Financial Stability, Monetary Policy and the Payment-Intermediary Share *

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Abstract

The payment intermediary share is the share of fixed income claims held by financial intermediaries with money-like liabilities. It is higher in times of higher risk premia, such as during the 1970s and in recent recessions. This paper proposes a model of a modern monetary economy that accounts for the valuation of fixed income claims as well as their allocation inside vs outside the payment intermediaries. While all assets are valued for their risk and return properties, those held inside payment intermediaries are also valued as collateral that backs inside money. The payment-intermediary share depends on the transactions demand for inside money as well as portfolio responses to uncertainty shocks. It determines the quantitative impact of monetary policy and macro-prudential regulation on asset prices.

1 Introduction

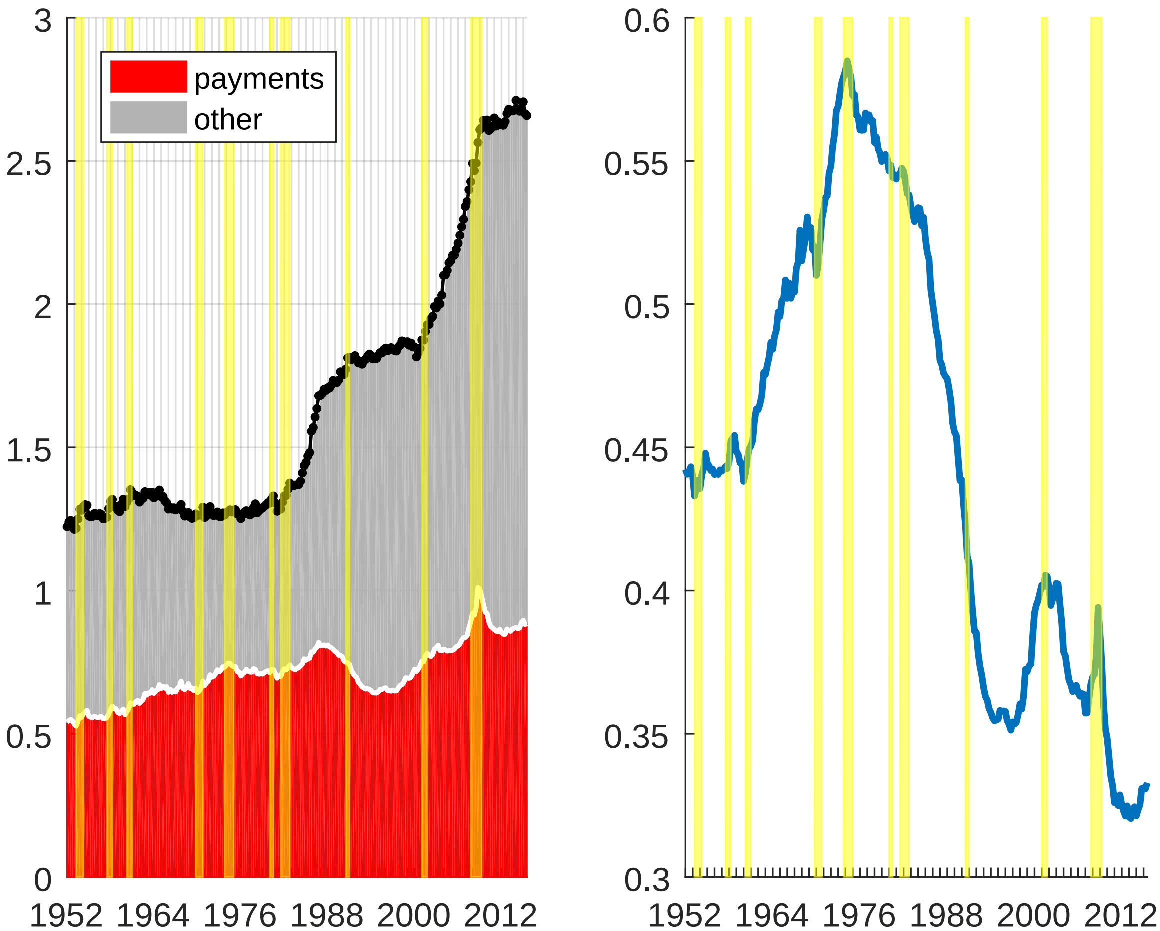
The payment-intermediary share is the share of fixed income claims held by financial intermediaries with money-like liabilities. Figure 1 shows holdings of fixed income instruments for the United States over the postwar period. In the left hand panel, the black line is the total amount of fixed income instruments issued by nonfinancial sectors, measured relative to GDP. The red area is the amount of fixed income instruments held by banks and money market mutual funds. In the right hand panel, we plot the ratio of the red area to the total, the payment intermediary share. It was high in the 1970s and spiked up during financial turmoil preceding recent recessions.

This paper provides a model of the payment intermediary share based on the portfolio choice of a competitive intermediary sector facing financial frictions. Intermediaries buy fixed income

*Preliminary and incomplete.

instruments as collateral that back inside money. The key margin is how many assets society finds optimal to place inside intermediaries, as opposed to holding them directly. This choice depends on the demand for payments, as well as the costs of intermediation and how they interact with the properties of the available collateral assets. Since payments instruments are nominal, it also depends on monetary policy.

Figure 1: Payment intermediary share



2 Model

We study the transmission of monetary policy in a discrete time, infinite horizon endowment economy. Households derive utility from consumption. Comsumption incurs a liquidity cost which is decreasing in the household's deposit holdings. Banks act as investment vehicles that are financed by bank equity and deposits. On the asset side, banks hold output claims and central bank reserves. We consider two tools of monetary policy, permanent changes in the quantity of outstanding reserves and interest payments on banks' reserve holdings.

2.1 Aggregate output and household preferences

Aggregate output Y_t follows the exogenous stochastic process

$$Y_t = G_t Y_{t-1},$$

where G_t is the real growth rate of the economy.

Households have Epstein-Zin preferences over consumption, so utility is recursively defined as

$$U_t = \left[(1-\beta)\, C_t^{1-1/\sigma} + \beta \mathbb{E} \left(U_{t+1}^{1-\gamma} \right)^{\frac{1-1/\sigma}{1-\gamma}} \right]^{\frac{1}{1-1/\sigma}}.$$

Aggregate output is non-storable, meaning that, in equilibrium, aggregate consumption C_t equals aggregate output.

2.1.1 Household problem

Households choose consumption and asset holdings to maximize utility. Each household is endowed with one claim to the aggregate endowment stream Y_t. He can trade two kinds of assets, deposits D_t and financial securities A_t^j, where $j \in J$ denotes the type of security. Deposits R_{t+1}^D nominal unit in the next period and are purchased for one nominal unit today. The other assets are characterized by their respective nominal dividend stream $\{P_s \delta_s^j\}_{s=t+1}^{\infty}$, and are traded at nominal prices Q_t^j.

Households pay a liquidity charge on their payment needs, which are defined as the sum of consumption and changes in their portfolio value,

$$\Delta_t = P_t C_t + D_t + \sum_j Q_t^j A_t^j - R_t^D D_{t-1} - \sum_j \left(Q_t^j + P_t \delta_t^j \right) A_{t-1}^j - P_t \tau_t \tag{1}$$

The liquidity charge h_t,

$$h_t = h\left(\frac{D_t}{\Delta_t}\right) \quad \text{with} \;\; h' < 0 \text{ and } h'' > 0$$

is a decreasing and convex function of real deposits relative to payment needs. Total liquidity cost are given as the producy of liquidity charge and payment needs.

