

# Escaping the Losses from Trade: The Impact of Heterogeneity on Skill Acquisition\*

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February 2018

## Abstract

While trade openness generates aggregate welfare gains, it can have unequal effects on the wage of skilled relative to unskilled workers. In this paper, we ask how heterogeneous the welfare gains of trade openness can be in the short and the long-run. To do so, we build a dynamic heterogeneous-household life-cycle model of international trade with incomplete credit markets. The model incorporates an endogenous costly skill acquisition, which allows unskilled workers to invest in education and escape the short-run losses of trade openness. We calibrate the model to match trends in trade openness in the United States between the late 1980s and 2010. We find that poor households take the longest to acquire skills and are therefore the last to experience positive gains from trade openness, which in some cases may not realize within a life-time. We also argue that welfare for all workers increases in the long-run, and that physical capital accumulation is essential for this result.

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# 1 Introduction

Trade openness affects worker groups unevenly. While the positive aggregate welfare gains from trade have been extensively documented—both theoretically and empirically—it is not until recently that studies have provided clear evidence of the unequal effects of trade openness on groups of worker.<sup>1</sup> In particular, trade openness has contributed to wage inequality over the last few decades, most notably to the increase in wages of workers with more education/skills relative to those with less.<sup>2</sup>

There are two key issues that arise when studying the effects of trade openness on welfare across groups of workers with different skill levels. First, workers’ skill levels are the result of endogenous decisions and adjust to changes in the economy environment. For instance, a high skill premium resulting from trade openness will incentivize unskilled workers to invest in education and eventually benefit from the high-skill wages.<sup>3</sup> Hence, unskilled workers that initially experience an adverse effect of trade openness could escape this losses by investing in skill acquisition.<sup>4</sup> The second issue, tightly linked to the previous one, is that even if workers would find it beneficial to invest in education, they may lack the resources to do so. Basically, since earnings risk is pervasive and only partially insurable, imperfections in insurance and credit markets will distort skill acquisition decisions.<sup>5</sup> Thus, workers with similar ability but different wealth may pursue alternative education investment decisions and experience heterogeneous benefits of trade openness.<sup>6</sup> It is also because of these factors that initial inequality can shape the effects of trade openness, as well as inequality going forward.

In this paper, we explore the effects of trade openness on welfare and inequality by taking into account these two issues. To carry out our analysis we build a dynamic life-cycle model of international trade with heterogeneous agents and incomplete credit markets. The model allows for costly endogenous skill acquisition which, jointly with idiosyncratic risk, induces an equilibrium distribution of wealth. These features result key to understand the short and long-run implications of trade openness. In line with previous work, trade openness results in an increase in the wage premium which induces households to invest more in education.<sup>7</sup> However, the pace at which they invest depends not only on their productivity, but also on their wealth: poor households will take the longest to acquire skills and, therefore, will be the

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<sup>1</sup>See Autor et al. (2016).

<sup>2</sup>See, for example, Burstein et al. (2013); Burstein et al. (2016); Burstein and Vogel (2017)

<sup>3</sup>See Findlay and Kierzkowski (1983) and Danziger (2017) among other studies.

<sup>4</sup>Introducing the possibility to acquire skill into the analysis of the effects of trade on different groups of workers already poses important challenges associated with the intrinsic dynamics associated with the intertemporal tradeoff of consumption entailed in acquiring costly education.

<sup>5</sup>This imperfections can lead to less than socially optimal educational attainment. (Abbott et al., 2013)

<sup>6</sup>The importance of liquidity constraints on education attainment has long been well understood. (Becker, 1975) Belley and Lochner (2007) show that parental financial resources matter significantly for college attendance in the 2000s. See Heckman and Mosso (2014) for a survey of the literature.

<sup>7</sup>See Findlay and Kierzkowski (1983) and Danziger (2017) among others.

last to experience positive gains from trade openness, and in some cases might never do so. Thus, wealth heterogeneity result key to understand why, even allowing for endogenous skill acquisition, workers might lose from trade.

We calibrate the model to the U.S. economy and analyze the dynamics of a trade shock that reduces the cost of importing manufacturing goods, which are domestically produced by the unskilled-intensive sector. We choose the trade shocks as to match the decrease in home bias of the manufacturing sector observed in the U.S. between the late 1980s and 2010. We derive three main results from our analysis. First, the possibility to acquire skill together with endogenous accumulation of physical capital implies that, even though certain workers lose from trade openness in the short-run, all workers experience gains from trade in the long-run. A key insight from our model is that capital accumulation plays a key role in reaching a new steady state in which all workers are better off. Second, the workers that suffer the greatest losses from trade in the short-run are those who were initially unskilled and poor. That is, in line with the insights from the *Stolper-Samuelson* theorem stating that unskilled-workers lose at impact, the initial distribution of wealth across unskilled workers also determines how long it will take for these workers to experience gains from trade. Third, because heterogeneity of wealth matters for skill acquisition, we find that heterogeneity in wealth amplifies the effects of trade openness on between-group inequality. In particular, trade openness sharply increases inequality in the short run, and it eventually converges to smaller level of inequality although larger than the one the economy started with.

We carry out a detailed analysis of the model to derive the three main results of this paper. First we analyze the two steady states of the economy—before and after a shock to trade barriers, which we label as closed and open economies respectively. We show that the drop in trade costs leads to an increase in the wage premium of 1.15 percent across steady states. The higher wage premium induces households to invest more in education and the measure of skilled workers increases by 9.8 percent. This increase in skilled workers results from a change in optimal education policies, such that poorer households are more likely to acquire skills in the open economy. Interestingly, we find that the wealth distributions for both skilled and unskilled workers shift to the right, implying that in the open economy every worker is richer than in the closed economy. Importantly, the increase in wealth for unskilled workers is largely due to accumulation of physical capital. Hence, allowing for endogenous skill acquisition and dynamic capital accumulation implies that every worker—independently of skill—experiences welfare gains from trade in the long-run.

Despite this long-run results, the gains of trade are very different when considering the transition dynamics of the model. Along the transition, the wage premium initially overshoots the open economy level and then converges gradually as more workers decide to invest in education. On impact, the college-wage premium increase by approximately 15 times the increase in such premia across steady states. The share of workers with a college degree

increases gradually and also overshoots its new steady state level.<sup>8</sup> The drop in trade barriers also leads to an increase in the marginal product of capital that raises the interest rate on impact by about 2 percentage points.<sup>9</sup> Consequently, trade openness in the short-run is beneficial for skilled-wealthy households but detrimental for unskilled-poor ones, a finding that is line with the *Stolper-Samuelson* theorem.

Why doesn't unskilled workers invest in education and benefit from high skilled wages? Because education investment is costly and poor households cannot afford it, thus they endure the transition towards the open economy. This is the key mechanism in the model, which lies in the interaction of endogenous skill acquisitions and an equilibrium wealth distribution. As we show, losses from trade for poor households can be substantial: up to 5 percent of life-time consumption and lasting as long as ten years.

**Related Literature** This paper is related to multiple strands of literature in International Trade and Macroeconomics. First, the paper is related to the quantitative literature on the effects of trade between different groups of workers. [Burstein et al. \(2016\)](#) quantify the impact of computers, occupations, and international trade in US between-group inequality and show that moving to autarky in equipment goods and occupation services in 2003 reduces the skill premium by 2.2 and 6.5 percentage points, respectively; [Burstein and Vogel \(2017\)](#) introduce firm and sector heterogeneity into the standard Heckscher-Ohlin framework and find that, quantitatively, reductions in trade costs increase the skill premium in almost all countries. We contribute to this literature not only by taking into account changes in skill acquisition induced by initial changes in the skill premium caused by lower trade costs, but also by adding the important dimension of wealth heterogeneity in order to understand the impact of trade.

There exists a relatively scarce literature on the effects of trade on skill acquisition. [Findlay and Kierzkowski \(1983\)](#) incorporate the formation of human capital into the two-factor, two-good model of international trade and show that the implications of the model are consistent with empirical evidence on the role of human capital in explaining patterns of comparative advantage. [Atkin \(2016\)](#) presents empirical evidence that the growth of export manufacturing in Mexico during a period of major trade reforms altered the distribution of education. [Danziger \(2017\)](#) considers a dynamic two-symmetric-country Melitz-type trade model with endogenous skill demand and supply and calibrates it to show that ignoring adjustments in skill supply leads to a substantial bias in the quantitative assessment of trade liberalization. We contribute to this literature by considering a quantitative trade model with heterogeneous agents whose consumption can differ from earnings and where physical capital can be accumulated over time.

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<sup>8</sup>This overshooting in prices and quantities are in line with previous studies such as in [Danziger \(2017\)](#).

