Designing Stress Scenarios*

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Abstract

We study the optimal design of scenarios by a risk-averse principal (e.g, a risk officer, a regulator) who seeks to learn about the exposures of agents (e.g., traders, banks) to a set of risk factors. We decompose the problem into a learning part, and a design part. Conditional on the stress scenarios, we show how to apply a Kalman filter to solve the learning problem. The design of optimal scenarios is then a function of what the regulator wants to learn and of how she intends to intervene if she uncovers excessive exposures. We show how the optimal design depends on ex-ante leverage, the correlation of exposures within and across agents, and the non-linearities in potential losses.

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1 Introduction

Stress tests are ubiquitous in risk management and financial supervision. They are used extensively in banking but also in insurance and market infrastructure. They are used by risk officers in private businesses to set and monitor risk limits, as well as by financial regulators to assess the health of financial institutions.

Despite the growing importance of stress testing in many areas, there is little theoretical guidance on exactly how one should design the stress scenarios. For instance, there is a growing literature on the disclosure of supervisory information, reviewed by Goldstein and Sapra (2014). The literature has focused on the trade-offs involved in disclosing information, from the reputation of the regulator (Shapiro and Skeie, 2015) to the importance of having a fiscal backstop (e Castro et al., 2014).

That literature, however, does not actually model *stress testing*. Instead, it focuses on *asset quality reviews*. An asset quality review aims to measure the health of a balance sheet at a point in time. The existing literature is concerned with asset quality reviews and their disclosure. It is silent about the design of forward looking scenarios, which are the hallmarks of stress testing.

The goal of our paper is to fill this void. We model stress testing as a learning mechanism. We consider one principal (regulator) and a (potentially large) number of agents (banks). Banks are exposed to a set of systematic risk factors, but their exposures are unknown to the regulator. The regulator is risk averse and worried that banks might incur significant losses in some states of the world. The regulator then designs a set of scenarios and asks the banks to report what their losses would be under these scenarios. From their responses, the regulator learns about the underlying exposures of the banks. Based on this information, the regulator can decide to intervene, i.e., she can ask a set of banks to reduce their exposures to some factors.

The main contribution of our paper is to frame the design problem in tractable manner. Our main insight is that we can rewrite the learning problem in a way that allows us to apply a Kalman filter. We can then obtain a closed-form solutions for learning as a function of scenarios.

Our framework allows us to study how the design of scenarios depends on

- the precision of regulatory information
- the health of the banks and the risk aversion of the regulator

- the correlation of exposures within and across banks

Literature

Much of the work on stress tests focuses on banking. Stress tests have become a major tool of banking supervision in the United States and in Europe. Regulatory stress tests are used to set bank capital levels and validate dividend policies. Stress testing exercises mobilize large human and financial resources among regulators and bankers. Regulators need to design the scenarios, run their own models, and monitor closely the answers provided by the banks. The banks, on the other hand, allocate considerable time and effort to comply with the guidelines and estimate their losses at a fairly granular level.

Several recent papers study specifically the trade-offs involved in revealing information about banks' balance sheets. Goldstein and Leitner (2013) focus on the Hirshleifer (1971) effect: revealing too much information destroys risk-sharing opportunities between risk neutral investors and (effectively) risk averse bankers. These risk-sharing arrangements also play an important role in Allen and Gale (2000). Shapiro and Skeie (2015) study the reputation concerns of a regulator when there is a trade-off between moral hazard and runs. e Castro et al. (2014) study a model of optimal disclosure where the government trades off Lemon market costs with bank run costs, and show that a fiscal backstop allows government to run more informative stress tests. Schuermann (2012) analyzes the design and governance (scenario design, models and projection, and disclosure) for more effective stress test exercises. Schuermann (2016) particularly determines how wartime stress testing can be adapted to peacetime concerns in order to insure adequate lending capacity and other key financial services.

Most empirical papers on stress tests focus on the information content at the time of disclosure, using an event study methodology to determine whether stress tests provide valuable information to investors. Petrella and Resti (2013) assess the impact of the 2011 European stress test exercise. For the 51 banks with publicly traded equity, they find that the publication of the detailed results provided valuable information to market participants. Similarly, Donald et al. (2014) evaluate the 2009 U.S. stress test conducted on 19 bank holding companies and find significant abnormal stock returns for banks with capital shortfalls. Candelon and Sy (2015), Bird et al. (2015), and Fernandes et al. (2015) also find significant average cumulative abnormal returns for stress tested

BHCs around many of the stress test disclosure dates. Flannery et al. (2016) find that U.S. stress tests contain significant new information about assessed BHCs. Using a sample of large banks with publicly traded equity, the authors find significant average abnormal returns around many of the stress test disclosures dates. They also find that stress tests provide relatively more information about riskier and more highly leveraged bank holding companies. Glasserman and Tangirala (2016) evaluate one aspect of the relevance of scenario choices. They show that the results of U.S. stress tests are somewhat predictable, in the sense that rankings according to projected stress losses in 2013 and 2014 are correlated. Similarly, the rankings across scenarios in a given year are also correlated. They argue that regulators should experiment with more diverse scenarios, so that it is not always the same banks that project the higher losses. Acharya et al. (2014) compare the capital shortfalls from the stress tests with the capital shortfalls predicted using the systemic risk model of Acharya et al. (2016) based on equity market data. Our paper differs from the existing literature in that we attempt to directly evaluate the accuracy of the tests using granular data on loan losses. Camara et al. (2016) study the quality of the 2014 EBA stress tests using the actual micro data from the tests.

2 Technology and Preferences

We consider a model of banking in which bank losses depend on macroeconomic variables. A regulator can take actions to intervene in the banking sector and limit the banks' exposure to these macroeconomic risks. The regulator does not know the risk exposures of the banks and elicits information from the banks in the form of stress tests.

2.1 Banks and Risks

There are N banks exposed to systematic and idiosyncratic risks. The systematic risk of bank $i \in [1:N]$ is measured by a row-vector of exposures $x_i = [x_{i,1}, .., x_{i,J}]$ to a row-vector s of size J of factors $s = [s_1, ..s_j, ..s_J]$ which captures the macroeconomic state of the economy. The (demeaned) losses of bank i are given by

$$y_i = x_i \cdot s + \eta_i = \sum_{j=1}^{J} x_{i,j} s_j + \eta_i, \tag{1}$$

where η_i is a random idiosyncratic shock. We normalize the baseline scenario to $s = 0$, so that our scenarios should be interpreted as deviations from the baseline.

In the general case, we do not restrict the factors to be independent, or even to have mean zero. For instance, we can allow for a non-linear mapping by setting $s_{j+1} = s_j^2$. Our factors can be thought of as fundamental shocks in a macroeconomic model or as traditional macroeconomic variables such as GDP, unemployment, or house prices are functions of the factors. In a linear interpretation, factors can be negative (good state) or positive (bad case), and $x_{i,j}$ are positive numbers for most banks, although a particular bank could, in principle, have a short exposure. We consider several special cases below.

The net worth of bank i is given by

$$w_i = \bar{w}_i - y_i, \tag{2}$$

where $\bar{w}_i$ is the mean level of profits net of debt. For instance, we could have $\bar{w}_i = r_i - d_i$ where r_i would be average revenues and d_i outstanding debt. Given Eq. (1) and Eq. (2), the aggregate net worth of the banking system is

$$W = \sum_{i=1}^{N} w_i = \bar{w} - \overline{x} \cdot s - \overline{\eta},$$

where $\overline{x}$ is the sum of the vectors of risk exposures to the macro factors of the N banks in the economy, x_i, and $\overline{\eta}$ is the sum of all random shocks η_i.

2.2 Regulatory Interventions and Preferences

A regulator has preferences over the total net worth of the banking system[1]

$$U(W).$$

The regulator can intervene in the banking sector to force the banks to change their risk exposures. An intervention on bank i is vector $a_i = [a_{i,1}, .., a_{i,J}]$ that describes the change in the exposure of bank i to factor j. Upon intervention on (i, j), the exposure becomes $(1 - a_{i,j}) x_{i,j}$. The cost of

[1]As in Acharya et al.'s systemic risk paper, we impose the restriction that only W matters. More generally, we would have $U([w_i]_{1..N})$. This would capture the case where the idiosyncratic failure of bank i matters regardless of the health of the banking sector as a whole.

the intervention is convex and for simplicity we make it quadratic and equal to $\frac{1}{2}a_{ij}^2$. This cost captures the losses incurred by the banks when they are forced to change their positions, as well as the direct costs of the intervention for the regulator and the banks. To shorten the notation, we define the cell-by-cell multiplication operator $\circ$ as

$$(\mathbf{1} - a_i) \circ x_i \equiv [(1 - a_{i,1}) x_{i,1}, .., (1 - a_{i,J}) x_{i,J}] .$$

Let $\mathcal{I}$ denote the information set of the regulator. The regulator's problem is to maximize

$$\mathbb{E}\left[U\left(\bar{w} - \overline{\eta} - \left(\sum_{i=1}^{N} (\mathbf{1} - a_i) \circ x_i \right) \cdot s \right) - \frac{1}{2} \sum_{i=1}^{N} \|a_i\|^2 \middle| \mathcal{I} \right].$$

Example. Suppose that there is only one bank.[2] Then, the regulator chooses his intervention policy to maximize

$$\mathbb{E}\left[U\left(\bar{w} - \overline{\eta} - \sum_{j=1}^{J} (1 - a_j) x_j s_j \right) - \frac{1}{2} \sum_{j=1}^{J} a_j^2 \middle| \mathcal{I} \right].$$

The first order condition (FOC) for this problem is

$$a_j = \mathbb{E}\left[x_j s_j U'(W) \middle| \mathcal{I} \right]. \tag{3}$$

The FOC equates the marginal cost of an action to its expected marginal benefit. The expected marginal benefit of intervening along the dimension of risk factor j depends on the covariance between the marginal social utility $U'(W)$ and the marginal increase in the bank's net worth that can be attained by intervening, $x_j s_j$. The action is positive if we expect that U' is large when s is positive. In the model below, we study the case where the planner does not know the vector of risk exposures x and learns about x by running a stress test.

3 Stress Tests

3.1 Definitions

The regulator knows the banks' debt positions. However, he does not know the risk exposures of the banks. The regulator has a prior belief over the distribution of exposures, both within banks,

[2] Equivalently, all banks are identical and have the same risk exposures.

and across banks. We stack the exposures in one NJ vector as

$$\mathbf{x} \equiv \begin{bmatrix} [x_1'] \\ .. \\ [x_i'] \\ .. \\ [x_N'] \end{bmatrix}$$

and we summarize the prior as

$$\mathbf{x} \sim N\left(\overline{\mathbf{x}}, \Sigma_x\right),$$

where

$$\overline{\mathbf{x}} = \begin{pmatrix} \overline{x}_1 \\ \vdots \\ \overline{x}_N \end{pmatrix} \quad \text{and } \Sigma_x = \begin{bmatrix} \Sigma_x^1 & \Sigma_x^{12} & \cdots & \Sigma_x^{1N} \\ \Sigma_x^{12} & \Sigma_x^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & \Sigma_x^{(N-1)N} \\ \Sigma_x^{1N} & \cdots & \Sigma_x^{(N-1)N} & \Sigma_x^N \end{bmatrix}$$

with $\Sigma_x^i = \mathbb{V}(x_i)$ for all i and $\Sigma_x^{ij} = \mathbb{C}\mathrm{ov}(x_i, x_j)$ for all $i \neq j$. If $\Sigma_x^{ij} = 0$, the regulator's prior is that the risk exposures of bank i and j are independent. To learn about the banks' risk exposures, the regulator can ask the banks to estimate their losses under scenario $\hat{s}$

Definition 1. A scenario $\hat{s}$ is a realization of the vector of states s.

