

Sovereign Debt Restructurings*

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Abstract

Sovereign debt crises generally involve debt restructurings characterized by a mix of face-value haircuts and debt maturity extensions. We develop a quantitative model of endogenous sovereign debt maturity choice and restructuring that captures key stylized facts of debt over the business cycle and during restructuring episodes, including the variation of haircuts, maturity extensions and default duration found in the data. We also find that policy interventions implementing minimum haircuts and redistributing losses away from holders of short term debt improve the outcome of distressed debt restructurings and reduce the frequency of debt distress events. Methodologically, the use of dynamic discrete choice solution methods allows us to smooth decision rules on default and debt portfolio choices, rendering the problem tractable.

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1 Introduction

Debt restructurings are a salient feature of sovereign defaults. Restructuring operations involve the maturity extension of the original debt instruments and are sometimes accompanied by a reduction in the face value of debt, i.e., a face value haircut. We develop a quantitative small open-economy model of sovereign debt maturity choice, default, and restructuring, that not only captures the business cycle dynamics of key debt statistics but crucially also mimics the key debt maturity and payment dynamics surrounding distressed debt restructurings.

In our framework, the borrowing government selects the size and the maturity of its debt portfolio, where the decisions on whether to default and which debt-maturity portfolio to select are affected by the current level of debt and its maturity, the country's income, and the expected terms of the restructuring. Sovereign debt is restructured in the context of a default. In a restructuring, lenders receive a new debt instrument that may differ from the original liabilities due to a combination of changes in the face-value of the debt and a different repayment period. The size of a debt haircut and maturity extension from the restructuring are determined as the equilibrium result of a debt negotiation process where the lenders and the borrowing country make alternating offers. Specifically, each period while in default, either the lender or the borrower will offer a restructuring deal, where any one party may initiate the process with some probability. A restructuring offer specifies a value for the new debt portfolio. Thus, when a lender makes an offer, the borrower will consider its income, the amount of the debt in default, and the value of the restructured debt portfolio demanded by the lender. The borrower may prefer not to enter the deal and remain in default that period, but if it decides to enter, it will face a portfolio choice that involves selecting the level and maturity of the new debt. If instead, it is the borrower that extends the restructuring offer, the borrower picks a value for the new debt portfolio that is just enough to make the lender accept it.

Our renegotiation framework allows the model to rationalize several stylized facts of sovereign debt restructurings, while also capturing key debt dynamics over the business cycle. Specifically, our work contributes to the literature on debt and default on several fronts. First, our model mimics the procyclical behavior of debt maturity, duration, and the yield spread curve at different

maturities documented in the data, as well as the dynamics of key macro variables preceding a default event, including the decrease in debt duration.

Second, our study characterizes two fundamental dimensions of sovereign debt restructurings, namely the size of the debt haircut and the maturity extension. The model-implied distributions of these variables across restructurings are quite close to the empirical distributions. Consistent with the data, the study also shows that countries that enter debt restructurings with larger debt burdens tend to experience larger debt haircuts and larger maturity when exiting the restructurings. Moreover, we find that borrowers with a higher income at the time of default experience smaller haircuts and a shorter exit maturity.

In the model, this occurs because lenders know that higher-income countries are less likely to default again, so they demand a higher recovery value. To repay a higher exit value, it is less costly for the borrower to opt for a shorter debt maturity that pays a lower exit yield spread. In addition to explaining the variation in key restructuring features by country characteristics, our baseline model captures the distribution of losses across holders of different bond maturities, in particular, the higher net present value haircuts experienced by short-term bondholders compared to creditors holding the debt of longer tenors.

A third contribution of our analysis draws from the observation that restructurings generally involve some participation of a multilateral financial organization such as the International Monetary Fund (IMF), which often includes conditionalities that are reflected in policy interventions. Our work contributes to the literature by providing new insights into the role of different policy interventions on debt restructuring outcomes. We first consider the effects of imposing a minimum haircut, and then we separately assess the impact of schemes associated to different distributions of losses across lenders holding bonds of different maturities, i.e., schemes that compare creditor rights in different ways. Our results for both types of interventions suggest that policy strategies that lead to the largest maturity extensions in debt restructurings also generate the largest welfare improvements. During the 80s, most restructurings involved changes in maturity and no face value haircuts. Our results indicate that policies that lead to longer maturity and moderate haircuts should be given a second look, a view that seems consistent with a recent IMF proposal on restructurings that emphasizes a more active use of reprofiling,

i.e., restructurings that rely on maturity extensions.

A fourth contribution of our study is that it helps account for important differences in restructuring events observed over time, especially some of the changes in debt restructurings pre vs. post-1980s. In particular, our model helps explain the observed increase in debt haircut size and the lower duration of exclusion in default by considering a higher probability of the borrower starting the restructuring deal, which is consistent with the emergence in the 1990s of “take-it-or-leave-it” country offers for restructurings.

Finally, we provide a new method to solve sovereign default models with maturity choice. Quantitative studies have found it quite challenging to solve for the optimal default, debt and maturity portfolio choices, and equilibrium prices of bonds of different characteristics. Using methods from dynamic discrete choice based on idiosyncratic shocks affecting the borrower, we show that borrower decision rules on default and debt portfolio choices are smooth functions of the state variables, making the problem tractable and easier to solve quantitatively. The intuition is that the idiosyncratic shocks introduce an additional source of randomness to the problem that can be integrated out. Thus, the analysis can be performed in terms of conditional choice probabilities. Using standard assumptions on the distribution of these shocks we can characterize these choice probabilities almost in closed form.

Our analysis borrows from different strands of the literature on sovereign debt default, maturity, and restructuring. Following the seminal work in international sovereign debt by [Eaton and Gersovitz \(1981\)](#), a large portion of the literature on quantitative models of sovereign debt default has used only one-period debt ([Arellano, 2008](#); [Aguiar and Gopinath, 2006](#); [D’Erasmus, 2008](#); [Yue, 2010](#); [Mendoza and Yue, 2012](#), among others). Models with long debt duration, such as [Chatterjee and Eyigungor \(2012\)](#) and [Hatchondo and Martinez \(2009\)](#), feature exogenous maturity. In contrast, our quantitative model features endogenous sovereign debt maturity and repayment under debt dilution. The work of [Arellano and Ramanarayanan \(2012\)](#) also includes the choice of maturity, but there are several important differences (see also [Hatchondo and Martinez, 2013](#); [Hatchondo et al., 2016](#); [Bai et al., 2014](#)). Our modeling of debt maturity follows that in [Sánchez et al. \(2018\)](#), which uses a quantitative model that incorporates debt maturity as the choice of the number of periods (a discrete number between 1 and 20 years) in order to

understand maturity choice and the term structure of interest rate spreads but does not consider debt restructurings.

Importantly, our work is also related to recent models on sovereign default and restructurings. [Yue \(2010\)](#), [Benjamin and Wright \(2013\)](#), [Asonuma and Trebesch \(2016\)](#) and [Asonuma and Joo \(2017\)](#) study different aspects of sovereign debt restructurings in the context of one-period bond models. More recent work by [Mihalache \(2017\)](#) explores sovereign debt restructurings and maturity extensions appealing to political economy considerations. In contrast to our work, Mihalache models debt duration instead of maturity itself and considers a Nash bargaining debt restructuring mechanism following [Yue \(2010\)](#) with an exogenous length of negotiation instead of a restructuring mechanism like [Benjamin and Wright \(2013\)](#) that delivers endogenous delays, as in our model. The latter feature is relevant because the haircut measures and the economic problem faced by the negotiating parties in a restructuring take into account delays. Also differently to our model, it assumes that the characteristics of the defaulted debt do not matter at all for the restructuring, which means for example that there may be recovery rates exceeding 100 percent during periods of high enough endowment draws, an undesirable feature that can lead to artificially lower spreads in non-default periods. The recent study of [Passadore and Xu \(2018\)](#) introduces bonds with long maturity and debt restructuring in a sovereign default model, but focuses on illiquidity in the secondary markets of sovereign debt and considers both the maturity and the outcomes of debt restructurings to be exogenous.

Our work is also related to other types of default resolution or prevention mechanisms. The work of [Bianchi \(2016\)](#) and [Roch and Uhlig \(2016\)](#) study the desirability of bailouts. They show that in some cases bailouts may induce additional borrowing offsetting its potential benefits. [Hatchondo et al. \(2014\)](#) and [Hatchondo et al. \(2017\)](#) find a similar result analyzing the introduction of contingent convertible bonds (cocos) and voluntary debt exchanges, respectively. The work of [Chatterjee and Eyigungor \(2015\)](#) is also related, as it proposes the introduction of a seniority rule for sovereign bonds such that in default creditors are to be paid off in the order in which they lent.

Our method to solve the numerical challenges presented by this setup follows a similar intuition as [Chatterjee and Eyigungor \(2012\)](#), who introduces a random i.i.d. shock to income

that smooths the default decision rule. With a binary choice, like that of defaulting or not, it is possible to resort to a wide range of statistical distributions to model the shock while keeping the model tractable. In our more complex environment, we consider i.i.d. shocks affecting the default decision and the characteristics of the debt portfolio, including the maturity. In this case, the Generalized Extreme Value distribution (McFadden, 1978) provides a tractable way to characterize agents’ decision rules and to smooth the problem along the other dimensions, i.e., maturity and asset choices.¹

In summary, we develop a quantitative sovereign debt model that not only captures the business cycle dynamics of several debt statistics but that crucially also mimics key features of sovereign debt restructurings and delivers key policy implications associated with these distressed debt resolution mechanisms. Importantly, our ability to conduct our quantitative analysis is largely made possible by the use of novel solution techniques.

The rest of the paper is organized as follows. In Section 2, we review the last 40 Years of restructurings. In Section 3, we present the economic environment, the baseline model and define the equilibrium. In Section 4, we describe our solution method. In Section 5 we explain the calibration of our model with endogenous restructuring offers. In Section 6, we evaluate the performance of our model by comparing its predictions with the data. In Section 7 we account for changes in restructuring over time, and in Section 8, we present two policy exercises. In Section 9 we conclude.

2 A Review of 40 Years of Restructurings

Using data from Cruces and Trebesch (2013) expanded to 187 distressed sovereign debt restructuring events between 1970 and 2013, we recover the maturity extensions using different definitions of haircuts from the literature.² The sample mean of the net present value (NPV) of haircuts for this period based on the measure proposed by Sturzenegger and Zettelmeyer (2006b), SZ henceforth, is 37 percent, and the average maturity extension is about 4 years.

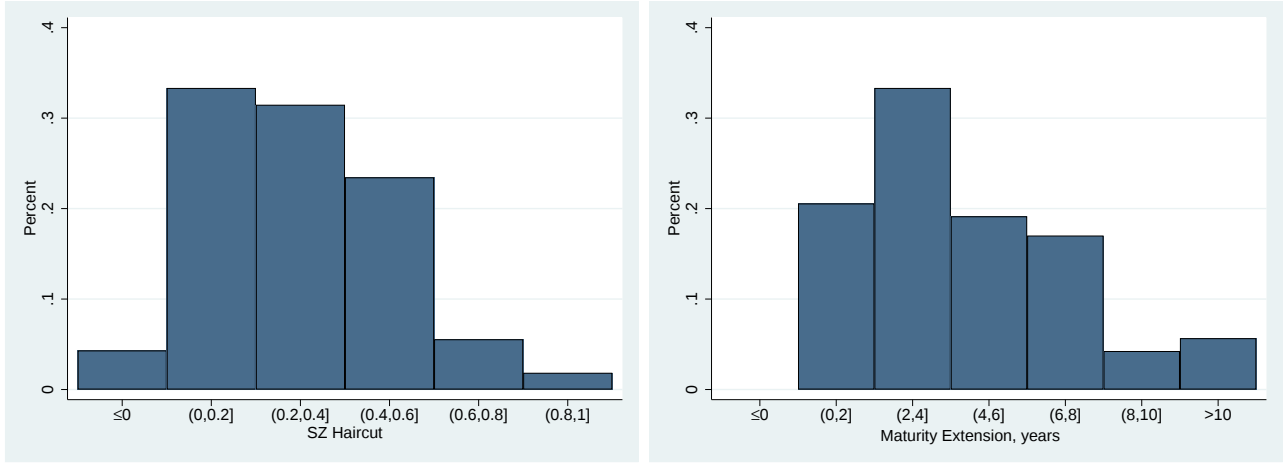
¹See for instance Mihalache and Wiczer (2018) for a recent application of our approach in the sovereign default and maturity literature.

²See Appendix A for details.

Average statistics provide only a partial description of the data, so the distributions of haircuts and maturity extensions are presented in Figure 1. The figure indicates important variations in haircuts and maturity extensions across restructuring episodes over the past several decades (see also Makoff, 2015). It is possible to identify different types of debt restructurings based on the varying degrees of payment rescheduling and reductions in the face value of principal or coupon payments, and the associated SZ haircuts.

Distressed debt exchange events such as the one in Pakistan in 1999, or Uruguay in 2003 among others, involved the rescheduling of debt payments and little or no face-value reductions either in the principal or in coupon payments. The NPV haircuts (≤ 15 percent) and creditor losses tend to be the lowest in these cases. The distribution of SZ haircuts in the figure shows that these events, also known as reprofiling, correspond to a substantial share of all restructurings in the data. Reprofilings were most frequent in the 1980s and regained significant attention in international financial markets in recent years, in part following an IMF proposal aiming to provide the institution with a broader range of policy responses during sovereign debt distress episodes and to emphasize the use of debt reprofiling operations as a partial substitute for bailouts. Debt crises like that of Ukraine in 2000 were resolved with somewhat larger maturity extensions and some debt value reduction in coupons or principal, with associated SZ haircuts generally below 30 percent. These soft restructurings are also quite prevalent in the data, as shown in the figure. Debt resolution operations, like those for Ecuador in 2000 or the Brady restructurings for countries like Mexico or Philippines among others, are characterized by longer maturity extensions that may exceed 10 years and larger reductions in coupons and principal that, when combined, amount to moderate but permanent capital losses for creditors, with NPV haircuts reaching between 30 and 50 percent. The hard restructurings implemented in the larger and more severe debt crises, like Argentina in 2005 and Greece in 2012, were generally associated with 20-30 year maturity extensions and deep face-value reductions in both principal and coupons, which translated into NPV losses ranging from 50 to about 80 percent for lenders.

Figure 1: Haircuts and maturity extensions



Source: Authors' calculations based on [Cruces and Trebesch \(2013\)](#). Sample excludes donor-supported restructurings.

Lastly, we ask what accounts for the significant variations in haircuts, maturity extensions, and duration of defaults. Table 1 presents regressions of H_{SZ} haircuts, face value haircuts, maturity extensions, and duration of defaults, for the “Full” sample of more than 150 default episodes. For robustness, we also consider results for a “Restricted” set of restructurings that are not donor-supported, i.e., we exclude restructurings that involve debt relief to highly indebted low-income economies, following the classification of restructurings in [Cruces and Trebesch \(2013\)](#).³

The first row in Table 1 indicates that countries that enter default with a larger debt burden exhibit larger haircuts, longer maturity extensions and a longer period of default. The effect of income on haircuts, maturity extensions shown in the second row, is negative and significantly different from zero. The dummy variables for the 1990s and 2000s shown in the last two rows suggest that restructurings in these decades involved larger NPV and face value haircuts compared to earlier decades. The results for haircuts are comparable to the findings in [Cruces and Trebesch \(2013\)](#). They find average haircuts were about 25 percentage points higher during the 1990s and 2000s compared to deals implemented during the 1970s and 1980s.

³Donor-supported restructurings are those co-financed by the World Banks Debt Reduction Facility.

Table 1: Determinants of haircuts and maturity extensions in the data

	SZ Haircut		Face-Value Haircut		log(Maturity Ext)		log(Default Dur)	
	Full	Restricted	Full	Restricted	Full	Restricted	Full	Restricted
Debt-to-GDP ratio	0.328 (0.094)	0.358 (0.099)	0.334 (0.104)	0.274 (0.090)	0.878 (0.508)	1.161 (0.467)	0.844 (0.451)	0.734 (0.443)
log(income/pop)	-0.124 (0.021)	-0.056 (0.020)	-0.132 (0.026)	-0.015 (0.019)	-0.211 (0.093)	-0.216 (0.093)	-0.150 (0.106)	.0155 (0.122)
1990s	0.279 (0.038)	0.114 (0.034)	0.353 (0.048)	0.088 (0.029)	0.607 (0.165)	0.641 (0.166)	-0.192 (0.251)	-0.329 (0.337)
2000s	0.406 (0.061)	0.223 (0.065)	0.497 (0.072)	0.200 (0.061)	0.850 (0.378)	0.727 (0.386)	-0.801 (0.252)	-1.009 (0.270)
# observations	168	144	168	144	128	124	120	102
R-squared	0.428	0.205	0.456	0.301	0.148	0.149	0.142	0.139

Note: Standard errors are in parenthesis. Restricted sample does not include restructurings that [Cruces and Trebesch \(2013\)](#) classify as “donor,” the variable “Maturity Ext” is the extension of debt maturity that results from the restructuring, “Default Dur” is the length, in years, of the period between the default and the end of the restructuring, and the variables “1990s” and “2000s” are dummy variables that are 1 if the restructuring was in that decade and 0 otherwise. The variable “2000s” also includes two episodes available after the year 2010. In the last two columns, the year of reference for the decades dummies is the year of default because that facilitates the comparison with [Das et al. \(2012\)](#). All regressions also have a constant and dummy variables for the continent of the country.

The last column in Table 1 shows the results for the time between default and restructuring. The length of that period is also increasing in indebtedness and decreasing in income—the latter only in the full sample. The coefficients on the dummy variables indicate, as pointed out by [Das et al. \(2012\)](#), that there has been considerable variation in the length of the sovereign restructuring process, with post-1980s restructurings taking less time to be resolved. This was due in part to the emergence since the late 1990s of take-it-or-leave-it bond exchange offers that were preceded by informal discussions with creditors, but rarely formal negotiations ([Sturzenegger and Zettelmeyer, 2006a](#)).

3 The Model

3.1 Environment

We consider a small open endowment economy à la [Eaton and Gersovitz \(1981\)](#) with a benevolent government. The country participates in international credit markets, where lenders are risk neutral. The country cannot commit to repaying its obligations, so given an outstanding amount

of assets, b (debt if $b < 0$), it has two actions to choose from. The first option is to pay its obligations and hence keep its good credit status. Alternatively, the country may decide not to make its debt payment, i.e., default.

A default brings immediate financial autarky and a direct output loss to the defaulting country. After the initial default decision, the country may have the opportunity to return to international debt markets, but only after restructuring its debt. The restructuring of the debt may entail a haircut and a different maturity from the original defaulted portfolio.

