

A Theory of Foreign Exchange Interventions*

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Abstract

This paper develops a theory of foreign exchange interventions in a small open economy with limited capital mobility. Home and foreign bond markets are segmented and intermediaries are limited in their capacity to arbitrage across markets. As a result, the central bank can implement nonzero spreads by managing its portfolio. Crucially, spreads are inherently costly, over and above the standard costs from distorting households' consumption profiles. The extra term is given by the carry-trade profits of foreign intermediaries, is convex in the spread—as more foreign intermediaries become active carry traders—and increasing in the openness of the capital account—as foreign intermediaries find it easier to take larger positions. Optimal interventions balance these costs with terms of trade benefits. We show that they lean against the wind of global capital flows to avoid excessive currency appreciation. Due to the convexity of the costs, interventions should be small and spread out, relying on credible promises (forward guidance) of future interventions. By contrast, excessive smoothing of the exchange rate path may create large spreads, inviting costly speculation. Finally, in a multi-country extension of our model, we find that the decentralized equilibrium features too much reserve accumulation and too low world interest rates, highlighting the importance of policy coordination.

JEL codes: F31, F32, F41, F42

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1 Introduction

Foreign exchange interventions are among the most important macroeconomic policy tools, yet among the least understood. Countries mainly use them for two purposes: To manage their

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exchange rate, when relying on monetary policy alone is infeasible or undesired, and to accumulate reserves as insurance against sudden stops. Both roles are of crucial importance. There is ample evidence that many countries intervene to dampen exchange rate volatility, slow down exchange rate adjustments or lean against capital flows.¹ And reserve accumulation has gone so far that now \$12 trillion, or 80% of US GDP, are being saved by the world’s central banks. The effect of this reserve hoarding on global imbalances, world interest rates and exchange rates can hardly be overstated.

In light of the popularity of foreign exchange interventions among policymakers, it might almost come as a surprise that many important questions remain unanswered. When should foreign exchange interventions be deployed? How costly are they and what is the right, welfare-relevant way to measure these costs? How should interventions be designed to maximize their effectiveness, and how does that depend on the specific goal of the intervention? What are the implications of the increasingly common usage of interventions for the world economy? Should countries coordinate their interventions?

In this paper, we propose a tractable and microfounded framework that speaks to all these questions. We base our analysis on a canonical real small open economy model augmented with limited capital mobility, in which the country faces endowment and interest rates shocks. As is well-known in this type of model,² the economy has market power as exporter of its endowment, generating a desire for terms-of-trade management. In the model’s financial markets, domestic and foreign intermediaries can arbitrage between domestic and foreign bond markets, but arbitrage is limited due to a fixed transaction cost and position limits. Under these conditions, a portfolio balance channel emerges: changes in the portfolio of the central bank induce short-lived interest rate spreads—that is, exchange rate adjusted or “UIP” spreads³—between domestic and foreign bonds, as in Kouri (1976), Branson and Henderson (1985), or more recently Gabaix and Maggiori (2015). We analyze this model through the lens of the small open economy’s central bank as social planner and ask: How should it optimally manage its holdings of foreign bonds?

Our first contribution is to show that this central bank planning problem can be entirely framed in terms of the UIP spreads that the central bank’s portfolio choice generates. The logic is straightforward: If a country seeks to depreciate its exchange rate, it sells home bonds and purchases foreign ones, generating a positive UIP spread. Crucial to our analysis, UIP spreads are inherently *costly*, over and above the standard costs from distorting (domestic) households’ consumption profiles. The reason is intuitive: UIP spreads invite foreign intermediaries to take profitable carry trade positions, and hence the country as a whole is losing money at an amount equal to the carry trade

¹This holds for emerging markets and advanced economies alike. For emerging markets, see for example the “fear of floating” literature around Calvo and Reinhart (2002), Levy-Yeyati and Sturzenegger (2005a) and McKinnon and Schnabl (2004), among many others. Among advanced economies, recent interventions have for example been conducted by Denmark, Switzerland, or the Czech Republic to depreciate their respective currencies. See Section 2 below for more examples.

²see e.g. Costinot et al. (2014), or Farhi and Werning (2012, 2013).

³UIP is short for the uncovered interest parity condition which is satisfied if the expected excess return from investing in local bonds is exactly zero.

profits made by foreign intermediaries. These additional costs are naturally *convex* in the level of the spread—as more foreign intermediaries become active carry traders when spreads are higher—and *increasing* in the openness of the capital account—as foreign intermediaries then find it easier to take larger positions. It is worth stressing that our costs are the welfare-relevant costs identified by our model and apply to the whole country; as such, our costs can, and often will, be different from the central banks’ own quasi-fiscal cost of holding reserves, which do not include costs incurred by the country’s private economy from nonzero UIP spreads.⁴

The formulation of the planning problem in terms of UIP spreads highlights an interesting connection with the recent literature on optimal capital controls (see, for example, [Farhi and Werning, 2013](#)). In this literature, the planner typically chooses proportional taxes on capital flows, which also manifest themselves as UIP spreads. The crucial difference to this literature is that in our model, the planner faces an extra cost from nonzero UIP spreads, coming from the carry-trading activities of foreign intermediaries. Indeed, we show that in the limit of zero private capital mobility (financial autarky), our planning problem becomes essentially analogous to one of optimal capital controls. This seems to suggest that capital controls and foreign exchange interventions are substitutes. Yet, this is not the case: Precisely in the limit of zero capital mobility, capital controls are meaningless. Instead, as we explain below, our theory suggests that capital controls and foreign exchange interventions are complements: the former enhance the effectiveness of the latter.

Our second contribution is to fully characterize the optimal policy. We find that it should lean against the wind of capital inflows by implementing positive UIP spreads, and thus depreciate the exchange rate (vice versa when capital flows reverse direction). When faced with positive endowment or wealth shocks, e.g. productivity boosts in the export sector or natural resource discoveries, the central bank should intervene to avert any excessive exchange rate movements and replicate the frictionless equilibrium (zero UIP spreads).

For intermediate degrees of capital mobility, the additional cost term delivers several new insights about the optimal design of foreign exchange interventions. First, interventions should be smooth. Since nonzero spreads lead to costly carry trade activities and households are forward-looking, it is undesirable to adjust the spread immediately after a shock hits. Second, the convexity of the cost term generates a desire for spreading out interventions over time. In particular, this includes promising future interventions, a form of “forward guidance” of foreign exchange interventions. As a consequence of this, interventions should be highly inertial, possibly lasting significantly longer than the shock itself. This, however, leads to natural and novel type of time inconsistency: After the shock has subsided the central bank would like to revert to zero UIP spreads, if allowed to reoptimize. Thus, central bank credibility turns out to be an essential input into a successful conduct of intervention policies. To sum up, we find that foreign exchange interventions should be small, frequent, persistent and credible. None of these properties can be obtained in the special case with no private capital mobility.

⁴For a recent study on the quasi-fiscal cost of reserve holdings, see [Adler and Mano \(2016\)](#).

Our lessons are not limited to the canonical model in which management of a country's terms-of-trade is the underlying motive. We present three extensions that each exhibit a different rationale to conduct foreign exchange interventions. In the first extension, we explain how an economy with an exchange rate peg and sticky prices in the domestic good optimally intervenes to smooth the path of output gaps.⁵ We show that in this case the planner still leans against the wind of capital inflows but for a different reason: By accumulating reserves it is able to raise the domestic interest rate and shift household spending to the future in order to avoid excessive consumption of the domestic good in the present. The qualitative properties of interventions—size, frequency, persistence and credibility—go through unaltered. In our second extension, we allow for taste shocks in intermediaries' demand for home bonds. This allows us to capture, albeit in reduced form, aspects such as liquidity premia or heterogeneous beliefs. Analyzing this case is particularly important given that many central banks allegedly intervene when the exchange rate moves “away from fundamentals”. In such a scenario, there is an incentive for the home country to behave as a monopolist in the supply of its own bond, optimally selling, but not fully accommodating, foreigners' demand for the home bond. Unlike before, the central bank can now make profits, behaving as a speculator in the sense of [Friedman \(1953\)](#). Aside from this profit opportunity, we show that the dynamic properties of optimal interventions are still qualitatively the same as before. Finally, as our third extension, we present an economy which is pursuing a “managed float” policy in which the exchange rate is required to follow a smooth path. We show that the slow exchange rate adjustments at the core of this kind of policy—which seems to be very common among EMEs—may cause significant costs by creating large UIP spreads, which invite foreign intermediaries to enter carry trades against the central bank.

Our third contribution is to characterize the positive and normative consequences of widespread foreign exchange interventions for the international monetary system. We embed our baseline model in a world composed of two continua of small open economies: a continuum of “emerging market economies” (EMEs), which are subject to limited capital mobility as before; and a continuum of “advanced economies” (AEs), which have perfect capital mobility. We hit AEs with a savings shock to capture recent trends like population aging or debt overhang. We show that in response to the capital inflows from AEs, EMEs engage in “reserve wars”, as each EME tries to manipulate its terms-of-trade in its favor—an effort which turns out to be self-defeating in the world equilibrium. Interestingly, such behavior by EMEs leads to public flows flowing upstream and private flows flowing downstream, which is what the evidence in [Gourinchas and Jeanne \(2013\)](#) and [Aguilar and Amador \(2011\)](#) suggests. In addition, such reserve wars depress the world interest rate, making the incentive for other EMEs to intervene even stronger. We explain that for reasonable calibrations,

⁵Countries that face(d) this kind of problem are for example advanced economies during the Bretton Woods era, or European countries that peg to the Euro such as Denmark or the Czech Republic. Even Switzerland can be counted into this category during the time they had in place an exchange rate floor, which was effectively a peg, from 2011 to 2015. Our analysis abstracts from an effective lower bound on policy rates and instead stresses that foreign exchange interventions can help a pegging country regain some control over its interest rate more generally. For an analysis with a binding zero lower bound see [Amador et al. \(2016\)](#).

both AEs and EMEs would be better off if interventions were ruled out altogether. In fact, the model suggests that even a “unilateral” move by EMEs to coordinate their interventions would significantly reduce their volumes, possibly all the way to zero, again with welfare gains for both AEs and EMEs.

Literature First and foremost, our paper is part of a nascent literature that incorporates a portfolio balance channel into a general equilibrium framework. Importantly, [Gabaix and Maggiori \(2015\)](#) show how limits to arbitrage in global asset markets may offer an explanation of several international finance puzzles, such as the forward premium puzzle. Furthermore, they consider a perturbation of the equilibrium to suggest that interventions may be effective and welfare-enhancing. We share their modelling of limits to arbitrage (although our microfoundation differs somewhat) but focus on optimal policy choice. [Cavallino \(2015\)](#) and [Liu and Spiegel \(2015\)](#) pursue a similar route in the context of a linear-quadratic New Keynesian framework. [Cavallino \(2015\)](#) shows analytically that a central bank may use interventions to profit from the irrationality of noise traders, while [Liu and Spiegel \(2015\)](#) show numerically that the central bank may respond to negative (rational) risk premium shocks by accumulating reserves. [Chang and Velasco \(2016\)](#) build a model with borrowing constraints on the financial sector and show that foreign exchange interventions may be useful when those constraints are binding. In contemporaneous work, [Amador et al. \(2016\)](#) consider an environment similar to ours in which foreigners have position limits. They show that implementing an exchange rate peg when monetary policy hits the lower bound is particularly costly, as this generates an arbitrage opportunity for foreigners. Using the model, they find that Switzerland incurred costs to defend its peg in the order of 1% of GDP. Other papers, such as [Benes et al. \(2013\)](#), [Blanchard et al. \(2015\)](#), [Devereux and Yetman \(2013\)](#) and [Ostry et al. \(2012\)](#) study the effects of interventions but lack a fully microfounded model.⁶ To the best of our knowledge, our paper is the first in this literature to derive general principles of foreign exchange interventions, clearly stressing: its costs and benefits; the relationship with existing studies on related topics, such as capital controls; the importance of the nationality of arbitrageurs (that is, whether arbitrageurs are foreign-based or home-based); and the importance of coordination in a world equilibrium.

As we mentioned in the introduction, our paper is related to the burgeoning literature on optimal capital controls.⁷ In an environment similar to ours but with perfect financial markets, [Farhi and Werning \(2012, 2013\)](#) find that optimal capital controls are used to lean against the wind after interest rate shocks while they are not used against endowment shocks. In our baseline model, we derive an analogous result for foreign exchange interventions but show that the additional costs associated with foreign exchange interventions are crucial for the optimal policy. In a model with two large economies, [Costinot et al. \(2014\)](#) show that a country might want to use capital controls

⁶[Devereux and Yetman \(2013\)](#) has a microfounded new Keynesian model but their modeling of capital immobility is ad hoc, which makes the results somewhat harder to interpret. In particular, the model does not predict a cost of sterilization, which is a feature of microfounded models.

⁷See, among many others, [Bianchi \(2011\)](#); [Magud et al. \(2011\)](#); [Farhi and Werning \(2012, 2013\)](#); [Heathcote and Perri \(2014, 2015\)](#).

to depress the international interest rate if the endowment is growing over time. This is related to some of our results in Section 6.

There is a large literature documenting EMEs' reserve accumulation in the past decades.⁸ The main goal of these papers has been to quantify the contribution of different potential explanations, such as building buffers against sudden stops or “neo-mercantilist” strategies of real exchange rate undervaluation. However, most papers do not tackle the question of how the country actually manages to manipulate the net foreign asset position of the country to achieve this objectives. Closest to us in this literature is Jeanne (2012), which emphasizes the interaction of public and private flows. That paper allows domestic households to access foreign markets subject to a transaction cost, which allows the planner to costlessly manipulate the UIP within certain limits. However, since capital is otherwise perfectly mobile, there is no region in which the planner balances “costs” and “benefits” of foreign exchange interventions.

Finally, our study of a world equilibrium with reserve accumulation in Section 6 is related to Obstfeld (2011), who emphasizes the dangers of currency wars through reserve accumulation and its consequences for the global interest rate. Models of low global interest rates are also put forth by Coeurdacier et al. (2015) and Caballero and Farhi (2015). In our case, lower interest rates are a consequence of reserve accumulation and an increasing share of EMEs in world markets (see Section 6).

2 Background on Foreign Exchange Interventions

Foreign exchange interventions are defined as changes in a central bank's holdings of reserve assets, where reserve assets are defined as “those external assets that are readily available to and controlled by, the monetary authorities for meeting balance of payments financing needs, for intervention in exchange markets to affect the currency exchange rate, and for other related purposes” (IMF, 2011). In this section, we briefly describe the history of such interventions, followed by a discussion of the most commonly cited benefits and costs of interventions. Finally, we summarize the debate on the optimal way of trading off costs and benefits. Throughout this section, we will both draw on existing papers as well as establish new facts. A reader mainly interested in our theoretical analysis may skip this section.

2.1 A brief history

Foreign exchange interventions have been a key part of the international monetary system in the last century. During the times of the gold standard, and after its collapse, during the Bretton Woods system, interventions have routinely been used to “break” the trilemma and generate some degree of monetary independence (Bordo et al., 2015). After the collapse of Bretton Woods, exchange rates

⁸See, for instance, Aizenman and Lee (2007); Alfaro and Kanczuk (2009); Benigno and Fornaro (2012); Bianchi et al. (2012); Hur and Kondo (2014); Jeanne and Rancière (2011); Jeanne (2012); Korinek and Serven (2010).

were allowed to float, with one of the promises being that this would reduce the need of large scale foreign exchange interventions. Yet, this promise soon turned out to be false as advanced economies continued regular, and often coordinated, interventions until the turn of the century. For example, as the United States struggled with a strong dollar, the five most important central banks negotiated the “Plaza Accord” in September 1985, following which they engaged in massive interventions to depreciate the dollar. In the 1980s and 1990s, European countries’ central banks intervened to limit exchange rate volatility in the context of the European Exchange Rate Mechanism, a fixed exchange rate system which preceded the introduction of the Euro. Furthermore, Japan was intervening heavily until 2004, mostly in order to moderate the appreciation of the Yen. After that, however, interventions by advanced economies did begin to fall out of favor—at least until a few years ago, when, faced with the limitations of monetary policy at very low interest rates, some European countries, such as Switzerland or Denmark, have resorted once again to foreign exchange interventions in order to achieve their policy objectives.

In contrast with the recent decline among advanced economies, interventions have become a very important policy tool for many *emerging market economies* (EMEs). Since the famous “sudden stop” episodes of the 1990s, one of the main objectives of EMEs’ interventions has been to build a “war chest” of reserves to insure against sudden stops.⁹ As a result, EMEs often engage in reserve-accumulation strategies when their current level of reserves is perceived to be inadequate.¹⁰ Furthermore, unlike the early interventions by advanced economies after the collapse of Bretton Woods, these reserve accumulation programs are conducted unilaterally by each EME central bank and generally tend to rely on comparisons with peers, raising concerns about coordination failures and amplification of reserve hoarding behavior IMF (2011). In addition, Obstfeld (2011) emphasizes the spillovers excessive reserve accumulation may have on the world equilibrium through the world interest rate. We address these issues in Section 6.

This type of policies has drastically altered the landscape of the international monetary system. Figure 1 plots in Panel (a) the reserve holdings of EMEs and AEs, relative to world GDP, and in Panel (b) EMEs’ and AEs’ shares of world GDP. Before the 1990s, most of global reserves were owned by AEs—as one would expect given their dominant share of world GDP. Since then, however, the rate of reserve accumulation by EMEs has been staggering. As of 2011, EMEs’ reserves increased to 9% of world GDP, twice those held by AEs, despite the fact that EMEs only account for half as much GDP (Panel (b)).¹¹

⁹A sudden stop is a quick reversal in capital flows that leads to a shortage of international liquidity.

¹⁰For example, in 2011 Chile started a reserve accumulation program to raise its reserves from 13.3% to 17% of GDP within a single year.

¹¹The lion share of this increase of course came from China, which held reserves worth 4% of world GDP in 2011. Nevertheless, even without China, EME reserves over world GDP still rose *eightfold* between 1990 and 2011, compared to twelfold when China is included.

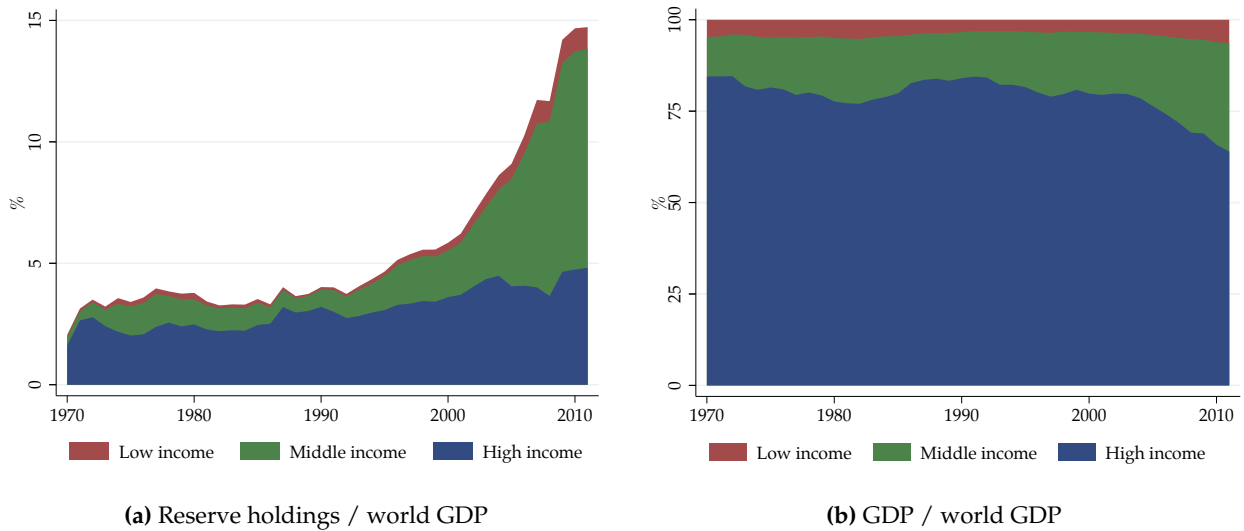


Figure 1: Reserve holdings and GDP as shares of world GDP, by income group. *Note:* These figures show the stock of reserves (Panel (a)) and GDP (Panel (b)) over world GDP. The data comes from the 2011 version of [Lane and Milesi-Ferretti \(2007\)](#). Details on these figures can be found in Appendix A.1.

