

The Macroeconomics of Sorting and Turnover in a Dynamic Assignment Model*

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Abstract

We build a tractable assignment model to characterize the matching and separation patterns of CEOs and their employers. Managers learn about their own type by observing a sequence of public signals (productivity shocks). The sorting is *ex ante* perfect across managers of a given cohort who are not currently matched (either because they were unmatched in the previous period or because they decided to split from their previous match), but is not typically so *ex post*. Moreover, in the special case with frictionless matching, perfect *ex ante* sorting occurs across managers of a given cohort regardless of their assignment history. We calibrate the model to match empirical targets from a large matched employer-employee dataset covering the Danish labor force between 2000 and 2009. We have a particular interest in the degree of complementarity between the attributes of the manager and those of the firm in the production function and our results fill a gap in the literature on the aggregate effects of a particular form of misallocation, *mismatch*, the size of which depends critically on this elasticity. Moreover, in ongoing work our model connects turnover to systematic differences in the cross-sectional size distribution of firms across countries at different income levels, which has been shown to be correlated with aggregate TFP in earlier work.

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1 Introduction

A sizeable literature addresses the allocation of skills across firms or jobs. Beginning with [Lucas \(1978\)](#) it has been argued that idiosyncratic differences in technologies or organization capital, for instance, are key to understanding the size distribution of firms and establishments. Similarly, individuals are endowed with different innate abilities, which, by way of investment or experience, manifest themselves as variations in skill or human capital (as in [Bhattacharya et al., 2013](#), for instance). While the idea of heterogeneity in productivities and skills is uncontroversial, the extent to which complementarities govern their interaction remains an open question. [Abowd et al. \(1999\)](#) find no empirical evidence for worker-job complementarities. [Lopes de Melo \(2018\)](#), on the other hand, argues that the estimates from fixed effects regressions may be biased due to the non-monotonicity of wages in firm types, a feature that [Eeckhout and Kircher \(2011\)](#) also emphasize. [Lise et al. \(2013\)](#) find evidence for positive complementarity, at least for relatively well educated workers in the United States.

But why should we care about complementarities at all? With complementarities, a worker's marginal product is a function of her own skill as well as the characteristics of the job she is assigned to. Clearly then, the prices that decentralize the optimal assignment of workers to jobs depend on the degree to which skills and productivities complement each other. In such an environment, maximizing aggregate productivity is not just a matter of making sure everyone has *a* job but requires that everybody ends up with the *right* kind of job.

The intuition is quite simple and to fix ideas consider the planner's problem of assigning a manager to a firm. In the absence of complementarities, a CEO's marginal product is only a function of the efficiency units she embodies. Her contribution to the total surplus does not vary across firms and the assignment of managerial talent is of no consequence for efficiency. Only the *aggregate* supply of talent matters. In contrast, if managers and firms are gross complements in production, a competent CEO's marginal product is a steeper function of firm productivities compared to the profile of an incompetent manager and positive sorting is optimal. The empirically relevant degree of complementarity, however, is still an open question.

Consider, for example, a Canadian street performer by the name of Guy Laliberté. He started his career as an accordion player, fire breather, and stiltwalker with a small group of colorful characters in the small Canadian town of Baie-Saint-Paul. In due time he founded a circus and named it *Cirque du Soleil*, which is now a global enterprise with several thousand employees. While Guy reportedly was creative from an early age, his compensation – and thus arguably his marginal product – rose sharply over the course of his career. It was low while he was entertaining pedestrians on the sidewalks of Baie-Saint-Paul. Soon, however, he quit

the sidewalks and moved into the first *Cirque du Soleil* tent seating 800 spectators in 1984. After several upgrades, the circus is now on multiple simultaneous tours in addition to resident shows in Las Vegas and Orlando. Guy’s creativity is now paired with a technology that allows him to reach a much larger audience than the few dozen or hundred pedestrians who saw him on a street corner in his early days. Back then, he probably never imagined that he would one day spend \$35 million for an eleven-day vacation on the *International Space Station*.

In the absence of talent-technology complementarities, Guy’s marginal product and compensation are proportional to his skills and that, in turn, would suggest that his performance talent increased by a few orders of magnitude over the course of three decades. While we acknowledge the importance of (endogenous) skill accumulation, we also believe that technology-skill complementarities are critical features of talent markets in general and the market for entrepreneurs and senior managers more specifically. Once we admit such complementarities, identifying the empirically relevant degree turns into a first-order concern.

If managerial abilities and firm productivities were observable attributes, one could identify the extent to which they complement each other simply by matching the salient empirical price moments to their model-generated counterparts. Alas, we cannot observe them directly and have no choice but to back out the distribution of firm and manager attributes as well as the substitution elasticity from other data moments. Given the data limitations of the time, earlier assignment models like [Gabaix and Landier \(2008\)](#) and [Terviö \(2008\)](#) sidestepped the challenges associated with identifying the relevant degree of complementarity altogether. While moments from widely available cross-sectional data are *necessary* to calibrate an assignment model of this kind, previous work highlighted the importance of the data’s *time series* dimension to pin down the substitution elasticity between firms and managers more definitively ([Alder, 2016](#)). Consequently, we need panel data with enough variation in the time series dimension and a *dynamic* assignment model with endogenous separation and turnover to make further progress. With these two ingredients, data and theory, we can overcome many of the identification problems and calibrate the parameters of a dynamic assignment model to match population moments in a panel data set.

The basic idea is the following. To learn something about Guy Laliberté’s creative talent separately from the contribution of the technologies he operates, we need to know the 1980s busker/entrepreneur of Baie-Saint-Paul as well as the successful CEO from the 1990s and 2000s. Not only that, we also need a population of entrepreneurs and managers, some of whom have trajectories similar to Guy’s, while others have fairly unremarkable careers, and yet another group may experience some sort of failure.

Our theory accomplishes this goal in a tractable way and we parameterize our model using matched employer-employee data from three different sources that cover the Danish la-

bor force from 2000 to 2009. The first of the data sources are register data that contain socioeconomic variables, employment information, and a complete employer-employee link for the entire population between 2000 and 2009. The second source is the *Købmændstaden's Oplysningsbureau* (KOB) dataset that contains firm accounting data for all Danish limited liability firms from 1991 to the present. Lastly, the third dataset, *Erhvervs- og Selskabstyrelsen* (ES) data, identifies CEOs and board members of limited liability firms from 2000 to 2010. Unique firm and worker identifiers enable us to merge information from these three sources into a single dataset and to construct employment histories for virtually all individuals of interest, namely CEOs and entrepreneurs. This rich panel structure delivers the additional identification we need to jointly characterize the distribution of managerial abilities and productivities as well as the parameter that governs substitutability in the technology that generates the match surplus.

In our model, agents do not know their own type but receive a sequence of public signals (productivity shocks) that enables them to update their beliefs using Bayes' rule (as in [Groes et al., 2015](#)). In contrast to [Jovanovic \(1979\)](#) agents learn about their own abilities rather than about an *ex ante* unknown match quality.¹ Conditional on age – and possibly the managers' most recent assignment – our model features perfect sorting in the agents' *priors*, but not *ex post*. The planner separates CEOs from their previous match whenever the gap between their prior and posterior is sufficiently wide. In the special case where matching is costless, the sorting of CEOs is *ex ante* perfect conditional on age, but does not depend on her assignment history. Moreover, in this frictionless version of the model, the dynamic program “collapses” to a simple static optimization problem. The aggregate state (i.e. the aggregate distribution of beliefs) is stationary by construction since it does not depend on any previous assignment decision and the only relevant idiosyncratic state variable is an individual manager's expected ability.

When matching is costly, however, both the aggregate and idiosyncratic states have additional elements. To begin with, the aggregate state is now the joint distribution of beliefs (about the managers' abilities) and qualities that reflects last period's assignments adjusted for a one-period Bayesian update on all the managers' expected abilities. Similarly, the manager's idiosyncratic state has two elements: yesterday's match and the manager's belief about her type today. Theoretically, this allows for the possibility of sufficiently – but not excessively – persistent mismatch in the planner's constrained efficient assignment of CEOs to firm. Yet it also preserves enough variation in the time-series or life-cycle dimension to structurally uncover the strength of the CEO-firm complementarities. Similar to [Bandiera et al. \(2017\)](#), our empirical evidence suggests that assignment frictions play a crucial role. In addition,

¹See also [Nagypal \(2007\)](#) for a similar distinction. [Jovanovic \(2015\)](#) considers the implications of *private* output histories.

the dynamics of our model and the panel structure of our dataset enable us to recover the importance of these frictions from a structural calibration and – in ongoing work – we are connecting our results to the extensive literature on mismatch and aggregate productivity.²

To begin with, we solve the planner’s problem without fixed costs and discuss the cross-sectional and life-cycle participation and assignment patterns. Since this simple model features excessive turnover, we introduce a fixed (deadweight) cost for match formation and parameterize this second version to match the salient stylized fact in the data, which includes age- and ability-specific entry, exit, and employer-to-employer transitions. In ongoing work we show that the constrained optimal allocation can be decentralized by way of a fairly simple contract.

Our theory has potential applications beyond the CEO-to-project assignments considered here. For instance, it is a natural building block toward a dynamic theory of entrepreneurship. In our model, the returns to entrepreneurship are uncertain while the outside option is associated with a constant flow payment.³ Occupational choice theories like [Johnson \(1978\)](#), [Jovanovic \(1979\)](#), or [Miller \(1984\)](#) suggest that individuals select risky occupations when they are young. [Evans and Jovanovic \(1989\)](#), in contrast, argue that liquidity constraints are tighter for younger individuals since they have, on average, accumulated less wealth. This constraint acts as a countervailing force to the rookies’ appetite for risk. [Evans and Leighton \(1989\)](#) find that the hazard rate into self-employment is constant in age, which is consistent with the conclusion that liquidity constraints are binding in the data.

