

The Role of Firm Heterogeneity in the Earnings Inequality

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Abstract

Over the past three decades, individual earnings inequality has seen rising alongside increases in the concentration of firm employment and revenue in the U.S. This paper studies the factors underlying these trends and their macroeconomic impacts. I extend a canonical uninsurable earnings risks model with heterogeneous firms and labor market search friction as in Lucas and Prescott (1974). There are a large number of spatially distinct labor markets - island or firms - and workers' earnings becomes a product of their own labor productivity and a function of employers' productivity. Workers may leave an island by paying search cost or can be exogenously separated. Once leave, they move to a nearby island following a transition process.

Through searching, better workers move to better firms. Better workers can afford search cost since they tend to be wealthy. Also once they move to a better firm, their wage increase is larger than the increase of low productivity workers. Thus productive workers engage searching, have higher chance to move up while low productive workers tend to stay where they are.

The model replicates earnings distribution, firm size distribution and wealth distribution successfully. With the quantitatively disciplined model, transitional dynamics exercise is designed to measure individual and firm component in rising inequality. It shows that the individual component in wages explains the most of the rise in earnings concentration. The majority of the firm concentration is driven by the changes in firm productivity distribution. The model suggests that shifts in the productivity distributions and changes in the worker-firm matching pattern which are driving rising inequality, have important implications: They explain 22% of output growth, 15% of capital growth and a quarter of the decline in the interest rate since the 1990's.

1. Introduction

Over the past three decades there has been a significant rise in the U.S. earnings inequality. While most macroeconomic theories about earnings inequality frequently assume that individuals' earnings' depend on their ability alone, a recent study by Song, Price, Guvenen, Bloom, and Von Wachter (2015) shows that the rise in earnings inequality among workers has primarily been a between-firm phenomenon. Using earnings data from W-2 records held by the Social Security Administration (SSA), they find that over two-thirds of the increase in earnings inequality from 1981 to 2013 can be accounted for by the rising variance of earnings between firms. At the same time, we know that the firm size (in terms of employment, revenue) distribution is highly left skewed. Autor, Dorn, Katz, Patterson, and Van Reenen (2017) show that the concentration of firms is extreme and industries become increasingly dominated by a few superstar firms.¹

These facts lead to interesting questions. How does rising firm inequality affect trends in earnings inequality? If there is a systemic relationship, what is the mechanism behind these observations? What will be the effect of this trend on the growth? This paper seeks to understand earnings inequality and firms inequality together, find a coherent mechanism which accounts these trends, quantify the factors affecting the trends and their macroeconomic consequences.

To answer these questions, I construct a model with uninsurable earnings risk as in Huggett (1993) and Aiyagari (1994) but I add heterogeneous firms and search friction in the labor market. In Huggett-Aiyagari type models there is a representative firm that hires all workers. Given a lack of firm heterogeneity, all earnings differences are results of workers' skill. While convenient, this abstracts away the rich heterogeneity in firms and as a result, misses an important determinant of earnings differences.

The search friction follows Lucas and Prescott (1974). There is a continuum of islands which differ by productivity. In the beginning of a period, workers in each island decide whether they stay and work, or move to other islands. If a worker searches, the worker must pay search cost and moves to a nearby island following a job transition process.

With heterogeneity in both workers and firms, I am able to study how firms and

¹Further, they relate this trend to falling labor share. Since superstar firms tend to have high profits and a low share of labor in their value added and sales, as the importance of these firms increases, the aggregate labor share tends to fall. Using manufacturing data, they show that industries where concentration rose the most were also those where the labor share fell by the most. Interestingly, while the labor share fell, average wages were not systematically falling.

workers affect the earnings distribution and how has it changed over time. In this environment, workers' earnings depend not only on their skills but also on firms' that they are working for. Thus earnings risks come from various sources: labor productivity shocks, shocks to their employers and separations, like firing.

To see causes and consequences of rising inequalities, I perform a transitional dynamics experiment from 1992 to 2013. To do so, I calibrate the model twice: targeting the data circa 1992 and 2013. The model has calibrated to match the earnings concentration, individual and firm component in a variance of earnings and the firm distribution. Although I do not target moments of the wealth distribution, the model matches the wealth distribution fairly well, except at the very top 1%. Also the model captures features of earnings dynamics that are described in Guvenen, Karahan, Ozkan, and Song (2015) reasonably well.

Then I feed the paths of stochastic processes that shapes the economy over the transition periods and compute a non stationary equilibrium. Labor productivity process, firm productivity process, job transition process and search cost vary. The quantitative exercises show the changes in labor productivity distribution - individual component - explain the most of the rising earnings concentration. Variance decomposition of earnings into *between firm* and *within firm* components follows Song et al. (2015) shows the majority of the rise in the variance can be explained by the between firm component. Also positive assortative sorting plays a major role in rising the between firm component, which both are consistent with Song et al.'s (2015) findings. These two results - individual component explain the concentration and between firm component accounts the majority of rise in variance - do not contradict each other but show the individual characteristic matters to firm-worker matching pattern, worker allocation among firms which compose the between firm component.

In the transitional dynamics, the available resources - total effective labor and technology - remain constant over the simulation periods. This allows me to assess implications of rising inequality to macroeconomic variables such as output growth. The model suggests: the rising inequality accounts for 22% of the output growth, 15% of the capital growth and a quarter of decline of the interest rate since 1990's. The output growth is driven by capital growth and increasing concentration to large, productive firms. Among other factors, positive sorting plays a role: the number of workers increases 5.8% but the effective labor rises 10.9% for these firms. As productive workers are increasingly matched with productive firms, total output increases.

The capital growth and the decline of the interest rate are caused by changes in workers' saving decisions. From 1992 to 2013, the variances of labor productivity

and firm productivity increase. This means higher earnings risk for workers hence the majority of them try to increase saving. Comparing the 1992 economy and 2013 economy given a same interest rate², the aggregate capital demand also rises, but not as much as the rise in the supply. Thus the interest rate falls.

I begin in Section 2 by introducing the model. Section 3 presents calibration results, the accounting and the effect of rising inequalities to the U.S. economy is described in Section 4. Section 5 concludes and discusses avenues for future research.

2. Model

The economy runs infinitely. There are a measure of household and a continuum of islands. On each island there is a continuum of firms. Each island has different productivity but firms are identical within an island. Households differ by their labor productivities, assets and locations.

2.1. Households

A household is characterized by her labor productivity, ε , asset, a and island, z . The labor productivity vary according to Markov processes, $\Gamma_{\varepsilon, \varepsilon'}$; $\varepsilon \in \mathbf{E} \equiv \{\varepsilon_1, \dots, \varepsilon_{N_\varepsilon}\}$, where $Pr(\varepsilon' = \varepsilon_j | \varepsilon = \varepsilon_i) = \pi_{ij} \geq 0$ and $\sum_{j=1}^{N_\varepsilon} \pi_{ij} = 1$ for each $i = 1, \dots, N_\varepsilon$. $\varepsilon_{N_\varepsilon+1} = 0$, which captures retirees with zero labor income³. Workers with labor productivity from ε_1 to $\varepsilon_{N_\varepsilon-1}$ retire with the probability p_r and retirees get transfer from the government. After retirement an agent die with the probability p_d . Once dead the agent is born in the same island where the ancestor retired with the wealth inherited from the deceased and a labor productivity which is drawn from the ergodic distribution of ε .

At the beginning of the period, a working household may choose to leave the island. If she chooses to leave, she has to pay search cost s and will be allocated to an island at the end of the period. The household cannot choose the destination, but will arrive the next island following job transition process which depends on the household's labor productivity. In addition to searching decision, the household decides how much to save, a' . A household solves the following problem when she decides to leave the island.

²mid value of the equilibrium interest rates

³In Survey of Consumer Finance, substantial portion of the sample has no labor income. For example, 22.5% of the sample has zero or negative earnings in 1998. (Rodriguez, Díaz-Giménez, Quadrini, and Ríos-Rull (2002))

$$W_s(\varepsilon, a, z; \mu) = \max_{a'} u(c) + \beta \int_x \pi(z|\varepsilon) \left(\sum_{z'} \Gamma_{zz'} \sum_{\varepsilon'} \Gamma_{\varepsilon\varepsilon'} W(\varepsilon', a', x'_{z'}; \mu') \right) dx$$

$$c + a' + s = (1 + r(\mu))a + w(z; \mu)\varepsilon - T(w(z; \mu)\varepsilon), \quad a' > \underline{a}$$

Workers pay income taxes according to a progressive income tax system. As in Heathcote, Storesletten, and Violante (2017), I assume a tax function takes the form :

$$T(y) = y - \tau_i(y)^{1-\gamma}$$

where γ indicates progressivity of the tax system: a larger γ means more progressive income tax.

If the household stays, she works, earns labor income and decides how much to save or borrow. Although the worker decides to stay, she can be exogenously moved to another island with probability δ at the end of the period. Therefore the worker starts the next period with (ε', a', z') with probability $1 - \delta$ or starts with (ε', a', z') with probability δ . Even if the worker stays, the firm could hit by productivity shock, z' so wage rate can be changed in the next period. This worker's problem is,

$$\begin{aligned} W_e(\varepsilon, a, z; \mu) = \max_{a'} u(c) + \beta & \left((1 - \delta) \sum_{z'} \Gamma_{zz'} \sum_{\varepsilon'} \Gamma_{\varepsilon\varepsilon'} W(\varepsilon', a', z'; \mu') \right. \\ & \left. + \delta \int_x \pi(z|\varepsilon) \left(\sum_{z'} \Gamma_{zz'} \sum_{\varepsilon'} \Gamma_{\varepsilon\varepsilon'} W(\varepsilon', a', z'; \mu') \right) dx \right) \end{aligned}$$

$$\begin{cases} c + a' = (1 + r(\mu))a + \tau, & a' > \underline{a}, & \text{if } \varepsilon = \varepsilon_{N_\varepsilon} \\ c + a' = (1 + r(\mu))a + w(z; \mu)\varepsilon - T(w(z; \mu)\varepsilon), & a' > \underline{a}, & \text{otherwise} \end{cases}$$

Retirees do not have a search option and the location does not affect their income. The government makes the transfer, τ .

