

Idiosyncratic Shocks and the Role of Granularity in Business Cycles

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ABSTRACT

Idiosyncratic shocks faced by large firms in the U.S. appears to be volatile. Such idiosyncratic shocks may not average out in the cross-section and thus can even generate aggregate fluctuations. In this paper, we first construct a panel of US firms using data from Compustat and show a set of stylized facts about cross-sectional and cyclical features of idiosyncratic shocks faced by large firms. Our panel data reveals that the mean and kurtosis of idiosyncratic shocks decrease with firm size, while the standard deviation and skewness increase with firm size. In particular, the distribution of idiosyncratic shocks faced by the largest firms has a negative mean and positive skew. We also show that the mean of idiosyncratic shocks faced by the largest firms is significantly countercyclical. To examine the quantitative importance of such features of idiosyncratic shocks faced by large firms, we then develop an equilibrium business cycle model wherein idiosyncratic shocks can alone alter the shape of the distribution of firms and thus can drive aggregate fluctuations. We develop a general framework to study such models, wherein the law of large number does not hold and the distribution of firms over productivities becomes a random object, rendering infeasible the use of a standard numerical method. The flexibility of this new approach allows us to isolate the two channels through which the idiosyncratic movements of the firms generate aggregate volatility: *average productivity* and *dynamic inefficiency*. In addition we quantify the relative importance of the shocks to large firms in driving the cycle. The model is estimated to match micro-level moments of firm size distribution and idiosyncratic shocks, together with standard macro moments. Consistent with existing studies, our results show that idiosyncratic shocks are a quantitatively important micro-origin of aggregate fluctuations, accounting for 25 percent of output volatility relative to the data. Large firm movements account for 11 percent of aggregate volatility, wherein 63 percent reflects *dynamic inefficiency* channel.

Keywords: idiosyncratic shocks; business cycles; heterogeneous firm models; large firms

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I. Introduction

Idiosyncratic shocks to large firms appears to play a non-trivial role in driving business cycles. In this paper, we study a new equilibrium business cycle model wherein idiosyncratic shocks do not average out in the cross-section; hence, the distribution of firms stochastically changes over time, potentially driving aggregate fluctuations. To quantify the contribution of such granular shocks to generate aggregate fluctuations, we first construct a panel data of firms and show evidence that the distribution of idiosyncratic shocks faced by the largest firms in the U.S. has a negative mean, which is significantly countercyclical. The model is estimated to account for these regularities and used to study the quantitative role of granular shocks in driving business cycles. We show that granular shocks that make the distribution of firms vary stochastically over time can be an important origin of aggregate fluctuations, accounting for percent of output volatility relative to the data. Our empirical investigation has its chief objective to show a set of stylized facts about cross-sectional and cyclical features of idiosyncratic shocks faced by large firms. In a different line of research, Guvenen, Karahan, Ozkan, and Song (2016) found a highly skewed earnings shock distribution and it varies substantially by worker’s characteristics such as age and the previous year’s earnings. However, little is known about the cross-sectional features of idiosyncratic shocks faced by large firms, often blamed for generating aggregate fluctuations (Gabaix, 2011).¹ In this paper, we construct an annual panel of firm-level idiosyncratic shocks based on the Compustat database. The use of Compustat is ideal for our purposes because: (1) almost all large firms that can potentially account for aggregate fluctuations are covered, (2) basic accounting data items such as sales and employment used in our paper are available in other alternative datasets, ensuring a straightforward direct comparison to other works, and (3) the moments obtained can be fairly directly used to discipline the model that we newly build. Our panel data reveals that the mean and kurtosis of idiosyncratic shocks decrease with firm size, while the standard deviation and skewness increase with firm size. In particular, the distribution of idiosyncratic shocks faced by the largest firms is positively skewed with a negative mean, which is also significantly countercyclical.

Our quantitative exercise includes a methodologically distinct approach in that we model granular shocks as part of the model economy’s aggregate state.

Unlike a standard heterogeneous firm model wherein a distribution of firms over productivities is time-invariant, that distribution changes stochastically over time. A granular shock each period determines the shape of the distribution of firms over productivities. Agents in our model economy know the probability distribution of such granular shocks, and they take

¹ Davis, Haltiwanger, Jarmin, and Miranda (2006) showed

the conditional expectations according to the probability space defined. Therefore, the solution of our model economy can rely on standard stochastic dynamic programming. This allows us to base our business cycle analysis on a standard recursive method under a rational expectation framework; thus, our approach is applicable to a wide variety of existing business cycle models to study granular shocks.

Computing an equilibrium business cycle model with heterogeneous agents under aggregate uncertainty is not trivial and is of interest to many researchers in the literature. A common numerical method is to approximate the firm distribution with a finite set of moments such as the mean of asset holdings, following Krusell and Smith's (1997, 1998) heterogeneous household model, or of capital stock as in Khan and Thomas' (2008) heterogeneous firm model. Recently, the shape of the distribution of micro-level agents lies at the heart of the debate on the aggregate implications, as, for example, demonstrated by Krueger, Mitman, and Perri (2016). They show that as long as the movement of cross-sectional distributions lines up with the movement of the mean of the distribution, then quasi-aggregation works even for economies with highly skewed distributions. However, this quasi-aggregation method is generally ill-suited for our model for the following two reasons. First, the presence of granular shocks make the distribution of firms random and time-varying. Second, our model economy replicates a fat-tailed distribution of the firm size, consistent with data. It is not obvious that the mean can capture the aggregate dynamics well; and, indeed, it cannot, as it turns out. This paper shows that our approach of introducing granular shocks into the model economy's aggregate state explicitly, along with the finiteness of the distribution of firms, makes the quasi-aggregation applicable, which is a useful result for further studies of granular shocks in a variety of business cycle models.

One of the main contributions of this work is to provide a new methodological approach suitable for different model frameworks that are characterized by random distribution of firms over productivities. The flexibility of this new approach allows us to isolate the two channels through which the idiosyncratic movements of the firms generate aggregate volatility: *average productivity* and *dynamic inefficiency*. Idiosyncratic movements of the firms can mimic the effect of the aggregate productivity shock by altering the shape of the distribution over productivities they change the average productivity in the economy. Moreover the idiosyncratic shocks contribute to generate a time-varying dynamic inefficiency due to the predetermined capital and the firm random movements. Finally this new methodological approach allows us to quantify the relative importance of the shocks to large firms in driving the cycle. We take a frictionless version of Khan and Thomas' (2008) model and extend it as follows. As in their original model, each firm faces persistent shocks to its own idiosyncratic productivity. While their original model has a stationary distribution of productivity across

firms, we relax the assumption of the law of large numbers as in Carvalho and Grassi (2017), and thus firms’ idiosyncratic productivity shocks do not wash-out in the cross-section. This leaves the potential role of granular shocks in driving aggregate fluctuations, as such shocks make the firm distribution vary over time. We estimate the model parameters using indirect inference, minimizing the gap between moments generated from the model and the empirical moments. Our approach is not targeting business cycle moments and thus it is not necessary for us to repeat Krusell and Smith’s iterative steps when performing estimation. Our model is well-matched against both micro and macro moments, targeting real employment and employment share of the largest firm, as well as the average growth rate shock among the top 100 firms. The estimated model is then used to study the business cycle implications of granular shocks. Our main findings are as follows. First, consistent with existing business cycle studies, granular shocks that result in a time-varying distribution of firms are an important micro-origin of aggregate fluctuations, accounting for about 25 percent of output fluctuations and 32 percent of investment fluctuations relative to the empirical counterpart. Large firm movements account for 11 percent of aggregate volatility, wherein 63 percent reflects *dynamic inefficiency* channel.

Related work

Recent studies have found idiosyncratic shocks to be important in accounting for business cycles. The seminal contribution of Gabaix (2011) introduced the “granular hypothesis,” showing that if the firm size distribution is fat-tailed and the central limit theorem breaks down, then idiosyncratic shocks to large firms can cause sizeable aggregate fluctuations.² Recent studies including, di Giovanni and Levchenko (2012) and di Giovanni, Levchenko, and Mejean (2014) provide empirical foundations for this hypothesis, showing that idiosyncratic shocks to large firms are an important driver of aggregate fluctuations.³ The theoretical underpinnings are further established by Carvalho and Grassi (2017) upon which our work builds. Relative to sharp analytical results derived in their work, we place emphasis on the computational results, and our model, together with estimation and numerical solution methods, may be useful for applying granular shocks directly to other business cycle models.