In times of good credit status, the country may face a “debt rollover” shock, a , where $a = 1$ if the country is facing a disruption in its access to financial markets and is hence impeded from rolling over or changing its debt portfolio, and 0 otherwise. When the country experiences this sudden stop event, world financial markets cease to lend to the economy, so the country may only choose between repaying or repudiating its obligations. Thus, the decision to default is influenced by the current level of debt and its maturity, by the country’s income, y , which fluctuates exogenously over time, by the costs of default, by the debt rollover shock, and by the expected terms of the restructuring.

If the country decides not to default, it selects the maturity of the new portfolio, m' , and the debt level, b' . The optimal choices of maturity and asset levels are influenced by the current level of income, the current level of debt and its maturity, and the debt rollover shock. There is also a cost of adjusting the portfolio, which would be omitted from the equations in the main body of the paper for simplicity.⁴

The conditions of the debt restructuring are endogenously determined via an alternating-offers mechanism that resembles that of [Benjamin and Wright \(2013\)](#). That is, each period in default, either the lender or the borrower has a chance to make a restructuring offer to the other party. In both cases, the offer consists of a menu of possible portfolios that the borrower may choose from. All these portfolios deliver the same value to the lender, although the value may differ depending on who is making the offer. A restructured portfolio in the proposed menu has yearly payments b^R and maturity m^R . If the lender is making the offer, the lender selects the

⁴We explain in the calibration section the role of this adjustment cost. In [Appendix C](#), where we present all the equations, we do display this adjustment cost.

set of (b^R, m^R) pairs that, if accepted, maximizes the market value of the new portfolio. The restructuring proposal takes into account the incentives of the borrower to accept or not. If the borrower is the one proposing a deal, the borrower will choose the offer that makes the lender indifferent. However, if this value is sufficiently large, the borrower may choose not to make a restructuring offer at all.

To make the problem tractable, we make a few assumptions on the support of the assets and we introduce additive preference shocks to choices. We then show how to simplify the problem to be solved computationally.

First, we assume that the maturity of the new asset portfolio can be a natural number $m' \in \{1, 2, \dots, \mathcal{M}\}$. In addition, we assume that assets can only take a value in a discrete support. This discrete grid has a total of $\mathcal{N}$ points. While the first assumption is uncontroversial, the last assumption can be interpreted as some units for debt or assets. For example, in the real world, people choose savings or debt in multiples of cents or dollars. What we have in mind, however, is more sparse (and bounded) support for sovereign debt, for example, millions of dollars, or one-tenth of a percent of GDP. The assumption of a discrete and bounded support for debt is usual in the literature on sovereign default ([Chatterjee and Eyigungor, 2012](#)).

With this assumption, we can characterize the problem of the government as either choosing to default or choosing the optimal debt and maturity combination. This decision boils down to choosing one out of $\mathcal{J} + 1$ possible alternatives. In writing down the problem, it is convenient to define vectors b and m , where (b_j, m_j) are the j th element of each vector, respectively. These vectors has $\mathcal{J} = \mathcal{M} \times \mathcal{N}$ elements and have the following structure:

$$b = \left[\underbrace{b_1, b_2, \dots, b_{\mathcal{N}}}_{\text{grid for } b}, \underbrace{b_1, b_2, \dots, b_{\mathcal{N}}}_{\text{grid for } b}, \dots, \underbrace{b_1, b_2, \dots, b_{\mathcal{N}}}_{\text{grid for } b} \right]^T$$

$$m = \left[\underbrace{m_1, m_1, \dots, m_1}_{\text{repeated } \mathcal{N} \text{ times}}, \underbrace{m_2, m_2, \dots, m_2}_{\text{repeated } \mathcal{N} \text{ times}}, \dots, \underbrace{m_{\mathcal{M}}, m_{\mathcal{M}}, \dots, m_{\mathcal{M}}}_{\text{repeated } \mathcal{N} \text{ times}} \right]^T,$$

where the operator T represents the transpose.

Second, we assume that there is random vector ϵ of size $\mathcal{J} + 1$, which is the number of all possible combinations of b and m , captured by $\mathcal{J} = \mathcal{M} \times \mathcal{N}$, and one additional element that captures the choice of default. We call the elements of the random vector ϵ as ϵ_j , and the one associated with the choice of default is $\epsilon_{\mathcal{J}+1}$. The introduction of these $\mathcal{J} + 1$ shocks is useful to solve our model numerically using the tools of dynamic discrete choice. As we show below, these shocks play a modest role in the decisions, with a slightly larger impact in determining the choice of maturity in those cases for which the country is almost indifferent among several alternatives.

We assume ϵ is drawn from a multivariate distribution with joint cumulative density function $F(\epsilon) = F(\epsilon_1, \epsilon_2, \dots, \epsilon_{\mathcal{J}+1})$ and joint density function $f(\epsilon) = f(\epsilon_1, \epsilon_2, \dots, \epsilon_{\mathcal{J}+1})$. To simplify notation in what follows, we use the following operator to denote the expectation of any function $Z(\epsilon)$ with respect to all the elements of ϵ ,

$$E_{\epsilon}Z(\epsilon) = \int_{\epsilon_1} \int_{\epsilon_2} \dots \int_{\epsilon_{\mathcal{J}+1}} Z(\epsilon_1, \epsilon_2, \dots, \epsilon_{\mathcal{J}+1}) f(\epsilon_1, \epsilon_2, \dots, \epsilon_{\mathcal{J}+1}) d\epsilon_1 d\epsilon_2 \dots d\epsilon_{\mathcal{J}+1}.$$

Additionally, on some occasions we use the function $q^*(m_i, r^\ell)$ to reflect the discounted sum of payments of one unit for m_i periods, using the interest rate r^ℓ to discount those payments, $q^*(m_i, r^\ell) = \sum_{n=1}^{m_i} \frac{1}{(1+r^\ell)^n}$.

3.2 Borrower's Problem in Good Credit Standing

Under the economic setup described above, the country's choice

$$V^G(y, a, b_i, m_i, \epsilon) = \max \left\{ V^D(y, b_i, m_i, \epsilon_{\mathcal{J}+1}), V^P(y, a, b_i, m_i, \epsilon) \right\},$$

where V^D and V^P are the values if the country chooses to default and repay, respectively. The policy function $D(y, a, b_i, m_i, \epsilon)$ is 1 if default is preferred and 0 otherwise.

In case of default the problem is simply

$$V^D(y, b_i, m_i, \epsilon_{\mathcal{J}+1}) = u(y^D) + \beta E_{y'|y^D} E_{\epsilon'} V^R(y', b_i, m_i, \epsilon') + \epsilon_{\mathcal{J}+1},$$

where $y^D = \min \{y, \pi\}$ represents the income of the country net of the default punishment.

In case of repayment, the value depends on the rollover shock, a . In normal times (i.e., no debt rollover shock, $a = 0$), the value is

$$V^P(y, 0, b_i, m_i, \epsilon) = \max_j u(c_{ij}(y)) + \beta E_{y', a'|y, 0} E_{\epsilon'} V^G(y', a', b_j, m_j, \epsilon') + \epsilon_j,$$

subject to,

$$c_{ij}(y) = y + b_i + q(y, 0, b_j, m_j; m_i - 1)b_i - q(y, 0, b_j, m_j; m_j)b_j \text{ and } j \in \{1, 2, \dots, \mathcal{J}\}.$$

The constraint implies that consumption is equal to income, y , net of debt payments, b_i , plus the net resources that are obtained from or paid to international markets, captured by the next two summands. The first of these two summands depends on the market price of outstanding obligations, $q(y, 0, b_j, m_j; m_i - 1)$, which takes into account the current income, y , the debt rollover shock, $a = 0$, and the obligations that the country will have from the beginning of the next period, (b_j, m_j) . These four variables determine the risk of default. The market price also depends on $m - 1$, which is the remaining number of years of payments of the outstanding debt after the current year's payment. The term $q(y, 0, b_j, m_j; m_i - 1)$ captures the price per unit of resources promised per year, it is multiplied by b_j to reflect the market value of the total outstanding obligations at the beginning of the present period. With a negative value of b , the term represents the gross resources leaving the country. Similarly, the term $-q(y, 0, b_j, m_j; m_j)b_j$ is the value of the outstanding debt at the end of the current period and, therefore, represents the gross resources obtained from international markets. The combination of both terms captures the net resources obtained from international markets.

The policy functions for the amount of assets and maturity choices are $B(y, a, b_i, m_i, \epsilon)$ and $M(y, a, b_i, m_i, \epsilon)$, respectively. Notice that when a country makes only its debt payment, the policies are $B(y, a, b_i, m_i, \epsilon) = b$ and $M(y, a, b_i, m_i, \epsilon) = m - 1$, respectively. This will be the case, for example, when there is a debt rollover shock.

In the case where the country cannot rollover the debt ($a = 1$), the value of repayment is

$$V^P(y, 1, b_i, m_i, \epsilon) = u(y + b_i) + \beta E_{y', a'|y, 1} E_{\epsilon'} V^G(y', a', b_i, m_i - 1, \epsilon') + \epsilon_i.$$

The country does not have the option to change the debt portfolio, and the choice reduces to simply making the promised payment and continuing next period with a debt characterized by the same payment and one-period shorter maturity or defaulting.

3.3 Default, Renegotiation, and Restructuring

We follow [Benjamin and Wright \(2013\)](#) and assume that, after a default the borrower and lenders have the opportunity to make a restructuring offer. This opportunity alters stochastically between the borrower and lenders, and only one party can make an offer each period.

In default, with probability λ the lender (L) offers a restructuring deal, and the sovereign borrower (S), the country, decides whether to accept. Similarly, with probability $(1 - \lambda)$ the sovereign has the option to make a restructuring offer to the lender. In both cases, the offer specifies a value that the new restructured portfolio must attain, W .

The characteristics of the restructuring are always chosen by the country, and that includes yearly payment, b^R , maturity, m^R , and a transfer of fresh money from the lenders to the country, τ^R , which is determined as

$$\tau^R(y, W, j) = \underbrace{q^R(y, b_j, m_j; m_j) \times (-b_j)}_{\text{resources raised}} - \underbrace{W}_{\text{amount agreed in the restructuring}},$$

where the price of the debt being restructured, q^R , is slightly different from the value of debt in normal times, q , because we assume the period immediately after restructuring the country has no access to credit market and thus cannot change its debt portfolio that period.⁵

With this in mind, we call $\Omega(y, W)$ the set of possible portfolios that attain that value after

⁵Technically, the only difference is that the probability of the debt rollover shock for the next period is 1.

restructuring,

$$\Omega(y, W) = \{j \in \{1, 2, \dots, \mathcal{J}\} : \tau^R(y, W, j) \geq 0\}.$$

Now, consider a function $\tilde{V}^R$ reflecting both alternatives: (i) a country with an offer of W deciding whether to accept it or not, and (ii) a country considering whether to make an offer of W or not; i.e.,

$$\begin{aligned} \tilde{V}^R(y, b_i, m_i, \epsilon, W) = \max \bigg\{ & V^D(y, b_i, m_i, \epsilon_{\mathcal{J}+1}); \max_{j \in \Omega(y, W)} u(y^D + \tau^R) \\ & + \beta E_{y'|y^D} E_{\epsilon'} V^G(y', 1, b_j, m_j, \epsilon') + \epsilon_j \bigg\}. \end{aligned}$$

The policy on whether the offer is made or accepted is $\tilde{H}(y, b_i, m_i, \epsilon, W)$, which takes value 1 if the offer is made or accepted and 0 otherwise.

Now, how is W determined? If the borrower were to make the offer, the borrower would pick the value that leaves the lender just indifferent between accepting or not. In this case, the offer is,

$$W^S(y, b_i, m_i) = -b_i q^D(y, b_i, m_i; m_i),$$

where q^D is the price of debt in default given the debt in default and current income y . We assume that if this is the offer, the lender always accepts it. Accordingly, there is no point for the borrower to offer any higher value, and any value smaller will be rejected for sure by the lenders. Note, however, that borrowers are not required to make the offer when they have the opportunity.

For the case in which the lender makes the offer, it is clear that the lender's choice of a restructuring offer, which is made before the values of ϵ are realized, is

$$\begin{aligned} W^L(y, b_i, m_i) = \arg \max_{x \leq -b_i q^*(m_i, r^L)} \bigg\{ & x \times E_{\epsilon'} \tilde{H}(y, b_i, m_i, \epsilon, x) \\ & + \left(1 - E_{\epsilon'} \tilde{H}(y, b_i, m_i, \epsilon, x)\right) \times \left(-b_i q^D(y, b_i, m_i; m_i)\right) \bigg\}, \end{aligned}$$

where we impose the constraint that the market value of the new debt portfolio selected by

the borrower cannot be larger than the value of the debt in default discounted using r^L . This constraint is similar to that made by [Benjamin and Wright \(2013\)](#) and is the reason the past values of the debt in default are a state variable in the value of default and the acceptance policy function. Note that if $r^L = 0$, this constraint would use the face value of debt, which would be in line with bond clauses that establish that all future payments become due at the time of default.

Finally, the value of a country in restructuring is simply,

$$V^R(y, b_i, m_i, \epsilon) = \lambda \tilde{V}^R(y, b_i, m_i, \epsilon, W^L(y, b_i, m_i)) + (1 - \lambda) \tilde{V}^R(y, b_i, m_i, \epsilon, W^S(y, b_i, m_i)).$$

The policy functions $H^L(y, b_i, m_i, \epsilon) = \tilde{H}(y, b_i, m_i, \epsilon, W^L(y, b_i, m_i))$ captures the acceptance of a restructuring offer made by the lender.

3.4 Equilibrium

Given the world interest rate r and lenders' risk neutrality, the price of the country's debt must be consistent with zero expected discounted profits. The price of a non-defaulted bond of maturity $m_i > 0$ of a country with income y , yearly debt payment $-b_j$, and portfolio maturity $m_j > 0$, can be represented by $q(y, a, b_j, m_j; m_i) =$

$$\frac{E_{y', a' | y, a} E_{\epsilon'}}{1 + r} \left\{ (1 - D(y', a', b_j, m_j, \epsilon')) (1 + q(y', a', B(y', a', b_j, m_j, \epsilon'), M(y', a', b_j, m_j, \epsilon'); m_i - 1)) \right. \\ \left. + D(y', a', b_j, m_j, \epsilon') q^D(y', b_j, m_j; m_i) \right\}.$$

After the country repays 1 unit today, the valuation of debt maturing in $m_i - 1$ periods depends on the expectation about future payoffs associated with repayments, reflected in future prices when $D = 0$, and future payoffs in default states, in which case the relevant price will be q^D , which will be explained below. Similarly, the price of debt used in restructuring is $q^R(y, b_j, m_j; m_i) =$

$$\frac{E_{y' | y} E_{\epsilon'}}{1 + r} \left\{ (1 - D(y', 1, b_j, m_j, \epsilon')) (1 + q(y', 1, B(y', 1, b_j, m_j, \epsilon'), M(y', 1, b_j, m_j, \epsilon'); m_i - 1)) \right. \\ \left. + D(y', 1, b_j, m_j, \epsilon') q^D(y', b_j, m_j; m_i) \right\}.$$

Recall that q^D is the price per unit of yearly payment b_j in default, which is

$$q^D(y', b_j, m_j; m_i) = \frac{E_{y'|y^D}}{1+r} \left\{ q^D(y', b_j, m_j; m_i) + \lambda E_{\epsilon'} H^L(y', b_j, m_j, \epsilon') \left[\frac{1}{-b_j} \frac{q^*(m_i, r^R)}{q^*(m_j, r^R)} W^L(y', b_j, m_j) - q^D(y', b_j, m_j; m_i) \right] \right\}.$$

Notice that this function illustrates the fact that a lender with promises up to m_i years would obtain $q^D(y', b_j, m_j; m_i)$ per dollar of yearly promises that she holds. Note also that this depends on the total debt defaulted upon, which in this case is b_j yearly payments for m_j years. One key aspect here affecting the cost of borrowing at different maturities is how the total repayment made by the country, $W^L(y', b_j, m_j)$, it is divided across bondholders. The simplest part is reflected in the fraction $\frac{1}{-b_j}$. A bondholder having one unit of yearly payments receives one over the total yearly payments promised. The most interesting part of how W^L is divided across bondholders with bonds of different maturity. For this, we use the fraction $\frac{q^*(m_i, r^R)}{q^*(m_j, r^R)}$. Note that alternatively, we could have used the fraction $\frac{m_i}{m_j} = \frac{q^*(m_i, 0)}{q^*(m_j, 0)}$, but this expression would not take into account the timing of payments. Later, we will change r^R in policy exercises to show the effect of different rules to distribute losses across lenders holding debt of different maturity.

4 Discrete Choices and Extreme Value Shocks

4.1 The Ex-Ante Problem

It is important to highlight that the shocks ϵ make the default decision, from an ex-ante point of view, stochastic. In this model, a single borrower that has observed her own state variables and the realization of the ϵ shocks, makes a unique deterministic decision on whether to default or not. However, by taking expectations over the ϵ shocks, we can view the default decision as probabilistic. We call $\mathbf{D}(y, a, b_i, m_i) = E_{\epsilon} D(y, a, b_i, m_i, \epsilon)$ the probability of default. In a similar fashion, the random component ϵ makes the debt and maturity choice decision random, from an ex-ante perspective. Similarly, we call $\mathbf{G}_{y,a,b_i,m_i}(b_j, m_j)$ the probability distribution of choosing an amount of debt $-b_j$ and maturity m_j for next period, *conditional on not defaulting* and on

the current levels of income, asset and maturity of the portfolio.

Proposition 1. *Using the ex-ante policy functions $\mathbf{D}$ and $\mathbf{G}$, the price of the bond can be written as,*

$$q(y, a, b_j, m_j; m_i) = \frac{E_{y', a' | y, a}}{1 + r} \left\{ (1 - \mathbf{D}(y', a', b_j, m_j)) \left[1 + \sum_{k=1}^{\mathcal{J}} q(y', a', b_k, m_k; m_i - 1) \mathbf{G}_{y', a', b_j, m_j}(b_k, m_k) \right] + \mathbf{D}(y', a', b_j, m_j) q^D(y', b_j, m_j; m_i) \right\}.$$

Proof. See Appendix C. □

Note that we can similarly define $\mathbf{H}^L(y, b_i, m_i) = E_{\epsilon} H^L(y, b_i, m_i, \epsilon)$ as the probability that a restructuring offer made by the lender is accepted. Finally, we can let $\mathbf{V}^G(y, a, b_i, m_i) = E_{\epsilon} [V^G(y, a, b_i, m_i, \epsilon)]$ and $\mathbf{V}^R(y, b_i, m_i) = E_{\epsilon} [V^R(y, b_i, m_i, \epsilon)]$ be the ex-ante (before observing the ϵ shocks) lifetime utilities in good credit status and in renegotiation, respectively.

