

Structural Estimation of Dynamic Macroeconomic Models using Higher-Frequency Financial Data*

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Abstract

In this paper we show how high-frequency financial data can be used in a combined macro-finance framework to estimate the underlying structural parameters. Our formulation of the model allows for substituting macro variables by asset prices in a way that enables casting the relevant estimation equations partly (or completely) in terms of financial data. We show that using only financial data allows for identification of the majority of the relevant parameters. Adding macro data allows for identification of all parameters. In our simulation study, we find that it also improves the accuracy of the parameter estimates. In the empirical application we use interest rate, macro, and S&P500 stock index data, and compare the results using different combinations of macro and financial variables.

JEL classification: C13; E32; O40

Keywords: Structural estimation; High-Frequency financial data; AK-Vasicek model; Martingale Estimation Function; Generalized Method of Moments

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1 Introduction

One important lesson from the financial crisis of 2007/2008 is the need for a joint framework, which overcomes the traditional separation of macroeconomic and finance. Over the last decade, a large literature developing models at the intersection of macroeconomics and finance has emerged (among others Rudebusch and Swanson 2012, Gürkaynak and Wright 2012). Most of the papers, however, focus on the interaction of macro variables, fiscal and monetary policy, and their implications for the term structure of interest rates. In a macro-finance framework, the asset pricing kernel is consistent with the macroeconomic dynamics. So the open question is to what extent financial data can be used to replace macroeconomic variables in structural estimation.

In this paper we exploit asset pricing implications of a simple macro-finance model to cast the relevant estimation equations partly (or completely) in terms of financial data. This allows us to estimate the structural parameters using only financial data, only macroeconomic data, or a combination of these. Our motivation for doing so is that macro data, in contrast to financial data, are usually available at lower frequencies and subject to substantial revisions. Given the relatively high volatility of financial data, we investigate the informational content of various financial data including interest rates, bond and stock prices for the dynamics of macroeconomic aggregates. Our aim is to provide new insights into the use of financial data in a simple macro-finance model, and to derive implications for the estimation of more elaborated models.

The structure of our approach is as follows. We start by describing the model, similar to Christensen, Posch and van der Wel (2016), and derive the stochastic discount factor (SDF), i.e., the asset pricing kernel which allows to price any financial asset in the economy. In a second step we define various financial variables, compute their price dynamics, and cast the model's equilibrium dynamics in terms of financial data alone or combined with macro data, and the structural parameters. In a third step, we estimate the structural parameters of the model using the Generalized Method of Moments (GMM) and Martingale Estimation Function (MEF), with different specifications and different types of financial data. We study the identification of parameters both in a simulation study and empirically using

interest rate, macro, and S&P500 stock index data.

Our results obtained from both simulation study and empirical estimation indicate that using a combined macro-finance framework not only improves the identification of structural parameters but also the accuracy of the estimates. Another important feature of our macro-finance estimation approach is that it drastically reduces the upward-bias typically encountered in similar mean-reverting interest rate models in the literature.

The rest of the paper is structured as follows. Section 2 describes the used macro framework and derives the general equilibrium price for the claim on future dividends. The following section is devoted to the derivation of the systems of equilibrium equations and the derivation of the different estimators. Furthermore, the interdependencies between macro and finance dynamics as suggested by the model are discussed. Before turning to the empirical estimation in section 5, we run various simulation studies in section 4 to evaluate small sample properties and to test identification and different parameter restrictions.

2 The Macro Framework and Asset-Pricing

2.1 The Model

In this paper we use the same continuous time stochastic AK-model as in Christensen, Posch & van der Wel (2016). Thus, we will only summarize the main properties of the model and refer to this paper for a more detailed overview.

At each instance in time, output Y_t is generated by combining capital, factor productivity and a constant amount of labor

$$Y_t = A_t F(K_t, L)$$

Here the aggregate capital stock is given by K_t , total factor productivity (TFP) is represented by A_t while L is the constant population size. In this economy TFP is driven by B_t , a standard Brownian motion, with $\mu(A_t)$ representing the generic drift- and $\eta(A_t)$ the generic volatility function.

$$dA_t = \mu(A_t)dt + \eta(A_t)dB_t$$

If gross investments, I_t , are higher than capital depreciation, K_t increases according to

$$dK_t = (I_t - \delta K_t)dt + \sigma K_t dZ_t$$

with σ being the volatility of stochastic depreciation, δ representing the depreciation rate and Z_t being another standard Brownian motion.

The equilibrium conditions in this economy are standard: production factors are rewarded with their marginal products $r_t = Y_K$ and $w_t = Y_L$. The good market clearing condition is given by $Y_t = C_t + I_t$ which together with the law of motions of capital and TFP implies for the evolution of output

$$\begin{aligned} dY_t &= Y_A dA_t + Y_K dK_t + \frac{1}{2} Y_{KK} \sigma^2 K_t^2 dt \\ &= (\mu(A_t) Y_A + (I_t - \delta K_t) Y_K + \frac{1}{2} Y_{KK} \sigma^2 K_t^2) dt + Y_A \eta(A_t) dB_t + \sigma Y_K K_t dZ_t \end{aligned}$$

Households in this economy are represented by a representative household. This stand-in consumer exhibits additively separable utility and maximizes expected life time utility according to

$$U_0 \equiv E_0 \int_0^\infty e^{-\rho t} u(C_t, A_t) dt, \quad \text{with} \quad u_C > 0, \quad u_{CC} < 0$$

subject to

$$dK_t = ((r_t - \delta) K_t + w_t L - C_t) dt + \sigma K_t dZ_t$$

where the rental rate of capital is denoted by r_t and w_t denotes the labor wage rate.

2.2 The Euler Equation

In this section we will derive the Euler equation which will later be needed to obtain an expression for the stochastic discount factor in the economy. In the above model specification the state variables are technology, A_t , and capital K_t .

$$V(K_0, A_0) = \max_{\{C_t\}_{t=0}^\infty} U_0 \quad \text{s.t. (6) and (2)}$$

The first-order condition is

$$U_C(C_t, A_t) = V_K(K_t, A_t)$$

The Euler-equation is given by

$$\begin{aligned} \frac{dU_C}{U_C} = & (\rho - (r_t - \delta))dt - \frac{U_{CC}(C_t, A_t)}{U_C(C_t, A_t)} C_K \sigma^2 K_t dt + \frac{U_{CC}(C_t, A_t)}{U_C(C_t, A_t)} C_A \eta(A_t) dB_t \\ & + \frac{U_{CA}(C_t, A_t)}{U_C(C_t, A_t)} \eta(A_t) dB_t + \frac{U_{CC}(C_t, A_t)}{U_C(C_t, A_t)} C_K \sigma K_t dZ_t \end{aligned}$$

Applying conditional expectation we obtain

$$\rho - \frac{1}{dt} E_t \left[\frac{dU_C}{U_C} \right] = r_t - \delta + \frac{U_{CC}(C_t, A_t)}{U_C(C_t, A_t)} C_K \sigma^2 K_t \equiv r_t^f$$

Using the inverse marginal utility function and $U(C_t, A_t) = U(C_t)$ we obtain an expression for the path of consumption

$$\begin{aligned} dC_t = & \frac{U'(C_t)}{U''(C_t)} (\rho - (r_t - \delta))dt - \sigma^2 C_K K_t dt - \frac{1}{2} (C_A^2 \eta(A_t)^2 + C_K^2 \sigma^2 K_t^2) \frac{U'''(C_t)}{U''(C_t)} dt \\ & + C_A \eta(A_t) dB_t + C_K \sigma K_t dZ_t \end{aligned}$$

with $U' > 0$ and $U'' < 0$ The equilibrium dynamics of the economy are given by

$$\begin{aligned} d \ln C_t = & \left(\frac{U'(C_t)(\rho - r_t + \delta)}{U''(C_t)C_t} - \frac{C_K K_t \sigma^2}{C_t} - \frac{1}{2} \frac{C_A^2 \eta(A_t)^2 + C_K^2 \sigma^2 K_t^2}{C_t^2} \frac{U'''(C_t)C_t + U''(C_t)}{U''(C_t)} \right) dt \\ & + \frac{C_A \eta(A_t)}{C_t} dB_t + \frac{C_K \sigma K_t}{C_t} dZ_t \\ d \ln Y_t = & \left(\frac{\mu(A_t)}{A_t} + \left(\frac{Y_t - C_t}{K_t} - \delta \right) \frac{K_t Y_K}{Y_t} + \frac{1}{2} \sigma^2 \frac{K_t^2 Y_{KK}}{Y_t} \right) dt - \frac{1}{2} \frac{Y_A^2 \eta(A_t)^2 + \sigma^2 Y_K^2 K_t^2}{Y_t^2} dt \\ & + \frac{Y_A \eta(A_t)}{Y_t} dB_t + \frac{\sigma Y_K K_t}{Y_t} dZ_t \\ d \ln K_t = & (r_t - \delta + w_t/K_t - C_t/K_t - \frac{1}{2} \sigma^2) dt + \sigma dZ_t \end{aligned}$$

Now using the model specification

$$d \ln C_t = (r_t - \rho - \delta - \frac{1}{2} \sigma^2) dt + \sigma dZ_t \quad (1a)$$

$$d \ln Y_t = (\mu(r_t)/r_t + r_t - \rho - \delta - \frac{1}{2} \eta(r_t)^2/r_t^2 - \frac{1}{2} \sigma^2) dt + \eta(r_t)/r_t dB_t + \sigma dZ_t \quad (1b)$$

$$dr_t = \mu(r_t) dt + \eta(r_t) dB_t \quad (1c)$$

Since the central objective of this paper is the parameter estimation of macro economic models using financial data, we want to keep the model as simply as possible while maintaining the capability of explaining both macroeconomic and financial dynamics. Thus, we are again following the model by Christensen, Posch & van der Wel (2016), and use the Vasicek specification for the interest rate. Despite its simplicity the Vasicek interest rate model still plays a crucial role in the finance literature and turns out to be a reasonable interest rate process for our model without relying on unreasonable assumptions. Thus, we use the below Vasicek specification with $\mu(r_t) = \kappa(\gamma - r_t)$ and $\eta(r_t) = \eta$

$$dr_t = \kappa(\gamma - r_t)dt + \eta dB_t \quad (2)$$

The equilibrium dynamics of the AK-Vasicek model can then be summarized by

$$d \ln C_t = (r_t - \rho - \delta - \frac{1}{2}\sigma^2) dt + \sigma dZ_t \quad (3a)$$

$$d \ln Y_t = (\kappa\gamma/r_t - \frac{1}{2}\eta^2/r_t^2 + r_t - \kappa - \rho - \delta - \frac{1}{2}\sigma^2) dt + \eta/r_t dB_t + \sigma dZ_t \quad (3b)$$

$$dr_t = \kappa(\gamma - r_t)dt + \eta dB_t, \quad \text{where} \quad r_t = r_t^f + \delta + \sigma^2 \quad (3c)$$

This is the system of equilibrium equations used in the estimations in the paper by Christensen, Posch & van der Wel (2016). Note that the system depends on mixed-frequency macro and finance data. Similarly, to the baseline paper we also want to estimate the six structural parameters of the model that are given by the vector

$$\phi = (\kappa, \gamma, \eta, \rho, \delta, \sigma)^T \quad (4)$$

However, in our case we are using various different combinations of financial and macro data to cast the relevant estimation equations partly or completely in terms of financial data. In order to obtain the asset pricing implications of the model we derive the stochastic discount factor in the following section.

2.3 The Stochastic Discount Factor

Following Hansen and Scheinkman(2009), the stochastic discount factor for $s > t$ can be derived from the Euler equation as the process

$$\frac{\Lambda_s}{\Lambda_t} = e^{-\rho(s-t)} \frac{V_K(K_s, A_s)}{V_K(K_t, A_t)} \quad (5)$$

For $U(C_t, A_t) = U(C_t)$ and by using the analytical solution $C_t = \rho K_t$

$$\frac{dU_c}{U_c} = (\rho - (r_t - \delta))dt - \frac{U_{cc}(C_t)}{U_c(C_t)}C_K\sigma^2K_tdt + \frac{U_{cc}(C_t)}{U_c(C_t)}C_K\sigma K_t dZ_t$$

Hence, the SDF reads

$$d\Lambda_t = (-(r_t - \delta))\Lambda_t dt - \frac{U_{cc}(C_t)}{U_c(C_t)}C_K\sigma^2K_t\Lambda_t dt + \frac{U_{cc}(C_t)}{U_c(C_t)}C_K\sigma K_t\Lambda_t dZ_t \quad (6)$$

Analogously to Christensen, Posch & van der Wel (2016), we can derive the certainty equivalent rate of return by

$$-\frac{1}{dt}E_t \left[\frac{d\Lambda_t}{\Lambda_t} \right] = r_t - \delta + \frac{U_{cc}(C_t)}{U_c(C_t)}C_K\sigma^2K_t \equiv r_t^f \quad (7)$$

Applying the same AK-specification $U(C_t) = \ln(C_t)$ and $C_t = \rho K_t$ to the evolution of Λ_t we obtain

$$d\Lambda_t = (-(r_t - \delta) + \sigma^2)\Lambda_t dt - \sigma\Lambda_t dZ_t \quad (8)$$

In the appendix we show how to apply Ito's formula to obtain the stochastic discount factor as the process:

$$\frac{\Lambda_s}{\Lambda_t} = e^{-\int_t^s (r_v - \delta - \frac{1}{2}\sigma^2)dv - \sigma \int_t^s dZ_v} \quad (9)$$

Recall that the Vasicek specification for the rental rate of physical capital is given by (2). As shown in the appendix this is an Ornstein-Uhlenbeck process, allowing us to find a solution for $s > t$ by using a standard technique for differential equations.

$$r_s = e^{-\kappa(s-t)}r_t + (1 - e^{-\kappa(s-t)})\gamma + \eta e^{-\kappa(s-t)} \int_t^s e^{\kappa(u-t)} dB_u \quad (10)$$

Using (10) we show in the appendix how to obtain the expected value of the SDF and arrive at

$$E_t \left[\frac{\Lambda_s}{\Lambda_t} \right] = e^{-\left(\frac{r_t - \delta}{\kappa} + \frac{\eta^2}{\kappa^3}\right)(1 - e^{-\kappa(s-t)}) - \left(\gamma - \delta - \sigma^2 - \frac{\eta^2}{2\kappa^2}\right)(s-t) + \frac{\eta^2}{4\kappa^3}(1 - e^{-2\kappa(s-t)})} \quad (11)$$

