

# On the Assignment of Workers to Occupations and the Human Capital of Countries\*

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February, 2018

## Abstract

A worker's human capital determines his absolute and relative productivity across the different occupations. For a country as a whole, the cross-section distribution of workers determines the country's equilibrium assignment of workers to jobs and the resulting aggregate human capital. We consider a tractable general equilibrium Roy model and use it to infer, from observed data across countries and time: (i) the absolute and comparative advantage components of the different workers, (ii) the occupation intensity of a country's human capital, and (iii) the distortions in the allocation of workers to jobs. Contrary to the standard measure that implicitly assumes that human capital entails only absolute advantage, we show that the data implies that: (a) human capital has a strong comparative advantage component; (b) a higher distribution in the human capital distribution of workers lead to skill-upgrading across occupations and to a higher skill intensity of the overall human capital of countries; (c) cross-country differences in aggregate human capital explains a much larger fraction of the cross-country income differences from the standard model. We also find substantial costs from the distortions in the allocation of workers to jobs, especially for the less developed countries. Finally, we use the model to assess whether a country should direct its efforts to enhance the skills at the higher- or lower-end of the skill distribution.

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\*The views expressed here are those of the authors and do not necessarily reflect the opinion of the Federal Reserve Bank of St. Louis or the Federal Reserve System.

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# 1 Introduction

We develop a simple general equilibrium Roy model in which a set of workers with different human capital levels are assigned to a set of different occupations. The model draws on recent work by Acemoglu and Autor (2011), Costinot and Fogel (2014), Burstein et al (2016) and Hsieh et al. (2016). Those papers focus on the equilibrium wage inequality arising in equilibrium as workers with different characteristics are allocated across occupations with different skills requirements. Our focus is instead on the country’s aggregate effective supply of human capital services resulting from the endogenous assignment of workers to occupations.

In our setting, a worker’s human capital determines his absolute and relative productivity across the different occupations. For a country as a whole, the cross-section distribution of workers determines the country’s equilibrium assignment of workers to jobs and the resulting aggregate human capital. We consider a tractable general equilibrium Roy model and use it to infer, from observed data across countries and time: (i) the absolute and comparative advantage components of the different workers, (ii) the occupation intensity of a country’s human capital, and (iii) the distortions in the allocation of workers to jobs. The standard measure commonly used in growth and development accounting is encompassed as the special case when human capital entails only absolute advantage. However, using data on the observed assignment of workers to jobs, we show that: (a) human capital has a strong comparative advantage component; (b) a higher distribution in the human capital distribution of workers lead to skill-upgrading across occupations and to a higher skill intensity of the overall human capital of countries; (c) cross-country differences in aggregate human capital explains a much larger fraction of the cross-country income differences from the standard model. We find substantial costs from the distortions in the allocation of workers to jobs, especially for the less developed countries. Finally, we use the model to assess whether a country should direct its efforts to enhance the skills at the higher- or lower-end of the skill distribution.

## 2 Workers and Occupations Across Time and Countries

In this section, we describe a number of patterns in the assignment of workers to occupations across countries with very different levels of development. We also explore the time evolution in the assignment of workers to jobs for a handful of countries. Albeit from a limited set of countries and years, our sample includes countries in very different levels of development. We first describe our data sources, describing countries for which we have available data. We describe the types of jobs and occupations and the different variables available to group the workers of those countries. Then, we describe how the assignment of workers to the different jobs varies across those country-year pairs.

### 2.1 Data

Our source of cross-country comparisons on demographic and economic outcomes at the micro level is the *Integrated Public Use Microdata Series, International (IPUMS)*, which aggregates microdata samples from international census records. The records are converted into a consistent format that allows for cross-country comparisons. While the universe of international census records included in IPUMS is very large, the need for reliable data on earnings by occupations and worker’s characteristics reduces our sample to only a handful of country-year pairs. Table 1 describes the countries and years for which we have data.

**Table 1: Country-Years Available**

| Country       | Years       |
|---------------|-------------|
| Brazil        | 2000        |
| Canada        | 1971 & 2001 |
| India         | 1999        |
| Indonesia     | 1976 & 1995 |
| Mexico        | 1960 & 1990 |
| Panama        | 2000        |
| United States | 1960 & 2000 |

For each of these country-year pairs, we collect individual level census data on age, gender, occupation, annual labor income (in 2000 USD PPP), and schooling attainment for all occupied workers 25 or older<sup>1</sup>. Occupations are directly comparable across countries, and are coded according to the major categories in the 1988 International Standard Classification of Occupations (ISCO) scheme. Therefore, we consider nine,  $j = 1, \dots, 9$ , broad occupation categories:

1. Legislators, senior officials and managers
2. Professionals
3. Technicians and associate professionals
4. Clerks
5. Service workers and shop and market sales
6. Skilled agricultural and fishery workers
7. Crafts and related trades workers
8. Plant and machine operators and assemblers
9. Elementary occupations

As for workers, we define 42 groups, i.e.  $e = 1, \dots, 42$  according to:

- *gender*: male and female;
- *age*: 25-34, 35-50, 51+ years of age; and
- *schooling*: no-schooling, primary incomplete, primary complete, secondary incomplete, secondary complete, tertiary incomplete and tertiary complete or more.

For each country-year, we use all these combinations to construct the vector containing the cross-section distribution of workers

$$S_t = [S_t(1); \dots; S_t(e); \dots S_t(42)],$$

for the country in that year. We also generate  $42 \times 9$  matrices indicating the fractions of workers type  $e$  in occupation  $j$ , and the average incomes of those workers. We indicate those proportions and average incomes as  $p_t(e, j)$  and  $y_t(e, j)$  respectively.

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<sup>1</sup>Details are presented in the appendix

### 3 Model

In this section, we describe the economic environment that we use to model the human capital of countries. We first lay out the environment and then define equilibrium and prove its existence and uniqueness. Then, we discuss how the aggregate human capital of a country is shaped from the endowment of workers and the technology of the country.

#### 3.1 The Environment

As in the standard growth model, a composite final good  $Y_t$  is produced in each period  $t$  according to a production function  $Y_t = Z_t F(K_t, H_t)$ . Here,  $F(\cdot, \cdot)$  is a constant returns to scale production function,  $Z_t$  is an exogenous Hicks-neutral productivity term (TFP),  $K_t$  is the flow of services from the stock of physical capital in the country, and  $H_t$  is the aggregate flow of human-capital augmented labor services. We extend the standard model by considering a human capital  $H_t$  that arise in equilibrium from the allocation of workers to tasks.

First,  $H_t$  is the resulting bundle of human capital services is given and is given by

$$H_t = G_t[H_t(1), \dots, H_t(J)],$$

where  $G_t(\cdot)$  is a constant returns to scale function defined over the vector  $H_t(j)$  of effective labor services provided in occupations or ‘jobs’  $j = 1, \dots, J$  in period  $t$ . Second, we consider the allocation of workers with different human capital ‘groups’, indexed by  $e = 1, \dots, E$  across the different occupations  $j = 1, \dots, J$ . The aggregate supply levels  $H_t(j)$  arise from optimal occupation choices, i.e. the general equilibrium assignments, of workers in groups  $e$  to occupations  $j$ , as explained below.

Much of our analysis can proceed with generic functions  $F_t(\cdot)$  and  $G_t(\cdot)$ , but for concreteness we will focus on commonly used functional forms. Specifically, we adopt a Cobb-Douglas aggregate production function:

$$Y_t = Z_t (K_t)^\alpha (H_t)^{1-\alpha}, \quad (1)$$

where  $0 < \alpha < 1$  is the physical capital share of output. Similarly, the human capital aggregator is given by a CES, i.e.

$$H_t = \left[ \sum_{j=1}^J M_t(j) [H_t(j)]^\rho \right]^{\frac{1}{\rho}}, \quad (2)$$

where  $\rho$  is a parameter that indicates the degree of complementarity across the different occupations and can entertain values anywhere between  $-\infty$ , Leontieff, i.e. extreme complements, and  $+1$ , i.e. perfect substitutes.<sup>2</sup> Here,  $M_t(j) \geq 0$  are the distributional factors that determine the share of human capital services in occupations  $j$  on the overall flow of human capital services  $H_t$  in the economy. We adopt the normalization  $\sum_{j=1}^J M_t(j) = 1$  so that the TFP of the country  $Z_t$  captures all the Hicks neutral productivity shifts and  $M_t(j)$  are the CES distributional parameters in the production function of  $H_t$ .

