

A Model of Endogenous Debt Maturity with Heterogeneous Beliefs*

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Abstract

This paper studies optimal debt maturity when firms issue non-contingent claims and investors disagree about repayment probabilities. The optimal debt maturity choice is a mix of long- and short-term debt securities. Multiple maturity issuances allow firms to best leverage scarce collateral by intertemporally catering risky promises to investors most willing to hold risk. Heterogeneous investors directly contrasts theories of debt predicated on agency costs and liquidity risk and provide a novel explanation for why large and mature companies typically issue debt with multiple maturities. Lastly, we show that non-financial covenants aimed at preventing debt dilution do not affect real outcomes because they simply reallocate collateral from short-term to long-term debt holders.

Keywords: Collateral, Debt Maturity, Investment, Cost of Capital, Debt Covenants, Seniority

JEL Codes: D92, G11, G12, G31, G32, E22

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1 Introduction

Many large and mature corporations typically raise capital by issuing debt of various maturities. For example, IBM tapped the bond market 5 times in the first 6 months of 2017 with a different maturity offering each time. In July 2017, AT&T raised \$22.5 billion issuing maturities ranging from 5.5 years to 41 years. In 2013, Verizon raised \$49 billion issuing debt through 6 different maturities. Microsoft offered 7 different maturities when it raised \$19.75 billion in 2016. In fact, virtually every recent major public debt offering involves multiple maturities.

What is most striking about the regularity with which firms issue multiple maturity debt is that they are all large, mature and well established firms. This is puzzling because existing explanations for why a firm would issue multiple maturities rely on balancing inefficient liquidation risk from short-term debt and maintaining control rents or optimal continuation policies from long-term debt (Diamond (1991), (1993), and Houston and Venkataraman (1994)). Yet, empirical and survey evidence suggests that liquidation and information asymmetries are not likely to have significant effects on large, mature corporations with abundant public information and analyst coverage. For example, Graham and Harvey (2001) find that listed firms are typically not concerned with information asymmetries and the mis-pricing of securities due to agency problems. Johnson (2003) finds that the liquidity risk effects of short-term debt matter almost exclusively for unrated firms, and are virtually non-existent for rated firms. Billet, King, and Mauer (2007) do not find evidence that liquidity risk drives short-term debt use at all for rated firms.

This paper presents a novel theory to explain why issuing multiple debt maturities is cost minimizing and value maximizing for firms in the absence of agency conflicts or liquidity risk. Firms use debt maturity to inter-temporally cater risky claims on cash-flows to investors most willing to hold them. For any given investment amount, a firm has

three ways to structure its debt maturity: 1) use long-term debt that matches the timing of assets and liabilities. Long-term debt requires paying positive credit spread due to default risk, but it insulates the firm from changes in the cost of issuing debt in the future; 2) issue short-term debt that needs to be rolled over. Short-term debt is initially safe without liquidity risk, but completely exposes the firm to price fluctuations in the future; or 3) issue a combination of the two maturities in which only the short-term component is subject to price changes.¹

Debt maturity choice is analyzed as a tradeoff between the cost of risky debt over time due to changes in expected firm cash-flows and heterogeneity in the price investors are willing to pay to hold risky debt claims on those cash flows. For a given investment amount, issuing more long-term debt requires offering higher compensation to less optimistic investors. At the same time, more long-term debt also reduces the amount of short-term debt that needs to be rolled over, which results in more optimistic investors determining the market value of short-term debt. Therefore, changes in the relative supply of debt issued across time leads to changes in the relative cost of debt financing during those times. The main result is that a debt issuance strategy that involves multiple maturities allows the firm to issue to optimal amount of risky debt in each time period and capitalize on the fact that some investors are more willing to hold risk than others. By contrast, a single debt maturity strategy only permits the optimal supply of risky debt in a single time period and forces the firm to raise a larger portion of its capital from investors who seek higher compensation. The firm is able to raise additional capital on better terms by catering its debt securities through maturity choice.

The intuition for the optimality of multiple debt maturities is the following. Con-

¹Recent studies of Hugonnier, Malamud, and Morellec (2014 and 2015) have used search frictions to highlight the point that capital supply frictions can generate new predictions for firm financial policy. The simple collateral constraints and investor heterogeneity generate produce a similar environment in which one can easily study how debt maturity interacts with investment decisions.

sider all long-term debt funding. There are two benefits from substituting a portion of the long-term debt for short-term debt. First, short-term debt is risk free in the initial period. Second, because less long-term debt is issued, a more optimistic investor prices it in equilibrium, raising long-term bond prices. The costs of partially substituting into short-term debt are the following. First, the firm must pay expected refinancing costs in bad states. The second cost is the dilution cost to long-term debt due to the rising face value of short-term debt claims needed to ensure short-term debt is safe. In general, the benefits always outweigh the costs. The reason is twofold. Substituting risky for risk free debt is always cheaper, and short-term refinancing costs are paid in expectation rather than with certainty as with long-term debt. Moreover, the dilution effect on long-term debt prices is mitigated by the fact that increasingly optimistic investors finance long-term debt as the supply of long-term bonds is reduced. This means that the marginal investor cares less about the dilution costs the more the firm substitutes away from long-term debt

By similar reasoning, multiple debt maturities generally dominate all short-term debt financing. The benefits of substituting a portion of debt financing into long-term debt are the following: Raising one dollar long-term is cheaper than raising one dollar short-term conditional on bad news because short-term debt is information sensitive. In addition, the reduction in short-term debt supply reduces the dilution costs, which further increases long-term debt prices. The only cost of substituting into long-term debt is paying a positive credit spread with certainty and giving up the opportunity to finance short-term debt risk free.

Our model is an incomplete markets economy with binomial states and three-periods: 1) an initial state, 2) intermediate states, and 3) terminal states. The key frictions are investor heterogeneity and repayment enforcement problems.² Creditors cannot coerce

²Our focus is on limited enforcement frictions as in Rampini and Vishwanathan (2010) and Fostel and Geanakoplos (2015, 2016). Distinct from these papers, we ask how a firm should optimally structure its debt maturity in an economy with collateral constraints and heterogeneous investors. Rampini and Vishwanathan

debtors into repayment, and collateral is used as enforcement. When a debtor fails to honor its debts, the creditor has the right to seize collateral to be made whole, but no more.³ We consider risk neutral creditors with heterogeneous beliefs over the expected value of repayment. Investors are willing to pay different prices to hold risky debt, and these prices change in intermediate states as uncertainty is either resolved in good states or grows in bad states.

After establishing that a multiple maturity structure is optimal, we explore the effects of non-financial debt covenants that prevent debt dilution. Specifically, we allow long-term debt to be secured by one of the most common covenants found in long-term corporate debt indentures—the negative pledge covenant. A negative pledge covenant is a distinct way to earmark specific assets for creditors if a debtor defaults on one debt obligation before another debt obligation comes due. In particular, the negative pledge stipulates that a debtor cannot use any of its assets as security for subsequent debt obligations without securing the current issuance or issue claims more senior than those under consideration. Negative pledges are almost ubiquitous in corporate indentures but their impact, to our knowledge, has not been rigorously studied by economists.^{4,5}

To model the negative pledge, we assume long-term debt holders are entitled to receive a *pro rata* share of firm assets that are stipulated in their indenture, irrespective of what

(2010) focus on how a firm should allocate scarce collateral between investment and risk management. Fostel and Geanakoplos (2016) focus on how collateral constraints and financial innovation affect investment efficiency.

³Our choice to highlight a collateral friction is supported by recent empirical evidence suggesting that collateral plays an important role in the design of debt contracts and the provision of credit (Cerqueiro, Ongena, and Roszbach (2016)).

⁴By contrast, corporate legal scholars view negative pledge covenants are highly important. Legal scholar Philip Wood (2007) states that the negative pledge clause is “one of the most fundamentally important covenants in an unsecured term loan agreement.” Hart and Moore (1995) mention negative pledges in their discussion. In fact, we show that the negative pledge is equivalent as a hard senior long-term claim in the spirit of Hart and Moore (1995) if the defining characteristic of seniority is being impervious to dilution.

⁵See section 4, table 2 to see a coarse breakdown of negative pledge data in primary public debt indentures obtained from Mergent-FISD. In essence, these covenants are more likely to appear in medium-to-long-term debt contracts and in debt contracts issued by non-financial sector firms.

happens with its short-term obligations. We show that the delivery on long-term debt with the covenant is equivalent to the delivery in an economy in which long-term debt is the only maturity traded. In other words, the covenant protects long-term debt holders from short-term debt collateral dilution. *Ceteris paribus*, the price any investor is willing to pay for a protected long-term bond rises when their claims cannot be diluted. In equilibrium, the firm responds by re-optimizing its maturity structure away from short-term debt toward long-term debt. Consistent with our model's predictions, Billet, King, and Mauer (2007) find that debt covenants and short-term debt act as substitutes; firms with more covenants in their public debt indentures tend to issue relatively more long-term debt than firms with few or no covenants. More interestingly, the covenant does not affect equilibrium investment or credit spreads because it simply reallocates the otherwise diluted collateral back to long-term debt holders. This allows the firm to raise more long-term debt because there is more collateral promised to long-term debt holders. The equilibrium price of long-term debt is unaffected because it reflects both higher collateral but also the additional claims against it. Likewise, short-term debt claims are reduced as is the collateral available for short-term debt holders so that equilibrium prices are unaffected.

The model matches many empirical aspects of debt markets. Choi, Hackbarth, and Zechner (2016) show that corporations typically issue debt into, on average, more than 3 distinct maturity bins, and that large and mature corporations are more likely to issue multiple maturity debt. Norden, Rooenboom, and Wang (2016) show that borrowing costs are lower and leverage is positively associated with debt granularity. Our model also rationalizes why the largest corporations, who are the least likely to be subject to information asymmetries and liquidation risk, have active commercial paper programs (Kahl, Shivdasani, and Wang (2015)). Issuing commercial paper acts as a substitute for more expensive long-term debt because it reduces the amount of long-term promises made for a fixed amount of collateral. Rolling over the short-term claims by using collateral to-

morrow is marginally less expensive than issuing additional risky long-term claims today. Furthermore, pledgeable collateral jointly determines maturity, leverage, and the “growth option” of investment in our setting. Empirical studies typically treat only leverage and maturity as jointly determined and growth options as exogenous. Our model suggests that existing identification of the impact of growth on maturity and leverage is insufficient. Finally, the model generates cross sectional predictions on maturity, leverage, and growth that differ based on firm or industry specific collateral values or on the recovery rate of the debt securities.

The organization of the paper is as follows: related literature is below. Section 2 introduces the model, agents, the different debt contracts considered. Section 3 characterizes the equilibrium debt liability structure and comparative static results. Section 4 introduces the covenant and provides a numerical example of the model. Section 5 concludes. All proofs that are not obvious from the text are contained in the appendix.

Related literature

In a series of papers, Hart and Moore (1994, 1995, 1998) show that debt is an optimal contract to resolve agency problems and constrain management to either payout cash flows or prevent inefficient investment. These models either only examine long-term debt (Hart and Moore (1995)), or characterize repayment paths as either the fastest or slowest (Hart and Moore (1994) and (1998)), but never a combination of the two. Zwiebel (1996) show that multiple repayment paths will constrain empire building and prevent control takeovers, but only for the *most risky firms* for whom debt is a possible financing source, which is at odds with the empirical observations. While agency frictions certainly explain why management may use debt in its capital structure, they do not appear to adequately describe why multiple maturities are simultaneously used to raise capital (see also Jensen and Meckling (1986), Bolton and Scharfstein (1990, 1998)).

Private information can affect the types of debt securities firms issue. When firms have inside information, Flannery (1986) shows that firms will use short-term debt to signal quality. Diamond (1991 and 1993) and Houston and Venkataraman (1994) show that liquidity risk breaks the reliance on short-term debt and generates different debt maturity choices in the cross section based on credit ratings.⁶ We show that multiple maturity debt is optimal without liquidation risk and firms do not use private information in any way.

There are many models in which debt affects firm value. For example, the maturity of debt can improve investment incentives due to debt overhang (Myers (1977), He and Diamond (2014)), optimal default timing (He and Milbradt (2016)), and information asymmetries (Flannery (1986), Kale and Noe (1990)). Debt maturity affects corporate financial policy our model because the same risky debt claim will be priced differently in different periods when investors disagree about repayment probabilities.⁷ Another distinction in our model are non-exclusive relationships between debtors and creditors. Non-exclusive relationships are quite natural in practice because large corporations typically raise capital many public creditors and private private bank syndicates. For example, Dass and Massa (2014), using Lipper eMAXX data, highlight that the average corporation has 17 institutional investors acting as creditors at any point in time (see also Detragiache, Garella, and Guiso (2000)).⁸

Diamond and He (2014) highlight the subtle effects of debt maturity on debt overhang

⁶Proposition 2 of Diamond (1991) shows that short-term debt is the unique funding outcome in the model absent liquidation risk or loss of control rents. The agency problem is therefore necessary in his model to obtain an equilibrium with multiple maturities. Firm borrowing is fixed in Diamond (1991) and firms can borrow up to the fixed amount at the same interest rate.