Workers compare two values and decide whether to search or not, and the retirees' value is $W_e(\varepsilon, a, z; \mu)$.

$$\begin{cases} W(\varepsilon, a, z; \mu) = W_e(\varepsilon, a, z; \mu), & \text{if } \varepsilon = \varepsilon_{N_\varepsilon} \\ W(\varepsilon, a, z; \mu) = \max[W_e(\varepsilon, a, z; \mu), W_s(\varepsilon, a, z; \mu)], & \text{otherwise} \end{cases} \quad (1)$$

Let $\chi(\varepsilon, a, z)$ be households' decision rule for searching and $A(\varepsilon, a, z)$ be the decision rule for saving.

2.2. Firms

An island is characterized by its productivity, z , which follows a Markov Process. In an island there is a continuum of identical firms thus the labor market is competitive in each island. Firms hire workers and rent capital, k , with price $r + \delta_d$. Using labor and capital, they produce goods with CRS technology. A firm's problem in an island is

$$V(z; \mu) = \max_{\bar{\varepsilon}, k} f(z, \bar{\varepsilon}, k) - w(z; \mu)\bar{\varepsilon} - (r(\mu) + \delta_d)k \quad (2)$$

$$f(z, \bar{\varepsilon}, k) = zk^\alpha \bar{\varepsilon}^{1-\alpha}$$

$$\bar{\varepsilon} = \int_j \varepsilon dj$$

where j indicates the island and $\bar{\varepsilon}$ is sum of effective labor in the island. Since the labor market in the island is competitive, wage rate becomes the marginal productivity of labor, $w(z, \mu) = f_{\bar{\varepsilon}}(z, \bar{\varepsilon}, k) = (1 - \alpha)z(\frac{k}{\bar{\varepsilon}})^\alpha$. I assume frictionless capital market, then firms will rent a capital where $r(\mu) = f_k(z, \bar{\varepsilon}, k) = \alpha z(\frac{k}{\bar{\varepsilon}})^{1-\alpha}$. Rearrange the equation to find $\frac{k}{\bar{\varepsilon}} = (\frac{r}{\alpha z})^{\frac{1}{\alpha-1}}$ and substitute $\frac{k}{\bar{\varepsilon}}$ in the wage equation, the wage rate becomes a function of productivity and interest rate, $w(z, \mu) = (1 - \alpha)z(\frac{r}{\alpha z})^{\frac{\alpha}{\alpha-1}}$. From the equation it is clear that the wage rate increases as the productivity increases.

At the end of the period some workers leave islands exogenously and new workers arrive from other islands. Also an island may experience a technology shock, which occurs with Poisson frequency δ_x . If the island is hit by the shock, it draws new productivity from a bounded Pareto distribution. Firms start each period with a new set of workers and new productivity if a shock occurs.

Let $L(z)$ be the firms' decision rule for labor and let $K(z)$ be the decision rule for capital rental.

2.3. Government

The government levies the labor income tax to the workers to fund pension to the retirees and a government spending, G . Their budget is balanced at each period.

2.4. Equilibrium

An equilibrium is a set of functions

$$(r, w, V, W, L, K, \chi, A, G)$$

that solves workers' and firms' problem and clear markets for labor, asset and goods.

1. W solves (1), (S, A) are the associated decision rules
2. V solves (2), (L, K) are the associated decision rules
3. $L(z) = \int_s \varepsilon \mu(\varepsilon, a, z) ds$
4. $\sum_{x=1}^{n_z} k_x = \int_s A(\varepsilon, a, z) \mu(\varepsilon, a, z) ds$
5. $\int_s T(w(z; \mu) \varepsilon) \mu(\varepsilon, a, z) ds \geq \int_s \tau \mu(\varepsilon, a, z) ds + G$
6. $\mu'(B, \varepsilon_j, z_k) = (1 - \delta)(1 - \delta_x) \int_{\{(a, \varepsilon_i, z_k) | A(a, \varepsilon_i, z_k) \in B\}} \pi_{ij}(1 - \chi(a, \varepsilon_i, z_k)) \mu ds_0 +$
 $\delta(1 - \delta_x) \int_{\{(a, \varepsilon_i, z_l) | A(a, \varepsilon_i, z_l) \in B\}} \pi(k|\varepsilon_i) \pi_{ij}(1 - \chi(a, \varepsilon_i, z_l)) \mu ds_1 +$
 $(1 - \delta) \delta_x \int_{\{(a, \varepsilon_i, z_l) | A(a, \varepsilon_i, z_l) \in B\}} \pi_{lk} \pi_{ij}(1 - \chi(a, \varepsilon_i, z_l)) \mu ds_1 +$
 $\delta \delta_x \int_{\{(a, \varepsilon_i, z_l) | A(a, \varepsilon_i, z_l) \in B\}} \pi(k|\varepsilon_i) \pi_{lk} \pi_{ij}(1 - \chi(a, \varepsilon_i, z_l)) \mu ds_1 +$
 $\int_{\{(a, \varepsilon_i, z_l) | A(a, \varepsilon_i, z_l) \in B\}} \pi(k|\varepsilon_i) \pi_{ij} \chi(a, \varepsilon_i, z_l) \mu ds_1$
 $s_0 = (a \times \varepsilon_i \times z_k), s_1 = (a \times \varepsilon_i \times z_l)$

$$\forall (B, \varepsilon_j, z_k) \in \mathbf{S}$$

where $a \in \mathbf{B} \subset \mathbf{R}$, $\varepsilon \in \mathbf{E} \equiv \{\varepsilon_1, \dots, \varepsilon_{n_\varepsilon}\}$, $z \in \mathbf{Z} \equiv \{z_1, \dots, z_{n_z}\}$, $\mathbf{S} = \mathbf{B} \times \mathbf{E} \times \mathbf{Z}$

3. Calibration

The model period is a year. I divide parameters into two groups: parameters in the first group are exogenously set using data and rest of the parameters are jointly calibrated. To study the role of firm heterogeneity in the earnings inequality and the effect of inequality to aggregate output, capital and interest rate, I do a transitional dynamics experiment from 1992 to 2013 in the following section. To do so, I calibrate the model twice: targeting data of early 1990's (period 1) and early 2010's (period 2)⁴. Later I will impose paths of parameters for the transitional dynamics experiment

⁴I used long term average for some of the data if a series is stable over time. In case the data are not available for certain period, I used the closest period data.

and compute the contribution of each components to the rise in inequality and their impacts to the economy.

3.1. *Parameters set ex ante*

Preference and production. I set the household discount factor β to 0.94. Utility function is $u(c) = \log(c)$. Production technology is CRS and the capital share, α is set to 0.36. The depreciation rate, δ_d , is set to 0.076 to match the capital-output ratio around 3.0.

Separation rate. Separation rate, δ , for period 1 is 6.1%. Since there is no unemployment in the model⁵ I use job to job flow in LEHD. Average job to job flow in a quarter from 2000 to 2001⁶ is 4.3%. Assuming a worker experience job transition once a year on average, job to job flow is 17.1% in a year. From JOLTS, layoff out of all separation is 45.7% (2001-2016 average), and I pinned down separation rate as 45.7% of all job to job flows. The rest of flows (7.2%) will be filled with job searchers. By the same fashion, separation rate for the period 2 is 4.1% and the target for job searcher is 4.8%.

Life cycle. Retirement probability, p_d , is set to 2.2%, implies average working life is 45 years (American Community Survey, average of 2005-2016). Probability of dying for the period 1 is 6.8% to target 24.5% of population who have zero or negative earnings in SCF 1992 (Diaz-Gimenez, Quadrini, and Ríos-Rull (1997)). Dying probability for the period 2 is 6.1% to target 26.6% of zero or negative earnings households in SCF 2013 (Kuhn and Ríos-Rull (2016)).

Tax and transfer⁷ The parameter governs tax progressivity, γ , is set to 0.159 following Straub (2017) and τ_i is set to 0.9 to balance government budget.

⁵Job flow from employed to unemployed (and vice versa) is an important aspect of a labor market. In U.S. mean and median unemployment duration have been rising over time. However it is not clear whether there is a gradual decline or rise in the unemployment rate. Given there is no noticeable trend in the unemployment rate, the longer duration implies less workers experience unemployment, but once an worker is fired, she remains longer as an unemployed. In other words, unemployment shock has become less frequent but larger, and it may affects agents' choices. I have tried the version of a model with unemployment state (employed, partially employed, unemployed) but it has a little impact to the results while computationally costly.

⁶The publicly available data starts from 2000.

⁷Parameters govern tax and transfer are fixed over the transition period in my model but it is known that tax progressivity has been dropped in the U.S. Hubmer, Krusell, and Smith Jr (2016) shows the drop in tax progressivity is an important driver of the rise in wealth inequality.

3.2. Calibrated parameter

Parameters in this group are jointly calibrated to match moments from various sets of the U.S. data with corresponding steady state moments that are obtained from the model solutions. I describe targets with certain parameters, but since the parameters are jointly determined this association is heuristic.

Labor productivity. Parameters related to the labor productivity process are chosen to match the earnings distribution. I target earnings Gini index, shares of earnings by quintile and top group (top 90-95%, 95-99%, top 1%) from Survey of Consumer Finances (SCF) 1992 and 2013 ⁸. To match these statistics, I assume a worker's productivity is drawn from a bounded Pareto distribution. With probability δ_ε , workers lose their current labor productivity and draw new one from the distribution. The lower bound ($\underline{\varepsilon}_1$), upper bound ($\bar{\varepsilon}_1$) and shape parameter ($\eta_{\varepsilon,1}$) for the period 1 are set to 0.2, 25.0 and 1.92 respectively. For the period 2, $\underline{\varepsilon}_2$, $\bar{\varepsilon}_2$ and $\eta_{\varepsilon,2}$ are 0.1, 80.0 and 1.86. Shock probability, δ_ε , is set to 0.12 for both periods. I chose the number of grid point to 7, and the 7th point is assigned to the retirees hence $\varepsilon_7 = 0$. From 1st to 6th values are set to represents [10.0, 20.0, 30.0, 37.4, 2.55, 0.05] (%) of working population respectively. For example, ε_2 is a median value between x_1 and x_2 such that $f(x_1) = \frac{1-(\underline{\varepsilon}_1/x_1)^{\eta_\varepsilon}}{1-(\underline{\varepsilon}_1/\bar{\varepsilon}_1)^{\eta_\varepsilon}} = 0.1$, $f(x_2) = \frac{1-(\underline{\varepsilon}_1/x_2)^{\eta_\varepsilon}}{1-(\underline{\varepsilon}_1/\bar{\varepsilon}_1)^{\eta_\varepsilon}} = 0.1 + 0.2$ where $f(x_i)$ is the cdf of the bounded Pareto distribution.