² The role of input-output linkages has been extensively studied as a mechanism by which idiosyncratic shocks can propagate. See, for this line of studies, Acemoglu et al. (2012) and Carvalho, Nirei, Saito, and Tahbaz-Salehi (2016), among others.

³ See, for example, Hogen, Miura, and Takahashi (2017), Magerman, Bruyne, Dhyne, and Hove (2016), and Arroyo and Alfarano (2016) for empirical applications to other countries.

Organization

The rest of the paper is organized as follows. Section II shows empirical results on idiosyncratic shocks from our panel data. In Section III, the model of heterogeneous firms with granular shocks is developed. Section IV presents the business cycle results in the presence of granular shocks as well as estimation and numerical solution methods. Section V concludes.

II. Empirics

This section empirically explores cross-sectional and cyclical features of idiosyncratic shocks faced by large firms. Because our focus is on large firms, we use the Compustat data, which includes most major large firms in the U.S. for a long period of time, a ideal dataset for our purposes.⁴ Our panel data reveals that the mean and kurtosis of idiosyncratic shocks decrease with firm size. In contrast, the standard deviation and skewness increase with firm size. Focusing on the largest 100 firms in the U.S., we show that the cross-sectional mean of idiosyncratic shocks are negative and significantly countercyclical. Next sub-sections explain our empirical strategy, construction of our panel data, and empirical findings about idiosyncratic shocks faced by large firms.

A. Data and sample selection

We follow Gabaix (2011) closely to sample firms from the Compustat data, although we use data from 1951 to 2017, the most recently available one. Using Standard Industry Classification (SIC) codes, we exclude firms in the oil, energy and financial sectors.⁵ We eliminate sample firms with missing data items to ensure that sales and employment data are valid for all the sample. Finally, to eliminate some large outliers in the sample, we winsorize the extreme idiosyncratic shocks at 90%.⁶ The resulting dataset is an unbalanced panel of 13,660 firms between 1951 and 2017, with the average of 11.3 observations per firm. The basic descriptive statistics are reported in Table I.

⁴ All publicly listed firms are available in the data. Since the largest firms in the U.S. tend to be public listed firms, our choice of data is arguably better than other alternatives. They account for 41 percent of the total US sales (Asker, Farre-Mensa, and Ljungqvist, 2014) and one-thirds of the total US employment (Davis, Haltiwanger, Jarmin, and Miranda, 2007).

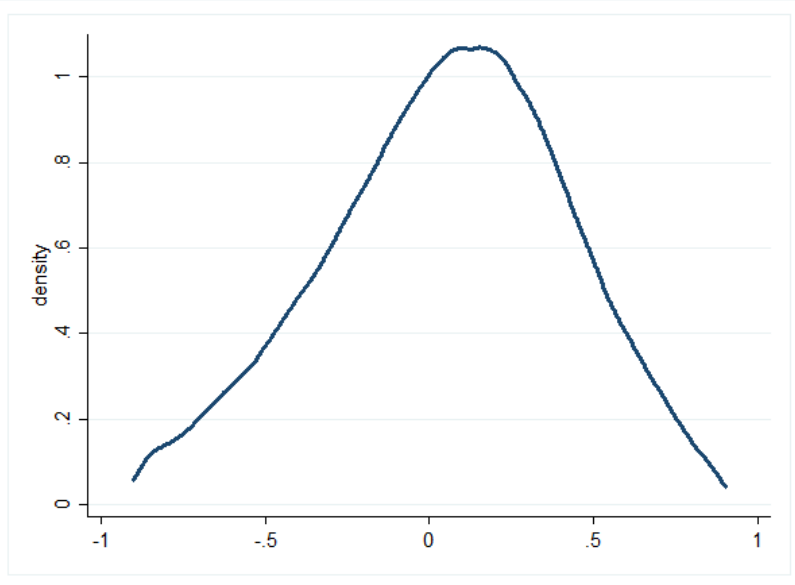
⁵ Specifically, we exclude oil and oil-related firms with SIC codes 2911, 5172, 1311, 4922, 4923, 4924, and 1389; energy firms with SIC code between 4900 and 4940; financial firms with SIC code between 6000 and 6999.

⁶ We set the threshold relatively larger than Gabaix (2011), who chose 20%, although our results are not influenced by the choice of the threshold.

B. Idiosyncratic shocks

We follow Gabaix (2011) closely in that we use only basic accounting information from Compustat such as sales and employment. This not only makes our exercise transparent but also makes our results comparable to other works, as sales and employment are often available in different datasets.

Figure 1. Distribution of idiosyncratic shocks



Note: Density of $\varepsilon_{i,t}$ is plotted after estimating $\ln z_{i,t} = \beta_0 + \beta_1 \ln z_{i,t-1} + \eta_i + \lambda_t + \varepsilon_{i,t}$.

Our baseline analysis uses sales and we isolate idiosyncratic shocks using log growth of sales as follows. For firm i in period t , we let $z_{i,t}$ denote $\text{sales}_{i,t}$. We then estimate Equation (1),

$$\ln z_{i,t} = \beta_0 + \beta_1 \ln z_{i,t-1} + \eta_i + \lambda_t + \varepsilon_{i,t}, \quad (1)$$

where $z_{i,t-1}$ is a lagged sales, η_i is firm fixed effect, and λ_t is a time-dummy.⁷ Estimating Equation (1), we then define idiosyncratic shocks as the estimated $\tilde{\varepsilon}_{i,t}$. Figure 1 shows the distribution of $\varepsilon_{i,t}$. It exhibits a substantial heterogeneity across firms and it is not far from Gaussian-looking distribution. However, distributions of idiosyncratic shocks vary with firm size and we discuss this major finding in the next sub-section.

⁷ We follow Bloom et al. (2011) in estimating without industry fixed effects, and thus idiosyncratic shocks estimated here are considered to reflect supply and demand side shocks altogether.

C. Cross-section: large firms face negative but positively skewed shocks

To see how distributions of idiosyncratic shocks vary with firm size, we sort firms based on the previous year’s sales. For each year t , we re-sort firms based on the previous year’s sales, $z_{i,t-1}$. For all percentile groups of firms in each year t , we calculate distributions of $\varepsilon_{i,t}$. Averaging distributions for each group of firms across years between 1951 and 2016, we plot Figure 2. As seen in Figure 2, as firm size increases the mean and kurtosis of idiosyncratic shocks decrease. In contrast, as firm size increases the standard deviation and skewness increase. On the distribution of individual worker’s earning growth, a recent study by Guvenen et al. (2016) show that earnings shocks faced by the top earners exhibit large variance and negative skewness. There, it appears that the top earners face a rare but substantial negative tail risk of earning shocks. While a direct comparison of large firms and high earners is not straightforward, we find different pictures, displaying that the large firms face negative idiosyncratic shocks mostly but there is a substantial upside potential.

These patterns can be seen even if we group firms into finer bins. Figure 3 shows how the first to fourth moments of idiosyncratic shocks vary with firm size. As can be seen in Figure 3, the first and fourth moments decrease with the percentile of sales of the previous year, while the second and third moments increase with the percentile of sales of the previous year.

