

Information Aggregation in Dynamic Markets with Adverse Selection

Vladimir Asriyan, William Fuchs, and Brett Green*

PRELIMINARY DRAFT

January 28, 2017

Abstract

How effectively does a decentralized marketplace aggregate information that is dispersed throughout the economy? We study this question in a dynamic setting, in which sellers have private information that is correlated with an unobservable aggregate state. We first characterize equilibria with an arbitrary (but finite) number of informed sellers. A common feature is that each seller's trading behavior provides an informative and conditionally independent signal about the aggregate state. We then ask whether the state is revealed as the number of informed sellers goes to infinity. Perhaps surprisingly, the answer is no. We provide conditions under which the amount of information revealed is necessarily bounded and does not reveal the aggregate state. When these conditions are violated, there may be coexistence of equilibria that lead to full revelation with those that do not. We discuss the implications for policies meant to enhance information dissemination in markets.

*Asriyan: CREi-UPF and Barcelona GSE, vasriyan@crei.cat. Fuchs: Haas School of Business, UC Berkeley, wfuchs@haas.berkeley.edu. Green: Haas School of Business, UC Berkeley, greenb@berkeley.edu.

1 Introduction

Since the seminal work of Hayek (1945), the question of whether markets effectively aggregate dispersed information has been a central one in economics. Formal investigations of this question are typically conducted in a setting with a single (perhaps divisible) asset about which traders have dispersed information. Whether information is aggregated then usually boils down to whether the equilibrium price reveals the value of the asset conditional on the union of traders information.¹ This broad class of models is quite natural for many applications and trading environments from static common-value auctions to dynamic trading of securities in financial markets. For other applications (e.g., real estate), dispersed information arises due to dispersion in ownership and one is interested in the extent to which aggregate trading behavior across many different assets reveals information about the underlying state of the economy. In this paper, we explore such a setting.

More specifically, we investigate the question of information aggregation in a dynamic setting with many assets, whose values are independently and identically drawn from a distribution that depends on an underlying aggregate state. The value of each asset is privately observed by its seller, who receives offers each period from competitive buyers. We ask whether the history of all transactions reveals the aggregate state as the number of informed sellers in the economy (denoted by N) grows large.

To answer this question, we begin by characterizing the set of equilibria for arbitrary but finitely many N . Due to a complementarity between the amount of information collectively revealed by others and the optimal strategy of an individual seller, multiple equilibria can exist. A feature common to all is that each individual seller's trading behavior provides an informative and conditionally independent signal about the aggregate state. Intuitively, one might expect that, by the law of large numbers, the state would then be revealed as the number of sellers tends to infinity.

Our first main result (Theorem 1) shows that this intuition is incorrect. We provide necessary and sufficient conditions under which there does not exist a sequence of equilibria that reveal the state as $N \rightarrow \infty$. The reason why aggregation fails is that the information content of each individual seller's behavior tends to zero at a rate of $1/N$, just fast enough to offset the additional number of observations. As a result, some information is revealed by the limiting trading behavior, but not enough to precisely determine the underlying state. Roughly speaking, the conditions for non-aggregation require that the correlation of asset values is sufficiently high and that agents are sufficiently patient. Intuitively, these conditions guarantee

¹Seminal works on this topic include Grossman (1976), Wilson (1977) and Milgrom (1979). More recent progress on this question has been made by Pesendorfer and Swinkels (1997), Kremer (2002), Lauermaun and Wolinsky (2013), Siga (2013), Bodoh-Creed (2013), and Axelson and Makarov (2014).

that if the aggregate state were to be revealed with certainty tomorrow, then the option value of delaying trade is large enough that a seller will prefer not to trade today.

When these conditions are not satisfied, there exists a sequence of equilibria such that information about the state is aggregated as $N \rightarrow \infty$. However, even when the conditions are not satisfied, information aggregation is not guaranteed. Our second main result shows that there exists a region of the parameter space in which there is coexistence of equilibria that reveal the state with equilibria that do not. The key difference across the two types of equilibria is the rate at which trade declines as the number of informed sellers grows. In the non-aggregating equilibria, trade declines at rate $1/N$ whereas in aggregating equilibria, the rate of trade declines slower than $1/N$.

Beyond the theoretical contribution, our results are important when thinking about market regulation. Recently, there has been a strong regulatory push towards making markets more transparent. For example, one of the stated goals of the Dodd-Frank Act of 2010 is to increase transparency and information dissemination in the financial system. The European Commission is considering revisions to the Markets in Financial Instruments Directive (MiFID), in part to improve the transparency of European financial markets. Such actions reflect a widely held belief that transparency and information dissemination are welfare enhancing because it is necessary for perfect competition, it decreases uncertainty, and it increases public trust. The introduction of benchmarks, which reveal some aggregate trading information has also received recent attention by policy makers and academics. In contemporaneous work, Duffie et al. (2014) analyze the role of benchmarks (e.g., LIBOR) in revealing information about fundamentals and suggest that the introduction of benchmarks is welfare enhancing. Our analysis highlights an important consideration absent in their setting, which is that due to the endogenous response of trade the informational content of the benchmark may change once it is published. Thus, our paper highlights that when considering the introduction of a benchmark, it is important to understand the extent to which its information content is endogenous.

1.1 Related Literature

Our paper contributes to two strands of literature. The first is the literature on information aggregation cited earlier. Our model differs from this literature along several important dimensions. First, we consider a setting with many indivisible assets with heterogeneous values rather than a single asset. Second, our model is dynamic and each seller has an incentive to delay the sale of their asset to signal quality.

Kyle (1985) studies a dynamic insider trading model and shows that the insider fully reveals his information as time approaches the end of the trading interval. Foster and Viswanathan (1996) and Back et al. (2000) extend this finding to a model with multiple strategic insiders with different information. Ostrovsky (2012) further generalized these findings to a broader class of securities and information structures. He considers a dynamic trading model with finitely many partially informed traders and provides necessary and sufficient conditions on security payoffs for information aggregation to obtain. Our paper differs from these works in that we study a setting with heterogeneous but correlated assets owned by privately informed sellers. We ask whether information aggregates as the number of sellers becomes arbitrarily large. Despite the fact that we look at the limit as $N \rightarrow \infty$, the strategic considerations do not vanish in our model due to the fact that there is an idiosyncratic component to the value of each asset.

Golosov et al. (2014) consider an environment in which a fraction of agents has private information about an asset while the other fraction are uninformed. Agents trade in a decentralized anonymous market through bilateral matches, i.e., signaling with trading histories is not possible. They find that information aggregation obtains in the long run. In contrast, in our setting observing trading histories plays a crucial role: signaling through delay diminishes the amount of trade thus reducing the information content of the market, and potentially allowing for the possibility that information aggregation fails.

Lauermann and Wolinsky (2016) study information aggregation in a search market, in which an informed buyer sequentially solicits offers from sellers who have noisy information about the buyer's value. They provide conditions under which information aggregation fails, and they trace this failure to a strong form of winner's curse that arises in a search environment. Although our setups are substantially different, we share the common feature that the fear of adverse selection hinders trade and thus reduces information generation in markets.

Finally, our paper is related to a growing literature that studies dynamic markets with adverse selection (e.g., Janssen and Roy (2002), Hörner and Vieille (2009), Fuchs and Skrzypacz (2012), Fuchs et al. (2015), Daley and Green (2012, 2015)). Our innovation is the introduction of asset correlation into an otherwise standard framework, which allows us to study the information aggregation properties of these markets. This paper builds upon our previous work, Asriyan et al. (2017). Indeed, we share the same setup, but the analysis there is focused on a setting with two informed agents, whereas in this paper we consider the case of many informed agents whose asset values are related to some 'aggregate' state of the economy and we focus on the information aggregation properties of markets.

2 The Model

There are $N + 1$ sellers indexed by $i \in \{1, \dots, N + 1\}$, with $N \geq 1$. Each seller is endowed with an indivisible asset and is privately informed of her asset's type, denoted by $\theta_i \in \{L, H\}$. Seller i has a value c_{θ_i} for her asset, where $c_L < c_H$. The value of a type- θ asset to a buyer is v_θ and there is common knowledge of gains from trade, $v_\theta > c_\theta$. One can interpret c_θ and v_θ as the present value of the flow payoffs from owning the asset to the seller and the buyer respectively.

There are two trading periods: $t \in \{1, 2\}$.² In each period, multiple competing buyers make offers to each seller.³ A buyer whose offer is rejected gets a payoff of zero and exits the game.⁴ The payoff to a buyer who purchases an asset of type θ at price p is $v_\theta - p$. Sellers discount future payoffs by a factor $\delta \in (0, 1)$. The payoff to a seller with an asset of type θ , who agrees to trade at a price p in period t is

$$(1 - \delta^{t-1})c_\theta + \delta^{t-1}p.$$

If the seller does not trade at either date, his payoff is c_θ . All players are risk neutral.⁵

The key feature of the model is that asset values are positively correlated, which is due to correlation with an unobservable underlying state, S , that takes values in $\{l, h\}$. The unconditional distribution of θ_i is $\mathbb{P}(\theta_i = H) = \pi \in (0, 1)$, asset types are mutually independent conditional on the state, but their conditional distributions are given by $\mathbb{P}(\theta_i = L|S = l) = \lambda \in (1 - \pi, 1)$.⁶ To allow for arbitrarily high level of correlation, we set $\mathbb{P}(S = h) = \pi$.

Importantly, asset correlation introduces the possibility that a trade of one asset contains relevant information about the aggregate state and therefore the value of other assets. To capture this possibility, we assume that all transactions are observable. Therefore, prior making offers in the second period, buyers observe the set of assets that traded in the first period. For convenience, we assume that offers are made privately; the level of rejected offers is not observed by other buyers.⁷

²We extend the analysis to an arbitrary number of trading periods in Section 4.

³Perfect competition among buyers is not essential for our results.

⁴The assumption that buyers are "short-lived" (i.e., make offers in only one period) is, by now, fairly standard in this literature (e.g., Swinkels, 1999; Kremer and Skrzypacz, 2007; Hörner and Vieille, 2009). If offers are publicly observable, our results remain unchanged even with long-lived buyers. However, if offers are not observable, then a long-lived buyer has incentive to engage in a form of experimentation.

⁵One can interpret c_θ and v_θ as the present value of the flow payoffs from owning the asset to the seller and buyer respectively. That is, $c_\theta = \sum_{t=1}^{\infty} \delta^{t-1} x_\theta$, where $x_\theta = (1 - \delta)c_\theta$ is thus the seller's expected flow payoff from owning a type- θ asset for one period.

⁶With perfect correlation (i.e., $\lambda = 1$), the set of equilibria is sensitive to the specification of off-equilibrium path beliefs. Nevertheless, Asriyan et al. (2017) show that with two assets the set of equilibria with perfect correlation is the limit of the set of equilibria as $\lambda \rightarrow 1$.

⁷Fuchs et al. (2015) show that this specification is without loss in a setting with a single asset.

Notice that by virtue of knowing her asset quality, each seller has a private and conditionally independent signal about the aggregate state of nature. Thus, if each seller were to report her information truthfully to a central planner, then the planner would learn the aggregate state with probability one as $N \rightarrow \infty$. Our interest is to explore under what conditions the same information can be gleaned from the transaction data of a decentralized market.⁸ To rule out trivial cases, we focus on primitives which satisfy the following assumptions.

Assumption 1. $\pi v_H + (1 - \pi)v_L < c_H$.

Assumption 2. $v_L < (1 - \delta)c_L + \delta c_H$.

The first assumption, which we refer to as the “lemons” condition, asserts that the adverse selection problem is severe enough to rule out the first-best efficient equilibrium in which all sellers trade immediately. In this equilibrium, trade is uninformative about the underlying state (regardless of N) and information aggregation trivially fails. The second assumption implies a lower bound on the discount factor and ensures that dynamic considerations remain relevant. Our main results do not rely on this assumption but it simplifies exposition and rules out “fully separating” equilibrium (also independent of N) in which all low types trade in the first period, all high types trade in the second period. In the fully separating equilibrium, all private information is revealed and thus information is trivially aggregated as $N \rightarrow \infty$.⁹

2.1 Strategies and Equilibrium Concept

A strategy of a buyer is a mapping from his information set to a probability distribution over offers. In the first period (i.e., at $t = 1$), a buyer’s information set is empty. In the second period, buyers know whether each asset traded in the first period. If asset i trades in the first period, then it is efficiently allocated and it is without loss to assume that buyers do not make offers for it in the second period (Milgrom and Stokey, 1982). The strategy of each seller is a mapping from her information set to a probability of acceptance. Seller i ’s information includes her type, the set of previous and current offers as well as the information set of buyers.

