_i} a \pi_{is} + \sum_{j \in \mathcal{T}} n_j R_j \right) = \max \left\{ \sum_{i \in \mathcal{T}} \sum_{s=1}^n \pi_{is} - \sum_{i \in \mathcal{T}} c(\lambda_i) \sum_j n_j g_{ji} \right. \\ \left. + \sum_{i \in \mathcal{T}} \lambda_i \sum_{j \in \mathcal{T}} n_j g_{ji} (a \bar{\pi}_i + R_i) \right\} \end{aligned}$$

The above equation holds if and only if:

$$a = \frac{1}{r} \tag{14}$$

$$r \sum_{j \in \mathcal{T}} n_j R_j = \max_{\{\lambda_i\}_i} \left\{ \sum_{i \in \mathcal{T}} \lambda_i \sum_{j \in \mathcal{T}} n_j g_{ji} (a \bar{\pi}_i + R_i) - \sum_{i \in \mathcal{T}} c(\lambda_i) \sum_j n_j g_{ji} \right\} \tag{15}$$

First order condition with respect to λ_i gives:

$$c'(\lambda_i) \sum_j n_j g_{ji} = \sum_j n_j g_{ji} (a \bar{\pi}_i + R_i), \quad \forall i \in \mathcal{T}$$

which can be simplified to be:

$$c'(\lambda_i) = a \bar{\pi}_i + R_i$$

Combined with $a = 1/r$ and the cost function, we have the optimal innovation rate for each sector as follows::

$$\lambda_i^* = \theta \left(\frac{\bar{\pi}_i/r + R_i}{\epsilon} \right)^{\frac{1}{\epsilon-1}}$$

Substitute the optimal innovation rates into (15)

$$r \sum_{j \in \mathcal{T}} n_j R_j = \sum_{i \in \mathcal{T}} \lambda_i^* \sum_{j \in \mathcal{T}} n_j g_{ji} (a \bar{\pi}_i + R_i) - \sum_{i \in \mathcal{T}} c(\lambda_i^*) \sum_j n_j g_{ji}$$

which holds if:

$$rn_j R_j = \sum_i \lambda_i^* n_j g_{ji} (a\bar{\pi}_i + R_i) - \sum_i c(\lambda_i^*) n_j g_{ji}$$

After some algebra, we have:

$$\begin{aligned} rR_j &= \sum_i \lambda_i^* g_{ji} (\bar{\pi}_i/r + R_i) - \sum_i \lambda_i^* \theta^{1-\epsilon} g_{ji} \\ &= \sum_i \theta \left(\frac{\bar{\pi}_i/r + R_i}{\epsilon} \right)^{\frac{1}{\epsilon-1}} g_{ji} (\bar{\pi}_i/r + R_i) - \sum_i \theta^\epsilon \left(\frac{\bar{\pi}_i/r + R_i}{\epsilon} \right)^{\frac{\epsilon}{\epsilon-1}} \theta^{1-\epsilon} g_{ji} \\ &= \theta \sum_i \left(\frac{\bar{\pi}_i/r + R_i}{\epsilon} \right)^{\frac{1}{\epsilon-1}} g_{ji} \left[\bar{\pi}_i/r + R_i - \frac{\bar{\pi}_i/r + R_i}{\epsilon} \right] \\ &= \theta \sum_i \left(\frac{\bar{\pi}_i/r + R_i}{\epsilon} \right)^{\frac{1}{\epsilon-1}} g_{ji} \left[(1 - 1/\epsilon) \left(\frac{\bar{\pi}_i}{r} + R_i \right) \right] \quad \forall j \end{aligned}$$

This is what is required by the proposition. □

Proposition 2. In the long run, the growth rates of different sectors are the same and are equal to the aggregate growth rate of the whole economy:

$$\begin{aligned} \dot{N}_t/N_t &= \dot{N}_{i,t}/N_{i,t} = \tau^* \text{ if } \eta < \tau^* \\ \dot{N}_t/N_t &= \dot{N}_{i,t}/N_{i,t} = \eta \text{ if } \eta > \tau^* \end{aligned}$$

where τ^* is the dominant eigenvalue of the matrix Φ .

Proof. Take the limit of $\Delta t \rightarrow 0$ of equation (10):

$$\begin{aligned} \dot{N}_{i,t} &= L_t \eta p_i + \lambda_i \sum_{j \in \mathcal{T}} N_{j,t} g_{ji} \\ &= L_0 e^{\eta t} \eta p_i + \lambda_i \sum_{j \in \mathcal{T}} N_{j,t} g_{ji} \quad \forall i \end{aligned}$$

Normalize $L_0 = 1$ and the above equations in matrix form:

$$\begin{bmatrix} \dot{N}_{1,t} \\ \dot{N}_{2,t} \\ \vdots \\ \dot{N}_{M,t} \end{bmatrix} = \eta e^{\eta t} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_M \end{bmatrix} + \begin{bmatrix} \lambda_1 g_{11} & \lambda_1 g_{21} & \cdots & \lambda_1 g_{M1} \\ \lambda_2 g_{12} & \lambda_2 g_{22} & \cdots & \lambda_2 g_{M2} \\ \vdots & \vdots & \ddots & \vdots \\ \lambda_M g_{1M} & \lambda_M g_{2M} & \cdots & \lambda_M g_{MM} \end{bmatrix} \begin{bmatrix} N_{1,t} \\ N_{2,t} \\ \vdots \\ N_{M,t} \end{bmatrix}$$