The optimal consumption and portfolio choice is subject to the budget constraint

$$P_t C_t + \sum_{j=1}^{J} Q_t^j A_t^j + D_t + h\left(\frac{D_t}{\Delta_t}\right)\Delta_t \leq P_t Y_t + \sum_{j=1}^{J}(P_t \delta_t^j + Q_t^j) A_{t-1}^j + R_t^D D_{t-1} + P_t \tau_t \tag{2}$$

where τ_t denotes any real transfer that the household may receive.

This optimization problem yields two Euler equations, one for deposits and one for other assets

$$\begin{aligned} 1 + \tilde{h}'\left(\frac{D_t}{\Delta_t}\right) &= \mathbb{E}_t\left[M_t \frac{P_t}{P_{t+1}} R_t^D\right] \\ 1 &= \mathbb{E}_t\left[M_t \frac{P_t}{P_{t+1}} R_t^j\right] \end{aligned}$$

where

$$M_t = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-1/\sigma}\left(\frac{V_{t+1}}{E_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\sigma}-\gamma}$$

is the standard Epstein-Zin pricing kernel, and

$$\tilde{h}'\left(\frac{D_t}{\Delta_t}\right) = \frac{h'\left(\frac{D_t}{\Delta_t}\right)}{1 + h\left(\frac{D_t}{\Delta_t}\right) - h'\left(\frac{D_t}{\Delta_t}\right)\frac{D_t}{\Delta_t}},$$

is a negative function, which is increasing in its argument. The equations are derived in Appendix A.

2.2 Banks

Deposits are issued by banks, which are shareholder maximizing investment vehicles. They hold output claims and central bank reserves on the asset side of their balance sheet, and finance these assets through the issuance of deposits and bank equity to households. Banks face two different costs of operation. First, banks pay a reserve requirement cost that is larger the smaller the value of reserves is relative the amount of issued deposits. Second, banks pay a leverage cost that is larger the larger the value of assets is relative to banks' equity.

The central bank sells reserves in period t for one nominal unit. Each unit pays R_{t+1}^M nominal units in period $t+1$. The nominal interest rate on reserves is one of the two tools of monetary policy.

The bank faces a deposit cost that is decreasing in its reserve ratio m_t, which is defined as the value of reserves over promised deposit payments

$$m_t = \frac{A_t^M}{R_t^D D_t}$$

When issuing one unit of deposits, the bank only receives a fraction $(1 - f(m_t))$ of the proceeds. The deposit cost $f(m_t)$ is decreasing and convex in m_t and goes to zero as $m_t \to \infty$

The bank also faces a leverage cost, which reduces the payoffs of its asset holdings. This cost $x(\ell_{t+1})$ is increasing in ℓ_{t+1}, which is the liabilities-to-asset ratio when entering $t+1$,

$$\ell_{t+1} = \frac{R_{t+1}^D D_t}{\Pi'_{t+1} A_t},$$

where $\Pi'_t A_t = \sum_{j=1}^J A_t^j (P_{t+1}\delta_t^j + Q_{t+1}^j) + R_t^M A_t^M$ is the nominal payoff of the bank's portfolio. Both cost, $x(\ell_t)$ and $f(m_t)$, are redistributed to the household through transfers τ_t.

2.2.1 Banks' optimization problem

Banks maximize shareholder profits and therefore solve

$$\max_{D_t, A_t^j, A_t^M} E_t \left[M_{t+1} \left(\left(1 - x\left(\ell_{t+1}\right)\right) \frac{\Pi'_t A_t}{P_{t+1}} - \frac{R_t^D D_t}{P_{t+1}} \right) \right] - \left(\frac{Q'_t A_t}{P_t} - \left(1 - f(m_t)\right) \frac{D_t}{P_t} \right). \tag{3}$$

This maximization problem is not well defined, because the scale of the banking sector will only be pinned down in equilibrium through a zero-profit condition that arises from free entry. To make progress, we reformulate the problem in terms of leverage and portfolio shares. We denote the portfolio share of asset j as $\alpha_t^j = \frac{Q_t^j A_t^j}{Q'_t A_t}$ and write $\alpha_t^m = 1 - \sum_j \alpha_t^j$. Furthermore, we define nominal returns $R_{t+1}^j = \frac{P_{t+1}\delta_{t+1}^j + Q_{t+1}^j}{Q_t^j}$ and denote the value of the bank's asset holdings as $Q'_t A_t = \sum_{j=1}^J Q_t^j A_t^j + A_t^M$.

Using the new notation, we rewrite the payoff of the bank's asset holdings as

$$\begin{aligned}
\Pi'_t A_t &= \sum_j A_t^j (P_{t+1}\delta_{t+1}^j + Q_{t+1}^j) + A_t^M \\
&= \sum_j Q_t^j A_t^j R_{t+1}^j + A_t^M R_{t+1}^M \\
&= Q'_t A_t \left(\sum_j \alpha_t^j R_{t+1}^j + \alpha_t^m R_{t+1}^M \right) \\
&= Q'_t A_t R_{t+1}^\alpha
\end{aligned}$$

We define the pre-return liabilities-to-asset ratio as $\bar{\ell}_t = \frac{R^D_{t+1} D_t}{Q'_t A_t}$ and rewrite the reserve ratio as $m_t = \alpha^m_t / \bar{\ell}_t$ and the liabilities to asset ratio as $\ell_{t+1} = \bar{\ell}_t / R^\alpha_{t+1}$.

The optimization problem of the bank can then be rewritten as

$$\max_{D_t, \alpha^j_t, \bar{\ell}_t} \frac{D_t}{P_t} \left(E_t \left[M_{t+1} \frac{P_t}{P_{t+1}} \left(\left(1 - x\left(\bar{\ell}_t / R^\alpha_{t+1}\right)\right) \frac{R^\alpha_{t+1}}{\bar{\ell}_t} - R^D_{t+1} \right) \right] - \left(\frac{1}{\bar{\ell}_t} - \left(1 - f(\alpha^m_t / \bar{\ell}_t)\right) \right) \right)$$

subject to $\alpha^m_t = 1 - \sum_t \alpha^j_t$. This notation shows that the scale of the banking sector, as determined by the amount of produced real balances $\frac{D_t}{P_t}$, is not defined by the optimization problem. In equilibrium, the free-entry condition will pin down real balances and ensure that the bracket equals zero.

The first order conditions with respect to $\frac{1}{\bar{\ell}_t}$ and α^j_t are

$$\begin{aligned} E_t \left[M_{t+1} \frac{P_t}{P_{t+1}} R^\alpha_{t+1} \left(1 - x\left(\frac{\bar{\ell}_t}{R^\alpha_{t+1}} \right) + x'\left(\frac{\bar{\ell}_t}{R^\alpha_{t+1}} \right) \frac{\bar{\ell}_t}{R^\alpha_{t+1}} \right) \right] &= \left(1 + \alpha^m_t f'(\alpha^m_t / \bar{\ell}_t) \right) \\ E_t \left[M_{t+1} \frac{P_t}{P_{t+1}} \left(R^j_{t+1} - R^M_{t+1} \right) \left(1 - x\left(\frac{\bar{\ell}_t}{R^\alpha_{t+1}} \right) + x'\left(\frac{\bar{\ell}_t}{R^\alpha_{t+1}} \right) \frac{\bar{\ell}_t}{R^\alpha_{t+1}} \right) \right] &= -f'(\alpha^m_t / \bar{\ell}_t) \quad \forall j \in J. \end{aligned}$$

An increase in $\frac{1}{\bar{\ell}_t}$ signifies a shift from deposit to equity financing. The left-hand-side of the first equation equalizes the benefits of more equity financing, namely asset returns and collateral benefits, with the additional funding cost, which are however reduced as reserve requirement cost go down. The second equation equalizes payoff benefits of holding relatively more of asset j and less reserves, namely excess returns and collateral effects, with the increase in reserve requirement cost.