The population of workers inside the country in period  $t$  is described by a discrete distribution

$$S_t = [S_t(1), \dots, S_t(e), \dots, S_t(E)].$$

Here,  $S_t(e) \geq 0$  for all groups  $e$ , and we normalize the population measure to one, so that  $\sum_{e=1}^E S_t(e) = 1$ . Thus,  $S_t(\cdot)$  is simply a discrete probability distribution, describing the cross-section of workers groups or types that populate the country at period  $t$ . In light of our data, we

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<sup>2</sup>Values of  $\rho$  above 1 are ruled out to keep the aggregate  $H_t$  to be a concave function of  $H_t(j)$ .

will assume that the number of human capital groups  $E$  is finite. In our baseline exercises, we think of each  $e$  as indexing education attainment levels. We then extend our groupings of human capital to include work experience and gender.

As in the data, workers in all the different groups  $e$  could potentially provide labor services in any of the  $j = 1, \dots, J$  occupations. The human capital type  $e$  of a worker, however, determines the proclivity of those workers to choose the different occupations. In our model, the human capital  $e$  of a worker determines not only his absolute advantage in the different occupations, but also his comparative advantage relative to other workers.

Specifically, the assignment of workers to occupations is potentially driven by four factors: (a) the *unitary skill price* in each occupation,  $w_t(j)$ ; it applies to all workers, regardless of their type  $e$ , entering in that occupation; (b) a productivity component  $T_t(e, j) > 0$  that determines the *average potential productivity* of workers with human capital  $e$  in occupation  $j$ ; and (c) a *random* component,  $\eta(j)$ , of the different workers for each possible occupation  $j$ ; and (d) distortions or *wedges*  $D_t(e, j)$  such as barriers and compensatory variations that can be specific to the pairings  $j$  and  $e$ . We take factors (b)-(d) as exogenously given but unitary skill prices (a) will be determined endogenously by the equilibrium of the economy.

We follow Burstein et. al (2016), Hsieh et. al (2016) and others, by assuming that each worker draws a random  $1 \times J$  vector,

$$\eta = [\eta(1), \dots, \eta(J)] \in \mathbb{R}_+^J,$$

from a continuous joint distribution described by a p.d.f.  $Q(\eta)$ , where each  $\eta(j)$  is drawn identically and independently, both, (i) across all individuals, and (ii) across all occupations for the same individual. In particular, we assume that the distribution is given by a multidimensional Frechet distribution:

$$Q(\eta) = \prod_{j=1}^J \exp \left\{ -[\eta(j)]^{-\theta} \right\},$$

where  $\theta > 1$  is a dispersion parameter that drives the degree of comparative advantage in the economy. We use a multidimensional Frechet distribution, a form of extreme value distribution, to exploit of a number of substantial analytical aspects in our quantitative analysis. In our formulation,  $T_t(e, j)$  governs the absolute and comparative advantage of workers in group  $e$  for the provision of skills in occupations or jobs  $j$ . Indeed, the expected amount of labor units that each of the workers with human capital  $e$  supplies to occupation  $j$  is equal to  $T_t(e, j) \Gamma(1 - \theta^{-1})$  where  $\Gamma(\cdot)$  is the Gamma function.<sup>3</sup>

### 3.2 Equilibrium Assignment

We consider competitive equilibria where the equilibrium assignment of workers to jobs is driven by the income maximization of workers, the profit maximization of firms and the clearing of all labor markets. The price system is simply a vector of unitary prices for each skill  $j$ ,  $w_t(j)$ , and the rental rate of physical capital, denominated  $R_t$ , all in units of the final good. Taking those prices as given, workers of all groups  $e$  will be assigned to occupations  $j$ , generating a matrix  $p_t(e, j)$  that indicates the fraction of workers in group  $e$  that work in occupation  $j$ . Here,  $p_t(e, j) \geq 0$  and  $\sum_{j=1}^J p_t(e, j) = 1$  for all  $e$ . Finally, taking as given the vector of unitary *occupation* or *skill* prices  $w_t(j)$ , the implied aggregate supply  $H_t(j)$  must be equal to the demand of those human capital services from all the firms in this economy for the markets to clear.

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<sup>3</sup>That is,  $\Gamma(1 - \theta^{-1}) = \int_0^\infty x^{-(1/\theta)} e^{-x} dx$ .

The optimization conditions for firms and workers are straightforward. First, consider the firms. Because of constant returns to scale the firm size distribution is not pinned-down. Yet, any firms' hiring of services from the different forms of human capital services  $H_t(j)$  and of physical capital  $R_t$  can be characterized by a stand-in firm that maximizes profits taking  $w_t(j)$  and  $R_t$  as given, i.e.:

$$\max_{\{H_t(j), K_t\}} \left\{ Z_t(K_t)^\alpha \left( \left[ \sum_{j=1}^J M_t(j) [H_t(j)]^\rho \right]^{\frac{1}{\rho}} \right)^{1-\alpha} - \sum_{j=1}^J w_t(j) H_t(j) - R_t K_t \right\}.$$

As expected, the first order conditions simply equate the wages  $w_t(j)$  to the marginal products of the different forms of human capital,  $H_t(j)$ , i.e.:

$$w_t(j) = \bar{w}_t \times M_t(j) [H_t(j)]^{\rho-1}, \quad (3)$$

where  $\bar{w}_t \equiv (1-\alpha) Z_t(K_t/H_t)^\alpha \times (H_t)^{1-\rho}$  is an economywide component that is common across in the price of all skills  $j$ .

Second, consider workers. Each worker realizes a vector  $\eta$  and chooses the occupation that maximizes his net income,

$$\max_{i \in \{1, \dots, J\}} \left\{ \eta(i) T_t(e, i) \frac{w_t(i)}{D_t(e, i)} \right\}.$$

This is a worker may opt for an occupation either because his particular human capital  $e$  usual leads to a *relatively* high value for the average  $T_t(e, j)$  or because he happened to get a *relatively* high realization  $\eta_t(j)$ . However, with many ex-ante identical workers in each group  $e$ , the share of workers of the group that opts to each of the occupations  $j$  is equal to the ex-ante probability that any of them chooses them. Under the Frechet distribution, such probability is given by

$$p_t(e, j) = \frac{\left[ w_t(j) \frac{T_t(e, j)}{D_t(e, j)} \right]^\theta}{\sum_{i=1}^J \left[ w_t(i) \frac{T_t(e, i)}{D_t(e, i)} \right]^\theta}. \quad (4)$$

In what follows, we examine the equilibrium skill prices  $w_t(j)$  and the allocations  $p_t(e, j)$  that arise in equilibrium, given the cross-section of workers  $S_t$ , their productivity across occupations  $T_t$  and  $Q$ , the productivity shifts  $M_t$  and the wedges  $D_t$ . The equilibrium outcomes also determine the value of human capital levels  $H_t(j)$  and  $H_t$

### 3.2.1 Undistorted Equilibrium

In an undistorted equilibrium,  $D(e, j) = 1$ , for all  $e$  and  $j$ . All workers get the same wage  $w_t(j)$  for each unit of effective labor provided in occupation  $j$ . The assignment condition (4), becomes,

$$p_t(e, j) = \frac{[w_t(j) T_t(e, j)]^\theta}{\sum_{i=1}^J [w_t(i) T_t(e, i)]^\theta}. \quad (5)$$

Given the fractions  $\{S_t(e)\}_{e=1}^E$ , the total mass of workers in from education  $e$  in occupation  $j$  will be given by  $q_t(e, j) = S_t(e) p_t(e, j)$  and the total fraction of individuals in occupation  $j$  in the country will be  $q_t(j) = \sum_{e=1}^E S_t(e) p_t(e, j)$ . More importantly, in terms of effective labor units, the total labor from workers with human capital  $e$  supplied to occupation  $j$ ,  $H_t(j, e)$ , is given by

$$H_t(j, e) = [S_t(e) p_t(e, j)] \left[ \Gamma(1 - \theta^{-1}) T_t(e, j) p_t(e, j)^{-1/\theta} \right], \quad (6)$$

where the first term in brackets is the total mass of workers with human capital  $e$  working in occupations  $j$  and the second term is the average effective labor units provided by the group. Notice that this average goes down with the probability of entry, as the marginal worker entering has, on average, lower skills realizations  $\eta(j)$ . Summing over all human capital types  $e$ :

$$H_t(j) = \Gamma(1 - \theta^{-1}) \sum_{e=1}^E S_t(e) [p_t(e, j)]^{(\theta-1)/\theta} T_t(e, j). \quad (7)$$

From here, we can write the total human capital  $H_t$  in the economy as

$$H_t = \Gamma(1 - \theta^{-1}) \left[ \sum_{j=1}^J M_t(j) \left[ \sum_{e=1}^E S_t(e) [p_t(e, j)]^{(\theta-1)/\theta} T_t(e, j) \right]^\rho \right]^{\frac{1}{\rho}}.$$

After plugging expression (4) for  $p_t(e, j)$ , the aggregate human capital  $H_t$  becomes

$$H_t = \Gamma(1 - \theta^{-1}) \left[ \sum_{j=1}^J M_t(j) \left[ \sum_{e=1}^E \frac{S_t(e) w_t(j)^{\theta-1} T_t(e, j)^\theta}{\left[ \sum_{i=1}^J [w_t(i) T_t(e, i)]^\theta \right]^{(\theta-1)/\theta}} \right]^\rho \right]^{\frac{1}{\rho}},$$

which is driven by the cross-section  $S_t(e)$ , the average productivities  $T_t(i, e)$ , and the occupation productivities  $M_t(j)$ , but also by the given wages  $w_t(j)$ .