2.2 Benefits and costs of interventions

Over the years, policymakers have intervened based on a wide variety of different reasons. In this subsection, we briefly summarize those reasons and the costs associated with foreign exchange interventions.

Benefits. The policy debate around foreign exchange interventions has identified the following three broad reasons why a country might engage in foreign exchange interventions: (i) exchange rate management, either to reduce exchange rate volatility, to smooth out exchange rate adjustments over time, or to improve the terms of trade; (ii) reserve accumulation, to insure against future sudden stops; (iii) regaining some monetary independence despite fixed exchange rates. Our baseline model in Section 3 features a canonical terms of trade management motive, finding that the central bank leans against the wind.

Actual policy implementation in the data seems to share this property with the optimal policy. [Chang \(2007\)](#) documents that many EME central banks have indeed been leaning against the wind of private capital flows—even those countries that appear to follow a policy of inflation targeting. Figure 2 shows a similar picture. Panel (a) plots the time series of quarterly net private flows into a large sample of 50 EMEs (blue, dashed) and flows into reserves (red, solid), aggregated over the countries in our sample. The comovement is striking, and not driven by aggregation: 49 of 50 in our sample show a similarly strong positive comovement, with an average correlation of 0.51. It is particularly interesting that there is not just a strong positive correlation between the two lines, but their volatilities are also of similar size. Figure 2(b) shows the two volatilities in a scatter plot across

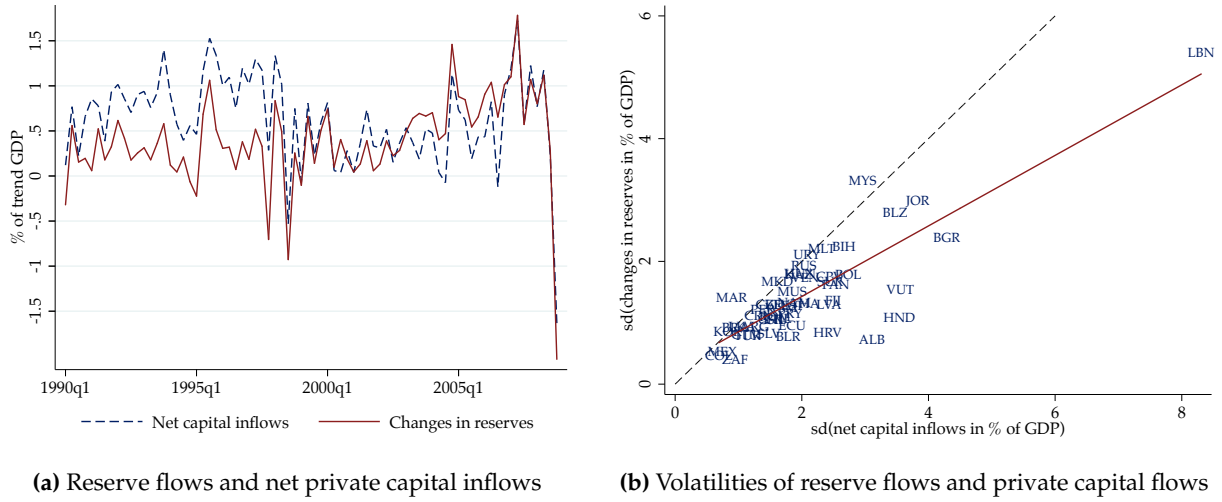


Figure 2: Note: Panel (a) shows the aggregate reserve flows over aggregate GDP and aggregate net private capital flows over aggregate GDP. Panel (b) compares the volatility of reserve flows over GDP with the volatility of net private capital flows over GDP across countries. The time period underlying both plots is from 1990:1 to 2008:4 and the sample consists of 50 emerging markets. The signs in Panel (a) are such that positive reserve flows reflect reserve accumulation and positive private capital flows reflect inflows. The data is from the IMF’s Balance of Payments statistics. Details on this figure can be found in Appendix A.2.

the same set of countries. While volatilities do not necessarily exactly line up along the 45° line (dashed), the volatility of reserves is significant at around one half of the volatility of net private flows (red, solid). By comparison, the volatility of reserves in the US is only 3.5% of the volatility of net private capital flows.

In Section 5 below, we also discuss how our model can be used to analyze different motives for interventions than to manage the terms of trade.

Costs. There are many ways economists have been measuring the costs associated with foreign exchange interventions.¹² Our model takes a particularly practical stance on this issue: The relevant economic costs are transfers that the country implicitly pays to foreign arbitrageurs running successful carry trades against central bank interventions. Critical to this story is that there is an empirical connection between UIP spreads and foreign exchange interventions. To provide evidence that there is, we combine data from Lustig et al. (2011) with the IMF’s Balance of Payments Statistics and run the following OLS regression,¹³

$$UIPspread_{it} = \alpha_i + \delta_t + \beta ResFlows_{it} + X_{it}\gamma + \epsilon_{it}$$

¹²For example, Adler and Mano (2016) measures the costs as quasi-fiscal costs the central bank incurs when it invests in low-yielding assets, selling high-yielding ones.

¹³We thank Adrien Verdelhan for sharing his data with us.

	UIP wedge			
	(1)	(2)	(3)	(4)
Res. flows / GDP (t)	0.723** (0.34)	0.713** (0.33)	0.666** (0.30)	0.535** (0.24)
Res. flows / GDP (t-1)		0.117 (0.20)	0.148 (0.19)	0.215 (0.21)
Int. rate spread (t-1)			2.258* (1.16)	1.931* (1.11)
Constant	0.353*** (0.10)	0.321** (0.14)	-0.250 (0.36)	-5.100*** (0.73)
Year dummies	No	No	No	Yes
Country FE	Yes	Yes	Yes	Yes
Observations	1528	1528	1528	1528
R-squared	0.011	0.011	0.033	0.163

Table 1: Do increases in reserves coincide with positive wedges in the uncovered interest parity condition (UIP)? Quarterly reserve flows are taken from the IMF’s Balance of Payments statistics, merged with quarterly UIP wedges and interest rate spreads computed based on data from [Lustig et al. \(2011\)](#). Details on the table can be found in Appendix A.3. *Note:* Standard errors, corrected for heteroskedasticity, are in brackets. * $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

where α_i and δ_t are country and time fixed effects; and X_{it} is a set of controls, in which we include past reserve flows as well as the past interest rate spread (which is well known to predict excess returns, see e.g. [Lustig et al. \(2011\)](#)). The outcomes, in Table 1, show a strong significant relationship between reserve purchases and positive UIP spreads, that is, expected excess returns from investing in the intervening country’s bond market.

2.3 Empirical evidence on effectiveness

Convincing empirical evidence about the effectiveness of foreign exchange interventions is very rare. This is mainly due to a simple simultaneity problem: Interventions influence exchange rates but also respond to shocks to exchange rates. One of the more solid papers that attempts to go around this issue is [Kearns and Rigobon \(2005\)](#), exploiting a “natural experiment”, in which Japan and Australia changed their foreign exchange policy for arguably exogenous reasons. They show that in both countries, interventions seem to have a significant effect on the exchange rate, with most of the effect occurring during the day of the intervention.

It is worth pointing out that, despite the shortage of convincing formal evidence on interventions, policymakers’ own experiences with them seem to suggest that they are indeed effective. For example, a survey by the BIS shows that 90% of the respondents believe were their foreign exchange interventions were completely or partially successful.

One of the least controversial points regarding the effectiveness of interventions, is that the effectiveness should be tightly linked to the (in)ability of capital to flow freely across borders. This

is important for our theory since the assumption of limited capital mobility is at the core of our analysis. It is important to notice that such limited capital mobility need not necessarily come from limits to arbitrage in the financial sector: [Canales-Kriljenko \(2003\)](#) documents that central banks in EMEs are large players in their foreign exchange markets, partially due to consciously taken policy measures designed to inhibit free cross-border capital flows.¹⁴ As already previewed in the introduction, this points to a *complementarity* between capital controls and macroprudential measures on the one side and foreign exchange interventions on the other.¹⁵

2.4 Implementation

There are roughly three important degrees of freedom in the implementation of foreign exchange interventions: Frequency vs. size, rules vs. discretion, and exchange rate rules vs. quantity-based rules.¹⁶ We discuss each of them in turn.

Frequency vs size. Countries are divided as to the optimal size and frequency of interventions. For example, [Kearns and Rigobon \(2005\)](#) document that Australia and Japan abandoned their small and frequent interventions in favor of large and infrequent ones in an effort to maximize their impact on the exchange rate. In contrast, other countries have had a very persistent presence in foreign exchange markets. [Adler and Tovar \(2011\)](#) document that Brazil and Uruguay intervened two-thirds of the days between 2004 and 2010.

Rules vs discretion. Another source of debate refers to whether interventions should be secret or public information. In a well-known survey of EME central banks, [Canales-Kriljenko \(2003\)](#) documents that about one half of the respondents carry out their interventions in secret. Among advanced economies, public interventions are more common. However, the historically largest intervener—the Bank of Japan—has frequently favored secret interventions. Note, however the line separating secret and public interventions becomes blurred in shallower markets, where the central bank’s presence hardly goes undetected.

Exchange rate rules vs quantity-based rules. In addition, even if one agrees that transparency and predictability are desirable, there is heterogeneity among countries regarding the kind of rules that they implement. On the one hand, some countries follow quantity-based rules. For

¹⁴In a sample of 90 countries, he finds that 36% have surrender requirements, 90% have some form of position limits, 50% prohibit usage of foreign currency for some domestic transactions and in 45% both legs of foreign exchange transactions are settled at the central bank. [Mohanty and Berger \(2013\)](#) confirms this observation in a more recent survey.

¹⁵Even in the case of advanced economies, this seems to be the case. [Kearns and Rigobon \(2005\)](#) find that Australian interventions create more “bang for the buck” due to a smaller foreign exchange market. And the central bank of New Zealand stated that interventions are more “likely to be effective” when “there is a relative absence of capital flows that might offset the intervention”.

¹⁶Arguably, whether to intervene in spot or forward markets is another degree of freedom. However, most, i.e. 70-80%, of the interventions are carried out in spot markets, since forward markets are generally more illiquid. There are some notable exceptions, such as Brazil.

example, Chile’s reserve accumulation program of 2011 consisted of buying USD 12 billion in pre-announced daily amounts at an average of USD 50 million per day. On the other hand, some countries follow exchange-rate based rules, mostly aimed at smoothing the exchange rate path. For example, Colombia had a rule that authorized the central bank to auction put options up to a specific amount whenever the exchange rate fell more than 5% below its average of the previous 20 days.

Perspective of our model. Our model provides simple yet powerful guidance on these questions. If the goal is to minimize the aforementioned costs coming from foreign arbitrageurs running carry trades (and hence speculating) against central bank interventions, then interventions should be frequent but small in size and pre-announced. Moreover, following a quantity-based rule is generally found to be closer to the optimal policy than an exchange-rate based rule that guarantees smooth exchange rate movements.

Frequent but small interventions are powerful due to two reasons: First, they span over a significant time period, so they are likely to affect interest rates for longer, amplifying the initial response of economic agents, especially when interventions are pre-announced.¹⁷ Second, the relatively small size of any specific intervention is associated with relatively minor UIP spreads and hence limits the room for foreign arbitrageurs to take advantage of the central bank action. Exchange rate rules are found to do the exact opposite: As we show in Section 5, by slowing down the exchange rate adjustment, central banks *invite* foreign arbitrageurs to take bets and trade against the central banks’ interventions.

3 Baseline Model

In this section, we present our baseline model. The model is a real small open economy (SOE) model in continuous time. The model is stylized as we strive to focus on the two essential model ingredients. These are on the one hand a finitely elastic foreign demand for home bonds that allows the home central bank to change home interest rates via a portfolio balance channel;¹⁸ and on the other a terms-of-trade management motive, which gives the central bank a reason for such interventions.¹⁹ We first describe the model and then discuss the equilibrium dynamics without interventions. Optimal interventions are then characterized in great detail in Section 4.

¹⁷In an empirical setting, our model would predict that the exchange rate should respond the moment the policy is announced. Interestingly, [Tapia and Tokman \(2004\)](#) finds that the announcement of intervention in Chile in 2001 had a large and significant effect on the exchange rate.

¹⁸This is similar to papers by [Lahiri and Végh \(2003\)](#), [Gabaix and Maggiori \(2015\)](#), and [Liu and Spiegel \(2015\)](#) among others.

¹⁹This is similar to the recent literature on capital controls by [Farhi and Werning \(2012, 2013\)](#) and [Costinot et al. \(2014\)](#), among others, which is based on the framework by [Gali and Monacelli \(2005\)](#).

3.1 Model setup

There are four agents in our model: Domestic households and a domestic central bank, and, foreign and domestic intermediaries. In line with our SOE assumption we also introduce an export demand curve of foreign households. There are two goods markets (a “home good” and a “foreign good”) as well as two asset markets (“home bonds” and “foreign bonds”). Throughout, we use “home” and “domestic”, as well as “UIP spread” and “UIP wedge” interchangeably. We start by describing domestic households and the two goods markets.²⁰

Households. There is a continuum of households in the home country, maximizing a common utility function $\int_0^\infty e^{-\rho t} \log(c_t) dt$, with c_t being a consumption bundle defined as $c_t = \kappa c_{Ht}^{1-\alpha} c_{Ft}^\alpha$. Here, c_{Ht} and c_{Ft} denote home’s consumption of home and foreign goods, respectively, and $\kappa \equiv (1-\alpha)^{-(1-\alpha)} \alpha^{-\alpha} > 0$ is a positive normalization constant. Throughout our analysis, we normalize the foreign good’s price to 1 and refer to that numeraire as “dollars”. The relative price of the home good is denoted by p_t . The per-period dollar budget constraint of the household is given by

$$\dot{b}_{Ht} = p_t y_{Ht} + y_{Ft} - p_t c_{Ht} - c_{Ft} + r_t b_{Ht} + t_t + \pi_t, \quad (1)$$

where y_{Ht} is home’s endowment of the home good, y_{Ft} is home’s endowment of the foreign good, b_{Ht} is the households’ position in home bonds, t_t are transfers from the central bank, and π_t are profits from domestic financial intermediaries. Both t_t and π_t are specified below. We denote by $q_t \equiv p_t^{-(1-\alpha)}$ the country’s real exchange rate, following the convention that high values correspond to depreciated exchange rates. Here, domestic households are only allowed to trade home bonds with a real interest rate of r_t . Later, we introduce financial intermediaries, who may access both home and foreign bond markets. We wish to stress that in this environment, domestic households’ own a nontrivial share of the home good and exhibit home bias in their preferences. These two assumptions are essential in generating the terms of trade management motive in our environment.

Maximizing utility subject to this budget constraint yields the following Euler equation,

$$\frac{\dot{c}_t}{c_t} = r_t - \rho + \frac{\dot{q}_t}{q_t}. \quad (2)$$

Finally, home’s total dollar expenditure is given by $q_t^{-1} c_t = p_t c_{Ht} + c_{Ft}$, which henceforth we denote by $\theta_t \equiv q_t^{-1} c_t$ due to its prominent role the analysis to come. The optimal demand for home and foreign goods is then

$$\begin{aligned} c_{Ht} &= (1-\alpha) \frac{\theta_t}{p_t} \\ c_{Ft} &= \alpha \theta_t. \end{aligned} \quad (3)$$

²⁰This presumes trade taxes are infeasible, as is standard in this literature, so terms-of-trade management is a second best tool. See, e.g. Costinot et al. (2014).

By symmetry, foreign's demand for home goods is

$$c_{Ht}^* = \alpha \frac{c_t^*}{p_t} \quad (4)$$

where in the following we assume foreign's consumption c_t^* to be equal to 1.

Foreign intermediaries. There are two types of intermediaries in our model: Foreign intermediaries and domestic intermediaries. Both types of intermediaries will behave similarly but they differ in their ownership structure, which will play a key role in our analysis. We describe foreign intermediaries in detail in this section and their domestic counterparts in the next.

The key ingredient in our model that makes foreign exchange interventions effective is a finite elasticity of demand for home bonds. As a result, a change in the portfolio of the central bank has an effect on the expected return of domestic assets r_t relative to their foreign counterpart r_t^* , i.e. the UIP wedge.²¹ [Backus and Kehoe \(1989\)](#) pointed out that these portfolio balance effects are muted in general equilibrium in a frictionless world in which Ricardian equivalence holds, as any actions by the central bank would be perfectly undone by the private sector. We break away from this result by modeling limited asset market participation, in the spirit of [Bacchetta and Van Wincoop \(2010\)](#) and [Gabaix and Maggiori \(2015\)](#). In particular, we assume that there exists a continuum of intermediaries owned by foreigners, labeled by $j \in [0, \infty)$, which can trade in both foreign and domestic bond markets. Foreign intermediaries' investment decisions are subject to three important restrictions.

First, each intermediary is subject to a net open position limit $X > 0$.²² Second, we follow [Alvarez et al. \(2009\)](#) in assuming that intermediaries face heterogeneous participation costs. In particular, each intermediary j active in the domestic bond market at time t is obliged to pay a participation cost of exactly j .²³

Putting these two ingredients together, intermediary j optimally invests an amount x_{jt} , solving

$$\max_{x_{jt} \in [-X, X]} x_{jt} (r_t - r_t^*) - 1_{\{x_{jt} \neq 0\}} j.$$

Intermediary j 's present value of net profits *conditional on investing* is $X |r_t - r_t^*|$, so investing is optimal for all intermediaries $j \in [0, \bar{j}]$ with the marginal intermediary $\bar{j}$ given by $\bar{j} = X |r_t - r_t^*|$. This gives an aggregate investment volume of

$$b_t^I = \bar{j} X \cdot \text{sign}(r_t - r_t^*).$$

²¹Since our model is deterministic, there is no difference between realized and expected returns. Hence, any UIP violation would also lead to a violation of the *covered* interest parity (CIP). In reality, foreign exchange interventions deal with assets of different risk characteristics. We deal with this in an extension with risk premium shocks in Section 5.

²²It is worth noting that, as discussed in Section 2, many emerging market central banks in fact do impose position limits on intermediaries' investments as a form of capital controls, hence artificially decreasing X .

²³It is straightforward to relax this assumption of linear costs to a more general monotonic function $f(i)$. We show in Appendix E.2 that assuming linear costs is inessential for our results.

Defining $\Gamma_F \equiv (X^2)^{-1}$ and substituting $\bar{j}$, we obtain

$$b_{It} = \frac{1}{\Gamma_F} [r_t - r_t^*]. \quad (5)$$

Equation (5) embodies that the foreign intermediaries' demand for home bonds has a *finite elasticity* to the return spread.²⁴ This equation is crucial to our analysis because it implies that changes in b_{It} , as for example induced by foreign exchange interventions, can indeed affect home interest rates. The key parameter in (5) is the inverse demand elasticity Γ_F .

When Γ_F is large, e.g. if position limits X are very small, intermediation is obstructed as evidenced by both, small levels of b_{It} and a small sensitivity of b_{It} to the interest rate spread. The case where $\Gamma_F = \infty$ corresponds to *financial autarky*. In this case, $b_{It} = 0$, which shuts down any sort of private financial intermediation in this baseline model. When Γ_F is small, e.g. if position limits X are very high, this leads to an equilibrium with less imperfect intermediation. In fact, if position limits were infinite, Γ_F would equal to zero and we would recover the infinite elasticity, $r_t = r_t^*$. We call this case the *frictionless* economy.

Home intermediaries. In addition to foreign intermediaries, we shall also assume there is a similar continuum of home intermediaries generating an analogous bond demand schedule,

$$b_{It}^H = \frac{1}{\Gamma_H} [r_t - r_t^*]. \quad (6)$$

There are two main differences between domestic and foreign intermediaries. First, domestic intermediaries are allowed to have a different elasticity Γ_H , which can be anywhere in $(0, \infty]$. Second, home intermediaries are owned by the representative household at home, whereas foreign intermediaries are owned by foreign households. To compute intermediaries' profits we need to take a stance on the way transaction costs are being paid. To keep the model tractable, we assume that domestic intermediaries pay the transaction costs as transfers to each other, ultimately reaching domestic households through profits.²⁵ This means, domestic intermediaries' total profits are just given by their total revenues, and so households receive the following stream of per-period profits (in dollars),

$$\pi_t = b_{It}^H (r_t - r_t^*). \quad (7)$$

Central bank. The home central bank is the home country's social planner in our model. It chooses a foreign exchange intervention (FXI) policy $\{b_{Gt}, b_{Gt}^*, t_t\}$ consisting of home bond investments b_{Gt} , foreign bond investments b_{Gt}^* , and transfers t_t to home households, subject to the central bank

²⁴Strictly speaking, Γ_F is only a semi-elasticity. For simplicity, we abuse the terminology and call it an "elasticity" in the remainder of this paper.