Table 1: Targets and models values: Earnings distribution

Gini		Quintiles					Top(%)		
		1q	2q	3q	4q	5q	90-95	95-99	99-100
1992									
Data	0.63	-0.40	3.19	12.49	23.33	61.39	12.38	16.37	14.76
Model	0.62	0.00	4.61	10.79	22.47	62.12	10.26	17.27	14.48
2013									
Data	0.67	-0.10	3.00	10.40	20.20	66.50	12.40	18.40	18.80
Model	0.68	0.00	2.80	9.37	19.16	68.67	10.23	17.43	21.57

Data: SCF

Table 1 shows the targets and the model results. The model does a good job resembling the overall distributions.

Firm productivity. The firm productivity process is pinned down to match statistics from the variance decomposition of earnings into person and firm component. The

⁸Statistics in the table are taken from Castaneda, Diaz-Gimenez, and Rios-Rull (2003) and Kuhn and Ríos-Rull (2016).

variance decomposition comes from the regression model introduced by Abowd, Kramarz, and Margolis (1999).

$$y_t^{i,j} = \nu^i + \psi^j + X_t^i \xi + e_t^{i,j}$$

$y_t^{i,j}$ is a real log earnings of a worker i at firm j at time t . ν^i captures fixed worker characteristics such as innate ability and ψ^j captures firm component in earnings (eg. compensation differentials). ξ captures time varying worker characteristics and $e_t^{i,j}$ captures transitory earnings fluctuation. Ignoring time varying component, $X_t^i = 0$ and rewrite $y_t^{i,j}$ as

$$y_t^{i,j} = (\nu^i - \bar{\nu}^j) + \bar{\nu}^j + \psi^j + \varepsilon_t^{i,j}$$

From the above equation, we define variances only depend on firm j component as *between firm* components and rest of the part as *within firm* components.

$$\text{var}(y_t^{i,j}) = \underbrace{\text{var}(\bar{\nu}^j) + \text{var}(\psi^j) + \text{cov}(\bar{\nu}^j, \psi^j)}_{\text{between-firm component}} + \underbrace{\text{var}(\nu^i - \bar{\nu}^j) + \text{var}(\varepsilon_t^{i,j})}_{\text{within firm component}}$$

In the model, a worker's earning is $w(z, \mu)\varepsilon$. Convert it to log, it becomes $\log(\varepsilon) + \log(w(z, \mu))$ so $\log(\varepsilon)$ corresponds to $\nu + X\xi^9$ and ψ correspond to $\log(w(z, \mu))$.

I set the lower bound($\underline{z}_1, \underline{z}_2$) and the upper bound($\bar{z}_1, \bar{z}_2$) of a Pareto distribution as 0.5 and 1.4 for both periods. The shape parameter for the period 1 is 1.6 and for the period 2 is 3.4. As the shape parameter increases, the distribution becomes more right skewed. Therefore larger shape parameter implies relatively higher wage rate premium of productive firms. I choose 10 grid points for the firm productivity, so the number of island in the economy is 10. From 1st to 10th values are set to represents [40.0, 20.0, 10.0, 8.0, 7.0, 5.0, 5.0, 3.6, 0.3, 0.1] (%) of total firms in the economy respectively. Like the labor productivity, z_2 is median of two values that returns 0.4 and (0.4+0.2) from the cdf of the bounded Pareto distribution.

I target the variance decomposition results in Song et al. (2015). They used the U.S. Social Security Administration (SSA) data which connects employee - employer. They repeated the variance decomposition for five adjacent seven-year intervals from 1980 to 2013, and I target results using 1987-1993 and 2007-2013 data. Table 2 presents the targets and the model results.

Job transition probability and search cost. If a worker decides to search or being

⁹Since there is no time varying worker characteristics in the model and yet X is a worker's characteristic in the decomposition, I consider $\nu + X\xi$ as the worker factor.

Table 2: Targets and models values: Variance of earnings

	y	ν	Between firm			Within firm
			$\bar{\nu}$	ψ	$2cov(\bar{\nu}, \psi)$	$\nu - \bar{\nu}$
1992*						
Data**	0.70	0.52	0.12	0.12	0.08	0.38
Model	0.79	0.52	0.04	0.15	0.11	0.49
2013*						
Data**	0.78	0.58	0.15	0.11	0.11	0.40
Model	0.93	0.58	0.03	0.21	0.14	0.55

* 1992: results using 1987-1993 data, 2013: 2007-2013 data

** Data values are adjusted to be consistent with the model. First, $var(e)$ (1992: 0.16, 2013: 0.14) is subtracted from $var(y)$ because there is no transitory shock to the earnings in the model. Second, $var(X\xi)$ is added to $var(\nu)$, $var(X\xi - \bar{X}\bar{\xi})$ and $2cov(\nu - \bar{\nu}, (X\xi - \bar{X}\bar{\xi}))$ are added to $var(\nu - \bar{\nu})$ since there is no time varying worker characteristic in the model.

Data: Song et al. (2015)

exogenously separated from a firm she will move to the next island following the job transition probability. I use Beta distribution to set the transition probability over the islands. The island distribution is mapped to a support of a Beta cdf, $F_x(a, b)$, and the probability of moving to i th island is simply $F_{x_i}(a, b) - F_{x_{i-1}}(a, b)$ where x_i is cumulative measure of islands up to the i th point. Transition probabilities depend on worker types, $\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$, $\{\varepsilon_4\}$ and $\{\varepsilon_5, \varepsilon_6\}$ follows a same transition process. As labor productivity increases, a worker has a higher chance to arrive at a high productivity island and vice versa¹⁰. The shape parameters for Beta distributions are listed in the Table 6 where $a_{i,j}$ is a shape parameter for i th worker group in period j ¹¹. Search cost which affects the number of searchers, are set to 0.18 and 0.45 for the period 1 and 2.

Job transition probability and the workers' search decisions shape the firm size distribution.¹² Thanks to the search option, once a worker happen move to a low pro-

¹⁰In other words, a transition probability associated with a high labor productivity has a first-order stochastic dominance over a process associated with a low productivity.

¹¹ a is inversely related to the mass on the left side and b is inversely related to the mass on the right side. From the period 1 to 2, I increase a and decrease b to match higher employment concentration to large firms.

¹² Models with heterogeneous firm mostly use decreasing return to scale technology to have non degenerate firm distribution.(For example, Khan and Thomas (2008), Khan and Thomas (2013)). Having homogeneous workers and frictionless labor market, firms hire workers where wage rate equals labor productivity. Given wage rate and capital, size of employment becomes a function of firm productivity and it is increasing in firms productivity and capital. In my model, firms also hire workers where marginal labor productivity equals wage rate. However, without decreasing return to scale, it is optimal to hire every available workers on the island as long as workers are willing to accept the wage rate. Therefore firm size distribution determined by the job transition process and workers' decisions.

ductive island, she may leave the place. Thus some islands ends up with less workers than the job transition probability implies and some islands hire more workers in a stationary equilibrium. Table 3 shows firm size distribution and share of workers who searches over total working population.

Table 3: Target and models values: Firm size distribution* and worker flow

	Firm size distribution						Searcher (%)
	1-4	5-19	20-49	50-499	500-4999	5000+	
1992							
Data	5.87	14.94	10.73	21.43	17.64	29.41	7.20
Model	5.76	12.11	10.86	17.89	24.11	29.27	7.22
2013							
Data	5.08	12.68	9.76	20.84	18.23	33.41	4.80
Model	6.46	13.60	5.85	12.71	27.06	34.30	4.78

* Share of workers who employed to firms who hire n workers

Data: BDS

Table 4: Wealth distribution

	Gini	Quintiles					Top(%)		
		1q	2q	3q	4q	5q	90-95	95-99	99-100
1992									
Data	0.78	-0.39	1.74	5.72	13.43	79.49	12.62	23.95	29.55
Model	0.77	-0.58	1.29	5.65	15.05	78.59	14.26	31.15	19.39
2013									
Data	0.85	-0.70	0.60	3.20	9.80	87.00	12.10	27.40	35.50
Model	0.83	-0.61	0.33	4.05	11.60	84.64	11.73	29.62	31.91

Data: SCF

3.3. Validation

The model matches not targeted moments. First, the model matches the wealth distribution quite well except at the very top. Also it captures the rise in concentration between 1992 and 2013 pretty well which implies labor income could be one of main driver of rise in wealth inequality.

