

Dynamic Compensation under Uncertainty Shocks and Limited Commitment^{*}

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Abstract

This paper studies dynamic compensation and risk management when firms face cash flow volatility shock. Back-loaded compensation with a penalty upon the arrival of the shock is used to incentivize effort and prudence from managers. Thus, implications of the volatility shock depend critically on firms' ability to commit to future compensation: firms with full commitment power impose high pay-performance sensitivity and large penalties to implement low risk, and defer compensation more when volatility becomes higher. In contrast, firms with limited commitment ability optimally allow excessive risk-taking from managers in exchange for a low pay-performance sensitivity; they expedite compensation when volatility is higher because commitment to future payments becomes less feasible. These predictions shed light on empirical observations particularly the controversial compensation practices during the recent financial crisis.

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“It is apparently going to take more than shrinking bank profits to put a big dent in Wall Street bonuses.”

—*The New York Times*, February 29th, 2012

1 Introduction

Contemporary firms rely heavily on incentive pay, which constitutes the majority of overall firm cost as well as total employee compensation. Extant research on managerial compensation usually focuses on its relationship with profitability, yet the very foundation of incentive pay stems from the uncertainty of how profitability reflects a manager’s true effort. Moreover, with growing research identifying uncertainty shocks as the key contributing factor to financial crises and economic downturns,¹ and the mounting public attention scrutinizing compensation practices during those times, the need to understand theoretically how compensation and uncertainty interact with each other has critical value for both academics as well as policy makers.

The current paper takes a step towards such goal. In a continuous-time dynamic principal-agent model, a manager is hired to run a firm, whose cash flow is subject to volatility shocks. The manager privately chooses the project productivity and its risk, that is, the arrival probability of the volatility shock. The manager’s preference for not working hard means he must be given sufficient incentives to achieve high productivity and low risk. The contract uses promised delayed compensation, or continuation utility, as incentives. Continuation utility increases if the realized cash flow is high *and* the volatility shock does not occur, and decreases if the opposite happens. The manager is paid with cash bonuses after his cumulative performance exceeds a certain hurdle (i.e. the bonus hurdle) and is fired if performance deteriorates sufficiently much.

I show that, absent of any other contracting friction, the optimal contract incentivizes *low* risk for managers with good performance history. When the volatility shock arrives, the contract *raises* the bonus hurdle so that the manager is less likely to receive bonuses

¹See for instance [Bloom \(2009\)](#), [Di Tella \(2013\)](#), [He and Krishnamurthy \(2013\)](#), [Atkeson et al. \(2014\)](#) and [Brunnermeier and Sannikov \(2014\)](#).

unless he can produce series of strong performance. In other words, the contract prescribes more *back-loaded* compensation in the high uncertainty state. Intuitively, when cash flow uncertainty is higher, the manager’s continuation utility becomes more volatile. It is then optimal to defer compensation more so that the manager is neither paid too easily or fired too early. Large deferred compensation also means the contract can impose sufficiently large penalty when the volatility shock occurs, allowing the implementation of low risk.

This benchmark theoretical prediction, however, does not reconcile with certain empirical findings on how managerial compensation varies with firm risk. For example, [Peters and Wagner \(2014\)](#) show that higher industry-level equity volatility is strongly associated with *higher* current compensation, while [Cheng et al. \(2015\)](#) find a similar relationship for firms in the financial sector. In particular, the higher compensation in riskier firms mainly takes the form of incentive pay such as cash bonuses.² The financial sector’s behavior in the recent crisis is also at odds with the benchmark result. Despite being in a period of heightened uncertainty, some firms actually *front-loaded* their employees’ compensation and expedited payments.³ Overall, huge losses of company wealth notwithstanding, many bankers and executives who were accused responsible for leading their firms into financial turmoil still received substantial amount of cash bonuses and other incentive pays.⁴ Unsurprisingly, such practices raised much public attention and scholarly debate over the efficiency of the current compensation structure.

In light of these observations, I re-examine the implication of uncertainty on compensation, and argue that the discrepancy between the benchmark predictions and the actual observations stems from an overlooked but critical market friction: the firms’ ability to fully commit to long-term incentive contracts. Although often assumed in theoretical studies of contracts, complete commitment is generally difficult to maintain in practice given the prevalence of at-will employment.⁵ Importantly, the concern over firms’ commitment vary

²[Frydman and Jenter \(2010\)](#) show that performance insensitive compensation (e.g., salary) makes up less than 20% of top managers’ total compensation.

³For example, Bank of America announced in 2009 that it would accelerate the vesting period of their employees’ stocks from 2012 to 2011, claiming that such action is “critical for retention of talent” during volatile times. Deloitte reported instances of financial firms lowering performance hurdles during the crisis so employees would still be entitled to bonuses. See [Forbes Insights \(2009\)](#) and [Management Today \(2010\)](#).

⁴See [The Wall Street Journal \(2009\)](#). Moreover, [Kaplan and Rauh \(2010\)](#) and [Philippon and Reshef \(2012\)](#) show that the high-level incentive payments are not exclusive to a handful of managerial elites.

⁵Under US labor laws, firms can fire employees without having to establish just cause or give warning.

with the level of uncertainty, and it is likely the greatest when uncertainty is high and firm value is low.

I find the theoretical predictions of the model drastically different if firms has only *limited commitment*, that is, if they can unilaterally terminate contracts any time. Limited commitment restricts firms' ability to use deferred compensation as incentive. As a result, commitment-strapped firms are forced to *lower* their bonus hurdles such that their managers are more likely to receive cash bonuses. Meanwhile, limited commitment exacerbates the cost of incentive provision, leading firms to optimally forgo prudent risk management in order to lower the incentive needed to maintain high productivity. In other words, under limited commitment, firms may intentionally allow a high risk exposure to the volatility shock in exchange for a lower pay-performance sensitivity; when the shock arrives, they offer managers more *front-loaded* compensation and pay cash bonuses immediately after small amounts of good performance. The managers are, however, simultaneously subject to higher probability of turnover, and the overall effect of the volatility shock on their continuation utility is still negative, implying that those bonuses are not to be regarded as "rewards" for their excessive risk-taking behaviors.

Contributions of this paper are two-fold: on the academic side, to my best knowledge, it is the first to jointly consider dynamic moral hazard and risk management in the presence of volatility shock and limited commitment. It bridges several strand of literature that studies a subset of those topics. First, it follows DeMarzo and Sannikov (2006), Biais et al. (2007), Sannikov (2008), and Zhu (2013) in modeling the cash flow process and the moral hazard problem.⁶ Second, the volatility shock is modeled as regime-switching of model parameters, applying the technique of Hoffmann and Pfeil (2010), Piskorski and Tchisty (2010), Narita (2011), Li (2012) and DeMarzo et al. (2012). However, these studies assume exogenous probability of the regime-switch, whereas my paper features endogenous probability and

Firms can also liquidate anytime, after which they are no longer liable for any future compensation promised to employees.

⁶It thus differs from most of the long-standing limited commitment literature, which focuses on the optimal risk sharing contract between a risk-neutral principal and a risk-averse agent. In such an environment, termination is never part of an optimal compensation scheme. By contrast, I study a setting where a risk-neutral agent is protected by limited liability. In this environment, incentives are optimally provided in part through the threat of contract termination. Given the high frequency of managerial turnover in financial firms, this is a critical feature of my study.

hence endogenous risk-taking incentives. Third, the interaction of dynamic risk management and moral hazard of this paper shares similarities with DeMarzo et al. (2013) and Wong (2017). A central message of those studies is that incentives to eliminate moral hazard increases the agent’s desire for excessive risk-taking and, as a result, high effort and efficient risk management may not be simultaneously feasible. In turn, the incentive to prevent excessive risk-taking in my model leads to stronger incentives required to eliminate moral hazard. Consequently, the optimal contract may forgo efficient risk management even when it is jointly feasible with implementing high effort.⁷ Finally, all of the aforementioned studies assume full commitment power of the principal, while my paper demonstrates the sharp contrast of implications of different levels commitment power.

In terms of policy implication, results of this paper serve as a caveat to the popular perception that the high compensation observed in the financial crisis is a sign of managerial entrenchment and suboptimal contracts. In the aftermath of the recent financial crisis, many blamed the managerial compensation scheme in place for not aligning managers’ interests with those of investors, and hence the policy recommendations to propagate the use of delayed payment as a solution.⁸ However, the effectiveness of such recommendation hinges on the credibility that future payment promised to managers will be delivered at full value. If managers believe that when firms are in distress, investors will withdraw their investment by selling shares, they would worry more about firm’s long-term commitment ability and require even higher and more immediate compensation at the time of distress. Such compensation also implies limited penalty for risk-taking behaviors and therefore may expose firms to *ex-ante* even higher risks.

The paper proceeds as the following: Section 2 describes the main model and the optimal contract. Section 3 derives the model implications and discusses their empirical relevance. Section 4 explores several extensions. Section 5 concludes. All proofs are in the Appendix.

⁷A recent study by Li and Williams (2017) demonstrates numerical examples where costly risk management could also lead to optimally forgone productivity to shirking. Details of such difference are further elaborated in Section 3.

⁸For instance, the Troubled Asset Relief Program (TARP) limits the ability of the executives of TARP firms to cash out their restricted stock until the government is repaid in full. See TARP Standards for Compensation and Corporate Governance, 74 Fed. Reg. 28,394, 28,410 (June 15, 2009).

2 Model

In this section I describe the model. I start with a principal-agent environment with volatility regime switching and solve for the optimal contract, assuming the principal has full commitment power. Then, I discuss the effect of considering the principals limited commitment and the different optimal contract it implies.

2.1 Basic Environment

Time is continuous. A principal (she), representing a firm, must hire an agent, representing a manager (he), to run a project. Both the principal and the agent are risk neutral. The cash flow Y_t of the project follows

$$dY_t = (\mu - s_t)dt + \sigma_t dZ_t, \quad (1)$$

where Z_t is a standard Brownian motion. The agent controls the cash flow growth rate via a binary variable $s_t \in \{0, S\}$ ($S > 0$), representing “working” and “shirking”, or high and low productivity, respectively. Shirking results in lower expected cash flow, but yields a positive personal benefit (specified later) for the agent only. The principal can observe Y_t but not s_t . She discounts future cash flows by r and the agent by $\gamma > r$, i.e. the agent is more impatient.⁹ The agent is also protected by limited liability with an outside option whose value is normalized to 0. The principal has an outside option $\underline{V} > 0$.¹⁰

The volatility of the cash flow, σ_t , is stochastic, taking one of two values: σ_l or σ_h , respectively representing “normal” or “crisis” time, with $\sigma_l < \sigma_h$. For simplicity, I assume the economy starts with σ_l , and transitions to σ_h whose arrival is characterized by a Poisson (jump) process N_t . The σ_h state is absorbing (i.e. no further jump back to σ_l), but the main results of this paper remain true if states were recurring, which is discussed in the Appendix.

⁹The asymmetry in the discount rates is a standard, essential requirement for a non-trivial incentive-compatible contract to exist in this type of model. However, once I impose the principal’s commitment constraint, this additional constraint leads to the existence of an optimal contract even for the case where $r = \gamma$, which I describe at the end of this section.

¹⁰One can interpret contract termination here as a firm’s liquidation and exit from the market permanently or, equivalently, as a firm’s replacement of managers, where $\underline{V}$ is the normalized net profit from contracting with a new manager.

Because there is at most one state transition in equilibrium, I refer to it as “the volatility shock”. This shock is publicly observable and thus can be contracted on.

While the exact timing of the volatility shock is exogenously, its arrival intensity is an endogenous choice of the agent. I assume the arrival intensity is $\pi + a$, where $\pi > 0$, and $a \in \{0, A\}$ is a binary choice representing a “safe” action and a “risky” action, respectively. The agent chooses a privately, and he derives a personal benefit from the “risky” action. Because the volatility shock arrives with probability $\pi > 0$ even when $a = 0$, the principal cannot directly infer the agent’s risk choice. Therefore, a can be understood as risk management, and $a = A$ is *excessive* risk-taking that is undesirable to the principal but can only be eliminated via the agent’s costly effort (in the form of forgone personal benefit).¹¹ Finally, because the σ_h state is absorbing, there is no risk management choice available in that state.

In sum, the agency problem (in the low volatility state) is *two-dimensional*: the level of shirking/effort s and the level of risk management a . I assume the agent’s personal benefit, denoted as $B(s, a)$, has the following properties:

Assumption 1. $B(0, 0) = 0$; $B(S, 0) = \lambda S$, $\lambda \in (0, 1)$; $B(0, A) = \delta A$, $\delta \in (0, 1)$; and

$$\max\{\lambda S, \delta A\} < B(S, A) < \lambda S + \delta A$$

In words, the agent prefers not to “work hard” in general, whether the work is to maintain a high level of productivity ($s = 0$), or a low level of risk ($a = 0$). He derives no private benefit from the $(s = 0, a = 0)$ action profile, and enjoys maximal private benefit if he shirks *and* allows excessive risks. However, the personal leisure from shirking and not managing risk are (imperfect) substitutes. If the action space is continuous, this is equivalent to the private benefit function being jointly concave in the two choice variables.¹² This is economically reasonable: concave private benefits are necessary conditions for a well-defined socially optimal choice of actions.¹³

¹¹One can also interpret π as the systemic risk, and a as the firm’s idiosyncratic risk. The idiosyncratic risk can be eliminated but it requires the manager’s due diligence, which is personally costly to him.