<sup>9</sup>The increase in the marginal product of capital is driven by the reallocation of workers to the sector with the highest wage index because of the more intensive use of skilled workers.

The previous contribution puts this paper very close to the recent literature exploiting heterogeneous agents macro models to understand the effects of trade shocks. [Lyon and Waugh \(2017\)](#) and [Lyon and Waugh \(2018\)](#) embed the trade model of [Dornbusch et al. \(1977\)](#) in which each sector represents a local labor market into a small open economy setting with heterogeneous agents and incomplete markets to study redistribution policies and quantify the losses from trade, respectively. This paper contributes to the uprising strand of literature by considering difference in endogenously determined skill levels in a life-cycle setting.

The paper also contributes to the large literature on the effects of trade shocks on labor markets. [Autor et al. \(2013\)](#) and [Pierce and Schott \(2016\)](#) provide empirical evidence on the effects of trade shocks on labor markets. [Autor et al. \(2013\)](#) provide evidence of a negative effect on earnings and employment in labor markets relatively more exposed to import competition shocks. [Pierce and Schott \(2016\)](#) also show the industries that are relatively more exposed to import competition from China also saw greater declines in employment after China joined the WTO. Another strand of this literature has focused on structural trade models with labor dynamics like [Artuç et al. \(2010\)](#), [Coçar et al. \(2016\)](#), [Dix-Carneiro \(2014\)](#) and [Caliendo et al. \(2015\)](#). We contribute to this literature by bringing in the wealth heterogeneity dimension into the picture and showing that the initial distribution of wealth matters for how trade shocks affect workers differently. In this sense this paper also relates to the more general literature on trade and inequality ([Helpman et al., 2010, 2017](#); [Burstein et al., 2013](#); [Antràs et al., 2017](#)).

**Roadmap** The rest of the paper is organized as follows. In [Section 2](#) we lay down the model and provide for our particular environment some qualitative results that are well-known in the international trade literature. [Section 3](#) considers the baseline calibration of the model and provides the quantitative results of our main exercise. [Section 4](#) concludes.

## 2 The Model

Consider a small open economy (SOE) populated by a continuum of finitely-lived heterogeneous households. A household lives for  $J_R$  periods, and then retires to be replaced by a newborn. Households are altruistic and care about the life-time utility of the newborn, which generates a bequest motive. At age one, during the first period of life, households make an education investment decision, which is an irreversible one-time decision. We refer as *college* to the households who make the education investment, and *non-college* to the ones who don't. Each household corresponds to a worker in the economy.

The production side of the economy consists of two sectors: services and manufacturing, indexed by  $i \in \{s, m\}$ . Within each sector, there is a representative firm producing intermediate tradable goods, and a representative firm producing the non-tradable final good. Intermediate

goods are produced using a constant return to scale technology in capital, *college* workers, and *non-college* workers, all three factors which are supplied by households. Production technologies for intermediate firms across sectors differs because manufacturing is more intensive in *non-college* workers. The final good of each sector is produced by combining the domestic intermediate good with the imported one. The small open economy assumption implies that demand for imports is always met at exogenously given world prices. Finally, both intermediate goods can be traded across borders subject to iceberg-type trade barriers.

We start by discussing firms, and then move to households.

## 2.1 Firms

**Final Non-Tradable Goods** Final goods producers bundle intermediate goods in an Armington fashion to produce non-tradable goods for final use. The technology for firms in sector  $i$  is given by

$$Q_i = \left[ \omega_i^{\frac{1}{\eta_i}} (D_i)^{\frac{\eta_i-1}{\eta_i}} + (1 - \omega_i)^{\frac{1}{\eta_i}} (D_i^*)^{\frac{\eta_i-1}{\eta_i}} \right]^{\frac{\eta_i}{\eta_i-1}} \quad i = s, m \quad (1)$$

where  $D_i$  and  $D_i^*$  denote the amount of tradable intermediate goods produced domestically and imported respectively, required to produce  $Q_i$  units of the final good. Here,  $\eta_i$  denotes the elasticity of substitution across intermediate inputs which, as we will show later, also provides a sector-specific trade elasticity. The value of  $\omega_i$  determines the country home bias.

The profit maximization problem of the final good firm reads

$$\begin{aligned} \max_{D_i, D_i^*} \{ & q_i Q_i - (p_i D_i + \tau_i p_i^* D_i^*) \} \\ \text{subject to } & (1) \end{aligned} \quad (2)$$

where  $q_i$  is the price of the bundle  $Q_i$ ,  $p_i$  and  $p_i^*$  denote the price of the intermediate domestic and imported good respectively, and  $\tau_i \geq 1$  is the iceberg cost of importing the good. From the firm's problem in (2), we obtain the conditional demands for intermediates

$$D_i = \omega_i \left( \frac{p_i}{q_i} \right)^{-\eta_i} Q_i \quad (3)$$

$$D_i^* = (1 - \omega_i) \left( \frac{\tau_i p_i^*}{q_i} \right)^{-\eta_i} Q_i \quad (4)$$

The price of the non-tradable  $q_i$  is then given as

$$q_i = \left[ \omega_i (p_i)^{1-\eta_i} + (1 - \omega_i) (\tau_i p_i^*)^{1-\eta_i} \right]^{\frac{1}{1-\eta_i}}. \quad (5)$$

From (3) we immediately obtain a measure of *home bias* in each sector  $i$ ,  $hm_i = \omega_i (p_i/q_i)^{1-\eta_i}$ , where the *home bias* in sector  $i$  is defined as the share of total expenditure by final good producers on goods produced domestically,  $p_i D_i / q_i Q_i$ .

**Intermediate Tradable Goods** The tradable intermediate good in sector  $i \in \{s, m\}$  is produced according to the production function

$$F_i(K_i, L_{ic}, L_{in}) = K_i^{\alpha_i} \left( \gamma_i L_{ic}^{\frac{\sigma_i-1}{\sigma_i}} + (1-\gamma_i) L_{in}^{\frac{\sigma_i-1}{\sigma_i}} \right)^{(1-\alpha_i) \frac{\sigma_i}{\sigma_i-1}}. \quad (6)$$

where  $K_i$  stands for capital used in sector  $i$ ,  $L_{ic}$  for the *college* labor used, and  $L_{in}$  for the *non-college* labor.

Let  $L_i$  denote the labor composite used in production, given as

$$L_i \equiv \left( \gamma_i L_{ic}^{\frac{\sigma_i-1}{\sigma_i}} + (1-\gamma_i) L_{in}^{\frac{\sigma_i-1}{\sigma_i}} \right)^{\frac{\sigma_i}{\sigma_i-1}}.$$

It can be immediately appreciated that we are assuming a unity elasticity of substitution between capital and the labor composite, and allowing the elasticity of substitution across labor types, parametrized by  $\sigma_i$ , to be different from one and sector specific.

Intermediate goods firm's profit maximization reads

$$\begin{aligned} \max_{K_i, L_{ic}, L_{in}} \quad & \{p_i F_i(K_i, L_{ic}, L_{in}) - w_c L_{ic} - w_n L_{in} - (r + \delta) K_i\} \\ \text{subject to} \quad & (6) \end{aligned} \quad (7)$$

where  $\delta$  is the capital depreciation rate. From optimality conditions we obtain

$$\frac{w_c}{w_n} = \frac{\gamma_i}{1-\gamma_i} \left( \frac{L_{ic}}{L_{in}} \right)^{-\frac{1}{\sigma_i}} \quad (8)$$

$$\frac{w_c}{r + \delta} = \frac{1-\alpha_i}{\alpha_i} \gamma_i \frac{K_i}{L_i} \left( \frac{L_{ic}}{L_i} \right)^{-\frac{1}{\sigma_i}} \quad (9)$$

$$\frac{w_n}{r + \delta} = \frac{1-\alpha_i}{\alpha_i} (1-\gamma_i) \frac{K_i}{L_i} \left( \frac{L_{in}}{L_i} \right)^{-\frac{1}{\sigma_i}} \quad (10)$$

Equation (8) shows that the wage premium  $-w_c/w_n$  is pinned down entirely by relative supply of skill levels. However, the level of wages is affected by the supply of capital. In turn, a model that accounts for the dynamics of capital accumulation is essential for a comprehensive assessment of the long-run welfare implications of trade openness across these groups of workers.

## 2.2 Households

Households derives utility of consuming a bundle of the non-tradable service good  $c_s$ , and the non-tradable manufacturing good  $c_m$ . Let  $c = \mathcal{C}(c_s, c_m)$  denote this bundle. Each household is endowed with  $\bar{h}$  hours of work, and an idiosyncratic labor productivity  $x$  which evolves stochastically as a Markov process  $\pi_x(x', x)$ . They have access to a one period risk-free bond, subject to a borrowing limit.

