Rollover Crises and Currency Unions

Javier Bianchi *
Federal Reserve Bank of Minneapolis

Jorge Mondragon †
University of Minnesota

February 12, 2018

PRELIMINARY AND INCOMPLETE

Latest version here

Abstract

This paper examines how an economy that belongs to a currency union is more vulnerable to a rollover crisis. We study a model of endogenous sovereign default with self-fulfilling rollover crisis and downward nominal wage rigidity. We show that a recession triggered by self-fufilling beliefs about a rollover crisis pushes the economy towards more frequent default.

Keywords: sovereign debt, rollover crisis, downward nominal wage rigidity.
JEL Codes: E62, F34, F41, F44, H50.

*Email: `javieribianchi@gmail.com`
†Email: `jorge.mondragon.minero@gmail.com`

1 Introduction

The recent European sovereign debt crisis saw several economies experiencing dramatic increases in sovereign spreads. Many observers have attributed the emergence of these dynamics to self-fulfilling expectations. As investors became pessimistic about the solvency of several governments in Southern Europe, bond prices deteriorated limiting the access of the government to credit markets, and pushing these governments to the verge of default.

These events have triggered a debate about what made countries in Southern Europe vulnerable to self-fulfilling rollover crisis, and how does this relate to the fact that these economies are part of a currency union. A common explanation, indeed, has to do with the inability of central banks in these economies to provide a backstop to the central government and potentially inflate away the debt.[1] In this paper, we provide an alternative argument of why being a member of a currency union makes an economy vulnerable to a rollover crisis.

We study a model of sovereign default with rollover crisis à la Cole and Kehoe (2000). We consider an economy that is part of a currency union and that is subject to downward wage rigidity. Unable to have an independent monetary policy, shocks either to fundamentals or non-fundamentals, trigger variations in the real wage, that can expose the economy to a severe recession. The key insight is that off-equilibrium pessimistic beliefs about a recession that would be triggered by a rollover crisis pushes the government to experience on the equilibrium path more frequents rollover crisis.

The paper contributes to the literature on sovereign default, in particular the one dealing with rollover crisis that started with Cole and Kehoe (2000)) (e.g. Bocola and Dovis, 2016, Roch and Uhlig (2016), Conesa and Kehoe, 2017 Aguiar, Chatterjee, Cole, and Stangebye, 2016). A distinctive features of our paper is that we study a production economy, and consider how nominal rigidities affect the exposure to rollover crisis. We show that for small levels of wage rigidity, the economy does not experience unemployment, yet the off equilibrium threat of high unemployment leads to a drastic increase in unemployment.

1.1 Households

Households' preferences over private and public consumption are given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \, \left[u(c_t) \right], \tag{1}$$

where u is a CRRA utility function, c_t denotes private consumption in period t, $\beta \in (0,1)$ is the subjective discount factor, and $\mathbb{E}_t$ denotes expectation operator conditional on the information set available at time t.

[1] See Krugman and De Grawe for these arguments; and work by Aguiar et al. and Corsetti.

The consumption good is assumed to be a composite of tradable (c^T) and nontradable goods (c^N), with a constant elasticity of substitution (CES) aggregation technology:

$$c = C(c^T, c^N) = [\omega(c^T)^{-\mu} + (1-\omega)(c^N)^{-\mu}]^{-1/\mu},$$

where $\omega \in (0,1)$ and $\mu > -1$. The elasticity of substitution between tradable and nontradable consumption is therefore $1/(1+\mu)$.

Each period households receive tradable endowment y_t^T, which is stochastic and follows a stationary first-order Markov process. In addition, households receive labor income, $w_t h_t$, and profits from the ownership of firms producing nontradable goods ϕ_t^N.

Households' sequential budget constraint, expressed in terms of tradables, is therefore given by

$$c_t^T + p_t^N c_t^N = y_t^T + \phi_t^N + w_t h_t - T_t, \tag{2}$$

where p_t^N denotes the relative price of nontradables in terms of tradables, w_t denotes the wage rate in terms of tradable goods.

Households inelastically supply $\overline{h}$ hours of work to the labor markets. Due to the presence of the wage rigidity (discussed in detail in the next subsections), households will only be able to sell $h_t \leq \overline{h}$ hours in the labor markets. The actual hours worked h_t is determined by firms' labor demand and taken as given by the households. As usual in the sovereign debt literature (see, for example, Aguiar and Gopinath, 2006 and Arellano, 2008), we assume that households are hand-to-mouth and that the government can distribute proceedings from external borrowing to the households using lump-sum taxes or transfers T_t, expressed in tradable units.

The households' problem consists of choosing c_t^T and c_t^N to maximize (1) given the sequence of prices $\{p_t^N, w_t\}$, endowments $\{y_t^T\}$, profits $\{\phi_t^N\}$, and taxes $\{T_t\}$. The optimality condition of this problem yields the equilibrium price of nontradable goods as a function of the ratio between tradable and nontradable consumption:

$$p_t^N = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{c_t^N}\right)^{\mu+1}. \tag{3}$$

That is, the relative price of nontradables is equal to the marginal rate of substitution between tradables and nontradables.

1.2 Firms

Firms are competitive and have access to a decreasing-returns-to-scale technology to produce nontradable goods with labor:

$$y_t^N = F\left(h_t^d\right),$$

where y_t^N denotes output of nontradable goods in period t, $F(.)$ is a continuous, differentiable, increasing and concave function. Firms' profits each period are then given by

$$\phi_t^N = p_t^N y_t^N - w_t h_t^d. \tag{4}$$

The optimal choice of labor h_t equates the value of the marginal product of labor and the wage rate, all expressed in tradable units,

$$p_t^N F'\left(h_t^d\right) = w_t. \tag{5}$$

1.3 Government

The government determines, taxes, borrowing, and repayment decisions to maximize households' lifetime utility. The government lacks commitment to all future policies.