Combining the equations we find the bank's Euler equation for non-reserve assets as

$$E_t \left[M_{t+1} \frac{P_t}{P_{t+1}} R^j_{t+1} \left(1 - x\left(\frac{\bar{\ell}_t}{R^\alpha_{t+1}} \right) + x'\left(\frac{\bar{\ell}_t}{R^\alpha_{t+1}} \right) \frac{\bar{\ell}_t}{R^\alpha_{t+1}} \right) \right] = 1,$$

which shows that the bank has its own pricing kernel, given by the household's pricing kernel reduced by leverage cost and increased by collateral benefits.

2.3 The central bank

The central bank issues an aggregate reserve quantity $\bar{A}^m_t$ to the banks. Monetary policy sets the process

$$\bar{A}^m_t = \Gamma_t \bar{A}^m_{t-1},$$

where Γ_t is the growth rate of reserve quantities.

As a second tool of monetary policy, the central bank can also pay interest on reserves which are nominally risk-free. It does so by selling reserves at price Q_t^m. Any budget deficit $A_{t-1}^m - Q_t^m A_t^m$ is financed through lump sum taxes that are part of the household transfers τ_t.

2.4 Equilibrium

Having defined the environment of the economy, the household problem and the banks' maximization problem, we can define the stochastic equilibrium of the economy.

Definition 2.1 (Equilibrium)**.** Given initial A_0^m and Y_0, and given a process of real and nominal growth rates, G_t and Γ_t, and reserve rates R_{t+1}^m, an equilibrium is defined through processes of prices R_{t+1}^D and Q_t^j $\forall j \in J$, as well as processes of the price level P_t and deposit quantities D_t, such that

- households solve their maximization problem (49),
- banks solve their optimization problem (3),
- all asset markets clear,
- banks make zero profits (free entry).

To solve for the equilibrium, we collect the first order conditions of the household, the first order conditions of the bank, as well as the zero profit condition for banks. In equilibrium, the bank holds all issued reserves, such that $A_t^m = \bar{A}_t^m$. Market clearing of deposits can then be written as

$$R_t^D D_t = \bar{\ell}_t \frac{\bar{A}_t^m}{\alpha_t^m},$$

which uses

$$\alpha_t^m Q' A_t = Q_t^m \bar{A}_t^m.$$

For other assets that can be held both by households and the bank, we use subscripts H and B, respectively, and impose $A_{H,t}^j + A_{B,t}^j = 0$.

Collecting all equilibrium conditions yields the following system of equations:

$$\mathbb{E}_t\left[M_t\frac{P_t}{P_{t+1}}R^D_{t+1}\right]=1+\tilde{h}'\left(\frac{D_t}{\Delta_t}\right) \tag{4}$$

$$\mathbb{E}_t\left[M_t\frac{P_t}{P_{t+1}}R^j_{t+1}\right]=1 \tag{5}$$

$$E_t\left[M_{t+1}\frac{P_t}{P_{t+1}}R^\alpha_{t+1}\left(1-x\left(\frac{\bar{\ell}_t}{R^\alpha_{t+1}}\right)+x'\left(\frac{\bar{\ell}_t}{R^\alpha_{t+1}}\right)\frac{\bar{\ell}_t}{R^\alpha_{t+1}}\right)\right]=\left(1+\alpha^m_t f'(\alpha^m_t/\bar{\ell}_t)\right) \tag{6}$$

$$E_t\left[M_{t+1}\frac{P_t}{P_{t+1}}R^j_{t+1}\left(1-x\left(\frac{\bar{\ell}_t}{R^\alpha_{t+1}}\right)+x'\left(\frac{\bar{\ell}_t}{R^\alpha_{t+1}}\right)\frac{\bar{\ell}_t}{R^\alpha_{t+1}}\right)\right]=1 \quad \forall j\in J \tag{7}$$

$$E_t\left[M_{t+1}\frac{P_t}{P_{t+1}}\left(\left(1-x\left(\bar{\ell}_t/R^\alpha_{t+1}\right)\right)\frac{R^\alpha_{t+1}}{\bar{\ell}_t}-R^D_{t+1}\right)\right]=\left(\frac{1}{\bar{\ell}_t}-\left(1-f(\alpha^m_t/\bar{\ell}_t)\right)\right) \tag{8}$$

$$D_tR^D_{t+1}=\bar{\ell}_t\frac{\bar{A}^m_t}{\alpha^m_t} \tag{9}$$

$$\tag{10}$$

2.4.1 Stationarity

Denote with $X_t=\prod_{j=1}^t G_j$ the stochastic trend in consumption, and with $N_t=\prod_{j=1}^t\Gamma_j$ the stochastic trend in the growth of nominal quantities in the economy, such that $Y_t=\bar{Y}X_t$ and $\bar{A}^m_t=N_t\bar{A}^m$. Conjecture that the price level is $P_t=\frac{1}{v_t}N_t/X_t$, where v_t is a stationary measure for the value of money. Furthermore, define $d_t=\frac{D_t}{\Delta_t}$ as the ratio of nominal deposits to nominal payment needs. The system of equilibrium equation can then be written in terms of stationary variables as

$$\mathbb{E}_t\left[M_t\frac{G_t}{\Gamma_t}\frac{v_{t+1}}{v_t}R^D_{t+1}\right]=1+\tilde{h}'\left(d_t\right) \tag{11}$$

$$\mathbb{E}_t\left[M_t\frac{G_t}{\Gamma_t}\frac{v_{t+1}}{v_t}R^j_{t+1}\right]=1 \tag{12}$$

$$E_t\left[M_t\frac{G_t}{\Gamma_t}\frac{v_{t+1}}{v_t}R^\alpha_{t+1}\left(1-x\left(\frac{\bar{\ell}_t}{R^\alpha_{t+1}}\right)+x'\left(\frac{\bar{\ell}_t}{R^\alpha_{t+1}}\right)\frac{\bar{\ell}_t}{R^\alpha_{t+1}}\right)\right]=\left(1+\alpha^m_t f'(\alpha^m_t/\bar{\ell}_t)\right) \tag{13}$$

$$E_t\left[M_t\frac{G_t}{\Gamma_t}\frac{v_{t+1}}{v_t}R^j_{t+1}\left(1-x\left(\frac{\bar{\ell}_t}{R^\alpha_{t+1}}\right)+x'\left(\frac{\bar{\ell}_t}{R^\alpha_{t+1}}\right)\frac{\bar{\ell}_t}{R^\alpha_{t+1}}\right)\right]=1 \quad \forall j\in J \tag{14}$$

$$E_t\left[M_t\frac{G_t}{\Gamma_t}\frac{v_{t+1}}{v_t}\left(\left(1-x\left(\bar{\ell}_t/R^\alpha_{t+1}\right)\right)\frac{R^\alpha_{t+1}}{\bar{\ell}_t}-R^D_{t+1}\right)\right]=\left(\frac{1}{\bar{\ell}_t}-\left(1-f(\alpha^m_t/\bar{\ell}_t)\right)\right) \tag{15}$$

$$\frac{d_t}{1-h(d_t)}\frac{R^D_{t+1}}{v_t}\bar{Y}=\bar{\ell}_t\frac{\bar{A}^m}{\alpha^m_t}, \tag{16}$$

where the last equation uses the household's budget constrain (eq. 2) and the equation for Δ_t (eq. 1), to find that $\Delta_t = \frac{P_t Y_t}{1-h(d_t)}$