A newborn household inherits the wealth of her parents, and draws a random labor productivity which is correlated with the parents' one. During the first period of life, newborns can decide to invest in a college degree. The average cost of college is  $\kappa$ , but the actual cost paid is  $\kappa u$ , where  $u$  is a bounded shock with mean one.<sup>10</sup> The education investment does not require time but only money, and can only be done during the first period of life. If a household decides to go to college, the borrowing limit is looser for a few periods (students debt). Let  $\underline{a}_{j,e}$  denote the borrowing limit for a household with education  $e$  and age  $j$ .

Let  $V_j(a, x, e)$  be the maximum attainable life-time utility to a household of age  $j$ , with wealth  $a$ , productivity  $x$ , and education  $e$ . Thus, the value for a newborn ( $j = 1$ ) who goes to college ( $e = c$ ) is given by

$$\begin{aligned} V_1(a, x, u, c) &= \max_{c_s, c_m, a'} \{U(c) + \beta \mathbb{E} [V_2(a', x', c)|x]\} \\ q_s c_s + q_m c_m + a' + q_s \kappa u &\leq w_c x \bar{h} + (1 + r)a \\ c &= \mathcal{C}(c_s, c_m) \\ a' &\geq \underline{a}_{1,c} \end{aligned} \tag{11}$$

where that education cost is in units of the services final good. Analogously, the value for a newborn that doesn't go to college is given as

$$\begin{aligned} V_1(a, x, n) &= \max_{c_s, c_m, a'} \{U(c) + \beta \mathbb{E} [V_2(a', x', n)|x]\} \\ q_s c_s + q_m c_m + a' &\leq w_n x \bar{h} + (1 + r)a \\ c &= \mathcal{C}(c_s, c_m) \\ a' &\geq \underline{a}_{1,n} \end{aligned} \tag{12}$$

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<sup>10</sup>Adding the shock  $u$  makes the household problem quantitatively more tractable.

The value for ages  $j \geq 2$  are given as

$$\begin{aligned}
V_j(a, x, e) &= \max_{c_s, c_m, a'} \{U(c) + \beta \mathbb{E}[V_{j+1}(a', x', e)|x]\} \\
q_s c_s + q_m c_m + a' &\leq w_e x \bar{h} + (1+r)a \\
c &= \mathcal{C}(c_s, c_m) \\
a' &\geq \underline{a}_{j,e}
\end{aligned} \tag{13}$$

where  $w_e$  is the wage to a worker with education  $e$ . The terminal condition in (13) is  $V_{J_R+1}(a, x, e) = \mathbb{E}_{u,x}[\max\{V_1(a, x, u, c), V_1(a, x, n)\}]$ , reflecting the optimal education choice of the newborn next period.

Let  $\mathbf{c}_{js}(a, x, e)$ ,  $\mathbf{c}_{jm}(a, x, e)$  and  $\mathbf{a}'_j(a, x, e)$  denote households' optimal policies for services consumption, manufactures consumption, and saving respectively.

### 2.3 Equilibrium Definition

We assume that there is no international financial markets for lending/saving, and thus the trade balance must be equal to zero every period. This is, while the domestic consumption of a good can be greater or smaller than its domestic production, the value of imports must equal the value of exports.

Finally, we need to determine the foreign demand of the domestic tradable (intermediate) goods. Because of the SOE assumption, we simply assume an iso-elastic foreign demand:  $B_i^* (\tau_i^* p_i)^{-\eta_i^*}$ . The term  $\tau_i^*$  can be understood as the iceberg cost that foreigners pay to purchase the domestic goods, while  $B_i^*$  it's a level shifter.

Next, we provide a formal definition of the economy equilibrium.

**Definition** A *recursive stationary equilibrium* for this economy is given by: factor prices  $\{w_c, w_n, r\}$ ; goods prices  $\{q_i, p_i, p_i^*\}_{i=s,m}$ ; value functions for households  $V_j(a, x, e)$ ; policies for households  $\{\mathbf{c}_{js}(a, x, e), \mathbf{c}_{jm}(a, x, e), \mathbf{a}'_j(a, x, e)\}$ ; policies for intermediate goods firms  $\{L_{ic}, L_{in}, K_i\}_{i=s,m}$ ; policies for final goods firms  $\{Q_i, D_i, D_i^*\}_{i=s,m}$ ; a measure across households states  $\mu_j(a, x, e)$ ; such that, given prices and trade costs  $\{\tau_i, \tau_i^*\}_{i=s,m}$ : (i) Households' policies solve his problem and achieves value  $V_j(a, x, e)$ ; (ii) Firms policies maximize their profits; (iii) Labor markets clear:  $\sum_{i=s,m} L_{ie} = \sum_j \int x d\mu_j(a, x, e)$  for  $e = c, n$ ; (iv) capital markets clear:  $\sum_{i=s,m} K_i = \sum_{j,e} \int a d\mu_j(a, x, e)$ ; (v) final good market clears:  $Q_i = \sum_{j,e} \int \mathbf{c}_{ji}(a, x, e) d\mu_j(a, x, e)$  for  $i = s, m$ ; (vi) intermediate goods market clears:  $Y_i = D_i + B_i^* (\tau_i^* p_i)^{-\eta_i^*}$  for  $i = s, m$ ; (vii) trade balance is zero:  $\sum_{i=s,m} p_i(Y_i - D_i) = 0$ ; and (viii) the measure  $\mu_j(a, x, e)$  is stationary and consistent with households policies.

## 2.4 Calibration

We consider a period to be a year. We set the discount factor to  $\beta = 0.956$  which deliver an annual risk-free rate of 4%. We assume that the household consumption bundle is given by  $\mathcal{C}(c_s, c_m) = \left( \sum_{i=s,m} \nu_i^{1/\rho} c_i^{\frac{\rho-1}{\rho}} \right)^{\frac{\rho}{\rho-1}}$ ; and set  $\nu_s = 1 - \nu_m = 0.4$  and  $\rho = 0.5$ . For the idiosyncratic productivity of the household  $x$ , we assume an AR(1) process in logs with auto-regressive coefficient  $\rho_x = 0.85$  and innovation standard error  $\sigma_x = 0.25$ ; and discretize the process using [Tauchen \(1986\)](#). Finally, we assuming a working span of  $J_R = 30$  years.

We use standard values for technology parameters. For intermediate goods, in both sectors we assume  $\alpha_i = 0.33$  and  $\sigma_i = 2$ . We assume that services is more intensive in *college* workers while manufactures is more intensive in *non-college* workers, and thus set  $\gamma_s = 1 - \gamma_m = 0.7$ . For the final good, we assume identical technologies:  $\omega_i = 0.7$  and  $\eta_i = 4$ . Finally, we assume a depreciation rate of  $\delta = 0.1$ .

## 2.5 Trade Shocks and Skill Acquisition

The rich structure of the model we built in the previous subsection will allow us to carry out a quantitative analysis of how trade shocks affect workers over time. However, it is worth developing some intuition about the main mechanisms at play in the model before proceeding to the quantitative analysis. In order to do so, we will focus on a simplified version of the static block of the model without capital as factor of production and same elasticities of substitution between skills across sectors, that is, assuming that households' dynamic decisions and skill levels are fixed, that  $\alpha_i = 0$  for  $i \in \{m, s\}$  and that  $\sigma \equiv \sigma_m = \sigma_s$ . This will allow us to rely on two of the main theorems in International Trade to develop intuition, while only referencing to the simple dynamic mechanism telling us that an increase in the return to skill will increase the number of workers that decide to acquire an education.

**How do changes in import prices affect the skill premium?** Consider a decline in the trade costs that domestic final good producers in sector  $m$  pay for intermediate goods produced abroad. Assume that the decline in the price paid by producers leads to expenditure switching across countries and a decline in the relative price of sector  $m$  intermediate goods produced in the home country,  $p_m$ . The following is a version of the Stolper-Samuelson theorem for this experiment in our model.

**Proposition 2.1 (Stolper-Samuelson)** *Given a distribution of skills across workers, a decrease in the relative price of the intermediate good produced domestically in sector  $m$  will decrease the wage of non-educated workers and increase that of educated workers if non-educated workers are used more intensively in the production of the intermediate good in sector  $m$ , that is, whenever the following condition holds given the wage premium,  $\frac{w_c}{w_n}$ , before the*

price change:

$$\left(\frac{1 - \gamma_m}{1 - \gamma_s}\right)^{\sigma-1} > \frac{\gamma_m \left(\frac{w_c}{w_n}\right)^{1-\sigma} + (1 - \gamma_m)}{\gamma_s \left(\frac{w_c}{w_n}\right)^{1-\sigma} + (1 - \gamma_s)}. \quad (14)$$

**Proof** See Appendix A.

Consider the case of the United States, for which there is evidence that the manufacturing sector is intensive in non-educated workers. Then, according to Proposition 2.1 a decline in the price that final goods producers pay for imported manufacturing goods would lead to an increase in the skill premium given a distribution of skills across workers.