Table 5: Earnings dynamics

	Std.		Skewness		Kurtosis		$P(\Delta y) < \mathbf{x}^*$		
	1y	5y	1y	5y	1y	5y	$\mathbf{x} = 0.2$	0.5	1.0
Data	0.51	0.78	-1.07	-1.25	14.93	9.51	0.67	0.83	0.93
Model 1**	0.50	0.86	0.07	0.17	14.11	6.08	0.78	0.84	0.90
	(0.43)	(0.88)	(-0.07)	(0.05)	(17.77)	(5.51)	(0.84)	(0.89)	(0.92)
Model 2**	0.51	0.89	0.02	0.17	16.25	6.74	0.80	0.85	0.90
	(0.46)	(0.91)	(-0.04)	(-0.03)	(20.96)	(5.95)	(0.84)	(0.89)	(0.91)

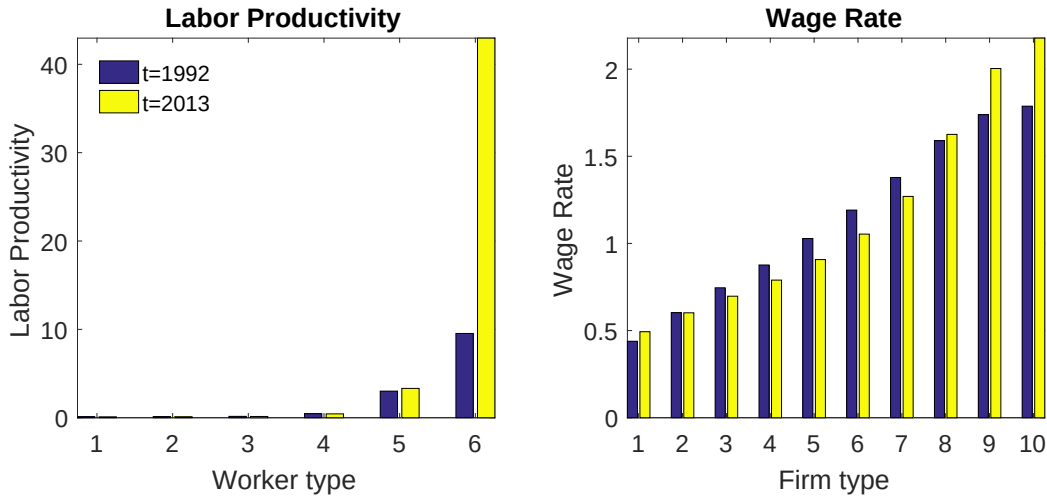
* $|\Delta y|$: Absolute log earnings change less than a threshold x

** Model 1: Results with the period 1 parameters, Model 2: Results with the period 2 parameters

Data: Guvenen et al. (2015)

Second, the model captures features of earnings dynamics. Using SSA data, Guvenen et al. (2015) reports various moments of the U.S. earnings dynamics¹³. They show earnings dynamics are very different from a log normal process which has been the standard assumption in the incomplete market literature. I simulate the model¹⁴ to compute the moments. My model does a fairly good job generating moments which are consistent with the data except skewness and kurtosis of 5 year earnings growth. Table 5 shows moments from the data and the model. Numbers in parenthesis are the results from the earning processes without the search option.

Figure 1. Labor productivity and wage rate



¹³ Moments are computed with 1994-2013 data.

¹⁴ Panel size is 2000 and simulation period 1200. First 200 periods data are discarded when computing the statistics. Increasing panel size or periods has a marginal effect to the results.

Table 6: Parameter values

Parameter	Value	Description
Preferences and production		
β	0.94	Discount rate
α	0.36	Capital share
δ_d	0.076	Depreciation rate
Life cycle		
p_r	0.022	Retirement probability
$p_{d,1}$	0.0681	Dying probability
$p_{d,2}$	0.0607	Dying probability
Tax and transfer		
τ_i	0.88	Labor income tax
τ_w	0.6973	Transfer
γ	0.159	Tax progressivity
Labor productivity		
$\underline{\varepsilon}_1$	0.2	Lower bound of Pareto distribution
$\bar{\varepsilon}_1$	25.0	Upper bound of Pareto distribution
$\eta_{\varepsilon,1}$	1.92	Shape of Pareto distribution
$\underline{\varepsilon}_2$	0.1	Lower bound of Pareto distribution
$\bar{\varepsilon}_2$	80.0	Upper bound of Pareto distribution
$\eta_{\varepsilon,2}$	1.86	Shape of Pareto distribution
δ_ε	0.12	Shock probability
Firm productivity		
$\underline{z}_1, \underline{z}_2$	0.5	Lower bound of Pareto distribution
$\bar{z}_1, \bar{z}_2$	1.4	Upper bound of Pareto distribution
η_1	1.6	Shape of Pareto distribution
η_2	3.4	Shape of Pareto distribution
δ_x	0.05	Shock probability
Job transition		
$a_{1,1}$	0.9	Shape of Beta distribution
$b_{1,1}$	0.285	Shape of Beta distribution
$a_{2,1}$	1.9	Shape of Beta distribution
$b_{2,1}$	0.2	Shape of Beta distribution
$a_{3,1}$	3.2	Shape of Beta distribution
$b_{3,1}$	0.19	Shape of Beta distribution
$a_{1,2}$	4.2	Shape of Beta distribution
$b_{1,2}$	0.28	Shape of Beta distribution
$a_{2,2}$	4.4	Shape of Beta distribution
$b_{2,2}$	0.195	Shape of Beta distribution
$a_{3,2}$	4.5	Shape of Beta distribution
$b_{3,2}$	0.18	Shape of Beta distribution
s_1	0.18	Search cost
s_2	0.45	Search cost
δ_1	0.061	Separation rate
δ_2	0.041	Separation rate

Figure 2. Parameter paths

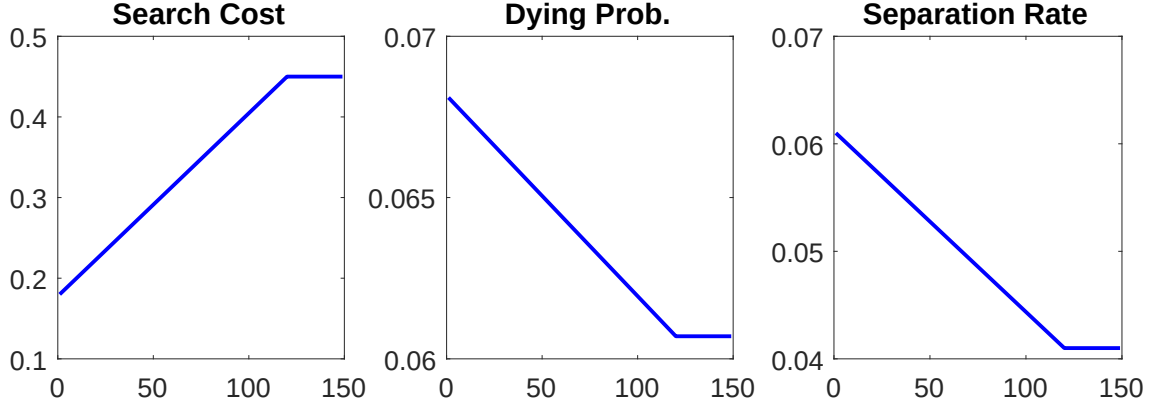
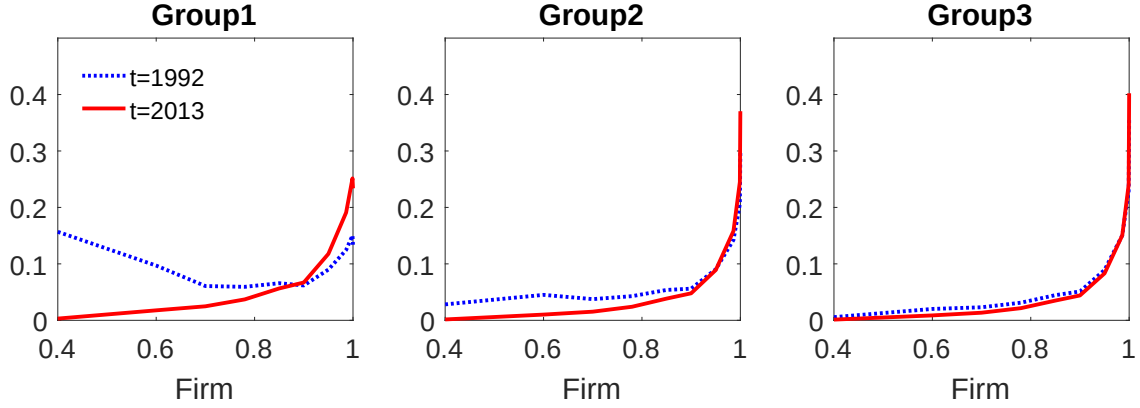


Figure 3. Transition probability



4. Results

The Section 3 demonstrates the ability of the model to capture earnings distribution, earnings dynamics as well as firm distribution. This section explores the causes and implications of rising inequality. In Section 4.1, I explain the details of the transitional dynamics experiment. Section 4.2 accounts factor affecting the rising earnings inequality and firm inequality, Section 4.3 investigates quantitative implications of the rising inequality to the aggregate variables: output, capital and interest rate.

4.1. Transitional Dynamics

To quantify the factors affecting the inequality trends and the implications of rising inequalities, I compute the general equilibrium transitional dynamics induced by

the change in the parameter values that shape the economy. As explained in section 3, I calibrate the model twice targeting the economies circa 1990 and 2010¹⁵. Some of the parameters are common for both periods: Parameters regarding preference, production, tax, transfer and productivity shock probabilities. Varying parameters are: Parameters define labor productivity process, firm productivity process, dying (and birth) probability, job transition process and search cost. I linearly interpolate the parameters over the transition periods. Figure 1 shows productivity by type at the period 1 and 2. Both have become more right skewed. Figure 2 shows paths of the search cost, dying probability and separation rate. Increasing search cost and decreasing separation rate together reduce job flows and decreasing dying probability raises the retiree population. Figure 2 depicts the job transition probability for each group.¹⁶ It shows a worker is more likely to be allocated to productive islands as her labor productivity increases and as time goes by. The reasons all workers have higher probability to move to large, productive firms in the period 2 are employment share of large firms increases overtime, while worker flow decreases. Except exogenous separation, workers search and leave when they arrive to a low productivity firm. If job transition probability does not vary, decreasing searchers means more workers in small, low productivity islands and less employment concentration to large firms.

