Optimal fiscal policy with heterogeneous agents and aggregate shocks*

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Abstract

We show that allocations in incomplete insurance-market economies can be represented as the solution of the program of a constrained planner. This representation allows for solving Ramsey programs in incomplete-market economies with aggregate shocks. We apply this framework to derive optimal fiscal policy and public debt dynamics in an economy with capital after persistent technology shocks, when the planner can use distorting taxes on capital and labor and positive lump-sum transfers. Average capital taxation is a simple function of the tightness of credit constraints. In a quantitative exercise, it is shown that private savings increase too much after a technology shock, and are absorbed by an increase in public debt and a decrease in capital taxes. Simulations of these optimal solutions can be obtained by simple perturbation methods.

Keywords: Incomplete markets, optimal policy, public debt.

JEL codes: E21, E44, D91, D31.

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1 Introduction

Heterogeneous agent models provide a framework to think about many relevant aspects of inequalities and individual risk in general equilibrium. In these models, infinite-lived agents face incomplete insurance markets and borrowing limits that prevent them from perfectly insuring themselves against idiosyncratic risk, in the tradition of the Bewley-Huggett-Aiyagari literature (Bewley, 1983, Imrohoroğlu 1989, Huggett 1993, and Aiyagari 1994). These models frameworks become increasingly popular now widely used, since they fill a gap between micro- and macroeconomics and enable to include aggregate shocks and a number of additional frictions on both the goods and labor markets. However, considering normative analysis, little is known about optimal policies in these environments with capital, due to the difficulties generated by the large and time-varying heterogeneity across agents. This is unfortunate, since a vast literature, reviewed below, suggests that the interaction between capital accumulation, income and wealth inequalities have first-order implications for the optimal design of time-varying fiscal policies.

This paper provides a theoretical and quantitative analysis of optimal fiscal policy in incomplete insurance-market economies with both capital and aggregate shocks. We derive new results about the optimal dynamics of public debt, distorting capital and labor taxes and transfers, considering rich tradeoffs about redistribution, insurance and incentives. These new results are based on a methodological contribution that offers a general and tractable representation of incomplete insurance-market economies. In these economies, it is indeed known that heterogeneity increases as time goes by, because agents differ according to the whole history of the realization of their idiosyncratic risk. Huggett (1993), using results of Hoppenhayn and Prescott (1992), and Aiyagari (1994) have shown that these economies without aggregate risk have a recursive structure when the distribution of wealth is introduced as a state variable. Unfortunately, the distribution of wealth has an infinite support, which is the root of many analytical difficulties. Furthermore, in presence of aggregate shocks, the nature of the state vector is still an open question after the seminal work of Krusell and Smith (1998) (see Miao 2006).

We show that incomplete-market economies can be consistently represented as the limit of an economy with finite support. More precisely, we construct an environment where the heterogeneity across agents depends only on a finite but arbitrarily large number, denoted N, of consecutive past realizations of the idiosyncratic risk. As a theoretical outcome, agents having the same realization of idiosyncratic risk for the previous N periods thus choose the same consumption and wealth levels. The interest of this *truncated* representation of incomplete insurance-market economies lies in four properties. First, the allocation can be represented as the solution of a family-head program, which ensures existence of the equilibrium with aggregate shocks. Second, the standard-incomplete market model is indeed the limit of our truncated representation when N increases, under general conditions. Third and more importantly, as our representation has a finite state-space, one can use the tools derived in Marcet and Marimon (2011) to study Ramsey programs. These tools rely on the extensive use of Lagrange coefficients. Finally, the finite state-space simplifies to a large extent the simulation of the model, as standard perturbation methods can be used.

We take advantage of our limited-heterogeneity environment to study a public debt application, where we solve for the Ramsey policy in an incomplete-market economy with technological shock. The planner has four instruments: linear taxes on capital and labor, public debt and transfers./ These instruments are standard in the Ramsey literature. We derive three sets of results. The first two concern the theoretical characterization of optimal fiscal policy.

First, we show that the average optimal long-run capital tax is directly related to the equilibrium severity of credit constraints. More formally, tax on capital is proportional to the sum of the Lagrange multipliers on agents' credit constraints. As a direct consequence, capital tax is always non-negative (confirming the result of Aiyagari, 1995) and is positive if and only if credit constraints bind for some agents in equilibrium. Oppositely, if credit constraints do not bind, then optimal long-run capital taxes are zero. This is the case when the credit limit is below the natural borrowing limit in the sense of Aiyagari (1994). This new result contributes to clarify the deviations from the Chamley-Judd no-capital tax result found in the literature. In addition, pre-tax marginal return on capital is uniquely pinned down by the planner discount factor, as originally found by Aiyagari (1995). Finally, labor taxes are shown to directly depend on the elasticity of labor supply.

Second, the dynamics of the fiscal policy is mainly driven by the difference in two valuations of liquidity. The first one is planner valuation of government liquidity, which is measured by the Lagrange multiplier of the government budget constraint. The second one is social valuation of the liquidity of private agents. Since it internalizes agents' saving incentives, this valuation differs from their own private valuation, which is simply equal to their marginal utility. The difference in the social valuations of government and private liquidities is denoted the *liquidity-valuation gap*, and is key to understand optimal fiscal policy.

Third, we simulate the model using standard values for preferences, household income process, and public spending. Our simulated equilibrium features, at the steady-state equilibrium, an optimal public debt-to-GDP ratio of 148%, a labor tax rate of 36%, and a capital tax rate of 11%, while 3% of households are credit constrained. The equilibrium dynamics feature capital taxes that are much more volatile than labor taxes, which confirms earlier results in complete insurance-market economies (Aiyagari, Marcet, Sargent and Seppala, 2002, or Farhi 2010 for a recent discussion). A final property of the equilibrium dynamics is that public debt is procyclical, while capital taxes are counter-cyclical. The dynamic behavior of public debt comes from the fact that after a positive *technology* shock, household savings are too high compared to the constrained efficient allocation. The planner absorbs these excess savings by a higher public debt, and redistributes wealth by cutting capital taxes.

Related literature. This paper first contributes to the literature on the theory of incomplete insurance-market economies with aggregate shocks, by providing a new tractable framework. Some environments have already been developed to show equilibrium existence and to derive theoretical properties when insurance markets are incomplete. This is the case of no-trade equilibria with permanent idiosyncratic shocks (Constantinides and Duffie, 1996), for instance used in Heathcote, Storesletten and Violante (2016). More recently, Krusell, Mukoyama and Smith (2011) study a class of no-trade equilibrium in an economy without capital and with a tight-enough credit constraint. This environment is used by Ravn and Sterk (2015) in a model with endogenous idiosyncratic risk. Departing from no-trade, a class of incomplete-market models allowing for “small trades” has been studied. The equilibrium in these models features “reduced heterogeneity” with a finite number of equilibrium wealth levels, instead of a continuous distribution as in Bewley-Aiyagari-Hugget environments. This reduced heterogeneity relies on a specific assumption, such as quasi-linearity in Challe and Ragot (2016) or LeGrand and Ragot, (2016) use a quasi-linearity assumption, or partial insurance mechanisms in Challe, Matheron, Ragot and Rubio-Ramirez (2015). This framework allows one to study optimal policies, as in Ragot (2016) with limited financial-market participation, or in Bilbiie, Challe and Ragot (2017) with nominal frictions. The current paper extend these previous works and show that partial insurance can provide a general theory of truncated representations of Bewley economies. In addition, it derives new tools to study optimal policies, based on the dynamic structure of Lagrange coefficients.

Second, our paper contributes to the literature on distortions and optimal policies in incomplete insurance-market models. Many contributions identify relevant tradeoffs, but the general case with capital accumulation and aggregate shocks has not been studied yet, to the best of our knowledge. In economies without aggregate shocks, Aiyagari (1995) shows that the capital tax is non-negative. Aiyagari and McGrattan (1998) compute the optimal steady-state level of debt. Dávila, Hong, Krusell, and Ríos-Rull (2012) identify constrained-inefficiencies in incomplete-insurance market models with production, showing that the capital stock can be too low. Açikgöz (2015) solve for the Ramsey program to obtain the steady-state optimal level of public debt and compare it with results of Aiyagari and McGrattan (1998). Dyrda and Pedroni (2016) solve numerically for the optimal policies along the transition between an initial and a final steady state. Gottardi, Kajii, and Nakajima (2014) solve for the Ramsey allocation in an incomplete-market model with human capital accumulation. Nuño and Moll (2017) use a continuous-time approach and mean-field games to characterize difference in inequalities in constrained-efficient and in constrained-inefficient economies. Some papers consider a finite number of agents to derive additional results. Shin (2006) studies a two-agent economy, where the government can issue public debt and distorting taxes on labor. Recently, Bhandari, Evans, Golosov, and Sargent (2013) consider an environment without capital with a finite number of agents facing uncorrelated risks and incomplete insurance-markets. They show that when the planner has access to non-distorting tools (positive or negative lump-sum transfers), then the level of public debt is irrelevant. The truncated approach used in our paper allows to derive new results in an economy with capital, based on an extensive use of Lagrange multipliers. Some papers have introduced incomplete insurance markets in overlapping generation models to investigate quantitatively the optimal fiscal policies (Imrohoroğlu 1998 and Conesa, Kitao, and Krueger 2009). Finally, many papers have considered incomplete-insurance markets to analyze the effect of fiscal policies. The positive effects are studied in Heathcote (2005), who consider aggregate shocks. A recent contribution is Kaplan and Violante (2014), who consider transaction costs on some assets.

Third, this paper is also related to the vast literature on optimal fiscal policy with aggregate shocks. Seminal contributions consider a complete-market representative agents (Barro 1979; Lucas and Stokey 1983, surveyed in Chari and Kehoe 1999). More recent contributions consider incomplete market for the aggregate risk, introducing non state-contingent public debt (Aiyagari, Marcet, Sargent and Seppala, 2002; Farhi, 2010). The New Dynamic Public Finance literature focuses on optimal fiscal policy in environments with asymmetric information, using tools of mechanism design. We use here a Ramsey approach, limiting the number of instruments. The gain of this restriction is to allow for the analysis of an economy with capital and aggregate shocks (see Farhi and Werning 2013 and Golosov, Tsyvinski, and Werquin 2016 for recent contributions and discussions of the difference in methods).

Fourth, this paper is related to the computational literature studying incomplete-insurance markets with perturbation methods. Reiter (2009) uses a perturbation method to solve for aggregate dynamics around a steady-state equilibrium of a Bewley model, with idiosyncratic shock but no aggregate shock. Mertens and Judd (2012) use perturbation method around a steady-state with neither aggregate nor idiosyncratic shock. They use a penalty function to pin down equilibrium saving decision. Other papers using perturbation method but restricting exogenously the state-space are Preston and Roca (2007) and Kim, Kollmann, and Kim (2010). In our model, we use perturbation method around a steady state with idiosyncratic shocks and without aggregate shocks, in an economy which delivers a finite-dimensional state-space as a theoretical outcome. This last property is key to be able to derive optimal policies using Lagrangian techniques.

The rest of the paper is organized as follows. In Section 2, we present the environment. We describe the central planner problem and derives the associated allocation in Section 3. We then show in Section 4 how the planner allocation can be decentralized and we provide convergence properties. We then take advantage in Section 5 of the finite equilibrium structure to solve the Ramsey program. We discuss in Section 6 the implications of the Ramsey program for the fiscal policy. In Section 7, we provide a numerical application illustrating our findings. Finally, conclusions are given in Section 8.

2 The environment

Time is discrete, indexed by $t \geq 0$. The economy is populated by a continuum of agents, distributed on a segment J following a non-atomic measure ℓ. The population is of size 1: $\ell(J) = 1$.[1]

2.1 Risk

Aggregate risk. The aggregate risk is represented by a probability space $(\mathcal{S}^\infty, \mathcal{F}, \mathbb{P})$. In any period t, the aggregate state, denoted s_t takes values in the state space $\mathcal{S} \subset \mathbb{R}^+$. The aggregate state at date 0, denoted $s_0 \in \mathcal{S}$, is given. The history of aggregate shocks up to time t is summarized by the sequence of aggregates shocks from date 1 to date t and denoted $s^t = \{s_1, \ldots, s_t\} \in \mathcal{S}^t$. The aggregate state follows a first-order Markov process. Finally, the period-0 probability density function of any history s^t is denoted $m^t(s^t)$. Loosely speaking, $m^t(s^t)$ is the probability in period 0 that history s^t occurs in period t.

Idiosyncratic risk. At the beginning of each period, agents face an uninsurable idiosyncratic labor productivity shock e_t that can take E values in the set $\mathcal{E} = \{1, \ldots, E\} \in \mathbb{R}_+^E$. Households in state $e \in \mathcal{E}$ have a labor productivity $\theta_e > 0$. The productivity shock e_t follows a discrete first-order Markov process with transition matrix $M(s_t) \in [0,1]^{E \times E}$. The probability $M_{e,e'}(s_t)$ is the probability for an agent to switch from state e at date t to state e' at date $t+1$, when aggregate state is s_t in period t. The history of idiosyncratic shocks up to date t is denoted $e^t = \{e_0, \ldots, e_t\} \in \mathcal{E}^{t+1}$.

Remark 1 (Notations) *For the sake of clarity, for any random variable $X_t : \mathcal{S}^t \to \mathbb{R}$, we will denote X_t, instead of $X_t(s^t)$, the realization of the random variable X_t in state s^t and for any random variable $Y_t : \mathcal{S}^t \times \mathcal{E}^t \to \mathbb{R}$, we will denote Y_{t,e^t} the realization of the random variable Y_t in state (s^t, e^t).*

[1] Among others, Feldman and Gilles (1985) have identified issues when applying the law of large number to a continuum of random variables. Green (1994) describes a construction of the set J and of the measure ℓ to ensure that our statements hold. Feldman and Gilles (1985), Judd (1985), and Uhlig (1996) also propose other solutions to this issue. From now on, we assume that the law of large numbers applies.