4.2 Key Additional Assumptions

We assume that the vector ϵ is i.i.d over time and has the following joint cumulative density function:

$$F(\mathbf{x}) = \exp \left[- \left(\sum_{j=1}^{\mathcal{J}} \exp \left(- \frac{x_j - \mu}{\rho \sigma} \right) \right)^{\rho} - \exp \left(- \frac{x_{\mathcal{J}+1} - \mu}{\sigma} \right) \right],$$

where μ is a parameter such that shocks have mean zero, σ is a parameter that scales the variance of the shocks, and ρ is a constant related to the correlation of the shocks in the debt/maturity choice. This distribution, the Generalized Extreme Value distribution, was pioneered by [McFadden \(1978\)](#) in the context of discrete choice models with random utility.⁶ By using the

⁶This type of distributional assumption has been extended to dynamic models and is widely used in different fields in economics, particularly structural labor, industrial organization and international trade. The seminal works of [Rust \(1987\)](#), [Pakes \(1986\)](#), [Wolpin \(1984\)](#) and [Miller \(1984\)](#), have extended discrete choice models to dynamic settings. See also [Caliendo et al. \(2015\)](#) for a recent quantitative application using a large dynamic general equilibrium model on the effects of international trade on labor markets.

Generalized Extreme Value distribution we model the decision problem as a Nested Logit, where the first nest captures the default decision and the second the debt portfolio choice.⁷

The next proposition shows how the additional assumptions simplify the problem before observing the shocks ϵ .

Proposition 2. *Under the assumptions described above, the value functions $\{\mathbf{V}^{\mathbf{G}}, \mathbf{V}^{\mathbf{R}}\}$ and policy functions $\{\mathbf{D}, \mathbf{H}^{\mathbf{L}}, \mathbf{H}^{\mathbf{S}}, \mathbf{G}\}$ can be written solving the expectation over ϵ in closed form.*

Proof. Appendix C. □

The equations with most economic interpretation are shown next to illustrate the method. Appendix C has the expressions for all the functions mentioned in Proposition 2.

The probability of default can be expressed as,

$$\mathbf{D}(y, 0, b_i, m_i) = \frac{\exp\left(u(y^D) + \beta E_{y'|y^D} \mathbf{V}^{\mathbf{R}}(y', b_i, m_i)\right)^{1/\sigma}}{\left(\sum_{j=1}^{\mathcal{J}} \exp\left(u(c_{ij}(y)) + \beta E_{y', a'|y, 0} \mathbf{V}^{\mathbf{G}}(y', a', b_j, m_j)\right)^{\frac{1}{\rho\sigma}}\right)^{\rho} + \exp\left(u(y^D) + \beta E_{y'|y^D} \mathbf{V}^{\mathbf{R}}(y', b_i, m_i)\right)^{1/\sigma}}.$$

The probability of default adopts the logistic form that is common in dynamic discrete choice models. According to these expressions, a default is more likely when the value of default is larger relative to the value of repaying. The variance of the shocks plays a role in this probability. When σ is very large, the i.i.d. shocks will largely determine the choice, and economic conditions will not affect the default decision much. In the limit, there will be a 50% chance of default. On the other hand, when the shocks are very small, the default decision will be almost completely determined by economic conditions and all borrowers with the same state variables y, b_j, m_j will make the same decision.

Similarly, the probability of choosing a new debt level b_j and maturity m_j conditional on not defaulting, $\text{Prob}(b' = b_j, m' = m_j | y, a, b_i, m_i) \equiv \mathbf{G}(b_j, m_j | y, a, b_i, m_i)$ is

$$\mathbf{G}_{y, 0, b_i, m_i}(b_j, m_j) = \frac{\exp\left(u(c_{ij}(y)) + \beta E_{y', a'|y, 0} [\mathbf{V}^{\mathbf{G}}(y', a', b_j, m_j)]\right)^{\frac{1}{\rho\sigma}}}{\sum_{k=1}^{\mathcal{J}} \exp\left(u(c_{ik}(y)) + \beta E_{y', a'|y, 0} [\mathbf{V}^{\mathbf{G}}(y', a', b_k, m_k)]\right)^{\frac{1}{\rho\sigma}}},$$

⁷Clearly, it would be possible to create additional nests, but without additional information, it would be difficult to discipline this choice.

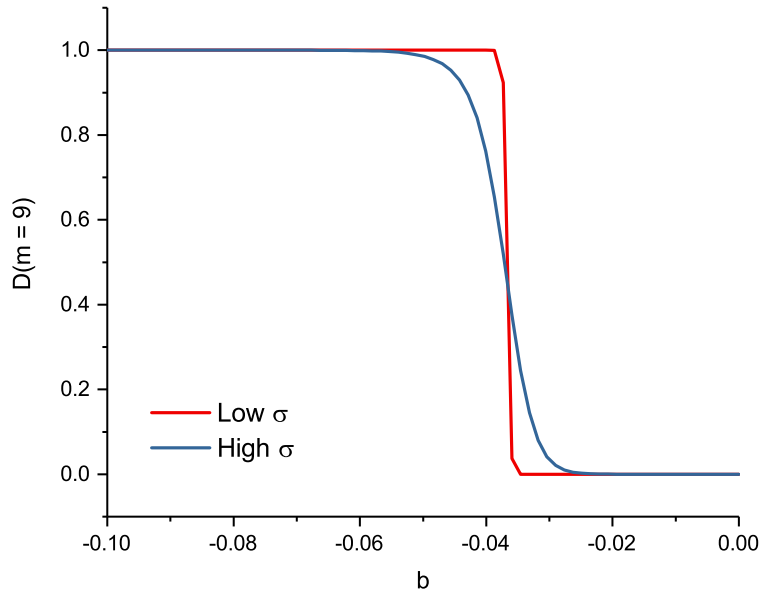
which again says that the probability that a borrower selects a new debt-maturity portfolio j is larger the larger is the value associated with of that particular portfolio.

4.3 The Role of ρ and σ

We now provide some intuitions on the effect of the i.i.d. ϵ shocks on the problem. The goal is to understand how the variance of the shocks modifies the original problem.⁸

The effect of the shocks can be clearly seen in Figure 2 which shows the probability of default for the same income level but different variance of the shock. With a small variance, the borrower tends to follow a single cutoff rule, defaulting with probability 1 for debt levels that are above a threshold. However, as the variance of the shock increases, this probability changes gradually and smoothly with the levels of debt. Since the default probabilities, as well as the other policy function of borrowers, enter the equilibrium price equations, the smooth changes in the decision rules imply smooth changes in prices.

Figure 2: Probability of default for different variance of ϵ shocks

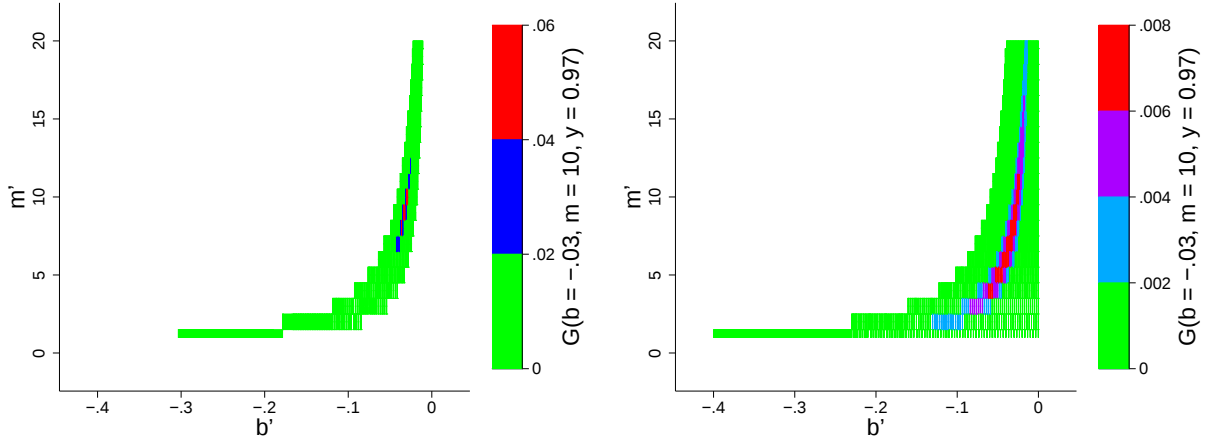


⁸The numerical results in this section are just for illustration. The calibration of the model is explained in the next section.

More interestingly, the decision rules on portfolio choices for borrowers in good standing are shown in Figure 3. It depicts the probability of choosing a new debt maturity portfolio for a borrower as a color map. The shape of the figure resembles an indifference curve map, indicating that borrowers try to achieve a value of total borrowing, and they can do this with low payments for longer periods of time (longer maturity), or higher payments for shorter periods (shorter maturity). The intensity of the colors indicate that borrowers prefer a particular combination, which is in the center, but there may be a dispersion around it due to the i.i.d. ϵ shocks.

The comparison of the two panels in Figure 3 highlights how the portfolio choice is affected by the variance of the shocks. With a smaller variance, the probability of choosing a certain set of portfolios is highly concentrated, as shown in the left panel. As the variance increases, the choices are more dispersed.

Figure 3: Debt-maturity portfolio probability for different variance of ϵ shocks



The benefit of the smooth decision rules means that in the algorithm that searches for an equilibrium, small changes from one iteration to the next should not cause large changes in the demands for different debt portfolios or prices. In this way, the iterative procedure to solve for equilibrium will tend to converge without major oscillations. Recall that these decision rules enter the valuation of a particular portfolio in the pricing equation. The ϵ shocks generate a smooth demand for a large variety of portfolios, even for borrowers in the same state.

5 Calibration

We solve the model numerically, calibrating the parameters based on the literature and available data and in the remaining cases selecting them jointly to reproduce key features.

We calibrate the model to a yearly frequency. Households in the economy have a constant relative risk aversion (CRRA) utility with risk aversion coefficient γ , which is set at 2, a standard value in the literature. We set the maximum possible maturity to 20 years, which is significantly larger than the maturity observed for emerging markets.⁹ We set the yearly risk-free interest rate to 0.042, which matches the long-run average of 10-year U.S. Treasury bonds. The standard deviation of the income shock is set to 0.019, and the persistence is set to 0.86 to replicate the detrended GDP per capita process for Colombia as estimated in [Sánchez et al. \(2018\)](#). For the empirical estimation of the debt rollover shocks, the paper considers the list of events documented in [Comelli \(2015\)](#) to derive a 2×2 transition probability matrix, where the probability of transitioning from a non-shock state to a shock state is 11.8%, and the probability of transitioning from a shock state to a shock state is 46.8%.¹⁰ In addition, [Richmond and Dias \(2009\)](#) document that *after a restructuring*, access to financial markets is limited and countries regain partial market access after 2-3 years on average. To capture this, we assume that the period immediately after restructuring the country cannot rollover or change its debt (i.e. experiences a debt-rollover shock), and after has a 53% probability of facing normal financial markets. Note that this last assumption affects access to financial markets post-restructuring. However, the duration of the default period is endogenous in our model, and we calibrate some of the model parameters to target this statistic, as explained next.

We set the interest rate used to limit how much the lenders can ask for in restructuring to $r^L = 0$. Note that with this value the constraint would use the face value of the debt in default as the maximum that the lenders can ask the country to repay. This is in line with bond clauses that establish that all future payments become due at the time of default. We set the rate used

⁹Our results are robust to allowing for longer maximum maturities.

¹⁰This is also the same calibration used in [Sánchez et al. \(2018\)](#). Alternative ways of modeling exogenous variation in the availability of credit would be to add risk-averse pricing kernels, as proposed, for instance by [Lizarazo \(2013\)](#), or to introduce exogenous variations in the risk-free rate. As far as these factors can be modeled as two-state Markov chains, the model would have exactly the same number of state variables.

to discount payments of different maturity at the time of default equal to the risk-free rate; i.e., $r^R = r$. As we show below, with this value the model is able to reproduce recent evidence presented by [Asonuma et al. \(2015\)](#), who argue that in present value terms, creditors with short-term securities suffer significantly more than creditors with long-term securities during sovereign debt restructuring episodes.

We introduce adjustment costs for changing the debt portfolio in normal times to capture issuance costs. We assume that both changes in maturity and changes in the size of yearly payments are costly. As a consequence, there are two parameters in that function, α_1 and α_2 .¹¹ Similarly, we introduce adjustment costs in restructuring the portfolio to capture that holder of sovereign debt, which are often financial institutions, dislike face value reductions due to regulations. In particular, we assume that the adjustment costs in restructuring apply only if the implied face value haircut is positive, and increase exponentially with this haircut.¹²

The parameters that were not preset above are jointly calibrated to minimize the distance of the model from the targets. Table 2 summarizes the model parameters and the fit of their target statistics.

The parameters in the adjustment cost function in normal times are calibrated jointly with the remaining parameters. The level of this cost is calibrated to available data on the cost of issuing debt. The curvature prevents large increases in debt and a consumption boom in the period before default, so it is calibrated to the average level of debt before default.¹³ The parameters in the adjustment cost function in restructuring are also calibrated such that the model replicates some moments in the data. The level of this cost is calibrated to the average maturity extension. If face value haircuts are more costly, restructurings will involve longer maturity extensions. The curvature of this function is associated with the calibration with the duration of default. We target the duration of a default episode to be 2.3 years. This is in line with the findings of [Das](#)

¹¹We use the following function form $\chi(b, m, b', m') = \alpha_1 \exp\left(\alpha_2 \left(\frac{m+m'}{2}|b-b'| - \frac{b+b'}{2}|m-m'|\right)\right) - \alpha_1$, where $-b$ and m are the level and maturity of the debt portfolio, respectively, after making the current payment, and $-b'$ and m' are those of the newly issued debt.

¹²We use the following function form, $\chi^R(b \times m, b^R \times m^R) = \alpha_1^R \exp(\alpha_2^R \max\{0, -(b \times m) + (b^R \times m^R)\}) - \alpha_1^R$, where $b \times m$ is the face value of the pre-restructuring debt, and $b^R \times m^R$ is that after the restructuring.

¹³See the discussion in [Hatchondo et al. \(2016\)](#). While they impose an upper limit on the spread, we prevent this behavior with the curvature of the adjustment cost function.

et al. (2012), who document that “The average total duration from the start of debt distress to the finalization of a restructuring is 28 months.”

As shown in the Table 2, we calibrate these four parameters in the adjustment cost functions together with the discount factor, β , the threshold of income in the default loss function, ϕ , the probability of lenders making an offer after default, λ , and the variance of the extreme value shocks ρ and σ . As is standard in the literature, β and ϕ are calibrated to replicate the debt-to-output ratio and the default rate. The probability of lenders making an offer after default, λ , affects directly the value of the haircut, so it is linked to that particular statistic. The values of ρ and σ must be larger than zero such that the benefits of using this method applies. We calibrate these parameters to match the standard deviation of duration and the standard deviation of the debt-to-output ratio because, as we show in Table 8 in Appendix D, they are directly affected by these parameters. More importantly, we show in Appendix D that with this calibration the ϵ shocks are not significant in generating defaults or shaping the distribution of maturity and debt choices (see Table 9 and Figure 11).

Table 2: Calibrated parameters and fit of targeted statistics

Parameter	Value	Basis	Data	Model
Discount factor, β	0.94	Debt/output	31%	31%
Corr. parameter, ρ	0.25	Std. dev. duration	0.89	1.27
Variance parameter, σ	0.002	Std. dev. debt/output	7.97	9.97
Output loss in default, π	0.94	Default rate	3%	3.1%
Lender’s offer probability, λ	0.50	Average SZ haircut	37%	36.1%
Portfolio adj. cost, α_1	0.00005	Average issuance costs	0.2%	0.16%
Portfolio adj. cost, α_2	20	Debt/output at default	50%	44%
Restr. adj. cost α_1^R	0.0008	Maturity extension, years	4.77	4.77
Restr. adj. cost α_2^R	10	Duration of default, years	2.33	2.74

Note: The three data moments not directly related to default or restructuring, which are at the top of this table, are for Colombia and the data sources are in Appendix A. Default rate in the data is based on Tomz and Wright (2013), p.257. Average haircut is based on Cruces and Trebesch (2013), Table 1. We compute the extension measures using the statistics provided in that paper. Issuance costs are taken as conservative estimates based on the statistics from Joffe (2015), Figure 1. Debt-to-output at default is the median reported in Mendoza and Yue (2012), Figure 1. Duration of default is taken from Das et al. (2012), p.27. Details on our computations are also in Appendix A.

6 Model Evaluation

In the previous sections, we introduced and calibrated a new model of maturity choice and debt restructuring in default. In this section, we evaluate the performance of the model using additional data. The model does remarkably well along with several dimensions.

Our model can closely match several key non-targeted empirical stylized facts of emerging markets. For exposition purposes, we divide these statistics into three groups. First, as illustrated in Table 3 for a sample of three well-known emerging market economies, our model closely captures the business cycles moments usually discussed in the literature of quantitative sovereign default models, such as the volatility of consumption relative to output volatility, which exceeds a value of 1 both in the data and the model, the volatility of the trade balance relative to output, the correlation of consumption with output, which is high and positive both in the model and the data, and the correlation of the trade balance with output, which is mild both in the model and in the sample data.

Second, our model statistics also mimic closely the median sovereign debt maturity and duration from the data, as well as their cyclical behavior (Table 3).¹⁴ Overall, the model delivers a maturity of 7.7 years and a duration of 4.1 years, close to the average values of the sample. Additionally, the model generates the lower debt maturity and duration found in the data during bad times. Specifically, during bad times, both debt maturity and duration in the model are about 10 percent lower than their averages, with maturity declining to 7 years and duration declining to 3.7 years, consistent with similar declines documented for the sample. Our model is also able to capture the positive correlation between maturity and duration with output generally observed in the data.

Third, as our study focuses on sovereign default risk, we also analyze sovereign bond yield spreads over risk-free debt instruments.¹⁵ The results in Table 3 suggest that our framework underpredicts the level of the spreads, especially on the short end of the yield spread curve, which is

¹⁴Consistent with the data, we use the Macaulay definition to derive the duration of sovereign debt. See Appendix B for definitions of debt duration and yield spreads.

¹⁵The spread at each maturity is the difference between the yield on a zero-coupon bond with default risk, and the yield on a bond with the same characteristics but with no default risk. We present the details of the model computations in Appendix B.