⁷Heterogeneity is at the heart of Jung and Subramanian (2014), but an agency problem gives maturity a role in their model. Specifically, heterogeneous beliefs between managers and equity holders leads to a tradeoff between manager optimism and long-term debt issuance. Our model also has a flavor of this effect, but heterogeneity between the firm and investors is only material for determining what portion long-term debt constitutes of total debt issuance.

⁸Large firms typically raise capital from a syndicate of creditors rather than a single creditor even when considering private loan markets. Using supervisory data on bank holding companies, Caglio, Darst, and Parolin (2016) show that larger corporates borrow from, on average, 8 banks compared to small firms that tend to borrow from one.

and investment incentives. The optimal debt maturity balances the symptoms of short- and long-term debt overhang; respectively, earlier default versus reduced investment incentives. However, they consider different debt maturities with equivalent market values and a fixed asset. We do not consider overhang effects because how debt maturities are structured in our model affects the *ex ante* value of the asset/project the firm undertakes.

Dynamic debt maturity models in continuous time focus on refinancing policies, optimal leverage ratios, and target average debt maturity (see Leland (1994, and 1998)). He and Milbradt (2016) bring debt maturity to the forefront of these models. They emphasize the joint determination of default and maturity by showing that a firm actively manages maturing debt depending on the firm's distance to default. The firm issues short- or long-term debt and commits to a constant book leverage policy, *i.e.* maintains a constant aggregate face value of outstanding debt. Rollover losses arise as equity holders must absorb any cash flow shortfall when maturing bonds are refinanced when credit conditions deteriorate. The rollover losses feed back to the default decision by equity holders, leading to earlier default. Our model characterizes the firm's optimal debt financing strategy with multiple maturity issuances. Though we do not focus on the timing of default, our model has a similar feedback mechanism in which the anticipated short-term debt rollover losses induce the firm to substitute toward more initial long-term debt. We view our paper as complementary.

Brunnermeier and Oehmke (2013) show that financial firms' inability to commit to a maturity structure leads short-term debt to dilute long-term debt. In their model with a fixed supply of assets, the firm increases aggregate debt liabilities when new debt is issued to repay expiring claims, diluting the per-claim value of existing debt. Equity holders cannot absorb losses in their model as they can in our model and He and Milbradt (2016). He and Xiong (2012b), with a fixed maturity structure, show how short-term debt can amplify default risks when liquidity risk is present because equity holders will default at

earlier valuation thresholds. Default timing is fixed in our model, but maturity is allowed to adjust.

Geanakoplos (2009) and He and Xiong (2012a) study debt financing in incomplete asset markets with heterogeneous agents. In their models, short-term debt is the unique equilibrium because a sequence of short-term claims allows agents to take maximum leverage. The difference in their models is that all agents own risky assets, some of whom have higher valuations than others, and agents can borrow against the assets by issuing safe promises to obtain leverage. Issuing consecutive short-term claims allows optimists to borrow against the lowest value of the asset one period in the future, while a long-term claim only allows an agent to borrow against the final period worst case outcome. We adopt the same uncertainty structure to highlight that our result is not a special case of what one assumes about uncertainty, but that introducing an optimizing agent with production into a heterogeneous agent framework delivers the multiple maturity phenomenon that we commonly see in practice.

2 Model

2.1 Time and uncertainty

The model is a dynamic three-period production economy with incomplete asset markets. Time is denoted $t = \{0, 1, 2\}$. Uncertainty is given by a tree of state events $s \in S$ with root s_0 , intermediate states $s \in S$ that take values $\{U, D\}$, and a set of terminal nodes denoted $S_T = \{UU, UD, DU, UU\} \subset S$. Let state realization U be up or a “good” state and D be down or a “bad” state.

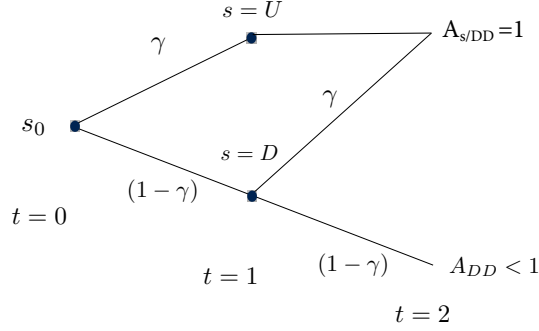
The economy receives a technology shock denoted A_{s_T} that affects output at $t = 2$. The expected value of the technology shock is conditional on the information revealed at $t = 1$.

We assume for simplicity that good news at $t = 1$ resolves uncertainty at $t = 2$ and there is no production shock: $A_{UU} = A_{UD} = 1$. Bad news at $t = 1$ raises uncertainty at $t = 2$ about the ability of the firm to repay debts, akin to “scary bad news” in Geanakoplos (2009). Specifically, there is no technology shock at terminal node $s = DU$, but there is a shock at terminal node $s = DD$, $A_{DD} < A_{DU} = 1$. Note that this uncertainty structure is the same as the simplification made in the continuous time version of Diamond and He (2014).⁹ Figure 1 depicts the economy’s state tree.

The assumptions about time and uncertainty are made for simplicity. We show in the appendix that equilibrium is *qualitatively* unaffected under different assumptions. For example, one may assume that good news is more likely to follow good news rather than bad news, which alleviates the concern that bad news is effectively not as bad. This would change only *quantitatively* how much long- versus short-term debt the firm would issue. Alternatively, one could allow for uncertainty conditional on $s = U$ and that $A_{UD} \neq A_{DU}$. This alternative would also only affect the relative amounts of long- and short-term debt issued, and would not change the result that issuing both maturities is optimal. The latter alternative would equate the model’s uncertainty structure with what He and Xiong (2012a) consider. Moreover, Fostel and Geanakoplos (2010) prove that agents have the incentive to produce projects that become more volatile in bad times. The reason is simple: uncertainty following bad news is not very informative, which implies that price declines in bad intermediate periods are relatively small. Alternatively, if uncertainty was completely resolved after bad news, then prices in bad intermediate periods would fall much further and reflect the certain bad outcome in the final period. Lower intermediate prices would limit *ex ante* how much agents could leverage and borrow against their projects.

⁹In example 2 of Diamond and He (2014), they assume that asset volatility is state-contingent. Specifically, $\sigma_H = \varepsilon > 0 = \sigma_L$ where σ_i is asset volatility conditional on state i . Clearly uncertainty is resolved when $\sigma_L = 0$.

Figure 1: Economy State Tree



Tree3.pdf

2.2 Debt contracts

The key friction in our model is that agents cannot be coerced to repay debts. As in Rampini and Vishwanathan (2010 and Fostel and Geanakoplos (2016), collateral serves as the payment enforcement mechanism. Specifically, creditors have the right to confiscate debtor collateral up to the value of the promise but nothing more. “Collateral” in our economy will be the firm itself, and can be thought of as the physical assets it produces from its investment decision. The collateral value of a debt contract is given by a state-contingent delivery function, $d_{ST}(\cdot)$. Implicitly we are assuming there are no collateral cash flow problems in that all agents anticipate the state-contingent value of collateral (see Fostel and Geanakoplos (2015)). We return to the debt delivery functions in section 2.4.

There is a single durable consumption good available in the economy at $t = 0$, which is the numeraire. There are two types of promises that can be made, each with different maturity. A promise that matures after one period is called short-term and a promise that matures after two periods is called long-term. All promises are non-contingent, pay zero-coupons. For simplicity, we normalize the repayment value of each honored promise

to 1. Promises can be interpreted as debt contracts. Let the quantity of debt issued at any state and time be q_{s/S_T} . The quantity of long-term debt issued at $t = 0$ is denoted q^ℓ and the market price denoted by p^ℓ . Short-term debt may be issued at $t = 0, 1$. The quantity of short-term debt issued at $t = 0$ is given by q_0^ζ and the quantity of short-term debt issued at $t = 1$ is given by q_s^ζ , $s = U, D$. The prices of short-term debt at $t = 0, 1$ are respectively p_0^ζ , p_U^ζ , and p_D^ζ . Following much of the literature, we assume equal seniority between short- and long-term debt. All market prices of debt securities will be determined through equilibrium market clearing conditions

2.3 Agents

We first describe the firm and its objective, followed by the investors problem.

2.3.1 Firm

We assume a representative firm is owned and operated by a manager (equity claimant) with access to a two-period decreasing returns to scale production technology. Clearly all agency problems are resolved with the assumption that the firm is owned and operated by the same agent. This is done to contrast our maturity results from extant agency-based models. The production function is denoted by $f(I; \alpha, A_s) = A_s I^\alpha$, $\alpha < 1$ where I is the amount of capital the manager raises and puts into production. We assume the firm has no cash endowment, does not generate cash flow at $t = 1$, and that new promises issued at $t = 1$ do not scale the project's original size.¹⁰

The firm's objective is to maximize expected profits by choosing how much capital

¹⁰Alternatively, one could assume that there is an extreme form limited commitment at the interim date in which no cash flows can be verified at a reasonable cost so debt repayments cannot come from cash flow. Under this alternative, what would be important would be that the cash-flow is independent of how the project is financed. The debt maturity mix will affect the investment cost that generates the cash-flow. Even if management could abscond with all intermediate cash, they would still issue the types of debt securities that would reduce costs to make higher profits in the final period.

to raise and the maturity of the debt contracts is issues.¹¹ Let ρ denote the portion of investment capital that is raised by issuing long-term debt, $\rho = \frac{p^\ell q^\ell}{I}$, and let γ denote the probability of good news.¹² Formally, the firm maximizes the following problem:

$$\left\{ \begin{array}{l} \max_{I, \rho} \Pi = \left\{ \gamma(I^\alpha - q^\ell - q_U^\zeta) + (1 - \gamma)\gamma(I^\alpha - q^\ell - q_D^\zeta), 0 \right\} \\ \text{s.t.} \quad I = p^\ell q_0^\ell + p_0^\zeta q_0^\zeta \\ \quad \quad p_s^\zeta q_s^\zeta = q_0^\zeta, s = U, D \\ \quad \quad 0 \leq \rho = \frac{p^\ell q^\ell}{I} \leq 1 \end{array} \right. \quad (1)$$

Because there is no production shock at $s = U$ and the firm can always repay, γ characterizes the firm's decision for both terminal states, $s = \{UU, UD\}$. At $t = 1$ the firm must decide whether it is beneficial to roll-over the short-term component of its debt portfolio. The firm repays short-term debt holders by raising $p_s^\zeta q_s^\zeta = q_0^\zeta, s = \{U, D\}$. At $s = U$, the firm can always repay debts and $p_0^\zeta, p_U^\zeta = 1$. The firm simply owes $q_U^\zeta + q^\ell$ at $t = 2$. At $s = D$, there is uncertainty regarding whether the firm can repay debt at $t = 2$. In this case, $p_D^\zeta < p_0^\zeta = 1$ and the firm owes $q_D^\zeta + q^\ell$ at $t = 2$. In the event of default, the firm makes no profits and all assets are distributed to creditors pro rata. The maximization problem is subject to the following constraints: the amount of capital the firm can use for production has to be raised by issuing bonds at $t = 0$. Conditional on rolling over short-term debt at $t = 1$, the firm issues new short-term debt, $q_0^\zeta = p_s^\zeta q_s^\zeta, s = \{U, D\}$. Lastly, the portion of the firm's investment that is raised through long-term debt, ρ , is naturally bound between

¹¹We restrict the analysis to debt issuance and do not allow for equity financing. This allows us to focus the analysis entirely on the endogenous composition of debt issuance in terms of the debt liability structure. Incorporating equity is a natural extension to the model.

¹²We will show that γ does not determine then general existence of multiple maturities as an equilibrium outcome. The fact that γ is known to the firm and may not be equivalent to the marginal investor's expectation of good news is not completely without loss of generality. γ will determine the relative amount of long-versus short-term claims, $0 < \rho < 1$, that makeup the optimal debt liability structure. However, one can solve the model by restricting γ to almost surely equal the marginal buyer's expectation so that there is a "true" state probability. This approach will pin down a unique ρ for all A_{DD} rather than have a state-space consisting of (A_{DD}, γ) -pairs.