2.2 Preferences

In each period t, the economy has two goods: a consumption-capital good and labor. Households rank consumption and labor according to a period utility function $U(c, l)$ which is increasing and concave in consumption c, decreasing and convex in labor l, and twice continuously differentiable in (c, l). The derivative with respect to consumption and labor are denoted U_c and U_l, respectively.

As is standard in this class of model, we consider the class of Greenwood-Hercowitz-Huffman (GHH) utility function which does not exhibit wealth effect for the labor supply:[2]

$$U(c, l) = u\left(c - \chi^{-1}\frac{l^{1+1/\varphi}}{1 + 1/\varphi}\right), \tag{1}$$

where $\varphi > 0$ is the Frisch elasticity of labor supply, $\chi > 0$ is a parameter scaling the labor disutility, and u a function that is twice continuously derivable, increasing, and concave, satisfying the Inada conditions. The discount factor is denoted $\beta \in (0, 1)$. Each household ranks consumption and labor streams, denoted respectively as $(c_t)_{t\geq 0}$ and $(l_t)_{t\geq 0}$, according to the intertemporal criterion $\sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$.

2.3 Production and assets

In any period t, a production technology with constant-returns-to-scale (CRS) transforms capital K_{t-1} and labor L_t into $F(K_{t-1}, L_t, s_{t-1})$ units of output. Capital must be installed one period before production, and the productivity depends the aggregate state at the date of capital installation. Labor L_t is measured in efficient units. This formulation allows for capital depreciation, which is subsumed by the production function F as in Farhi (2010). The production function is smooth in K and L and satisfies the standard Inada conditions. The good is produced by a unique profit-maximizing representative firm. We denote as $\tilde{w}_t$ the real before-tax wage rate in period t and as $\tilde{r}_t$ the real before-tax rental rate of capital in period t. Profit maximization yields in each period $t \geq 1$:

$$\tilde{r}_t = F_K(K_{t-1}, L_t, s_{t-1}) \quad \text{and} \quad \tilde{w}_t = F_L(K_{t-1}, L_t, s_{t-1}). \tag{2}$$

[2] All our results can be derived with a general utility function $U(c, l)$. A GHH utility function simplifies slightly the algebra, especially when deriving the Ramsey program in Section 5.

Importantly, previous assumptions imply that the return on the capital $\tilde{r}_t$ in period t is known in period $t-1$. Indeed, (i) the productivity of firms is known in period $t-1$; (ii) the labor transition process between periods $t-1$ and t is known in period $t-1$; (iii) the labor supply exhibits no wealth effect. As shown in Section 3.3 below, this property simplifies the state space.

2.4 Government, fiscal tools and resource constraints

In each period t, the government has to finance an exogenous public good expenditure G_t and a positive lump-sum transfer $T_t > 0$ to all agents. These expenditures are financed by levying distorting taxes on capital income τ_t^K, or on labor income τ_t^L or by issuing an amount B_t of a riskless one-period public bond.[3] The same tax rate τ_t^K applies to all capital incomes, both from public bonds or from capital shares. Since the rate of return on capital at date t is known at date $t-1$, the absence of arbitrage implies that public debt and capital have to pay the same pre-tax interest rate $\tilde{r}_t$ for any aggregate history $s^t \in \mathcal{S}^t$. Positive lump-sum transfers $T_t > 0$ are allowed because Heathcote, Storesletten, and Violante (2016) show that a positive lump-sum transfer is needed to properly approximate the current US fiscal system.

In consequence, following the tradition of Lucas and Stokey (1983), we assume that the taxation scheme only allows for distorting taxes, while lump-sum taxes (or negative transfers T_t) are not available.[4] As it is standard in the literature with capital taxes, we also assume that the date-0 tax rate, bearing on initial capital, is exogenously set, so as to limit the amount of revenues the government can extract in the initial period. Indeed, taxing capital in the first period is non-distorting and the government would heavily tax the initial capital stock (see Farhi 2010 or Sargent and Ljungqvist 2012 Section 16.7 for discussion and references).

We can thus deduce that the period-t budget constraint of the government is

$$G_t + (1+\tilde{r}_t)B_{t-1} + T_t \leq \tau_t^L \tilde{w}_t L_t + \tau_t^K \tilde{r}_t A_{t-1} + B_t. \tag{3}$$

[3] The question of the optimal mix of these financing tools will be the focus of the second part of the paper and in particular of the Ramsey program studied in Section 5.

[4] The absence non-distorting taxes ($T_t < 0$) is easy to rationalize, as it is sufficient to introduce a positive –but small– mass of households with very low market income (for instance living out of home production). These households do not affect the Ramsey program below, except by preventing lump-sum taxes. Bhandari et al. (2016) consider an economy without capital where negative lump-sum taxes are allowed.

We denote

$$r_t = (1 - \tau_t^K)\tilde{r}_t \quad \text{and} \quad w_t = (1 - \tau_t^L)\tilde{w}_t \tag{4}$$

the after-tax real interest and wage rates respectively. Using the CRS property of the production function implying that $F(K, L, s) = KF_K(K, L, s) + LF_L(K, L, s)$, the budget constraint (3) becomes:

$$G_t + r_t K_{t-1} + w_t L_t + (1 + r_t)B_{t-1} + T_t \le F(K_{t-1}, L_t, s_{t-1}) + B_t. \tag{5}$$

Finally, if C_t^{tot} denotes the total consumption in period t, the economy-wide resource constraint is $G_t + C_t^{tot} + K_t \le F(K_{t-1}, L_t, s_{t-1}) + K_{t-1}$.

3 The island economy

In general, the previous economy features a growing heterogeneity in wealth levels over time, because agents with different idiosyncratic histories will choose different savings. This heterogeneity can be represented by a time-varying distribution of wealth levels with infinite support, which raises considerable theoretical and computational challenges. We now present an environment, in which the savings of each agent depend on the realizations of the idiosyncratic risk for only a given number of consecutive past periods –and not on the whole productivity history. As an endogenous outcome, the heterogeneity among the population is summarized by a finite (but possibly large) number of agents types.

To simplify the exposition, we present this economy using the family and island metaphor (see Lucas 1975 and 1990 for seminal references and Heathcote, Storesletten, and Violante 2016 for a more recent reference). The gain of this presentation strategy is that equilibrium existence can be proved using standard techniques. In Section 4 below, we show that the island allocation can be decentralized.

3.1 The islands

We denote by $N \ge 0$ the length of the truncation for idiosyncratic histories. In other words, we construct an equilibrium in which agents with the same realization of idiosyncratic risk for the last N periods, will choose the same consumption and savings.

Island description. There are E^N different islands, where we recall that the cardinal of the set $\mathcal{E}$ of idiosyncratic risks realizations is E. Agents with the same history for the last N periods are located on the same island. Any island is represented by a vector $e^N = (e_{N-1}, \ldots, e_0) \in \mathcal{E}^N$ summarizing the previous N-period idiosyncratic history of all island inhabitants. At the beginning of each period, agents face a new idiosyncratic shock. Agents with history $\hat{e}^N \in \mathcal{E}^N$ in the previous period, are therefore endowed in the current period with history e^N and we denote $e^N \succeq \hat{e}^N$ whenever e^N is a possible continuation of $\hat{e}^N$. The specification $N = 0$ corresponds to the full insurance case (only one island and one agent type), and thus to the standard representative-agent assumption. Symmetrically, the case $N \to \infty$ corresponds to a standard incomplete market economy with aggregate shocks, à la Krusell and Smith (1997).

The family head. The family head maximizes the welfare of the whole family, attributing an identical weight to all agents. The family head can freely transfer resources among agents within the same island, but cannot do so across islands. All agents belonging to the same island are treated identically and will therefore receive the same allocation, as it is consistent with welfare maximization. The family head will therefore choose in each period the per-capita consumption level c_{t,e^N}, the labor supply l_{t,e^N}, and the end-of-period savings $a_{t,e^N} \geq -\bar{a}$ for agents in each island $e^N \in \mathcal{E}^N$. As both capital and public debt are substitutes, there is no portfolio choice and a_{t,e^N} is simply total wealth.

Note that agents face borrowing constraints, such that individual total asset holdings must be higher than a given constant quantity $-\bar{a}$.[5] Furthermore, some proofs below require that households cannot save more than $a^{\max}$. This maximal amount can be chosen to be arbitrarily large, in particular such that it is never a binding constraint in any equilibrium under consideration.[6] Finally, and without loss of generality, we assume that all households enter the economy with an initial amount of wealth denoted a_{-1}.

Island sizes. The probability $\Pi_{t,\hat{e}^N,e^N}$ that an household with history $\hat{e}^N = (\hat{e}_{-N+1}, \ldots, \hat{e}_0)$ in period t, experiences history $e^N = (e_{-N+1}, \ldots, e_0)$ in period $t+1$ is the probability to switch

[5] See Aiyagari (1994) for a discussion of the relevant values for of $\bar{a}$, called the natural borrowing limit in an economy without aggregate shocks. See Shin (2006) for a discussion with aggregate shocks. A standard value in the literature is $\bar{a} = 0$, which ensures that consumption is positive in all states of the world.

[6] As for instance in Szeidl (2013), the assumption on the maximal bound $a^{\max}$ enables us to consider general utility function. Indeed, an alternative option is to assume that the periodic utility function u is bounded, as in Miao (2006).

from state $\hat{e}_0$ at t to state e_0 at $t+1$, provided that histories $\hat{e}^N$ and e^N are compatible. More formally:

$$\Pi_{t,\hat{e}^N,e^N} = 1_{e^N \succeq \hat{e}^N} M_{\hat{e}_0,e_0}(s_t), \tag{6}$$

where $1_{e^N \succeq \hat{e}^N} = 1$ if e^N is a possible continuation of history $\hat{e}^N$ and 0 otherwise. From the expression (6), we can deduce the low of motion of island sizes $(S_{t,e^N})_{t\geq 0, e^N \in \mathcal{E}^N}$:

$$S_{t+1,e^N} = \sum_{\hat{e}^N \in \mathcal{E}^N} S_{t,\hat{e}^N} \Pi_{t,\hat{e}^N,e^N}, \tag{7}$$

where the initial size of each island $(S_{-1,e^N})_{e^N \in \mathcal{E}^N}$, with $\sum_{e^N \in \mathcal{E}^N} S_{-1,e^N} = 1$, is given. The law of motion (7) is thus valid from period 0 onwards.

Timing. At the beginning of each period, agents move from an island $\tilde{e}^N$ to another island e^N, by taking with them their wealth, equal to the per-capita saving $a_{t-1,\tilde{e}^N}$ on island $\tilde{e}^N$. On island e^N, wealths of all agents coming from island $\tilde{e}^N$, (equal to $S_{t-1,\tilde{e}^N}\Pi_{t-1,\tilde{e}^N,e^N} a_{t-1,\tilde{e}^N}$) –and for all islands $\tilde{e}^N$– are pooled together and then equally divided among the S_{t,e^N} agents of island e^N, such that each of them holds, in the beginning of period t, the wealth $\tilde{a}_{t,e^N}$ amounting to

$$\tilde{a}_{t,e^N} = \sum_{\tilde{e}^N \in \mathcal{E}^N} \frac{S_{t-1,\tilde{e}^N}}{S_{t,e^N}} \Pi_{t-1,\tilde{e}^N,e^N} a_{t-1,\tilde{e}^N}. \tag{8}$$

Let us summarize the timing of events. 1) In the beginning of period t, the agent coming from island $\hat{e}^N$ learns her new individual status e_t and aggregate risk realization s_t. 2) She moves to the island $e^N \succeq \hat{e}^N$ with her individual asset holding $a_{t-1,\hat{e}^N}$. 3) She obtains the beginning-of-period wealth $\tilde{a}_{t,e^N}$ after the wealth pooling operation described in equation (8). 4) The family head then decides upon the common consumption of e^N-island inhabitants c_{t,e^N}, their labor supply l_{t,e^N}, and their savings a_{t,e^N}.

3.2 Program of the family head

The program of the family head can now be expressed as follows:[7]

$$\max_{\left(a_{t,e^N},c_{t,e^N},l_{t,e^N}\right)_{t\geq 0,e^N\in\mathcal{E}^N}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\sum_{e^N\in\mathcal{E}^N} S_{t,e^N} U\left(c_{t,e^N}, l_{t,e^N}\right)\right], \tag{9}$$

$$a_{t,e^N} + c_{t,e^N} = w_t \theta_{e^N} l_{t,e^N} + (1+r_t)\tilde{a}_{t,e^N} + T_t, \text{ for all } e^N \in \mathcal{E}^N, \tag{10}$$

$$c_{t,e^N},\, l_{t,e^N} \geq 0, a_{t+1,e^N} \geq -\bar{a}, \text{ for all } e^N \in \mathcal{E}^N, \tag{11}$$

$$(S_{-1,e^N})_{e^N\in\mathcal{E}^N} \text{ and } (a_{-1,e^N})_{e^N\in\mathcal{E}^N} \text{ are given,} \tag{12}$$

and subject the law of motion (7) for $(S_{t,e^N})_{t\geq 0}^{e^N\in\mathcal{E}^N}$, and to the definition (8) of $(\tilde{a}_{t,e^N})_{t\geq 0}^{e^N\in\mathcal{E}^N}$.[8]

The family head maximizes the aggregate welfare (9) subject to the budget constraints (10) on all islands, to positivity and borrowing constraints (11), and to initial conditions (12). As the objective function is concave, constraints are linear (such that the admissible set is convex), and allocations are constrained to be bounded (the assumption about the existence of $a_{\max}$, guaranteeing a compact admissible set), existence of the equilibrium can be proved using standard techniques, such as the tools of Stokey and Lucas (1989).[9] If $\beta^t \nu_{t,e^N} m(s^t)$ denotes the Lagrange multiplier of the credit constraint $a_{t,e^N} \geq \bar{a}$ on island e^N, the first order conditions are

$$U_c(c_{t,e^N}, l_{t,e^N}) + \nu_{t,e^N} = \beta \mathbb{E}_t \left[\sum_{\tilde{e}^N \succeq e^N} \Pi_{t,e^N,\tilde{e}^N} U_c(c_{t+1,\tilde{e}^N}, l_{t+1,\tilde{e}^N})(1+r_{t+1})\right], \tag{13}$$

$$l_{t,e^N} = \left(w_t \theta_{e^N}\right)^{\varphi}, \tag{14}$$

$$\nu_{t,e^N}(a_{t,e^N} + \bar{a}) = 0 \text{ and } \nu_{t,e^N} \geq 0. \tag{15}$$

The Euler equations (13) and (14), with respect to consumption and labor respectively, have the same flavor as in a Aiyagari-Bewley-Huggett economy, while equation (15) is a standard complementary slackness condition. We discuss these equations more formally in Section 4 below.