**How does an increase in the skill premium affect the distribution of skills across workers and production?** Let us briefly turn to the dynamic block of the model. The model tells us that an increase in the skill premium will make the acquisition of education more attractive for new workers. This will in principle lead new generations of workers to become educated, gradually shifting the distribution of skill in the economy towards a more educated economy. This change in the distribution will in turn affect the comparative advantage of the home country, and therefore production, in line with Rybczynski's theorem.

**Proposition 2.2 (Rybczynski)** *A shift in the distribution of skills in the economy towards more educated workers will increase the output of domestic intermediate goods produced in sector  $s$  and decrease the output of the other sector.*

**Proof** See Appendix B.

**How do changes in output feed back into prices?** From preferences we know that in equilibrium

$$\frac{q_m Q_m}{\nu_m} = \frac{q_s Q_s}{\nu_s}.$$

We also know that  $p_i Y_i = p_i D_i = \omega_i \left(\frac{p_i}{q_i}\right)^{1-\eta_i} q_i Q_i$ . Hence, we obtain that in equilibrium the following condition must hold

$$\frac{Y_m}{Y_s} = \frac{\omega_s}{\omega_m} \frac{\nu_m}{\nu_s} \frac{p_s^{\eta_s}}{p_m^{\eta_m}} \frac{q_s^{1-\eta_s}}{q_m^{1-\eta_m}}.$$

For simplicity, let us assume that  $\eta \equiv \eta_m = \eta_s$ . Then, from the previous condition we obtain that

$$\hat{Y}_m - \hat{Y}_s = \eta (\hat{p}_s - \hat{p}_m) + (1 - \eta) (\hat{q}_s - \hat{q}_m)$$

and if  $p_s = 1$  and world prices are given we obtain that

$$\begin{aligned}\eta \hat{p}_m + (1 - \eta) \hat{q}_m &= -(\hat{Y}_m - \hat{Y}_s) \Leftrightarrow \\ \hat{p}_m (\eta + (1 - \eta) \phi) &= -(\hat{Y}_m - \hat{Y}_s)\end{aligned}$$

where  $\phi$  is positive. Therefore, if  $\hat{Y}_m - \hat{Y}_s > 0$ , then  $\hat{p}_m < 0$  which will counteract the initial Stolper-Samuelson forces.

### 3 Quantitative Exercises

We study a case of trade openness as a reduction in the iceberg cost of importing manufacturing goods,  $\tau_m$ . In particular, we start the economy at a steady-state with a high  $\tau_m$ , and analyze the effect of an (unexpected) drop in  $\tau_m$ . We refer to the high- $\tau_m$  equilibrium as a "closed economy", and the low- $\tau_m$  equilibrium as the "open economy". We select  $\tau_m$  in each economy to match the home bias in the manufacturing sector.

We first analyze how the closed and open economy compare in terms of welfare an inequality. We then analyze these measures along the transition from the closed to the open economy.

#### 3.1 Steady State Analysis

We select  $\tau_m$  to match a home bias in the manufacturing sector of roughly 90% in the closed economy, and about 70% in the open economy. This is similar to the change in home bias observed in the United States between the late 1980s and the year 2010. Table 1 shows results for the open and closed economies, including the home bias.

A drop in the trade costs induces an increase of the wage premium  $w_c/w_n$  of 1.15%. When accounting for households' labor productivity, the labor income of *college* relative to *non-college* workers also increases, by about 1%. The increased college-premium induces more households to invest in education, and the measure of *college* workers increases by 9.82%. Interestingly, despite the increase in wage inequality, the purchase power of *non-college* wage  $-w_n/q-$  increases by 2.2%. Overall, welfare increases for both type of households in the open economy.

Wealthier and more productive households are more likely to invest in education, as Figure 1 shows. Very productive households benefit the most from higher wages, and thus decide to invest in education even if they are very poor. As productivity decreases, only wealthier households invest in find it beneficial to forgo consumption today and invest in education. In the open economy, wage premium increases and so does incentives to obtain a college degree. Consequently, as Figure 1 shows, education policies shift to the left along the wealth dimension implying that it is more likely for workers to acquire an education in the open

Table 1: Comparison of Main Statistics Across Steady States

|                                    | Closed Economy | Open Economy | Difference |
|------------------------------------|----------------|--------------|------------|
| Home Bias in Sector $m$            | 0.913          | 0.697        | -0.2155    |
| Home Bias in Sector $s$            | 0.986          | 0.986        | -0.0001    |
| Wage Premium                       | 1.46           | 1.48         | 1.15%      |
| Wage Premium (adjusted)            | 1.65           | 1.6701       | 1.00%      |
| Share of College Workers           | 0.43           | 0.4715       | 9.82%      |
| Non-college real wage $w_n/q$      | 0.41           | 0.42         | 2.2%       |
| Average Utility <i>College</i>     | -23.45         | -21.19       | 9.63%      |
| Average Utility <i>Non-college</i> | -32.54         | -29.51       | 9.37%      |

economy.

The increase in the measure of *college* workers, jointly with higher wages, makes the economy wealthier. This can be seen in 2: in the open economy, both *college* and *non-college* workers are born in wealthier households. Hence, allowing for endogenous skill acquisition, as well as capital accumulation, implies that every—independently of skill—worker has positive welfare gains from trade in the long-run.

The key insights of comparing the steady-states are two fold. First, households make an effort to save and invest in a college education. This increases their wealth and welfare in the long-run. Second, a wealthier economy accumulates more capital, which increases all wages in the economy. Thus, in the long-run, the common wisdom that trade openness increases everyone welfare holds. However, as we discuss next, it can take several years to experience these benefits, and the transition towards an open economy may actually be detrimental for several households in the economy.

### 3.2 Transition

In the short-run, trade openness –induced by a drop in  $\tau_m$ – reduces the demand for domestic manufactures, which also reduces the demand for *non-college* workers who are more intensively used in the manufacturing sector. Correspondingly, the existing capital in the economy relocates towards the services sector. This induces an increase in the real wage for *college* workers, and a decline in the *non-college* wage, as Figure 3 shows. As a result, the wage premium overshoots in the short-run increasing by approximately 15 percent, 15 times the increase in such premia across steady states (see Figure 4). Along the transition, households respond to the higher wage-premium by investing more in education, and thus the measure of *college* workers increases, as shown in Figure 5. Eventually, this reduces college wages, and the wage premium. Furthermore, the drop in trade barriers also leads to an increase in the marginal product of capital that raises the interest rate on impact by about 2 percentage points.<sup>11</sup>

<sup>11</sup>The increase in the marginal product of capital is driven by the reallocation of workers to the sector with the highest wage index because of the more intensive use of skilled workers.