0 and 1. To derive the firm's first order conditions for a maximum, first use the definition of $\rho = \frac{p^\ell q^\ell}{I}$ to write the problem in terms of choice variables I and ρ :

$$\max_{I_0, \rho} \Pi = \left\{ \gamma \left(I_0^\alpha - \frac{\rho I}{p^\ell} - \frac{(1-\rho)I}{p_U^\xi} \right) + (1-\gamma) \gamma \left(I^\alpha - \frac{\rho I}{p^\ell} - \frac{(1-\rho)I}{p_D^\xi} \right), 0 \right\}.$$

If an interior maximum for ρ exists, the first order necessary conditions with respect to I and ρ respectively, are

$$\alpha I^{\alpha-1} \left[1 - (1-\gamma)^2 \right] = \frac{\rho \left[1 - (1-\gamma)^2 \right]}{p^\ell} + \frac{(1-\rho)}{p_U^\xi} \left[\gamma + \frac{\gamma(1-\gamma)}{p_D^\xi} \right] \quad (2)$$

$$\frac{\left[1 - (1-\gamma)^2 \right]}{p^\ell} = \frac{1}{p_U^\xi} \left[\gamma + \frac{\gamma(1-\gamma)}{p_D^\xi} \right]. \quad (3)$$

The necessary conditions for the corner solutions are easily obtained by plugging either $\rho = 0$ or 1 into the firm's maximization problem—there will be no first order condition with respect to ρ . Equation (2) says that the marginal product of capital in states where the firm makes profits—which occurs with probability $1 - (1-\gamma)^2$ —must equal the maturity-weighted expected marginal cost of debt. The marginal cost of long-term debt is given by $\frac{[1-(1-\gamma)^2]}{p^\ell}$ and the marginal cost of a sequence of short-term bonds is given by $\frac{1}{p_U^\xi} \left[\gamma + \frac{\gamma(1-\gamma)}{p_D^\xi} \right]$. With probability γ , the firm will issue a sequence of risk-free bonds ($p_U^\xi = 1$). With probability $\gamma(1-\gamma)$, the firm will face an increase in the cost of debt and have to pay $p_D^\xi < p_U^\xi = p_0^\xi = 1$ per bond to roll over existing claims. Equation (3) says that in an interior debt maturity optimum, the marginal cost of a long-term bond must equal the marginal cost of a sequence of short-term bonds. Intuitively, if marginal cost of one maturity is lower than the other, the firm will find it optimal to always issue the less expensive maturity, raise more capital, and make higher profits. But since the marginal costs must be equivalent in any interior optimum, we can combine equations (2) and (3)

into a simplified version of either long- or short-term debt respectively:

$$\alpha I^{\alpha-1} = \frac{1}{p^\ell}, \quad (4)$$

$$\alpha I^{\alpha-1} \left(1 - (1 - \gamma)^2\right) = \frac{1}{p_U^\xi} \left[\gamma + \frac{\gamma(1 - \gamma)}{p_D^\xi} \right]. \quad (5)$$

The equilibrium debt maturity the firm chooses will be determined by the relative market prices of risky debt securities, (p^ℓ, p_D^ξ) . In order to find debt prices and characterize equilibrium, we must solve the investors' problem with the debt delivery functions for the different maturity strategies.

2.3.2 Investors

There exists at $t = 0$ a continuum of uniformly distributed investors with unit mass, $h \in H \sim U[0, 1]$, each of whom is endowed with a unit of the durable consumption good in all non-terminal states, $e^h, e_s^h, s \neq S_T$. The uniform distribution allows one to rank investors according to the likelihood each places on the subsequent state being good, denoted by h . Investors are risk-neutral, expected utility maximizers that consume at $t = 2$, and do not discount the future. Without loss of generality, we assume investors have different priors (see Fostel and Geanakoplos (2015)).¹³

Investors also have access to a riskless storage technology and form portfolios consisting of cash and bonds purchased from the firm. The von-Neumann-Morgenstern preferences are given by:

$$U^h(x_{UU}, x_{UD}, x_{DU}, x_{DD}) = h^2 x_{UU} + h(1 - h)x_{UD} + (1 - h)hx_{DU} + (1 - h)^2 x_{DD} \quad (6)$$

¹³The specific reason for heterogeneity is not relevant. One could equally assume investors differ in a measure of risk aversion; have different endowments across states, which produces different marginal utilities across states; or have different degrees of "patience." The critical assumption is the heterogeneity of marginal utilities across investors. We choose to think about beliefs because it is most familiar in these models.

We now characterize the investors' budget sets. Given debt prices, $(p_0^\ell, p_0^\xi, p_U^\xi, p_D^\xi)$, each investor, $h \in H$, chooses cash holdings, $\{x_0^h, x_D^h, x_U^h\}$, debt holdings, $\{q^{\ell,h}, q_0^{\xi,h}, q_U^{\xi,h}, q_D^{\xi,h}\}$, and final period consumption decisions, $\{x_s^h\}, s \in S_T$, to maximize utility given by (6) subject to the budget set defined by:

$$\begin{aligned}
B^h(p_0^\ell, p_0^\xi, p_U^\xi, p_D^\xi) = & \left\{ (x_0, x_D, x_U, q_0^\ell, q_0^\xi, q_U^\xi, q_D^\xi, x_s)_{h \in H} \right. \\
& x_0^h + p^\ell q^{\ell,h} + p_0^\xi q_0^{\xi,h} = e_0^h, \\
& x_U^h + p_U^\xi q_U^\xi = e_1^h + q_U^\xi \times d_U(q_0^\xi) \\
& x_D^h + p_D^\xi q_D^\xi = e_1^h + q_D^\xi \times d_D(q_0^\xi) \\
& \left. x_s^h = x_0^h + x_{U,D}^h + q^\ell \times d_s(q^\ell) + q^\xi \times d_s(q^\xi), s \in S_T \right\}. \quad (7)
\end{aligned}$$

Each investor may use their initial cash endowment to purchase either type of debt at $t = 0$. The endowment received at $t = 1$ plus any returns from short-term debt holdings are used to either purchase short-term debt at $t = 1$ or held for final consumption. All cash that is not used to purchase debt is carried forward to consume at $t = 2$. All final period consumption comes from debt purchases and cash holdings.

It is clear that each investor will choose the debt maturity that delivers repayment in the state that the investor finds most likely. And because the contracts that investors purchase are debt contracts that pay out 1 in repayment states, the most optimistic investors simply purchase the debt security that they can purchase in the largest quantity. Put differently, the optimists purchase the cheapest debt securities that pay the highest yield.

Consider a conjectured equilibrium with both long- and short-term debt: at $t = 0$, optimists will use all of their endowment to purchase long-term bonds, while relative pessimists will hold a mix of short-term debt and cash. If the firm issues short-term debt that it rolls over at $t = 1$, the optimists will use their $t = 1$ endowment to purchase risky short-term debt at $s = D$ as well. All investors will hold a portfolio of safe short-term debt

and cash at $s = U$. Alternatively, consider a conjectured short-term only debt equilibrium: all investors at $t = 0$ will hold a portfolio of safe short-term debt and cash, and optimists will use all of their $t = 1$ endowment to purchase risky short-term debt at $s = D$. Lastly, consider a conjectured long-term only debt equilibrium: optimistic investors at $t = 0$ will purchase long-term debt and all other investors will remain in cash. All investors will use their $t = 1$ endowment for consumption because no further bonds are issued.

In order to close the model and characterize which type of debt securities will trade, we must determine exactly how investors price long- and short-term debt based the recovery value of debt given default.

2.4 Debt repayment

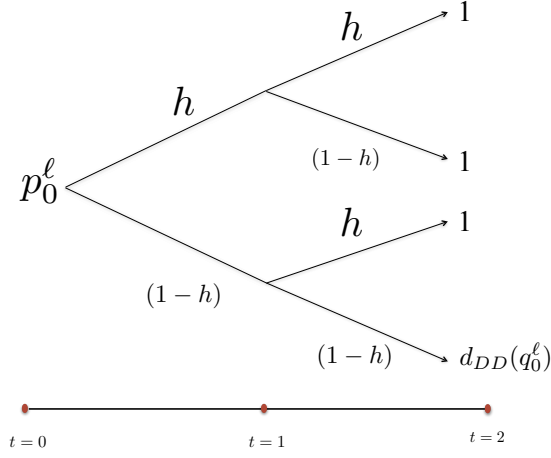
Due to the repayment enforcement friction, debt is effectively collateralized by future output as in Rampini and Vishwanathan (2010) and Fostel and Geanakoplos (2016). The implicit assumption underlying these models is that there are no collateral “cash-flow problems.”¹⁴

Short-term Debt

Let $d_{U,D}(q_0^\zeta)$ describe short-term debt delivery at time 1. Short-term debt repayment is conditional on whether the firm rolls over debt at $t = 1$. Specifically, short-term debt will be “safe” if it is always rolled over, and both recovery and prices are equal to 1; otherwise, short-term debt will be risky due to potential liquidation. To highlight the novelty of investor heterogeneity rather than liquidation risk ala Diamond (1991), we assume there is no inefficiency from default in any state. Risky debt, regardless of its maturity, is always fairly priced. As such, considering the case in which short-term debt is not rolled over

¹⁴Traditional macro/finance models such as Kiyotaki and Moore (1993) assume that creditors can confiscate land, but not the fruit produced by the land. Corporate finance models following Holmstrom and Tirole (1997) assume an information asymmetry between borrowers and lenders. Borrowing too much in these models reduces cash-flow and reduces incentives to work hard to produce good cash flows.

Figure 2: Long Term Financing



does not deliver any new insight, and we can proceed considering only the case in which debt is always rolled over.¹⁵

The firm must issue q_s^ζ one-period debt contracts to rollover expiring claims. Short-term debt delivery in the final period is state-contingent with default only at $s = DD$.¹⁶

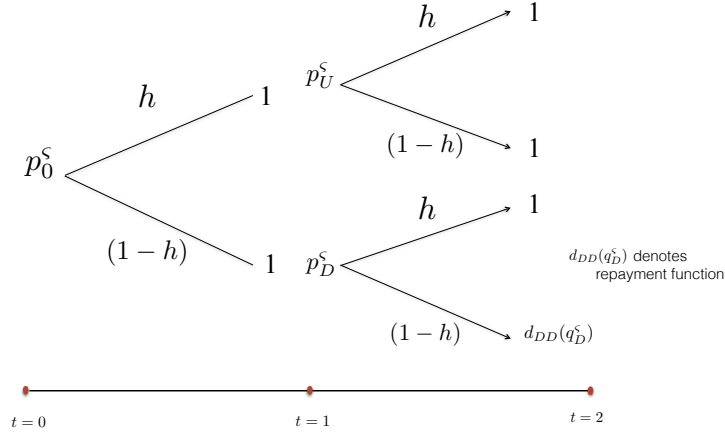
$$d_{DD}(q^\zeta) = \begin{cases} 1, & s \neq DD \\ \frac{A_{DD}I^\alpha}{q_D^\zeta + q^\ell}, & s = DD \end{cases}. \quad (8)$$

Equation (8) says that all debts are honored as long as two periods of bad news do not occur. Firms default after two periods of bad news due to the technology shock and all firm assets are divided pro rata to debt holders. The sequence of short-term debt contract payouts is depicted in figure 3.

¹⁵Flannery (1986) considers only safe short-term debt at $t = 0$ as well. His analysis highlights the importance on asymmetric information in determining debt maturity.

¹⁶This is simply restating absolute priority ala Merton (1974) via a collateral constraint. Equity receives nothing when debt holders are not repaid *ex post*, but collateral delivery is required to obtain debt *ex ante*. We will show that $A_{DD} < \alpha$ is sufficient for $d_{DD}(\cdot) < 1$, and consider this parametrization throughout the paper to focus on risky debt.

Figure 3: Short-Term Financing



Long-term Debt

Long-term debt is very simple to describe as it matches the maturity of assets with liabilities. Let $d_s(q^\ell)$ denote the long-term debt delivery function. Given our assumptions on short-term debt being rolled over, both long- and short-term deliveries are given by (8), $d_s(q^\ell) = d_s(q^\varsigma)$, or generically $d_s(\cdot), s \in S_T$.¹⁷ Equation (8) gives the recovery value of firm assets for any of the possible debt maturity equilibria by simply setting either $q_D^\varsigma = 0$ for a long-term only equilibrium or $q^\ell = 0$ for a short-term only equilibrium. We now characterize equilibrium.

2.5 Equilibrium

1 *Equilibrium is a collection of the following non-negative items: debt prices, firm investment decision, investor cash holdings, debt holdings, and final consumption decisions that maximize firm profits given by (1), investor utility given by (6) subject to their budget constraints in (7), and both the goods and debt markets clear in all periods.*

¹⁷We are implicit assuming that the collateral value of the firm is not being split as explicit collateral pieces for long- versus short-term debt. We return to this assumption in section 4 of the paper when we consider restrictive negative covenants.

To highlight the importance of heterogeneity and catering debt securities to investor needs, we begin by solving the well known special case of the model with homogeneous investors. We then show how moving to heterogeneous investors delivers both similar predictions as Diamond (1991) and the new prediction that issuing both a combination of debt maturities is generally the least cost financing option, even without liquidity risk.