Definition 2. A stress test is a collection of scenarios $\left\{\hat{s}^{(m)}\right\}_{m=1}^{M}$ and reported losses $\{\hat{y}_i\}_{i=1}^{N}$for each bank under the scenarios $\left\{\hat{s}^{(m)}\right\}_{m=1}^{M}$..

For each bank i and each scenario $\hat{s}^{(m)}$, the bank reports its net losses $\hat{y}_i^{(m)}$, or equivalently, its net worth $\hat{w}_i^{(m)}$. For simplicity, we will express all of our analysis in terms of the net losses reported by the banks $\hat{y}_i = \left[\hat{y}_i^{(1)}, ..., \hat{y}_i^{(M)}\right]$.

Banks use imperfect models to try and predict what their losses would be under the stress test scenario. More precisely, we assume that bank i estimates its losses under scenario $\hat{s}_m$ as

$$\hat{y}_i^{(m)} = \hat{s}^{(m)} \cdot x_i' + e_i^{(m)} \tag{4}$$

where $e_i^{(m)} \sim N\left(0, \Sigma_{e,i}^{(m)}\right)$. This formulation captures the idea that the banks themselves do not know perfectly their true exposures and/or that the scenarios are not perfectly well defined. $\Sigma_{e,i}^{(m)}$

determines the precision of bank i's report under scenario m. We specify the following functional form for the precision with which banks predict their revenues for a particular scenario $\hat{s}^{(m)}$:

$$\Sigma_{e,i}^{(m)} \equiv \left(1 + \lambda_i (M) \left\| \alpha_0^i \cdot \hat{s}^{(m)} \right\|^{\alpha_1^i}\right) \sigma_\varepsilon^2, \tag{5}$$

where α_0^i is an $J \times 1$ vector of positive scalars, $\alpha_1^i > 2$, and $\lambda_i (M)$ is a positive and increasing function of M. This specification captures the idea that extreme scenarios are harder to estimate, and that asking for more scenarios reduces the precision of the responses. Eq. (5) implies that bank i's report is more precise (less noisy) for less extreme scenarios and for less complex stress tests (with fewer scenarios). Our measure of how extreme a scenario is $\left\| \alpha_0^i \cdot \hat{s}^{(m)} \right\|$. The vector of weights α_0^i captures the idea that some dimension might be easier to learn than others. The curvature α_1^i determines the sensitivity of the precision of bank i's report to extreme scenarios. We assume that it is greater than 2 to ensure an interior solution. Finally, $\lambda_i (M)$ represents the increasing cost of computing expected losses for a bank with limited information processing capacity. Note that (5) can also be interpreted as the result of an entropy constraint.

The structure of the bank's internal models are the outcome of learning about risk exposures from historical data. Therefore, the mistakes a bank makes in computing its expected revenues for each scenario may be correlated. The variance-covariance matrix of the errors made by bank i in computing its stress test results is given by

$$\boldsymbol{\Sigma}_e^i = \begin{bmatrix} \Sigma_{e,i}^{(1)} & \Sigma_{e,i}^{(1,2)} & \cdots & \Sigma_{e,i}^{(1,M)} \\ \Sigma_{e,i}^{(1,2)} & \Sigma_{e,i}^{(2)} & \ddots & \vdots \\ \vdots & \cdots & \ddots & \Sigma_{e,i}^{(M-1,M)} \\ \Sigma_{e,i}^{(1,M)} & \cdots & \Sigma_{e,i}^{(M-1,M)} & \Sigma_{e,i}^{(M)} \end{bmatrix},$$

where

$$\Sigma_{e,i}^{(m,\ell)} = \mathbb{C}ov \left[e_i^{(m)}, e_i^{(\ell)} \right] \quad \forall m, \ell = 1, ..., M.$$

Differences in $\boldsymbol{\Sigma}_e^i$ across banks reflect differences in information (priors), amount or quality of data available, or in the bank's information processing capacity. We assume that x_i and $e_i = \left[e_i^{(1)}, ..., e_i^{(M)} \right]$ are independent, but we allow banks to make correlated mistakes. The correlation among the mistakes made by bank i and bank j is given by $\boldsymbol{\Sigma}_e^{ij}$. The results of the stress test can be summarized in the NM vector

$$\hat{\mathbf{y}} \equiv \begin{bmatrix} [\hat{y}_1'] \\ .. \\ [\hat{y}_i'] \\ .. \\ [\hat{y}_N'] \end{bmatrix},$$

and similarly for $\mathbf{e}$.

3.2 Learning

The information available to the regulator after seeing the results of the stress test can be summarized in the following state space representation

$$\hat{\mathbf{y}} = \left(\mathbf{I}_N \otimes \hat{S}\right) \mathbf{x} + \mathbf{e}, \tag{6}$$

where $\mathbf{I}_N$ is the identity (diagonal) matrix of size N,

$$\hat{S} \equiv \begin{bmatrix} \hat{s}^{(1)} \\ .. \\ \hat{s}^{(m)} \\ .. \\ \hat{s}^{(M)} \end{bmatrix},$$

is the $M \times J$ matrix of size scenarios. The covariance matrix of the error terms $\mathbf{e}$ is

$$\boldsymbol{\Sigma}_{\mathbf{e}} \left(\hat{S}\right) \equiv \begin{bmatrix} \boldsymbol{\Sigma}_e^1 & \boldsymbol{\Sigma}_e^{12} & \cdots & \boldsymbol{\Sigma}_e^{1N} \\ \boldsymbol{\Sigma}_e^{12} & \boldsymbol{\Sigma}_e^2 & \ddots & \vdots \\ \vdots & \cdots & \ddots & \boldsymbol{\Sigma}_e^{(N-1)N} \\ \boldsymbol{\Sigma}_e^{1N} & \cdots & \boldsymbol{\Sigma}_e^{(N-1)N} & \boldsymbol{\Sigma}_e^N \end{bmatrix}$$

which is of size $NM \times NM$ and depends on the stress test scenarios. The key insight is that we can apply the Kalman filter to the state space representation in equation (6) and, as a result, we can characterize the posteriors beliefs of the regulator after observing the results of the stress test.

Proposition 1. *After observing the results* $\hat{\mathbf{y}}$ *of the test, the regulator has a posterior regarding the banks' risk exposures*

$$\mathbf{x}|\,\hat{\mathbf{y}} \sim N\left(\hat{\mathbf{x}}, \hat{\Sigma}_{\mathbf{x}}\right),$$

where

$$\hat{\mathbf{x}} = \left(\mathbf{I}_{NJ} - K\left(\mathbf{I}_N \otimes \hat{S}\right)\right)\bar{\mathbf{x}} + K\hat{\mathbf{y}} \tag{7}$$

$$K = \Sigma_{\mathbf{x}}\left(\mathbf{I}_N \otimes \hat{S}\right)'\left(\left(\mathbf{I}_N \otimes \hat{S}\right)\Sigma_{\mathbf{x}}\left(\mathbf{I}_N \otimes \hat{S}\right)' + \boldsymbol{\Sigma}_{\mathbf{e}}\left(\hat{S}\right)\right)^{-1} \tag{8}$$

$$\hat{\Sigma}_{\mathbf{x}} = \left(\mathbf{I}_{NJ} - K\left(\mathbf{I}_N \otimes \hat{S}\right)\right)\Sigma_{\mathbf{x}} \tag{9}$$

The $NJ \times MN$ *matrix* K *is the Kalman gain,* $\hat{\mathbf{x}}$ *is the posterior mean of exposures, and* $\hat{\Sigma}_{\mathbf{x}}$ *the posterior covariance matrix.*

These are the standard Kalman filter formulas applied to the context of our model. The matrix $\hat{\Sigma}_{\mathbf{x}}$ will play a critical role in our analysis. It is worth emphasizing that $\hat{\Sigma}_{\mathbf{x}}$ measures the residual uncertainty that persists after the regulator observes the results of the stress test. Intuitively, the planner will try to reduce this residual uncertainty as much as possible, along the dimensions that depend on its objective function and its priors about the risk exposures of the banks. The weight K_j that the regulator puts on the outcome of the stress test to update the risk exposure to factor j is the Kalman gain along dimension j and it is a measure of the amount of information about the exposure to risk factor j contained in the stress test result.

3.3 Scenario choice as signal design

In the standard state-space representation in Equation (6), the reported losses by the banks are signals about linear combinations of the banks' exposure to the risk factors. The scenarios that define the stress test determine the structure of the signals observed by the regulator by establishing these weights. In addition, as Eq. (5) shows, the scenarios determine the precision of the banks' reported revenues. When choosing the set of scenarios that define the stress test, the regulator takes into account how different scenarios affect the information contained in the stress test results.

On the one hand, increasing the relative weight of risk factor j in a scenario makes the stress test results more informative about the banks' exposure to risk factor j. On the other hand, increasing this weight makes the scenario more extreme and reduces the precision of the banks' report. Therefore, both the relative weights of the risk factors in the scenario and the absolute size of the scenario are relevant in determining the informativeness of the stress test results.

When designing the stress scenarios, the regulator anticipates that she will learn from the results of the test. The extent to which learning will take place is captured by the *ex-ante distribution of the posterior mean*, given by

$$\hat{\mathbf{x}} \sim N\left(\overline{\mathbf{x}}, \Sigma_{\hat{\mathbf{x}}}\right), \tag{10}$$

where the ex-ante variance of posterior beliefs is given by

$$\Sigma_{\hat{\mathbf{x}}} \equiv \Sigma_{\mathbf{x}} - \hat{\Sigma}_{\mathbf{x}} = K\left(\mathbf{I}_N \otimes \hat{S}\right)\Sigma_{\mathbf{x}}$$

The matrix $\Sigma_{\hat{\mathbf{x}}}$ represents the expected amount of learning from stress test $\hat{S}$. Note, of course, that the Kalman K gain itself is a function of the scenario, given by equation (8). The goal of the regulator is to maximize her learning $\Sigma_{\hat{\mathbf{x}}}$ by minimizing the residual uncertainty $\hat{\Sigma}_{\mathbf{x}}$.

For instance, the regulator might want to increase the stress along factor j by making s_j larger. However, by doing so, the regulator would increase the signal about x_j and the noise in the stress test.