We consider long-term debt, as in Arellano and Ramanarayanan (2012), Hatchondo and Martinez (2009), and Chatterjee and Eyigungor (2012). A bond issued in period t promises an infinite stream of coupons that decreases at an exogenous constant rate δ. In particular, a bond issued in period t promises to pay $\delta(1-\delta)^{j-1}$ units of the tradable good in period $t+j$, for all $j \geq 1$. Hence, debt dynamics can be represented by the following law of motion:

$$b_{t+1} = (1-\delta)b_t + i_t, \tag{6}$$

where b_t is the stock of bonds due at the beginning of period t, and i_t is the stock of new bonds issued in period t. The government can trade this long-term bond with atomistic international lenders not only to smooth consumption and allocate it optimally over time, but to boost employment through the keynesian channel as well. Debt contracts cannot be enforced and the government may decide to default at any point of time.

The government's sequential budget constraint each period in which it has access to debt markets is given by

$$\delta b_t = T_t + q_t i_t, \tag{7}$$

where q_t is the equilibrium price of the bond. The budget constraint indicates that tax revenues and new debt issuance have to the repayment of outstanding debt obligations.

Government's default entails two punishments. First, the government switches to financial autarky and cannot borrow for a stochastic number of periods. Second, there is an output loss function $\phi(y^T)$, which we assume to be increasing in tradable income. We think of this endowment loss as capturing various default costs related to reputation, sanctions, or misallocation of resources.

1.4 Foreign Lenders

Sovereign bonds are traded with atomistic, risk-neutral foreign lenders. In addition to investing through the defaultable bonds, lenders have access to a one-period risk-less security paying a net interest rate r. By a no-arbitrage condition, equilibrium bond prices are then given by

$$q_t = \frac{1}{1+r}\mathbb{E}_t[(1-d_{t+1})(\delta + (1-\delta)q_{t+1})]. \tag{8}$$

Equation (8) indicates that in equilibrium an investor has to be indifferent between selling a government bond in period t at price q_t and keeping the bond until next period bearing the risk of default. In case of repayment next period, the payoff is given by the coupon δ plus the market value q_{t+1} of the non-maturing fraction of the bonds. In case of default, the price q_{t+1} is equal to zero since we assume no recovery of defaulted bonds.

1.5 Equilibrium

In equilibrium the market for nontradable goods clears:

$$c_t^N = F(h_t). \tag{9}$$

For the labor markets, it is assumed that nominal wages have a lower bound $\bar{w}$, by which $W_t \geq \bar{w}$ for all t, following Schmitt-Grohé and Uribe (2016).[2] Given that the economy is under a currency peg and assuming that the law of one price holds for tradable goods and that the price of foreign tradable goods is constant and normalized to one, the wage rigidity can be expressed as

$$w_t \geq \bar{w}, \tag{10}$$

where w_t is the real wage and $\bar{w}$ is the wage lower bound, both in terms of the tradable good.

Actual hours worked cannot exceed the inelastically supplied level of hours:

$$h_t \leq \overline{h}. \tag{11}$$

Labor market equilibrium implies that the following slackness condition must hold for all dates and states:

$$(w_t - \bar{w})\left(\overline{h} - h_t\right) = 0. \tag{12}$$

This condition implies that when the nominal wage rigidity binds, the labor market can exhibit involuntary unemployment, given by $\overline{h} - h_t$. Similarly, when the nominal wage rigidity is not

[2] In Schmitt-Grohé and Uribe (2016), $\bar{w}$ depends on the previous period wage. For numerical tractability, we take $\bar{w}$ as an exogenous (constant) value as in Bianchi, Ottonello, and Presno (2015).

binding, the labor market must exhibit full employment.

Combining the equilibrium price equation (3) with resource constraint (9), the relative price p_t^N can be expressed as

$$p_t^N = \mathcal{P}^N(c_t^T, h_t) = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{F(h_t)}\right)^{\mu+1} \tag{13}$$

Using the firm's first order condition (5) and the equilibrium relative price (13), the wage rigidity condition can be expressed as

$$\mathcal{W}_t(c_t^T, h_t) = \frac{1-\omega}{\omega}\left(\frac{c_t^T}{F(h_t)}\right)^{\mu+1} F'(h_t) \geq \bar{w}. \tag{14}$$

Using the households' budget constraint (2) and the definition of the firms' profits and market clearing condition (9), the resource constraint of the economy can be rewritten as

$$c_t^T = y_t^T - d_t\phi(y_t^T) + (1-d_t)[\delta b_t - q_t i_t] \tag{15}$$

A competitive equilibrium given government policies in our economy is then defined as follows:

Definition 1 (Competitive Equilibrium)**.** *Given initial debt b_0, an exogenous process $\{y_t^T, \zeta_t\}_{t=0}^{\infty}$, government policies $\{T_t, b_{t+1}, d_t\}_{t=0}^{\infty}$, a competitive equilibrium is a sequence of allocations $\{c_t^T, c_t^N, h_t\}_{t=0}^{\infty}$ and prices $\{p_t^N, w_t, q_t\}_{t=0}^{\infty}$ such that:*

1. *Allocations solve household's and firms' problems at given prices.*

2. *Government policies satisfy the government budget constraint (7).*

3. *Bond pricing equation (8) holds.*

4. *The market for nontradable goods clears.*

5. *The labor market satisfies conditions*

1.6 Recursive Government Problem

We consider the optimal policy of a benevolent government with no commitment, that chooses public spending, external borrowing, and taxes to maximize households welfare, subject to the implementability conditions. We focus on the Markov recursive equilibrium in which all agents choose sequentially.