2.5.1 Examples of investor beliefs and debt maturity choice

1 *Homogeneous investors and debt maturity irrelevance.*

Assume a unit mass of competitive investors all share the common prior that the up state occurs with probability, γ . The only change a common belief makes to the model is to how investors value the different debt securities. We will use the following parameters throughout the examples: $A_{DD} = .5$, $\alpha = 0.8$, $\gamma = 0.7$. There is no risk of liquidation at $t = 1$ so that all short-term debt issued at $t = 0$ is risk free.

Long-term—If long-term debt is the only security traded, $\rho = 1$. A competitive equilibrium requires that investors make zero profits in expectation. If γ is the probability of $s = U$, then long-term debt is valued by all investors according to

$$\left(1 - (1 - \gamma)^2\right) \times 1 + (1 - \gamma)^2 \times d_{DD}(q^\ell) = p^\ell. \quad (9)$$

With $\rho = 1$, equation (8) simply becomes $\frac{A_{DD}I^\alpha}{q^\ell}$. Using equation (4) and the funding condition $I = p^\ell q^\ell \Rightarrow \frac{1}{q^\ell} = \frac{p^\ell}{I}$, we see that the recovery value of debt in an all long-term funding regime is purely driven by economy primitives:

$$d_{DD}(q^\ell) = \frac{A_{DD}}{\alpha}. \quad (10)$$

Intuitively, the less production is affected by the technology shock (high A_{DD}), the more assets are available for investors to recover. Likewise, the more output a firm generates

per unit of investment capital (low α), the larger the per claim value investors have in recovery. The recovery value of long-term debt is $\frac{A_{DD}}{\alpha} = \frac{0.5}{0.8} = .625$ and long-term bond price is .9663. From (4), the marginal cost of issuing long-term debt is $\frac{1}{p^\ell} = 1.034$. The level of investment, $I = (\alpha p^\ell)^{\frac{1}{1-\alpha}} = .2760$, and the value of the firm, $V_\gamma^\ell = I^\alpha = .3570$. Lastly, firm expected profits are $\Pi_\gamma^\ell = (1 - [1 - \gamma]^2) (V_\gamma^\ell - q^\ell) = 0.0650$.

Short-term—If short-term debt is the only security traded, $\rho = 0$. Equations (9) and (10) pin the equilibrium bond price for long-term debt contracts. Similarly, all risky short-term debt purchased at $s = D$ must yield zero profit to investors in expectation

$$\gamma \times 1 + (1 - \gamma) \times d_{DD}(q_D^\xi) = p_D^\xi. \quad (11)$$

The debt delivery function from (8) becomes

$$d_{DD}(q_D^\xi) = \frac{A_{DD}}{\alpha} \left(\frac{p_D^\xi \gamma + \gamma(1 - \gamma)}{1 - (1 - \gamma)^2} \right). \quad (12)$$

Solving (11) and (12) simultaneously, gives $p_D^\xi = .8685$, and a recovery rate of .5242. From (5), the marginal cost to the firm of issuing term debt is $\frac{1}{1 - (1 - \gamma)^2} \left[\gamma + \frac{\gamma(1 - \gamma)}{p_D^\xi} \right] = 1.034$. Since the marginal cost of issuing short-term debt is the same across funding structures, the value of the firm though short-term funding is equivalent to the value through long-term funding: $V_\gamma^\xi = V_\gamma^\ell$.

From the example, clearly $d_{DD}(q_D^\xi) < d_{DD}(q^\ell)$ when $p_D^\xi < 1$ and short-term debt dilutes the per-claim collateral value of the firm in default. However, because all information is common to all agents and there is no inefficiency from default, the fact that $d_{DD}(q_D^\xi) < d_{DD}(q^\ell)$ is irrelevant as debt is fairly priced. A firm will not be able to save on short-term costs by substituting into more long-term debt or vice versa. Hence, the Modigliani-Miller irrelevance theorem holds.

2 Debt maturity with heterogeneous investors

Now consider the heterogeneous investor case described in section 2.3.2. All of the firms first order conditions for an optimal are the same. The only thing that changes are the debt pricing equations.

Long-term—The marginal long-term bond buyer at $t = 0$ must be indifferent between buying the bond and cash. The break even condition for the marginal long-term bond buyer is

$$1 - (1 - h_0)^2 + (1 - h_0)^2 d_{DD}(q^\ell) = p^\ell. \quad (13)$$

In equilibrium, long-term bond prices are determined by market clearing in the bond market. Specifically, investor demand for risky bonds must equal supply of bonds the firm issues:

$$\frac{1 - h_0}{p^\ell} = q^\ell, \text{ Long-term debt market clearing} \quad (14)$$

In order to solve for equilibrium, we must determine who the marginal long-term bond buyer is, h_0 .¹⁸ There are four unknowns, (I, p^ℓ, q^ℓ, h_0) , four equations, (4), (13), (14), and $I = q^\ell q^\ell$, and debt recovery is the same as the homogeneous case, $d_{DD}(q^\ell) = \frac{A_{DD}}{\alpha} = 625$. The equilibrium is $[p^\ell = .9702, h_0 = .7182, q^\ell = .2903, I = .2817]$. Expected firm profit is $\Pi_h^\ell = .9375 \times (.3629 - .2904) = 0.0660$. Notice the value of the firm and hence firm profits, are higher under the heterogeneous regime: $\Pi_h^\ell = 0.0660 > \Pi_\gamma^\ell = 0.0650$ and $V_h^\ell = .3629 > V_\gamma^\ell = .3570$. This is because the marginal investor's prior is higher than the common belief $h = .7182 > \gamma = .70$, which leads to lower interest rates and more investment.

Short-term—By similar reasoning, the marginal short-term debt holder at $t = 1$ must be

¹⁸There is no marginal buyer in the homogeneous beliefs equilibrium and all prices were determined by γ .

indifferent between risky short-term debt and cash:

$$h_1 + (1 - h_1) d_{DD} (q_D^\xi) = p_D^\xi. \quad (15)$$

Market clearing for short-term debt requires clearing at both $t = 0, 1$. All short-term debt is initially riskless without liquidity risk, $p_0^\xi = 1$. The firm must issue $q_U^\xi = q_0^\xi$ in the good state and $q_D^\xi = \frac{q_0^\xi}{p_D^\xi}$ in the bad state to ensure short-term debt is initially riskless. Because debt is initially riskless, all investors can hold a portfolio of short-term debt and cash. Market clearing in the good state is same because all debts are repaid in full conditional on $s = U$. Lastly, market clearing conditional on $s = D$ requires that investor demand for risky bonds equals the supply of bonds the firm issues:

$$\frac{1 - h_1}{p_D^\xi} = q_D^\xi, \text{ Short-term debt market clearing.} \quad (16)$$

The short-term system can be boiled down to four unknowns, $(I, p_D^\xi, q_D^\xi, h_1)$, and four equations:

$$\begin{aligned} h_1 + (1 - h_1) d_{DD} (q_D^\xi) &= p_D^\xi \\ \frac{1 - h_1}{p_D^\xi} &= q_D^\xi \\ \alpha I^{(\alpha-1)} &= \frac{1}{1 - (1 - \gamma)^2} \left[\gamma + \frac{\gamma(1 - \gamma)}{p_D^\xi} \right] \\ I &= p_D^\xi q_D^\xi. \end{aligned}$$

The equilibrium values are $[p_D^\xi = .8786, h_1 = .7199, q_D^\xi = .3253, I = .2800]$ and $d_{DD} (q_D^\xi) = .5286$. The value of the firm equals $V_h^\xi = .3612$ and expected profits equal $\Pi_h^\xi = 0.0657$. The value of the firm is lower under the short-term debt regime with heterogeneous investors, but is still higher than the homogenous case. The reason why short-term debt is more expensive than long-term debt in this example is because the firm has to significantly

increase the face value of short-term debt conditional on bad news, $q_D^\zeta > q^\ell$, which occurs with probability $1 - \gamma$. Market clearing at $s = D$ requires raising capital from investors who are increasingly pessimistic about repayment probabilities, which tends to increase interest rates and borrowing costs. At the same time, the likelihood of having to pay the higher interest rates is high enough that the firm prefers its debt repayments to insulated from intermediate signals. But this reasoning suggests that it's possible that the firm prefers to be exposed to intermediate signals and issue short-term debt for high enough γ . The following example shows that this is indeed the case.

3 Short-term debt with heterogeneous investors

Keep everything about the economy the same as example 2, but let γ rise to 0.80. It is clear that endogenous variables under long-term financing are unchanged because γ does not affect pricing and investment: $[p^\ell = .9702, h_0 = .7182, q^\ell = .2903, I = .2817.]$ Profits and firm value also remain fixed: $\Pi_h^\ell = .0660$ and $V_h^\ell = .3629$. By contrast, the short-term variables are now $[p_D^\zeta = .8717, h_1 = .7097, q_D^\zeta = .3330, I = .2903]$. The value of the firm is $V_h^\zeta = .3712$ and firm profits are $\Pi_h^\zeta = .0713$. Now the firm strictly prefers to issue short-term debt over long-term debt, $\Pi_h^\zeta > \Pi_h^\ell$. The intuition is simple. A high likelihood of good news at $t = 1$ increases the chances the firm borrows risk free in the short-term debt market for two consecutive periods. The firm is willing to bear the risk of having to repay higher borrowing costs conditional on bad news because that risk is relatively unlikely to occur.

Notice in the example that long-term bond prices are substantially higher than the equilibrium risky short-term bond prices, $p^\ell = .9702 > p_D^\zeta = .8717$. The reason is that the marginal long-term bond buyer at $t = 0$ places very little weight on default, $(1 - h_0)^2 = .0793\%$, while the marginal risky bond buyer conditional on bad news places a much higher likelihood of default, $(1 - h_1) = .2903\%$. The following example shows that the firm can increase its value even more by offering investors a combination of debt securities.

The firm can offer investors at $t = 0$ long-term debt and take advantage of the fact that investors at $t = 1$ would also like to purchase risky debt, but can only do so if the firm issues short-term debt.

4 Optimal debt maturity mix with heterogeneous investors

Keep everything the same as the example above, but now allow the firm to choose $0 < \rho < 1$ rather than a corner solution. We now have eight unknowns $(I, \rho, p^\ell, q^\ell, p_D^\xi, q_D^\xi, h_0, h_1)$ and eight equations: (2),

$$\begin{aligned} \alpha I^{\alpha-1} [1 - (1 - \gamma)^2] &= \frac{\rho [1 - (1 - \gamma)^2]}{p^\ell} + \frac{(1 - \rho)}{p_D^\xi} \left[\gamma + \frac{\gamma(1 - \gamma)}{p_D^\xi} \right], \text{ first order w.r.t } I \\ 1 - (1 - h_0)^2 + (1 - h_0)^2 d_{DD}(q^\ell) &= p^\ell, \text{ long-term debt pricing} \\ h_1 + (1 - h_1) d_{DD}(q_D^\xi) &= p_D^\xi, \text{ short-term debt pricing} \\ \frac{1 - h_0}{p^\ell} &= q^\ell, \text{ long-term debt market clearing} \\ \frac{1 - h_1}{p_D^\xi} &= q_D^\xi, \text{ short-term debt market clearing.} \\ I &= p^\ell q^\ell + p_D^\xi q_D^\xi, \text{ firm funding condition} \\ \rho &= \frac{p^\ell q^\ell}{I}, \text{ long-term debt portion.} \end{aligned}$$

The solution to this problem is found in table 1. The first thing to note is that all bond pricing is significantly higher under the multiple debt maturity regime and either of the respective corner solutions in the above examples. The risky short-term bond price is now $p_D^\xi = .941$ compared to .871, while the long-term bond price is now $p^\ell = .989$ compared to .970. Moreover, the firm is raising debt on better prices *and raising more capital for investment*, $I = .311$ compared to .281 under the long-term regime or .290 under the short-term regime. This of course means that the firm has increased its value from .3712 to .3929 and its profitability from .0713 to .0750. How can this be? The answer is that the firm has increased the size of investment by using both long- and short-term debt to offer debt securities to investors most willing to buy them at both points in time. The two

Table 1: Optimal maturity mix

	p_D^S	p_0^L	q_0^S	q_D^S	q_0^L	I	ρ	h_0	h_1	V^P	Π
$(\alpha, A_{DD}, \gamma) = (.8, .5, .8)$	.941	.989	.149	.158	.163	.311	.519	.8341	.8548	.3929	.075

marginal buyers in the multiple debt maturity regime are $h_0 = .8341$ for long-term bonds and $h_1 = .8548$ for short-term bonds conditional on a bad state. Both marginal buyers are significantly more optimistic about firm cash flows than either of their counterparts in the corner solution regimes. When markets are incomplete and investors have different expectations over firm payouts, a firm can cater its debt securities to different investors across time to receive the highest price in the market.