2.4 Claim on Future Dividends

We use the stochastic discount factor together with equations (12) and (13) to price the below defined claim on future dividends. Since we are interested in equilibrium prices we use the stochastic discount factor in order to be able to price any asset in this economy consistently with macro dynamics. Note that we assume that there are markets for contingent claims that are all in zero-supply in equilibrium. To find the equilibrium prices we compute (see e.g. Cochrane, 2005)

$$P_t = E_t \left[\frac{\Lambda_s}{\Lambda_t} X_s \right] \quad (12)$$

that is for $s > t$ the pricing equation states that the equilibrium price P of an asset at time t is given by the conditional expectation of the product of the stochastic discount factor and the future payoff X_s . Furthermore we obtain equilibrium returns by using (e.g. Cochrane, 2005)

$$R_s = \frac{X_s}{P_t} \quad (13)$$

Starting from (12) consider a claim on all future dividends (in an endowment economy this is equivalent to a claim on the tree, *not* only on the next period's fruit), that is we have the price

$$P_{d,t} = E_t \left[\int_t^\infty \frac{\Lambda_s}{\Lambda_t} Y_s ds \right] \quad (14)$$

To find the equilibrium price of this asset we have to compute an expression for Y_s , the output in period s . Note that in our model specification we have that $Y_t = A_t K_t$, implying that $Y_s = r_s K_s$. Note that we have already defined r_s in (9). In appendix (A.4) we show how to use the model properties together with Ito's formula to obtain an expression for the capital stock in period $s > t$

$$K_s = K_t e^{\int_t^s (r_v - \rho - \delta - \frac{1}{2}\sigma^2) dv + \sigma \int_t^s dZ_v} \quad (15)$$

Note that we can cast Y_s analogously to (15) in Christensen, Posch & van der Wel (2016) and obtain (see appendix)

$$Y_s = Y_t e^{\kappa\gamma \int_t^s \frac{1}{r_v} dv - \frac{\eta^2}{2} \int_t^s \frac{1}{r_v^2} dv + \int_t^s (r_v - \delta - \rho - \kappa - \frac{1}{2}\sigma^2) dv + \sigma \int_t^s dZ_v + \int_t^s \frac{\eta}{r_v} dB_v} \quad (16)$$

However, here we use a slightly different formulation using (9) together with (15). Hence, multiplying the two equations we arrive at

$$Y_s = \left[K_t r_t + K_t \gamma e^{\kappa(s-t)} - K_t \gamma + K_t \eta \int_t^s e^{\kappa(u-t)} dB_u \right] e^{\int_t^s (r_v - \delta - \rho - \kappa - \frac{1}{2} \sigma^2) dv + \sigma \int_t^s dZ_v} \quad (17)$$

To find the price of the claim on future dividends we use (14) together with (9) and (17) and obtain

$$\begin{aligned} P_{d,t} &= E_t \left[\int_t^\infty \left[K_t r_t + K_t \gamma e^{\kappa(s-t)} - K_t \gamma + K_t \eta \int_t^s e^{\kappa(u-t)} dB_u \right] e^{-\int_t^s (\rho + \kappa) dv} ds \right] \\ &= E_t \left[\int_t^\infty \left[K_t r_t + K_t \gamma e^{\kappa(s-t)} - K_t \gamma + K_t \eta \int_t^s e^{\kappa(u-t)} dB_u \right] e^{-(\rho + \kappa)(s-t)} ds \right] \\ &= \int_t^\infty [K_t r_t - K_t \gamma] e^{-(\rho + \kappa)(s-t)} ds + \int_t^\infty K_t \gamma e^{-\rho(s-t)} ds \end{aligned}$$

Solving the integrals yields we arrive at

$$P_{d,t} = K_t \left[\frac{r_t - \gamma}{(\rho + \kappa)} + \frac{\gamma}{\rho} \right] \quad (18)$$

This is an intuitive result. The price of the claim is based on the sum of two annuities. Recall that in the AK-Vasicek model the parameter γ can be interpreted as the long-term mean of the interest rate, or, since $A_t = r_t$, the long-term mean of total factor productivity. Therefore, the price of the claim is based on the current capital stock times γ plus the current capital stock times the current level of A_t minus γ . Since this term can be either positive, zero or negative, the price of the claim raises when current total factor productivity A_t is higher than its long-term level and decreases if the current level of technology lies below its equilibrium level.

As shown in the appendix we can take the derivative of the equilibrium price equation (3.2) and obtain

$$dP_{d,t} = P_{d,t} [r_t - \rho - \delta] dt + P_{d,t} \sigma dZ_t + P_{d,t} \frac{[\rho \kappa (\gamma - r_t)]}{\rho r_t + \kappa \gamma} dt + P_{d,t} \frac{\rho \eta}{\rho r_t + \kappa \gamma} dB_t \quad (19)$$

or applying Ito's formula to find an expression for the log price change of the claim

$$\begin{aligned} d \ln P_{d,t} &= \left[r_t - \delta - \rho - \frac{1}{2} \sigma^2 + \frac{\rho \kappa (\gamma - r_t)}{[\rho r_t + \kappa \gamma]} - \frac{1}{2} \frac{(\rho \eta)^2}{[\rho r_t + \kappa \gamma]^2} \right] dt \\ &\quad + \frac{\rho \eta}{[\rho r_t + \kappa \gamma]} dB_t + \sigma dZ_t \end{aligned} \quad (20)$$

or

$$d \ln P_{d,t} = d \ln C_t + \left[\frac{\rho \kappa (\gamma - r_t)}{[\rho r_t + \kappa \gamma]} - \frac{1}{2} \frac{(\rho \eta)^2}{[\rho r_t + \kappa \gamma]^2} \right] dt + \frac{\rho \eta}{[\rho r_t + \kappa \gamma]} dB_t \quad (21)$$

3 Estimation

3.1 Taking the Model to the Data

Our objective is the estimation of the structural parameters of our dynamic macroeconomic model using higher-frequency financial data. In this section we describe how to take our model to the data and offer economic intuition for using the price of the claim on future dividends. As shown in the web appendix we considered various alternative asset prices and returns. Those asset prices fully reflect the macro dynamics of our model but are unsuitable when it comes to consistently taking the model to the data. In this context the central problem is to find asset prices in the model that have a real world analogs.

It is straight forward to incorporate output, consumption and interest rate data in our model estimation. To get the intuition for a real world analogue for the dividend claim, we will start with a simple example. Consider that an investors buys a broad defined stock index (a market portfolio) at period t . If he sells the stock index in the next period, his return is given by the sum of accumulated dividends up to this period, plus the price change of this index. In this context, the index across stocks represents the average production of firms in the economy and consists of stocks paying dividends. Like a stock index, the above defined claim on capital does not have an expiration date and gives the owner the right to all future dividends. Hence, to take our model to the data we turn to one of the most important indices worldwide, the S&P500 index and use this rich financial data from the stock market in our parameter estimation. As shown in the web appendix, we have to use the price rather than the return of the claim on future dividends to match it with stock index data.

3.2 Specifying the Estimation Equations

In order to obtain robust results we use alternative estimation methods and also compare the resulting parameter estimates. As already pointed out in the intro-

duction, the central procedures used in this paper are GMM and MEF estimation techniques. We apply optimal GMM as well as optimal MEF estimation and consider different numbers of conditional moment and parameter restrictions. A complete specification of the estimation equations can be found in the web appendix.

The central system of equilibrium equations that we estimate is given by

$$d \ln C_t = (r_t - \rho - \delta - \frac{1}{2}\sigma^2) dt + \sigma dZ_t \quad (22a)$$

$$d \ln P_{d,t} = \left[r_t - \rho - \delta - \frac{1}{2}\sigma^2 + \frac{\rho\kappa(\gamma - r_t)}{[\rho r_t + \kappa\gamma]} - \frac{1}{2} \frac{(\rho\eta)^2}{[\rho r_t + \kappa\gamma]^2} \right] dt + \frac{\rho\eta}{[\rho r_t + \kappa\gamma]} dB_t + \sigma dZ_t \quad (22b)$$

$$dr_t = \kappa(\gamma - r_t)dt + \eta dB_t, \quad \text{where } r_t = r_t^f + \delta + \sigma^2 \quad (22c)$$

Alternatively, we also estimate the above system but add or remove the differential on consumption by the one obtained for output

$$d \ln Y_t = (\kappa\gamma/r_t - \frac{1}{2}\eta^2/r_t^2 + r_t - \kappa - \rho - \delta - \frac{1}{2}\sigma^2) dt + \eta/r_t dB_t + \sigma dZ_t \quad (23)$$

Finally, by noting that the differential for consumption is contained in both the claim as well as the output differential equation, we also consider the case where we substitute parameters by data. If we explicitly use consumption data in the above equations, we denote them by alternative claim and alternative output formulation, respectively. That is we estimate:

$$d \ln \left(\frac{P_{d,t}}{C_t} \right) = \left[\frac{\rho\kappa(\gamma - r_t)}{[\rho r_t + \kappa\gamma]} - \frac{1}{2} \frac{(\rho\eta)^2}{[\rho r_t + \kappa\gamma]^2} \right] dt + \frac{\rho\eta}{[\rho r_t + \kappa\gamma]} dB_t \quad (24a)$$

$$d \ln \left(\frac{Y_t}{C_t} \right) = (\kappa\gamma/r_t - \frac{1}{2}\eta^2/r_t^2 - \kappa) dt + \eta/r_t dB_t \quad (24b)$$

When considering the alternative claim formulation, consumption and output are still driven by the interest rate, highlighting the impact of macro on financial dynamics. At the same time, however, the dynamics of the claim are given by financial data and indirectly by consumption data. Hence, this specification allows for an additional feedback channel between macroeconomic and financial variables. The formulation of the dividend claim gives some important insights into the

behaviour of macro and financial variables as suggest by our model. Equation 18 states that the price of the dividend claim consists of a combination of macro and finance data as well as model parameters.

$$P_{d,t} = K_t \left[\frac{r_t - \gamma}{(\rho + \kappa)} + \frac{\gamma}{\rho} \right]$$

Rearranging terms

$$K_t = P_{d,t} \left[\frac{\rho^2 + \rho\kappa}{\rho r_t + \kappa\gamma} \right]$$

That is, in our macro-finance model, macroeconomic variables can be expressed completely in terms of financial data and parameters. Multiplying both sides of the above equation by ρ we can deduce an expression for consumption or by multiplying by r_t we obtain a financial expression for output. In a similar manner we can also derive an expression for the rental rate on physical capital, expressed in terms of macro-finance data and parameters.

$$r_t = \frac{P_{d,t}}{C_t}(\rho + \kappa) - \frac{\gamma\kappa}{\rho}$$

While this formulation yields an expression for the rental rate of physical capital in terms of both macro and financial variables, the availability of consumption data limits the frequency.

Thus, in our model setting, given financial data and parameter values we can derive time series for macro economic variables at any desired frequency. Even though, this simple economic model is probably misspecified it still shows how macro-finance linkages and especially how macro and finance data can be evaluated in a joint and consistent framework.

3.3 Discrete-time Version of the Model

To account for the discrete-time character of the data we start from the systems of differential in section 2, integrate over $t \geq (t - \Delta)$, use exact solution whenever possible and arrive at exact discrete-time analogs based on observable variables.

The six structural parameters of the model are given by the vector

$$\phi = (\kappa, \gamma, \eta, \rho, \delta, \sigma)^T$$

We begin with the system given by the equilibrium equations (32a), (32b) and (32c). The derivations to the following results are shown in the appendix. The discrete version of this system reads.

$$\ln(C_t/C_{t-\Delta}) = \int_{t-\Delta}^t r_v^f dv - (\rho - \frac{1}{2}\sigma^2)\Delta + \varepsilon_{C,t} \quad (25a)$$

$$\ln(P_{d,t}/P_{d,t-\Delta}) = \int_{t-\Delta}^t r_v^f dv - (\rho - \frac{1}{2}\sigma^2)\Delta \quad (25b)$$

$$\begin{aligned} & + \rho\kappa \int_{t-\Delta}^t \left(\frac{\gamma - \delta - \sigma^2}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv \\ & - \rho\kappa \int_{t-\Delta}^t \left(\frac{r_v^f}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv \\ & - \frac{1}{2}(\rho\eta)^2 \int_{t-\Delta}^t \frac{1}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]^2} dv + \varepsilon_{P_d,t} \\ r_t^f & = e^{-\kappa\Delta} r_{t-\Delta}^f + (1 - e^{-\kappa\Delta})(\gamma - \delta - \sigma^2) + \varepsilon_{r,t} \end{aligned} \quad (25c)$$

We define m_t the (3x1) vector of martingale difference sequence as

$$m_t = \begin{pmatrix} \ln(C_t/C_{t-\Delta}) - \int_{t-\Delta}^t r_v^f dv + (\rho - \frac{1}{2}\sigma^2)\Delta \\ \ln(P_{d,t}/P_{d,t-\Delta}) - \int_{t-\Delta}^t r_v^f dv + (\rho - \frac{1}{2}\sigma^2)\Delta - \rho\kappa \int_{t-\Delta}^t \left(\frac{\gamma - \delta - \sigma^2}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv \\ + \rho\kappa \int_{t-\Delta}^t \left(\frac{r_v^f}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv + \frac{1}{2}(\rho\eta)^2 \int_{t-\Delta}^t \frac{1}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]^2} dv \\ r_t^f - e^{-\kappa\Delta} r_{t-\Delta}^f - (1 - e^{-\kappa\Delta})(\gamma - \delta - \sigma^2) \end{pmatrix} \quad (26)$$

Where the (3x1) vector of martingale increments, ε_t , is given by

$$\varepsilon_t = \begin{pmatrix} \varepsilon_{C,t} \\ \varepsilon_{d,t} \\ \varepsilon_{r,t} \end{pmatrix} = \begin{pmatrix} \sigma(Z_t - Z_{t-\Delta}) \\ \rho\eta \int_{t-\Delta}^t \frac{1}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]} dB_v + \sigma(Z_t - Z_{t-\Delta}) \\ \eta e^{-\kappa\Delta} \int_{t-\Delta}^t e^{\kappa(v-(t-\Delta))} dB_v \end{pmatrix} \quad (27)$$