Plugging the first order condition for wages (3) into (5) and taking out the common factor  $\bar{w}_t$ , we obtain

$$p_t(e, j) = \frac{[M_t(j) [H_t(j)]^{\rho-1} T_t(e, j)]^\theta}{\sum_{i=1}^J [M_t(i) [H_t(i)]^{\rho-1} T_t(e, i)]^\theta}. \quad (8)$$

Next, plug this expression (8) into the formula (7) for the human capital services in occupation  $j$  to obtain the fixed-point conditions

$$H_t(j) = \left\{ \Gamma(1 - \theta^{-1}) \sum_{e=1}^E \frac{S_t(e) M_t(j)^{(\theta-1)} T_t(e, j)^\theta}{\left[ \sum_{i=1}^J (M_t(i) [H_t(i)]^{\rho-1} T_t(e, i))^\theta \right]^{(\theta-1)/\theta}} \right\}^{\frac{1}{1-(\rho-1)(\theta-1)}}. \quad (9)$$

Closed form solutions can be obtained when  $\rho = 1$  and the different forms of human capital services  $H_t(j)$  are perfect substitutes with each other. In this case,  $w_t(j) = \bar{w}_t \times M_t(j)$  and the shares of workers  $e$  into occupations  $j$  are given by

$$p_t(e, j) = \frac{[M_t(j) T_t(e, j)]^\theta}{\sum_{i=1}^J [M_t(i) T_t(e, i)]^\theta}, \quad (10)$$

and the implied aggregate human capital levels per occupation,  $H_t(j)$ , are given by

$$H_t(j) = \Gamma(1 - \theta^{-1}) \sum_{e=1}^E S_t(e) \frac{[M_t(j)]^{(\theta-1)} [T_t(e, j)]^\theta}{\left( \sum_{i=1}^J [M_t(i) T_t(e, i)]^\theta \right)^{(\theta-1)/\theta}}. \quad (11)$$

For the general case of  $\rho \leq 1$ , we have this simple but general result:

**Proposition 1** *Consider an economy with a given configuration of workers  $S_t(e)$ , technological output shares  $M_t(j)$ , and average skills of workers  $e$  in jobs  $j$ ,  $T_t(e, j)$ , with shape parameter  $\theta > 1$ . Then, for all  $\rho \leq 1$  there exists a unique vector of occupation human capital levels  $H_t(j)$  that solves the undistorted equilibrium fixed point condition (9.)*

As the others, the proof of this proposition is in the appendix. In any event, given the solutions to this fixed point problem in  $\{H_t(j)\}_{j=1}^J$ , we could readily compute the value of  $H_t$ , the equilibrium wages  $w_t(j)$ , and, of course the assignment of workers  $p_t(e, j)$ . Computing an equilibrium is straightforward to implement, which facilitates our quantitative exercises below.

### 3.2.2 Distorted Equilibria

We now consider the case in which different workers may receive different compensations for the same units of skills. To this end, we allow for wedges  $D_t(e, j) \geq 1$  that reduce the effective supply of services of workers of type  $e$  into occupations  $j$ . With those wedges, the effective wages for workers in group  $e$  operating in occupation  $j$  are scale down by  $1/D_t(e, j)$ , i.e.

$$w_t(e, j) = w_t(j) / D_t(e, j),$$

where  $w_t(j)$  is the marginal product to aggregate human capital services  $H_t(j)$ .

In all our analysis, we take the wedges as exogenously given, and consider two different formulations. In the first case, which we call pure wedges, the distortions  $D_t(e, j)$  reduces the income of workers and their effective supply of skills in equal measure. In the second case, the distortions  $D_t(e, j)$  only reduce the earnings or utility of the workers, not his or her supply of skills. From the point of view of an individual workers, they are equivalent, but the two cases are different enough for the general equilibrium to merit a separate treatment.

**$D_t(e, j)$  as Pure Wedges** Besides scaling by  $1/D_t(e, j)$  the earnings of workers  $e$  in occupations  $j$ , a wedge can also impact in the same proportion the effective supply of labor services received by the firm. The equilibrium supply of skills from group  $e$  is

$$H_t(e, j) = \Gamma(1 - \theta^{-1}) \frac{S_t(e) [p_t(e, j)]^{(\theta-1)/\theta} T_t(e, j)}{D_t(e, j)}. \quad (12)$$

The equilibrium conditions (3) and (4) remain valid. Therefore, the wedges  $D_t(e, j)$  not only impact the effective supplies directly, but also indirectly by affecting  $p_t(e, j)$ .

It is evident that the analysis for the undistorted equilibrium carries through as long as the terms  $T_t(e, j)$  are replaced for  $T_t(e, j) / D_t(e, j)$ . Then, as with the undistorted case, we can take out the common factor  $\bar{w}_t$  from (4) and obtain

$$p_t(e, j) = \frac{\left[ M_t(j) [H_t(j)]^{\rho-1} \frac{T_t(e, j)}{D_t(e, j)} \right]^\theta}{\sum_{i=1}^J \left[ M_t(i) [H_t(i)]^{\rho-1} \frac{T_t(i, j)}{D_t(i, j)} \right]^\theta}. \quad (13)$$

Next, plug this expression into the formula for (7) for the human capital services in occupation  $j$  to obtain a very similar set of fixed-point conditions as above:

$$H_t(j) = \left\{ \Gamma(1 - \theta^{-1}) \sum_{e=1}^E \frac{S_t(e) M_t(j)^{(\theta-1)} \left[ \frac{T_t(e, j)}{D_t(e, j)} \right]^\theta}{\left[ \sum_{i=1}^J \left( M_t(i) [H_t(i)]^{\rho-1} \frac{T_t(i, j)}{D_t(i, j)} \right)^\theta \right]^{(\theta-1)/\theta}} \right\}^{\frac{1}{1-(\rho-1)(\theta-1)}}.$$

Needless to say, when  $D_t(e, j)$  are pure wedges, a distorted equilibrium with  $T_t$  is observable equivalent to an undistorted equilibrium with  $T_t/D_t$ . These wedges do not reflect discrimination or direct distortions in the labor market (as skills are paid their marginal product), but could reflect earlier forms of discrimination that occur when workers are acquiring their skills, such as the quality of education received by women relative to men. In the quantitative exercises of the next sections, allowing for distortions  $D_t(e, j)$  can be quite useful for the countries and years for which reliable wage data allows us to obtain an independent measure of the wedges.

**$D_t(e, j)$  as Implicit Taxes to Workers** As a variation, we can think of  $D_t(e, j)$  as implicit taxes. In that case, even if there is not a reduction in the effective supply of services that each worker of type  $e$  delivers to the firms, the effective income for the workers is  $w_t(e, j) = w_t(j) / D_t(e, j)$ . This introduces a wedge between the contribution of the worker to the firm and the payoff he actually receives.

If wedges  $D_t(e, j)$  distort the workers payoff but not their actual output, they can arise from different forms of distortions, e.g. discrimination, as analyzed by Hsieh et al. (2016). Wedges can be group specific, i.e. depending only on  $e$ , or occupation specific, i.e. depending only on  $j$ . Simple but extreme versions of wedges would be (a) a caste system in which for each group  $e$ , there are pre-assigned occupations  $\hat{J}(e)$  such  $D_t(e, j) = 1$  if  $j \in \hat{J}(e)$  and  $D_t(e, j) = \infty$  otherwise; or (b) an uniform barrier to only one occupation if  $D_t(e, j^*) = \bar{D} > 1$  for  $j^*$ . These wedges can reflect actual taxes, or utility costs associated to things like harrasment or poor working conditions. At any rate, any arbitrarily patterns across  $e$  and  $j$  can be considered in this setting.