¹²A example of $B(s, a)$ that satisfies Assumption 1 is $B(s, a) = \sqrt{(\lambda s)^2 + (\delta a)^2}$

¹³Alternatively, one can write the model in terms of costly effort, and the assumption would be that the cost function is jointly convex: the marginal cost of effort maintaining high productivity is increasing with the level of effort exerted on maintaining low risk.

A key result to be derived in this paper is that although both shirking and excessive risk-taking are socially suboptimal, it may not be always optimal for the principal to prevent the agent from taking both action. To reduce the complexity of the optimal contract, for most part of the paper I impose the following assumption:

Assumption 2. *S is sufficiently large such that the principal always prefers to incentivize high productivity, or $s = 0$.*

That is, I limit the attention to contracts without shirking in equilibrium.¹⁴ This is a standard simplification in most studies of dynamic moral hazard problem, particularly those that also involve risk management such that DeMarzo et al. (2013) and Wong (2017). Later in Section 4, I extend the model and demonstrate that if S is moderate, then higher volatility may cause the principal to optimally implement shirking, which provides additional justification for interpreting the high volatility state as the “crisis” state.

2.2 Incentive Compatibility

To find the optimal contract, it is necessary to establish the incentive compatibility (IC) conditions. Formally, let $(\Omega, \mathcal{F}, P)$ denote the probability space and $\{\mathcal{F}_t\}_{t \geq 0}$ the filtration generated by the cash-flow history $\{Y_t\}_{t \geq 0}$ and the state transition history $\{N_t\}_{t \geq 0}$. A contract specifies a non-decreasing payment process $\{C_t\}_{t \geq 0}$ to the agent, a stopping time τ when the contract is terminated, and a sequence of recommended actions $\{s_t\}_{t \geq 0}, \{a_t\}_{t \geq 0}, \{C_t\}_{t \geq 0}$.¹⁵ Given the contract, the agent chooses a given set of actions $\{\hat{s}_t, \hat{a}_t\}_{t \geq 0}$. His objective function is the expected discounted value of consumption plus private benefits

$$W_t^{\hat{s}, \hat{a}} = E_{\hat{s}, \hat{a}} \left[\int_t^\tau e^{-\gamma(u-t)} (dC_u + B(\hat{s}_u, \hat{a}_u) du) \middle| \mathcal{F}_t \right], \quad (2)$$

As in the previous literature, W_t is referred to as the agent’s continuation utility. I define a contract to be *incentive compatible* if $(s_t, a_t) = (\hat{s}_t, \hat{a}_t)|_{t \geq 0}$. The following Proposition

¹⁴The precise lower bound for S depends on other parameter values. However, as it becomes clearly after Proposition 3, if this assumption is true, the principal’s payoff under the optimal contract does not depend on S , which can then be assumed to be arbitrarily large. This assumption also rule out complicated contracts near the boundaries (defined later), which is the focus of Zhu (2013).

¹⁵All quantities are assumed to be integrable and measurable under the usual conditions.

describes the dynamics of W_t and the IC condition:

Proposition 1. *Given any contract and any sequence of the agent's choices, there exist predictable, finite processes β_t ($0 \leq t \leq \tau$) and ϕ_t ($0 \leq t \leq \tau$) such that, in the low volatility state, W_t evolves according to*

$$dW_t = \gamma W_t dt - B(s_t, a_t) dt - dC_t + \beta_t (dY_t - (\mu - s_t) dt) - \phi_t (dN_t - (\pi + a_t) dt) \quad (3)$$

The incentive compatibility condition is

$$\{s_t, a_t\} = \arg \max_{\hat{s}_t, \hat{a}_t} B(\hat{s}_t, \hat{a}_t) - \beta_t \hat{s}_t - \phi_t \hat{a}_t \quad (4)$$

If $(s = 0, a = 0)$ is implemented, then

$$dW_t = \gamma W_t dt - dC_t + \beta_t \sigma_l dZ_t - \phi (dN_t - \pi dt) \quad (5)$$

If $(s = 0, a = A)$ is implemented, then

$$dW_t = \gamma W_t dt - \delta A dt - dC_t + \beta_t \sigma_l dZ_t - \phi (dN_t - (\pi + A) dt) \quad (6)$$

dW_t and the IC condition in the high volatility state follow (3) and (4) with $\phi_t = a_t = 0$.

Equation (3) can be derived using standard martingale methods. The first three terms on its right hand side reflect dynamic promise keeping. The last two term reflect the agent's incentives. β_t is the agent's pay-performance sensitivity, or PPS: for every dollar of extra cash flow $(dY_t - (\mu - s_t) dt)$, the agent's continuation utility changes by β_t dollars. ϕ_t is the adjustment of agent's continuation utility when the volatility shock occurs. If $\phi > 0$ it acts as a punishment to the agent. Before the volatility shock hits, the agent is compensated with $(\pi + a_t)\phi_t$ to keep W a martingale.

The IC condition (4) is the solution to a maximization over the agent's flow payoff. Given any β and ϕ , define $\underline{w} = \beta\mu - \phi\pi$ as a benchmark utility, the agent's set of available actions and their corresponding payoff can be summarized in the following two-by-two matrix:

Agent's action space and instantaneous payoff

	$a = 0$ (low risk)	$a = A$ (high risk)
$s = 0$ (working)	$\underline{w}$	$\underline{w} + \delta A - \phi A$
$s = S$ (shirking)	$\underline{w} + \lambda S - \beta S$	$\underline{w} + B(S, A) - \beta S - A\phi$

For the contract to implement any (s, a) combination from the matrix above, it must be that the agent does not derive higher payoff from any other combinations. This thus allow me to derive the required incentive that eliminates shirking in equilibrium (Assumption 2) while implementing the desirable level of risk, summarized as the following:

Proposition 2. $\{s_t = 0, a_t = 0\}$ are implemented if and only if

$$\beta_t \geq \lambda, \quad \phi_t \geq \delta \quad (\text{IC: } a = 0)$$

and $\{s_t = 0, a_t = A\}$ are implemented if and only if

$$\beta_t \geq b \equiv \max \left\{ \lambda - (\delta - \phi) \frac{A}{S}, \frac{B(S, A) - \delta A}{S} \right\}, \quad \phi_t < \delta \quad (\text{IC: } a = A)$$

In particular, $\phi < \delta$ implies $b < \lambda$

Intuitively, because the agent enjoys private benefit from shirking and from excessive risk-taking, to implement $(s = 0, a = 0)$, the contract must provide the agent sufficient skin-in-the-game ($\beta \geq \lambda$) and punish the agent with a sufficiently large reduction in his continuation utility ($\phi \geq \delta$) whenever the volatility shock occurs. In contrast, to implement $(s = 0, a = A)$, the principal cannot punish the agent too much ($\phi < \delta$).

Importantly, if the contract allows high risk, the minimal degree of PPS (β) needed to prevent shirking is lower than that needed when the contract incentivizes low risk. That is because the agent's marginal private benefit from shirking is λ when $a = 0$ and $(B(S, A) - \delta A)/S < \lambda$ when $a = A$. In other words, since shirking and neglecting risk management are somewhat substitutes to the agent, by freeing him from the risk management duty, the principal effectively reduces the agent's marginal utility from shirking. This result plays a critical role in later analysis about which level of risk the optimal contract implements.

2.3 Optimal Contract with Full Commitment

I now proceed to solve the optimal contract, first assuming with the principal has full power of commitment. The principal's objective function is the expected discounted value of the cash flow minus payments to the agent, with where the expectations taken under the probability measure associated with the agent's choices $(\hat{s}, \hat{a})$:

$$V_t^{\hat{s}, \hat{a}} = E_{\hat{s}, \hat{a}} \left[\int_t^\tau e^{-r(u-t)} (dY_u - dC_u) + e^{-r\tau} \underline{V} \middle| \mathcal{F}_t \right]. \quad (7)$$

The *optimal contract* is defined as a contract that (1) maximizes the principal's objective function over the set of contracts, (2) satisfies the IC conditions, and (3) grants the agent non-negative initial level of utility W_0 and $W_t^{s,a}$. Because there are two volatility states, there are two value functions for the principal, both are functions of the agent's continuation utility W .

2.3.1 High Volatility State

I begin with characterizing the value function and the optimal contract in the high volatility state since there is no more state transition and no risk management.

Proposition 3. *The optimal contract in the high volatility state is summarized by a concave, twice differentiable value function $V_h(W)$ defined on $W \in [0, W_h^C]$ and solves the principal's Hamiltonian-Jacobian-Bellman (HJB) equation:*

$$rV_h(W) = \max_{\beta \geq \lambda} \left[\mu + \gamma W V_h'(W) + \frac{1}{2} \beta^2 \sigma_h^2 V_h''(W) \right] \quad (8)$$

W_h^C is chosen such that $V_h'(W_h^C) = -1$ and $V_h''(W_h^C) = 0$. Payments to the agent $dC_t = \max\{W_t - W_h^C, 0\}$. The contract terminates when $W_t = 0$, at which point $V_h(0) = \underline{V}$.

Such optimal contract is the same as that in [DeMarzo and Sannikov \(2006\)](#). Nevertheless, several features are worth mentioning: first, because the value function is concave, and β only appears on the $V_h''(W) < 0$ term, it is optimal to choose $\beta = \lambda$ always. Intuitively, β is set at the minimal level allowed by the IC condition because the cost of preventing shirking is captured by the likelihood of the contract hitting the termination boundary $W = 0$ (i.e. the

likelihood of a costly separation). Conditional on the contract being incentive compatible, the principal prefers the lowest possible variability in dW , which comes from the lowest possible level of PPS. This is a key mechanism in play throughout the model.

The second feature worth discussing is the payment boundary W_h^C . It is a reflecting boundary such that once crossed, an instant payment $dC_t = W_t - W_h^C$ is made to bring W_t back to W_h^C . Thus, while C_t can be interpreted as bonuses to the agent after sufficiently good history of performance, W_h^C can be conveniently interpreted as a *bonus hurdle*. This hurdle is optimally set based on two conditions, a “smooth-pasting” condition $V'_h(W_h^C) = -1$ and a “super-contact” condition. $V''_h(W_h^C) = 0$.¹⁶ Substituting those boundary conditions back to the HJB equation yields

$$rV_h(W_h^C) + \gamma W_h^C = \mu \quad (9)$$

Intuitively, the principal delays cash payments until the expected cash flow is fully distributed between her and the agent. I thus refer to (9) as the “optimal payment frontier”.¹⁷ The position of W_h^C in relation to this frontier relies crucially on the principal having full commitment power, and thus will be re-examined later under limited commitment.

2.3.2 Low Volatility State

The optimal contract in the low volatility state is substantially more complicated. It takes into account the volatility shock and its arrival intensity which is a private choice of the agent. The following proposition characterizes the contract:

Proposition 4. *The optimal contract in the low volatility state is summarized by a concave, twice differentiable value function $V_l(W)$ defined on $W \in [0, W_l^C]$ and solves the principal’s*

¹⁶Specifically, “smooth-pasting” exists because the principal can always make a lump-sum payment of dC to the agent, moving the agent from W to $W - dC$ and changing the principal’s value by $V(W - dC) - dC$; “super-contact” exists since W_C is optimally chosen and the principal is risk-neutral.

¹⁷This frontier is “second-best” compared to the “first-best” frontier $r(V + W) = \mu$. The efficiency loss from the second-best frontier stems from the fact that the agent is impatient and discount his promised utility by $\gamma > r$.

HJB equation:

$$rV_l(W) = \max_{\beta, \phi, a} \mu + [\gamma W - \delta a + (\pi + a)\phi]V_l'(W) + \frac{1}{2}\beta^2\sigma_l^2V_l''(W) + (\pi + a)[V_h(W - \phi) - V_l(W)] \quad (10)$$

subject to the IC condition (*IC: $a = 0$*) if $a = 0$, or (*IC: $a = A$*) if $a = A$. W_l^C is chosen such that $V_l'(W_l^C) = -1$ and $V_l''(W_l^C) = 0$. Payments to the agent $dC_t = \max\{W_t - W_l^C, 0\}$. The contract terminates when $W = 0$, at which point $V_l(0) = \underline{V}$.

The contract in the low volatility state has one major difference from that in the high volatility state: instead of one control β , the principal now has two additional control ϕ and a . Compared to $V_h(W)$ defined in (8), the additional terms $[-\delta a + (\pi + a)\phi]V_l'(W)$ comes from the promise keeping terms in dW (equation (3)), and the additional term $(\pi + a)[V_h(W - \phi) - V_l(W)]$ reflects the expected change in principal's valuation due to the volatility shock.

The optimal choices of β , ϕ , and a naturally depend on the state variable W .¹⁸ The following set of properties summaries how they are determined. First, as in the case of high volatility state, β only appears on the $V_l'' < 0$ term. Therefore, it is optimal to set β to its minimal value allowed by the IC condition:

Property 1. β is always chosen such that either (*IC: $a = 0$*) or (*IC: $a = A$*) binds. That is $\beta = \lambda$ if $a(W) = 0$, and $\beta = b \equiv \max\left\{\lambda - (\delta - \phi)\frac{A}{S}, \frac{B(S, A) - \delta A}{S}\right\} < \delta$ if $a(W) = A$.