Rewrite the equations in a compact form:

$$\dot{\tilde{N}}_t = \eta e^{\eta t} \tilde{p} + \Phi \tilde{N}_t$$

where

$$\dot{\tilde{N}}_t = \begin{bmatrix} \dot{N}_{1,t} \\ \dot{N}_{2,t} \\ \vdots \\ \dot{N}_{M,t} \end{bmatrix}, \quad \tilde{p} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_M \end{bmatrix}, \quad \tilde{N}_t = \begin{bmatrix} N_{1,t} \\ N_{2,t} \\ \vdots \\ N_{M,t} \end{bmatrix}$$

By the fundamental theorem of Picard and Lindelof, the above system admits a solution as follows:

$$\tilde{N}_t = e^{\Phi t} \tilde{N}_0 + \int_0^t e^{(t-s)\Phi} \eta e^{\eta s} \tilde{p} ds$$

Assume that Φ has distinct real eigenpairs $(\tau_1, V_1), \dots, (\tau_M, V_M)$, then $e^{\Phi t}$ can be decomposed to be as follows:

$$e^{\Phi t} = V e^{Tt} V^{-1}$$

where

$$T = \begin{bmatrix} \tau_1 & 0 & \cdots & 0 \\ 0 & \tau_2 & \cdots & 0 \\ \vdots & & \ddots & \vdots \\ 0 & 0 & \cdots & \tau_M \end{bmatrix}, \quad V = \text{aug}(V_1, V_2, \dots, V_M)$$

With the new notation, we have:

$$\tilde{N}_t = V e^{Tt} V^{-1} \tilde{N}_0 + \int_0^t V e^{(t-s)T} V^{-1} \eta e^{\eta s} \tilde{p} ds$$

Define $\Gamma = V^{-1} \tilde{N}_0$, $H = \eta V^{-1} \tilde{p}$, we have for each sector:

$$\begin{aligned} N_{i,t} &= V_{i1} e^{\tau_1 t} \Gamma_1 + \cdots + V_{iM} e^{\tau_M t} \Gamma_M + \int_0^t e^{\eta s} [V_{i1} e^{\tau_1(t-s)} H_1 + \cdots + V_{iM} e^{\tau_M(t-s)} H_M] ds \\ &= V_{i1} e^{\tau_1 t} \Gamma_1 + \cdots + V_{iM} e^{\tau_M t} \Gamma_M + \sum_{j=1}^M \int_0^t V_{ij} e^{\tau_j t + (\eta - \tau_j)s} H_j ds \\ &= V_{i1} e^{\tau_1 t} \Gamma_1 + \cdots + V_{iM} e^{\tau_M t} \Gamma_M + \sum_{j=1}^M \frac{1}{\eta - \tau_j} V_{ij} e^{\tau_j t} [e^{(\eta - \tau_j)t} - 1] H_j \\ &= V_{i1} e^{\tau_1 t} \Gamma_1 + \cdots + V_{iM} e^{\tau_M t} \Gamma_M + \sum_{j=1}^M \frac{1}{\eta - \tau_j} V_{ij} (e^{\eta t} - e^{\tau_j t}) H_j \end{aligned}$$

Define $\tau^* = \max_i \{\tau_i\}_i$. If $\eta > \tau^*$:

$$\begin{aligned} \lim_{t \rightarrow \infty} N_{i,t} &= \lim_{t \rightarrow \infty} e^{\eta t} \sum_j V_{ij} e^{(\tau_j - \eta)t} \Gamma_j + \sum_{j=1}^M \frac{1}{\eta - \tau_j} V_{ij} e^{\eta t} [1 - e^{(\tau_j - \eta)t}] H_j \\ &= e^{\eta t} \sum_{j=1}^M \frac{V_{ij}}{\eta - \tau_j} H_j \end{aligned}$$

From which we have:

$$\frac{\dot{N}_{i,t}}{N_{i,t}} = \eta, \quad \forall i$$

If $\eta < \tau^*$:

$$\begin{aligned}\lim_{t \rightarrow \infty} &= \lim_{t \rightarrow \infty} e^{\tau^* t} \sum_{j=1}^M V_{ij} e^{(\tau_j - \tau^*)t} \Gamma_j + \sum_{j=1}^M \frac{1}{\eta - \tau_j} V_{ij} e^{\tau^* t} [e^{(\eta - \tau^*)t} - e^{(\tau_j - \tau^*)t}] H_j \\ &= e^{\tau^* t} V_{ii^*} (\Gamma_{i^*} + \frac{V_{i^*}}{\tau^* - \eta} H_{i^*})\end{aligned}$$

and

$$\frac{\dot{N}_{i,t}}{N_{i,t}} = \tau_{i^*}, \quad \forall i$$

Since all sectors grow at the same rate, so does the aggregate growth rate of the economy:

$$\begin{aligned}\frac{\dot{N}_t}{N_t} &= \eta, \quad \text{if } \eta > \tau^* \\ \frac{\dot{N}_t}{N_t} &= \tau^*, \quad \text{if } \eta < \tau^*\end{aligned}$$

□

Proposition 3. The sector shares approach to Q in the long run, which satisfy the following system of equations:

$$\psi^* Q = \Psi Q$$

where $\psi^* = \tau^* + 1$.