²⁵It would make absolutely no difference if transaction costs would directly flow to households, it would just require an additional term in the representative household's budget constraint.

budget constraint²⁶

$$\dot{b}_{Gt} + \dot{b}_{Gt}^* = r_t b_{Gt} + r_t^* b_{Gt}^* - t_t. \quad (8)$$

The central bank's FXI policy must also ensure that the country satisfies a no-Ponzi condition,

$$\lim_{t \rightarrow \infty} e^{-\int_0^t r_s^* ds} \text{nfa}_t = 0 \quad (9)$$

where $\text{nfa}_t \equiv b_{Ht} + b_{Gt}^* + b_{Gt}$ is the net foreign asset position of the country. Note that in this economy, it is without loss to set $b_{Gt}^* + b_{Gt} = 0$ due to the availability of transfers between the central bank and households.

Competitive equilibrium. The model is closed with a goods market clearing condition,

$$c_{Ht} + c_{Ht}^* = y_{Ht} \quad (10)$$

and a bond market clearing condition,

$$b_{Ht} + b_{It} + b_{It}^H + b_{Gt} = 0. \quad (11)$$

We can now formally define a competitive equilibrium in this environment.

Definition 1. Given initial debt positions $(b_{H0}, b_{I0}, b_{I0}^H, b_{G0}, b_{G0}^*)$, paths for shocks $\{y_{Ht}, y_{Ft}, r_t^*\}$, and a central bank FXI policy $\{b_{Gt}, b_{Gt}^*, t_t\}$, an allocation $\{c_t, c_{Ht}, c_{Ft}, c_{Ht}^*, b_{Ht}, b_{It}, b_{It}^H, \pi_t\}$ together with prices $\{q_t, r_t\}$ is a *competitive equilibrium* iff they solve (1)–(11).

Next, we characterize the competitive equilibrium, with the goal to derive “implementability conditions” that describe the set of competitive equilibria that can be attained through different FXI policies. Substituting consumption demands (3) and (4) into the goods market clearing condition (10) gives us an expression for the dollar value of the endowment of home goods,

$$q_t^{-1/(1-\alpha)} y_{Ht} = (1 - \alpha)\theta_t + \alpha. \quad (12)$$

Using the households' dollar budget constraint (1), we then obtain

$$\dot{b}_{Ht} = \alpha(1 - \theta_t) + y_{Ft} + r_t b_{Ht} + t_t + \pi_t. \quad (13)$$

Here, the policy variable t_t , can be eliminated after adding the central bank's budget constraint (8),

²⁶Note we implicitly assumed that the relevant interest rate for marginal changes of reserves is r_t^* . One might argue that negative levels of b_{Gt}^* should be associated with a different, higher interest rate. In reality however, reserves are (almost) always positive and so marginal changes in reserves are associated with the foreign interest rate on savings, r_t^* .

which allows us to rewrite the households' budget constraint as a country-wide budget constraint,

$$\dot{nfa}_t = \alpha(1 - \theta_t) + y_{Ft} + (r_t - r_t^*)(b_{Ht} + b_{Gt}) + r_t^* nfa_t + \underbrace{\pi_t}_{b_{It}^H(r_t - r_t^*)}. \quad (14)$$

In this equation, policy variable b_{Gt} can be expressed as $-b_{Ht} - b_{It} - b_{It}^H$ using home bond market clearing (11), where intermediaries' bond demand b_{It} is given by (5). Then, the country-wide budget constraint (14) simplifies to

$$\dot{nfa}_t = \underbrace{\alpha(1 - \theta_t) + y_{Ft}}_{\text{net exports}} + \underbrace{r_t^* nfa_t}_{\text{interest income}} - \underbrace{\frac{1}{\Gamma_F} (r_t - r_t^*)^2}_{\text{costs from UIP deviations}}. \quad (15)$$

Up to the last term, equation (15) is nothing more than a standard open economy budget constraint. It implies that home's net foreign asset position improves if the trade balance (net exports) $\alpha(1 - \theta_t) + y_{Ft}$ is large, or interest income $r_t^* nfa_t$ from existing foreign assets is large. The last term, however, is new. It captures the costs the country incurs if the interest rate spread $r_t - r_t^*$, which is the same as a UIP deviation in our context, is different from zero.

Why does the country face costs from UIP deviations? Suppose the spread $r_t - r_t^*$ is positive. This invites foreign intermediaries to come in and take a position $b_{It} = \frac{1}{\Gamma_F} (r_t - r_t^*)$ in the domestic bond market, taking home revenues

$$b_{It} \cdot (r_t - r_t^*) = \frac{1}{\Gamma_F} (r_t - r_t^*)^2. \quad (16)$$

These carry trades represent economic costs to home as they are paying a premium to foreign investors over and above the world interest rate r_t^* . Naturally, the costs increase when foreign intermediaries become more elastic to the UIP wedge, that is, when Γ_F is lower. In that case, for a given UIP wedge, intermediaries take larger positions, generating larger costs. Vice versa, if there are no active foreign intermediaries, $\Gamma_F = \infty$, the country does not incur any costs.

Noticeably, the costs in (16) are *independent* of the degree of domestic intermediation Γ_H . The reason is straightforward: While domestic intermediaries, similar to foreign ones, take a position that is proportional to the UIP wedge, their revenues do not leave the country and are instead rebated to domestic households.²⁷

Next we study the set of equilibria that are implementable by choosing a given path of foreign exchange interventions. For this result and the remainder of the paper, we introduce as notation for the UIP wedge $\tau_t \equiv r_t - r_t^*$.

²⁷Even though the assumption that domestic intermediaries' revenues entirely enter the representative household's budget constraint seems like a strong one, the model can easily cope with less extreme situations where only a fraction of those revenues fall to agents that enter the government's welfare considerations. In that case, one would merely relabel "domestic" and "foreign" intermediaries as "those who enter the government's welfare considerations" and "those who do not".

Rewriting the budget constraint (15) in present value terms we obtain the following implementability result.

Proposition 1 (Implementability conditions.). *Suppose $\Gamma_F > 0$ and $\Gamma_H > 0$. Let $\theta_t = q_t^{-1}c_t$ be the dollar value of home consumption and $\tau_t = r_t - r_t^*$ be the “wedge” in the uncovered interest parity (UIP) condition. Then, given an initial net foreign asset position nfa_0 and shocks $\{y_{Ht}, y_{Ft}, r_t^*\}$, the paths $\{c_t\}$ and $\{q_t, r_t\}$ are part of a competitive equilibrium iff the corresponding $\{\theta_t, \tau_t\}$ solve the following two conditions: The Euler equation,*

$$\frac{\dot{\theta}_t}{\theta_t} = r_t^* + \tau_t - \rho \quad (17a)$$

and the country-wide present value budget constraint,

$$\int_0^\infty e^{-\int_0^t r_s^* ds} \left[\alpha(\theta_t - 1) + y_{Ft} + \frac{1}{\Gamma_F} \tau_t^2 \right] dt = nfa_0. \quad (17b)$$

Proposition 1 gives us a simple characterization of the set of competitive equilibria as it is commonly used in models of optimal Ramsey taxation (see, e.g., Lucas and Stokey, 1983 or Chari and Kehoe, 1999). A key difference with this literature, however, is that the planner in our model does not choose a path of taxes, but rather an FXI policy as defined above. The importance of Proposition 1 is that it shows that setting FXI policies—which are paths of asset positions—in fact is equivalent to setting wedges τ_t in the UIP condition—which behave like taxes.

As a side remark, we would like to stress that in addition to the costs $\frac{1}{\Gamma_F} \tau_t^2$ coming from foreign intermediation, setting a path of nonzero UIP wedges τ_t is, of course, already “costly” in that it distorts the consumption choices of domestic households. This will be the reason a planner in our economy only cares to deviate from $\tau_t = 0$ if there is an additional reason, like managing the terms of trade, that makes such deviations beneficial. There, the costs $\frac{1}{\Gamma_F} \tau_t^2$ coming from foreign intermediation will be an additional resource cost that the country incurs, and that, as it turns out, critically changes the optimal policy.

A simple corollary of Proposition 1 is that the set of implementable allocations is independent of the degree of domestic intermediation.

Corollary 1. *In Proposition 1 the set of implementable allocations is independent of the degree of domestic intermediation Γ_H (as long as $\Gamma_H > 0$).*

We next set up the planning problem of choosing the optimal FXI policy.

3.2 Planning problem

We think of the central bank as the home economy’s social planner. Thus, the central bank maximizes the welfare of domestic households across all competitive equilibria it can possibly implement using foreign exchange interventions. Domestic households’ utility is given by $\int_0^\infty e^{-\rho t} \log c_t dt$. Since the

dollar value of home consumption, $\theta_t = q_t^{-1}c_t$, is slightly more convenient to use, we express utility in terms of θ_t and state the planning problem as,²⁸

$$\max_{\{\theta_t, \tau_t\}} \int_0^\infty e^{-\rho t} \{\log \theta_t - (1 - \alpha) \log ((1 - \alpha)\theta_t + \alpha)\} dt \quad (18)$$

subject to the two implementability conditions (17a) and (17b).

In the planning problem (18), the freedom of setting different FXI policies is completely embodied in the choice of the UIP wedge τ_t . When the central bank desires to raise consumption in period t relative to the next, it lowers τ_t . Such a policy would then be implemented by selling reserves and purchasing home bonds, which, due to a finitely elastic foreign demand function, affects the domestic interest rate r_t and thus τ_t .

One possibility for the central bank in this baseline model is to set $\tau_t = 0$ in all periods, in which case it implements an allocation that would prevail if Γ_F were equal to zero, that is, it “undoes” the imperfect intermediation friction. This is clearly possible in this model since the central bank can freely access both bond markets, and thus it may always create the right kind of bond supply to ensure that $r_t = r_t^*$, in which case foreign intermediaries’ positions b_{It} are zero. We wish to emphasize that, however, there is no simple relationship between τ_t and the country’s reserve position b_{Gt}^* in this baseline model. In particular, $\tau_t = 0$ does not necessarily correspond to zero reserves, and the relationship between τ_t and reserves more generally depends on Γ_F and Γ_H .²⁹ We discuss how a $\tau_t = 0$ policy can be implemented in Section 3.4.

As a benchmark, we now characterize the *first best* allocation. We define this to be the optimum to the planning problem (18) subject only to the resource constraint (17b).³⁰ For ease of notation, we abbreviate the planner’s per-period objective as $V(\theta) \equiv \log \theta - (1 - \alpha) \log ((1 - \alpha)\theta + \alpha)$.

Lemma 1 (First best.). *When only the resource constraint (17b) is binding, the optimal (first best) allocation $\{\theta_t, \tau_t\}$ and the corresponding shadow resource cost λ satisfy (i) $\tau_t = 0$, (ii) the implicit equation,*

$$\underbrace{e^{-\rho t} V'(\theta_t)}_{\text{marg utility at time } t} = \underbrace{e^{-\int_0^t r_s^* ds} \lambda \alpha}_{\text{marg resource cost at time } t}$$

for each $t \geq 0$, and (iii) the resource constraint (17b).

Lemma 1 states the obvious: Absent any incentive compatibility conditions, the planner equates the marginal utility of (dollar) consumption in any period t to the corresponding resource cost. Still, Lemma 1 will prove to be a useful benchmark later on.

²⁸See Appendix B.2 for a derivation.

²⁹Even though Γ_H does not enter the planning problem directly, it turns out to matter for the reserve accumulation policy that implements the optimal paths for θ_t and τ_t .

³⁰There are other ways to define “first best” here. For instance, one could allow the planner to set optimal tariffs on exports of the home good. In that case, however, the SOE can extract an unlimited amount of resources from the rest of the world, so this alternative definition is rather meaningless.

A general advantage of writing the planning problem in terms of the UIP spread τ_t is that it provides us with a convenient link to the large literature on optimal capital controls. We now explore this link.

3.3 Connection to literature on capital controls

Our planning problem (18) is related to a recent literature on capital controls (see, e.g., [Bianchi \(2011\)](#); [Farhi and Werning \(2012, 2013\)](#); [Heathcote and Perri \(2014\)](#); [Jeanne \(2012\)](#)). In that literature, capital controls are typically modeled as a proportional tax on capital flows, which directly induces a spread in the uncovered interest parity equation, i.e. a spread between r_t and r_t^* —just like in our model, where foreign exchange interventions induce such a spread. The key difference with our framework is the additional cost term $\frac{1}{\Gamma_F} \tau_t^2$ in the resource constraint, capturing net losses from foreign intermediation.

To better understand the connection to the literature on capital controls, consider a model with the same real structure as our paper but frictionless financial markets. Suppose the planner is allowed to pick a path for taxes on capital inflows, which immediately show up as UIP wedges τ . Then, the planning problem in that economy,³¹ casted in terms of UIP wedges τ , is exactly the same as our planning problem, except for the extra resource costs $\frac{1}{\Gamma_F} \tau_t^2$. Indeed, our planning problem and the one described above become formally equivalent when $\Gamma_F = \infty$. As will become clear in Section 4, the new term will shape the response of optimal policy and deliver several new insights.

However, we would like to stress that the economic interpretation is very different. Capital flow taxes are powerful to the extent that they shape the response of the private sector. Therefore, they are *ineffective* in the absence of private intermediation, which is precisely when FXI policies are most effective. The converse is true when private intermediation is frictionless. More generally, the two tools can be viewed as complements: capital controls may effectively put sand in the wheels of private intermediation, which increases Γ_F and thereby relaxes the planner’s FXI problem.³²

3.4 Zero-reserves and zero UIP wedge allocations

Before we move on to study how the central bank should optimally use foreign exchange interventions, we analyze two specific implementable allocations: the “laissez-faire” equilibrium, where the central bank keeps reserves at zero, and a $\tau_t = 0$ economy, where the central bank undoes the financial friction. These two allocations will help gauge the optimal interventions we study in the next section. In both cases, we characterize the response of the economy to three perfect foresight shocks: one to the world interest rate r_t^* , one to the endowment of the home good y_{Ht} , and one to the endowment of the foreign good y_{Ft} .³³ We choose the signs of the shocks so that they imply a

³¹[Farhi and Werning \(2013\)](#) analyze a version of this planning problem that includes labor.

³²This may explain why policymakers often put taxes on both inflows and outflows ([Fernández et al., 2015](#)) or put in place position limits on foreign exchange positions ([BIS, 2005, 2013](#)).

³³Since this is a deterministic economy, we refer to “shocks” as the deterministic response of an economy previously in steady state to a change in the deterministic path of a parameter such as r_t^* , y_{Ht} or y_{Ft} .

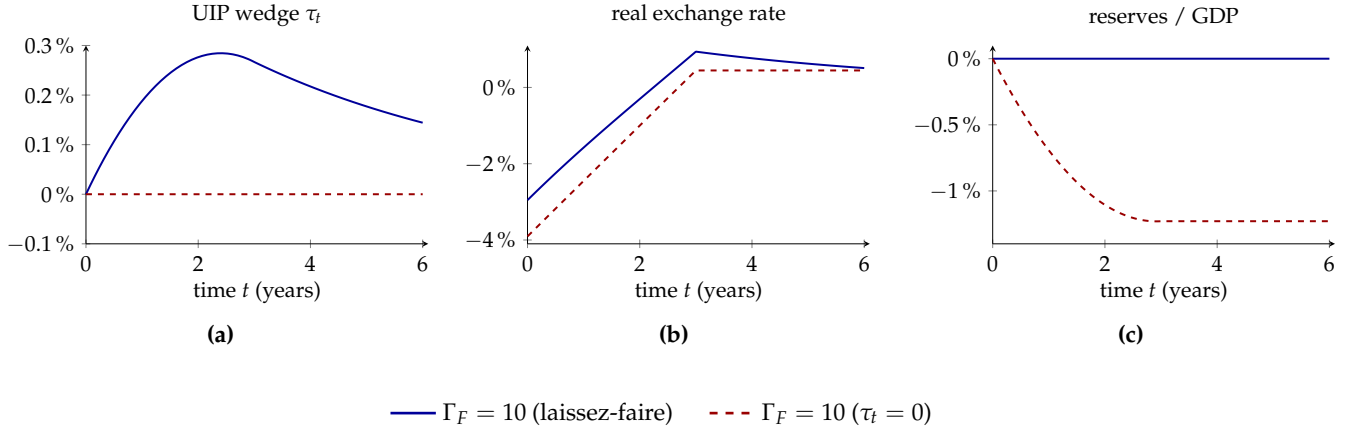


Figure 3: Response of the laissez-faire economy (blue, solid) and the $\tau_t = 0$ economy (red, dashed) to a negative r^* shock in a deterministic model economy calibrated to Brazil, with $\Gamma_F = 10$. The shock lowers r^* by 2% for $t \in [0, 3]$.

positive response of home consumption. To be consistent with our exercise in Section 4 below, we use the same simple calibration for the graphs shown in this section. In order to avoid repetition, we refer the reader to Section 4.1 for details on calibrated parameter values and a discussion of our calibration strategy.

Negative shock to the world interest rate r_t^* . Figure 3 illustrates the effects of a negative, 3-period-long shock to r_t^* on the two allocations. First, consider the laissez-faire allocation, the solid blue line. There, the shock increases foreigners' demand for home bonds, which pushes down r_t , but less than one-for-one with the shock to r_t^* due to the finite demand elasticity. This means, τ_t rises over time (Panel (a)). The lower rate r_t then has the following consequences. It leads to higher domestic consumption c_t , an appreciated real exchange rate (Panel (b)) and a worsening of the net foreign asset position. As shown in Panel (c), reserves are zero by design.

The response of the $\tau_t = 0$ economy (dashed red line) is quite different from that. First, as a direct consequence of $\tau_t = 0$, interest parity holds, $r_t = r_t^*$. Therefore, the shock to r_t^* is passed through to r_t one-for-one. This leads to a more pronounced uptick in consumption and hence a more pronounced real exchange rate appreciation (Panel (a)), compared to the laissez-faire economy. Finally, note that since zero spreads reduce intermediation profits to zero, the central bank must do all the intermediation, selling reserves to reflect the country's desire to borrow (Panel (c)).

Why does the central bank need to intervene in the same direction as the shock in order to achieve $\tau_t = 0$? Notice that any friction inhibiting the free flow of international capital *always* exhibits the following two properties: First, there is the “elasticity” property, discussed in detail above: By taking a certain foreign exchange position, the central bank has the ability to influence domestic real interest rates and exchange rates. But there is also an “underreaction” property: In response to shocks to the foreign interest rate r_t^* , the level of domestic interest rates underreacts

and moves *less* than one for one, giving rise to a positive UIP spread $\tau_t > 0$ (Panel (a)). The real exchange rate underreacts as well, since the expected UIP spreads are all positive. It is precisely this “underreaction” aspect of limited capital mobility that seems to suggest reserve decumulation is necessary to achieve $\tau_t = 0$ after a negative r_t^* shock.

Seemingly contrary to our model’s prediction, policymakers believe that domestic real interest rates r_t are more likely to overreact rather than underreact to global liquidity shocks, absent any intervention.³⁴ However, in practice, global liquidity shocks do not only affect r^* but also the risk and liquidity properties of local bonds vis-a-vis foreign bonds. At the end of Section 5.2 we discuss how shocks with these features that comove with r^* shocks might eliminate any “underreaction”, without affecting the “elasticity” aspect.³⁵ For expositional clarity, in Section 4’s plots we show the reserve purchases or sales the central bank needs to make at the optimum *relative* to any interventions it might have to conduct (if any) to achieve $\tau_t = 0$. This avoids confusion as to whether paths for reserves are determined by the “underreaction” aspect or rather by the optimal policy itself.

Shocks to endowments y_{Ht} and y_{Ft} . Assuming $r_t^* = \rho$, it follows immediately that—for any path $\{y_{Ht}, y_{Ft}\}$ —a constant path of dollar consumptions $\{\theta_t\}$ achieves the first-best outcome described in Lemma 1. Note that when θ_t is constant, equation (12) directly links higher values for y_{Ht} to a depreciated real exchange rate in a way that exactly compensates y_{Ht} , leaving the exported value in dollars, and hence the current account, constant. This is a direct consequence of assuming Cole-Obstfeld preferences. This implies that reacting to y_{Ht} does not actually require any action by the central bank.