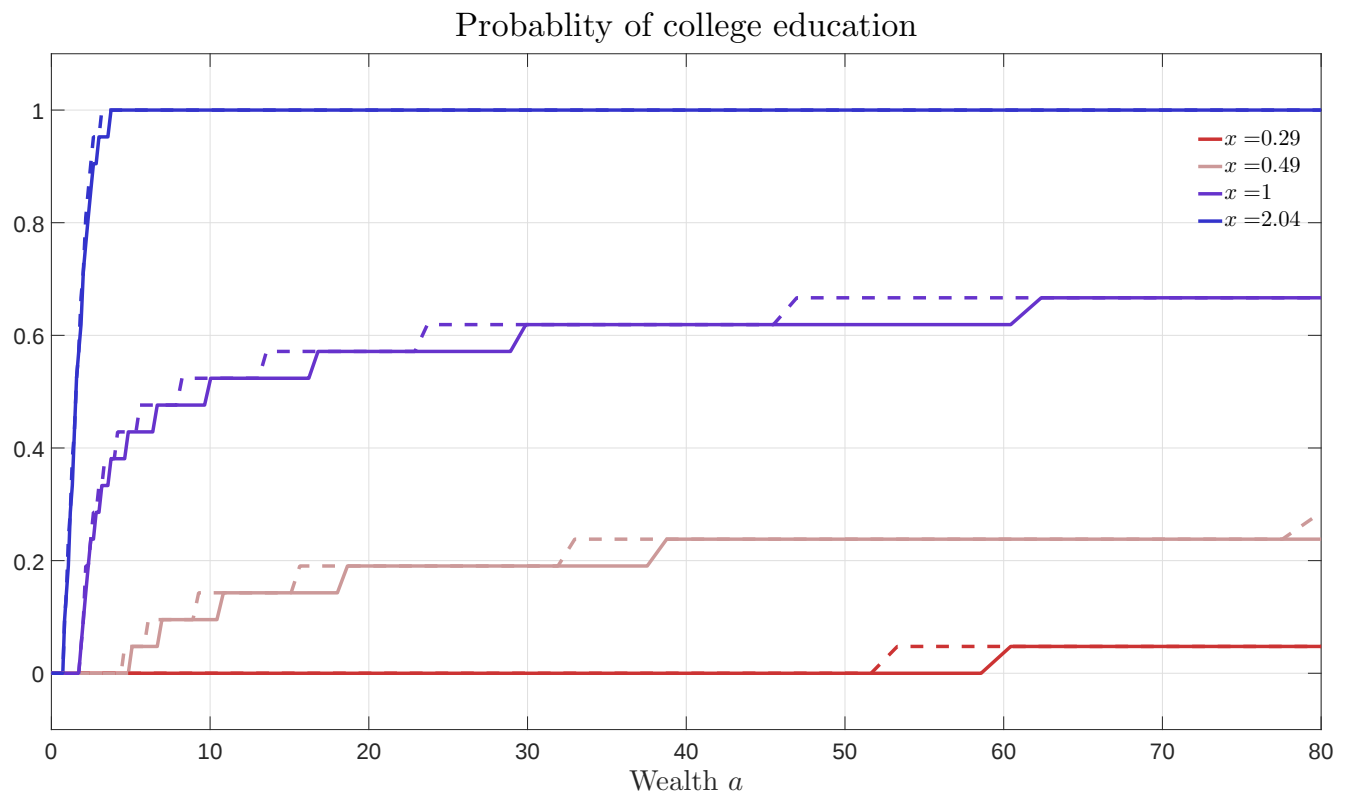


Figure 1: Steady State Education Policies: Closed (solid line) and Open (dashed line) Economies