Proof.

$$\tilde{N}_{t+1} = L_t \eta \tilde{p} + \Psi \tilde{N}_t$$

Divide both sides of the equation by N_t :

$$\frac{1}{N_t} \tilde{N}_{t+1} = \frac{L_t \eta}{N_t} \tilde{p} + \Psi Q_t$$

Which can be rewritten as:

$$\frac{1}{N_{t+1}} \frac{N_{t+1}}{N_t} \tilde{N}_{t+1} = \frac{L_t \eta}{N_t} \tilde{p} + \Psi Q_t$$

Note that:

$$\begin{aligned}\frac{N_{t+1}}{N_t} &= 1 + \frac{L_t \eta + \sum_i \lambda_i \sum_j N_{j,t} g_{ji}}{N_t} \\ &= 1 + \frac{L_t \eta}{N_t} + \frac{\sum_i \lambda_i \sum_j N_{j,t} g_{ji}}{N_t}\end{aligned}$$

As shown in Proposition 2, as $t \rightarrow \infty$, the innovation from the existing knowledge stocks dominates the innovation from the entrants so we have:

$$\lim_{t \rightarrow \infty} \frac{\sum_i \lambda_i \sum_j N_{j,t} g_{ji}}{N_t} = \tau^*$$

Plug the above result to the previous equation:

$$\frac{N_{t+1}}{N_t} = 1 + \frac{L_t \eta}{N_t} + \tau^*$$

With the result and note the fact that $\frac{1}{N_{t+1}} \tilde{N}_{t+1} = Q_{t+1}$, we have:

$$(1 + \frac{L_t \eta}{N_t} + \tau^*) Q_{t+1} = \frac{L_t \eta}{N_t} \tilde{p} + \Psi Q_t$$

Rearrange the above equation to get:

$$(1 + \tau^*) Q_{t+1} = \frac{L_t \eta}{N_t} (\tilde{p} - Q_{t+1}) + \Psi Q_t$$

Take the limit of $t \rightarrow \infty$ for the above equation:

$$\lim_{t \rightarrow \infty} (1 + \tau^*) Q_{t+1} = \lim_{t \rightarrow \infty} \frac{L_t \eta}{N_t} (\tilde{p} - Q_{t+1}) + \lim_{t \rightarrow \infty} \Psi Q_t$$

Note that $\lim_{t \rightarrow \infty} \frac{L_t \eta}{N_t} = 0$ when $\tau^* > \eta$, so we have:

$$\psi^* Q = \Psi Q$$

Where $\psi^* = 1 + \tau^*$. □

Proposition 4. Given fixed sectoral innovation rates and $(1 + \varepsilon_{Q_{jt}g_{ji}}) > 0 \forall t, i, j$, higher inter-sectoral knowledge spillovers lead to higher economic growth.

$$\frac{\partial \tau^*}{\partial g_{ji}} > 0, \quad \forall i \neq j$$

where $\varepsilon_{Q_{jt}g_{ji}}$ is the elasticity of Q_{jt} with respect to g_{ji} .

For any two sectors j and j' where $Q_{j'} > Q_j$, if

$$\lim_{t \rightarrow \infty} Q_{j't} (1 + \varepsilon_{Q_{j't}g_{j'i}}) > \lim_{t \rightarrow \infty} Q_{jt} (1 + \varepsilon_{Q_{jt}g_{ji}})$$

then

$$\frac{\partial \tau^*}{\partial g_{j'i}} > \frac{\partial \tau^*}{\partial g_{ji}}$$

For any two sectors i and i' where $\lambda_{i'} > \lambda_i$, if

$$\lim_{t \rightarrow \infty} \lambda_{i'}' (1 + \varepsilon_{Q_{jt}g_{j'i'}}) > \lim_{t \rightarrow \infty} \lambda_i (1 + \varepsilon_{Q_{jt}g_{ji}})$$

then

$$\frac{\partial \tau^*}{\partial g_{j'i'}} > \frac{\partial \tau^*}{\partial g_{ji}}$$

Proof.

$$\dot{N}_t = L_t \eta + \sum_i \lambda_i \sum_j N_{j,t} g_{ji}$$

Define the aggregate growth rate as $\mathbf{g}_{N_t} = \frac{\dot{N}_t}{N_t}$, take the derivative of the above equation with respect to g_{ji} :

$$\begin{aligned} \frac{\partial \dot{N}_t}{\partial g_{ji}} &= \lambda_i N_{j,t} + \lambda_i \frac{\partial N_{j,t}}{\partial g_{ji}} g_{ji} \\ \frac{\partial \dot{N}_t}{N_t \partial g_{ji}} &= \lambda_i \frac{N_{j,t}}{N_t} + \lambda_i \frac{\partial N_{j,t}}{N_t \partial g_{ji}} g_{ji} \\ \frac{\mathbf{g}_{N_t}}{\partial g_{ji}} &= \lambda_i Q_{jt} + \lambda_i \frac{\partial Q_{jt}}{\partial g_{ji}} g_{ji} \\ \frac{\mathbf{g}_{N_t}}{\partial g_{ji}} &= \lambda_i Q_{jt} (1 + \varepsilon_{Q_{jt}, g_{ji}}) \end{aligned}$$