[7] To avoid multiplying indices in notations, for any agent on island $e^N = (e_{-N+1}, \ldots, e_0)$, we denote θ_{e^N} her productivity instead of θ_{e_0}.

[8] Note that $\mathbb{E}_t[\cdot]$ in (9) is the expectation operator at date $t \geq 0$ over all future aggregate histories.

[9] Due to the finite heterogeneity representation, we can also prove the existence of a recursive equilibrium. However, we do not present the recursive formulation to save some space.

3.3 Market clearing conditions

Labor market. On island e^N, the labor supply in efficient units at a date t amounts to $\theta_{e^N} S_{t,e^N} l_{t,e^N}$. Summing across all islands yields the total labor supply $L_t = \sum_{e^N \in \mathcal{E}^N} \theta_{e^N} S_{t,e^N} l_{t,e^N}$. Using the law of motion (7) of the island size, the equation (14) determining the individual labor supply l_{t,e^N} and the equation (2) determining the individual wage, we obtain

$$L_t = \sum_{e^N \in \mathcal{E}^N} \theta_{e^N} \sum_{\hat{e}^N \in \mathcal{E}^N} S_{t-1,\hat{e}^N} \Pi_{t-1,\hat{e}^N,e^N} \left(F_L(K_{t-1}, L_t, s_{t-1})\theta_{e^N}\right)^{\varphi}, \tag{16}$$

which shows that L_t does not depend on the current aggregate shock s_t. Together with equation (2), we deduce that the interest rate on capital r_t is determined as date $t-1$, which justifies the unique interest rate for all interest-bearing assets.

Financial market. The total end-of-period savings of all agents, denoted A_t at date t is

$$A_t = \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} a_{t,e^N} = \sum_{e^N \in \mathcal{E}^N} S_{t+1,e^N} \tilde{a}_{t+1,e^N}, \tag{17}$$

where the last equality stems from equation (8) defining the relationship between $(\tilde{a}_{t+1,e^N})_{e^N \in \mathcal{E}^N}$ and $(a_{t,e^N})_{e^N \in \mathcal{E}^N}$. The clearing of the financial market at date t implies that at any date t, the following equality holds

$$A_t = B_t + K_t. \tag{18}$$

3.4 Sequential equilibrium definition

Definition 1 (Sequential equilibrium) *A sequential competitive equilibrium is a collection of individual allocations $\left(c_{t,e^N}, l_{t,e^N}, \tilde{a}_{t,e^N}, a_{t,e^N}\right)_{t\geq 0, e^N \in \mathcal{E}^N}$, of island population sizes $\left(S_{t,e^N}\right)_{t\geq 0, e^N \in \mathcal{E}^N}$, of aggregate quantities $(L_t, A_t, B_t, K_t)_{t\geq 0}$ and of price processes $(w_t, r_t, \tilde{r}_t, \tilde{w}_t)_{t\geq 0}$ such that, for a given fiscal policy $(T_t, \tau^K_{t+1}, \tau^L_t, B_t)_{t\geq 0}$, for an initial distribution of island population and wealth $\left(S_{-1,e^N}, a_{-1,e^N}\right)_{e^N \in \mathcal{E}^N}$, for initial values of the capital stock $K_{-1} = \sum_{e^N \in \mathcal{E}^N} S_{-1,e^N} a_{-1,e^N}$, of the public debt B_{-1}, of the capital tax τ_0, and of the initial aggregate shock s_{-1}, we have:*

1. *given prices, individual strategies $\left(a_{t,e^N}, c_{t,e^N}, l_{t,e^N}\right)_{t\geq 0, e^N \in \mathcal{E}^N}$ solve the agents' optimization program in equations (9)–(12);*

2. *island sizes and beginning of period individual wealth* $\left(\tilde{a}_{t,e^N}, S_{t,e^N}\right)_{t\geq 0, e^N\in\mathcal{E}^N}$ *are consistent with law of motions (7) and (8);*

3. *labor and financial markets clear at all dates: for any* $t \geq 0$, *equations (16)–(18) hold;*

4. *the government budget constraint (5) holds at any date;*

5. *factor prices* $(w_t, r_t, \tilde{r}_t, \tilde{w}_t)_{t\geq 0}$ *are consistent with (2) and (4).*

The equilibrium is a finite-state equilibrium defined at each date by $6E^N + 8$ variables and $6E^N + 8$ equations. The equilibrium is defined for a given fiscal policy $(T_t, \tau_{t+1}^K, \tau_t^L, B_t)_{t\geq 0}$. This representation of incomplete-market economies may be considered as a little bit abstract. The gain of such representation is that first-order conditions are easy to derive and the equilibrium existence can be proved using standard techniques. Before deriving optimal policies in such an environment (i.e., optimizing over $(T_t, \tau_{t+1}^K, \tau_t^L, B_t)_{t\geq 0}$), we show that the previous economy is a consistent representation of the standard incomplete-market economy.

4 Decentralization and convergence properties

We now show that the previous allocation can be decentralized and prove that it converges to the Bewley economy as N increases and under general conditions. We start with given factor prices, and without aggregate shocks. Two main reasons motivate these restrictions. First, dropping aggregate shocks implies that we have existence proof of a recursive representation in this case (see Huggett, 1993). Second, fixing factor prices avoid issues related to equilibrium multiplicity, that may otherwise emerge as proved in Açikgöz (2016). We then discuss the introduction of aggregate shocks. This section can be skipped if the reader is convinced by the relevance of the island economy and wants to directly consider Ramsey policy in this environment.

The economy is similar to the one described in Section 2, except a couple of differences. First, we consider as given an after-tax interest rate r –such that $\beta(1 + r) < 1$– an after-tax wage w, and a constant transfer T. Second, no family head imposes allocations, and agents hold financial assets for standard self-insurance motives. Finally, each agent receives at each date a lump-sum transfer $\Gamma_{N+1}(e^{N+1})$, which is contingent on her individual history $e^{N+1} = (e_N, \ldots, e_1, e)$ over the previous $N + 1$ periods. This last point is the actual difference with a standard incomplete market framework.

Using standard techniques, the program of the agents can be written recursively as[10]

$$V_{N+1}(a, e^{N+1}) = \max_{a',c,l} U(c,l) + \beta\mathbb{E}\left[\sum_{\tilde{e}^N \succeq e^N} \Pi_{e^N,(e^{N+1})'} V_{N+1}(a', \left(e^{N+1}\right)')\right], \tag{19}$$

$$a' + c = w\theta_{e^N} l + (1+r)a + T + \Gamma_{N+1}(e^{N+1}), \tag{20}$$

$$c,\, l \geq 0, a' \geq -\bar{a}, \tag{21}$$

where $V_{N+1} : [-\overline{a}, a^{max}] \times \mathcal{E}^{N+1} \to \mathbb{R}$ is the value function of a given agent. Compared to the economies studied by Huggett (1993) and Aiyagari (1994), the individual history e^{N+1} is an idiosyncratic state variable as it determines the transfer $\Gamma_{N+1}(e^{N+1})$. The Lagrange coefficient of the credit constraint $a' \geq -\bar{a}$ is denoted as ν and the solution of this program consists of the policy rules g_c^{N+1}, $g_{a'}^{N+1}$, g_l^{N+1} and g_ν^{N+1} –defined over $[-\overline{a}, a^{max}] \times \mathcal{E}^{N+1}$– determining respectively consumption, savings, and the Lagrange multiplier of individual budget constraint.

Note that the standard Bewley economy is simply defined as the previous program where we further impose $\Gamma_{N+1}(e^{N+1}) = 0$, for all periods. In this case, we only need the current idiosyncratic state as a state variable (instead of the whole history e^{N+1}). The value function is then denoted $V^{Bewley} : [-\overline{a}, a^{max}] \times \mathcal{E} \to \mathbb{R}$. We can now state our convergence result.

Proposition 1 (Finite state space) *There exists a balanced transfer Γ^*_{N+1}, such that:*

1. *the solution of any agent's program (19)–(21) is characterized by the set of policy functions $(g_{a'}^{*,N}, g_c^{*,N}, g_l^{*,N}, g_\nu^{*,N})$, where each function is defined on $[-\overline{a}, a^{max}] \times \mathcal{E}^N$;*
2. *the equilibrium is fully characterized by only E^N different wealth levels, consumption levels, labor efforts, and Lagrange coefficients that are denoted $(a'_{e^N}, c_{e^N}, l_{e^N}, \nu_{e^N})_{e^N \in \mathcal{E}^N}$;*

[10] Following the literature, we denote as a' the saving choice in the current period. The value a is thus the beginning-of-period wealth.

3. for any $e^N \in \mathcal{E}^N$, the solution $(a'_{e^N}, c_{e^N}, l_{e^N}, \nu_{e^N})$ verifies the first-order conditions:

$$U_c\left(c_{e^N}, l_{e^N}\right) + \nu_{e^N} = \beta\mathbb{E}\left[\sum_{e'\in\mathcal{E}} M_{e,e'} U_c\left(c_{(e^N)'}, l_{(e^N)'}\right)(1+r)\right], \tag{22}$$

$$l_{e^N} = (\chi w \theta_e)^{\varphi}, \tag{23}$$

$$a'_{e^N} + c_{e^N} = w\theta_e l_{e^N} + (1+r_t)\hat{a}_{e^N} + T, \tag{24}$$

$$\hat{a}'_{e^N} = \sum_{\tilde{e}^N \in \mathcal{E}^N} \frac{S_{\tilde{e}^N}}{S'_{e^N}} \Pi_{\tilde{e}^N, e^N} a'_{\tilde{e}^N}, \tag{25}$$

$$\nu_{e^N}(a'_{e^N} + \bar{a}) = 0 \text{ and } \nu_{e^N} \geq 0. \tag{26}$$

The previous proposition shows that there exists a transfer such that the conditions (22)–(15) are exactly the same as the ones of the program of the family head, given by (13)–(15) and the pooling equation (8) written in a recursive form (Item 3 of the Proposition). The wealth $\hat{a}'_{e^N}$ in equation (25) that results from the transfer Γ^*_{N+1},– that could also be defined as $\hat{a}' = a + \Gamma^*_{N+1}$– has the same expression as the after-pooling wealth in the island economy in equation (8). In particular, although $N+1$ periods are needed to construct the transfer Γ^*_{N+1}, the after-transfer wealth only depends on the last N periods. The transfer therefore reduces wealth heterogeneity and implies that there are only have E^N different wealth levels in each period (Item 2). The transfer consists in pooling resources of all agents having the same idiosyncratic history for $N+1$ periods, and redistributes the same amount to agents having the same idiosyncratic history for N periods. The formal expression of the specific transfer Γ^*_{N+1} is given in the proof in Appendix A. Because of this limited heterogeneity, policy rules only depend on the last N –and not $N+1$– previous shocks (Item 1), which explains why they are denoted $g^{*,N}$. As a consequence, for given factor prices and when the transfer is set to Γ^*_{N+1}, the solution of the program of the family head (9) is the same as the solution of the decentralized program (19). We can now state the main proposition of this section.

Proposition 2 (Convergence of the transfer) *For given factor prices and for the transfer Γ^*_{N+1}, if there exists $\kappa \in (0,1)$ and $\overline{N} \geq 1$, such that for all $N \geq \overline{N}$, $e^N \in \mathcal{E}^N$ and $a_1, a_2 \in [-\bar{a}, a^{max}]$:*

$$\left|\tilde{g}^N_{a'}(a_1, e^N) - \tilde{g}^N_{a'}(a_2, e^N)\right| < \kappa\left|a_1 - a_2\right|, \tag{27}$$

then $\lim_{N\to\infty} |\Gamma^*_{N+1}| = 0$, *and for all* a *and* $e^N = (e_{-N+1}, \ldots, e_1, e_0)$*:*

$$\lim_{N\to\infty} V_{N+1}(a, e^{N+1}) = V^{Bewley}(a, e_0).$$

Condition (27) states that if we increase the beginning-of-period wealth of agents having the same history e^N (for $N \geq \overline{N}$) by an amount $\triangle a$, then these agents will always save less than $\triangle a$. In other words, the marginal propensity to save is always smaller than 1 for all agents, as soon as N is high enough. If this condition is fulfilled, then the transfer tends toward 0 as the length of idiosyncratic history N increases. Indeed, if the saving propensity is strictly lower than one, initial differences in wealth vanish and agents experiencing the same history of idiosyncratic shocks end up having the same wealth as time goes by. As a consequence, the wealth pooling generated by the transfer Γ^*_{N+1} concerns wealth levels which tend to be closer to each other, and the transfer Γ^*_{N+1} tends toward 0. In this case, it is then easy to show that the value function of the truncated economy converges toward the value function of the Bewley economy, which only depends on the current idiosyncratic shock.