Example. Let us consider a simple case with one bank, two factors, and one scenario: $N = 1$, $J = 2$, $M = 1$. When there is only one scenario, $M = 1$, and two macroeconomic factors, $J = 2$, the unconditional distribution of the expected risk exposures after observing the stress test results, $\hat{x}$, for the only bank in the economy is

$$\hat{x} \sim N\left(\overline{x}, \Sigma_{\hat{x}}\right),$$

where we define $\Sigma_{\hat{x}} \equiv \Sigma_x - \hat{\Sigma}_x$. The overall effect of the scenario choice on the amount that can be learned from the stress test is

$$\frac{d\Sigma_{\hat{x}}}{d\hat{s}_j} = \left[\frac{dK_l}{d\hat{s}_j}\hat{s}_h + K_l \mathbb{I}\left\{j = h\right\}\right]_{lh} \Sigma_x, \tag{11}$$

where $\mathbb{I}\{j = h\}$ is 1 if $j = h$ and 0 otherwise. Increasing the weight of risk factor j in scenario m affects the amount of information that can be obtained from the stress tests by affecting the

Kalman gain K. The change in the information gain in the dimension of the risk exposure to factor j is given by

$$\frac{dK_l}{d\hat{s}_j} = \frac{\partial K_l}{\partial \hat{s}_j} + \frac{\partial K_l}{\partial \mathbf{\Sigma_e}\left(\hat{S}\right)} \frac{\partial \mathbf{\Sigma_e}\left(\hat{S}\right)}{\partial \hat{s}_j}. \tag{12}$$

As can be seen from Equation (12), the weight of factor j in the scenario affects the Kalman gain in two ways. First, directly by shifting the amount of information about the exposure to factor j contained in the stress test result. Second, indirectly by changing the precision of the bank's report.

When the regulator has a prior such that the bank's exposures to the risk factors are uncorrelated, i.e., if $\Sigma_{x,12} = 0$, we have that the direct effect of changes in $\hat{s}_j$ on $\Sigma_{\hat{x},jj}$ given by

$$\operatorname{sgn}\left(\left(\frac{\partial K_j}{\partial \hat{s}_j}\hat{s}_j + K_j\right)\Sigma_{x,jj}\right) = \operatorname{sgn}\left(\hat{s}_j\right). \tag{13}$$

Equation (13) shows that when $\hat{s}_j$ is positive (negative), making the scenario more extreme by increasing (decreasing) $\hat{s}_j$ increases the amount of information the stress test results contain about the bank's risk exposure to risk factor j. Similarly, the direct effect of changes in $\hat{s}_j$ on $\Sigma_{\hat{x},hh}$ depends on the how the weight $K_h$that the regulator puts on the stress test result to learn about x_h changes with $\hat{s}_j$. When $\Sigma_{x,12} = 0$ we have

$$\operatorname{sgn}\left(\frac{\partial K_h}{\partial \hat{s}_j}\hat{s}_h\Sigma_{x,hh}\right) = -\operatorname{sgn}\left(\hat{s}_j\hat{s}_h^2\right). \tag{14}$$

As Equation (14) shows, making the scenario more extreme in the dimension of one risk factor reduces the amount of information that the stress test result contains about the bank's exposure to the other risk factor $h \neq j$. When $\Sigma_{x,12} = 0$, learning about the risk exposure of the bank to one risk factor contains no information about the bank's exposure to the other risk factor. Therefore, increasing the amount that can be learned in one dimension by making the scenario more extreme in this dimension decreases the informational gain in the other dimension.

The indirect effect of changing $\hat{s}_j$ on the amount that can be learned from the stress test depends on how the scenario affects the precision of the bank's report. A lower precision of the bank's report, i.e. a higher $\mathbf{\Sigma_e}\left(\hat{S}\right)$, always decreases the amount of information that the stress test contains about the bank's risk exposures. Formally,

$$\frac{\partial K_l}{\partial \boldsymbol{\Sigma}_{\mathbf{e}}\left(\hat{S}\right)} \leq 0. \tag{15}$$

Moreover, given our assumption in Equation (5),

$$\operatorname{sgn}\left(\frac{\partial \boldsymbol{\Sigma}_{\mathbf{e}}\left(\hat{S}\right)}{\partial \hat{s}_j}\right) = \operatorname{sgn}\left(\hat{s}_j\right). \tag{16}$$

The expression for $\boldsymbol{\Sigma}_{\mathbf{e}}\left(\hat{S}\right)$ in Equation (5) captures the idea that more extreme scenarios are harder for the bank to predict accurately. Hence, though increasing $|\hat{s}_j|$ makes the stress test results relatively more informative about the bank's exposure to factor j, it also decreases the total amount of information available by reducing the overall precision of the stress test result.

Using Equations (14), (15), and (16), the total effect of $\hat{s}_j$ on the amount amount of information about the exposure to risk factor i contained in the stress test result is

$$\operatorname{sgn}\left(\frac{d\Sigma_{\hat{x},hh}}{d\hat{s}_j}\right) = -\operatorname{sgn}\left(\hat{s}_j\hat{s}_h^2\right).$$

When the regulator has a prior belief that the risk exposures of the bank are uncorrelated, she faces a trade-off between learning about one factor or the other. However, when the regulator's prior is such that $\Sigma_{x,12} \neq 0$, this result may be overturned and putting more weight on $\hat{s}_j$ in the scenario can directly increase the weights K_1 and K_2 the regulator puts on the stress test result to update her beliefs on both risk exposures.

3.4 Stress test design

Running stress tests and processing information is costly for the banks and for the regulator. The cost for the banks is captured by the precision of the bank's reports being decreasing in the number of scenarios. We can also incorporates a cost for the regulators for creating additional scenarios: choosing M scenarios for the stress test has a cost $\mathscr{C}(M)$, where $\mathscr{C}'(\cdot) > 0$ and $\mathscr{C}''(\cdot) \geq 0$. When designing a stress test, the regulator has to choose how many and which scenarios to include. The regulator's problem can be written as

$$\max_{M\in\mathbb{N}\cup\{0\}} V(M) - \mathscr{C}(M),$$

where

$$V(M) = \max_{\{s_m\}_{m=1}^{M}} \mathbb{E}\left[U\left(W\left(a^{\star}\left(\mathcal{I}_M\right)\right)\right) - \frac{1}{2}\left\|a^{\star}\left(\mathcal{I}_M\right)\right\|^2\right] \tag{17}$$

and

$$V(0) = \mathbb{E}\left[U\left(W\left(a^{\star}\left(\mathcal{I}_0\right)\right)\right) - \frac{1}{2}\left\|a^{\star}\left(\mathcal{I}_0\right)\right\|^2\right],$$

where $a^{\star}(\mathcal{I})$ is the optimal intervention policy of the regulator given his information set $\mathcal{I}$ given by Eq. (3) and

$$W(a) = \sum_{i=1}^{N}\left(\omega_i - \eta_i - \sum_{j=1}^{J}\left(1 - a_{ij}\right)x_{ij} \cdot s_j\right). \tag{18}$$

The information set on which the regulator conditions his intervention policy depends on his scenario choice. The choice of scenario affects the actual choice of policy $a^{\star}$ and the ex-ante distribution of this policy. However, since the regulator is choosing $a^{\star}$ optimally, the envelope theorem implies we can ignore the direct effect the scenario choice has on $a^{\star}$ and focus on the effect on its ex-ante distribution. To fix ideas, we explore this dependence in an application in the next section.

4 Linear quadratic preferences

We consider the case where the regulator has linear quadratic preferences over the aggregate net worth of the banking sector. In this case, the regulator's utility is given by

$$U(W) = W - \frac{\gamma}{2}W^2.$$

We normalize our model in such a way that the factors are orthogonal, mean zero and unit variance: $\mathbb{E}[s] = 0$ and $\mathbb{V}[s] = I_J$, where $\mathbb{V}$ denotes the variance-covariance matrix and I_J is the identity matrix of size J. We start by looking at the optimal action choice. Then, analyze the optimal scenario choice. To do this we first focus on how the scenario choice affects the structure of the information available to the regulator and then proceed to illustrate how the optimal scenario choice depends on the primitives of the model. To simplify the analysis, we assume that the cost function $\mathscr{C}(\cdot)$ is such that it is too costly for the regulator to choose more than one scenario.

4.1 Intervention policy

When there are N banks and J factors the banking sector's wealth is given by $W = \bar{\omega} - \bar{\eta} - \sum_{i=1}^{N}\sum_{j=1}^{J}\left(1 - a_{i,j}^{\star}(\hat{y})\right) x_{i,j}s_j$. With linear quadratic preferences, the FOC that characterizes the optimal action choice in Equation (3) becomes

$$a_{i,j}^{\star}(\hat{y}) = \mathbb{E}\left[x_{i,j}s_j \left(1 + \gamma \sum_{n=1}^{N}\sum_{h=1}^{J}\left(1 - a_{n,h}^{\star}(\hat{y})\right) x_{n,h}s_h\right) \middle| \hat{y}\right] \quad \forall i = 1, ..., N \forall j = 1, ..., J. \tag{19}$$

Moreover, given our normalization of the risk factors, the intervention policy problem is separable in the dimension of the risk factor. More specifically, the regulator chooses $a_j^{\star} \equiv \left(a_{1,j}^{\star}(\hat{y}), a_{2,j}^{\star}(\hat{y}), ..., a_{N,j}^{\star}(\hat{y})\right)$ for each dimension j separately. In this case, the optimal intervention policy in Equation (19) can be written as

$$a_j^{\star} = \left(\mathbb{I}_N + \gamma\left(\hat{\Sigma}_{x,j} + \hat{x}_j\hat{x}_j'\right)\mathbb{E}\left[s_j^2\right]\right)^{-1}\gamma\left(\hat{\Sigma}_{x,j} + \hat{x}_j\hat{x}_j'\right)\mathbf{1}_{N\times 1}\mathbb{E}\left[s_j^2\right] \quad \forall j = 1, \ldots, J. \tag{20}$$

Equation (20) shows that the optimal intervention policy of the regulator only depends on $\hat{y}$ through $\hat{\Sigma}_{x,j} + \hat{x}_j\hat{x}_j'$. This is consistent with the regulator designing the stress test to extract information from the banks. The only way in which the stress test results affect the regulator's behaviour is by changing her perceived risk in the economy, which is measured by $\left\{\gamma\mathbb{E}\left[x_{i,j}x_{n,j}\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]\right\}_{i,n,j}$. Moreover, since the regulator's intervention problem is separable in the dimension of the risk factor, the regulator only cares about $\hat{\Sigma}_{x,jj} \equiv \left\{\hat{\Sigma}_{x,jj}^{in}\right\}_{in} \forall j \in J$ when she chooses her intervention policy.