Where $\varepsilon_{Q_{jt}, g_{ji}} = \frac{g_{ji}}{\frac{N_{j,t}}{N_t}} \frac{\partial \frac{N_{j,t}}{N_t}}{\partial g_{ji}}$ is the elasticity of Q_{jt} with respect to g_{ji} . Given that $(1 + \varepsilon_{Q_{jt}, g_{ji}}) > 0 \forall t, i, j$, we have:

$$\frac{\partial \mathbf{g}_{N_t}}{\partial g_{ji}} > 0$$

and

$$\frac{\partial \tau^*}{\partial g_{ji}} = \lim_{t \rightarrow \infty} \frac{\partial \mathbf{g}_{N_t}}{\partial g_{ji}} > 0$$

Now take the derivative of $\mathbf{g}_{N_t}$ with respect to $g_{j'i}$:

$$\frac{\partial \mathbf{g}_{N_t}}{\partial g_{j'i}} = \lambda_i \frac{N_{j',t}}{N_t} (1 + \varepsilon_{Q_{j't}, g_{j'i}})$$

Divide $\frac{\partial \mathbf{g}_{N_t}}{\partial g_{j'i}}$ by $\frac{\partial \mathbf{g}_{N_t}}{\partial g_{ji}}$, we have:

$$\frac{\frac{\partial \mathbf{g}_{N_t}}{\partial g_{j'i}}}{\frac{\partial \mathbf{g}_{N_t}}{\partial g_{ji}}} = \frac{N_{j',t} (1 + \varepsilon_{Q_{j't}, g_{j'i}})}{N_{j,t} (1 + \varepsilon_{Q_{jt}, g_{ji}})} = \frac{Q_{j't} (1 + \varepsilon_{Q_{j't}, g_{j'i}})}{Q_{jt} (1 + \varepsilon_{Q_{jt}, g_{ji}})}$$

If

$$\lim_{t \rightarrow \infty} Q_{j't} (1 + \varepsilon_{Q_{j't}, g_{j'i}}) > \lim_{t \rightarrow \infty} Q_{jt} (1 + \varepsilon_{Q_{jt}, g_{ji}})$$

then

$$\frac{\frac{\tau^*}{\partial g_{j'i}}}{\frac{\tau^*}{\partial g_{ji}}} = \lim_{t \rightarrow \infty} \frac{Q_{j't} (1 + \varepsilon_{Q_{j't}, g_{j'i}})}{Q_{jt} (1 + \varepsilon_{Q_{jt}, g_{ji}})} > 1$$

From which:

$$\frac{\partial \tau^*}{\partial g_{j'i}} > \frac{\partial \tau^*}{\partial g_{ji}}$$

Follow the similar approach as above we can show that:

$$\frac{\partial \mathbf{g}_{N_t}}{\partial g_{j'i}} = \lambda_{i'} \frac{N_{j,t}}{N_t} (1 + \varepsilon_{Q_{jt}, g_{j'i'}})$$

and

$$\frac{\frac{\partial \mathbf{g}_{N_t}}{\partial g_{ji'}}}{\frac{\partial \mathbf{g}_{N_t}}{\partial g_{ji}}} = \frac{\lambda_{i'}(1 + \varepsilon_{Q_{j't}g_{j'i}})}{\lambda_i(1 + \varepsilon_{Q_{jt}g_{ji}})}$$

If

$$\lim_{t \rightarrow \infty} \lambda_{i'}(1 + \varepsilon_{Q_{j't}g_{j'i}}) > \lim_{t \rightarrow \infty} \lambda_i(1 + \varepsilon_{Q_{jt}g_{ji}})$$

then

$$\frac{\partial \tau^*}{\partial g_{ji'}} > \frac{\partial \tau^*}{\partial g_{ji}}$$

□

Proposition 5. The right tail of the accumulative firm size distribution $F(n)$ is given by:

$$F(n) = 1 - \alpha n^{-\frac{\eta}{\tau^*}}$$

where α is a constant.

Proof. Conditional on a firm entering sector i at the beginning, start from equation (13) and take the limit of $\Delta t \rightarrow 0$,

$$\dot{n}_t = \sum_i \lambda_i \sum_j n_{j,t} g_{ji}$$

where $n_{i,0} = 1$ and $n_{j,0} = 0 \forall j \neq i$. We already know from Proposition 2 that the aggregate number of patents N_t grows a rate τ^* in the long run. This will be true for individual firms as well because of the constant return to scale of production function.

$$\lim_{t \rightarrow \infty} \dot{n}_t = \tau^* n_t$$

The above equation implies that firm size for the same cohort of firms that enter the same sector at the beginning is a deterministic function of age:

$$n_t = \alpha_i e^{\tau^* t}$$

where α_i is a constant specific to initial sector that firms enter. Revert the above equation to express a firm's age in terms of size:

$$t = \frac{1}{\tau^*} \log \frac{n_t}{\alpha_i}$$

Recall the population of firms grows at the rate η , so the distribution of firm age is an exponential distribution. The proportion of firms older than a is thus:

$$Prob(\text{firms older than } a) = e^{-\eta a}$$

Substitute firm age a in terms of firm size n , we have:

$$Prob(\text{firms larger than } n) = e^{-\frac{\eta}{\tau^*} \log \frac{n}{\alpha_i}} = \left(\frac{n}{\alpha_i}\right)^{-\frac{\eta}{\tau^*}}$$