As shown in the appendix, Ψ_t , the (3x3) conditional covariance matrix reads

$$\Psi_t = \begin{pmatrix} \sigma^2\Delta & \sigma^2\Delta & 0 \\ \sigma^2\Delta & (\rho\eta)^2\Delta/[\rho(r_{t-\Delta}^f + \delta + \sigma^2) + \kappa\gamma]^2 + \sigma^2\Delta & \rho\eta^2 e^{-\kappa\Delta}\Delta/[\rho(r_{t-\Delta}^f + \delta + \sigma^2) + \kappa\gamma] \\ 0 & \rho\eta^2 e^{-\kappa\Delta}\Delta/[\rho(r_{t-\Delta}^f + \delta + \sigma^2) + \kappa\gamma] & \eta^2(1 - e^{-2\kappa\Delta})/2\kappa \end{pmatrix} \quad (28)$$

Finally, the conditional mean of the parameter derivatives, using a first order deterministic Taylor expansion reads

$$\psi_t^T = \begin{pmatrix} 0 & \psi_{12} & \Delta e^{-\kappa\Delta}(r_{t-\Delta} - \gamma) \\ 0 & \psi_{22} & -(1 - e^{-\kappa\Delta}) \\ 0 & \psi_{32} & 0 \\ \Delta & \psi_{42} & 0 \\ 0 & \psi_{52} & (1 - e^{-\kappa\Delta}) \\ -\sigma\Delta & \psi_{62} & 2\sigma(1 - e^{-\kappa\Delta}) \end{pmatrix}$$

where the conditional mean of the parameter derivatives for the price of the claim on future dividends in the middle column is given by

$$\begin{aligned} \psi_{12} &= -\rho(\gamma - \delta - \sigma^2) \mathbb{C}_1 + \rho\kappa\gamma(\gamma - \delta - \sigma^2) \mathbb{C}_2 - \gamma(\rho\eta)^2 \mathbb{C}_3 \\ &\quad + \rho \mathbb{C}_4 - \rho\kappa\gamma \mathbb{C}_5 \\ \psi_{22} &= -\rho\kappa \mathbb{C}_1 + \rho\kappa^2(\gamma - \delta - \sigma^2) \mathbb{C}_2 - \kappa(\rho\eta)^2 \mathbb{C}_3 \\ &\quad - \rho\kappa^2 \mathbb{C}_5 \\ \psi_{32} &= \eta\rho^2 \mathbb{C}_2 \\ \psi_{42} &= \Delta - \kappa(\gamma - \delta - \sigma^2) \mathbb{C}_1 + \rho\eta^2 \mathbb{C}_2 + \kappa \mathbb{C}_4 \\ &\quad + \rho\kappa(\gamma - \delta - \sigma^2) \mathbb{C}_6 - (\rho\eta)^2 \mathbb{C}_7 - \rho\kappa \mathbb{C}_8 \\ \psi_{52} &= \rho\kappa \mathbb{C}_1 + \rho^2\kappa(\gamma - \delta - \sigma^2) \mathbb{C}_2 - \rho^3\eta^2 \mathbb{C}_3 \\ &\quad - \rho^2\kappa \mathbb{C}_5 \\ \psi_{62} &= -\sigma\Delta + 2\rho\kappa\sigma \mathbb{C}_1 + 2\rho^2\kappa\sigma(\gamma - \delta - \sigma^2) \mathbb{C}_2 - 2\rho^3\eta^2\sigma \mathbb{C}_3 \\ &\quad - 2\rho^2\kappa\sigma \mathbb{C}_5 \end{aligned}$$

with the terms $\mathbb{C}_i$ above defined as in appendix A.4.

As shown in the appendix, when replacing consumption with output the dis-

crete version of the system reads

$$\ln(Y_t/Y_{t-\Delta}) = \int_{t-\Delta}^t r_v^f dv + \kappa\gamma \int_{t-\Delta}^t 1/(r_v^f + \delta + \sigma^2) dv \quad (29a)$$

$$\begin{aligned} & -\frac{1}{2}\eta^2 \int_{t-\Delta}^t 1/(r_v^f + \delta + \sigma^2)^2 dv \\ & -(\kappa + \rho - \frac{1}{2}\sigma^2)\Delta + \varepsilon_{Y,t} \end{aligned} \quad (29b)$$

$$\begin{aligned} \ln(P_{d,t}/P_{d,t-\Delta}) &= \int_{t-\Delta}^t r_v^f dv - (\rho - \frac{1}{2}\sigma^2)\Delta \\ &+ \rho\kappa \int_{t-\Delta}^t \left(\frac{\gamma - \delta - \sigma^2}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv \\ &- \rho\kappa \int_{t-\Delta}^t \left(\frac{r_v^f}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv \\ &- \frac{1}{2}(\rho\eta)^2 \int_{t-\Delta}^t \frac{1}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]^2} dv + \varepsilon_{P_d,t} \\ r_t^f &= e^{-\kappa\Delta} r_{t-\Delta}^f + (1 - e^{-\kappa\Delta})(\gamma - \delta - \sigma^2) + \varepsilon_{r,t} \end{aligned} \quad (29c)$$

We define m_t the (3x1) vector of martingale difference sequence as

$$m_t = \begin{pmatrix} \ln(Y_t/Y_{t-\Delta}) - \int_{t-\Delta}^t r_v^f dv - \kappa\gamma \int_{t-\Delta}^t 1/(r_v^f + \delta + \sigma^2) dv \\ \quad + \frac{1}{2}\eta^2 \int_{t-\Delta}^t 1/(r_v^f + \delta + \sigma^2)^2 dv + (\kappa + \rho - \frac{1}{2}\sigma^2)\Delta \\ \ln(P_{d,t}/P_{d,t-\Delta}) - \int_{t-\Delta}^t r_v^f dv + (\rho - \frac{1}{2}\sigma^2)\Delta - \rho\kappa \int_{t-\Delta}^t \left(\frac{\gamma - \delta - \sigma^2}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv \\ \quad + \rho\kappa \int_{t-\Delta}^t \left(\frac{r_v^f}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv + \frac{1}{2}(\rho\eta)^2 \int_{t-\Delta}^t \frac{1}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]^2} dv \\ r_t^f - e^{-\kappa\Delta} r_{t-\Delta}^f - (1 - e^{-\kappa\Delta})(\gamma - \delta - \sigma^2) \end{pmatrix} \quad (30)$$

with the (3x3) conditional covariance matrix

$$\Psi_t = \begin{pmatrix} \eta^2\Delta/(r_{t-\Delta}^f + \delta + \sigma^2)^2 + \sigma^2\Delta & \Psi_{t,12} & \Psi_{t,13} \\ \Psi_{t,21} & \Psi_{t,22} & \Psi_{t,23} \\ \Psi_{t,31} & \Psi_{t,32} & \eta^2(1 - e^{-2\kappa\Delta})/2\kappa \end{pmatrix} \quad (31)$$

where

$$\begin{aligned} \Psi_{t,12} &= \Psi_{t,21} = (\rho\eta^2\Delta)/[\rho(r_{t-\Delta}^f + \delta + \sigma^2)^2 + \kappa\gamma(r_{t-\Delta}^f + \delta + \sigma^2)] + \sigma^2\Delta \\ \Psi_{t,13} &= \Psi_{t,31} = \eta^2 e^{-\kappa\Delta} \Delta / (r_{t-\Delta}^f + \delta + \sigma^2) \\ \Psi_{t,22} &= (\rho\eta)^2 \Delta / [\rho(r_{t-\Delta}^f + \delta + \sigma^2) + \kappa\gamma]^2 + \sigma^2\Delta \\ \Psi_{t,23} &= \Psi_{t,32} = \rho\eta^2 e^{-\kappa\Delta} \Delta / [\rho(r_{t-\Delta}^f + \delta + \sigma^2) + \kappa\gamma] \end{aligned}$$

3.4 GMM Estimation

Applying GMM estimation to our model is relatively straightforward. The vector of Instruments, z_t , used in the GMM estimation consists of lagged right-hand variables. We consider both GMM using only first moments as well as second moments and additionally check our results by considering different parameter restrictions. Finally, since using the dividend claim in this context is a new approach, we also test different time series for the dividend claim.

4 Simulation Study

In order to examine the small sample properties of our estimation procedures we conduct simulation studies for the different versions of the model. We simulate 25 years of data for the short rate, consumption, output and the dividend claim from the model. The median estimates as well as the interquartile range for 1000 replications are reported in the tables below. Additionally, the parameter values used in the data generating process are given in the first columns. These values are also highlighted by red bars in the histograms of this section. Above each table a short description of the used system settings can be found. At the beginning of each column the used building blocks of the model are specified. Here, the terms *Int*, *Claim*, *AltClaim*, *Cons* and *Out* denote the used estimation equations for the interest rate, the dividend claim, the alternative dividend claim formulation, consumption and output respectively.

We start our estimations from the most simple system of the model and successively add or remove additional macro and financial variables. By doing so we are able to show how the parameter estimates behave when incorporating macro economic dynamics into our financial systems of estimation equations. Additionally, we examine identification and small properties of different macro-finance formulations.

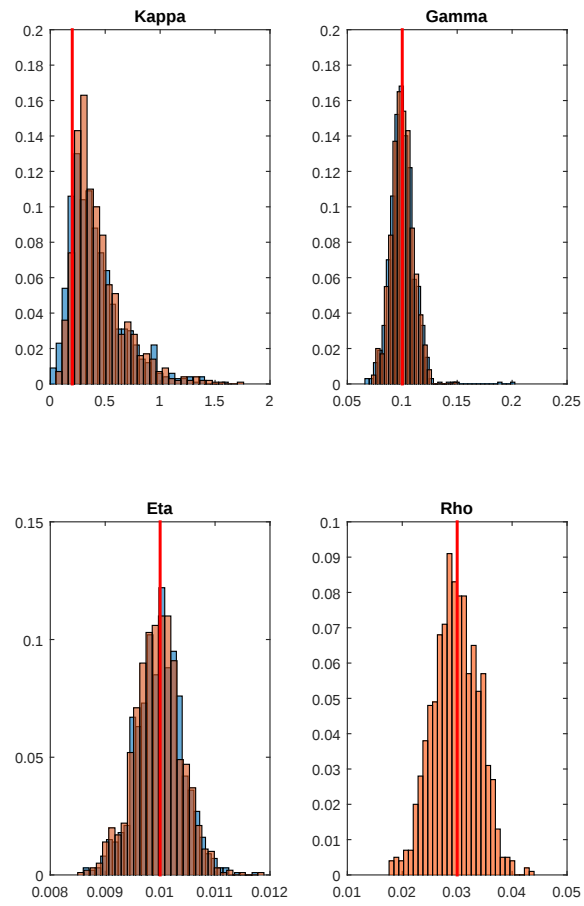
4.1 Simulation Study: GMM Using Second Moments

Simulation Study GMM Estimation						
Median estimate						
	Sytem1 Int	System2 Int Claim	System3 Int Cons	System4 Int Cons Out	System5 Int Cons Claim	System6 Int Cons AltClaim
$\kappa = 0.2$	0.3563 (0.2859)	0.3749 (0.2631)	0.3554 (0.2927)	0.3117 (0.2790)	0.2121 (0.0641)	0.2083 (0.0643)
$\gamma = 0.1$	0.0996 (0.0127)	0.0998 (0.0125)	0.0100 (0.0125)	0.0997 (0.0144)	0.1000 (0.0129)	0.0999 (0.0132)
$\eta = 0.01$	0.0100 (0.0006)	0.0100 (0.0005)	0.0100 (0.0005)	0.0097 (0.0006)	0.0099 (0.0006)	0.0098 (0.0007)
$\rho = 0.03$		0.0297 (0.0059)	0.0300 (0.0055)	0.0301 (0.0056)	0.0297 (0.0059)	0.0296 (0.0060)
$\delta = 0.05$	0.05	0.05	0.05	0.05	0.05	0.05
$\sigma = 0.02$	0.02	0.02	0.0197 (0.0011)	0.0194 (0.0012)	0.0198 (0.0013)	0.0199 (0.0013)
<i>The interquartile range is given below the estimates</i>						

When performing GMM estimation of the small-scale finance version of our model, the simulation study suggest that we can identify 3 of the 5 model parameters contained in the estimation equation for the interest rate. When fixing δ and σ at their true values we obtain the results reported in the second column of the table above. While the estimates for γ and η are extremely close to their values used in the data generating process, the parameter capturing the speed of mean-reversion, κ is heavily upward biased. Nevertheless, this bias, as extensively discussed by Christensen, Posch & van der Wel (2016), is a common feature in the estimation of mean-reverting models.

Figure (1) plots the histograms for the first two systems of the above table. The blue shaded histogram correspond to the case where only the interest rate equation is used in the estimation, while the orange histogram is obtained from additionally considering the estimation equation for the claim. Note that both systems are completely cast in terms of financial variables. When adding the differential for the dividend claim to the small-scale finance system we still rely on two parameter restrictions but are now able to accurately estimate the parameter of the subjective rate of time preference, ρ . As can be seen in figure (1), incorporating the claim does not significantly alters the estimation results for the remaining two parameters.

Figure 1: Simulation Study, GMM, Histograms System1 and System2



To highlight the effects of using additional macro economic variables in our estimation, figure (2) plots the histograms for system2 and system5. By adding consumption as a macro economic variable the GMM approach is able to identify 5 of the 6 structural model parameters. Hence, in our framework adding macro dynamics to the finance systems improves the identification. The estimates for γ , η and ρ are again not significantly altered by increasing the number of estimation equations. Most interestingly, however, is the behaviour of κ . When combining the claim and consumption estimation equations the upward bias nearly vanishes. As can be seen below this observation holds true for both GMM as well as MEF estimation. Even when substituting parameters with consumption data, as done in the alternative claim formulation in system6, we only obtain marginally upward biased estimates. In fact, the macro-finance system consisting of the interest rate, the (alternative) claim and consumption is the only combination of considered estimation equations with this property. The complete finance as well as all other macro-finance systems are all characterized by an substantially upward biased κ . Finally, figure (3) shows the histograms for systems 3 and 4 of above table.