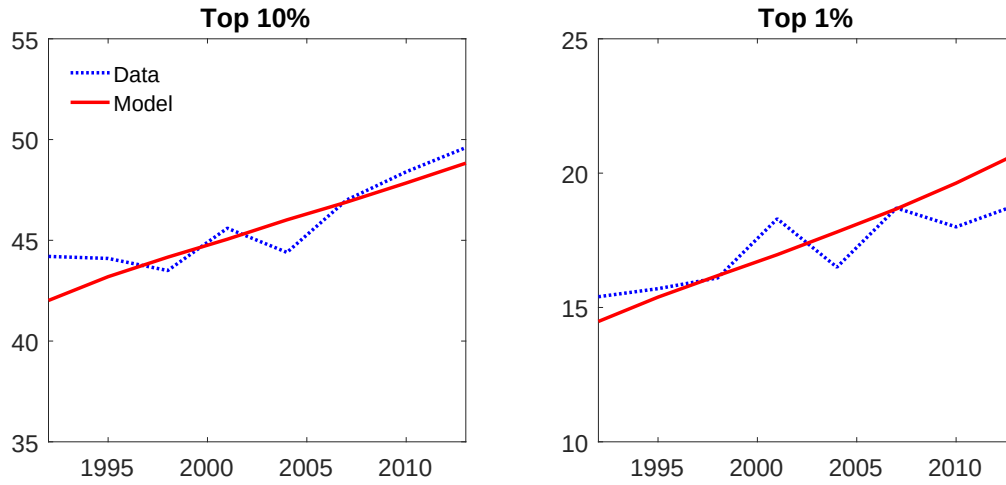
To compute the transitional dynamics, I initiate the model in a steady state with the 1992 distribution then feed in exogenous paths of varying parameters and stochastic processes. I define a non stationary equilibrium and describe how I solve it in Appendix A. Importantly, I keep the total available resources (effective labor and average (firm measure weighted) firm productivity) constant over the transition periods. Simulation periods are 150, parameters vary from 1st - 120th periods and remain constant thereafter. Finally, I assume agents have perfect foresight over the entire path.

After computing the transition with varying parameters, I fix a certain set of parameters at the period 1 value while varying others to measure the contribution of that component to inequality trends. I will quantify the role of changes in labor productivity and firm productivity process.

¹⁵Considering earnings inequality rose rapidly in 1980's, it would be interesting to do the transitional dynamics from 1980. But I choose 1990 because (although it is still in the process of applying) I plan to use Longitudinal Employer-Household Dynamics (LEHD) data to estimate the productivity processes and the job transition probability and the most states' data are available from early 1990 or later.

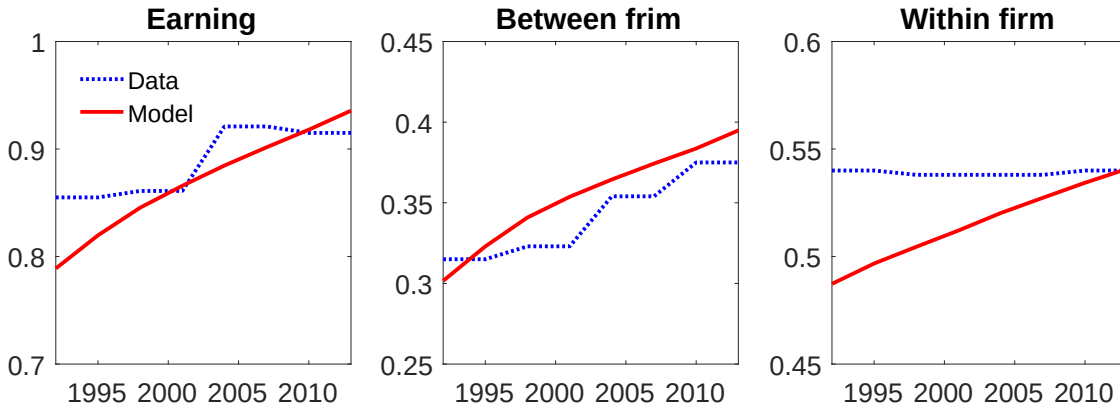
¹⁶Group 1: $\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$, Group 2: $\{\varepsilon_4\}$, Group 3: $\{\varepsilon_5, \varepsilon_6\}$

Figure 4. Share of earnings, top group



Kuhn and Ríos-Rull (2016) uses SCF data which is triannual. From 1992 to 2013 there are 8 data points, I save distributions every 15 periods so the first period and the last period (120th) in transition match 1992 and 2013 respectively.
Data: Kuhn and Ríos-Rull (2016)

Figure 5. Variances

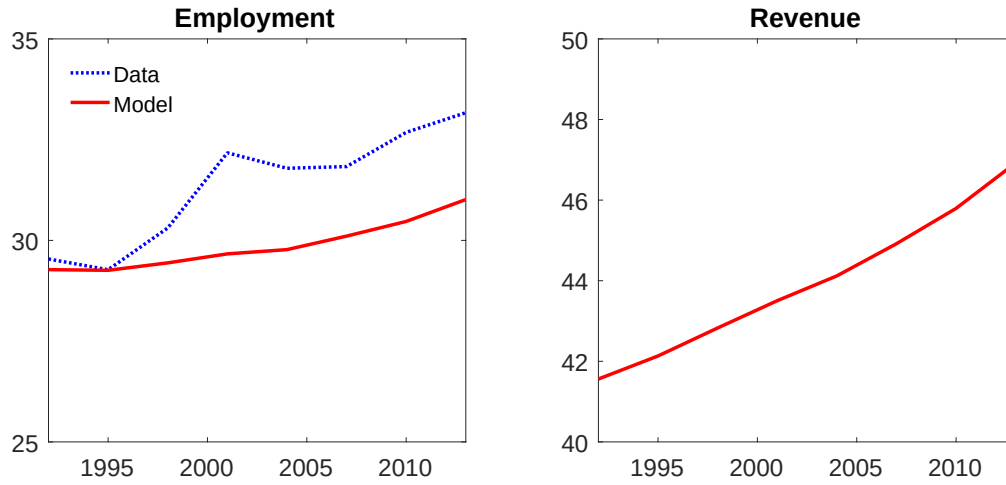


Song et al. (2015) split 1980-2013 data into 5 groups and regress the regression model in the Section 3 to see the changes in variance of earnings. I use 2nd - 5th estimation results (4 data points, represents 1987-1993, 1994-2000, 2001-2007 and 2007-2013 respectively). Out of 120 transition periods, I keep distributions every 15 periods. To plot these graphs I map one data point to 2 model data points.
Data: Song et al. (2015)

4.2. Accounting for earnings inequality

First I present the transitional dynamics results with all varying parameters in Figure 1 and 2. Figure 4 depicts the share of earning by top 10% and 1% of popula-

Figure 6. Employment and revenue share of large firms*



* Firms with more than 5000 workers.
Data: BDS

tion¹⁷. The model captures the rising trend well. Figure 9 shows variance of earnings and its decomposition results. The model misses the flat within firm variance schedule over the period and overshoots increases in earnings variance, yet it follows the overall trend.¹⁸ Figure 6 shows employment and revenue share of firms who hires more than 5000 workers and the model captures rising dominance of large firms.¹⁹

In the rest of this section I quantify the factors affecting the earnings inequality and the firm inequality by computing the transitional dynamics while i) fixing labor productivity at the period 1 level ii) by fixing firm productivity, transition probability and search cost at the period 1 level.

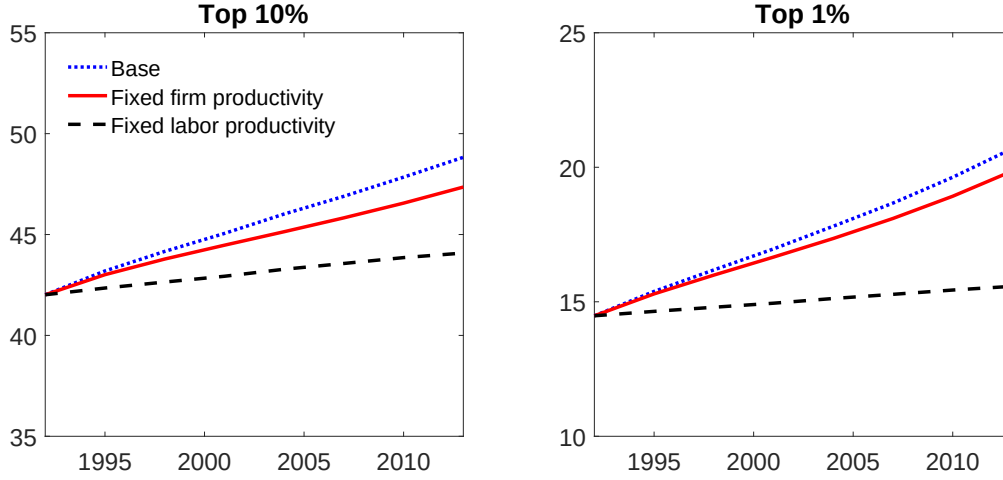
Earnings inequality. Concentration of earnings to the top is mostly driven by the changes in labor productivity distribution. Figure 7 shows share of earnings held by top 10% and 1%, the case with changing all parameters (*base* henceforth), the case with constant firm productivity and the case with constant labor productivity. When labor productivity remains constant, concentration to the top drops significantly. Top

¹⁷Figure 10 depicts the share of wealth by top groups. The model misses the high concentration of top 1% wealth share but closely follows the trend.

¹⁸ The statistics are presented in Table 7. Although majority of rise in earnings variance is explained by *between firm* component as reported in Song et al. (2015), its composition does not align with the data.

¹⁹ There is no public data to compute large firms' revenue share but Dorn, Katz, Patterson, and Van Reenen (2017) reports changes in revenue share of largest 20 firms by industry using U.S. Economic Census data. From 1992 to 2012, it increased 3.0% in manufacturing, 10.76% in retail, 4.12% in wholesale, 3.8% in service and 14.4% in finance on average.

Figure 7. Share of earnings, top group



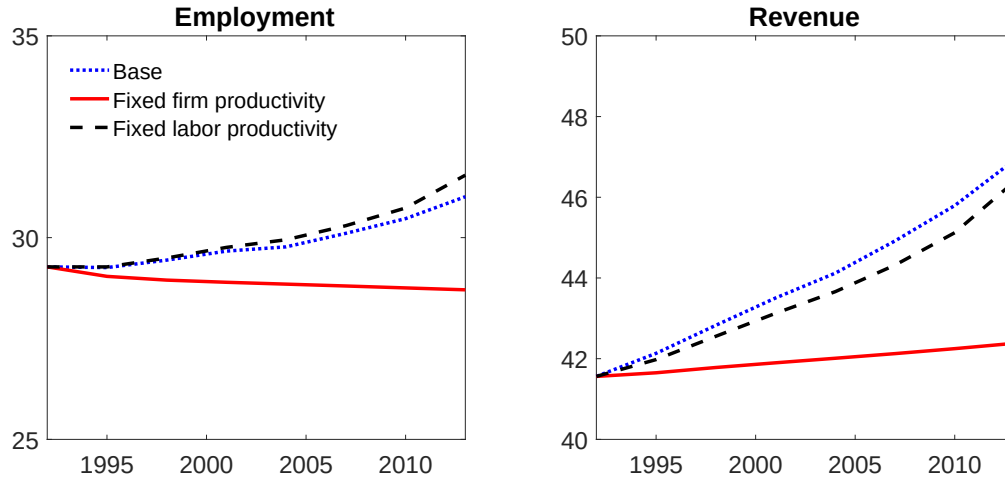
10% share rises 30.3% and top 1% share only rises 17.6% of the base case.²⁰ In contrast, when fixing firm productivity the top groups' earning shares rise close to the base case (top 10%: 78.3%, top 1%: 86.6% of the base case). Also Gini coefficient rises from 0.62 to 0.67 while it rises to 0.65 with constant labor productivity.