We use Perfect Bayesian Equilibria (PBE) as our solution concept. This has three implications. First, each seller’s acceptance rule must maximize her expected payoff at every information set taking buyers’ strategies and the other sellers’ acceptance rules as given (*Seller Optimality*). Second, any offer in the support of the buyer’s strategy must maximize his

⁸Though we do not model them explicitly, one can imagine a variety of reasons why information aggregation is a desirable feature (e.g., better allocation of capital).

⁹Strictly speaking, to rule out fully separating equilibria we only need $v_L < (1 - \delta)c_L + \delta v_H$; this stronger condition simplifies exposition without affecting our main results.

expected payoff given his beliefs, other buyers' strategy and the sellers' strategy (*Buyer Optimality*). Third, given their information set, buyers' beliefs are updated according to Bayes' rule whenever possible (*Belief Consistency*).

2.2 Updating

Let σ_i^θ denote the probability that the type- θ , seller i trades in the first period. There are two ways in which the prior about seller i is updated between the first and second periods. First, conditional on rejecting the offer in the first period, buyers' interim belief is given by

$$\pi_{\sigma_i} \equiv \mathbb{P}(\theta_i = H | \text{reject at } t = 1) = \frac{\pi(1 - \sigma_i^H)}{\pi(1 - \sigma_i^H) + (1 - \pi)(1 - \sigma_i^L)} \quad (1)$$

Second, before making offers in the second period, buyers learn about any other trades that took place in the first period. How this information is incorporated into the posterior depends on the trading strategy of the other sellers (i.e., σ_j^θ , $j \neq i$). Let $z^j \in \{0, 1\}$ denote the indicator for whether seller j trades in the first period, and let $\mathbf{z} = (z^j)_{j=1}^{N+1}$ and $\mathbf{z}_{-i} = (z^j)_{j \neq i}$. Denote the probability of $\mathbf{z}_{-i}$ conditional on seller i being of type θ by $\rho_\theta^i(\mathbf{z}_{-i})$, which can be written as

$$\rho_\theta^i(\mathbf{z}_{-i}) \equiv \sum_{s \in \{L, H\}} \mathbb{P}(S = s | \theta_i = \theta) \cdot \prod_{j \neq i} \mathbb{P}(z^j | S = s), \quad (2)$$

where $\mathbb{P}(z^j = 1 | S = s) = \sum_{\theta \in \{L, H\}} \sigma_j^\theta \cdot \mathbb{P}(\theta_j = \theta | S = s)$ is the probability that seller j traded in state s . Provided there is positive probability that i rejects the bid at $t = 1$ and $\mathbf{z}_{-i}$ is realized, we can use equations (1) and (2) to express the posterior probability of seller i being high type conditional on these two events:

$$\pi_i(\mathbf{z}_{-i}; \sigma_i, \sigma_{-i}) \equiv \mathbb{P}(\theta_i = H | z^i = 0, \mathbf{z}_{-i}) = \frac{\pi_{\sigma_i} \cdot \rho_H^i(\mathbf{z}_{-i})}{\pi_{\sigma_i} \cdot \rho_H^i(\mathbf{z}_{-i}) + (1 - \pi_{\sigma_i}) \cdot \rho_L^i(\mathbf{z}_{-i})}. \quad (3)$$

To conserve on notation, we often suppress arguments of π_i .

2.3 Equilibrium Properties

Asriyan et al. (2017) analyze the model with two sellers (i.e., $N = 1$), and they establish several properties that must hold in any equilibrium. These properties extend to the model studied here with an arbitrary number of sellers.

In order to introduce them, we will use the following definitions and notation. We refer to the *bid* for asset i at time t as the maximal offer made across all buyers for asset i at time t . Let $V(\tilde{\pi}) \equiv \tilde{\pi}v_H + (1 - \tilde{\pi})v_L$ denote buyer's expected value for an asset given an arbitrary

belief $\tilde{\pi}$. Let $\bar{\pi} \in (\pi, 1)$ be such that $V(\bar{\pi}) = c_H$, and recall that π_i denotes the probability that buyers assign to $\theta_i = H$ prior to making offers in the second period.

Property 1 (Second period) *If seller i does not trade in the first period, then in the second period:*

- (i) *If $\pi_i > \bar{\pi}$ then the bid is $V(\pi_i)$, which the seller accepts w.p.1.*
- (ii) *If $\pi_i < \bar{\pi}$ then the bid is v_L , which the high type rejects and the low type accepts w.p.1.*
- (iii) *If $\pi_i = \bar{\pi}$, then the bid is $c_H = V(\pi_i)$ with some probability $\phi_i \in [0, 1]$ and v_L otherwise.*

Note that a high type will only accept a bid higher than c_H . When the expected value of the asset is below c_H (as in (i)), buyers cannot attract both types without making a loss. Thus, only the low type will trade and competition pushes the bid to v_L . When the expected value of the asset is above c_H (as in (ii)), competition forces the equilibrium offer to be the expected value. Finally, when the expected value of the asset is exactly c_H (as in (iii)), buyers are indifferent between offering c_H and trading with both types or offering v_L and only trading with the low type.

Property 2 (First period) *In the first period, the bid for each asset is v_L . The high-type rejects the first period bid with probability 1. The low-type seller accepts with probability $\sigma_i \in [0, 1]$.*

Any offer that is acceptable to a high type in the first period is accepted by the low type w.p.1. Assumption 1 implies that any such offer yields negative profits for the buyers. Hence, in equilibrium only low types trade in the first period and competition pushes the bid to v_L . Finally, if $\sigma_i = 1$, then the bid in the second period must be v_H (Property 1). But then the low-type seller i would strictly prefer to delay trade to the second period (Assumption 2), a contradiction.

Notice that Property 1 implies a second period payoff to a type- θ seller i as a function of (π_i, ϕ_i) , which we denote by $F_\theta(\pi_i, \phi_i)$, where

$$F_H(\pi_i, \phi_i) \equiv \max \{c_H, V(\pi_i)\}, \quad (4)$$

and

$$F_L(\pi_i, \phi_i) \equiv \begin{cases} v_L & \text{if } \pi_i < \bar{\pi} \\ \phi_i c_H + (1 - \phi_i) v_L & \text{if } \pi_i = \bar{\pi} \\ V(\pi_i) & \text{if } \pi_i > \bar{\pi}. \end{cases} \quad (5)$$

Properties 1 and 2 also imply that an equilibrium can be characterized by $\{\sigma_i, \phi_i\}_{i=1}^{N+1}$.

From seller i 's perspective, the strategy of seller $j \neq i$ in the first period is relevant because it influences the distribution of news $\mathbf{z}_{-i}$ and therefore the distribution of π_i . In particular, the (expected) continuation value of a seller from rejecting an offer in the first period can be written as

$$Q_\theta^i(\sigma_i, \sigma_{-i}, \phi_i) \equiv (1 - \delta)c_\theta + \delta \sum_{\mathbf{z}_{-i}} \rho_\theta^i(\mathbf{z}_{-i}) F_\theta(\pi_i(\mathbf{z}_{-i}), \phi_i). \quad (6)$$

where σ_{-i} denotes the vector of $\{\sigma_j\}_{j \neq i}$. It is worthwhile to note that because the posterior conditional on rejection, π_i , is increasing in σ_i , so does Q_L^i . Moreover, due to imperfect correlation, π_i (and therefore Q_L^i) is more sensitive to σ_i than it is to σ_j for all $j \neq i$. These observations naturally lead to a third useful property that equilibria must satisfy.

Property 3 (Symmetry) *In any equilibrium, $\sigma_i = \sigma > 0$ for all i . If buyer mixing is part of the equilibrium then $\phi_i = \phi$ for all i .*

The key step to prove symmetry is to show that if $\sigma_i > \sigma_j \geq 0$, then $Q_L^i > Q_L^j$. This follows from the fact mentioned above that the expected continuation value, Q_L^i , is more sensitive to i 's own trading probability than it is to that of the other players. Note that if $Q_L^i > Q_L^j$, then the low-type seller i strictly prefers to wait, which contradicts $\sigma_i > 0$ being consistent with an equilibrium. That there must be strictly positive probability of trade then follows: if $\sigma_i = 0$ for all i , then no news arrives and buyers' in the second period would have the same beliefs as buyer's in the first period. This would imply that the second period bid is v_L but in that case the low-type sellers would be strictly better off by accepting v_L in the first period, which contradicts $\sigma_i = 0$.

2.4 Equilibria

Given Properties 1–3, we can drop the subscripts and denote a candidate equilibrium by the pair (σ, ϕ) . Because all equilibria are symmetric, any information about seller i that is contained in news $\mathbf{z}_{-i}$ does not depend on the identity of those who sold but only on the number (or fraction) of other sellers that traded. For example, suppose that $\mathbf{z}_{-i} = \mathbf{z}(K)$ where event $\mathbf{z}(K)$ is such that $\sum_{j \neq i} z^j = K \leq N$. Then

$$\rho_\theta^i(\mathbf{z}(K)) = \sum_{s \in \{l, h\}} p_s^K \cdot (1 - p_s)^{N-K} \cdot \mathbb{P}(S = s | \theta_i = \theta),$$

where $p_s \equiv \sigma \cdot \mathbb{P}(\theta_i = L | S = s)$ is the probability that any given seller trades in state s . Naturally, the probability of observing K trades among sellers $j \neq i$ is $\binom{N}{K} \cdot \rho_\theta^i(\mathbf{z}(K))$.

Furthermore, since any equilibrium involves $\sigma \in (0, 1)$, a low-type seller must be indifferent between accepting v_L in the first period and waiting until the second period. The set of equilibria can thus be characterized by the solutions to

$$Q_L(\sigma, \sigma, \phi) = v_L. \quad (7)$$

Asriyan et al. (2017) show that with two sellers, there can be multiple solutions to (7) and hence multiple equilibria. The following result extends this finding to an arbitrary $N \geq 1$.

Proposition 1 (Existence and Multiplicity) *Fix N . An equilibrium always exists. If λ and δ are sufficiently large, there are multiple equilibria. If correlation is sufficiently small, the equilibrium is generically unique.*

Intuitively, a higher σ has two opposing effects on the seller's continuation value. On the one hand, the posterior beliefs and thus prices in the second period are increasing in σ , which increases the expected continuation value Q_L . On the other hand, as other low types trade more aggressively, the distribution over buyers' posterior shifts towards lower posteriors, thus decreasing Q_L . The latter force generates complementarities in sellers' trading strategies, which results in multiple equilibria when the correlation between assets is high. In contrast, when correlation is low, these complementarities are weak and the equilibrium is unique.

When multiple equilibria exist, they can be ranked in terms of trade and welfare. As we pointed out in Asriyan et al. (2017), the existence of multiple equilibria can help reconcile the mixed empirical evidence as to the effects of post-trade transparency and illuminate the debate about introducing reporting requirements in otherwise opaque markets. In this paper, we focus on a different aspect of the environment, namely the information content of trading behavior. More specifically, we ask whether information about the underlying state is aggregated as the number of informed participants grows large.

To understand the essence of this question, first notice that the trading behavior of each seller provides an informative signal about the aggregate state. If the seller trades in the first period, then she reveals her asset's type is L , which is more likely when the aggregate state is l than when it is h . Conversely, if the seller does not trade in the first period, then buyers update their beliefs about the asset toward H and their belief about the aggregate state toward h . Moreover, the amount of information revealed in the first period is increasing in the low-type's trading probability, which we now denote by σ_N (in order to explicitly indicate its dependence on the number of other informed participants).