Next, the optimal choice of ϕ depends on three conditions: first, implementing low risk requires $\phi \geq \delta$, while implementing high risk requires $\phi < \delta$; secondly, the agent's limited liability restricts $\phi \geq W$. Lastly, if neither the IC constraint or the limited liability constraint is binding, then ϕ satisfies its first order condition from (10). That is, ϕ is set by matching the marginal value of agent's continuation utility to the principal before and after the volatility shock. These conditions are summarized as the following:

Property 2. Define $\phi_1(W)$ as the solution to $V_h'(W - \phi_1) = V_l'(W)$, $\phi_2(W)$ as the solution to $V_h'(W - \phi_2) = V_l'(W) + Ab\sigma_l^2F''(W)/S$,¹⁹ and $\hat{\phi} = (B(S, A) - \lambda S)/A$. Then:

¹⁸Thus, they should always be understood as concise representation of $\beta(W), \phi(W), a(W)$.

¹⁹Provided such ϕ_1 and ϕ_2 exists within $[0, W_l^C]$.

- (1) If $a(W) = 0$, then $\phi(W) = \max\{\delta, W, \phi_1\}$
(2) If $a(W) = A$, then $\phi = \phi_1$ if $\phi_1 \in [W, \hat{\phi}]$, and $\phi = \phi_2$ if $\phi_2 \in [\hat{\phi}, \delta]$. Otherwise,
 $\phi = \min\{\delta, W\}$

Finally, the optimal level of risk management a depends simply on comparing the payoff from $a = 0$ with $a = A$.

Property 3. *The contract implement low risk $a(W) = 0$ if and only if $W \geq \delta$ and:*

$$\frac{1}{2}\lambda^2\sigma_l^2V_l''(W) \geq (\phi - \alpha)AV_l'(W) + \frac{1}{2}b^2\sigma_l^2V_l''(W) + A[V_h(W - \phi) - V_l(W)] \quad (11)$$

with ϕ defined according to the rules in Property 2.

The left-hand-side is the cost of implementing low risk ($V_l''(W) < 0$). As stated in Proposition 2, incentivizing lower risk requires stronger incentives ($\beta = \lambda$) to prevent shirking. The right-hand-side is the cost of implementing right risk, which is comprised of three parts: first, the $(\phi - \alpha)A$ term coming from the drift of dW in equation (6); secondly, the cost of incentive to prevent shirking, which is now lower than that on the left-hand-side ($\beta = b < \lambda$); third, the increasing probability of the volatility shock and the loss of firm value $V_h(W - \phi) - V_l(W)$.

To sum up, the optimal contract when the principal has full commitment power is a pair of value functions $V_l(W)$ and $V_h(W)$, both defined when W is between two boundaries: the termination boundary, and the payment boundary; the latter can also be interpreted as a bonus hurdle because cash payments are made once W exceeds such boundary. β is set to the minimal value required to prevent shirking. ϕ is set based on a slope-matching condition, subject to the IC constraint and the agent's limited liability.

An example of the optimal contract is illustrated in Figure 1. Note that the value function in the high volatility state is lower for any given W , which is one reason the high volatility state can be interpreted as the “crisis” state. High volatility implies lower firm value because more volatile cash flows increase the likelihood of contract termination which is a necessary but costly action for the principal to provide proper incentives. Another important observation is the the payment boundaries is higher in the high volatility state

($W_h^C > W_l^C$). This result is examined in more depth in Section 3 and is subject to significant changes when limited commitment is introduced.

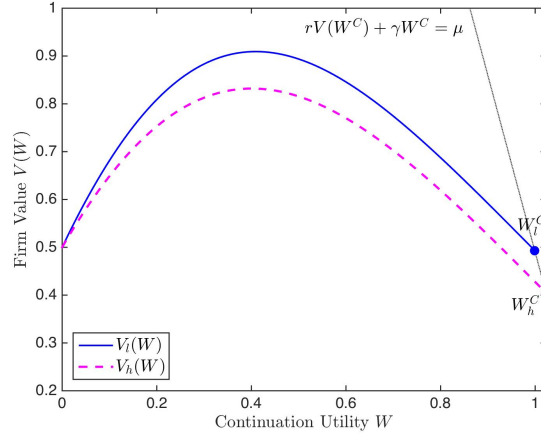


Figure 1 – Optimal Contract: Full Commitment

This figure plots firm value functions under different cash flow volatility σ_s and with full commitment on the principal's side. The parameter values are $\underline{V} = 0.5$, $\gamma = 0.04$, $r = 0.02$, $\mu = 0.5$, $\lambda = 0.2$, $\sigma_l = 0.28$, $\sigma_h = 0.306$, $\pi = 0.001$, and $\delta = 0.1$. W_l^C and W_h^C indicate the payment boundaries (i.e. bonus hurdles) in low and high volatility states, respectively. $rV(W^C) + \gamma W^C = \mu$ indicates the “optimal payment frontier”.

2.4 Optimal Contract with Limited Commitment

So far, the structure of the optimal contract relies on the principal's commitment to all future payments once the contract is signed. However, before the agent's continuation utility W hits the payment boundary W^C , the agent is not actually paid. His continuation utility measures the present value of the total amount of payment he expects to receive in the future, only if the principal honors the contract. Just as the agent is tempted to quit his job when W approaches his reservation utility R , the principal will likewise be tempted to exercise her outside option, which, in this model, is liquidating the project and receiving $\underline{V}$ if firm value V drops below the liquidation value before W reaches the payment boundary. If enforcement is not perfect and commitment becomes a binding constraint before the cash payment boundary is reached, the optimal contract will consequently be different.

To consider the impact of principal's commitment while maintaining tractability, I consider a simple yet standard form of limited commitment: the principal can terminate the

contract anytime.²⁰ This is quite reasonable given firms generally are free to fire managers or liquidate projects at any time in practice. Once the contract is terminated, both parties receive the value of their outside options: $\underline{V}$ for the principal and 0 for the agent.²¹

The limited commitment introduced implies a participation constraint for the principal:

$$V_s \geq \underline{V} \quad \forall s \in \{l, h\} \quad (12)$$

Considering only the high volatility state for a moment, the principal's value function must therefore lie in the area bounded by $W \geq 0$, $V \geq \underline{V}$ and the “optimal payment frontier” (9). Consequently, the boundary condition of the payment boundary W^C depends on whether the value function crosses the “optimal payment frontier” $rV(W) + \gamma W = \mu$ or $V(W) = \underline{V}$ first. If the value function meets former first, the “super-contact” condition holds, and $V''(W^C) = 0$ is still true. In that case, the principal's commitment constraint is not binding. If the value function reaches $V(W) = \underline{V}$ first, the principal cannot delay cash payment to the agent as much as she wants without violating her participation constraint. As a result, the “super contact” condition is replaced with a physical boundary condition $V(W^C) = \underline{V}$.²²

Let $\tilde{X}$ represent a variable X in the limited commitment environment. The following proposition summarizes the optimal contract:

Proposition 5. *The optimal contract under volatility regime switching with limited commitment on the principal's side defines a pair of value functions $\tilde{V}_j(W)$ and payment boundaries $\tilde{W}_j^C$, $j \in \{l, h\}$, such that $\tilde{V}_j(W)$ satisfies the same system of ODEs (8) and (10), and*

²⁰In other words, the principal cannot commit to *not* to terminate the contract unless $W = 0$.

²¹Conditional on the continuation of the contract, the principal can still commit to all payments once the payment boundary is reached. More precisely, the principal can commit to a *continuous* path of dC_t , but not a discrete one. This is mainly a technical assumption to ensure that a reflecting payment boundary is still well-defined. One may also be worried that a limited commitment principal should not be able to “commit” to terminating the contract immediately when the agent's participation constraint binds either. The principal can “reset” the contract once $W = 0$, for instance through renegotiation. I address these concerns in Section 4 and show that the main results of this paper remain intact under different versions of limited commitment.

²²Meanwhile, the “smooth-pasting condition” $V'(W^C) = -1$ is still true under limited commitment as the principal can still make a lump-sum payment of dI to the agent, moving the agent from W to $W - dI$ and changing the principal's value by $V(W - dI) - dI$, as long as $V(W) \geq \underline{V}$ is satisfied.

boundary conditions $\tilde{V}_j(0) = \underline{V}$, $\tilde{V}'_j(\tilde{W}_j^C) = -1$, and

$$\tilde{V}_j''(\tilde{W}_j^C) = 0 \text{ if } \tilde{V}_j(\tilde{W}_j^C) \geq \underline{V} \text{ and} \quad (13)$$

$$\tilde{V}_j(\tilde{W}_j^C) = \underline{V}, \text{ otherwise.} \quad (14)$$

The optimal choice of β , ϕ and a follow the same rules described in Property 1, 2, and 3.

The boundary conditions specified in this Proposition imply that the optimal contract can be categorized into three different types depending on when the limited commitment constraint (14) is binding: type one, it is *not* binding in either volatility state; type two, it binds in one state but not the other, and, because higher volatility implies lower firm value, the state in which it is binding must be the high volatility state; type three, it binds in both states. Clearly, type one is equivalent to the contract with full commitment, while type three offers the sharpest contrast. Thus for the remainder of the paper, I will concentrate discussion on the third type only. That is, unless stated otherwise, I assume that the parameter space is such that under full commitment, $V_j(W^C) < \underline{V}$ for both $j = l$ and $j = h$.²³

A numerical example of the optimal contract under limited commitment is shown in Figure 2 using the same parameter values as in Figure 1. Note that in Figure 1, firm value in both states is below the liquidation value $\underline{V}$ and therefore payment boundaries in neither state can be sustained without principal's full commitment. That is, Figure 2 falls into case in which the limited commitment constraint binds in both states. Comparing to Figure 1, both value functions are lower than their previous counterparts, due to the binding commitment constraint. Moreover, there is a significant change to the position of the payment boundaries: the boundary is lower in the high volatility state. The implication of which is discussed in details in Section 3.

Before moving on, note that limited commitment also expands the space of parameters in which the optimal contract exist—the case when $r = \gamma$. In the full commitment model, $r = \gamma$ means the principal can costlessly delay payments to the agent. The payment boundary is therefore infinity, and the optimal contract does not exist. In contrast, the limited commit-

²³This assumption simplifies the analysis but is not necessary for most of the results to be presented shortly to hold. In the Appendix, I provide examples in which a contract where (14) binds only in the high volatility (i.e. type two contract) state produce very similar results.

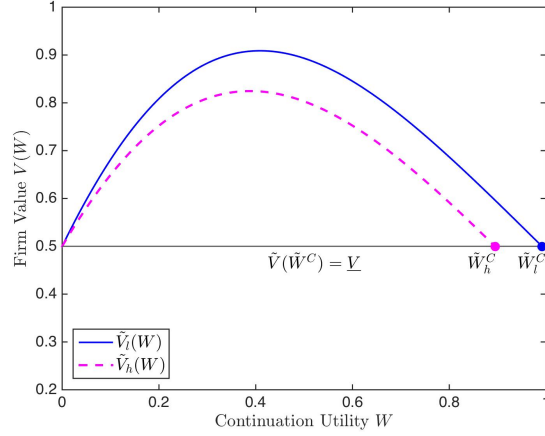


Figure 2 – Optimal Contract: Limited Commitment

This figure plots firm value functions under different cash flow volatility σ_s and with limited commitment on the principal's side. Parameter values are the same as in Figure 1. $\tilde{W}_l^C$ and $\tilde{W}_h^C$ indicate the payment boundaries (i.e. bonus hurdles) in low and high volatility states, respectively. $\tilde{V}(\tilde{W}^C) = \underline{V}$ indicates the principal's participation constraint due to her limited commitment.

ment constraint puts a physical bound on the payment boundary such that the payment boundary is where the limited commitment constraint binds.

3 Implications

In this section, I derive implications of the optimal contract under different level of commitment and discuss their empirical relevance. Two sets of implications are presented: one regarding the optimal level of risk management and PPS, the other regarding the agent's compensation in the presence of the volatility shock.

3.1 Optimal Risk Management and PPS

The main implication on optimal risk management and PPS can be summarized as the following:

Prediction 1. *High risk is always implemented near the termination boundary. Away from that boundary, there exists a set of parameters such that, the optimal contract under full commitment implement low risk and thus high PPS, while the optimal contract under limited*

commitment implement high risk but a lower PPS.

The fact that the contract cannot prevent high risk near the termination boundary is straightforward: when W is too low, in particular if $W < \delta$, then the agent’s limited liability implies $\phi \leq W < \delta$, and high risk-taking cannot be prevented. Such intuition is similar to that featured in previous studies such as DeMarzo et al. (2013) and Wong (2017) regarding why socially suboptimal risk-taking is inevitable even under the optimal contract. Put differently, the principal is “constrained” in her risk management policy, as incentivizing low risk is simply not feasible when W is too low.

Away from the termination boundary, the principal becomes “unconstrained” in her risk management choice: it is possible to incentivize either low risk or to allow high risk. Her exact choice depends on the trade-off summarized in Property 3: most importantly, there is a benefit for allowing high risk from the lower PPS (β) required to prevent shirking. Since the cost of PPS is captured by the concavity of the value function $V''(W)$, the benefit from a lower PPS is higher precisely under limited commitment, when a lower the payment boundary W^C increases concavity of the value function.