Under this form of wedges, the solution for  $p_t(e, j)$  would be given by (13), but the expression for  $H_t(e, j)$  is not (12) but the undistorted one, (6). Plugging the distorted  $p_t(e, j)$  into the undistorted (6) and summing over all  $e$ , the supply of human capital services into occupation  $j$ , we get that the equilibrium allocation of  $H_t(j)$  must solve the fixed point conditions

$$H_t(j) = \left\{ \Gamma (1 - \theta^{-1}) \sum_{e=1}^E S_t(e) \frac{\left[ \frac{M_t(j)}{D_t(e, j)} \right]^{(\theta-1)} [T_t(e, j)]^\theta}{\left[ \sum_{i=1}^J \left[ M_t(i) [H_t(i)]^{\rho-1} \frac{T_t(e, j)}{D_t(e, j)} \right]^\theta \right]^{(\theta-1)/\theta}} \right\}^{\frac{1}{1-(\rho-1)(\theta-1)}}.$$

With the solution to  $H_t(j)$ , we compute the equilibrium value of  $H_t$  using (2), wages  $w_t(j)$  using (3) and the equilibrium  $p_t(e, j)$  using (4.)

### 3.3 A Convenient Decomposition

It is convenient to separate the role of pure absolute advantages across all occupations from those from comparative advantages for specific occupations. Consider a decomposition in the form  $T_t(e, j)$  as  $T_t(e, j) = A_t(e) C_t(e, j)$  where  $A_t(e)$  is *uniform absolute productivity term of group  $e$*  across all *across all occupations  $j$*  and  $C_t(e, j)$  is the comparative advantage term.<sup>4</sup> For brevity, we will discuss the pure wedges case only here.

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<sup>4</sup>Any productivity shifts specific to an occupation but common to all groups  $e$  would be captured by the terms  $M_t(j)$ , and, because of rescaling, in the TFP term  $Z_t$ .

Under such a decomposition, the assignment of workers becomes

$$p_t(e, j) = \frac{\left[ w_t(j) \frac{C_t(e, j)}{D_t(e, j)} \right]^\theta}{\sum_{i=1}^J \left[ w_t(i) \frac{C_t(e, i)}{D_t(e, i)} \right]^\theta}.$$

Two immediate implications follow. First, given wages, pure absolute advantage terms  $A_t(e)$  do not affect the allocation of workers across occupations. Second, the general equilibrium determination of the wages  $w_t(j)$  is how the aggregate equilibrium determines the assignment of different workers  $e$  across occupations  $j$ .

Similarly, the formula for the country's overall human capital services becomes:

$$H_t = \Gamma (1 - \theta^{-1}) \left[ \sum_{j=1}^J M_t(j) \left[ \sum_{e=1}^E S_t(e) A_t(e) \frac{w_t(j)^{\theta-1} \left[ \frac{C_t(e, j)}{D_t(e, j)} \right]^\theta}{\left[ \sum_{i=1}^J \left[ w_t(i) \frac{C_t(e, i)}{D_t(e, i)} \right]^\theta \right]^{(\theta-1)/\theta}} \right]^\rho \right]^{\frac{1}{\rho}},$$

which is strictly increasing in  $A_t(\cdot)$  and homogeneous of degree one in  $A_t(\cdot)$  and  $S_t(\cdot)$ . Solving for the general equilibrium determination of wages, and human capital across occupations, the fixed point that solves the equilibrium allocations becomes

$$H_t(j) = \left\{ \Gamma (1 - \theta^{-1}) \sum_{e=1}^E S_t(e) A_t(e) \frac{M_t(j)^{(\theta-1)} [C_t(e, j) / D_t(e, j)]^\theta}{\left[ \sum_{i=1}^J \left[ M_t(i) [H_t(i)]^{\rho-1} \left[ \frac{C_t(e, i)}{D_t(e, i)} \right]^\theta \right]^{(\theta-1)/\theta}} \right\}^{\frac{1}{1 - (\rho-1)(\theta-1)}}.$$

### 3.4 A Simple Benchmark: Absolute Advantage Only

Before characterizing the equilibrium assignment of workers to occupations of the model, it is convenient to consider the underlying, simpler case of absolute advantage only. Such a case boils down to the aggregate efficiency human capital units underlying in most the standard growth- and development-accounting analyses in the literature.

Assume **absolute but no comparative advantage**, across human capital types, i.e., for some  $A_t(e) \geq 0$ , we can write  $T_t(e, j) = A_t(e)$  for all  $e$  and  $j$ . That is, the effective units of labor that a worker can provide is shifted *uniformly across all occupations* by the absolute advantage term  $A_t(e)$ .

For now, consider a random and uniform assignment in which, regardless their human capital type  $e$  workers are randomly allocated to each occupation  $j = 1, \dots, J$  with probability  $p(j)$ , where  $\sum p(j) = 1$ . Then, the total human capital services  $H_t(j)$  provided by group  $e$  to occupation  $j$  is simply  $p(j) S_t(e) \Gamma (1 - \theta^{-1}) B_t(j) A_t(e)$ . Summing over all groups  $e$ , we obtain

$$H_t(j) = \Gamma (1 - \theta^{-1}) p(j) \sum_{e=1}^E S_t(e) A_t(e).$$

Plugging this in the aggregator (2), the combination of all  $H_t(j)$  leads to an aggregate human capital for the country in the form

$$H_t = \Gamma (1 - \theta^{-1}) \left( \sum_{j=1}^J M_t(j) [p(j)]^\rho \right)^{\frac{1}{\rho}} \left( \sum_{e=1}^E S_t(e) A_t(e) \right),$$

The term inside brackets becomes completely indistinguishable from any other underlying the TFP term  $Z_t$  in the aggregate production function (1). Accounting for human capital boils down to the traditional measurement of the population's distribution of human capital levels,  $S_t(e)$ , and the absolute enhancements on productivity,  $A_t(e)$ , of the different types of human capital, as traditionally done using Mincer estimates.

Notice that this separation is independent of the particular choice of  $p(j)$ .<sup>5</sup> More interestingly, this form of indeterminacy is much more general. As long as human capital only shifts absolute advantage, i.e.  $T_t(e, j) = A_t(e)$ , the term  $\sum_{e=1}^E S_t(e) A_t(e)$  enters multiplicatively in the production function. The equilibrium assignment of human capital types  $e$  to occupations  $j$  is undetermined, and many different configurations of  $p_t(e, j)$  would deliver the same  $H_t(j)$  and  $H_t$  levels. The details for this argument are in the appendix. In any case, if human capital levels  $e$  only drive absolute advantage across all occupations, observing data on the allocation of those human capital groups across occupations,  $p_t(e, j)$  would be uninformative about the human capital  $H_t$  of a country and the aggregate return of expanding the human capital endowments of countries,  $S_t(\cdot)$  across the different levels  $e$ . This irrelevance result is overturned once we look into economies where comparative advantage drives the allocation of workers.

## 4 Inference from Observed Data

In this section we show how we can use the general equilibrium conditions of the model to infer the underlying distribution of skills of the different human capital groups. We first show how the conditions of undistorted equilibria can be used to deliver a simple characterization of the comparative advantage components  $C_t(e, j)$  from the observed allocation  $p_t(e, j)$ . Next, we show how to extend the inference to include both the components of  $T_t(e, j)$  and the wedges  $D_t(e, j)$  when, besides the assignment data  $p_t(e, j)$ , we also observe earnings data  $y_t(j, e)$ . We should how to use these inferred values and general equilibrium assignment conditions to infer the aggregate human capital of countries.