Figure 2: Steady State Wealth Distributions

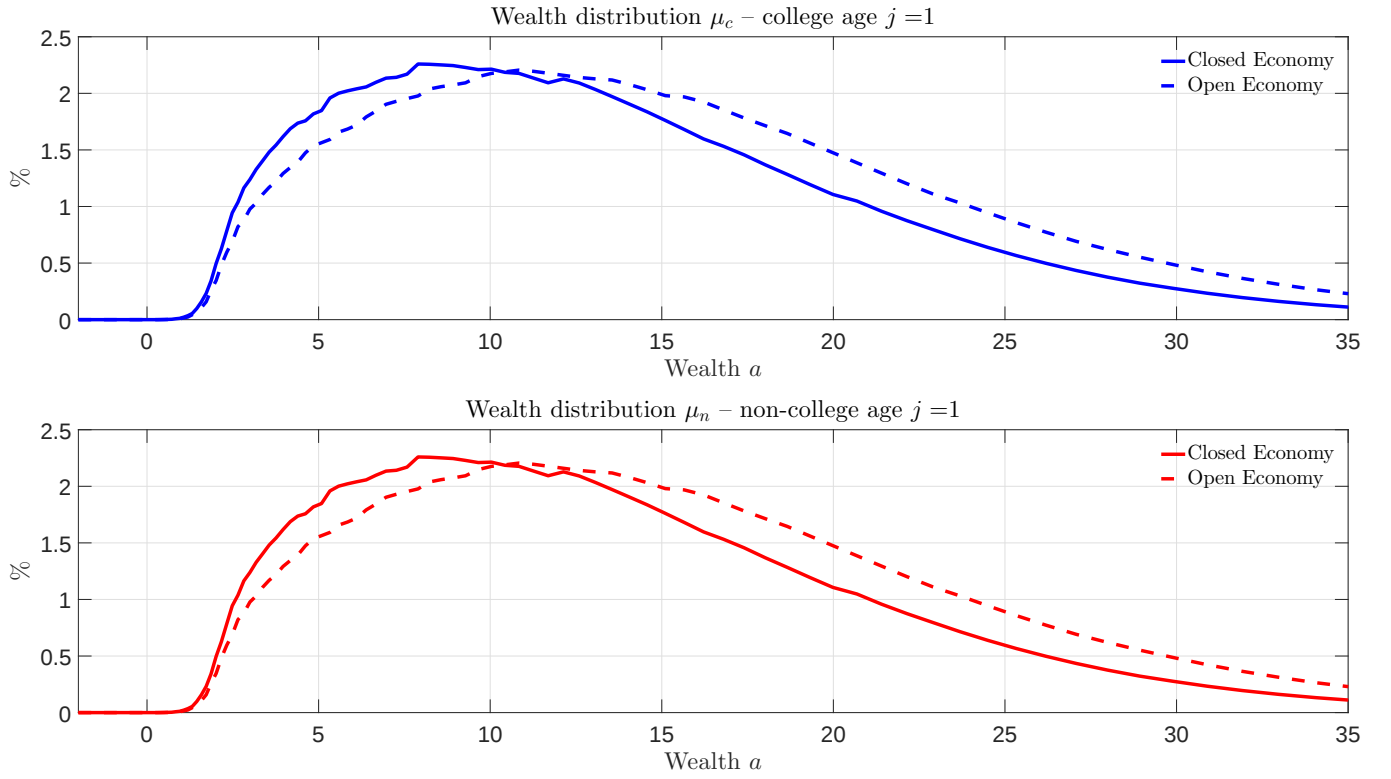


Figure 3: Factor Prices and Capital Stock

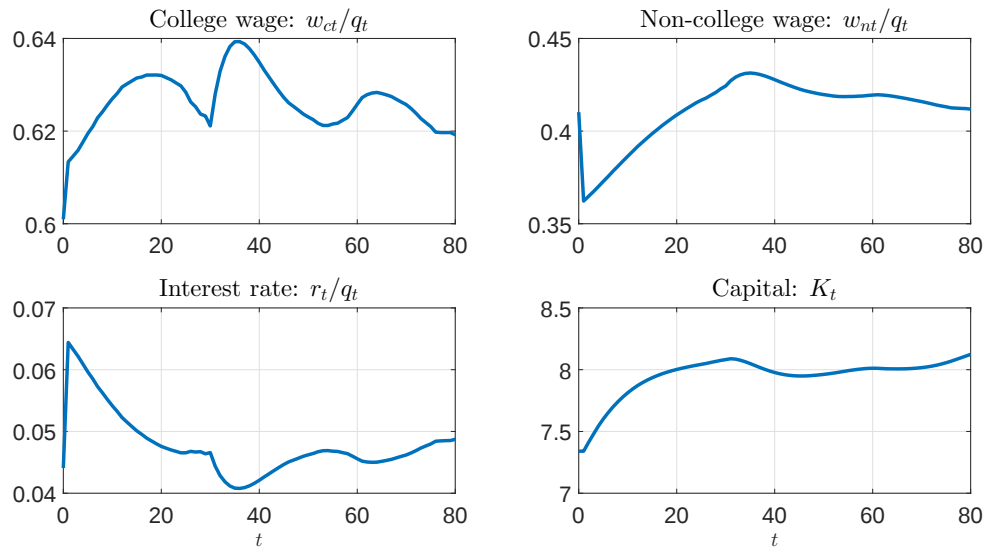


Figure 4: Wage Premium

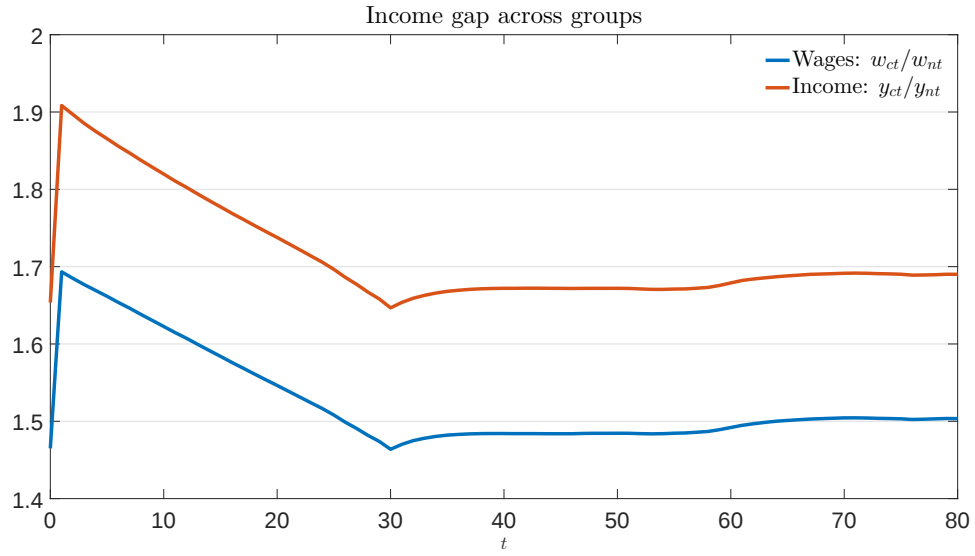
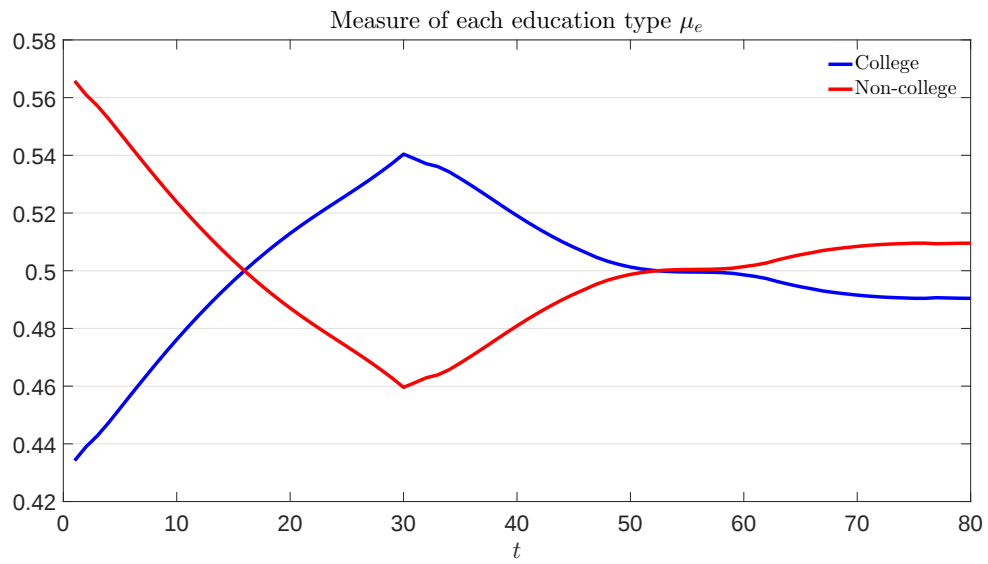


Figure 5: Share of Workers Across Education Types



Consequently, trade openness in the short-run is beneficial for *college*-wealthy household but detrimental for *non-college*-poor households, a result that is in line with *Stolper-Samuelson Theorem*. Why doesn't newborns invest in education and benefit from higher college wages? Because a college degree is expensive and newborns of poor households cannot afford to invest in education, who will then suffer from the lower non-college wages. This can be seen in Figure 6, that plots the evolution of the wealth distribution for both type of households. The wealth dist for *non-college* workers in the first period of the transition is substantially more skewed to the left than the one in closed economy: the only households who don't invest in an education are the poor ones, and many wealthy households that wouldn't have invested in education in the closed economy now decide to do so. Interestingly, after the transition is almost over (150 years later), the wealth distribution for *non-college* workers is almost identical to the one in the closed economy, and most of the wealth gains go to the *college* households whose wealth distribution shifts to the right.

A quantification of the heterogeneous gains and losses of trade openness is Figure 7, that computes the consumption equivalent for several households along the transition relative to the closed economy. For *college* workers, the welfare gains are substantial: they gain between 5 and 10 percent of consumption terms on impact.<sup>12</sup> On the other hand, *non-college* poor households loose the most and it takes them about ten years to recover their life-time utility.

## 4 Conclusion

We argued that trade openness can have unequal effects on heterogeneous households, especially in the short-run. An increase in the skill-premium induces incentivizes households to invest in education, but this decision may be constrained by the household's wealth. In turn, poor-unskilled workers take the longest to acquire skills and are therefore the last to experience positive gains from trade openness. When calibrating the model to the United States, we argued that several household found trade openness detrimental.

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<sup>12</sup>A 10% consumption equivalent means that consumption should have been increased by 10% every period in the closed economy to leave him indifferent relative to the welfare gains of the transition.

Figure 6: Evolution of Wealth Distribution

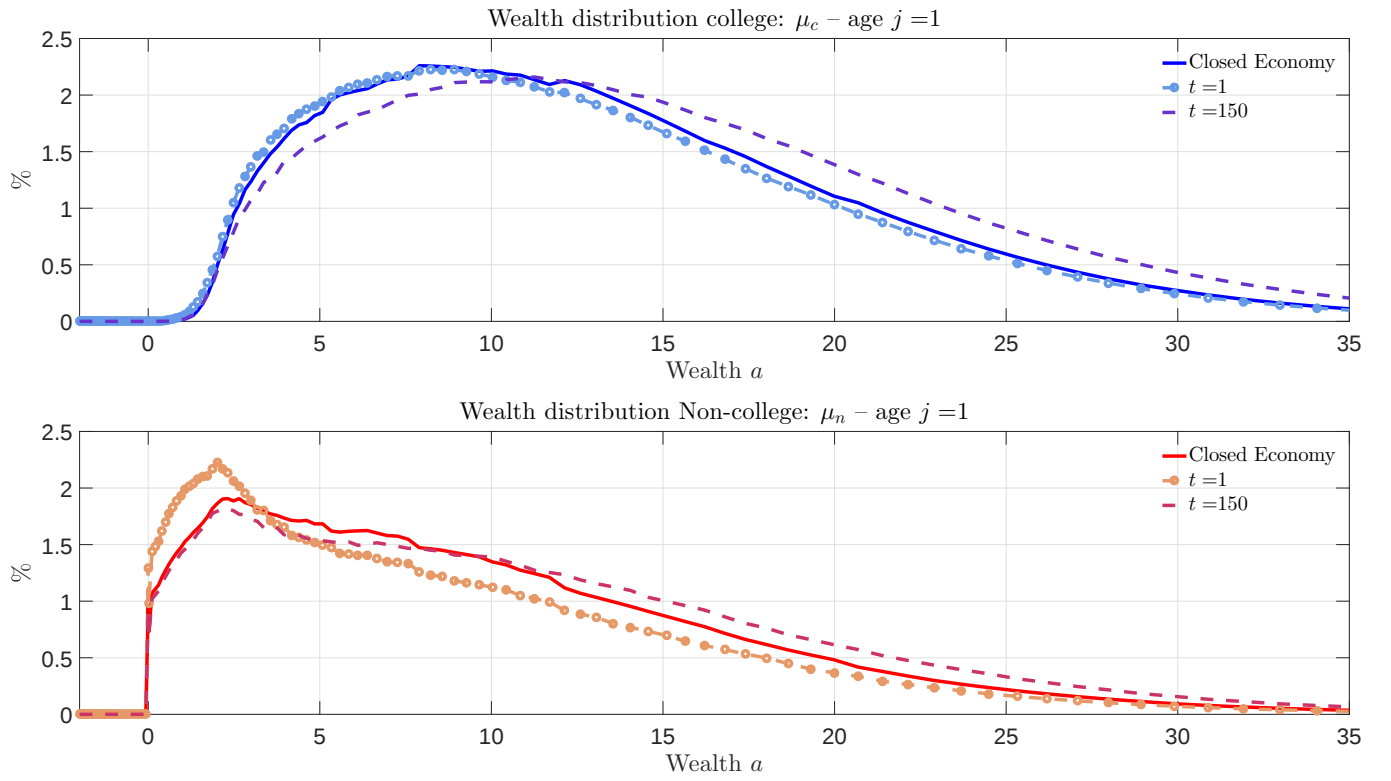
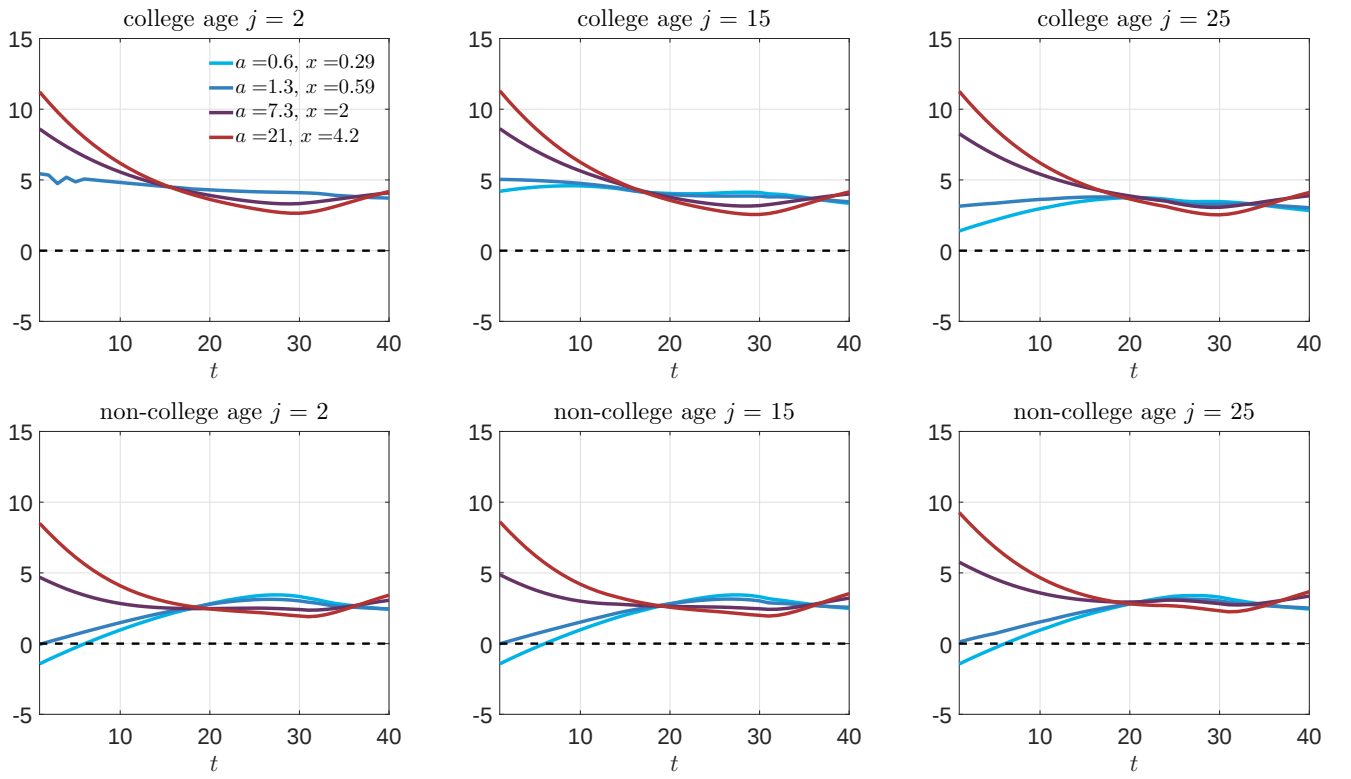