Figure 2: Simulation Study, GMM, Histograms System2 and System5

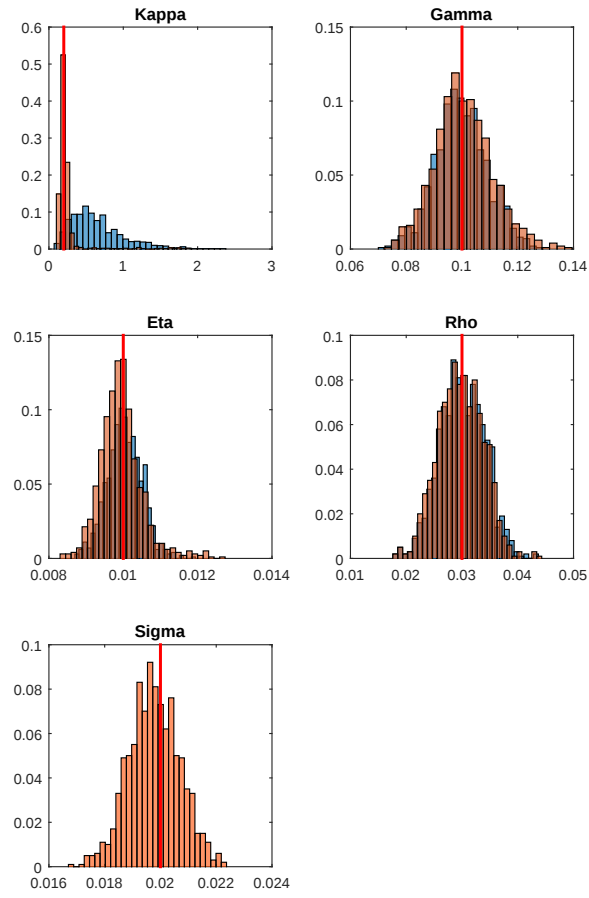
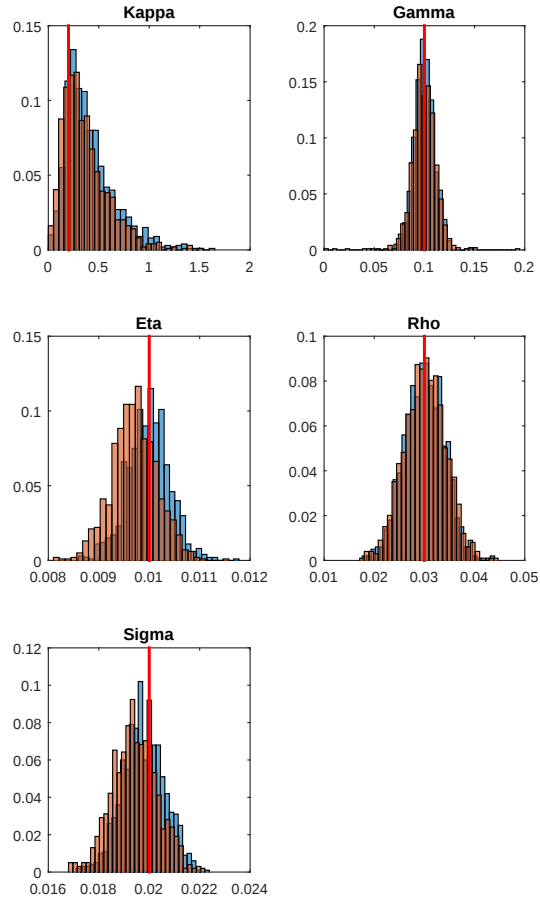


Figure 3: Simulation Study, GMM, Histograms Sytem3 and System4



The blue shaded histogram corresponds to the small-scale macro-finance system build around the interest rate and consumption equations, while the orange one belongs to system4. In this system the small-scale macro-finance system 3 is extended by the differential for output. As can be seen above, incorporating output as additional estimation equation does not significantly alter the estimation results when it comes to GMM estimation including second moments. There appears to be only a marginal downward bias in the median estimates for the two variance parameters η and σ .

4.2 Simulation Study: MEF Using Second Moments

Following exactly the same structure as in the previous section the tables and histograms below report the results from the simulation study for the MEF estimation approach exploiting second moments.

Simulation Study MEF Estimation						
Median estimate						
	System2	System3	System4	System5	System7	System8
	Int Claim	Int Cons	Int Cons Out	Int Cons Claim	Int Cons Out Claim	Int Cons AltOut AltClaim
$\kappa = 0.2$	0.3334 (0.2768)	0.3454 (0.2685)	0.2983 (0.3768)	0.2018 (0.0462)	0.3382 (0.2776)	0.3506 (0.2722)
$\gamma = 0.1$	0.0989 (0.0133)	0.0992 (0.0127)	0.01001 (0.0148)	0.0994 (0.0134)	0.1012 (0.0142)	0.0993 (0.0128)
$\eta = 0.01$	0.0100 (0.0005)	0.0101 (0.0005)	0.0101 (0.0006)	0.0100 (0.0005)	0.0101 (0.0005)	0.0101 (0.0005)
$\rho = 0.03$	0.0300 (0.0054)	0.0299 (0.0054)	0.0300 (0.0055)	0.0301 (0.0054)	0.0300 (0.0055)	0.0300 (0.0055)
$\delta = 0.05$	0.05	0.05	0.0500 (0.0019)	0.05	0.0521 (0.0034)	0.0500 (0.0016)
$\sigma = 0.02$	0.0200	0.0200 (0.0011)	0.0200 (0.0011)	0.0200 (0.0011)	0.0200 (0.0011)	0.0200 (0.0011)
<i>The interquartile range is given below the estimates</i>						

5 Data and Results

In order to estimate the different systems of equilibrium equations of our model we need data on consumption, the short-term interest rate and on the price of

the claim on future dividends. We consider the time period from January 1982 to December 2012. Data on consumption and the short rate is obtained from the Federal Reserve Economic Dataset (FRED). The monthly level of real Personal Consumption Expenditures (PCE) is used as a proxy for consumption. Following Christensen, Posch & van der Wel (2016), we use the 3-month interest rate, derived from US treasury bonds as proxy for the risk-free rate. For the claim on future dividends we use monthly data on the S&P500 obtained from the Center for Research in Security Prices (CRSP). This rich data set offers time series for differently computed returns and index levels ranging from 1925 up to January 2016. For our purpose return data with dividends either in- or excluded are of great interest, since they allow us to pin down suitable real world analogues for the price equation of the dividend claim. For the empirical estimation we consider 3 different time series constructed from the CRSP data set. The first time series consists of the equal-weighted return of the S&P500 including dividends, the second time series is constructed as the change of the market value (price) of stocks used in the S&P500. The third time series is used mainly as a robustness check and is computed as the difference of the return of the equal-weighted return including and excluding dividends.

In the empirical estimation we follow the same approach as in the simulation study. We start by estimating our two complete finance versions and successively add and remove additional macro and finance variables. The tables below show the results from the empirical GMM estimations. Above each table a short description of the used system settings as well as the chosen claim data can be found. At the beginning of each column the used building blocks of the model are specified. Again, the terms *Int*, *Claim*, *AltClaim*, *Cons* and *Out* denote the used estimation equations for the interest rate, the dividend claim, the alternative dividend claim formulation, consumption and output respectively. Additionally, asymptotic t-statistics are given below the estimates. We start by exploiting second moments in the GMM estimation.

Empirical GMM Estimation Results						
Claim Data: Equal weighted Return Inc. Dividends						
	System1	System2	System3	System4	System5	System6
	Int	Int Claim	Int Cons	Int Cons Out	Int Cons Claim	Int Cons AltClaim
κ	0.0431 (0.0344)	0.0229 (101.2)	0.2046 (0.149)	0.0385 (0.2444)	0.0587 (164.9)	0.0909 (6.1388)
γ	0.0378 (0.0211)	0.11 (98.82)	0.0745 (0.5768)	0.051 (0.3195)	0.0401 (168.1)	0.0327 (5.3258)
η	0.0084 (0.9)	0.0119 (1.1134)	0.0092 (1.0735)	0.0051 (0.7124)	0.0095 (3.5107)	0.0101 (23.36)
ρ		0.0011 (>200)	0.0083 (0.2048)	0.005 (0.1408)	0.0005 (>200)	0.0044 (5.8864)
δ	0.05	0.05	0.05	0.05	0.05	0.05
σ	0.02	0.02	0.0149 (0.8068)	0.0129 (0.9532)	0.0152 (14.667)	0.01673 (49.693)
<i>Asymptotic t-statistics are given below the estimates</i>						

One important observation that is present in all of the considered systems of estimation equations is the relatively low value of the parameter of time preference, ρ . The highest values of ρ are observed when using the artificially constructed time series for the dividend claim. Nevertheless, this low value of ρ is inline with the empirical estimation results of

As shown in the simulation study, the GMM estimation approach is able to identify κ , γ and η when considering only the equilibrium equation for the interest rate. Hence, we have to restrict the remaining 2 parameters of the model. As before, the parameter ρ is not contained in this small-scale finance system. The standard errors are quite high in this setting, causing the asymptotic t-statistics to be low. When adding the equilibrium equation of the claim on future dividends to our

system of equilibrium equations, we still rely on two parameter restrictions but are now able to identify ρ . Estimating this complete finance version of the model yields plausible parameter estimates with very low asymptotic standard errors. Nevertheless, γ appears to be a little high but still in line with similar estimates in the literature. The parameter estimate for ρ on the other hand seems to be unrealistically low, as it suggest a time preference rate well below one percent. In column 4 we are adding consumption instead of the claim as an additional macroeconomic variable to the small-scale finance system. By doing so, the simulation study suggest that all 6 parameters except for δ can be identified by our GMM estimation approach. The estimation of this macro-finance model yields plausible parameter estimates for nearly all 5 parameters. Nevertheless, there are again problems resulting from low asymptotic standard errors. When adding the estimation equation for output to this macro-finance model the results do not change much, but imply a more plausible log-term mean of the interest rate, γ . This behaviour is also backed by simulation study results. Even the slightly lower estimates of the two variance terms η and σ are present in the empirical estimates as well.

Finally, the last two columns show the effects of adding an additional finance variable to the small-scale macro-finance model consisting of the estimation equations for consumption and the interest rate. Both claim formulation drastically reduce the estimated value of γ while at the same time keeping nearly all other parameters at plausible values. Furthermore, the asymptotic standard errors are considerably lower compared to the previous macro-finance estimations in columns 4 and 5. When it comes to plausibility as well as accuracy as measured by the magnitudes of asymptotic standard errors, the macro-finance formulations of the last two columns outperform the previous estimation approaches. When using the other two time series for the dividend claim, we obtain similar results. This appears to be not that surprisingly since both time-series are based on the performance of the S&P500. Finally, our results are also robust to alternative parameter restrictions. The following tables show these cases.

6 Empirical Results: MEF

7 Conclusion

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A Appendix A

A.1 Properties and Derivations for the Stochastic Discount Factor

Starting from equation (8) we apply Ito's formula to obtain the evolution of $\ln(\Lambda_t)$:

$$\begin{aligned} d\ln(\Lambda_t) &= \frac{1}{\Lambda_t}(d\Lambda_t) - \frac{1}{2} \frac{1}{\Lambda_t^2} (d\Lambda_t)^2 \\ &= -(r_t - \delta - \frac{1}{2}\sigma^2)dt - \sigma dZ_t \end{aligned}$$

Integrating yields:

$$\int_t^s d\ln(\Lambda_v)dv = - \int_t^s (r_v - \delta - \frac{1}{2}\sigma^2)dv - \sigma \int_t^s dZ_v$$

From which we obtain the stochastic discount factor as the process given by equation (9).

Now to compute the expected value of the SDF we start from equation (2). Since this is an Ornstein-Uhlenbeck process we can find the solution by using a standard technique in differential equations as shown below.

$$\begin{aligned} e^{\kappa t}(dr_t + \kappa r_t)dt &= e^{\kappa t}\kappa\gamma dt + e^{\kappa t}\eta dB_t \\ \int_t^s (dr_t e^{\kappa u}) &= \int_t^s (d\gamma e^{\kappa u}) + \eta \int_t^s e^{\kappa u} dB_u \\ e^{\kappa s}r_s - e^{\kappa t}r_t &= e^{\kappa s}\gamma - e^{\kappa t}\gamma + \eta \int_t^s e^{\kappa u} dB_u \\ r_s &= e^{-\kappa(s-t)}r_t + (1 - e^{-\kappa(s-t)})\gamma + \eta e^{-\kappa(s-t)} \int_t^s e^{\kappa(u-t)} dB_u \end{aligned}$$

Note that in order to obtain the expected value of the stochastic discount factor we employ log-normality and compute

$$\ln E_t [e^{\ln(\Lambda_s) - \ln(\Lambda_t)}] = E_t[\ln(\Lambda_s) - \ln(\Lambda_t)] + \frac{1}{2} \text{Var}_t[\ln(\Lambda_s) - \ln(\Lambda_t)] \quad (32)$$

We can now plug our solution for r_s into our log expression for the stochastic discount factor and obtain

$$\begin{aligned}
\ln(\Lambda_s) - \ln(\Lambda_t) &= - \int_t^s r_v dv + \int_t^s (\delta + \frac{1}{2}\sigma^2) dv - \sigma \int_t^s dZ_v \\
&= - \int_t^s (e^{-\kappa(v-t)} r_t + (1 - e^{-\kappa(v-t)})\gamma - \delta - \frac{1}{2}\sigma^2) dv \\
&\quad - \eta \int_t^s e^{-\kappa(v-t)} \int_t^v e^{\kappa(u-t)} dB_u dv - \sigma \int_t^s dZ_v
\end{aligned}$$

Reversing the order of integration and evaluating the ds integrals yield

$$\ln(\Lambda_s) - \ln(\Lambda_t) = -\frac{r_t - \gamma}{\kappa}(1 - e^{-\kappa(s-t)}) - (\gamma - \delta - \frac{1}{2}\sigma^2)(s-t) - \frac{\eta}{\kappa} \int_t^s (1 - e^{-\kappa(s-u)}) dB_u - \sigma \int_t^s dZ_v$$

Inspection of the last two integrals give rise to a normally distributed random variable with mean zero and variance

$$\begin{aligned}
Var_t[\ln(\Lambda_s) - \ln(\Lambda_t)] &= \int_t^s \left(\frac{\eta}{\kappa}(1 - e^{-\kappa(s-u)}) \right)^2 du + \int_t^s \sigma^2 du \\
&= \left(\left(\frac{\eta}{\kappa} \right)^2 + \sigma^2 \right) (s-t) - 2\frac{\eta^2}{\kappa^3}(1 - e^{-\kappa(s-t)}) + \frac{\eta^2}{2\kappa^3}(1 - e^{-2\kappa(s-t)})
\end{aligned}$$

And

$$E_t[\ln(\Lambda_s) - \ln(\Lambda_t)] = -\frac{r_t - \gamma}{\kappa}(1 - e^{-\kappa(s-t)}) - (\gamma - \delta - \frac{1}{2}\sigma^2)(s-t)$$

Thus by plugging in we conclude

$$\ln E_t [e^{\ln \Lambda_s - \ln \Lambda_t}] = - \left(\frac{r_t - \gamma}{\kappa} + \frac{\eta^2}{\kappa^3} \right) (1 - e^{-\kappa(s-t)}) - \left(\gamma - \delta - \sigma^2 - \frac{1}{2} \frac{\eta^2}{\kappa^2} \right) (s-t) + \frac{\eta^2}{4\kappa^3} (1 - e^{-2\kappa(s-t)})$$

From which we obtain (11).