3 Steady state analysis

The steady state is defined by

$$\begin{aligned}
\beta \frac{1}{\bar{\Pi}} \bar{R}^D &= 1 + \tilde{h}'\left(\bar{d}\right) \\
\beta \frac{1}{\bar{\Pi}} \bar{R}^j &= 1 \\
\beta \frac{1}{\bar{\Pi}} \bar{R}^M &= 1 + f'(\bar{\alpha}^m/\bar{\ell}) \\
x\left(\frac{\bar{\ell}}{\bar{R}^\alpha}\right) &= x'\left(\frac{\bar{\ell}_t}{\bar{R}^\alpha}\right)\frac{\bar{\ell}_t}{\bar{R}^\alpha} \\
\beta \frac{1}{\bar{\Pi}}\left(\left(1 - x\left(\bar{\ell}/\bar{R}^\alpha\right)\right)\frac{\bar{R}^\alpha}{\bar{\ell}} - \bar{R}^D\right) &= \frac{1}{\bar{\ell}} - \left(1 - f(\bar{\alpha}^m/\bar{\ell})\right) \\
\frac{\bar{d}}{1-h(\bar{d})}\frac{\bar{R}^D}{\bar{v}}\bar{Y} &= \bar{\ell}\frac{\bar{A}^m}{\bar{\alpha}^m},
\end{aligned}$$

where $\bar{\Pi} = \frac{\bar{\Gamma}}{\bar{G}}$. This system has five unknowns α_m, $\bar{\ell}$, $\bar{R}^D$, $\bar{R}^j$ and $\bar{d}$.

The last equation can be rewritten as

$$\beta \frac{1}{\bar{\Pi}} \bar{R}^D - 1 = \; f'(\bar{\alpha}^m/\bar{\ell}_t)\frac{\bar{\alpha}^m}{\bar{\ell}} - f\left(\bar{\alpha}^m/\bar{\ell}\right) - \beta\frac{1}{\bar{\Pi}}x'\left(\bar{\ell}/\bar{R}^\alpha\right) = \; \tilde{h}'\left(\bar{d}\right),$$

which says that the negative excess return of deposis is equal to the sum of liquidity cost and collateral benefits, which again is equal to the households liquidity benefits.

Small return approximation system becomes

$$r^D - \pi = \delta + \tilde{h}'\left(\bar{d}\right) \tag{17}$$

$$r^j - \pi = \delta \tag{18}$$

$$r^M - \pi = \delta + f'(\bar{\alpha}^m/\bar{\ell}) \tag{19}$$

$$r^D - \pi = \delta - f(\bar{\alpha}^m/\bar{\ell}) + f'(\bar{\alpha}^m/\bar{\ell})\frac{\bar{\alpha}^m}{\bar{\ell}} - x'(\tilde{\ell}) \tag{20}$$

$$x\left(\tilde{\ell}\right) = x'\left(\tilde{\ell}\right)\tilde{\ell}, \tag{21}$$

$$\bar{v} = \frac{\bar{d}}{1-h(\bar{d})}\frac{\bar{Y}}{\bar{A}^m}\frac{\bar{\alpha}^m}{\bar{\ell}}(1+r^D), \tag{22}$$

where

$$\tilde{\ell} = \frac{\bar{\ell}}{1 + \bar{\alpha}^m r^M + (1 - \bar{\alpha}^m) r^j}.$$

3.1 Conventional monetary policy in steady state

In steady state, monetary policy is summarized by three values, the growth rate of nominal quantities $\bar{\Gamma}$, the nominal interest rate on reserves $\bar{R}^m$ and the nominal quantity of reserves $\bar{A}^m$. Changes in $\bar{\Gamma}$ translate directly into changes in the inflation rate $\pi = \log\left(\bar{\Gamma}/\bar{G}\right)$ and therefore the nominal short rate $r^f = \delta + \pi$, which we label the federal funds rate, as it is the short safe rate that banks earn on non-reserve assets. We define as conventional monetary policy, a change in the federal funds rate, keeping fixed the value of money $\bar{v}$. The central bank can implement this policy through a change in the nominal growth rate and a sterilizing adjustment in nominal quantities according to equation 22.

Proposition 3.1 (Policy pass through)**.** A percentage point *increase* in the federal funds rate *increases* the deposit rate r^d by $\left(1 - \frac{\bar{\alpha}^m}{\bar{\ell}}\right)$ percentage points and *lowers* the quantity of deposits relative to consumption by approximately $-\frac{1-h'(\bar{d})}{\tilde{h}''\left(\bar{d}\right)} \frac{\bar{\alpha}^m}{\bar{\ell}} \Delta\pi$.

Proof. Following equation 19, an increase in inflation by $\Delta\pi$ lowers $f'(\bar{\alpha}^m/\bar{\ell})$ by the same amount. f' is negative and increasing, such that $\bar{\alpha}^m/\bar{\ell}$ has to fall, approximately by $\Delta\left(\frac{\bar{\alpha}^m}{\bar{\ell}}\right) = -\frac{\Delta\pi}{f''(\bar{\alpha}^m/\bar{\ell})}$. From equation 20, it follows that the deposit rate changes by

$$\Delta(r^D) = \Delta\pi + \Delta\left(\frac{\bar{\alpha}^m}{\bar{\ell}}\right) \cdot \left[-f'\left(\frac{\bar{\alpha}^m}{\bar{\ell}}\right) + f'\left(\frac{\bar{\alpha}^m}{\bar{\ell}}\right) + f''\left(\frac{\bar{\alpha}^m}{\bar{\ell}}\right)\frac{\bar{\alpha}^m}{\bar{\ell}}\right] = \left(1 - \frac{\bar{\alpha}^m}{\bar{\ell}}\right)\Delta(\pi).$$

From equation 4, this change translates into a decrease in the ratio of deposits to payments needs by

$$\Delta\bar{d} = -\frac{1}{\tilde{h}''\left(\bar{d}\right)} \frac{\bar{\alpha}^m}{\bar{\ell}} \Delta\pi.$$

Since desposits relative to consumption are given by $\bar{d}/(1 - h(\bar{d}))$, the decrease in real deposits is approximately $(1 - h'(\bar{d})) \cdot \Delta\bar{d}$. □

Proposition 3.1 implies that the pass through from changes in the federal funds rate on the deposit rate is larger, the smaller the banks' portfolio share of reserves is. On the other hand, the effects of deposit quantities are larger when the reserve share is larger, and when the households elasticity with respect to deposit quantity changes is large. The intuition is that if reserves are a large fraction of banks' porfolio, changes in the federal funds rate have small effect on their asset returns, and banks are therefore unable to increase deposit rates in line with the increase in the inflation rate. This drives a wedge between deposit returns and other non-payment assets. To what extent households respond to that wedge with quantity adjustments, depends on the elasticity of their liquidity cost.

From the above, we also learn that the ratio of reserves to deposits, $\bar{m} = \bar{\alpha}^m/\bar{\ell}$, decreases in response to an increase in the federal funds rate. As long as the return difference between reserves and the federal funds rate is small, this adjustment is largely driven by a decrease in the reserve share, not a decrease in bank leverage, as $\tilde{\ell}$ remains constant. To the extent that real deposits are decreasing, an increase in the federal funds rate is then associated with a reduction in bank equity (larger dividends/buybacks).

