# Managing a Housing Boom

Jason Allen, Daniel L. Greenwald*

February 16, 2018

(Preliminary and Incomplete Draft, Not for Circulation)

**Abstract**

We investigate how macroprudential policies intended to dampen rises in debt and house prices are influenced by segmentation in the housing and mortgage market. We develop a modeling framework with two mortgage submarkets: a government-insured sector with loose LTV limits and tight PTI limits, and an uninsured sector displaying the reverse pattern. This form of heterogeneity is modeled after the Canadian mortgage system, but is common in countries around the world. We find that this segmentation has important consequences for the effectiveness of macroprudential policy. While tightening payment-to-income (PTI) limits is highly effective at dampening a housing boom in a one-sector system, tightening these limits in the insured sector *only* is much weaker, due to substitutions into the uninsured sector. In contrast, the effect of tightening loan-to-value (LTV) limits in the uninsured sector is *strengthened* by market segmentation, causing price-rent ratios to fall, while the same tightening in the insured sector would counterproductively cause price-rent ratios to rise.

*Allen: Bank of Canada, email: jallen@bank-banque-canada.ca. Greenwald: Massachussetts Institute of Technology Sloan School; email: dlg@mit.edu.

# 1 Introduction

The prevalence in recent years of large booms in house prices, and the disruptive consequences of large crashes, have drawn increased attention to the role of macroprudential policy in stabilizing housing and mortgage markets. But despite this uptick in salience, much remains to be learned theoretically about the effective conduct of these policies. For example, Jácome and Mitra (2015) note that while limits on loan-to-value (LTV) and payment-to-income (PTI) ratios are widespread around the world, there are few theoretical frameworks available to guide regulators in their application.

In this paper, we develop a realistic yet tractable general equilibrium framework, extending Greenwald (2017), that can be used to evaluate the effects of the essential macroprudential tools available to policymakers. Our key innovation is to capture an important but largely overlooked feature of many mortgage markets around the world: a segmented market, with different submarkets offering different credit standards. We design this framework to reflect the Canadian mortgage system, which is divided between a government-insured sector that offers high LTV ratios (max of 95%) but imposes tight PTI requirements (max of 44%), and an uninsured sector that uses much tighter LTV limits (max of 80%) but leaves PTI ratios essentially unlimited. While we focus on Canada, it is worth noting that such segmentation is common, and can be seen for example in the US, where the FHA, the GSEs, and private label securitizers all employed different underwriting policies during the US housing boom.

Using this framework, we find that market segmentation has a first-order influence on the consequences of macroprudential policy, whose effects depend crucially on which submarket-constraint combination is targeted. While a single sector model would show a tightening of PTI limits to be highly effective at dampening a rise in debt and house prices, the tightening these limits in the government-insured sector *only* — a relatively easy policy for regulators to implement — is substantially weakened. Essentially, borrowers react to this tightening by switching into the uninsured sector where PTI limits are loose but housing collateral is scarce, putting countervailing upward pressure on housing demand and house prices. Tightening PTI limits in the uninsured sector, in contrast, does not trigger such a countervailing force, since borrowers with this submarket-constraint pairing are far from the insured-uninsured boundary and therefore do not exhibit sector switching behavior.

A reverse intuition holds for shifts in LTV limits. Tightening LTV limits in the insured sector has a similar effect as in a single-sector model. As found in Greenwald (2017), this policy counterproductively causes price-rent ratios to *rise*, due to a "constraint switching effect" where an increase in the fraction of LTV-constrained borrowers — who can use additional housing to relax their borrowing limits — increases the demand for housing collateral. Perhaps surprisingly, this result does *not* hold when LTV limits are tightened in the uninsured sector. Importantly, this policy switches marginal borrowers from the uninsured sector, where they are LTV-constrained, to the insured sector, where they become PTI-constrained. This switch reduces those borrowers' collateral needs, dampening housing demand, and ensuring that price-rent ratios fall instead of rise.

Looking ahead toward the complete draft, we intend to explore further macroprudential policies, such as adjustments to the amortization rates in the two sectors, and the differential implications of monetary policy in segmented markets. Moreover, the availability of high-quality Canadian data, combined with the implementation of a number of different macroprudential policies in recent years offers excellent opportunities for model validation, by checking the observed responses to these policies against the implied responses in the model. We also aim to capture an additional key feature of modern housing booms — that they are often concentrated in specific regions, while other regions have much more muted trends in house prices. Our investigation will focus on how policies differentially impact these two types of regions, and in particular whether they widen or bridge gaps between them.

**Related Literature.** To be completed.

**Overview.** The rest of the draft is organized as follows.

## 2 Background: The Canadian Mortgage Market

The Canadian mortgage market is divided into two sectors. First, there is the "insured" sector, in which default risk on mortgages is guaranteed by the government. This sector borrowers to obtain loans with up to 95% LTV ratios, but restricts them to a maximum PTI ratio of 44% (also known as total debt service, or "TDS"). Second, there is an "uninsured" sector, in which default risk on mortgages is not guaranteed. Mortgages in this sector are required to have an LTV ratio below 80%, but do not have any formal cap on PTI limits.

The resulting system is therefore captured by the diagram in Figure 1. This figure uses "I" to denote the insured submarket, "U" to denote the uninsured market, and $\theta$

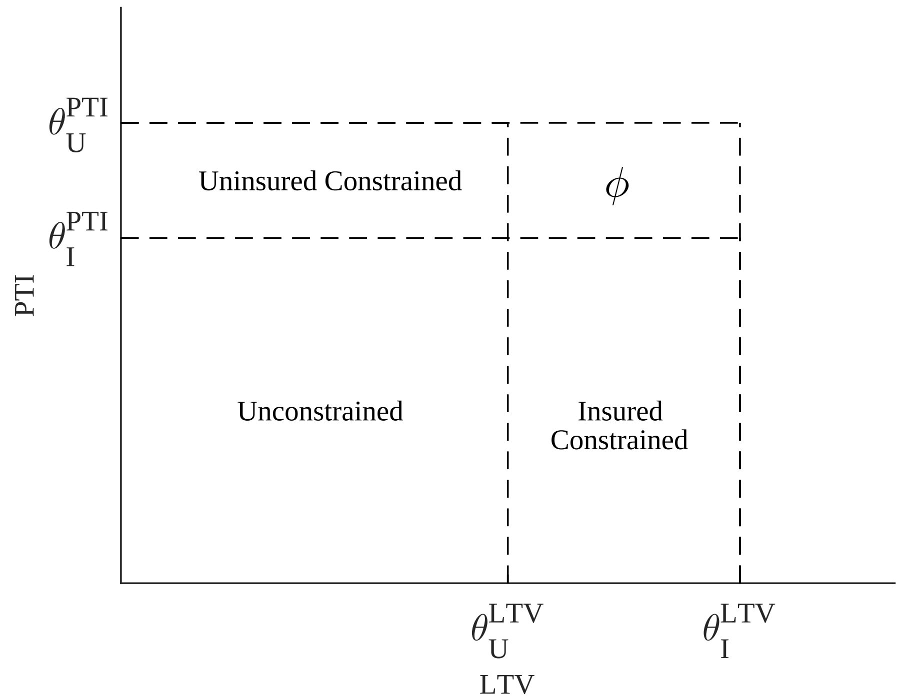
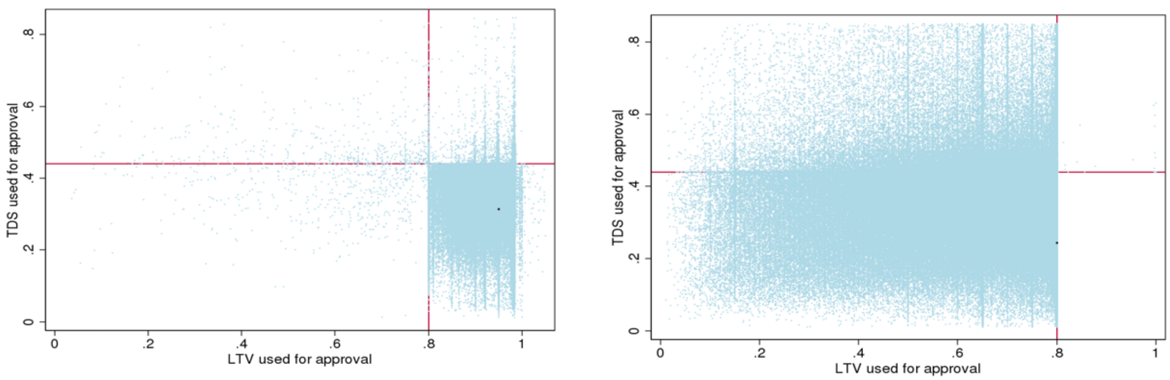