A.2 Properties and Derivations for the Claim on Future Dividends

To obtain the expression for period k 's capital stock given by (15) we use the SDF given by (9) together with the basic pricing equation (12). We start with

the equation for the evolution of capital, where we substitute I_t to express dK_t in terms of C_t and r_t

$$\begin{aligned} dK_t &= (I_t - \delta K_t) + \sigma K_t dZ_t, \\ &= (r_t K_t - C_t - \delta K_t)dt + \sigma K_t dZ_t \end{aligned}$$

where $I_t = Y_t K_t - C_t = r_t K_t - C_t$.

Now we use Ito's formula to derive an expression for $d \ln(K_t)$

$$\begin{aligned} d \ln(K_t) &= \frac{1}{K_t} dK_t - \frac{1}{2} \frac{1}{K_t^2} (dK_t)^2 \\ &= \frac{1}{K_t} (r_t K_t - C_t - \delta K_t) + \frac{1}{K_t} \sigma K_t dZ_t - \frac{1}{2} \frac{1}{K_t^2} \sigma^2 K_t^2 dt \\ &= (r_t - \frac{C_t}{K_t} - \delta - \frac{1}{2} \sigma^2) dt + \sigma dZ_t \\ &= (r_t - \rho - \delta - \frac{1}{2} \sigma^2) dt + \sigma dZ_t \end{aligned}$$

Where we used the closed-form solution $\rho = C_t/K_t$.

Now, to obtain an expression for K_s integrate over t to s :

$$\begin{aligned} \int_t^s d \ln(K_t) dt &= \int_t^s (r_v - \rho - \delta - \frac{1}{2} \sigma^2) dv + \sigma \int_t^s dZ_v \\ \ln(K_s) - \ln(K_t) &= \int_t^s [r_v - \rho - \delta - \frac{1}{2} \sigma^2] dv + \sigma \int_t^s dZ_v \\ K_s &= K_t e^{\int_t^s (r_v - \rho - \delta - \frac{1}{2} \sigma^2) dv + \sigma \int_t^s dZ_v} \end{aligned}$$

which is the same as (15).

Now to obtain an expression for the capital stock in period s recall that in the AK-Vasicek model A_t is equal to r_t . Thus, the evolution of TFP is captured by the evolution of r_t . Furthermore, due to the AK-specification we have $Y_t = A_t K_t$. The evolution of Y_t , can be described in terms of dr_t and dK_t

$$\begin{aligned} dK_t &= (r_t K_t - C_t - \delta K_t)dt + \sigma K_t dZ_t \\ dr_t &= \kappa(\gamma - r_t)dt + \eta dB_t \end{aligned}$$

together with the Euler equation, or using the analytical solution $C_t = \rho K_t$.

Now, using Ito's formula to derive an expression for $d \ln(K_t r_t)$ we arrive at (16) by computing

$$\begin{aligned} d \ln(K_t r_t) &= \frac{1}{K_t}(dK_t) - \frac{1}{2} \frac{1}{K_t^2}(dK_t)^2 + \frac{1}{r_t}(dr_t) - \frac{1}{2} \frac{1}{r_t^2}(dr_t)^2 \\ d \ln(Y_t) &= (r_t - \delta - \rho - \frac{1}{2}\sigma^2 + \frac{\kappa\gamma}{r_t} - \kappa - \frac{1}{2} \frac{\eta^2}{r_t^2})dt + \sigma dZ_t + \frac{\eta}{r_t} dB_t \\ Y_s &= Y_t e^{\kappa\gamma \int_t^s \frac{1}{r_v} dv - \frac{\eta^2}{2} \int_t^s \frac{1}{r_v^2} dv + \int_t^s (r_v - \delta - \rho - \kappa - \frac{1}{2}\sigma^2) dv + \sigma \int_t^s dZ_v + \int_t^s \frac{\eta}{r_v} dB_v} \end{aligned}$$

The last equation is analogous to (15) in Christensen, Posch & van der Wel (2016). In the derivation of (17), however, we will exploit the Ornstein Uhlenbeck specification. Considering the Vasicek specification of the interest rate we know that:

$$A_s = r_s = e^{-\kappa(s-t)} \left(r_t + (e^{\kappa(s-t)} - 1)\gamma + \eta \int_t^s e^{\kappa(u-t)} dB_u \right)$$

While period's s capital stock is given by:

$$K_s = K_t e^{\int_t^s (r_v - \rho - \delta - \frac{1}{2}\sigma^2) dv + \sigma \int_t^s dZ_v}$$

Hence, Y_s is given by

$$A_s K_s = \left[K_t r_t + K_t \gamma e^{\kappa(s-t)} - K_t \gamma + K_t \eta \int_t^s e^{\kappa(u-t)} dB_u \right] e^{\int_t^s (r_v - \delta - \rho - \kappa - \frac{1}{2}\sigma^2) dv + \sigma \int_t^s dZ_v}$$

Starting from equation (3.2) note that

$$\begin{aligned} P_{d,t} &= K_t \left[\frac{r_t - \gamma}{(\rho + \kappa)} + \frac{\gamma}{\rho} \right] \\ &= dK_t \left[\frac{r_t - \gamma}{(\rho + \kappa)} + \frac{\gamma}{\rho} \right] + \frac{K_t}{(\rho + \kappa)} dr_t \\ &= P_{d,t} \frac{dK_t}{K_t} + \frac{K_t}{(\rho + \kappa)} dr_t \\ &= P_{d,t} [r_t - \rho - \delta] dt + P_{d,t} \sigma dZ_t + \frac{K_t}{(\rho + \kappa)} [\kappa(\gamma - r_t)] dt + \frac{K_t}{(\rho + \kappa)} \eta dB_t \end{aligned}$$

To obtain an expression for the last term in terms of $P_{d,t}$ note that:

$$\frac{K_t}{\rho + \kappa} = \frac{\rho P_{d,t}}{\rho r_t + \kappa \gamma}$$

or

$$\frac{K_t}{\rho + \kappa} = P_{d,t} \left[\frac{1}{r_t + \frac{\kappa\gamma}{\rho}} \right]$$

Plugging in yields:

$$dP_{d,t} = P_{d,t}[r_t - \rho - \delta]dt + P_{d,t}\sigma dZ_t + P_{d,t} \frac{[\rho\kappa(\gamma - r_t)]}{\rho r_t + \kappa\gamma} dt + P_{d,t} \frac{\rho\eta}{\rho r_t + \kappa\gamma} dB_t$$

or in logs:

$$d \ln P_{d,t} = d \ln K_t + \left[\frac{\rho\kappa(\gamma - r_t)}{[\rho r_t + \kappa\gamma]} - \frac{1}{2} \frac{(\rho\eta)^2}{[\rho r_t + \kappa\gamma]^2} \right] dt + \frac{\rho\eta}{[\rho r_t + \kappa\gamma]} dB_t$$

A.3 Derivations of the Discrete time formulations

In this section we will derive the discrete time formulations for the system of equilibrium equations given by (24). We start from the baseline model and substitute the equation for log output by the equation for the log price of the claim on future dividends

Hence we have for the claim on future dividends

$$\begin{aligned} d \ln P_{d,t} &= d \ln K_t + \left[\frac{\rho\kappa(\gamma - r_t)}{[\rho r_t + \kappa\gamma]} - \frac{1}{2} \frac{(\rho\eta)^2}{[\rho r_t + \kappa\gamma]^2} \right] dt + \frac{\rho\eta}{[\rho r_t + \kappa\gamma]} dB_t \\ &= \left[\frac{\rho\kappa(\gamma - r_t^f - \delta - \sigma^2)}{[\rho(r_t^f + \delta + \sigma^2) + \kappa\gamma]} - \frac{1}{2} \frac{(\rho\eta)^2}{[\rho(r_t^f + \delta + \sigma^2) + \kappa\gamma]^2} \right] dt \\ &\quad + (r_t^f - \rho + \frac{1}{2}\sigma^2)dt + \frac{\rho\eta}{[\rho(r_t^f + \delta + \sigma^2) + \kappa\gamma]} dB_t + \sigma dZ_t \end{aligned}$$

now integrating over $(t - \Delta)$ to t we obtain

$$\begin{aligned} \ln(P_{d,t}/P_{d,t-\Delta}) &= \int_{t-\Delta}^t r_v^f dv - (\rho - \frac{1}{2}\sigma^2)\Delta + \rho\kappa \int_{t-\Delta}^t \frac{\gamma - r_v^f - \delta - \sigma^2}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} dv \\ &\quad - \frac{1}{2}(\rho\eta)^2 \int_{t-\Delta}^t \frac{1}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]^2} dv \\ &\quad + \rho\eta \int_{t-\Delta}^t \frac{1}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]} dB_v + \sigma \int_{t-\Delta}^t dZ_v \end{aligned}$$

Since we just want to substitute this equation in the baseline model we obtain the discrete version of (24) as

$$\begin{aligned}
\ln(C_t/C_{t-\Delta}) &= \int_{t-\Delta}^t r_v^f dv - (\rho - \frac{1}{2}\sigma^2)\Delta + \varepsilon_{C,t} \\
\ln(P_{d,t}/P_{d,t-\Delta}) &= \int_{t-\Delta}^t r_v^f dv - (\rho - \frac{1}{2}\sigma^2)\Delta + \rho\kappa \int_{t-\Delta}^t \left(\frac{\gamma - r_v^f - \delta - \sigma^2}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv \\
&\quad - \frac{1}{2}(\rho\eta)^2 \int_{t-\Delta}^t \frac{1}{\left[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma \right]^2} dv + \varepsilon_{P_d,t} \\
r_t^f &= e^{-\kappa\Delta} r_{t-\Delta}^f + (1 - e^{-\kappa\Delta})(\gamma - \delta - \sigma^2) + \varepsilon_{r,t}
\end{aligned}$$

where the martingale increments are defined by

$$\begin{aligned}
\varepsilon_{C,t} &\equiv \sigma(Z_t - Z_{t-\Delta}) \\
\varepsilon_{d,t} &\equiv \rho\eta \int_{t-\Delta}^t \frac{1}{\left[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma \right]} dB_v + \sigma \int_{t-\Delta}^t dZ_v \\
\varepsilon_{r,t} &\equiv \eta e^{-\kappa\Delta} \int_{t-\Delta}^t e^{\kappa(v-(t-\Delta))} dB_v
\end{aligned}$$

Using the above calculations together with the expression for log output of the discrete version of system (??) we obtain the discrete version of system (23) as

$$\begin{aligned}
\ln(Y_t/Y_{t-\Delta}) &= \int_{t-\Delta}^t r_v^f dv + \kappa\gamma \int_{t-\Delta}^t 1/(r_v^f + \delta + \sigma^2) dv \\
&\quad - \frac{1}{2}\eta^2 \int_{t-\Delta}^t 1/(r_v^f + \delta + \sigma^2)^2 dv \\
&\quad - (\kappa + \rho - \frac{1}{2}\sigma^2)\Delta + \varepsilon_{Y,t} \\
\ln(P_{d,t}/P_{d,t-\Delta}) &= \int_{t-\Delta}^t r_v^f dv - (\rho - \frac{1}{2}\sigma^2)\Delta \\
&\quad + \rho\kappa \int_{t-\Delta}^t \left(\frac{\gamma - \delta - \sigma^2}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv \\
&\quad - \rho\kappa \int_{t-\Delta}^t \left(\frac{r_v^f}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv \\
&\quad - \frac{1}{2}(\rho\eta)^2 \int_{t-\Delta}^t \frac{1}{\left[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma \right]^2} dv + \varepsilon_{P_d,t} \\
r_t^f &= e^{-\kappa\Delta} r_{t-\Delta}^f + (1 - e^{-\kappa\Delta})(\gamma - \delta - \sigma^2) + \varepsilon_{r,t}
\end{aligned}$$

and define the vector of martingale increments as

$$\varepsilon_t = \begin{pmatrix} \varepsilon_{Y,t} \\ \varepsilon_{d,t} \\ \varepsilon_{r,t} \end{pmatrix} = \begin{pmatrix} \int_{t-\Delta}^t \frac{\eta}{[r_v^f + \delta + \sigma^2]} dB_v + \sigma \int_{t-\Delta}^t dZ_v \\ \rho\eta \int_{t-\Delta}^t \frac{1}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]} dB_v + \sigma \int_{t-\Delta}^t dZ_v \\ \eta e^{-\kappa(s-t)} \int_{t-\Delta}^t e^{\kappa(v-(t-\Delta))} dB_v \end{pmatrix}$$