The risk choice at the payment boundary best highlights the role of different levels of commitment. Under full commitment, the “super-contact” condition $V''(W^C) = 0$ implies the cost of prevent shirking through β is zero at that point. Therefore, low risk is always preferred. In contrast, under limited commitment when the principal’s participation constraint is binding, $V(W^C) = \underline{V}$ replaces the “super-contact” condition, implying $V''(W^C) < 0$. Thus, the contract does not completely eliminate the cost of β even at the payment boundary, and the benefit of implementing high risk in exchange for a lower level of β may dominate the increasing probability of the volatility shock.

Figure 3 provide an illustration of the intuition stated above. Each graph plots three value functions: $\{V_l, a = 0\}$, $\{V_l, a = A\}$, and V_h . Here, $\{V_l, a = 0\}$ should be understood as implementing $a = 0$ whenever it is “feasible”²⁴. The value function corresponds to implementing low risk is higher than that corresponding to high risk under full commitment. In contrast, under limited commitment, while the commitment constraint is binding for both level of risk choice, the one corresponding to high risk is less severely affected by the com-

²⁴That is, $W \geq \phi$, where ϕ follows the slope-matching procedure specified in 2

mitment constraint and thus is higher, thanks to the smaller β , or variation in dW when high risk is allowed.

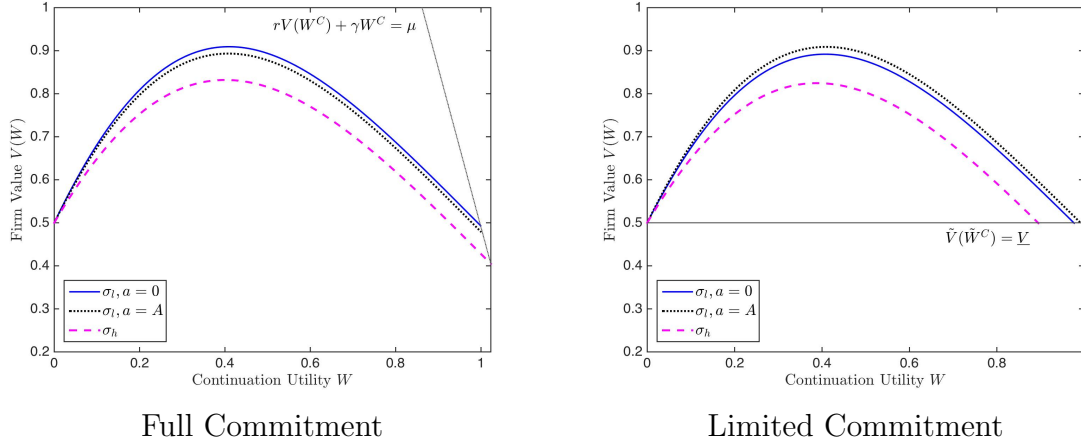


Figure 3 – Value Functions under Different Risk Choices

This figure presents the value functions under different risk choices. The solid line (blue) represents the value function of the contract that implements low risk ($a = 0$) whenever it is feasible (i.e. $W \geq \phi$, where ϕ follows the slope-matching procedure specified in 2). The dotted line (black) represents the value function of the contract that always implements high risk ($a = A$). The dashed line (magenta) represents the value function in the high volatility state. Parameter values are the same as those in Figures 1 and 2.

Importantly, the reason why the limited commitment optimal contract implements high risk when W is high is different from why full commitment contract implements high risk when W is low. The implementation under full commitment is a *constrained* choice, that is, because implementing low risk requires $\phi \geq \delta$, when $W < \delta$, the principal is simply unable to implement low risk. In other words, preventing shirking and preventing excessive risk-taking are not jointly feasible when the agent's continuation utility is low, the same conclusion as in DeMarzo et al. (2013) and Wong (2017). In contrast, the limited commitment optimally implement high risk even in the *unconstrained* region when $W > \delta$, or when preventing excessive risk-taking is compatible with preventing shirking. This is a novel result compared to most existing studies, in which the first-best policies are almost always implemented near the payment boundary. In this model, the combination of a two-dimensional agency frictions and the existence of limited commitment imply that the principal must trade-off the incentive for one friction with the other even at the payment boundary, and thus first-best policies may never be implemented.

3.2 Bonus Payment and Turnover Probability

The main implication on the dynamics of compensation in the presence of the volatility shock can be summarized in the following two Predictions:

Prediction 2. *When volatility becomes higher, the optimal contract under full commitment provides more back-loaded while the optimal contract under full commitment provides more front-loaded compensation.*

Whether the contract back-load or front-load compensation in the high volatility state depends on the position of the payment boundaries. As seen in Figure 1, under full commitment the payment boundary is *higher* when volatility is higher. That is, $W_h^C > W_l^C$. Intuitively, since the cost of providing incentives to the agent is the possibility of early termination after sufficiently poor performance, it is higher when volatility is higher, as higher uncertainty of cash flows increases the likelihood of sufficiently poor performance and the subsequent early termination. The principal adjusts the contract optimally by giving the agent more financial slack.²⁵ More financial slack lowers the possibility of costly early termination and is therefore optimal for the principal under higher volatility.

In contrast, as seen in Figure 2, under limited commitment the payment boundary is *lower* when volatility is higher. That is, $\tilde{W}_h^C < \tilde{W}_l^C$. The reason is that limited commitment implies the contract must guarantee the principal at least $\underline{V}$, which restricts the amount of future cash flow generated by continuing the project that can be credibly promised as compensation to the agent. Higher volatility increases the likelihood of early contract termination and thus reduces the total value generated by the contract. A principal lacking the ability to commit to future payments is forced to lower the performance hurdle because she can credibly promise less compensation in the future.

Prediction 1 and 2 together imply the next result:

Prediction 3. *After entering the high volatility state, conditional on surviving, agents under full commitment contracts receive cash bonuses only after producing significantly strong series*

²⁵Here, financial slack, defined as $W_s^C - R$, measures how much loss the principal is willing to bear before terminating the agent's contract.

of performance; while agents under limited commitment contracts receive cash bonuses as long as they produce a small amount of good performance.

This result is a combination of the position of the payment boundary and the level of agent's continuation utility after the volatility shocks hits, which is captured by $W - \phi$. Under full commitment, the payment boundary is higher in the high volatility state (Prediction 2). Moreover, since the contract implements low risk near the payment boundary (Prediction 1), the punishment ϕ when the volatility shock occurs is large. Together, they imply that the agent's continuation utility immediately after the volatility shock ($W - \phi$) is far away from the payment boundary. As a result, the agent must produce strong cumulative performance that increases W sufficiently high in order to receive actual payments. On the contrary, under limited commitment, the payment boundary is lower in the high volatility state. Since the contract implements high risk near the payment boundary, the punishment ϕ cannot be too large. Consequently, $W - \phi$ is close to the payment boundary, and a small success that increases W marginally is enough for the agent to receive actual payments.²⁶

Figure 4 illustrates the intuition above by demonstrating how ϕ varies with the agent's cumulative performance. In both panels, $W \leq \hat{W}$ represents the region in which the contract is terminated once the volatility shock arrives. Conditional on survival (i.e. $W > \hat{W}$), each dotted segment links the agent's continuation utility before the volatility shock (the circles on $V_l(W)$, the solid line) with that after the shock (the circles on $V_h(W)$, the dashed line). The projection of those segments on the W -axis measures the size of ϕ . Under full commitment, the optimal contract implements low risk; thus ϕ must be large enough, and the agent's continuation utility after the volatility shock is substantially lower than the payment boundary W_h^C .²⁷ Meanwhile, the optimal contract implements high risk under limited commitment; agents are more likely to receive cash bonuses with a small amount of good performance after the shock, because ϕ is small and $\tilde{W}_h^C$ is low.²⁸

²⁶While high risk is always implemented for low W regardless of the commitment level, contracts with low W are most likely terminated if the volatility shock occurs.

²⁷For example, if the volatility shock occurs at $W = W_l^C$, then $\phi \geq \delta$ implies that the agent's continuation utility after the volatility shock is $W_l^C - \delta$, and its distance to the high volatility state payment boundary is $(W_h^C - W_l^C) + \delta$.

²⁸For example, if the volatility shock occurs at $W = \tilde{W}_l^C$, as long as $\tilde{W}_h^C - \tilde{W}_h^C < \delta$, his continuation utility after the volatility shock is very close to $\tilde{W}_h^C$ since $\tilde{V}_l'(\tilde{W}_l^C) = \tilde{V}_h'(\tilde{W}_h^C) = -1$.

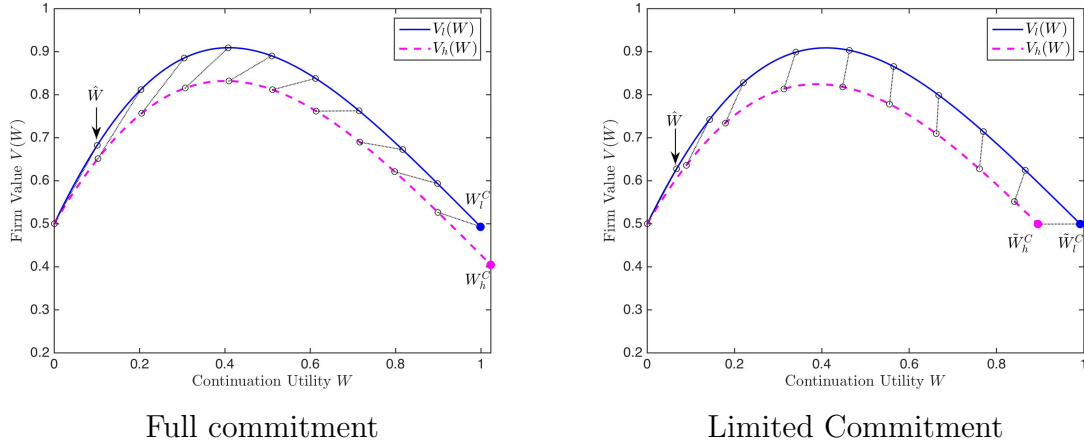


Figure 4 – Changes in W upon Arrival of the Volatility Shock

This figure displays ϕ , the changes in the agent's continuation utility at the time of the volatility shock, determined according to Property 2. Each dotted segment (black) links the agent's continuation utility before the volatility shock (the circles on $V_l(W)$, the solid line) with that after the shock (the circles on $V_h(W)$, the dashed line). $\hat{W}$ represents the cutoff point where contracts with $W < \hat{W}$ are terminated when the volatility shock arrives. Parameter values are the same as those in Figures 1 and 2.

To further demonstrate Prediction 3 visually, I also present in Figure 5 results from simulating the cash flow process and the compensation around the time of the volatility shock.²⁹ The plot on the left presents the probability of payments.³⁰ As it shows, the probability of payments in the few periods immediately after the uncertainty shock is much higher than the probability under the full commitment contract. Meanwhile, the plot on the right presents the fraction of active projects (managers) at each given time. Payment probability under the limited commitment contract quickly diminishes to zero due to a higher rate of contract termination, while it is much more persistent under the full commitment contract. In other words, there is a higher likelihood of contract termination, or managerial turnover, under limited commitment. Such shortened contract length is a result of a lower

²⁹Specifically, I segment the continuous-time model into discrete time intervals. The economy starts with low volatility, and the agent's initial wealth W_0 is drawn uniformly from the interval $(0, W_l^C)$. I simulate N different paths of cash flows and for each allow the state to switch to σ_h following a Poisson arrival process, representing the transition into the crisis time. For each period before and after the uncertainty shock, I calculate the probability of cash payments by taking the average number of recorded payments among all surviving firms after the crisis. I repeat this simulation procedure for both the full commitment and limited commitment contract.

³⁰Since the per-period shock to cash flow is i.i.d and the variance of the shock is constant within each volatility regime, the probability of cash payments has a one-to-one correspondence to the *average amount* of payment.

payment boundary leads to more front-loaded payments, which in turn also implies more aggressive managerial replacement following negative performance.

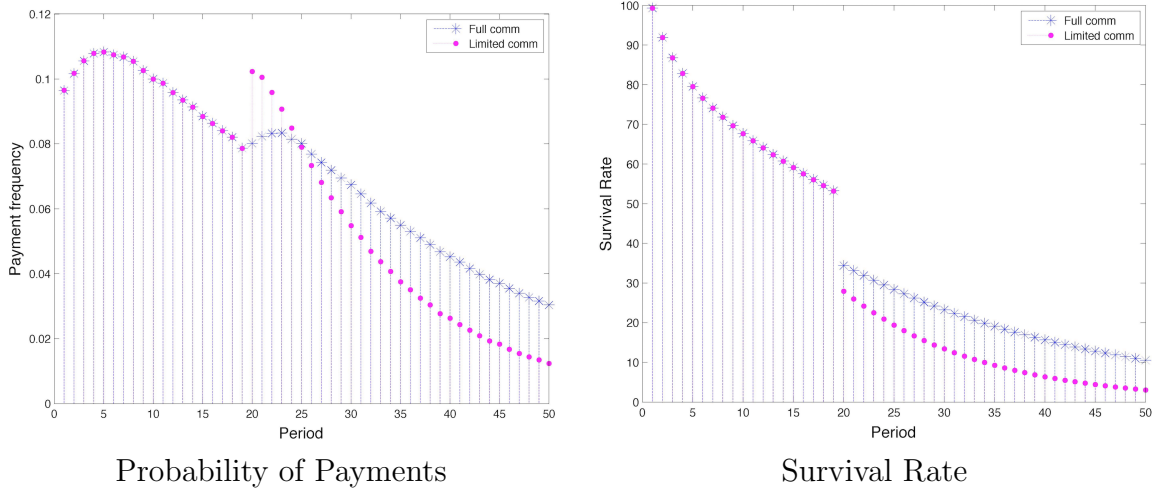


Figure 5 – Simulation Results

This figure presents the probability of cash compensation (bonuses) and the fraction of active projects from simulating 1,000,000 paths of cash flows. Period 20 corresponds to the volatility shock. Parameter values are the same as those in Figures 1 and 2.