Every period the government enters with access to financial markets, it evaluates the lifetime utility of households if debt contracts are honored against the lifetime utility of households if they are repudiated. We use $\mathbf{S} = (b, y^T, \zeta)$ to denote the vector of all states. This includes an endogenous state b, and a vector of exogenous $\mathbf{s} = (y^T, \zeta)$. The variable ζ is a sunspot variable to index for the possibility of multiplicity of equilibria as in Cole-Kehoe. Different from the equilibrium according to the timing in Eaton-Gersovitz, the possibility of rollover crisisl imply that the bond price is a function of the initial debt position and the sunspot, in addition the debt choice and current income shock.

The government problem with access to financial markets can be formulated in recursive form as follows:

$$V(\mathbf{S}) = \max_{d\in\{0,1\}} \{(1-d)V^r(b,\mathbf{s}) + dV^d(\mathbf{s})\}, \tag{16}$$

where $V^r(b,\mathbf{s})$ and $V^d(\mathbf{s})$ denote, respectively, the value of repayment, given by the Bellman equation

$$V^r(b,y^T) = \max_{b',h} \left\{u(c^T, F(h_t)) + \beta\mathbb{E}\left[V(b',\mathbf{s}')\right]\right\} \tag{17}$$

subject to

$$\begin{aligned} c^T &= y^T + q(\mathbf{S},b')(b' - (1-\delta)b) - \delta b \\ \bar{w} &\leq \mathcal{W}_t(c^T, h) \\ h &\leq \bar{h} \end{aligned}$$

where $q(\mathbf{S},b')$ denotes the bond price schedule, taken as given by the government.

The value of default is given by

$$V^d(y^T) = \max_{h} \left\{u\left(c^T, F(h_t)\right) + \beta\mathbb{E}\left[\psi V(0,\mathbf{s}'^d(y^{T\prime}) + (1-\psi)V^d(y^{T\prime})\right]\right\} \tag{18}$$

subject to

$$\begin{aligned} c^T &= y^T - \phi(y^T) \\ \bar{w} &\leq \mathcal{W}(c^T, h) \\ h &\leq \bar{h} \end{aligned}$$

where $\psi \in [0,1]$ is the probability of re-entering financial markets after a default.

Let $\left\{d(b,\mathbf{s}), \hat{c}^T(b,\mathbf{s}), \hat{b}(b,\mathbf{s}), \hat{h}(b,\mathbf{s})\right\}$ be the optimal policy rules associated with the government problem. A Markov perfect equilibrium is then defined as follows.

Definition 2 (Markov perfect equilibrium)**.** *A Markov perfect equilibrium is defined by value functions* $\{V(b,\mathbf{s}), V^r(b,\mathbf{s}), V^c(b,\mathbf{s}), V^d(y^T)\}$*, policy functions* $\{d(b,\mathbf{s}), \hat{c}^T(b,\mathbf{s}), \hat{b}(b,\mathbf{s}), \hat{h}(b,\mathbf{s})\}$*, and a bond price schedule* $q(\mathbf{S},b')$ *such that*

1. *Given the bond price schedule, policy functions solve problems (16), (17), (19), and (18),*

2. The bond price schedule satisfies the bond pricing equation,

$$q(\mathbf{S}, b') = \frac{1}{1+r}\mathbb{E}[(1-d')(\delta + (1-\delta)q(\mathbf{S}', b''))|y^T].$$

where

$$\begin{aligned} b'' &= \hat{b}(b', \mathbf{s}') \\ d' &= d(b', \mathbf{s}') \end{aligned}$$

1.7 Multiplicity of Equilibrium

As in Cole-Kehoe, the government is subject to rollover crisis. For now, we follow the game described in Chatterjee and Eyigungor (2012), which is a slightly simplified version of Cole-Kehoe.

Now the government can face a state in which the foreign lender is unwilling to issue any new debt if the government has a current outstanding debt position. This state will be denoted as a crisis, with superscript c, and can be expressed as

$$V^c(b, \mathbf{s}) = \max_{b', h} \left\{ u(c^T, h) + \beta \mathbb{E}\left[V(b, \mathbf{s}')\right] \right\} \tag{19}$$

subject to

$$\begin{aligned} & c^T = y^T + q(\mathbf{S}, b')(b' - (1-\delta)b) - \delta b \\ & \bar{w} \leq \mathcal{W}(c^T, h) \\ & h \leq \bar{h} \\ & b' \leq (1-\delta)b \qquad \text{if } b > 0 \end{aligned}$$

Realise that the maximization problems V^r and V^c are exactly the same with the only difference that the choice set of sovereign debt is more restricted for the last one. Therefore, we can conclude that $V^r \geq V^c$.

The government and the international lenders play in the beginning of each period a game contingent the beliefs to the actions of their counterparts. The action set of the government is to repay and to default $\{R, D\}$, meanwhile the action set of the international lenders is to lend or not $\{L, N\}$. This game can be better represented as

Sovereign \ Lender	Lending	Not Lending
Repay	(V^r, ε)	$(V^c, 0)$
Default	$\left(V^d, \frac{-rb}{1+r}\right)$	$(V_d, 0)$

where $\varepsilon > 0$ is a small positive number denoting a small utility benefit derived by the international lender from lending to the government when repayment is expected. Under this game, when $V^c > V^d$ or $V^d > V^r$, there is only one Nash equilibrium $\{R, L\}$ and $\{D, N\}$, respectively. But, when $V^r \geq V^d \geq V^c$, both previous strategies are Nash equilibria. The sunspot ζ chooses which Nash equilibrium is going to happen. If the sunspot is not triggered $\zeta = 0$, then the players will pick the good Nash equilirbium $\{R, L\}$. If the sunspot is triggered $\zeta = 1$, then the players will pick the bad Nash equilibrium $\{D, N\}$. With this, we are able to construct the government with access to financial makets value function as

$$V(\mathbf{S}) = \begin{cases} V^r(b, y^T) & \text{if } V^d(y^T) \leq V^c(b, y^T) \\ V^r(b, y^T) & \text{if } V^d(y^T) \in \left[V^c(b, y^T), V^r(b, y^T)\right] \quad \& \quad \zeta = 0 \\ V^d(y^T) & \text{if } V^d(y^T) \in \left[V^c(b, y^T), V^r(b, y^T)\right] \quad \& \quad \zeta = 1 \\ V^d(y^T) & \text{if } V^d(y^T) > V^r(b, y^T) \end{cases}$$