In contrast, y_{Ft} shocks require an active portfolio management by the central bank. After a positive y_{Ft} shock, the central bank needs to accumulate reserves in order to save on behalf of households, undoing the financial friction. In the laissez-faire equilibrium, households can only save by paying a premium, which lets dollar expenditure become procyclical. In other words, the laissez-faire equilibrium has inefficient real exchange rate fluctuations in response to wealth shocks $\{y_{Ft}\}$.

As we have just seen, for y_{Ht} and y_{Ft} shocks, the first best is the optimal policy, as there is no terms-of-trade management motive in this case. This is not the case for world interest rate shocks r_t^* , in response to which we know study the optimal foreign exchange interventions.

³⁴To check the prediction of our model we also conducted a (preliminary) analysis of the response of various countries’ UIP spreads to identified US monetary policy shocks. We could not find any statistically significant evidence for a nonzero response of UIP wedges.

³⁵In a recent paper Engel (2016) makes a related point, arguing that multiple comoving shocks are necessary to explain the covariance between interest rate differentials and UIP spreads.

4 Optimal foreign exchange interventions

In this section, we present our main results about the normative behavior of foreign exchange interventions. The fundamental trade-off that determines the optimal use of foreign exchange intervention in our model is between two forces. The first is the desire to minimize the additional cost term (16). The second is a terms-of-trade management motive, which appears when the economy is subject to r_t^* shocks. More generally, however, our analysis should carry over to any sort of “macroeconomic stabilization” motive, which might lead the central bank to use foreign exchange interventions as a second best tool to influence the exchange rate or home interest rates. We explore this trade-off in three steps in this section. In Section 4.1 we briefly introduce the calibration underlying the plots shown in this section and the previous one. Section 4.2 characterizes the optimal intervention policy without any motive to manage the terms-of-trade. In this case, the planner would like best *not* to distort private consumption decisions. And in a final step in Section 4.3, we include our terms-of-trade management motive and study the optimal policy under the full trade-off. Before this, we briefly introduce the calibration underlying the plots in this section and the previous one.

4.1 Calibration

For our illustrative simulations, we calibrate the model parameters to Brazil. For the discount rate we pick $\rho = 0.075$, corresponding to the average 5yr treasury yield from 2000–2015 plus the average J.P. Morgan EMBI+Brazil return over the same time period. For the openness α of the economy, we choose $\alpha = 0.15$ matching a 15% imports to GDP ratio in 2013. We normalize y_H to 1 and y_F to 0. Our results do not really depend on the initial net foreign asset position, so for simplicity, we set it to zero at the beginning of our figures. To get an idea of the relative size of domestic compared to foreign intermediation, we note that Brazil’s domestic banks operated balance sheets roughly five times the size of foreign banks’ subsidiaries in Brazil. Interpreting balance sheet size as rough proxy for portfolio constraints (corresponding to X in our microfoundation), this leads us to calibrate $\Gamma_H/\Gamma_F = 5^2$.

Unfortunately, there is no easy way to calibrate Γ_F . Therefore, we provide three values for Γ_F , $\Gamma_F \in \{1, 10, \infty\}$, wherever it does not clutter up figures; elsewhere we use a single value, mostly $\Gamma_F = 10$, as illustration. Notice that when varying Γ_F , we also vary Γ_H according to our calibration of the ratio Γ_H/Γ_F . As will become clear below, choices for Γ_F of 1 or 10 approximately imply that a real exchange rate depreciation of 1% for one year requires a peak accumulation of reserves relative to GDP of 1.5% for $\Gamma_F = 10$, and of 7% for $\Gamma_F = 1$. This seems to be in the ballpark of empirical estimates. Kearns and Rigobon (2005) use structural breaks in the intervention policies of two advanced economies, Japan and Australia, to identify the effectiveness of interventions. Converting their findings into this context reveals that the same 1% depreciation requires a reserve accumulation of 4% over GDP for both economies. Equivalent numbers for emerging market economies are most

certainly lower than these. De Gregorio (2013) mentions that practitioners in those countries often use a reserve accumulation of 1-2% (over GDP) as benchmark.

Finally, we consider in our plots r^* shocks that temporarily lower r^* by 2% for 3 years before they return to the steady state value of 7.5%.

4.2 Optimal interventions without terms-of-trade management

We can shut off the macro stabilization motive in our planning problem (18) in two ways. First, we can set $\alpha = 1$. In this case, the country loses home bias for its own good and hence the central bank loses the ability to influence the price of its own good by reallocating consumption over time. Thus, there is no longer a motive to manage the terms-of-trade. Second, it turns out that there is also no such motive when the world interest rate is constant and equal to home's discount factor, i.e. $r_t^* = \rho$ at all times. We now investigate both of these cases, showing that in both of them the optimal UIP wedge τ_t is equal to zero. After that, we discuss the implications for reserves b_{Gt}^* and actual interventions.

Proposition 2. *Suppose $\alpha = 1$ or $r_t^* = \rho$ for all $t \geq 0$. Then, the optimal allocation coincides with the first best. In particular, $\tau_t = 0$ at all times t .*

Proposition 2 identifies two cases for which the planner sees no need for nonzero UIP wedges and hence chooses not to distort the economy. The arguments behind the two cases are distinct. When $\alpha = 1$, the home economy has no home bias and therefore its consumption is unable to affect the real exchange rate. Therefore, interventions are completely ineffective in this case. When $r_t^* = \rho$ at all times, even if α is possibly less than 1 or there are endowment shocks $\{y_{Ht}\}$, home's consumption θ_t is constant over time, and so are home's exports. Thus, there is no reason to manipulate the terms of trade over time.

In the next subsection, we explore deviations from this “neutrality” result. In particular, when $\alpha < 1$ and the economy faces r_t^* shocks, it turns out that the central bank has a macroeconomic stabilization motive and generally finds it optimal to implement nonzero UIP wedges τ_t . For the remainder of Section 4, we set $\alpha < 1$.

4.3 Optimal interventions with terms-of-trade management

In this subsection, we study the solution to the planning problem (18) in the presence of a motive manage the terms of trade. This motive is also at the core of many papers on capital controls (see e.g. Costinot et al., 2014 or Farhi and Werning, 2012, 2013). In Section 5.1 below, we study an alternative motive for intervention, based on fixed exchange rates and sticky prices.

We focus on r^* shocks since we have already seen in the previous section that time-varying $\{r_t^*\}$ is crucial for an intervention motive. Specifically, we refer to paths $\{r_t^*\}$ such that $r_t^* > \rho$ for all $t \in [0, T)$ and $r_t^* = \rho$ thereafter as *positive interest rate shocks*; and to paths $\{r_t^*\}$ such that $r_t^* < \rho$ for all $t \in [0, T)$ and $r_t^* = \rho$ thereafter as *negative interest rate shocks*. We assume that $\{r_t^*\}$ is

integrable throughout this section. As before, we first characterize the optimal foreign exchange intervention policy in terms of the path of induced UIP wedges $\{\tau_t\}$. Subsequently, we discuss the implications for reserves and exchange rates, relative to the “undoing” benchmark, $\tau_t = 0$ for all t (see the discussion in Section 3.4).

4.3.1 The benchmark of financial autarky, $\Gamma_F = \infty$

We begin the analysis by studying the special case of our model in which the private sector is in financial autarky ($\Gamma_F = \infty$). This is useful to isolate the motive for intervention by the central bank.

Proposition 3. *Suppose $\Gamma_F = \infty$. Then, the optimal intervention after a positive interest rate shock hits is to set $\tau_t < 0$ for any $t \in [0, T)$ and $\tau_t = 0$ thereafter. In particular, the central bank only intervenes during the time of the interest rate shock. Analogously, $\tau_t > 0$ for $t \in [0, T)$ and $\tau_t = 0$ thereafter in response to a negative interest rate shock.*

Similar to Costinot et al. (2014) our model embeds a terms-of-trade management motive: Individual agents do not internalize the effect of their consumption decisions on the price of the exported good. This effect is nonzero as a result of the assumption of home bias. To fix ideas, suppose the foreign interest rate is temporarily low at time t , $r_t^* < \rho$. Since this implies that exports are relatively low at time t (borrowing against future income), the planner would like to lower the export price, or equivalently depreciate the real exchange rate. Setting a positive UIP wedge, $\tau_t > 0$ then reduces current consumption, which in turn, achieves the desired real exchange rate depreciation.

This is also what we see in Figure 4, which shows the economy’s reaction to the simple negative interest rate shock described in our calibration in Section 4.1. The red line shows the response in case of financial autarky, $\Gamma_F = \infty$. It is evident that the UIP wedge jumps up as the shock hits and back down to zero as the shock fades (Panel (b)), thereby reallocating domestic demand into the future and depreciating the real exchange rate (Panel (c)). The economy executes this intervention by accumulating additional reserves during the period of the shock (Panel (d)). Such a policy is often referred to as “leaning against the wind” of international capital flows.

4.3.2 Intervention smoothing

Compared to the special case of $\Gamma_F = \infty$, studying optimal policy with intermediate degrees of capital mobility delivers three key new insights. Given that setting wedges is costly, one may expect that the optimal policy would lie somewhere between the $\Gamma_F = 0$ solution, that is $\tau_t = 0$, and the $\Gamma_F = \infty$ solution characterized in Proposition 3. The following result shows this intuition is fundamentally wrong.

Proposition 4 (Smoothing.). *Suppose $\Gamma_F \in (0, \infty)$. Then, at the optimum, τ_t is continuous in $t \in (0, \infty)$, with $\tau_0 = 0$.*

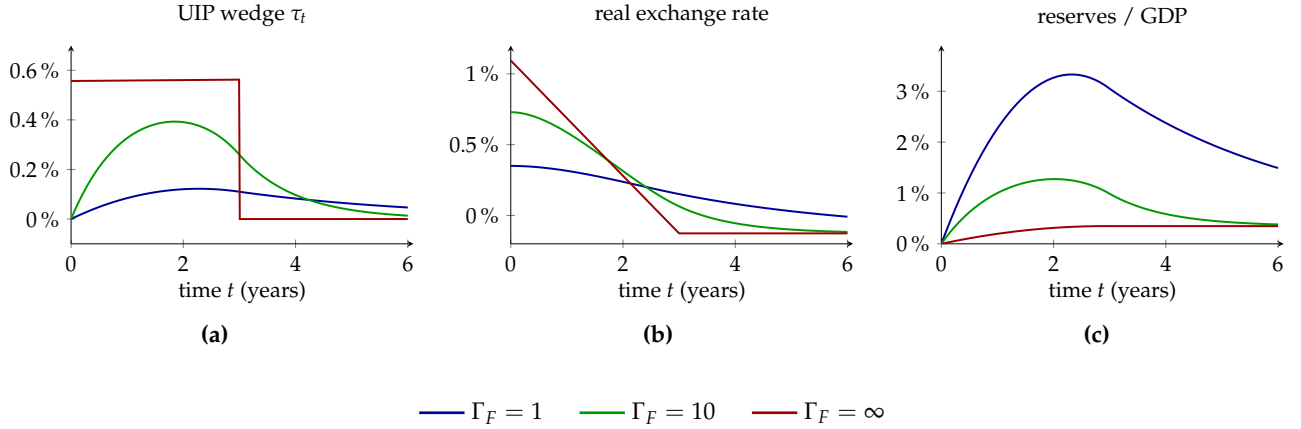


Figure 4: Optimal intervention after a negative r^* shock in a deterministic model economy calibrated to Brazil, for various degrees of capital market imperfection Γ_F . The shock lowers r^* by 2% for $t \in [0, 3]$. The results are relative to the zero UIP wedge economy ($\tau_t = 0$).

Proposition 4 highlights a property of the model that is only present for intermediate degrees of capital mobility: the central bank chooses a *smooth* path for τ_t .³⁶ Contrast this with the $\Gamma_F = \infty$ solution: There, τ_t jumps whenever r_t^* jumps. The reason for the “smoothing” result is very natural: With $\Gamma_F \in (0, \infty)$, each deviation of τ_t away from zero incurs convex costs (16). Hence, it is optimal to spread out interventions over time, optimally making interventions small and long lived, rather than large and short lived.

This result follows from a helpful lemma. To state the lemma, we introduce

$$\mathcal{T}_t = \underbrace{e^{-\int_0^t r_u^* du} \lambda}_{\text{marg resource cost at time } t} - \underbrace{e^{-\rho t} V'(\theta_t)}_{\text{marg utility at time } t}$$

as the deviation of time t (dollar) consumption θ_t from first best levels. $\mathcal{T}_t > 0$ whenever consumption θ_t is too large relative to first best, and $\mathcal{T}_t < 0$ whenever θ_t is too small relative to first best.³⁷ Here, $V(\theta)$ is the planner’s per-period objective. Using this notation, the lemma can be stated.

Lemma 2. Suppose $\Gamma_F \in (0, \infty)$. Let $V(\theta)$ be the planner’s per period objective, as defined before Lemma 1. Then, under the optimal foreign exchange intervention policy, the interest rate spread τ_t satisfies the following first order condition

$$\underbrace{e^{-\int_0^t r_s^* ds} \lambda \frac{2}{\Gamma_F} \tau_t}_{\text{marginal cost}} = \underbrace{\int_0^t \mathcal{T}_s ds}_{\text{marginal benefit}} . \quad (19)$$

Lemma 2 is a straightforward consequence of the first order conditions of the planning problem and immediately implies Proposition 4. First, since the integrals of r_t^* and $\mathcal{T}_t$ are continuous

³⁶This is reminiscent of a number of “tax smoothing” results in the optimal taxation literature, spawned by Barro (1979).

³⁷Notice that this is not a mathematically rigorous statement since λ here is not the same as in the first best problem. $\mathcal{T}_t$ still turns out to be a very useful object.

functions, it follows that τ_t is continuous. And second, since the right hand side of (19) is zero at $t = 0$, it must be that $\tau_0 = 0$.

The first order condition (19) has a useful intuition. Suppose the planner increased τ_t by a marginal unit. Increased carry trades by foreign intermediaries would then consume an extra $\frac{2}{\Gamma_F}$ of the economy's resources, valued at shadow price λ . This is captured as the marginal cost term on the left hand side of (19). However, such an intervention would also lower θ_s in all previous periods $s \leq t$, due to the forward looking nature of the Euler equation. In each such period s , it saves one unit of resources and increases marginal utility, whose joint effect on utility is precisely captured by $\mathcal{T}_s$. This explains the right hand side of (19).

Panel (b) of Figure 4 illustrates our “intervention smoothing” result. Even though the path for the interest rate shock $\{r_t^*\}$ is discontinuous, and in stark contrast with the optimal UIP wedges when $\Gamma_F = \infty$ (the red line), for finite positive values of Γ_F the optimal UIP wedges are continuous and start at zero. Their sign is the same as the one for $\Gamma_F = \infty$, so here again, the planner leans against the wind and accumulates reserves (Panel (c)) to depreciate the real exchange rate (Panel (a)). It is worth pointing out that the reason for why the lowest value for Γ_F , $\Gamma_F = 1$, is associated with the largest accumulation of reserves comes from the fact that we calibrate the ratio Γ_H/Γ_F to the data, and so lower values for Γ_F , capturing more foreign intermediation, automatically lead to lower values for the Γ_H as well, capturing more domestic intermediation. This leads to two countervailing forces: Lower Γ_F pushes for less aggressive and smoother interventions, while lower Γ_H means the central bank needs to accumulate more reserves to achieve a given spread τ_t .

The fact that in Panel (b) of Figure 4, optimal UIP wedges for $\Gamma_F < \infty$ are still positive well beyond the end of the shock at $t = 3$ is the subject of our next subsection.

4.3.3 Forward guidance

The smoothness of the intervention τ_t has two interesting indirect consequences: The first one, described in this subsection, could be described as “FXI forward guidance”. Since interventions are smoothed out over time, the planner in fact has an interest in promising to keep intervening—that is, creating nonzero wedges $\tau_t \neq 0$ —even at times $t > T$, after the shock subsided. We formalize this in the following result.

Proposition 5. *Suppose $\Gamma_F \in (0, \infty)$. Then, after a positive interest rate shock, $\tau_t < 0$ at all times t (including $t > T$). Analogously, after a negative interest rate shock, $\tau_t > 0$ at all times t (including $t > T$).*

To see the intuition behind this result, consider the first order condition (19). While the marginal cost of a marginal intervention at time t is born at that current time due to increased private carry trade activity, the marginal benefits $\mathcal{T}_s$ of influencing future interest rates accrue at all times $s \leq t$ before t . In that sense, the logic is analogous to forward guidance in a New Keynesian model (see, e.g., Eggertsson and Woodford 2003 or Werning, 2011), where marginal benefits of low rates after the zero lower bound stops binding also propagate back in time through the Euler equation.

This result also speaks to the ongoing debate over whether the likely channel through which foreign exchange interventions work is a portfolio balance channel or some kind of signaling channel. While the core of our model consists of a portfolio balance channel—foreign intermediaries only imperfectly react to the interest rate spread $r - r^*$, well in the spirit of the old portfolio balance literature (see, e.g. Kouri, 1976, Branson and Henderson (1985) or Kenen (1987))—our microfoundations and rational expectations weave a natural signaling channel into our model. Since future interventions are effective through a portfolio balance channel in the future and agents are forward-looking, signaling future interventions has the power to affect agents' actions today. In Panel (b) of Figure 4, the forward guidance aspect of optimal interventions is clearly visible. For finite, positive Γ_F , the optimal UIP wedges are not only smooth over time, but also stretch well into the future, beyond the period of the shock.

4.3.4 Time inconsistency

Clearly, in contrast with the direct effect through current portfolio choices, the effectiveness of signaling future interventions critically depends on the credibility of the central bank. This naturally opens the door to problems of credibility and time inconsistency. We formulate this in the next proposition.

Proposition 6. *Suppose $\Gamma_F \in (0, \infty)$. The optimal policy is time-inconsistent and re-optimization at any time $t_0 \geq T$ yields $\tau_t = 0$ for all $t > t_0$. Moreover, a planner without any commitment power can only achieve the no-intervention outcome, $\tau_t = 0$ at all times t .*

The argument behind the first part of Proposition 6 is quite straightforward. We already saw above that in an environment without shocks (see Lemma 2) the optimal UIP wedge is zero at all times. The time after a shock has faded, that is, $t > T$ for the interest rate shocks we have focused on in this section, is precisely such a time of no more shocks. Yet the optimal policy as described in Proposition 5 requires nonzero UIP wedges in all periods. The planning problem thus is time inconsistent.

The second part is more involved. Even if a planner without any commitment power will set $\tau_t = 0$ for all $t > T$, why would he do so *during* the time of the shock as well? The answer to this question lies in the fact that even during the time of the shock, interventions derive their effect from affecting *earlier* consumption decisions, see e.g. the first order condition in (19). Yet, since those consumption decisions are in the past, a planner without any commitment power does not take them into account and instead chooses as optimal policy $\tau_t = 0$.