3.3 Empirical Relevance

Theoretical predictions of the model presented in this section can help reconcile empirical observations regarding risk management and compensation in practice. Since the high risk in this model is socially undesirable to the principal but generates private benefit to the agent, one can interpret the agent’s tendency to pick $a = A$ as a *risk-shifting* behavior. While both canonical agency theory (e.g. [Jensen and Meckling \(1976\)](#)) and modern dynamic agency models (e.g. [DeMarzo et al. \(2013\)](#) and [Wong \(2017\)](#)) both predict risk-shifting among distressed firms, the empirical evidence on when risk-shifting takes place has been largely mixed. On the one hand, [Eisdorfer \(2008\)](#) and [Landier et al. \(2015\)](#) find firms increase risk-taking following poor performance. On the other hand, [Andrade and Kaplan \(1998\)](#), [Gilje \(2016\)](#) and [Aretz et al. \(2017\)](#) conclude with no evidence that risk-shifting occurs only among distress firms. Studies such as [Rauh \(2008\)](#) and [Huang et al. \(2011\)](#) paint a similarly mixed picture in the asset management industry.

Prediction 1 offers an explanation to this discrepancy by revealing a relationship between risk-shifting and firm performance different from existing models. Among firms with strong commitment power, the model generates a time-series predictions whereby firms engaging in risk-shifting only after series of negative shocks; however, between firms with different levels of commitment, the model makes a cross-sectional prediction whereby even firms with strong history of performance may optimally choose to engage in risk-shifting if they are commitment strapped. Since most empirical studies utilize sample of firms within certain industries or over certain time-period,³¹ the mixed empirical evidence regarding risk-shifting and firms' performance history may be attributed to the average commitment power of the specific industry or time-period studied.

In terms of managerial compensation and turnover, Predictions 2 and 3 offer results consistent with empirical findings (e.g. Jenter and Kanaan (2010), Kaplan and Minton (2012) and Eisfeldt and Kuhnen (2013)) and anecdotal evidence regarding the compensation practice during market downturns. In particular, observations from the recent crisis suggests that the high level of cash bonuses can be attributed to firms setting lower bonus hurdles. Importantly, the model distinguish pay-for-performance sensitivity (PPS, measured by β_t) from the actual payments (measured by dC_t). While commitment-constrained firms allow risk-shifting in exchange for lower PPS, they are more likely to make larger immediate payments conditional on negative uncertainty shocks. These results provide theoretical support for studies based on observed compensation: for example, Peters and Wagner (2014), who find higher industry-level equity volatility is strongly associated with *higher* managerial compensation, and Cheng et al. (2015), who find a similar relationship for firms in the financial sector.

It should be emphasized that while the majority of existing empirical studies usually focus on CEO compensation, this paper applies to a broader range of employees. Moreover, those existing studies often do not explicitly control for firms' ability to commit to long-term payments. Empirical work that examines CEO compensation or general labor costs while taking into account firms' commitment power can help better test the predictions made in

³¹For example Landier et al. (2015) focus on financial industry while Gilje (2016) on energy firms.

this paper.³²

4 Discussions

This section provides additional discussions regarding three key elements of the model: first, the interpretation of the high volatility state as the “crisis” state; second, the specific form of the limited commitment constraint; third, the possibility of state-varying outside options.

4.1 Real Impact of Volatility Shock

The main focus of this paper is regime-switching between low and high volatility states. The high volatility is interpreted as the “crisis” state out of two considerations: first, firm value is indeed lower in the high volatility state; secondly, such interpretation helps link the theoretical predictions with empirical evidence, in particular the debate of compensation in the recent financial crisis. While a crisis can also be modeled as a state with low average productivity, that is, the “drift” of the cash flow process μ , I hereby demonstrate that a volatility shock in this model can induce an endogenous decrease of drift as well. Therefore, modeling the crisis as a volatility shock provides more generality than modeling it as a drift/productivity shock.

The key to transform the volatility shock into a drift shock is to relaxes the assumption that no shirking is preferred by the principal regardless of the level of uncertainty. This is true as long as S , the social cost of shirking measured by the reduction in average cash flow, is high. However, preventing shirking through imposing pay-performance sensitivity β is also costly because it leads to the possibility of early contract termination. Moreover, the cost to incentives is higher when cash flow volatility is high. Thus, if S is somewhat moderate, the optimal contract may allow shirking in equilibrium, particularly in the high volatility state.

Consider the high volatility state under full commitment. A contract that involves no

³²Although beyond the scope of this paper, there are certainly measures empirical studies can use as proxies for commitment. For example, legal protection at the regional or country level. In the state of New York, judges usually favor employers during labor disputes involving bonus payments. The burden is on the employees who need to provide evidence that they are entitled for bonuses. In contrast, California is more lenient to employees in similar disputes. More broadly, European countries have in general stronger protection for employees relative to the US.

payment but simply allows the agent to shirk forever defines a pair of payoffs (W_S, V_S) for the agent and the principal respectively, where

$$W_S = \frac{\lambda S}{\gamma}; \quad (15)$$

$$V_S = \frac{1}{r} \left(\mu - \frac{\lambda S}{\gamma} \right). \quad (16)$$

Whether the optimal contract should allow shirking or not depends on the level of V_S . When S is sufficiently low, the principal is better off stopping incentive provision³³. The agent will choose to shirk, receive no payment from the principal and instead be compensated by his private benefit from shirking. The optimal contract is static, unrelated to the agent's performance, and therefore involves no termination.

Define $V_j^* = \max V_j(W)$ as the maximal firm value in state j . If $V_l^* > V_S > V_h^*$, the optimal contract may prevent shirking in the low volatility state, and switches to the static contract in the high volatility state. The dynamics of the optimal contract follow the ODEs described in Proposition 4 and 5, while the value function V_h is replaced by the static payoff given by equation (16). Formally:

Proposition 6. *Suppose S is or σ_h is sufficiently high such that the optimal contract induces $s_t = 0$ under σ_l but $s_t = S$ under σ_h . The principal's value function $V_l(W)$ and payment boundaries W_l^C satisfy the HJB equation (10) subject to boundary conditions $V_l(0) = \underline{V}$; $V_l'(W_l^C) = -1$; and $V_l''(W_l^C) = 0$. Furthermore, $\phi_l(W) = W - W_S = W - \frac{\lambda S}{\gamma}$, and V_h is given by (16).*

The case under limited commitment is similar with the appropriate boundary conditions described in Proposition 5.

The shirking equilibrium reveals that while a crisis is (rightfully so) usually defined as a period of low productivity, the underlying change in the economic regime could be a shift in volatility. In other words, there exists a potential endogeneity problem between profitability

³³More specifically, the contract that allows shirking forever is optimal only when $V_S > B(W)$, where $B(W)$ is a V-shaped function that extends above V_h^* . See Zhu (2013) for the detailed characterization of $B(W)$. Here I avoid the complicated situations by assuming that S is either high enough or low enough so that either working or shirking permanently is the optimal effort.

and volatility. Existing empirical work that studies compensation often considers profitability and volatility as independent factors. However, fluctuation in profitability can be driven by the change in volatility through the channel of managerial effort, raising empirical challenges since such effort is normally difficult to measure. It also provides further evidence in addition to previous work that uncertainty is the key to understanding the recent financial crisis.

Certain features of the optimal contract that involves shirking in equilibrium have important policy implications. Since the manager is compensated through the private benefit of shirking when uncertainty is high, no cash payment is made under that regime. This implies the possibility of observing little or no bonuses during a recession. However, though much to the media or public's liking, this equilibrium is actually worse in terms of total welfare, because productivity, measured by mean cash flow, is now lower due to less effort from managers. This is true as long as $\lambda < 1$ so that there is deadweight loss associated with managerial shirking. This result highlights the importance of compensation in keeping managers properly incentivized, even though the exact timing of their compensation may not match their overall performance at the time when a large negative shock occurs.

Last but not least, it is worth noting that the change in the agent's equilibrium effort is a feature of increasing volatility but not necessarily of decreasing profitability. While lower average cash flow μ does bring down firm value under an incentive compatible contract, it also lowers V_S , firm value under a static contract that allows shirking. As a result, working can still be the optimal effort to induce if $V_S < V_h^*$. In contrast, V_S does not depend on σ , but V_h^* does. When cash flow volatility becomes higher, V_h^* becomes lower until falling below V_S , and the incentive compatible contract is dominated by the static contract, a unique feature of stochastic volatility.

4.2 Alternative Commitment Mechanism

Economic research has long recognized that firms do not possess full commitment power over labor contracts.³⁴ Recent research shows that firms can also default on labor contracts

³⁴There are many papers that study risk sharing in labor contracts which emphasize the lack of commitment from the firm side, for example [Thomas and Worrall \(1988\)](#), [Abreu et al. \(1990\)](#), [Ray \(2002\)](#), [Berk et al. \(2010\)](#), [Grochulski and Zhang \(2011\)](#) and [Miao and Zhang \(2014\)](#). More generally, the relational contract literature, including [Atkeson \(1991\)](#), [Levin \(2003\)](#), [Grochulski and Zhang \(2013\)](#) and [Opp and Zhu \(2015\)](#),

because of limited or costly access to financial markets.³⁵ Furthermore, most of these studies also assume that firms, in addition to not being able to commit to intertemporal compensation, also cannot commit to terminal payments such as severance pay.³⁶ In practice, severance pay, especially for incentive purposes, is rarely paid in the event of firm liquidation. It is difficult to specify severance pay in contracts such that it is fully contingent. The execution of severance pay is also subject to changes in the external enforcement environment.³⁷

While mechanisms that provide managers with payments at the time of contract termination exist, it is unlikely that their implementation can avoid the limited commitment problem. Imagine that a firm sets up for the agent an escrow account of balance W and allow the agent to withdraw the balance in the event of contract termination. This mechanism for implementing the terminal payment, however, must necessarily give the principal access to the escrow account in order to update W according to the agent's performance and at the same time forbid the agent access to the account to prevent balance manipulation. This is essentially giving firms the control right to that account and asking firms to "commit" to paying the agent out of the balance of that account. This is effectively no different than giving firms the control right to cash flows, which is what this paper describes. The same difficulty of commitment remains, as it is intrinsically difficult to prevent the principal from reducing the balance of any account to which she has control right before the agent can withdraw from it.

Moreover, based on the security implementation of the optimal contract, I provide a novel justification for the limited commitment constraint. Details of such argument can be found in the Appendix. In short, the implementation involves standard securities such as equity and debt and hence potential tension between their holders. Following an increase in uncertainty, the face value of long-term debt must decline, implying the redemption of debt which entails a wealth transfer from equity holders. Under high volatility, firm value is low, in which case equity holders may find it optimal to default on rather than recall the

has studied the lack of commitment from both contracting parties.

³⁵See, for example, [Ellul et al. \(2014\)](#) and [Palacios and Stomper \(2014\)](#).

³⁶For instance, [Berk et al. \(2010\)](#), [Ai and Li \(2014\)](#) and [Bolton et al. \(2015\)](#). Even when severance pay is included in managerial contracts, as long as it is not fully guaranteed, limited commitment remains and the argument of this paper still applies

³⁷For example, during the recent bankruptcies of Hostess Brands Inc. and Hawker Beechcraft Corp., the US Justice Department blocked proposals to grant extra bonuses to the executives of those companies.

debt. Therefore, in the implementation of the contract, assuming that the principal has full ability to commit to payments is equivalent to assuming that equity holders have full ability to commit to maintaining a certain capital structure. The latter assumption is largely unrealistic as equity holders usually can default on debt without considering the effect on the firm as a whole.

4.3 Outside Options

In the baseline model, the assumption that both the principal and agent receive constant outside value whenever the contract terminates is critical to obtaining a closed-form solution. It is not so, however, to establishing the key findings of this paper regarding the difference in payment and termination likelihood under the full versus limited commitment contract. There are many ways to endogenize firms' outside option $\underline{V}$ and therefore make it state-contingent. I discuss only one straightforward case here and show how the main results still hold. Suppose firms can, at any time, replace the incumbent manager with a new manager by paying a search cost C^R . The contract with the new manager will specify an initial W , which maximizes firm value in that state. That is, $\underline{V}_j = V_j(W^*) - C^R$, where $W^* = \arg \max V_j(W)$. Figure 6 illustrates the value functions and payment boundaries under this assumption.