With this setup, let us characterize the fundamental default and crisis default sets as

$$\begin{aligned} \mathcal{F}(b) &= \left\{ y^T \in \mathbb{R}_+ \middle| \quad V^d(y^T) > V^r(b, y^T) \right\} \\ \mathcal{C}(b) &= \left\{ y^T \in \mathbb{R}_+ \middle| \quad V^d(y^T) \in \left[V^c(b, y^T), V^r(b, y^T)\right] \right\}, \end{aligned}$$

and the default set as

$$\mathcal{D}(b, \zeta) = \begin{cases} \mathcal{F}(b) \cup \mathcal{C}(b) & \text{if } \zeta = 1 \\ \mathcal{F}(b) & \text{if } \zeta = 0 \end{cases}$$

Proposition 1 (Wage rigidity and crisis region)**.** *Suppose an environment where tradable endowment shocks have no persistency, no return to financial markets, no output cost when default* $\kappa(Y_T) = Y_T$*, and there is no rigidity in wages* $\bar{w}$*. For every sovereign debt* b *such that* $\mathcal{D}(b, \zeta) \neq \emptyset$ *and for any sunspot shock realization* $\zeta \in \{0, 1\}$*, the real wages of the states follow* $w^d \geq w^r \geq w^c$*.*

Proof: See Appendix A

This is the main novel mechanism this paper tackle. The key insight of this proposition is that as wage rigidities increases, the economy can be in crisis with lower positions of debt. Moreover, for small increases of wage rigidities, the crisis region increases. Hence, a sovereign government than cannot depreciate its currency to reduce the real wage can become more exposed to rollover crisis.

This tells us a tradeoff when introducing floor wage rigidities. In a default state, the possible wage is the highest, making it unlikely to bind. Nevertheless, when choosing to repay there is a risk of having a wage that can bind with the wage constraint. Moreover, the state of repayment when there is no lending by the bankers is the one with the lowest wage. Even though, this state will never happen in equilibrium, realise that it does affect the default crisis zone. This introduces an effect regarding the threat of unemployment in a non-lending state. Because the crisis zone increases due to the threat, the sovereign will adjust its amount of sovereign debt issued to protect itself against bad sunspot realizations.

1.8 Analytical Example

Let us start with a simple example to illustrate how downward wage rigidity interacts affects the vulnerability to a self-fulling rollover crisis. First, let us assume some simple functional forms for the model. Suppose that the nontradable production function is linear with respect of labor $F(h) = h$, the utility function is Cobb-Douglas $u(C) = \ln(C)$, and the output loss function is $\kappa(y^T) = \bar{\kappa} y^T$ where $\bar{\kappa} \in [0, 1]$. Also, suppose there is no return to financial markets once in a default state $\psi = 0$, there is only one possible level of output in the economy $y^T \in \{\bar{y}\}$, there is a unitary elasticity of substitution between tradable and nontradable goods $\mu = 0$, there is a unit of labor inelastically supplied $\bar{h} = 1$, and the maturity of the debt is of only one period $\delta = 1$.

To start, let us describe an environment in which wages are perfectly flexible. In this environment the problem simplifies greatly. Firstly, realise that the value of a default state is constant because there is only one possible tradable endowment realization. This is,

$$V^d = \left(\frac{\omega}{1-\beta}\right) \ln(y^T) + \left(\frac{1-\omega}{1-\beta}\right) \ln\left(\bar{h}\right) + \bar{U}_P;$$

where $\bar{U}_P \equiv \left(\frac{\omega}{1-\beta}\right) \ln(1-\bar{\kappa})$ is the penalty for defaulting.

The repayment value functions are also simplified considerably. They can be expressed as

$$V^r(b) = \max_{b'} \left\{\omega \ln\left(\bar{y} - b + q(b', b)b'\right) + (1-\omega)\ln\left(\bar{h}\right) + \beta\left(\pi V(b', 1) + (1-\pi)V(b', 0)\right)\right\}$$

and

$$\begin{aligned} V^c(b) = \max_{b'} & \left\{\omega \ln\left(\bar{y} - b + q(b', b)b'\right) + (1-\omega)\ln\left(\bar{h}\right) + \beta\left(\pi V(b', 1) + (1-\pi)V(b', 0)\right)\right\} \\ \text{s.t. } & b' \leq 0 \end{aligned}$$

Again, realise that the problem V^r and V^c are the same except the choice set options of how much debt the government can incur. Because the choice set of the crisis value function is more restrictive, we can conclude that $V^r \geq V^c$.

Finally, the value function when the government has access to financial markets follows (**??**)

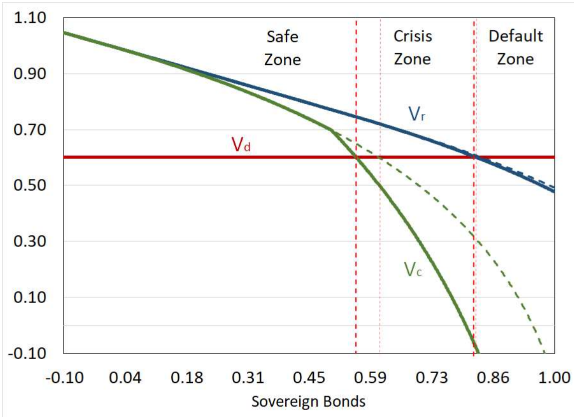
and the bond pricing can be expressed as

$$q(b', b) = \begin{cases} \frac{1}{1+r} & \text{if } V^d \leq V^c(b) \\ \frac{1-\pi}{1+r} & \text{if } V^d \in (V^c(b), V^r(b)] \\ 0 & \text{if } V^d > V^r(b) \end{cases}$$