Figure 1: Borrowing Constraints by Sector

to denote a maximum ratio, so that e.g., $\theta_I^{LTV}$ indicates the maximum LTV ratio in the insured sector. According to our definitions of these submarkets, borrowers wishing to take out loans with a PTI ratio below $\theta_I^{PTI}$ and with an LTV ratio below $\theta_U^{LTV}$ are "unconstrained" meaning they do not hit either credit limit in either submarket, and are free to borrow from either. However, any borrower seeking to go beyond this rectangle will be restricted to a single market. In particular, borrowers seeking a PTI ratio higher than $\theta_I^{PTI}$ can only obtain such a loan in the uninsured sector, while borrowers seeking an LTV ratio higher than $\theta_U^{LTV}$ can only obtain such a loan in the insured sector. No loans can be given simultaneously violating both of these limits.

To make sure that this definition fits with real-world behavior, Figure 2 displays the LTV and PTI (also known as TDS) ratios for new purchase loans by sector. Corresponding almost perfectly with our definitions in Figure 1, these data confirm that nearly all high-LTV borrowers go to the insured sector, and among these, extremely few surpass the typical 44% PTI limit. Similarly, nearly all high-PTI borrowers go to the uninsured sector, and among these, virtually none take on more than 80% LTV. This pattern therefore validates our stylized definition of the two sectors, allowing us to proceed with construction of the model.




(a) Insured Sector (b) Uninsured Sector

Figure 2: Borrower Ratios: Insured vs. Uninsured Sectors

# 3 Model

This section constructs the modeling framework. The overall approach follows Greenwald (2017), with the main innovations appearing in the treatment of the mortgage submarkets.

**Demographics.** The economy is populated by two families of agents — borrowers and savers — who are denoted by the subscripts $b$ and $s$, respectively. Agents are infinitely lived and types are permanent, with a fixed measures $\chi_b$ of borrowers and $\chi_s = 1 - \chi_b$ of savers. A complete set of contracts over consumption and housing services can be traded within each family, but not across families.

**Preferences.** Each agent of type $j \in \{b, s\}$ maximize expected utility of the form

$$V_j = \mathbb{E}_t \sum_{k=0}^{\infty} \beta_j^k u(c_{j,t+k}, h_{j,t+k}, n_{j,t+k}) \tag{1}$$

where the inputs to the utility function are: nondurable consumption $c$, housing services $h$, and labor supply $n$. Period utility takes the separable form

$$u_j(c, n, h) = \log(c) + \xi \log(h) - \eta_j \frac{n^{1+\varphi}}{1 + +\varphi}.$$

For notation, let e.g., $u^c_{j,t}$ denote the derivative of the utility function of agent $j$ with respect to $c$, and let

$$\Lambda_{j,t+1} = \beta_j \frac{u^c_{j,t+1}}{u^c_{j,t}}$$

denote the stochastic discount factor of an agent of type $j$.

**Mortgages.** The mortgage sector consists of two submarkets: one for government insured mortgages, denoted $I$, and one for uninsured mortgages, denoted $U$. Borrowers obtaining new loans choose freely which submarket they prefer to enter. A mortgage in sector $j$ is a nominal perpetuity with a fixed interest rate and geometrically decaying coupon. This means that the borrower in submarket $j$ pays back a constant fraction $\nu_j$ of the remaining principal balance in each period, so that the payment at time $t+k$ on \$1 of debt issued at time $t$ is $\$(1-\nu_j)^k(r^*_t + \nu_j)$ for all $k$ until the loan is renewed, where $r^*_t$ is the fixed interest rate. Each mortgage in submarket $j$ is renewed each period with probability $\rho_j$, at which time the borrower prepays her existing balance and can take out a new loan.

To induce changes in real mortgage rates similar to shifts in mortgage spreads or term premia, I introduce a proportional tax $\Delta_{q,t}$ on all future mortgage payments received by the saver on loans in submarket $j$ originated at time $t$, subject to the process

$$\Delta_{q,t} = (1-\phi_q)\mu_q + \phi_q \Delta q, t-1 + \varepsilon_{q,t}.$$

This tax does not correspond to any real-world policy, but is instead a parsimonious way to create a wedge between long rates and average future short rates, which needed to match the large and volatile discrepancy between these two items in the data. As a result, I rebate the proceeds from this tax back to the savers in lump sum fashion each period.

The size of a new loan is limited by both an LTV constraint and a PTI constraint, defined by

$$\frac{m^*_{i,t}}{p^h_t h^*_{i,t}} \leq \theta^{LTV}_{,t} j, t, \qquad\qquad \frac{(r^*_{j,t} + \nu_j + \alpha) m^*_{i,t}}{w_t n_{i,t} e_{i,t}} + \omega \leq \theta^{PTI}_{,t} j, t$$

respectively. The LTV constraint caps the ratio of the balance on the new loan $m^*_{i,t}$, against that borrower's housing collateral $p^h_t h^*_{i,t}$, where $h^*_{i,t}$ is the quantity of newly purchased housing. These constraints are applied at origination only.

The PTI constraint caps the ratio of the borrower's debt and related payments to her income. The numerator on the left hand side is the initial mortgage payment, where the offset term $\alpha$ is used in calibration to adjust for property taxes and insurance payments, as well as to adjust for the difference in amortization between true fixed-rate mortgages and the model's geometrically decaying coupon loans. The denominator is equal to labor income, which is shifted for each borrower by the idiosyncratic productivity shock $e_{i,t}$, drawn i.i.d. across borrowers and time from a distribution with c.d.f. $\Gamma_e$. Due to complete markets within the borrower family, this income shock has no impact on borrower consumption allocations, but is instead used to create variation among borrowers, allowing for endogenous sorting into the two submarkets, and ensuring that endogenous fractions are limited by each constraint. Finally, $\omega$ is used in calibration to adjust for debt obligations other than mortgages.

These constraints imply the maximum loan balances

$$\bar{m}^{LTV}_{i,j,t} = \theta^{LTV}_{,t} j, t p^h_t h^*_{i,t} \qquad \bar{m}^{PTI}_{i,j,t} = \frac{(\theta^{PTI}_{,t} j, t - \omega) w_t n_{i,t} e_{i,t}}{r^*_{j,t} + \nu_j + \alpha}$$

which define the maximum that borrower $i$ can obtain in submarket $j$ under each limit. Since both limits must be satisfied simultaneously, the maximum loan balance in submarket $j$ is defined by $\bar{m}_{i,j,t} = \min(\bar{m}^{LTV}_{i,j,t}, \bar{m}^{PTI}_{i,j,t})$.

At equilibrium, the optimal policy will be to choose the insured space — which has looser income-based constraints and tighter collateral-based constraints — if and only if $e_{i,t}$ exceeds an endogenous threshold $e^*_t$. We define $F_{j,t}$ to be the fraction of borrowers who choose sector $j$, so that $F_{U,t} = \Gamma_e(e^*_t)$ and $F_{I,t} = 1 - \Gamma_e(e^*_t)$.

**Macroprudential Shocks.** To allow for experimentation with macroprudential policies, we can allow the maximum LTV and PTI ratios for each submarket to vary. In particular, we specify AR(1) processes

$$\begin{aligned}
\log \theta^{LTV}_{j,t} &= (1 - \phi_\theta) \log \bar{\theta}^{LTV}_j + \phi_\theta \log \theta^{LTV}_{j,t} + \varepsilon^{LTV}_{j,t} \\
\log \theta^{PTI}_{j,t} &= (1 - \phi_\theta) \log \bar{\theta}^{PTI}_j + \phi_\theta \log \theta^{PTI}_{j,t} + \varepsilon^{PTI}_{j,t}
\end{aligned}$$

that generate time variation in debt limits.