The accumulative firm size distribution, conditional on firms entering sector i at the beginning, is thus:

$$F_i(n) = 1 - \left(\frac{n}{\alpha_i}\right)^{-\frac{\eta}{\tau^*}}$$

The unconditional firm size distribution is therefore:

$$\begin{aligned} F(n) &= \sum_i p_i F_i(n) \\ &= \sum_i p_i \left(1 - \left(\frac{n}{\alpha_i}\right)^{-\frac{\eta}{\tau^*}}\right) \\ &= 1 - \sum_i p_i \left(\frac{1}{\alpha_i}\right)^{-\frac{\eta}{\tau^*}} n^{-\frac{\eta}{\tau^*}} \\ &= 1 - \alpha n^{-\frac{\eta}{\tau^*}} \end{aligned}$$

where $\alpha = \sum_i p_i \left(\frac{1}{\alpha_i}\right)^{-\frac{\eta}{\tau^*}}$ □

Proposition 6. If an economy is represented by a reducible technology network, $\tilde{G}$, then the long-run growth rates of each technology cluster as well as the economy are given by:

$$\dot{N}_t^i / N_t^i = \tau_i^*$$

$$\dot{N}_t / N_t = \tau^{max} = \max_i \{\tau_i^*\}$$

where τ_i^* is the dominant eigenvalue of $\tilde{\Phi}^i$ and $\tau^{max} = \max_i \{\tau_i^*\}$.

Proof. Within each cluster, the proof is the same as the proof of Proposition 2. Since each cluster is a irreducible network, all sectors within a cluster grow at the same rate, which is determined by the eigenvalue τ_i^* of the network of every cluster Φ_i .

Since different clusters grow at different rate, as time passes on, the fastest growing cluster will dominate and the aggregate growth of the economy is thus determined by the growth of the dominant cluster. □

Proposition 7. The conditional sector shares within cluster i , Π^i , satisfy the following system of equations:

$$\psi_i^* \Pi^i = \Psi^i \Pi^i \quad \forall i \in (1, 2, \dots, \mathcal{N}),$$

where ψ_i^* is the dominant eigenvalue of Ψ^i . Except the fastest growing cluster, the shares of all other clusters shrink and eventually vanish:

$$\lim_{t \rightarrow \infty} \hat{Q}_t^i \rightarrow 0, \quad \forall i \neq i_{max}.$$

Proof. From results in Proposition 6, we know that:

$$N_t^i = c_i e^{\tau_i^* t} \quad \forall i \in (1, \dots, \mathcal{N})$$

$$N_t = c e^{\tau_{max} t}$$

where c_i and c are constants specific to cluster i and the whole economy respectively. Since $\tau_i^* < \tau_{max}$, we have:

$$\lim_{t \rightarrow \infty} \frac{N_t^i}{N_t} = \lim_{t \rightarrow \infty} \frac{c_i}{c} e^{(\tau_i^* - \tau_{max})t} = 0$$

so

$$\lim_{t \rightarrow \infty} \hat{Q}_t^i = \lim_{t \rightarrow \infty} \frac{\sum_h N_{h,t}^i}{N_t} = \lim_{t \rightarrow \infty} \frac{N_t^i}{N_t} = 0 \quad \forall i \neq i_{max}$$

It is easy to see from the above results that sectors other than those in the fastest growing cluster shrink in terms of the relative size. However, we can still explore the relative size distribution of sectors within each cluster. Due to the fact that each cluster consists of an irreducible network, we can examine the within cluster sector size distribution separately for each cluster. Specifically, for cluster i :

$$\tilde{N}_{t+1}^i = L_t \eta \tilde{p}^i + \Psi^i \tilde{N}_t^i$$

Following the similar procedure in the proof of Proposition 3, we have:

$$(1 + \frac{L_t \eta \sum_{h=1}^{M_i} p_h^i}{N_t^i} + \tau_i^*) \Pi_{t+1}^i = \frac{L_t \eta}{N_t} \tilde{p}^i + \Psi^i \Pi_t^i$$

Note that:

$$\lim_{t \rightarrow \infty} \frac{L_t \eta}{N_t} = 0$$

Therefore, when $t \rightarrow \infty$, the above system of equations simplify to:

$$\psi_i^* \Pi^i = \Psi^i \Pi^i$$

where $\psi_i^* = 1 + \tau_i^*$. □

Proposition 8. The right tail of the accumulative firm size distribution $F_i(n)$ for cluster i is given by:

$$F_i(n) = 1 - \alpha_i n^{-\eta/\tau_i^*} \quad \forall i \in (1, 2, \dots, \mathcal{N})$$

where τ_i^* is the dominant eigenvalue of $\tilde{\Phi}^i$ and α_i is a cluster specific constant.