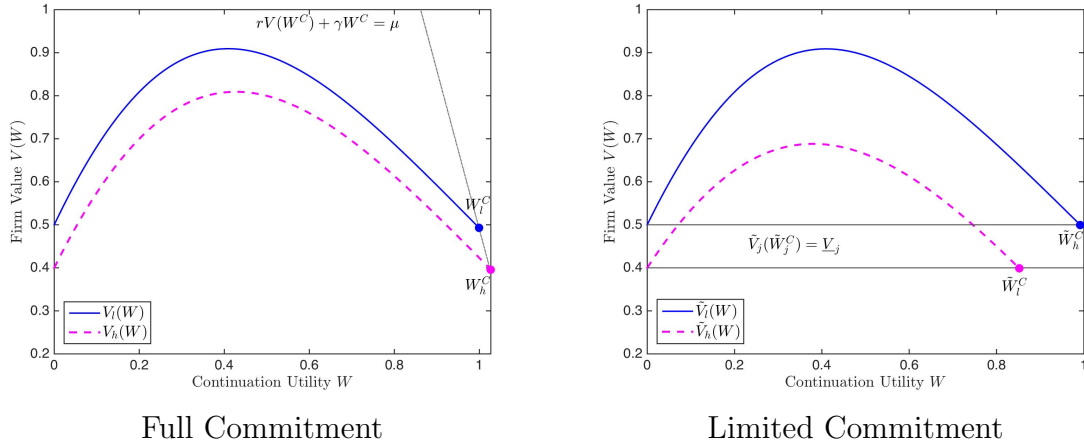


Figure 6 – Endogenous State-varying Liquidation Value

This figure plots the value functions with endogenous $\underline{V}$ defined by $\underline{V}_j = V_j(W^*) - C^R$, where $W^* = \arg \max V_j(W)$. The full commitment case is shown in the left panel. The limited commitment case is shown in the right panel. Parameter values are the same as in Figures 1 and Figure 2.

Under both full commitment and limited commitment, the liquidation value is lower in the high volatility state, because $V_h^*(W)$ is lower. However, the bonus hurdles still resemble the baseline case shown in Figures 1 and Figure 2, that is, the hurdle is higher in the high volatility state under full commitment and lower in the high volatility state under limited commitment. Intuitively, higher volatility implies a lower value for the principal's outside option, which reduces the tightness of the commitment constraint. Therefore, whether the payment boundary in the high volatility state is higher or lower simply depends on whether the decrease in $\underline{V}$ or the increase in σ dominates, which is to be determined by future empirical work. In short, large variation in volatility relative to small changes in outside options would produce results consistent with those of the present model.³⁸

5 Conclusion

It has been argued that the build up of large risks within the financial industry and the sudden release of such risk is the most prominent feature of financial crises. Moreover, bankers and executives that were seemingly responsible for such aggressive risk-taking behavior still receive substantial compensation in the wake of the crisis. To understand such phenomena, this paper derives the optimal compensation contract in a principal-agent model where the agent has private control over both productivity and the probability of a volatility shock. Uncertainty and agency frictions require the provision of incentives in the form of back-loaded compensation, which increases with good performance and the absent of the volatility shock. The agent receives cash bonuses only if his cumulative performance exceeds a bonus hurdle. Thus, the specific features of the optimal compensation depends crucially on the degree to which firms can credibly pledge future payments. When managers can privately influence both firms' productivity and risk, firms that lack sufficient commitment power may allow managers to take on large risks in exchange for a lower pay-performance sensitivity needed to

³⁸It is also unclear whether the outside option value for sure decreases in crisis times. In the above case, the search cost C^R can also be made state contingent, reflecting the likely different difficulty in finding a replacement manager in good versus bad times. How it is different, however, is not immediately clear. In bad times, more managers are laid off. The search for a replacement could therefore be less difficult due to a larger managerial talent pool, but it could also be more difficult due to greater information asymmetry regarding a potential manager's true ability.

maintain high productivity. When uncertainty is high, the value of continuing operations is low, those commitment-constrained firms could pledge even less future payments and must therefore provide their managers with more immediate compensation by lowering their bonus hurdles, seemingly rewarding their excessive risk-taking behavior.

There are several directions in which this model can be fruitfully extended. One example is liquidity management in response to volatility shocks. Recent studies such as [Campello et al. \(2011\)](#) examine cross-sectional liquidity management along different dimensions. A slightly modified dynamic model à la [Bolton et al. \(2011\)](#) would be equipped to provide theoretical insights into these observations.

Appendix A. Proofs

Proof of Proposition 1:

Consider dW_t in the low volatility state. Define the agent's total expected utility received under a contract conditional on his information at time t as:

$$G_t = E^{\hat{s}, \hat{a}} \left[\int_0^\tau e^{-\gamma x} dC_u + \int_0^\tau e^{-\gamma u} B(s_u, a_u) dx \mid \mathcal{F}_t \right],$$

where The process $G = \{G_t, \mathcal{F}_t; 0 \leq t < \tau\}$ is an $\mathcal{F}_t$ -martingale. The expectation is taken with respect to the probability measure induced by $\{\hat{s}, \hat{a}\}$. Then, by the martingale representation theorem, there exist $\mathcal{F}_t$ -predictable, integrable processes β and ϕ such that

$$G_t = G_0 + \int_0^t e^{-\gamma x} \beta_u \sigma_l dZ_u^{\hat{s}, \hat{a}} - \int_0^t e^{-\gamma u} \phi(dN_u^{\hat{a}} - (\pi + \hat{a}_u) du). \quad (17)$$

Differentiating (17), and substituting in the definition of $W_t^{\hat{s}, \hat{a}}$ yields

$$dG_t = e^{-\gamma t} dC_t + e^{-\gamma t} B(\hat{s}, \hat{a}) dt - \gamma e^{-\gamma t} W_t^{\hat{s}, \hat{a}} dt + e^{-\gamma t} dW_t^{\hat{s}, \hat{a}},$$

therefore

$$dW_t^{\hat{s}, \hat{a}} = \gamma W_t^{\hat{s}, \hat{a}} dt - dC_t - B(\hat{s}, \hat{a}) dt + \beta_t \sigma_l dZ_t^{\hat{s}, \hat{a}} + \phi(dN_t^{\hat{a}} - (\pi + \hat{a}_t) dt).$$

which implies the evolution of the agent's continuation utility given in (3), the evolution given in (5) for $\{\hat{s}, \hat{a}\} = \{0, 0\}$ and the evolution given in (6) for $\{\hat{s}, \hat{a}\} = \{0, A\}$. Proof of dW_t in the high volatility state follows the above argument without the terms associated with dN_t and ϕ .

To prove the IC condition, let $\hat{G}_t$ denote the payoff to a strategy $\{\hat{s}, \hat{a}\}$ until time $t < \tau$ and then $\{s, a\}$ thereafter, then

$$\hat{G}_t = \int_0^t e^{-\gamma u} dC_u + \int_0^t e^{-\gamma u} B(\hat{s}, \hat{a})_u du + e^{-\gamma t} W_t^{s, a}.$$

Differentiating $\hat{G}_t$ yields

$$\begin{aligned} e^{\gamma t} d\hat{G}_t &= B(\hat{s}, \hat{a}) dt - B(s, a) dt + \beta_t (\hat{s}_t - s_t) dt + \beta_t \sigma_l dZ_t^{\hat{s}, \hat{a}} \\ &\quad - \phi_t (\hat{a}_t - a_t) dt + \phi_t (dN_t^{\hat{a}} - (\pi + \hat{a}_t - a_t) dt) \end{aligned}$$

If (4) does not hold on a set of positive measure, then the agent could chose $\{\hat{K}, \hat{\sigma}\}$ such that

$$B(\hat{s}, \hat{a}) + \beta_t \hat{s}_t - \phi_t \hat{a}_t > B(s, a) + \beta_t s_t - \phi_t a_t$$

Therefore to implement $\{\hat{s}, \hat{a}\} = \{s, a\}$ requires (4). $\square$

Proof of Proposition 2:

Given equation (4), implementing $\{s, a\} = \{0, 0\}$ requires that

$$\lambda S - \beta S \leq 0 \quad (18)$$

$$\delta A - \phi A \leq 0 \quad (19)$$

$$B(S, A) - \beta S - \phi A \leq 0 \quad (20)$$

The first two lines imply $\beta \geq \lambda$ and $\phi \geq \delta$. Together they imply the third line since $B(S, A) < \lambda S + \delta A \leq \beta S + \phi A$.

Similarly, implementing $\{s, a\} = \{0, A\}$ requires that

$$B(S, A) - \delta A - \beta S \leq 0 \quad (21)$$

$$\delta A - \phi A > 0 \quad (22)$$

$$\lambda S + \phi A - \beta S - \delta A \leq 0 \quad (23)$$

The first line implies $\beta \geq \frac{B(S, A) - \delta A}{S}$; the second line implies $\phi < \delta$. The third line implies $\beta \geq \frac{\lambda S - (\delta - \phi)A}{S}$. Thus $\phi < \delta$ and $B(S, A) < \delta A + \lambda S$ together imply

$$\beta \geq b \equiv \max \left\{ \frac{\lambda S - (\delta - \phi)A}{S}, \frac{B(S, A) - \delta A}{S} \right\}$$

□

Proof of Proposition 3, 4 and 5:

The proof of Proposition 3 is identical to that of Proposition 1 in DeMarzo and Sannikov (2006). Before proving Proposition 4, I first discuss the case of limited commitment in the high volatility state (so that there is no risk management choice yet, and $\beta = \lambda$.) Define the social benefit function as $F(W) = W + V(W)$, which satisfies

$$F''(W^C) = \frac{rF(W^C) + (\gamma - r)W^C - \mu}{\frac{1}{2}\lambda^2\sigma_s^2}.$$

Recall that $\tilde{\cdot}$ denotes a variable under limited commitment. When the principal's participation constraint is binding, $\tilde{F}(W^C) = \underline{V} + \tilde{W}^C$ implies

$$\tilde{F}''(\tilde{W}^C) = \frac{r\underline{V} + \gamma\tilde{W}^C - \mu}{\frac{1}{2}\lambda^2\sigma_s^2}.$$

Suppose $\tilde{F}''(\tilde{W}^C) > 0$; that is, $r\underline{V} + \gamma\tilde{W}^C > \mu$. This implies that $\tilde{V}''(\tilde{W}^C) > 0$, since $\tilde{V}(\tilde{W}^C) = \underline{V}$ and $r\tilde{V}(\tilde{W}^C) + \gamma\tilde{W}^C > \mu$. Compare this result to the case of full commitment, where $rV(W^C) + \gamma W^C = \mu$. If $\tilde{W}^C < W^C$, since $rV(W) + \gamma W < \mu$ for all $W < W^C$, it must be that $rV(\tilde{W}^C) + \gamma\tilde{W}^C < \mu$, which implies $V(\tilde{W}^C) < \tilde{V}(\tilde{W}^C)$. However, this is a contradiction since $V(W) \geq \tilde{V}(W)$ for every W . If, on the other hand, $\tilde{W}^C > W^C$ but $\tilde{V}(\tilde{W}^C) = \underline{V}$ and $V(W^C) < L$, then $\tilde{V}(\tilde{W}^C) > V(\tilde{W}^C)$, which is again a contradiction. Therefore, $\tilde{F}''(W) < 0$ in the neighborhood of $\tilde{W}^C$.

The rest of the argument, about $\tilde{F}$ being also concave besides in the neighborhood of $\tilde{W}^C$, follows the standard argument. The proof also implies immediately that $r\tilde{V}(W) + \gamma W \leq \mu$ for all W if the boundary condition $\tilde{V}''(\tilde{W}^C) = \underline{V}$ is true. This conclusion is used in the following verification theorem.

Verification Theorem:

for any incentive compatible contract, define an auxiliary gain process $\mathcal{G}$ as

$$\mathcal{G}_t = \int_0^t e^{-ru}(dY_u - dC_u) + e^{-rt}V(W_t) ,$$

where W_t evolves according to dW_t . By Ito's lemma,

$$\begin{aligned} e^{rt}\mathcal{G}_t = & \left(\mu + \gamma W_t V'(W_t) + \frac{1}{2}\beta_t^2 \sigma^2 V''(W_t) - rV(W_t) \right) dt \\ & - (1 + V'(W_t))dI_t + (1 + \beta_t V(W_t))\sigma dZ_t . \end{aligned} \quad (24)$$

The first two terms are negative, therefore $\mathcal{G}_t$ is a supermartingale. Now, evaluating the principal's payoff for this contract,

$$\begin{aligned} E \left[\int_0^\tau e^{-ru}(dY_u - dC_u) + e^{-r\tau}\underline{V} \right] = \\ E(\mathcal{G}_{t \leq \tau}) + e^{-r\tau} E \left[1_{\{t \leq \tau\}} \left(E_t \left(\int_t^\tau e^{-r(u-t)}(dY_u - dC_u) + e^{-r(\tau-t)}\underline{V} \right) - V(W_t) \right) \right] . \end{aligned}$$

First, $E(\mathcal{G}_{t \leq \tau}) \leq \mathcal{G}_0$, since $\mathcal{G}_t$ is a supermartingale. Then, $E_t \left(\int_t^\tau e^{-r(u-t)}(dY_u - dC_u) + e^{-r(\tau-t)}\underline{V} \right) \leq \frac{\mu}{r} - W_t$, since by the argument above, $rV(W) + \gamma W \leq \mu$ for all W . Letting $t \rightarrow \infty$ implies that

$$E \left(\int_0^\tau e^{-ru}(dY_u - dC_u) + e^{-r\tau}\underline{V} \right) \leq V_0(W) .$$

Moving on to the two the low volatility states. Suppose $a = 0$, differentiate the corresponding social benefit function with respect to W . Substituting in the boundary conditions and evaluating the equation at the payment boundary W_l^C implies

$$F_l'''(W_l^C) = \frac{(\gamma - r) + (\gamma W_l^C - \pi \phi(W_l^C)) F_l''(W_l^C)}{\frac{1}{2}\lambda^2 \sigma_l^2} ,$$

where $F_l''(W_l)$ is given by

$$F_l''(W_l^C) = \frac{rF_l(W_l^C) + (\gamma - r)W_l^C - \mu + \pi (F_h(W_l^C - \phi(W_l^C)) - F_l(W_l^C))}{\frac{1}{2}\lambda^2 \sigma_l^2} .$$

Hoffmann and Pfeil (2010) and Piskorski and Tchisty (2010) show the optimality conditions for the full commitment case. Under limited commitment, if the commitment constraint is not binding in the low volatility state, the proof is identical to theirs. Now, suppose that it is binding, which implies that it must also be binding in the high volatility state.