**Housing.** The final asset in the economy is housing, which is divisible in fixed total supply $\bar{H}$ and produces a service flow equal to its stock. To focus on the use of housing as a collateral asset, We fix the saver's demand for housing at $h_{s,t} = \bar{H}_{s,t}$, which ensures that the borrower is the marginal pricer of housing at equilibrium.

**Taxation.** Each household's labor income is subject to proportional taxation at rate $\tau$. Tax revenues are returned to borrowers and savers in lump-sum transfers equal to the average amount paid by that type.

**Bonds.** We introduce a risk-free one-period bond that can be used as a policy instrument by the central bank. An investment of \$1 at time $t$ yields a guaranteed nominal payoff of \$$R_t$ at time $t+1$. This bond is in zero net supply and cannot be shorted, which means that it will be held by savers only at equilibrium.

**Representative Borrower's Problem.** The individual borrower's problem aggregates to that of a representative borrower. This representative borrower's endogenous state variables are the principal balances $m_{j,t-1}$ and promised interest payments $x_{j,t-1}$ and promised payments $x_{j,t-1}$ for each submarket $j \in \{I, U\}$, as well as total borrower housing $h_{b,t-1}$, which follow the laws of motion

$$m_{j,t} = \rho F_{j,t} m^*_{j,t} + (1-\rho)(1-\nu_j)\pi_t^{-1} m_{j,t-1} \tag{2}$$

$$x_{j,t} = \rho F_{j,t} r^*_{j,t} m^*_{j,t} + (1-\rho)(1-\nu_j)\pi_t^{-1} x_{j,t-1} \tag{3}$$

where $F_{j,t}$ is the fraction of borrowers taking out new loans who choose submarket $j$. The representative borrower chooses nondurable consumption $c_{b,t}$, labor supply $n_{b,t}$, the size of newly purchased houses $h^*_{b,t}$, and the face value of newly issued mortgages in each sector $m^*_t$, and the income threshold at which to send borrowers to the insured sector $e^*_t$ to maximize (1) using the aggregate utility function

$$u_b(c_{b,t}, h_{b,t}, n_{b,t}) = \log(c_{b,t}/\chi_b) + \xi \log(h_{b,t}/\chi_b) + \eta_j \frac{n_{b,t}^{1+\varphi}}{1+\varphi}$$

subject to the budget constraint

$$c_{b,t} \leq \underbrace{(1-\tau_y) w_t n_{b,t}}_{\text{labor income}} - \underbrace{\rho p^h_t \left(h^*_{b,t} - h_{b,t-1}\right)}_{\text{housing purchases}} - \underbrace{\delta p^h_t h_{b,t-1}}_{\text{maintenance}} + \underbrace{T_{b,t}}_{\text{transfers}}$$
$$+ \sum_{j \in \{I,U\}} \Bigg\{ \underbrace{\rho\left(m^*_{j,t} - (1-\nu_j)\rho \pi_t^{-1} m_{j,t-1}\right)}_{\text{net new debt issuance}} - \underbrace{\pi_t^{-1}(1-\tau_y) x_{j,t-1}}_{\text{interest payment}} - \underbrace{\pi_t^{-1} \nu_j m_{j,t-1}}_{\text{principal payment}} \Bigg\}$$

and the aggregate borrowing constraints

$$m_{U,t}^* \leq \int^{\bar{e}_{U,t}} \bar{m}_{U,t}^{PTI} e_i \, d\Gamma_e(e_i) + \bar{m}_{U,t}^{LTV} \left(\Gamma_e(e_t^*) - \Gamma_e(\bar{e}_{U,t})\right)$$
$$m_{I,t}^* \leq \int_{e_t^*}^{\bar{e}_{I,t}} \bar{m}_{I,t}^{PTI} e_i \, d\Gamma_e(e_i) + \bar{m}_{I,t}^{LTV} \left(1 - \Gamma_e(\bar{e}_{I,t})\right).$$

where

$$\bar{e}_{j,t} = \frac{\bar{m}_{j,t}^{LTV}}{\bar{m}_{j,t}^{PTI}}, \quad j \in \{U, I\}$$

represent the threshold income levels in each submarket at which a borrower crosses from being PTI-constrained to being LTV-constrained.

**Representative Saver's Problem.** The individual saver's problem similarly aggregates to that of a representative saver. This representative saver's problem largely parallels the representative borrower's problem. The state variables are the balances and promised payments on each type of loan $m_{j,t}$ and $x_{j,t}$, again following (2) and (3). The representative saver chooses nondurable consumption $c_{s,t}$, labor supply $n_{s,t}$, and the average balance on newly issued mortgages $m_t^*$. [TBC]

**Productive Technology.** The productive technology follows the standard New Keynesian assumptions of a competitive final goods producer and a continuum of monopolistically competitive intermediate goods producers. Both types of firm are owned by the saver at equilibrium. The final goods producer solves the static problem

$$\max_{y_t(i)} P_t \left( \int y_t(i)^{\frac{\lambda-1}{\lambda}} \, di \right)^{\frac{\lambda}{\lambda-1}} - \int P_t(i) y_t(i) \, di$$

where $y_t(i)$ is the intermediate good produced by firm $i$, $P_t(i)$ is the price of that good, and $P_t$ is the price of the final good.

Intermediate firms choose their price $P_t(i)$, and operate the linear production function

$$y_t(i) = a_t n_t(i)$$

to meet the demand of the final good producer, where $n_t(i)$ denotes labor hours employed by firm $i$, and $a_t$ is total factor productivity (TFP), which follows an AR(1) in logs:

$$\log a_{t+1} = (1-\phi_a)\mu_a + \phi_a a_t + \varepsilon_{a,t+1}.$$

Intermediate goods producers are subject to pricing frictions of the Calvo-Yun form, meaning that fraction $\zeta$ of firms cannot adjust their price in a given period, while the other $1-\zeta$ fraction are free to do so.

**Monetary Authority.** The monetary authority follows a Taylor rule similar to the specification in Smets and Wouters (2007), given by

$$\begin{aligned}\log R_t = \log \bar{\pi}_t &+ \phi_r \left(\log R_{t-1} - \log \bar{\pi}_{t-1}\right) \\ &+ (1-\phi_r)\left[\left(\log R_{ss} - \log \pi_{ss}\right) + \psi_\pi \left(\log \pi_t - \bar{\pi}_t\right)\right]\end{aligned}$$

where the subscript "ss" indicates the steady state value. The inflation target $\bar{\pi}_t$ follows an AR(1) in logs

$$\log \bar{\pi}_t = (1-\phi_{\bar{\pi}}) \log \pi_{ss} + \phi_{\bar{\pi}} \log \bar{\pi}_{t-1} + \varepsilon_{\bar{\pi},t}.$$

These shocks correspond to near-permanent changes in inflation that can cause nominal rates to move relative to real rates.

## 3.1 Model Solution

This section provides the key equilibrium conditions of the model. The full set of conditions will be available in the complete draft of the paper.

For the dynamics of debt and the allocation of borrowers across submarkets, the key equilibrium relation is the borrower's optimality condition for $e_t^*$

$$\mu_{U,t}\bar{m}_{U,t}^{LTV} = \mu_{I,t}\bar{m}_{I,t}^{PTI} e_t^*$$

which sets the benefit to the threshold borrower of taking out a loan to be equal in both submarkets, under the assumption (verified at equilibrium) that the threshold borrower would be LTV-constrained in the uninsured space, but PTI-constrained in the insured space. This net benefit is equal to the product of the size of the loan that could be taken out and the benefit of an additional dollar of debt in that submarket, which is equal to the multiplier on the debt limit for that submarket. Solving yields the threshold value

$$e_t^* = \frac{\mu_{U,t}\bar{m}_{U,t}^{LTV}}{\mu_{I,t}\bar{m}_{I,t}^{PTI}}.$$

In the special case that the amortization rates $\nu_j$ are identical across submarkets, we have $\mu_{I,t} = \mu_{U,t}$. In this case, optimality implies that that borrowers should simply take whichever submarket offers them a bigger maximum loan size, yielding $e_t^* = \bar{m}_{U,t}^{LTV}/\bar{m}_{I,t}^{PTI}$.