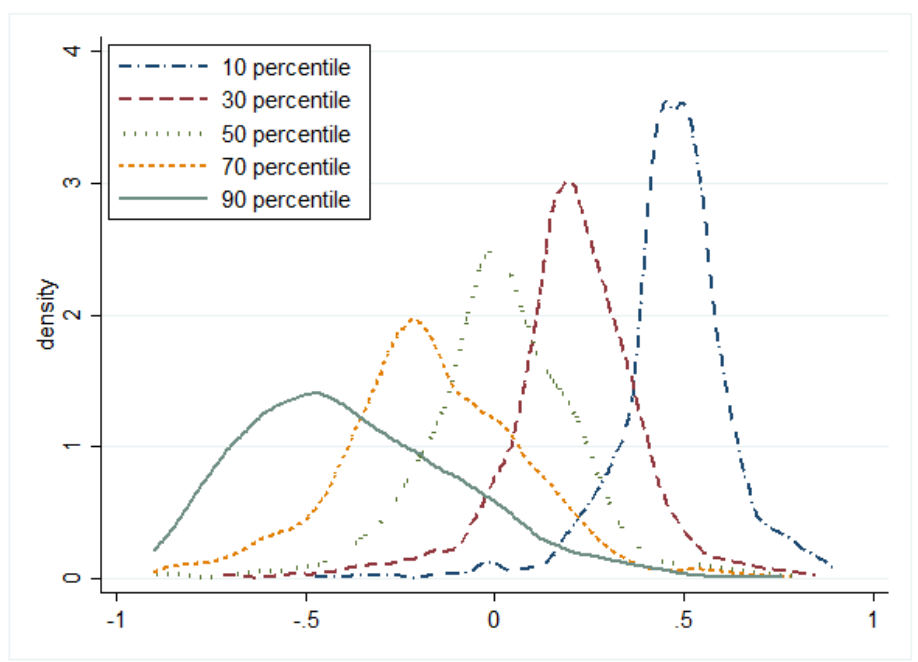
D. Business cycles: idiosyncratic shocks faced by large firm are countercyclical

In this sub-section, we next examine cyclical properties of the cross-sectional moments of idiosyncratic shocks faced by the large firms. Following Gabaix (2011), we focus on the largest 100 firms in the U.S. Figure 4 plots historical series of GDP growth and the cross-sectional average of idiosyncratic shocks among the top 100 firms, which is a HP-filtered series using smoothing parameter 6.25.

As seen in Figure 4, idiosyncratic shocks faced by large firm appears to be countercyclical, correlation between GDP growth and the average idiosyncratic shocks faced by the largest 100 US firms is significantly positive (0.48).

Periods with low GDP growth tend to be associated with the low average idiosyncratic shocks faced by the largest 100 firms in U.S. Gabaix (2011) provides the narrative of these episodes. For instance, the General Motors (GM) used to contribute to aggregate fluctuations; and, 1969 and 1970 are periods when GM had a chain of industrial strike actions and we had low GDP growth during these two years. In contrast, Gabaix (2011) did not find evidence that 1958 and 1982 are “granular year,” when low GDP appeared to be due to tightening monetary policy. While Gabaix (2011) did not include the recent Great Recession

Figure 2. Distribution of idiosyncratic shocks by firm size groups



Note: Firms are grouped based on the previous year sales. For each percentile group: 10, 30, 50, 70, and 90 percentile of the previous year sales, density of $\varepsilon_{i,t}$ is plotted after estimating $\ln z_{i,t} = \beta_0 + \beta_1 \ln z_{i,t-1} + \eta_i + \lambda_t + \varepsilon_{i,t}$.

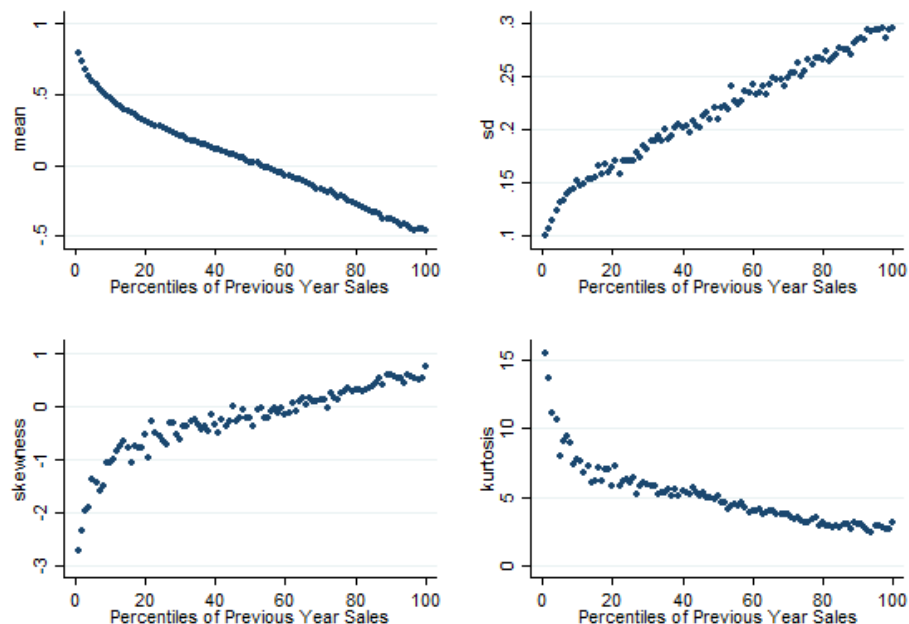
episode, our exercise here shows that the largest 100 firms in U.S. experienced a substantial negative idiosyncratic shocks in 2009.

All in all, stylized facts on idiosyncratic shocks emerging from our exercise is that the cross-sectional mean of idiosyncratic shocks for the largest 100 firms in the U.S. is negative and countercyclical. We also show a robust relationship between cross-sectional moments of idiosyncratic shocks and firm size: the mean and kurtosis of idiosyncratic shocks increase with the percentile of sales of the previous year, while the standard deviation and skewness decrease with the percentile of sales of the previous year.

III. Model

We build a standard equilibrium business cycle model with heterogeneous firms and extend it as follows. First, we take a frictionless version of Khan and Thomas' (2008) model in a setting wherein each firm faces persistent shocks to its own idiosyncratic productivity. In particular we assume that firms' idiosyncratic productivity embeds a base and a temporary component and both are directly observable. Moreover, each firm faces an exogenous shock

Figure 3. Cross-sectional moments of idiosyncratic shocks by firm size groups



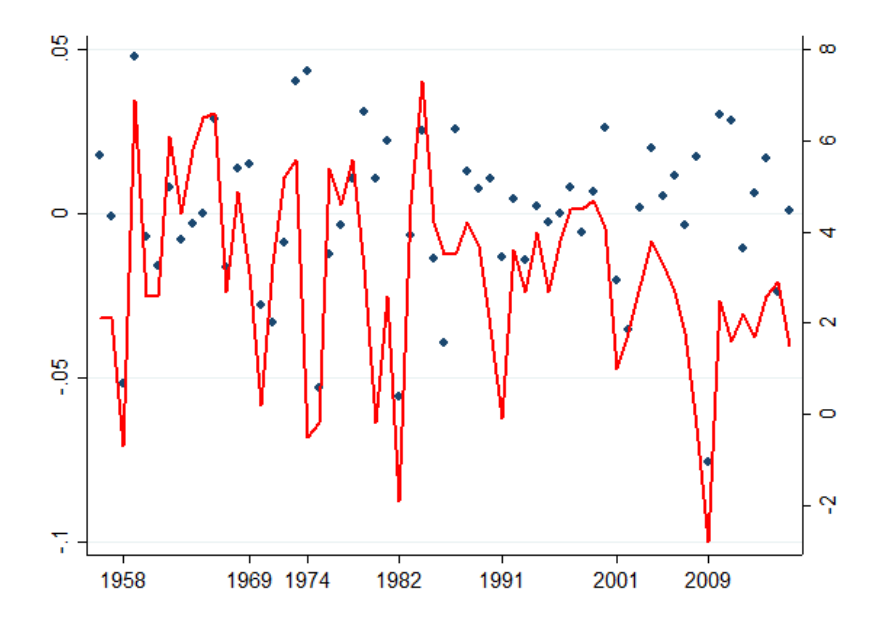
Note: Firms are grouped based on the previous year sales. For each percentile group from 0 to 100 according to the previous year sales, the first, second, third, and fourth moments are plotted against each percentile of firms sorted according to the previous year sales.

to its base component: in each period the firm maintains its base component with probability $1 - \pi$ and draw a new base component with probability π . While their original model has a stationary distribution of productivity across firms, we relax the assumption of the law of large numbers as in Carvalho and Grassi (2017) and thus firms' idiosyncratic productivity shocks do not wash-out in the cross-section. This leaves the potential role of idiosyncratic shocks in driving aggregate fluctuations. Second, we introduce “granular shocks” as part of the economy's aggregate state. As will becomes clear below, we construct a set of possible idiosyncratic productivity distribution and define its probability space consistent with the data. Agents know the probability distribution of granular shocks and form expectations accordingly, rendering a standard rational expectation framework and recursive methods applicable to our business cycle analysis.