Table 3: Fit of key non-targeted moments

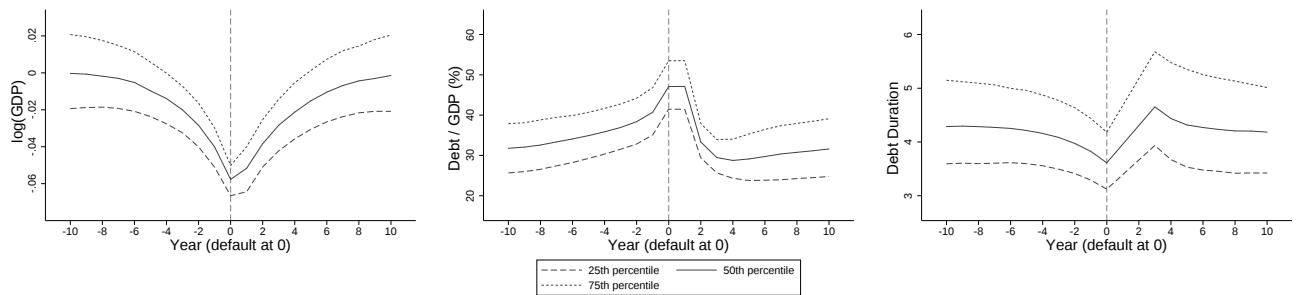
Statistics	Brazil	Colombia	Mexico	Model
$\sigma(\log(c))/\sigma(\log(y))$	1.15	1.15	1.65	1.08
$\sigma(\log(TB/y))/\sigma(\log(y))$	0.57	1.36	1.35	0.58
$\rho(\log(c), \log(y))$	0.75	0.90	0.69	0.85
$\rho(TB/y, \log(y))$	0.16	-0.08	-0.09	0.16
Duration (years)	3.41	5.42	5.61	4.09
Duration (years, bad times)	3.18	4.26	5.57	3.71
Maturity (years)	5.98	9.69	10.90	7.71
Maturity (years, bad times)	5.64	7.34	10.68	7.01
$\rho(mat, \log(y))$	0.65	0.93	0.03	0.32
$\rho(dur, \log(y))$	0.69	0.93	-0.20	0.22
1-year spread (%)	1.01	0.96	0.64	0.01
1-year spread (% , bad times)	1.78	1.42	0.80	0.22
10-year spread (%)	2.19	2.33	1.38	1.07
10-year spread (% , bad times)	7.08	4.32	1.78	1.73
$\rho(1YS, \log(y))$	-0.43	-0.61	-0.14	-0.32
$\rho(10YS, \log(y))$	-0.74	-0.89	-0.17	-0.61
$10YS - 1YS(\%)$	1.07	1.59	0.79	0.78

Note: See appendix for computational details and data sources.

to be expected in a model where bond prices are derived assuming risk-neutral lending. However, note that our spreads are lower than those in models with exogenous maturity because, especially for periods of high spreads, the country chooses a debt maturity that is shorter than the fixed maturity set by the standard calibration of those models. Nevertheless, our model captures very well the dynamics of yield spreads for different bond maturities over the business cycle. Finally, as summarized in Table 3, yield spreads for 1-year and 10-year instruments are countercyclical, and spreads for short-term bonds are lower than those for longer-term instruments.

We then ask: What are the dynamics of the model leading to default episodes? The behavior of output and the debt-to-output ratio, shown in Figure 4, are similar to those documented in the data by [Mendoza and Yue \(2012\)](#) in their Figure 1, where output falls in the years prior to a default and, on average, countries default when output is 5% below normal and then activity stays depressed for several years, even after restructuring, and gradually returns to normal values. With income depressed before default, countries increase the level of debt, which increases from around 30% of GDP in normal times to around 45% at the time of default. At the same time, and consistent with the data, duration in our model decreases before default, in part due to the increase in spreads resulting from the higher default risk as output falls, and then duration increases after restructuring.

Figure 4: Behavior around default

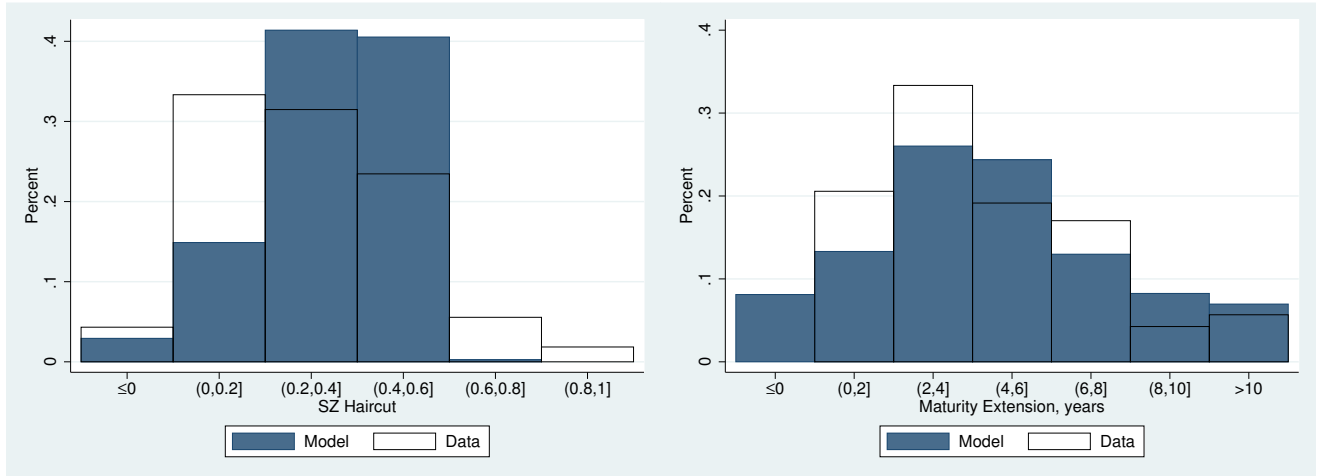


Note: To construct the figures, we first isolate the corresponding statistics for $x = \{0, 1, 2, \dots, 10\}$ before and after default episodes, and then take the medians across these for each x .

Finally, our framework rationalizes the haircuts and maturity extensions experienced by the defaulting countries during distressed debt restructurings, so we compare the distribution of these key variables with the data. Haircuts and maturity extensions are endogenously determined in

our setup as functions of the current income, and the debt and its maturity at default. Recall that the model is calibrated to reproduce the average haircut, H_{SZ} , and the average extension of maturity. Figure 5 shows the distribution of these variables both in our model and in the data. The model generates a distribution of haircuts that is more concentrated around the mean haircut than that in the data, while the distribution of maturity extensions is remarkably similar to that in the data. In the next section, we study what accounts for the variation in the outcomes of debt restructurings. In particular, we detail the model’s predictions regarding restructurings and we compare them with the data.

Figure 5: Endogenous restructurings: haircut and maturity extension



Note: The figure compares the distribution of SZ haircut and maturity extension measures in the data (sample excludes donor-supported restructurings) and model simulated data from the baseline economy.

7 Accounting for Differences across Restructurings

We now study how different key economic features shape the size of the haircuts and the maturity extensions in a restructuring, comparing the model’s predictions with the stylized facts from the data. As discussed in Section 2, the data shows large differences in haircuts and maturity extensions across time and geography. These differences are in part explained by the heterogeneity in the indebtedness and in the state of the economy -measured by output per capita- of the countries at the time of restructuring. Additionally, looking at different decades, [Cruces and Trebesch \(2013\)](#) “find a notable increase in haircut size over time. Haircuts were about 25

percentage points higher in the 1990s and 2000s compared to deals in the 1970s and 1980s. One reason is that deals during the 1980s mostly implied maturity extensions only, thus postponing the day of reckoning that many debtor countries had deep-rooted solvency problems.”

Table 4 presents regressions of *SZ* haircut, face-value haircut, maturity extension and duration of the default period using model-simulated data. Columns 1, 3, 5 and 7, use simulated data from the baseline model only. The first row indicates that countries that enter default with larger debt burdens exhibit larger haircuts and longer maturity extensions upon restructuring, but also tend to experience somewhat longer default periods. In our model, the key determinant of the value of restructured debt is the country’s ability to pay, which is independent of the past. Therefore, holding other things constant, countries with more debt in the past obtain larger haircuts. Maturity extensions from model-simulated data, shown in column 5 of the table, are similar to haircuts from the point of view of the lenders, so they are increasing in the country’s debt-to-output ratio for the same reasons. Recall also that a country’s debt is the upper bound of what lenders can ask for in a restructuring, so restructuring deals are accepted faster by countries with smaller debts.

Table 4 also shows that, in the model, countries with higher income receive smaller haircuts and shorter maturity extensions. Lenders ask for a higher market value of debt in restructuring, W , from countries with higher income because these countries are less likely to default again, and because the probability that an offer is accepted for a given W increases with income, given that the cost of exclusion is increasing in income.

To account for the important differences in the restructurings that occurred during the 1980s and early 1990s, relative to the more recent restructuring episodes discussed earlier, we perform an exercise in our model where we consider changes in the probability that the lender makes the restructuring offer. In more recent years, countries are more likely to make the restructuring offer, as nowadays sovereign debt is held by a large variety of lenders, while in the 1980s most of the debt was mostly held by large banks and other financial institutions. Thus, the country is usually the one making the restructuring offer. In this vein, there is evidence that since the late 1990s there has been an increase in the use of take-it-or-leave-it bond exchange offers that were preceded by informal discussions with creditors, but rarely formal negotiations (Sturzenegger

and Zettelmeyer, 2006a).

Columns 2, 4, 6 and 8 in Table 4 show the outcomes of the same type of regressions but now the sample is constructed using simulated data from three different models. The first model exhibits a high probability that lenders make an offer ($\lambda = 0.75$), which we relate to the situation in the 1980s based on our previous discussion about renegotiation trends in the data. The second model has a higher probability that the borrower makes an offer ($\lambda = 0.5$), which coincides with our baseline model and is the calibrated average across the full sample period. Finally, the third model has an even higher probability that borrowers make the restructuring offer ($\lambda = 0.25$) which we relate to the more recent restructuring episodes in the data. In the regressions, the omitted category is the case for $\lambda = 0.75$.

Table 4: Determinants of haircuts and maturity extensions in the model

	SZ Haircut		Face-Value Haircut		log(Maturity Ext)		log(Default Dur)	
	Baseline	All	Baseline	All	Baseline	All	Baseline	All
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Debt $\times$ Maturity / y	0.870 (0.108)	0.906 (0.112)	0.878 (0.130)	0.913 (0.136)	1.153 (0.437)	1.162 (0.449)	0.079 (0.298)	0.077 (0.295)
$\log(y)$	-1.115 (0.438)	-0.992 (0.450)	-1.385 (0.461)	-1.300 (0.465)	-2.502 (2.255)	-2.523 (2.304)	-5.157 (1.633)	-5.184 (1.618)
Higher prob. countries make an offer ($\lambda = 0.5$)		0.105 (0.023)		0.104 (0.022)		0.0704 (0.106)		-0.067 (0.074)
Much higher prob. countries make an offer ($\lambda = 0.25$)		0.272 (0.025)		0.276 (0.027)		0.161 (0.118)		-0.131 (0.080)

Note: Regression computed using simulated data from the model. Bootstrap standard errors shown in parenthesis computed using random samples (with replacement) with equal size as the one in the data in Cruces et al. (2002), which we use in Section 2. Columns 1, 3, 5, and 7, show the coefficients of the regression using the baseline model, where the probability that lenders make an offer is $\lambda = 0.5$. Columns 2, 4, 6, and 8, show the coefficients of the regression using pooled simulated data from three different models ($\lambda = \{0.25, 0.5, 0.75\}$). The omitted category is $\lambda = 0.75$.

The effects of the debt-to-output ratio and income in these regressions are very similar to those with simulated data from the baseline model. Higher values of the debt in default lead to higher haircuts, longer extensions and somewhat longer durations of default. Higher levels of output lead to lower haircuts, shorter maturity extensions and shorter periods in default.

The effect of the difference in the probability that lenders make an offer, λ , is captured by the two dummies in the regressions. As the probability of countries making an offer increases,

haircuts are larger because they offer the minimum value that lenders would accept. In addition, lower values of λ also lead to larger maturity extensions because reducing the value of W allows countries to pick portfolios with longer debt maturity.¹⁶ Finally, when borrowers are more likely to make a restructuring offer, the duration of the default period is shorter. Since the offer is more favorable to borrowers, in the model the borrowers will be more willing to make an offer when they have an opportunity to do so and will be more willing to accept when lenders are offering, thus shortening the time the countries spend in default.¹⁷

Overall, the directions of the effects are in line with those found in the data in Table 1, even when the magnitude of the coefficients may differ in some cases. In the case of haircuts, the elasticities with respect to the debt-to-output ratio are larger in the model, but for maturity extension and the duration of default, the model's regressions have elasticities that are similar to those found in the regressions in Table 1, which in the case of the duration of default are small in the model and non-significant in the data. Comparing the effects of output, we find that the coefficients are much larger in the model. One possible reason is that variation in the data comes from comparisons across countries with substantially different levels of income per capita, and we cannot control by country fixed effects (there are only a few countries with more than one restructuring event), while the model captures the same country at different points of the business cycle. Finally, the effects of changes in the probability of lenders making an offer, which we link to changes across different decades, are somewhat similar for the haircuts, but smaller in the model for maturity extensions and duration of default. While in the model we can fully control for the change in only one parameter, in the data the comparison is less transparent.

To analyze more closely the effects of the changes in λ in the model, we present a larger set of moments obtained by considering different values of this parameter. In Table 5, these changes can be seen by comparing columns 2, 3, 4 and 5 for the probability of lenders making an offer at 0.1, 0.25, 0.75 and 0.9, respectively. Intuitively, as lenders are more likely to make the restructuring offer, face-value haircuts become much lower than in the baseline economy. In

¹⁶This effect is carefully discussed in the next section.

¹⁷The results of the regressions are robust to using ($\lambda = \{0.1, 0.5, 0.9\}$), with the coefficients for the regression of the dummy variables quantitatively larger.

particular, the haircuts are 19% in the model with $\lambda = 0.9$, as opposed to 30% in the baseline. In contrast, when countries are more likely to make the restructuring offer, face-value haircuts become significantly larger than in the baseline (59% in the model with $\lambda = 0.1$, as opposed to 30% in the baseline).¹⁸ Thus, changes in λ help account for the increase in face value haircuts observed during restructurings over time.

Table 5: Effects of alternative parameter values on restructuring

	Baseline				
	$\lambda = 0.5$	$\lambda = 0.1$	$\lambda = 0.25$	$\lambda = 0.75$	$\lambda = 0.9$
Default (%)	3.14	3.06	3.13	2.99	2.88
(Debt $\times$ Maturity)/GDP (%)	31.42	20.64	26.31	34.29	36.01
Maturity (years)	7.71	9.66	8.12	7.54	7.55
1-year spread (% , bad times)	0.22	1.08	0.49	0.10	0.06
10-year spread (% , bad times)	1.73	2.68	2.05	1.51	1.38
$\sigma(\log(c))/\sigma(\log(y))$	1.08	1.02	1.05	1.11	1.12
Avg. haircut, face value (%)	29.95	58.65	41.50	23.00	19.41
Avg. haircut, SZ (%)	36.12	63.18	46.59	28.94	25.02
Median extension (years)	4.77	4.67	4.60	4.75	4.77
Duration of Default (years)	2.74	2.36	2.50	3.01	3.20

Finally, note that, when lenders are more likely to make the restructuring offer, the duration of the default is higher in the model. Specifically, in the case of $\lambda = 0.9$, the duration of default is 3.2 years, while it is 2.7 years in the baseline model and 2.4 years when $\lambda = 0.1$. Thus, the model is also consistent with the stylized fact highlighted by [Das et al. \(2012\)](#), that there has been a reduction in the length of the sovereign restructuring processes after the 1980s.

8 Policy Interventions in Restructurings

Restructurings are usually associated with policy interventions, which often involve some IMF participation. Of the 17 arrangements reviewed in an IMF report for the period from 1998 to 2014

¹⁸Columns 3 and 4 indicate that these patterns are also monotone as we change λ from 0.1 to 0.9.

(IMF, 2014), 11 included conditionality related to the restructuring. In this section, we study policy interventions that affect the output of private negotiations between lenders and borrowers. Technically, different behavioral and market features, such as agents' lack of commitment and incomplete markets, may justify policy interventions. Interestingly, we find that some of those policies may be ex-ante optimal. We compare welfare across different policy interventions using two different measures. On the one hand, we use the value function of a country with no debt and median income. Thus, it can be thought of as a country choosing which type of bond to issue from that period onward. Since this hypothetical country has no debt, a default is an event that may occur only in the remote future, so welfare gains or losses of restructuring are in general small. On the other hand, we compute the welfare gains of a policy change starting at the baseline equilibrium and taking into account both the transition. This last measure also takes into account that with the policy reform, lenders holding the debt at the time of the reform may gain or lose as the value of the debt suddenly changes. We compute the welfare measure net of the lender's compensation.¹⁹

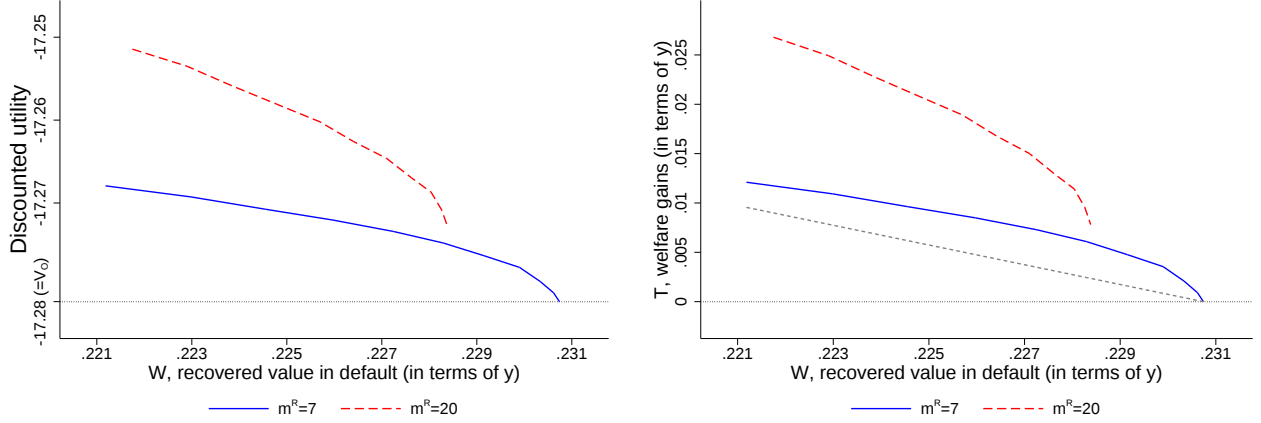
8.1 Imposing Minimum Market-Value Haircuts

A purely market-based restructuring outcome is associated with a high recovery value, W , which can only be achieved with a low maturity of the new debt. The left panel of Figure 6 shows the expected discounted utility that the country would obtain with deals that required to repay a value of W . As expected, the higher W , the lower is the value of utility obtained by the country. The figure shows two lines, which correspond to delivering the same W with alternative debt maturity—20 years in the case of the red dashed line, and 7 years in the case of the solid blue line.