For the dynamics of house prices, we can turn to the borrower's optimality condition for new housing:

$$p_t^h = \frac{u_{b,t}^h/u_{b,t}^c + \mathbb{E}_t\left\{\Lambda_{b,t+1}p_{t+1}^h\left[1-\delta-(1-\rho)\mathcal{C}_{t+1}\right]\right\}}{1-\mathcal{C}_t}$$

The term $\mathcal{C}_t$ represents the marginal collateral value of housing, defined by

$$\mathcal{C}_t = \sum_{j\in\{I,U\}} \mu_{j,t}F_{j,t}^{LTV}\theta_{j,t}^{LTV} \tag{4}$$

where $F_{j,t}^{LTV}$ is the fraction of borrowers who are in submarket $j$ and constrained by LTV, computed as

$$\begin{aligned} F_{U,t}^{LTV} &= \Gamma_e(e_t^*) - \Gamma_e(\bar{e}_{U,t}) \\ F_{I,t}^{LTV} &= 1 - \Gamma_e(\bar{e}_{I,t}). \end{aligned}$$

This term $\mathcal{C}_t$ equals the value to the borrower from an extra dollar of average housing wealth due to the extra debt it allows her to obtain. An extra dollar of housing wealth relaxes LTV constraints in submarket $j$ by $\theta_{j,t}^{LTV}$. But since not all borrowers are LTV-constrained, this relaxation only loosens overall credit limits by $F_{j,t}^{LTV}\theta_{j,t}^{LTV}$ in each submarket. Finally, each additional dollar of credit is valued at rate $\mu_{j,t}$, the multiplier on the credit constraint in submarket $j$. In the special case with only an insured mortgage sector, we would obtain the case of Greenwald (2017), with

$$\mathcal{C}_t = \mu_t F_t^{LTV}\theta_{I,t}^{LTV} \tag{5}$$

where $\mu_t$ is the multiplier on the single budget constraint, and $F_t^{LTV}$ is the total LTV-constrained fraction of the population.

An increase in the collateral value $\mathcal{C}_t$ raises house prices and price-rent ratios. This gives rise to a "constraint switching" effect, as in Greenwald (2017). Specifically, since LTV-constrained borrowers are willing to pay a premium for additional housing collateral, which they can use to relax their borrowing limits, an increase in the fraction of LTV-constrained borrowers — here $F^{LTV}_{U,t}$ or $F^{LTV}_{I,t}$ — increases housing demand. However, this submarket structure opens the door to additional dynamics through the cutoff value $e^*_t$. Recall that borrowers with income shocks just below $e^*_t$ will be LTV-constrained in the uninsured market, while borrowers with income shocks just above $e^*_t$ will be PTI-constrained in the insured market. As a result, when $e^*_t$ increases, meaning that borrowers are switching from the insured to the uninsured space, those borrowers will also be switching from being PTI-constrained to being LTV-constrained. This increases $\mathcal{C}_t$, pushing up housing demand. Through this force, flows into the uninsured space to increase house prices and price-rent ratios, while flows out of the uninsured space reduce them.

## 3.2 Calibration

The calibration is a work in progress, but largely follows the methodology of Greenwald (2017) adapted to Canadian data. Calibrated values can be found in Table 1.

# 4 Results

This section presents the numerical results of the paper.

## 4.1 Simple Numerical Example

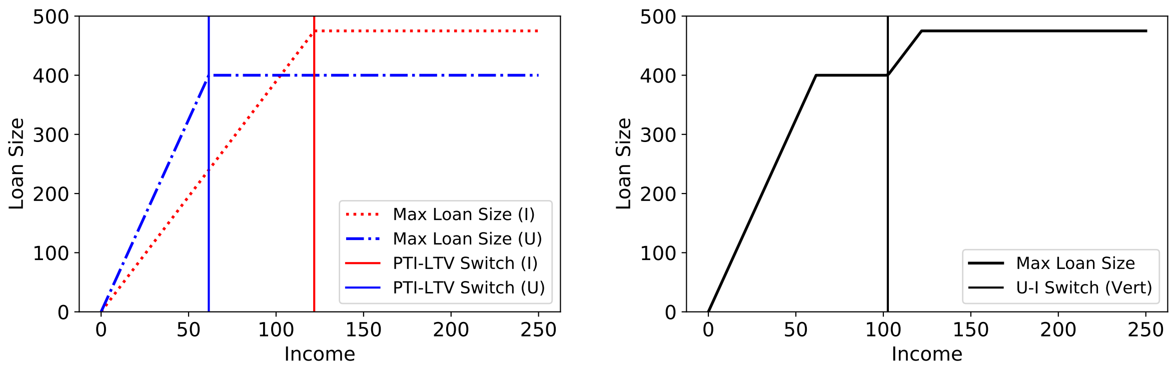
For intuition, it may be helpful to first consider the borrowing limit of an individual borrower, displayed in Figure 3. This plot fixes a single borrower's house value at $500k, and shows how this borrower's debt limit varies with her income. Figure 3a shows how this limit varies by submarket. First, the blue line labeled "Max Loan Size (U)" shows the debt limit in the uninsured space. For a low enough level of income, the borrower is PTI-constrained. As the borrower's income increases, she is able to obtain more and more credit in the uninsured market, until finally her maximum loan size under her PTI constraint is equal to her maximum loan size under her LTV limit. At this point, she becomes LTV-constrained and cannot obtain additional credit in this submarket no matter

| Parameter | Name | Value | Internal | Target/Source |
|---|---|---|---|---|
| | | *Demographics and Preferences* | | |
| Fraction of borrowers | $\chi_b$ | 0.319* | N | 1998 Survey of Consumer Finances |
| Income dispersion | $\sigma_e$ | 0.411* | N | Fannie Mae Loan Performance Data |
| Borr. discount factor | $\beta_b$ | 0.981 | Y | Value-to-income ratio (Survey) |
| Saver discount factor | $\beta_s$ | 0.990 | N | Avg. 10Y rate |
| Housing preference | $\xi$ | 0.25 | N | Davis and Ortalo-Magné (2011) |
| Borr. labor disutility | $\eta_b$ | 8.796 | Y | $n_{b,ss}/\chi_b = 1/3$ |
| Saver labor disutility | $\eta_s$ | 5.838 | Y | $n_{s,ss}/\chi_s = 1/3$ |
| Inv. Frisch elasticity | $\varphi$ | 1.0 | N | Standard |
| | | *Housing and Mortgages* | | |
| Mortgage amortization (U) | $\nu_U$ | 0.522% | N | See text |
| Mortgage amortization (I) | $\nu_I$ | 0.522% | N | See text |
| Income tax rate | $\tau_y$ | 0.204 | N | Elenev, Landvoigt, and Van Nieuwerburgh (2016) |
| Max LTV ratio (I) | $\bar{\theta}_I^{LTV}$ | 0.95 | N | See text |
| Max LTV ratio (U) | $\bar{\theta}_U^{LTV}$ | 0.8 | N | See text |
| Max PTI ratio (I) | $\bar{\theta}_I^{PTI}$ | 0.44 | N | See text |
| Max PTI ratio (U) | $\bar{\theta}_U^{PTI}$ | 0.7 | N | See text |
| Renewal Rate | $\rho$ | 0.05 | N | 5-Year Contract |
| PTI offset (taxes, etc.) | $\alpha$ | 0.313% | Y | $q^*_{ss} + \alpha$ = 9.97% (annualized) |
| PTI offset (other debt) | $\omega$ | 0.05 | N | See text |
| Term premium (mean) | $\mu_q$ | 0.124% | Y | Avg. mortgage rate |
| Term premium (pers.) | $\phi_q$ | 0.852* | N | Autocorr. of (mort. rate - 1Y rate), US data |
| Log housing stock | $\log \bar{H}$ | 2.178 | Y | $p^h_{ss} = 1$ |
| Log saver housing stock | $\log \bar{H}_s$ | 1.867 | Y | Canadian Survey Data |
| Housing depreciation | $\delta$ | 0.005 | N | Standard |
| | | *Productive Technology* | | |
| Productivity (mean) | $\mu_a$ | 1.099 | Y | $y_{ss} = 1$ |
| Productivity (pers.) | $\phi_a$ | 0.964 | N | Garriga, Kydland, and Sustek (2015) |
| Variety elasticity | $\lambda$ | 6.0 | N | Standard |
| Price stickiness | $\zeta$ | 0.75 | N | Standard |
| | | *Monetary Policy* | | |
| Steady state inflation | $\pi_{ss}$ | 1.005 | N | Avg. infl. expectations |
| Taylor rule (inflation) | $\psi_\pi$ | 1.5 | N | Standard |
| Taylor rule (smoothing) | $\phi_r$ | 0.89 | N | Campbell, Pflueger, and Viceira (2014) |
| Infl. target (pers.) | $\phi_{\bar{\pi}}$ | 0.994 | N | Garriga et al. (2015) |
| Macropru. policy (pers.) | $\phi_\theta$ | 0.99 | N | |

*Note:* The model is calibrated at quarterly frequency. Parameters denoted "Y" in the "Internal" column are internally calibrated, meaning that they are not set explicitly in closed form, but are instead chosen implicitly to match a particular moment at steady state. A star (*) indicates that the parameter is currently taken from Greenwald (2017) but needs to be recalibrated for the Canadian environment.