If the information content of each individual trade were to converge to some positive level (i.e., $\lim_{N \rightarrow \infty} \sigma_N = \bar{\sigma} > 0$), then information about the state would aggregate. The reason is

that by the law of large numbers the fraction of assets traded would concentrate around its population mean $\bar{\sigma} \cdot \mathbb{P}(\theta_i = L | S = s)$, which is strictly greater when the aggregate state is l than when it is h . If, on the other hand, σ_N decreases to zero at a rate weakly faster than $1/N$ (i.e., $\lim_{N \rightarrow \infty} N \cdot \sigma_N < \infty$), then information would not aggregate. In this case, despite having arbitrarily many signals about the state, the informativeness of each signal goes to zero fast enough that the overall amount of information does not reveal the true state.

Of course, the equilibrium trading behavior of each individual seller is determined endogenously. Therefore, in order to establish information aggregation properties of equilibria, we need to understand how the set of equilibrium values of σ_N changes with N . Moreover, since different equilibria have different σ_N , the limiting information aggregation properties could be different for different sequences of equilibria. As we will see in the next section, neither of the two cases mentioned above is pathological.

3 Information Aggregation

We begin by studying the information aggregation properties of equilibria in the first period. Consider a sequence of economies indexed by N (standing for $N + 1$ assets), and let σ_N denote an equilibrium trading probability in the first period and π_N^{State} be the buyers' posterior belief that the aggregate state is h , conditional on having observed the outcome of trade in the first period. That is, given a first period trading history $\mathbf{z} = (z^j)_{j=1}^{N+1}$, $\pi_N^{State}(\mathbf{z}) \equiv \mathbb{P}(S = h | \mathbf{z})$. We say that:

Definition 1 *There is information aggregation along a given sequence of equilibria if along this sequence $\pi_N^{State} \rightarrow^p 1_{\{S=h\}}$ as $N \rightarrow \infty$.*

Our notion of information aggregation requires that, upon observing the trading history, buyers (or the econometrician, who observes only whether and when an asset trades) learn all the information available in the market that is relevant to infer the aggregate state. Asymptotically, this is equivalent to asking whether agents' beliefs about the aggregate state become degenerate at the truth.¹⁰

To understand the conditions under which information is aggregated, it is useful to consider a 'fictitious' economy in which buyers learn the state of nature before making second period offers. Conditional on knowing the true state, the information revealed by other sellers is irrelevant for buyers when forming beliefs about seller i . This implies that seller i 's continuation value $Q_L^{i, fict}(\sigma_i, \phi_i)$ is independent of the trading strategies of the other sellers and, thus,

¹⁰That our definition involves convergence in probability is standard in the literature (see e.g., Kremer (2002)).

the fictitious economy has a unique equilibrium. With exogenous arrival of information, it is possible that a low-type seller i will prefer not to trade in the first period (see Daley and Green (2012)). This occurs when $Q_L^{i,fict}(0, \phi_i) > v_L$.

Note that when $\sigma_i = 0$ and buyers learn the state is l then their posterior in the low state, $\pi_i(l)$, is lower than $\bar{\pi}$ and hence the second period price would be v_L . On the other hand, if buyers learn the state is high then their posterior in the high state, $\pi_i(h)$, is higher than $\bar{\pi}$ and, as a result, the bid would be $V(\pi_i(h))$ if and only if $1 - \frac{(1-\lambda)(1-\pi)}{\pi} > \bar{\pi}$. In this case,

$$Q_L^{i,fict}(0, \phi_i) = (1 - \delta) \cdot c_L + \delta \cdot (\lambda \cdot v_L + (1 - \lambda) \cdot V(\pi_i(h))).$$

This motivates our introduction of the following two conditions which will be used to establish our main results:

Condition 1. $\pi_i(h) > \bar{\pi}$.

Condition 2. $v_L < Q_L^{i,fict}(0, \phi_i)$.

We now establish our main result, which shows that conditions (1) and (2) are necessary and sufficient in characterizing the information aggregation properties of equilibria.

Theorem 1 (Information Aggregation) *If conditions (1) and (2) hold, there does not exist a sequence of equilibria along which information aggregates in the first period. On the other hand, if either condition is reversed, there exists a sequence of equilibria along which information aggregates in the first period.*

The proof of the first part uses the observation that if information were to aggregate, then for N large enough the continuation payoffs of the sellers in the economy with finitely many assets become arbitrarily close to the continuation payoffs in the fictitious economy, thus making delay also optimal when there are a large but finite number of assets. But this contradicts Property 3, which states that no trade in the first period cannot be part of an equilibrium. Indeed, in Proposition 3 in the Appendix, we show that the trading probability σ_N must go to zero at a rate proportional to $1/N$, which is fast enough to prevent information from aggregating, but slow enough to ensure that the market does not become completely uninformative in the limit, as is required by the equilibrium.

On the other hand, when the fictitious economy has an equilibrium with positive trade, which is the case when either of the two conditions is reversed, we can explicitly construct a sequence of equilibria in which the trading probability σ_N is bounded away from 0, and information is trivially aggregated along such a sequence of equilibria. However, as we demonstrate

next, a violation of either condition (1) or (2) is not sufficient to ensure information aggregation along *every* sequence of equilibria.

Proposition 2 (Coexistence) *There exists a $\bar{\delta} < 1$ such that, for any $\delta > \bar{\delta}$, if λ is sufficiently large, then there exist two sequences of equilibria such that along one sequence information aggregation fails, while along the other information about the state aggregates.*

To prove this result, we show that under the stated conditions there exists a sequence of equilibria in which the probability of the event that *no seller has traded in the first period* remains bounded away from zero, in *both* states of nature. Thus, even if N grows to ∞ , the uncertainty about the state of nature does not vanish. The coexistence of the two (aggregating and non-aggregating) types of equilibria then follows immediately from Theorem 1 by noting that for λ large enough, condition (2) becomes violated.

Thus far, we considered the information aggregation properties of equilibria conditional on the trading history in the first period. To consider aggregation in the second period, we simply extend Definition 1 by requiring that the convergence of buyers' beliefs be conditional on the history of trade over two periods (rather than the first period). Clearly, if information were already aggregated by the first period, it would also be aggregated in the second period. But what if information does not aggregate in the first period? Will trading behavior in the second period provide the additional information necessary to identify the true state?

The answer is that such an outcome is indeed possible; there can exist sequences of equilibria in which information is not aggregated based only on first period trading behavior, but is successfully aggregated from both first and second period trading behavior.¹¹

A key force underlying this possibility is that there are no further opportunities to trade after the second period. Because it is the last opportunity to trade, a low-type seller accepting an offer of v_L w.p.1 in the second period (thus revealing her type and information about the aggregate state) is consistent with an equilibrium. If there were additional trading opportunities, then such behavior could not be part of an equilibrium. As we show next, the possibility of information aggregation in the second period (when it is not aggregated in the first) is indeed an artifact of the second period also being the last opportunity to trade. That

¹¹To illustrate this possibility in more detail, suppose that there exist a sequence of equilibria that achieves the lower bound (i.e., $\pi_i(\mathbf{z}(0)) = \bar{\pi}$) for all N . Clearly, $\pi_i(z(k)) < \bar{\pi}$ for all $k > 0$. Hence, if at least one seller trades in the first period, then the second-period bid will be v_L for all sellers and all the remaining low-type sellers will reveal their type by accepting, which aggregates information (since all and only low types trade over the two periods). Now suppose that no sellers trade in the first period (i.e., $\mathbf{z}_{-i} = \mathbf{z}(0)$). Then, in the second period, buyers will mix between a pooling bid with probability ϕ_N and a separating one with probability $(1 - \phi_N)$. If the buyer mixing is independent across sellers the fraction of sellers who trade in the second period will converge to $\lim_{N \rightarrow \infty} \phi_N + (1 - \phi_N)\mathbb{P}(\theta = L|S)$, which also reveals the state provided $\lim_{N \rightarrow \infty} \phi_N < 1$.

is, our main results regarding the information aggregation properties of equilibria hold for all periods save for the last.

4 Longer Trading Horizon

In this section, we extend our main results to a setting with an arbitrary number $T > 1$ of trading periods. In particular, we show that the analogues of Theorem 1 and Proposition 2 hold when considering information aggregation over all trading periods but the last.

The analysis of equilibria of this economy is considerably more involved, as we need to keep track of the evolution of not only the buyers' beliefs about asset qualities, but also of agents' beliefs about the aggregate state as well as the number of assets remaining on the market. For brevity, we sketch the main argument here and relegate a formal treatment to the Appendix. But first, we extend our notion of information aggregation to this setting. Let $\pi_{t,N}^{State}$ denote the buyers' posterior belief that the state is high, conditional on having observed trading history in periods 1 through t . We say that:

Definition 2 *There is information aggregation along a given sequence of equilibria in period t if along this sequence $\pi_{t,N}^{State} \xrightarrow{p} 1_{\{S=h\}}$ as $N \rightarrow \infty$.*

In the previous section we argued that, because the second period was also the last trading opportunity, information in that period could aggregate independently of whether conditions (1) and (2) are satisfied. The following theorem shows that, given an arbitrary trading horizon $T > 1$, the conditions (1) and (2) are necessary and sufficient to characterize the information aggregation properties of equilibria in all periods but the last.

Theorem 2 *If conditions (1) and (2) hold, there does not exist a sequence of symmetric equilibria along which information aggregates in any period $t < T$. On the other hand, if either condition is reversed, there exists a sequence of equilibria along which information aggregates in the first period.*

Furthermore, there exists a $\bar{\delta} < 1$ such that, for any $\delta > \bar{\delta}$, if λ is sufficiently large, then there exist two sequences of equilibria such that along one sequence information aggregation fails for any $t < T$, while along the other information about the state aggregates in the first period.

The proof that our results extend to longer trading horizons hinges on arguments similar to those for the two period economy. For the non-existence result, we show that the first (earliest) period in which information about the state is supposed to aggregate is essentially

like the first period in a two period economy. Thus, given that conditions (1) and (2) are satisfied, the option value of waiting for the state to be revealed is sufficiently high to make it profitable for the seller to prefer not to trade at all. The proof itself is more involved since with longer horizon we must pin down the equilibrium continuation values and this is harder with many remaining periods. Furthermore, the belief about the remaining seller types is also evolving with time. Thus, Assumption 1 may fail to hold in some histories on the equilibrium path of play, which can potentially give rise to other types of equilibria (absent in the two period model). Nevertheless, we are able to show that, even if other equilibria arise, they must still fail to aggregate information as long as conditions (1) and (2) hold.

In order to establish the existence and co-existence results, we construct a class of equilibria that essentially share the information aggregation properties of the two period model. This is achieved by requiring that once the belief about the seller weakly exceeds $\bar{\pi}$, all future bids be pooling. When either condition (1) or (2) is violated, we show that such equilibria exist and that there is at least one equilibrium sequence within this class along which information aggregates. Then following arguments similar to those for the proof of Proposition 2, we show that there can also exist another sequence of equilibria in which aggregation fails.

5 Optimal Reporting of Trades

Assuming a planner or platform designer observes all the realized trades, a natural question to ask next is, what reporting mechanism would the planner want to put in place to convey some or all of this information to the remaining traders in the market? This is an information design question.¹² Relative to the recent Bayesian Persuasion literature following Kamenica and Gentzkow (2011) (henceforth, KG) our problem has several additional complexities. First, the information available to the planner is endogenous; thus, when agents change their trading behavior, this could either increase or reduce the information available to the planner. In particular, if the planner’s posterior about the aggregate state becomes less precise, then this puts an endogenous constraint on how precise a signal about the aggregate state the planner can convey back to the market. In contrast, in KG, the set of distributions over posteriors that can be chosen is independent of the reporting choice. The endogenous reaction of the seller’s trading strategy also affects the interim belief the market has before receiving the planner’s report and this needs to be accounted for when calculating posteriors, while in KG the prior beliefs are held fixed and thus one can more directly calculate possible distributions of posteriors that can be generated with a given information structure. Finally, in our setting

¹²Note that the question of information design is relevant for asset markets where trade occurs in an opaque environment, i.e., where traders on one ‘platform’ do not necessarily observe trading behavior on others.

the belief determines a market price that can take a continuum of values and determine payoffs rather than payoffs being simply determined by a binary choice.