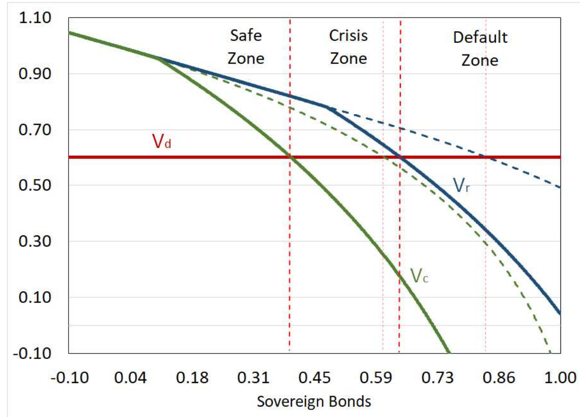
In Figure 2(c) we can see how the safe, crisis, and default zone are constructed. As the optimal policy function for debt yields a positive amount of assets, the value of repayment and crisis are the same. As soon as the optimal level of debt starts to become positive, then these two value functions start to differ. The crisis and default zones are characterized as thresholds when the two previous curves cross the fixed value of default. The pricing function of the debt can be seen in Figure 3(c). The debt pricing is a step function because there is no uncertainty in the level of tradable endowment for the next period. In the safe zone the pricing is the inverse of the gross interest rate, in the crisis zone is the inverse of the gross interest rate adjusted by the probability of a sunspot not happening, and in the default zone the price is zero. Finally, realize that the wages in Figure 4(c). The wages for the default state is a constant, but for the repayment and crisis states they are stictly decreasing with respect of debt. Moreover, realise that the wages for the repayment state are never lower than the wages for the crisis state. Also, when there are positive debt position, the wages for default are always higher than any of the other states. This helps us visualize that the wage floor rigidities are going to affect first the crisis state, then the reapayment state, and finally the default state; as they increase in level.

Let us study now what happens when there exists a wage floor rigidty. In Figure 2(a) we can see how the zones change when there is a small wage floor rigidity. The most significant movement is the fall of the value function of the crisis state as soon as the wage constraint starts to activate. This increases the crisis zone by reducing the safe zone. On the other hand, in Figure 2(b) we can see these changes when there is a high wage floor rigidity. Here not only the value of the crisis state falls, but also the value of the repayment state. This undoubtedly increases the default zone, but it is unsure if the crisis zone increases or decreases. So even though the crisis zone increases by decreasing the safe zone, the default zone increases by decreasing the crisis zone. The bond pricing adjustments can be appreciated in Figures 3(a) and 3(b) for these two different levels of wage rigidities. Finally, the wages become fixed as soon as the wage constraint is activated, 4(a) and 4(b).

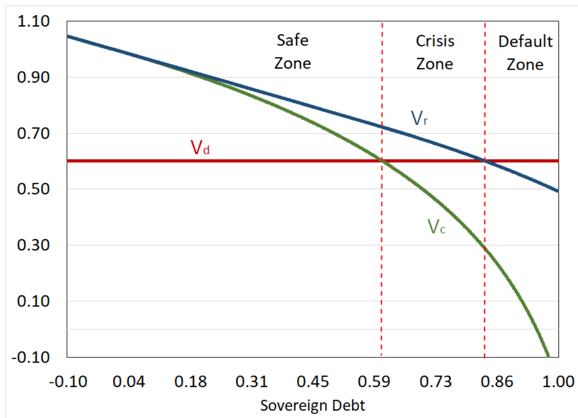
The main cost of having the wage constraint activated in any state is that it induces unemployment. Given that the state where wages are the highest when having positive debt is default, this creates another benefit for the government to not honor its debt. Defaulting on its sovereign debt, the government can reduce unemployment. When wages are perfectly flexible, there is no unemployment issue. Figure 5(a) shows what happens to unemployment for the repayment, crisis, and default states when there is a low level of wage rigidity. Realise



(a) Low Wage Floor Rigidity ($\bar{w} = 0.6$)



(b) High Wage Floor Rigidity ($\bar{w} = 1.0$)



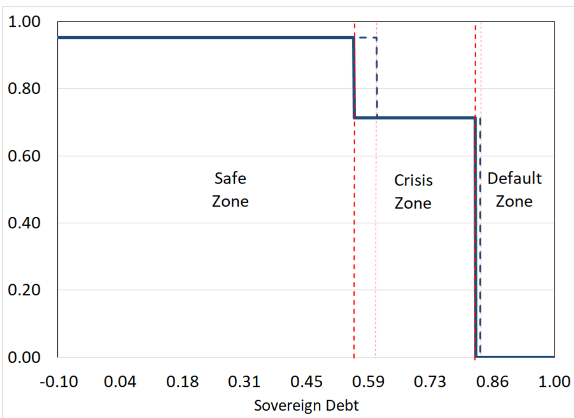
(c) Perfect Flexible Wages ($\bar{w} = 0.0$)

Figure 1: Value Functions under different wage rigidities ($\omega = 0.5$, $\bar{y} = 1.10$, $\beta = 0.95$, $r = 0.05$, $\pi = 0.25$)