Proposition 2. *The regulator intervenes more in bank i along dimension j when the perceived risks coming from bank i being exposed to risk factor j, $\left\{\gamma\mathbb{E}\left[x_{i,j}x_{n,j}\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]\right\}_n$, is higher.*

As proposition 2 shows, a risk averse regulator will intervene more to reduce risk along dimension j for bank i the higher her perceived risks associated with bank i being exposed to risk factor j. This risks have four components: risk aversion γ; the variance of risk factor j, $\mathbb{E}\left[s_j^2\right]$; the expected exposures of the banks to risk factor j, $\{\hat{x}_{n,j}\}_n$; and the precision with which the regulator estimates these exposures, $\hat{\Sigma}_{x,jj}$. Intuitively, a higher γ implies that the regulator dislikes risk more and, therefore, prefers the banking sector to be exposed to less risk overall. Therefore, a more risk averse regulator intervenes more in all dimensions, in particular she intervenes more in bank i along dimension j. Similarly, a higher uncertainty about the net worth of the banking sector coming from bank i being exposed to factor j, measured by $\sum_{n=1}^{N} \mathbb{E}\left[x_{i,j}x_{n,j}\middle|\, \hat{y}\right] \mathbb{E}\left[s_j^2\right]$, also induces the regulator to intervene more. This uncertainty can be driven by the volatility of the macroeconomic factor j—$\mathbb{E}\left[s_j^2\right]$—by imprecise estimates of the banks' risk exposures—$\hat{\Sigma}_{x,jj}$—or by high expected exposure to the risk factor j—high $\hat{x}_{n,j}$.

4.2 Scenario choice

Suppose that the regulator optimally chooses one scenario for the stress test. In this case, the regulator chooses a scenario $\hat{s} = [\hat{s}_1, \hat{s}_2, \ldots, \hat{s}_J]'$ to solve

$$\max_{\hat{s}\in\mathbb{R}^J} \mathbb{E}_{\hat{y},\eta}\left[W - \frac{\gamma}{2}W^2 - \frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{J}\left(a_{i,j}^{\star}\left(\hat{y}\right)\right)^2\right], \tag{21}$$

which is the same as

$$\min_{\hat{s}\in\bar{\mathbb{R}}^J} \mathbb{E}_{\hat{y}}\left[\gamma\left(\sum_{j=1}^{J}\left(\mathbf{1}_{1\times N} - a_j^{\star}\left(\hat{y}\right)'\right)\mathbb{E}\left[x_j x_j'\middle|\,\hat{y}\right]\left(\mathbf{1}_{1\times N} - a_j^{\star}\left(\hat{y}\right)'\right)'\mathbb{E}\left[s_j^2\right]\right)^2 + \sum_{j=1}^{J} a_j^{\star}\left(\hat{y}\right)' a_j^{\star}\left(\hat{y}\right)\right],$$

where $a_j^{\star}\left(\hat{y}\right)$ is given by Equation (20). Although $\hat{s}$ does not appear explicitly in Equation (21) it determines the distribution of $\hat{x}$ and, as can be seen from Equation (19), the optimal intervention policy choice $a_j^{\star}\left(\hat{y}\right)$. More specifically, as we argue in the next Subsection, it only affects the precision of the information contained in the stress test.

4.2.1 Scenario choice as information precision choice

The regulator's problem in Equation (21) only depends on the scenario $\hat{s}$ indirectly through the distribution of $\hat{x}$ and the optimal intervention policy choice $a_{i,j}^{\star}\left(\hat{x}\right)$. Recall from Equation (10) that $\hat{x} \sim N\left(\overline{x}, \Sigma_{\hat{x}}\right)$ where $\Sigma_{\hat{x}} = \Sigma_x - \hat{\Sigma}_x$. Therefore, the scenario $\hat{s}$ only affects the distribution of $\hat{x}$ through $\hat{\Sigma}_x$. As can be seen from Equation (20), the scenario choice $\hat{s}$ only affects the regulator's intervention policy through the posterior variance $\hat{\Sigma}_x$. Therefore, choosing a scenario $\hat{s}$ is equivalent to choosing a posterior covariance matrix $\hat{\Sigma}_x$. Each scenario $\hat{s}$ determines the structure of the signals about the banks' risk exposures contained in the stress test results $\hat{y}$. This signal structure determines the amount of information that can be extracted from the stress test results and, hence, the amount of residual uncertainty that the regulator faces after observing $\hat{y}$, which is measured by $\hat{\Sigma}_x$.

The regulator's problem in Equation (21) can be rewritten in terms of choosing a posterior covariance matrix$\hat{\Sigma}_x \in \mathbf{\Sigma}$. The set $\mathbf{\Sigma}$ from which the regulator can choose maps a set of posterior precisions $\left\{\hat{\Sigma}_{x,jj}\right\}_j$ for each feasible set of stress test scenarios $\hat{s}$ and is given by Equation (9). Moreover, the set $\mathbf{\Sigma}$ is the set of feasible posterior variances and it plays the role of a budget set in models in which agents choose the precision of their signals under information processing constraints.

The regulator cares about the stress test result because it contains information about the banks' exposures to the risk factors. As we discussed in the previous section, the regulator's choice of scenario determines the signals that she has available to choose her intervention policy. Hence, it is natural that the regulator's problem can be cast in terms of choosing the precision of the information available to the regulator. Formulating the regulator's problem in this way unfolds the problem in two. First, we can focus on the information acquisition aspect embedded in the stress scenario design. Once one understands the choice of information, we can use the insights of section (Kalman) to understand the optimal scenario choice.

5 Optimal stress scenario

The amount of learning from the stress test that can be attained by the regulator depends on the regulator's prior distribution over the banks' risk exposures and on the distribution of the errors in the banks' reps. In the next sections, we explore how these parameters affect the optimal scenario choice. We start by analyzing the case in which there is only one bank to focus on how the prior mean of the risk exposures and the correlation among them within a bank determine the scenario choice. Then, we consider the case with two banks and concentrate on how having systemic factors impacts the stress scenario design.

5.1 One bank

When there is only one bank in the economy, the net worth of the banking system is given by $W = \bar{w} - \eta - \sum_{j=1}^{J} \left(1 - a_j^\star\left(\hat{y}\right)\right) x_j s_j$. Then, the optimal intervention policy in Equation (20) becomes

$$a_j^{\star}(\hat{y}) = \frac{\gamma\left(\hat{\Sigma}_{x,jj} + \hat{x}_j^2\right)\mathbb{E}\left[s_j^2\right]}{1+\gamma\left(\hat{\Sigma}_{x,jj} + \hat{x}_j^2\right)\mathbb{E}\left[s_j^2\right]}. \tag{22}$$

As in the general case in Equation (20), conditional on an expected risk exposure, the regulator wants to intervene more the more uncertain she is about that risk exposure. The regulator is willing to bear a higher exposure to factor j when the exposure to factor j is more precisely estimated.

Moreover, the regulator's problem can be rewritten as minimizing losses as follows

$$\min_{\left\{\hat{\Sigma}_{x,jj}\right\}_j \in \boldsymbol{\Sigma}} \sum_{j=1}^{J} \mathbb{E}_{\hat{y}}\left[a_j^{\star}(\hat{y})\right].$$

Since ex-post interventions are costly for the regulator, she chooses posterior variances to minimize the total amount of expected interventions. Recall that $a_j^{\star}(\hat{y})$ is increasing in $\hat{\Sigma}_{x,jj}$ and in $\hat{x}_j$ and that

$$\hat{x}_j = \left(\Sigma_{x,jj} - \hat{\Sigma}_{x,jj}\right)^{\frac{1}{2}} z + \bar{x}_j$$

where $z \sim N(0,1)$. The derivative of the expected intervention in dimension j with respect to $\hat{\Sigma}_{x,jj}$ is

$$\frac{d\mathbb{E}\left[a_j^{\star}(\hat{y})\right]}{d\hat{\Sigma}_{x,jj}} = \mathbb{E}_{\hat{y}}\left[\frac{\partial a_j^{\star}(\hat{y})}{\partial\left(\hat{\Sigma}_{x,jj} + \hat{x}_j^2\right)\mathbb{E}\left[s_j\right]^2}\frac{\partial\left(\hat{\Sigma}_{x,jj} + \hat{x}_j^2\right)}{\partial\hat{\Sigma}_{x,jj}}\mathbb{E}\left[s_j\right]^2\right]. \tag{23}$$

Equation (23) represents the change in precautionary interventions by the regulator as the posterior precision changes. When the uncertainty about the bank's net worth coming from factor j, measured by $\left(\hat{\Sigma}_{x,jj} + \hat{x}_j^2\right)\mathbb{E}\left[s_j\right]^2$, increases, the regulator intervenes more along dimension j. The second term in Equation (23) represents the change in this uncertainty as the residual uncertainty about the bank's exposure to risk factor j changes. Figure (**??**) shows the regulator's loss as a function of residual uncertainty about factor 1. As one would expect, the regulator's loss is increasing in residual uncertainty.

5.1.1 Comparative statics

In this Subsection we provide numerical examples to illustrate how the primitives of the model affect the regulator's optimal scenario choice. The parameters used to compute the examples are

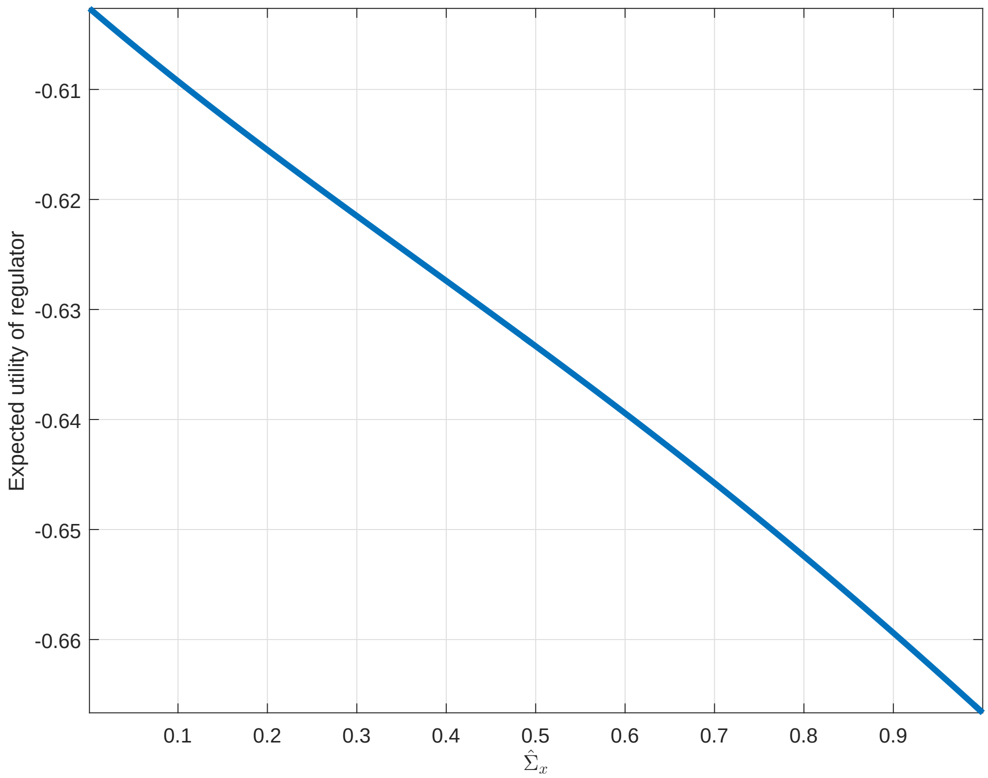
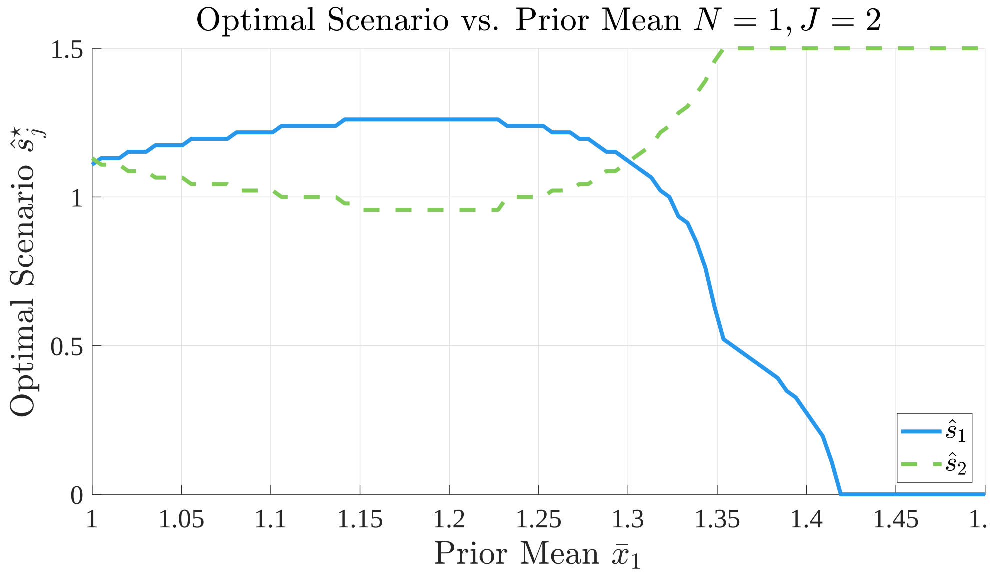