Given the fact that $\tilde{V}'_l(\tilde{W}_l^C) = \tilde{V}'_h(\tilde{W}_h) = -1$, the slope matching procedure that pins down ϕ implies $\phi_l(\tilde{W}_l^C) = \tilde{W}_h - \tilde{W}_l^C$. Given that $r\tilde{F}_l(\tilde{W}_l^C) + \gamma\tilde{W}_l^C < \mu$ if $\tilde{W}_h < \tilde{W}_l^C$, then $\phi(\tilde{W}_l^C) < 0$ and $\tilde{F}_l'''(\tilde{W}_l^C) > 0$. If $\tilde{W}_h > \tilde{W}_l^C$, then $\gamma\tilde{W}_l^C - \pi\phi(\tilde{W}_l^C) > 0$ as $\pi < \frac{\gamma\tilde{W}_l^C}{\phi(\tilde{W}_l^C)}$. Since $\phi(\tilde{W}_l^C) < \tilde{W}_h < \frac{\mu-rV}{\gamma}$, $\gamma\tilde{W}_l^C - \pi\phi(\tilde{W}_l^C) > 0$ as long as $\pi < \frac{\tilde{W}_l^C}{\mu-rV}$. Note that for a non trivial contract, $\tilde{W}_l^C > R = 0$, there is always π small enough such that $\pi < \frac{R}{\mu-rV}$ is satisfied. The subsequent verification is similar to that used above and is therefore omitted here. $\square$

Appendix B. Recurring States

In the main body of the paper, I assume that the transition probability from the high to the low uncertainty state is zero, that is the crisis state is absorbing. This assumption greatly simplifies the verification of the optimality of the contract, but is unnecessary for the results of this paper to hold. In this appendix, I provide a full characterization of the optimal contract when this assumption is relaxed. Now suppose the economy switches between normal and crisis times stochastically. Let $\pi_j, j \in \{l, h\}$ denote the transition probability. [Li \(2012\)](#) establishes the technical details required to study this case. The following proposition summarizes the result:

Proposition 7. *Suppose $\pi_l > 0$ and $\pi_h > 0$. Let N_t be the total number of state transitions at time t . The agent's continuation utility W_t in state $j \in \{l, h\}$ follows*

$$dW_t = \gamma W_t dt - B(s_t, a_t) - dC_t + \beta_t(dY_t - (\mu - s_t)dt) - \phi_t(dN_t - (\pi_j + a_t)dt). \quad (25)$$

The optimal contract is a pair of value functions $V_j(W)$ and payment boundaries W_j^C such that

$$\begin{aligned} rV_j(W) = & \mu - s + (\gamma W - B(s, a) + (\pi + a)\phi) V'_j(W) + \frac{1}{2}\beta^2\sigma_j^2 V''_j(W) \\ & + (\pi_j + a) \left(V_j(W - \phi) - V_j(W) \right), \end{aligned} \quad (26)$$

subject to boundary conditions $V_j(0) = \underline{V}$; $V'_j(W_j^C) = -1$; and

$$V''_j(W_j^C) = 0,$$

where ϕ is determined by Property 2. If the principal has only limited commitment, the optimal contract is a pair of value functions $\tilde{V}_j(W)$ and payment boundaries $\tilde{W}_j^C, j \in \{l, h\}$, such that $\tilde{V}_j(W)$ satisfies the same system of ODEs (26) and boundary conditions $\tilde{V}_j(0) = \underline{V}$; $\tilde{V}'_j(\tilde{W}_j^C) = -1$, and

$$\begin{aligned} \tilde{V}_j''(\tilde{W}_j^C) &= 0, \quad \text{if } \tilde{V}_j(\tilde{W}_j^C) \geq \underline{V}, \\ \tilde{V}_j''(\tilde{W}_j^C) &= \underline{V}, \quad \text{otherwise.} \end{aligned}$$

Proof: The proof builds on the iteration procedure described in [Li \(2012\)](#) and is therefore omitted in the interest of space. $\square$

The optimal contract characterized under the recurring state is qualitatively identical to the one summarized in Section 2 under a one-time shock. In fact, the principal’s value functions of the contract under recurring states converge to value functions under a one-time shock when $\pi_h \rightarrow 0$. Given π_j are assumed to be small numbers, the case of a one-time shock provides a good approximation for the general case of recurring states and does not lose any important result.

All the remaining results discussed in the main body regarding the position of payment boundaries, the probability of cash payment and expected termination time are preserved in the recurring state contract, as long as the parameters are that once the limited commitment constraint is imposed, it is binding in both states. The discussion of ϕ can be expanded to not only “negative” volatility shocks ($\sigma_l \rightarrow \sigma_h$) but also to “positive” shocks ($\sigma_h \rightarrow \sigma_l$) by reversing Prediction 3.

Appendix C. Contracts with Limited Commitment Binding in One State Only

Section 2 introduces three types of contracts based on when the limited commitment constraint is binding. While the main text focuses on the third type, here I also provide some discussion of the second type: the contract where the limited commitment constraint is binding only in the high volatility state. In general, this type of contract can behave like contracts with either full commitment or limited commitment, but the commitment constraint is binding in both states depending on the parameter value σ in each state.

Figure 7 demonstrates the difference between two levels of volatility in the high volatility state. For the same level of σ_l , the relative positions of W_l^C and $\tilde{W}_h^C$ are similar to the full commitment case when σ_h is moderate but converge to the case in which the commitment constraint is binding in both states when σ_h becomes high enough.

The finding of this section greatly expands the domain of contracts to which Predictions 1, 2 and 3 can apply. Immediate bonuses in crisis times could be possible if the abrupt volatility increase is substantial enough that many firms that operate smoothly during normal times suddenly become constrained in the amount they can credibly pledge to pay their managers in the long run. The greater the increase of market risk during the crises, the greater is this concern. Future research that calibrates or empirically investigates the real scope of this commitment constraint will be helpful in determining the proportion of firms that are subject to limited commitment contracts and firms for which the dynamics of bonuses follow the predictions of this paper.

Appendix D. Renegotiation-Proof Contracts

In this paper, the principal terminates the manager’s contract whenever $W = 0$. This assumption, however, is not required to deliver the qualitative results of this paper. One way to consider the principal’s lack of commitment when $W \rightarrow 0$ is renegotiation, which is derived in this section. Despite the principal’s having only limited power of commitment, renegotiation-proofness is not a necessary feature of the resulting equilibrium contract. In

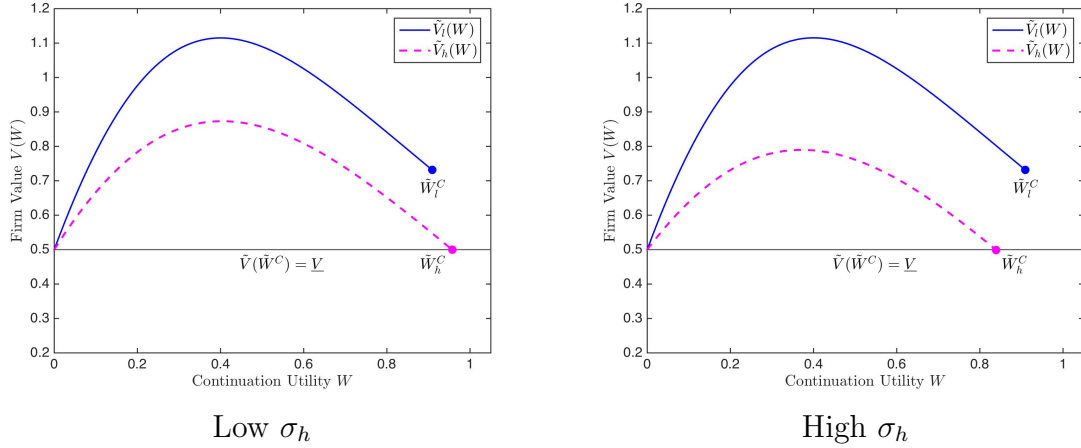


Figure 7 – Contracts with the Commitment Constraint Binding in One State Only

This figure plots firm value functions when the limited commitment constraint is binding only in the high volatility state. Parameter values are the same as those in Figures 1 and 2, except $\sigma_l = 0.22$ and $\sigma_h = 0.29$ for the left panel and $\sigma_h = 0.31$ for the right panel.

the limited commitment literature,³⁹ the off-equilibrium strategy for a defaulting party is usually autarky or complete exclusion from re-entering any contracting relationship. In practice, modifying the contract, via renegotiation or replacing the incumbent agent, is usually costly. Allowing for costly contract modification preserves the key predictions of the model, as explained further in the following paragraphs.

A renegotiation-proof contract requires the slope of the principal's value function to be non-positive. Such condition is ruled out in the main context of this paper because $\tilde{V}(0) = \tilde{V}(\tilde{W}^C) = \underline{V}$ when the limited commitment constraint binds, hence a non-trivial contract must have a region where the principal's valuation is increasing in the agent's continuation value W . To allow renegotiation-proof contracts, I modify the assumption about the principal's commitment ability. I assume now that the principal will only withdraw the investment when firm value is below a certain threshold, say zero. This assumption is similar to the one made by [Ai and Li \(2014\)](#) and [Bolton et al. \(2015\)](#). Formally, I assume the corresponding constraint on the payment boundary as

$$V(W^C) \geq 0.$$

The dynamics of the agent's continuation value under renegotiation-proof contracts follow

$$dW_t = \gamma W_t dt - B(s_t, a_t) - dC_t + \beta_t(dY_t - (\mu - s_t)dt) - \phi_t(dN_t - (\pi + a_t)dt) + dX_t \quad (27)$$

The new term dX_t defines a reflecting termination boundary $\underline{W}$ which satisfies the boundary condition $V(\underline{W}) = \underline{V}$ and $V'(\underline{W}) = 0$. Termination is stochastic at this boundary with probability $dX_t/\underline{W}$.

³⁹See for instance, [Thomas and Worrall \(1988\)](#), [Abreu et al. \(1990\)](#), [Ray \(2002\)](#), [Levin \(2003\)](#) and [Miao and Zhang \(2014\)](#).

Figure 8 shows an example of the value functions for the renegotiation-proof contracts, where both the endogenous renegotiation boundaries as well as the payment boundaries are displayed. Critically, the relative propositions of the payment boundaries for both the full and limited commitment contracts are still the same as those summarized in Section 3. Therefore, the renegotiation-proof contracts lead to the same predictions.

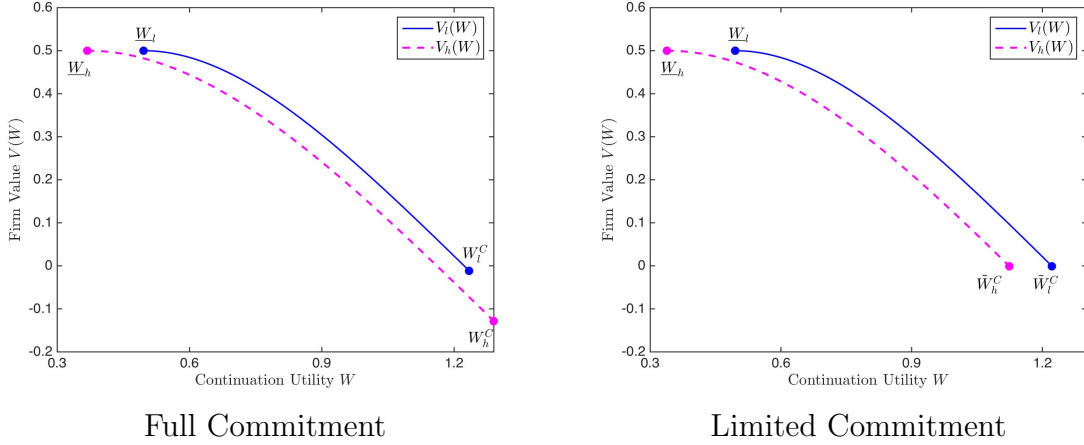


Figure 8 – Renegotiation-Proof Contracts

This figure plots firm value functions of renegotiation-proof optimal contracts. Parameter values are the same as those in Figures 1 and 2, except $\sigma_l = 0.36$ and $\sigma_h = 0.42$. $\underline{W}$ represents the renegotiation boundary given by $V(\underline{W}) = \underline{V}$ and $V'(\underline{W}) = 0$.