3 The general debt maturity solution with heterogeneous investors

3.1 The tradeoff between long- and short-term debt

The examples in the previous section show that some debt dilution is actually desirable when investors are heterogeneous. For example, the highest recovery rate on debt is for a long-term debt regime when the timing of assets and liabilities are matched, but issuing both long- and short-term debt actually yields higher profits and market value. It turns out that issuing both long- and short-term debt simultaneously is generally to cost minimizing and value maximizing strategy for the firm. For any given investment amount, moving from an all long-term debt financing regime to a mix of short- and long-term debt has two benefits. First, it allows a more optimistic buyer to price risk long-term risk at $t = 0$, which lowers the cost of long-term capital. Second, substituting some long-term for short-term debt allows the firm to borrow risk free at $t = 0$ and gives the firm the opportunity to borrow risk free at $t = 1$, both of which lower overall financing costs. The cost of substituting all long to both long- and short-term debt is that the per claim value investors receive must

be diluted due to increasing the face value of debt conditional on bad news at $t = 1$. Debt dilution lowers the price any investor is willing to pay for hold a risky debt claim on firm cash flows. In general, the two forces pushing equilibrium debt prices up outweigh the dilution cost. The reason is that investors pricing risk are generally optimistic and do not place much weight on the value of debt in default. Hence they are generally not concerned with the dilution cost.

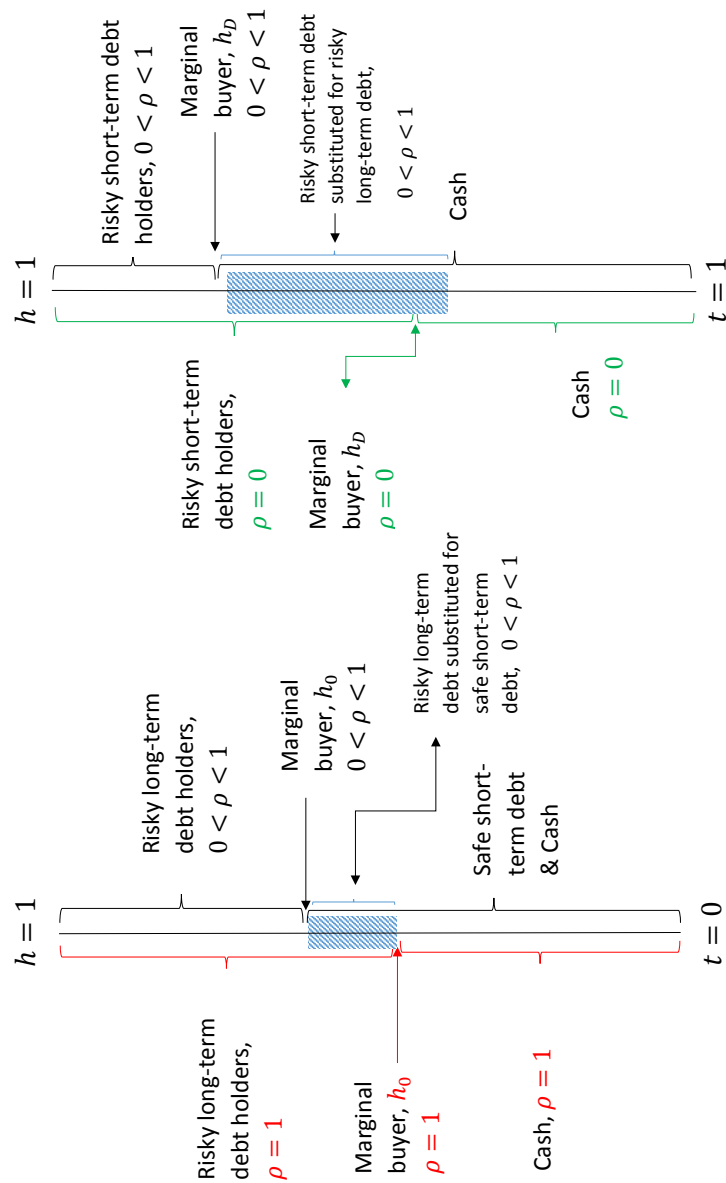
The alternative is to issue all short-term debt. This maximizes the dilution cost, which tends to lower debt prices. Cutting back on short-term debt and replacing it with long-term debt has two benefits as well. First, it concentrates risky short-term debt to more optimistic investors at $t = 1$, which raises prices. Second, it reduces the cost of dilution, which further raises prices. The cost of substitution is that the firm must pay a positive credit spread on long-term debt at $t = 0$ while all short-term debt is risk free. However, because the firm is selling risky long-term debt to optimistic investors, the initial bond prices are very high and close to risk free. Thus, on the margin, the cost of substituting into long-term debt is very small, while the benefits of higher risky short-term bond prices conditional on bad news are quite large. This intuition is formally stated in the following proposition.

1 *Multiple debt maturity structure:* *When there is no liquidity risk and investors are heterogeneous, the optimal debt financing strategy is characterized by a maturity choice set with all positive quantities, $Q \equiv (q_0^\ell, q_0^\zeta, q_s^\zeta) > 0, s = \{U, D\}$.*

Marginal buyers price risky assets in expectation. For example, at $t = 0$, the expected return to holding a risky long-term bond to maturity must be equivalent to holding cash, either because cash is the only other asset available in the economy, ($\rho = 1$), or because safe short-term debt is also issued, $0 < \rho < 1$.

$$\frac{1 - (1 - h_0)^2 + (1 - h_0)^2 d_s(\cdot)}{p_0^\ell} = 1. \quad (17)$$

Figure 4: Marginal buyer regimes



It is clear that with a fixed collateral value given by $d_s(\cdot)$ in (??), issuing more debt requires paying a higher borrowing cost as the equilibrium marginal buyer puts more weight on default and recovery states, $s = DD$. Note that the combination of a collateral constraint and investor heterogeneity generates an upward sloping supply of capital curve which contrasts the Modigliani and Miller assumption of a perfectly elastic supply curve. Likewise, an increase in the collateral value of the firm's assets raises risky debt prices because all investors receive more collateral delivery in default states. Similarly, at $t = 1$, a marginal buyer prices risky short-term debt by equating the expected return to holding cash:

$$\frac{h_1 + (1 - h_1) d_s(\cdot)}{p_s^\xi} = 1. \quad (18)$$

Short-term debt will be “safe” at $t = 0$ if and only if it is unconditionally rolled over at $t = 1$. The condition for rollover is profits must be greater than or equal to zero after repaying both long- and short-term debts:

$$I_0^\alpha \geq q_0^\ell + q_D^\xi. \quad (19)$$

Note that state probabilities, γ , do not factor in this decision because the firm only retains profits when it fully repays all debts in coinciding states. It is clear that the price of short-term debt issued at $t = 0$ must be $p_0^\xi = 1$ if the firm continues and produces at $t = 2$. The following proposition succinctly states when the firm will issue short-term debt with full commitment to always repay rather than relinquishing control of its assets through default.

2 Short-term Debt Rollover: Suppose $Q = (q_0^\ell, q_0^\xi, q_s^\xi) > 0$ $s = \{U, D\}$. Short-term debt at $t = 0$ is safe if and only if

$$\varepsilon \equiv \alpha \frac{1 - \rho}{(1 - \alpha \rho)} \leq \frac{p_D^\xi}{p_0^\ell} < 1. \quad (20)$$

The gist of proposition 2 is that issuing safe short-term debt is possible as long as there is a balance between the price of risky promises issued today versus tomorrow, $\frac{p_D^\xi}{p_0^\ell}$. Equations (17) and (18) show that the ratio of risky debt prices, $\frac{p_D^\xi}{p_0^\ell}$, the firm balances over time is fundamentally related to how different investors price risky debt securities through marginal buyer priors, h_0 and h_D . In a competitive equilibrium, risky debt prices are pinned down through market clearing; the supply of risky claims the firm issues must equal investor demand to hold risky debt: $\frac{(1-h_0)}{p_0^\ell} = q_0^\ell$ for long-term debt at $t = 0$ and $\frac{(1-h_D)}{p_D^\xi} = q_D^\xi$ for risky short-term debt issued at $t = 1$. In short, the more risky claims the firm issues at a single point in time, the lower the equilibrium price it receives in the market. If it becomes too expensive to rollover short-term debt (issue long-term debt,)— p_D^ξ is low (high) relative to p_0^ℓ —then the firm should switch to all long-term (short-term) funding.

We provide a sketch of the proof that (20) *almost surely* holds.¹⁹ For any given investment demand, I_0 , ρ goes to 1 as the firm substitutes long-term debt for short-term debt. The substitution into more long-term debt does two things: 1) it results in lower long-term debt prices at $t = 0$ as more pessimistic investors are needed to finance investment, and 2) it reduces the amount of risky short-term debt needed to ensure the safety of expiring claims at $t = 1$, which raises short-term debt prices. Thus (20) holds trivially. Alternatively, ρ goes to 0 when short-term debt is substituted for long-term debt. The substitution into short-term debt has the opposite effect on relative debt prices across time as substituting into long-term debt; short-term debt prices at $t = 1$ fall as long-term debt prices at $t = 0$ rise. As long as the risky-debt price ratio is greater than a measure of firm collateral, $\frac{p_D^\xi}{p_0^\ell} > \alpha$, (20) holds. Clearly, as α falls the condition holds. The intuition is that α measures the return to a unit of capital input (firm marginal productivity rises as α falls, $0 < \alpha < 1$). The more productive the firm is for each unit of input, the higher the expected

¹⁹We can show numerically it holds $0 < \alpha \lesssim .9$.

returns to production and the more likely it is the firm will always repay short-term debt to avoid collateral forfeiture.

We provide a sketch of the proof for intuition. For any given fundamental collateral value, $\frac{A_{DD}}{\alpha}$, and subsequent investment amount, I_0 , financed with multiple maturity debt, issuing safe short-term debt substitutes for risky long-term debt at $t = 0$, which lowers financing costs as long as the amount of short-term debt that needs to be issued in bad times is not too high. On the margin, short-term debt reduces financing costs the more long-term debt is initially issued because long-term credit spreads rise because the firm raises each marginal unit of capital from a more pessimistic investor. Likewise, the more short-term debt initially issued that needs to be rolled over at $t = 1$, the more beneficial using some long-term debt at $t = 0$ becomes.

Figure 4 captures the substitutability of debt maturity based on the different price of risk the firm faces in different time periods. Multiple maturity debt concentrates fewer total long-term promises to investors most willing to hold risk at $t = 0$ than a maturity with no short-term debt, $q_0^\zeta = 0$. Concurrently, in a multiple maturity equilibrium, the debt needed to ensure short-term debt is rolled over at $t = 1$ is also more concentrated to investors with higher willingness to hold risk than if the firm only issued long-term debt, $q_0^\zeta = 0$. A multiple debt maturity structure reallocates risky debt away from investors today, who require more collateral to borrow at a given interest rate, to investors tomorrow who require less collateral, and vice versa.

Before continuing, we draw the key distinction between our model and the heterogeneous agent models of Geanakoplos (2003 and 2009), Fostel and Geanakoplos (2008, 2010), and He and Xiong (2012a) where a sequence of short-term debt contracts is optimal. In these models, all agents are endowed with both a risk-less and risky asset. Some agents (optimists) want to hold more risky assets than others due to different marginal utilities. Leveraged optimists buy all the risky assets by issuing riskless promises equal to the

risky asset’s value in the worst state. Issuing short-term claims allows agents to borrow against the asset’s intermediate-state and terminal-state value, but long-term claims only allow agents to borrow against the terminal-state value. Thus, for optimists who price the asset in equilibrium, short-term debt always dominates long-term debt. These models are best suited to describe debt financing of financial assets for which the use of leverage is paramount. Banks, hedge funds, and institutional investors typically use leverage to make their asset purchases.

By contrast, the “firm” in our model is endowed with a risky production technology and issues debt claims using its technology as collateral to produce and consume—similar to Fostel and Geanakoplos (2016) and Rampini Vishwanathan (2010)—while investors have riskless assets that they use to purchase firm debt. Optimists, or natural bond buyers, use their riskless asset to buy risky debt claims. The firm maximizes its expected equity value by concentrating its risky claims to optimists. Using multiple debt maturities allows the firm to smooth debt financing cost across time by issuing risky claims to natural bond buyers rather than consolidating costs into a single maturity bucket that places debt into the hands of investors with lower willingness to purchase bonds.