TO BE CONTINUED...

References

- Asriyan, V., W. Fuchs, and B. Green (2017). Information spillovers in asset markets with correlated values. *American Economic Review* (*forthcoming*).
- Axelson, U. and I. Makarov (2014). Informational Black Holes in Auctions. *Working Paper*.
- Back, K., C. Cao, and G. Willard (2000). Imperfect competition among informed traders. *Journal of Finance* 55(5), 2117–2155.
- Bodoh-Creed, A. (2013). Efficiency and Information Aggregation in Large Uniform-Price Auctions. *Journal of Economic Theory* 146(6), 2436–2466.
- Daley, B. and B. Green (2012). Waiting for News in the Market for Lemons. *Econometrica* 80(4), 1433–1504.
- Daley, B. and B. Green (2015). An Information-Based Theory of Time-Varying Liquidity. *Journal of Finance* (*forthcoming*).
- Duffie, D., P. Dworczak, and H. Zhu (2014). Benchmarks in Search Markets. *Working Paper*.
- Foster, F. and S. Viswanathan (1996). Strategic trading when agents forecast the forecasts of others. *Journal of Finance* 51(4), 1437–1478.
- Fuchs, W., A. Öery, and A. Skrzypacz (2015). Transparency and Distressed Sales under Asymmetric Information. *Cowles Foundation Working Paper*.
- Fuchs, W. and A. Skrzypacz (2012). Costs and benefits of dynamic trading in a lemons market. *Working Paper*, 1–33.
- Golosov, M., G. Lorenzoni, and A. Tsyvinski (2014). Decentralized trading with private information. *Econometrica* 82(3), 1055–1091.
- Grossman, S. (1976). On the efficiency of competitive stock markets where trades have diverse information. *The Journal of Finance* 31(2), 573–585.
- Hörner, J. and N. Vieille (2009). Public vs. Private Offers in the Market for Lemons. *Econometrica* 77(1), 29–69.

- Janssen, M. C. and S. Roy (2002). Dynamic trading in a durable good market with asymmetric information. *International Economic Review* 43(1), 257–282.
- Kamenica, E. and M. Gentzkow (2011). Bayesian persuasion. *The American Economic Review* 101(6), 2590–2615.
- Kremer, I. (2002). Information Aggregation in Common Value Auctions. *Econometrica* 70(4), 1675–1682.
- Kremer, I. and A. Skrzypacz (2007). Dynamic Signaling and Market Breakdown. *Journal of Economic Theory* 133, 58–82.
- Kyle, A. S. (1985). Continuous auctions and insider trading. *Econometrica: Journal of the Econometric Society*, 1315–1335.
- Lauermann, S. and A. Wolinsky (2013). A Common Value Auction with Bidder Solicitation. *Working Paper*.
- Lauermann, S. and A. Wolinsky (2016). Search with Adverse Selection. *Econometrica* 84(1), 243–315.
- Milgrom, P. and N. Stokey (1982). Information, trade and common knowledge. *Journal of Economic Theory* 26(1), 17–27.
- Milgrom, P. R. (1979). A Convergence Theorem for Competitive Bidding with Differential Information. *Econometrica* 47(3), 679–688.
- Ostrovsky, M. (2012). Information Aggregation in Dynamic Markets with Strategic Traders. *Econometrica* 80(6), 2595–2649.
- Pesendorfer, W. and J. M. Swinkels (1997). The Loser’s Curse and Information Aggregation in Common Value Auctions. *Econometrica* 65(6), 1247–1281.
- Siga, L. (2013). Complex signals and information aggregation in large markets. Technical report, mimeo, New York University.
- Swinkels, J. M. (1999). Education Signaling with Preemptive Offers. *Review of Economic Studies* 66(4), 949–970.
- Wilson, R. (1977). A Bidding Model of Perfect Competition. *The Review of Economic Studies* 44(3), 511–518.

Appendix A

Proof of Property 1. See Lemma 1 in Asriyan et al. (2017). ■

Skimming Property. The proof extends Lemma 2 in Asriyan et al. (2017) to the case of $N + 1$ assets. Since $c_H > c_L$ and $F_H \geq F_L$, the continuation value of the low type seller from rejecting the bid v_L in the first period satisfies:

$$\begin{aligned} Q_L^i &= (1 - \delta) \cdot c_L + \delta \cdot \mathbb{E}_L\{F_L(\pi_i, \phi_i)\} \\ &< (1 - \delta) \cdot c_H + \delta \cdot \mathbb{E}_L\{F_L(\pi_i, \phi_i)\} \\ &\leq (1 - \delta) \cdot c_H + \delta \cdot \mathbb{E}_L\{F_H(\pi_i, \phi_i)\}. \end{aligned}$$

Therefore, in order to prove that $Q_H^i > Q_L^i$, it is sufficient to show that $\mathbb{E}_H\{F_H(\pi_i, \phi_i)\} \geq \mathbb{E}_L\{F_H(\pi_i, \phi_i)\}$. Recall that F_H is increasing in π_i and independent of ϕ_i . Hence, the desired inequality is implied by proving that conditional on $\theta_i = H$, the random variable π_i (weakly) first-order stochastically dominates π_i conditional on $\theta_i = L$.

Note that the distribution of π_i in the second period is a function of the trading probabilities of the seller i and of the realization of news from sellers $j \neq i$, $z_i^j \in \{0, 1\}$. For $\theta \in \{L, H\}$ and $\mathbf{z}_{-i} = (z_i^j)_{j \neq i}$ define $\rho_\theta^i(\mathbf{z}_{-i}) \equiv \mathbb{P}(\mathbf{z}_{-i} | \theta_i = \theta)$. Fix the interim belief π_i^{Int} , and consider news $\mathbf{z}'_{-i}$ and $\mathbf{z}''_{-i}$ (which occur with positive probability) such that the posterior π_i satisfies $\pi_i(\mathbf{z}'_{-i}) \geq \pi_i(\mathbf{z}''_{-i})$, i.e., $\mathbf{z}'_{-i}$ is “better news” for seller i than $\mathbf{z}''_{-i}$. But note that:

$$\frac{\pi_i^{Int} \cdot \rho_H^i(\mathbf{z}'_{-i})}{\pi_i^{Int} \cdot \rho_H^i(\mathbf{z}'_{-i}) + (1 - \pi_i^{Int}) \cdot \rho_L^i(\mathbf{z}'_{-i})} = \pi_i(\mathbf{z}'_{-i}) \geq \pi_i(\mathbf{z}''_{-i}) = \frac{\pi_i^{Int} \cdot \rho_H^i(\mathbf{z}''_{-i})}{\pi_i^{Int} \cdot \rho_H^i(\mathbf{z}''_{-i}) + (1 - \pi_i^{Int}) \cdot \rho_L^i(\mathbf{z}''_{-i})}.$$

Combining with the fact that $\sum_{\mathbf{z}_{-i}} \rho_\theta^i(\mathbf{z}_{-i}) = 1$ for $\theta \in \{L, H\}$, we have $\rho_H^i(\mathbf{z}'_{-i}) \geq \rho_L^i(\mathbf{z}'_{-i})$ and $\rho_L^i(\mathbf{z}''_{-i}) \geq \rho_H^i(\mathbf{z}''_{-i})$, which establishes the result. ■

Proof of Property 2. See Lemma 3 in Asriyan et al. (2017). ■

Proof of Property 3. The proof extends Proposition 2 in Asriyan et al. (2017) to the case of $N + 1$ assets. The proof that all equilibria involve strictly positive probability of trade in the first period is in the text. We show here that all equilibria must be symmetric. In search of a contradiction, assume there exists an equilibrium in which $\sigma_A > \sigma_B \geq 0$ for some $A, B \in \{1, \dots, N\}$. We establish the result by first showing that the beliefs for seller A are more favorable than for seller B , following all news realizations; then we show that good news about seller A are more likely to arrive than good news about seller B .

Consider the posterior belief about seller $i \in \{A, B\}$ following some news $\mathbf{z}_{-i} = (z_i^j)_{j \neq i}$:

$$\pi_i(\mathbf{z}_{-i}) = \frac{\pi_{\sigma_i} \cdot \rho_H^i(\mathbf{z}_{-i})}{\pi_{\sigma_i} \cdot \rho_H^i(\mathbf{z}_{-i}) + (1 - \pi_{\sigma_i}) \cdot \rho_L^i(\mathbf{z}_{-i})}$$

where we can express $\rho_\theta^i(\mathbf{z}_{-i})$ as:

$$\rho_\theta^i(\mathbf{z}_{-i}) = \sum_{s \in \{l, h\}} \mathbb{P}(S = s | \theta_i = \theta) \cdot \mathbb{P}((z_i^j)_{j \neq i, i'} | S = s) \cdot \mathbb{P}(z_i^{i'} | S = s)$$

for $i, i' \in \{A, B\}$ and $i' \neq i$. Note that $\rho_\theta^i(\mathbf{z}_{-i})$ depends on $\sigma_{i'}$ only through the term $\mathbb{P}(z_i^{i'} | S)$.

We now show that $\sigma_A > \sigma_B$ implies that:

$$\frac{1 - \pi_{\sigma_A}}{\pi_{\sigma_A}} \cdot \frac{\rho_L^A(\mathbf{z}_{-i})}{\rho_H^A(\mathbf{z}_{-i})} < \frac{1 - \pi_{\sigma_B}}{\pi_{\sigma_B}} \cdot \frac{\rho_L^B(\mathbf{z}_{-i})}{\rho_H^B(\mathbf{z}_{-i})}, \quad (8)$$

which will establish that $\pi_A(\mathbf{z}_{-i}) > \pi_B(\mathbf{z}_{-i})$ for all news $\mathbf{z}_{-i}$. There are two cases to consider, depending on whether $z_i^{i'} = 0$ or $z_i^{i'} = 1$.

If $z_i^{i'} = 1$, then $\mathbb{P}(z_i^{i'} = 1 | S = s) = \sigma_{i'} \cdot \mathbb{P}(\theta_{i'} = L | S = s)$ and the likelihood ratio $\frac{1 - \pi_{\sigma_i}}{\pi_{\sigma_i}} \cdot \frac{\rho_L^i(\mathbf{z}_{-i})}{\rho_H^i(\mathbf{z}_{-i})}$ decreases in σ_i but is independent of $\sigma_{i'}$. Intuitively, if seller i' traded, her type is revealed to be low, and the intensity with which she trades is irrelevant for updating. But then inequality (8) follows because π_{σ_i} is increasing in σ_i .

If $z_i^{i'} = 0$, then $\mathbb{P}(z_i^{i'} = 0 | S = s) = 1 - \sigma_{i'} \cdot \mathbb{P}(\theta_{i'} = L | S = s)$, and now the likelihood ratio $\frac{1 - \pi_{\sigma_i}}{\pi_{\sigma_i}} \cdot \frac{\rho_L^i(\mathbf{z}_{-i})}{\rho_H^i(\mathbf{z}_{-i})}$ decreases in both σ_i and $\sigma_{i'}$. However, given that both i and i' did not trade (both are good news for i), inequality (8) follows because the assets i and i' are imperfectly correlated and $\frac{1 - \pi_{\sigma_i}}{\pi_{\sigma_i}} \cdot \frac{\rho_L^i(\mathbf{z}_{-i})}{\rho_H^i(\mathbf{z}_{-i})}$ is more sensitive to trading probability σ_i than to $\sigma_{i'}$.

Finally, note that $\sigma_A > \sigma_B$ also implies that the probability that seller B trades and releases bad news about seller A is lower than the probability that seller A trades and releases bad news about seller B . Since the posteriors following good news are higher than following bad news, this establishes the result. ■

Proof of Proposition 1. The proof extends Theorem 1 in Asriyan et al. (2017) to the case of $N + 1$ assets. To prove existence of an equilibrium, it suffices to show there exists a $(\sigma, \phi) \in [0, 1]^2$ such that equation (7) holds, i.e., $Q_L(\sigma, \sigma, \phi) = v_L$ where the second argument states that all other sellers also trade with intensity σ . Note that by varying σ from 0 to 1, Q_L ranges from $[(1 - \delta)c_L + \delta v_L, (1 - \delta)c_L + \delta v_H]$. By continuity of Q_L and Assumption 2, the intermediate value theorem gives the result.