Last but not least, renegotiation-proofness is not a necessary feature for the contract to be optimal despite limited commitment. The principal is still able to rule out further renegotiation since the only action she cannot commit to is not to withdraw when firm value is negative. In particular, the principal can commit to the random termination schedule described above, which is crucial in keeping proper managerial incentives (See [Anderson et al. \(2016\)](#) for discussions regarding the optimality of committing to random termination.) Further, the assumption that investors withdraw their investment when firm value drops below zero replaces the earlier assumption of liquidation at any time. Therefore, the value of the firm at the termination boundary is still the liquidation value, since it is determined by the agent's effective limited liability constraint and the principal is able to commit to termination once that boundary is reached.

Appendix E. Contract Implementation, Capital Structure and the Commitment Constraint

In this section, I establish a novel equivalence between firms' commitment to compensation contracts and their commitment to capital structure. This equivalence provides a justification for the limited commitment assumption made throughout this paper, as firms' commitment to capital structure is known to be implausible.

Implementing the full commitment contract involves the use of debt and equity and therefore creates a conflict between debt and equity holders that leads to potential commitment issues. When the regime switches from low to high volatility, the face value of debt must be brought down at the expense of equity holders. Such implementation imposes an implicit assumption that equity holders must commit to maintaining a certain capital structure, which is generally implausible since equity holders do not always act for the benefit of the entire firm. In contrast, contracts with limited commitment do not require firms to make such commitment to capital structure and should therefore be more prevalent in practice.

Implementation in this paper follows the standard literature in using a set of common securities with limited liability: equity, long-term debt and credit line. Equity can be held by both the manager as well as outside investors who receive dividend payments and can decide the firm's capital structure. The long-term debt is a callable consol bond that pays a fixed rate and has a fixed face value. The firm can issue more long-term debt or call it back at its face value.⁴⁰ Finally, the credit line provides the manager with limited liquidity. The manager decides both the dividend and the credit line balance, but incentive compatibility under the optimal contract renders irrelevant who makes dividend and credit line decisions.

There is more than one implementation of the optimal contract. The following proposition provides a standard result. Since the focus is the commitment requirement, for simplicity I assume there is no rick choice (i.e. the arrival rate of the volatility shock is a constant π):

Proposition 8. *Both the full and limited commitment contract can be implemented by*

- (a) *the manager holding inside equity share λ ;*
- (b) *the face value of the callable debt satisfying $D_j = V_j(W_j^C)$;*
- (c) *the credit line balance M_t and credit limit L^* satisfying $W_t = \lambda(L_j^* - M_t)$ and $\lambda L_j^* = W_s^C$.*

Dividend I_t is paid when $M_t = 0$. Liquidation occurs when M_t reaches L_j^ .*

Proof: The proof follows the security implementation in models that combine Brownian motion and Poisson jumps, such as [Moreno-Bromberg and Roger \(2016\)](#), with exogenous state-transitions. $\square$

Security implementation of the optimal contract implies a certain capital structure which can potentially raise questions under the regime switching environment: the boundary conditions of the full commitment contract implies $V_h(W_h^C) < V_l(W_l^C)$. Since $V_h(W_h^C)$ and $V_l(W_l^C)$ also correspond to the face value of the callable debt in the high and low volatility states, the implementation of the full commitment contract requires that the face value of long-term debt be brought down when volatility increases. In other words, some portion of the long-term debt must be called back, hence the usage of callable debt here. These callbacks induce a transfer of wealth out of equity holders' pockets while debt holders are paid in full.

To further investigate this problem, I compute the value of the aforementioned securities, in particular that of equity. To simplify the analysis, I assume that $\underline{V} = 0$, so that there will be no residual value in the event of firm liquidation, eliminating the need to specify the priority of residual claims among equity holders and debt holders. Let V_s^E denote the equity

⁴⁰Although callable debt is usually redeemed at a premium, the specific value of the premium does not play a role in this model and is therefore without loss of generality assumed to be zero.

value, which is defined by

$$V^E(W_t) = E \left(\int_t^\tau e^{-rs} dI_s | W_t \right) ,$$

where W_t , the manager's continuation utility, can be transferred to M_t , the credit line balance, through the relationship defined in Proposition 8.

The implementation requires equity holders to commit to the particular capital structure specified in the optimal contract by redeeming the outstanding debt at the time of the uncertainty shock. Do they always find it preferable to recall debt when uncertainty is high? The answer is hardly yes, as equity holders can usually withdraw investment in practice and default on any debt obligation. In this model, let $\hat{D} - D$ measure the value of debt redemption at the time of the volatility shock. Equity holders will find it optimal to default when $\hat{V}^E$, the value from maintaining the firm, is lower than $\hat{D} - D$, the cost of doing so. On the contrary, under the limited commitment contract, $\tilde{V}(\tilde{W}^C) = \underline{V}$ for both states, implying an identical face value of long-term debt before and after regime switching. That is, the capital structure of the limited commitment contract can be maintained without a tendency on the part of equity holders to default ex post.

Equity holders making ex post default decisions is common in both financial research and in practice. There is a large body of literature studying the endogenous default decision of equity holders and the conflict with debt holders, notably [Leland \(1994\)](#), [Leland and Toft \(1996\)](#) and [He and Xiong \(2012\)](#). In this paper, I do not characterize the exact default boundary of equity holders; rather, the point I want to make is that equity holders cannot (credibly) commit to not defaulting for the sake of the entire firm when there is a chance of their finding default preferable ex post.

In addition to justifying the prevalence of limited commitment contracts in practice, the equivalence between the commitment to contract termination time and the commitment to capital structure also offers an empirically testable hypothesis: the investors of more distressed firms are more likely to withdraw their investment, default on firms' debt and alter firms' capital structure. The degree of distress can be a potential proxy for the commitment power firms have over their labor contracts which is difficult to observe.

References

- Abreu, D., Pearce, D., and Stacchetti, E. 1990. Toward a theory of discounted repeated games with imperfect monitoring. *Econometrica*, 58:1041–1063.
- Ai, H. and Li, R. 2014. Investment and CEO compensation under limited commitment. *Journal of Financial Economics*, forthcoming.
- Anderson, R. W., Bustamante, M. C., Guibaudo, S., and Zervos, M. 2016. Agency, firm growth and managerial turnover. Working paper.
- Andrade, G. and Kaplan, S. N. 1998. How costly is financial (not economic) distress? evidence from highly leveraged transactions that became distressed. *The Journal of Finance*, 53(5):1443–1493.
- Aretz, K., Banerjee, S., and Pryshchepa, O. 2017. In the path of the storm: Does distress cause non-financial firms to risk-shift? Working paper.
- Atkeson, A. 1991. International lending with moral hazard and risk of repudiation. *Econometrica*, 59:1069–1089.
- Atkeson, A., Eisfeldt, A., and Weill, P.-O. 2014. Measuring the financial soundness of us firms 1926-2012. Working paper. University of California-Los Angeles.
- Berk, J., Stanton, R., and Zechner, J. 2010. Human capital, bankruptcy, and capital structure. *Journal of Finance*, 65:891–926.
- Biais, B., Mariotti, T., Plantin, G., and Rochet, J.-C. 2007. Dynamic security design: Convergence to continuous time and asset pricing implications. *Review of Economic Studies*, 74:345–390.
- Bloom, N. 2009. The impact of uncertainty shocks. *Econometrica*, 77:623–685.
- Bolton, P., Chen, H., and Wang, N. 2011. A unified theory of tobin’s q, corporate investment, financing, and risk management. *Journal of Finance*, 66:1545–1578.
- Bolton, P., Wang, N., and Yang, J. 2015. A theory of liquidity and risk management based on the inalienability of risky human capital. Working paper. National Bureau of Economic Research.
- Brunnermeier, M. K. and Sannikov, Y. 2014. A macroeconomic model with a financial sector. *American Economic Review*, 104(2):379–421.
- Campello, M., Giambona, E., Graham, J., and Harvey, C. 2011. Liquidity management and corporate investment during a financial crisis. *Review of Financial Studies*, 24:1944–1979.
- Cheng, I.-H., Hong, H., and Scheinkman, J. A. 2015. Yesterday’s heroes: Compensation and risk at financial firms. *The Journal of Finance*, 70(2):839–879.
- DeMarzo, P., Fishman, M., He, Z., and Wang, N. 2012. Dynamic agency and the q theory of investment. *Journal of Finance*, 67.
- DeMarzo, P. and Sannikov, Y. 2006. Optimal security design and dynamic capital structure in a continuous-time agency model. *Journal of Finance*, 61:2681–2724.

- DeMarzo, P. M., Livdan, D., and Tchistyi, A. 2013. Risking other people’s money: Gambling, limited liability, and optimal incentives. Working paper.
- Di Tella, S. 2013. Uncertainty shocks and balance sheet recessions. Working paper. Stanford University.
- Eisdorfer, A. 2008. Empirical evidence of risk shifting in financially distressed firms. *The Journal of Finance*, 63(2):609–637.
- Eisfeldt, A. and Kuhnen, C. 2013. CEO turnover in a competitive assignment framework. *Journal of Financial Economics*, 109:351–372.
- Ellul, A., Pagano, M., and Schivardi, F. 2014. Employment and wage insurance within firms: Worldwide evidence. Working paper. Indiana University, University of Naples Federico II, and LUISS.
- Frydman, C. and Jenter, D. 2010. CEO compensation. *Annual Review of Financial Economics*, 2:75–102.
- Gilje, E. P. 2016. Do firms engage in risk-shifting? empirical evidence. *Review of Financial Studies*, 29(11):2925–2954.
- Grochulski, B. and Zhang, Y. 2011. Optimal risk sharing and borrowing constraints in a continuous-time model with limited commitment. *Journal of Economic Theory*, 146:2356–2388.
- Grochulski, B. and Zhang, Y. 2013. Market-based incentives. Working paper. Federal Reserve Bank of Richmond and University of Texas-Austin.
- He, Z. and Krishnamurthy, A. 2013. Intermediary asset pricing. *American Economic Review*, 103:732–770.
- He, Z. and Xiong, W. 2012. Rollover risk and credit risk. *Journal of Finance*, 67:391–430.
- Hoffmann, F. and Pfeil, S. 2010. Reward for luck in a dynamic agency model. *Review of Financial Studies*, 23:3329–3345.
- Huang, J., Sialm, C., and Zhang, H. 2011. Risk shifting and mutual fund performance. *Review of Financial Studies*, 24(8):2575–2616.
- Jensen, M. C. and Meckling, W. H. 1976. Theory of the firm: Managerial behavior, agency costs and ownership structure. *Journal of Financial Economics*, 3(4):305–360.
- Jenter, D. and Kanaan, F. 2010. CEO turnover and relative performance evaluation. Working paper. Stanford University.
- Kaplan, S. and Minton, B. 2012. How has CEO turnover changed. *International Review of Finance*, 12:57–87.
- Kaplan, S. and Rauh, J. 2010. Wall street and main street: What contributes to the rise in the highest incomes. *Review of Financial Studies*, 23:1004–1050.
- Landier, A., Sraer, D. A., and Thesmar, D. 2015. The risk-shifting hypothesis: Evidence from subprime originations. Working paper.

- Leland, H. E. 1994. Corporate debt value, bond covenants, and optimal capital structure. *Journal of Finance*, 49:1213–1252.
- Leland, H. E. and Toft, K. B. 1996. Optimal capital structure, endogenous bankruptcy, and the term structure of credit spreads. *Journal of Finance*, 51:987–1019.
- Levin, J. 2003. Relational incentive contracts. *American Economic Review*, 93:835–857.
- Li, R. 2012. Optimal contract design over the business cycle. Working paper, Purdue University.
- Li, R. and Williams, N. 2017. Optimal contracts with hidden risk. Working paper.
- Miao, J. and Zhang, Y. 2014. A duality approach to continuous-time contracting problems with limited commitment. *Journal of Economic Theory*.
- Moreno-Bromberg, S. and Roger, G. 2016. Leverage and risk taking. Working paper.
- Narita, F. 2011. Hidden actions, risk-taking, and uncertainty shocks. Working paper, University of Minnesota.
- Opp, M. M. and Zhu, J. Y. 2015. Impatience versus incentives. *Econometrica*, 83(4):1601–1617.
- Palacios, M. and Stomper, A. 2014. Training the doubtful and timid. Working paper, Vanderbilt University and Humboldt University.
- Peters, F. and Wagner, A. 2014. The executive turnover risk premium. *Journal of Finance*, 69(4):1529–1563.
- Philippon, T. and Reshef, A. 2012. Wages and human capital in the u.s. financial industry: 1909–2006. *Quarterly Journal of Economics*, 127:1551–1609.
- Piskorski, T. and Tchistyi, A. 2010. Optimal mortgage design. *Review of Financial Studies*, 23:3098–3140.
- Rauh, J. D. 2008. Risk shifting versus risk management: Investment policy in corporate pension plans. *Review of Financial Studies*, 22(7):2687–2733.
- Ray, D. 2002. The time structure of self-enforcing agreements. *Econometrica*, 70:547–582.
- Sannikov, Y. 2008. A continuous-time version of the principal-agent problem. *Review of Economic Studies*, 75:957–984.
- Thomas, J. and Worrall, T. 1988. Self-enforcing wage contracts. *Review of Economic Studies*, 61:541–554.
- Wong, T.-Y. 2017. Dynamic agency and endogenous risk-taking. Working paper.
- Zhu, J. Y. 2013. Optimal contracts with shirking. *Review of Economic Studies*, 80:812–839.