Figure 1: Regulator's expected utility as a function of posterior variance of x_1, $\hat{\Sigma}_{x,11}$.

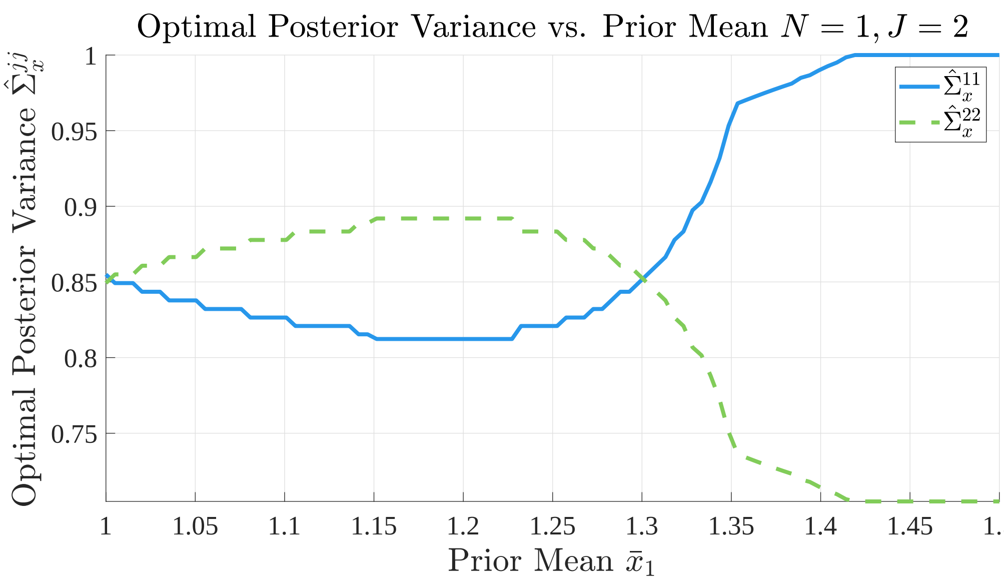
Note: Figure1 illustrate the regulator's expected utility as a function of the posterior variance of the bank's exposure to risk factor 1. The parameters used are $N = 1$, $J = 2$, $\bar{x}$=$[1, 1]$,$\lambda (M) = M$, $\alpha_0 = [1, 1]$, $\alpha_1 = 2.5$,$\sigma_\varepsilon^2 = 1$, and $\Sigma_x = \boldsymbol{I}_2$.

$J = 2$, $\bar{x}$=$[1, 1]$,$\lambda (M) = M$, $\alpha_0 = [1, 1]$, $\alpha_1 = 5$,$\sigma_\varepsilon^2 = 1$, and $\Sigma_x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$.

Prior mean risk exposure The prior mean of the exposure of the bank to each factors affects the how much the regulator wants to learn about each risk exposure. When the prior expected risk exposure to a factor is large, the risk along dimension j becomes more important and the regulator wants to learn more about the bank's exposure which he expects to be bigger. When risk exposures are uncorrelated, this is achieved by increasing factor j in the stress test scenario. Panel (a) in Figure 2 shows the weights of risk factors 1 and 2 in the optimal scenario as $\bar{x}_1$ changes. Panel (b) in the same figure shows the posterior precisions for the bank's risk exposures as a function of the expected risk exposure to factor 1.



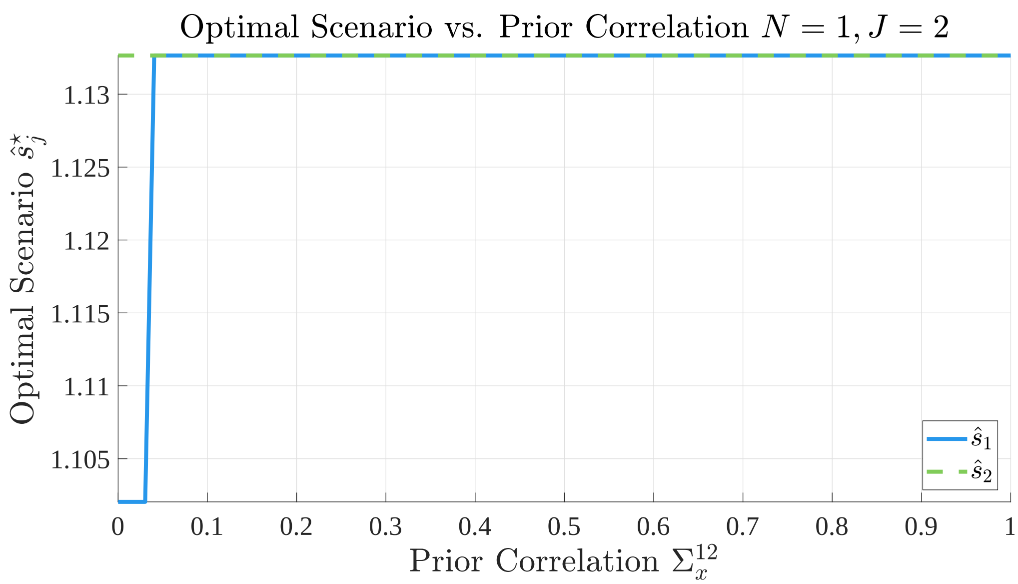
(a) Optimal scenario choice as a function of the prior mean $\bar{x}_1$.



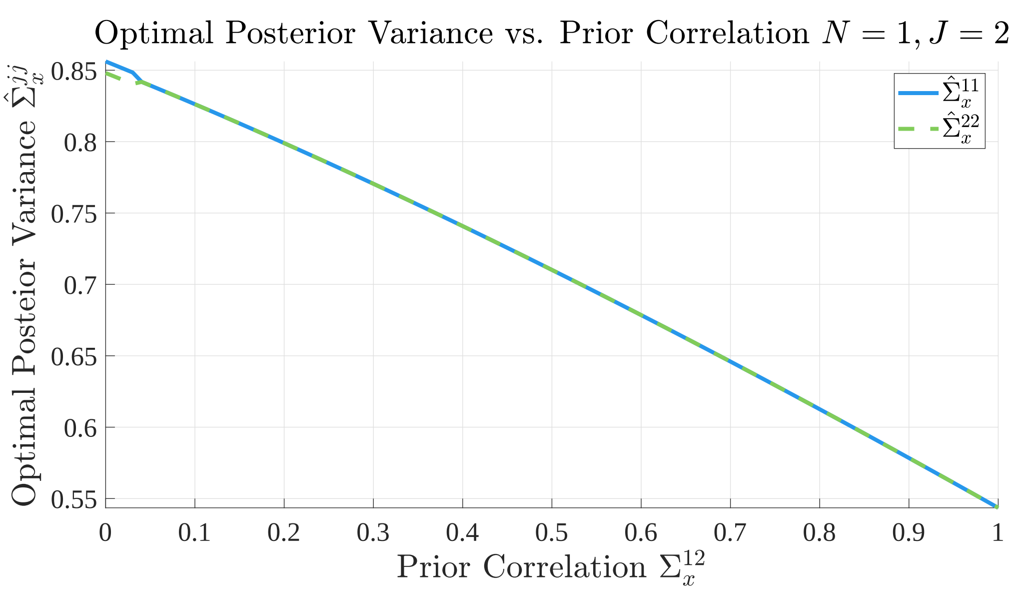
(b) Optimal posterior variances as a function of of the prior mean $\bar{x}_1$.

Figure 2: Optimal scenario and information choice as a function of the prior mean $\bar{x}_1$.

Note: Figure 2 illustrates the regulator's optimal choice of scenario and the implied posterior variance as a function of the prior mean of the risk exposure to factor 1. The parameters used are $N = 1$, $J = 2$, $\bar{x}$=$[1, 1]$,$\lambda(M) = M$, $M = 1$, $\alpha_0 = [1, 1]$, $\alpha_1 = 2.5$,$\sigma_\varepsilon^2 = 1$, and $\Sigma_x = \boldsymbol{I}$.



(a) Optimal scenario choice as a function of prior correlation between risk exposures, $\Sigma_{x,12}$.



(b) Optimal posterior variance as a function of prior correlation between risk exposures, $\Sigma_{x,12}$.

Figure 3: Optimal scenario and information choice as a function of the prior correlation in risk exposures, $\Sigma_{x,12}$.

Note: Figure 3 illustrates the regulator's optimal choice of scenario and the implied posterior variance as a function of prior correlation in risk exposures. The parameters used are $N = 1$, $J = 2$, $\bar{x}$=[1, 1],$\lambda(M) = M$, $M = 1$, $\alpha_0 = [1, 1]$, $\alpha_1 = 2.5$,$\sigma_\varepsilon^2 = 1$, and $\Sigma_x = \boldsymbol{I}$.

Prior correlation in risk exposures within banks The prior correlation among risk exposures plays a crucial role in determining how much the regulator can learn from a stress test. When the prior correlation among risk exposures is zero, the regulator faces a trade-off between learning about one risk exposure or the other. Scenarios that provide a lot of information about the bank's risk exposure to factor 1 contain very little information about the bank's risk exposure to factor 2. For example, an extreme scenario that puts all the weight on risk factor 1 and no weight on risk factor 2 contains no information about the bank's risk exposure to factor 2 at all. When the correlation between risk exposures is non-zero, this trade-off is attenuated as signals about one risk exposure always contain some information about the other. If scenarios are symmetric, the regulator always chooses a symmetric scenario.