Table 1: Parameter Values: Baseline Calibration

how high her income goes. The threshold income level at which she switches from being PTI- to LTV-constrained is shown as the vertical line "PTI-LTV Switch (U)."




(a) Debt Limit by Submarket (b) Overall Debt Limit

Figure 3: Debt Limits

The red line "Max Loan Size (I)" similarly plots the borrower's maximum loan balance in the insured submarket. As in the uninsured case, the borrower is PTI-constrained when her income is low, but once her income reaches a high enough level, labeled "PTI-LTV Switch (I)," she becomes LTV-constrained and her debt limit plateaus. The key difference between the two submarkets is that the insured space has a tighter PTI limit and a looser LTV limit relative to the insured space. This means that debt limits in the insured market (i) grow more slowly with income in the PTI-constrained region of the insured submarket, (ii) plateau at a higher maximum loan size under the LTV constraint, and (iii) switch from being PTI- to LTV-constrained at a higher level of income.

Since a borrower can take a loan in either submarket, the borrower's overall debt limit is the upper envelope of the limits in the two submarkets, shown in Figure 3b. Because of the difference in shapes, the submarket offering the largest debt opportunity switches from the uninsured space (to the left) to the insured space (to the right) at the threshold income level shown in the vertical line "U-I Switch (Vert)." In the special case $\nu_U = \nu_I$, so that borrowers always choose the submarket offering the largest loan, the population of borrowers who are LTV-constrained and in the uninsured market are those whose incomes fall in the flat part of Figure 3b to the left of the "U-I Switch" line, while the population of borrowers who are both LTV-constrained and in the insured market are those whose incomes fall in the flat region of Figure 3b to the right of the "U-I Switch" line.

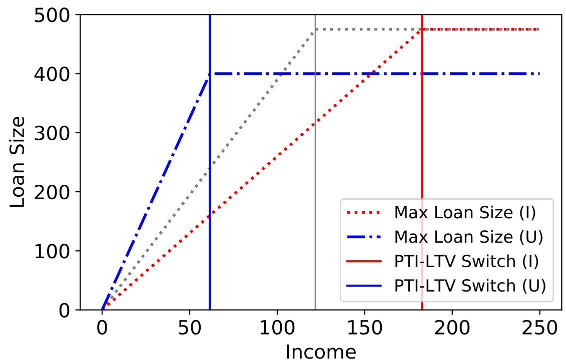
## 4.2 Responses: Macroprudential Policy

This section explores how the economy reacts to changes in the credit limit parameters $\theta^{LTV}_{j,t}$ and $\theta^{PTI}_{j,t}$, which we can interpret as shifts in macroprudential policy. To this end, we compute impulse responses after linearizing the model around the deterministic steady state. To isolate the importance of the two-submarket setup, these responses are contrasted to those from an alternative version of the model without an uninsured sector, where all mortgages must satisfy the credit standards of the insured sector. Since this paper considers macroprudential policy aimed at containing a housing boom, we consider negative shocks (credit tightenings).

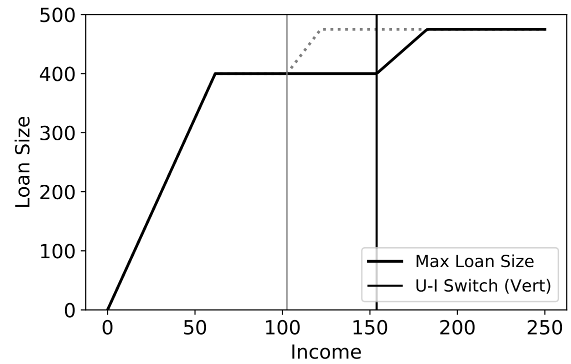
**Shocks to PTI Limits.** First, we can consider shocks to the PTI limits of each economy. To begin, Figure 5 shows the response to a -1% shock to the PTI limit in the insured space. The figure reveals that the paths for the Benchmark and No Uninsured models are dramatically different, demonstrating the importance of the submarket structure for macroprudential policy. In the No Uninsured model, this policy delivers substantial reductions in debt and price-rent ratios.[1] This is due to the constraint switching effect discussed at length in Greenwald (2017) — a tightening of PTI limits reduces the fraction of borrowers who are LTV-constrained, diminishing collateral demand and pushing down house prices (i.e., reducing $F^{LTV}_t$ in equation (5)). This fall in house prices in turn drives debt limits down further by tightening LTV limits. Thus, in environments with only a single submarket, tightening PTI limits is an effective way to dampen house prices and debt balances.

In contrast, the Benchmark model displays much smaller reductions in debt, debt limits, and price-rent ratios. To understand this result, we can augment the impulse responses with intuition from the simple numerical example defined in Section 4.1, whose response to a tightening of $\theta^{PTI}_{I,t}$ can be seen in Figure 4. Specifically, Figure (4a) shows that following this shock, debt limits grow more slowly with income in the insured submarket. This pushes the PTI-LTV switch to the right, reducing the LTV-constrained fraction in the insured space, which should depress house prices. However, there is also a countervailing force, shown in Figure 4b, as the U-I switching threshold is also shifted to the right, pushing more borrowers to switch into the uninsured space to escape tighter limits in the insured space. Importantly, these borrowers become LTV-constrained upon enter-

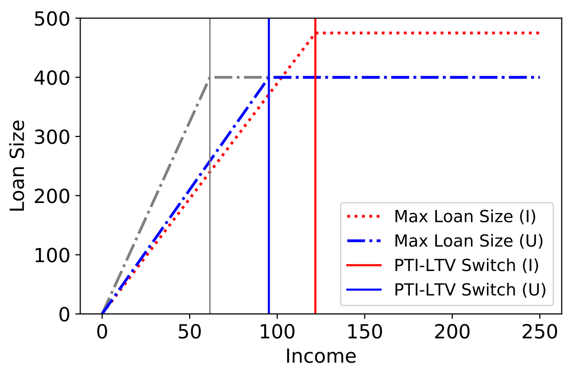
[1] For magnitudes, note that the size of the shock is 1%, not one percentage point. This means that relatively moderate policy like returning the PTI limit from its current level of 44% to its traditional level of 40% would correspond to a 9.1% fall in $\theta^{PTI}_{I,t}$.