A. *Production*

In our model economy, there are a large number of firms with a unit mass. We allow persistent heterogeneity in productivity and it together with capital stock generates a rich

Figure 4. GDP growth and mean idiosyncratic shocks faced by largest 100 firms in the U.S.



Note: The solid line plots GDP growth and the dots represents the cross-sectional average of $\varepsilon_{i,t}$ among the largest 100 firms in the U.S., after estimating $\ln z_{i,t} = \beta_0 + \beta_1 \ln z_{i,t-1} + \eta_i + \lambda_t + \varepsilon_{i,t}$. We detrend the latter with an HP filter with smoothing parameter $\lambda = 6.25$ following Ravn and Uhlig (year).

distribution of firms. We assume that each heterogeneous firm undertakes production via the Cobb-Douglas production function: $z\varepsilon k^\alpha n^\nu$. The total factor productivity common across firms, z , follows a Markov chain $z \in \{z_1, \dots, z_{N_z}\}$ and the transition matrix is denoted by Π^z with each element of $\pi_{i,j}^z = \Pr(z' = z_j \mid z = z_i) \geq 0$ and $\sum_{j=1}^{N_z} \pi_{i,j}^z = 1$.⁸ Analogously, firms' idiosyncratic productivity, ε , follows a Markov chain $\varepsilon \in \{\varepsilon_1, \dots, \varepsilon_{N_\varepsilon}\}$ and the transition matrix is denoted by Π^ε with each element of $\pi_{i,j}^\varepsilon = \Pr(\varepsilon' = \varepsilon_j \mid \varepsilon = \varepsilon_i) \geq 0$ and $\sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon = 1$. In particular, we characterize the firms' idiosyncratic productivity, ε , as the sum of two component: a persistent, ϵ , and a transitory, ξ :

$$\varepsilon = \epsilon + \xi$$

As anticipated above, the base component is lost with probability π and the firm must draw, independently from its past state, another value from a Pareto distribution, $\mathcal{P}(em, e)$, with multiplier em and parameter of the curvature e . The transitory component, ξ is *i.i.d* normal

⁸For expositional purposes, we present the general framework wherein firms share a common productivity component. In the model simulation we shut down it and the only source of aggregate volatility comes from the Granularity

distributed, $\mathcal{N}(0, \sigma_\xi^2)$, with zero mean and variance σ_ξ^2 .

In a standard heterogeneous firm model that relies on the law of large numbers, a stationary distribution of productivity values implied by Π^ε exists. The distribution of idiosyncratic productivities is time-invariant as idiosyncratic shocks to each firm's productivity offset each other in the cross-section. Therefore, idiosyncratic shocks alone cannot matter for aggregate dynamics in this standard setting. In our model, however, we deviate from the assumption of the law of large numbers and thus the distribution of productivities at the firm level is time-variant and changes over time stochastically. In particular, we make the dynamics of the firm-level productivity distribution stochastic in our model so that agents are aware the possibility that idiosyncratic shocks to each individual firms may not be offset in the cross-section.

For this purpose, we first define the Granular shock s as the random variable that determines the distribution of firms' idiosyncratic productivity shocks that pins down the distribution of the firms over productivities. Second, we introduce the granular shocks into the model economy's aggregate state, such that agents know the probability distribution over the distributions of productivity shocks⁹. To make expressions compact in the section below, we define $\theta = (z, s)$ to summarize the economy's exogenous aggregate state¹⁰ and Π^θ with elements $\pi_{h,k}^\theta = \Pr(\theta' = \theta_k \mid \theta = \theta_h) \geq 0$ and $\sum_{k=1}^{N_\theta} \pi_{h,k}^\theta = 1$.

In each period, each firm produces a homogeneous goods using predetermined capital stock k , and labor n , via an increasing and concave production function, $y = z\varepsilon F(k, n)$. Taking the current aggregate state of the economy as given, each firm chooses its current level of employment. Production occurs subsequently and the firm undertakes investment at the end of the period, so the accumulation of capital stock follows as $k' = (1 - \delta)k + i$.

In addition to the exogenous aggregate state variables, z and s , our model's aggregate state vector involves a non-trivial distribution of firms, denoted by μ . μ is the probability measure that we use to summarize the distribution of firms over (ε, k) . μ evolves over time following stochastic transitions of the exogenous aggregate state together with each firm's investment, and, as such, the law of motion of the firm distribution can be written as $\mu' = \Gamma(\theta, \mu)$.

Below, we state the dynamic optimization problem for each firm and define $v(\varepsilon, k; \theta, \mu)$ as the expected discounted value of a firm at the beginning of each period with firm-level productivity ε and predetermined capital stock k , when the aggregate state of the economy is (θ, μ) . We denote the discount factor for the firm as $d_k(\theta_h, \mu)$ to its next period expected value if the exogenous aggregate state is θ_k .

⁹ This is a distinct approach. See Carvalho and Grassi (2017).

¹⁰ In the section IV.B.1 we explain how we discretize this process.

We now turn to a functional equation to describe the problem faced by the firm.

$$v(\varepsilon_i, k; \theta_h, \mu) = \max_{n, k'} \left[z\varepsilon F(k, n) - \omega(\theta_h, \mu)n + (1 - \delta)k - k' \right. \\ \left. + \sum_{k=1}^{N_\theta} \pi_{h,k}^\theta d_k(\theta_h, \mu) \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon v(\varepsilon_j, k'; \theta_k, \mu') \right] \quad (2)$$

subject to $\mu' = \Gamma(\theta, \mu)$, $k' \in \mathcal{R}_+$, and $n \in \mathcal{R}_+$.

Given (ε, k) and the equilibrium wage rate $\omega(\theta, \mu)$, the firm's choice of its current level employment is a simple static problem. We denote $N(\varepsilon, k; \theta, \mu)$ as the choice of the current level of employment. Next, the firm chooses its capital stock for the next period and we denote $K(\varepsilon, k; \theta, \mu)$ as the choice of such.

B. Households

Our model economy is populated by a unit measure of identical households. Their life time expected utility maximization problem is written as

$$W^h(\lambda; \theta_h, \mu) = \max_{c, n^h, \lambda'} \left[U(c, 1 - n^h) + \beta \sum_{k=1}^{N_\theta} \pi_{h,k}^\theta W^h(\lambda'; \theta_k, \mu') \right] \quad (3)$$

$$\text{subject to:} \quad c + \int_{\mathbf{S}} \rho_1(\varepsilon', k'; \theta_h, \mu) \lambda'(d[\varepsilon' \times k']) \\ \leq w(\theta_h, \mu) n^h + \int_{\mathbf{S}} \rho_0(\varepsilon, k; \theta_h, \mu) \lambda(d[\varepsilon \times k]). \quad (4)$$

Households hold one-period shares in firms, which is denoted by λ . Given the prices—the real wage, $w(\theta, \mu)$, and the prices of shares, $\rho_0(\varepsilon, k; \theta, \mu)$ and $\rho_1(\varepsilon', k'; \theta, \mu)$, households choose their current consumption, c , hours worked, n^h , and the numbers of new shares, λ' . Let $C(\lambda; \theta, \mu)$ and $N(\lambda; \theta, \mu)$ represent the household decision rules for consumption, hours worked, and let $\Lambda(\varepsilon', k', \lambda; s, \mu)$ be the household decision rule for shares purchased in firms that will begin the next period with (ε', k') .

C. Recursive Equilibrium

A recursive competitive equilibrium is a set of functions:

$$\begin{aligned} \text{prices} & : (\omega, d, \rho_0, \rho_1) \\ \text{quantities} & : (N, K, C, N^h, \Lambda^h) \\ \text{values} & : (V, W) \end{aligned}$$

that solve firm and household problems and clear the markets for assets, labor, and output as follows:¹

1. V satisfies equation (2), and (N, K) are the associated policy functions for firms.
2. W satisfies equation (3) - (4), and (C, N^h, Λ^h) are the associated policy functions for households.
3. $\Lambda^h(\varepsilon, k, \mu; \theta, \mu) = \mu(\varepsilon, k)$ for each $(\varepsilon, k) \in \mathcal{S}$.
4. The labor and goods market clear.

$$N^h(\mu; \theta, \mu) = \int_{\mathbf{s}} N(\varepsilon, k; \theta, \mu) \cdot \mu(d[\varepsilon \times k])$$

$$C(\mu; \theta, \mu) = \int_{\mathbf{s}} [z\varepsilon F(k, N(\varepsilon, k; \theta, \mu)) - (K(\varepsilon, k; \theta, \mu) - (1 - \delta)k)] \cdot \mu(d[\varepsilon \times k])$$

5. the resulting individual decision rules for firms and households are consistent with the aggregate law of motion, Γ , where Γ defines the mapping from μ to μ' .