For convenience, let us classify equilibria into the following three categories. For $K \in \{0, 1, \dots, N\}$, let $\mathbf{z}(K)$ denote the event that K sellers other than i have traded in the first

period. We will refer to an equilibrium in which the posterior belief about the seller satisfies $\pi_i(\mathbf{z}(0)) = \bar{\pi}$ as a *low trade* equilibrium, and when the posterior belief about the seller satisfies $\pi_i(\mathbf{z}(N)) = \bar{\pi}$ as a *high trade* equilibrium. Finally, we label the equilibria in which the posterior ranking is $\pi_i(\mathbf{z}(N)) < \bar{\pi} < \pi_i(\mathbf{z}(0))$ as *medium trade* equilibria. It is straightforward to rule out the remaining posterior rankings. We will now show that these equilibria coexist when λ and δ are large enough.

1. Low trade. That there is at most one low trade equilibrium follows from the fact that the trading intensity σ in this category is fully pinned down by the requirement that $\pi_i(\mathbf{z}(0)) = \bar{\pi}$. Let x be the value of σ such that $\pi_i(\mathbf{z}(0); x, x) = \bar{\pi}$ (x denotes the trading probability of all $N + 1$ sellers). As ϕ varies from 0 and 1, $Q_L(x, x, \phi)$ varies continuously from $(1 - \delta)c_L + \delta v_L$ to $(1 - \delta)c_L + \delta(\rho_L^i(\mathbf{z}(0))v_L + (1 - \rho_L^i(\mathbf{z}(0)))c_H)$ where $\rho_L^i(\mathbf{z}(0)) > 0$. Hence, there exists a $\bar{\delta}_\lambda < 1$, such that $Q_L(x, x, 1) = v_L$. Clearly, a low trade equilibrium exists if $\delta > \bar{\delta}_\lambda$. Moreover, it is easy to show that $\sup_\lambda \rho_L^i(\mathbf{z}(0)) < 1$. Hence, this equilibrium exists if δ is larger than $\bar{\delta} \equiv \sup_{\lambda \in (1-\pi, 1)} \hat{\delta}_\lambda < 1$.
2. High trade. That there is at most one high trade equilibrium follows because σ is pinned down by the requirement that $\pi_i(\mathbf{z}(N)) = \bar{\pi}$. Let y be the value of σ such that $\pi_i(\mathbf{z}(N); y, y) = \bar{\pi}$. As ϕ varies from 0 to 1, Q_L varies continuously from

$$(1 - \delta)c_L + \delta \left(\rho_L^i(\mathbf{z}(N))v_L + \sum_{\mathbf{z}_{-i}} \rho_{i,L}(\mathbf{z}_{-i})V(\pi_i(\mathbf{z}_{-i}; y, y)) \right)$$

to

$$(1 - \delta)c_L + \delta \left(\rho_L^i(\mathbf{z}(N))c_H + \sum_{\mathbf{z}_{-i}} \rho_L^i(\mathbf{z}_{-i})V(\pi_i(\mathbf{z}_{-i}; y, y)) \right).$$

Hence, we have $\lim_{\lambda \rightarrow 1} \rho_L^i(\mathbf{z}(N)) = 1$, and it follows that the range of Q_L converges to the interval $((1 - \delta)c_L + \delta v_L, (1 - \delta)c_L + \delta c_H]$ as λ goes to 1. By Assumption 2, v_L is inside this interval. This establishes the existence of the threshold $\bar{\lambda}_\delta$ such that the high trade equilibrium exists whenever $\lambda > \bar{\lambda}_\delta$.

Notice that we can already conclude that the low and the high trade equilibria coexist when $\delta > \bar{\delta}$ and $\lambda > \bar{\lambda}_\delta$. We next show that there is also at least one medium trade equilibrium when these parametric conditions hold.

3. Medium trade. Let r be a candidate trading probability in a medium trade equilibrium (i.e., such that $\pi(\mathbf{z}(N); r, r) < \bar{\pi} < \pi(\mathbf{z}(0); r, r)$). Let x, y be defined by

$$\pi_i(\mathbf{z}(N); y, y) = \bar{\pi} = \pi_i(\mathbf{z}(0); x, x).$$

From the monotonicity of posteriors in the trading probability, we have $r \in (x, y)$. Since the low and the high trade equilibria coexist, there exist $\phi', \phi'' \in (0, 1)$ such that $Q_L(x, x, \phi') = Q_L(y, y, \phi'') = v_L$.¹³ Now, note that $\lim_{r \downarrow x} Q_L(r, r, \cdot) = Q_L(x, x, 1)$ and $\lim_{r \uparrow y} Q_L(r, r, \cdot) = Q_L(y, y, 0)$. But then, because $Q_L(x, x, 1) > Q_L(x, x, \phi') = Q_L(y, y, \phi'') > Q_L(y, y, 0)$, the intermediate value theorem implies that there must exist an r' such that $Q_L(r', r', \cdot) = v_L$.

In equilibrium, it must be the case $\pi_i(\mathbf{z}(N)) \leq \bar{\pi} \leq \pi_i(\mathbf{z}(0))$ (see discussion above). Hence, by continuity of posteriors in λ and σ , we have $\lim_{\lambda \rightarrow 1-\pi} \pi_i(\mathbf{z}(N)) = \lim_{\lambda \rightarrow 1-\pi} \pi_i(\mathbf{z}(0)) = \lim_{\lambda \rightarrow 1-\pi} \pi_\sigma = \bar{\pi}$, and therefore $\lim_{\lambda \rightarrow 1-\pi} \sigma = \bar{\sigma}$. Let us now consider the uniqueness argument. For $K \in \{1, \dots, N\}$, consider $\delta_K \in (0, 1)$ defined by

$$v_L = (1 - \delta_K) c_L + \delta_K \left(\rho_L^i(\mathbf{z}(K))|_{\sigma=\bar{\sigma}} \cdot v_L + (1 - \rho_L^i(\mathbf{z}(K))|_{\sigma=\bar{\sigma}}) \cdot c_H \right).$$

We now show that a sufficient condition for the equilibrium to be unique for λ close to $1 - \pi$ is that $\delta \notin \{\delta_0, \dots, \delta_N\}$ (hence our qualifier that the equilibrium is generically unique). As λ approaches $1 - \pi$, in equilibrium posteriors converge as well. In particular, we have that for r satisfying $\pi(\mathbf{z}(N); r, r) < \bar{\pi} < \pi(\mathbf{z}(0); r, r)$ and $\pi(\mathbf{z}(K); r, r) \neq \bar{\pi}$:

$$\lim_{\lambda \rightarrow 1-\pi} Q_L(r, r, \phi) = (1 - \delta) c_L + \delta \left[\rho_L^i(\mathbf{z}(K))|_{\sigma=\bar{\sigma}} \cdot v_L + (1 - \rho_L^i(\mathbf{z}(K))|_{\sigma=\bar{\sigma}}) \cdot c_H \right]$$

for some $K \in \{0, \dots, N\}$. But if $\delta \notin \{\delta_0, \dots, \delta_N\}$, then for λ close to $1 - \pi$, a medium trade equilibrium cannot exist, because otherwise we cannot have $Q_L = v_L$ as required by (7). Thus, we have ruled out medium trade equilibria. But from our earlier arguments for the existence of the medium trade equilibria, the low and the high trade equilibrium generically do not coexist when the medium trade equilibrium does not exist. ■

For the proof of Theorem 1, it will be useful to reference the following lemma, which is straightforward to verify so the proof is omitted. Let $\pi_i(s; \sigma_N)$ denote the buyers' posterior posterior about seller i , conditional on buyers knowing that the state is s and the trading probability being σ_N . Then, for $s \in \{l, h\}$, we have:

$$\pi_i(s, \sigma) = \frac{\mathbb{P}(S = s | \theta_i = H) \cdot \pi_\sigma}{\mathbb{P}(S = s | \theta_i = H) \cdot \pi_\sigma + \mathbb{P}(S = s | \theta_i = L) \cdot (1 - \pi_\sigma)},$$

where as before π_σ is the interim belief and $\mathbb{P}(S = l | \theta_i = L) > \mathbb{P}(S = l | \theta_i = H)$ are derived by Bayes' rule from the primitives.

¹³We ignore the non-generic cases where either $\phi' = 1$ or $\phi'' = 0$, which are ruled out when $\delta > \bar{\delta}$ and $\lambda > \bar{\lambda}_\delta$.

Lemma 1 *Given a sequence $\{\sigma_N\}_{N=1}^\infty$ of trading probabilities along which information aggregates, we also have convergence of posteriors: $\pi_i(z; \sigma_N) \rightarrow^p \pi_i(S; \sigma_N)$ as $N \rightarrow \infty$.*

Proof of Theorem 1. We first establish our *Non-aggregation result*. Assume that conditions (1) and (2) hold and recall that in equilibrium, for any N , we must have:

$$v_L = Q_L^i(\sigma_N, \phi_i) = (1 - \delta) c_L + \delta \sum_{k=0}^N \rho_L^i(z_k) \cdot F_L(\pi_i(z_k; \sigma_N), \phi_i)$$

where

$$\begin{aligned} \sum_{k=0}^N \rho_L^i(z_k) \cdot F_L(\pi_i(z_k; \sigma_N), \phi_i) &= \sum_{s=l, h} \mathbb{P}(S = s | \theta_i = L) \sum_{k=0}^N \mathbb{P}(Z = z_k | S = s) \cdot F_L(\pi_i(z_k; \sigma_N), \phi_i) \\ &> \lambda \cdot v_L + (1 - \lambda) \cdot \sum_{k=0}^N \mathbb{P}(Z = z_k | S = h) \cdot F_L(\pi_i(z_k; \sigma_N), \phi_i) \end{aligned}$$

Since by Lemma 1, $\pi_i(z; \sigma_N) \rightarrow^p \pi_i(h; \sigma_N)$ when the state is h , and because condition (1) implies that $\pi_i(h; \sigma_N) > \pi_i(h; 0) > \bar{\pi}$, we have that for a given $\epsilon > 0$, if N is large enough, then:

$$\sum_{k=0}^N \mathbb{P}(Z = z_k | S = h) \cdot F_L(\pi_i(z_k; \sigma_N), \phi_i) > V(\pi_i(h; \sigma_N)) - \epsilon.$$

Therefore, we conclude that:

$$\begin{aligned} v_L = Q_L^i(\sigma_N, \phi_i) &> (1 - \delta) c_L + \delta \cdot (\lambda \cdot v_L + (1 - \lambda) \cdot V(\pi_i(h; \sigma_N))) - \delta \cdot (1 - \lambda) \cdot \epsilon \\ &> (1 - \delta) c_L + \delta \cdot (\lambda \cdot v_L + (1 - \lambda) \cdot V(\pi_i(h; 0))) - \delta \cdot (1 - \lambda) \cdot \epsilon. \end{aligned}$$

Since ϵ was arbitrary, we conclude that:

$$v_L \geq (1 - \delta) c_L + \delta \cdot (\lambda \cdot v_L + (1 - \lambda) \cdot V(\pi_i(h; 0))),$$

which violates condition (2).