### 4.1 Undistorted Equilibrium: Inferring $T_t(e, j)$ and $M_t(j)$ from observed $p_t(e, j)$

From the equation (5), we get that, for any two occupations  $j, j'$  and human capital groups,  $e, e'$ , we can define a “ratio of ratios”

$$\left[ \frac{p_t(e, j) / p_t(e, j')}{p_t(e', j) / p_t(e', j')} \right] = \left[ \frac{T_t(e, j) / T_t(j', e)}{T_t(e, j) / T_t(j', e')} \right]^\theta. \quad (14)$$

This simple condition leads to three very useful implications about  $A_t(e)$ ,  $C_t(e, j)$  and  $M_t(j)$ . First, the comparative advantage terms  $C_t(e, j)$  are proportional to  $p_t(e, j)^{\frac{1}{\theta}}$ . Therefore, only the distribution parameter  $\theta$  is needed to infer the underlying comparative advantage terms from

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<sup>5</sup>With some simple algebra it can be shown that, under the two conditions stated above, the optimal uniform allocation  $p^*(\cdot)$  across occupations is simply

$$p^*(i) = \frac{[T_t^J(i)]^{\frac{\rho}{1-\rho}}}{\sum_{j=1}^J [T_t^J(j)]^{\frac{\rho}{1-\rho}}},$$

for all  $i = 1, \dots, J$ .

observed assignment data  $p_t(e, j)$ . Second, the occupation shifters  $M_t(j)$  are direct drivers of  $w_t(j)$  and therefore, can be inferred from the allocation of total human capital across occupations. Third, since absolute productivity components  $A_t(e)$  have no bearing on the allocation of workers across occupations, then data on  $p_t(e, j)$  provides no information regarding  $A_t(e)$ . Absolute advantage can not be recovered from allocations, but inferred directly from income data, i.e.  $A_t(e) = y_t(e)$ . Formally we state the following:

**Proposition 2** *Adopt the decomposition  $T_t(e, j) = A_t(e) C_t(e, j)$  explained above and let  $p_t(e, j)$  be the observed assignment of workers of human capital types  $e = 1, \dots, E$  into jobs  $j = 1, \dots, J$ , and  $A_t(e)$  an estimate of the absolute productivities of workers in group  $e$ . Then, if the underlying equilibrium of the economy is undistorted: (a) the comparative advantage term is given by*

$$C_t(e, j) = \bar{C}_t p_t(e, j)^{\frac{1}{\theta}},$$

*for some positive  $\bar{C}_t$  uniform across  $e$  and  $j$ ; (b) the pure relative occupation productivity terms  $M_t(j)$  are given by*

$$M_t(j) = \frac{\left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, j) \right]^{1-\rho}}{\sum_{i=1}^J \left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, i) \right]^{1-\rho}}.$$

**Proof.** See Appendix. ■

From these simple results, we can obtain a very straightforward characterization of the aggregate human capital of a country.

**Proposition 3** *Let  $p_t(e, j)$  be the observed assignment of workers of human capital types  $e = 1, \dots, E$  into jobs  $j = 1, \dots, J$ , and  $A_t(e)$  an estimate of the absolute productivities of workers in group  $e$ . If the underlying equilibrium of the economy is undistorted, then the aggregate human capital of the country is given by*

$$H_t = \Gamma(1 - \theta^{-1}) \frac{\left[ \sum_{e=1}^E S_t(e) A_t(e) \right]^{\frac{1}{\rho}}}{\left\{ \sum_{i=1}^J \left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, i) \right]^{1-\rho} \right\}^{\frac{1}{\rho}}}.$$

**Proof.** See Appendix. ■

An important result is that the resulting aggregate human capital is independent of the comparative advantage parameter  $\theta$ . The key parameter is  $\rho$ , which governs the degree of substitution between occupations. The data on  $p_t(e, j)$  determines the value of  $H_t$  unless  $\rho = 1$  and occupations are perfect substitutes. If so,  $H_t = \sum_{e=1}^E S_t(e) A_t(e)$ , as in traditional measurements. Finally, as expected, notice that the equilibrium assignment of workers into occupations results in a country's that is increasing and homogeneous of degree 1 both in the quantity  $S_t(e)$  of their workers as well as in their quality,  $A_t(e)$ .

## 4.2 Distorted Equilibrium: Inferring $\{T_t(e, j), M_t(j), D_t(e, j)\}$ from observed $\{p_t(e, j), y_t(e, j)\}$

In the previous section we showed how to use data on the assignment of workers to occupations,  $p_t(e, j)$ , and the average income  $y_t(e)$  of each type of workers, and the conditions of an indistorted

equilibrium to infer the comparative and absolute advantage components of the skills of workers, and the productivity parameters of each occupation in the aggregate human capital. In this section, we extend those inference exercises to economies in which frictions and/or compensating differentials can distort the assignment of workers to occupations. Such distortions generate income differences in conditional mean incomes each type  $e$  of workers across occupations  $j$ .

First, we show how to perform a similar inference as the one before, for any form of wedges  $D_t(e, j)$ . Here, we show several simple results: (a), if the distortions are uniform across occupations  $j$ , i.e.  $D_t(e, j) = \hat{D}_t(e)$  for all  $j$ , even if they vary across human capital groups  $e$ , they do not distort the allocation of workers. Those distortions only alter the absolute advantage of workers, and the inference of the previous section remain valid since they do not distort assignments; (b), if the conditional average income differences in the data are driven by pure wedges (in the sense defined above), the formula for aggregate human capital remains unchanged; (c) when the distortions are in the form of implicit taxes (in the sense defined above), the inferred values of aggregate human capital are affected, but simple formulas can still be obtained. In all those cases, changes in the wedges  $D_t(e, j)$ , would result in changes in the effective comparative advantage of workers, and in changes in the value of aggregate human capital of countries. Second, we show our simple method for using  $y_t(e, j)$  data to infer wedges  $D_t(e, j)$ .

**Inference of  $C_t(e, j)$ ,  $M_t(j)$  and  $H_t$  given  $D_t(e, j)$ :** Consider having already gotten (inferred) data on  $D_t(e, j)$  and on the average income  $y_t(e)$  for workers of type  $e$ . First, consider that the wedges vary across workers, but not across occupations, i.e.  $D_t(e, j) = \hat{D}_t(e) > 1$ . In this case, the terms  $\hat{D}_t(e)$  only distort the absolute advantage of workers. The true  $A_t(e)$  underlying in the economy is the counterfactual income  $y_t(e) \hat{D}_t(e)$  that group  $e$  would accrue in an undistorted economy. Other than that, the inferred comparative advantage terms  $C_t(e, j)$  would not change.

Next consider the pure wedges case, where the average earnings  $y_t(e, j)$  differences terms  $D_t(e, j)$  reduce the effective supply of skills. In this case, the equilibrium assignment is given by the expression (4) leading to a ratio-of-ratios, for any pairs  $e, e'$  and  $j, j'$  of the form

$$\left[ \frac{p_t(e, j) / p_t(e, j')}{p_t(e', j) / p_t(e', j')} \right] = \left[ \frac{\frac{C_t(e, j) / D_t(e, j)}{C_t(e, j') / D_t(e, j')}}{\frac{C_t(e', j) / D_t(e', j)}{C_t(e', j') / D_t(e', j')}} \right]^\theta.$$

Therefore, the observed assignment of workers to occupations can be driven by either the underlying comparative advantages of workers or by the wedges they face in each occupation. From the previous expression, the only possible solution for  $C_t(e, j)$  must necessarily be of the form

$$C_t(e, j) = \bar{C}_t D_t(e, j) [p_t(e, j)]^{\frac{1}{\theta}} \quad (15)$$

for any constant  $\bar{C}_t$ , which we normalize so as to the weight of the distortions are subsumed

into the absolute advantage  $A_t(e)$ , as discussed below. When plugging expression (15) into (4) the  $D_t(e, j)$  terms cancel each other. Similarly, when plugging (15) into (6) those terms cancel out. Therefore, the same expressions as in the undistorted equilibrium for  $H_t(e, j)$ ,  $M_t(j)$ , and  $H_t$  attain when  $D_t(e, j)$  are pure wedges that reduce the effective supply of skills from workers.

Finally, consider the case where  $D_t(e, j)$  are pure taxes that reduce the earnings of the worker but not the services received by firms. First, notice that the only solution for  $C_t(e, j)$  is still (15) since occupational choices, from the point of view of the worker, are the same as the pure wedges case. Now, plugging  $T_t(e, j) = A_t(e) \bar{C}_t D_t(e, j) [p_t(e, j)]^{\frac{1}{\theta}}$  into the undistorted expression  $H_t(j, e) = [S_t(e) p_t(e, j)] \Gamma(1 - \theta^{-1}) T_t(e, j) p_t(e, j)^{-1/\theta}$ , normalizing  $\bar{C}_t = 1$ , simplifying and

then summing over  $e$ , we obtain

$$H_t(j) = \Gamma(1 - \theta^{-1}) \sum_{e=1}^E S_t(e) A_t(e) p_t(e, j) D_t(e, j). \quad (16)$$

That is, the effective aggregate supplies of skills  $j$  include the taxes  $D_t(e, j)$  on workers. Since aggregate skill prices,  $w_t(j) = \bar{w}_t \times M_t(j) [H_t(j)]^{\rho-1}$ , are equalized, then

$$M_t(j) \left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, j) D_t(e, j) \right]^{\rho-1} = M_t(i) \left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, i) D_t(e, i) \right]^{\rho-1}.$$

Then, imposing  $\sum_{i=1}^J M_t(i) = 1$  and solving,

$$M_t(j) = \frac{\left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, j) D_t(e, j) \right]^{1-\rho}}{\sum_{i=1}^J \left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, i) D_t(e, i) \right]^{1-\rho}}, \quad (17)$$

i.e. the tax distortions would enter in our inference for the distributional occupational weights  $M_t(j)$ .