IV. Quantitative Analysis

A. Numerical Method

In this subsection, we describe our solution method that can be applied to a broad class of models wherein idiosyncratic shocks do not wash out in the cross-section so that the role of granularity in driving the business cycle is not negligible.

An equilibrium business cycle model with heterogeneous firms add a non-trivial distribution of firms as an additional state variable. Thus, solution methods are more involved than that used in other models with representative firms.¹¹ To handle this complexity, we adopt

¹¹ Terry (2016) compares several computational methods.

the method of Krusell and Smith (1998) extended to a heterogeneous firm environment by Khan and Thomas (2008). There, it is assumed that agents approximate the distribution of firms with a vector of moments. For example, such as $\mu(\varepsilon, k)$ in equation 2 can be replaced by a vector of the aggregate capital stock $m = (m_1, \dots, m_I)$. This quasi-aggregation with the first moment of capital stock is often used in existing studies to handle heterogeneity and works well.¹² Agents only need to know two time-varying elements as the aggregate states, aggregate capital stock and aggregate productivity. As we show below, a key reason behind this result is that a distribution of capital share among agents is time-invariant. However, this quasi-aggregation result fails in our model economy with granular shocks. Below, we show how it fails and propose a fix that renders the standard quasi-aggregation method compatible with the granular shocks in a heterogeneous firm business cycle model.

First, we show that a large object, $\mu(\varepsilon, k)$, can be reduced to a N_ε by N_ε grid point object in the absence of investment frictions. Second, the presence of granular shocks requires past information about the distribution of firms but this makes the first-moment quasi aggregation work well.

For convenience, we re-write the problem of equation 2, but removing total factor productivity and leaving granular shocks $s = s_1, \dots, s_{N_s}$ as the only aggregate shocks,

$$v(\varepsilon_i, k; s_l, \mu) = \max_{n, k'} \left[\varepsilon_i k^\alpha n^\nu - \omega(s_l, \mu)n + (1 - \delta)k - k' \right. \\ \left. + \sum_{m=1}^{N_s} \pi_{l,m}^s d_m(s_l, \mu) \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon v(\varepsilon_j, k'; s_m, \mu') \right] \quad (5)$$

$$\text{subject to} \quad \mu' = \Gamma(s_l, \mu), \quad k' \in \mathcal{R}_+, \text{ and } n \in \mathcal{R}_+.$$

Notice that the choice of the current level of employment is static in that a firm solves $N(\varepsilon_i, k; s_l, \mu) = \arg \max_n [\varepsilon_i k^\alpha n^\nu - \omega(s_l, \mu)n]$, yielding $N(\varepsilon_i, k; s_l, \mu) = [\nu z \varepsilon_i k^\alpha / \omega(s_l, \mu)]^{1/(1-\nu)}$. Using this decision rule for employment, we can replace the first and second terms in equation 5 so that

$$\varepsilon_i k^\alpha n^\nu - \omega(s_l, \mu)n = (1 - \nu) \varepsilon_i^{1/(1-\nu)} k^{\alpha/(1-\nu)} \left(\frac{\nu}{\omega(s_l, \mu)} \right)^{\nu/(1-\nu)}. \quad (6)$$

¹² See, for example, Khan and Thomas(2008), Mitman, Kruger and Perri(2016), among others.

Therefore, we can simplify the problem,

$$v(\varepsilon_i, k; s_l, \mu) = \max_{k'} \left[(1 - \nu) \varepsilon_i^{1/(1-\nu)} k^{\alpha/(1-\nu)} \left(\frac{\nu}{\omega(s_l, \mu)} \right)^{\nu/(1-\nu)} + (1 - \delta)k - k' \right. \\ \left. + \sum_{m=1}^{N_s} \pi_{l,m}^s d_m(s_l, \mu) \sum_{j=1}^{N_\varepsilon} \pi_{i,j}^\varepsilon v(\varepsilon_j, k'; s_m, \mu') \right]. \quad (7)$$

This problem yields the optimal investment decision,

$$G(\varepsilon_i, k; s_l, \mu) = L_0(\varepsilon_i) L_1(s_l, \mu) \quad (8)$$

$$L_0(\varepsilon_i) = \left(\sum_{m=1}^{N_\varepsilon} \pi_{ij}^\varepsilon \varepsilon_i^{1/(1-\nu)} \right)^{(1-\nu)/(1-(\alpha+\nu))} \quad (9)$$

$$L_1(\mu) = \left(\frac{1 - (1 - \delta) \sum_{m=1}^{N_s} \pi_{lm} d_m(s_l, \mu)}{\alpha \sum_{m=1}^{N_s} \pi_{lm} d_m(s_l, \mu) \left(\frac{\nu}{\omega(s_l, \mu')} \right)^{\nu/(1-\nu)}} \right)^{\frac{1-nu}{\alpha+\nu-1}}. \quad (10)$$

This shows that the investment decision is independent on the current capital stock, k . It is then immediate that there are only N_ε points in the dimension of k of $\mu(\varepsilon, k)$ with positive mass of firms. Therefore, $\mu(\varepsilon, k)$ is a N_ε by N_ε grid point object.

For $s = 1, \dots, N_s$, let $H(s) = (h_1(s), \dots, h_{N_\varepsilon}(s))$ as the vector of mass of firms over idiosyncratic productivity values, which is time-varying due to the granular shocks. We also define $\Phi_{i,j}(s)$ as a two-dimensional array with elements of $\phi_{i,j}(s)$ for $i = 1, \dots, N_\varepsilon$, $j = 1, \dots, N_\varepsilon$, representing the mass of firm moving from ε_i to ε_j when the granular shock is s . It is then immediate that $h_j(s) = \sum_{i=1}^{N_\varepsilon} h_i(s_{t-1}) \phi_{i,j}(s_t)$. Without granular shocks, $\Phi_{i,j}(s)$ collapses to Π_ε , which is time-invariant. Moreover, $H(s)$ becomes $H = (h_1, \dots, h_{N_\varepsilon})$ as the stationary distribution of productivities implied by Π_ε .

Turning back to the case with granular shocks, we can show that the distribution of firms over idiosyncratic productivity and capital stock, when the granular shock is s , $\mu_s(\varepsilon_j, k_i)$, can be summarized as follows. For each ε_i , with $i = 1, \dots, N_\varepsilon$ and $j = 1, \dots, N_\varepsilon$,

$$\mu_s(\varepsilon_j, k_i) = h_i(s_{t-1}) \phi_{i,j}(s_t) \quad (11)$$

$$\text{where } k_i = \chi_i(s_{t-1}) K \quad (12)$$

$$\text{and } \chi_i(s_{t-1}) = \frac{h_i(s_{t-1}) L_0(\varepsilon_i)}{\sum_{m=1}^{N_\varepsilon} h_m(s_{t-1}) L_0(\varepsilon_m)}. \quad (13)$$

From equation 12, granular shocks make the aggregate capital stock insufficient to determine

the grid points of k in $\mu(\varepsilon, k)$ that have positive mass. To precisely determine capital shares across productivities, we need more disaggregated information that tells us the size of mass for firms that transit from ε_i to ε_j , which depends on the granular shocks in the previous period s_{t-1} . Furthermore, equation 11 reveals an obvious fact that the current granular shock s_t affects how many firms move between productivity grid points in the current period. These results indicate that we need to track a time-varying capital share across firms over two-dimensions, ε_{t-1} and ε_t . When the agents' information set include the granular shocks in the previous period, s_{t-1} , agents can accurately predict the future aggregate capital stock (the unconditional mean of the distribution of capital stock across firms) and therefore future prices (interest rates and wages).