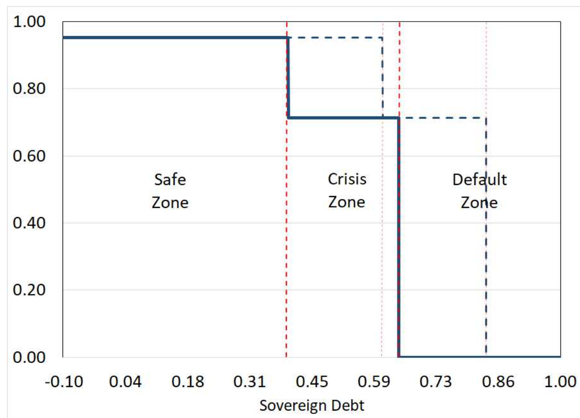
that unemployment will never be an equilibrium pheonomenon because in the safe and crisis zones repayment state unemployment is zero and in the crisis and default zone default state unemployment is zero also. But what is important is that the crisis state unemployment is well positive even in the safe zone. This induces a higher danger threat for the sovereign to have high levels of sovereign debt because the crisis zone is larger. Figure 5(b) shows what happens when the wage rigidity is high. Here we can see that there will be unemployment in equilibrium because the wage floor is so high that in the crisis zone the repayment state incurs in it. Moreover, the reason that the default zone increases is because as debt accrues, the more attractive default appears because unemployment will disappear. This suggests that unemployment spells can trigger default scenarios because of the opportunity of increasing labor hiring.

2 Quantitative Analysis

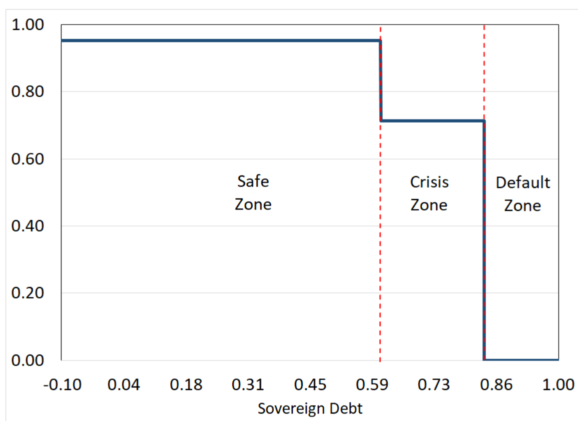
TO BE COMPLETED



(a) Low Wage Floor Rigidity ($\bar{w} = 0.6$)

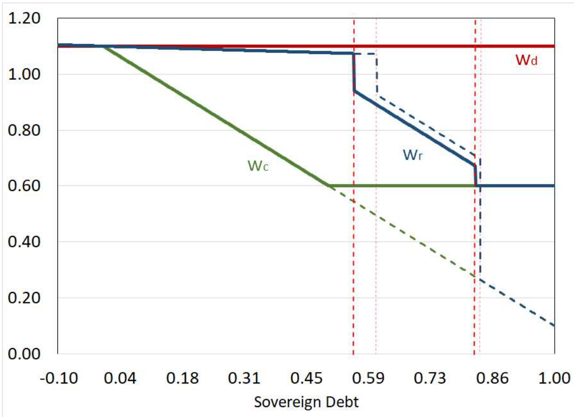


(b) High Wage Floor Rigidity ($\bar{w} = 1.0$)

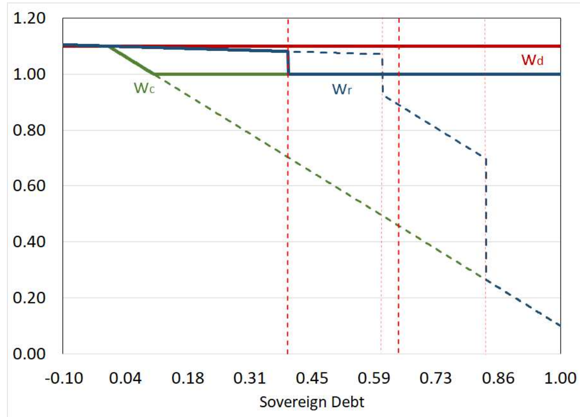


(c) Perfect Flexible Wages ($\bar{w} = 0.0$)

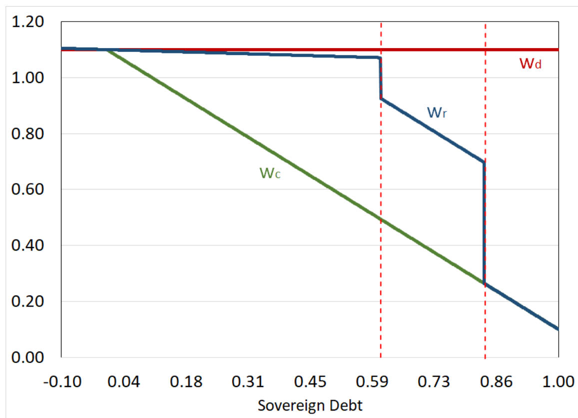
Figure 2: Debt Pricing under different wage rigidities ($\omega = 0.5$, $\bar{y} = 1.10$, $\beta = 0.95$, $r = 0.05$, $\pi = 0.25$)



(a) Low Wage Floor Rigidity ($\bar{w} = 0.6$)

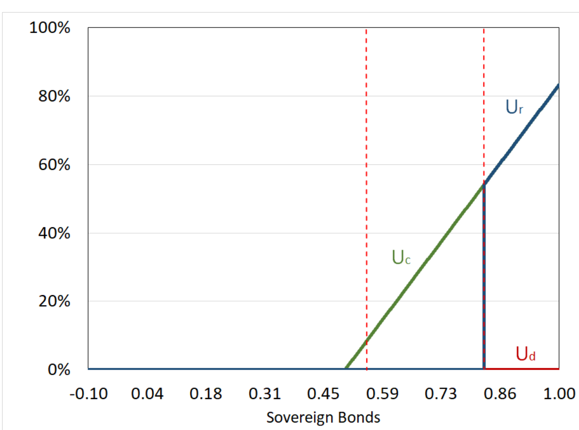


(b) High Wage Floor Rigidity ($\bar{w} = 1.0$)