Our model is particularly relevant for large corporations where liquidity risk and information asymmetries are likely second order concerns. Propositions 2 and 3 imply that safe short-term debt should be used *in conjunction* with risky-long term debt because it will help lower aggregate risky financing costs. This intuition rationalizes the existence of corporate commercial paper (CP) programs. In our model short-term CP are the safe issued at $t = 0$. The CP issuance is refinanced by a potentially risky debt issuance at $t = 1$. This interpretation is consistent with the “bridge financing” story of Kahl, Shivdasani, and Wang (2015), but is fundamentally based on issuing debt across time using the same underlying collateral. Moreover, safe-debt is *sufficient* for a debt maturity mix, which contrasts the liquidation risk stories underpinning Diamond (1991) and Houston and Venkataraman

(1994).

Lastly, investor heterogeneity and collateral constraints generate differences in cost of capital through time and helps reconcile survey evidence firms try to time the market when choosing debt financing. For example Graham and Harvey (2001) and Servaes and Tufano (2006) find that global CFO respondents largely issue debt maturity to time market interest rates and limit the amount of debt that needs to be refinanced at any point in time. Economist typically view a market timing response with circumspect. Our model suggests that debt maturity choice it is not so much about market timing *per se*. Rather, debt maturity is used to smooth financing costs by limiting the amount of risky debt that firms issue at any point in time. Lastly, both surveys find very little support for information asymmetries (Flannery 1986) and debt overhang (Meyers (1977) as the main factors driving maturity choice, while credit ratings are important insofar as they affect the terms of borrowing, but expectations about credit rating changes are low order at best (Diamond (1991) and Houston Venkatarman (1994)).

3.2 Debt maturity optimization and comparative statics

This section briefly discusses the model's comparative static results related to how the maturity profile is optimized toward long- or short-term debt depending on model parameters. The predictions of our model are broadly consistent with existing empirical studies, but are based on using collateral to make promises across time rather than asymmetric or private information.

Let $0 < \rho^*(\alpha, \gamma, A_{DD}) < 1$ denote the equilibrium amount of long-term debt issued for any given set of state parameters. Specifically, γ is the likelihood that good news arrives in the following period, from the firm's perspective. A_{DD} determines the amount of collateral the firm can pledge at $s = DD$ and is a measure of down risk, while α is the returns to scale parameter.

More short-term debt is issued the more likely good news arrives in $t = 1$, $\frac{\partial \rho}{\partial \gamma} < 0$. The reason is that the likelihood of rolling over short-term debt at the risk-free rate increases, which lowers expected rollover costs relative to long-term funding. We can interpret γ as a measure of management “optimism” which is consistent with the empirical findings of Landier and Thesmar (2008) and Graham et. al (2013) that management optimism leads to more short-term debt issuance, controlling for firm risk factors and leverage.

More short-term debt is issued the more collateral the firm can pledge at $s = DD$, $\frac{\partial \rho}{\partial A_{DD}} < 0$. The reason is that risky short-term debt prices at $t = 1$ are more responsive to movements in A_{DD} than risky long-term debt prices at $t = 0$. To see why, consider any given investor, h . This investor puts more weight on $s = DD$ at time $t = 1$ than she does at $t = 0$ due to scary bad news, $(1 - h) > (1 - h)^2$. The value of an investor’s claim at $s = DD$, irrespective of maturity, is the delivery rate given by (??). Investor h therefore values the recoverable claim more at $t = 1$ than at $t = 0$. The interpretation is that more short-term debt is issued the higher are expected cash-flows, or the lower is down risk, because of higher collateral values, which lower expected short-term rollover costs.

Lastly, there are two interpretations for α : 1) a measure of firm productivity (higher curvature), and 2) the firm’s “growth option.” The more productive the firm (lower α), the more long-term debt the firm is utilized (higher ρ). The reason is related to the observations in Diamond and He (2014). Mainly, the value of long-term debt responds more to firm fundamentals than short-term debt when bad news increases uncertainty relative to good news ($p_U^S = 1, p_D^S < 1$). Thus, changes in fundamentals affect short-term debt values only at $t = 1$ when short-term debt issued at $t = 0$ is risk free. Conversely, changes in fundamentals always affect risky long-term debt. Thus, $\left(\frac{\partial \rho}{\partial \alpha}\right) < 0$.

The returns to scale parameter α is also a measure of the firm’s “growth option.” Empirical studies measure growth options as the market-to-book value of assets. In the model,

the market value of the firm's assets is simply the amount it produces because there is only one asset whose price is normalized to 1. The book value of the firm's asset is the amount of capital it raises to produce, or the book value of its liabilities. The market-to-book value of the firm is given by²⁰

$$\text{market-to-book} = \frac{I_0^\alpha}{I_0} = I_0^{(\alpha-1)} = \frac{1}{\alpha p_0^\ell(\alpha, A_{DD}, \gamma)}. \quad (21)$$

Notice that the growth option of the firm is inextricably linked to the exogenous parameters of the model through the market price of debt. Therefore, growth options are endogenously determined along with the firm's maturity choice and leverage through an asset's fundamental collateral value via α . Empirical treatment of the growth options as exogenous to leverage and maturity choices is not justified within the framework of our model. The joint endogeneity of growth options and maturity choice may help explain why the empirical literature reports mixed results (Barclay and Smith (1995) and Guedes and Opler (1996) find that growth options and maturity are negatively related. Stohs and Mauer (1996) and Johnson (2003) find a positive relationship, while Billet et. al. (2007) find no relationship when controlling for covenants).

To sum up,

3 Debt maturity is optimized more long-term:

- *the lower the likelihood of good states, low γ , $\frac{\partial p}{\partial \gamma} < 0$;*
- *the lower are expected cash-flows or the higher is down risk, low A_{DD} , $\frac{\partial p}{\partial A_{DD}} < 0$.*

²⁰We use the first order conditions (2) and (3) to derive the market-to-book in terms of the long-term bond price, p_0^ℓ . It can also be expressed in terms of short-term bond prices since the expected costs across maturities must be the same in an interior maturity equilibrium.

4 Protected debt with endogenous maturity

In this section we show that protective debt covenants do not affect real output or the value of the firm's equity claim when collateral is required to issue debt. To anticipate the result, protecting long-term debt raises any individual investor's expected per claim value. The firm responds by re-optimizing its maturity more toward the relatively cheap long-term debt and away from the relatively more expensive short-term debt. The re-optimized firm changes the relative supply of different debt maturities it offers, which brings relative equilibrium prices back to pre-protected levels. No *aggregate real* outcomes are affected when the firm simply substitutes one maturity for another.

Our treatment of protected long-term debt can be thought either as an explicit collateral pledge or earmark, or the inclusion of a negative pledge covenant that explicitly spells out how long-term debt is secured from short-term debt dilution. The benefit of thinking about negative pledge covenants, as detailed below, is two fold: 1) negative pledges are among the most common covenants found in public debt indentures, 2) given their prominence, surprisingly little is known in the academic literature of their impact. We thus attempt to fill this void with the support of strong practical relevance.

4.1 Protected debt and the negative pledge covenant

We begin by describing the negative pledge covenant. Negative pledges are among the most common covenants found in public debt indentures and widely recognized by the law and economics profession (see Bjerre (1999), Wood (2007, 2008)). The covenant stipulates that the firm cannot issue secured debt in the future without securing the current debt issue. For example, Billet et. al. (2007) classify negative pledge covenants as "Secured Debt Restrictions" because they restrict the security of future debt issues. Table III in their paper shows that negative pledges are typically the 3rd or 4th most common

Table 2: Negative pledge covenant

	Negative pledge covenant	
	Yes	No
Non-financial	14,783	11,424
Financial	3,117	4,825
< 5yr	2,244	2,376
5yr - 30 yr	15,284	13,401
Total	17,900	16,249

covenant, behind cross default or acceleration, asset sale, and merger clauses. Negative pledges are more common than leverage, dividend, and share repurchase restrictions. Table 2 gives a general sense for the basic statistics on types of bonds that contain a negative pledge covenant. They are more prone in medium-to-long-term non-financial corporate indentures.

We assume that the negative pledge ensures that ρ portion of the firm assets are used as exclusive collateral for long-term debt, irrespective of short-term debt financing at $t = 1$. These assets cannot be used as collateral for short-term debt without violating the pledge and opening the firm up to costly litigation. The remainder of the assets, $(1 - \rho)$, are used as collateral to secure short-term rollover financing.

With the covenant, the recovery values given by (8) now become

$$\begin{cases} d_{DD}(q_0^\ell) = \frac{\rho A_{DD} I_0^\alpha}{q_0^\ell}, \text{ long-term recovery} \\ d_{DD}(q_D^\xi) = \frac{(1-\rho) A_{DD} I_0^\alpha}{q_D^\xi}, \text{ short-term recovery} \end{cases}.$$

The collateral, or future output, is split between long-term creditors protected by the pledge and short-term creditors who fund the short-term debt rollover at $t = 1$. Using the first order conditions for an interior maximum, (2) and (3), along with the funding constraints

in program (1) that relate q_0^ℓ and q_1^ζ to I_0 and ρ , the debt delivery functions can be written as:

$$\begin{cases} \hat{d}_{DD}(\hat{q}_0^\ell) = \frac{A_{DD}}{\alpha} \\ \hat{d}_{DD}(\hat{q}_D^\zeta) = \frac{A_{DD}}{\alpha} \left(\frac{\hat{p}_D^\zeta}{\hat{p}_0^\ell} \right) \end{cases}. \quad (22)$$

The hats represent variables in an economy with the covenant. With the negative pledge, the maturity specific collateral rates in (22) behave as if $\rho = 1$ for long-term debt and $\rho = 0$ for short-term debt in (23), even though $0 < \hat{\rho} < 1$. By contrast, one can easily show that the recovery rates for all debt holders in the non-covenant economy are given by

$$d_{DD}(\cdot) = \frac{A_{DD}}{\alpha} \left(\frac{p_D^\zeta}{(1-\rho)p_0^\ell + \rho p_D^\zeta} \right). \quad (23)$$

The difference between the recovery rates in the non-covenant and covenant economy are that all debt holders receive a proportion of the fundamental recovery value, $\frac{A_{DD}}{\alpha}$, in the non-covenant economy, while only short-term debt holders receive a proportion of the fundamental recovery value with the covenant. All long-term debt holder are protected from debt dilution. Note also that the proportional recovery rates across the two economies are generally different.

4 Consider any $\hat{d}_{DD}(\hat{q}_0^\ell)$ defined by (22) with a secured covenant and corresponding $d_{DD}^*(\cdot)$ defined by (23) without a covenant. In any debt financing strategy for which $Q \equiv (q_0^\ell, q_0^\zeta, q_s^\zeta) > 0, s = \{U, D\}$, the following hold

- $\hat{d}_{DD}(\hat{q}_0^\ell) > d_{DD}^*(\cdot)$. Moreover, $\hat{d}_{DD}(\hat{q}_0^\ell) = d_{DD}(q_0^\ell)|_{q_0^\zeta=0}$ —long-term debt with secured covenants are protected from short-term debt dilution.
- $\frac{1-(1-h_0)^2+(1-h_0)^2[\hat{d}_{DD}(q_0^\ell)]}{p_0^\ell} > \frac{1-(1-h_0)^2+(1-h_0)^2[d_{DD}(\cdot)]}{p_0^\ell}$ —any given investor is willing to pay more for long-term debt with a secured covenant than without.

An immediate implication of proposition 5 is that the firm's debt maturity will be

optimized more toward long-term financing when long-term creditors are protected from dilution.

1 *The debt maturity mix is optimized more long-term when long-term creditors are protected with secured covenants. Let $\hat{\rho}$ be the equilibrium portion of long-term debt when collateral values are determined by (22). Let ρ^* be the equilibrium portion of debt issued long-term debt when collateral values are determined by (23). Then, $\hat{\rho} > \rho^*$.*

The result that the firm substitutes away from short-term debt toward more protected long-term debt is consistent with the empirical findings of Billet et. al. (2007). Interestingly, substitution effects arise with collateral purely through relative prices even without agency conflicts. Instead, it is the ability of the firm to split its collateral to back different debt maturities that generates the substitution effect. Secured debt covenants reallocate scarce collateral and lowering protected debt credit spreads (higher debt prices), which incentivizes shifting risky debt issuance more heavily toward the protected maturity. Furthermore, because a larger portion of debt financing occurs through long-term debt that is not information sensitive at $t = 1$, the volatility of investment returns will also fall. To see this, note that the firm can repay both short- and long-term debt at $t = 2$ conditional on $s = U$ at $t = 1$, while there is default risk conditional on $s = D$. Issuing debt securities at $s = D$ necessarily involves paying higher credit spreads to rollover expiring claims and lowers the project's returns at $t = 2$.²¹ Protective covenants shift more of the financing to less information sensitive long-term debt helping stabilize the unconditional returns at $t = 0$. The immediate implication the return volatility is endogenous to both maturity choice as well as to the inclusion of protective covenants.