The time inconsistency issue raises the question of how much the effectiveness of interventions depends on the credible signaling of future interventions. We explore this question in our simulations in Figure 5. Here we compare a full commitment (FC) policy with $\Gamma = 10$ to a limited commitment (LC) policy, where the central bank can only credibly commit to interventions until the shock fades at time $T = 3$. We see in Panel (b) that the LC planner uses the limited commitment

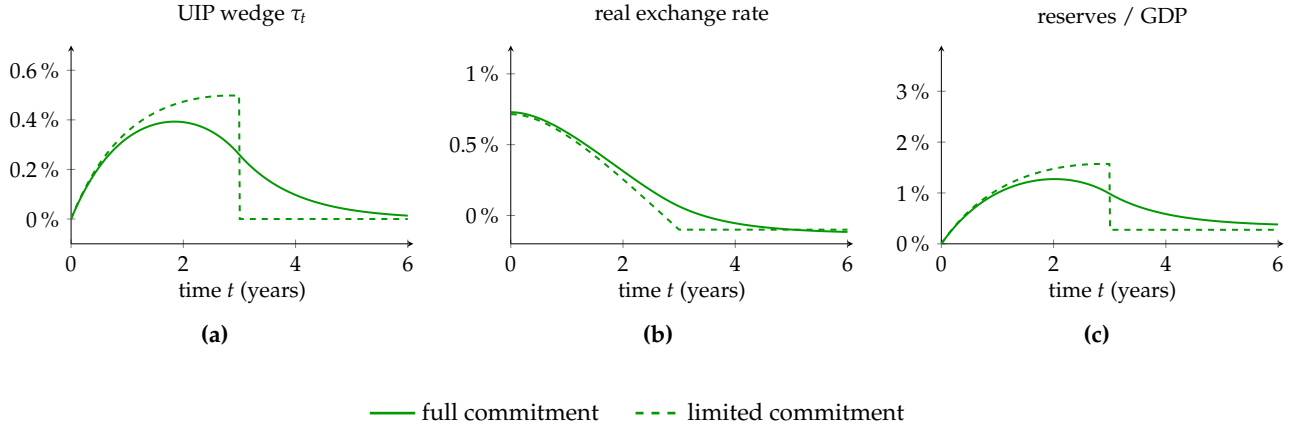


Figure 5: Comparing interventions with full commitment to interventions with limited commitment (only commitment until $t = 3$). The deterministic model economy is calibrated to Brazil with $\Gamma_F = 10$. The shock lowers r^* by 2% for $t \in [0, 3]$. The results are relative to the zero UIP wedge economy ($\tau_t = 0$).

to promise strong interventions during the time of the shock, to make up for the interventions the FC planner promises after $t = T$. While the increased interventions until $t = T$ are costly to the LC planner, they do achieve almost exactly the same extent of real exchange rate depreciation as the FC planner. Panel (d) shows that the LC planner accumulates more reserves than the FC counterpart until $t = T$. At that time, however, when the UIP wedge τ_t drops to zero and home as well as foreign intermediaries close their carry trade positions, the LC planner balances this by repatriating a large fraction of the accumulated reserves.

We should note that the type of time inconsistency is different from the standard time inconsistency coming from the tendency to depreciate one's (real or nominal) exchange rate when foreigners hold domestic local currency bonds. Since we analyzed our model in terms of domestic bonds measured in dollars, this type of time inconsistency does not appear in our setup and hence does not get entangled with the novel type of time inconsistency that we describe in Proposition 6. Of course, were we to denote initial positions in terms of the local price index, the more standard time inconsistency would re-emerge.

In sum, Sections 4.3.3 and 4.3.4 highlight that foreign exchange interventions are more powerful when they are coupled with signaling and when the central bank has at least some amount of commitment power.

5 Extensions

In this section, we present and discuss three extensions to our baseline model. In a first one, in Section 5.1, we introduce a model with an alternative motive for interventions. There, the central bank faces constraints on exchange rate movements—to make it stark we assume a fixed exchange

rate—and yet seeks to use foreign exchange interventions to stabilize the output gap. We find that all our previous analytical results go through in this economy. In Section 5.2 we enrich our model somewhat to include liquidity and risk premia, at least in some abstract form. We show that when a country sells “safe haven” bonds, it might actually earn money from foreign exchange interventions, rather than pay for them. Finally, Section 5.3 presents an economy which is pursuing a “managed float” policy in which the real exchange rate is required to follow a smooth path. We show that this kind of policy, which resonates well with many exchange-rate based rules of EMEs, may significantly backfire, inviting costly speculation.

In Appendix E we provide two additional extensions, one on whether the time inconsistency of our baseline model can be fixed by the planner itself using assets of multiple maturities a la Lucas and Stokey (1983); and the other generalizing intermediaries’ asset demand to nonlinear demand schedules.

5.1 Sticky prices and fixed exchange rates

So far, our analysis focused on foreign exchange interventions driven by a terms-of-trade management motive. One possible interpretation of that model is that there is a monetary authority in the background choosing the nominal interest rate to close the output gap at all times. To see this, consider a simple extension in which the home good is actually produced with a simple technology $y_{Ht} = n_t$, and the household experiences disutility of labor given by $v(n_t)$, so preferences are given by

$$\int_0^\infty e^{-\rho t} \{\log(c_t) - v(n_t)\} dt. \quad (20)$$

In addition, to have a meaningful monetary policy problem, assume the home currency price of the home good is fixed at $p_{Ht} = 1$ and the nominal exchange rate is given by e_t .³⁸ This implies that the dollar value of y_{Ht} is given by

$$e_t^{-1} y_{Ht} = (1 - \alpha) \theta_t + \alpha c^*, \quad (21)$$

where we re-introduced c^* from Section 3.1. Compare this to the flexible price allocation: There, output is given as $y_{Ht}^f = n_t^f$, pinned down jointly with the flexible price exchange rate $e_t = e_t^f$ by combining (21) with

$$v'(n_t^f) = \theta_t^{-1} (e_t^f)^{-1}.$$

Thus in our previous analysis, we can imagine that monetary policy is implementing $e_t = e_t^f$ at all times.³⁹ In this sense, the output gap objective takes priority over the real exchange rate objective. In this subsection, we explore the polar opposite: We assume that—for some unmodeled reason—the

³⁸We use the convention that lower values of e_t reflect a more appreciated home currency.

³⁹Here, we take a shortcut and loosely describe monetary policy as choosing a path for the nominal exchange rate. This can be made more formal by assuming that there is a nominal interest rate i_t such that $r_t = i_t + \dot{e}_t/e_t$. This interest rate can then be implemented using a standard interest rate rule. Note that, according to the standard definition, our interventions in the baseline model are not fully sterilized: The nominal interest rate automatically adjusts in response to interventions to replicate the flexible price allocation, and is therefore not constant.

monetary authority has some exchange rate objective e_t . To make it stark, we assume a fixed exchange rate regime, $e_t \equiv \bar{e}$, and ask: How can the planner use foreign exchange interventions to regain some monetary independence and mitigate the impact on the domestic economy? Examples of interventions of this sort arguably include recent interventions by Euro neighbors like Denmark, Switzerland, or the Czech Republic, which try to fend off appreciations and at the same time avoid being pushed into the zero, or effective, lower bound for interest rates.

In a first step, we ask which allocations can be implemented by central bank policies. Fortunately, it is straightforward to show that, in fact, when stated in terms of $\{\theta_t, r_t\}$, the same implementability conditions as in Proposition 1 continue to hold in this economy.⁴⁰ The reason is that sticky prices let labor supply and the home endowment depend on θ_t , yet neither enters the implementability conditions. They do, however, enter the objective function. Replacing labor with (21) in the utility function (20) and following the same steps as before, we find the planning problem to be

$$\max_{\{\theta_t, \tau_t\}} \int_0^\infty e^{-\rho t} \{ \log \theta_t - (1 - \alpha) \log((1 - \alpha)\theta_t + \alpha c^*) - v((1 - \alpha)\theta_t + \alpha c^*) \} dt \quad (22)$$

subject to (17a) and (17b).

The crucial difference to our previous planning problem is the objective function. The reason for this is that the rationale for intervening has changed.⁴¹ Before, the central bank was intervening to manage the country's terms of trade. Now a second rationale emerges: Regaining monetary independence despite the fixed exchange rate. Suppose the world interest rate decreases temporarily. The flexible exchange rate response would be to let the currency appreciate today and depreciate in the future. Since this is impossible with a fixed exchange rate, the economy experiences a boom and a subsequent recession. In this situation, by accumulating reserves and hence generating a positive UIP spread, the planner is able to shift expenditure into the future. This mitigates both the boom and the subsequent recession.

To see whether this “leaning against the wind” property, as well as our other results in Section 4, carry over to this fixed exchange rate environment, notice that the planning problem (22) is almost unchanged: It still involves a strictly concave per period objective function and the maximization is subject the exact same constraints. Therefore, the results in Lemmas 1 and 2 and Propositions 2–6 carry over to this alternative environment, one for one. In particular, optimal interventions in this fixed exchange rate environment are still small, frequent, persistent and credible. This example illustrates that while the reason for intervening may differ across applications, the way interventions are implemented does not. In this sense, the results of Section 4 are robust.

⁴⁰Strictly speaking, with $c^* \neq 1$, the term $\alpha(\theta_t - 1)$ in (17b) needs to be replaced by $\alpha(\theta_t - c^*)$ but everything else is unchanged.

⁴¹To see this most clearly, one may set $c^* = 0$. In that case, there is no “terms-of-trade management” motive and, yet, the planner would like to intervene to stabilize the output gap.

5.2 Safe havens

While the fact that our analysis is deterministic has several advantages, in particular in terms of clarity and tractability, it lacks risk or “safety” premia. In recent years, some advanced economies, most notably Switzerland, have conducted foreign exchange interventions in a setting where domestic bond yields are typically *lower* than the world interest rate, despite the interventions. This raises the question of whether these interventions are costly to the central bank at all, or if they might actually benefit from them.⁴² Here, by cost we mean our cost term $1/\Gamma_F \tau^2$ which can never be negative (i.e. a profit) in our analysis without risk premia so far. In this subsection, we add a simple modification to our existing model that seeks to capture the key implications of risk premia for foreign exchange interventions, without giving up our tractable deterministic framework.⁴³

The reason why risk is important to study in our context is that agents in the domestic economy might value bonds using a different stochastic discount factor than foreign intermediaries.⁴⁴ To capture this idea in our deterministic model, we now explore what happens if intermediaries perceive an additional benefit of ξ_t for each additional unit of the domestic bond held,⁴⁵

$$b_{It}^H = \frac{1}{\Gamma_H} (r_t + \xi_t - r_t^*) \quad \text{and} \quad b_{It}^F = \frac{1}{\Gamma_F} (r_t + \xi_t - r_t^*).$$

These equations only affect the previous intermediary bond demands (5) and (6) but leave our definition of competitive equilibrium otherwise unchanged. In particular, intermediary profits are still given by $b_{It}^j (r_t - r_t^*)$ for intermediary $j = H, F$. While it turns out that, as before, domestic intermediaries’ bond demand does not enter the implementability conditions, the new foreign intermediaries’ demand function changes the costs of UIP wedges $\tau_t = r_t - r_t^*$ to

$$\tau_t b_{It}^F = \frac{1}{\Gamma_F} \tau_t^2 + \frac{1}{\Gamma_F} \xi_t \tau_t = \frac{1}{\Gamma_F} \left(\tau_t + \frac{\xi_t}{2} \right)^2 + \frac{1}{\Gamma_F} \frac{\xi_t^2}{4}. \quad (23)$$

⁴²Of course, our analysis cannot speak to the political circumstances that engulf such large-scale interventions. These were among the reasons that led the Swiss central bank to stop its interventions in January 2015, causing a large appreciation of the Swiss Franc and a corresponding valuation loss on the SNB’s foreign exchange holdings. See, e.g., <http://www.wsj.com/articles/swiss-national-bank-scraps-minimum-exchange-rate-1421315392> and <http://www.wsj.com/articles/swiss-national-bank-reports-23-5-billion-loss-in-2015-1457099558>.

⁴³We also would like to highlight a different but highly complementary perspective by Amador et al. (2016) to understanding Swiss interventions, using CIP rather than UIP violations: If one interprets the risk of the Swiss National Bank (SNB) abandoning the peg as an exogenous event that is outside the SNB’s control, then the costs of interventions are indeed characterized by CIP violations. Due to expected appreciation in the forward markets at the time, these “CIP costs” were positive for the SNB. Notice that of course, such CIP costs only realize eventually, after the peg is abandoned. To judge, however, whether such a peg is costly if kept *permanently* (i.e. under perfect commitment, irrespective of beliefs in the forward market), it is advisable to consider the actual, UIP related, costs the country incurs instead. We argue in this subsection that for Switzerland, these “UIP costs” may well have been *negative*.

⁴⁴Risks that symmetrically affect all bond market participants (intermediaries and the central bank) can be captured by movements in r_t^* in our model.

⁴⁵A simple microfoundation of these benefits would be a latent risk that materializes with some Poisson intensity $\lambda \rightarrow 0$ and intermediaries that are Knightian to different degrees. See Caballero and Farhi (2015) for a microfoundation of safety along those lines in a deterministic economy.

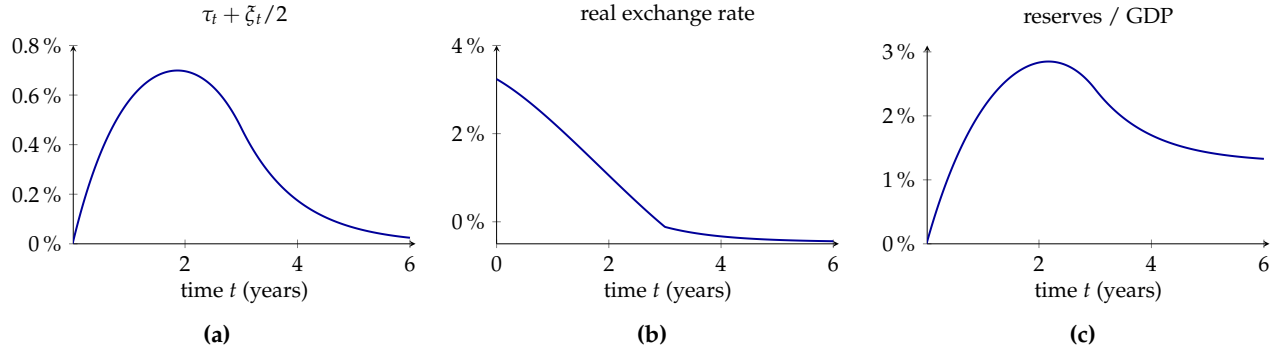


Figure 6: Optimal intervention after a positive ξ_t shock to intermediaries' preferences for home bonds. The deterministic model economy is calibrated to Brazil, with $\Gamma_F = 1$. The shock increases ξ_t by 2% for $t \in [0, 3]$. The results are relative to the zero UIP wedge economy ($\tau_t = 0$).

In this environment, it is evident that not all interventions are costly. If—as one could argue applies to safe havens like Denmark or Switzerland—the domestic bond is considered a particularly safe asset to investors, captured by a positive ξ_t , then intervening in the foreign exchange market to generate positive UIP wedges τ_t can leave the country with a net profit. The intuition for this is straightforward: In the case of $\xi_t > 0$, the country is the sole producer of an asset that outside investors value higher than the asset's producer (the home country). Thus, it can supply the market with the asset and charge a premium in form of a positive τ for it. This generates profits.

Apart from the fact that interventions can be profitable, (23) also reveals that the myopically optimal UIP wedge τ_t is no longer zero, and instead equal to $-\xi_t/2$. That is, the planner now seeks to smooth out $\tau_t + \xi_t/2$ over time, rather than τ_t , since those are the deviations from the myopically optimal policy. This is essentially the key difference to our previous analysis. Figure 6 illustrates the optimal intervention in response to a ξ_t shock, again relative to the non-intervention economy. It can be seen that qualitatively, the plots look very similar to their counterparts in Figure 4.

Intermediary preference shocks are also useful in a different way. In Section 3.4 we argued that limited capital mobility plays two roles in our analysis. On the one hand, they determine the *elasticity* with which reserve flows are able to affect domestic interest rates. On the other, they cause *underreaction* of domestic interest rates in the laissez-faire competitive equilibrium in response to shocks to the world interest rate r_t^* . The preference shocks ξ_t can help disentangle the elasticity role—which we are ultimately interested in—from the underreaction. To provide a clean argument, consider a situation where ξ_t only affects home intermediaries. When ξ_t increases as r_t^* falls, or in other words intermediaries associate a smaller risk premium with domestic bonds, it is possible that changes in r_t^* are passed through one-for-one to changes in r_t (rather than less than one-for-one). In this case, both the equilibrium UIP spread and equilibrium reserve holdings are exactly zero. In this interpretation, since home intermediaries' demands do not affect the planning problem, Figures 4

and $\bar{r}$ represent the actual holdings of reserves.

5.3 Smooth exchange rates vs. smooth UIP wedges

There is ample evidence by now that some emerging market policymakers, especially East-Asian ones, seem to conduct policies aimed at smoothing *exchange rates*, at short to medium horizons.⁴⁶ This is sometimes referred to as a “managed float” and should not be confused with the kind of “intervention smoothing” policy that we found to be optimal in our model: Here, *UIP wedges* τ_t are smooth, while exchange rates jump initially, albeit by less than without intervention if the optimal policy is employed. This is the case even though an exchange rate that is smooth at $t = 0$ would certainly lie in the space of implementable allocations. This raises the natural question of why our optimal policy problem did not select such a “managed float” allocation.

To explore this question, we simulate the optimal policy under an additional ad-hoc “smooth exchange rate” constraint, namely that the initial real exchange rate q_0 be the same as in the steady state, $q_0 = q_{ss}$.⁴⁷ Since the real exchange rate in the model moves one-to-one with dollar consumption θ_t , this is equivalent to adding the constraint

$$\theta_0 = \theta_{ss} \equiv 1 + \alpha^{-1} r^* n f a_0. \quad (24)$$

We thus solve our planning problem (18) subject to (17a), (17b) and (24). It is worth stressing that when $\Gamma_F \in (0, \infty)$ this additional constraint does *not* make the problem trivial, in the sense that it is impossible to approximate the solution of our original problem arbitrarily closely. The reason for this comes from the fact that this would require infinitely large, and infinitely costly, UIP wedges τ_t for t close to zero.

We plot the optimal managed float intervention in Figure 7 and compare it to our unconstrained optimal policy, computed without (24) as constraint. Panel (b) shows the two economies’ real exchange rate paths. It is clearly visible that while the unconstrained optimal policy features a sharp and sudden exchange rate appreciation (solid), the managed float policy generates a much slower appreciation that stretches even beyond the first year of the shock (dashed). As a consequence of the slow and predictable appreciation, the country must bear the cost of an excessive UIP spread (Panel (a)), inviting a significant amount of carry trade activity both by foreign and by domestic intermediaries. To implement this policy, the domestic central bank acts as a “shock absorber”: it accumulates reserves at a rapid pace initially, and then slowly decumulates them over the years.

The reader may wonder why the central bank trying to implement a peg in Section 5.1 did not encounter such heightened carry-trade activity in the beginning of the intervention period. The

⁴⁶See the voluminous “fear of floating” literature, e.g., Calvo and Reinhart (2002), Levy-Yeyati and Sturzenegger (2005b) or McKinnon and Schnabl (2004).

⁴⁷To be clear, by “smooth exchange rate” we mean continuity of the exchange rate, not any kind of differentiability. Furthermore, note we can interpret this a smooth nominal exchange rate requirement, together with the commitment to pick a flex-price allocation, as explained in 5.1.

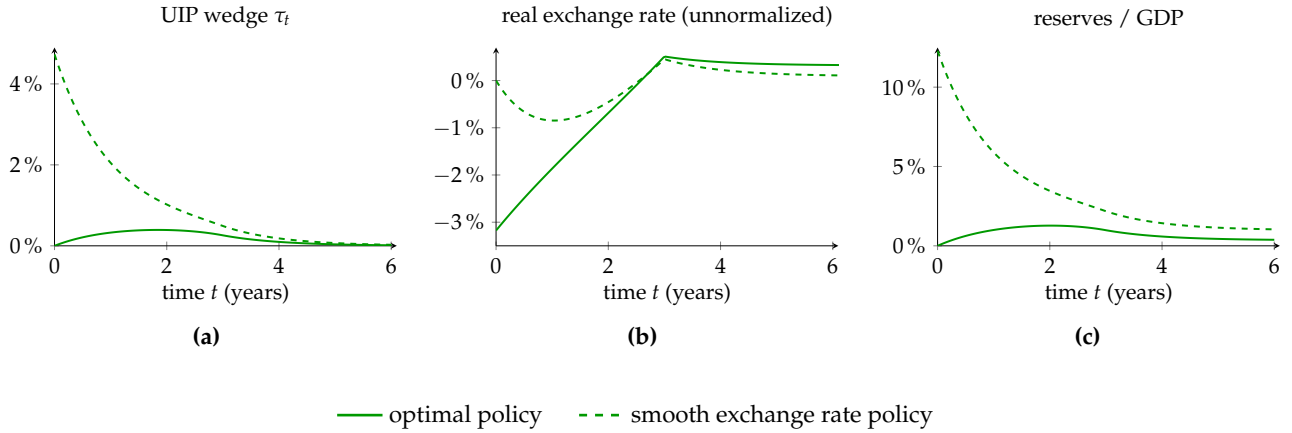


Figure 7: Comparing the optimal policy (solid) to a “managed float” policy (dashed) Panel (a) shows the UIP wedge, which in the model is proportional to carry trade activity by foreigners. Panel (b) shows the unnormalized real exchange rate, not the one relative to the competitive equilibrium. Panel (c) shows the country’s reserve positions. The deterministic model economy is calibrated to Brazil, with $\Gamma_F = 10$. The r_t^* shock hits for $t \in [0, 3]$.

reason for this is that in Section 5.1 we allowed for a non-zero output gap while in this subsection we assumed the output gap is zero at all times, i.e. the central bank is choosing one among the flex-price allocations. Thus, this subsection emphasizes that smoothing the exchange rate path may be very costly if domestic objectives are perceived by agents to be more important than external objectives. Put differently, a policymaker seeking to implement a smooth exchange rate path may avoid speculation by foregoing some domestic stability, as in Section 5.1.