From Proposition 2, we deduce that the truncated economy will be a consistent representation of the corresponding Bewley economy (i.e., without any transfers Γ^{*N+1}), if the propensity to save is always strictly less than one for all agents in the Bewley economy. In this case, one can show that the property about saving propensity is shared by all truncated economies whenever N is high enough N. To our knowledge, all calibrated Bewley models found in the literature share this property, such that one can be confident about the general relevance of this truncated representation of incomplete-market economies.

Introducing aggregate shocks. In the economy with aggregate shocks, the limit of the truncated economy can be proved to exist, but it is difficult to compare this limit with other models, such as the one of Krusell and Smith (1998) for instance, as, to our knowledge, there is no proof of the existence of a recursive representation for incomplete market economies with aggregate shocks (see Kubler and Schmedders 2002 and 2003, and Miao 2006 for some contributions). The current construction of a truncated economy could be the foundation of such a proof, but we let this possibility for future research.[11]

[11] In this section, we achieved decentralization through a fiscal transfer Γ_{N+1}, but this is not the only option. Indeed, following the constructions of Alvarez et al. (2009) and Khan and Thomas (2015), it would be possible

5 Optimal fiscal policy: The Ramsey problem

5.1 The Ramsey problem

We now solve the Ramsey program in our incomplete market island-economy with aggregate shocks. The Ramsey program consists for the government to choose a sequence of lump-sum taxes and of distorting taxes on capital and labor as well as a path of public debt levels that maximize the aggregate welfare. This aggregate welfare computed using an utilitarian perspective is simply the objective of the family head in equation (9). The government assumes that individual agents behave rationally. It also faces constraints on the economy-wide resources, as well as on its budget. In other words, the government has to select, subject to several constraints, the competitive equilibrium associated to the largest aggregate welfare. The following definition formalizes this statement, following the notations of Section 3.

Definition 2 (Ramsey program for a truncated economy) *Let $N > 0$. Given initial conditions about the wealth distribution $\left(S_{-1,e^N}, a_{-1,e^N}\right)_{e^N \in \mathcal{E}^N}$, the initial public debt B_{-1}, the initial capital tax τ_0^K, and the initial aggregate state s_{-1}, the Ramsey program consists in choosing, at date 0, a fiscal policy made of lump-sum, capital, and labor tax paths $(T_t, \tau_{t+1}^K, \tau_t^L)_{t\geq 0}$, and of public debt paths $(B_t)_{t\geq 0}$, that maximize the aggregate welfare defined in (9) among the set of competitive equilibria characterized in Definition 1.*

Note that since the initial capital tax rate is given, the tax rate path starts at date 1. Equation (4) implies that the government can equivalently decide the post-tax interest rate $(r_t)_{t\geq 1}$ and the post-tax wage rate $(w_t)_{t\geq 0}$ instead of the distorting taxes $(\tau_t^K)_{t\geq 1}$ and $(\tau_t^L)_{t\geq 0}$.

to provide a sequential decentralization of the island economy. Indeed, islands are devices to pool income at each date t (and as such provide insurance) for idiosyncratic risks occurring before date $t-(N+1)$. As a consequence, if all agents are ex ante identical, it would possible to achieve decentralization using insurance or financial contracts perfectly hedging at any date t the risks occurring before date $t-(N+1)$.

As a consequence, we can formalize the Ramsey program as follows:

$$\max_{\left(T_t, r_{t+1}, w_t, B_t, (a_{t,e^N}, c_{t,e^N}, l_{t,e^N})_{e^N \in \mathcal{E}^N}\right)_{t\geq 0}} \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} U(c_{t,e^N}, l_{t,e^N}) \right], \tag{28}$$

$$B_t + F(K_{t-1}, L_t, s_{t-1}) \geq G_t + (1+r_t)B_{t-1} + r_t K_{t-1} + w_t L_t + T_t, \tag{29}$$

$$\text{for all } e^N \in \mathcal{E}^N: \tag{30}$$

$$a_{t,e^N} + c_{t,e^N} + T_t = w_t \theta_{e^N} l_{t,e^N} + (1+r_t)\tilde{a}_{t,e^N}, \tag{31}$$

$$U_c\left(c_{t,e^N}, l_{t,e^N}\right) + \nu_{t,e^N} = \beta \mathbb{E}_t \left[\sum_{\tilde{e}^N \in \mathcal{E}^N} \Pi_{t+1,e^N,\tilde{e}^N} U_c\left(c_{t+1,\tilde{e}^N}, l_{t+1,\tilde{e}^N}\right)(1+r_{t+1}) \right], \tag{32}$$

$$l_{t,e^N} = (\chi w_t \theta_{e^N})^{\varphi}, \tag{33}$$

$$\nu_{t,e^N}(a_{t,e^N} + \overline{a}) = 0, \tag{34}$$

$$A_t = \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} a_{t,e^N}, \; L_t = \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} l_{t,e^N}, \; K_t = A_t - B_t, \tag{35}$$

$$c_{t,e^N}, l_{t,e^N}, (a_{t,e^N} + \overline{a}) \geq 0. \tag{36}$$

with the law of motion (7) of $(S_{t,e^N})_{t\geq 0, e^N \in \mathcal{E}^N}$, and the definition (8) of $(\tilde{a}_{t,e^N})_{t\geq 0, e^N\in\mathcal{E}^N}$. All constraints (29)–(36) should be understood, unless specified, for all $s^t \in \mathcal{S}^t$ and all $e^N \in \mathcal{E}^N$.[12]

The objective (28) corresponds to the maximization of the aggregate welfare, which amounts to the addition of individual welfares among islands. Maximization devices are on one hand individual quantities: consumption level, labor effort, and asset holdings and on the other hand fiscal instruments: public debt, lump-sum taxes and post-tax interest and wage rates. Equation (29) is the government budget constraint, while the individual budget constraint is given in equation (31). The individual Euler equations for consumption and labor are provided in equations (32) and (33), respectively. The Lagrange multiplier ν_{t,e^N} appears in the slackness condition (34), stating that either the borrowing constraint multiplier ν_{t,e^N} on island e^N is null, or the borrowing constraint for island e^N binds: $a_{t,e^N} = -\overline{a}$. Equation (35) gathers the aggregation for individual wealth and labor supply, as well as the financial market clearing. Finally, positivity and borrowing constraints appear in equation (36).

[12] $\mathbb{E}_t[\cdot]$ is the conditional expectation at date t with respect to aggregate shocks.

5.2 Simplification of the Ramsey program

In this section, we simplify the formulation of the Ramsey program exposed in equations (28)–(36), following Marcet and Marimon (2011). We first denote λ_{t,e^N} the discounted Lagrange multiplier of the Euler equation of agent e^N in state s^t, normalized by the size of the island and the probability to be in any state s_t in period t. Hence, the period 0 Lagrange coefficient of period t Euler equation for the state s_t is $\beta^t m^t(s^t) S_{t,e^N} \lambda_{t,e^N}$. We also define for all $e^N \in \mathcal{E}^N$:

$$\Lambda_{t,e^N} = \frac{\sum_{\hat{e}^N \in \mathcal{E}^N} S_{t-1,\hat{e}^N} \lambda_{t-1,\hat{e}^N} \Pi_{t,\hat{e}^N,e^N}}{S_{t,e^N}}, \tag{37}$$

which can be interpreted as the average for agents of island e^N of their previous period Lagrange multiplier for the Euler equation. Indeed, in equation (37), the quantities $\lambda_{t-1,\hat{e}^N}$ for any $\hat{e}^N \in \mathcal{E}^N$ are weighted by the size of population transiting from state $\hat{e}^N$ to e^N. Finally, we can notice that we have $\lambda_{t,e^N} = 0$ if $a_{t,e^N} = \overline{a}$. We can therefore drop the product of Lagrange multipliers $\lambda_{t,e^N} \nu_{t,e^N}$ (for any t and any e^N) that is actually always equal to 0. It should indeed be noted that λ_{t,e^N} has a different interpretation of the Lagrange multiplier ν_{t,e^N} on credit constraint: it is null when the credit-constraint is binding and nonzero when agents actually save or borrow. The following lemma summarizes our simplification of the Ramsey program.

Lemma 1 (Simplified Ramsey program) *The Ramsey program in equations (28)–(36) can be simplified into:*

$$\max_{\left(r_{t+1},w_t,B_t,T_t,(a_{t,e^N},c_{t,e^N},l_{t,e^N})_{e^N\in\mathcal{E}^N}\right)_{t\geq 0}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \Big(U(c_{t,e^N}, l_{t,e^N}) \tag{38}$$

$$+ U_c(c_{t,e^N}, l_{t,e^N}) \Big(\Lambda_{t,e^N}(1+r_t) - \lambda_{t,e^N} \Big) \Big),$$

$$s.t.\ \lambda_{t,e^N} = 0 \textit{ if } a_{t,e^N} = \overline{a}, \tag{39}$$

and subject to equations (7), (8), (29)–(33), (35)–(36), and (37).

The proof is relegated in Appendix C.[13] This technical simplification is based on a re-writing of the Lagrangian to introduce Lagrange coefficients into the objective, as done by Marcet and Marimon (2011) in a first reformulation of the program, toward a recursive formulation. This

[13] Açikgöz (2015) uses the same methodology to find a recursive formulation in a model without aggregate shocks. Our proof carefully considers the effect of truncation on Lagrange multipliers.

considerably simplifies the solution of the maximization problem. To save some space, we do not provide such a recursive formulation here, as the sequential representation allows us to simply derive first-order conditions.

6 Understanding fiscal policy

6.1 First-order conditions

An understanding of optimal fiscal policy can be obtained from the first-order conditions of the program (38), which are necessary conditions. These conditions relate three different concepts, which are three different valuations of liquidity. The first one is the *social valuation of government liquidity*, defined as a marginal valuation of additional amount of goods for the government. It is measured by the (normalized) Lagrange multiplier of the government budget constraint (29), denoted as μ_t in period t. The second valuation of liquidity is the marginal value of one unit of goods for agents e^N in period t when saving incentives are internalized. This value, called the *social valuation of liquidity for agents* e^N and denoted ψ_{t,e^N}, is formally defined as

$$\psi_{t,e^N} = U_c(c_{t,e^N}, l_{t,e^N}) - U_{cc}(c_{t,e^N}, l_{t,e^N}) \left(\lambda_{t,e^N} - \Lambda_{t,e^N}(1 + r_t)\right). \tag{40}$$

This social valuation of liquidity for agents e^N, ψ_{t,e^N}, is thus different from the marginal utility of consumption $U_c(c_{t,e^N}, l_{t,e^N})$, which is the third valuation of liquidity called *the private valuation of liquidity for agents* e^N. Indeed, the valuation ψ_{t,e^N} takes into consideration the Euler equations from periods $t-1$ to t and from periods t to $t+1$. An extra consumption unit makes the agent more willing to smooth out her consumption between period t to $t+1$ and thus her Euler equation more "binding". This more "binding" constraint decreases the utility by the algebraic quantity $U_{cc}(c_{t,e^N}, l_{t,e^N})\lambda_{t,e^N}$, where λ_{t,e^N} is the Lagrange multiplier of the agent's Euler equation at date t. The extra consumption unit at t also makes the agents less willing to smooth her consumption at date $t-1$ and therefore "relaxes" the date $t-1$ constraint. This is reflected by the quantity Λ_{t,e^N} which is, for an agent at date t on island e^N, the average Lagrange multipliers of the Euler equation at date $t-1$, when the agent was on other islands.

It is easy to show that if the government could implement island-specific lump-sum transfers (such as unconstrained T_{t,e^N}), it would implement $\mu_t = \psi_{t,e^N}$ for all $e^N \in \mathcal{E}^N$. As a consequence, the difference $\mu_t - \psi_{t,e^N}$ is a measure of the cost for island e^N of imperfect and distorting policy tools. We will call the difference $\mu_t - \psi_{t,e^N}$ the *liquidity-valuation gap for agents* e^N, as it is equal to the marginal gain of transferring resources from the budget of island e^N to the budget of the government. Note that the liquidity-valuation gap $\mu_t - \psi_{t,e^N}$ can be either positive or negative depending on the island, but, as shown below, the sum of social values over all islands is nonnegative.

We now present and discuss the first-order conditions of the planner using these concepts. These conditions define the joint dynamics of the *liquidity valuation gaps* $\mu_t - \psi_{t,e^N}$ and the *valuation of government liquidity,* μ_t. We derive them formally in Appendix D.

The dynamics of the social valuation of government liquidity, μ_t. We begin with the first-order condition related to the social valuation of government liquidity μ_t :

$$\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1} \left(1 + \tilde{r}_{t+1} \right) \right]. \tag{41}$$

Equation (41) sets equal the marginal benefit of one additional extra unit of debt at date t to the marginal extra cost at date $t+1$, using the before-tax return $\tilde{r}_t$ to value the next period. On the one hand, the extra debt unit relaxes the government budget constraint at date t by one unit and thus implies a benefit that amounts to the Lagrange multiplier of the government budget constraint, μ_t. On the other hand, the extra debt unit implies debt reimbursement and interest payment in the next payment, i.e. a total payment of $1 + \tilde{r}_{t+1} = 1 + F_K(A_t - B_t, L_{t+1})$ that makes next period government budget constraint stricter.

Dynamics of the liquidity-valuation gaps, $\mu_t - \psi_{t,e^N}$. We begin with defining $\mathcal{C}_t$ as the set of islands on which agents are credit-constrained at date t. Formally:

$$\mathcal{C}_t = \{ e^N \in \mathcal{E}^N, \ \lambda_{t,e^N} = 0 \}. \tag{42}$$

Then, for all islands in which credit constraints do not bind, the first-order condition for savings yields

$$\forall e^N \in \mathcal{E}^N \setminus \mathcal{C}_t,\ \mu_t - \psi_{t,e^N} = \beta \mathbb{E}_t \left[\sum_{\tilde{e}^N \in \mathcal{E}^N} (1+r_{t+1}) \Pi_{t+1,e^N,\tilde{e}^N} \left(\mu_{t+1} - \psi_{t+1,\tilde{e}^N} \right) \right], \tag{43}$$

Equation (41) can be interpreted as a *modified* Euler equation for agents, who are not credit-constrained. It equalizes the current *liquidity-valuation gap* $\mu_t - \psi_{t,e^N}$ to its discounted value tomorrow. It is noteworthy that the Euler equation for the *liquidity-valuation gap* is similar to the Euler equation for the *private valuation of liquidity* for the same agents (equation 32). Both the agents and the planner perceive that the marginal gain to transfer resources to the next period is r_t.