Our theory offers alternative support for this Knightian view according to which risk-bearing is a defining trait of entrepreneurship ([Knight, 1921](#)). In our model, the optimal assignment of entrepreneurs to projects takes into account the trade-off between someone’s expected ability and the variance of her contribution to the match surplus. In a nutshell, younger cohorts exhibit more moderate cross-sectional variance in the expected abilities of its members, but any one individual’s beliefs are less precise. When these features are combined with diminishing marginal returns to the match surplus, the model delivers a rich pattern of entrepreneurial or managerial dynamics and selection that we can calibrate to corresponding moments in our matched employer-employee dataset.

We emphasize that our model is a building block rather than a full-fledged theory of entrepreneurship. Most importantly, we abstract from entry, exit, and other *firm* dynamics that are distinct from those of the managers and entrepreneurs themselves. This being said, our model can connect variations in turnover – and hence the strength of sorting between the abilities of the managers and the productivities of the technologies they operate – to differences

²See, for instance, [Bhattacharya et al. \(2013\)](#) or [Guner et al. \(2017\)](#), in addition to some of our own earlier work.

³One may think of the outside option as the wage associated with supplying a raw unit of labor, as in Lucas’ canonical span-of-control model ([Lucas, 1978](#)).

in the tail shape of the cross-sectional size distribution of firms, which has been shown to be correlated with aggregate TFP in earlier work (Alder, 2016).⁴

The remainder of the paper is organized as follows. Section 2 details the model environment and introduces the dynamic planning problem. Section 4 characterizes the patterns of CEO mobility. We describe the data in section 5 and the specifics of our calibration strategy in section 6. The counterfactuals in section 7 quantify the effects of mismatch (to be completed). Section 8 concludes (to be completed).

2 Model

2.1 Population and Endowment

The economy is populated by a unit measure of individuals who are endowed (at birth) with skill a . This ability to manage a firm is drawn from a known normal distribution with mean zero and precision ϕ . The draws from this distribution are i.i.d. across agents.

Individuals live for R periods. Each period, a new cohort of size $\frac{1}{R}$ enters the economy, while the oldest cohort (of the same size) retires. New entrants do not know their type. Instead, they observe a *public* signal $a_0 = a + \epsilon_0$. The innovation ϵ_0 is drawn from a normal distribution with mean zero and precision ψ . At age $r \in \{1, \dots, R\}$, a manager's contemporaneous ability is given by $a_r = a + \epsilon_r$. The uncertainty associated with the CEOs' type, ϵ_r , is drawn from the same distribution as ϵ_0 .⁵ The innovations are independent cross-sectionally and over time. Individuals have a periodic unit endowment of time and they can split it between performing managerial tasks and an "outside" occupation or activity that generates a surplus w per unit of time.⁶

The economy is also endowed with a measure $N \in (0, 1]$ of long-lived projects with idiosyncratic attributes denoted by q . The qualities or productivities are drawn from a c.d.f. $G(\cdot)$ with corresponding density $g(\cdot)$. Each individual owns one such project for the duration of her lifetime. When generation g dies after R periods, the "orphaned" projects are bequested to randomly chosen individuals of the cohort with birth date $g + R$.

⁴Moreover, in Alder et al. (2017) we introduce firm dynamics into a multi-dimensional assignment model.

⁵Relaxing this assumption generates the same qualitative results but requires heavier notation.

⁶In Lopes de Melo (2018), unemployed workers receive a flow value $b(x)$ that depends on their type x . Similarly, we could allow for the flow surplus of an unmatched agent to depend on her type, but we do not explore this possibility here. While nothing prevents the planner from making part-time assignments, doing so turns out to be inefficient. In the planner's solution, agents are either managing full time or they are not managing at all.

2.2 Individual Preferences and Technology

Individuals have linear preferences over their discounted consumption stream. At age r , their valuation is:

$$U(\{c_t\}) = \sum_{t=r}^R \beta^{t-r} c_t.$$

Since there is no consumption-saving tradeoff c_t , is simply the sum of an agent's income at age t in the decentralized equilibrium: the flow value associated with the outside option, if taken, her managerial income, and the flow profit associated with ownership of a project. The social planner simply values the sum of (integral over) the individual households' valuations.

A CEO of age r matched to a project q generates a surplus x_r , which is a function of her contribution $a_r = a + \epsilon_r$ and q :

$$x_r = x(a + \epsilon_r, q) \quad (1)$$

The technology is continuous and increasing in both the manager's effective ability a_r and the firm's productivity (or the project's quality) q . Moreover, $x(\cdot, \cdot)$ satisfies the (weakly) increasing differences property:

$$x(a'_r, q') - x(a_r, q') \geq x(a'_r, q) - x(a_r, q), \text{ for } a'_r \geq a_r \text{ and } q' \geq q. \quad (2)$$

In addition, we allow for the possibility that the formation of a *new* match entails a dead-weight cost $\mu \geq 0$, which does not depend on the agent's belief or the project's productivity. This is a particularly parsimonious way to model the effect of frictions such as time-consuming or costly search, as well as moving or start-up costs associated with a change in employer (employee). Importantly, paying the cost μ yields no information.

2.3 Beliefs

The distribution of true, but unobserved managerial talent is stationary. Each cohort of size $\frac{1}{R}$ draws types from the same normal distribution with mean zero and variance ϕ^{-1} . Clearly then, the aggregate distribution of talent follows that same distribution. In contrast, individual realizations of a are not observable. Instead, at age r an agent receives a public signal (or managerial ability shock) $a_r = a + \epsilon_r$ and forms a belief about her own type. Since a_r is public information, everybody has the same belief about any one individual's type. Recall that ϵ is drawn from a mean-zero normal distribution with precision ψ , regardless of age:

$$\epsilon_t \sim N(0, \psi^{-1}), \text{ for all } t \quad (3)$$

Each signal enables individuals to update their prior using a Kalman filter. Since the innovation is normal, the distribution of posteriors, denoted by $\tilde{a}'$, of someone who is r years old is:

$$\hat{a}' \equiv \mathbb{E}(\tilde{a}') = \frac{\phi_i + r\psi}{\phi_i + (r+1)\psi} \hat{a} + \frac{\psi}{\phi_i + (r+1)\psi} a_{rthelaw}, \quad (4)$$

where $\hat{a}$ denotes the mean of the agent's distribution of priors $\tilde{a}$. The posteriors' variance is independent of the realization of a_t and given by:

$$\text{Var}(\tilde{a}') = \frac{\psi}{(\phi + r\psi)(\phi + (r+1)\psi)}. \quad (5)$$

Note that the variance of the posteriors is independent of the individuals' life-time history of shocks and decreasing in age: older workers have a more precise idea about their own abilities. Since there is no private information, this implies that everybody in this economy has more precise beliefs about the abilities of older workers compared to younger ones. Moreover, since $\frac{\phi + r\psi}{\phi + (r+1)\psi} + \frac{\psi}{\phi + (r+1)\psi} = 1$ and the second term is decreasing in r , it is easy to see that agents put more and more weight on their prior $\hat{a}_r$ at the expense of the innovation a_r as they approach retirement at age R .

One can also show that the mean priors in the cohort of age r agents are distributed normally,

$$\hat{a}_r \sim N\left(0, \frac{r\psi}{\phi(\phi + r\psi)}\right),$$

and we will write $\hat{a}_r \sim F_r(A)$.

Since we are, at least initially, interested in the *efficient* allocation of managerial talent to technologies with different productivities, we begin by writing down the planner's problem. In future research, we will characterize the types of contract(s) that are needed to decentralize the economy and verify whether or not they can replicate the planner's constrained efficient assignments.

3 Planner's Problem

Before we can specify the planner's Bellman equation, we need to introduce some additional notation. Let $A \subseteq \mathbb{R}$ and $Q \subseteq \mathbb{R}^+$ denote the domains for the agents' abilities and the projects' productivities, respectively, and let $\Delta(A \times Q)$ be the set of all feasible matches. For each $r \in \{1, \dots, R\}$, let a feasible assignment be characterized by the joint density $H_r \in \Delta(A \times Q)$ with marginal $H_r^A(a) = F_r(a)/R$. The set of joint densities $\{H_r(\hat{a}, q)\}_{r=1}^R$ describes the (endogenous) state of the economy. Feasibility requires that the sum of the marginals over q , which are

denoted by H_r^Q , coincide with the exogenous density of productivities:

$$\sum_{r=1}^R H_r^Q(q) = g(q). \quad (6)$$

The planner's policy is a set of feasible matches (joint densities) $\{M_r(\hat{a}, q)\}_{r=1}^R \in \Delta(A \times Q)$ that satisfy $M_r^A(a) = F_r(a)/R$ for all r and $\sum_{r=1}^R M_r^Q(q) = G(q)$.