The changes in variance of earnings are explained by the two factors more evenly. Figure 11 depicts the changes in variance of earnings and its decomposition. When shut off varying labor productivity, earnings variance rises from 0.78 to 0.86 and when shut off varying firm productivity and it rises up to 0.88. In all cases, *between firm* component explains majority of rise in the variance of earnings, which is consistent with the Song et al.'s (2015) findings. It is interesting even if the firm productivity distribution remains constant the between firm component contributes the most to the change. Table 8 reports the variance decomposition results, and it shows the rises in $cov(\bar{\nu}, \psi)$ which will be explained in the next subsection, mainly drives the changes in between firm component.

Firm inequality. Not surprisingly, the employment and revenue concentration to large firms is driven by the changes in firm productivity distribution. In Figure 8, with constant firm productivity, the employment share of large firms decreases. This happens because although search cost remains at a low level, expected benefit of moving is not high enough because firm productivity is more equally distributed in the period 1. Also workers are less likely arrive at productive firms. The number of searchers

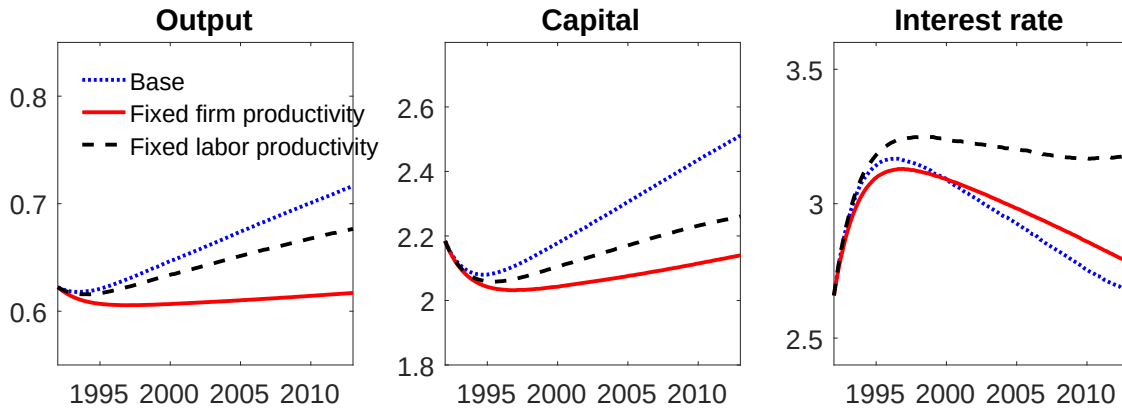
²⁰Figure 12 shows share of wealth held by top 10% and 1%. Top 10% wealth share decreases when labor productivity distribution remain constant, and top 1% wealth share barely moves. On the other hand, when keep firm productivity constant the wealth concentration becomes more extreme.

Figure 8. Employment and revenue share of large firms*



* Firms with more than 5000 workers.

Figure 9. Aggregate variables



decreases overtime, hence less concentration. However revenue share rises thanks to the increase in average effective labor in large firms: Workers and firms tend to be more positively sorted as the number of searchers decreases. Since workers pay the fixed cost to search, at the same wealth level more productive workers search who expects large wage increase once move to productive firms. Therefore as the number of searcher decreases, the average labor productivity of searchers increases.

When fixing labor productivity, the firm concentration rises as much as the rise in the base case.

4.3. *The effect of inequality to the U.S. economy*

On top of accounting earnings and firm inequality trends, I explore the implications of the rise in inequality to the U.S. economy. As mentioned in section 4.1, I keep total effective labor and average firm productivity constant over the transition periods²¹ so if there are changes in aggregates, they come from changes in the distributions not from the quantity of resources. Keeping firm productivity constant might seem strange with CRS technology. In fact, output is the largest when the most productive firm hires all workers in the economy. However with the islands assumption and worker flow that is controlled by transition probability its average level have an impact to the aggregate variables. The model implied transitional dynamics points to the following outcomes.

Output. The output increases by 15.1%. Considering the U.S. GDP has grown 68.4% between 1992 and 2013, it is a substantial amount. Table 9 shows the growth rate of the output by firm productivity group. Bottom 90% firms' output decreases by 13.5% due to drop in productivity and labor. 90-99.9% firms' output increases by 3.2%. Although effective labor (as well as the number of workers) decreases, but rise in productivity dominates the drop in labor. The most productive firms' output increases by 35%. Because these firms' output share is above 40%, total output increases. One thing to mention is that positive sorting and worker - firm complementarity increase the output: effective labor rises 10.9% while the number of workers increases 5.8% for these firms. As shown in Song et al. (2015), sorting plays a major role in the rise in inequality but contributes to the growth. Between firm and labor productivity, changes in firm productivity distribution plays a larger role for the growth. With constant firm productivity, output decreases by 0.8%.

Capital. The capital increased by 14.9% while it rises 101.3% in the data.²² Like output, changes in firm productivity distribution is crucial for the capital growth. With constant firm productivity and constant transition process, there is a little change in the worker - firm matching hence capital is stable as well as the output. It is interesting that firm productivity explains a little of rising earnings and wealth concentration but it has an larger impact to the aggregate variables than labor productivity.

²¹ In other words, $\int_s \varepsilon_t \mu_t(\varepsilon_t, a_t, x_{zt}) ds = \int_s \varepsilon_{t+1} \mu_{t+1}(\varepsilon_{t+1}, a_{t+1}, x_{zt+1}) ds$, $\int_s z_t \mu_{x,t}(\bar{\varepsilon}_t, k_t, z_t) ds = \int_s z_{t+1} \mu_{x,t+1}(\bar{\varepsilon}_{t+1}, k_{t+1}, z_{t+1}) ds$ for all t .

²²I use 'Household Wealth' in Piketty, Saez, and Zucman (2016) to compute the capital growth rate because the aggregate capital is aggregate wealth held by household in the model.

Interest rate. When compare the steady state in 1992 and in 2013, equilibrium interest rates are 3.24% and 2.53% respectively. However in the transitional dynamics it is 2.6% at $t = 1$ and rise up to 3.1% then falls again. Why interest rate is lower at $t = 1$ than the stationary equilibrium? It implies aggregate saving is larger in the transitional dynamics since capital demand is same given an interest rate.²³ Would it be the case for all workers? Figure 13 plots the difference between saving decision of the period 1 stationary equilibrium and saving decision of the transitional dynamics at $t = 1$ given a same interest rate (equilibrium interest rate of the period 1 economy). It is clear from the figure that low labor productivity workers who work for low productivity firms increases saving the most. They increase saving around 4% while the most productive workers at the most productive firms reduce saving by 0.14%. This is because productivity of workers and firms at the bottom decreases in non stationary economy. Recall the variances of the productivity processes rise between the period 1 and 2, but the averages stay the same: The low wage workers see lower wages going forward, so save more to make up for it. Overall, the majority of workers see the not so bright future hence interest rate needs to be dropped to discourage savings.

The interest rate shows downward trend except the initial hump while the capital increases. To see why, I set $r = 2.95\%$ ²⁴ and compare capital supply and demand of 1992 and 2013 economy. Figure 14 depicts the difference in saving rates, wealth and earnings between the two economies. The majority of workers tend to save more in 2013, hence wealth of workers in most category rises. As earnings variance increases, workers face higher earnings risk, try to save more to insure themselves. From the demand side, the aggregated capital demand also increases by 17%. However the aggregate supply increases more (63%) than the demand, interest rate falls.

This suggests rising inequality can be a reason of the decline in real interest rates over the past 25 years (Holston, Laubach, and Williams (2017), Carvalho, Ferrero, Nechio, et al. (2017)²⁵). Based on Holston et al.'s (2017) estimates, real interest in the U.S. declines from around 3% to 0.5% since 1990's, and the transitional dynamics result accounts for a quarter of the decline.

²³Firms' capital demand is a function of its productivity and an interest rate.

²⁴a mid value between the period 1 and 2 equilibrium interest rates

²⁵ Holston et al. (2017) shows the trend GDP growth and natural rates trends are related. Carvalho et al. (2017) focuses on the role of demographic shift: As life expectancy increases individuals' retirement period becomes longer, generating additional incentives to save.

5. Conclusion

Motivated by empirical evidences that show the rising inequality in individuals' earnings and firms' employment and revenue, I propose a model with two side heterogeneity and labor market friction to study these trends. I have calibrated the model to match the earnings concentration, individual and firm components in the variance of earnings and firm distribution. The model also matches the moments in earnings dynamics and the wealth distribution. From the transitional dynamics experiment, I find the changes in labor productivity explain the most of the rising concentration in earnings and the majority of the firm concentration is driven by the changes in firm productivity distribution. The shifts in productivity distributions and worker-firm matching have important implications: They explain 22% of the output growth, 15% of the capital growth and a quarter of decline of the interest rate since 1990's.

There are several avenues related to this work that deserves further study. First, a model that can incorporate a more flexible wage rate to different workers within a firm and a micro data that discipline the model are necessary. The technology assumption in the model results a same wage *rate* to all workers *within* a firm. Also the calibration results to match firm concentration implies increasing a large firm wage premium, which is against a recent finding by Bloom, Guvenen, Smith, Song, and Wachter (2018). Estimating earnings process allowing for two-sided worker-firm unobserved heterogeneity in a rich environment such as Bonhomme, Lamadon, and Manresa (2015) would allow me to take a model with richer wage setting to the data.