Figure 7: Consumption Equivalents Along the Transition



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## Appendix

### A Proof of Proposition 2.1

For the Armington model consider a shock to  $p_m^*$  that leads to expenditure switching and a decline in the price produced at home.

Consider the unit-cost functions:

$$c_i(w_c, w_n, r) = \min_{L_{i,c}, L_{i,n}, K_i} \{w_c L_{i,c} + w_n L_{i,n} + r K_i | F_i(L_{i,c}, L_{i,n}, K_i) \geq 1\},$$

where

$$F_i(L_{i,c}, L_{i,n}, K_i) = \left( \gamma_i^{\frac{1}{\sigma_i}} L_{i,c}^{\frac{\sigma_i-1}{\sigma_i}} + (1 - \gamma_i)^{\frac{1}{\sigma_i}} L_{i,n}^{\frac{\sigma_i-1}{\sigma_i}} \right)^{\left( \frac{\sigma_i}{\sigma_i-1} \right) (1-\alpha_i)} K_i^{\alpha_i}.$$

Then we know that in this particular case

$$c_i(w_c, w_n, r) \propto \left( \gamma_i w_c^{1-\sigma_i} + (1 - \gamma_i) w_n^{1-\sigma_i} \right)^{(1-\alpha_i)} (r)^{\alpha_i}$$

and that in general by the "envelope theorem"

$$\begin{aligned} \frac{\partial c_i(w_c, w_n, r)}{\partial w_e} &= a_{i,L_e}(w_c, w_n, r) \\ \frac{\partial c_i(w_c, w_n, r)}{\partial r} &= a_{i,K}(w_c, w_n, r) \end{aligned}$$

for  $e \in \{c, n\}$  where  $a_{i,x}$  denotes the optimal choice for factor  $x$  as a function of factor prices to produce one unit of the good.

The zero-profit conditions imply that in equilibrium

$$\begin{aligned} p_m &= c_m(w_c, w_n, r) = \kappa_m \left( \gamma_m w_c^{1-\sigma_m} + (1 - \gamma_m) w_n^{1-\sigma_m} \right)^{\left( \frac{1-\alpha_m}{1-\sigma_m} \right)} (r)^{\alpha_m}, \\ p_s &= c_s(w_c, w_n, r) = \kappa_s \left( \gamma_s w_c^{1-\sigma_s} + (1 - \gamma_s) w_n^{1-\sigma_s} \right)^{\left( \frac{1-\alpha_m}{1-\sigma_m} \right)} (r)^{\alpha_s}. \end{aligned}$$

By totally differentiating these conditions we obtain

$$\begin{aligned} dp_i &= a_{i,L_c} dw_c + a_{i,L_n} dw_n + a_{i,K} dr \Rightarrow \\ \frac{dp_i}{p_i} &= \frac{w_c a_{i,L_c}}{c_i} \frac{dw_c}{w_c} + \frac{w_n a_{i,L_n}}{c_i} \frac{dw_n}{w_n} + \frac{r a_{i,K}}{c_i} \frac{dr}{r}. \end{aligned}$$

Define cost shares by  $\theta_{i,L_e} \equiv \frac{w_e a_{i,L_e}}{c_i}$  for  $e \in \{c, n\}$  and  $\theta_{i,K} \equiv \frac{r a_{i,K}}{c_i}$ . Then we obtain that

$$\begin{aligned} \begin{pmatrix} \hat{p}_m \\ \hat{p}_s \end{pmatrix} &= \begin{pmatrix} \theta_{m,L_c} & \theta_{m,L_n} & \theta_{m,K} \\ \theta_{s,L_c} & \theta_{s,L_n} & \theta_{s,K} \end{pmatrix} \begin{pmatrix} \hat{w}_c \\ \hat{w}_n \\ \hat{r} \end{pmatrix} \\ &= \begin{pmatrix} \theta_{m,L_c} & \theta_{m,L_n} \\ \theta_{s,L_c} & \theta_{s,L_n} \end{pmatrix} \begin{pmatrix} \hat{w}_c \\ \hat{w}_n \end{pmatrix} + \begin{pmatrix} \theta_{m,K} \\ \theta_{s,K} \end{pmatrix} \hat{r} \end{aligned}$$

which implies that

$$\begin{pmatrix} \hat{w}_c \\ \hat{w}_n \end{pmatrix} = \begin{pmatrix} \theta_{m,L_c} & \theta_{m,L_n} \\ \theta_{s,L_c} & \theta_{s,L_n} \end{pmatrix}^{-1} \begin{pmatrix} \hat{p}_m - \theta_{m,K} \hat{r} \\ \hat{p}_s - \theta_{s,K} \hat{r} \end{pmatrix}.$$

**Assumption 1** Assume that only the two types of labor are factors of production, that is,  $\alpha_i = 0$  for  $i \in \{m, s\}$ . Hence,  $\theta_{m,K} = \theta_{s,K} = 0$  and  $\kappa_i = 1$  for  $i \in \{m, s\}$ .

We now have that

$$\begin{aligned} \begin{pmatrix} \hat{w}_c \\ \hat{w}_n \end{pmatrix} &= \begin{pmatrix} \theta_{m,L_c} & \theta_{m,L_n} \\ \theta_{s,L_c} & \theta_{s,L_n} \end{pmatrix}^{-1} \begin{pmatrix} \hat{p}_m \\ \hat{p}_s \end{pmatrix} \\ &= \frac{1}{\det \theta} \begin{pmatrix} \theta_{s,L_n} & -\theta_{m,L_n} \\ -\theta_{s,L_c} & \theta_{m,L_c} \end{pmatrix} \begin{pmatrix} \hat{p}_m \\ \hat{p}_s \end{pmatrix} \end{aligned}$$

where

$$\begin{aligned} \det \theta &= \theta_{m,L_c} \theta_{s,L_n} - \theta_{m,L_n} \theta_{s,L_c} \\ &= \theta_{m,L_c} (1 - \theta_{s,L_c}) - (1 - \theta_{m,L_c}) \theta_{s,L_c} \\ &= \theta_{m,L_c} (1 - \theta_{s,L_c}) - (1 - \theta_{m,L_c}) \theta_{s,L_c} \\ &= \theta_{m,L_c} - \theta_{s,L_c} = \theta_{s,L_n} - \theta_{m,L_n}. \end{aligned}$$

Therefore, we have that

$$\begin{aligned} \hat{w}_c &= \frac{\hat{p}_m \theta_{s,L_n} - \hat{p}_s \theta_{m,L_n}}{\theta_{s,L_n} - \theta_{m,L_n}} \\ &= \frac{(\theta_{m,L_n} - \theta_{s,L_n}) \hat{p}_s + \theta_{s,L_n} (\hat{p}_s - \hat{p}_m)}{\theta_{m,L_n} - \theta_{s,L_n}} \end{aligned}$$

and

$$\begin{aligned}\hat{w}_n &= \frac{\hat{p}_s \theta_{m,L_c} - \hat{p}_m \theta_{s,L_c}}{\theta_{m,L_c} - \theta_{s,L_c}} \\ &= \frac{(\theta_{s,L_c} - \theta_{m,L_c}) \hat{p}_m - (\hat{p}_s - \hat{p}_m) \theta_{m,L_c}}{\theta_{s,L_c} - \theta_{m,L_c}}\end{aligned}$$

**Assumption 2** WLOG, assume that the manufacturing sector is intensive in low skilled workers, that is,  $\theta_{m,L_n} - \theta_{s,L_n} > 0$ , which implies that  $\theta_{s,L_c} - \theta_{m,L_c} > 0$  given that  $\theta_{i,L_c} + \theta_{i,L_n} = 1$  for  $i \in \{m, s\}$ .