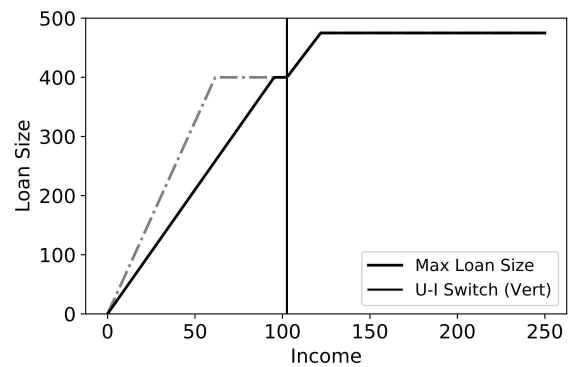

(a) Debt Limit by Submarket ($\theta_I^{PTI} \downarrow$)




(b) Overall Debt Limit ($\theta_I^{PTI} \downarrow$)




(c) Debt Limit by Submarket, ($\theta_U^{PTI} \downarrow$)




(d) Overall Debt Limit, ($\theta_U^{PTI} \downarrow$)

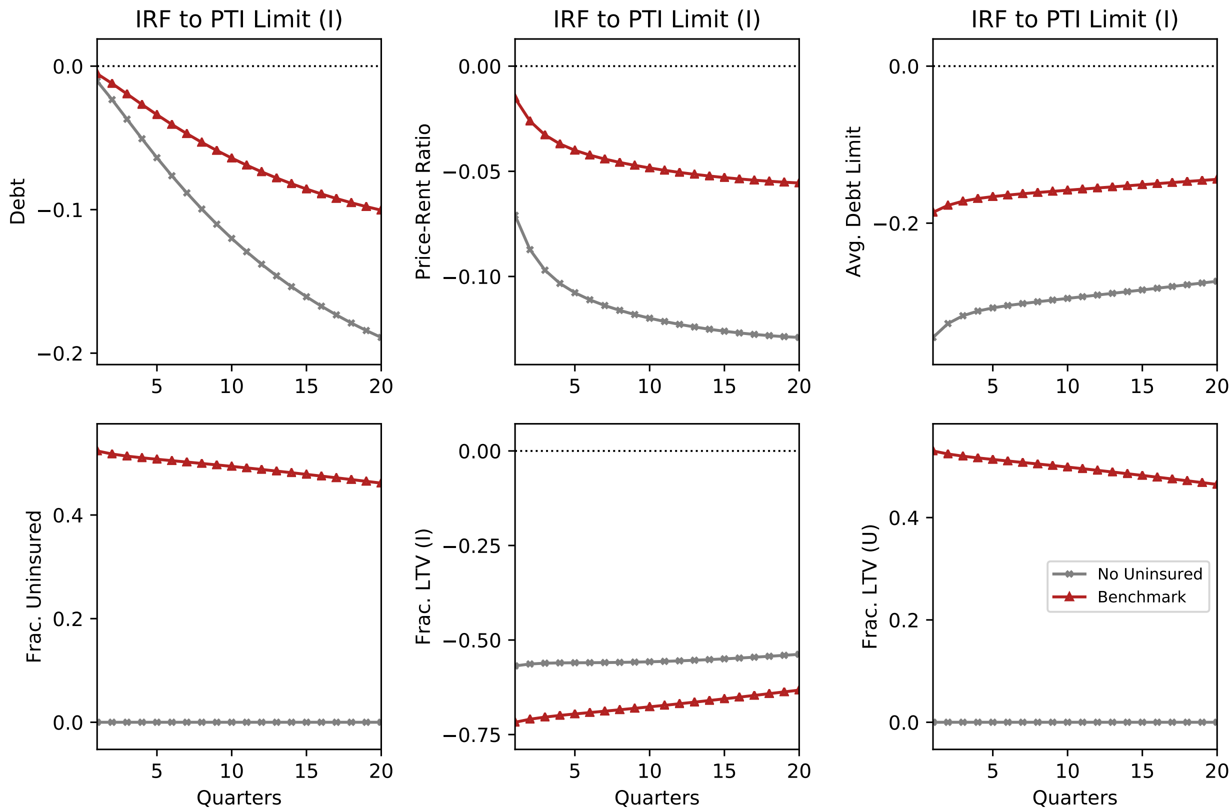
*Note:* Grey lines indicate the baseline case of Figure 3b, while the red, blue, and black lines indicate the responses following a reduction in $\theta_{j,t}^{PTI}$. The size of the shift in $\theta_{j,t}^{PTI}$ is exaggerated relative to the impulse reponses for visual clarity.

Figure 4: Example, PTI Limit Tightenings by Submarket

ing the uninsured space. This increase in $F_{U,t}^{LTV}$, seen clearly in Figure 5, boosts housing demand, leading to much smaller overall decreases in debt and price-rent ratios.

These results therefore indicate that the effects of tightening PTI limits in the submarket in which PTI limits are *already* relatively tight can be substantially weakened as borrowers exercise their option to switch submarkets. Since the other submarket has relatively tight LTV limits, this switch not only lessens the reduction in borrower debt limits, but also substantially counteracts the fall in house prices by pushing up collateral demand through equation (4).

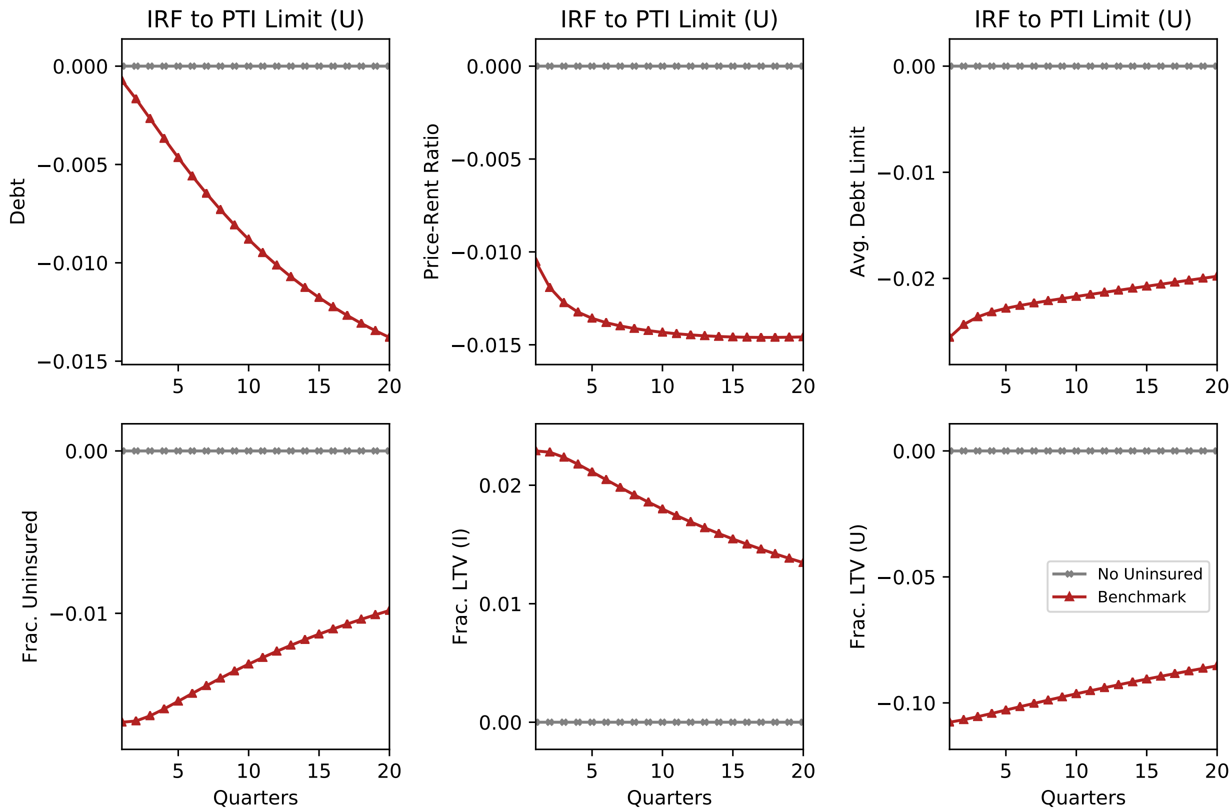
In contrast, tightening the PTI limit in the uninsured sector depresses debt and house prices through a more standard intuition, shown in Figure 6. To understand this, we can again turn to the debt limit example, now shown in Figures 4c and 4d. In this case,




*Note:* A value of 1 represents a 1% increase relative to steady state, except for Frac. Uninsured, Frac. LTV (I), and Frac. LTV (U), which are measured in percentage points. Debt ($m_t$) and Avg. Debt Limit are measured in real terms.