Proof. The proof here follows that in Proposition 5. The population growth rate of firms is the same across clusters, so we have the same firm age distribution over different clusters. The only difference is that we have cluster specific growth rate, so firms in the same age cohort can have different sizes

depending on which cluster they belong to. This feature determines the distribution of firm sizes in different clusters. In particular, $\frac{\eta}{\tau_i^*}$ is the shape parameter for the firm size distribution of each cluster, which give rises to our results in the proposition. $\square$

TABLE A.1
SUMMARY OF TECHNOLOGY CATEGORIES

Category by HJB	Relabel	Name	Number
11	1	Agriculture,Food,Textiles	7808
12	2	Coating	21257
13	3	Gas	6733
14	4	Organic Compounds	49041
15	5	Resins	53567
19	6	Miscellaneous-Chemical	141561
21	7	Communications	101276
22	8	Computer Hardware & Software	86433
23	9	Computer Peripherals	30084
24	10	Information Storage	44174
25	11	Electronic business methods and software	13882
31	12	Drugs	97507
32	13	Surgery & Med Inst.	53455
33	14	Genetics	4480
39	15	Miscellaneous-Drug& Med	9035
41	16	Electrical Devices	52782
42	17	Electrical Lighting	23333
43	18	Measuring & Testing	46449
44	19	Nuclear & X-rays	22562
45	20	Power Systems	51666
46	21	Semiconductor Devices	55702
49	22	Miscellaneous-Elec	32256
51	23	Mat. Proc & Handling	56574
52	24	Metal Working	35482
53	25	Motors & Engines + Parts	36162
54	26	Optics	16837
55	27	Transportation	30674
59	28	Miscellaneous-Mechanical	49743
61	29	Agriculture,Husbandry,Food	21358
62	30	Amusement Devices	8817
63	31	Apparel & Textile	11825
64	32	Earth Working & Wells	21464
65	33	Furniture,House Fixtures	18616
66	34	Heating	13527
67	35	Pipes & Joints	10707
68	36	Receptacles	20888
69	37	Miscellaneous-Others	98724

The first column corresponds to the original numeric technology category defined by Hall, Jaffe and Trajtenberg(HJT). The second column is the relabelled category number in the paper and the last column shows the number of patents in each technology category.

TABLE A.2

PROBIT REGRESSIONS ACROSS SECTORS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$Mproximity_{f,t-1}^x$	49.21***	22.26***	37.60***	17.99***	17.49***	14.97***	14.82***	8.05***
	9.33	2.1	7.93	1.11	1.04	0.51	0.71	0.4
$y_{f,t-1}^x$	1.78***	1.23***	1.51***	1.74***	1.61***	1.52***	1.62***	1.30***
	0.1	0.04	0.08	0.05	0.04	0.03	0.04	0.03
$N_{f,t-1}$	0.01*	0.03***	0.01	0	0.01*	-0.02***	0.01	0.01
	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
<i>YearDummy</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Firm-sector FE</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Constant</i>	-3.05***	-2.61***	-2.96***	-2.66***	-2.70***	-1.86***	-2.37***	-2.49***
	0.07	0.05	0.07	0.05	0.05	0.04	0.04	0.04
Observations	55,194	55,194	55,194	55,194	55,194	55,194	55,194	55,194

TABLE A.2 Continued

	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
$Mproximity_{f,t-1}^x$	23.54***	18.07***	20.08***	8.75***	12.86***	233.11***	32.34***	23.42***
	1.78	1.26	1.76	0.45	0.97	24.73	2.92	1.63
$y_{f,t-1}^x$	1.49***	1.71***	1.28***	2.30***	2.12***	2.04***	1.96***	1.48***
	0.06	0.06	0.06	0.05	0.05	0.1	0.08	0.04
$N_{f,t-1}$	0.01	0	0.02**	-0.01*	-0.01	0.03***	0.01	0.01
	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
<i>YearDummy</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Firm-sector FE</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Constant</i>	-3.00***	-2.87***	-3.29***	-2.48***	-2.23***	-3.50***	-2.82***	-2.49***
	0.06	0.06	0.08	0.05	0.05	0.12	0.07	0.05
Observations	55,194	55,194	55,194	55,194	55,194	55,194	55,194	55,194

TABLE A.2 Continued

	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
$Mproximity_{f,t-1}^x$	39.98***	14.24***	24.61***	20.92***	14.13***	47.67***	26.08***	43.16***
	4.66	1.28	2.82	1.38	1.33	3.07	1.18	2.42
$y_{f,t-1}^x$	1.57***	1.20***	1.35***	1.44***	1.74***	1.36***	1.41***	1.36***
	0.05	0.03	0.05	0.04	0.06	0.05	0.03	0.04
$N_{f,t-1}$	0.03***	0.03***	0.03***	0.02***	0.03***	0.02***	0.01*	0.02***
	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
<i>YearDummy</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Firm-sector FE</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Constant</i>	-2.69***	-2.18***	-2.47***	-2.48***	-2.93***	-2.52***	-1.99***	-2.26***
	0.06	0.04	0.05	0.05	0.06	0.05	0.04	0.04
Observations	55,194	55,194	55,194	55,194	55,194	55,194	55,194	55,194

TABLE A.2 Continued

	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)
$Mproximity_{f,t-1}^x$	27.63***	23.76***	49.76***	34.52***	43.78***	182.38***	75.24***	21.80***
	1.88	2.66	3.07	1.52	3.25	17.95	7.24	2.52
$y_{f,t-1}^x$	1.63***	1.52***	1.95***	1.52***	2.07***	2.32***	1.84***	2.15***
	0.05	0.06	0.05	0.04	0.06	0.13	0.07	0.07
$N_{f,t-1}$	0.01	0.04***	0	-0.01	-0.01**	0.01	0.02***	-0.01
	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
<i>YearDummy</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Firm-sector FE</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Constant</i>	-2.50***	-2.88***	-2.63***	-1.98***	-2.54***	-2.97***	-2.72***	-2.64***
	0.05	0.07	0.06	0.04	0.06	0.1	0.06	0.06
Observations	55,194	55,194	55,194	55,194	55,194	55,194	55,194	55,194