Choice of labor taxes. The first-order condition for the post-tax real wage w_t can be written as

$$\sum_{e^N \in \mathcal{E}^N} \frac{S_{t,e^N} l_{t,e^N} \theta_{e^N}}{L_t} \left(\mu_t - \psi_{t,e^N} \right) = \mu_t \varphi \left(\frac{1}{1-\tau_t^L} - 1 \right). \tag{44}$$

Equation (44) sets equal the social gain of financing the government budget using labor tax τ_t^L, which is a weighted sum of all liquidity valuation gaps (the left hand side) to its cost (the right hand side). More precisely, the left hand side is the marginal gain of transferring resources for all islands $e^N \in \mathcal{E}^N$ to the budget of the government using an increase in labor tax τ_t^L. For every island e^N, this implies a liquidity valuation gap $\mu_t - \psi_{t,e^N}$ and each island is weighted by its share in total labor effort $\frac{S_{t,e^N} l_{t,e^N} \theta_{e^N}}{L_t}$, expressed in efficient units. For every island, this weight is thus proportional to the labor tax base. The right hand side is the cost of labor tax distortion, which mainly implies a reduction in the base of the labor tax. The magnitude of distortions depend positively on the fiscal wedge generated by labor tax τ_t^L, the Frisch elasticity of labor supply φ –which determines how agents adapt their labor effort to the tax distortion–, and on the government liquidity valuation μ_t.

Choice of the tax on capital. The first-order condition, corresponding to the post-tax interest rate r_t, provides insight on the equilibrium tax on capital. It can be written as:

$$\sum_{e^N \in \mathcal{E}^N} \left[S_{t,e^N} \tilde{a}_{t,e^N} \left(\mu_t - \psi_{t,e^N} \right) \right] = \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} U_c(c_{t,e^N}, l_{t,e^N}) \Lambda_{t,e^N}. \tag{45}$$

where $\tilde{a}_{t,e^N}$ is given by (8). Equation (45) sets equal the social gain of financing the government budget using the distorting capital tax τ_t^K, which is again a weighted sum of all liquidity-valuation gaps (the left hand side) to its cost (the right hand side). More precisely, the left hand side is the average liquidity valuation gap weighted by the before-tax wealth on each island, which is the tax base. The right hand side is the sum of individual distortions of a higher capital tax that affects individual Euler equations, and more precisely, consumption smoothing between the previous period and today. Therefore, individual distortions are measured by Λ_{t,e^N}, which assesses the tightness of Euler equations between the previous period and today, and thus the willingness to smooth out consumption between both periods.

Choice of the lump-sum transfer T_t. The first-order condition relative to the lump-sum transfer T_t is

$$\sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \left(\mu_t - \psi_{t,e^N} \right) \geq 0, \text{ with equality when } T_t > 0. \tag{46}$$

This equation states that, when the positivity constraint on the lump-sum transfer is not binding, the government sets the population-weighted sum of liquidity valuation gaps to 0. In particular, this would be also the case in absence of any positivity constraint. However, when the constraint $T_t \geq 0$ is binding and when the government would actually like to tax some island using the lump-sum instrument, the constraint $T_t \geq 0$ binds, and the social benefit of liquidity for the government is higher than its average cost over all islands: $\mu_t > \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \psi_{t,e^N}$.[14]

6.2 Steady-state fiscal policy

Using first-order conditions, we can derive theoretical implications about the steady-state optimal fiscal policy. Main results are summarized in the next proposition.[15]

Proposition 3 (steady-state) *In the steady state of the Ramsey equilibrium,*

[14] Allowing for negative transfers does not imply that the constraint (46) holds with equality. Indeed, the ability of the lowest income agents to pay lump-sum taxes may provide a binding bound on negative taxes. It this bound does not bind, it is easy to show that full insurance can be implemented, using only lump-sum taxes to pay for G and for the interest payment on the amount of public debt which allows for full insurance. This is the result of Woodford (1990) in a simpler environment.

[15] To denote steady-state variables, we simply drop the subscript t.

1. *the marginal productivity of capital is pinned down by the discount factor β:*

$$1 + F_K(K, L) = \frac{1}{\beta}, \tag{47}$$

2. *the capital tax is non-negative; it is positive if and only if credit-constraints bind for some agents, or more formally:*

$$\tau^K = \frac{\sum_{e^N \in \mathcal{C}} S_{e^N} \nu_{e^N}}{\beta \sum_{e^N \in \mathcal{E}^N} S_{e^N} U_c\left(c_{e^N}, l_{e^N}\right)}, \tag{48}$$

where we recall that $\mathcal{C}$ is the set of islands where credit constraints bind at the steady state, and ν_{e^N} is the Lagrange multiplier of credit constraint for island e^N;

3. *the labor tax is determined by the net average liquidity valuation gap:*

$$\mu\varphi \frac{\tau^L}{1 - \tau^L} = \sum_{e^N \in \mathcal{E}^N} \frac{S_{e^N} l_{e^N} \theta_{e^N}}{L} \left(\mu - \psi_{e^N}\right). \tag{49}$$

The first item in equation (47) of Proposition 3 is a direct implication of the government Euler equation (41). As a consequence, the marginal productivity of capital is determined by the discount factor β only, as was originally explained by Aiyagari (1995).

To the best of our knowledge, the second item in Proposition 3 is new in the literature. We prove that the capital tax is always nonnegative and that its value is determined by the severity of credit constraints. If credit constraints do not bind for any agent –for instance if they are chosen to be below the natural borrowing limit, as defined by Aiyagari (1994)–, then the equilibrium capital tax[16] will be 0. Conversely, the steady-state capital tax will be positive if and only if some agents are credit-constrained. Indeed, when credit constraints are binding for some agents, on the one hand, credit constrained agents cannot borrow as much as they would like to, while on the other hand, non-constrained agents save too much to self-insure. Both effects contribute to create an oversupply of liquidity. To correct this oversupply, the government raises the capital tax, which decreases the private interest rate and thus the incentives to save.[17]

[16] The denominator in equation (49) is indeed bounded away from zero, as the economy is finite, i.e. not all marginal utilities can be zero.

[17] This logic is already present in Woodford (1990), and discussed in Davila et al. (2012). The exercise in this last paper differs from ours, because the authors analyze the optimality of the capital stock in a situation where the planner can change agents' saving decisions without distortion, while we specifically focus on distorting fiscal

The third item in equation (49) of the proposition is a restatement of (44) in steady state, which determines the labor tax rate as a function of distortions and of the average liquidity valuation gap.

These three relationships in equations (47)–(49) determine the marginal product of capital, as well as labor and capital taxes as a function of the heterogeneous valuation of liquidity across islands. The level of public debt is then determined as a residual of the government budget constraint. Public debt can be either positive or negative, and one of the main factor affecting it is the Frisch elasticity of the labor supply. As can be seen from equation (49) and confirmed in numerical simulations, when labor supply is very elastic, the labor tax is very small. Then, if agents are patient, they will hold a large quantity of savings for precautionary motives and the credit constraint will rarely be binding in equilibrium. From equality (48) in Proposition 3, we know that capital taxes will also be low. In consequence, the government can hardly rely on taxation to finance public spending and it must therefore accumulate resources to eventually finance public spending out of interest payment. Public debt is therefore very likely to be negative in that case. On the opposite, if labor supply is inelastic, labor taxes will be high and the government will be able to debt finance interest payment using taxation. Public debt will then be positive and used by agents to self-insure.

7 Quantitative investigation of the optimal tax system

We now simulate the model using standard parameter values, to study quantitative properties of the optimal fiscal policies.

7.1 Parametrization

The period is a year. The discount factor is set equal to $\beta = 0.96$ as Heathcote (2005) to obtain an annual discount rate close to 4%. The curvature of the utility function is set to $\sigma = 2$ (Hall

2010), while the Frisch elasticity of labor supply is set to $\phi = 0.5$, which is consistent with empirical estimates (Chetty, Guren, Manoli, and Weber (2011)). We nevertheless provide sensitivity analysis for these last two parameters as many different values can be found in the

instruments.

literature. The scaling parameters of labor supply is $\chi = 1$ as a benchmark and we use as an adjustment factor for matching the same steady state in comparative statics.

The aggregate state is assumed to follow an AR(1) process $s_t = \rho_s s_{t-1} + \varepsilon_t^s$, where the innovation $(\varepsilon_t^s)_{t\geq 0}$ is a white-noise process with a normal distribution, $\mathcal{N}(0, \sigma_s^2)$. The production function has a Cobb-Douglas functional form with a constant capital depreciation δ:

$$F(K, L, s) = A(s)K^{\alpha}L^{1-\alpha} - \delta K,$$

where $A(s) = \exp(s)$ is the technology level. The quantity α is the capital share, which is set to 0.36, while δ is the annual depreciation rate, set to 10% (Heathcote, 2005). Public consumption is assumed to be constant and matches the average ratio of public spending over GDP in the US for the period 2000-2016, which is 19%. This implies $G = 0.30$. The autocorrelation of the technology shock ρ_s is 0.81, which corresponds to a standard quarterly correlation of 0.95 for the TFP shock, while the standard deviation is $\sigma_s = 0.01$. Table 1 summarizes the calibration of model parameters.

N	β	ϕ	σ	χ	δ	α	G	ρ_s	σ_s
4	0.96	0.5	2	1	0.1	0.36	0.30	0.815	0.01

Table 1: Parameter calibration

For the labor process, we use the specification of Domeij and Heathcote (2004), which is a simple three-state process defined to reproduce an AR(1) wage process with an annual autocorrelation of 0.90 and a standard deviation for the innovation equal to 0.224. These two values correspond to their US empirical counterparts, estimated on PSID data. The three levels for the hourly productivity are respectively $e_1 = 0.213$, $e_2 = 0.848$, and $e_3 = 3.940$ and the corresponding transition matrix, which is constant, is:[18]

$$M = \begin{bmatrix} 0.9 & 0.099 & 0.001 \\ 0.006 & 0.988 & 0.006 \\ 0.001 & 0.099 & 0.001 \end{bmatrix}$$

[18] Compared to Domeij and Heathcote (2004), we introduce a small probability to switch from states 1 to 3, and from states 3 to 1. This does not change the properties of the AR(1) process, but it speeds up the algorithm, as all islands are visited.

Choice of N. Quantitative investigation of the property of the model suggests that an economy featuring $N = 4$, and thus $3^4 = 81$ different agents, is a good benchmark. Indeed, we have also solved the model for $N = 6$ (729 agents) and checked that steady-states were quantitatively similar. It turns out that in the economy with $N = 6$, the tax on capital amounts to 10%, compared to 11% when $N = 4$, while the labor tax is 37%, compared to 36% for $N = 4$. These differences in steady state tax values between economies with $N = 4$ and $N = 6$ are one order of magnitude lower than the tax changes implied by reasonable variations in preference parameters, on which there is no strong consensus. We are thus confident that an economy with $N = 4$ captures the key properties of the dynamics of the fiscal system in our model.

Solution method. Our truncated equilibrium enables us to solve the model using a standard perturbation method. First, we start with determining the steady-state allocation without aggregate shocks, using an iteration over the post-tax interest rate, as described in Appendix E. Second, we linearize all equations around the steady-state (including first-order conditions of the Ramsey problem), to obtain the dynamics of the fiscal system and constrained-optimal allocation after a small technology shock. The technology shock is small enough such that, for all simulations, credit constrained islands remain credit constrained, while no other island becomes credit constrained along the business cycle.

For $N = 4$, the dynamic system is composed of 521 variables (including all Lagrange coefficients), which is simulated with Dynare in 7 seconds. Gains of the perturbation method are twofold. First, we can choose a continuous state-space for the technology shock. Second, we can use the same mature methodology as the one used in the DSGE literature to compute IRFs and second moments. For future works, this means that we can rely on tools (such as Dynare) that have already been developed and make our results directly comparable with a large branch of the literature.

7.2 Steady state results and sensitivity analysis

Steady state output. We start with describing the steady-state allocation and fiscal system. As shown theoretically in Proposition 3, the pre-tax interest rate $\tilde{r}$ is pinned-down by the discount factor β. Since $\tilde{r} = F_K(K, L)$, this implies that the capital-output ratio is $K/Y = 2.5$, and that the equilibrium pre-tax wage $\tilde{w} = F_L(K, L) = 1.1$. The labor supply and total output are then easy to derive as there is no wealth effect due to GHH utility function.

The optimal capital tax amounts to $\tau^K = 11\%$, while the labor tax is $\tau^L = 36\%$. The debt-to-GDP ratio amounts to 148%. Although different from actual taxes in the US (reported to be $\tau^K_{US} = 39.7\%$ and $\tau^L_{US} = 26.9\%$, by Domeij and Heathcote, 2004), these values are not very far away from their empirical counterparts. In addition, public debt is high, but does not reach unrealistically high or negative values.

How do our results compare with the literature studying the optimal seady-state debt level? Aiyagari and McGrattan (1998) find an optimal debt over GDP of 60%, maximizing steady-state welfare. Açikgöz (2015) shows that results are quantitatively different if one solves for the Ramsey program instead of maximizing steady-state welfare. He finds an optimal quantity of debt above 300% of GDP. Dyrda and Pedroni (2016) find that a negative optimal debt-to-GDP ratio, below -300% of GDP. As is shown in sensitivity analysis below, these substantial quantitative differences can partly be explained by parameter choices, such as the elasticity of labor supply or the income process, for which there is no strong consensus.