This figure also shows that the maximum market value that the country can attain with the issued debt is $W = 0.231$. This deal would give the country a value of expected discounted utility of -17.28 with 7 years of debt maturity, which is the only maturity at which the country can offer such a debt portfolio. For illustration, the red line shows that with maturity 20 the maximum

¹⁹The technical details are given in Appendix B.3.

Figure 6: Possible gains from haircuts at different maturities



Note: The figure corresponds to income level $y = 0.945$ for the current period. The left panel plots the discounted utility for the borrower implied by different values for the lender W to satisfy in a restructuring. The right panel shows the transfer (in goods) to be made to the borrower facing the highest W offer, to bring it to the value implied by alternative W levels. The short-dashed line in the right panel gives a 45° angle. We give the associated formulas in Appendix B.5.

value of debt that can be obtained is less than 0.229. This implies that if the country must repay 0.231, it will be forced to restructure the debt with maturity 7. Thus, in market-based solutions where the lender is not interested in maturity per se and makes a take-it-or-leave-it offer that extracts maximum rents, the result entails equilibrium solutions with high recovery values (a low market-value haircut) and low exit maturities.

The right panel of Figure 6 shows the same function but now the expected discounted utility has been expressed in dollars using a transfer such that for the restructuring period the country is indifferent between that deal and the deal described before, in which the expected discounted utility for the country was -17.28 . This transformation is useful because now both axes have the same units. In there, imagine introducing a minimum haircut, or directly reducing the value of W that the lender would recover. In particular, consider a minimum haircut of 5%, which would set $W = 0.221$. Keeping prices constant, the lenders would lose 0.01. Keeping the maturity of the restructured debt at 7 years, the country gains only slightly more than 0.01, since the blue line in the right panel of Figure 6 is a bit above the dashed gray line that starts at $W = 0.231$ and makes an angle of 45° with the x axis. Instead, if this was the policy experiment in partial equilibrium, the country could choose maturity 20 years for the restructured debt and the gains would more than double. Why? Introducing a minimum haircut would allow the country to

choose a longer debt maturity at the time of restructuring, which has significant implications for the borrower's debt sustainability and welfare.

We study the effects of a policy that imposes a minimum market-value haircut during a debt restructuring. Formally we modify the constraint on the possible choices of restructuring offers by the lender in equation (1) to be $x \leq \theta(-bq^*(m, r^L))$, and we consider minimum haircut values from 10% to 100%.²⁰ The median change in welfare relative to the baseline as a result of the implementation of this policy is shown in the last row of Table 6. As shown in the table, imposing a minimum haircut would induce the borrower to choose a longer optimal maturity for its exit debt portfolio. The welfare gain, expressed as percentage increase in equivalent consumption, is maximized when imposing a 40% minimum haircut value. As shown in the table, in this case, the gains relative to the baseline setup for an economy with no debt, i.e., an ex-ante measure of gains, is 0.040%, and the median welfare gain in the ergodic distribution of the baseline economy is 0.047%, as shown in the last two rows of the table. While these gains are small, they are larger than the welfare gains of reducing business cycles, of 0.01%, obtained by Lucas (1987), and are comparable to the 0.14% estimate of Krusell and Smith (1999) that considers an economy with heterogeneous consumers. Note that for this minimum haircut, the median debt maturity extension is at 10 years, the SZ haircut has actually increased slightly (from 36.1% to 37.4%), and the average annual default rate decreases about 10 basis points to 3.0%. While the slightly lower default risk is associated to lower spreads, the lower recovery ratio in restructurings means that in bad times the lenders may require somewhat higher spreads. In bad times the 1-year spread is about 37 basis points higher, while the 10-year spreads are 22 basis points lower, so long-term debt becomes relatively less expensive than in the setup without a minimum haircut. Consistent with this price shift, the country now relies on debt with a maturity that is on average almost 0.7 years longer.

The welfare change experienced by the country as a result of the imposition of a minimum haircut policy in restructuring depends on the initial state of the economy. To analyze the welfare impact across the different economies, consider that initially there are no minimum haircuts, and then a 40% minimum haircut policy is imposed. The resulting decline in sovereign bond prices

²⁰That is, in these experiments we consider $\theta \in \{0.9, \dots, 0.1\}$, while $\theta = 1$ in the baseline model.

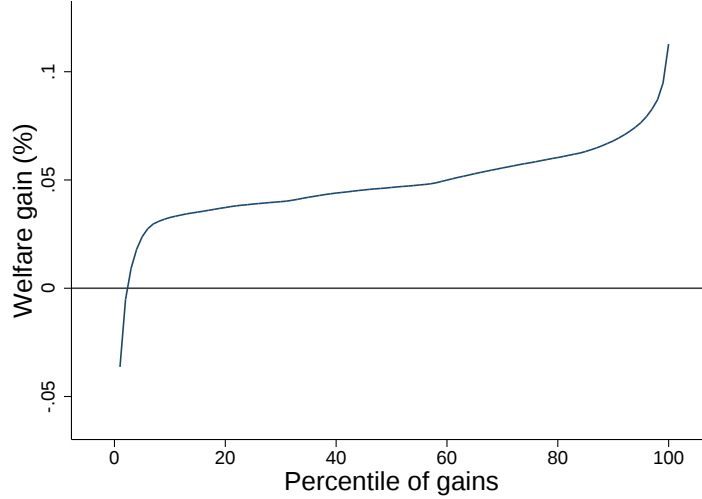
Table 6: Imposing minimum market value haircuts

	Minimum Haircut										
	Baseline	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
Default (%)	3.14	3.10	2.96	2.80	3.01	3.44	4.07	3.89	3.51	3.16	2.78
(Debt $\times$ Maturity)/GDP (%)	31.42	31.12	29.71	28.66	32.24	37.65	47.24	39.73	29.19	21.83	16.09
Maturity (years)	7.71	7.65	7.38	7.32	8.45	10.39	13.47	14.28	13.70	13.18	12.51
1-year spread (% , bad times)	0.22	0.23	0.24	0.30	0.59	1.17	2.33	2.76	2.73	2.64	2.35
10-year spread (% , bad times)	1.73	1.68	1.48	1.29	1.51	1.97	2.68	3.16	3.45	3.75	3.84
$\sigma(\log(c))/\sigma(\log(y))$	1.08	1.08	1.09	1.09	1.06	1.03	1.01	1.03	1.03	1.02	1.02
Avg. haircut, face value (%)	29.95	29.64	27.70	21.63	13.92	14.51	20.22	34.77	54.96	75.43	91.55
Avg. haircut, SZ (%)	36.12	35.65	34.39	33.74	37.40	40.47	40.94	51.45	66.78	82.53	98.02
Median extension (years)	4.77	4.76	4.90	6.55	10.09	11.54	10.50	10.18	9.45	7.26	4.45
Duration of Default (years)	2.74	2.75	2.74	2.68	2.66	2.81	3.06	2.95	2.69	2.41	2.19
Welfare gains (CE%) starting from no debt	-	0.001	0.006	0.021	0.040	0.038	0.015	-0.040	-0.079	-0.104	-0.119
Median Welfare gains (CE%) starting at baseline	-	0.001	0.007	0.022	0.047	0.045	0.011	-0.081	-0.173	-0.267	-0.373

Note: The computation of welfare gains is described in detail in Appendix B.3.

generates losses to bondholders, and the borrowing country compensates them with transfers so that they remain indifferent to the policy change. The amount of the transfers differs with the initial conditions, and the welfare gains after making the transfer vary by country. Figure 7 shows the distribution of welfare gains for different economies in the case of a 40% minimum haircut, which, for the median economy, is the minimum-haircut size associated to the largest welfare gain, as previously shown in Table 6. In the figure, the countries' welfare gains are ordered from the lowest to the largest. The countries with the lowest gains actually experience losses of about 0.03% in equivalent consumption terms, while the countries that benefit the most experience welfare gains of about 0.12%. Roughly 90% of the economies gain between 0.04% and 0.1%, with a median gain of just above 0.06%.

Figure 7: Welfare gains distributions from a 40% minimum haircut



Note: The computation of welfare gains is described in detail in Appendix B.3.

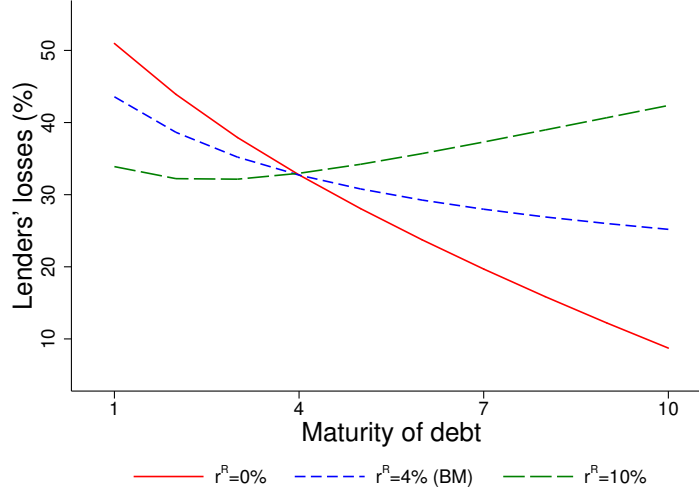
8.2 Redistributing Losses among Bondholders with Bonds of Different Maturities

In the model, the proceedings of a restructuring are distributed to lenders with bonds of different maturities according to a discounting rule of $\frac{q^*(n, r^R)}{q^*(m, r^R)}$, where n is the maturity of a bond and m is that of the entire portfolio. Here, $q^*(n, r^R)$ is the present value of n payments, discounted at rate r^R , which in turn shapes the relative returns for each maturity. In our baseline, we set $r^R = 4\%$.

Accordingly, this rule determines the losses to bondholders with bonds of different maturities, which are in general non-uniform. In order to illustrate this variation, we compute the losses of lenders holding zero-coupon bonds of different maturity.²¹ In Figure 8, we show for our baseline the losses and compare the results with two alternative models that are only different in r^R .

²¹In defining these losses, we compare the expected return to a unit of a zero-coupon bond with maturity n if the debt is in default to that if the debt is in good standing. We give the corresponding formula in Appendix B.4.

Figure 8: Losses of bondholders with bonds of different maturities, alternative redistribution schemes



Note: The figure uses $y = 0.97$, $b = -0.05$, $m = 8$ as states. We give the exact formula used to compute the losses in Appendix B. The figure compares three different models with three alternative discounting rules in redistributing the restructured debt among holders of different maturities.

The dashed blue line shows that in our baseline case losses are larger for holders of short-term bonds. This is consistent with the new evidence presented by [Asonuma et al. \(2015\)](#), who argue that in present value terms, creditors with short-term securities suffer significantly more than creditors with long-term securities during sovereign debt restructuring episodes. In the alternative cases shown in Figure 8, we set $r^R = 0$ and $r^R = 10\%$. Clearly, a lower r^R reduces the losses of holders of long-term debt and increases the losses of holders of short-term debt.

In Table 7 we present key statistics for economies with different values of r^R . Clearly, as the losses of holders of long-term debt are reduced (lower r^R), the equilibrium maturity of the model increases. Interestingly, changing r^R has little effect on overall haircuts, which vary around 35%, but has a large effect on face-value haircuts and maturity extensions. Setting $r^R = 0$, which minimizes the losses of holders of long-term debt, the face value haircut is 32.8% and the maturity extension is 3.8 years. In contrast, in the economy in which losses are relatively flat across different maturities ($r^R = 10\%$), which minimizes the losses of holders of short-term debt, the face-value haircut is 25.5% and the maturity extension is 5.7 years. As shown in the table, the higher r^R , the lower the default rate and hence the lower the yield spreads paid by the

borrower. Both in terms of ex-ante welfare and relative to the baseline, economies with higher r^R are preferred.

Table 7: Alternative redistribution schemes for lenders of different maturity

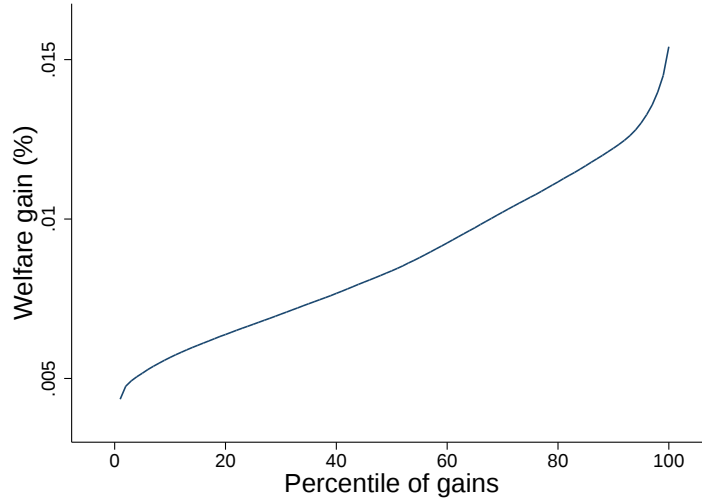
	Baseline ($r^R = 4\%$)	$r^R = 0\%$	$r^R = 2\%$	default bonds valuation		
				$r^R = 6\%$	$r^R = 8\%$	$r^R = 10\%$
Default (%)	3.14	3.57	3.38	3.00	2.91	2.86
(Debt $\times$ Maturity)/GDP (%)	31.42	34.52	33.02	30.81	30.42	30.12
Maturity (years)	7.71	8.70	8.20	7.50	7.35	7.24
1-year spread (% , bad times)	0.22	0.57	0.38	0.16	0.11	0.08
10-year spread (% , bad times)	1.73	2.15	1.93	1.68	1.66	1.67
$\sigma(\log(c))/\sigma(\log(y))$	1.08	1.03	1.05	1.09	1.10	1.11
Avg. haircut, face value (%)	29.95	32.76	30.96	28.60	26.93	25.50
Avg. haircut, SZ (%)	36.12	36.68	36.69	35.71	35.27	34.74
Median extension (years)	4.77	3.78	4.63	5.06	5.37	5.70
Duration of Default (years)	2.74	2.88	2.78	2.72	2.72	2.72
(Debt $\times$ Maturity)/GDP at default (%)	43.99	54.74	49.12	41.65	40.26	39.38
Welfare gains (CE%) starting from no debt	-	-0.006	-0.003	0.002	0.004	0.005
Median Welfare gain (CE%) starting at baseline	-	-0.008	-0.004	0.003	0.006	0.008

Note: The computation of welfare gains is described in detail in Appendix B.3.

The welfare change experienced by the country from alternative restructuring redistribution schemes for bondholders of different maturity depends on the state of the economy. To analyze the welfare effect of an alternative scheme, consider a change from the 4% discount rate in the baseline economy to a 10% rate that is associated to the largest median gains, as previously shown in Table 7. The borrowing country compensates bondholders with transfers so that they remain indifferent to the policy change. Figure 9 shows the distribution of welfare gains for different economies after making the transfers. In the figure, the countries' welfare gains are ordered from the lowest to the largest. The countries with the lowest gains actually experience losses of about 0.005% in equivalent consumption terms, while the countries that benefit the most experience welfare gains of about 0.015%. Roughly 90% of the economies gain between 0.006% and 0.012%, with a median gain of about 0.008%, which is about the same gain documented in

the literature from eliminating the cost of business cycle fluctuations.

Figure 9: Welfare gains distributions from $r^R = 10\%$

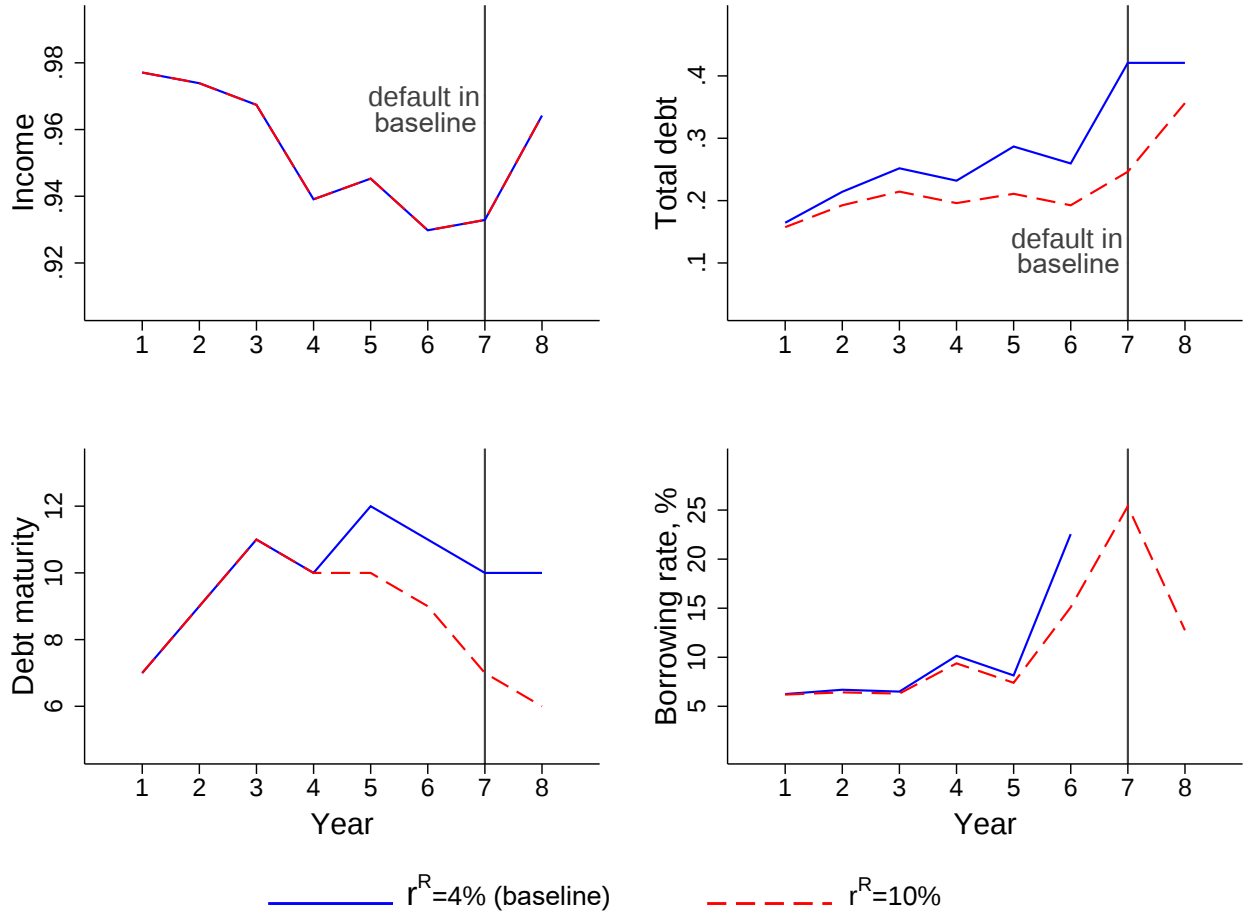


Note: The computation of welfare gains is described in detail in Appendix B.3.