The planner assigns managers to projects in order to maximize the total match surplus net of the deadweight costs associated with the formation of a new match.⁷ Formally, the planner solves:

$$\begin{aligned} V(\{H_r\}_{r=1}^R) = & \max_{\{M_r\}_{r=1}^R} \sum_{r=1}^R \int_{A \times Q} \left(\mathbb{E}_{\hat{a}, r}[x(a, q)] - \mu \mathbb{1}[M_r(\hat{a}, q) - H_r(\hat{a}, q) > 0] \right) dM_r(\hat{a}, q) \\ & + \delta V(\{H'_r\}_{r=1}^R), \end{aligned} \quad (7)$$

where

$$\mathbb{E}_{\hat{a}, r}[x(a, q)] = \int_A x(a, q) (2\pi\sigma_r^2)^{-\frac{1}{2}} \exp\left[-\frac{(a - \hat{a})^2}{2\sigma_r^2}\right] da$$

and

$$\sigma_r^2 = \frac{\phi + (r+1)\psi}{\psi(\phi + r\psi)}.$$

The law of motion for H is an application of the Kalman filter in equations (4) and (5) to the set of joint densities $\{M_r\}_{r=1}^R$, with all the uncertainty over innovations integrated out. Formally, we write $H'_{r+1} = \kappa(M_r)$, for $r \in \{1, \dots, R-1\}$. Since agents exit the economy deterministically after R periods, we know that $H_r(\hat{a}, q) = 0$ for any $q \in Q$ and $r > R$. We also assume that all agents in the age cohort $r = 1$ are unmatched and therefore we have $H_1(\hat{a}, q) = 0$.

Before we attempt to characterize the solution of the dynamic program more sharply, it is worth mentioning that the planner will sort agents and projects positively whenever he *can*, provided μ is sufficiently small.⁸ In particular, due to the supermodularity of $x(\cdot, \cdot)$ all pairs

⁷To be precise, the planner aggregates the utility valuations of all agents in the economy. Since individual preferences are linear, there is no consumption-saving decision and aggregate income equals aggregate surplus. This implies that the planner simply maximizes the total surplus net of total matching costs.

⁸We have not yet derived sufficient conditions for positive sorting in the presence of matching costs, except in the simple $R = 2$ case.

that are separated will be re-matched assortatively. In the special case with $\mu = 0$, $\{M_r\}_{r=1}^R$ will implement perfect sorting *ex ante*. When μ is positive, the gain from separating and reassigning a CEO may not be big enough to offset the deadweight losses associated with the formation of new matches and the planner's policy will not implement the unconstrained first best.

Since the youngest agents in the economy are unattached, the planner can and will sort them positively. Put differently, the assignment of age-1 managers to projects is a bijection $G_1 : A \rightarrow Q$. This eliminates, to the best of our knowledge, potential multiplicities in the optimal assignment of CEOs to projects.

To build some intuition for the optimal assignment in this overlapping generations model, it helps to first consider the frictionless case.

3.1 Special Case with $\mu = 0$

In the special case with zero matching costs, the planner's dynamic program collapses to a sequence of static problems. The efficient assignment that solves the Bellman equation (7) with $\mu = 0$ is such that:

1. For each $r \in \{1, \dots, R\}$, the assignment of managers to firms is a strictly increasing one-to-one correspondence $m_r : q \rightarrow \hat{a}$ from project qualities to beliefs.⁹
2. For each q , optimality implies

$$\mathbb{E}_{\hat{a}_r, r}[x(a, q)] = \mathbb{E}_{\hat{a}_{r'}, r'}[x(a, q)], \text{ for } r, r' \in \{1, \dots, R\}.$$

3. Lastly, the resource constraint imposes

$$\frac{1}{R} \sum_{r=1}^R \left[F_r(m_r(q)) - F_r(m_r(\min(q))) \right] = G(q).$$

3.2 General Case with $\mu \geq 0$

The planner's Bellman equation for the more general $\mu \geq 0$ case provides only limited intuition for the constrained optimal allocation of CEOs to projects. To fix ideas, consider a version of this economy with $R = 2$. When managers – both current and prospective – reach age R , the planner decides whether to leave them in place (i.e., they remain with their cur-

⁹The proof relies on the fact that (2) implies that $\mathbb{E}_{\hat{a}_r, r}[x(\cdot, \cdot)]$ also exhibits weakly increasing differences.

rent match q or unmatched), assign them to a different project q' (re-assignment), drop them altogether (exit), or “promote” them to a managerial position for the first time (entry).

For the time being, we’re only considering stationary solutions to the planner’s problem. The constrained-efficient transition dynamics are beyond the scope of this paper.

Since everybody retires at age R , the value of (always) forming a new match between a manager of that age with belief $\hat{a}$ to a technology q is:

$$v(\hat{a}, q; R) = \frac{\mathbb{E}_{\hat{a}_R, R}[x(a, q)] - \mu}{1 - \delta}. \quad (8)$$

Since $v(\hat{a}, q; R)$ is super-modular, the planner will sort positively between the available managers and technologies. In contrast to the special case with $\mu = 0$, the distribution of beliefs and projects in this “assignment pool” is not exogenous. It depends on last period’s matches (when they were aged r) and the planner’s choice to break up or preserve them once they reach age $r + 1$.

To decide whether or not to break up an existing match, the planner needs to know the efficient assignment $m_R(q)$. The reason is that the value of a recently separated manager depends on the “best” available assignment today and that in turn depends on how many other CEOs are being released from the current match.

To fix ideas, consider the case of a project q that “inherited” a CEO with updated belief $\check{a}$ and age R .¹⁰ To characterize the optimal break-up decision, we need to consider the possible alternatives. If q was in the assignment pool, the planner would assign the project to a CEO with belief $\hat{a}_R(q) = m_R(q)$. Similarly, if $\check{a}$ was single and the image of some q in the domain Q he would assign her to a project of quality $m_R^{-1}(\check{a}) = q(\check{a})$. Otherwise she remains unmatched (since $0 < N < 1$ by assumption).

The planner is indifferent between preserving and breaking up the $(\check{a}, q)$ match if there exists a belief $\dot{a}$ such that:

$$v(\dot{a}, q; R) + v(\hat{a}(q), q(\dot{a}); R) = v(\dot{a}, q(\dot{a}); R) + v(\hat{a}(q), q; R) - \left(1 + \mathcal{I}(q(\dot{a}) \in Q)\right)\mu. \quad (9)$$

In fact, there are two beliefs that satisfy this indifference condition and we denote them by $\underline{a}_R(q)$ and $\bar{a}_R(q)$. The former is too *incompetent* to remain in the same match while the latter is too *competent*. The planner preserves all the matches between q and CEOs with beliefs $\underline{a}_R(q) \leq \hat{a} \leq \bar{a}_R(q)$ and he splits up the remainder.

¹⁰The planner matched this CEO to q yesterday since she had belief $m_1(q) = \hat{a}$. Between yesterday and today, she observed one more productivity shock and updated her belief from $\hat{a}$ to $\check{a}$. She also “updated” her age from 1 to 2 and this matters because of (5).

Now that we know what the planner does with CEOs when they reach age R , we can characterize the planner's optimization problem for young CEOs. By assumption, individuals of age 1 enter the economy as singles and the planner anticipates his optimal assignment policy for older CEOs when he forms the initial assignments. In particular, using the separation thresholds we derived earlier he can derive the probability of future separations and the manager's posterior distribution of beliefs conditional on her match status. Once we put all the pieces back together, the value of assigning a CEO with belief $\hat{a}$ when she is young (i.e. $r = 1$) to a project q is:

$$v(\hat{a}, q; 1) = \frac{\mathbb{E}_{\hat{a}_1, 1}[x(a, q)] - \mu + \delta \int_{\underline{a}_2(q)}^{\bar{a}_2(q)} \mathbb{E}_{\hat{a}_2, 2}[x(a, q)] dF_{\hat{a}_2|\hat{a}_1}(\hat{a}_2)}{1 - \delta \Pr(\hat{a}_2|\hat{a}_1 \notin [\underline{a}_2(q), \bar{a}_2(q)])}.$$

The assignment that solves (7) in this two-period problem is such that:

1. The assignment functions are strictly increasing one-to-one correspondences $\{m_r : q \rightarrow \hat{a}\}_{r=1}^2$.
2. For each q , the planner is indifferent between CEOs of different ages. That is, $\hat{a}_1(q) = m_1(q)$ and $\hat{a}_1(q) = m_2(q)$ are such that:

$$v(\hat{a}_1(q), q; 1) = v(\hat{a}_2(q), q; 2).$$

3. The matching functions $m_1 : q \rightarrow \hat{a}$ and $m_2 : q \rightarrow \hat{a}$ are feasible, that is:

$$\tilde{G}(q) \geq \frac{1}{2} \left[F_1(m_1(q)) - F_1(\min(q \in \mathbb{Q})) \right] + \tilde{F}_2(m_2(q)) - \tilde{F}_2(\min(q \in \mathbb{Q})) \Big],$$

where

$$\begin{aligned} \tilde{G}(q) &= \int_{m_1(\min(q \in \mathbb{Q}))}^{m_1(q)} g(m_1^{-1}(\hat{a}_1)) \\ &\quad \times \left(\Pr(\hat{a}_2|\hat{a}_1 \notin [\underline{a}_2(m_1^{-1}(\hat{a}_1)), \bar{a}_2(m_1^{-1}(\hat{a}_1))]) \right) \frac{1}{2} dF_{\hat{a}_1, 1}(\hat{a}_1) \\ &\quad + \int_{m_2(\min(q \in \mathbb{Q}))}^{m_2(q)} g(m_2^{-1}(\hat{a}_2)) \frac{1}{2} dF_{\hat{a}_2, 2}(\hat{a}_2) \\ \tilde{F}(\hat{a}) &= \int_{-\infty}^{m_1(\min(q \in \mathbb{Q}))} F_{\hat{a}_2|\hat{a}_1}(\hat{a}) dF_{a_1}(\hat{a}_1) \\ &\quad + \int_{q \in \mathbb{Q}} \left[\int_{-\infty}^{\min\{\hat{a}, \underline{a}_2(q)\}} dF_{\alpha|m_1(q)}(\alpha) + \int_{\min\{\hat{a}, \bar{a}_2(q)\}}^{\hat{a}} dF_{\alpha|m_1(q)}(\alpha) \right] dF_{\hat{a}}(m_1(q)). \end{aligned}$$

The distribution of projects in the assignment pool, denoted by $\tilde{G}(\cdot)$, is stationary or,

put differently, the distribution of projects that is “stuck” in a matched formed in the previous period is the same every period.