Concerning the capital tax, it has been shown in Proposition 3, that it is directly related to the severity of credit constraints. In steady state, only 4% of agents are credit-constrained and all of them have the smaller productivity level e_1. This is sufficient to induce a positive capital tax, though quite small. Indeed, more than 90 of the government steady-state revenues comes from labor taxes. Table 2 summarizes steady-state quantities.

$\tilde{r}(\%)$	$\tilde{w}$	$\tau^K(\%)$	$\tau^L(\%)$	$B/Y(\%)$	$K/Y(\%)$	L	Y
4.2	1.1	11	36	148	2.5	0.9	1.6

Table 2: Steady-state outcome

Sensitivity analysis. We perform a sensitivity analysis for some key parameters. For all experiments, we recalibrate the labor supply parameter χ to obtain the same public spending-to-GDP ratio. All results are gathered in Table 3. The first line of the table recalls the model outcome for the benchmark calibration. The second line increases the Frisch elasticity of the labor supply to 0.75. As expected, labor tax decreases, while capital tax increases to balance the budget of the government. Public debt decreases, due to the increased distortions implied by raising capital tax. Note that if we further increase the Frisch elasticity up to 2, we obtain a negative optimal public debt (not reported in Table 3), as we explained in Section 6.2. The third line of Table 3 presents the result when the concavity of the utility function decreases from 2 to 1. In this case, agents save more and the "severity" of credit constraint decreases, as the concavity of the utility function is lower. The capital tax decreases to 5% and the labor tax increases to 39%. Public debt slightly increases as total savings increase (while the capital stock remains barely unaffected).

	Parameters				Output		
	ϕ	σ	χ	N	$\tau^K(\%)$	$\tau^L(\%)$	$B/Y(\%)$
Benchmark calibration	0.50	2	1.00	4	11	36	148
Change in ϕ	0.75	2	0.90	4	18	32	103
Change in σ	0.50	1	1.03	4	5	39	155

Table 3: Sensitivity analysis

7.3 Dynamic properties

We now investigate the dynamic properties of the optimal fiscal system after a technology shock. Before providing second-order moments, we first present impulse response functions.

Figure 1 provides aggregate outcome after an unexpected increase of 1% in TFP. All variables are provided as proportional deviations from steady state, except the tax rates τ_t^L, τ_t^K, which are given in level deviations from steady state.

The first panel in Figure 1 provide the TFP shock. With our timing convention, agents learn at the moment of the shock (i.e., in period 1) that TFP will increase in the next period, and adjust their investment accordingly. GDP increases in period 2 (as it is predetermined at the moment of the shock), by 1.2%. Aggregate consumption (Ctot) increases on impact to almost 1% and then decreases smoothly toward its steady-state value.

The second line in Figure 1 provides the dynamics of financial variables. The capital stock (K) increases progressively, with a maximal increase below 1%, whereas the aggregate savings of private agents (A) increases rapidly, by 1.25% at the maximum. As a consequence, public debt (B) increases (up to 2%) to absorb excess savings. The private sector saves too much

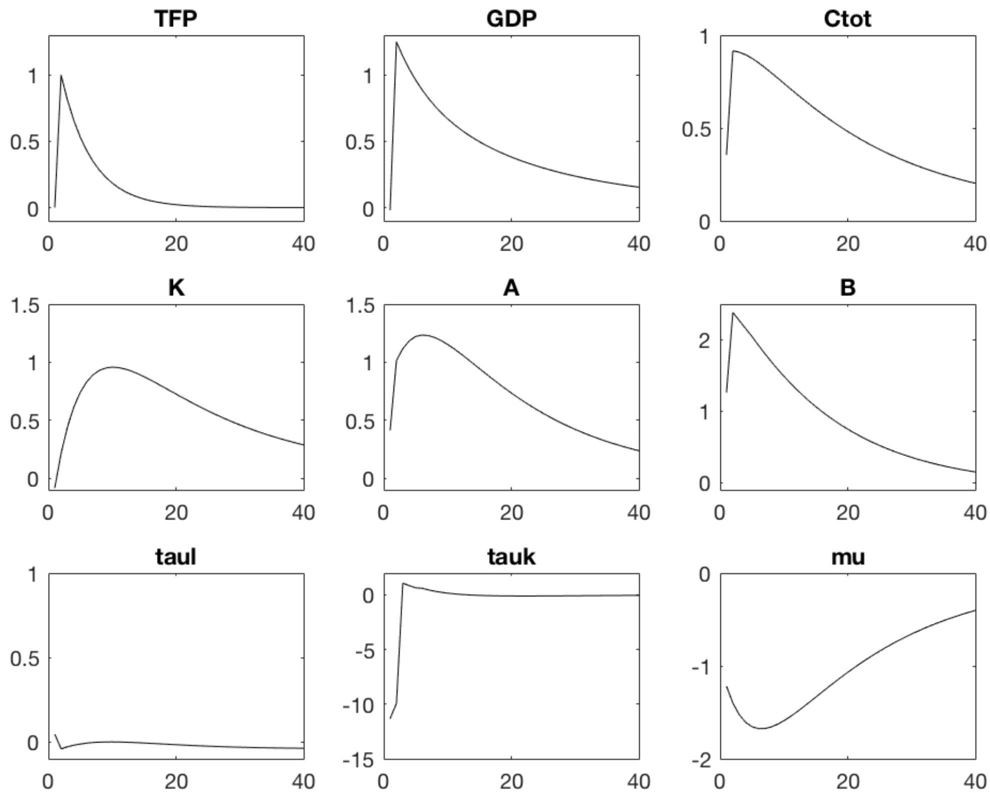

Figure 1: Aggregate IRF after a 1% increase in TFP

after a technology shock. Indeed, wage inequality increases after such a shock (as the real wage increases), which generates additional incentives to save for high-productivity agents to smooth out consumption in case of switching to low productivity. The government absorbs these excess savings by an increase in public debt –and not by an increase in capital tax as we explain below.

The last line plots the labor and capital tax rates. The labor tax (taul) decreases a little bit after the technology shock, while the capital tax (tauk) decreases a lot for two periods before converging rapidly back to its steady-state value. Cutting capital taxes at the impact enables the government to make all agents with savings wealthier, and thus triggers an immediate increase in aggregate consumption (in fact for all agents having non-zero savings). The government prefers to use capital taxes rather than labor taxes, since public debt enables the government to absorb excess savings generated by tax cuts. There is no such equivalent tool for smoothing out the distortions implied by labor taxes. After two periods, the effect of the TFP shock are fully reflected in wages and interest rates and the capital tax returns rapidly to its steady state value. Capital taxes are therefore much more volatile than labor taxes. This result is also found in representative-agent model (see Farhi, 2010 for a discussion), as it does not crucially depend on agents' heterogeneity. Finally, the technology shock relaxes the budget constraint of the government, because the economy is wealthier. This can be seen by the decrease in the Lagrange multiplier of the budget of the government (mu).

Table 4 provides unconditional second-order moments generated by the simulated model. As consistent with our analysis of IRFs, the volatility of the labor tax is low and the volatilities of capital tax and of public debt are high compared to output. Public debt is volatile because it reverts slowly to its mean. Concerning autocorrelations, optimal fiscal policy adds some persistence to output, as its autocorrelation (0.93) is higher than the one of the exogenous process for TFP (0.81). The persistence of capital tax is much lower than the one of labor taxes and public debt, as observed discussing IRFs.

Standard deviations			
$sd(Y)$	$sd(\tau^K)$	$sd(\tau^L)$	$sd(B)$
0.06	0.15	0.003	0.17

Autocorrelations			
$corr(Y, Y_{-1})$	$corr(\tau^K, \tau^K_{-1})$	$corr(\tau^L, \tau^L_{-1})$	$corr(B, B_{-1})$
0.93	0.45	0.94	0.97

Table 4: Second-order moments

The investigation of the quantitative properties of the model shows that some results are robust to reasonable parameter values variations. This is the case of the high volatility of capital tax, the slow mean-reversion dynamics of public debt, and the range of capital and labor taxes. Other results, such as the steady-state level of public debt, are not precisely pinned down in this class of model at this stage, as already discussed by Aiyagari and McGrattan (1998).

8 Concluding remarks

We have proved that the competitive equilibrium in an incomplete insurance-market economy with aggregate shocks can be represented as the allocation of a family-head program. The gain of this representation is that it generates a finite-dimensional state-space equilibrium, in which the Ramsey outcome can be studied with aggregate shocks and with various fiscal tools. We apply this framework to study optimal fiscal policy, when positive transfer, distorting taxes on capital and labor, and public debt are available.

The methodology presented in this paper could be applied in different settings. First, additional heterogeneity, such as the structure of qualification, could be introduced to use more extensively empirical estimates of key parameters. Second, we could also include other distortions, such as limited participation, search-and-matching frictions on the labor market or nominal frictions, which are all extensively used in macroeconomic models. The tools developed in the present paper, based on the dynamic structure of Lagrange coefficients, could allow us to derive new theoretical and quantitative results for optimal monetary policy.

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Appendix

A Proof of Proposition 1

Consider an agent endowed with the $N+1$-period history $e^{N+1} = (\hat{e}^N, e) \in \mathcal{E}^{N+1}$ that can be written as $e^{N+1} = (e_N, e^N)$. In the former notation, e^{N+1} is seen as the history $\tilde{e}^N$ with the successor state e, while in the latter notation, e^{N+1} is seen as the state e_N followed by history e^N. The agent also pays a lump-sum transfer $\Gamma^N_{e^{N+1}}(z)$, depending on a set z of aggregate state variables, that we specify below.

The solution to the maximization program (19)–(21) are the policy rules denoted $c = g_c(a, e^{N+1})$, $a' = g_{a'}(a, e^{N+1})$, $l = g_l(a, e^{N+1})$ and the multiplier $\tilde{\eta}$ satisfying the following first order conditions:

$$\xi_{e^N} u'\left(c - \frac{l^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right) U_c\left(c,l\right) + \nu = \beta \mathbb{E}\left[\sum_{e' \in \mathcal{E}} M_{e,e'} \xi_{e'^N} \left(c' - \frac{l'^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}\right)(1+r')\right], \tag{50}$$

$$l = (w\theta_e)^{\varphi}, \text{ if } e > 0, \tag{51}$$

$$l = \delta \text{ if } e = 0, \tag{52}$$

$$\nu(a' + \bar{a}) = 0 \text{ and } \eta \geq 0. \tag{53}$$

We can now construct the transfer scheme following a guess-and-verify strategy. The transfer is constructed such that all agents with the same individual history over the last N periods will have the same after-transfer wealth. Denoting the measure of agents with history e^N the previous period as $S_{e^N,-1}$. The current measure of agents with history e^N is

$$S_{e^N} = \sum_{\hat{e}^N \in \mathcal{E}^N} S_{\hat{e}^N,-1} \Pi_{\hat{e}^N, e^N}, \tag{54}$$

This is the parallel of the law of motion (7) in the island economy. If we assume that all agents with the same history $\hat{e}^N$ have the same beginning-of-period wealth $a_{\hat{e}^N}$, the average welfare (before transfer) of agents with history $e^N \succeq \hat{e}^N$ will be

$$\tilde{a}_{e^N} = \sum_{\tilde{e}^N \in \mathcal{E}^N} \frac{S_{\hat{e}^N,-1}}{S_{e^N}} \Pi_{\hat{e}^N, e^N} a_{\hat{e}^N}, \tag{55}$$

$\tilde{a}_{e^N}$ is the average wealth of agents having history $\hat{e}^N$, the previous period with $e^N \succeq \hat{e}^N$. By construction, $\tilde{a}_{e^N}$ is similar to the after-pooling wealth $\tilde{a}_{e^N}$ of equation (8) in the island economy.

The transfer that we define as

$$\Gamma^{*N+1}(e^{N+1}) = (1+r)\left(\tilde{a}_{e^N} - a_{\tilde{e}^N}\right) \tag{56}$$

swaps the beginning-of-period wealth $(1+r)a_{\tilde{e}^N}$ by the average wealth $(1+r)\tilde{a}_{e^N}$. The idea is that all agents with current history e^N will have an identical after-transfer wealth, independently of the previous-period history $\hat{e}^N$.

It is easy to checked that the transfer scheme is balanced in each period. In addition, all agents take the transfer Γ^{*N+1} as given. Indeed, there is a continuum with mass $S_{\tilde{e}^N}$ of agents with history $\tilde{e}^N$, in which each individual agent is atomistic. As a consequence, an individual agent has no effect on the aggregate wealth of agents with history $\tilde{e}^N$ and therefore no effect on the transfer Γ^{*N+1}.

The impact of transfer on agents' wealth. We consider the impact of transfer $\Gamma^{*N+1}(e^{N+1})$ for an agent with history $e^{N+1} = (\hat{e}^N, e) = (e_N, e^N)$ and beginning-of-period wealth $a_{\hat{e}^N}$. Her budget constraint (20) becomes using transfer expression (56):

$$a' + c + T = w\theta_e l + \delta 1_{e=0} + (1+r)\tilde{a}_{e^N}, \tag{57}$$

which in fact only depends on the N-period history e^N. Equation (19) implies that agents with the same history e^N have the same expected continuation utility. Therefore, agents with the same e^N behave similarly: they consume the same level, they supply the same labor quantity, and hold the same wealth. The solution to the maximization program (19)–(21) are policy rules $c = \tilde{g}_c^N(a, e^N)$, $a' = \tilde{g}_{a'}^N(a, e^N)$, $l = \tilde{g}_l^N(a, e^N)$ and $\nu = \tilde{g}_\nu^N(a, e^N)$, that only depend on the N-period history e^N. This policy rules are solution to (50- (53). This concludes the proof.