3.2 Interest on reserves

The other tool of monetary policy are interest on reserves (IOR). While they have been historically zero in the data, the Federal Reserve started paying IOR in late 2008. In steady state, a change in interest on reserves also directly affects the reserve share that the bank chooses, but different from an increase in the inflation rate it has no immediate impact on other nominal rates. We again assume that the central bank keeps the value of money constant by adjusting the base level of reserves $\bar{A}^m$.

Proposition 3.2 (Policy pass through)**.** A percentage point *increase* in the interest on reserves r^m, *increases* the deposit rate r^d by $\frac{\bar{\alpha}^m}{\bar{\ell}}$ percentage points and *increases* the quantity of deposits relative to consumption by approximately $\frac{1-h'(\bar{d})}{\tilde{h}''(\bar{d})}\frac{\bar{\alpha}^m}{\bar{\ell}}\Delta\pi$.

Proof. Following equation 19, an increase in the interest on reserves r^m increases $f'(\bar{\alpha}^m/\bar{\ell})$ by the same amount. f' is negative and increasing, such that $\bar{\alpha}^m/\bar{\ell}$ has to increase, approximately by $\Delta\left(\frac{\bar{\alpha}^m}{\bar{\ell}}\right) = \frac{\Delta\pi}{f''(\bar{\alpha}^m/\bar{\ell})}$. From equation 20, it follows that the deposit rate changes by

$$\Delta(r^D) = \frac{\bar{\alpha}^m}{\bar{\ell}}\Delta(\pi).$$

From equation 4, this change translates into an increase in the ratio of deposits to payments needs by

$$\Delta\bar{d} = \frac{1}{\tilde{h}''\left(\bar{d}\right)}\frac{\bar{\alpha}^m}{\bar{\ell}}\Delta\pi.$$

Since desposits relative to consumption are given by $\bar{d}/(1-h(\bar{d}))$, the increase in real deposits is approximately $(1-h'(\bar{d}))\cdot\Delta\bar{d}$. □

Different from conventional monetary policy, the pass through from changes of the interest rate on reserves is larger, the larger the banks' portfolio share of reserves is. Also different from changes in the federal funds rate, increases in interest on reserves increase the quantity of real deposits. As adjustments in the reserve share relative to deposits are again largely driven by the reserve portfolio share, not by leverage adjustments, an increase in interest on reserves should trigger an increase in equity issuance of banks (a reduction in dividend payments).

4 Steady state with a risky asset

Now we consider a setting, in which there is no aggregate risk, but the bank can only hold a single asset subject to idiosyncratic risk. This is equivalent to a setting with aggregate risk in which households are risk-neutral. We denote the stochastic return of the risky asset $\tilde{R}^C$. The risky asset is sold to the bank by the households.

The equilibrium is defined by the household's first order conditions

$$\frac{\bar{M}}{\bar{\Pi}} R^D = 1 - h(d) \tag{23}$$

$$\frac{\bar{M}}{\bar{\Pi}} \mathbb{E}\left[\tilde{R}^C\right] = 1, \tag{24}$$

as well as the bank's first order conditions

$$\frac{\bar{M}}{\bar{\Pi}} E\left[\tilde{R}^C\left(1 - x\left(\frac{\bar{\ell}}{\tilde{R}^\alpha}\right) + x'\left(\frac{\bar{\ell}}{\tilde{R}^\alpha}\right)\frac{\bar{\ell}}{\tilde{R}^\alpha}\right)\right] = 1, \tag{25}$$

$$\frac{\bar{M}}{\bar{\Pi}} E\left[R^M\left(1 - x\left(\frac{\bar{\ell}}{\tilde{R}^\alpha}\right) + x'\left(\frac{\bar{\ell}}{\tilde{R}^\alpha}\right)\frac{\bar{\ell}}{\tilde{R}^\alpha}\right)\right] = 1 + f'(\alpha^m/\bar{\ell}), \tag{26}$$

$$\frac{\bar{M}}{\bar{\Pi}} E\left[R^F\left(1 - x\left(\frac{\bar{\ell}}{\tilde{R}^\alpha}\right) + x'\left(\frac{\bar{\ell}}{\tilde{R}^\alpha}\right)\frac{\bar{\ell}}{\tilde{R}^\alpha}\right)\right] = 1, \tag{27}$$

and the zero profit condition

$$\frac{\bar{M}}{\bar{\Pi}} E\left[\left(1 - x\left(\frac{\bar{\ell}}{\tilde{R}^\alpha}\right)\right)\frac{\tilde{R}^\alpha}{\bar{\ell}} - R^D\right] = 1/\bar{\ell} - \left(1 - f(\alpha/\bar{\ell})\right), \tag{28}$$

where

$$\tilde{R}^\alpha = \alpha R^M + (1-\alpha)\tilde{R}^C.$$

We define the household's safe shadow rate as $\bar{R}^S = \frac{\bar{\Pi}}{\bar{M}}$. Since the household is seller of the risky asset,

$$E[\tilde{R}^C] = \bar{R}^S.$$

4.1 Bank portfolio

Define $\tilde{X} = x'\tilde{\ell} - x$. This is the net collateral benefit per unit of asset invested, the same for all assets. To obtain the actual net collateral benefit, we must multiply by the return.

The equations boil down to

$$\begin{aligned} E\left[\tilde{R}^C \tilde{X}\right] &= 0 & (29) \\ \frac{R^S}{R^M}\left(1 + f'(\alpha^m/\bar{\ell})\right) &= 1 + E\left[\tilde{X}\right], & (30) \\ \frac{R^S}{R^F} &= 1 + E\left[\tilde{X}\right] & (31) \end{aligned}$$

4.1.1 Interpretation

Since the household prices the tree, the bank chooses a portfolio with a zero expected net collateral benefit on the tree. The spread between reserves and the clean short rate must reflect bank liquidity and average net collateral benefits (the latter since reserves are riskless). The spread between the shadow rate and the fedfund rate only reflects collateral benefits.

So we want to tweak the random variable $\tilde{X}$ s.t. it is orthogonal to $\tilde{R}^C$ and its mean is equal to some function (without liquidity benefits simply an exogenous spread).

Ignoring liquidity benefits, we can rewrite the first two equations as

$$\begin{aligned} -cov\left(\tilde{R}^C, \tilde{X}\right) &= E\left[\tilde{R}^C\right] E\left[\tilde{X}\right] = R^S\left(\frac{R^S}{R^M} - 1\right) & (32) \\ E\left[\tilde{X}\right] &= \frac{E\left[\tilde{R}^C\right]}{R^M} - 1 = \frac{R^S}{R^M} - 1 & (33) \end{aligned}$$

4.1.2 Solution using functional form assumptions

Suppose x is a power function plus a constant $x = b\left(\bar{x} + \tilde{\ell}^{\xi}\right)$. Then $\tilde{X} = b\left((\xi - 1)\tilde{\ell}^{\xi} - \bar{x}\right)$. The first order conditions become

$$\begin{aligned} (\xi - 1)\bar{\ell}^{\xi} E\left[\tilde{R}^C / \left(\tilde{R}^{\alpha}\right)^{\xi}\right] &= \bar{x} E\left[\tilde{R}^C\right] \\ \frac{R^S}{R^M}\left(1 + f'(\alpha^m/\bar{\ell})\right) &= 1 + b\left((\xi - 1)\bar{\ell}^{\xi} E\left[1/\left(\tilde{R}^{\alpha}\right)^{\xi}\right] - \bar{x}\right) \\ \frac{R^S}{R^F} &= 1 + b\left((\xi - 1)\bar{\ell}^{\xi} E\left[1/\left(\tilde{R}^{\alpha}\right)^{\xi}\right] - \bar{x}\right) \end{aligned}$$