Figure 5: Response to -1% Shock to $\theta_{I,t}^{PTI}$

the reduction in $\theta_{U,t}^{PTI}$ has driven more borrowers in the uninsured space to hit their PTI limits, lowering the fraction of uninsured borrowers constrained by LTV, and depressing house prices through $\mathcal{C}_t$. However, because the marginal borrower indifferent between submarkets would not be PTI-constrained in the uninsured space, this tightening does not directly shift the U-I switch point. Due to a general equilibrium effect from house prices falling, there is a small shift of borrowers from the uninsured to the insured space, as well as a small increase in the fraction of insured submarket borrowers who are LTV-constrained, but these offsetting responses are relatively mild. Overall, these results imply that tightening PTI limits in the submarket with looser PTI constraints is likely to be an effective policy that avoids the offsetting responses found from tightening $\theta_{I,t}^{PTI}$ above.

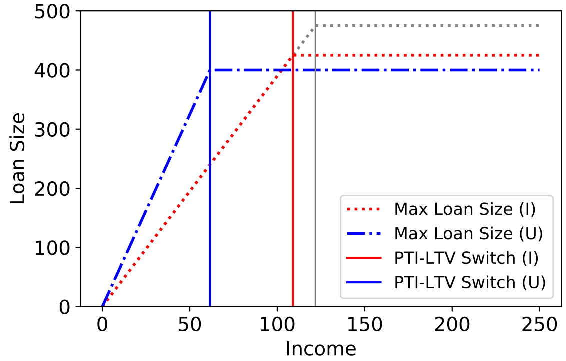



*Note:* A value of 1 represents a 1% increase relative to steady state, except for Frac. Uninsured, Frac. LTV (I), and Frac. LTV (U), which are measured in percentage points. Debt ($m_t$) and Avg. Debt Limit are measured in real terms.

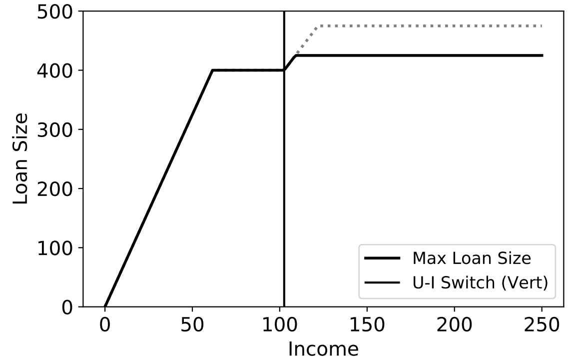
Figure 6: Response to -1% Shock to $\theta^{PTI}_{U,t}$

**Shocks to LTV Limits.** Turning now to LTV limits, we can begin with an experiment tightening the LTV limit in the insured submarket, shown in Figure 8. The impulse responses show that the effect of tightening LTV limits are very similar in both the No Uninsured and Benchmark cases, and generally follow the findings of Greenwald (2017), generating a drop in debt balances, but actually pushing price-rent ratios *upward* due to an increase in the fraction of LTV-constrained borrowers due to a tightening in these very constraints.

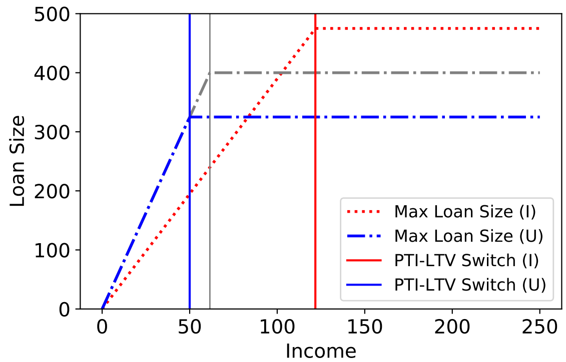
The intuition for the similarity of the responses across models can again be seen using the debt limit example, now shown in Figures 7a and 7b. Similar to the logic from the example tightening the uninsured sector PTI limit, this policy influences which constraint binds within the insured submarket, but does not directly influence the boundary between the uninsured and insured sectors, since the threshold borrower would be PTI-



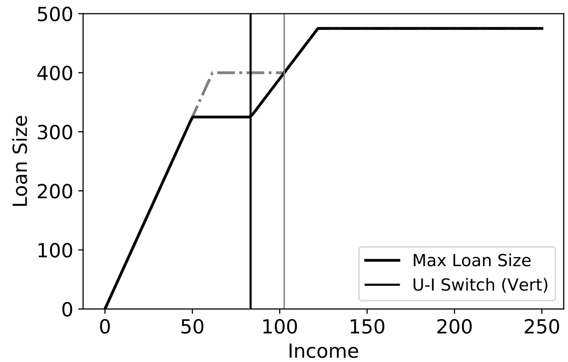

(a) Debt Limit by Submarket ($\theta_I^{LTV} \downarrow$)




(b) Overall Debt Limit ($\theta_I^{LTV} \downarrow$)




(c) Debt Limit by Submarket, ($\theta_U^{LTV} \downarrow$)



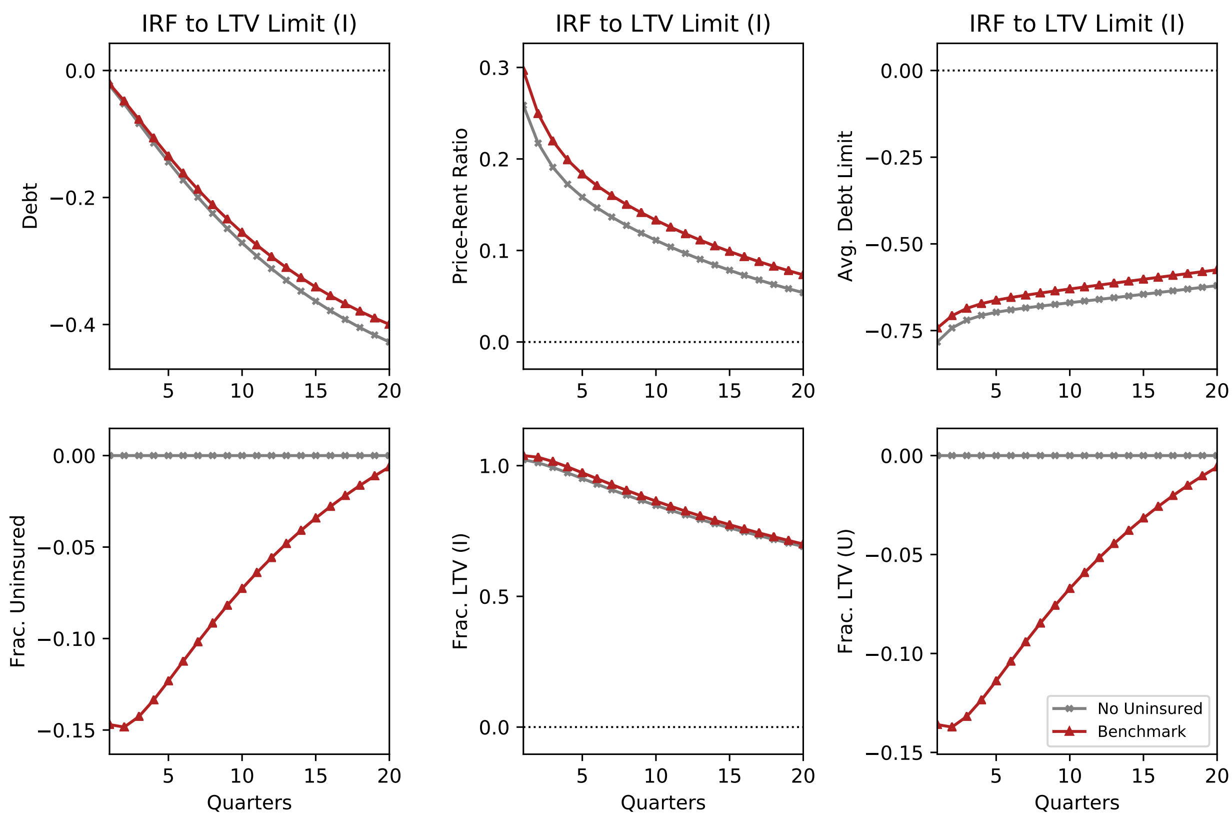

(d) Overall Debt Limit, ($\theta_U^{LTV} \downarrow$)

*Note:* Grey lines indicate the baseline case of Figure 3b, while the red, blue, and black lines indicate the responses following a reduction in $\theta_{I,t}^{PTI}$. The size of the shift in $\theta_{I,t}^{PTI}$ is exaggerated relative to Figure 5 for visual clarity.