B Proof of Proposition

Proposition 4 (Convergence of the transfer) *For given factor prices and for the transfer Γ^{*N+1}, if there exists $\kappa \in (0,1)$ and $\overline{N} \geq 1$, such that for all $N > \overline{N}$, $e^N \in \mathcal{E}^N$ and $a_1, a_2 \in [-\overline{a}, a^{max}]$:*

$$\left|\tilde{g}_{a'}^N(a_1, e^N) - \tilde{g}_{a'}^N(a_2, e^N)\right| < \kappa\left|a_1 - a_2\right|, \tag{58}$$

then $\lim_{N\to\infty} |\Gamma^{*N+1}| = 0$*, and*

$$\lim_{N\to\infty} V_{N+1}(a, e^{N+1}) = V^{Bewley}(a, e),$$

We denote by $Conv(A)$ the convex hull of the set $A \subset \mathbb{R}$, and μ the Lebesgue measure on $\mathbb{R}$. We start with the following lemma.

Lemma 2 (contraction lemma) *Assume that $A \subset [-\bar{a}, a^{max}]$, and that conditions of Propo-*

sition 2) are fulfilled. For $(e_N, \ldots, e_0) \in \mathcal{E}^{N+1}$*, define*

$$B = \left\{ \tilde{g}_{a'}^N(a, (e_{N-1}, \ldots, e_0)), a \in A \right\}$$

then $\mu\left(Conv(B)\right) \leq \kappa \times \mu\left(Conv(A)\right)$.

The proof is obvious as g^{N+1} is a Lipschitz function with coefficient κ: The measure of image of a set is less than κ the measure of the set. The general proof is now done in two steps.

B.1 Step 1

For any $N-$economy, in any period, for $e^N \in \mathcal{E}^N$, define for $1 \leq k \leq N$

$$A_k^{(N)}(e^N) = \{a_{f^N} | f^N \in \mathcal{E}^N, (f_{N-1}, ..f_{N-k}) = (e_{N-1}, ..e_{N-k})\}$$

In words, $A_k^{(N)}(e^N)$ is the set of beginning-of-period wealth values of all agents having a k period history of idiosyncratic shock $(e_{N-1}, \ldots, e_{N-1-(k-1)})$, starting N period ago and finishing $N-k$ periods ago. Note that

$$A_{N-1}^{(N)}(e^N) = \{a_{(e_{N-1}, \ldots, e_1, f)} | f \in \mathcal{E}\} \tag{59}$$

is the set of beginning-of-period wealth levels of agents who can have the history $e^N \in \mathcal{E}^N$ in the current period, after the idiosyncratic shock is revealed. As a consequence, if $e^N \succeq \tilde{e}^N$, then $a_{\tilde{e}^N} \in A_{N-1}^{(N)}(e^N)$.

We have

$$A_{k+1}^{(N)}(e^N) \subset \{\tilde{g}_{a'}^N(a, (e_{N-1}, \ldots, e_{N-1-(k-1)}, e_{N-k+1}, f_{N-k}, ..., f_0)) \\ |(f_{N-k}, ..f_0) \in \mathcal{E}^{N-k+1}, a \in A_k^{(N)}(e^N)\}.$$

In words, the set $(A_{k+1}^{(N)}(e^N))$ of wealth values of all agents having a $k+1$ period history of idiosyncratic shock $(e_{N-1}, \ldots, e_{N-k+1})$ (starting N period ago and finishing $N-k+1$ periods ago) is the set of set of wealth values chosen by all agents having a k period history of idiosyncratic shock $(e_{N-1}, \ldots, e_{N-k})$ (starting N period ago and finishing $N-k$ periods ago) experiencing a value e_{N-k+1} of their idiosyncratic shock $N-k+1$ periods ago.

For $\bar{N} \leq k < N$, we deduce, applying Lemma (2)

$$\mu\left(Conv(A_{k+1}^{(N)}(e^N))\right) \leq \kappa\mu\left(Conv(A_k^{(N)}(e^N))\right)$$

Iterating from $k = \bar{N}$ to $k = N-1$, one finds

$$\mu\left(Conv(A_{N-1}^{(N)}(e^N))\right) \leq \kappa^{N-1-\bar{N}}\mu\left(Conv(A_{\bar{N}}^{(N)}(e^N))\right) \leq \kappa^{N-\bar{N}-1}[a^{max} - \bar{a}]$$

The last inequality is obtained as $[a^{max} - \bar{a}]$ is the highest set of possible wealth levels.

B.2 Step 2

It is easy to show that $\tilde{a}(e^N) \in Conv(A_{N-1}^{(N)}(e^N))$. In words, the post-tax beggining-of-periods wealth of agents having an history e^N is an average of the before-tax wealth of agents having history e^N. Indeed, recall that

$$\tilde{a}_{e^N} = \sum_{\tilde{e}^N \in \mathcal{E}^N} \frac{S_{\tilde{e}^N,-1}}{S'_{e^N}} \Pi\left(\tilde{e}^N, e^N\right) a_{\tilde{e}^N}, \text{ for all } e^N. \tag{60}$$

Using the definition of $\Pi\left(\tilde{e}^N, e^N\right)$ given in (6), $\tilde{a}_{e^N}$ can be written as

$$\tilde{a}_{e^N} = \sum_{f \in \mathcal{E}} \frac{S_{(f,e_{N-1},\ldots,e_1),-1}}{S_{(e_{N-1},\ldots,e_0)}} M_{e_1,e_0} a_{(f,e_{N-1},\ldots,e_1)}, \text{ for all } e^N \in \mathcal{E}^N \tag{61}$$

Using (7), one finds $\sum_{f \in \mathcal{E}} \frac{S((f,e_{N-1},\ldots,e_1))}{S'((e_{N-1},\ldots,e_0))} M_{e_1,e_0}(s_t) = 1$ for all $e^N \in \mathcal{E}^N$. As a consequence, the expression (7) shows that $\tilde{a}_{e^N}$ is a weighted sum (with positive weights summing to one), of elements of $A_{N-1}^{(N)}(e^N)$, which is defined by (59). As a consequence, $\tilde{a}_{e^N} \in Conv(A_{N-1}^{(N)}(e^N))$.

In addition whatever $\tilde{e}^N \in \mathcal{E}^N$, $a_{\tilde{e}^N} \in A_{N-1}^{(N)}(e^N) \subset Conv(A_{N-1}^{(N)}(e^N))$, if $e^N \succeq \tilde{e}^N$, as a consequence

$$|\tilde{a}_{e^N} - a_{\tilde{e}^N}| \leq \mu\left(Conv(A_{N-1}^{(N)}(e^N))\right) \leq \kappa^{N-\bar{N}-1}[a^{max} - \bar{a}]$$

From the previous inequality and the definition (56), we deduce, that

$$\lim_{N\to\infty}|\Gamma^{*N+1}| = 0$$

In words, the transfer can me made uniformly arbitrarily close to 0 when N increases.

B.3 Convergence of the value function

The convergence of the value function is now easy to prove. Indeed, for all $\epsilon > 0$, there exists a $\bar{N}$ such that for all $N \geq \bar{N}$, such that $|\Gamma^{*N+1}| < \epsilon$. Define two distorted value functions for the original Bewley economy.

$$V^{(+\epsilon)}(a,e) = \max_{a',c,l} U(c,l) + \beta\mathbb{E}\left[\sum_{\tilde{e}^N \succeq e^N} M_{e,e'} V_{N+1}(a',e')\right], \tag{62}$$

$$a' + c = w\theta_e l + (1+r)a + T + \epsilon, \tag{63}$$

$$c,\, l \geq 0, a' \geq -\bar{a}. \tag{64}$$

Similarly, $V^{(-\epsilon)}(a,e)$ is the value function where the budget constraint is diminished by $-\epsilon$ (Technically, ϵ has to be low enough for the resource to be positive for all agents, what is possible as $\theta_e > 0$).

Using standard dynamic programming arguments with bounded returns (see Stokey, Lucas, and Prescott 1989 Section 9.2), one has for all $a \in [a^{max}; \bar{a}]$, $e \in \mathcal{E}$, then $V^{(+\epsilon)}(a,e) \leq V^{Bewley}(a,e) \leq V^{(-\epsilon)}(a,e)$ and $V^{(+\epsilon)}(a,e) \leq V_{N+1}(a, e^{N+1}) \leq V^{(-\epsilon)}(a,e)$. The last inequality coming from the boundary of the transfers Γ^{*N+1}. As a consequence, for all $a \in [a^{max}; \bar{a}]$, $e \in \mathcal{E}$

$$\left|V^{Bewley}(a,e) - V_{N+1}(a, e^{N+1})\right| \leq \left|V^{(+\epsilon)}(a,e) - V^{(-\epsilon)}(a,e)\right|$$

It is then easy to show that the right hand can be made arbitrarily small as ϵ tends toward 0, as the set $[a^{max}; \bar{a}] \times \mathcal{E}$ is compact and $V^{(+\epsilon)}$ is continuous in ϵ.

C Proof of Lemma 1

There are two steps in our simplification –that follows Marcet and Marimon (2011) used by Açikgöz (2015). First, we substitute some expressions: after-pooling savings $\tilde{a}$ using (8), capital using the financial market clearing equation (at any t, $K_t + B_t = A_t$). Second, we express the Lagrangian and include the individual consumption Euler equation (32) and credit constraints (34).

The Ramsey program (28)–(36) can be rewritten as

$$
\begin{aligned}
\max_{\left(r_t, w_t, B_t, (a_{t,e^N}, c_{t,e^N}, l_{t,e^N})_{e^N \in \mathcal{E}^N}\right)} \quad & \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} U(c_{t,e^N}, l_{t,e^N}) && (65)\\
& -\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \sum_{e \in \mathcal{E}^N} S_{t,e^N} \lambda_{t,e^N} \Big(U_c(c_{t,e^N}, l_{t,e^N}) + \nu_{t,e^N} \\
& \qquad -\beta \mathbb{E}_t \Big[\sum_{\hat{e}^N \in \mathcal{E}^N} \Pi_{t+1,e^N,\hat{e}^N} U_c(c_{t+1,\hat{e}^N}, l_{t+1,\hat{e}^N})(1 + r_{t+1}) \Big] \Big) \\
\text{s.t.} \qquad B_t + F(A_{t-1} - B_{t-1}, L_t) &= G_t + (1 + r_t) B_{t-1} && (66)\\
& \quad + r_t (A_{t-1} - B_{t-1}) + w_t L_t \\
a_{t,e^N} + c_{t,e^N} &= w_t n_{e^N} l_{t,e^N} && (67)\\
& \quad + (1 + r_t) \sum_{\hat{e} \in \mathcal{E}^N} \frac{S_{t-1,\hat{e}^N}}{S_{t,e^N}} \Pi_{t,\hat{e}^N,e^N} a_{t-1,\hat{e}^N} \\
l_{t,e^N} &= (w_t n_{e_t^N})^{\varphi}, && (68)\\
\lambda_{t,e^N} (a_{t,e^N} + \overline{a}) &= 0 \quad e^N \in \mathcal{C} && (69)
\end{aligned}
$$

The Ramsey problem simplifies into maximizing the expression (65) subject to the government budget constraint (66) and individual budget constraints (67) –that should hold for any $e^N \in \mathcal{E}^N$. Equality (68) is the Euler equation determining the individual labor supply.

The proof boils now down to simplifying the following expression

$$
\begin{aligned}
J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} U(c_{t,e^N}, l_{t,e^N}) \\
-\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \lambda_{t,e^N} \Big(U_c \left(c_{t,e^N}, l_{t,e^N} \right) + \nu_{t,e^N} \\
-\beta \mathbb{E}_t \left[\sum_{\hat{e}^N \in \mathcal{E}^N} \Pi_{t+1,e^N,\hat{e}^N} U_c \left(c_{t+1,\hat{e}^N}, l_{t+1,\hat{e}^N} \right) (1 + r_{t+1}) \right] \Big)
\end{aligned}
$$

Distributing the sum in λ_{t,e^N}, we obtain:

$$
\begin{aligned}
J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \Big(U(c_{t,e^N}, l_{t,e^N}) - \lambda_{t,e^N} U_c(c_{t,e^N}, l_{t,e^N}) \Big) \\
+\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^{t+1} \sum_{e^N \in \mathcal{E}^N} \sum_{\hat{e}^N \in \mathcal{E}^N} \Big(S_{t,e^N} \lambda_{t,e^N} \Pi_{t+1,e^N,\hat{e}^N} \Big) U_c(c_{t+1,\hat{e}^N}, l_{t+1,\hat{e}^N})(1 + r_{t+1})
\end{aligned}
$$

or after an harmless renormalization

$$
\begin{aligned}
J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \Big(U(c_{t,e^N}, l_{t,e^N}) - \lambda_{t,e^N} U_c(c_{t,e^N}, l_{t,e^N}) \Big) \\
+\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^{t+1} \sum_{e^N \in \mathcal{E}^N} \sum_{\hat{e}^N \in \mathcal{E}^N} \Big(S_{t,\hat{e}^N} \lambda_{t,\hat{e}^N} \Pi_{t+1,\hat{e}^N,e^N} \Big) U_c(c_{t+1,e^N}, l_{t+1,e^N})(1 + r_{t+1})
\end{aligned}
$$

We define

$$
\Lambda_{t,e^N} = \frac{\sum_{\hat{e}^N \in \mathcal{E}^N} S_{t-1,\hat{e}^N} \lambda_{t-1,\hat{e}^N} \Pi_{t,\hat{e}^N,e^N}}{S_{t,e^N}},
$$

which implies

$$
\begin{aligned}
J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \Big(U(c_{t,e^N}, l_{t,e^N}) - \lambda_{t,e^N} U_c(c_{t,e^N}, l_{t,e^N}) \Big) \\
+\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^{t+1} \sum_{e^N \in \mathcal{E}^N} S_{t+1,e^N} \Lambda_{t+1,e^N} U_c(c_{t+1,e^N}, l_{t+1,e^N})(1 + r_{t+1})
\end{aligned}
$$

Remarking that $\Lambda_{0,e^N} = 0$, we have

$$J = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \Big(U(c_{t,e^N}, l_{t,e^N}) \\ + \Big(\Lambda_{t,e^N}(1+r_t) - \lambda_{t,e^N} \Big) U_c(c_{t,e^N}, l_{t,e^N}) \Big)$$

D Derivation of first-order conditions for the Ramsey program

We compute the first-order conditions of the simplified Ramsey program (38)–(39). Denoting $\beta^t m^t(s^t)\mu_t$ the Lagrange multiplier associated to the government budget constraint, we obtain the following expression for the Lagrangian:

$$\begin{aligned}\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \sum_{e^N \in \mathcal{E}^N} & S_{t,e^N} \Big(U(c_{t,e^N}, l_{t,e^N}) \\ & + U_c(c_{t,e^N}, l_{t,e^N}) \Big(\Lambda_{t,e^N}(1+r_t) - \lambda_{t,e^N} \Big) \Big) \\ & - \mu_t \beta^t \left(G_t + B_{t-1} + r_t A_{t-1} + w_t L_t - B_t - F(A_{t-1} - B_{t-1}, L_t) \right),\end{aligned} \tag{70}$$

where $c_{t,e^N} = (w_t n_{e_t^N}) l_{t,e^N} - a_{t,e^N} + (1+r_t) \sum_{\hat{e}^N \in \mathcal{E}^N} \frac{S_{t-1,\hat{e}^N}}{S_{t,e^N}} \Pi_{t,\hat{e}^N,e^N} a_{t-1,\hat{e}^N}$ and $l_{t,e^N} = (w_t n_{e_t^N})^{\varphi}$.