Second, the interaction between investment and inequality needs further study. In this framework, firms' capital demand is a function of its productivity and an interest rate given the frictionless capital market assumption. It allowed me to decompose the factors affecting the inequality trends neatly, but misses the interaction between the firms' investment decisions and its effect to the inequalities. For example, if firms have to choose the capital in advance, it may lessen the effect of the productivity shock. Assuming persistent productivity, a firm with high productivity today will choose high level of capital for the future. If the firm hit by a shock, having large capital may prevent the firm experiencing large drop of output and wage cut. At the same time it may interfere with small firms' growth. Given the highly concentrated firm distribution, a capital friction may affect earnings distribution as well as the aggregate variables going forward.

Lastly, this framework is suitable to study policy implications. Including a precise tax system would allow us to measure its effect to the wealth distribution as in

Hubmer et al. (2016), further its impact on the aggregates such as growth. Different type of taxes can be compared and evaluated, to answer a such question, how to design a tax system that does not interfere with the growth but mitigate the extreme concentration?

Appendix A. Transitional dynamics

First, I present the modified agents' problems and define a non stationary equilibrium. In section A.2, I describe how I compute the transitional dynamics.

A.1. Non stationary economy

A.1.1. Households

When a household searches,

$$W_{s,t}(\varepsilon_{it}, a_t, z_{mt}; \mu_t) = \max_{a_{t+1}} u(c_t) + \beta \int_x \pi(z_{nt}|\varepsilon_{it}) \left(\sum_{z_n} \Gamma_z \sum_{\varepsilon_j} \Gamma_\varepsilon W_{t+1}(\varepsilon_{jt+1}, a_{t+1}, z_{nt+1}; \mu_{t+1}) \right) dx$$

$$c_t + a_{t+1} + s_t = (1 + r_t(\mu_t))a_t + w_t(z_{mt}; \mu_t)\varepsilon_{it} - T(w_t(z_{mt}; \mu_t)\varepsilon_{it}), \quad a_{t+1} > \underline{a}$$

The main differences are, the distribution of ε , z as well as search cost, job transition probability vary over time hence t shows up.

When a household stays,

$$\begin{aligned} W_{e,t}(\varepsilon_{it}, a_t, z_{mt}; \mu_t) = & \max_{a_{t+1}} u(c_t) + \beta((1 - \delta_t) \sum_{z_m} \Gamma_z \sum_{\varepsilon_j} \Gamma_\varepsilon W_{t+1}(\varepsilon_{jt+1}, a_{t+1}, z_{nt+1}; \mu_{t+1}) \\ & + \delta_t \int_x \pi(z_{nt}|\varepsilon_{it}) \left(\sum_{z_n} \Gamma_z \sum_{\varepsilon_j} \Gamma_\varepsilon W_{t+1}(\varepsilon_{jt+1}, a_{t+1}, z_{nt+1}; \mu_{t+1}) \right) dx) \end{aligned}$$

$$\begin{cases} c_t + a_{t+1} = (1 + r_t(\mu_t))a_t + \tau_t, & a_{t+1} > \underline{a}, & \text{if } \varepsilon = \varepsilon_{N_\varepsilon} \\ c_t + a_{t+1} = (1 + r_t(\mu_t))a_t + w_t(z_{mt}; \mu_t)\varepsilon_{it} - T(w_t(z_{mt}; \mu_t)\varepsilon_{it}), & a_{t+1} > \underline{a}, & \text{otherwise} \end{cases}$$

Workers compare two values and decide whether to search or not, and the retirees' value is $W_{e,t}(\varepsilon_t, a_t, z_{mt}; \mu_t)$.

$$\begin{cases} W_t(\varepsilon_{it}, a_t, z_{mt}; \mu_t) = W_{e,t}(\varepsilon_{it}, a_t, z_{mt}; \mu_t), & \text{if } \varepsilon = \varepsilon_{N_\varepsilon} \\ W_t(\varepsilon_{it}, a_t, z_{mt}; \mu_t) = \max[W_{e,t}(\varepsilon_{it}, a_t, z_{mt}; \mu_t), W_{s,t}(\varepsilon_{it}, a_t, z_{mt}; \mu_t)], & \text{otherwise} \end{cases}$$

Let $\chi_t(\varepsilon_{it}, a_t, z_{mt})$ be a household's decision rule for searching and $A_t(\varepsilon_{it}, a_t, z_{mt})$ be the decision rule for saving.

A.1.2. *Firms*

A firm's problem in an island is

$$V(z_t; \mu_t) = \max_{\bar{\varepsilon}_t, k_t} f(z_t, \bar{\varepsilon}_t, k_t) - w_t(z_t, \mu_t)\bar{\varepsilon}_t - r_t(\mu_t)k_t \quad (3)$$

$$f(z_t, \bar{\varepsilon}_t, k_t) = z_t k_t^{\alpha-1} \bar{\varepsilon}_t^{1-\alpha}$$

$$\bar{\varepsilon}_t = \int_j \varepsilon_t dj$$

Let $L_t(z_t)$ be the a firm's decision rule for labor and let $K_t(z_t)$ be its decision rule for capital rental.

A.1.3. *Government*

The government levies labor income tax to the workers to fund pension to the retirees and government spending, G_t .

A.1.4. *Equilibrium*

An equilibrium is a set of functions

$$(r_t, w_t(z_t), V_t, W_t, L_t, K_t, \chi_t, A_t, G_t)_{t=1}^T$$

that solves workers' and firms' problem and clear markets for labor, asset and goods.

1. W_t solves (A.1.1), (S_t, A_t) are the associated decision rules
2. V_t solves (3), (L_t, K_t) are the associated decision rules
3. $L_t(z_t) = \int_s \varepsilon_t \mu_t(\varepsilon_t, a_t, z_t) ds$
4. $\sum_{x=1}^{n_z} k_{xt} = \int_s A_t(\varepsilon_t, a_t, z_t) \mu_t(\varepsilon_t, a_t, z_t) ds$
5. $\int_s T(w_t(x_{zt}; \mu_t) \varepsilon_t) \mu_t(\varepsilon_t, a_t, z_t) ds \geq \int_s \tau_t \mu_t(\varepsilon_t, a_t, z_t) ds + G_t$

$$\begin{aligned}
6. \mu_{t+1}(B, \varepsilon_{jt+1}, z_{kt+1}) &= (1-\delta_t)(1-\delta_x) \int_{\{(a_t, \varepsilon_{it}, z_{kt}) | A_t(a_t, \varepsilon_{it}, z_{kt}) \in B\}} \pi_{ij}(1-\chi(a_t, \varepsilon_{it}, z_{kt})) \mu_t ds_{0t} + \\
&\quad \delta_t(1-\delta_x) \int_{\{(a_t, \varepsilon_{it}, z_{lt}) | A_t(a_t, \varepsilon_{it}, z_{lt}) \in B\}} \pi(k|\varepsilon_i) \pi_{ij}(1-\chi_t(a_t, \varepsilon_{it}, z_{lt})) \mu_t ds_{1t} + \\
&\quad (1-\delta_t)\delta_x \int_{\{(a_t, \varepsilon_{it}, z_{lt}) | A_t(a_t, \varepsilon_{it}, z_{lt}) \in B\}} \pi_{lk} \pi_{ij}(1-\chi_t(a_t, \varepsilon_{it}, z_{lt})) \mu_t ds_{1t} + \\
&\quad \delta_t \delta_x \int_{\{(a_t, \varepsilon_{it}, z_{lt}) | A_t(a_t, \varepsilon_{it}, z_{lt}) \in B\}} \pi(k|\varepsilon_i) \pi_{lk} \pi_{ij}(1-\chi_t(a_t, \varepsilon_{it}, z_{lt})) \mu_t ds_{1t} + \\
&\quad \int_{\{(a_t, \varepsilon_{it}, z_{lt}) | A_t(a_t, \varepsilon_{it}, z_{lt}) \in B\}} \pi(k|\varepsilon_i) \pi_{ij} \chi_t(a_t, \varepsilon_{it}, z_{lt}) \mu_t ds_{1t} \\
s_{0t} &= (a \times \varepsilon_{it} \times x_{kt}), s_{1t} = (a \times \varepsilon_{it} \times x_{lt}) \\
\forall t &= 1, \dots, T, \forall (B, \varepsilon_{jt}, x_{kt}) \in \mathbf{S}_t \\
\text{where } a_t &\in \mathbf{B} \subset \mathbf{R}, \varepsilon_t \in \mathbf{E}_t \equiv \{\varepsilon_{1t}, \dots, \varepsilon_{n_{\varepsilon}t}\}, z_t \in \mathbf{Z}_t \equiv \{z_{1t}, \dots, z_{n_zt}\}, \mathbf{S}_t = \mathbf{B} \times \mathbf{E}_t \times \mathbf{Z}_t
\end{aligned}$$

A.2. Solving transitional dynamics

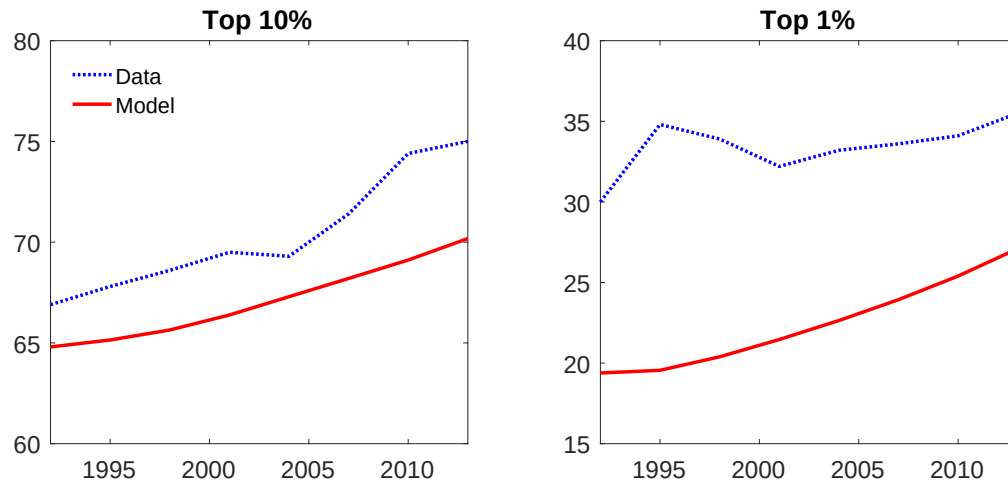
The transitional dynamics is computed as a non stationary equilibrium as defined in section A.1. Solving an equilibrium given parameters' paths and given a path of r_t ²⁶ follows:

1. Solve agents' problem from $t = T$ to $t = 1$. At $t = T$, use the W of the stationary equilibrium with period 2 parameters to compute expected value. Save $(K_t, \chi_t, A_t)_{t=1}^T$.
2. Update μ_t with paths of decision rules that are saved in the previous step. At $t = 1$, use the distribution from the stationary equilibrium with period 1 parameters.
3. Adjust $(r_t)_{t=1}^T$. For each t ,
 - If $|K_{demand,t} - K_{supply}| < tol$, $r_t = r_t$
 - If $K_{demand} > K_{supply}$, then $r_t = r_t - \epsilon$. If $K_{demand,t} < K_{supply,t}$, then $r_t = r_t + \epsilon$
4. Iterate until $(r_t)_{t=1}^T$ stop updating.