A.4 MEF

The matrix of parameter derivatives reads

$$\Phi_t = \begin{pmatrix} 0 & \Phi_{12} & \Delta e^{-\kappa\Delta}(r_{t-\Delta} - \gamma) \\ 0 & \Phi_{22} & -(1 - e^{-\kappa\Delta}) \\ 0 & \Phi_{32} & 0 \\ \Delta & \Phi_{42} & 0 \\ 0 & \Phi_{52} & (1 - e^{-\kappa\Delta}) \\ -\sigma\Delta & \Phi_{62} & 2\sigma(1 - e^{-\kappa\Delta}) \end{pmatrix}$$

where

$$\begin{aligned} \Phi_{12} = \frac{\partial m_2}{\partial \kappa} &= -\rho(\gamma - \delta - \sigma^2) \int_{t-\Delta}^t \left(\frac{1}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv - \gamma(\rho\eta)^2 \int_{t-\Delta}^t \frac{1}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]^3} dv \\ &\quad + \rho\kappa\gamma(\gamma - \delta - \sigma^2) \int_{t-\Delta}^t \left(\frac{1}{(\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma)^2} \right) dv \\ &\quad + \rho \int_{t-\Delta}^t \left(\frac{r_v^f}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv - \rho\kappa\gamma \int_{t-\Delta}^t \left(\frac{r_v^f}{(\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma)^2} \right) dv \\ \Phi_{22} = \frac{\partial m_2}{\partial \gamma} &= -\rho\kappa \int_{t-\Delta}^t \left(\frac{1}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv + \rho\kappa^2(\gamma - \delta - \sigma^2) \int_{t-\Delta}^t \left(\frac{1}{(\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma)^2} \right) dv \\ &\quad - \rho\kappa^2 \int_{t-\Delta}^t \left(\frac{r_v^f}{(\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma)^2} \right) dv - \kappa(\rho\eta)^2 \int_{t-\Delta}^t \frac{1}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]^3} dv \\ \Phi_{32} = \frac{\partial m_2}{\partial \eta} &= \eta\rho^2 \int_{t-\Delta}^t \frac{1}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]^2} dv \end{aligned}$$

$$\begin{aligned}
\Phi_{42} = \frac{\partial m_2}{\partial \rho} &= -\kappa(\gamma - \delta - \sigma^2) \int_{t-\Delta}^t \left(\frac{1}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv \\
&+ \rho\kappa(\gamma - \delta - \sigma^2) \int_{t-\Delta}^t \left(\frac{r_v^f + \delta + \sigma^2}{(\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma)^2} \right) dv \\
&+ \kappa \int_{t-\Delta}^t \frac{r_v^f}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]} dv - \rho\kappa \int_{t-\Delta}^t \frac{r_v^f(r_v^f + \delta + \sigma^2)}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]^2} dv + \Delta \\
&+ \rho\eta^2 \int_{t-\Delta}^t \frac{1}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]^2} dv - (\rho\eta)^2 \int_{t-\Delta}^t \frac{r_v^f + \delta + \sigma^2}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]^3} dv
\end{aligned}$$

$$\begin{aligned}
\Phi_{52} = \frac{\partial m_2}{\partial \delta} &= \rho\kappa \int_{t-\Delta}^t \left(\frac{1}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv + \rho^2\kappa(\gamma - \delta - \sigma^2) \int_{t-\Delta}^t \left(\frac{1}{(\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma)^2} \right) dv \\
&- \rho^2\kappa \int_{t-\Delta}^t \frac{r_v^f}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]^2} dv - \rho^3\eta^2 \int_{t-\Delta}^t \frac{1}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]^3} dv
\end{aligned}$$

$$\begin{aligned}
\Phi_{62} = \frac{\partial m_2}{\partial \sigma} &= 2\rho\kappa\sigma \int_{t-\Delta}^t \left(\frac{1}{\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma} \right) dv + 2\rho^2\kappa\sigma(\gamma - \delta - \sigma^2) \int_{t-\Delta}^t \left(\frac{1}{(\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma)^2} \right) dv \\
&- 2\rho^2\kappa\sigma \int_{t-\Delta}^t \frac{r_v^f}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]^2} dv - 2\rho^3\eta^2\sigma \int_{t-\Delta}^t \frac{1}{[\rho(r_v^f + \delta + \sigma^2) + \kappa\gamma]^3} dv - \sigma\Delta
\end{aligned}$$

$$\mathbb{C}_1 = \left(\frac{\Delta}{\rho r_{t-\Delta} + \kappa\gamma} - \left(\frac{\kappa(\gamma - r_{t-\Delta})\rho}{(\rho r_{t-\Delta} + \kappa\gamma)^2} - \frac{\eta^2 \rho^2}{(\rho r_{t-\Delta} + \kappa\gamma)^3} \right) \frac{1}{2} \Delta^2 \right)$$

$$\mathbb{C}_2 = \left(\frac{\Delta}{(\rho r_{t-\Delta} + \kappa\gamma)^2} - \left(\frac{\kappa(\gamma - r_{t-\Delta})2\rho}{(\rho r_{t-\Delta} + \kappa\gamma)^3} - \frac{\eta^2 3\rho^2}{(\rho r_{t-\Delta} + \kappa\gamma)^4} \right) \frac{1}{2} \Delta^2 \right)$$

$$\mathbb{C}_3 = \left(\frac{\Delta}{(\rho r_{t-\Delta} + \kappa\gamma)^3} - \left(\frac{\kappa(\gamma - r_{t-\Delta})3\rho}{(\rho r_{t-\Delta} + \kappa\gamma)^4} - \frac{\eta^2 6\rho^2}{(\rho r_{t-\Delta} + \kappa\gamma)^5} \right) \frac{1}{2} \Delta^2 \right)$$

$$\mathbb{C}_4 = \left(\frac{(r_{t-\Delta} - \delta - \sigma^2)\Delta}{(\rho r_{t-\Delta} + \kappa\gamma)} + \left(\frac{\kappa(\gamma - r_{t-\Delta})(\kappa\gamma + \rho\delta + \rho\sigma^2)}{(\rho r_{t-\Delta} + \kappa\gamma)^2} - \frac{\eta^2(\rho\kappa\gamma + \rho^2\delta + \rho^2\sigma^2)}{(\rho r_{t-\Delta} + \kappa\gamma)^3} \right) \frac{1}{2} \Delta^2 \right)$$

$$\mathbb{C}_5 = \left(\frac{(r_{t-\Delta} - \delta - \sigma^2)\Delta}{(\rho r_{t-\Delta} + \kappa\gamma)^2} + \left(\frac{\kappa(\gamma - r_{t-\Delta})(\kappa\gamma - \rho r_{t-\Delta} + 2\rho\delta + 2\rho\sigma^2)}{(\rho r_{t-\Delta} + \kappa\gamma)^3} + \frac{\eta^2(\rho^2 r_{t-\Delta} - 2\rho\kappa\gamma - 3\rho^2\delta - 3\rho^2\sigma^2)}{(\rho r_{t-\Delta} + \kappa\gamma)^4} \right) \frac{1}{2} \Delta^2 \right)$$

$$\mathbb{C}_6 = \left(\frac{r_{t-\Delta}\Delta}{(\rho r_{t-\Delta} + \kappa\gamma)^2} + \left(\frac{\kappa(\gamma - r_{t-\Delta})(\kappa\gamma - \rho r_{t-\Delta})}{(\rho r_{t-\Delta} + \kappa\gamma)^3} + \frac{\eta^2(\rho^2 r_{t-\Delta} - 2\rho\kappa\gamma)}{(\rho r_{t-\Delta} + \kappa\gamma)^4} \right) \frac{1}{2} \Delta^2 \right)$$

$$\mathbb{C}_7 = \left(\frac{r_{t-\Delta}\Delta}{(\rho r_{t-\Delta} + \kappa\gamma)^3} + \left(\frac{\kappa(\gamma - r_{t-\Delta})(\kappa\gamma - 2\rho r_{t-\Delta})}{(\rho r_{t-\Delta} + \kappa\gamma)^4} + \frac{3\eta^2(\rho^2 r_{t-\Delta} - \rho\kappa\gamma)}{(\rho r_{t-\Delta} + \kappa\gamma)^5} \right) \frac{1}{2} \Delta^2 \right)$$

$$\begin{aligned}\mathbb{C}_8 = & \frac{(r_{t-\Delta}^2 - r_{t-\Delta}\delta - r_{t-\Delta}\sigma^2)\Delta}{(\rho r_{t-\Delta} + \kappa\gamma)^2} \\ & + \left(\frac{\kappa(\gamma - r_{t-\Delta})(r_{t-\Delta}(\rho\delta + \sigma^2\rho + 2\kappa\gamma) - \delta\kappa\gamma - \sigma^2\kappa\gamma)}{(\rho r_{t-\Delta} + \kappa\gamma)^3} + \frac{\eta^2(\kappa\gamma(\kappa\gamma + 2\rho\delta + 2\sigma^2\rho) - \rho r_{t-\Delta})}{(\rho r_{t-\Delta} + \kappa\gamma)^4} \right)\end{aligned}$$

B Simulation Study Plots

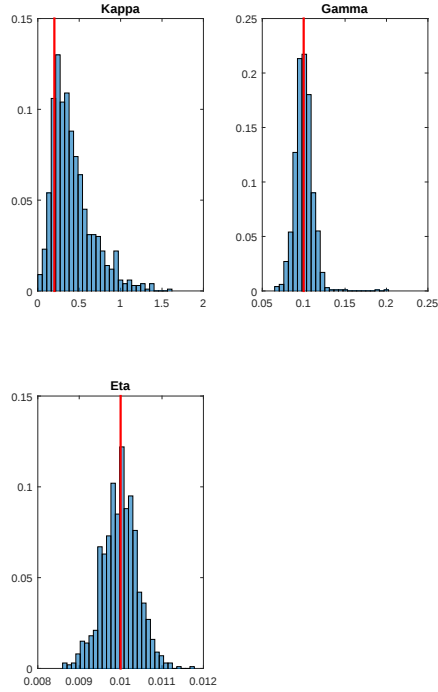
In order to examine the small sample properties of our estimation procedures we conduct simulation studies for the different versions of the model. We simulate 25 years of data for the short rate, consumption, output and the dividend claim from the model. The median and mean estimates as well as the interquartile range for 1000 replications are reported in the tables below. For reasons of clarity let GMM5 and MEF5 denote the cases where second moments for consumption and the interest rate are used in the estimation.

B.1 Complete-Finance Formulations

B.1.1 Interest Rate Only

$$dr_t = \kappa(\gamma - r_t)dt + \eta dB_t, \quad \text{where} \quad r_t = r_t^f + \delta + \sigma^2$$

Figure 4: GMM5: Interest Rate Only



GMM5: Interest Rate Only					
	κ	γ	η	δ	σ
Median	0.3563	0.0996	0.0100	0.0500	0.0200
Mean	0.4173	0.1000	0.0100		
IQR	(0.2859)	(0.0127)	(0.0006)		

B.1.2 Interest Rate and Dividend Claim

Our complete finance estimation setting consists of the two equilibrium equations for the interest rate and the price of the dividend claim.

$$\begin{aligned}
d \ln P_{d,t} &= \left[r_t - \rho - \delta - \frac{1}{2}\sigma^2 + \frac{\rho\kappa(\gamma - r_t)}{[\rho r_t + \kappa\gamma]} - \frac{1}{2} \frac{(\rho\eta)^2}{[\rho r_t + \kappa\gamma]^2} \right] dt \\
&\quad + \frac{\rho\eta}{[\rho r_t + \kappa\gamma]} dB_t + \sigma dZ_t \\
dr_t &= \kappa(\gamma - r_t)dt + \eta dB_t, \quad \text{where } r_t = r_t^f + \delta + \sigma^2
\end{aligned}$$

Again all six parameters are contained in the model. Nevertheless, there are identification problems using the GMM approach.

GMM5: Interest Rate and Dividend Claim						
	κ	γ	η	ρ	δ	σ
Median	0.3749	0.0998	0.0100	0.0297	0.0500	0.0200
Mean	0.4384	0.1000	0.0100	0.0297		
IQR	(0.2631)	(0.0125)	(0.0005)	(0.0059)		

B.2 Macro-Finance Model Estimation

To evaluate the effects of using a combined macro-finance framework we consider additional macro variables. Starting from the complete finance version we add the differentials for consumption and output, both separately as well as combined.

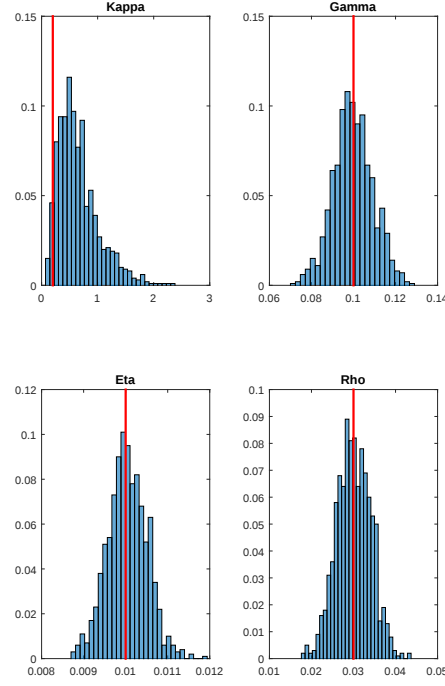
B.2.1 Interest Rate and Consumption

We now estimate the small-scale macro-finance version of the model consisting of the equilibrium equation for the interest rate and consumption.

$$\begin{aligned}
d \ln C_t &= (r_t - \rho - \delta - \frac{1}{2}\sigma^2) dt + \sigma dZ_t \\
dr_t &= \kappa(\gamma - r_t)dt + \eta dB_t, \quad \text{where } r_t = r_t^f + \delta + \sigma^2
\end{aligned}$$

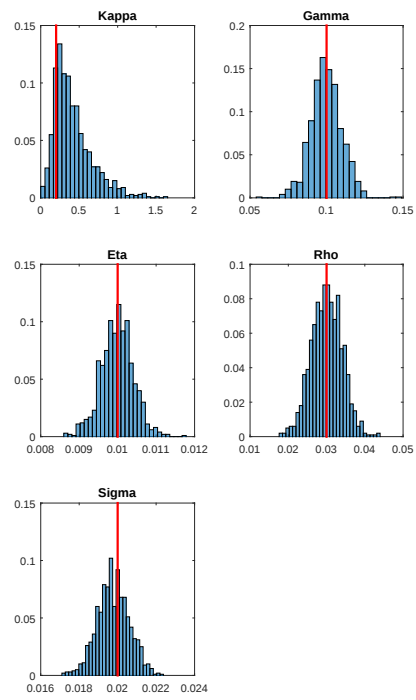
The model contains all 6 structural parameters. Nevertheless, as encountered above, the GMM estimation approach is unable to identify all parameters. Thus, we again set δ equal to its true value and estimate the remaining 5 parameters.