In contrast, the absence of granular shocks makes the aggregate capital stock a sufficient information to underpin the distribution of firms, as shown by Khan and Thomas (2008), and we can write as follows. For each ε_i , with $i = 1, \dots, N_\varepsilon$ and $j = 1, \dots, N_\varepsilon$,

$$\mu(\varepsilon_j, k_i) = h_i \pi_{i,j}^\varepsilon \quad (14)$$

$$\text{where } k_i = \chi_i K \quad (15)$$

$$\text{and } \chi_i = \frac{h_i L_0(\varepsilon_i)}{\sum_{m=1}^{N_\varepsilon} h_m L_0(\varepsilon_m)}. \quad (16)$$

This establishes, without the granular shocks, that the aggregate state vector contains only the aggregate capital stock, K . This means that the standard quasi-aggregation works well.

Our solution method is iterative as in Krusell and Smith (1997, 1998) and Khan and Thomas (2008). We conjecture that the law of motion in the aggregate capital and prices depends on $(z, s, s_{-1}, \bar{K})$. Forecasting rules are conditional on the current aggregate total factor productivity and granular shocks in both current and previous periods, (z, s, s_{-1}) . It takes the form of $\log(y) = \alpha(z, s, s_{-1}) + \beta(z, s, s_{-1})\log(\bar{K})$, where $y = \bar{K}'$ or p . We then simulate the economy for a long time period to generate the time series of equilibrium prices and quantities. We use the data generated from the simulation to regress the forecasting rules and keep the process until it converges. In this way, agents can still accurately predict the future aggregate capital stock as indicated by the R^2 equal to 1 and small standard errors. The inclusion of granular shocks in the previous period is a general approach in that it may be applied to many other models with granular shocks as well as additional frictions such as collateral constraints.

B. Structural Estimation

We set the length of a period in the model to be one year and calibrate the household discount factor, β , to imply an average real interest rate of 4 percent, as reported by Gomme, Ravikumar, and Rupert (2011). We calibrate the labor share, ν , to obtain an average labor share of income at 0.60 (Cooley and Prescott, 1995) and the depreciation rate of capital, δ , to match the average 0.07 of the investment-to-capital ratio. We set n_f , the total number of firms, equal to 4.5 millions (Business Dynamics Statistics, BDS). With these 4 pre-set parameters, set outside of the model, we estimate the 6 remaining parameter values for technology and preferences to match several salient moments from both micro and macro data by solving the model repeatedly over the parameter space defined as below. We estimate the model parameters so that the model's steady state can match several salient moments from both micro and macro data. A parameter set, Ω , involves (1) the capital share, α , (2) the preference parameter, η , (3) the curvature of the Pareto distribution, e , (4) the multiplier of the Pareto distribution, em , (5) the probability of resetting the base component, π , and (6) the number of productivity grid points, N_ϵ . These 6 parameters are estimated against the following 6 data moments, $\mathbf{m}$, including (1) the aggregate output to capital ratio: 2.3 (the average private capital-to-output ratio between 1954 and 2002 reported by Khan and Thomas, 2013), (2) the aggregate total hours worked; one-third, (3) the largest firm's employment share: 0.01 (Walmart's domestic employment size of 1.4 million as in Carvalho and Grassi, 2017), (4) the largest firm's population share: one over 4.5 million (Business Dynamics Statistics, BDS), (5) the ratio of the largest firm's employment size to that of the smallest one: 1.0×10^6 (Walmart's domestic employment size), (6) average growth rate shock for the top 100 largest firms: -0.48 (estimated from Compustast, 2017).

Our estimation will pick the set of estimated parameters, $\hat{\mathbf{m}}(\Omega)$, by minimizing the distance between the set of moments generated by the model, $\hat{\mathbf{m}}(\Omega)$ and those from data $\mathbf{m}$. Formally, we state this minimization problem as follows,

$$\hat{\mathbf{m}}(\Omega) = \arg \min_{\Omega} (\mathbf{m} - \hat{\mathbf{m}}(\Omega))' W (\mathbf{m} - \hat{\mathbf{m}}(\Omega)) \quad (17)$$

where W represents the weighting matrix derived as the variance-covariance matrix of moments from data.

B.1. Calibration of the Granular shock

Having a set of parameters concerning the idiosyncratic productivity process, em , e , and N_ϵ estimated from the above indirect inference, we can create and discretize the process for s . As already mentioned above, we identify the Granular shock as the distribution of the firms'

idiosyncratic shocks of productivity that pins down the distribution of firms over productivities. The main problem is that the number of the possible distributions of firms' idiosyncratic shocks of productivity is almost infinite¹³. For this reason we need to find a criteria that leads us to pick up a set of distributions, $s \in \{s_1, \dots, s_{N_s}\}$, and allows us to create a consistent the transition matrix Π^s .

Since the ‘‘Granular hypothesis’’ investigates the role of idiosyncratic shocks to large firms and we find a strong countercyclicality of the average productivity shock of the largest 100 firms, we decide to identify a sampling criteria of the distributions that focus on some moments characteristics of the distribution of idiosyncratic shocks to the largest 100 firms. For this purpose and to be consistent with the underlined process of productivity, we decide to pick up a set of shock distributions based on the moments of mean of the average idiosyncratic shocks among the largest 100 firms. In particular, we can formalize s , the average of the idiosyncratic shock among the large 100 firm, as an AR(1) as follows: $s' = \mu_s + \rho_s s + \eta'_s$ with $\eta'_s \sim \mathcal{WN}(0, \sigma_{\eta_s}^2)$ where $\rho_s (= 0)$ and μ_s and σ_{η_s} are respectively set equal to 0, -0.48 and 0.03 as in Compustat. In this way, we sample the set of distributions of the shocks $s \in \{s_1, \dots, s_{N_s}\}$ such that the expected average idiosyncratic shocks among the largest 100 firms is μ_s with σ_{η_s} standard deviation. Finally, we construct the transition matrix Π^s where there is no time dependence of the the average of idiosyncratic shocks among the largest 100 firms since $\rho_s = 0$.

C. Aggregate Results

In this section, we presents the business cycle results in the presence of the granular shock. These results are obtained by iteratively picking a set of distribution $s \in \{s_1, \dots, s_{N_s}\}$, associated with an expected average idiosyncratic shocks among the largest 100 firms equal to μ_s with σ_{η_s} standard deviation, and simulating the model. The final results are calculate as the average of the results associated with a drawn set of distributions. In particular, we decided to sample sets composed by 3 distributions for 1000 times. In this way, we can mimic the simulation of a model with 3000 kinds of different distributions consistent with the micro moments of the distribution of the productivity shocks of the largest 100 firms, with the stochastic process of productivity embedded in the model and with a rational expectation framework. In the first subsection we present the business cycle results of the model simulations, to gauge the relative importance of the Granularity in generating aggregate volatility and in particular of the Granularity associated with the movements of top 100 largest firms.

¹³ N_ε^m

In the second part, we study the two channels of granularity through which the idiosyncratic movements of the firms drive the cycle.

C.1. Business cycle moments and top 100 largest firms

We begin with Table III to examine business cycle results in our model economy. We simulate the model for 5,000 period and the standard business cycle moments filtered by an HP-filtered are presented in Table III.

As Table III shows, granular shocks in our model economy play an substantial role in driving the business cycle. Relative to their empirical counterpart, granular shocks drive about 25% of output, 32% of investment and 20% of hours worked. The relative volatilities of investment and hours worked are somewhat higher than the empirical counterpart. We can note that the model can almost exactly replicate the positive correlation between average growth rate shock among the top 100 largest firms, *Avgr100*, and output that we find in the data. Moreover, consistent with Gabaix (2011), the model predicts a very strong positive correlation between the Gabaix's Granular residuals and output. The purpose of the paper is not explaining the US business cycle with only granular shocks and there are other factors, from which we abstract, that play an important roles for the business cycle. Some omitted factors may amplify or dampen the overall the business cycle. Nonetheless, we emphasize a success of our model economy in reproducing a familiar set of business cycle moments as well as the sizeable role of granular shocks in accounting for the US business cycle. In what fallows, we simulate the model wherein we allow Granularity to act just for the top 100 largest firms while the rest of the distribution of productivities is maintained at its stationary shape. The purpose of this experiment is to isolate the effect of Granularity that is directly generated by the productivity shocks that hit the top 100 largest firms. In a perfect competition framework, without strategic interactions, productivity shock to large firms can affect the aggregate output through general equilibrium effect. In this environment, a negative productivity shock that hits one of largest firm decreases the demand of labour and so the wage, affecting the demand of labor and production of the less productive firms. As Table IV shows that large firm movements can account for 11 percent of the output, 15 percent of the investment and 8 percent of the hours volatility.