The general equilibrium effects are more subtle. Specifically, the firm increases the supply of risky long-term bonds it issues but reduces the supply of risky short-term bonds.

²¹This will in general be the case as long as $s = D$ is characterized by more uncertainty than $s = U$. We have assumed away uncertainty at $s = U$ for simplicity.

In equilibrium, the relative prices of the two debt maturities must be equivalent in expectation (see equation (2)). The firm substitutes between maturities such that equilibrium debt prices are unchanged.

5 Let $Q^* \equiv (q_0^{\ell*}, q_0^s, q_s^s)$, $s = \{U, D\}$, $\forall q \in Q^* > 0$ be an equilibrium debt financing strategy that corresponds to aggregate debt financing terms $(I_0^*, p_0^{\ell*}, p_0^s, p_s^s)$ as the solution to program (1) with debt deliveries given by (23). When secured debt alters deliveries via (22),

1. The optimal debt financing strategy is given by $\hat{Q} \equiv (\hat{q}_0^{\ell}, \hat{q}_0^s, \hat{q}_s^s)$, $s = \{U, D\}$, $\forall q \in \hat{Q} > 0$ such that $q_0^{\ell*} > \hat{q}_0^{\ell}$ and $q_s^{s*} < \hat{q}_s^s$, $s = \{0, U, D\}$; and
2. $(I_0^*, p_0^{\ell*}, p_0^s, p_s^s)$ is unchanged.

The intuition is the following. Collateral is already required to issue debt because of the payment enforcement concern. Making the collateral in one debt maturity even more “secure” by explicitly preventing dilution does not impact equilibrium borrowing terms because the overall amount of collateral the firm can pledge is unchanged. The covenant simply reallocates a portion of the firm’s collateral toward protected debt ensuring the recovery value is as if there were no dilution due to a concurrent short-term debt issuance. The notion that the negative pledge prevents dilution is equivalent to what Hart and Moore (1995) consider to be a *hard claim* on firm cash flows. They show that hard claims on the value of assets in place prevent managers with empire building motives from undertaking negative net present value projects because the resources needed to fund intermediate investment projects are encumbered by existing long-term debt claims and cannot be diluted. Senior long-term debt is an efficient tool to implement first best behavior. In our model, protected debt also prevents dilution, and could be interpreted as senior to short-term debt issued at $t = 1$. However, there is no equilibrium effect on investment levels or profitability because seniority/the covenant does not increase the total collateral value of the firm;

it simply reallocates what would be diluted by short-term debt to senior long-term debt holders.

The following numerical example highlights the major comparative static results from proposition 4 and covenant results of corollary 1 and proposition 6.

4.2 Numerical example

This section provides a numerical illustration of the economy with and without the negative pledge covenant. We choose the following parameters

$\{\alpha = 0.8, 0 < A_{DD} \leq \alpha, 0 < \gamma \leq 1\}$. Figure (5) shows the equilibrium marginal investor regime for $A_{DD} = 0.5$ and $\gamma = 0.8$. What is important to note is that all risky debt financing is done by relative optimists. At $t = 0$ optimists use their cash to buy long-term bonds while relative pessimists buy safe short-term bonds. At $t = 1$, all risky short-term debt is also purchased by a subset of optimists. The more long-term debt the firm issues, the more investors it must seek to finance its risky long-term debt issuance pushing the marginal buyer at $t = 0$ down further. Issuing safe debt to relative pessimists at $t = 0$ enables the firm to issue its risky long-term promises to more optimists at $t = 0$ and roll those claims through relative optimists at $t = 1$.

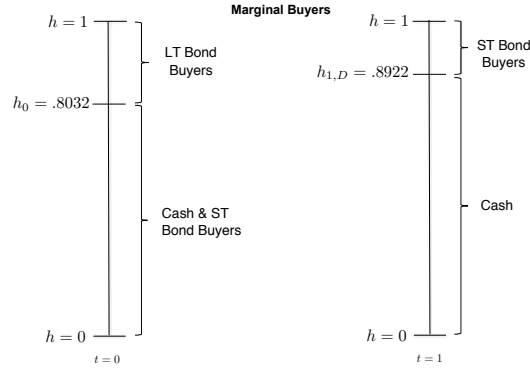
Table 3 highlights the major effects of the secured debt covenant for various (A_{DD}, γ) -pairs. The top (bottom) panel contains the endogenous variables for the economy with (without) the covenant. The numbers in red highlight the key changes. First, note that all debt prices, investment levels and profits are unchanged across the two panels. More (Less) long-term (short-term) debt is issued in the economy with the covenant. The covenant simply tilts the maturity in favor of long-term debt, $\rho \uparrow$, and the firm substitutes away from short-term debt.

The last set of results are the comparative statics for changes in down risk, A_{DD} , and

Table 3: Endogenous Variables

Covenant	p_0^ζ	p_D^ζ	p_0^ℓ	q_0^ζ	q_D^ζ	q_0^ℓ	I_0	ρ	Π
$(A_{DD}, \gamma) = (.5, .8)$	1	.941	.989	.145	.154	.167	.311	.533	.075
$(A_{DD}, \gamma) = (.2, .8)$	1	.894	.980	.136	.153	.163	.297	.539	.072
$(A_{DD}, \gamma) = (.5, .5)$	1	.957	.985	.107	.112	.199	.304	.646	.057
No Covenant	p_0^ζ	p_D^ζ	p_0^ℓ	q_0^ζ	q_D^ζ	q_0^ℓ	I_0	ρ	Π
$(A_{DD}, \gamma) = (.5, .8)$	1	.941	.989	.149	.158	.163	.311	.519	.075
$(A_{DD}, \gamma) = (.2, .8)$	1	.894	.980	.138	.154	.162	.297	.534	.072
$(A_{DD}, \gamma) = (.5, .5)$	1	.957	.985	.113	.115	.197	.304	.638	.057

Figure 5: Regime: Portfolio - Rollover



good state probability, γ . The first two rows of either panel show how variables change as A_{DD} decreases for a fixed γ , while the third row shows change as γ falls for the same A_{DD} as the first row. First consider risky debt prices, p_D^ζ and p_0^ℓ . More down risk at $t=2$ lowers all risky debt prices, resulting in lower investment and profits. The firm re-optimizes its debt maturity more toward long-term debt, $\rho \uparrow$. Second consider a decrease in good state probability for the same A_{DD} as in the first row. The bottom row shows that the firm re-optimizes its debt maturity more toward long-term debt, $\rho \uparrow$, resulting in lower long-term debt prices, but higher risky short-term debt prices. The firm also invests less and is less profitable. The comparative statics confirm the major predictions of the model.

5 Conclusion

This paper characterizes optimal debt maturity in an economy where borrowers must use collateral to borrow from heterogeneous lenders. We have shown that a multiple debt maturity offering consisting of both long- and short-term debt is generally the least costly debt financing strategy when the price of issuing risky debt changes over time. Using multiple debt maturities allows firms to cater risky debt securities to investors most willing to hold risk, which facilitates a reduction in borrowing costs and an increase in production and output. Moreover, a combination of debt maturities arises naturally when firms issue “safe” short-term debt and rationalizes why large corporates use commercial paper as bridge financing to finance long-term investment projects. Further, the model predicts that firms will use more short-term debt when managers operating in shareholders’ best interest are optimistic about investment returns, or when expected cash flows are high, or down-side risk is low. We also show how growth options and leverage are endogenous to the firm’s debt maturity choice because the price of the securities issued are affected by maturity, which in turn affects investment returns. Finally, we show that protective debt covenants or senior long-term debt that prevent dilution simply leads to more long-term financing and a substitution away from short-term financing. However, protective covenants do not affect real outcomes in general equilibrium with collateral requirements because they simply reallocate collateral claims among long- and short-term debt holders.

Our model also rationalizes one of Myers’ (1993) most striking findings. Myers notes that almost all leverage increasing actions are good news and leverage decreasing activities are bad news. In our model, the firm uses debt maturity to reduce financing costs and borrow and invest more against any given asset collateral value, which raises the value of the firm. In the paper we have abstracted away from agency concerns to highlight that the mechanism operates through collateral constraints and investor heterogeneity rather than liquidation risk or private information. In so doing, our model suggests that leverage, debt

maturity and proxies for growth options are all jointly determined, which may help explain the different findings of various empirical studies.

An important assumption we have made is that there are no collateral cash flow problems, which means that the future value of the firm that serves as collateral can be rationally anticipated. One outcome of this assumption is that investors will always demand to hold risky claims on firm cash flows even following bad news. Altering the model to allow for investor coordination failure and self-fulfilling debt runs with collateral may produce new and interesting interactions between debt maturity and run risk and the design of corporate securities to prevent such outcomes.

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A Appendix Omitted Proofs

Proof of Proposition 1: Suppose $A_{DD} = \alpha$. Issuing only long-term debt permits financing at the risky free rate, $p_0^\ell = 1$. Short-term debt co-exists iff $p_D^\zeta = 1$ as well because otherwise the l.h.s of (2) fails to equal the r.h.s. Maturity becomes irrelevant if there is no downside risk to collateral. Thus, $A_{DD} < \alpha$ being sufficient for $p_0^\ell < 1$ is also sufficient for (??) to be less than 1 because $p_D^\zeta < p_0^\ell < 1 \forall 0 < \rho < 1$. Thus, it is clear from (??) that $d_{DD}(\cdot)|_{0 < \rho < 1} < d_{DD}(q_0^\ell)|_{\rho=1}$ for all (A_{DD}, α) -pairs and risky short-term debt dilutes existing long-term debt. *Q.E.D.*

Proof of Proposition 2: Combining (2) and (3) and plugging into (19) immediately gives (20). Note that $\varepsilon(\rho; \alpha) \in (\alpha, 0)$, $0 < \rho < 1$ and clearly decreases in the arguments that increase ρ . Proposition 5 shows that $\frac{\partial \rho}{\partial \gamma} < 0$ and $\frac{\partial \rho}{\partial A_{DD}} < 0$, meaning that $\frac{\partial \varepsilon(\rho; A_{DD}, \gamma)}{\partial A_{DD}} \Big|_\alpha < 0$ and $\frac{\partial \varepsilon(\rho; A_{DD}, \gamma)}{\partial \gamma} \Big|_\alpha < 0$. Therefore, $A_{DD} \downarrow 0$ and $\gamma \downarrow 0 \implies \varepsilon \xrightarrow{\lim} 0$. Any risky bond price ratio $\frac{p_D^\zeta}{p_0^\ell} > 0$ will satisfy (20) for small values of A_{DD} and γ because $\rho \xrightarrow{\lim} 1$. It is less obvious that (20) is always satisfied when $\rho \xrightarrow{\lim} 0$ because $\varepsilon \xrightarrow{\lim} \alpha$. The reason is that moving from all short- to an interior solution involves reducing the safe short-term debt issued at $t = 0$ in favor of risky long-term debt which is always costly at $t = 0$. By contrast, moving from all long to an interior involves issuing less risky long-term for safe short-term at $t = 0$, for which the cost benefits are always clear. $\varepsilon \xrightarrow{\lim} \alpha$ as $\gamma \uparrow 1$ because $\rho \rightarrow 0$. As long as $\frac{p_D^\zeta}{p_0^\ell} \geq \alpha$ as $\gamma \uparrow 1$, condition (20) will hold for all γ because $\frac{p_D^\zeta}{p_0^\ell} \uparrow$ as $\gamma \downarrow 0$ and $\varepsilon \uparrow$. Similarly, if $\frac{p_D^\zeta}{p_0^\ell} \geq \alpha$ holds for $A_{DD} \rightarrow 0$, then it will hold for all $A_{DD} \rightarrow \alpha$ because $\frac{p_D^\zeta}{p_0^\ell} \uparrow$ as $A_{DD} \downarrow$. For the numerical example in Table 1 of appendix B, $\frac{p_D^\zeta}{p_0^\ell} \cong 0.95$, with $\alpha = 0.7$, $\gamma = 0.8$, and $A_{DD} = 0.5$. We can show numerically that (20) does indeed hold $\forall (A_{DD}, \gamma)$ -pairs. *Q.E.D.*