The solution to the yet more general case with $R > 2$ satisfies analogous conditions, but is numerically more demanding to implement. This is currently work in progress. In the meantime, it is informative to compare the constrained efficient assignments in the two-period version of the model with different values of μ .

3.3 Effect of μ in Two-Period Version of Model

There isn’t much we can characterize in closed form so instead we impose functional forms for G (the distribution of projects) and $x(\cdot, \cdot)$, plus parameters that govern the distribution of beliefs (namely ϕ and ψ), and the aggregate measure of projects N .

We assume that projects follow a unit-scale Generalized Pareto distribution with tail index $\xi > 0$. The match surplus technology is CES with substitution elasticity σ , and share parameters λ^a and λ^q for the CEO and the project, respectively. Moreover, the manager’s contribution is exponentiated and is thus effectively log-normal.¹¹

$$G(q) = 1 - (1 + \xi q)^{-\frac{1}{\xi}} \quad (10)$$

$$x(a, q) = \left(\lambda^a \exp(a + \epsilon)^{\frac{\sigma-1}{\sigma}} + \lambda^q q^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} \quad (11)$$

Figure 1 shows the age-specific assignments for different values of the matching cost μ together with the frictionless matches (dashed line). Two patterns emerge. Since it’s optimal to assign some young CEOs to projects, some of them will not be in the assignment pool one period later. The drop in the availability of projects affects the two age cohorts differentially.

All else equal, young CEOs are less affected by the presence of matching costs since they remain in their original assignment with positive probability. Old ones, on the other hand, will be separated from their project with probability one and hence incur the cost μ every period. The left panels 1a, 1c, and 1e show that $m_1(\mu') < m_1(\mu)$ for $\mu' > \mu$. Put differently, the presence of matching costs favors the participation and assignment of young CEOs. The effect on older managers is more subtle. Since μ is more onerous relative to the value for low-productivity / low-ability matches, old and incompetent CEOs tend to lose out while

¹¹The CES technology is convenient since a single parameter governs the substitution elasticity. An alternative functional form would be bilinear as in [Lindenlaub \(2017\)](#). To deal with the possibility of non-positive realizations of $a + \epsilon$, we exponentiate the managers contributions and effectively assume that managerial ability follows a log-normal distribution. This has been done elsewhere (see [Alder et al., 2017](#), for example).

high-ability types benefit. Although it's not immediately obvious from these plots, positive matching costs tend to discourage *entry* by old CEOs and we will return to this in the discussion of our – admittedly preliminary – calibration results. In addition, the aggregate cost of this matching friction in terms of foregone surplus is quite sizeable. Figure 2a shows the drop in aggregate surplus for μ between 0 and 0.125, assuming that the matching cost itself is deadweight. If, in contrast, we only take the loss associated with *mismatch* into account, the corresponding drop in the aggregate surplus shown in Figure 2b (coming very soon!) is driven by the *mismatch* associated with μ . The output gap discrepancy between 2a and 2b captures the deadweight loss from μ .

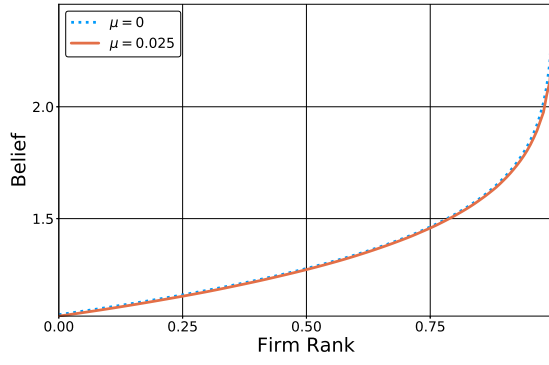
4 Entry, Exit, and Assignment Patterns in the Cross-Section and over the Life-Cycle

The mobility patterns of CEOs in this model are governed by the evolution of the age-specific cross-sectional distribution of mean beliefs, the precision (i.e., the inverse of the variance) of the individuals' beliefs, the size of the matching cost μ , and the amount of curvature in the production technology $x(\cdot, \cdot)$. While matching is assortative in beliefs across members of a given age cohort who have their most recent assignment (including the outside option) in common, the matching cost generates assignment “overlap” when we consider the various decisions of an entire cohort. The planner may assign CEOs with identical beliefs and of the same age to different projects and this complicates the CEOs' participation, separation, and exit decisions. In fact, we have to rely on numerical solutions to describe these lifecycle patterns.

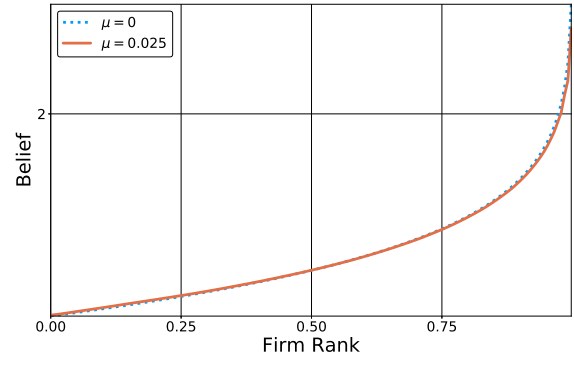
When the fixed matching cost is positive, the sorting of agents across productivities *within* age cohorts is not perfect and it is a bit of a challenge to highlight the role of the remaining parameters transparently. To build some intuition for the various forces that shape sorting patterns in the economy, let us consider a very stylized model endowed with a unit measure of potential managers, costless matching ($\mu = 0$), and three age cohorts.

4.1 Illustrative Parameterizations with $\mu = 0$

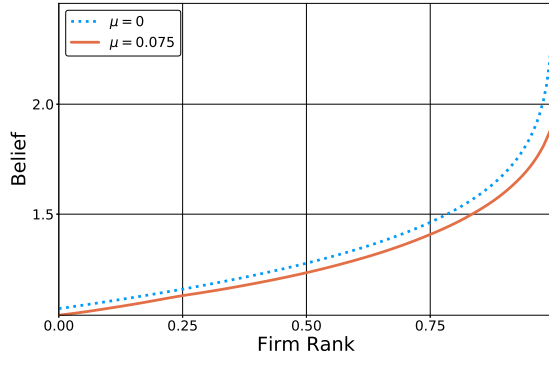
We start this exercise with the following set of parameter values: $\psi \rightarrow \infty$, $\rho \rightarrow \infty$, and $N = 1$. Put differently, the agents' beliefs coincide with the distribution of true types in the economy (and hence do not evolve with the agents' age), the surplus technology is (almost) linear, and



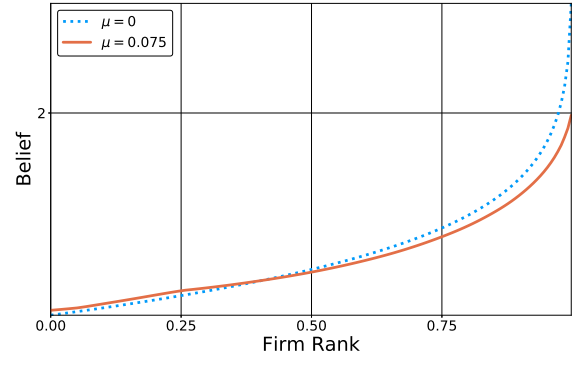
(a) $\mu = 0.025, r = 1$



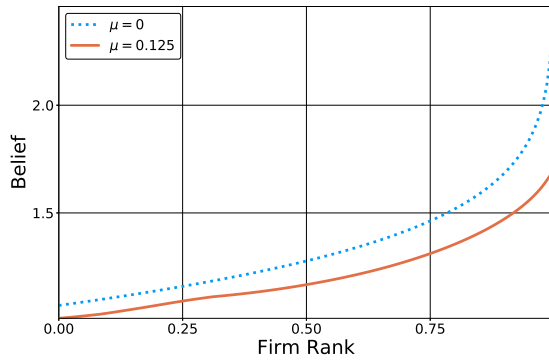
(b) $\mu = 0.025, r = 2$



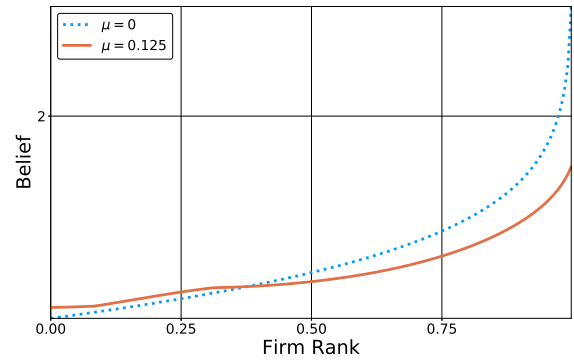
(c) $\mu = 0.075, r = 1$



(d) $\mu = 0.075, r = 2$

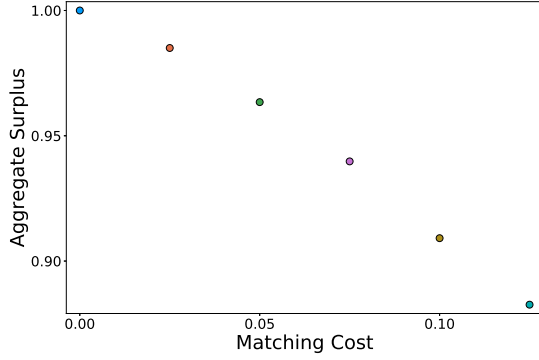


(e) $\mu = 0.125, r = 1$



(f) $\mu = 0.125, r = 2$

Figure 1: Optimal Assignments under Different Values of μ



(a) When μ is a Deadweight Loss

(b) When μ is Rebated Lump Sum

Figure 2: Effect of μ on Aggregate Surplus

the planner can match all the individuals to a project.¹² We plot all the results by quintile of the distribution of productivities. As Figure 3a illustrates, the assignment of managers to projects is identical across the three age cohorts.