## 5 The Aggregate Human Capital of Countries

We now take the theoretical framework to the data, to evaluate its quantitative implications for cross-country differences in human capital and income. As discussed below, the model's assignment structure, while conceptually simple, is empirically demanding, as data on the allocation of workers across occupations that also includes information on their wages and education is not readily available for most countries. As a consequence, we are unable to build the large datasets with more than a hundred countries that are sometimes used in income/accounting exercises. Despite this limitation, we are able to build a set of 7 countries, both developed and developing, which allows us to fully exploit the richness of our setup. We use the data to compute measures of aggregate human capital under different assumptions, illustrating the model's mechanisms and their implications for cross-country differences. We then perform standard income accounting exercises, and compare the success ratios of our model's measures of human capital in explaining income differences across time and countries to those of the standard human capital model. Finally, we use the model to perform several counterfactual exercises, assessing the relative importance of the distribution of human capital types and comparative advantages across occupations in explaining differences across countries.

As discussed in the previous section, we can use the model's observables to infer comparative and absolute advantage components of the skills of workers, and the productivity parameters of each occupation in the aggregate human capital. This requires data on the economy's exogenous distribution of human capital groups,  $S_t$ , as well as the equilibrium allocations of different groups across occupations,  $p_t(e; j)$ , and the equilibrium average market incomes for each occupation/group allocation,  $y_t(e; j)$ . Additionally, cross-country comparisons on the relative importance of aggregate human capital in explaining aggregate income differences require macroeconomic data on aggregate physical capital,  $K_t$ , and aggregate income,  $Y_t$ .

While information on  $S_t, K_t$ , and  $Y_t$  is readily available for a large set of countries, information on  $p_t(e; j)$  and  $y_t(e; j)$  that is directly comparable across countries is much scarcer, as it requires detailed income information for multiple worker types across a set of comparable occupations.

## 5.1 Calculating human capital from the data: Undistorted case

Recall from the previous section that, if only absolute advantage exists, a measure of homogeneous human capital can be defined as:

$$H_{hom} = \Gamma(1 - \theta^{-1}) Z_t \sum_e A(e) S(e)$$

In this case,  $A(e)$  can be directly interpreted as a measure of the average wage for each human capital group. Therefore, aggregate human capital in a given sample is the simple weighted average of wages across groups, multiplied by the gamma function and an exogenous TFP term  $Z_t$ .  $\theta$  corresponds to the degree of comparative advantage in the economy. Table YY presents measures of human capital with only absolute advantages for all samples, relative to the US in 2000, assuming  $\theta = 3.44$ , which results in  $\Gamma(1 - \theta^{-1}) \approx 1.28$ .

If there are comparative advantages, aggregate human capital can be written as:

$$H = \Gamma(1 - \theta^{-1}) \left[ \sum_j \left( \left[ \sum_e p(e, j) \right]^{\frac{1}{\rho} - 1} \left[ \sum_e A(e) p(e, j) \right] \right)^\rho \right]^{\frac{1}{\rho}},$$

where  $\rho$  is a measure of complementarity between human capital from different kinds of occupations.<sup>6</sup> Table ZZ presents measures of human capital with comparative advantages, allowing for different values for  $\rho$ : the standard value is  $\rho = 1/3$ , a high value of  $\rho$  is  $2/3$ , and a low value of  $\rho$  is  $-1$ . The second and third cases are those of high substitution (low complementarity) and low substitution (high complementarity) among different occupations respectively.

### 5.1.1 Introducing distortions

- Distortions are calculated as:

$$D(e; j) = \frac{w_{\max}(e)}{w(e, j)}$$

- This is, for each worker group  $e$ , distortions are measured as how much lower is the group maximum relative to the wage in occupation  $j$ . A higher index for group  $e$  in occupation  $j$  indicates a large penalty for that occupation.
- We then recalculate relative measures of human capital...

### 5.1.2 Results: Human Capital and Cross-Country Income Differences

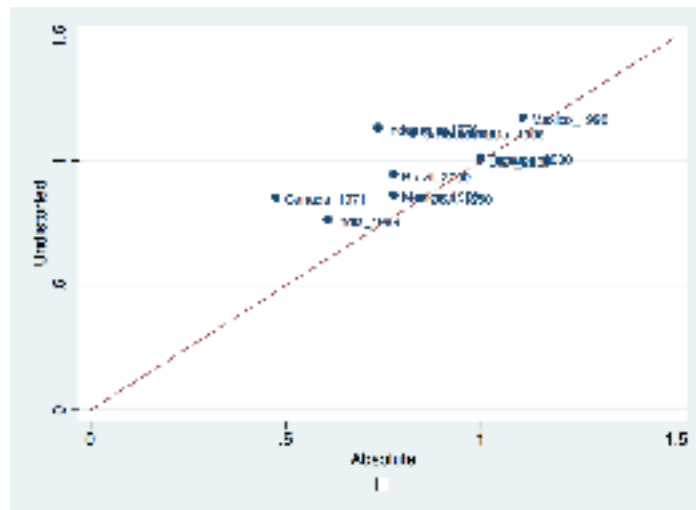
- We now use the measures of human capital for each sample calculated in the previous section, and combine them with data on aggregate physical capital,  $K_t$ , and aggregate income,  $Y_t$ , to build success ratios in the spirit of Caselli (2005):

$$success_i = \frac{Y_{\text{model}}^{US,2000} / Y_{\text{model}}^i}{Y_{\text{actual}}^{US,2000} / Y_{\text{actual}}^i}$$

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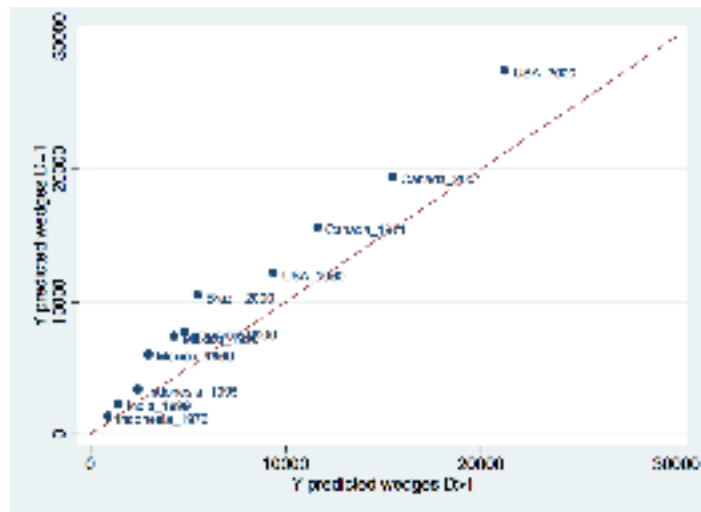
<sup>6</sup>  $\frac{1}{1-\rho}$  is the elasticity of substitution, as the human capital aggregator is a CES function of the different occupations.

- Success ratios in the exercises are typically very high, in many cases close to 1 or even above it (which means that human capital more than explains income differences across countries; this is, if  $\text{success} > 1$ , TFP would be smaller in the US). This is a far cry from the results in Caselli, where human capital success ratios for comparisons of relative income between the 90th and 10th income deciles are 0.35
- Unfortunately, this is not (only) an indication of our model's strength. Human capital measures use the level of wages, so human capital differences across countries not only reflect differences in the distribution of education and relative advantages, but absolute differences in wages reflecting differences in TFP and capital. Thus, it is not surprising that these measures of human capital have a great explanatory power.
- Therefore, the exercise must be redone using measures of relative wages (relative to a given workers group). In any case, relative to the standard model, our benchmark model has higher success ratios, which I think should also hold when using relative wages (as we will shift HK measures by a constant). Success ratios are also larger in the model that interprets distortions as wedges rather than pure taxes.
- As we have discussed earlier, it is not trivial to compare human capital across countries. Relying on the level of wages exacerbates differences in favor of rich countries as we include complementarities with technology and physical capital in the human capital index. On the other hand, measures of relative wages do not take into account differences in the quality of education across country: a country where the skill premium is larger than in the US (such as Chile), can end up with a larger measure of human capital.
- The second exercise computes gains from eliminating distortions (making  $D(e, j) = 1$  for all  $e, j$ ). I think this exercise is not affected by the use of absolute wages, as comparisons are not made across countries. Gains from eliminating wedges (which probably reflect technological limitations, such as differences in schooling quality across groups or discrimination in human capital investments, and reflect actual productivity differences) are much larger than gains from eliminating taxes, which reflect discrimination at the work place or policy distortions that affect workers payoffs but not their actual productivity).