Next, we establish our *Aggregation result*. If either condition (1) or (2) is reversed, then in the fictitious economy, the equilibrium trading probability in the first period must satisfy $\sigma^* > 0$. We now find an equilibrium sequence $\{\sigma_N\}$ of the actual model such that the sequence is uniformly bounded below by a positive number, which clearly implies that information aggregates along this sequence of equilibria. First, consider a sequence $\{\hat{\sigma}_N\}$, not necessarily an equilibrium one, such that $\hat{\sigma}_N = \hat{\sigma} \in (0, \sigma^*)$, i.e., this is a sequence of constant trading probabilities that are positive but below σ^* . Along such a sequence, clearly information

aggregation holds and, by Lemma 1, $\pi_i(z, \hat{\sigma}_N) \rightarrow^p \pi_i(s, \hat{\sigma}_N)$ in state s . Therefore, combined with the fact that $\pi_i(z, \hat{\sigma}_N) = \pi_i(z, \hat{\sigma}) < \pi_i(z, \sigma^*)$, there exists an N^* such that for $N > N^*$, we have:

$$\mathbb{E}_L \{F_L(\pi_i(z, \hat{\sigma}_N), \phi_i)\} < \mathbb{E}_L^{fict} \{F_L(\pi_i(s, \sigma^*))\} = \frac{v_L - (1 - \delta) \cdot c_L}{\delta},$$

where the last equality holds since $\sigma^* > 0$ implies that the low type must be indifferent to trading at $t = 1$ and delaying trade to $t = 2$. The correspondence $\mathbb{E}_L \{F_L(\pi_i(z, \sigma), \cdot)\}$ is upper hemicontinuous in σ for each N , and has a maximal value of v_H that is strictly greater than $\mathbb{E}_L^{fict} \{F_L(\pi_i(s, \sigma^*))\}$. Hence, for each $N > N^*$, we can find a σ_N such that $\sigma_N \geq \hat{\sigma}_N > 0$ and $\mathbb{E}_L \{F_L(\pi_i(z, \sigma_N), \phi_i)\} = \frac{v_L - (1 - \delta) \cdot c_L}{\delta}$. This delivers the desired equilibrium sequence $\{\sigma_N\}$ that is uniformly bounded below by a positive number. ■

Lemma 2 *If conditions (1) and (2) hold, there exists a $\gamma > 0$ such that $\sigma_N N < \gamma$ for all N .*

Proof. Suppose to the contrary that there is a subsequence of equilibria indexed by N_m such that $\sigma_{N_m} \cdot N_m$ goes to ∞ as m goes to ∞ . Let X_i denote the indicator that which takes value of 1 if seller i has traded in the first period. Define $Y_{N_m} = N_m^{-1} \cdot \sum_{i=1}^{N_m} X_i$ be the fraction of sellers who have traded in the first period, and note that conditional on the state being s , Y_{N_m} has a mean p_{s, N_m} and variance $N_m^{-1} \cdot p_{s, N_m} \cdot (1 - p_{s, N_m})$, where recall that $p_{s, N_m} = \sigma_{N_m} \cdot \mathbb{P}(\theta_i = L | S = s)$. Next, since $p_{l, N_m} > p_{h, N_m}$, using Chebyshev's inequality we have that:

$$\begin{aligned} \mathbb{P} \left(Y_{N_m} \geq \frac{p_{h, N_m} + p_{l, N_m}}{2} | S = h \right) &= \mathbb{P} \left(Y_{N_m} - p_{h, N_m} \geq \frac{p_{l, N_m} - p_{h, N_m}}{2} | S = h \right) \\ &\leq \mathbb{P} \left((Y_{N_m} - p_{h, N_m})^2 \geq \left(\frac{p_{l, N_m} - p_{h, N_m}}{2} \right)^2 | S = h \right) \\ &\leq \frac{\mathbb{E} \{ (Y_{N_m} - p_{h, N_m})^2 | S = h \}}{\left(\frac{p_{l, N_m} - p_{h, N_m}}{2} \right)^2} \\ &= \frac{N_m^{-1} \cdot p_{h, N_m} \cdot (1 - p_{h, N_m})}{\left(\frac{p_{l, N_m} - p_{h, N_m}}{2} \right)^2} \\ &= 4 \cdot \frac{\sigma_{N_m} \cdot \mathbb{P}(\theta_i = L | S = h) - \sigma_{N_m}^2 \cdot \mathbb{P}(\theta_i = L | S = h)^2}{N_m \cdot \sigma_{N_m}^2 \cdot (\mathbb{P}(\theta_i = L | S = l) - \mathbb{P}(\theta_i = L | S = h))^2} \end{aligned}$$

which by our assumption tends to 0 as m tends to ∞ . By a similar reasoning, we have that:

$$\begin{aligned}
\mathbb{P}\left(Y_{N_m} < \frac{p_{h,N_m} + p_{l,N_m}}{2} | S = l\right) &= \mathbb{P}\left(p_{l,N_m} - Y_{N_m} > \frac{p_{l,N_m} - p_{h,N_m}}{2} | S = l\right) \\
&\leq \mathbb{P}\left((p_{l,N_m} - Y_{N_m})^2 > \left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2 | S = l\right) \\
&\leq \frac{\mathbb{E}\{(Y_{N_m} - p_{l,N_m})^2 | S = l\}}{\left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2} \\
&= \frac{N_m^{-1} \cdot p_{l,N_m} \cdot (1 - p_{l,N_m})}{\left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2} \\
&= 4 \cdot \frac{\sigma_{N_m} \cdot \mathbb{P}(\theta_i = L | S = l) - \sigma_{N_m}^2 \cdot \mathbb{P}(\theta_i = L | S = l)^2}{N_m \cdot \sigma_{N_m}^2 \cdot (\mathbb{P}(\theta_i = L | S = l) - \mathbb{P}(\theta_i = L | S = h))^2}
\end{aligned}$$

which again tends to 0 as m tends to ∞ . Combining these two observations, we immediately conclude that information about the state aggregates along the sequence of equilibria $\{\sigma_{N_m}\}$, a contradiction to Theorem 1. ■

Lemma 3 *If Conditions (1) and (2) hold, then $\mathbb{P}(Y_N = 0 | S = s)$ is bounded away from zero uniformly over N for $s \in \{l, h\}$.*

Proof. We have that $\mathbb{P}(Y_N = 0 | S = s) = (1 - p_{s,N})^N$ for $s \in \{l, h\}$. By Lemma 2, there exists a $\gamma \in \mathbb{R}$ such that $p_{s,N} \leq N^{-1} \cdot \gamma \cdot \mathbb{P}(\theta_i = L | S = s)$. Hence, we have that:

$$\begin{aligned}
\mathbb{P}(Y_N = 0 | S = s) &\geq (1 - N^{-1} \cdot \gamma \cdot \mathbb{P}(\theta_i = L | S = s))^N \\
&= \left(\frac{N - \gamma \cdot \mathbb{P}(\theta_i = L | S = s)}{N}\right)^N \\
&\rightarrow e^{-\gamma \cdot \mathbb{P}(\theta_i = L | S = s)}
\end{aligned}$$

as N tends to ∞ . ■

Proof of Proposition 2. We know that for any N , the low trade equilibrium (if it exists) is unique. Let us conjecture that along such equilibria $\sigma_N = \kappa_N \cdot N^{-1} \in (0, 1)$ where κ_N converges to some $\kappa > 0$. Note that in such an equilibrium, we must have:

$$\pi_i(0; \sigma_N) = \bar{\pi}$$

$$\Longleftrightarrow$$

$$\frac{1}{1 + \frac{\sum_{s=l,h} \mathbb{P}(S=s|\theta_i=L) \cdot (1-\kappa_N \cdot N^{-1} \cdot \mathbb{P}(\theta_i=L|S=s))^N}{\sum_{s=l,h} \mathbb{P}(S=s|\theta_i=H) \cdot (1-\kappa_N \cdot N^{-1} \cdot \mathbb{P}(\theta_i=L|S=s))^N} \cdot \frac{(1-\kappa_N \cdot N^{-1})(1-\pi)}{\pi}} = \bar{\pi},$$

which implicitly defines κ_N . Hence, by Lemma 4 below, κ must be given by:

$$\frac{\sum_{s=l,h} \mathbb{P}(S=s|\theta_i=L) \cdot e^{-\kappa \cdot \mathbb{P}(\theta_i=L|S=s)}}{\sum_{s=l,h} \mathbb{P}(S=s|\theta_i=H) \cdot e^{-\kappa \cdot \mathbb{P}(\theta_i=L|S=s)}} = \frac{1-\bar{\pi}}{\bar{\pi}} \cdot \frac{\pi}{1-\pi} \in (0,1)$$

We have thus defined the sequence of candidate low trade equilibria $\{\sigma_N\}$. These equilibria exist for large enough δ , uniformly over N , if the probability of the best news conditional on the seller's type being low is bounded away from 0 along this sequence. But note:

$$\begin{aligned} \mathbb{P}(Y_N = 0|\theta_i = L) &= \sum_{s=l,h} \mathbb{P}(S=s|\theta_i=L) \cdot (1 - \sigma_N \cdot \mathbb{P}(\theta_i=L|S=s))^N \\ &= \sum_{s=l,h} \mathbb{P}(S=s|\theta_i=L) \cdot (1 - \kappa_N \cdot N^{-1} \cdot \mathbb{P}(\theta_i=L|S=s))^N \\ &\rightarrow \sum_{s=l,h} \mathbb{P}(S=s|\theta_i=L) \cdot e^{-\kappa \cdot \mathbb{P}(\theta_i=L|S=s)} > 0, \end{aligned}$$

where the last limit follows from Lemma 4.

Next, note that condition (1) is independent of δ ; hence, if it is violated and δ is large enough, we have the coexistence of the two (aggregating and non-aggregating) sequences of equilibria. On the other hand, condition (2) depends on δ . However, note that the last expression is bounded away from 0 uniformly over λ whenever λ is above some $\bar{\lambda} \in (1-\pi, 1)$; the reason is that κ is continuous in λ and:

$$\lim_{\lambda \rightarrow 1} \sum_{s=l,h} \mathbb{P}(S=s|\theta_i=L) \cdot e^{-\kappa \cdot \mathbb{P}(\theta_i=L|S=s)} = \frac{1-\bar{\pi}}{\bar{\pi}} \cdot \frac{\pi}{1-\pi}.$$

Hence, $\inf_{\lambda \in (\bar{\lambda}, 1)} \mathbb{P}(Y_N = 0|\theta_i = L) > 0$. Thus, we can choose δ large enough so that the above equilibrium exists, and then we can independently choose λ large enough so that condition (2) is violated, in order to have coexistence of the two sequences of equilibria. ■

Lemma 4 *Let $\{\alpha_x\}$ be a net of positive real numbers such that $\alpha_x \rightarrow \alpha$ as $x \rightarrow \infty$ where $\alpha \in (0, 1)$. Then $\left(\frac{x-\alpha_x}{x}\right)^x \rightarrow e^{-\alpha}$ as $x \rightarrow \infty$.*

Proof. Let us suppose that for any $\gamma \in (0, 1)$, $\left(\frac{x-\gamma}{x}\right)^x \rightarrow e^{-\gamma}$ as $x \rightarrow \infty$. Then, given $\epsilon > 0$ so that $\epsilon < \alpha < 1 - \epsilon$, if x is large enough then $|\alpha_x - \alpha| < \epsilon$, $\left(\frac{x-\alpha-\epsilon}{x}\right)^x \geq e^{-\alpha-\epsilon} - \epsilon$, and

$\left(\frac{x-\alpha+\epsilon}{x}\right)^x \leq e^{-\alpha+\epsilon} + \epsilon$. This in turn implies that:

$$e^{-\alpha-\epsilon} - \epsilon \leq \left(\frac{x-\alpha-\epsilon}{x}\right)^x \leq \left(\frac{x-\alpha}{x}\right)^x \leq \left(\frac{x-\alpha+\epsilon}{x}\right)^x \leq e^{-\alpha+\epsilon} + \epsilon.$$

Since ϵ is arbitrary, we conclude that $\left(\frac{x-\alpha}{x}\right)^x \rightarrow e^{-\alpha}$ as $x \rightarrow \infty$. Next, we prove the supposition that for any $\gamma \in (0, 1)$, $\left(\frac{x-\gamma}{x}\right)^x \rightarrow e^{-\gamma}$ as $x \rightarrow \infty$. Note that $\left(\frac{x-\gamma}{x}\right)^x = e^{x \cdot \log\left(\frac{x-\gamma}{x}\right)}$ and by L'Hospital's rule:

$$\lim_{x \rightarrow \infty} x \cdot \log\left(\frac{x-\gamma}{x}\right) = \lim_{x \rightarrow \infty} \frac{\log\left(\frac{x-\gamma}{x}\right)}{x^{-1}} = \lim_{x \rightarrow \infty} -\frac{\gamma \cdot x}{x-\gamma} = -\gamma.$$