C.2. The channels: aggregate productivity and dynamic inefficiency

In this subsection we study and quantify the two channel through which the granularity generates business cycle fluctuations: *aggregate productivity* and *dynamic inefficiency*. The *aggregate productivity* channel coincides with the granularity effect already incorporated in

the static model, without capital, as in Carvalho and Grassi (2017). When more firms are hit by a positive than negative productivity shocks, the average productivity increases and the economy experiences an expansion phase. This channel mimics the effect of a positive aggregate productivity shock where the mass of the firms shifts to the right part of the productivity distribution. However *aggregate productivity* effect is just one of the two channels. In a dynamic environment, where firms have to choose capital in the previous periods, random productivity shocks and granularity can generate a time varying dynamic inefficiency. Every period we can face a different mass of firms that is hits by productivity shocks and, consequently, they own a capital stock that is not consistent with their level of productivity. As Table IV shows, regarding the top 100 largest firms, that half of the volatility generated by the granularity is attributable to the allocative inefficiency channel that can explain 6 percent of the output, 8 percent of the investment and 5 percent of the hours volatilities. While the *dynamic inefficiency* channel always has a negative effect on the aggregate output, the *aggregate productivity* channel can have positive or negative impact on the economy and the endogenous total factor productivity, TFP, reflects the net effect between these two channels.

To investigate the real effect of these channels, we run some experiments. The first experiment investigates the misallocation effect. Maintaining the average productivity constant, figure 5 shows that great churning in the distribution implies a deeper recession in the economy driven by a drop in both consumption and investment. In particular, in $s = 1$ two large firms and in $s = 3$ a large and a small firm are hit by productivity shocks of opposite signs¹⁴. The second and third experiment compare *dynamic inefficiency* and *average productivity effect*. In the first case, in $s = 1$ a very large firm experiences a positive productivity shock while in $s = 3$ a large and a small firms are hit by productivity shocks that maintains constant the average productivity and a large firm has a positive productivity shock. As it is shown in figure 6, in $s = 1$ the average productivity effect dominates the misallocation effect: consumption and investment increase on impact. On the contrary, in $s = 3$ the increase of average productivity can not support the growth of consumption because we raise the level of misallocation. Finally, in the third experiment we show how a very positive productivity shock, that brings the less productive firm to be the highest one and increases the average productivity, can lead a drop in consumption. As 7 shows the misallocation overcomes the average productivity effect: once experienced the positive shock the firm has to build a massive stock of capital draining a large amount of resources from the economy.

¹⁴ $s = 2$ is associated with the ergotic distribution.

V. Conclusion

The purpose of this work is to quantify the contribution of the granular shocks in generating aggregate volatility and to investigate the different channels through which granularity drives the business cycle. To quantify the contribution of such granular shocks to generate aggregate fluctuations, we first construct a panel data of firms and show evidence that the distribution of idiosyncratic shocks faced by the largest firms in the U.S. has a negative mean, which is significantly countercyclical. Following, we develop and estimate a new frictionless business cycle model with heterogeneous firms where we abandon the law of large number and introduce the granular shocks as part of the aggregate states such that we let the Granularity shape the firm productivity distribution. Indeed, one of the contribution of this paper is to work out a general solution method that can be applied to a broad class of models. We show how the firm distribution can not be directly approximated by using the well known quasi-aggregation technique because of the stochastic productivity distribution and we show how to adapt the quasi-aggregation technique with granular models by adding past information. Finally we evaluate the importance of granularity generated by shocks to large firms and we isolate and study the channels through which Granularity affects the aggregate volatility. Our main findings are as follows. Firstly, consistent with the previous literature, 'Granular shocks' that continuously make the firm productivity distribution time-varying is an important source of the aggregate fluctuations explaining around 25 percent of the output volatility, 32 percent of investment and 20 percent of hours worked. Secondly, large firm movements can account for 11 percent of the output, 15 percent of the investment and 8 percent of the hours worked. Thirdly, relative to top 100 largest firms, that half of the volatility generated by the Granularity is attributable to the misallocation channel that can explain 6 percent of the output, 8 percent of the investment and 5 percent of the hours volatilities.

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Table I. Descriptive Statistics

	mean	sd	skewness	kurtosis	p5	p25	p50	p75	p95
sales	1081.9	3980.3	11.6	213.6	5.2	33.1	136.2	596.4	4716.4
employment	8.0	24.6	10.8	200.3	0.0	0.3	1.4	5.5	37.0
age	16.0	11.6	1.5	5.2	4	7	12	21	40
life	27.5	15.8	0.8	2.9	7	15	24	37	60
lnsales_shock_ijt	0.1	0.4	-0.2	2.6	-0.61	-0.20	0.07	0.31	0.63

Table II. Parameter Values

Description	Parameter	Value
Discount factor	β	0.96
Labor share	ν	0.60
Capital share	α	0.245
Preference parameter	η	2.15
Depreciation rate	δ	0.07
Probability of reset	π	0.43
Pareto multiplier	em	8.0
Pareto curvature	e	5.947
The number of productivity grid points	N_ε	21
Total number of firms	m	4.5 millions

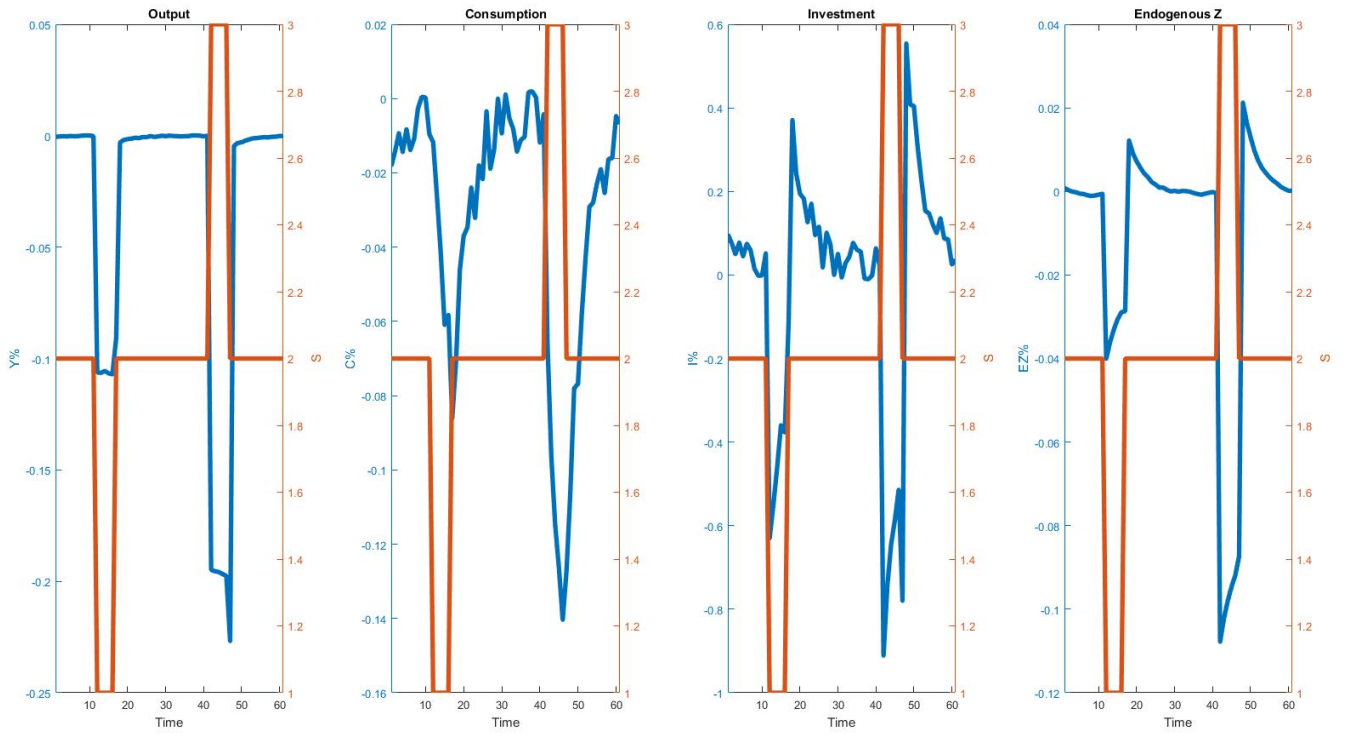
Table III. Business Cycle Moments

Variables	Model _{4.5mill.}			Data		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x, y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x, y)$
Output	0.408	1.000	1.000	1.57	1.0	1.0
Consumption	0.090	0.220	0.652	1.3	0.7516	0.9
Investment	2.366	5.803	0.987	7.0	4.5	0.91
Hours	0.356	0.873	0.981	2.0	1.3	0.9
TFP	0.201	0.493	0.992			
Avgr100	6.9	16.9	0.364	3.0	1.89	0.4
Γ	3.0	0.073	0.499			