We solve for the optimal α and $\bar{\ell}$ using the first order condition on reserves and the risky asset. For now assume that there are no liquidity benefits (abundant reserve regime), so that

$$\begin{aligned}\frac{1}{E\left[\tilde{R}^C/\left(\tilde{R}^\alpha\right)^\xi\right]} b\bar{x}E\left[\tilde{R}^C\right] &= b\left(\xi-1\right)\bar{\ell}^\xi \\ \frac{1}{E\left[1/\left(\tilde{R}^\alpha\right)^\xi\right]}\left(\frac{R^S}{R^M}-1+b\bar{x}\right) &= b(\xi-1)\bar{\ell}^\xi\end{aligned}$$

Get rid of $\bar{\ell}$

$$\frac{E\left[\tilde{R}^C/\left(\tilde{R}^\alpha\right)^\xi\right]}{E\left[1/\left(\tilde{R}^\alpha\right)^\xi\right]}\left(\frac{R^S}{R^M}-1+b\bar{x}\right) = b\bar{x}E\left[\tilde{R}^C\right]$$

Approximate $\tilde{R}^\alpha$ as $\exp\left(\alpha r_M + (1-\alpha)\tilde{r}_C\right)$ such that

$$E\left[1/\left(\tilde{R}^\alpha\right)^\xi\right] \approx E\left[\exp(-\xi\alpha r^M - \xi(1-\alpha)\tilde{r}^C)\right]$$

and

$$E\left[\tilde{R}^C/\left(\tilde{R}^\alpha\right)^\xi\right] \approx E\left[\exp(-\xi\alpha r^M - (\xi(1-\alpha)-1)\tilde{r}^C)\right]$$

Assume log-normality such that

$$\frac{E\left[\tilde{R}^C/\left(\tilde{R}^\alpha\right)^\xi\right)}{E\left[\tilde{R}^C\right]E\left[1/\left(\tilde{R}^\alpha\right)^\xi\right]} \approx \frac{\exp\left(\left(1-\xi(1-\alpha)\right)\bar{r}^C + 0.5\left(1-\xi(1-\alpha)+\xi^2(1-\alpha)^2\right)\sigma_c^2\right)}{\exp\left(\bar{r}^C+0.5\sigma_c^2\right)\exp(-\xi(1-\alpha)\bar{r}^C + 0.5\xi^2(1-\alpha)^2\sigma_c^2)} = \exp\left(-\xi(1-\alpha)\sigma_c^2\right),$$

which leads to

Result 4.1 (Risky asset share)**.** Given functional form assumptions and log-normal returns, the bank's portfolio share of the risk is approximately

$$1-\alpha = \frac{1}{\xi\sigma_c^2}\log\left(1+\frac{r^s-r^M}{b\bar{x}}\right)$$

The portfolio share of the risky asset is decreasing in riskiness of the asset, decreasing in risk aversion, increasing in shadow-reserve spread.

To get an idea on quantities: assume $r^s = 1\%$, $r^m = 0\%$, $b\bar{x} = 0.5\%$, $\sigma_c^2 = 0.1^2$ and $\xi = 120$, then $(1-\alpha) = 91.55\%$. If the spread was to double to 2% the risky asset portfolio share would go up to 134%. In the full model with liquidity benefits this is prevented by the liquidity cost, which lowers the effective spread between reserve rate and shadow rate as α goes down, thereby preventing α to turn negative.

To solve for optimal leverage, we can use one of the two equations

$$\begin{aligned} E\left[\tilde{R}^C\right]^{1/\xi} E\left[\tilde{R}^C/\left(\tilde{R}^\alpha\right)^\xi\right]^{-1/\xi}\left(\frac{\bar{x}}{\xi-1}\right)^{1/\xi} &= \bar{\ell} \\ E\left[1/\left(\tilde{R}^\alpha\right)^\xi\right]^{-1/\xi}\left(\frac{r_s-r_m+b\bar{x}}{b(\xi-1)}\right)^{1/\xi} &= \bar{\ell} \end{aligned}$$

To double-check, use both with log-normality, to get

$$\begin{aligned} \exp\left(\alpha r^m+(1-\alpha)\bar{r}^C+\frac{1}{\xi}\left(\xi(1-\alpha)-0.5\xi^2(1-\alpha)^2\right)\sigma_c^2\right)\left(\frac{\bar{x}}{\xi-1}\right)^{1/\xi} &= \bar{\ell} \\ \exp(\alpha r^m+(1-\alpha)\bar{r}^C-0.5\xi(1-\alpha)^2\sigma_c^2)\left(\frac{r_s-r_m+b\bar{x}}{b(\xi-1)}\right)^{1/\xi} &= \bar{\ell}. \end{aligned}$$

or to make them more comparable

$$\begin{aligned} \exp(\alpha r^m+(1-\alpha)\bar{r}^C-0.5\xi(1-\alpha)^2\sigma_c^2)\exp\left((1-\alpha)\sigma_c^2\right)\left(\frac{b\bar{x}}{b(\xi-1)}\right)^{1/\xi} &= \bar{\ell} \\ \exp(\alpha r^m+(1-\alpha)\bar{r}^C-0.5\xi(1-\alpha)^2\sigma_c^2)\left(\frac{r_s-r_m+b\bar{x}}{b(\xi-1)}\right)^{1/\xi} &= \bar{\ell}. \end{aligned}$$

Plugging in solution for $(1-\alpha)$ into $\exp\left((1-\alpha)\sigma_c^2\right)$ shows equations are the same. So continue with

$$\exp(\alpha r^m+(1-\alpha)\bar{r}^C-0.5\xi(1-\alpha)^2\sigma_c^2)\left(\frac{r_s-r_m+b\bar{x}}{b(\xi-1)}\right)^{1/\xi}=\bar{\ell}.$$

which can be rewritten as

$$\exp\left[-\left(-\alpha r^m-(1-\alpha)\bar{r}^c+0.5(1-\alpha)^2\sigma_c^2\right)\right]\exp\left[-0.5(\xi-1)(1-\alpha)^2\sigma_c^2\right]\left(\frac{r_s-r_m+b\bar{x}}{b(\xi-1)}\right).^{1/\xi}=\bar{\ell}.$$

The first part is $E\left[1/\tilde{R}^\alpha\right]^{-1}$, such that plugging in for $(1-\alpha)$ leads to

Result 4.2 (Expected leverage)**.** Expected realized leverage is given by

$$\exp\left(-\frac{1}{2\sigma_c^2}\frac{\xi-1}{\xi^2}\left(\log\left(1+\frac{r^s-r_m}{b\bar{x}}\right)\right)^2\right)\left(\frac{r_s-r_m+b\bar{x}}{b(\xi-1)}\right)^{1/\xi}=E\left[\tilde{\ell}\right].$$

So higher variance of the risky asset leads to lower portfolio share but higher expected leverage. Rough intuition: Keeping fixed the portfolio share, log-leverage is inversely related to risk, however the same is true for the portfolio share, which has a quadratic effect on risk and therefore dominates.