In Figure 3b, managers have imprecise beliefs and sorting with $\psi < \infty$ is affected by two distinct forces: First, the mean beliefs of young agents are more tightly clustered around the mean of the distribution and that generates the inverted *U*-shape of the blue bars. Second, since the managers' contributions are effectively log-normal, the assignment of young CEOs is skewed to the right of the size distribution of firms. Since all agents of all age cohorts participate in this assignment market, market clearing implies that assignment of older CEOs is skewed to the left of the size distribution.

When we tighten the degree of complementarity as in 3c, the curvature partially offsets the skewness of the managers' effective contributions to the match surplus. Young CEOs are more tightly clustered around the median productivity whereas older managers tend to be assigned more frequently to extreme project types in either tail of the size distribution.

Next, let's consider the case where the supply of potential managers exceeds the aggregate measure of projects, that is, $N < 1$. In the example illustrated in Figure 3d, $N = 0.1$. Since the distribution of mean beliefs varies by age, the measure of matched managers also differs across cohorts. Older agents are more likely to be managers than their more junior colleagues. This is, in fact, a pattern that we also observe in the data.

¹²In this stylized thought experiment, the substitution elasticity is high, but finite. When the technology is linear, the assignment problem is uninteresting since optimality does not require perfect sorting. Any assignment maximizes the total surplus. A tiny bit of curvature is enough, however, to restore sorting while marginal products remain largely functions of the manager's (or firm's) own type and this is exactly the point from where we want to build the intuition for the effects of alternative parameter values.

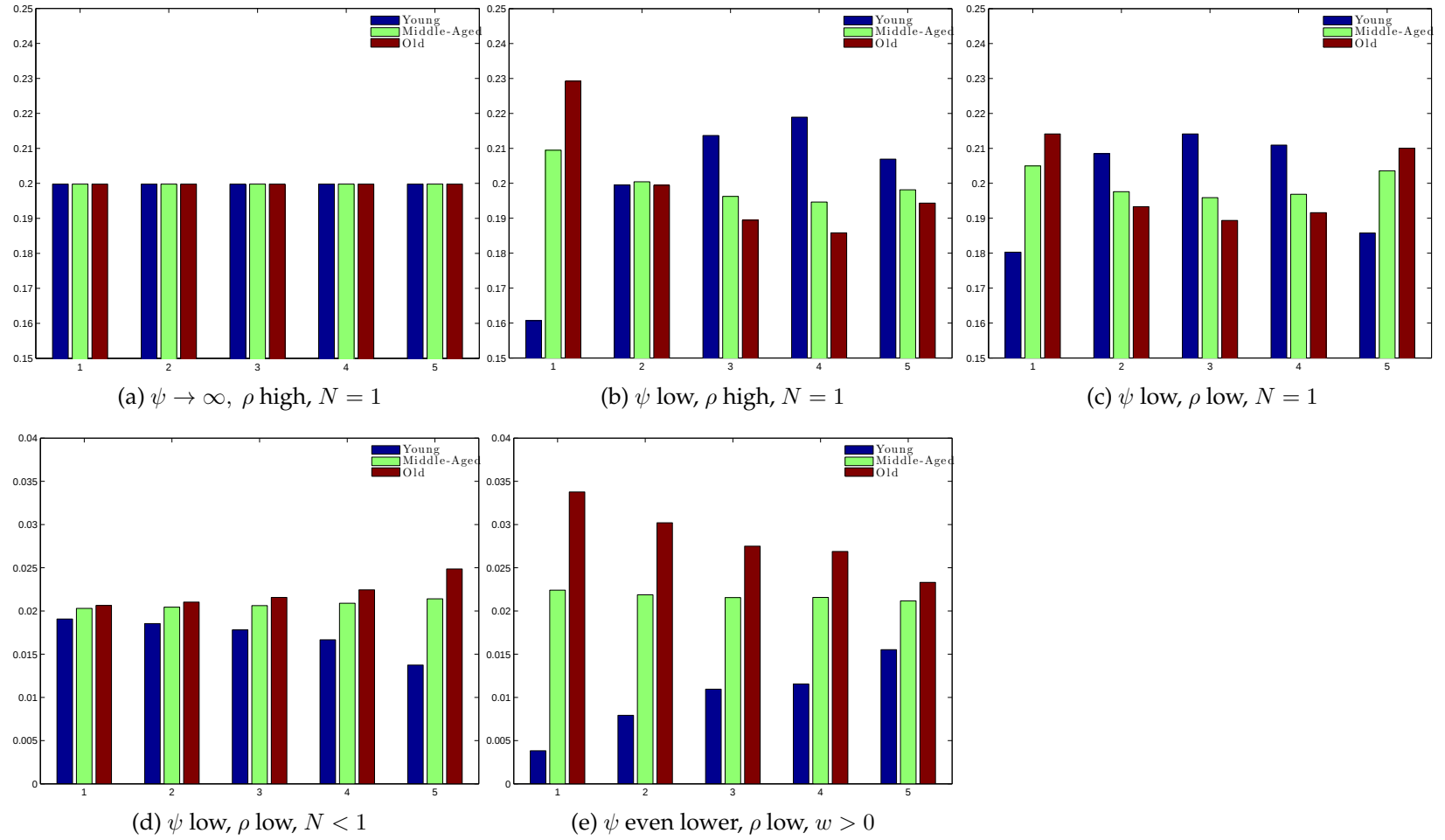


Figure 3: Stylized Sorting Patterns by Age and Size Quintile, $R = 3$, and $\mu = 0$

Lastly, if participation is not universal, the precision of the productivity shocks (ψ) affects the hazard rate into managerial work or entrepreneurship across different age groups. In Figure 3e, for instance, as the innovations become less precise, young managers are less likely to participate in the market, but there is a differential effect across the size distribution of firms. The planner is less averse to matching a young manager to a high productivity project compared to a low productivity one.

In sum, the three key parameters generate a fairly rich sorting pattern across the managerial talent and productivity distributions. Furthermore, the matching cost μ interacts with these three parameters in non-trivial ways and we are currently working on an analogous stylized example (in section 4.2).

As for the parameterization of the model, the challenge is to find a set of moments in the data with corresponding counterparts in the model in order to discipline the parameterization. Before we turn our attention to the calibration strategy, we discuss the data used in the quantitative exercise in some detail. This is, in fact, what we are turning our attention to in the next two sections.

4.2 Illustrative Parameterization with $\mu > 0$

TO BE COMPLETED.

5 Data

We use information on CEOs and firm accounting data from three different data sources during the period 2000 to 2009. The first is the Danish administrative register data covering 100 percent of the population of individuals and firms in the years 2000 to 2009. The second is from *Købmændstaden Oplysningsbureau* (KOB), which contains accounting data from the firm population dating back to 1991. The third is from the Danish Commerce and Companies Agency (*Erhvervs- og Selskabstilsynet*, or ES) at the Ministry of Economic and Business Affairs and has information on managers of firms from 2000 to 2010. Personal information on managers can be linked from the ES data to the register data through a personal id number and the information on firm accounting data can be linked from the KOB data to the register data through a firm identifier. The register data further contains a link between the firm and the manager identifiers. Using all three datasets we can identify the CEOs of all Danish publicly and privately held stock companies and link them to their respective firms' accounting data.

Our first dataset is administrative register data, from Statistics Denmark, covering 100 percent of the population in the years 2000 to 2009. The Statistics Denmark data is from the

Integrated Database for Labor Market Research (IDA), which contains annual information on socioeconomic variables (e.g., age, gender, education), characteristics of employment (e.g., wages, earnings, occupations, industries), and a employer-employee link for the population. The employer-employee links only exist for those individuals who held a job during the last week of November in any given year.

Clearly, we do not need prices to calibrate the planner's problem, yet we extract a fair amount of information on the managers' compensation from our dataset and we do so for two reasons. First, the data is of independent interest in that it offers a general description of the population we are focusing on. Moreover, since we plan to decentralize the planner's problem at a future stage, we decided to discuss prices and allocations in a single comprehensive data section.

For the managers in our dataset, we generate a measure of yearly earnings that contains yearly taxable salaries including perks, tax-free salaries, jubilee and severance payments, and the value of stock options and futures. These values are recorded when they are taxed, that is, when the options are exercised or the stock grants are vesting. The measure of earnings further contains payment for work on any board of directors. Payments for consulting work and other work related to giving presentations that are distinct from the tasks carried out in their capacity as CEOs of their employer are not included in the earnings variable.

The Statistics Denmark individual register data can be linked, through a firm identifier, to our second dataset, KOB, that contains firm accounting data, covering 100 percent of the publicly and privately held stock companies in the population since 1991. The KOB dataset is collected and digitized from scanned documents at the Ministry of Economic and Business Affairs by a private data collector called Experian. According to Danish law, all stock companies must provide the Ministry of Economic and Business Affairs with information about the firms' assets and measures of profitability such as operating cost and net income. Other types of firm information (e.g., the number of employees, total wage bill, sales) are provided on a voluntary basis by a majority of the firms. The firms' reported accounting data are subject to random auditing by external accountants and the reliability of the accounting data is accordingly high.