TABLE A.2 Continued

	(33)	(34)	(35)	(36)	(37)
$Mproximity_{f,t-1}^x$	41.57***	80.25***	93.52***	44.97***	15.37***
	2.78	8.48	9.01	2.89	0.52
$y_{f,t-1}^x$	1.76***	1.60***	1.37***	1.62***	1.40***
	0.05	0.07	0.06	0.05	0.03
$N_{f,t-1}$	-0.01	0.02**	0.02***	-0.01*	-0.01**
	0.01	0.01	0.01	0.01	0.01
<i>YearDummy</i>	Yes	Yes	Yes	Yes	Yes
<i>Firm-sector FE</i>	Yes	Yes	Yes	Yes	Yes
<i>Constant</i>	-2.47***	-2.73***	-2.87***	-2.30***	-1.70***
	0.05	0.06	0.06	0.05	0.03
Observations	55,194	55,194	55,194	55,194	55,194

The results in this table are based on the estimation of Probit regression (2) for each sector. Note that there is no sector dummies here since the regression itself is sector-specific.

TABLE A.3
NEGATIVE BINOMIAL REGRESSIONS ACROSS SECTORS

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$Wpatent_{f,t-1}^x$	0.58***	0.42***	0.46***	0.65***	0.55***	0.24***	0.40***	0.66***
	0.05	0.03	0.05	0.03	0.03	0.02	0.03	0.03
$I(x_{f,t-1} = 1)$	2.89***	1.86***	2.33***	1.77***	2.15***	1.70***	2.34***	1.70***
	0.2	0.1	0.16	0.1	0.08	0.05	0.08	0.07
$N_{f,t-1}$	-0.01	0.02**	0.01	-0.02***	-0.02***	0.02***	-0.02**	-0.02**
	0.02	0.01	0.01	0	0.01	0	0.01	0.01
$PatentCounts_{f,t-1}^x$	0.11***	0.07***	0.07	0.07***	0.10***	0.11***	0.09***	0.08***
	0.04	0.02	0.06	0.01	0.01	0.01	0.01	0.01
<i>YearDummy</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Firm-sector FE</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Constant</i>	-2.32***	-2.26***	-2.84***	-1.36***	-1.79***	-1.62***	-2.00***	-1.87***
	0.27	0.12	0.23	0.09	0.09	0.05	0.1	0.08
$ln\alpha$	0.81***	0.19**	0.88***	-0.26***	0.16**	-0.15***	0.36***	0.26***
	0.26	0.09	0.15	0.1	0.07	0.05	0.07	0.07
Observations	28,437	28,385	28,435	28,294	28,253	27,898	28,041	28,134

TABLE A.3 Continued

	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)
$Wpatent_{f,t-1}^x$	0.59***	0.60***	0.72***	0.32***	0.12***	0.63***	0.61***	0.42***
	0.06	0.06	0.07	0.02	0.03	0.03	0.04	0.04
$I(x_{f,t-1} = 1)$	2.59***	2.87***	1.86***	3.18***	2.92***	2.14***	2.31***	2.28***
	0.15	0.21	0.21	0.09	0.12	0.22	0.19	0.08
$N_{f,t-1}$	-0.02*	-0.04***	-0.02*	-0.04***	0.02**	-0.02*	-0.04***	-0.03***
	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
$PatentCounts_{f,t-1}^x$	0.07***	0.07***	0.07**	0.09***	0.09***	0.17**	0.18***	0.11***
	0.02	0.01	0.03	0.01	0.01	0.07	0.04	0.01
<i>YearDummy</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Firm-sector FE</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Constant</i>	-2.54***	-2.37***	-2.60***	-2.35***	-3.02***	-2.17***	-2.63***	-1.89***
	0.18	0.21	0.32	0.11	0.11	0.27	0.2	0.13
$\ln\alpha$	0.73***	0.70***	0.57**	-0.23***	0.14	0.65***	0.84***	0.33***
	-0.12	0.13	0.27	0.07	0.11	0.2	0.14	0.09
Observations	28,282	28,240	28,383	27,820	28,058	28,195	28,399	28,225

TABLE A.3 Continued

	(17)	(18)	(19)	(20)	(21)	(22)	(23)	(24)
$Wpatent_{f,t-1}^x$	0.30***	0.26***	0.33***	0.45***	0.39***	0.32***	0.19***	0.26***
	0.05	0.03	0.05	0.03	0.03	0.04	0.02	0.03
$I(x_{f,t-1} = 1)$	2.57***	1.89***	2.10***	1.93***	2.89***	1.93***	1.75***	1.87***
	0.11	0.07	0.11	0.07	0.14	0.1	0.06	0.08
$N_{f,t-1}$	0.03**	0.03***	0.04***	0	0.01	0.03***	0.02***	0.03***
	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
$PatentCounts_{f,t-1}^x$	0.04**	0.07***	0.06***	0.10***	0.09***	0.10***	0.12***	0.07***
	0.02	0.01	0.02	0.01	0.02	0.02	0.01	0.02
<i>YearDummy</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Firm-sector FE</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Constant</i>	-2.69***	-2.09***	-2.39***	-1.99***	-3.02***	-2.44***	-2.10***	-2.19***
	0.24	0.1	0.18	0.1	0.14	0.14	0.08	0.11
$\ln\alpha$	0.70***	0.13*	0.46***	0.32***	0.51***	0.59***	0.12*	0.25***
	0.14	0.08	0.14	0.07	0.12	0.1	0.06	0.09
Observations	28,369	28,286	28,371	28,254	28,226	28,325	28,224	28,331