6 Reserve wars

So far, we have analyzed the optimal policy of a small open economy (SOE) against a passive rest of the world. Now, we explore the strategic interaction among different SOEs’ foreign exchange interventions and their effects on the rest of the world. For this purpose, we consider a world with two kinds of countries: Advanced economies (AE), which are assumed to have frictionless financial markets, and emerging markets (EME), which are SOEs like the one described in Section 3. For ease of exposition, we focus on a two period version of our model.

The structure of this section is as follows. In Subsection 6.1, we describe the model. Then, we characterize the world equilibrium in a decentralized setting where each EME central bank chooses its own foreign exchange policy taking as given the actions of the other EMEs. Finally, we consider the world equilibrium where foreign exchange policies are set by a single “EME cooperative planner”, who can be interpreted as a stand-in for closer central bank cooperation.

6.1 Setup

The world consists of an interval $[0, 2]$ of small open economies and exists for two periods $t = 0, 1$. Economies $i \in [0, 1]$ are “advanced economies” (AE) while economies $i \in (1, 2]$ are “emerging market economies” (EME). Economies within each of the two regions are identical in all respects. A typical SOE $H \in [0, 2]$ has the same preferences as the SOE in Section 3 except for being two-period lived, that is, $U = \ln(C_0) + (1 + \rho_j)^{-1} \ln(C_1)$ with $C_t = \kappa c_{Ht}^{1-\alpha} c_{Ft}^\alpha$, where the discount factor ρ_j is allowed to depend on the region $j = AE, EME$. Here, c_{Ht} denotes consumption of H ’s own good, and c_{Ft} denotes consumption of a common foreign good. We assume that the foreign good is a composite good given by $c_{Ft} = c_{AEt} + c_{EMEt}$, where c_{AEt} and c_{EMEt} are themselves aggregates of varieties produced in each region,

$$\ln(c_{AEt}) = \int_{i \in [0, 1]} \ln(c_{it}) di \quad \text{and} \quad \ln(c_{EMEt}) = \int_{i \in (1, 2]} \ln(c_{it}) di.$$

This market structure captures that EMEs compete more with one another than with AEs in world good markets and allows us to obtain clean benchmark results. We normalize the price of the composite foreign good to 1 in each period. We assume that each SOE is endowed with its own good: EMEs with $\chi \in [0, 1]$ units of their own good and AEs with $1 - \chi$ units of their own good. Henceforth, we will exploit symmetry and label with a star “*” variables from a typical AE, and without stars variables from a typical EME.

EMEs are characterized by imperfect financial mobility. Residents in any given EME are only allowed to trade a bond in the jurisdiction of their own country, paying gross interest rate $1 + r$ (as before in units of the foreign good which we call “dollars”). In addition, there is a continuum of intermediaries located in AEs who can freely access each of the AEs’ bond markets, which pay a common interest rate r^* in units of the foreign good c_{Ft} . They can trade in each of the EMEs’ bond markets but only up to a position limit $X\sqrt{\chi}$, and only after paying an idiosyncratic transaction cost.⁴⁸ Following the same steps as in Section 3, this leads to a finitely elastic demand function for each EME bond, $b_I = \chi \Gamma_F^{-1}(r - r^*)$. As in the previous sections, central banks are assumed to have perfect access to AEs’ bond markets as well as their own bond market, but not other EMEs’ bond markets.⁴⁹ We define $\theta_t \equiv \chi^{-1} P_t C_t$ and $\theta_t^* \equiv (1 - \chi)^{-1} C_t^*$ as the (normalized) dollar consumptions of EMEs and AEs. As before, the UIP wedge is defined as $\tau = \frac{1+r}{1+r^*} - 1$.

We next characterize and define a competitive equilibrium in this economy. The home market clearing for a typical EME is

$$(1 - \alpha) p_{Ht}^{-1} \theta_t + p_{Ht}^{-1} C_{EMEt} = 1 \tag{25}$$

⁴⁸We normalize X by $\sqrt{\chi}$ to make sure the limit where EMEs are small, $\chi \rightarrow 0$, is well-defined.

⁴⁹If central banks could access other EMEs bond markets, they would behave like arbitrageurs, exploiting opportunities generated by other central banks. This may generate another motive for central bank coordination, independent of the one we focus on. In any event, our results would go through as long as trading in other EME bonds by central banks is costly.

where C_{EMEt} is the world demand for the EME aggregate good, again normalized by the endowment χ . Similarly, in AEs,

$$(1 - \alpha)p_{Ht}^{*-1}\theta_t^* + p_{Ht}^{*-1}C_{AEt} = 1. \quad (26)$$

The market clearing condition for the composite foreign good yields,

$$\alpha\chi\theta_t + \alpha(1 - \chi)\theta_t^* = \chi C_{EMEt} + (1 - \chi)C_{AEt}. \quad (27)$$

In what follows, we focus on a symmetric equilibrium with positive consumption of both aggregate goods, $P_{EMEt} = P_{AEt} = p_{Ft} = 1$.⁵⁰ By symmetry we mean that every EME central bank finds it optimal to carry out the same foreign exchange policy, implying that

$$p_{Ht} = p_{Ht}^* = 1.$$

Symmetry allows us to simplify (25), (26) and (27), yielding

$$(1 - \alpha)\theta_t + C_{EMEt} = (1 - \alpha)\theta_t^* + C_{AEt} = 1. \quad (28)$$

and

$$\chi\theta_t + (1 - \chi)\theta_t^* = 1. \quad (29)$$

In each SOE, the optimal solution of the consumers' problem implies the following Euler equations,

$$\theta_1 = \frac{(1 + r^*)(1 + \tau)}{1 + \rho_{EME}}\theta_0 \quad \text{and} \quad \theta_1^* = \frac{1 + r^*}{1 + \rho_{AE}}\theta_0^*. \quad (30)$$

Finally, consolidating the consumer's budget constraint with the central bank's yields the country-wide budget constraint,

$$\alpha\theta_0 - C_{EME0} + \frac{1}{1 + r^*}(\alpha\theta_1 - C_{EME1}) + \frac{1}{\Gamma_F}\tau^2 = 0,$$

which using (28) implies

$$(\theta_0 - 1) + \frac{1}{1 + r^*}(\theta_1 - 1) + \frac{1}{\Gamma_F}\tau^2 = 0. \quad (31)$$

For simplicity, we take initial net foreign asset positions to be zero.

We are ready to formally define a symmetric world competitive equilibrium in this economy. For ease of exposition and brevity, we state the foreign exchange policy directly in terms of τ .

Definition 2. A *symmetric world competitive equilibrium* given an EME central bank foreign ex-

⁵⁰This will always occur in equilibrium if discount factors are not too different. Note, however, that the following analysis would still be true if one of the aggregate goods had negative consumption, in the interpretation that then extra supply of that good would be created, using some of the other aggregate good as inputs.

change policy τ , is an allocation $\{\theta_t, \theta_t^*, C_{EMEt}, C_{AEt}\}_{t=0,1}$ together with an interest rate r^* , such that equations (28) – (31) hold.

This defines a competitive equilibrium given a (symmetric) set of EME foreign exchange policies. We now characterize the competitive equilibrium that occurs if EME central banks play a Nash equilibrium in their choice of foreign exchange policies. For our numerical illustrations, we stick with our baseline calibration and assume that $\Gamma_F = 10$ and $\alpha = 0.15$. Interpreting one period as the equivalent of 3 years, so that our 2-period model captures the same kind of shock as before, we set $1 + \rho_{EME} = 1.075^3$. We re-calibrate ρ_{AE} as we vary χ such that the annualized response of the world interest rate when $\tau = 0$ is 5.5%. All results below will be stated in annualized terms.

Noncooperative world equilibrium. The typical EME central bank maximizes its own welfare taking two objects as given: the world interest rate r^* and foreign expenditure levels $\{C_{EMEt}\}$. Proceeding exactly like in equation (18), we find that the problem of an individual EME central bank is

$$\max \sum_{t=0}^1 (1 + \rho_{EME})^{-t} \{\ln(\theta_t) - (1 - \alpha) \ln((1 - \alpha)\theta_t + C_{EMEt})\} \quad (32)$$

subject to

$$\alpha\theta_0 - C_{EME0} + \frac{1}{1 + r^*}(\alpha\theta_1 - C_{EME1}) + \frac{1}{\Gamma_F} \left(\frac{\theta_1(1 + \rho_{EME})}{\theta_0(1 + r^*)} - 1 \right)^2 = 0. \quad (33)$$

This is a two-period version of the problem analyzed in Section 4, where we had $C_{EMEt} = \alpha$ and $\chi = 1$. In (33) we also substituted out $\tau = \frac{\theta_1(1 + \rho_{EME})}{\theta_0(1 + r^*)} - 1$. The intuition behind this problem is the same as before: When the interest rate r^* is lower than the EME discount rate ρ_{EME} , or foreign expenditure C_{EMEt} is higher at $t = 0$ than at $t = 1$, the EME central bank optimally accumulates reserves and implements a positive UIP wedge τ . It is worth noting that the IC constraint (33) is a non-linear function of θ_0 and θ_1 for any $\Gamma_F \in (0, \infty)$. In Appendix F.1 we prove that it is without loss to relax (33) as inequality and that this inequality constraint describes a convex, bounded set in $(\theta_0, \theta_1) \in \mathbb{R}_{++}^2$. Thus, whenever $C_{EMEt} \geq 0$ for all t (to ensure the concavity of the objective) this is a well-behaved concave maximization problem with a convex constraint.

We next characterize the central bank Nash equilibrium, in which each central bank solves (32) taking $\{C_{EMEt}\}$ and r^* as given, but in equilibrium $\{C_{EMEt}\}$ is pinned down by (28), and r^* is pinned down by (29) and (30). To simplify the proofs, we assume that EMEs are small in the following formal result ($\chi = 0$). For $\chi > 0$, we verified numerically that the proposition still holds.

Proposition 7 (Reserve wars.). *Assume imperfect capital mobility, $\Gamma_F \in (0, \infty)$, and that emerging markets are small, $\chi = 0$. If emerging market central banks choose their foreign exchange policy in a non-cooperative way, then it holds that:*

1. *There exists a unique Nash equilibrium.*

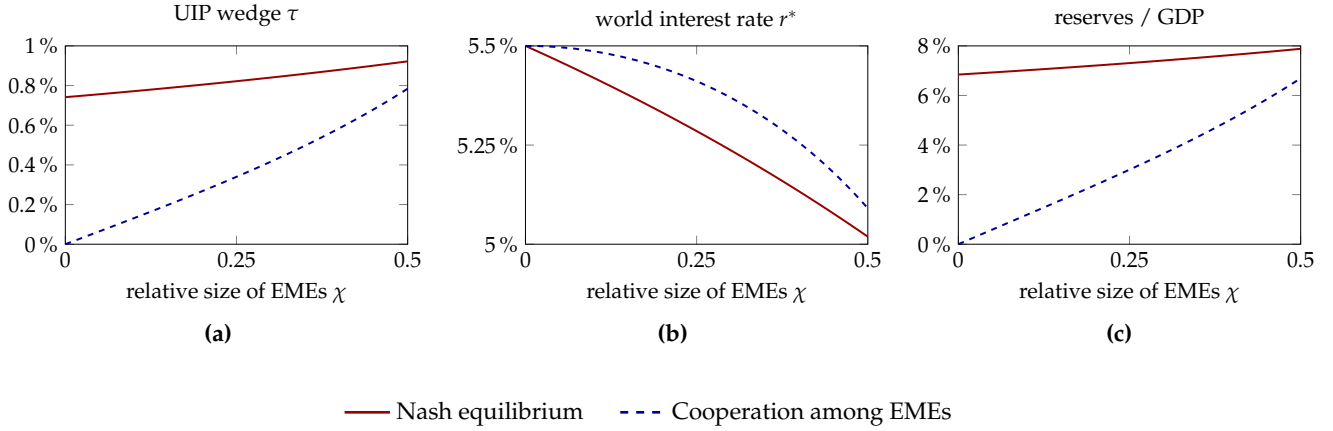


Figure 8: World equilibrium outcomes as a function of the size of emerging markets, with and without coordination among emerging market central banks.

2. In the Nash equilibrium, emerging markets accumulate reserves and the UIP wedge τ is strictly positive.
3. Compared to a no-intervention world, capital flows more upstream towards the advanced economies, driven by reserves, while private capital flows more downstream towards emerging markets.
4. Welfare of emerging markets is lower than without intervention.

Proposition 7 describes four key properties of the Nash equilibrium. We discuss them in turn. First, the existence of a unique Nash equilibrium follows because for $\chi = 0$, foreign exchange interventions in the model are strategic substitutes: When EMEs choose to accumulate reserves, they depreciate their real exchange rates leading to higher total consumption of the EME good $C_{EME,0}$ in the first period. Because the EME good is a Cobb-Douglas aggregate of all EMEs, this also raises the first period demand for any single EME, hence calling for a more appreciated real exchange rate to exert monopoly power. Notice that, by contrast, for larger values of χ , a force for complementarity emerges: Then, more reserve accumulation lowers world real interest rates even more, causing even stronger capital inflows into every single EME, and raising their desire to counter that with more reserve accumulation.

In the unique Nash equilibrium, the UIP wedge τ is strictly positive since advanced economies are attempting to save, $\rho_{AE} < \rho_{EME}$, causing private capital to flow into EMEs. The reserve accumulation by EMEs then pushes public funds upstream, while the positive UIP wedge lets intermediaries take larger downstream positions. This explains parts 2 and 3 of Proposition 7. We illustrate these outcomes in Figure 8 as function of the overall size of EMEs χ (red, solid line). Panel (a) shows that the equilibrium UIP wedges are positive throughout, and rise with χ as the feedback loop through lower world interest rates kicks in (Panel (b)). Analogously, reserves are positive and increase with χ relative to GDP, as shown in Panel (c).

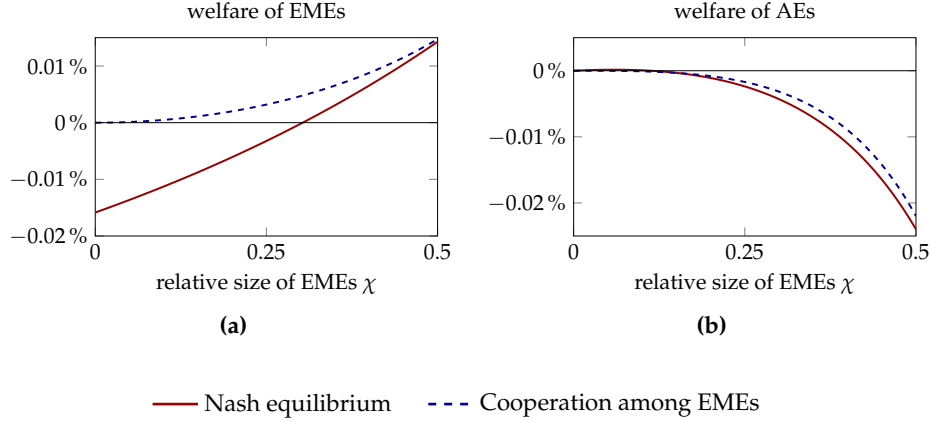


Figure 9: Welfare as a function of the size of emerging markets, with and without coordination among emerging market central banks.

In general equilibrium, interventions are self-defeating: Even if all EMEs accumulate reserves to depreciate their $t = 0$ real exchange rate, this does not happen. Foreign demand is infinitely elastic if $\chi = 0$, fixing the real exchange rate at 1, as can be inferred from equation (28). This means that, in contrast to their intended purpose, interventions cause welfare *losses* for emerging markets. In addition, if $\chi > 0$, interventions might also reduce the welfare of advanced economies by depressing the world interest rate. Put together, these kinds of noncooperative reserve wars can cause welfare losses for *all* countries. We illustrate this in Figure 9 as function of the overall size of EMEs χ (red, solid line). As Panel (a) shows, welfare of EMEs suffers due to the competitive devaluations up until EMEs are so large that their effect on the world interest rate compensates for the welfare losses associated with the devaluations. In Panel (b) we see that welfare of AEs rises ever so slightly for small χ due to intermediary profits from carry trades against emerging markets and then rapidly falls below zero as AEs try to save in an increasingly low interest rate environment.

Cooperation of emerging market central banks The self-defeating nature of interventions suggest that there may be gains from policy coordination among EME central banks. This is what we consider next. The world equilibrium can now be regarded as the outcome of a planning problem in which a single “EME planner” maximizes the objective (32) subject to the IC constraint (33), but now takes into account the endogeneity of $\{C_{EMEt}\}$ and r^* , coming from equilibrium conditions (28), (29) and (30). In the case where $\chi = 0$, we can prove the following result, standing in stark contrast with the noncooperative outcome.

Proposition 8 (Central bank cooperation.). *Assume imperfect capital mobility, $\Gamma_F \in (0, \infty)$, and that emerging markets are small in total, $\chi = 0$. If emerging market central banks cooperate, then it is optimal for emerging markets not to accumulate any reserves, implying a zero UIP wedge $\tau = 0$.*

We illustrate the contrast between the Nash equilibrium outcome (red line, solid) and the

cooperative solution (blue line, dashed) in Figures 8 and 9 as function of the overall size of EMEs χ . Figure 8 compares the equilibrium UIP wedge, the world interest rate and total reserves position as fraction of GDP. When $\chi = 0$, internalizing that competitive devaluations are self-defeating and with no possibility of manipulating the world interest rate in its favor, the cooperative planner sets $\tau = 0$. As χ increases, the planner boosts savings in an attempt to lower the interest rate thereby increasing τ . Panel (b) shows that the cooperative planner believes that EMEs' reserve wars let the world interest rate fall too low in the Nash equilibrium. Put differently, the Nash equilibrium has reserve over-accumulation even from the point of view of the cooperative planner (Panel (c)).

Figure 9 shows the welfare of EMEs and AEs under policy cooperation. Naturally, the welfare of EMEs is now always larger than with a $\tau = 0$ policy or with the Nash equilibrium policy, since both are feasible policies. Interestingly, although the cooperative policy is still of the "beggar-thy-neighbor" type—EMEs manipulate the interest rate in their favor at the expense of AEs—AEs are better off under this partial degree of cooperation than in the Nash equilibrium for reasonable values of χ . In other words, even partial cooperation can make everyone better-off with respect to decentralized foreign exchange interventions.⁵¹ Finally, note that given that international transfers are infeasible, a τ lying between this "partial" cooperative solution and $\tau = 0$ may be an indirect way of transferring resources to AEs for a global planner. It should be noted, however, that we abstracted from heterogeneity in initial NFA positions, which is important in reality to assess the consequences of world interest rate movements.

7 Conclusion

Foreign exchange interventions are one of most important policy tools for many countries around the world. Yet, many debates regarding their usefulness and the best implementation design persist. We believe this is partly due to the lack of a unified framework to analyze the optimal design jointly with the macroeconomic rationales behind interventions. In this paper, we provided such a framework. At the core of our model lies the assumption of limited capital mobility, which gives rise to a general equilibrium portfolio balance channel. We showed that interventions essentially manage a path of UIP spreads, and that each nonzero UIP spread represents a cost to the economy, coming from foreign intermediaries' carry trade activity. These costs, which are naturally convex as larger spreads invite further speculation, lie at the heart of our optimal policy design. In a nutshell, they make it optimal to spread out interventions.

Our findings pick a clear side in the debate. Interventions should be small and frequent to avoid inviting significant speculation. Furthermore, they should be highly inertial and pre-announced to maximize the impact on the contemporaneous exchange rate. Interventions were found to be more powerful if the monetary authority is more credible, as this allows it to spread out interventions even further into future, minimizing the overall cost of generating an exchange rate response today.