²⁶As explained in the section 2, wage rate of a firm is function of its productivity. Thanks to this result, $(w_t)_{t=1}^T$ that clears labor market can be calculated without solving agents' problem.

Appendix B. Additional figures and tables

Figure 10. Share of wealth, top group



Data: Kuhn and Ríos-Rull (2016)

Figure 11. Variances

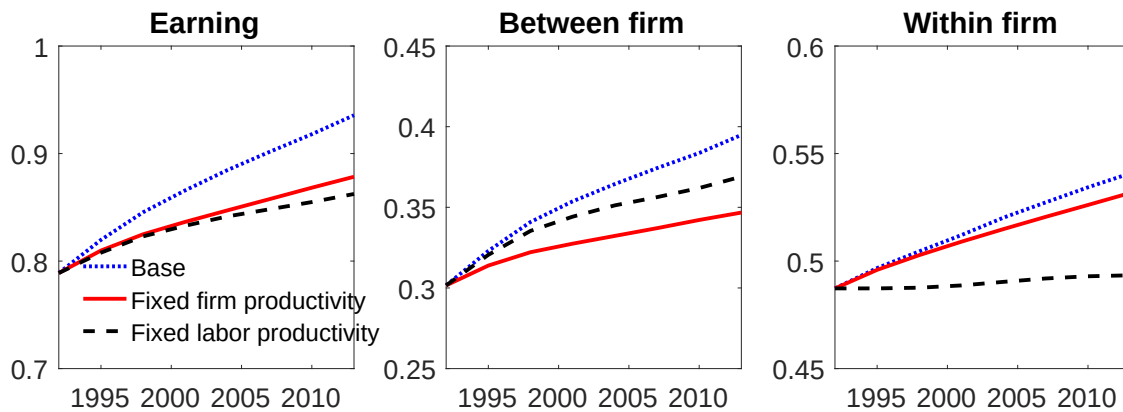


Figure 12. Share of wealth, top group

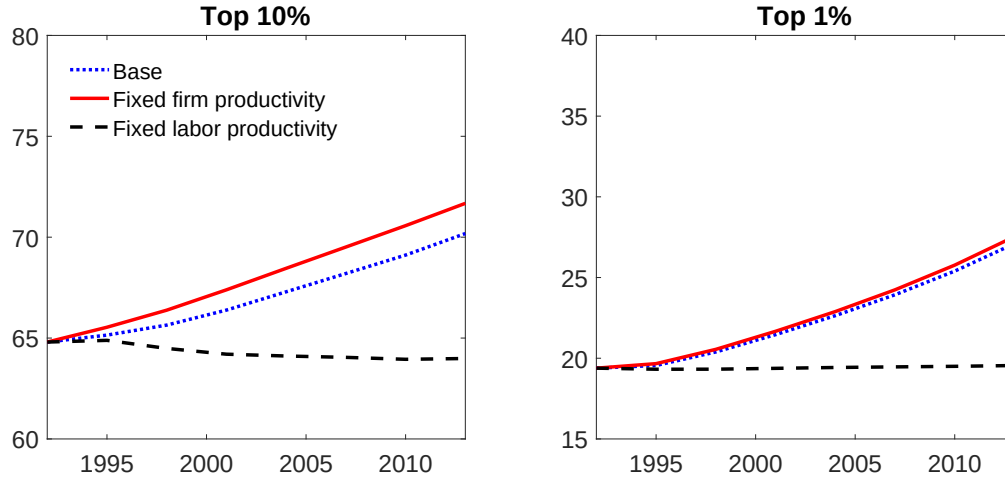
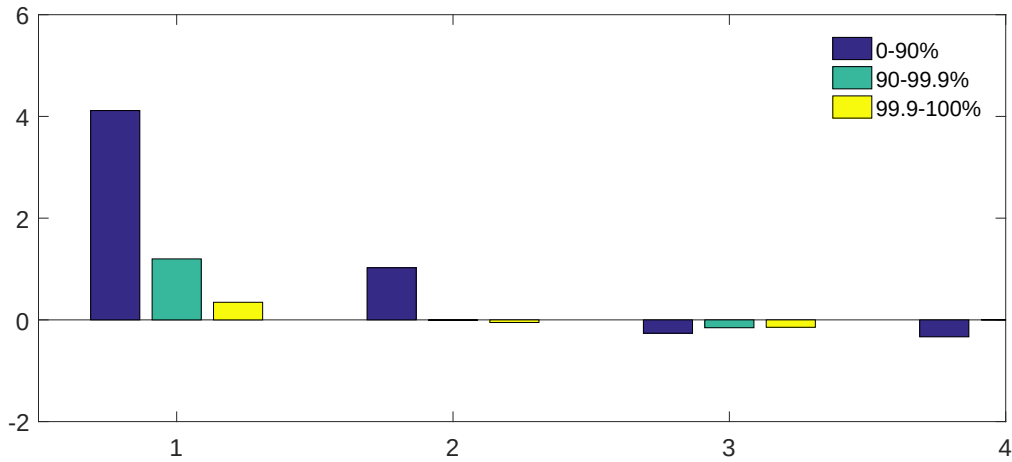
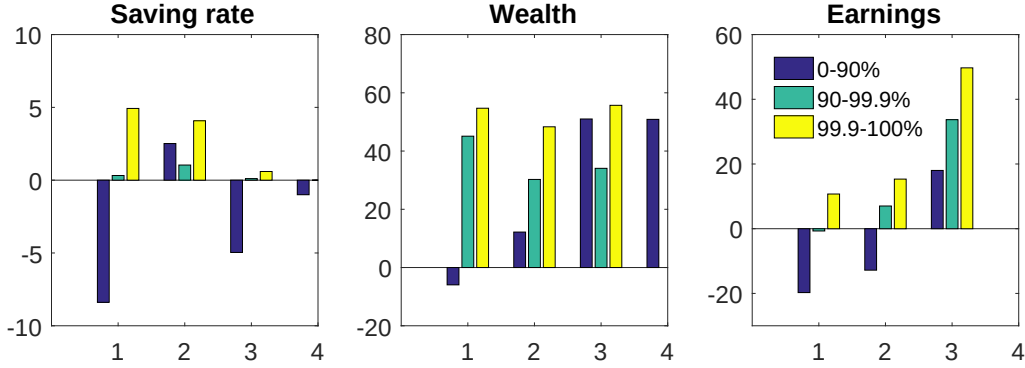


Figure 13. Difference in the saving decisions (%)



Difference between the saving rate, earnings and wealth of the period 1 and of period 2 given a same interest rate ($r = 2.95\%$). Horizontal axis indicates worker group, 1st: $\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$, 2nd: $\{\varepsilon_4\}$, 3rd: $\{\varepsilon_5, \varepsilon_6\}$, 4th: $\{\varepsilon_7\}$, and 0 – 90% (and rest of the legends) means firm group by productivity.

Figure 14. Difference in the saving, wealth and earnings (%)



Difference between the saving decision of the period 1 stationary equilibrium and the saving decision of transitional dynamics at $t = 1$ given a same interest rate (equilibrium interest rate of period 1 economy). Horizontal axis indicates worker group, 1st: $\{\varepsilon_1, \varepsilon_2, \varepsilon_3\}$, 2nd: $\{\varepsilon_4\}$, 3rd: $\{\varepsilon_5, \varepsilon_6\}$, 4th: $\{\varepsilon_7\}$, and 0 – 90% (and rest of the legends) means firm group by productivity.

Table 7: Variance of earnings

	Data			Model		
	1992	2013	Difference	1992	2013	Difference
Earnings	0.855	0.915	0.060	0.789	0.936	0.147
Between firm	0.315	0.375	0.060	0.301	0.394	0.093
$var(\bar{v})$	0.106	0.143	0.037	0.036	0.034	- 0.002
$var(\psi)$	0.115	0.108	-0.007	0.153	0.213	0.060
$2cov(\bar{v}, \psi)$	0.035	0.065	0.030	0.112	0.148	0.035
Within firm	0.540	0.540	0.000	0.487	0.540	0.053

Data: Song et al. (2015)

Table 8: Variance of earnings

	Fixed firm productivity			Fixed labor productivity		
	1992	2013	Difference	1992	2013	Difference
Earnings	0.789	0.878	0.089	0.789	0.862	0.027
Between firm	0.301	0.347	0.046	0.301	0.369	0.068
$var(\bar{v})$	0.036	0.043	0.007	0.036	0.029	-0.007
$var(\psi)$	0.153	0.169	0.016	0.153	0.204	0.051
$2cov(\bar{v}, \psi)$	0.112	0.135	0.023	0.112	0.135	0.023
Within firm	0.487	0.531	0.044	0.487	0.493	0.006

Table 9: Growth rate by firm productivity group

	0-90%	90-99.9%	99.9-100%
Output	- 13.5	3.2	35.0
Average productivity	- 5.1	2.4	13.5
Capital	- 13.6	3.1	34.8
Effective labor	- 3.7	- 7.0	10.9
The number of workers	- 3.8	-7.7	5.8

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