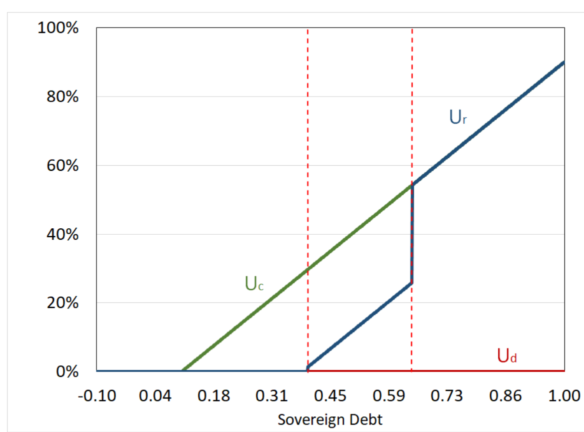


(c) Perfect Flexible Wages ($\bar{w} = 0.0$)

Figure 3: Wages under different wage rigidities ($\omega = 0.5$, $\bar{y} = 1.10$, $\beta = 0.95$, $r = 0.05$, $\pi = 0.25$)



(a) Low Wage Floor Rigidity ($\bar{w} = 0.6$)



(b) High Wage Floor Rigidity ($\bar{w} = 1.0$)

Figure 4: Unemployment under different wage rigidities ($\omega = 0.5$, $\bar{y} = 1.10$, $\beta = 0.95$, $r = 0.05$, $\pi = 0.25$)

3 Conclusions

TO BE COMPLETED

References

Aguiar, M., S. Chatterjee, H. Cole, and Z. Stangebye (2016): "Quantitative Models of Sovereign Debt Crises," Discussion paper, Elsevier.

Aguiar, M., and G. Gopinath (2006): "Defaultable Debt, Interest Rates and the Current Account," *Journal of International Economics*, 69(1), 64–83.

Arellano, C. (2008): "Default Risk and Income Fluctuations in Emerging Economies," *American Economic Review*, 98(3), 690–712.

Arellano, C., and A. Ramanarayanan (2012): "Default and the Maturity Structure in Sovereign Bonds," *Journal of Political Economy*, 120(2), 187–232.

Bianchi, J., P. Ottonello, and I. Presno (2015): "Unemployment, Sovereign Debt, and Fiscal Policy in a Currency Union," .

Bocola, L., and A. Dovis (2016): "Self-Fulfilling Debt Crises: A Quantitative Analysis," NBER Working Paper 22694.

Chatterjee, S., and B. Eyigungor (2012): "Maturity, indebtedness, and default risk," *American Economic Review*, 102(6), 2674–99.

Cole, H. L., and T. J. Kehoe (2000): "Self-Fulfilling Debt Crises," *Review of Economic Studies*, 67(1), 91–116.

Conesa, J. C., and T. J. Kehoe (2017): "Gambling for redemption and self-fulfilling debt crises," *Economic Theory*, 64(4), 707–740.

Hatchondo, J. C., and L. Martinez (2009): "Long-Duration Bonds and Sovereign Defaults," *Journal of International Economics*, 79(1), 117–125.

Roch, M. F., and H. Uhlig (2016): "The dynamics of sovereign debt crises and bailouts," International Monetary Fund.

Schmitt-Grohé, S., and M. Uribe (2016): "Downward nominal wage rigidity, currency pegs, and involuntary unemployment," *Journal of Political Economy*, 2.

A Propositions Proofs

Proposition 2 (Fundamental Default Set Monotonicity)**.** *Pick an arbitrary level of sovereign debt* b_1 *such that* $\mathcal{F}(b_1) \neq \emptyset$*, if* $b_2 \geq b_1$ *then* $\mathcal{F}(b_1) \subseteq \mathcal{F}(b_2)$*.*

Proposition 3 (Complete Default Set Monotonicity)**.** *Pick an arbitrary sunspsot realization* $\zeta \in \{0, 1\}$ *and a level of sovereign debt* b_1 *such that* $\mathcal{D}(b_1, \zeta) \neq \emptyset$*, if* $b_2 \geq b_1$ *then* $\mathcal{D}(b_1, \zeta) \subseteq \mathcal{D}(b_2, \zeta)$*.*

Proposition 4 (No Resources Inflows)**.** *Suppose an environment where tradable endowment shocks have no persistency, no return to financial markets, and no downward wage rigidity. For every sovereign bonds* b *such that* $\mathcal{D}(b, \zeta) \neq \emptyset$ *and for any sunspot shock realization* $\zeta \in \{0, 1\}$*, every feasible* b' *will yield no resources inflows,* $q(b')\,(b' - (1-\delta)b) - \delta b \leq 0$*.*

Results in propositions 2-4 are derived in Arellano (2008). They tell us that incentives to default are monotonic with respect of sovereign debt. In other words, given a level of tradable output, the sovereign debt due in the period acts only as a shifter in the available amount of tradable resources in the economy. Therefore, if for a level of sovereign debt a government finds optimal to default, then for a higher level of sovereign debt the default decision will still be optimal because the government will have less resources overall.

Default episodes happen when governments are unable to roll-over their debt. This idea is captured with Proposition 4. If the government is in a state where it chooses to default, all the feasible issuances of sovereign bonds must have not been enough to cover the sovereign bonds due in the period. Specifically, there was no feasible issuances of sovereign bonds that could have given a positive flux of resources from outside, $q(b')\,(b' - (1-\delta)b) - \delta b \not> 0$.