⁵¹The only exception is when χ is small enough that foreigners are better-off as a result of carry-trade profits.

Finally, we also showed that the optimal policy is better approximated by a quantity rule rather than a smooth exchange rate rule. In the case of the latter, speculative costs may become prohibitively costly if the monetary authority tries to close the output gap at the same time.

Our unified framework allowed us to derive these “micro” features of optimal interventions and at the same time to analyze the macroeconomic motives for interventions. We found that interventions lean against the wind after global interest rate shocks—either for a terms-of-trade manipulation motive or a “output gap stabilization” motive, and serve a market-making role after large commodity shocks. In addition, since our framework is embedded in a standard macroeconomic model, we also used the model to tackle the important question about the degree to which intervention policies should be coordinated across countries. We made the point that coordination is essential to avoid wasteful competitive devaluations and reserve over-accumulation. Such reserve over-accumulation was shown to have important amplification effects on the fall of the world interest rate, hurting advanced economies. As a result, committing to replicate a world with free capital mobility led to a strict Pareto improvement over the Nash equilibrium.

We believe there are several avenues for future research. Using a richer model with a realistic calibration seems necessary for a more serious quantification of the importance of the channels stressed in this paper. For example, one may add the friction of limit capital mobility to a medium-scale version of a standard New Keynesian dynamic stochastic general equilibrium model and estimate it. In addition, it may be interesting to use such a structural model to back out an estimation of the foreign exchange intervention reaction function and compare it to the model-predicted optimal policy.

In addition, there is still much progress to be made even from a purely theoretical side. For example, our paper assumes that there is Ricardian Equivalence between the central bank and domestic households. Yet this is certainly a strong assumption. More realistically, in a model with heterogeneous households and/or firms one could imagine that interventions have important redistributive effects that could both amplify or mitigate the effectiveness of interventions.

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Appendix

A Data used in Section 2

A.1 Time series of reserve holdings

The two plots in Figure 1 show data from the 2011 revision of **Lane and Milesi-Ferretti (2007)**. On the left, we aggregate paths for reserves over world GDP by the identifier “income_class”, which categorizes countries into low, middle, and high income. On the left, we show the shares of world GDP by “income_class”.

A.2 Time series and scatter plots of reserve flows

To construct Figure 2, we use IMF's quarterly Balance of Payments statistics from 1990:1 to 2008:4 and restrict the sample to only include emerging markets, that is, countries with "income_group" identifier of 2 or 3. We also keep Israel and South Korea since they only recently became recognized as advanced economies. The final sample is an unbalanced panel of 50 emerging markets. We focus on three variables: quarterly reserve flows (FX reserve flows "BFRAFX"), quarterly private capital flows (net financial account minus reserve assets, "BF" minus "BFRA"), (annualized) trend GDP.

For Figure 2(a), we then aggregate reserve flows and private capital flows across countries and plot their ratios with trend GDP. For Figure 2(b) we compute ratios over (trend) GDP by country and plot the standard deviations of reserve flows over GDP vs. the standard deviations of private capital flows over GDP.

A.3 Reserve flows and UIP wedges

To construct Table 1, we first create a quarterly version of the UIP wedge data from Lustig et al. (2011) by summing their monthly excess return measure over the months of each quarter. We also use their data on interest rate differentials, averaged for each quarter. This data is then merged the IMF's quarterly Balance of Payments statistics (this time across all countries and available times). We use the same measure of reserve flows over trend GDP as described in Appendix A.2.

B Proofs for Section 3

B.1 Implementability conditions

This section proves Proposition 1. It requires two directions. We start by showing that (17a) and (17b) are necessarily satisfied if $\{c_t, q_t, r_t\}$ belong to a competitive equilibrium with interest rate shocks $\{r_t^*\}$. The paragraph below Definition 1 already showed that the flow version (15) of the present value budget constraint (17b) holds along a competitive equilibrium. (17a) follows directly from the Euler equation (2) and the definitions of θ_t and τ_t .

Now, consider the reverse direction: Given paths $\{\theta_t, \tau_t\}$, $\{r_t^*, y_{Ht}\}$, and an initial net foreign asset position nfa_0 that satisfy (17a) and (17b), can we always find a competitive equilibrium consisting of initial debt positions $(b_{H0}, b_{I0}, b_{I0}^H, b_{G0}, b_{G0}^*)$, a central bank FXI policy $\{b_{Gt}, b_{Gt}^*, t_t\}$ and an allocation $\{c_t, c_{Ht}, c_{Ft}, c_{Ht}^*, b_{Ht}, b_{It}, b_{It}^H\}$ with prices $\{q_t, r_t\}$ such that (1)–(11) hold?

We first construct the equilibrium objects and then check optimality conditions. We can take the initial debt positions to be $b_{H0} = nfa_0$. Moreover, we define for any $T > 0$

$$b_{HT} = \int_T^\infty e^{-\int_T^t r_s^* ds} \left[\alpha(\theta_t - 1) + \frac{1}{\Gamma_F} \tau_t^2 \right] dt, \quad (34)$$

and thus construct $b_{It} = \frac{1}{\Gamma_F} \tau_t$, $b_{It}^H = \frac{1}{\Gamma_H} \tau_t$, $b_{Gt}^* = -b_{Gt} = b_{Ht} + b_{It} + b_{It}^H$, and $\pi_t = b_{It}^H(r_t - r_t^*)$ for each $t \geq 0$. Transfers are defined to be $t_t = r_t b_{Gt} + r_t^* b_{Gt}^*$. We let the real exchange rate be defined by $q_t^{-1/(1-\alpha)} y_{Ht} = (1-\alpha)\theta_t + \alpha$ and let consumption paths be $c_{Ft} = \alpha\theta_t$ and $c_{Ht} = (1-\alpha)q_t^{\alpha/(1-\alpha)} c_t$ and $c_{Ht}^* = \alpha q_t^{1/(1-\alpha)}$. This concludes our construction of a candidate equilibrium. We move on to checking the equilibrium conditions.

The Euler equation (2) is equivalent to (17a). Equations (3), (4), (5), (6), (7), (8), (9), and (11) hold by construction. It is straightforward to check that the home good market clears—that is,

equation (10) holds—given our definitions for c_{Ht} and c_{Ht}^* . Finally, reversing the steps in equations (12)–(15) shows that the differential (flow) version of the (34) (which is exactly (15)) implies the budget constraint (1).

B.2 Simplifying the planner's objective

In this section we derive the simplified objective function used in (18) from the original per period utility $\log c_t$. Using (12), we can express

$$c_t = q_t \theta_t = \left(\frac{y_{Ht}}{(1 - \alpha)\theta_t + \alpha} \right)^{1-\alpha} \theta_t$$

which then yields a per period utility of

$$\log c_t = \log \theta_t - (1 - \alpha) \log ((1 - \alpha)\theta_t + \alpha) + (1 - \alpha) \log y_{Ht}.$$

The y_{Ht} term in this expression is exogenous so it is without loss for our planning problem to drop it. This gives us the objective in (18).

B.3 First best

Here, we prove Lemma 1 that characterizes the first best allocation, that is, the optimal allocation when the Euler implementability condition (17a) does not bind. In that case the first order conditions with respect to θ_t and τ_t of the planning problem (18) read

$$\tau_t = 0 \quad \text{and} \quad e^{-\rho t} V'(\theta_t) = e^{-\int_0^t r_s^* ds} \lambda \alpha,$$

using the notation $V(\theta)$ for the planner's per period objective, as described in Lemma 1, and calling λ the multiplier on the resource constraint (17b). Due to the Inada conditions of this problem, the first order conditions are necessary. This proves Lemma 1.

C Proofs for Section 4

C.1 Model without macro stabilization motive

Here, we provide a proof to Proposition 2. Consider first the case where $\alpha = 1$. Let $\{\theta_t\}, \lambda$ be the first best path of dollar consumption and the corresponding first best shadow value of resources, respectively. We now show that the (17a) does not bind, that is, it is satisfied for the first best θ_t with $\tau_t = 0$. Lemma 1 describes θ_t as the solution to the first order condition

$$e^{-\rho t} V'(\theta_t) = e^{-\int_0^t r_s^* ds} \lambda \alpha, \tag{35}$$

where, with $\alpha = 1$, $V'(\theta_t) = 1/\theta_t$. Log-differentiating this first order condition yields $\dot{\theta}_t/\theta_t = r_t^* - \rho$ which is exactly what we needed to show.

Now consider the case where α is allowed to be less than 1, but $r_t^* = \rho$. Again, let $\{\theta_t\}, \lambda$ be as in the first best allocation, satisfying (35). Since r_t^* is constant, this implies that θ_t is constant too, trivially satisfying the Euler equation $\dot{\theta}_t/\theta_t = r_t^* - \rho = 0$. This concludes our proof of Proposition 2.

C.2 Financial autarky, $\Gamma_F = \infty$

Before we prove Proposition 3, we first derive very generally, for any $\Gamma_F > 0$, the (necessary) first order conditions to the planning problem (18) in the following lemma, a version of Lemma 2.

Lemma 3. Suppose $\Gamma_F \in (0, \infty)$. Let $V(\theta)$ be the planner's per period objective, as defined before Lemma 1, and let $\tilde{T}_t(\lambda, \theta_t)$ be defined by⁵²

$$\tilde{T}_t(\lambda, \theta_t) = \lambda - e^{\int_0^t (r_u^* - \rho) du} V'(\theta_t).$$

Then, under the optimal foreign exchange intervention policy, the interest rate spread τ_t and the (dollar) consumption θ_t satisfy the following first order condition

$$\dot{\tau}_t = r_t^* \tau_t + \frac{\Gamma_F}{2\lambda} \tilde{T}_t(\lambda, \theta_t) \quad (36a)$$

$$\frac{\dot{\theta}_t}{\theta_t} = r_t^* - \rho + \tau_t \quad (36b)$$

where the derivative of τ_t exists whenever r_t^* does not jump. When $\Gamma_F = \infty$, $\tilde{T}_t(\lambda, \theta_t) = 0$, instead of (36a).

Proof. The current value Hamiltonian of the planning problem (18) with $\Gamma_F \in (0, \infty)$ is given by

$$H(\theta, \tau, \lambda, \mu, t) = e^{-\int_0^t (\rho - r_s^*) ds} V(\theta) - \lambda \alpha(\theta - 1) - \lambda \frac{1}{\Gamma_F} \tau^2 + \mu(r^* + \tau - \rho).$$

This is an optimal control problem with a subsidiary condition, as in Gelfand and Fomin (1963). The state variable is θ and has a free initial value θ_0 . θ has costate μ . As before, λ is the multiplier on the resource constraint (17b), and τ is the control variable. Notice that when $\lambda > 0$, this is a strictly concave problem with a unique maximizer $\{\theta_t, \tau_t\}$ and unique multipliers $\{\mu_t\}, \lambda$, satisfying the first order conditions. And indeed λ has to be positive, or else shadow cost to scaling θ_t up indefinitely (which would not violate the Euler equation (17a)) is zero or even negative, which is not resource feasible according to (17b).⁵³

The first order condition for τ is simply $\mu_t = \lambda \frac{2}{\Gamma_F} \tau_t$, and the costate equation for θ is $\dot{\mu}_t = r_t^* \mu_t + \tilde{T}_t(\lambda, \theta_t)$. Putting these two equations together yields (36a). Equation (36b) is just the Euler equation (17a).

When $\Gamma_F = \infty$, the Hamiltonian is

$$H(\theta, \tau, \lambda, \mu, t) = e^{-\int_0^t (\rho - r_s^*) ds} V(\theta) - \lambda \alpha(\theta - 1) + \mu(r^* + \tau - \rho)$$

implying that the costate μ_t is equal to zero at all times, $\mu_t = 0$ and hence that $\tilde{T}_t(\lambda, \theta_t) = 0$. $\square$

With Lemma 3 under the belt, we now approach Proposition 3. For concreteness, we prove the proposition for positive interest rate shocks, that is, we assume that there is some $T > 0$ such that $r_t^* > \rho$ for all $t \in [0, T]$ and $r_t^* = \rho$ thereafter. We show that, at the optimum, $\tau_t < 0$ for all $t \in [0, T]$ and $\tau_t = 0$ for $t \geq T$, with a jump at $t = T$.

⁵² $\tilde{T}_t$ is the current value equivalent of $\mathcal{T}_t$, defined before Lemma 2.

⁵³It is also possible to prove directly that the constraint set of the planning problem is convex, analogous to the steps in Appendix F.1.

Define as an auxiliary object, $\hat{\theta}_t \equiv e^{\int_0^t (\rho - r_u^*) du} \theta_t$. The necessary conditions now become $\dot{\hat{\theta}}_t / \hat{\theta}_t = \tau_t$ and

$$0 = \tilde{\mathcal{T}}_t(\lambda, e^{\int_0^t (r_u^* - \rho) du} \hat{\theta}_t) = \lambda - \frac{1}{\hat{\theta}_t} + \frac{(1 - \alpha)^2}{(1 - \alpha)\hat{\theta}_t + \alpha e^{\int_0^t (\rho - r_u^*) du}},$$

which can be simplified to

$$\frac{\lambda}{\alpha} \hat{\theta}_t = 1 + \frac{1}{\hat{\theta}_t e^{\int_0^t (r_u^* - \rho) du} + \frac{\alpha}{1 - \alpha}}. \quad (37)$$

For any $t < T$, this equation has a unique solution for $\hat{\theta}_t$ that is strictly decreasing in t . Therefore, $\tau_t < 0$ for $t < T$. For $t \geq T$, there is no explicit time dependence in (37) and $\hat{\theta}_t$ is constant, which means $\tau_t = 0$ for $t \geq T$. At $t = T$, τ_t jumps since the only time varying element of (37), $\int_0^t (\rho - r_u^*) du$, has a kink at $t = T$. This concludes our proof of Proposition 3.

C.3 Intervention smoothing

Lemma 2 is a special case of Lemma 3 in the previous subsection. In particular, (19) is an integral version of (36a). Moreover, as explained in the text below Lemma 2, Proposition 4 is a direct consequence of Lemma 2.

C.4 Forward guidance

This section contains the proof of Proposition 4.3.3. As in Section C.2, we will only provide a proof for positive interest rate shocks $\{r_t^*\}$. The case of negative interest rate shocks is analogous. We start with the first order conditions from Lemma 3, (36a) and (36b), for the case where $\Gamma_F \in (0, \infty)$. Again, as in Section C.2, we work in terms of $\hat{\theta}_t$, leading to the first order conditions

$$\begin{aligned} \dot{\tau}_t &= r_t^* \tau_t + \frac{\Gamma_F}{2\lambda} \tilde{\mathcal{T}}_t(\lambda, e^{\int_0^t (r_u^* - \rho) du} \hat{\theta}_t) \\ \dot{\hat{\theta}}_t &= \hat{\theta}_t \tau_t. \end{aligned}$$

This is a 2-dimension system of ODEs, with initial condition $\tau_0 = 0$ and the terminal condition that $(\tau_t, \hat{\theta}_t)$ converge to the unique steady state given by $\tau^{ss} = 0$ and

$$\tilde{\mathcal{T}}_T(\lambda, e^{\int_0^T (r_u^* - \rho) du} \hat{\theta}^{ss}) = 0.$$

Since the system is stationary after $t = T$, the state at $t = T$, $(\tau_T, \hat{\theta}_T)$, has to lie on the stationary system's stable arm. Figure 10 illustrates the phase diagram and its stable arm. The green line that then merges into the red line depicts the shape of the optimal trajectory that we are trying to determine mathematically.

To do this, it turns out to be helpful to define the path $\{\hat{\theta}_t^\infty\}$ as the solution to $\tilde{\mathcal{T}}_t(\lambda, e^{\int_0^t (r_u^* - \rho) du} \hat{\theta}_t^\infty) = 0$, for all $t \geq 0$.⁵⁴ In a first step, we show that it can never be that $\tau_t \geq 0$ and $\hat{\theta}_t > \hat{\theta}_t^\infty$ for any $t > 0$. In Figure 10, this would be a state $(\tau_t, \hat{\theta}_t)$ that lies to the top right of the time- t τ -locus. In such a case, for any $s > t$, both $\hat{\theta}_s$ and τ_s are positive and bounded away from zero, and hence the state

⁵⁴We call this $\hat{\theta}_t^\infty$ since, given a certain λ , it corresponds to the optimal path for $\hat{\theta}_t$ when $\Gamma_F = \infty$. This is also why $\hat{\theta}_t^\infty$ is well-defined, continuous and piece-wise differentiable for each t .

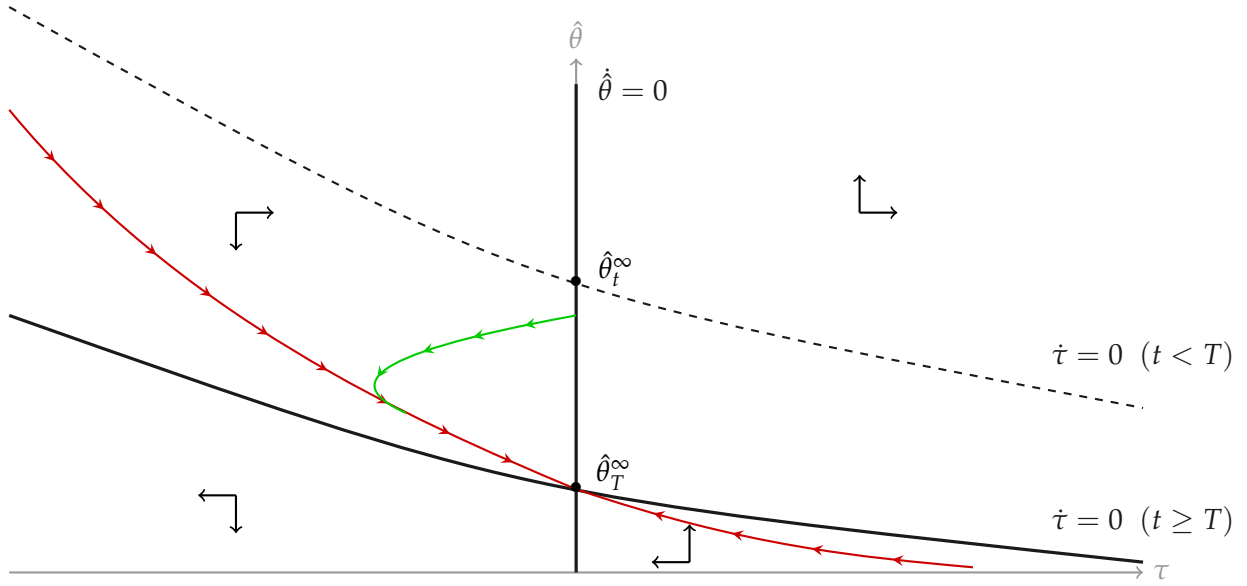


Figure 10: Describing the optimal policy in the state space for (τ, θ) .

$(\hat{\theta}_t, \tau_t)$ would diverge to ∞ . Translating the divergence back to θ_t , this would mean that the growth rate of θ_t , $\dot{\theta}_t/\theta_t = r_t^* - \rho + \tau_t$, diverges to infinity, violating the resource constraint (17b).

Second, consider the possibility that for some $t > 0$,

$$(\hat{\theta}_t, \tau_t) \in \{(\hat{\theta}, \tau) \mid \tau \geq 0 \text{ and } \hat{\theta} \leq \hat{\theta}_t^\infty\} \equiv \Theta_t.$$

Given $\hat{\theta}_t^\infty$ is decreasing in t , if $(\hat{\theta}_t, \tau_t) \in \Theta_t$, then $(\hat{\theta}_t, \tau_t) \in \Theta_s$ for any $s < t$ as well. In particular $(\hat{\theta}_t, \tau_t) \in \Theta_0$. Given no path satisfying the ODEs can ever enter Θ_0 (that is, Θ_0 is a “source” in the vector field sense), it must hold that $(\hat{\theta}_0, \tau_0) \in \text{int}\Theta_0$ (the interior of Θ_0). This contradicts the fact that $\tau_0 = 0$. Together, these two steps prove that $\tau_t \geq 0$ is impossible for any $t > 0$. This concludes the proof of the proposition.

C.5 Time inconsistency

This section proves the results claimed in Proposition 6. When there is re-optimization at any time $t_0 \geq T$, there is no more interest rate shock to r^* after t_0 . Thus, Proposition 2 applies to this re-optimization problem, making it optimal to set $\tau_t = 0$ for all $t \geq t_0$.