Table IV. Large firms and Channels

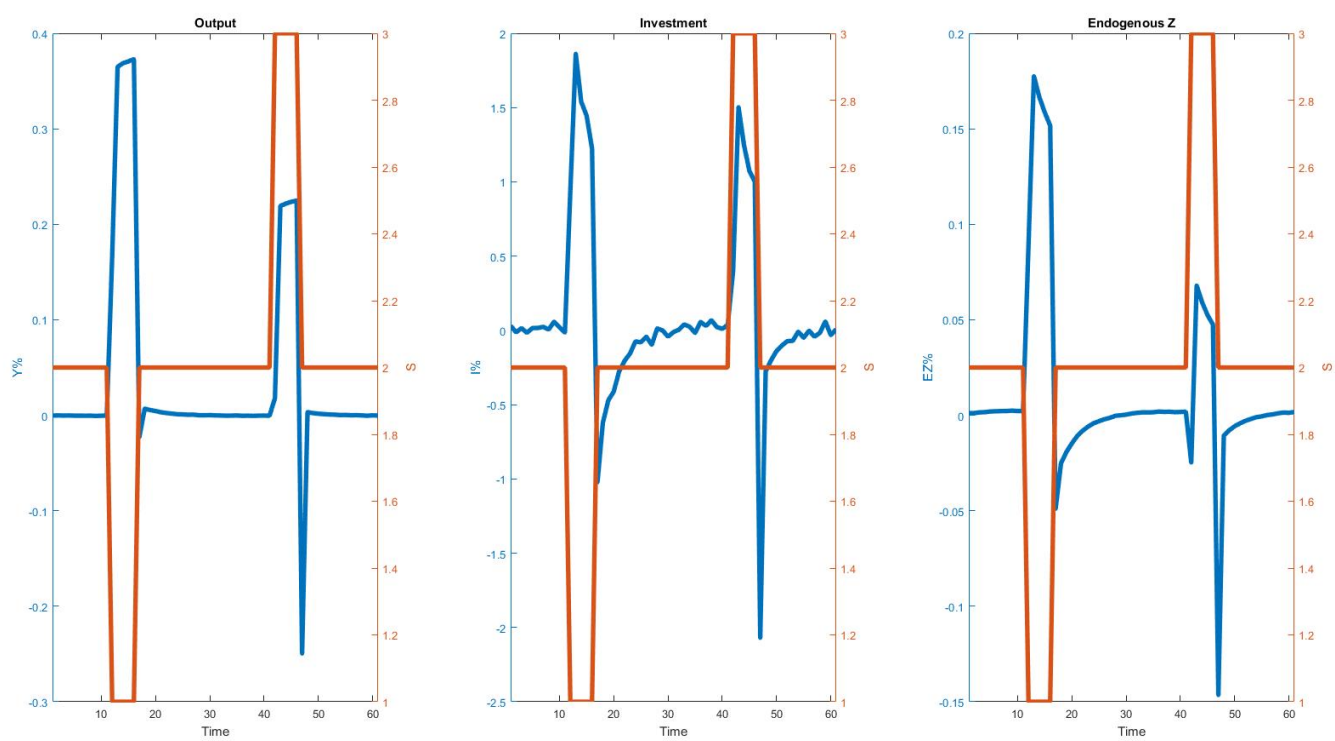
Variables	Model ₁₀₀ 1			Model ₁₀₀ 2		
	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x, y)$	$\sigma(x)$	$\frac{\sigma(x)}{\sigma(y)}$	$\rho(x, y)$
Output	0.173	1.000	1.000	0.109	1.0	1.0
Consumption	0.044	0.255	0.719	0.014	0.38	0.666
Investment	0.972	5.609	0.985	0.578	5.327	0.956
Hours	0.145	0.835	0.978	0.085	0.787	0.939
TFP	0.089	0.511	0.991	0.063	0.584	0.957
Avgr100	6.691	38.605	0.661	7.41	64.837	0.095
Γ	0.024	0.136	0.533	0.010	0.527	0.643

Figure 5. Comparing *dynamic inefficiency*



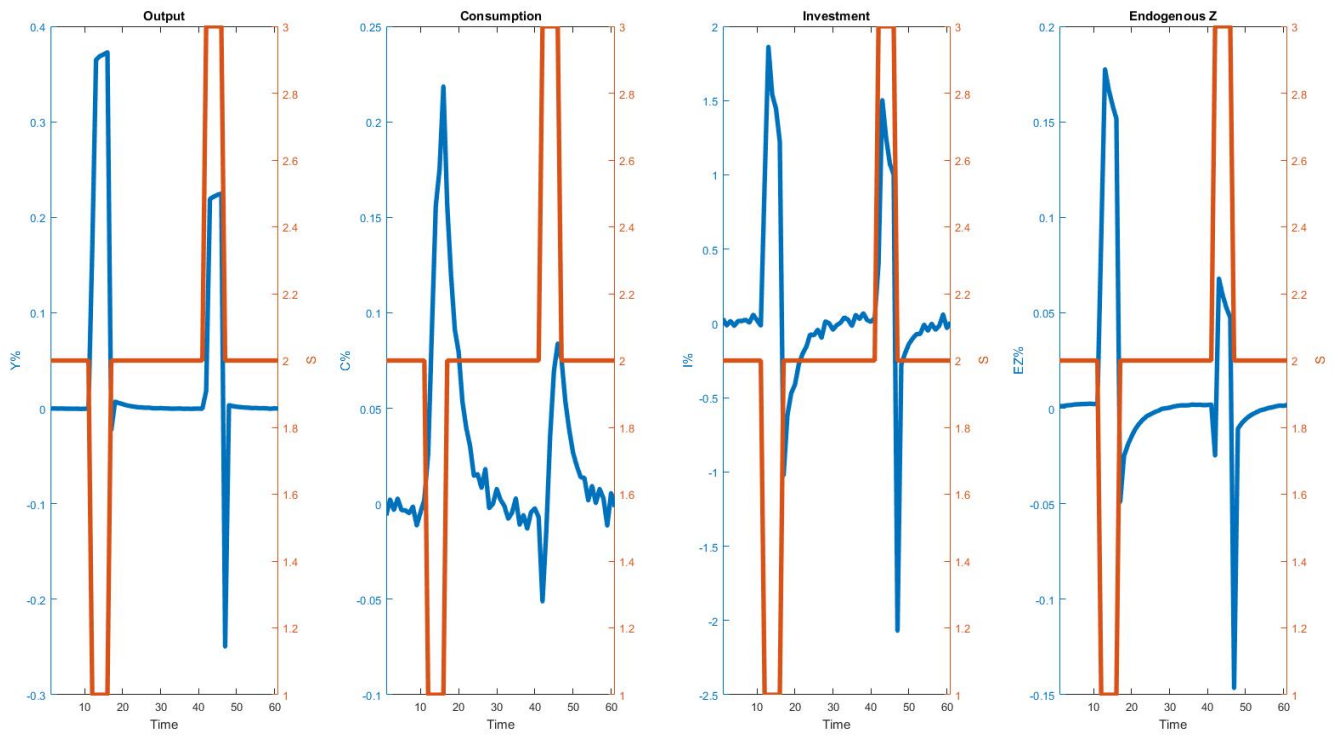
Note: For each state of s the average productivity is constant in the economy.

Figure 6. Comparing *dynamic inefficiency* and *average productivity*



Note: Economy with changing average productivity and allocative efficiency.

Figure 7. *Dynamic inefficiency stronger than average productivity effect*



Note: Economy with changing average productivity and allocative efficiency.