Figure 7: Example, LTV Limit Tightenings by Submarket

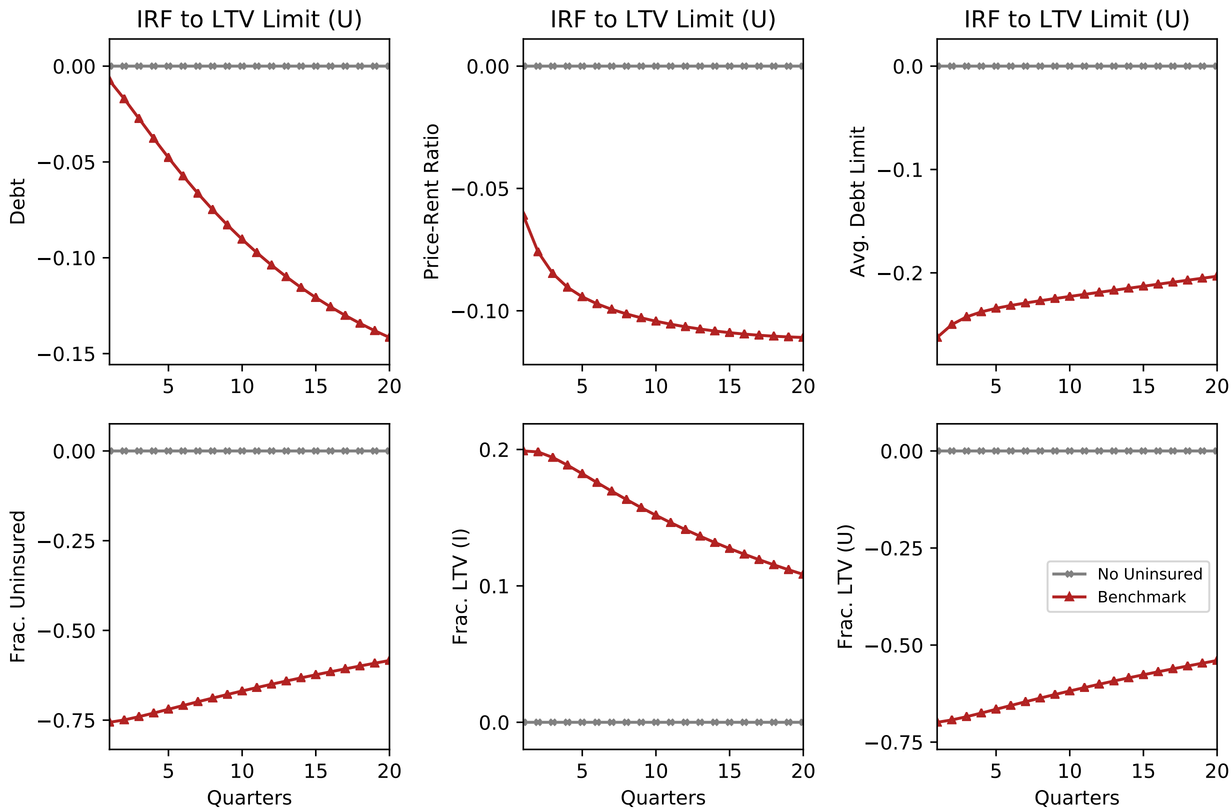
constrained, not LTV-constrained in the insured sector. This intuition, reinforced by the bottom row of 8, implies that tightening LTV limits in the insured (high-LTV) sector is a relatively ineffective policy, since it induces an offsetting increase in collateral and housing demand, potentially exacerbating the housing boom.

For the final experiment, we can consider a tightening of the LTV constraint in the uninsured submarket with responses shown in Figure 9, and plots for intuition in Figures 7c and 7d. This experiment has a strong impact on the fraction of uninsured borrowers, but unlike in the experiment tightening $\theta_{I,t}^{PTI}$, a decrease in the uninsured sector LTV limit reduces the income threshold at which borrowers switch from the uninsured to insured space, pushing pushes borrowers out of the uninsured sector. Since these borrowers switch from being LTV-constrained to being PTI-constrained upon entering the insured




*Note:* A value of 1 represents a 1% increase relative to steady state, except for Frac. Uninsured, Frac. LTV (I), and Frac. LTV (U), which are measured in percentage points. Debt ($m_t$) and Avg. Debt Limit are measured in real terms.

Figure 8: Response to -1% Shock to $\theta_{I,t}^{LTV}$




*Note:* A value of 1 represents a 1% increase relative to steady state, except for Frac. Uninsured, Frac. LTV (I), and Frac. LTV (U), which are measured in percentage points. Debt ($m_t$) and Avg. Debt Limit are measured in real terms.

Figure 9: Response to -1% Shock to $\theta^{LTV}_{U,t}$

sector, this puts downward pressure on housing demand. As a result, the effect of this policy on house prices actually takes the *opposite sign* relative to a tightening of LTV limits in the insured sector, implying that LTV limits should be reduced in the uninsured, not insured sectors if policymakers wish to dampen a rise in house prices.

# 5 Conclusion

In this paper, we studied the role of mortgage market segmentation in determining the effectiveness of macroprudential policies. We found that the ability to switch between submarkets seriously weakens the ability of PTI tightening in the insured (loose-LTV/tight-PTI) sector to cool overheated housing and mortgage markets. In contrast, switching between sectors *strengthened* the ability of tighter LTV constraints in the uninsured sector to lower housing demand, causing price-rent ratios to fall rather than rise — as they would in response to a tightening of the insured sector LTV limit, or as they would in a single sector model. Work in progress will evaluate the impacts of alternative macroprudential policies such as changes in amortization rates, and monetary policy, and investigate the differential impacts of these policies in booming vs. non-booming regions.

# References


ADELINO, M., A. SCHOAR, AND F. SEVERINO (2012): "Credit Supply and House Prices: Evidence From Mortgage Market Segmentation," Tech. rep., National Bureau of Economic Research.

BUCHAK, G., G. MATVOS, T. PISKORSKI, AND A. SERU (2017): "Fintech, Regulatory Arbitrage, and the Rise of Shadow Banks," Tech. rep., National Bureau of Economic Research.

CAMPBELL, J. Y., C. PFLUEGER, AND L. M. VICEIRA (2014): "Monetary Policy Drivers of Bond and Equity Risks," Tech. rep., National Bureau of Economic Research.

DAVIS, M. A. AND F. ORTALO-MAGNÉ (2011): "Household Expenditures, Wages, Rents," *Review of Economic Dynamics*, 14, 248–261.

DEFUSCO, A. A., S. JOHNSON, AND J. MONDRAGON (2017): "Regulating Household Leverage," Unpublished Paper, Kellogg School of Business, Northwestern University.

ELENEV, V., T. LANDVOIGT, AND S. VAN NIEUWERBURGH (2016): "Phasing Out the GSEs," *Journal of Monetary Economics, Forthcoming*.

GARRIGA, C., F. E. KYDLAND, AND R. SUSTEK (2015): "Mortgages and Monetary Policy," Working Paper 19744, National Bureau of Economic Research.

GREENWALD, D. L. (2017): "The Mortgage Credit Channel of Macroeconomic Transmission," MIT Sloan Working Paper No. 5184-16.

JÁCOME, L. I. AND S. MITRA (2015): "LTV and DTI Limits–Going Granular," Tech. rep., IMF Working Paper.

SMETS, F. AND R. WOUTERS (2007): "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach," *American Economic Review*, 97, 586–606.

# A Appendix

## A.1 Optimality Conditions

The borrower's optimality conditions with respect to $m^*_{j,t}$ are:

$$1 = \Omega^m_{b,j,t} + \Omega^x_{b,j,t} r^*_{j,t} + \mu_{j,t}$$

where $\Omega^m_{b,j,t}$ and $\Omega^x_{b,j,t}$ are the marginal continuation costs of an extra dollar of principal balance and promised interest payment on a sector $j$ mortgage, respectively, and $\mu_{j,t}$ is the multiplier on the sector $j$ borrowing constraint. [To be completed.]