Panel (a) in Figure 3 plots the weight on risk factor 1 in an optimal scenario as a function of the regulator's prior correlation among risk exposures. As one can see in this figure, the correlation in risk exposures does not affect the scenario choice. However, as panel (b) shows, the amount of information that the regulator can get from the same scenario changes as the prior correlation in risk exposures varies. As bank's risk exposures become more correlated in the bank's prior, the more informative the stress test.

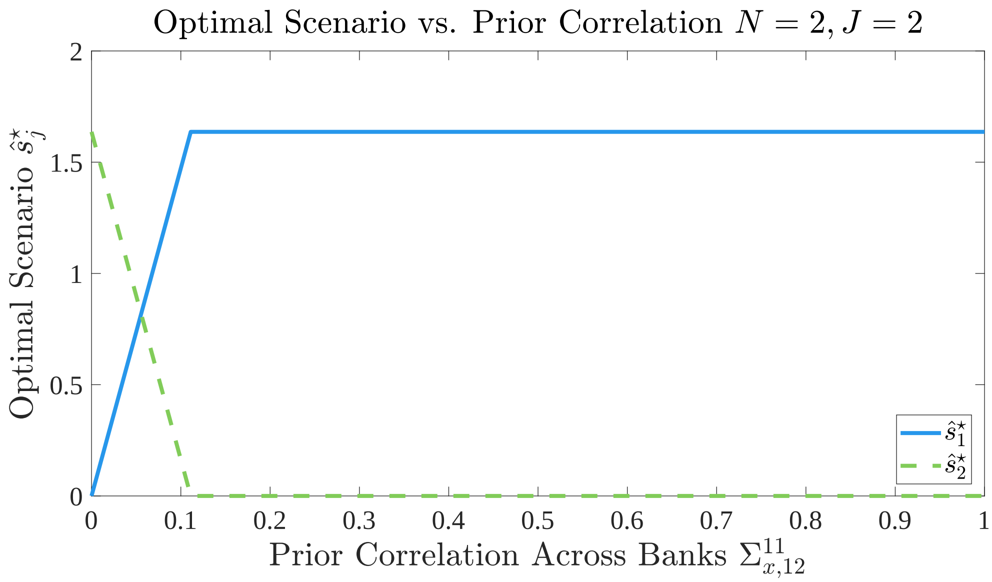
5.2 Two banks

Consider the case in which there are two banks and two risk factors. In this case, the banking sector's wealth is given by $W = \bar{\omega} - \bar{\eta} - \sum_{i=1}^{2} \left(\sum_{j=1}^{2} \left(1 - a^{\star}_{i,j} \left(\hat{y} \right) \right) x_{i,j} s_j \right)$. With linear quadratic preferences, the system that characterizes optimal action choices in Equation (20) becomes

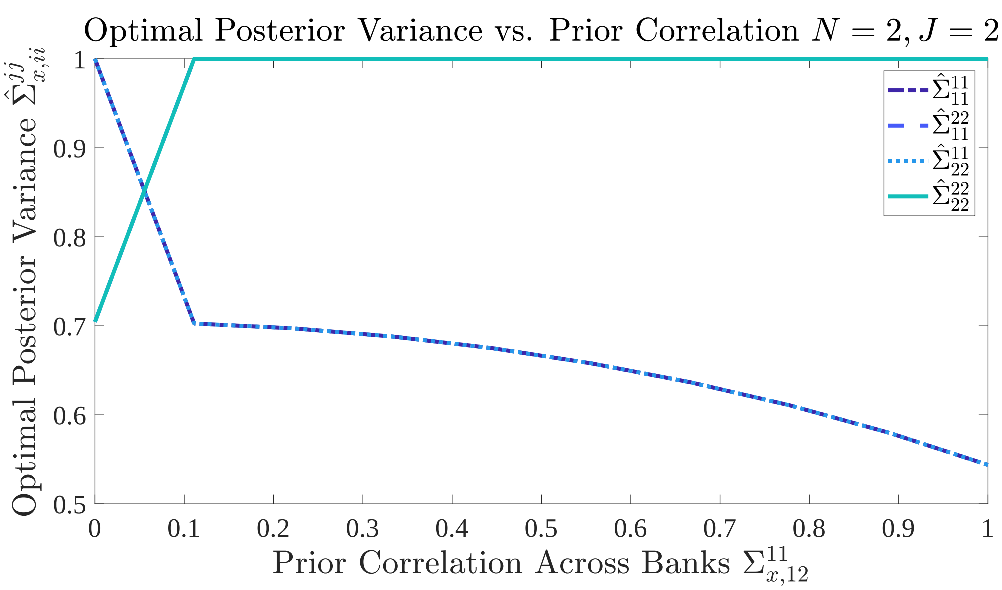
$$a^{\star}_{i,j} = \frac{\gamma \sum_{n=1}^{2} \mathbb{E} \left[x_{i,j} x_{n,j} \middle| \hat{y} \right] \mathbb{E} \left[s_j^2 \right]}{1 + \gamma \sum_{n=1}^{2} \mathbb{E} \left[x_{n,j}^2 \middle| \hat{y} \right] \mathbb{E} \left[s_j^2 \right]} \quad \forall i, j = 1, 2.$$

Correlated risk exposures across banks When there are multiple banks, the exposures to the risk factors are likely to be correlated among banks, even when the factors are orthogonal. Figure **??** shows the optimal scenario weights as a function of the correlation between the exposure to risk factor 1 for banks 1 and 2, when there are two banks, two risk factors and one scenario.

When the exposures to factor 1 are correlated across banks, the stress test results of bank i contain information about bank i's exposures to both risk factors and about bank j's exposure to



(a) Optimal scenario choice as a function of prior correlation between the banks' risk exposures to factor 1, $\Sigma^{12}_{x,11}$.



(b) Optimal posterior variance as a function of prior correlation between the bank's risk exposures to factor 1, $\Sigma^{12}_{x,11}$.

Figure 4: Optimal scenario and information choice as a function of the prior correlation between the bank's risk exposures to factor 1,, $\Sigma^{12}_{x,11}$.

Note: Figure 4 illustrates the regulator's optimal choice of scenario and the implied posterior variance as a function of prior correlation in risk exposures. The parameters used are $N = 2$, $J = 2$, $\bar{x}$=[1, 1],$\lambda(M) = M$, $M = 1$, $\alpha_0 = [1, 1]$, $\alpha_1 = 2.5$,$\sigma^2_\varepsilon = 1$, and $\Sigma_x = \boldsymbol{I}$.

factor 1, as long as the weight of factor 1 in the stress test's scenario is non-zero. Figure 4 shows that, in this case, learning about factor 1 becomes more valuable for the regulator to the point that the stress scenario only puts weight on factor 1. The decrease in the posterior variance of the exposure of the banks to factor 1 decreases mechanically with $\Sigma_{x,11}^{12}$since the the stress test of bank j becomes more informative of bank i's exposure to factor 1 as this correlation increases.

6 Evolution of stress scenarios

In this Section, we look at how stress scenarios evolve over time. New information contained in old stress tests shapes the scenario choice in the next stress test. [TBC]

7 Conclusion

We develop a model of stress testing as a learning mechanism and characterize the optimal scenario choice as a function of the ex-ante beliefs of the regulator, the precision of regulatory information, and the presence of systemic risk factors. Using the regulator's beliefs as inputs, the model can be used to design optimal stress scenarios.

[TBC]

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Appendix

The Appendix contains some auxiliary calculation for formulas in the text. It needs to be completed.

A Learning

A.1 Proof of Proposition 1

The following state space representation holds

$$\begin{aligned} \mathbf{x} &= \mathbf{x} \\ \hat{\mathbf{y}} &= \left(\mathbf{I}_N \otimes \hat{S}\right) \mathbf{x} + \mathbf{e} \end{aligned}$$

where $\mathbf{e} \perp \mathbf{x}$. Therefore, the standard Kalman filter formulas applied to the context of our model gives the result in the proposition.

A.2 Example

If $M = 1$ and $J = 2$, we have that for each bank i

$$\hat{x}_i = (I_2 - K_i S)\,\overline{x}_i + K_i \hat{y}_i \tag{A.1}$$

$$K_i = \begin{bmatrix} \Sigma^i_{x,1} & \Sigma^i_{x,12} \\ \Sigma^i_{x,12} & \Sigma^i_{x,2} \end{bmatrix} \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} \left(\begin{bmatrix} s_1 & s_2 \end{bmatrix} \left(\begin{bmatrix} \Sigma^i_{x,11} & \Sigma^i_{x,12} \\ \Sigma^i_{x,12} & \Sigma^i_{x,22} \end{bmatrix} \right) \begin{bmatrix} s_1 \\ s_2 \end{bmatrix} + \Sigma^i_e \right)^{-1} = \begin{bmatrix} \frac{\Sigma^i_{x,11} s_1 + \Sigma^i_{x,12} s_2}{\Sigma^i_{x,11}(s_1)^2 + 2\Sigma^i_{x,12} s_1 s_2 + \Sigma^i_{x,22} s_2^2 + \Sigma^i_e} \\ \frac{\Sigma^i_{x,12} s_1 + \Sigma^i_{x,22} s_2}{\Sigma^i_{x,11}(s_1)^2 + 2\Sigma^i_{x,12} s_1 s_2 + \Sigma^i_{x,22} s_2^2 + \Sigma^i_e} \end{bmatrix}$$

$$\hat{\Sigma}^i_x = \left(I_2 - \begin{bmatrix} K_1 \\ K_2 \end{bmatrix} \begin{bmatrix} s_1 & s_2 \end{bmatrix} \right) \begin{bmatrix} \Sigma^i_{x,11} & \Sigma^i_{x,12} \\ \Sigma^i_{x,12} & \Sigma^i_{x,22} \end{bmatrix} = \begin{bmatrix} (1 - K_1 s_1)\,\Sigma^i_{x,11} + K_1 s_2 \Sigma^i_{x,12} & (1 - K_1 s_1)\,\Sigma^i_{x,12} + K_1 s_2 \Sigma^i_{x,22} \\ K_2 s_2 \Sigma^i_{x,11} + (1 - K_2 s_2)\,\Sigma^i_{x,12} & K_2 s_2 \Sigma^i_{x,12} + (1 - K_2 s_2)\,\Sigma^i_{x,22} \end{bmatrix}.$$