Accounting Success Ratios: Standard Model (absolute adv. only) vs. Benchmark MMT:





**Gross Gains in AHK: From actual to D=1**

|                | Pure Wedges | Pure Taxes |
|----------------|-------------|------------|
| Brazil 2000    | 2.57        | 1.04       |
| Canada 1971    | 1.53        | 1.02       |
| Canada 2001    | 1.39        | 1.03       |
| India 1999     | 1.83        | 1.06       |
| Indonesia 1976 | 1.45        | 1.06       |
| Indonesia 1995 | 1.51        | 1.02       |
| Mexico 1960    | 2.80        | 1.05       |
| Mexico 1990    | 2.16        | 1.03       |
| Panama 2000    | 1.95        | 1.06       |
| USA 1960       | 1.44        | 1.04       |
| USA 2000       | 1.46        | 1.03       |

# A Proofs

**Proof of Proposition 1.** Take logs, and define  $h_t(j) = \ln(H_t(j))$ . Using expression (9) and for any  $h \in \mathbf{R}^J$ , define the mapping  $\mathcal{T}(\cdot)$  as

$$(\mathcal{T}h_t)(j) = \ln \left[ \left\{ \Gamma(1 - \theta^{-1}) \sum_{e=1}^E \frac{S_t(e) M_t(j)^{(\theta-1)} T_t(e, j)^\theta}{\left[ \sum_{i=1}^J \left( M_t(i) [e^{h_t(i)}]^{(\rho-1)} T_t(e, i) \right)^\theta \right]^{(\theta-1)/\theta}} \right\}^{\frac{1}{1 - (\rho-1)(\theta-1)}} \right]$$

It is easy to see that given  $S_t(e)$ ,  $M_t(j)$ ,  $T_t(e, j)$ ,  $\mathcal{T}$  maps vectors in the  $\mathbf{R}^J$  into vectors in  $\mathbf{R}^J$ . Pick any continuous norm  $\|\cdot\|$  in  $\mathbf{R}^J$  and define the distance  $d(h, g) = \|h - g\|$ . Obviously,  $(\mathbf{R}^J, d)$  is a Banach space. To check that  $\mathcal{T}$  is a contraction in  $(\mathbf{R}^J, d)$ , we simply verify that it satisfies Blackwell's sufficient conditions. To this end, consider any positive number  $a > 0$ . We have that  $\mathcal{T}(h + a)$  is simply given by

$$\begin{aligned} \mathcal{T}(h + a)(j) &= \frac{1}{1 - (\rho - 1)(\theta - 1)} \left\{ \ln[\Gamma(1 - \theta^{-1})] + \ln \left( \sum_{e=1}^E \frac{S_t(e) T_t(e, j) [M_t(j) T_t(e, j)]^{(\theta-1)}}{\left( \sum_{i=1}^J [M_t(i) [e^{h(i)+a}]^{(\rho-1)} T_t(e, i)]^\theta \right)^{(\theta-1)/\theta}} \right) \right\} \\ &= \mathcal{T}(h)(j) + \frac{(\theta - 1)(1 - \rho)}{1 + (1 - \rho)(\theta - 1)} a \end{aligned}$$

For  $\mathcal{T}$  to be *monotone* it is sufficient that  $(\theta - 1)(1 - \rho) > 0$ , which, since  $\theta > 1$  holds for all  $\rho < 1$ . Notice that  $\rho < 1$  also suffices for *discounting*. This completes our proof that  $\mathcal{T}$  is a contraction for  $\rho < 1$ . Moreover, since  $H_t(j) = \exp(h_t(j))$ , the implied solutions are valid since they are strictly positive. Finally, closed-form solution for the special case when  $\rho = 1$  is reported in the main body of the paper. ■

**Proof of Proposition ACM (U)** First, notice that for any pairs  $j, j'$  and  $e, e'$ , equation (5)

$$\left[ \frac{p_t(e, j)/p_t(e, j')}{p_t(e', j)/p_t(e', j')} \right] = \left[ \frac{[A_t(e) C_t(e, j)]/[A_t(e) C_t(e, j')]}{[A_t(e') C_t(e', j)]/[A_t(e') C_t(e', j')]} \right]^\theta.$$

The pure absolute terms  $A_t(e)$  and  $A_t(e')$  cancel out within the ratios in the numerator and denominator, respectively. Hence,

$$\left[ \frac{p_t(e, j)/p_t(e, j')}{p_t(e', j)/p_t(e', j')} \right] = \left[ \frac{C_t(e, j)/C_t(e, j')}{C_t(e', j)/C_t(e', j')} \right]^\theta. \quad (18)$$

Then, for any  $\bar{C}_t > 0$ , a constant across  $e$  and  $j$ , we have  $C_t(e, j) = \bar{C}_t * p_t(e, j)^{\frac{1}{\theta}}$  solves the solution and, without loss of generality, we can normalize  $\bar{C}_t = 1$ , so all absolute advantage terms are scaled in  $A_t(e)$ . To see that the only solutions are given by this form, assume that there is another solution of the form  $C_t(e, j) = \nu_t(e, j) * p_t(e, j)^{\frac{1}{\theta}}$  for some  $\nu_t(e, j) > 0$  that varies across  $e$  and/or  $j$ . Then, using (18), it has to be the case that, for all  $e$  and  $j$

$$1 = \left[ \frac{\nu_t(e, j)/\nu_t(e, j')}{\nu_t(e', j)/\nu_t(e', j')} \right]^\theta,$$

which can only hold if  $\nu_t(e, j) = \nu_t^E(e) \nu_t^J(j)$  for some  $\nu_t^E(e) > 0$  and  $\nu_t^J(j) > 0$ . Hence, without loss of generality, we can normalize  $A_t(e)$  and  $M_t(j)$  as respectively embedding those terms. This establishes part (a).

To establish part (b), plug  $T_t(e, j) = A_t(e) p_t(e, j)^{\frac{1}{\theta}}$  into  $p_t(e, j) = \frac{[w_t(j) T_t(e, j)]^\theta}{\sum_{i=1}^J [w_t(i) T_t(i, e)]^\theta}$  to obtain that for all  $e$  and  $j$ :

$$p_t(e, j) = \frac{[w_t(j) A_t(e)]^\theta p_t(e, j)}{\sum_{i=1}^J [w_t(i) A_t(e) p_t(e, i)^{\frac{1}{\theta}}]^\theta}.$$

Notice that the terms  $A_t(e)$  cancel out, as absolute advantages do not drive the allocation of workers across occupations. More interestingly,  $p_t(e, j)$  cancels across both sides of this equation and then, the conditions boil down to

$$1 = \frac{[w_t(j)]^\theta}{\sum_{i=1}^J [w_t(i)]^\theta p_t(e, i)}. \quad (19)$$

The only solution for these equations, as long as  $p_t(e, i) \neq p_t(e', i)$  for some  $e, e'$  and  $i$ , is that

$$w_t(j) = w_t(i), \quad (20)$$

for all  $i$  and  $j$ . To solve for the vector of unitary wages  $[w_t(1), \dots, w_t(J)]$ , plug  $T_t(e, j) = A_t(e) p_t(e, j)^{\frac{1}{\theta}}$  into the expression  $H_t(j, e) = S_t(e) p_t(e, j) \left\{ \Gamma(1 - \theta^{-1}) T_t(e, j) p_t(e, j)^{-1/\theta} \right\}$ , and then sum over  $e$  to obtain

$$H_t(j) = \Gamma(1 - \theta^{-1}) \sum_{e=1}^E S_t(e) A_t(e) p_t(e, j). \quad (21)$$

Since  $w_t(j) = \bar{w}_t \times M_t(j) [H_t(j)]^{\rho-1}$ , then, from (20)

$$M_t(j) \left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, j) \right]^{\rho-1} = M_t(i) \left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, i) \right]^{\rho-1},$$

or

$$M_t(i) = M_t(j) \frac{\left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, i) \right]^{1-\rho}}{\left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, j) \right]^{1-\rho}},$$