Suppose that  $\hat{p}_s - \hat{p}_m > 0$ .

Given the previous assumptions, we obtain Stolper-Samuleson's result that

$$\hat{w}_c > \hat{p}_s > \hat{p}_m > \hat{w}_n.$$

Now, when does the assumption that  $\theta_{m,L_n} - \theta_{s,L_n} > 0$  hold? In the case of Cobb-Douglas production functions this is clear. We have that  $\theta_{i,L_n} \equiv \frac{w_n a_{i,L_n}}{c_i}$  and

$$a_{i,L_n} = \frac{\partial}{\partial w_n} (\gamma_i w_c^{1-\sigma_i} + (1 - \gamma_i) w_n^{1-\sigma_i})^{\frac{1}{1-\sigma_i}} = (1 - \gamma_i) \left( \frac{c_i}{w_n} \right)^{\sigma_i}.$$

Hence,

$$\theta_{m,L_n} - \theta_{s,L_n} = (1 - \gamma_m) \left( \frac{c_m}{w_n} \right)^{\sigma_m-1} - (1 - \gamma_s) \left( \frac{c_s}{w_n} \right)^{\sigma_s-1}.$$

Now, notice that

$$\frac{c_i}{w_n} = \left( \gamma_i \left( \frac{w_c}{w_n} \right)^{1-\sigma_i} + (1 - \gamma_i) \right)^{\frac{1}{1-\sigma_i}}.$$

**Assumption 3** Skills are gross substitutes in production and their elasticity of substitution is the same across sectors, that is,  $\sigma_i > 1$  for  $i \in \{m, s\}$  and  $\sigma \equiv \sigma_m = \sigma_s$ .

Then notice that

$$\begin{aligned}
\frac{c_m}{w_n} &> \frac{c_s}{w_n} \Leftrightarrow \\
\frac{1}{\left(\gamma_m \left(\frac{w_c}{w_n}\right)^{1-\sigma} + (1-\gamma_m)\right)^{\frac{1}{\sigma-1}}} &> \frac{1}{\left(\gamma_s \left(\frac{w_c}{w_n}\right)^{1-\sigma} + (1-\gamma_s)\right)^{\frac{1}{\sigma-1}}} \Leftrightarrow \\
\left(\gamma_s \left(\frac{w_c}{w_n}\right)^{1-\sigma} + (1-\gamma_s)\right)^{\frac{1}{\sigma-1}} &> \left(\gamma_m \left(\frac{w_c}{w_n}\right)^{1-\sigma} + (1-\gamma_m)\right)^{\frac{1}{\sigma-1}} \Leftrightarrow \\
\gamma_s \left(\frac{w_c}{w_n}\right)^{1-\sigma} - \gamma_m \left(\frac{w_c}{w_n}\right)^{1-\sigma} &> (1-\gamma_m) - (1-\gamma_s) \Leftrightarrow \\
1 &> \left(\frac{w_c}{w_n}\right)^{\sigma-1}.
\end{aligned}$$

Therefore, the only way to assure that  $\theta_{m,L_n} - \theta_{s,L_n} > 0$  as long as  $\gamma_s > \gamma_m$  is if  $\frac{w_c}{w_n} < 1$ , which is counter-factual. Hence, if  $\frac{w_c}{w_n} > 1$  we need that

$$\frac{1-\gamma_m}{1-\gamma_s} > \frac{c_s}{c_m} = \left( \frac{\gamma_m \left(\frac{w_c}{w_n}\right)^{1-\sigma} + (1-\gamma_m)}{\gamma_s \left(\frac{w_c}{w_n}\right)^{1-\sigma} + (1-\gamma_s)} \right)^{\frac{1}{\sigma-1}}$$

which is equivalent to

$$\left( \frac{1-\gamma_m}{1-\gamma_s} \right)^{\sigma-1} > \frac{\gamma_m \left(\frac{w_c}{w_n}\right)^{1-\sigma} + (1-\gamma_m)}{\gamma_s \left(\frac{w_c}{w_n}\right)^{1-\sigma} + (1-\gamma_s)}.$$

## B Proof of Proposition 2.2

Let  $Y_i$  denote total production of good  $i$ . Notice that because of constant marginal costs, then total factors used in the production of good  $i$  are  $L_{i,c} = a_{i,L_c} Y_i$  and  $L_{i,n} = a_{i,L_n} Y_i$ . Hence, factor market clearing is given by

$$\begin{aligned}
a_{m,L_c} Y_m + a_{s,L_c} Y_s &= L_c, \\
a_{m,L_n} Y_m + a_{s,L_n} Y_s &= L_n.
\end{aligned}$$

By totally differentiating this system of equations we obtain

$$\begin{aligned}
a_{m,L_c} dY_m + a_{s,L_c} dY_s &= dL_c, \\
a_{m,L_n} dY_m + a_{s,L_n} dY_s &= dL_n,
\end{aligned}$$

where we have used the fact that  $a_{i,L_c}$  and  $a_{i,L_n}$  do not change if prices do not change. Hence, we obtain that

$$\begin{aligned}\frac{a_{m,L_c} Y_m}{L_c} \frac{dY_m}{Y_m} + \frac{a_{s,L_c} Y_s}{L_c} \frac{dY_s}{Y_s} &= \frac{dL_c}{L_c}, \\ \frac{a_{m,L_n} Y_m}{L_n} \frac{dY_m}{Y_m} + \frac{a_{s,L_n} Y_s}{L_n} \frac{dY_s}{Y_s} &= \frac{dL_n}{L_n},\end{aligned}$$

which we can rewrite as

$$\begin{aligned}\lambda_{m,L_c} \hat{Y}_m + \lambda_{s,L_c} \hat{Y}_s &= \hat{L}_c, \\ \lambda_{m,L_n} \hat{Y}_m + \lambda_{s,L_n} \hat{Y}_s &= \hat{L}_n,\end{aligned}$$

where  $\lambda_{i,L_e}$  measure the fraction of factor  $L_e$  employed in industry  $i$ .

Inverting this system of equations we obtain

$$\begin{aligned}\begin{pmatrix} \hat{Y}_m \\ \hat{Y}_s \end{pmatrix} &= \begin{pmatrix} \lambda_{m,L_c} & \lambda_{s,L_c} \\ \lambda_{m,L_n} & \lambda_{s,L_n} \end{pmatrix}^{-1} \begin{pmatrix} \hat{L}_c \\ \hat{L}_n \end{pmatrix} \\ &= \frac{1}{\det \lambda} \begin{pmatrix} \lambda_{s,L_n} & -\lambda_{s,L_c} \\ -\lambda_{m,L_n} & \lambda_{m,L_c} \end{pmatrix} \begin{pmatrix} \hat{L}_c \\ \hat{L}_n \end{pmatrix}\end{aligned}$$

where

$$\begin{aligned}\det \lambda &= \lambda_{m,L_c} \lambda_{s,L_n} - \lambda_{s,L_c} \lambda_{m,L_n} \\ &= \lambda_{m,L_c} (1 - \lambda_{m,L_n}) - (1 - \lambda_{m,L_c}) \lambda_{m,L_n} \\ &= \lambda_{m,L_c} - \lambda_{m,L_n} = \lambda_{s,L_n} - \lambda_{s,L_c}.\end{aligned}$$

Hence, assuming wlog that  $\hat{L}_n = 0$ , then

$$\hat{Y}_m = \frac{\lambda_{s,L_n}}{\lambda_{s,L_n} - \lambda_{s,L_c}} \hat{L}_c > \hat{L}_c > 0$$

and

$$\hat{Y}_s = \frac{-\lambda_{m,L_n}}{\det \lambda} \hat{L}_c < 0.$$