To better understand why a higher discount rate lowers the sovereign default rate, recall that in bad times, countries find it more expensive and difficult to borrow long-term in credit markets, which is reflected in lower debt maturity. Therefore, if during a crisis period the losses of short-term debt holders are raised relative to those of holders of long-term debt, the country, which relies on short-term funding in bad times, will be driven to an unsustainable debt situation, and hence, default. In contrast, a higher discount rate makes short-term borrowing less expensive, providing some relief to the borrower during a crisis.

Figure 10 provides insight on the effect of a higher r^R on the path of key macro variables and default incentives relative to the baseline. Following a sequence of adverse output realizations, shown on the upper right panel, both the economy with the baseline discount rate (BM) and the high discount (HD) economy increase the liabilities (upper right panel) to smooth consumption. Both economies initially increase debt maturity (lower left panel), which serves as a hedging strategy. With more debt and lower output, borrowing rates increase (lower right panel) and both economies reduce their debt maturity, as borrowing short term is substantially less expensive, but the BM economy borrows at a somewhat longer term than the HD economy. This occurs because the long-term bondholders in the BM economy face substantially lower losses than short-

Figure 10: Comparing dynamics between the baseline and $r^R = 10\%$



Note: The blue solid lines are from one particular default episode in the baseline model. The red dashed lines are from the simulation in the high discount economy ($r^R = 10\%$) that starts from the same initial states with the same realization of shocks for the entire period.

term bondholders upon default compared to the HD economy, which tends to increase the cost of short vs long-term debt (flatter yield curve). Crucially, the higher interest rates that make it particularly more costly to borrow short term, and the consequently somewhat longer maturity imply that the BM economy faces a larger debt service cost that, as shown in the lower right panel, quickly becomes unsustainable and leads to a default, an event that is avoided by the HD economy.

9 Conclusion

We present a novel model of sovereign debt, maturity choice, default, and restructuring that quantitatively mimics the key debt maturity and payment dynamics surrounding distressed debt restructurings while retaining the business cycle properties of debt and the yield spread curve observed in the data. Our ability to solve the model relies in good measure on the implementation of a novel solution method based on the dynamic discrete choice literature.

Our framework can rationalize the variation in haircuts and maturity extensions from restructuring across countries, and also provides insights on the changes in restructuring features documented over time, including the increase in face value haircuts, the decrease in maturity extensions, and the shorter duration of the restructurings.

Finally, our work provides a useful tool to discuss the implications of alternative policy interventions for restructuring outcomes. We show that some market-based restructuring solutions are associated with relatively high debt recovery values and low maturity extensions. In particular, our framework indicates that policy interventions imposing a minimum haircut or redistributing losses across lenders in restructurings may be welfare enhancing, especially when they lead to longer maturity extensions.

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Appendix A Data details

A.1 Haircuts and maturity extensions

Using data from [Cruces and Trebesch \(2013\)](#) expanded to 187 distressed sovereign debt restructuring events between 1970 and 2013, we recover the maturity extensions using the different definitions of haircuts provided. The data set contains the amount of debt restructured; haircut measure proposed by ([Sturzenegger and Zettelmeyer, 2006b](#), SZ henceforth), H_{SZ} ; market haircut, H_M ; face-value haircut, and underlying discount rate used to value future cash flows, where

$$H_{SZ} = 1 - \frac{\text{Present Value of New Debt}}{\text{Present Value of Old Debt}} \quad \text{and} \quad H_M = 1 - \frac{\text{Present Value of New Debt}}{\text{Face Value of Old Debt}}.$$

Note that the only difference between H_{SZ} and H_M is the valuation of the old debt. Now taking the ratio of complements of market haircut and SZ haircut, we obtain

$$\frac{1 - H_M}{1 - H_{SZ}} = \frac{\text{Present Value of Old Debt}}{\text{Face Value of Old Debt}} = \frac{1}{N} \sum_{i=0}^{N-1} \left(\frac{1}{1+r} \right)^i, \quad (1)$$

where the last step uses the assumption that debt is uniformly distributed across maturities. Given that H_M , H_{SZ} , and r are available in the dataset provided by [Cruces and Trebesch \(2013\)](#), we can use equation (1) to find the value of the maturity of the old debt, N . In the same way, we can take advantage of the face-value haircut and market haircut to compute the maturity for the new debt. The maturity extension is then calculated as the difference between the maturity of new debt and the maturity of old debt.

A.2 Rest of the empirical analysis

- GDP per capita: We use World Development Indicators (WDI) provided by the World Bank (constant 2005 US\$). For the volatility and correlations, we HP filter the data for the entire horizon with available data.
- Debt-to-GDP ratio: For debt-to-output ratios, we use external debt stocks (% of GNI) provided by the WDI for the entire period for which we have available data on spreads and maturity.
- Consumption: For the moments on consumption, we use households' final consumption expenditure per capita (constant 2005 US\$), provided by the WDI. For the volatility and correlations, the paper follows the same approach as for the GDP per capita, by HP filtering the log consumption per capita for the entire period. We also use this variable to construct the trade balance by subtracting consumption from output.
- Maturity: We use the external debt maturity. For Colombia (2001-2014) and Brazil (2005-2015) the data are from the HAVER database, and for Mexico (2007-2010) from the OECD database.

- Duration: We use the data available in the HAVER database for the duration of debt for Colombia, as we do for the maturity for this country. This measure of duration follows the Macaulay definition, as we use for our computations in the model. For Brazil and Mexico, we compute the duration using the maturity data described above for these countries, together with the official average interest on new external debt commitments provided by the International Debt Statistics, also following the Macaulay definition.
- Spreads: The yields are US dollar sovereign yields obtained from Bloomberg. The yield spreads are obtained by subtracting US yields from the same data source.

Appendix B Computational Details

B.1 Basics

We solve the model numerically with value function iteration on a discretized grid for debt and output. We use different debt grid for each maturity m_i , evenly spaced between 0 and $0.7q^*(m_i, r^R)$, where $q^*(m_i, r^R)$ is the risk-free price for a bond of maturity m_i . We use 121 points for the debt grid, and 51 points for the output. We solve the policy and value functions for all points on these grids, and discrete search is conducted to find the optimal debt policy also over these. The price function is solved for 41 equally-spaced points on this grid and the implied function is linearly interpolated in the other parts of the algorithm. Since the steeper regions of the price function is where default usually happens, we have an uneven grid for income that is finer below the median income. In particular, the income grid is spread evenly below the median income over 40 points, and evenly above the median income grid over 10 points. We use the Tauchen method to discretize the income process.

We solve for the lenders' offer, $W^L(y, b_i, m_i)$, through a discrete search over 501 points on a state-specific evenly-spaced W -grid. The lowest point on the grid is 0 and the highest is $\min[0.7, -b \times m_i]$. Since the borrowers' offer $W^S(y, b_i, m_i)$ is equal to $-b_i q^D(y, b_i, m_i; m_i)$ it is not necessary to follow the same discrete search as W^L for W^S .

For convergence, we use a measure of distance for the price function of debt in good standing

in a given iteration that takes into account the maximum absolute distance of the prices across two iterations relative to the level of the price in a given state. We declare convergence when this error is lower than 10^{-5} . We update the lenders' offer only when this error is $< 10^{-4}$.

After solving for the policy and value functions, we run the simulations for 1500 countries (paths) for 400 years and drop the first 100 periods. The model counterparts to the empirical correlation and standard deviation statistics are averages across samples. For the first-order moments, country-specific medians are taken before averaging across countries. This is consistent with our treatment of the data.

B.2 Computing duration and yield to maturity

Duration. Similar to [Hatchondo and Martinez \(2009\)](#) and [Sánchez et al. \(2018\)](#), we use the Macaulay definition to compute the duration of a bond as a weighted sum of future promised payments:

$$\frac{q(y, a, b_i, m_i; 1) + 2 \times (q(y, a, b_i, m_i; 2) - q(y, a, b_i, m_i; 1)) + \dots + n \times (q(y, a, b_i, m_i; m_i) - q(y, a, b_i, m_i; m_i - 1))}{q(y, a, b_i, m_i; m_i)}.$$

Yield to maturity. Consider a country with income y , debt rollover shock a , and a debt portfolio with maturity m and level b . The yield for a bond with maturity n is:

$$YTM(y, a, b_i, m_i; n) \equiv \left(\frac{1}{q(y, a, b_i, m_i; n) - q(y, a, b_i, m_i; n - 1)} \right)^{\frac{1}{n}} - 1.$$

Then the spread for maturity m is $YTM(y, a, b_i, m_i; n) - r$.

B.3 Computing the welfare gains from policy experiments

Median welfare gain in transition. In order to obtain the welfare gains in transition, we use the simulations coming out of our baseline economy.

For each observation in good financial standing at the end of period $t \geq \hat{t}$, we define the compensated value from switching to a world with an alternative restructuring rule at the end

of the current period as:²²

$$\mathcal{W}_{it}^{\text{POL}} = \frac{(c_{it} - x_{it})^{1-\gamma}}{1-\gamma} + \beta E_{y',a'|y_{it},a_{it}}[\mathbf{V}^{\text{G}^{\text{POL}}}(y', a', b_{it+1}, m_{it+1})].$$

Here, the variables c_{it} , b_{it+1} , m_{it+1} denote the consumption, asset and maturity choices of a country i in the baseline simulation at period t . x_{it} denotes the hypothetical losses of lenders from implementing the switch from the baseline to the alternative policy:

$$x_{it} \equiv q^{\text{POL}}(y_{it}, a_{it}, b_{it+1}, m_{it+1}; m_{it+1})b_{it+1} - q^{\text{BM}}(y_{it}, a_{it}, b_{it+1}, m_{it+1}; m_{it+1})b_{it+1}.$$

Notice that if the price becomes higher after the switch, the losses of lenders are negative, and we give additional resources to the country to compute the compensated value measure.

Meanwhile, the value from staying in the baseline is:

$$\mathcal{W}_{it}^{\text{BM}} = \frac{(c_{it})^{1-\gamma}}{1-\gamma} + \beta E_{y',a'|y_{it},a_{it}}[\mathbf{V}^{\text{G}^{\text{BM}}}(y', a', b_{it+1}, m_{it+1})].$$

For these countries, that are in good financial standing at the end of the current period, our measure of welfare gain from the switch is:

$$\text{Gain}_{it} = (\mathcal{W}_{it}^{\text{POL}}/\mathcal{W}_{it}^{\text{BM}})^{\frac{1}{1-\gamma}} - 1.$$

In this exercise, we set the threshold $\hat{t}$ at 200, in order to have the observations in the baseline simulations after this period to follow the corresponding ergodic distribution.

For the countries in financial exclusion at the end of the period $t \geq \hat{t}$, we follow a similar approach using the debt prices and expected values in default.

Welfare gain for a country with no debt Similar to above, we compute an alternative measure of welfare gains only computing the $\mathcal{W}_{it}$ for the case with $b_{it} = 0$, $m_{it} = 10$, $y_{it} = \bar{y}$, $a_{it} = 0$, and with good credit record.

²²We are using a CRRA utility function with parameter γ as assumed in our calibration.

Note that we can do this computation simply by using the policy, value and price functions of the two economies, without using the simulations.

B.4 Computing lenders' losses in Section 8.2.

In order to get the losses of lenders following a default, we focus on holders of zero-coupon bond of maturity n . Suppose that the sovereign has a portfolio with maturity m_i , debt level $-b_i$, and income y . We define the losses plotted in Figure 8 as:

$$\text{Loss}(y, b, m; n) = 1 - \frac{q^D(y^D, b_i, m_i; n) - q^D(y^D, b_i, m_i; n-1)}{q(y, 0, b, m; n) - q(y, 0, b, m; n-1)}.$$

In this formula, the nominator is the expected return to a unit of zero-coupon bond with maturity n if the debt is in default. The denominator is the price of the same bond if the debt is in good standing and the borrower has access to financial markets (no debt rollover shock, i.e. $a = 0$).

B.5 Computation for Figure 6

In Figure 6 of the main body, we consider alternative values that the lender can ask for in restructuring the debt in default, W . In our model, the borrower chooses the new debt level ($-b^R$), its maturity (n^R) and an initial transfer (τ^R) to satisfy this W . In the left panel of the figure, we show the implied value for the borrower of alternative combinations b^R and n^R to satisfy a W exactly, without any transfers needed, i.e. $\tau^R = 0$. Formally, the left panel shows:²³

$$\hat{V} \equiv \frac{(y^D)^{1-\gamma}}{1-\gamma} + \beta E_{y'|y^D} \mathbf{V}^G(y', a', b^R, m^R)$$

subject to $-b^R \times q^R(y, b^R, m^R; m^R) = W$. The two lines in the figure corresponds to different m^R 's.

Since the unit price of the restructured debt, $q^R(y, b^R, m^R; m^R)$, approaches to zero as the debt level increases, there is an upper bound of W that the borrower can match given a maturity

²³For this formula we set the ϵ shocks to zero for the current period and we use the CRRA utility function assumed in our calibration.

choice m^R . The highest W in the plotted case is achieved by $m^R = 7$, giving the country a value of $-17.28 (= V_0)$. The right panel of Figure 6 shows the gains for the borrower to satisfying alternative W levels given a maturity m^R , in terms of goods in the current period. In particular, we compute the immediate transfers in goods (T) to the country satisfying the highest feasible offer to match, to bring it to other $\hat{V}$ levels plotted in the left panel:

$$\hat{V} - V_0 = \frac{(c_0 + T)^{1-\gamma}}{1-\gamma} - \frac{(c_0)^{1-\gamma}}{1-\gamma},$$

where c_0 is the consumption corresponding to V_0 . Notice that as being transfers only in the period of restructuring, T does not affect the expected values in next period allowing us to compare only the flows in the current period.

Appendix C Proofs

C.1 Proposition 1

We begin by noting that the price of a non-defaulted bond does not directly depend on ϵ , but only indirectly through the borrowers' choices. $q(y, a, b_j, m_j; m_i) =$

$$\frac{E_{y', a' | y, a} E_{\epsilon'}}{1+r} \left\{ (1 - D(y', a', b', m', \epsilon')) (1 + q(y', a', B(y', a', b_j, m_j, \epsilon'), M(y', a', b_j, m_j, \epsilon'); m_i - 1)) \right. \\ \left. D(y', a', b_j, m_j, \epsilon') q^D(y, a, b_j, m_j; m_i) \right\}.$$

We can partition the sample space into countable, finite and mutually exclusive events. These consist of the realizations of ϵ that lead to default and those realizations that lead to a particular b_k, m_k choice. This is convenient since in this case we can write the expectation of these events (or a function of them) as the sum of the possible realizations times the probability of each of these events.

We denote by $\mathbf{D}(y, a, b_j, m_j)$ the probability that the realizations of ϵ are such that default is preferred in our model. In this case lenders holding the bond obtain $q^D(y, a, b_j, m_j; m_i)$. The

realizations of ϵ that lead to non-default and to the specific bond b_k, m_k being chosen have a probability which we denote by $\mathbf{G}_{y,a,b_j,m_j}(b_k, m_k)$. If borrowers make this particular choice the next period, they will face the bond price $q(y', a', b_k, m_k; m_i - 1)$. Taking into account all possible choices, we can characterize the expectation over ϵ in the previous equation to express the bond price as, $q(y, a, b_j, m_j; m_i) =$

$$\begin{aligned} \frac{1}{1+r} E_{y',a'|y,a} \left[(1 - \mathbf{D}(y', a', b_j, m_j)) \left[1 + \sum_{k=1}^{\mathcal{J}} q(y', a', b_k, m_k; m_i - 1) \mathbf{G}_{y',a',b_j,m_j}(b_k, m_k) \right] \right. \\ \left. + \mathbf{D}(y', a', b_j, m_j) q^D(y', b_j, m_j; m_i) \right]. \end{aligned}$$

C.2 Proposition 2

Using the notation introduced above, and taking expectation over the ϵ shocks, the problem of the borrower for $a = 0$ can be written as

$$\begin{aligned} \mathbf{V}^G(y, 0, b_i, m_i) &\equiv E_{\epsilon} [V^G(y, 0, b_i, m_i, \epsilon)] = E_{\epsilon} \max \{ [V^P(y, 0, b_i, m_i, \epsilon), V^D(y, 0, b_i, m_i, \epsilon_{\mathcal{J}+1})] \} \\ &= E_{\epsilon} \left[\max \left\{ \max_{j \in \{1, 2, \dots, \mathcal{J}\}} \left\{ \frac{(c_{ij}(y))^{1-\gamma}}{1-\gamma} + \beta E_{y',a'|y,0} \mathbf{V}^G(y', a', b_j, m_j) + \epsilon_j \right\}; \right. \right. \\ &\quad \left. \left. \frac{(y^D)^{1-\gamma}}{1-\gamma} + \beta E_{y'|y^D} \mathbf{V}^R(y', b_i, m_i) + \epsilon_{\mathcal{J}+1} \right\} \right] \end{aligned}$$

subject to

$$c_{ij}(y) = y + b_i - q(y, 0, b_j, m_j; m_i) b_j + q(y, 0, b_j, m_j; m_i - 1) b_i - \chi(b_i, m_i - 1, b_j, m_j),$$

and the problem of the borrower with a debt rollover shock can be written as

$$\begin{aligned}\mathbf{V}^{\mathbf{G}}(y, 1, b_i, m_i) &\equiv E_{\epsilon} [V^{\mathbf{G}}(y, 1, b_i, m_i, \epsilon)] = E_{\epsilon} \max \{ [V^P(y, 1, b_i, m_i, \epsilon), V^D(y, 1, b_i, m_i, \epsilon_{\mathcal{J}+1})] \} \\ &= E_{\epsilon} \left[\max \left\{ \frac{(y + b_i)^{1-\gamma}}{1-\gamma} + \beta E_{y', a' | y, 1} \mathbf{V}^{\mathbf{G}}(y', a', b_i, m_i - 1) + \epsilon_i; \right. \right. \\ &\quad \left. \left. \frac{(y^D)^{1-\gamma}}{1-\gamma} + \beta E_{y' | y^D} \mathbf{V}^{\mathbf{R}}(y', b_i, m_i) + \epsilon_{\mathcal{J}+1} \right\} \right].\end{aligned}$$