But, since  $M_t(j)$  are distributional shifts in the CES for  $H_t$ , it has to be the case that  $\sum_{i=1}^J M_t(i) = 1$ . Writing all  $M_t(i)$  in terms of a single  $M_t(j)$

$$\sum_{i=1}^J M_t(j) \frac{\left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, i) \right]^{1-\rho}}{\left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, j) \right]^{1-\rho}} = 1,$$

and taking  $j$  terms out of the summation and solving, we get

$$M_t(j) = \frac{\left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, j) \right]^{1-\rho}}{\sum_{i=1}^J \left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, i) \right]^{1-\rho}}, \quad (22)$$

as claimed. ■

**Proof of Proposition H (U)** With the expression for  $H_t$

$$H_t = \left[ \sum_{j=1}^J M_t(j) [H_t(j)]^\rho \right]^{\frac{1}{\rho}},$$

and plugging the expression for (22) and (21)

$$H_t = \left\{ \sum_{j=1}^J \frac{\left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, j) \right]^{1-\rho}}{\sum_{i=1}^J \left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, i) \right]^{1-\rho}} \left[ \Gamma(1 - \theta^{-1}) \sum_{e=1}^E S_t(e) A_t(e) p_t(e, j) \right]^\rho \right\}^{\frac{1}{\rho}},$$

which, after re-arranging, grouping and simplifying

$$H_t = \Gamma(1 - \theta^{-1}) \left\{ \frac{\sum_{j=1}^J \sum_{e=1}^E S_t(e) A_t(e) p_t(e, j)}{\sum_{i=1}^J \left[ \sum_{e=1}^E S_t(e) A_t(e) p_t(e, i) \right]^{1-\rho}} \right\}^{\frac{1}{\rho}},$$

as claimed. ■

# Data

In this appendix, we described in more detail the data we used for our calculations.

## A.1 Variable Definitions

All data is extracted from the *Integrated Public Use Microdata Series, International (IPUMS)*. IPUMS consists of microdata samples from international census records. The records are converted into a consistent format and made available to researchers through a web-based data dissemination system by the Minnesota Population Center. Based on data availability, we choose 11 specific country-year combinations: Brazil 2000; India 1999; Mexico 1960 and 1990; United States 1960 and 2000; Indonesia 1976 and 1995; Panama 2000; Canada 1971 and 2001.

From IPUMs, we collect the following variables:

- **age**: Respondent's age in years
- **sex**: Respondent's gender
- **edattain**: Respondent's educational attainment. Specifically, the highest level of schooling completed (degree or other milestone). The emphasis on completion is critical: a person enrolled in the final year of secondary school only receives the code for having completed lower secondary only – and in some samples only primary. **edattain** is an attempt to merge – into a single, roughly comparable variable – census sources that provide degrees, others that provide actual years of schooling, and those that have some of both. .
- **yrschool**: Respondent's completed years of schooling, regardless of schooling type. Only formal schooling is counted. Top coding is frequent.<sup>7</sup>
- **empstat**: Indicates whether or not the respondent was part of the labor force – working or seeking work – over a specified period of time prior to the census. This variable classifies all respondents into three groups: employed, unemployed, and inactive. The combination of employed and unemployed yields the total labor force.
- **occisco**: Respondent's primary occupation, coded according to the major categories in the 1988 International Standard Classification of Occupations (ISCO) scheme. For someone with more than one job, the primary occupation is typically the one in which the person had spent the most time or earned the most money. Occupation categories are:
  - Legislators, senior officials and managers
  - Professionals
  - Technicians and associate professionals
  - Clerks
  - Service workers and shop and market sales
  - Skilled agricultural and fishery workers
  - Crafts and related trades workers
  - Plant and machine operators and assemblers
  - Elementary occupations
  - Armed forces

More information on occupations can be obtained at <http://www.ilo.org/public/english/bureau/stat/isco/isco88/index.htm>

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<sup>7</sup>Top codes are figures that do not represent the real variable value, but instead are set because there is an upper boundary in the census data. For example, with **yrschool**, in Indonesia 1976 the variable does not take a value higher than 16, even though a person may have more than 16 years of formal education.

- **inccarn**: Respondent's total labor income in the previous month or year. Does not only include wages, but also income from businesses and farms. Most samples report data for the previous month. Data for Canada and the United States are annual figures. Amounts are expressed as they were reported at the time of the census in the currency of the respective country. Some samples report negative earnings for individuals. Top coding is frequent.<sup>8</sup>
- **incwage**: Reports the respondent's weekly, monthly or annual wage and salary income. The data are recorded in the currency of each country in that census year and are not adjusted for inflation. Topcodes are frequent, and vary across countries<sup>9</sup>. The data for the United States and Canada are annual figures; the data for Indonesia and Panama are monthly; other samples report weekly figures.

## A.2 Adjustments on the Raw Data and Other Calculations

Following the model, we adjust the data to obtain a sample that fits the model definitions, and that allows for comparisons across time and countries. This implies using the data to get measures of occupation, income, education attainment, gender, and age for all individuals.

To characterize occupations, we drop all individuals that do not fit in one the nine occupation categories: Managers, Professionals, Technicians, Clerks, Services, Agriculture, Traders, Operators and Elementary. These includes individuals with other occupations, individuals with unknown occupations, and individuals where the category is "not in universe" (NIU), such as children. We also exclude respondents occupied in the Armed Forces, as the market mechanisms that underlies the allocation of workers across occupation might not fully apply to the military, particularly in the presence of conscription.

For income, we use the variables **inccarn** and **incwage** to measure individual wages for each country. Unfortunately, for some samples **inccarn** was present but not **incwage**, and vice-versa. Therefore, the exact variable for each sample depends on availability, **incwage** being preferred when that both are available. Individuals with non-positive values, missing values, or top codes are eliminated from the database. The elimination of top-coded individuals introduces an inevitable bias in the data, as we exclude top earners from the sample. However, we think that is better than the alternative, as we would otherwise measure the income of top-coded individual with significant error, and introduce artificial equality across all top-coded respondents in a particular sample. In the particular case of Mexico 1960, a significant number of individuals had **inccarn** = 1. In regards to the issue, IPUMS states that "*A large number of cases in 1960 have a value of "1." It may indicate a low income value, but almost certainly does not literally mean 1 peso.*" Following the same logic as with top-coded earnings, these observations are eliminated, as we do not know the true value of these earnings. Therefore incomes in Mexico 1960 may be biased upwards.

Additionally, we transform income variables to annual equivalent year 2000 USD PPP figures. First, as incomes are reported in different frequencies, we multiply monthly (weekly) incomes by 12 (52). Second, we use CPI inflation<sup>10</sup> for each country to adjust nominal incomes in a given year into equivalent incomes for the year 2000. Finally, we convert equivalent incomes in local currency to equivalent USD adjusted by GDP PPP values using the 2000 PPP conversion factor<sup>11</sup> (also known as the PPP exchange rate)..

Education measures are derived from **edattain** and **yrschool**. From here we define seven education categories: (1) no schooling, (2) incomplete primary, (3) complete primary, (4) incomplete secondary, (5) complete secondary, (6) incomplete tertiary, and (7) complete tertiary. Individuals with missing observations are dropped. Minor adjustments must be done for some countries in which the denomination of education categories differs slightly.

For demographics, we define three age groups: "young" individuals aged between 25 and 35 years old, "middle aged" individuals between 36 and 50, and "old" individuals with ages above 50. Individuals below 25 or with missing data on age are dropped from the samples. Individuals with missing data on gender are also dropped.

Finally, as the model does not consider leisure, home production, or unemployment, we only keep observations for employed individuals, given by **empstat**. This filter was not applied for Mexico 1960, as that sample has no information for **empstat**. This may bias the data for Mexico 1960 by including unemployed or inactive individuals that still report positive income and the other variables.

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<sup>8</sup>In this case, top codes represent, where applicable, a determination by the Current Population Survey that some high values were too sparse and specific to be recorded as they were reported to the CPS without the possibility of identifying the respondents. In these cases, the CPS put numerous high value cases together under one particular high value to protect respondent anonymity.

<sup>9</sup>Same as **inccarn**

<sup>10</sup>Data from the World Bank: <http://data.worldbank.org/indicator/FP.CPI.TOTL.ZG>

<sup>11</sup>Data from the World Bank: <http://data.worldbank.org/indicator/PA.NUS.PPP?view=chart>

With the final data for each sample, all combinations of gender (two groups), age (three groups) and education attainment (seven groups) are used to create 42 human capital groups  $e$ . Those 42 groups are combined with the 9 occupation categories to construct 378 group-occupation combinations for each sample. In each sample, group-occupation combinations that have less than ten individuals are eliminated to avoid outliers which might bias the results.