Consider now the case of no commitment at all. For ease of notation, call b_t Home’s net foreign asset position nfa_t and let $v(t, b_t)$ be the no-commitment planner’s current time- t value function with current net foreign asset position b_t . The Hamilton-Jacobi-Bellman equation for the time- t planner is then

$$r_t^* v(t, b) = \max_{\tau, \theta} V(\theta) + v_t(t, b) + v_b(t, b) \left(\alpha(1 - \theta) + r_t^* b - \frac{1}{\Gamma_F} \tau^2 \right).$$

The first order condition for τ immediately yields the desired result, $\tau_t = 0$.

D Proofs for Section 5

D.1 Fixed exchange rate economy

In this section, we list the results we proved in the home-bias economy and how their proof can be modified to also apply to the fixed exchange rate economy. To do this, define the per period objective as follows,

$$V(\theta) \equiv \log \theta_t - (1 - \alpha) \log((1 - \alpha)\theta_t + \alpha c^*) - v((1 - \alpha)\theta_t + \alpha c^*).$$

Further, we assume that the multiplier on the resource constraint is positive, $\lambda > 0$.⁵⁵ Lemma 1 and Propositions 2—6 go through word by word as in the baseline model, given our assumption of a positive multiplier λ .

E Additional extensions

E.1 Lucas-Stokey and long term bonds

In a seminal contribution, Lucas and Stokey (1983) show how a planner without commitment power can achieve the full-commitment solution by essentially implementing a specific *composition* of household asset holdings across certain asset classes. Even though the asset classes are linearly dependent and hence redundant, they react different to policy changes, which lets a planner today select asset classes that would “punish” future planners upon deviations.

In our setup, we allow all agents—households, intermediaries and the central bank—to trade Arrow securities in addition to the existing short-term bonds. As in Lucas and Stokey (1983) the central bank can now control its asset composition across all asset classes. Crucially, however, this will *not* pin down the country’s asset composition since households are free to trade with intermediaries in whatever asset classes they like. Therefore, in our model, the central bank cannot implement a specific country’s asset composition. In this section, we sketch out a natural version of our economy with multiple types of bonds and show that they do not help the planner overcome his time consistency problem. As in the main body of our paper, we restrict our attention to the case where domestic bonds are also measured in dollars. This lets us focus on our novel type of time inconsistency, different from the standard exchange rate based time inconsistency.⁵⁶

Let $\pi_{t,s}^*$ denote the state price density for an international dollar payment at time s , measured at time t . That is, if a country promises a stream of payments $x_{t,s}$ in the foreign bond market at time t , and has no other external assets or liabilities, its time- t net foreign asset position is $\text{nfa}_t = \int_t^\infty \pi_{t,s}^* x_{t,s} ds$. Similarly, $\pi_{t,s}$ denotes the state price density for a promised payment in the domestic bond market at time s , measured at time t . Note that $\pi_{t,s}^* = e^{\int_t^s r_u^* du}$ and $\pi_{t,s} = e^{\int_t^s r_u du}$. To see the link to our previous short-term bonds, a single outstanding domestic short-term bond promises payments $x_{t,s} = r_s$ so that its value is exactly 1.

We allow all domestic and foreign bond market participants to trade all Arrow securities with each other, including short-term bonds. We choose the following notation: For each possible short

⁵⁵Compared to our baseline model, the per period objective $V(\theta)$ is no longer monotonically increasing in the fixed exchange rate economy. This means, under certain, arguably contrived, circumstances, it can occur that more resources hurt the economy. To focus on the “normal” case, we assume that $\lambda > 0$, which is always the case if the fixed exchange rate $\bar{e}$ is sufficiently low or the initial labor wedge sufficiently close to zero.

⁵⁶Here, we mean the tendency of a re-optimizing planner with external local currency liabilities to depreciate the currency.

term bond position in our previous notation $X_t = b_{Ht}, b_{It}, b_{It}^H, b_{Gt}, b_{Gt}^*$ we allow positions in Arrow securities denoted by $b_{Ht,s}, b_{It,s}, b_{It,s}^H, b_{Gt,s}, b_{Gt,s}^*$. In this notation, domestic bond market clearing requires that

$$b_{Ht,s} + b_{It,s} + b_{It,s}^H + b_{Gt,s} = 0$$

in addition to the short term bond market clearing condition (11). The main effect of the different Arrow securities in our model is that it creates an indeterminacy in intermediaries and households' positions. Using a natural extension of the intermediaries' maximization problem to this setup gives intermediary demand functions

$$b_{It} + \int_t^\infty \pi_{t,s} b_{It,s} ds = \frac{1}{\Gamma_F} (r_t - r_t^*) \quad \text{and} \quad b_{It}^H + \int_t^\infty \pi_{t,s} b_{It,s}^H ds = \frac{1}{\Gamma_H} (r_t - r_t^*).$$

Evidently, the demand functions do not pin down intermediary positions uniquely.

In this economy, the country's time t implementability conditions are given by (17a) and

$$\int_t^\infty e^{-\int_0^s r_u^* du} \left[\alpha(\theta_s - 1) + \frac{1}{\Gamma_F} \tau_s^2 \right] ds = \text{nfa}_t,$$

where the net foreign asset position is now given by

$$\text{nfa}_t = b_{Ht} + b_{Gt}^* + b_{Gt} + \int_t^\infty \pi_{t,s} (b_{Ht,s} + b_{Gt,s}) ds + \int_t^\infty \pi_{t,s}^* b_{Gt,s}^* ds.$$

Without loss of generality due to Ricardian equivalence between the central bank and domestic households, we let the central bank choose foreign and domestic bond portfolios whose values sum to zero. Then,

$$\text{nfa}_t = b_{Ht} + \int_t^\infty \pi_{t,s} b_{Ht,s} ds.$$

Here, $\pi_{t,s}$ is affected by central bank policies. So in order to fix a time inconsistency problem, the central bank would have to be able to control the positions $\{b_{Ht}, b_{Ht,s}\}$. Yet, for any change in the composition of the central bank position $\{b_{Gt}, b_{Gt,s}\}$ (leaving the total value of the position unchanged), there exist changes in intermediary portfolio compositions such that the household positions $\{b_{Ht}, b_{Ht,s}\}$ as well as the equilibrium quantities and prices $\{c_t\}$ and $\{q_t, r_t\}$ do not change. This makes it impossible for the central bank to commit themselves to future interventions through this mechanism.

E.2 Nonlinear intermediary demands

In our baseline model, intermediaries' demand functions for domestic bonds are linear functions of the UIP wedge τ_t . In this extensions, we explore the implications of nonlinear demand schedules

$$b_{It} = g^F(\tau_t) \quad \text{and} \quad b_{It}^H = g^H(\tau_t) \tag{38}$$

where g^F, g^H are strictly increasing and differentiable functions defined for all reals, with $g^k(0) = 0$ and $g^{k'}(0) = \frac{1}{\Gamma_k}$, for $k = F, H$. Such general demand schedules can be microfounded the same way we microfounded the linear demand schedules in Section 3.1, just with a more general transaction function: one where $f(j)$ is strictly increasing and continuous, with $f(0) = 0$. Using the nonlinear schedules (38), instead of the linear demands (5) and (7), Definition 1 defines the right notion of competitive equilibrium.

The key difference to the linear demand model is that now costs are no longer exactly quadratic, rather only locally so, given by

$$b_{It}\tau_t = g^F(\tau_t)\tau_t.$$

The new resource constraint implementability condition is then given by

$$\int_0^\infty e^{-\int_0^t r_s^* ds} [\alpha(\theta_t - 1) + \tau_t g^F(\tau_t)] dt = nfa_0.$$

To ease notation in this subsection, we introduce $G(\tau) \equiv g^F(\tau)\tau$. To ensure the associated planning problem with such an implementability condition is well defined, we further assume that $G(\tau) = \tau g^F(\tau)$ is strictly convex in τ . Since $\tau g^F(\tau)$ is always locally convex around $\tau = 0$, this essentially restricts $g^F(\tau)$ to not become too flat for large positive or negative values of τ .

Next, we argue that under these conditions, despite nonlinear demand functions, our key results remain true. Lemma 1 and Proposition 2 and their proofs go through without any changes. Lemma 3 in the appendix still holds when (36a) is replaced by

$$\dot{\tau}_t = r_t^* \frac{G'(\tau_t)}{G''(\tau_t)} + \frac{1}{G''(\tau_t)} \tilde{\mathcal{T}}_t(\lambda, \theta_t). \quad (39)$$

Proposition 3 trivially holds as well since it treats the special case where $\Gamma_F = \infty$, that is translated into this notation, $g^F(\tau) = 0$ for all τ . The first order condition in Lemma 2 is still the integral equivalent of (39) and reads

$$e^{-\int_0^t r_s^* ds} \lambda G'(\tau_t) = \int_0^t \mathcal{T}_s ds,$$

with the exact same interpretation as in Lemma 2. Based on this lemma, Proposition 4 carries over as well. We speculate that Proposition 5 goes through unchanged (there is no intuitive reason why not) but we have not worked out the formal proof. The partial commitment part of Proposition 6 holds conditional on Proposition 5 being correct, while the no-commitment part can be proven using the same argument as before.

F Proofs for Section 6

F.1 Convexity of the planning problem

To show that planning problem (32) is well-behaved, first note that the objective is strictly concave and strictly increasing in (θ_0, θ_1) as before. Therefore, if the constraint set were bounded and convex, the unique maximizer would necessarily lie on the constraint set's boundary. We now prove that the inequality version of (33),

$$B(\theta_0, \theta_1) \equiv (\alpha\theta_0 - C_{EME0}) + \frac{1}{1+r^*}(\alpha\theta_1 - C_{EME1}) + \frac{1}{\Gamma_F} \left(\frac{\theta_1(1+\rho_{EME})}{\theta_0(1+r^*)} - 1 \right)^2 \leq 0.$$

is indeed bounded and convex, thus validating our relaxation.

Boundedness is straightforward as the intermediary cost term $1/\Gamma_F(\dots)^2$ is always bounded from below by zero. For convexity, consider two points $\theta = (\theta_0, \theta_1)$ and $\theta' = (\theta'_0, \theta'_1)$ in $\mathbb{R}_{++}^2$, and choose $\lambda \in [0, 1]$. Define $\theta^\lambda = \lambda\theta + (1-\lambda)\theta'$. We now prove that $\lambda \mapsto B(\theta^\lambda)$ is a quasi-convex function, implying that $B(\theta^\lambda) \leq \max\{B(\theta), B(\theta')\} = 0$ as desired.

$B(\theta^\lambda)$ consists of two linear terms and a third, non-linear term. The nonlinear term is

$$\frac{1}{\Gamma_F} \left(\frac{(1 + \rho_{EME})}{(1 + r^*)} \frac{\lambda \theta_1 + (1 - \lambda) \theta'_1}{\lambda \theta_0 + (1 - \lambda) \theta'_0} - 1 \right)^2$$

which is a composition of a convex function $f(z) = \frac{1}{\Gamma_F} \left(\frac{(1 + \rho_{EME})}{(1 + r^*)} z - 1 \right)^2$ and a function $g(\lambda) = \frac{\lambda \theta_1 + (1 - \lambda) \theta'_1}{\lambda \theta_0 + (1 - \lambda) \theta'_0}$. Here, g is either strictly monotone or constant. Either way, the composition $f(g(\lambda))$ is quasiconvex, implying the result. Therefore, the constraint set $B(\theta_0, \theta_1) \leq 0$ is convex.

F.2 Characterizing the Nash equilibrium

F.2.1 Part 1

This subsection provides a proof for Proposition 7. We start with part 1 and rewrite the planning problem in a slightly more convenient form, by defining $\kappa_t = \frac{(1 + \rho_{EME})^t}{(1 + r^*)^t}$, $\hat{\theta}_t \equiv \kappa_t \theta_t$, and $\hat{C}_{EMEt} \equiv \kappa_t C_{EMEt}$. We also abbreviate $\rho = \rho_{EME}$ and $\hat{C} = \hat{C}_{EME}$ and let the per period objective be defined as

$$V(\hat{\theta}, \hat{C}) \equiv \log \hat{\theta} - (1 - \alpha) \log((1 - \alpha) \hat{\theta} + \hat{C})$$

and the constraint function be defined as

$$B(\hat{\theta}_0, \hat{\theta}_1, \hat{C}_0, \hat{C}_1) \equiv \sum_{t=0}^1 (1 + \rho)^{-t} (\alpha \hat{\theta}_t - C_t) + \frac{1}{\Gamma_F} \left(\frac{\hat{\theta}_1}{\hat{\theta}_0} - 1 \right)^2.$$

As before, $\hat{\theta}$ and $\hat{C}$ are complements in the utility function, that is $V_{\hat{\theta}, \hat{C}} > 0$. Using this notation, (32) becomes

$$\max \sum_{t=0}^1 (1 + \rho)^{-t} \{ V(\hat{\theta}_t, \hat{C}_t) - \alpha \log \kappa_t \} \quad (40)$$

subject to

$$B(\hat{\theta}_0, \hat{\theta}_1, \hat{C}_0, \hat{C}_1) \leq 0,$$

where we relaxed the constraint as inequality, applying the result from Appendix F.1. In that section we also mentioned that (for any $\hat{C}_t > 0$) the optimum choice for $(\hat{\theta}_0, \hat{\theta}_1)$ lies on a downward sloping, convex arc of the constraint set. On that arc, it holds that the marginal rate of substitution, $MRS(\hat{\theta}_0, \hat{\theta}_1) \equiv \frac{B_{\hat{\theta}_0}}{B_{\hat{\theta}_1}}$ is weakly increasing in $\hat{\theta}_0$ and weakly decreasing in $\hat{\theta}_1$. Notice that $MRS(\hat{\theta}_0, \hat{\theta}_1)$ is independent of $\{\hat{C}_t\}$. In a (pure strategy) Nash equilibrium, the demands $\hat{C}_t$ are then determined by

$$\hat{C}_t = \kappa_t - (1 - \alpha) \hat{\theta}_t \quad (41)$$

and the equilibrium interest rate is pinned down by AEs' discount rate, $r^* = \rho_{AE}$ since $\chi = 0$. As a third equilibrium condition, we need the resource constraint to hold, which, having substituted in (41) becomes

$$\bar{B}(\hat{\theta}_0, \hat{\theta}_1) \equiv \sum_{t=0}^1 (1 + \rho)^{-t} (\hat{\theta}_t - \kappa_t) + \frac{1}{\Gamma_F} \left(\frac{\hat{\theta}_1}{\hat{\theta}_0} - 1 \right)^2 \leq 0. \quad (42)$$

The Euler equation of (40) reads

$$V_{\hat{\theta}}(\hat{\theta}_0, \hat{C}_0) = MRS(\hat{\theta}_0, \hat{\theta}_1) V_{\hat{\theta}}(\hat{\theta}_1, \hat{C}_1)$$

and substituting in the Nash equilibrium condition (41) we find

$$V_{\hat{\theta}}(\hat{\theta}_0, 1 - (1 - \alpha)\hat{\theta}_0) = MRS(\hat{\theta}_0, \hat{\theta}_1) V_{\hat{\theta}}(\hat{\theta}_1, \kappa_1 - (1 - \alpha)\hat{\theta}_1). \quad (43)$$

Using the properties of V and MRS mentioned above, it is immediate that the left hand side of this equation strictly decreases in $\hat{\theta}_0$, while the right hand side increases in $\hat{\theta}_0$ and falls with $\hat{\theta}_1$. This implicit equation thus describes a strictly increasing, continuous function $\hat{\theta}_1 = h(\hat{\theta}_0)$, defined for any $\hat{\theta}_0 > 0$. Notice that the solution to (43) must necessarily generate a positive $MRS(\hat{\theta}_0, h(\hat{\theta}_0))$, and hence positive derivatives $B_{\hat{\theta}_0}$ and $B_{\hat{\theta}_1}$.⁵⁷ It is easy to see that the positivity of the two derivatives immediately implies that $h(\hat{\theta}_0)/\hat{\theta}_0 \rightarrow 1$ as $\hat{\theta}_0 \rightarrow 0$. A Nash equilibrium can be found as a solution to (42) with $\hat{\theta}_1 = h(\hat{\theta}_0)$ substituted in.

Before we show existence and uniqueness of such a solution, we establish a few helpful auxiliary results. First, note that $\bar{B}(\hat{\theta}_0, h(\hat{\theta}_0))$ is strictly increasing in $\hat{\theta}_0$. Suppose it were not: Then there has to be a point $(\hat{\theta}_0, h(\hat{\theta}_0))$ where either $\bar{B}_{\hat{\theta}_0}$ (which is larger than $B_{\hat{\theta}_0}$) or $\bar{B}_{\hat{\theta}_1}$ (which is larger than $B_{\hat{\theta}_1}$) is negative, contradicting the positivity of $MRS(\hat{\theta}_0, h(\hat{\theta}_0))$. Second, note that $\bar{B}(\hat{\theta}_0, h(\hat{\theta}_0))$ approaches ∞ as $\hat{\theta}_0 \rightarrow \infty$ and approaches a negative number as $\hat{\theta}_0 \rightarrow 0$. The former is straightforward since the quadratic cost term is bounded below by zero. The latter follows because $h(\hat{\theta}_0)/\hat{\theta}_0 \rightarrow 1$ as $\hat{\theta}_0 \rightarrow 0$, as explained above, and so the cost term vanishes as $\hat{\theta}_0 \rightarrow 0$. The intermediate value theorem then proves that there exists a unique number $\hat{\theta}_0$ such that $\bar{B}(\hat{\theta}_0, h(\hat{\theta}_0)) = 0$. This proves the existence and uniqueness of the symmetric world equilibrium as defined in Definition 2.

F.2.2 Part 2

To show that τ is positive, we now argue that $h(\hat{\theta}_0) > \hat{\theta}_0$ for all $\hat{\theta}_0$. Suppose this were not the case. Then, the right hand side of (43) can be bounded from above by

$$\begin{aligned} MRS(\hat{\theta}_0, \hat{\theta}_1) V_{\hat{\theta}}(\hat{\theta}_1, \kappa_1 - (1 - \alpha)\hat{\theta}_1) &\leq MRS(\hat{\theta}_0, \hat{\theta}_0) V_{\hat{\theta}}(\hat{\theta}_0, \kappa_1 - (1 - \alpha)\hat{\theta}_0) \\ &< V_{\hat{\theta}}(\hat{\theta}_0, 1 - (1 - \alpha)\hat{\theta}_0). \end{aligned}$$

This contradicts (43). Therefore, $h(\hat{\theta}_0) > \hat{\theta}_0$ and hence $\tau > 0$ in the Nash equilibrium.

F.2.3 Part 3

Compared to a no-intervention world, the EME consumption profile $(\hat{\theta}_0, \hat{\theta}_1)$ is tilted more towards the future, and hence EME's net foreign assets are larger after $t = 0$. Positivity of the UIP wedge, $\tau > 0$, however, means that private capital must flow more downstream, implying that the increase in net foreign assets is driven by rise of EME reserves.

F.2.4 Part 4

This follows from Proposition 8 below.

⁵⁷Notice that it can never be the case that both $B_{\hat{\theta}_0}$ and $B_{\hat{\theta}_1}$ are negative since in that case, there would have to exist some positive $\{\hat{C}_t\}$ such that the budget set $B \leq 0$ does not include any points close to zero. Yet, by scaling down any point $(\hat{\theta}_0, \hat{\theta}_1)$ in such a budget set, $(0, 0)$ can always be approximated arbitrarily closely.

F.3 Characterizing the cooperative outcome

This section proves Proposition 8. Utilizing the notation introduced in Appendix F.2 above, the cooperative planning problem can be written as

$$\max \sum_{t=0}^1 (1 + \rho)^{-t} \{ \log \hat{\theta}_t - \alpha \log \kappa_t \}$$

subject to the total EME resource constraint (42) which takes into account the endogeneity of $\{C_{EMEt}\}$. To analyze this problem, we guess and verify that it is without loss to relax the constraint (42) by setting $\Gamma_F = \infty$. In that case, the solution is simply given by

$$\hat{\theta}_0 = \hat{\theta}_1 = \frac{1 + (1 + \rho)\kappa_1}{2 + \rho}.$$

This “relaxed” features that $\hat{\theta}_1/\hat{\theta}_0 = 1$, that is, $\tau = 0$. Therefore, relaxing (42) by setting Γ_F to ∞ was without loss of generality, and we found the solution.