Similarly, the value of entering the restructuring stage is: $\mathbf{V}^{\mathbf{R}}(y, b_i, m_i) \equiv E_{\epsilon} [V^{\mathbf{R}}(y, b_i, m_i, \epsilon)] =$

$$\begin{aligned}&\lambda E_{\epsilon} \left[\max \left\{ \frac{(y^D)^{1-\gamma}}{1-\gamma} + \beta E_{y' | y^D} \mathbf{V}^{\mathbf{R}}(y', b_i, m_i) + \epsilon_{\mathcal{J}+1}; \right. \right. \\ &\quad \left. \left. \max_{j \in \Omega(y, W^L)} \left\{ \frac{(y^D - \tau_j^R b_j^R - \chi^R(b_i \times m_i, b_j^R \times m_j^R))^{1-\gamma}}{1-\gamma} + \beta E_{y' | y^D} \mathbf{V}^{\mathbf{G}}(y', 1, b_j^R, m_j^R) + \epsilon_j \right\} \right\} \right] + \\ &(1 - \lambda) E_{\epsilon} \left[\max \left\{ \frac{(y^D)^{1-\gamma}}{1-\gamma} + \beta E_{y' | y^D} \mathbf{V}^{\mathbf{R}}(y', b_i, m_i) + \epsilon_{\mathcal{J}+1}; \right. \right. \\ &\quad \left. \left. \max_{j \in \Omega(y, W^S)} \left\{ \frac{(y^D - \tau_j^R b_j^R - \chi^R(b_i \times m_i, b_j^R \times m_j^R))^{1-\gamma}}{1-\gamma} + \beta E_{y' | y^D} \mathbf{V}^{\mathbf{G}}(y', 1, b_j^R, m_j^R) + \epsilon_j \right\} \right\} \right].\end{aligned}$$

We now use properties of the distribution of ϵ to simplify the previous expressions further and also obtain expressions for the policy functions (portfolio choice and default probabilities).²⁴

For the case of $a = 0$,

$$\begin{aligned}\mathbf{V}^{\mathbf{G}}(y, 0, b_i, m_i) &= \sigma \log \left[\left(\sum_{j=1}^{\mathcal{J}} \exp \left(\frac{(c_{ij}(y))^{1-\gamma}}{1-\gamma} + \beta E_{y', a' | y, 0} \mathbf{V}^{\mathbf{G}}(y', a', b_j, m_j) \right)^{1/\rho\sigma} \right)^{\rho} + \right. \\ &\quad \left. + \exp \left(\frac{(y^D)^{1-\gamma}}{1-\gamma} + \beta E_{y' | y^D} \mathbf{V}^{\mathbf{R}}(y', b_i, m_i) \right)^{1/\sigma} \right].\end{aligned}$$

²⁴The next subsection contains the proofs on how the expectation of the maximum in these expressions is derived.

And for $a = 1$,

$$\mathbf{V}^{\mathbf{G}}(y, 1, b_i, m_i) = \sigma \log \left[\exp \left(\frac{(y + b_i)^{1-\gamma}}{1-\gamma} + \beta E_{y', a' | y, 1} \mathbf{V}^{\mathbf{G}}(y', a', b_i, m_i - 1) \right)^{1/\sigma} + \exp \left(\frac{(y^D)^{1-\gamma}}{1-\gamma} + \beta E_{y' | y^D} \mathbf{V}^{\mathbf{R}}(y', b_i, m_i) \right)^{1/\sigma} \right],$$

while the value of entering the restructuring stage is,

$$\begin{aligned} \mathbf{V}^{\mathbf{R}}(y, b_i, m_i) = & \lambda \sigma \log \left[\exp \left(\frac{(y^D)^{1-\gamma}}{1-\gamma} + \beta E_{y' | y^D} \mathbf{V}^{\mathbf{R}}(y', b_i, m_i) \right)^{1/\sigma} + \right. \\ & \left. + \left[\sum_{j \in \Omega(y, W^L(y, b_i, m_i))} \exp \left(\frac{(y^D - \tau_j^R b_j^R - \chi^R(b_i \times m_i, b_j^R \times m_j^R))^{1-\gamma}}{1-\gamma} + \beta E_{y' | y^D} \mathbf{V}^{\mathbf{G}}(y', 1, b_j^R, m_j^R) \right)^{1/\rho\sigma} \right]^\rho \right] + \\ & (1 - \lambda) \sigma \log \left[\exp \left(\frac{(y^D)^{1-\gamma}}{1-\gamma} + \beta E_{y' | y^D} \mathbf{V}^{\mathbf{R}}(y', b_i, m_i) \right)^{1/\sigma} + \right. \\ & \left. + \left[\sum_{j \in \Omega(y, W^S(y, b_i, m_i))} \exp \left(\frac{(y^D - \tau_j^R b_j^R - \chi^R(b_i \times m_i, b_j^R \times m_j^R))^{1-\gamma}}{1-\gamma} + \beta E_{y' | y^D} \mathbf{V}^{\mathbf{G}}(y', 1, b_j^R, m_j^R) \right)^{1/\rho\sigma} \right]^\rho \right]. \end{aligned}$$

Note that, relative to the previous expressions, now the ex-ante value functions do not have a max operator. Under the specific distributional assumptions, the expectation of the maximum over different choices results in the standard log-sum of exponentials widely used in dynamic discrete choice models. These expressions are sometimes referred to as the inclusive values.

In addition, we can characterize the probability of default as,

$$\mathbf{D}(y, 0, b_i, m_i) = \frac{\exp \left(\frac{(y^D)^{1-\gamma}}{1-\gamma} + \beta E_{y' | y^D} \mathbf{V}^{\mathbf{R}}(y', b_i, m_i) \right)^{1/\sigma}}{\left(\sum_{j=1}^{\mathcal{J}} \exp \left(\frac{(c_{ij}(y))^{1-\gamma}}{1-\gamma} + \beta E_{y', a' | y, 0} \mathbf{V}^{\mathbf{G}}(y', a', b_j, m_j) \right)^{\frac{1}{\rho\sigma}} \right)^\rho + \exp \left(\frac{(y^D)^{1-\gamma}}{1-\gamma} + \beta E_{y' | y^D} \mathbf{V}^{\mathbf{R}}(y', b_i, m_i) \right)^{1/\sigma}}.$$

Similarly, the probability of choosing a new debt level b_j and maturity m_j conditional on not defaulting, $\text{Prob}(b' = b_j, m' = m_j | y, a, b_i, m_i) \equiv \mathbf{G}_{y, a, b_i, m_i}(b_j, m_j)$ is,

$$\mathbf{G}_{y, 0, b_i, m_i}(b_j, m_j) = \frac{\exp \left(\frac{(c_{ij}(y))^{1-\gamma}}{1-\gamma} + \beta E_{y', a' | y, 0} \mathbf{V}^{\mathbf{G}}(y', a', b_j, m_j) \right)^{\frac{1}{\rho\sigma}}}{\sum_{k=1}^{\mathcal{J}} \exp \left(\frac{(c_{jk}(y))^{1-\gamma}}{1-\gamma} + \beta E_{y', a' | y, 0} \mathbf{V}^{\mathbf{G}}(y', a', b_k, m_k) \right)^{\frac{1}{\rho\sigma}}},$$

which again says that the probability that a borrower selects a new debt-maturity portfolio j is larger, the larger is the value of that particular portfolio. In a rollover shock, borrowers cannot change the portfolio, which is described by the following policy function:

$$\mathbf{G}_{y,1,b_i,m_i}(b_j, m_j) = \begin{cases} 1 & \text{for } b_j = b_i; \quad m_j = m_i - 1 \\ 0 & \text{otherwise} \end{cases}$$

The probability of choosing a restructured debt level b_j^R and maturity m_j^R conditional on restructuring has a very similar expression but the sum is only over those debt/maturity choices in the sets $\Omega(y, W^L(y, b_i, m_i))$ and $\Omega(y, W^S(y, b_i, m_i))$.

In addition, with probability λ a borrower in default receives a restructuring offer from lenders. The probability of choosing to exit default and reschedule the debt $\mathbf{H}^L(y, b_i, m_i) =$

$$\frac{\left[\sum_{j \in \Omega(y, W^L(y, b_i, m_i))} \exp \left(\frac{(y^D - \tau_j^R b_j^R - \chi^R(b_i \times m_i, b_j^R \times m_j^R))^{1-\gamma}}{1-\gamma} + \beta E_{y'|y^D} \mathbf{V}^G(y', 1, b_j^R, m_j^R) \right)^{1/\rho\sigma} \right]^\rho}{\exp \left(\frac{(y^D)^{1-\gamma}}{1-\gamma} + \beta E_{y'|y^D} \mathbf{V}^R(y', b_i, m_i) \right)^{\frac{1}{\sigma}} + \left[\sum_{j \in \Omega(y, W^L(y, b_i, m_i))} \exp \left(\frac{(y^D - \tau_j^R b_j^R - \chi^R(b_i \times m_i, b_j^R \times m_j^R))^{1-\gamma}}{1-\gamma} + \beta E_{y'|y^D} \mathbf{V}^G(y', 1, b_j^R, m_j^R) \right)^{1/\rho\sigma} \right]^\rho},$$

which depends on both the borrower's current state and the characteristics of the offer. Restructuring offers that provide a high value for the borrower have a greater chance of being accepted. With probability $(1 - \lambda)$ the borrower has the option to make a restructuring offer that leads to an almost identical expression for $\mathbf{H}^S(y, b_i, m_i)$ but with the set $\Omega(y, W^S(y, b_i, m_i))$ defining the possible debt/maturity choices.

Finally, the price of a bond in good standing is

$$q(y, a, b_j, m_j; m_i) = \frac{1}{1+r} E_{y',a'|y,a} \left[(1 - \mathbf{D}(y', a', b_j, m_j)) \left[1 + \sum_{k=1}^{\mathcal{J}} q(y', a', b_k, m_k; m_i - 1) \mathbf{G}_{y',a',b_j,m_j}(b_k, m_k) \right] \right. \\ \left. + \mathbf{D}(y', a', b_j, m_j) q^D(y', b_j, m_j; m_i) \right],$$

and the price of a bond in default

$$q^D(y, b_j, m_j, m_i) = \frac{E_{y'|y^D}}{1+r} \left[q^D(y', b_j, m_j, m_i) + \lambda \mathbf{H}^L(y', b_j, m_j) \left[\frac{1}{(-b_j)} \frac{q^*(m_i, r^R)}{q^*(m_j, r^R)} W^L(y', b_j, m_j) - q^D(y', b_j, m_j) \right] \right].$$

C.1 Expectations

We now show how to obtain the main expressions for the policy and value functions after taking expectations over the ϵ shocks. To avoid repetition, we only provide the proofs for a subset of the expressions. The rest can be obtained following almost identical steps.

Take the distribution for the shocks ϵ .

$$F(\mathbf{x}) = \exp \left[- \left(\sum_{j=1}^{\mathcal{J}} \exp \left(- \frac{x_j - \mu}{\rho \sigma} \right) \right)^{\rho} - \exp \left(- \frac{x_{\mathcal{J}+1} - \mu}{\sigma} \right) \right].$$

Define the partial derivative as $F_j(\mathbf{x}) = \partial F(\mathbf{x}) / \partial x_j$ as,

$$F_j(\mathbf{x}) = \begin{cases} \frac{1}{\sigma} \exp \left(- \left[\sum_{j=1}^{\mathcal{J}} \exp \left(- \frac{x_j - \mu}{\rho \sigma} \right) \right]^{\rho} - \exp \left(- \frac{x_{\mathcal{J}+1} - \mu}{\sigma} \right) \right) \left(\sum_{j=1}^{\mathcal{J}} \exp \left(- \frac{x_j - \mu}{\rho \sigma} \right) \right)^{\rho-1} \exp \left(- \frac{x_j - \mu}{\rho \sigma} \right) & \text{for } j = 1, \dots, \mathcal{J} \\ \frac{1}{\sigma} \exp \left(- \left[\sum_{j=1}^{\mathcal{J}} \exp \left(- \frac{x_j - \mu}{\rho \sigma} \right) \right]^{\rho} - \exp \left(- \frac{x_{\mathcal{J}+1} - \mu}{\sigma} \right) \right) \exp \left(- \frac{x_{\mathcal{J}+1} - \mu}{\sigma} \right) & \text{for } j = \mathcal{J} + 1 \end{cases}$$

in what follows, we omit the state variables y and a to simplify the notation. In addition, let

$$\Upsilon_{i,j} = \begin{cases} \frac{(c_{ij}(y))^{1-\gamma}}{1-\gamma} + \beta E_{y',a|y,a} \mathbf{V}^{\mathbf{G}}(y', a', b_j, m_j) & \text{for } j = 1, \dots, \mathcal{J} \\ \frac{(y^D)^{1-\gamma}}{1-\gamma} + \beta E_{y'|y^D} \mathbf{V}^{\mathbf{R}}(y', b_i, m_i) & \text{for } j = \mathcal{J} + 1. \end{cases}$$

Also, call $\mathbf{D}_i = \mathbf{D}(y, a, b_i, m_i)$, then The probability of default is,

$$\begin{aligned} \mathbf{D}_i &= \int_{-\infty}^{\infty} F_{\mathcal{J}+1}(\Upsilon_{i,\mathcal{J}+1} + \epsilon_{\mathcal{J}+1} - \Upsilon_{i,1}, \dots, \Upsilon_{i,\mathcal{J}+1} + \epsilon_{\mathcal{J}+1} - \Upsilon_{i,\mathcal{J}}, \epsilon_{\mathcal{J}+1}) d\epsilon_{\mathcal{J}+1} \\ &= \frac{1}{\sigma} \int_{-\infty}^{\infty} \exp \left(- \left[\sum_{j=1}^{\mathcal{J}} \exp \left(- \frac{\Upsilon_{i,\mathcal{J}+1} + \epsilon_{\mathcal{J}+1} - \Upsilon_{i,j} - \mu}{\rho \sigma} \right) \right]^{\rho} - \exp \left(- \frac{\epsilon_{\mathcal{J}+1} - \mu}{\sigma} \right) \right) \exp \left(- \frac{\epsilon_{\mathcal{J}+1} - \mu}{\sigma} \right) d\epsilon_{\mathcal{J}+1} \\ &= \frac{1}{\sigma} \int_{-\infty}^{\infty} \exp \left(- \exp \left(- \frac{\epsilon_{\mathcal{J}+1} - \mu}{\sigma} \right) \left[\sum_{j=1}^{\mathcal{J}} \exp \left(- \frac{\Upsilon_{i,\mathcal{J}+1} - \Upsilon_{i,j}}{\rho \sigma} \right) \right]^{\rho} - \exp \left(- \frac{\epsilon_{\mathcal{J}+1} - \mu}{\sigma} \right) \right) \exp \left(- \frac{\epsilon_{\mathcal{J}+1} - \mu}{\sigma} \right) d\epsilon_{\mathcal{J}+1}. \end{aligned}$$

Call $\exp(\phi_i) = 1 + \left[\sum_{j=1}^{\mathcal{J}} \exp\left(-\frac{\Upsilon_{i,\mathcal{J}+1}-\Upsilon_{i,j}}{\rho\sigma}\right) \right]^\rho$, then

$$\begin{aligned} \mathbf{D}_i &= \frac{\exp(-\phi_i)}{\sigma} \underbrace{\int_{-\infty}^{\infty} \exp\left(-\exp\left(-\frac{\epsilon_{\mathcal{J}+1}-\mu-\sigma\phi_i}{\sigma}\right)\right) \exp\left(-\frac{\epsilon_{\mathcal{J}+1}-\mu-\sigma\phi_i}{\sigma}\right) d\epsilon_{\mathcal{J}+1}}_{=\sigma} \\ &= \frac{1}{1 + \left[\sum_{j=1}^{\mathcal{J}} \exp\left(-\frac{\Upsilon_{i,\mathcal{J}+1}-\Upsilon_{i,j}}{\rho\sigma}\right) \right]^\rho}. \end{aligned}$$

After proper substitution of the definition of $\Upsilon_{i,j}$ and $\Upsilon_{i,\mathcal{J}+1}$, we get the expression for the probability of default described in the previous subsection.