Proof. Set a level of sovereign debt b_1 and level of tradable endowment such that $y^T \in \mathcal{F}(b_1)$. Then, it follows that $V^d(y^T) > V^r(b_1, y^T)$. Pick an arbitrary level of sovereign debt b_2 such that $b_2 \geq b_1$. Let us study the tradable resource constraint in the repayment scenario. Define the budget set of the government contingent to the amount of sovereign debt due as

$$\mathcal{B}^r(b) = \left\{(c^T, b') \in \mathbb{R}_+ \times (\infty, \mathbb{B}] \,\middle|\, \quad c^T \leq y^T + q(b', \mathbf{s})b' - (\delta + (1-\delta)q(b', \mathbf{s}))\, b\right\}.$$

By construction, acknowledge that $\mathcal{B}^r(b_2) \subseteq \mathcal{B}^r(b_1)$. Because the government is maximizing over a subset of a set, it follows that $V^r(b_1, y^T) \geq V^r(b_2, y^T)$. Joining the inequalities, we conclude that

$$V^d(y^T) > V^r(b_1, y^T) \geq V^r(b_2, y^T).$$

In other words, $y^T \in \mathcal{F}(b_2)$. Finally, because the level of sovereign debt b_2 was taken arbitrarily, we can conclude that $\mathcal{F}(b_1) \subseteq \mathcal{F}(b_2)$. ∎

Proof. Set a level of sovereign debt B_1 and a level of tradable endowment such that $y^T \in \mathcal{D}(b_1, \zeta)$. For a no sunspot realization $\zeta = 0$, using Proposition 2 it follows that $\mathcal{D}(b_1, 0) \subseteq \mathcal{D}(b_2, 0)$. For a sunspot realization $\zeta = 1$, it follows that $V^d(y^T) > V^c(b_1, y^T)$. Pick an arbitrary level of sovereign debt b_2 such that $b_2 \geq b_1$. Define the budget set of the government contingent to the amount of sovereign debt due as

$$\mathcal{B}^c(b) = \left\{(c^T, b') \in \mathbb{R}_+ \times (\infty, (1-\delta)b] \,\middle|\, \quad c^T \leq y^T + q(b', \mathbf{s})b' - (\delta + (1-\delta)q(b', \mathbf{s}))\, b\right\}.$$

By construction, acknowledge that $\mathcal{B}^c(b_2) \subseteq \mathcal{B}^c(b_1)$. Because the government is maximizing over a subset of a set, it follows that $V^c(b_1, y^T) \geq V^c(b_2, y^T)$. Joining the inequalities, we conclude that

$$V^d(y^T) > V^c(b_1, y^T) \geq V^c(b_2, y^T).$$

In other words, $y^T \in \mathcal{D}(b_2, 1)$. Hence, it follows that $\mathcal{D}(b_1, 1) \subseteq \mathcal{D}(b_2, 1)$ Finally, for any sunspot realization ζ and because the level of sovereign debt b_2 was taken arbitrarily, we can conclude that $\mathcal{D}(b_1, \zeta) \subseteq \mathcal{D}(B_2, \zeta)$. ∎

Proof. Pick an arbitrary $y^T \in \mathcal{D}(b, \zeta)$ and realise this implies $V^d(y^T) > V^r(b, y^T)$ if $\zeta = 0$ and $V^d(y^T) > V^c(b, y^T)$ if $\zeta = 1$. Because $V^r(b, y^T) \geq V^c(b, y^T)$, it only suffices to check the no sunpsot realization scenario. Acknowledge that the tradable resource constraints found in the repayment and in default states can be rewritten respectively as

$$c^r = y^T + [q(b')\,(b' - (1-\delta)b) - \delta b] \qquad \text{and} \qquad c^d = y^T \kappa(y^T).$$

Therefore,

$$\begin{aligned}
u\left(y^T, \bar{h}\right) + \beta \mathbb{E}\left[V^d(y^{T\prime})\right] &> u\left(y^T - \kappa(y^T), \bar{h}\right) + \beta \mathbb{E}\left[V^d(y^{T\prime})\right] \\
&> \max_{b'} \left\{u\left(y^T + [q(b')\,(b' - (1-\delta)b) - \delta b]\,, \bar{h}\right) + \beta \mathbb{E}\left[V(b', \mathbf{s})\right]\right\} \\
&\geq \max_{b'} \left\{u\left(y^T + [q(b')\,(b' - (1-\delta)b) - \delta b]\,, \bar{h}\right) + \beta \mathbb{E}\left[V^d(y^{T\prime})\right]\right\} \\
&\geq u\left(y^T + [q(b')\,(b' - (1-\delta)b) - \delta b]\,, \bar{h}\right) + \beta \mathbb{E}\left[V^d(y^{T\prime})\right],
\end{aligned}$$

for all feasible b'. Thus,

$$u\left(y^T, \bar{h}\right) > u\left(y^T + [q(b')\,(b' - (1-\delta)b) - \delta b]\,, \bar{h}\right).$$

Because $u(\cdot)$ is an increasing function,

$$y^T \geq y^T + [q(b')\,(b' - (1-\delta)b) - \delta b]\,.$$

Then, we arrive to

$$q(b')\,(b' - (1-\delta)B) - \delta b \leq 0.$$

Finally, because we picked an arbitrary $y^T \in \mathcal{D}(b, \zeta)$, we can conclude that for all feasible b' there are no resources inflows, $q(b')\,(b' - (1-\delta)b) - \delta b \leq 0$. ∎

Proof. Pick an arbitrary $y^T \in \mathcal{D}(B, \zeta)$. Using (14) we find that

$$
\begin{aligned}
w^d &= \frac{1-\omega}{\omega}\left(\frac{y^T}{F(\bar{h})}\right)^{1+\mu} F'(\bar{h}) \\
w^r &= \frac{1-\omega}{\omega}\left(\frac{y^T + q(b')(b' - (1-\delta)b) - \delta b}{F(\bar{h})}\right)^{1+\mu} F'(\bar{h}) \\
w^c &= \frac{1-\omega}{\omega}\left(\frac{y^T - \delta b}{F(\bar{h})}\right)^{1+\mu} F'(\bar{h})
\end{aligned}
$$

when the crisis and repayment scenario are different. Using Proposition 4, we can conclude that $w^d \geq w^r \geq w^c$. ∎