Take derivatives with respect to the reserve spread to find:

$$\frac{\partial E\left[\tilde{\ell}\right]}{\partial(r_s - r_m)} = E\left[\tilde{\ell}\right]\left(-(\xi-1)(1-\alpha^*) + \frac{1}{\xi}\right)\frac{1}{b\bar{x} + r^S - r^M}, \tag{34}$$

which is presumably negative for reasonable parameters.

To study $\bar{\ell}$, divide LHS and RHS by $E\left[\frac{1}{\tilde{R}^\alpha}\right]$ which is approximately the same as multiplying by

$$E\left[\tilde{R}^\alpha\right] = R^m + (1-\alpha)(R^S - R^M) = R^m + \frac{1}{\xi\sigma_c^2}\log\left(1 + \frac{r^s - r_m}{b\bar{x}}\right)(R^S - R^M)$$

So for $\bar{\ell}$ there are two opposing effects, more risk means lower portfolio share of the risky asset and therefore lower returns. For given expected cum-return leverage, that lowers pre-return leverage. However expected leverage goes up, which also increases pre-return leverage.

4.1.3 Deposit rate

Using the zero profit condition, we have that

$$R^S - R^D = \frac{1}{\bar{\ell}}\left(R^S - E\left[\left(1 - b\bar{x} - b\left(\frac{\bar{\ell}}{R^\alpha}\right)^\xi\right)R^\alpha\right]\right) + f\left(\alpha/\bar{\ell}\right)R^S \tag{35}$$

so the spread between deposit rate and shadow rate are losses from holding collateral in the bank, adjusted for leverage, plus the losses from paying liquidity cost on every deposit unit.

To solve in terms of parameters, ignore liquidity benefits, and write as

$$R^S - R^D = \frac{E\left[R^\alpha\right]}{\bar{\ell}}\left(\frac{R^S}{E\left[R^\alpha\right]} - (1 - b\bar{x}) + \frac{1}{E\left[R^\alpha\right]}E\left[b\left(\frac{\bar{\ell}}{R^\alpha}\right)^\xi R^\alpha\right]\right) \tag{36}$$

The multiplier in front of the bracket is

$$\frac{E\left[R^\alpha\right]}{\bar{\ell}} = \exp\left[0.5(\xi+1)(1-\alpha)^2\sigma_c^2\right]\left(\frac{r_s - r_m + b\bar{x}}{b(\xi-1)}\right)^{-1/\xi} \tag{37}$$

$$= \exp\left[0.5\frac{\xi+1}{\xi^2}\frac{1}{\sigma_c^2}\left(\log\left(1 + \frac{r^s - r^M}{b\bar{x}}\right)\right)^2\right]\left(\frac{r_s - r_m + b\bar{x}}{b(\xi-1)}\right)^{-1/\xi}. \tag{38}$$

which is decreasing in risk and decreasing in the spread for reasonable parameters (see above).

The first part inside the bracket is

$$
\begin{aligned}
\frac{R^S}{E\left[R^\alpha\right]} &= \exp\left(-\alpha(r_S - r_M)\right) && (39)\\
&= \exp\left[\left(1 - \frac{1}{\xi\sigma_c^2}\log\left(1 + \frac{r^s - r^M}{b\bar{x}}\right)\right)(r_S - r_M)\right] && (40)
\end{aligned}
$$

The last part inside the bracket is

$$
\begin{aligned}
\frac{1}{E\left[R^\alpha\right]}E\left[b\left(\frac{\bar{\ell}}{R^\alpha}\right)^\xi R^\alpha\right] =& b\exp\left(-0.5\xi(1-\alpha)^2\sigma_C^2\right)\left(\frac{r_s - r_m + b\bar{x}}{b(\xi-1)}\right) && (41)\\
=& b\exp\left(-0.5\frac{1}{\xi\sigma_C^2}\log\left(1 + \frac{r^s - r^M}{b\bar{x}}\right)^2\right)\left(\frac{r_s - r_m + b\bar{x}}{b(\xi-1)}\right) && (42)
\end{aligned}
$$

To get the latter use

$$
\begin{aligned}
E\left[R_\alpha^{-(\xi-1)}\right] &= E\left[\exp\left(-(\xi-1)\left(\alpha r_M + (1-\alpha)\tilde{r}_C\right)\right)\right]\\
&= \exp(-(\xi-1)r_M)E\left[\exp(-(\xi-1)(1-\alpha)(\tilde{r}_C - r_M)))\right]\\
&= \exp(-(\xi-1)r_M)\exp(-(\xi-1)(1-\alpha)(\bar{r}_C - r_M) + 0.5(\xi-1)^2(1-\alpha)^2\sigma_C^2))
\end{aligned}
$$

Multiply with

$$
\exp(\xi\alpha r^m + \xi(1-\alpha)\bar{r}^C - 0.5\xi^2(1-\alpha)^2\sigma_c^2)\left(\frac{r_s - r_m + b\bar{x}}{b(\xi-1)}\right) = \bar{\ell}^\xi.
$$

to find

$$
\begin{aligned}
&\exp(r_M)\exp((1-\alpha)(\bar{r}_C - r_M) - 0.5(\xi-1)(1-\alpha)^2\sigma_C^2))\left(\frac{r_s - r_m + b\bar{x}}{b(\xi-1)}\right)\\
=\ &\exp(r_M)\exp\left((1-\alpha)(r_S - r_M)\right)\exp\left(-0.5\xi(1-\alpha)^2\sigma_C^2\right)\left(\frac{r_s - r_m + b\bar{x}}{b(\xi-1)}\right)
\end{aligned}
$$

Combining everything

$$R^S - R^D = \exp\left[0.5\frac{\xi+1}{\xi^2}\frac{1}{\sigma_c^2}\left(\log\left(1+\frac{r^s-r^M}{b\bar{x}}\right)\right)^2\right]\left(\frac{r_s-r_m+b\bar{x}}{b(\xi-1)}\right)^{-1/\xi} \tag{43}$$

$$\cdot\left\{\exp\left[\left(1-\frac{1}{\xi\sigma_c^2}\log\left(1+\frac{r^s-r^M}{b\bar{x}}\right)\right)(r_S-r_M)\right]-(1-b\bar{x})\right. \tag{44}$$

$$\left.+\,b\exp\left(-0.5\frac{1}{\xi\sigma_C^2}\log\left(1+\frac{r^s-r^M}{b\bar{x}}\right)^2\right)\left(\frac{r_s-r_m+b\bar{x}}{b(\xi-1)}\right)\right\} \tag{45}$$

which is maybe easier to read as

$$R^S - R^D = \exp\left[0.5\frac{\xi+1}{\xi^2}\frac{1}{\sigma_c^2}\left(\log\left(1+\frac{r^s-r^M}{b\bar{x}}\right)\right)^2\right]\left(\frac{r_s-r_m+b\bar{x}}{b(\xi-1)}\right)^{-1/\xi} \tag{46}$$

$$\cdot\left\{\exp\left[\left(1-\frac{1}{\xi\sigma_c^2}\log\left(1+\frac{r^s-r^M}{b\bar{x}}\right)\right)(r_S-r_M)\right]-(1-b\bar{x})\right\} \tag{47}$$

$$+\,b\exp\left(0.5\frac{1}{\xi\sigma_C^2}\log\left(1+\frac{r^s-r^M}{b\bar{x}}\right)^2\right)\left(\frac{r_s-r_m+b\bar{x}}{b(\xi-1)}\right)^{1-1/\xi} \tag{48}$$

If alpha is nearly zero, the curly bracket reduces to $b\bar{x}$. Then the spread is decreasing in risk (and decreasing in the spread for reasonable parameters).