Also,

$$\hat{y}_i \sim N\left(\overline{x}_1 s_1 + \overline{x}_2 s_2, \sigma^2_{\hat{y}_i}\right),$$

where $\sigma^2_{\hat{y}i} = s_1^2\left(\Sigma^i_{x,11} + \Sigma^i_{e,11}\right) + 2 s_1 s_2 \left(\Sigma^i_{x,12} + \Sigma^i_{e,12}\right) + s_2^2 \left(\Sigma^i_{x,22} + \Sigma^i_{e,22}\right)$. Then, unconditionally of the realization of $\hat{y}_i$ the distribution of $\hat{x}_i$ is

$$\hat{x}_i \sim N\left(\overline{x}_i, K_i \sigma^2_{\hat{y}i} K_i^T\right).$$

Using the expressions for K_i we have that

$$\begin{aligned} \frac{\partial K_1}{\partial s_1} &= \frac{\Sigma^i_{x,11}\left(\Sigma^i_{x,11}(\hat{s}_1)^2 + 2\Sigma^i_{x,12}\hat{s}_1\hat{s}_2 + \Sigma^i_{x,22}(\hat{s}_2)^2 + \Sigma^i_e\right) - \left(\Sigma^i_{x,11}\hat{s}_1 + \Sigma^i_{x,12}\hat{s}_2\right)\left(2\Sigma^i_{x,11}\hat{s}_1 + 2\Sigma^i_{x,12}\hat{s}_2\right)}{\left(\Sigma^i_{x,11}(\hat{s}_1)^2 + 2\Sigma^i_{x,12}\hat{s}_1\hat{s}_2 + \Sigma^i_{x,22}(\hat{s}_2)^2 + \Sigma^i_e\right)^2} \\ &= \frac{-\left(\Sigma^i_{x,11}\hat{s}_1\right)^2 + \Sigma^i_{x,11}\Sigma^i_{x,22}(\hat{s}_2)^2 - 2\Sigma^i_{x,12}\hat{s}_2\left(\Sigma^i_{x,11} s_1 + \Sigma^i_{x,12} s_2\right) + \Sigma^i_{x,11}\Sigma^i_e}{\left(\Sigma^i_{x,11}(\hat{s}_1)^2 + 2\Sigma^i_{x,12}\hat{s}_1\hat{s}_2 + \Sigma^i_{x,22}(\hat{s}_2)^2 + \Sigma^i_e\right)^2} \end{aligned}$$

$$\frac{\partial K_2}{\partial s_2} = \frac{-\left(\Sigma^i_{x,22}\hat{s}_2\right)^2 + \Sigma^i_{x,22}\Sigma^i_{x,11}(\hat{s}_1)^2 - 2\Sigma^i_{x,12}\hat{s}_1\left(\Sigma^i_{x,12}\hat{s}_1 + \Sigma^i_{x,22}\hat{s}_2\right) + \Sigma^i_{x,22}\Sigma^i_e}{\left(\Sigma^i_{x,11}(\hat{s}_1)^2 + 2\Sigma^i_{x,12}\hat{s}_1\hat{s}_2 + \Sigma^i_{x,22}(\hat{s}_2)^2 + \Sigma^i_e\right)^2}$$

$$\frac{\partial K_2}{\partial s_1} = \frac{\Sigma^i_{x,12}\left(\Sigma^i_{x,11}\left(\hat{s}_1\right)^2 + 2\Sigma^i_{x,12}\hat{s}_1\hat{s}_2 + \Sigma^i_{x,22}\left(\hat{s}_2\right)^2 + \Sigma^i_e\right) - \left(\Sigma^i_{x,12}\hat{s}_1 + \Sigma^i_{x,22}\hat{s}_2\right)\left(2\Sigma^i_{x,11}\hat{s}_1 + 2\Sigma^i_{x,12}\hat{s}_2\right)}{\left(\Sigma^i_{x,11}\left(\hat{s}_1\right)^2 + 2\Sigma^i_{x,12}\hat{s}_1\hat{s}_2 + \Sigma^i_{x,22}\left(\hat{s}_2\right)^2 + \Sigma^i_e\right)^2}$$

$$\frac{\partial K_1}{\partial s_2} = \frac{\Sigma^i_{x,12}\left(\Sigma^i_{x,11}\left(\hat{s}_1\right)^2 + 2\Sigma^i_{x,12}\hat{s}_1\hat{s}_2 + \Sigma^i_{x,22}\left(\hat{s}_2\right)^2 + \Sigma^i_e\right) - \left(\Sigma^i_{x,11}\hat{s}_1 + \Sigma^i_{x,12}\hat{s}_2\right)\left(2\Sigma^i_{x,12}\hat{s}_1 + 2\Sigma^i_{x,22}\hat{s}_2\right)}{\left(\Sigma^i_{x,11}\left(\hat{s}_1\right)^2 + 2\Sigma^i_{x,12}\hat{s}_1\hat{s}_2 + \Sigma^i_{x,22}\left(\hat{s}_2\right)^2 + \Sigma^i_e\right)^2}$$

If the regulator has a prior such that the bank's exposures to the risk factors are uncorrelated, i.e., if $\Sigma_{x,12} = 0$, we have

$$\frac{\partial K_1}{\partial s_1} = \Sigma^i_{x,11}\frac{-\Sigma^i_{x,11}\left(\hat{s}_1\right)^2 + \Sigma^i_{x,22}\left(\hat{s}_2\right)^2 + \Sigma^i_e}{\left(\Sigma^i_{x,11}\left(\hat{s}_1\right)^2 + \Sigma^i_{x,22}\left(\hat{s}_2\right)^2 + \Sigma^i_e\right)^2}$$

$$\frac{\partial K_2}{\partial s_2} = \frac{-\left(\Sigma^i_{x,22}\hat{s}_2\right)^2 + \Sigma^i_{x,22}\Sigma^i_{x,11}\left(\hat{s}_1\right)^2 + \Sigma^i_{x,22}\Sigma^i_e}{\left(\Sigma^i_{x,11}\left(\hat{s}_1\right)^2 + \Sigma^i_{x,22}\left(\hat{s}_2\right)^2 + \Sigma^i_e\right)^2}$$

$$\frac{\partial K_2}{\partial s_1} = -\frac{2\Sigma^i_{x,22}\hat{s}_2\Sigma^i_{x,11}\hat{s}_1}{\left(\Sigma^i_{x,11}\left(\hat{s}_1\right)^2 + \Sigma^i_{x,22}\left(\hat{s}_2\right)^2 + \Sigma^i_e\right)^2}$$

$$\frac{\partial K_1}{\partial s_2} = -\frac{2\Sigma^i_{x,11}\hat{s}_1\Sigma^i_{x,22}\hat{s}_2}{\left(\Sigma^i_{x,11}\left(\hat{s}_1\right)^2 + \Sigma^i_{x,22}\left(\hat{s}_2\right)^2 + \Sigma^i_e\right)^2}$$

B Optimal Intervention policy

Under linear quadratic preferences, the first order condition that characterizes the regulator's optimal intervention policy is

$$a^\star_{i,j}\left(\hat{y}\right) = \mathbb{E}\left[x_{i,j}s_j\left(1-\gamma W\right)\middle|\,\hat{y}\right] \quad \forall i = 1,...,N, \quad \forall j = 1,...,J,$$

where $W = \sum_{i=1}^N\left(\sum_{j=1}^J\left(1-a^\star_{i,j}\left(\hat{y}\right)\right)x_{i,j}s_j + \eta_i - d_i\right)$. This is the same as

$$a^\star_{i,j}\left(\hat{y}\right) = \mathbb{E}\left[x_{i,j}s_j\left(1+\gamma\sum_{n=1}^N\sum_{h=1}^J\left(1-a^\star_{n,h}\left(\hat{y}\right)\right)x_{n,h}s_h + \gamma\sum_{i=1}^N d_i\right)\middle|\,\hat{y}\right].$$

Since $\mathbb{E}\left[s_j\right] = 0$ for all j, we have

$$a^\star_{i,j}\left(\hat{y}\right) = \gamma\sum_{n=1}^N\left(1-a^\star_{n,j}\left(\hat{y}\right)\right)\mathbb{E}\left[x_{i,j}x_{n,j}\middle|\,\hat{y}\right]\mathbb{E}\left[s_j^2\right].$$

Let $a^\star_j = \left(a^\star_{1,j},\ldots,a^\star_{N,j}\right)'$. Then, the FOCs can be written as

$$a^\star_j = -\gamma\begin{bmatrix} \mathbb{E}\left[x_{1,j}x_{1,j}\middle|\,\hat{y}\right] & \mathbb{E}\left[x_{1,j}x_{2,j}\middle|\,\hat{y}\right] & \ldots & \mathbb{E}\left[x_{1,j}x_{N,j}\middle|\,\hat{y}\right] \\ \mathbb{E}\left[x_{2,j}x_{1,j}\middle|\,\hat{y}\right] & \ddots & \ldots & \vdots \\ \vdots & \ldots & \ldots & \vdots \\ \mathbb{E}\left[x_{N,j}x_{1,j}\middle|\,\hat{y}\right] & \ldots & \ldots & \mathbb{E}\left[x_{N,j}x_{N,j}\middle|\,\hat{y}\right] \end{bmatrix}\mathbb{E}\left[s_j^2\right]a^\star_j + \begin{bmatrix} \gamma\sum_{n=1}^N\mathbb{E}\left[x_{1,j}x_{n,j}\middle|\,\hat{y}\right]\mathbb{E}\left[s_j^2\right] \\ \gamma\sum_{n=1}^N\mathbb{E}\left[x_{2,j}x_{n,j}\middle|\,\hat{y}\right]\mathbb{E}\left[s_j^2\right] \\ \vdots \\ \gamma\sum_{n=1}^N\mathbb{E}\left[x_{N,j}x_{n,j}\middle|\,\hat{y}\right]\mathbb{E}\left[s_j^2\right] \end{bmatrix},$$

which is the same as

$$a_j^\star = -\left(\hat{\Sigma}_{x,j} + \hat{x}_j\hat{x}_j'\right)\gamma\mathbb{E}\left[s_j^2\right]a_j^\star + \left(\hat{\Sigma}_{x,j} + \hat{x}_j\hat{x}_j'\right)\mathbf{1}_{N\times 1}\gamma\mathbb{E}\left[s_j^2\right],$$

where $\hat{x}_j = (\hat{x}_{i,j}, \ldots, \hat{x}_{N,j})'$. Then,

$$a_j^\star = \left(\mathbb{I}_N + \gamma\left(\hat{\Sigma}_{x,j} + \hat{x}_j\hat{x}_j'\right)\mathbb{E}\left[s_j^2\right]\right)^{-1}\gamma\left(\hat{\Sigma}_{x,j} + \hat{x}_j\hat{x}_j'\right)\mathbf{1}_{N\times 1}\mathbb{E}\left[s_j^2\right] \quad \forall j = 1, \ldots, J \tag{A.2}$$

B.1 Proof of Proposition 2

TBC. It follows from the system in Equation (A.2)

B.2 One bank

When there is only one bank, the optimal intervention policy in Equation (20) becomes

$$a^\star(\hat{y}) = -\gamma\begin{bmatrix} \mathbb{E}\left[x_1^2\middle|\hat{y}\right]\mathbb{E}\left[s_1^2\right] & 0 & \cdots & 0 & 0 \\ 0 & \mathbb{E}\left[x_2^2\middle|\hat{y}\right]\mathbb{E}\left[s_2^2\right] & \cdots & \cdots & 0 \\ \vdots & \ddots & \ddots & \cdots & \vdots \\ 0 & \cdots & \cdots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & \mathbb{E}\left[x_J^2\middle|\hat{y}\right]\mathbb{E}\left[s_J^2\right] \end{bmatrix} a^\star(\hat{y}) + \begin{bmatrix} \gamma\mathbb{E}\left[x_1^2\middle|\hat{y}\right]\mathbb{E}\left[s_1^2\right] \\ \gamma\mathbb{E}\left[x_2^2\middle|\hat{y}\right]\mathbb{E}\left[s_2^2\right] \\ \vdots \\ \gamma\mathbb{E}\left[x_{J-1}^2\middle|\hat{y}\right]\mathbb{E}\left[s_{J-1}^2\right] \\ \gamma\mathbb{E}\left[x_J^2\middle|\hat{y}\right]\mathbb{E}\left[s_J^2\right] \end{bmatrix}$$