Our third dataset, ES, is provided by the Danish Ministry of Economic and Business Affairs and contains information on all CEOs and board members of publicly and privately held stock companies in Denmark as well as information about the founders of the firms. The dataset contains information about the start and end dates (where applicable) of each spell for all CEOs, board members, and founders. Thanks to an individual ID number we can link the individuals in the ES data to their background characteristics in the Statistics Denmark dataset. The dataset also contains information on the type of firm the managers work for (e.g., the legal form of the firm) and the year the firm was founded in.

We deflate the labor earnings, assets, and value added to the 2000 level using Statistics Den-

mark's consumer price index.

5.1 Sample Selection

We use privately and publicly held stock market firms in Denmark containing employees who work either part time or full time during the period 2000-2009. We then select those firms where the CEO has wage work in the firm as his or her main occupation during the last week of November of a given year.

While we can match a CEO to 94 percent of all stock market firms in Denmark, only 74 percent of the firms have a CEO who has his or her main employment in the firm. Since our focus is on the match between CEOs and firms, we exclude all firms without a CEO with a main employment in the firm. The excluded firms are smaller than the average size since all included firms account for 80 percent of total employment. Of the matched firms, 8 percent have more than one registered CEO and for these firms, we identify the "unique" CEO as the individual with the highest occupational code (e.g., "Directors and chief executives" is a higher occupational code than "Productions and operations manager").¹³ If a firm has two or more registered CEOs who have the same occupation code, we select the individual with the highest earnings. In a small fraction, 0.2 percent, of the firms the co-CEOs have identical earnings and we exclude them from the sample for the time being.¹⁴

For our calibration we need accounting data and we therefore match the Statistics Denmark data to the accounting data from KOB.¹⁵ We match 95.5 percent of workers and 93.5 percent of the firms, where the unmatched firms are smaller than average or firms that terminate during our selected sample period. We exclude the firms with unmatched KOB accounting data and we also exclude firms in the years they have missing information on value added, total assets, or fixed assets. Less than 1 percent of firm-years have missing data on total assets or fixed assets and there are 4 percent of firm-years with missing value added. The firms with missing information are also primarily firms that are in their termination year or firms that have recently entered the sample. In total, this gives a base sample of 149,272 stock companies that all have non-missing accounting data and an identified CEO whose main occupation is as CEO of the company.

We think of a CEO as someone who manages people and we therefore select CEOs from firms with five or more employees. For the time being, we also exclude stock companies where

¹³The occupation codes are from the Danish version of the ISCO codes.

¹⁴We exclude them for the simple reason that we cannot structurally connect managerial "teams" to our theory. Eventually, we need to find a way to include them.

¹⁵In the calibration of the planner's problem, we only need employment and, to check for robustness, value added from the KOB data. In future work, we plan to use additional data (e.g. the value of the firms' intangible capital) to calibrate the decentralized equilibrium.

the CEO is the founder of the company because of problems separating company profit from CEO earnings when the CEO is also the founder. This reduces the sample of stock companies to 85,791. In the remaining sample of CEOs 7 percent are women and we exclude them to control for potential gender pay gap.¹⁶ We also exclude CEOs without a valid education, which leaves a sample of 79,995 male CEOs. In order to avoid complication with CEO retirement we further exclude all companies with CEOs of age 60 and above in 2009.¹⁷ We also exclude all CEOs below the age of 25 in 2000, which is less than 1 percent of the sample. Our final sample features 57,279 CEOs in stock companies employing 3.985 million workers. Descriptive statistics of the CEO and firm samples used in the analysis are in Tables 1 and 2 below.¹⁸

	Mean	Median	25 th percentile	75 th percentile
Number of CEOs	57,279			
CEO to CEO switchers across firms	0.0326			
Males	1			
Age	44.2	44	39	49
Potential experience	24.3	24	19	30
CEO tenure in firm	5.88	6	3	9
Hourly nominal wage in 2000 DKK	415.8	340	234	500
Total nominal earnings in 2000 DKK	709,811	571,824	390,309	848,562
Total nominal income in 2000 DKK	950,403	696,544	485,778	1,031,895

Table 1: Summary Statistics for CEOs

5.2 Empirical Moments

To calibrate our model, we match empirical moments to their model counterparts. To parameterize the planner’s problem, we rely on “assignment moments” exclusively. For completeness, we also discuss the moments we plan to use in a parameterization of the decentralized equilibrium, namely the cross-sectional and life-cycle patterns of CEO compensation. To start with, let us focus on the moments we use to parameterize the planner’s problem.

¹⁶Clearly, firms managed by their owners or founders and female CEOs are not much of a concern when we calibrate the planner’s problem where we need not worry about the split of the surplus between the firm’s owner and the firm’s CEO. We are in the process of reinstating these firms and then repeat the calibration exercise with updated targets.

¹⁷Selecting CEOs of age 59 or below in 2009 means that the oldest cohort of CEOs we include in the sample are the ones who are 50 in 2000 (and therefore 59 years old in year 2009).

¹⁸Note that between 2000 and 2009, the exchange rate fluctuated between 4.7 and 8.5 Danish Kronor per US Dollar.

	Mean	Median	25 percentile	75 percentile
Number of firms	57,279			
Number of employees	60	19	10	41
Total assets in 1,000 DKK	615,777	16,926	7,558	47,330
Intangible assets in 1,000 DKK	536,348	10,877	4,785	30,760
Value added in 1,000 DKK	44,262	9,838	5,037	22,837

Table 2: Summary Statistics for Firms

5.2.1 Allocations

We can generate the “net” assignment moments fairly directly from the data. In fact, the description of entry, exit, and assignment patterns (cross-sectionally and over the life-cycle) in section 4 is all we need. Since we work with only a handful of parameters, we will select only the most salient ones from the full set of moments. We will return to this point when we discuss our calibration strategy in section 6.

The more challenging task is the characterization of *gross* flows, particularly those related to the re-assignment of CEOs from one project to another. We calculate the CEO-to-CEO transition probability based on observed changes of employer, conditional on (plausibly) keeping the title of CEO. To the best of our abilities we are taking the effect of mergers, acquisitions, and spin-offs into account when we classify these separations. For our purposes a switch is a CEO-to-CEO transition that happens between year t and $t + 1$ or between t to $t + 2$, separated by a year of not being a CEO in the sample. The reason for allowing a transition year is a peculiarity with respect to occupational classifications in our sample. CEOs are labeled as such if their job in November is their *main* employment during the year. By allowing a transition year we can account for CEO-to-CEO transitions that take place toward the end of the year, but before the end of November. In this case, their November employment is not their main employment for that year. In the CEO-to-CEO transitions we therefore condition the CEO in year t on also being a CEO in year $t + 1$ or $t + 2$. The average CEO-to-CEO transition probability across firms is 2.6 percent.¹⁹

We estimate the quality distribution of firms in a number of different ways. One of the methods does not rely on prices and simply assumes that the firms’ productivities (which capture the joint contribution of the underlying project’s quality as well as the ability of the assigned

¹⁹To be classified as a CEO-to-CEO transition, the CEO in question must be in the CEO sample in the year $t + 2$. Clearly, then, the relevant comparison group are CEOs who show up twice in at least one $[t, t + 2]$ period in the sample. The corresponding sample size is 43,220 observations.

CEO) are proportional to employment.²⁰ Other proxies for size rely on value added, which has a direct counterpart in the planner’s problem, namely the match surplus, or the firms’ intangible capital stock. The shape parameter ξ that characterizes the tail shape of a Generalized Pareto distribution is somewhat sensitive to the choice of firm size proxy and we discuss the implications in section 6.

5.2.2 Prices

In addition to the allocation moments above, we generate three “price” moments in the data. The first one is *Roberts’ Law*, the (cross-sectional) elasticity of CEO pay with respect to firm size, which has been used in earlier work on CEOs (for instance, [Gabaix and Landier, 2008](#); [Alder, 2016](#)), and which we calculate by using either employment or value added as a proxy for firm size. We find that the elasticities are around 0.27 and are robust to conditioning on 1 to 4-digit industry fixed effects.

The average CEO share of the total match surplus is 0.33 and it is calculated by finding the average of CEO earnings over the sum of CEO earnings and the flow value of intangible assets.²¹

The annual wage growth of CEO compensation over the lifecycle is found by calculating the average growth rate in earning by age for each cohort of CEOs. To estimate this semi-elasticity we follow [Beaudry and Green \(2000\)](#) and regress log earnings on a cubic polynomial in age, a quadratic polynomial in cohort, and a linear interaction term between age and cohort while controlling for business cycle conditions.²² As in [Beaudry and Green \(2000\)](#) we perform this regression for the log of cohort-year averages in earnings, which, after taking averages, has 90 observations. The marginal effect of age on log earnings equals the excess annual growth rate of CEO compensation (i.e., the differential between growth in CEO vs. non-CEO pay), which is 1.93 percent.

6 Calibration

How do the parameters we choose affect the various moments the model generates and which we try to match to their empirical counterparts? We already know from the stylized parameterizations in section 4.1 that variations in individual parameter values can generate a fairly

²⁰The model underlying this assumption is a Lucas span-of-control economy as in [Alder \(2016\)](#).

²¹This is the same methodology that was used for publicly traded US firms in [Alder \(2016\)](#).