A Derivation of Euler equations

The optimization problem is

$$\max\ \left[(1-\beta)\,C_t^{1-1/\sigma}+\beta\mathbb{E}_t\left(U_{t+1}^{1-\gamma}\right)^{\frac{1-1/\sigma}{1-\gamma}}\right]^{\frac{1}{1-1/\sigma}} \tag{49}$$

subject to

$$P_tC_t+\sum_{j=1}^{J}Q_t^jA_t^j+D_t+h\left(\frac{D_t}{\Delta_t}\right)\Delta_t \leq Y_t+\sum_{j=1}^{J}(P_t\delta_t^j+Q_t^j)A_{t-1}^j+R_t^DD_{t-1}+P_t\tau_t$$

$$\Delta_t = P_tC_t+D_t+\sum_j Q_t^jA_t^j-R_t^DD_{t-1}-\sum_j\left(Q_t^j+P_t\delta_{t+1}^j\right)A_{t-1}^j$$

Writing p_{t+1} as a short-hand for $p(s_{t+1})$ and similarly x_{t+1} for $x(s_{t+1})$ for any stochastic variable x, the first order conditions are

$$
\begin{aligned}
\frac{\partial V_t}{\partial C_t} &= V_t^{1/\sigma}(1-\beta)C_t^{-1/\sigma} - \lambda_t P_t \left(1 + h_t - h_t' \cdot \frac{D_t}{\Delta_t}\right) \\
\frac{\partial V_t}{\partial C_{t+1}} &= p_{t+1} V_t^{1/\sigma} \beta \mathbb{E}_t \left(V_{t+1}^{1-\gamma}\right)^{\frac{\gamma-1/\sigma}{1-\gamma}} V_{t+1}^{1/\sigma-\gamma}(1-\beta)C_{t+1}^{-1/\sigma} - p_{t+1}\lambda_{t+1}P_{t+1}\left(1 + h_{t+1} - h_{t+1}' \cdot \frac{D_{t+1}}{\Delta_{t+1}}\right) \\
\frac{\partial V_t}{\partial A_t^j} &= -\lambda_t \left(1 + h_t - h_t' \cdot \frac{D_t}{\Delta_t}\right) Q_t^j + E_t \left[\lambda_{t+1}\left(1 + h_{t+1} - h_{t+1}' \cdot \frac{D_{t+1}}{\Delta_{t+1}}\right)\left(Q_{t+1}^j + P_{t+1}\delta_{t+1}^j\right)\right] \\
\frac{\partial V_t}{\partial D_t} &= -\lambda_t \left(1 + h_t - h_t' \cdot \frac{D_t}{\Delta_t} + h_t'\right) + E_t \left[\lambda_{t+1} R_{t+1}^D \left(1 + h_{t+1} - h_{t+1}' \cdot \frac{D_{t+1}}{\Delta_{t+1}}\right)\right]
\end{aligned}
$$

We get two Euler equations, one for deposits and one for other assets

$$
\begin{aligned}
1 + \tilde{h}_t' &= \mathbb{E}_t \left[M_t \frac{P_t}{P_{t+1}} R_t^D\right] \\
1 &= \mathbb{E}_t \left[M_t \frac{P_t}{P_{t+1}} R_t^j\right]
\end{aligned}
$$

where

$$
M_t = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-1/\sigma} \left(\frac{V_{t+1}}{E_t\left[V_{t+1}^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\sigma}-\gamma}
$$

is the standard Epstein-Zin pricing kernel.

In the above, we define

$$
\tilde{h}'\left(\frac{D_t}{\Delta_t}\right) = \frac{h_t'}{1 + h_t - h_t' \cdot \frac{D_t}{\Delta_t}},
$$

which is negative and increasing in its argument.

B Stationary equilibrium

To transform the set of equilibrium equations into stationary system, assume that the joint process of consumption growth and the growth rate of nominal quantities is a stationary Markov process. Denote the current state of the economy $s = \{G_t, \Gamma_t\}$.

Conjecture that the price level can be written as $P_t = \frac{1}{v_s}\frac{\prod_{\tau=0}^t \Gamma_\tau}{\prod_{\tau=0}^t G_\tau}$, where v_s is a stationary measure for the value of money that only depends on the current state. Furthermore, conjecture that all returns are stationary and write $R_{t+1}^j = R_{s'|s}^j$. Rewrite $d_t = \frac{D_t}{\Delta_t} = d(s)$.

$$\mathbb{E}\left[M_{s'|s}\frac{G_{s'}}{\Gamma_{s'}}\frac{v_{s'}}{v_s}R^D_{\cdot|s}\right] = 1+\tilde{h}'\left(d_s\right) \tag{50}$$

$$\mathbb{E}\left[M_{s'|s}\frac{G_{s'}}{\Gamma_{s'}}\frac{v_{s'}}{v_s}R^j_{s'|s}\right] = 1 \tag{51}$$

$$E\left[M_{s'|s}\frac{G_{s'}}{\Gamma_{s'}}\frac{v_{s'}}{v_s}R^\alpha_{s'|s}\left(1-x\left(\frac{\bar{\ell}_s}{R^\alpha_{s'|s}}\right)+x'\left(\frac{\bar{\ell}_s}{R^\alpha_{s'|s}}\right)\frac{\bar{\ell}_s}{R^\alpha_{s'|s}}\right)\right] = \left(1+\alpha^m_s f'(\alpha^m_s/\bar{\ell}_s)\right) \tag{52}$$

$$E\left[M_{s'|s}\frac{G_{s'}}{\Gamma_{s'}}\frac{v_{s'}}{v_s}R^j_{s|s'}\left(1-x\left(\frac{\bar{\ell}_s}{R^\alpha_{s|s'}}\right)+x'\left(\frac{\bar{\ell}_s}{R^\alpha_{s|s'}}\right)\frac{\bar{\ell}_s}{R^\alpha_{s|s'}}\right)\right] = 1 \;\; \forall j\in J \text{ with } a^j_{B,s}>0 \tag{53}$$

$$E\left[M_{s'|s}\frac{G_{s'}}{\Gamma_{s'}}\frac{v_{s'}}{v_s}\left(\left(1-x\left(\frac{\bar{\ell}_s}{R^\alpha_{s'|s}}\right)\right)\frac{R^\alpha_{s'|s}}{\bar{\ell}_s}-R^D_{\cdot|s}\right)\right] = \left(\frac{1}{\bar{\ell}_s}-\left(1-f(\alpha^m_s/\bar{\ell}_s)\right)\right) \tag{54}$$

$$d_s R^D_{\cdot|s} = \left(1-h(d_s)\right)v_s\frac{A^m_0}{Y_0}\frac{\bar{\ell}_s}{\alpha^m_s} \tag{55}$$

$$a^j_{H,s}+a^j_{B,s} = 0 \tag{56}$$

For the second to last equation, combine the budget constraint with the equation for Δ_t to find that

$$\Delta_t = \frac{P_tY_t}{1-h(d_t)} \tag{57}$$

and the fact that

$$\frac{\bar{A}^m_s}{P_tY_t} = \frac{\bar{A}^m_0\prod_{\tau=0}^t\Gamma_\tau}{\frac{1}{v(s)}\frac{\prod_{\tau=0}^t\Gamma_\tau}{\prod_{\tau=0}^t G_\tau}Y_0\prod_{\tau=0}^t G_\tau} = v_s\frac{A^m_0}{Y_0}.$$