Call, $\mathbf{G}_{i,j} = \mathbf{G}_{y,a,b_i,m_i}(b_j, m_j)$. Then, the probability of choosing the maturity/debt portfolio j , conditional on not defaulting is

$$\begin{aligned} \mathbf{G}_{i,j} &= \frac{1}{(1-\mathbf{D}_i)} \int_{-\infty}^{\infty} F_j(\Upsilon_{i,j} + \epsilon_j - \Upsilon_{i,1}, \dots, \Upsilon_{i,j} + \epsilon_j - \Upsilon_{i,\mathcal{J}}, \Upsilon_{i,j} + \epsilon_j - \Upsilon_{i,\mathcal{J}+1}) d\epsilon_j \\ &= \frac{1}{\sigma(1-\mathbf{D}_i)} \int_{-\infty}^{\infty} \exp\left(-\left[\sum_{k=1}^{\mathcal{J}} \exp\left(-\frac{\Upsilon_{i,j} + \epsilon_j - \Upsilon_{i,k} - \mu}{\rho\sigma}\right)\right]^\rho - \exp\left(-\frac{\Upsilon_{i,j} + \epsilon_j - \Upsilon_{i,\mathcal{J}+1} - \mu}{\sigma}\right)\right) \\ &\quad \left(\sum_{k=1}^{\mathcal{J}} \exp\left(-\frac{\Upsilon_{i,j} + \epsilon_j - \Upsilon_{i,k} - \mu}{\rho\sigma}\right)\right)^{\rho-1} \exp\left(-\frac{\epsilon_j - \mu}{\rho\sigma}\right) d\epsilon_j \\ &= \frac{1}{\sigma(1-\mathbf{D}_i)} \int_{-\infty}^{\infty} \exp\left(-\exp\left(-\frac{\epsilon_j - \mu}{\sigma}\right) \left[\left[\sum_{k=1}^{\mathcal{J}} \exp\left(-\frac{\Upsilon_{i,j} - \Upsilon_{i,k}}{\rho\sigma}\right)\right]^\rho + \exp\left(-\frac{\Upsilon_{i,j} - \Upsilon_{i,\mathcal{J}+1}}{\sigma}\right)\right]\right) \\ &\quad \exp\left(-\frac{\epsilon_j - \mu}{\rho\sigma}\right)^{\rho-1} \left(\sum_{k=1}^{\mathcal{J}} \exp\left(-\frac{\Upsilon_{i,j} - \Upsilon_{i,k}}{\rho\sigma}\right)\right)^{\rho-1} \exp\left(-\frac{\epsilon_j - \mu}{\rho\sigma}\right) d\epsilon_j \\ &= \frac{\left(\sum_{k=1}^{\mathcal{J}} \exp\left(-\frac{\Upsilon_{i,j} - \Upsilon_{i,k}}{\rho\sigma}\right)\right)^{\rho-1}}{\sigma(1-\mathbf{D}_i)} \int_{-\infty}^{\infty} \exp\left(-\exp\left(-\frac{\epsilon_j - \mu}{\sigma}\right) \left[\left[\sum_{k=1}^{\mathcal{J}} \exp\left(-\frac{\Upsilon_{i,j} - \Upsilon_{i,k}}{\rho\sigma}\right)\right]^\rho + \exp\left(-\frac{\Upsilon_{i,j} - \Upsilon_{i,\mathcal{J}+1}}{\sigma}\right)\right]\right) \\ &\quad \exp\left(-\frac{\epsilon_j - \mu}{\rho\sigma}\right) d\epsilon_j. \end{aligned}$$

Call $\exp(\eta_{i,j}) = \left[\sum_{k=1}^{\mathcal{J}} \exp\left(-\frac{\Upsilon_{i,j}-\Upsilon_{i,k}}{\rho\sigma}\right)\right]^\rho + \exp\left(-\frac{\Upsilon_{i,\mathcal{J}+1}-\Upsilon_{i,j}}{\sigma}\right)$, then

$$\mathbf{G}_{i,j} = \frac{\left(\sum_{k=1}^{\mathcal{J}} \exp\left(-\frac{\Upsilon_{i,j}-\Upsilon_{i,k}}{\rho\sigma}\right)\right)^{\rho-1}}{(1-\mathbf{D}_i)\sigma \exp(\eta_{i,j})} \underbrace{\int_{-\infty}^{\infty} \exp\left(-\exp\left(-\frac{\epsilon_j - \mu - \sigma\eta_{i,j}}{\sigma}\right)\right) \exp\left(-\frac{\epsilon_j - \mu - \sigma\eta_{i,j}}{\sigma}\right) d\epsilon_j}_{=\sigma}.$$

Since $(1 - \mathbf{D}_i) \exp(\eta_{i,j}) = \left(\sum_{k=1}^{\mathcal{J}} \exp\left(\frac{\Upsilon_{i,k} - \Upsilon_{i,j}}{\rho\sigma}\right) \right)^\rho$, we have that

$$\mathbf{G}_{i,j} = \frac{1}{\sum_{k=1}^{\mathcal{J}} \exp\left(\frac{\Upsilon_{i,k} - \Upsilon_{i,j}}{\rho\sigma}\right)}$$

which, after proper substitution of the definition of $\Upsilon_{i,j}$ and $\Upsilon_{i,k}$, gives the expression for the probability of choosing the maturity/debt portfolio j described in the previous subsection.

Finally, call $\mathbf{V}^{\mathbf{G}}_i = \mathbf{V}^{\mathbf{G}}(y, a, b_i, m_i)$, we can find $\mathbf{V}^{\mathbf{G}}_i = E_\epsilon [\max_{j \in \{1, \dots, \mathcal{J}+1\}} \{\Upsilon_{i,j} + \epsilon_j\}]$ in the following way,

$$\begin{aligned} \mathbf{V}^{\mathbf{G}}_i &= \sum_{j=1}^{\mathcal{J}+1} \int_{-\infty}^{\infty} (\Upsilon_{i,j} + \epsilon_j) F_j(\Upsilon_{i,j} + \epsilon_j - \Upsilon_{i,1}, \dots, \Upsilon_{i,j} + \epsilon_j - \Upsilon_{i,\mathcal{J}}, \Upsilon_{i,j} + \epsilon_j - \Upsilon_{i,\mathcal{J}+1}) d\epsilon_j \\ &= \frac{1}{\sigma} \sum_{j=1}^{\mathcal{J}} \int_{-\infty}^{\infty} (\Upsilon_{i,j} + \epsilon_j) \exp\left(-\left[\sum_{k=1}^{\mathcal{J}} \exp\left(-\frac{\Upsilon_{i,j} + \epsilon_j - \Upsilon_{i,k} - \mu}{\rho\sigma}\right)\right]^\rho - \exp\left(-\frac{\Upsilon_{i,j} + \epsilon_j - \Upsilon_{i,\mathcal{J}+1} - \mu}{\sigma}\right)\right) \\ &\quad \left(\sum_{k=1}^{\mathcal{J}} \exp\left(-\frac{\Upsilon_{i,j} + \epsilon_j - \Upsilon_{i,k} - \mu}{\rho\sigma}\right)\right)^{\rho-1} \exp\left(-\frac{\epsilon_j - \mu}{\rho\sigma}\right) d\epsilon_j + \\ &\quad \frac{1}{\sigma} \int_{-\infty}^{\infty} (\Upsilon_{i,\mathcal{J}+1} + \epsilon_{\mathcal{J}+1}) \exp\left(-\left[\sum_{k=1}^{\mathcal{J}} \exp\left(-\frac{\Upsilon_{i,\mathcal{J}+1} + \epsilon_{\mathcal{J}+1} - \Upsilon_{i,k} - \mu}{\rho\sigma}\right)\right]^\rho - \exp\left(-\frac{\epsilon_{\mathcal{J}+1} - \mu}{\sigma}\right)\right) \exp\left(-\frac{\epsilon_{\mathcal{J}+1} - \mu}{\sigma}\right) d\epsilon_{\mathcal{J}+1} \\ &= \sum_{j=1}^{\mathcal{J}} \exp(-\eta_{i,j}) \left(\sum_{k=1}^{\mathcal{J}} \exp\left(-\frac{\Upsilon_{i,j} - \Upsilon_{i,k}}{\rho\sigma}\right)\right)^{\rho-1} \\ &\quad \left[\Upsilon_{i,j} + \mu + \sigma\eta_{i,j} + \underbrace{\int_{-\infty}^{\infty} \left(\frac{\epsilon_j - \mu - \sigma\eta_{i,j}}{\sigma}\right) \exp\left(-\exp\left(-\frac{\epsilon_j - \mu - \sigma\eta_{i,j}}{\sigma}\right)\right) \exp\left(-\frac{\epsilon_j - \mu - \sigma\eta_{i,j}}{\sigma}\right) d\epsilon_{\mathcal{J}+1}}_{=\sigma\gamma} \right] + \\ &\quad \exp(-\phi_i) \left[\Upsilon_{i,\mathcal{J}+1} + \mu + \sigma\phi_i + \underbrace{\int_{-\infty}^{\infty} \left(\frac{\epsilon_{\mathcal{J}+1} - \mu - \sigma\phi_i}{\sigma}\right) \exp\left(-\exp\left(-\frac{\epsilon_{\mathcal{J}+1} - \mu - \sigma\phi_i}{\sigma}\right)\right) \exp\left(-\frac{\epsilon_{\mathcal{J}+1} - \mu - \sigma\phi_i}{\sigma}\right) d\epsilon_{\mathcal{J}+1}}_{=\sigma\gamma} \right], \end{aligned}$$

where $\gamma \approx 0.5772$ is Euler's constant. Making $\mu = -\sigma\gamma$, the previous expression simplifies to,

$$\mathbf{V}^{\mathbf{G}}_i = \sum_{j=1}^{\mathcal{J}} \left[\exp(-\eta_{i,j}) \left(\sum_{k=1}^{\mathcal{J}} \exp\left(-\frac{\Upsilon_{i,j} - \Upsilon_{i,k}}{\rho\sigma}\right)\right)^{\rho-1} [\Upsilon_{i,j} + \sigma\eta_{i,j}] \right] + \exp(-\phi_i) [\Upsilon_{i,\mathcal{J}+1} + \sigma\phi_i].$$

Note that,

$$\exp(-\phi_i) [\Upsilon_{i,\mathcal{J}+1} + \sigma \phi_i] = \frac{\Upsilon_{i,\mathcal{J}+1} + \sigma \log \left[1 + \left(\sum_{k=1}^{\mathcal{J}} \exp \left(-\frac{\Upsilon_{i,\mathcal{J}+1} - \Upsilon_{i,k}}{\rho\sigma} \right) \right)^\rho \right]}{1 + \left(\sum_{k=1}^{\mathcal{J}} \exp \left(-\frac{\Upsilon_{i,\mathcal{J}+1} - \Upsilon_{i,k}}{\rho\sigma} \right) \right)^\rho},$$

and also,

$$\begin{aligned} \exp(-\eta_{i,j}) \left(\sum_{k=1}^{\mathcal{J}} \exp \left(-\frac{\Upsilon_{i,j} - \Upsilon_{i,k}}{\rho\sigma} \right) \right)^{\rho-1} [\Upsilon_{i,j} + \sigma \eta_{i,j}] &= \\ &= \frac{\left(\sum_{k=1}^{\mathcal{J}} \exp \left(-\frac{\Upsilon_{i,\mathcal{J}+1} - \Upsilon_{i,k}}{\rho\sigma} \right) \right)^{\rho-1} \left[\Upsilon_{i,\mathcal{J}+1} + \sigma \log \left[1 + \left(\sum_{k=1}^{\mathcal{J}} \exp \left(-\frac{\Upsilon_{i,\mathcal{J}+1} - \Upsilon_{i,k}}{\rho\sigma} \right) \right)^\rho \right] \right]}{\exp \left(-\frac{\Upsilon_{i,j} - \Upsilon_{i,\mathcal{J}+1}}{\sigma} \right) \left[1 + \left(\sum_{k=1}^{\mathcal{J}} \exp \left(-\frac{\Upsilon_{i,\mathcal{J}+1} - \Upsilon_{i,k}}{\rho\sigma} \right) \right)^\rho \right]}, \end{aligned}$$

then, the value is,

$$\begin{aligned} \mathbf{v}_i^{\mathbf{G}} &= \left[\Upsilon_{i,\mathcal{J}+1} + \sigma \log \left[1 + \left(\sum_{k=1}^{\mathcal{J}} \exp \left(-\frac{\Upsilon_{i,\mathcal{J}+1} - \Upsilon_{i,k}}{\rho\sigma} \right) \right)^\rho \right] \right] \\ &\quad \frac{1}{1 + \left(\sum_{k=1}^{\mathcal{J}} \exp \left(-\frac{\Upsilon_{i,\mathcal{J}+1} - \Upsilon_{i,k}}{\rho\sigma} \right) \right)^\rho} \left(1 + \sum_{j=1}^{\mathcal{J}} \frac{\left(\sum_{k=1}^{\mathcal{J}} \exp \left(-\frac{\Upsilon_{i,\mathcal{J}+1} - \Upsilon_{i,k}}{\rho\sigma} \right) \right)^{\rho-1} \exp \left(\frac{\Upsilon_{i,\mathcal{J}+1} - \Upsilon_{i,j}}{\rho\sigma} \right)^{\rho-1}}{\exp \left(\frac{\Upsilon_{i,\mathcal{J}+1} - \Upsilon_{i,j}}{\rho\sigma} \right)^\rho} \right) \\ &= \Upsilon_{i,\mathcal{J}+1} + \sigma \log \left[1 + \left(\sum_{k=1}^{\mathcal{J}} \exp \left(-\frac{\Upsilon_{i,\mathcal{J}+1} - \Upsilon_{i,k}}{\rho\sigma} \right) \right)^\rho \right], \end{aligned}$$

which, after proper substitution of the definition of $\Upsilon_{i,k}$ and $\Upsilon_{i,\mathcal{J}+1}$, gives the expression for the ex-ante lifetime utility described in the previous subsection.

Appendix D Role of ϵ shocks

While these shocks are computationally convenient, they may also have important implications for the behavior of the model. In Table 8, we show how some statistics change as we modify the value of the variance of the extreme value shocks, ρ and σ . In particular, because these shocks affect the choices of debt and maturity, increasing their variance corresponds to increasing the standard deviation of the equilibrium duration, maturity, and debt-to-GDP ratio. Also, as these extreme value shocks are independent over time, increasing their variance lowers the

autocorrelation of equilibrium duration, maturity, and the debt-to-GDP ratio. Finally, since these shocks reduce the importance of the economic forces explicitly modeled, increasing their variance reduces the correlation of equilibrium maturity and duration with GDP. Thus, these moments are informative of how large the variance of these shocks should be in the model, and we use them in our calibration strategy. As described above, we used two of these moments in the calibration.

Table 8: Role of σ and ρ

	Baseline	$0.5 \times \sigma$	$2 \times \sigma$	$0.5 \times \rho$	$2 \times \rho$
<i>Std. Dev.</i> (Duration)	1.27	1.01	1.56	1.01	1.56
<i>Std. Dev.</i> (Maturity)	3.13	2.35	4.16	2.35	4.12
<i>Std. Dev.</i> (Debt/GDP)	9.97	8.93	12.91	8.83	13.08
<i>Autocorr.</i> (Duration)	0.91	0.90	0.91	0.90	0.91
<i>Autocorr.</i> (Maturity)	0.91	0.90	0.92	0.90	0.92
<i>Autocorr.</i> (debt-output)	0.91	0.90	0.92	0.90	0.92
<i>Corr.</i> (Maturity,log(y))	0.32	0.42	0.18	0.42	0.18
<i>Corr.</i> (Duration,log(y))	0.22	0.32	0.07	0.31	0.08

Given our calibration, how important are ϵ shocks? To answer this question we compute the default and portfolio choices in what we call the ϵ -zero model, where we set all realizations of every ϵ shock to zero.

In particular, consider a case with debt level $-b_i$, maturity m_i , income y , not experiencing a rollover shock, whose observed decision is to not default, and take a portfolio with $-b'_j$ and m'_j to the next period. In this case, the value of not defaulting, with a realization ϵ equal to zero is

$$\hat{V}^P(y, 0, b_i, m_i) = \max_{b_j, m_j} \{u(c_{ij}(y)) + \beta E_{y', a' | y, 0} E_{\epsilon'} V^G(y', a', b_j, m_j, \epsilon')\}$$

subject to

$$c_{ij}(y) = y + b_i + q(y, 0, b_j, m_j; m_i - 1)b - q(y, 0, b_j, m_j; m_j)b_j \text{ and } j \in \{1, 2, \dots, \mathcal{J}\}.$$

From this problem we obtain the policy functions of the ϵ -zero model.

If the economy is experiencing a rollover shock, the value of repayment with $\epsilon_i = 0$ is

$$\hat{V}^P(y, 1, b_i, m_i) = u(y - b_i) + \beta E_{y', a' | y, 1} E_{\epsilon'} V^G(y', a', b_j, m_j, \epsilon').$$

Similarly, the value of defaulting with $\epsilon_{\mathcal{T}+1} = 0$ in the current period would have been:

$$\hat{V}^D(y, b_i, m_i) = u(y^D) + \beta E_{y' | y^D} E_{\epsilon'} V^R(y', b_i, m_i, \epsilon'),$$

From here we obtain the policy function of default for the ϵ -zero model. In particular, the country defaults if $\hat{V}^P(y, 0, b_i, m_i) \leq \hat{V}^D(y, b_i, m_i)$.

Table 9 compares the baseline model with the ϵ -zero model. First, note that the default rate is very similar, with a value of 3.14% in the baseline model and 3.17% in the ϵ -zero model.

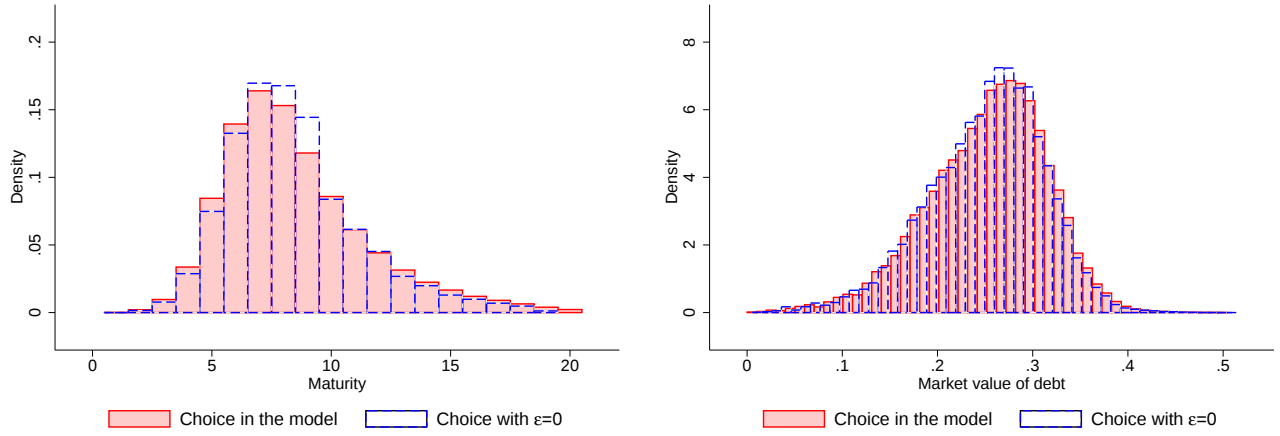
Table 9: Default rate in the model; with and without ϵ shocks in the current period (percent)

		Baseline model		
		Default	No default	Total
ϵ -zero model	Default	2.98	0.19	3.17
	No default	0.16	96.67	96.84
	Total	3.14	96.86	100.00

Second, we show that not only the default rates are similar but, in most cases, also the episodes of default are the same across models. The rate of default using the episodes in which there is default is 2.98% in both models. As shown in the table, in only 0.16% + 0.19% of the cases the decision of default is different across models, but because the changes are in the opposite direction, they offset, and the default rates are almost identical.

Finally, Figure 11 shows the statistical distributions of the choices of maturity and the market value of the debt of the portfolios taken to the next period in our benchmark model, and contrasts with those implied by shutting down ϵ shocks in the current period. The portfolio decisions are extremely similar.

Figure 11: Distributions of maturity and market value of debt with baseline and ϵ -zero models



Note: The figures only show the cases without a debt rollover shock, i.e. $a = 0$.