Proof of Proposition 3: We show for any investment amount I_0 , issuing $q_0^\ell > 0$ and $q_D^\zeta > 0$ is cost reducing relative to either $q_0^\ell = 0$ or $q_D^\zeta = 0$. First, note that (2) and (3) can be combined to express the firm's marginal product equal to either only the marginal cost of

long-term debt or the marginal cost of short-term debt. This relationship simply reflects the fact that an interior maximum must be characterized by maturity cost equivalence at the margin. Suppose all short-term debt is rolled over so that $p_0^\xi = 1$ always. Next, suppose maturity is irrelevant, and the firm can obtain the same terms of financing all long-term or via interior solution. Let I_0^* be the optimal investment amount for some parameter set $\Gamma(\alpha, A_{DD}, \gamma)$. If maturity is irrelevant, the firm must be indifferent to raising I_0^* by issuing all long-term debt, $Q = \tilde{q}_0^\ell$, at price $\tilde{p}_0^\ell$ or to issuing both long- and short-term debt, $Q = \hat{q}_0^\ell + \hat{q}_0^\xi$, at prices $\hat{p}_0^\ell$ and $\hat{p}_0^\xi = 1$. Clearly it must be the case that $\tilde{q}_0^\ell > \hat{q}_0^\ell, \forall \hat{q}_0^\xi > 0$, and since the firm takes prices as given, it must be the case that $\tilde{p}_0^\ell > \hat{p}_0^\ell$. Market clearing implies the supply of financing equals the firm's demand for financing. For only long-term debt, market clearing is given by $(1 - \tilde{h}_0) = \tilde{p}_0^\ell \tilde{q}_0^\ell = I_0^*$ and for both long- and short-term debt by $(1 - \hat{h}_0) + (1 - \hat{h}_D) = \hat{p}_0^\ell \hat{q}_0^\ell + \hat{p}_D^\xi \hat{q}_D^\xi = I_0^*$. Equating the two market clearing conditions for the same I_0^* gives $(1 - \tilde{h}_0) = (1 - \hat{h}_0) + (1 - \hat{h}_D)$. This can only hold if $\hat{h}_D = 1$ meaning that $\hat{q}_D = 0$ —no short-term debt is issued—or if $\tilde{h}_0 < \hat{h}_0$ —the marginal long-term bond buyer in an interior solution is more optimistic than the marginal bond buyer in the corner solution. But, the more optimistic the investor, the higher the price she is willing to pay $\implies \tilde{p}_0^\ell < \hat{p}_0^\ell$, which contradicts $\tilde{q}_0^\ell > \hat{q}_0^\ell, \forall \hat{q}_0^\xi > 0$. The same logic will also show that the firm can never be indifferent between all short-term financing and a combination of short- and long-term debt. *Q.E.D.*

Proof of Proposition 4: From (2) and a given set of risky debt prices $(p_0^{\ell*}, p_D^{\xi*})$, $\uparrow \gamma$ increases the l.h.s more than the right. If the firm issues more long-term debt, $\rho \uparrow$, long-term debt prices fall and short-term debt prices rise, causing further deviation from the necessary equality. Thus, the firm must issue more short-term debt, $\frac{\partial \rho}{\partial \gamma} < 0$, raising long-term debt prices and lowering short-term debt prices to a new set of equilibrium prices $(\tilde{p}_0^\ell, \tilde{p}_D^\xi)$. From (??) we know that long-term debt holders and risky short-term debt holders expect the same delivery at $s = DD$. For a given set of initial prices, $(p_0^{\ell*}, p_D^{\xi*})$, and correspond-

ing marginal buyers, (h_0^*, h_D^*) , raising A_{DD} increases deliveries. However, long-term debt holders at $t = 0$ place $(1 - h_0)^2$ weight on $s = DD$ while short-term debt holders at $t = 1$ place $(1 - h_D^\xi)$ weight on $s = DD$. Unless $h_0 \ll h_D$, in which case the firm is issuing almost all long-term debt and $\rho \xrightarrow{\lim} 1$, short-term debt buyers at $t = 1$ place more weight on deliveries than long-term debt holders at $t = 0$. But we know from Propositions 2 and 3 that if $\rho \xrightarrow{\lim} 1$ the firm will find it beneficial to issue more short-term debt to take advantage of safe debt financing at $t = 0$ and optimistic capital at $t = 1$. Thus almost surely, risky short-term debt holders at $t = 1$ are more responsive to changes in A_{DD} than long-term debt holders at $t = 0$. *Q.E.D.*

Proof of Proposition 5: A necessary condition for any $0 < \rho < 1$ in any collateral economy with or without the covenant is $p_0^\ell > p_D^\xi$ from $\frac{d\Pi}{d\rho} = 0$. Thus, the necessary condition also ensures $d_{DD}^\ell(\hat{q}_0^\ell) > d_{DD}^*(\cdot)$. Let $\rho^* = 1$ in which case $q_0^\xi = 0$. From (??) $d_{DD}^*(\cdot) = \frac{A_{DD}}{\alpha} = d_{DD}^\ell(\hat{q}_0^\ell)$ in (22). For the second item in the proof, it is clear from (17) and (18) that any given buyer with the same marginal utilities across states must pay a higher price for securities with higher deliveries. *Q.E.D.*

Proof of Corollary 1: From Proposition 5 and (22) we know that $d_{DD}(q_0^\ell) > d_{DD}(q_D^\xi)$ when long-term indentures include the covenant for a given (I_0^*, ρ^*) . Suppose the firm does not alter its debt structure and ρ^* is unchanged. Then, long-term debt prices must rise to a new level reflecting greater marginal valuations, $p_0^{\ell^c} > p_0^{\ell^*}$, where the superscript c denotes prices with the covenant. But if long-term debt is now cheaper in equilibrium, then the maturity structure for a given (I_0^*, ρ^*) cannot be optimizing and the firm must adjust. Thus the firm issues more long-term debt and reduces its short-term debt, leaving I_0^* unchanged and $\rho^c > \rho^*$ so lowering $\downarrow p_0^{\ell^c} = p_0^{\ell^*}$. *Q.E.D.*

Proof of Proposition 6: Follows immediately from the proof of Corollary 1 and investment optimality in (2) and (3). *Q.E.D.*

B Appendix

B.1 Multiple debt maturity funding

The ten endogenous variables are $(p_0^\varsigma, p_0^\ell, p_D^\varsigma, q_0^\varsigma, q_0^\ell, q_D^\varsigma, I_0, \rho, h_0, h_D)$. The system of equations, along with (2) and (3) is:

$$p_0^\varsigma = 1 \tag{24}$$

$$1 = \frac{1 - (1 - h_0)^2 + (1 - h_0)^2 d_{DD}(q_0^\ell)}{p_0^\ell} \tag{25}$$

$$1 = \frac{h_D + (1 - h_D) d_{DD}(q_D^\varsigma)}{p_D^\varsigma} \tag{26}$$

$$I_0 = p_0^\ell q_0^\ell + p_0^\varsigma q_0^\varsigma \tag{27}$$

$$\rho = \frac{p_0^\ell q_0^\ell}{I_0} \tag{28}$$

$$q_0^\varsigma = p_s^\varsigma q_s^\varsigma \tag{29}$$

$$1 - h_0 = p_0^\ell q_0^\ell \tag{30}$$

$$1 - h_D = p_D^\varsigma q_D^\varsigma \tag{31}$$

The first three equations are bond pricing equations. Equation (24) shows that short-term bonds issued at time 0 are risk free because all short-term debt is rolled over at time 1. Equation (25) states that long-term bonds are priced based on the time 0 marginal investor's expectations because he is indifferent between buying the bond and holding a cash equivalent asset. Similarly, equation (26) states that time 1 short-term bonds are priced based on the time 1 marginal investor's expectations because cash is the only other alternative asset. Equation (27) says that the amount of capital the firm raises in the bond market is equal to the investment it puts into its production technology. Equations (2) and (3) are the first order conditions *w.r.t.* the portfolio allocation ρ and investment level I_0 , respectively. The necessary condition for the firm to issue a portfolio of both long and

short-term bonds in (2) says that on the margin the expected cost of issuing either type of bond must be the same. The left hand side of (3) is the expected marginal product of capital irrespective of whether or not it is issued via long-term or short-term bonds. The right hand side is the expected-weighted marginal cost of capital. Equation (28) sets ρ equal to the portion of the firm's investment that is raised via long-term debt. Equation (29) shows that the firm will issue as many short-term bonds at time 1 as it takes to fully repay its time 0 short-term creditors. Equations (30) and (31) are, respectively, the long-term and time 1 short-term bond market clearing conditions.

C Appendix

Here we show that changing the uncertainty structure of the economy does not materially alter the optimal choice to issue both long- and short-term debt. Instead of the structure given by figure 1 where $\gamma = \gamma|_{s=D}$ let $\gamma_1 = \Pr(s = U) > \gamma_2 = \Pr(s = DU|_{s=D})$ so that the likelihood of receive a good state following a bad state is less than receiving an unconditional good state. Breaking the firm's problem given by (1) into its constituent pieces, we can write profits as

$$\max_{I_0, \rho} \Pi = \left\{ \gamma_1 \left[I_0^\alpha - \rho \frac{I_0}{p_0^\ell} - (1 - \rho) \frac{I_0}{1} \right] + (1 - \gamma_1) \gamma_2 \left[I_0^\alpha - \rho \frac{I_0}{p_0^\ell} - (1 - \rho) \frac{I_0}{p_D^\xi} \right] \right\}.$$

This profit expression simply states that conditional on good news at $t = 1$, both long- and short-term debt is repaid, and conditional on bad news at $t = 1$ long- and short-term debts are repaid only if good news arrives at $t = 2$. Notice that the only difference between this problem and the one presented in the main body of the paper is that $\gamma_2 < \gamma_1 = \gamma$. The first

order conditions for a maximum simply become

$$\begin{aligned}\frac{[\gamma_1 + \gamma_2(1 - \gamma_1)]}{p_0^\ell} &= \frac{1}{p_0^\xi} \left[\gamma_1 + \frac{\gamma_2(1 - \gamma_1)}{p_D^\xi} \right] \\ \alpha I_0^{\alpha-1} [\gamma_1 + \gamma_2(1 - \gamma_1)] &= \frac{\rho [\gamma_1 + \gamma_2(1 - \gamma_1)]}{p_0^\ell} + \frac{(1 - \rho)}{p_0^\xi} \left[\gamma_1 + \frac{\gamma_2(1 - \gamma_1)}{p_D^\xi} \right].\end{aligned}$$

Plugging into the other we obtain $\alpha I_0^{\alpha-1} = \frac{1}{p_0^\ell}$ which of course arises because in equilibrium the marginal cost of a long-term bond must equal the marginal cost of a short-term bond for $0 < \rho < 1$ allowing us to express the first order condition for a maximum as a function of either long- or short-term debt. Let $A \equiv \gamma + \gamma(1 - \gamma)$ when $\gamma = \gamma|_{s=D}$ and $B \equiv \gamma_1 + \gamma_2(1 - \gamma_1)$ from the restated problem above and $A > B$. Then, $\forall (I_0, \rho) : \alpha I_0^{(\alpha-1)} A > \alpha I_0^{(\alpha-1)} B$. This implies that $\frac{1}{p_0^\ell}|_A > \frac{1}{p_0^\ell}|_B \Rightarrow p_0^\ell|_B > p_0^\ell|_A$ at the optimum. In other words, for a given ρ , the firm will only raise the same amount of capital across the two economies if long-term bond prices are higher in the economy with more uncertainty at $s = D$, which is a contradiction because the firm is less likely to repay debt at $s = DU$ with in the more uncertainty case. Alternatively, the firm can raise less long-term debt and more short-term debt in the economy with more uncertainty at $s = D$, leaving total I_0 unchanged and tilting ρ more toward short-term debt. This results in lower short-term bond prices and higher long-term bond prices. And by proposition 3, starting from a corner solution, it will always be less costly to balance long- and short-term borrowing costs against one another rather than issuing all long- or short-term debt. The only thing that will change is the relative maturity tilt.

The same logic applies if we were to allow for uncertainty at $s = U$ and default at $s = UD$. For this, assume that firm deliver at $s = UD$ is higher than $s = DD$, where generically $d_{UD}(Q) = d_{DD}(Q) + \varepsilon < 1$. This simply reflects the fact that the ultimate shock to collateral is worse in two consecutive bad states than in an up state followed by a down

state. The firm's maximization problem can be split and written as follows:

$$\max_{I_0, \rho} \Pi = \left\{ \gamma^2 \left[I_0^\alpha - \rho \frac{I_0}{p_0^\ell} - (1 - \rho) \frac{I_0}{p_U^\xi} \right] + (1 - \gamma) \gamma \left[I_0^\alpha - \rho \frac{I_0}{p_0^\ell} - (1 - \rho) \frac{I_0}{p_D^\xi} \right] \right\}.$$

Only two things change in the problem. 1) Debts are no longer repaid conditional on $s = U$ so that now the first set of repayment states are given by γ^2 rather than γ . 2) $p_U^\xi \neq 1$ as it does with full repayment. Taking first order conditions for an interior maximum and plugging in, one can express the same marginal product equals marginal cost as $\alpha I_0^{\alpha-1} = \frac{1}{p_0^\ell}$. And by proposition 3, we know that for any given I_0 and a candidate corner solution, it is always be cheaper to fund a portion of the investment outlay by substituting into either long or short-term debt rather so that both debt maturities are utilized. *QED*.