Figure 5: GMM5: Interest Rate and Dividend Claim



GMM5: Interest Rate, Consumption						
	κ	γ	η	ρ	δ	σ
Median	0.3554	0.0995	0.0100	0.0300	0.0500	0.0197
Mean	0.4165	0.1000	0.0100	0.0300		0.0198
IQR	(0.2927)	(0.0125)	(0.0005)	(0.0055)		(0.0011)

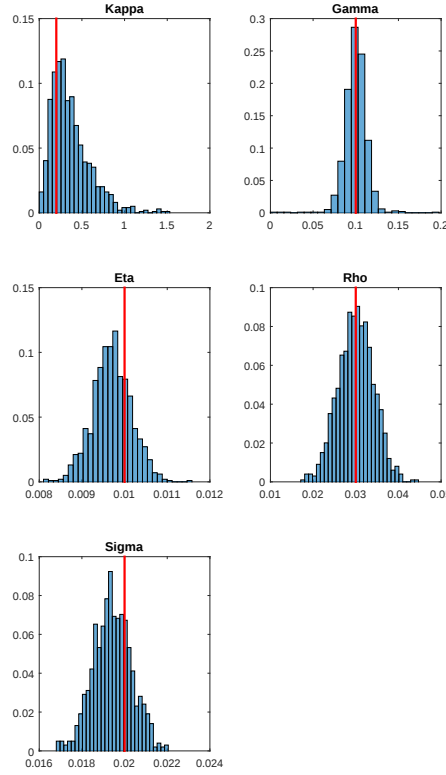
Figure 6: GMM5: Interest Rate, Consumption



B.2.2 Interest Rate, Consumption and Output

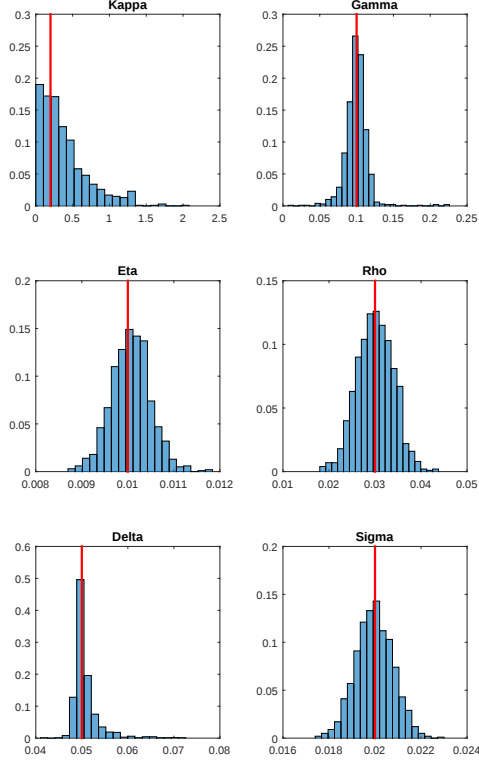
$$\begin{aligned}
d \ln C_t &= (r_t - \rho - \delta - \tfrac{1}{2}\sigma^2) dt + \sigma dZ_t \\
d \ln Y_t &= \left(\frac{\kappa\gamma}{r_t} - \tfrac{1}{2} \frac{\eta^2}{r_t^2} + r_t - \kappa - \rho - \delta - \tfrac{1}{2}\sigma^2 \right) dt + \frac{\eta}{r_t} dB_t + \sigma dZ_t \\
dr_t &= \kappa(\gamma - r_t)dt + \eta dB_t, \quad \text{where } r_t = r_t^f + \delta + \sigma^2
\end{aligned}$$

Figure 7: GMM5: Interest Rate, Consumption and Output



GMM5: Interest Rate, Consumption and Output						
	κ	γ	η	ρ	δ	σ
Median	0.3117	0.0997	0.0097	0.0301	0.0500	0.0194
Mean	0.3649	0.0992	0.0097	0.0301		0.0194
IQR	(0.2709)	(0.0144)	(0.0006)	(0.0056)		(0.0012)

Figure 8: MEF5: Interest Rate, Consumption and Output



MEF5: Interest Rate, Consumption and Output						
	κ	γ	η	ρ	δ	σ
Median	0.2893	0.1001	0.0101	0.0300	0.0500	0.0200
Mean	0.3768	0.1068	0.0101	0.0300	0.0508	0.0200
IQR	(0.3743)	(0.0148)	(0.0006)	(0.0055)	(0.0019)	(0.0011)

B.2.3 Interest Rate, Consumption and Dividend Claim

$$\begin{aligned}
d \ln C_t &= (r_t - \rho - \delta - \tfrac{1}{2}\sigma^2) dt + \sigma dZ_t \\
d \ln P_{d,t} &= \left[r_t - \rho - \delta - \tfrac{1}{2}\sigma^2 + \frac{\rho\kappa(\gamma - r_t)}{[\rho r_t + \kappa\gamma]} - \tfrac{1}{2} \frac{(\rho\eta)^2}{[\rho r_t + \kappa\gamma]^2} \right] dt \\
&\quad + \frac{\rho\eta}{[\rho r_t + \kappa\gamma]} dB_t + \sigma dZ_t \\
dr_t &= \kappa(\gamma - r_t)dt + \eta dB_t, \quad \text{where } r_t = r_t^f + \delta + \sigma^2
\end{aligned}$$

GMM5: Interest Rate, Consumption and Dividend Claim						
	κ	γ	η	ρ	δ	σ
Median	0.2121	0.1000	0.0099	0.0297	0.0500	0.0198
Mean	0.2193	0.1005	0.00098	0.0290		0.0198
IQR	(0.0641)	(0.0129)	(0.0006)	(0.0059)		(0.0013)

Figure 9: GMM5: Interest Rate, Consumption and Dividend Claim, Rho included

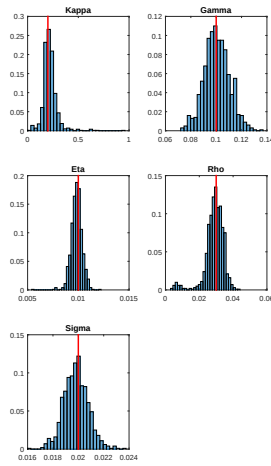
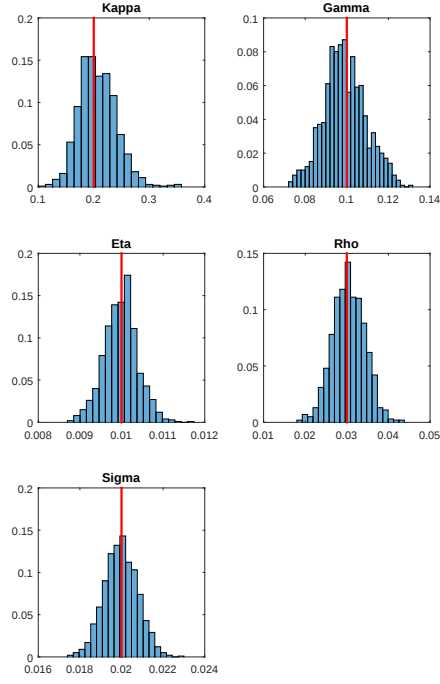
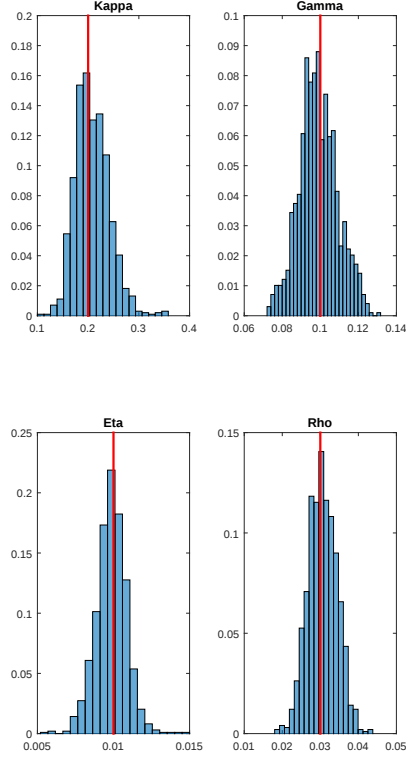


Figure 10: MEF5: Interest Rate, Consumption and Dividend Claim



MEF3: Interest Rate, Consumption and Dividend Claim						
	κ	γ	η	ρ	δ	σ
Median	0.2018	0.0994	0.0100	0.0301	0.0500	0.0200
Mean	0.2031	0.1001	0.0100	0.0302		0.0200
IQR	(0.0462)	(0.0134)	(0.0005)	(0.0054)		(0.0011)

Figure 11: MEF5: Interest Rate, Consumption and Dividend Claim

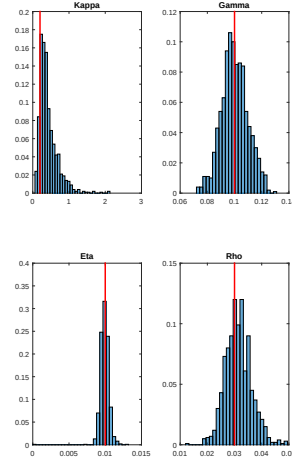


MEF3: Interest Rate, Consumption and Dividend Claim						
	κ	γ	η	ρ	δ	σ
Median	0.2062	0.0985	0.0099	0.0304	0.0500	0.0200
Mean	0.2085	0.0991	0.0099	0.0306		
IQR	(0.0450)	(0.0133)	(0.0012)	(0.0053)		

B.2.4 Interest Rate, Output and Dividend Claim

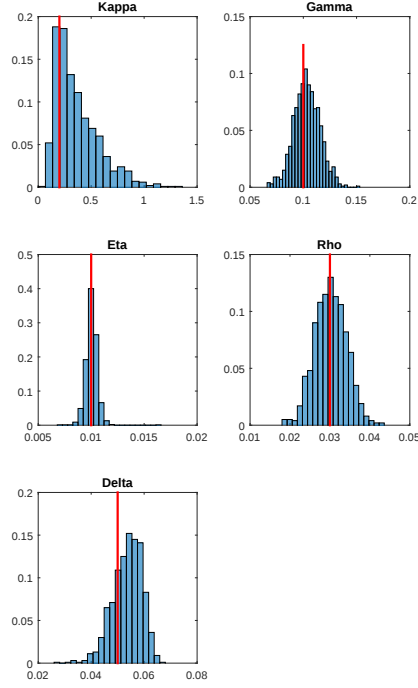
$$\begin{aligned}
d \ln Y_t &= \left(\frac{\kappa \gamma}{r_t} - \frac{1}{2} \frac{\eta^2}{r_t^2} + r_t - \kappa - \rho - \delta - \frac{1}{2} \sigma^2 \right) dt + \frac{\eta}{r_t} dB_t + \sigma dZ_t \\
d \ln P_{d,t} &= \left[r_t - \rho - \delta - \frac{1}{2} \sigma^2 + \frac{\rho \kappa (\gamma - r_t)}{[\rho r_t + \kappa \gamma]} - \frac{1}{2} \frac{(\rho \eta)^2}{[\rho r_t + \kappa \gamma]^2} \right] dt \\
&\quad + \frac{\rho \eta}{[\rho r_t + \kappa \gamma]} dB_t + \sigma dZ_t \\
dr_t &= \kappa (\gamma - r_t) dt + \eta dB_t, \quad \text{where } r_t = r_t^f + \delta + \sigma^2
\end{aligned}$$

Figure 12: GMM5: Interest Rate, Output and Dividend Claim



GMM5: Interest Rate, Output and Dividend Claim						
	κ	γ	η	ρ	δ	σ
Median	0.3587	0.0999	0.0099	0.0312	0.0500	0.0200
Mean	0.4253	0.1001	0.0099	0.0314		
IQR	(0.2650)	(0.0127)	(0.0007)	(0.0064)		

Figure 13: MEF5: Interest Rate, Output and Dividend Claim

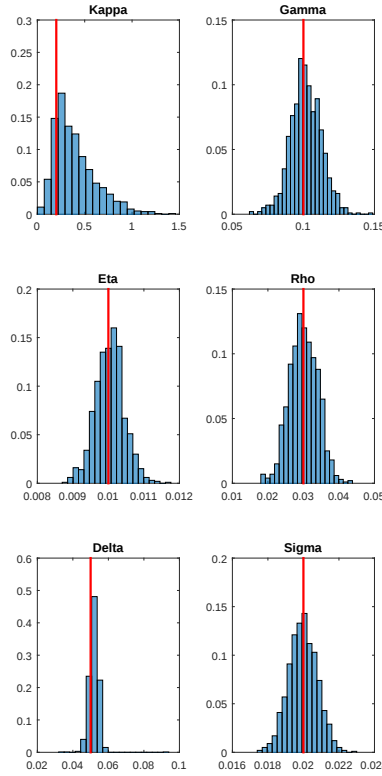


MEF5: Interest Rate, Output and Dividend Claim						
	κ	γ	η	ρ	δ	σ
Median	0.3109	0.1030	0.0100	0.0300	0.0542	0.0200
Mean	0.3641	0.1034	0.0100	0.0302	0.0536	
IQR	(0.2650)	(0.0165)	(0.0006)	(0.0056)	(0.0077)	

B.2.5 Interest Rate, Consumption, Output and Divided Claim

$$\begin{aligned}
d \ln C_t &= (r_t - \rho - \delta - \tfrac{1}{2}\sigma^2) dt + \sigma dZ_t \\
d \ln Y_t &= \left(\frac{\kappa\gamma}{r_t} - \tfrac{1}{2} \frac{\eta^2}{r_t^2} + r_t - \kappa - \rho - \delta - \tfrac{1}{2}\sigma^2 \right) dt + \frac{\eta}{r_t} dB_t + \sigma dZ_t \\
d \ln P_{d,t} &= \left[r_t - \rho - \delta - \tfrac{1}{2}\sigma^2 + \frac{\rho\kappa(\gamma - r_t)}{[\rho r_t + \kappa\gamma]} - \tfrac{1}{2} \frac{(\rho\eta)^2}{[\rho r_t + \kappa\gamma]^2} \right] dt \\
&\quad + \frac{\rho\eta}{[\rho r_t + \kappa\gamma]} dB_t + \sigma dZ_t \\
dr_t &= \kappa(\gamma - r_t)dt + \eta dB_t, \quad \text{where } r_t = r_t^f + \delta + \sigma^2
\end{aligned}$$