TABLE A.3 Continued

	(25)	(26)	(27)	(28)	(29)	(30)	(31)	(32)
$Wpatent_{f,t-1}^x$	0.36***	0.43***	0.18***	0.22***	0.19***	0.09	0.21***	0.25***
	0.03	0.04	0.03	0.03	0.03	0.09	0.05	0.04
$I(x_{f,t-1} = 1)$	2.42***	2.40***	2.88***	1.83***	3.03***	4.49***	2.63***	3.65***
	0.09	0.14	0.12	0.07	0.13	0.54	0.2	0.15
$N_{f,t-1}$	-0.01	0.03**	0.02**	0	0.01	0.08***	0.04***	0.01
	0.01	0.01	0.01	0.01	0.01	0.02	0.01	0.01
$PatentCounts_{f,t-1}^x$	0.10***	0.08***	0.08***	0.11***	0.10***	0.04	0.13**	0.11***
	0.02	0.02	0.02	0.01	0.03	0.13	0.06	0.02
<i>YearDummy</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Firm-sector FE</i>	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
<i>Constant</i>	-2.21***	-2.75***	-3.24***	-2.00***	-3.04***	-5.70***	-3.43***	-3.45***
	0.13	0.17	0.15	0.09	0.16	0.46	0.25	0.18
$\ln\alpha$	0.41***	0.48***	0.66***	0.34***	0.53***	1.32***	1.08***	0.32*
	0.1	0.16	0.14	0.07	0.18	0.43	0.16	0.19
Observations	28,318	28,393	28,300	28,246	28,337	28,412	28,382	28,352

TABLE A.3 Continued

	(33)	(34)	(35)	(36)	(37)
$Wpatent_{f,t-1}^x$	0.26*** 0.03	0.21*** 0.04	0.36*** 0.04	0.32*** 0.03	0.14*** 0.02
$I(x_{f,t-1} = 1)$	2.27*** 0.13	2.51*** 0.14	1.88*** 0.15	2.06*** 0.1	1.53*** 0.05
$N_{f,t-1}$	0.01 0.01	0.04*** 0.01	0.01 0.01	-0.01 0.01	0.02*** 0.01
$PatentCounts_{f,t-1}^x$	0.07** 0.03	0.05 0.04	0.09* 0.05	0.10*** 0.02	0.11*** 0.01
<i>YearDummy</i>	Yes	Yes	Yes	Yes	Yes
<i>Firm-sector FE</i>	Yes	Yes	Yes	Yes	Yes
<i>Constant</i>	-2.79*** 0.16	-3.19*** 0.22	-2.70*** 0.2	-2.15*** 0.13	-1.76*** 0.05
$\ln \alpha$	0.82*** 0.09	0.71*** 0.15	0.69*** 0.13	0.49*** 0.1	-0.18*** 0.05
Observations	28,386	28,422	28,427	28,387	28,134

The results in this table are based on the estimation of Negative Binomial regression (3) for each sector. Note that there is no sector dummies here since the regression itself is sector-specific.

TABLE A.4
SECTORAL ENTRY PROBABILITY AND INNOVATION INTENSITY

Name	Entry Probability	Innovation Intensity
Agriculture,Food,Textiles	0.001078	0.056423
Coating	0.003607	0.069459
Gas	0.001191	0.044832
Organic Compounds	0.002537	0.076724
Resins	0.003671	0.069619
Miscellaneous-Chemical	0.020506	0.062229
Communications	0.012182	0.105481
Computer Hardware & Software	0.010279	0.095151
Computer Peripherals	0.002868	0.108225
Information Storage	0.003202	0.121202
Electronic business methods and software	0.004119	0.097338
Drugs	0.014888	0.088518
Surgery & Med Inst.	0.011166	0.076118
Miscellaneous-Drug& Med	0.00267	0.65657
Electrical Devices	0.004401	0.08409
Electrical Lighting	0.003373	0.083668
Measuring & Testing	0.006807	0.072112
Nuclear & X-rays	0.002453	0.078219
Power Systems	0.005826	0.074447
Semiconductor Devices	0.00184	0.10364
Miscellaneous-Elec	0.005005	0.086706
Mat. Proc & Handling	0.013088	0.058572
Metal Working	0.006256	0.067428
Motors & Engines + Parts	0.005509	0.063773
Optics	0.002104	0.081174
Transportation	0.006851	0.062832
Miscellaneous-Mechanical	0.011863	0.065531
Agriculture,Husbandry,Food	0.006891	0.054318
Amusement Devices	0.00407	0.072859
Apparel & Textile	0.00418	0.057728
Earth Working & Wells	0.004055	0.039236
Furniture,House Fixtures	0.006618	0.063958
Heating	0.003005	0.048228
Pipes & Joints	0.00248	0.061579
Receptacles	0.006259	0.06207
Miscellaneous-Others	0.026874	0.066039

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