We use the ψ notation defined in (40) that we recall here for the sake of simplicity

$$\psi_{t,e^N} = U_c(c_{t,e^N}, l_{t,e^N}) + U_{cc}(c_{t,e^N}, l_{t,e^N}) \Big(\Lambda_{t,e^N}(1+r_t) - \lambda_{t,e^N} \Big) \tag{71}$$

Derivative with respect to a_{t,e^N}. We compute the derivative of the Lagrangian (70) wrt a_{t,e^N}. Note that c_{t,e^N} obviously depends on a_{t,e^N} but this is also the case of any $c_{t+1,\tilde{e}^N}$ where $s^{t+1} \succeq s^t$ and $\tilde{e}^N \in \mathcal{E}^N$. We obtain for all $e^N \in \mathcal{E}^N \setminus \mathcal{C}$:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial a_{t,e^N}} = & \beta^t S_{t,e^N} \frac{\partial c_{t,e^N}}{\partial a_{t,e^N}} \Big(U_c(c_{t,e^N}, (w_t n_{e_t^N})^{\varphi}) + U_{cc}(c_{t,e^N}, (w_t n_{e_t^N})^{\varphi}) \Big(\Lambda_{t,e^N}(1+r_t) - \lambda_{t,e^N} \Big) \Big) \\ & + \mathbb{E}_t \beta^{t+1} \sum_{\tilde{e}^N \in \mathcal{E}^N} S_{t+1,\tilde{e}^N} \frac{\partial c_{t+1,\tilde{e}^N}}{\partial a_{t,e^N}} \\ & \times \Big(U_c(c_{t+1,e^N}, (w_{t+1} n_{e_{t+1}^N})^{\varphi}) + U_{cc}(c_{t+1,e^N}, (w_{t+1} n_{e_{t+1}^N})^{\varphi} \Big(\Lambda_{t+1,\tilde{e}^N}(1+r_{t+1}) - \lambda_{t+1,\tilde{e}^N} \Big) \Big) \\ & - \mathbb{E}_t \beta^{t+1} S_{t,e^N} \mu_{t+1} (r_{t+1} - F_K(A_t - B_t, L_{t+1})),\end{aligned}$$

where we remind that $A_t = \sum_{e \in \mathcal{E}^N} S_{t,e^N} a_{t,e^N}$.

Noting that

$$\begin{aligned}
\frac{\partial c_{t,e^N}}{\partial a_{t,e^N}} &= -1, \\
\frac{\partial c_{t+1,\tilde{e}^N}}{\partial a_{t,e^N}} &= (1 + r_{t+1}) \frac{S_{t,e^N}}{S_{t+1,\tilde{e}^N}} \Pi_{t+1,e^N,\tilde{e}^N},
\end{aligned}$$

we deduce that $\frac{\partial \mathcal{L}}{\partial a_{t,e^N}} = 0$ iff, using the ψ notation in (71), for all $e^N \in \mathcal{E}^N \setminus \mathcal{C}$:

$$\begin{aligned}
\psi_{t,e^N} = {} & \beta \mathbb{E}_t \sum_{\tilde{e}^N \in \mathcal{E}^N} (1 + r_{t+1}) \Pi_{t+1,e^N,\tilde{e}^N} \psi_{t+1,\tilde{e}^N} \\
& - \beta \mathbb{E}_t \mu_{t+1} (r_{t+1} - F_K(A_t - B_t, L_{t+1})),
\end{aligned}$$

which is the modified Euler equation.

Derivative with respect to B_t. Computing the derivative of the Lagrangian (70) with respect to B_t yields

$$\frac{\partial \mathcal{L}}{\partial B_t} = \beta^t \mu_t - \beta^{t+1} \mathbb{E}_t \left[\mu_{t+1} \left(F_K(A_t - B_t, L_{t+1}) + 1 \right) \right]$$

We deduce the Euler equation for μ_t

$$\mu_t = \beta \mathbb{E}_t \left[\mu_{t+1} \left(F_K(A_t - B_t, L_{t+1}) + 1 \right) \right]. \tag{72}$$

Derivative with respect to r_t. Computing the derivative of the Lagrangian (70) with respect to r_t yields

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial r_t} = {} & \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} U_c(c_{t,e^N}, l_{t,e^N}) \Lambda_{t,e^N} \\
& + \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \frac{\partial c_{t,e^N}}{\partial r_t} \left(U_c(c_{t,e^N}, l_{t,e^N}) + U_{cc}(c_{t,e^N}, l_{t,e^N}) \left(\Lambda_{t,e^N}(1 + r_t) - \lambda_{t,e^N} \right) \right) \\
& - \mu_t \beta^t A_{t-1}
\end{aligned}$$

Noting that

$$\frac{\partial c_{t,e^N}}{\partial r_t} = \sum_{\hat{e}^N \in \mathcal{E}^N} \frac{S_{t-1,\hat{e}^N}}{S_{t,e^N}} \Pi_{t,\hat{e}^N,e^N} a_{t-1,\hat{e}^N},$$

we obtain that $\frac{\partial \mathcal{L}}{\partial r_t} = 0$ iff:

$$\begin{aligned}
\mu_t A_{t-1} = &\sum_{e \in \mathcal{E}^N} S_{t,e^N} U_c(c_{t,e^N}, l_{t,e^N}) \Lambda_{t,e^N} \\
&+ \sum_{e^N \in \mathcal{E}^N} \left(\sum_{\hat{e}^N \in \mathcal{E}^N} S_{t-1,\hat{e}^N} \Pi_{t,\hat{e}^N,e^N} a_{t-1,\hat{e}^N} \right) \psi_{t,e^N}
\end{aligned}$$

Derivative with respect to w_t. Computing the derivative of the Lagrangian (70) with respect to w_t yields:

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial w_t} = &\beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \left(\frac{\partial c_{t,e^N}}{\partial w_t} \left(U_c(c_{t,e^N}, l_{t,e^N}) + U_{cc}(c_{t,e^N}, l_{t,e^N}) \left(\Lambda_{t,e^N}(1+r_t) - \lambda_{t,e^N} \right) \right) \right) \\
&+ \beta^t \sum_{e^N \in \mathcal{E}^N} S_{t,e^N} \left(\frac{\partial l_{t,e^N}}{\partial w_t} \left(U_l(c_{t,e^N}, l_{t,e^N}) + U_{cl}(c_{t,e^N}, l_{t,e^N}) \left(\Lambda_{t,e^N}(1+r_t) - \lambda_{t,e^N} \right) \right) \right) \\
&- \beta^t \mu_t \left(L_t + w_t \frac{\partial L_t}{\partial w_t} - F_L(K_{t-1}, L_t) \frac{\partial L_t}{\partial w_t} \right)
\end{aligned}$$

Noting that $c_{t,e^N} = (w_t n_{e_t^N}) l_{t,e^N} - a_{t,e^N} + (1+r_t) \sum_{\hat{e}^N \in \mathcal{E}^N} \frac{S_{t-1,\hat{e}^N}}{S_{t,e^N}} \Pi_{t,\hat{e}^N,e^N} a_{t-1,\hat{e}^N}$ and $l_{t,e^N} = (w_t n_{e_t^N})^{\varphi}$.

$$\begin{aligned}
\frac{\partial c_{t,e^N}}{\partial w_t} &= (\varphi+1)(w_t n_{e_t^N})^{\varphi} n_{e_t^N}, \\
\frac{\partial l_{t,e^N}}{\partial w_t} &= \frac{\varphi}{w_t} (w_t n_{e_t^N})^{\varphi}, \\
U_l(c_{t,e^N}, l_{t,e^N}) &= -(w_t n_{e_t^N}) U_c(c_{t,e^N}, l_{t,e^N}) \\
U_{cl}(c_{t,e^N}, l_{t,e^N}) &= -(w_t n_{e_t^N}) U_{cc}(c_{t,e^N}, l_{t,e^N}) \\
\frac{\partial L_t}{\partial w_t} &= \frac{\varphi}{w_t} \sum_{e^N \in \mathcal{E}^N} n_{e_t^N} S_{t,e^N} l_{t,e^N} = \frac{\varphi}{w_t} L_t
\end{aligned}$$

We obtain

$$\mu_t\left(L_t+\varphi L_t-\varphi\frac{F_L(K_{t-1},L_t)}{w_t}L_t\right)=\frac{\varphi+1}{w_t}\sum_{e^N\in\mathcal{E}^N}S_{t,e^N}(w_tn_{e_t^N})^{\varphi+1}\psi_{t,e_t^N}$$
$$-\frac{\varphi}{w_t}\sum_{e^N\in\mathcal{E}^N}S_{t,e^N}(w_tn_{e_t^N})^{\varphi+1}\psi_{t,e_t^N}$$

and

$$\mu_tL_t\left(1+\varphi-\frac{\varphi}{w_t}F_L(K_{t-1},L_t)\right)=\sum_{e^N\in\mathcal{E}^N}S_{t,e^N}n_{e_t^N}(w_tn_{e_t^N})^{\varphi}\psi_{t,e^N}.$$

Additional derivative with respect to T_t.

$$\frac{\partial\mathcal{L}}{\partial T_t}=\beta^t\sum_{e^N\in\mathcal{E}^N}S_{t,e^N}\left(U_c(c_{t,e^N},(w_tn_{e_t^N})^{\varphi})+U_{cc}(c_{t,e^N},(w_tn_{e_t^N})^{\varphi})\left(\Lambda_{t,e^N}(1+r_t)-\lambda_{t,e^N}\right)\right)$$
$$-\beta^t\mu_t$$

Thus

$$\mu_t=\sum_{e^N\in\mathcal{E}^N}S_{t,e^N}\left(U_c(c_{t,e^N},(w_tn_{e_t^N})^{\varphi})+U_{cc}(c_{t,e^N},(w_tn_{e_t^N})^{\varphi})\left(\Lambda_{t,e^N}(1+r_t)-\lambda_{t,e^N}\right)\right)$$

Thus if

$$\mu_t>\sum_{e^N\in\mathcal{E}^N}S_{t,e^N}\psi_{t,e^N}$$

The government would choose a negative lump-sum taxes.

E Algorithm to solve the model

General method. For a given N, first rank the islands, according to their idiosynratic history. For instance, for island $e^t=\{e_{N-1},...,e_0\}\in\mathcal{E}^N$, $k=1+\sum_{i=0}^{N-1}(e_i-1)E^{N-1-i}$.

With this ranking we suspect that islands with a low k are more likely to be credit constrained.

1. For any given N we guess the islands which face binding credit constraint.
2. We write a code finding the steady-values of the Ramsey program (described below). We

verify that the set of islands facing credit constraints in 1. is correct.

3. We write a code which writes the set of dynamics equations in Dynare.

4. We use Dynare solver to double check the steady state and to simulate the model.

5. We check that the aggregate shock is small enough for the set of credit-constrained islands not to change.

Finding the steady-state. We now describe in more details the code to find the steady sate (step 2 of the general method).

1. Guess the set S^c of islands which are credit constrained, ans islands S^{nc} which are not.

2. Consider a post-tax interest rate r and take a value of T (a benchmark guess is $T = 0$)

   (a) Solve for labor supply for each island $k = 1...E^N$, using FOC of the households. Deduce aggregate Labor L

   (b) Derive aggregate capital K from $F_K(K, L) = \beta^{-1} - 1$.

   (c) Consider a w.

      i. Find consumption and assets using budget constraints, Euler equation and the definition of $\tilde{a}_k$, $k = 1, \ldots, E^N$. Public spending is given by $G = F(K, L) - rA - wL - T$.

      ii. Consider a value of μ. Guess values ψ_k, $k \in S^c$, then solve for ψ_k, $k \in S^{nc}$, using equations (43). Solve for $\lambda_k, k = 1, \ldots, E^N$. Check that λ_k are consistent with ψ_k, $k \in S^c$, otherwise iterate on ψ_k, $k \in S^c$.

      iii. Iterate over μ using (45).

   (d) Iterate over w using (44).

3. Iterate over r until G/Y has the targeted value. Check that the value of T is correct, otherwise iterate.

4. Check the Euler inequalities on each island to verify that the set S^c is indeed correct.

The whole procedure 3 takes less than 5 minutes.