By continuity, $\lim_{x \rightarrow \infty} e^{x \cdot \log\left(\frac{x-\gamma}{x}\right)} = e^{-\gamma}$. ■

Proposition 3 *If conditions (1) and (2) are satisfied, then there exist $0 < a_0 < a_1 < \infty$ such that $\sigma_N \cdot N \in (a_0, a_1)$.*

Proof. The fact that there exists an a_1 is in Lemma 5. Now, suppose to the contrary that there exists a subsequence $\{\sigma_{N_m} \cdot N_m\}$ where $\sigma_{N_m} \cdot N_m \rightarrow 0$ as $m \rightarrow \infty$. Then, given $\epsilon > 0$ and m large enough, we have:

$$(1 - \sigma_{N_m} \cdot \mathbb{P}(\theta_i = L | S = s))^{N_m} = \left(\frac{N_m - \sigma_{N_m} \cdot N \cdot \mathbb{P}(\theta_i = L | S = s)}{N}\right)^{N_m} \geq \left(\frac{N_m - \epsilon}{N_m}\right)^{N_m}$$

for $s \in \{l, h\}$, where the last expression converges to $e^{-\epsilon}$ by Lemma 9. Since ϵ is arbitrary, we have that $(1 - \sigma_{N_m} \cdot \mathbb{P}(\theta_i = L | S = s))^{N_m}$ goes to 1 as $m \rightarrow \infty$. Hence, we have that

$$\mathbb{P}(Y_{N_m} = 0 | \theta_i = L) = \sum_{s=l,h} \mathbb{P}(S = s | \theta_i = L) \cdot (1 - \sigma_{N_m} \cdot \mathbb{P}(\theta_i = L | S = s))^{N_m} \rightarrow 1$$

We know that the posterior belief following the event $Y_{N_m} = 0$ must be weakly greater than $\bar{\pi}$. But, by Assumption 2, if m is large enough, we cannot have the posterior belief following the event $Y_{N_m} = 0$ be greater than $\bar{\pi}$, since then the expected offer would be greater than c_H and the low type would strictly want to delay trade. Hence, for m large enough, we must have the posterior following the event $Y_{N_m} = 0$ be exactly equal to $\bar{\pi}$. But then note that σ_N would need to satisfy:

$$1 + \frac{1}{\frac{\sum_{s=l,h} \mathbb{P}(S=s|\theta_i=L) \cdot (1-\sigma_{N_m} \cdot \mathbb{P}(\theta_i=L|S=s))^{N_m}}{\sum_{s=l,h} \mathbb{P}(S=s|\theta_i=H) \cdot (1-\sigma_{N_m} \cdot \mathbb{P}(\theta_i=L|S=s))^{N_m}} \cdot \frac{(1-\sigma_{N_m} \cdot N_m^{-1})(1-\pi)}{\pi}} = \bar{\pi},$$

where $\sigma_{N_m} \cdot N_m \rightarrow 0$ implies by the argument above that $(1 - \sigma_{N_m} \cdot \mathbb{P}(\theta_i = L|S = s))^{N_m} \rightarrow 1$ for $s \in \{l, h\}$ and, thus, that the LHS converges to $\pi < \bar{\pi}$, a contradiction. ■

Appendix B

Proof of Theorem 2. We prove the result in two steps. First, we prove that if either condition (1) or (2) is reversed, there exist (sequences of) equilibria that aggregate information and, if δ is large, these equilibria coexist with others that do not aggregate information. We do this by considering a class of equilibria that we label class- P . The key property of equilibria in class- P is that once the posterior π_t^i about the seller is weakly above $\bar{\pi}$, the market stops revealing any information until the last period. Let $\pi_{i,t}$ denote the buyers' belief about the seller at time t . Then,

Definition 3 *An equilibrium is said to belong to class- P if it has the following features:*

1. *Equilibrium play at $t < T$ is given by:*

- (i) *If $\pi_{i,t} < \bar{\pi}$, then the bid is v_L , the low type accepts w.p. $\sigma_t \in (0, 1)$ whereas the high type rejects it.*
- (ii) *If $\pi_{i,t} > \bar{\pi}$, then the bid is $V(\pi_{i,t})$ and both types accept it.*
- (iii) *If $\pi_{i,t} = \bar{\pi}$, then the bid is $V(\pi_{i,t})$ w.p. ϕ_t (and both types accept it) and is v_L w.p. $1 - \phi_t$ (and both types reject it).*

2. *Equilibrium play at $t = T$ is given by:*

- (i) *If $\pi_{i,T} < \bar{\pi}$, then the bid is v_L , the low type accepts w.p.1 whereas the high type rejects it.*
- (ii) *If $\pi_{i,T} > \bar{\pi}$, then the bid is $V(\pi_{i,T})$ and both types accept it.*
- (iii) *If $\pi_{i,T} = \bar{\pi}$, then the bid is $V(\pi_{i,T})$ w.p. ϕ_T (and both types accept it) and is v_L w.p. $1 - \phi_T$ (and only low type accepts).*

Second, we establish the non-existence of (sequences of) aggregating equilibria when both conditions (1) and (2) hold. We use techniques that are similar to those in the proof of Theorem 1, with the exception that, in the longer horizon economy, in order to study the aggregation properties at any given date, we need to keep track of the dynamic incentives to trade since beliefs about the state evolve over time.

B.1 - Condition (1) or (2) is reversed

The following lemma states that the class- P of equilibria is non-empty and that it contains equilibria in which information about the state aggregates.

Lemma 5 *The class- P is non-empty. If either condition (1) or (2) is reversed, then there exists a sequence of equilibria in class- P along which information about the state aggregates in the first period.*

Proof. Let $\mathbf{z}^t$ denote the state variable that summarizes the history of trade up to period t , which we will define recursively as $\mathbf{z}^{t+1} = (\mathbf{z}_{t+1}, \mathbf{z}^t)$ where $\mathbf{z}_t$ summarizes the trading behavior in period $t - 1$, with $\mathbf{z}_1 \equiv \emptyset$ signifying that at $t = 1$ the information set is given by prior beliefs. The beliefs about the seller and about the state following history $\mathbf{z}^t$ are denoted by $\pi_{i,t}(\mathbf{z}^t) = \mathbb{P}(\theta_i = H | \mathbf{z}^t)$ and $\lambda_{\theta,t}(\mathbf{z}^t) = \mathbb{P}(S = l | \mathbf{z}^t, \theta_i = \theta)$ respectively. Note that these beliefs are computed using Bayes' rule and the equilibrium trading strategies of the low and the high type from periods 1 through t (i.e., $\{\sigma_s^\theta\}_{s=1, \theta=L,H}^{t-1}$); for brevity we omit the dependence on the latter when it is clear. Let $\mathbf{z}_{-i}^t$ denote the history of trade of sellers other than i up to period t and, analogously to equation (2), let $\rho_{\theta,t}(\mathbf{z}_{-i,t+1} | \mathbf{z}_{-i}^t) = \mathbb{P}(\mathbf{z}_{-i,t+1} | \theta_i = \theta, \mathbf{z}_{-i}^t)$ be the conditional distribution over other sellers' trading behavior (or news) in $t + 1$, given that the seller i is of type θ and given that the history of other sellers' trades up to date t is $\mathbf{z}_{-i}^t$ (as well as that seller i has not traded until period t).

Consider the candidate equilibria in Definition 3. We begin with period $t = T$. That the proposed play at $t = T$ is part of an equilibrium follows from the same reasoning as that for Property 1. From (2), the period- T payoffs to the low and the high type are (just as in equations (4) and (5) of Section 2) given by:

$$F_{H,T}(\pi_{i,T}, \phi_{i,T}) = \max \{c_H, V(\pi_{i,T})\} \quad (9)$$

and

$$F_{L,T}(\pi_{i,T}, \phi_{i,T}) = \begin{cases} V(\pi_{i,T}) & \text{if } \pi_{i,T} > \bar{\pi} \\ \phi_{i,T} c_H + (1 - \phi_{i,T}) v_L & \text{if } \pi_{i,T} = \bar{\pi} \\ v_L & \text{if } \pi_{i,T} < \bar{\pi} \end{cases} \quad (10)$$

We can use the payoffs in (9) and (10) to construct the expected continuation value $Q_{\theta,T-1}^i$ of the θ type seller from rejecting a bid v_L in $t = T - 1$. This continuation value depends on the posterior $\pi_{i,T-1}$. In particular, if $\pi_{i,T-1} > \bar{\pi}$, from (1) the bid is $V(\pi_{i,T-1})$ and both sellers' accept. In this case, we need to specify the off-equilibrium belief in case a seller deviates and rejects this bid. Setting $\pi_{i,T}$ so that $v_L = Q_{L,T-1}^i = (1 - \delta) c_L + \delta (\pi_{i,T} v_H + (1 - \pi_{i,T}) v_L)$

suffices to deter such deviations. If $\pi_{i,T-1} = \bar{\pi}$, the bid is $c_H = V(\bar{\pi})$ w.p. ϕ_{T-1} (and both types accept) and the bid is v_L w.p. $1 - \phi_{T-1}$ and both types reject. Setting ϕ_T to satisfy $v_L = Q_{L,T-1}^i = (1 - \delta) c_L + \delta (\phi_T c_H + (1 - \phi_T) v_L)$ suffices to ensure that the low type does not want to accept bid v_L at $T - 1$; clearly, the high type rejects v_L . Finally, if $\pi_{i,T-1} < \bar{\pi}$, the bid is v_L and only the low type accepts w.p. σ_{T-1} . In this case, the low type's expected continuation value upon rejection is:

$$Q_{L,T-1}^i = (1 - \delta) c_L + \delta \sum_{\mathbf{z}_{-i,T} | \mathbf{z}_{-i}^{T-1}} \rho_{L,T-1}(\mathbf{z}_{-i,T} | \mathbf{z}_{-i}^{T-1}) \cdot F_{L,T}(\pi_{i,T}(\mathbf{z}_{-i}^T), \phi_T), \quad (11)$$

which is the period- $T - 1$ analogue of equation (7). For the low type to accept bid v_L with interior probability, we must have $Q_{L,T-1}^i = v_L$. By arguments similar to those in the two period economy, we can show that there exists a pair (σ_{T-1}, ϕ_T) that solves equation (11): (a) $Q_{L,T-1}^i$ is upper hemicontinuous in σ_{T-1} , (b) $Q_{L,T-1}^i$ is lower than v_L when evaluated at $\sigma_{T-1} = 0$, and (c) $Q_{L,T-1}^i$ is greater than v_L when evaluated at $\sigma_{T-1} = 1$. We have thus verified that the proposed play at $T - 1$ is part of an equilibrium.

Using similar reasoning and working back, we can show that for any $t \in \{2, T - 1\}$: (a) $F_{\theta,t}$ is given by equations (9) and (10) but where we replace $(\pi_{i,T}, \phi_T)$ with $(\pi_{i,t}, \phi_t)$, and (b) the low type's expected continuation value from rejecting the bid at $t - 1$ satisfies $Q_{L,t-1}^i = v_L$: (i) if $\pi_{i,t-1} > \bar{\pi}$, the off-equilibrium belief is as before $\pi_{i,t} = \pi_{i,T}$, (ii) if $\pi_{i,t-1} = \bar{\pi}$, then ϕ_t satisfies $v_L = Q_{L,t-1}^i = (1 - \delta) c_L + \delta (\phi_t c_H + (1 - \phi_t) v_L)$ and, finally, (iii) if $\pi_{i,t-1} < \bar{\pi}$, the pair (σ_{t-1}, ϕ_t) satisfies:

$$v_L = Q_{L,t-1}^i = (1 - \delta) c_L + \delta \sum_{\mathbf{z}_{-i,t} | \mathbf{z}_{-i}^{t-1}} \rho_{L,t-1}(\mathbf{z}_{-i,t} | \mathbf{z}_{-i}^{t-1}) \cdot F_{L,t}(\pi_{i,t}(\mathbf{z}_{-i}^t), \phi_t). \quad (12)$$

At $t = 1$, the initial history is $\mathbf{z}_{-i}^1 = \emptyset$ and thus the belief about the seller is the prior $\pi_{i,1} = \pi < \bar{\pi}$, where the latter inequality holds by Assumption (1). This establishes the fact that class- P is non-empty. We next show that there is a sequence of equilibria within this class that aggregates information.