²²Business cycle conditions follow the same procedure as [Beaudry and Green \(2000\)](#). In a nutshell, they are the residuals from a linear regression of the unemployment rate of the population of men aged 25-60 on a quadratic time trend.

rich set of changes in model-generated moments. This is, of course, a feature that ours shares with many other model and we try to highlight the main “thrust” of an individual parameter as much as possible.

At this point, a few preliminary remarks are in order. First, we currently set $\mu = 0$ since it is computationally far less demanding. It will be quite obvious, however, that assignment frictions play a critical role and we discuss how and what they matter for below. Second, it is without loss of generality to assume $\lambda^q = 1 - \lambda^a$ (Alder, 2016). Lastly, since we cannot observe the managers’ marginal products, we cannot pin down the curvature of the surplus technology, which is parameterized by σ . The key to identifying this degree of complementarity is the variation in marginal products associated with imperfect sorting *ex post*. For example, instances where the same firm is run by two (or more) managers who turn out to have different managerial abilities is informative about this curvature, as are instances where the same manager is paired with different firms over the course of her career. Since the planner doesn’t (need to) keep track of the managers’ employment history to assign them optimally in the special case where $\mu = 0$, the degree of complementarity isn’t well identified. This being said, several of the parameters are jointly identified, namely ψ , λ^a , and σ and our numerical results imply that the surplus technology exhibits a less than unitary substitution elasticity. We can impose more empirical discipline on this elasticity when we use the evidence on managerial compensation and firm performance in the calibration of the decentralized equilibrium (similar to Groes et al., 2015; Alder, 2016), which is one of the next items on our to-do list. These limitations notwithstanding, let us now discuss our calibration strategy in some more detail.

6.1 Calibration Strategy

The distribution of firm sizes is a function of (1) the distribution of (true) managerial abilities, (2) the process governing the productivity shocks, and (3) the project qualities (or productivities). While the noise parameter ψ has some impact on the shape of the size distribution of firms, this moment is primarily driven by the precision of the normal distribution of true abilities ϕ and the tail index ξ associated with the Generalized Pareto distribution of projects in (10). Since we’re not splitting the surplus between managers and owners in any particular way, we set $\phi = 1$ and pick a value for ξ such that the tail index of distribution of match surpluses equals its empirical counterpart of 0.51 (see Figure 4); $\xi = 0.48$ does the job.

The precision of normal productivity shocks (or signal) is denoted by ψ . It determines, via equation (4), the relative importance of priors and innovations in the agents’ Bayesian updates of their posterior mean beliefs. All else equal, a low ψ generates more volatile beliefs and hence triggers more frequent separations, entry and exit. Moreover, it governs the rate at which the distribution of mean beliefs converges toward the true mean-zero normal with

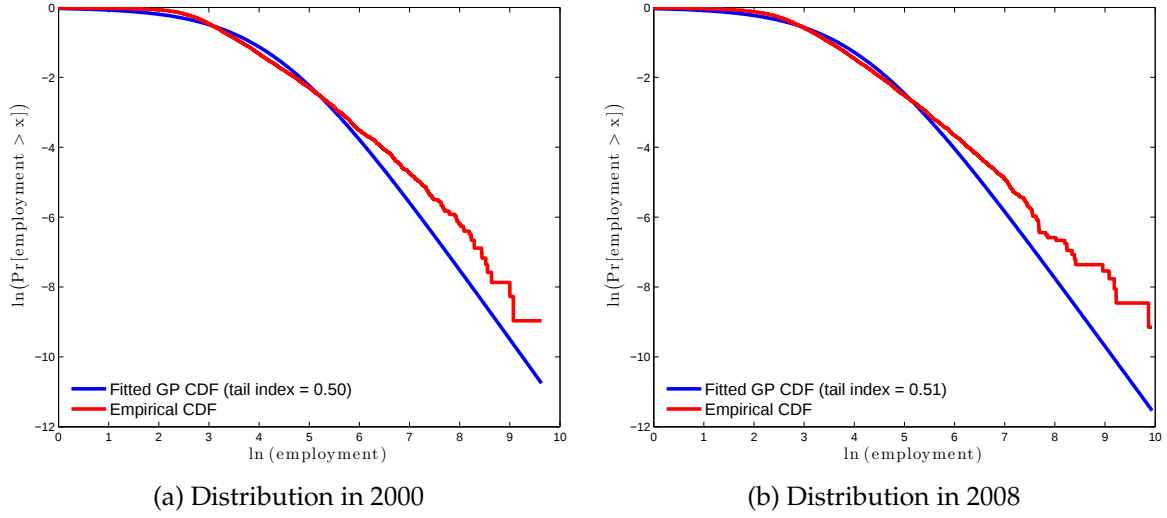


Figure 4: Danish Firm Sizes

variance ϕ^{-1} . But that it is not all. Since we exponentiate $a + \epsilon_t$ in the surplus technology, the distribution of the CEO's contribution is effectively log-normal and hence skewed.²³ The distribution of mean beliefs interacts with the skewness of effective contributions and with the curvature parameter σ to generate a rich pattern of participation and assignment across productivities.

The precision ψ interacts strongly with the curvature parameter σ associated with the match surplus function $x(\cdot, \cdot)$. We pick the value for ψ to match the fraction of young CEOs (age 25-29) in the data. It turns out that we need a fair amount of noise to match the lifecycle moments we observe in the data and we set $\psi = 0.035$. Figure 5 shows the aggregate age profile of CEOs in the data and in our model.

We choose λ^a to match the cross-sectional distribution of the largest group of managers in the data, those aged 40-44. In particular we match the 0.5 percentage point gap between the fraction of CEOs in the first and the tenth deciles of the firm size distribution (see the middle right panel in Figure 6). Since we don't have additional parameters we cannot target the hump-shaped cross-section separately but the model generates a pattern that mimics the data for this age group rather well.

Finally, we pick $N = 0.05$ to replicate the average firm size of 20 employees per firm. Although we do not model alternative occupations aside from managerial work explicitly, it would be fairly straightforward to do so using a Lucas span-of-control economy as in Alder

²³Any convex transformation of normally distributed beliefs generate such skewness. See Bolton et al. (2013) for a more detailed discussion.

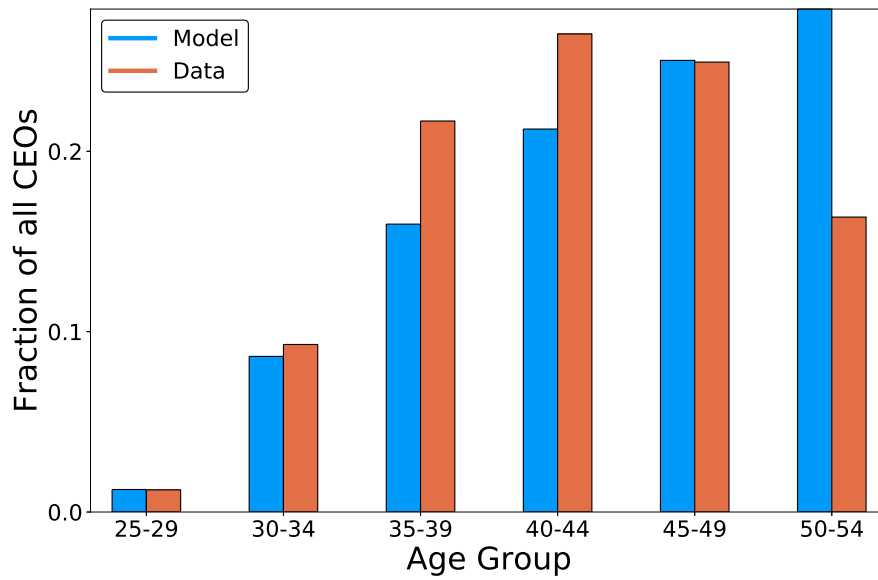


Figure 5: Age Distribution of CEOs

(2016). What we are trying to capture is that selection varies systematically with age and this requires a limited supply of management opportunities, which we can easily implement by picking $N < 1$.

6.2 Non-Targeted Moments

Since the number of parameters in the calibration exercise is quite small, we have a correspondingly small set of target moments. This being said, the model generates much richer assignment patterns across the age and productivity distribution. Remarkably, the model does a fairly good job in replicating them, as you can see in Figure 6.

What the model misses completely are the turnover patterns of CEOs. While the predictions for net entry and exit are in the ballpark, after all we indirectly used two of them as calibration targets, the gross flows are way off. This can be seen in Figure 7 (notice the difference in scale for the model and the data). We are particularly interested in transitions from one CEO assignment to another and these are actually quite low in the data. Moreover, they follow an inverted U-shape pattern. Young CEOs have high exit rates and don't switch to another executive job very often. Middle-aged managers are most likely to have a CEO spell followed by another one while the oldest tend to stay put or exit. Moreover, the latter have very low entry rates and the fraction of older CEOs actually drops once they are past their late 40s.