Figure 14: MEF5: Interest Rate, Consumption, Output and Dividend Claim



MEF5: Interest Rate, Consumption, Output and Dividend Claim						
	κ	γ	η	ρ	δ	σ
Median	0.3382	0.1012	0.0101	0.0300	0.0521	0.0200
Mean	0.3956	0.1061	0.0100	0.0301	0.0519	0.0200
IQR	(0.2776)	(0.0142)	(0.0005)	(0.0055)	0.0034	0.0011

B.2.6 Interest Rate, Consumption and Alternative Dividend Claim Formulation

Additionally, we consider an alternative macro-finance version, which also considers the "indirect" impact of macro on financial variables. To achieve this, we exploit the fact that the differential on consumption is explicitly contained in the equilibrium price equation for the claim on future dividends. That is we estimate

$$\begin{aligned}
d \ln C_t &= (r_t - \rho - \delta - \tfrac{1}{2}\sigma^2) dt + \sigma dZ_t \\
d \ln P_{d,t} - d \ln C_t &= \left[\frac{\rho\kappa(\gamma - r_t)}{[\rho r_t + \kappa\gamma]} - \frac{1}{2} \frac{(\rho\eta)^2}{[\rho r_t + \kappa\gamma]^2} \right] dt + \frac{\rho\eta}{[\rho r_t + \kappa\gamma]} dB_t \\
dr_t &= \kappa(\gamma - r_t)dt + \eta dB_t, \quad \text{where } r_t = r_t^f + \delta + \sigma^2
\end{aligned}$$

GMM5: Interest Rate, Consumption and Alt. Dividend Claim						
	κ	γ	η	ρ	δ	σ
Median	0.2063	0.0997	0.0098	0.0298	0.0500	0.0198
Mean	0.3438	0.1001	0.0100	0.0299		0.0199
IQR	(0.0670)	(0.0133)	(0.0007)	(0.0058)		(0.0012)

GMM5: Interest Rate, Consumption and Alt. Dividend Claim, Alt Rho						
	κ	γ	η	ρ	δ	σ
Median	0.2083	0.0999	0.0098	0.0296	0.0500	0.0199
Mean	0.2112	0.1002	0.098	0.0287		0.0199
IQR	(0.0643)	(0.0132)	(0.0007)	(0.0060)		(0.0013)

MEF3: Interest Rate, Consumption and Alt. Dividend Claim, Alt Rho						
	κ	γ	η	ρ	δ	σ
Median	0.2062	0.0985	0.0099	0.0304	0.0500	0.0200
Mean	0.2085	0.0991	0.0099	0.0306		
IQR	(0.0450)	(0.0133)	(0.0012)	(0.0053)		

Figure 15: GMM5: Interest Rate, Consumption and Alt. Dividend Claim

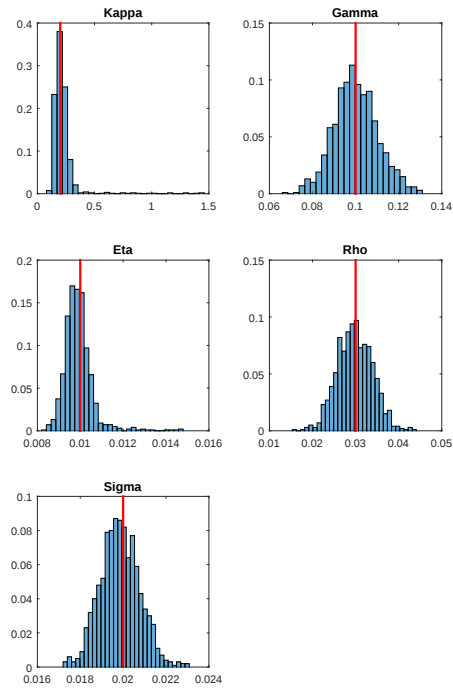


Figure 16: GMM5: Interest Rate, Consumption and Alt. Dividend Claim, Alt Rho

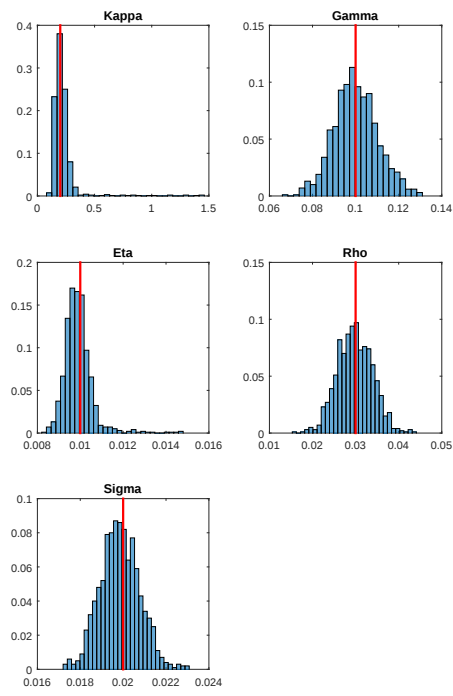
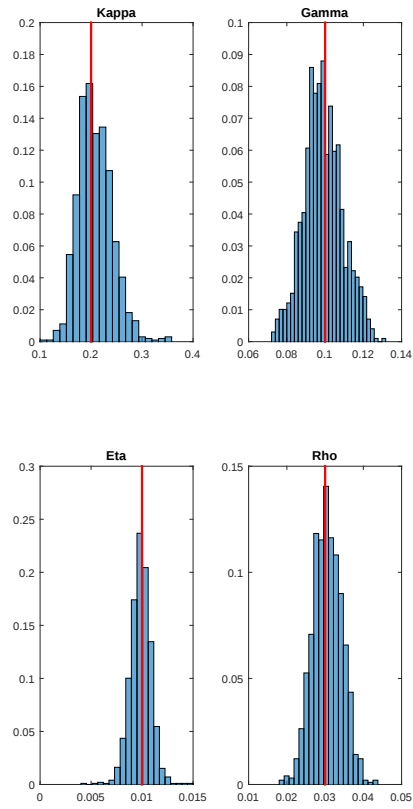


Figure 17: MEF3: Interest Rate, Consumption and Alt. Dividend Claim, Alt Rho



B.2.7 Interest Rate, Consumption, Alternative Output and Dividend Claim Formulations

The problems regarding the dependence on two shocks are also present in the differential for output. Since, the differential for consumption is also contained in the one for output, we do the same substitution and estimate

$$\begin{aligned}
d \ln C_t &= (r_t - \rho - \delta - \tfrac{1}{2}\sigma^2) dt + \sigma dZ_t \\
d \ln Y_t - d \ln C_t &= \left(\frac{\kappa\gamma}{r_t} - \tfrac{1}{2} \frac{\eta^2}{r_t^2} - \kappa \right) dt + \frac{\eta}{r_t} dB_t \\
d \ln P_{d,t} - d \ln C_t &= \left[\frac{\rho\kappa(\gamma - r_t)}{[\rho r_t + \kappa\gamma]} - \tfrac{1}{2} \frac{(\rho\eta)^2}{[\rho r_t + \kappa\gamma]^2} \right] dt + \frac{\rho\eta}{[\rho r_t + \kappa\gamma]} dB_t \\
dr_t &= \kappa(\gamma - r_t)dt + \eta dB_t, \quad \text{where} \quad r_t = r_t^f + \delta + \sigma^2
\end{aligned}$$

GMM5: Interest Rate, Consumption, Alt. Output and Alt. Claim						
	κ	γ	η	ρ	δ	σ
Median	0.3439	0.0993	0.0098	0.0326	0.0500	0.0198
Mean	0.4146	0.1000	0.00098	0.0344		0.0198
IQR	(0.3092)	(0.0128)	(0.0009)	(0.0087)		0.0014

Figure 18: GMM5: Interest Rate, Consumption, Alt. Output and Alt. Claim

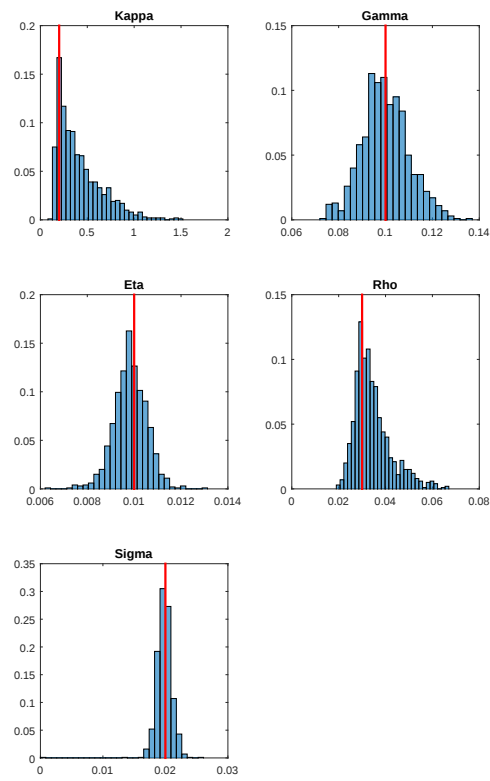
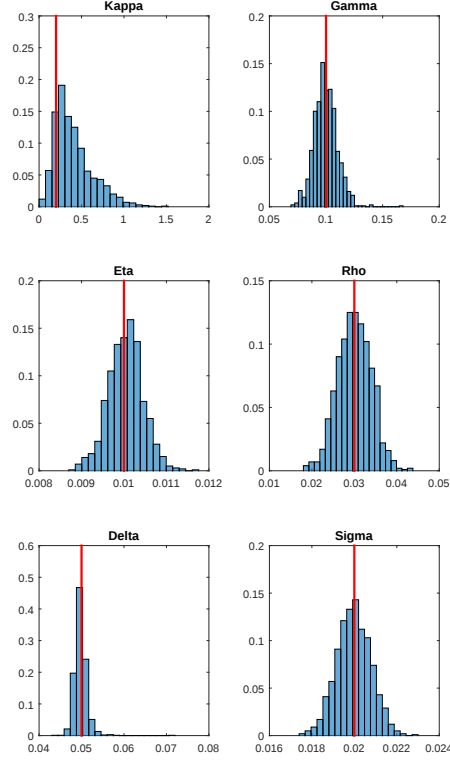


Figure 19: MEF5: Interest Rate, Consumption, Alt. Output and Alt. Claim



MEF5: Interest Rate, Consumption, Alt. Output and Alt. Claim						
	κ	γ	η	ρ	δ	σ
Median	0.3506	0.0993	0.0101	0.0300	0.0497	0.0201
Mean	0.4052	0.1000	0.0100	0.0300	0.0498	0.0200
IQR	(0.2722)	(0.0128)	(0.0005)	(0.0055)	0.0016	0.0011

C GMM Estimation: Additional Results and Robustness Checks

Empirical GMM Estimation Results						
Claim Data: Equal weighted Return Inc. Dividends						
	System1	System2	System3	System4	System5	System6
	Int	Int Claim	Int Cons	Int Cons Out	Int Cons Claim	Int Cons AltClaim
κ	0.0427 (0.0344)	0.0234 (181)	0.229 (0.1555)	0.0518 (0.2086)	0.2842 (4.6891)	9.47E-05 (3.1242)
γ	0.0873 (0.0603)	0.0754 (176)	0.1256 (1.1505)	0.1048 (0.5054)	0.1313 (22.3)	0.1536 (21.997)
η	0.0084 (0.9395)	0.0077 (0.5345)	0.0091 (1.0477)	0.0067 (0.8697)	0.0121 (23.447)	0.0082 (0.9707)
ρ		0.0005 (13709)	0.0075 (0.1775)	0.0083 (0.2349)	0.0104 (6.0852)	1.01E-06 (115.5)
δ	0.1	0.1	0.1	0.1	0.1	0.1
σ	0.02	0.02	0.015 (0.8661)	0.0139 (1.1144)	0.0365 (9.003)	0.0148 (0.8512)
<i>Asymptotic t-statistics are given below the estimates</i>						

Empirical GMM Estimation Results Claim Data: Change Market Value Stocks SP500						
	System1	System2	System3	System4	System5	System6
	Int	Int Claim	Int Cons	Int Cons Out	Int Cons Claim	Int Cons AltClaim
κ	0.0427 (0.0344)	0.0234 (181)	0.229 (0.1555)	0.0518 (0.2086)	0.12 (1.107)	3.35E-05 (0.919)
γ	0.0873 (0.0603)	0.0754 (176)	0.1256 (1.1505)	0.1048 (0.5054)	0.125 (3.624)	0.16 (19.81)
η	0.0084 (0.9395)	0.0077 (0.5345)	0.0091 (1.0477)	0.0067 (0.8697)	0.0079 (0.52)	0.008 (0.9656)
ρ		0.0005 (13709)	0.0075 (0.1775)	0.0083 (0.2349)	0.0006 (3.259)	6.68E-07 (294.5)
δ	0.1	0.1	0.1	0.1	0.1	0.1
σ	0.02	0.02	0.015 (0.8661)	0.0139 (1.1144)	0.0156 (0.9217)	0.0138 (0.6)
<i>Asymptotic t-statistics are given below the estimates</i>						

Empirical GMM Estimation Results Claim Data: Artificial Claim (Return SP500 Inc.-Ex Dividends)						
	System1	System2	System3	System4	System5	System6
	Int	Int Claim	Int Cons	Int Cons Out	Int Cons Claim	Int Cons AltClaim
κ	0.0427 (0.0344)	0.0234 (181)	0.229 (0.1555)	0.0518 (0.2086)	0.12 (1.107)	0.063 (12.05)
γ	0.0873 (0.0603)	0.0754 (176)	0.1256 (1.1505)	0.1048 (0.5054)	0.125 (3.624)	0.06 (14.914)
η	0.0084 (0.9395)	0.0077 (0.5345)	0.0091 (1.0477)	0.0067 (0.8697)	0.0079 (0.52)	0.0106 (1.748)
ρ		0.0005 (13709)	0.0075 (0.1775)	0.0083 (0.2349)	0.0006 (3.259)	0.011 (284.5)
δ	0.1	0.1	0.1	0.1	0.1	0.1
σ	0.02	0.02	0.015 (0.8661)	0.0139 (1.1144)	0.0156 (0.9217)	0.0246 (9.735)
<i>Asymptotic t-statistics are given below the estimates</i>						