This is the same as

$$a_j^\star(\hat{y}) = \frac{\gamma\mathbb{E}\left[x_j^2\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]}{1 + \gamma\mathbb{E}\left[x_j^2\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]} = \frac{\gamma\left(\hat{\Sigma}_{x,jj} + \hat{x}_j^2\right)\mathbb{E}\left[s_j^2\right]}{1 + \gamma\left(\hat{\Sigma}_{x,jj} + \hat{x}_j^2\right)\mathbb{E}\left[s_j^2\right]} \quad \forall j. \tag{A.3}$$

B.3 Two banks

When there are two banks and two factors, the regulator's optimal intervention policy is characterized by the following systems

$$a_j^\star = \left(\mathbb{I}_2 + \gamma\begin{bmatrix} \mathbb{E}\left[x_{1,j}x_{1,j}\middle|\hat{y}\right] & \mathbb{E}\left[x_{1,j}x_{2,j}\middle|\hat{y}\right] \\ \mathbb{E}\left[x_{2,j}x_{1,j}\middle|\hat{y}\right] & \mathbb{E}\left[x_{2,j}x_{2,j}\middle|\hat{y}\right] \end{bmatrix}\mathbb{E}\left[s_j^2\right]\right)^{-1}\begin{bmatrix} \gamma\sum_{n=1}^N \mathbb{E}\left[x_{1,j}x_{n,j}\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right] \\ \gamma\sum_{n=1}^N \mathbb{E}\left[x_{2,j}x_{n,j}\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right] \end{bmatrix},$$

or, equivalently,

$$a_j^\star = \left[\frac{\gamma\sum_{n=1}^2 \mathbb{E}\left[x_{1,j}x_{n,j}\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]}{1 + \gamma\sum_{n=1}^2 \mathbb{E}\left[x_{n,j}^2\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]}, \frac{\gamma\sum_{n=1}^2 \mathbb{E}\left[x_{2,j}x_{n,j}\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]}{1 + \gamma\sum_{n=1}^2 \mathbb{E}\left[x_{n,j}^2\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]}\right]' \quad \forall j.$$

C Scenario choice

Since $\mathbb{E}[s_j] = \mathbb{E}[s_j s_k] = 0$ for all $j \neq k$, the objective function of the regulator can be written as

$$
\begin{aligned}
O \equiv & \mathbb{E}_{\hat{x},s,\eta}\left[W - \frac{\gamma}{2}W^2 - \frac{1}{2}\sum_{j=1}^{J} a_j^\star(\hat{y})' a_j^\star(\hat{y})\right] \\
= & \mathbb{E}_{x,e,\eta,s}\left[\bar{\omega} - \bar{\eta} - \sum_{i=1}^{J}\left(\mathbf{1}_{1\times N} - a_j^\star(\hat{y})'\right) x_{,j} s_j\right] \\
& - \mathbb{E}_{x,e,\eta,s}\left[\frac{\gamma}{2}\left(\bar{\omega} - \bar{\eta} - \sum_{j=1}^{J}\left(\mathbf{1}_{1\times N} - a_j^\star(\hat{y})'\right) x_{,j} s_j\right)^2 + \frac{1}{2}\sum_{j=1}^{J} a_j^\star(\hat{y})' a_j^\star(\hat{y})\right] \\
= & \bar{\omega} - \frac{\gamma}{2}\left(\bar{\omega}^2 + \sigma_\eta^2\right) \\
& - \mathbb{E}_{x,e,s}\left[\sum_{i=1}^{J}\left(\mathbf{1}_{1\times N} - a_j^\star(\hat{y})'\right) x_{,j} s_j + \frac{\gamma}{2}\left(\sum_{j=1}^{J}\left(\mathbf{1}_{1\times N} - a_j^\star(\hat{y})'\right) x_{,j} s_j\right)^2 - \frac{1}{2}\sum_{j=1}^{J} a_j^\star(\hat{y})' a_j^\star(\hat{y})\right] \\
= & \bar{\omega} - \frac{\gamma}{2}\left(\bar{\omega}^2 + \sigma_\eta^2\right) \\
& - \mathbb{E}_{\hat{y}}\left[\frac{\gamma}{2}\left(\sum_{j=1}^{J}\left(\mathbf{1}_{1\times N} - a_j^\star(\hat{y})'\right) \mathbb{E}\left[x_j x_j' \middle| \hat{y}\right]\left(\mathbf{1}_{1\times N} - a_j^\star(\hat{y})'\right)' \mathbb{E}\left[s_j^2\right]\right)^2 - \frac{1}{2}\sum_{j=1}^{J} a_j^\star(\hat{y})' a_j^\star(\hat{y})\right] \\
= & \bar{\omega} - \frac{\gamma}{2}\left(\bar{\omega}^2 + \sigma_\eta^2\right) \qquad\qquad (A.4) \\
& - \frac{1}{2}\sum_{j=1}^{J}\mathbb{E}_{\hat{y}}\left[\gamma\left(\mathbf{1}_{1\times N} - a_j^\star(\hat{y})'\right) \mathbb{E}\left[x_j x_j' \middle| \hat{y}\right]\left(\mathbf{1}_{1\times N} - a_j^\star(\hat{y})'\right)' \mathbb{E}\left[s_j^2\right] + \sum_{j=1}^{J} a_j^\star(\hat{y})' a_j^\star(\hat{y})\right],
\end{aligned}
$$

where $a_j^\star = \left(a_{1,j}^\star, \ldots, a_{N,j}^\star\right)'$ and $\hat{x}_j = (\hat{x}_{i,j}, \ldots, \hat{x}_{N,j})'$.

C.1 One bank

When there is only one bank, the objective function of the regulator in Equation (A.4) becomes

$$O = \omega - \frac{\gamma}{2}\left(\omega^2 + \sigma_\eta^2\right) - \frac{1}{2}\sum_{j=1}^{J}\mathbb{E}_{\hat{y}}\left[\gamma\left(1 - a_j^\star(\hat{y})\right)^2 \mathbb{E}\left[x_j^2 \middle| \hat{y}\right]\mathbb{E}\left[s_j^2\right] + \left(a_j^\star(\hat{y})\right)^2\right].$$

Using the he optimal intervention policy in Equation (A.3) we have

$$
\begin{aligned}
O &= \omega - \frac{\gamma}{2}\left(\omega^2+\sigma_\eta^2\right) - \frac{1}{2}\sum_{j=1}^{J}\mathbb{E}_{\hat{y}}\left[\gamma\left(\frac{1}{1+\gamma\mathbb{E}\left[x_j^2\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]}\right)^2\mathbb{E}\left[x_j^2\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]+\left(\frac{\gamma\mathbb{E}\left[x_j^2\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]}{1+\gamma\mathbb{E}\left[x_j^2\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]}\right)^2\right] \\
&= \omega - \frac{\gamma}{2}\left(\omega^2+\sigma_\eta^2\right) - \frac{1}{2}\sum_{j=1}^{J}\mathbb{E}_{\hat{y}}\left[\left(\frac{1}{1+\gamma\mathbb{E}\left[x_j^2\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]}\right)^2\gamma\mathbb{E}\left[x_j^2\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]\left(1+\gamma\mathbb{E}\left[x_j^2\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]\right)\right] \\
&= \omega - \frac{\gamma}{2}\left(\omega^2+\sigma_\eta^2\right) - \frac{1}{2}\sum_{j=1}^{J}\mathbb{E}_{\hat{y}}\left[\frac{\gamma\mathbb{E}\left[x_j^2\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]}{1+\gamma\mathbb{E}\left[x_j^2\middle|\hat{y}\right]\mathbb{E}\left[s_j^2\right]}\right] \\
&= \omega - \frac{\gamma}{2}\left(\omega^2+\sigma_\eta^2\right) - \frac{1}{2}\sum_{j=1}^{J}\mathbb{E}_{\hat{y}}\left[\frac{\gamma\left(\hat{\Sigma}_{x,jj}+\hat{x}_j^2\right)\mathbb{E}\left[s_j^2\right]}{1+\gamma\left(\hat{\Sigma}_{x,jj}+\hat{x}_j^2\right)\mathbb{E}\left[s_j^2\right]}\right] \\
&= \omega - \frac{\gamma}{2}\left(\omega^2+\sigma_\eta^2\right) - \frac{1}{2}\sum_{j=1}^{J}\mathbb{E}_{\hat{y}}\left[a_j^\star\left(\hat{y}\right)\right]
\end{aligned}
$$

Since $\hat{x}_j \sim N\left(\bar{x}_j, \Sigma_{x,jj}-\hat{\Sigma}_{x,jj}\right)$ we have $\hat{x}_j = \left(\Sigma_{x,jj}-\hat{\Sigma}_{x,jj}\right)^{\frac{1}{2}} z + \bar{x}_j$ where $z \sim N(0,1)$. In this case, the objective function is separable in $\hat{\Sigma}_{x,jj}$.

Let,

$$
\mathbf{\Sigma} = \left\{\left\{\hat{\Sigma}_{x,jj}\right\} : \hat{\Sigma}_{x,jj} = \Sigma_{x,jj}\left(1-K_j\left(\hat{s}\right)\hat{s}_j\right) - \Sigma_{x,hj}K_j\left(\hat{s}\right)\hat{s}_h \quad \forall h, j = 1,..,J, h \neq j \quad \text{for some} \quad \hat{s}\in\Omega\right\}, \tag{A.5}
$$

where Ω is the set of scenarios. Then, the regulator's problem can be written in terms of choosing posterior variances as

$$
\min_{\left\{\hat{\Sigma}_{x,jj}\right\}\in\mathbf{\Sigma}} \frac{1}{2}\sum_{i=1}^{J}\mathbb{E}_{\hat{x},e,\eta,s}\left[a_j^\star\left(\hat{y}\right)\right].
$$

C.2 Two banks

When there are two banks, the objective function of the regulator in Equation (A.4) becomes

$$
O = \bar{\omega} - \frac{\gamma}{2}\left(\bar{\omega}^2+\sigma_\eta^2\right) - \mathbb{E}_{\hat{y}}\left[\frac{\gamma}{2}\left(\sum_{j=1}^{J}\left(\mathbf{1}_{1\times N}-a_j^\star\left(\hat{y}\right)'\right)\mathbb{E}\left[x_jx_j'\middle|\hat{y}\right]\left(\mathbf{1}_{1\times N}-a_j^\star\left(\hat{y}\right)'\right)'\mathbb{E}\left[s_j^2\right]\right)^2 - \frac{1}{2}\sum_{j=1}^{J}a_j^\star\left(\hat{y}\right)'a_j^\star\left(\hat{y}\right)\right]
$$