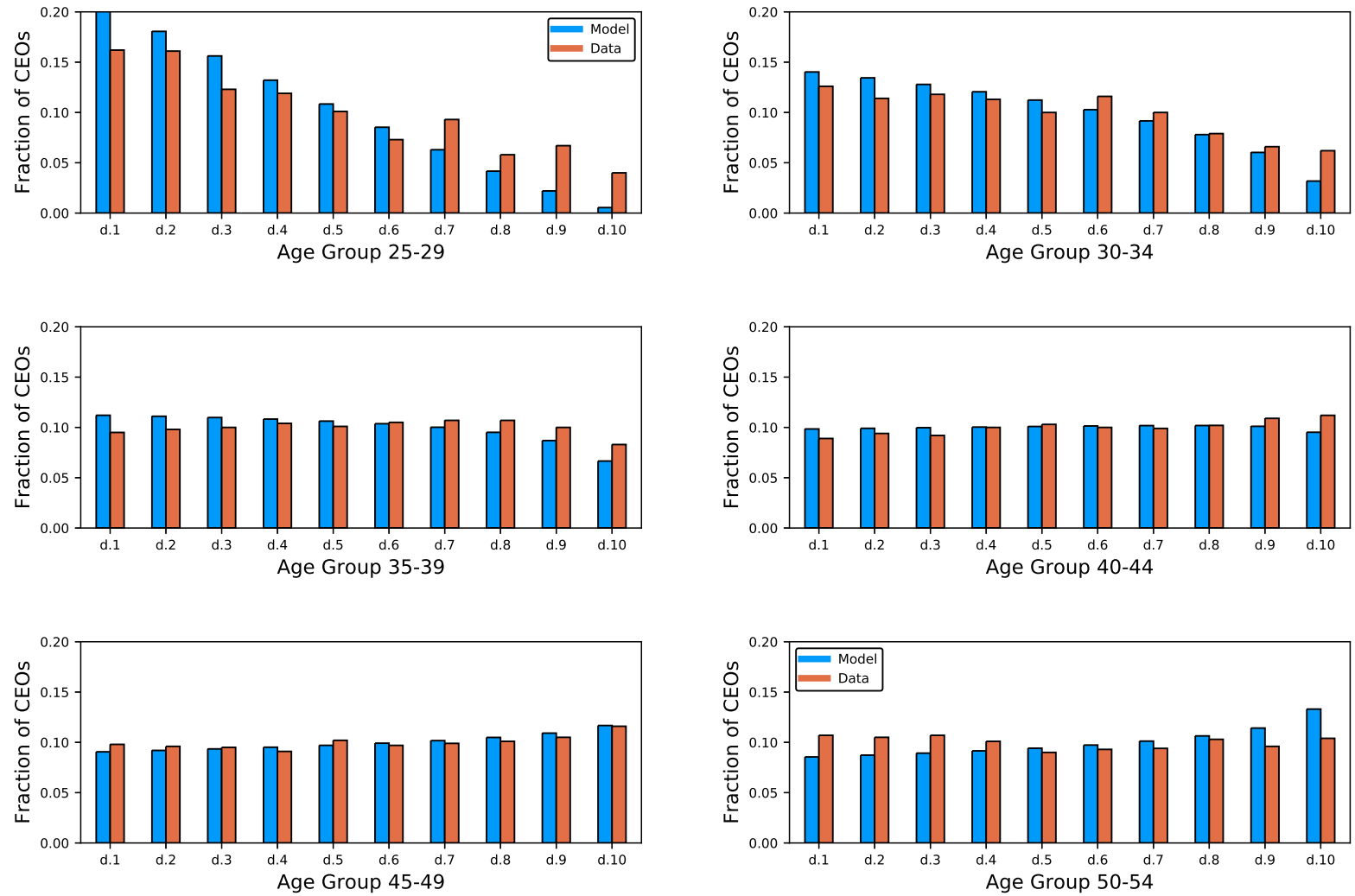


Figure 6: Distribution Across Firm Sizes by Age Group

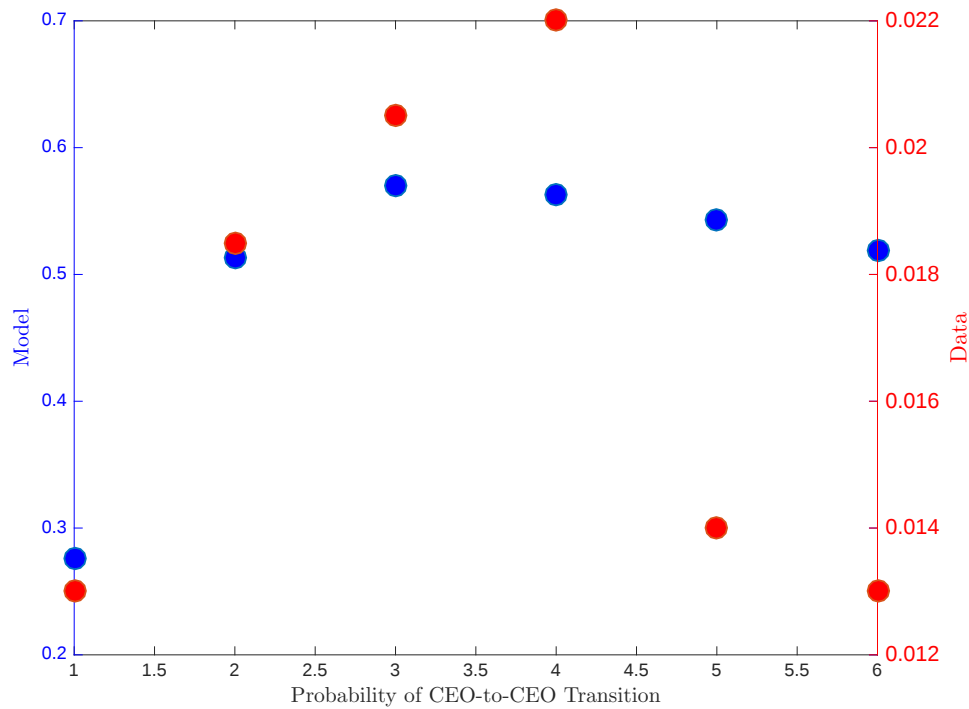


Figure 7: Probability of Transition from CEO to CEO

Our model generates an inverted-U-shaped transition pattern but with excessive churning and with a fairly strong inflow of new managers even as they approach retirement. This particular model prediction led us to introduce costly match formation in order to reduce the rate of turnover in general and to stem the entry of older managers in particular. At this point, this is still work in progress and we only have the stylized results from section 4.2.

7 Counterfactual Experiments

TO BE COMPLETED.

8 Conclusion

TO BE COMPLETED.

References

- ABOWD, J. M., F. KRAMARZ, AND D. N. MARGOLIS (1999): "High Wage Workers and High Wage Firms," *Econometrica*, 67, 251–333.
- ALDER, S. D. (2016): "In the Wrong Hands: Complementarities, Resource Allocation, and Aggregate TFP," *American Economic Journal: Macroeconomics*, 8, 199–241.
- ALDER, S. D., M. MEYER-TER VEHN, AND L. E. OHANIAN (2017): "Dynamic Sorting," Working Paper.
- BANDIERA, O., S. HANSEN, A. PRAT, AND R. SADUN (2017): "CEO Behavior and Firm Performance," *NBER Working Paper*, 1–59.
- BEAUDRY, P. AND D. A. GREEN (2000): "Cohort patterns in Canadian earnings: assessing the role of skill premia in inequality trends," *Canadian Journal of Economics*, 33, 907–936.
- BHATTACHARYA, D., N. GUNER, AND G. VENTURA (2013): "Distortions, Endogenous Managerial Skills and Productivity Differences," *Review of Economic Dynamics*, 16, 11–25.
- BOLTON, P., M. K. BRUNNERMEIER, AND L. VELDKAMP (2013): "Leadership, Coordination and Corporate Culture," *Review of Economic Studies*, 80, 512–537.
- ECKHOUT, J. AND P. KIRCHER (2011): "Identifying Sorting – In Theory," *Review of Economic Studies*, 78, 872–906.
- EVANS, D. S. AND B. JOVANOVIĆ (1989): "An estimated model of entrepreneurial choice under liquidity constraints," *Journal of Political Economy*, 97, 808–827.
- EVANS, D. S. AND L. S. LEIGHTON (1989): "Some empirical aspects of entrepreneurship," *American Economic Review*, 79, 519–535.
- GABAIX, X. AND A. LANDIER (2008): "Why Has CEO Pay Increased So Much?" *Quarterly Journal of Economics*, 123, 49–100.
- GROES, F., P. KIRCHER, AND I. MANOVSKII (2015): "The U-shapes of Occupational Mobility," *Review of Economic Studies*, 82, 659–692.
- GUNER, N., A. PARKHOMENKO, AND G. VENTURA (2017): "Managers and Productivity Differences," Working Paper, 1–64.
- JOHNSON, W. R. (1978): "A theory of job shopping," *Quarterly Journal of Economics*, 92, 261–278.

- JOVANOVIĆ, B. (1979): "Job Matching and the Theory of Turnover," *Journal of Political Economy*, 87, 972–990.
- (2015): "Learning and Recombination," *Working Paper*, 1–34.
- KNIGHT, F. H. (1921): *Risk, Uncertainty, and Profit*, New York: Houghton Mifflin.
- LINDENLAUB, I. (2017): "Sorting multi-dimensional Types: Theory and Application," *Review of Economic Studies*, 84, 718–789.
- LISE, J., C. MEGHIR, AND J.-M. ROBIN (2013): "Mismatch, Sorting and Wage Dynamics," *Working Paper*.
- LOPES DE MELO, R. (2018): "Firm Wage Differentials and Labor Market Sorting: Reconciling Theory and Evidence," *Journal of Political Economy*, 126, 313–346.
- LUCAS, R. E. (1978): "On the Size Distribution of Business Firms," *Bell Journal of Economics*, 9, 508–523.
- MILLER, R. A. (1984): "Job matching and occupational choice," *Journal of Political Economy*, 92, 1086–1120.
- NAGYPAL, E. (2007): "Learning by doing vs. learning about match quality: Can we tell them apart?" *Review of Economic Studies*, 74, 537–566.
- TERVIÖ, M. (2008): "The Difference That CEOs Make: An Assignment Model Approach," *American Economic Review*, 98, 642–68.