Since the beliefs $\pi_{i,2}(\mathbf{z}_{-i}^2)$ and $\rho_{\theta,1}(\mathbf{z}_{-i,2} | \mathbf{z}_{-i}^1)$ are as in equations (2) and (3) in Section 2, the expected continuation value of the low type seller from rejecting a bid v_L in the first period is exactly as in equation (6). Therefore, the trading behavior and the information aggregation properties of these equilibria in the first period are as in the first period of the two period economy, i.e., they are independent of trading horizon T . Thus, since when either condition (1) or (2) is reversed, in the two period economy, there exists a sequence of equilibria which aggregates information in the first period, this must also be true of the T period economy. ■

The following lemma states that if in addition δ is sufficiently large, then class- P also contains equilibria in which aggregation fails.

Lemma 6 *If either condition (1) or (2) is reversed, then there exists a sequence of equilibria in class- P along which information aggregation fails for any $t < T$.*

Proof. In Proposition 2, we showed that in the two period economy, when either condition is reversed and δ is sufficiently large, there exists a sequence of equilibria in which aggregation fails. From the class- P equilibria in Definition 3, we can conclude that if information does not aggregate in the first period of the two period economy, then it does not aggregate in the first period of the T period economy. In the T horizon economy, however, we must also show that information also does not aggregate in $t > 1$. But this is straightforward, since the equilibrium condition $v_L = Q_{L,t-1}^i$ implies that there must be a positive probability that the posterior in the second period is above $\bar{\pi}$: otherwise, from (1), the bid would be v_L in the second period implying that $Q_{L,t-1}^i < v_L$, a contradiction. But then, in equilibria in class- P , once the posterior is weakly above $\bar{\pi}$, the market stops revealing any additional information for any $t < T$. This establishes the result. ■

B.2 - Conditions (1) and (2) hold

When the buyers' belief about the seller $\pi_{i,t}$ is below $\bar{\pi}$, just as in class- P equilibria, the only possible equilibrium play in period t must involve a bid of v_L , the low type accepting it with some probability and the high type rejecting it.

On the other hand, when the belief is $\pi_{i,t} \geq \bar{\pi}$, the class- P equilibria featured a pooling bid that was accepted by both types, thus precluding any further information revelation. There is, however, another possibility. There can potentially exist equilibria in which (despite the fact that $\pi_{i,t} \geq \bar{\pi}$) the bid is still v_L and only the low type accepts it with some probability. For these type of equilibria to exist, it must be that the high type expects sufficiently good information to arrive from other sellers in the next period (i.e., there must be further trading opportunities and information arrival from other sellers); otherwise, there would a profitable deviation for buyers, who could offer slightly less than the pooling bid in the current period, attract both seller types and make positive profits. Together with the equilibria in class- P , these exhaust the set of all possible symmetric equilibria. Rather than prove the conditions under which these type of equilibria can arise, we show that even if such equilibrium play were possible, conditions (1) and (2) still guarantee information non-aggregation for any $t < T$.

Our strategy for the proof is the following. As in the proof of Theorem 1, we study the low type's incentive to delay trade from period t to period $t + 1$, when she anticipates information

about the aggregate state to be (essentially) revealed in period $t + 1$. As in Section 3, we compare the trading incentives in our economy with those of the ‘fictitious’ two period economy in which the state is revealed in the second period w.p.1. There are, however, two differences. First, since period $t + 1$ may not be the terminal date, the continuation value of the low type seller at t will depend on more than one period ahead payoffs. Second, conditions (1) and (2) use the prior beliefs about the seller and the state, but in the longer horizon economy these beliefs evolve as a function of past trading behavior. Nevertheless, we show that conditions (1) and (2) are sufficient to understand the information aggregation properties of equilibria in any period but the last. We prove this result by contradiction.

Consider any period $t < T$ and suppose that information has not aggregated until that period, but that it aggregates at t . This must be true if information is to aggregate in any period t . Clearly, there must still be sellers present at t to trade and reveal information; in fact, the number of sellers at this date must grow with N in order for aggregation to be possible. Furthermore, if the bid were pooling at this date and both seller types were to accept it with probability bounded away from zero as N grows large, then there would also be a positive probability (bounded away from zero) that no additional information gets revealed, which would contradict aggregation at t . Thus, it must be the case that with probability going to 1 as N grows large, the equilibrium play at t has the following feature: the bid is v_L , the low type accepts the bid w.p. $\sigma_t \in (0, 1)$ (we can again rule out $\sigma_t \in \{0, 1\}$), and the high type rejects. In these equilibria, it must be the case that v_L is equal to the expectation continuation value $Q_{L,t}^i$ of the low type seller upon rejecting the bid v_L at t . In what follows, our strategy is to show that information aggregation at t implies that the probability that the continuation value $Q_{L,t}^i$ strictly exceeds v_L remains bounded away from zero. But, this would lead to a contradiction, since in such histories for large but finite N , the sellers would strictly prefer not to trade at t , contradicting equilibrium requirement that σ_t be positive for all N .

The next lemma provides a lower bound on the low type’s continuation value as a function of low type’s beliefs about the state of nature.

Lemma 7 *Suppose that information has not aggregated before period $t < T$, but that it aggregates in period t . Then the probability that the low type’s expected continuation value $Q_{L,t}^i$ satisfies*

$$Q_{L,t}^i \geq \bar{Q}(\lambda_{L,t}) \equiv (1 - \delta) c_L + \delta \left(\lambda_{L,t} \cdot v_L + (1 - \lambda_{L,t}) \cdot V \left(1 - \frac{(1 - \lambda)(1 - \pi)}{\pi} \right) \right) \quad (13)$$

goes to 1 as N goes to ∞ , where $\lambda_{L,t}$ is the low type’s period t belief that the state is low.

Proof. We show in Lemma 9 below that, when the state is high, information aggregation at

t implies that if the seller were to reject bid v_L at t , the probability that the bid in $t + 1$ is pooling goes to 1 as N becomes large. The reason is that, conditional on the state being high, the probability that the seller is a high type drifts upwards over time. Because by condition (1) this probability exceeds $\bar{\pi}$ in the first period, it must exceed it for all t . In addition, since information aggregation at t implies that there are (essentially) no additional news arriving from other sellers in periods after t , equilibria with delay are no longer possible at $t + 1$. But, if the equilibrium were pooling in the high state, then by the convergence of posteriors the bid would converge to $V(\pi_{i,t+1}(h))$, which is greater than $V\left(1 - \frac{(1-\lambda)(1-\pi)}{\pi}\right)$, where $\pi_{i,t+1}(h)$ is the posterior about the seller conditional on the state being revealed to be high in $t + 1$. Proceeding along the same steps as in the proof of Theorem 1 and replacing the bids in the low state by the lowest possible bid v_L , we have the desired inequality. ■

Since $\lambda_{L,1} = \lambda$, condition (2) states that $\bar{Q}(\lambda_{L,1}) > v_L$. In the proof of Theorem 1, we showed that this implies that if information were to aggregate in the first period, then trade in the first period must have collapsed to zero for some large but finite N , which led to a contradiction. We use a similar argument in the longer horizon economy. While we cannot guarantee that $\bar{Q}(\lambda_{L,t}) > v_L$ for all $t < T$, we do not need to: we only need to find histories in which this inequality continues to hold. The following lemma shows that the probability that the lower bound $\bar{Q}(\lambda_{L,t})$ in (13) exceeds v_L is bounded away from zero.

Lemma 8 *Suppose that information has not aggregated before period $t < T$, but that it aggregates in period t . Then the probability of the event that $v_L < \bar{Q}(\lambda_{L,t})$ is bounded away from zero, uniformly over N .*

Proof. By condition (2), we have $v_L < \bar{Q}(\lambda_{L,1})$. Thus, information cannot aggregate in the first period. Lemma 3 shows that non-aggregation implies that the probability of the event that no seller trades before $t = 2$ remains bounded away from zero, uniformly over N in both states of nature. Since this event is good news about the state, we have $\lambda_{L,2} < \lambda_{L,1}$ and thus $v_L < \bar{Q}(\lambda_{L,2})$. But then information cannot aggregate in the second period either, and again by an argument similar to that in Lemma 3 the probability of the event that no seller trades before $t = 3$ must remain bounded away from zero, uniformly over N in both states of nature. Again, this event is good news about the state and we have $\lambda_{L,3} < \lambda_{L,2}$. Using this argument repeatedly, we can construct histories that occur with probability bounded away from zero, in which uncertainty about the aggregate state does not vanish for any $t < T$. ■

Thus, combining Lemmas 7 and 8, we can use arguments similar to those in the proof of Theorem 1 to show that if conditions (1) and (2) hold, then non-aggregation until period t but aggregation at t implies that trade at t must collapse to zero for some large but finite N ,

contradicting equilibrium requirement that trade be positive for any N . Hence, information aggregation must fail for any $t < T$, establishing the part of Theorem 2 that assumes conditions (1) and (2) hold. ■

The following lemma was used in the proof of Lemma 7.

Lemma 9 *Suppose that the state is h . Suppose that information has not aggregated before period $t < T$, but that it aggregates at t . Then the probability that after period t the bid is pooling and both types accept it goes to 1 as N goes to ∞ .*

Proof. We prove this by backwards induction. Assume that the state is high. Let us start in the final period and look at histories in which at least one seller arrives to T . If these histories had zero probability, it would imply that pooling occurred w.p.1 before T and we can simply consider the last period such that at least one seller is still present with positive probability. By reasoning analogous to that for Property 2, there is a unique equilibrium at this date, and it is given by (2) as in Definition 3. Next, let us go to period $t = T - 1$, and assume that information has aggregated before this period. We argue next that, if it were the case that the belief about the seller were $\pi_{i,T-1} > \bar{\pi}$, then the bid at $T - 1$ must (essentially) be $V(\pi_{i,T-1})$. To this end, consider histories z^{T-1} in which we have $\pi_{i,T-1}(z^{T-1}) > \bar{\pi}$, the bid is v_L and only the low type accepts with some probability. The low type's continuation value conditional on rejecting this bid is given by equation (11). We can rule out $\sigma_{T-1} = 1$ as before. We can rule out $\sigma_{T-1} = 0$, as then the belief would remain unchanged from $T - 1$ to T and there would be a profitable deviation for buyers at $T - 1$: a buyer could offer a bid slightly smaller than pooling, attract both types and make positive profits. Hence, we must have that $Q_{L,T-1}^i = v_L$ and $\sigma_{T-1} \in (0, 1)$. Next, note that the fact that information has aggregated before $T - 1$ implies that for large N , the posterior about the seller must be close (in probability) to the posterior about the seller conditional on the state being high, $\pi_{i,T}(h)$. But the latter posterior only drifts upwards over time as the low types trade with higher probability that the high type at any t . Since by condition (1) $\pi_{i,1}(h) > \bar{\pi}$, we must have that $V(\pi_{i,T}(h)) > c_H$ and therefore by the convergence of posteriors:

$$Q_{L,T-1}^i = (1 - \delta) c_L + \delta \cdot V(\pi_{i,T}(h)) + \epsilon \quad (14)$$

where ϵ can be made small by making N large. Since $V(\pi_{i,T}(h)) > c_H$, by Assumption 2, the low type would reject the bid v_L at $T - 1$ when N is sufficiently large but finite, a contradiction to the fact that for any N , any equilibrium with delay must feature $\sigma_{T-1} \in (0, 1)$. But also note that because the posteriors conditional on the high state drift upwards, $\pi_{i,T-1}(h) \geq \pi_{i,1}(h) > \bar{\pi}$, information aggregation before $T - 1$ also implies that, the probability

of histories in which $\pi_{i,T-1} > \bar{\pi}$ goes to one as N becomes large. Thus, we conclude that if information has aggregated before $T - 1$ and if the state is high, then the probability that the bid is pooling at $T - 1$ goes to 1 with N . Using the above argument repeatedly, we can conclude that if the state were high and information were to aggregate before t , then the probability that the bid after t would be pooling would go to 1 as N gets large. ■