Term structure of risk in macrofinance models

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Abstract

I propose a model-based approach to characterize the term structures of risk in cash flows and asset prices. I relax cross-equation restrictions in structural models and estimate the implied dynamics of macro fundamentals and asset prices. I use shock elasticities to characterize how risk propagates in asset prices. To account for time variation in the risk premium, I extend the theory of dynamic value decomposition (Hansen, 2012) to nonnormal shocks. I find that the leading models of time-varying risk premium fall short to account for the shape and level of the term structure of risk in equity returns and cash flows.

Keywords: term structure, shock elasticity, stochastic volatility, disaster risk, stochastic jump intensity

PRELIMINARY AND INCOMPLETE

1 Introduction

A great deal of recent asset pricing literature focuses on understanding the term structure of risk in various asset classes. Asset prices reflect how macroeconomic risk, such as monetary, fiscal shocks, or technological progress, is perceived by risk-averse investors. The great informational advantage of asset markets is the presence of observable prices of claims on similar cash flows (e.g., a dollar, dividend, inflation, volatility, etc) but with different maturities. Naturally, latest attempts to make a progress in understanding the link between asset prices and fundamental risk (e.g., Binsbergen, Brandt, and Koijen, 2012, Binsbergen, Hueskes, Koijen, and Vrugt, 2013, Dew-Becker, Giglio, Le, and Rodriguez, 2015, and Giglio and Kelly, 2016, among others) exploit alternative data sources. The authors provide a model-free evidence by computing the term structure of prices on various claims and investigate whether leading asset pricing models are in line with the data.

In this paper, I propose a complimentary model-based approach to describe the term structure of risk in asset prices. I develop this methodology in response to challenges inherent in the data driven approach. First, a model free approach has to rely on (only available) short samples of observable prices of cash flows with different maturities and assumes that liquidity of long-term instruments is as good as that of short-term instruments. Second, descriptive characteristics of the term structure of asset prices are not directly informative about which sources of risk are compensated most in financial markets.

My approach is similar in spirit to diagnostic methods of dynamic stochastic general equilibrium (DSGE) models in macroeconomics. Macroeconomists examine how economic shocks propagate in variables of interest across alternative horizons, that is they measure impulse response functions. Usually researchers have dogmatic views regarding how realistic effects of shocks on economic indicators look like. They develop these views by estimating empirical vector autoregressive (VAR) models. These flexible models deliver a set of stylized facts which structural models are expected to match. Every DSGE model can be (approximately) viewed as a restricted VAR. Thus, a comparison of how risk propagates in the structural model and in the data (empirical model) is an implicit test of cross-equation restrictions.

In this paper, I explore the term structure of macroeconomic risk in asset prices through the lens of realistic empirical models of the joint dynamics of macroeconomy and asset prices. The models are realistic because they (i) feature prominent sources of time varying risk premium as argued by the theoretical asset pricing literature and (ii) do not have cross-equation restrictions. I describe the term structure of risk by means of computing shock exposure elasticities also known as asset pricing counterparts to impulse response functions (Borovička and Hansen, 2016, and references therein) in these empirical models. I interpret the obtained term structure of risk as properties of the data. To confront a macro-based model of asset prices with the data, I analyze shock exposure elasticities under the null of the structural model and its empirical counterpart without cross-equation restrictions.

To characterize how risk propagates in asset prices and cash flows, I exploit discrete-time affine macro-based models with a time-varying risk premium. I consider alternative sources of time-variation in the price and quantity of risk: stochastic volatility and disaster risk with stochastic intensity. In discrete time, shocks driving stochastic volatility, stochastic intensity, and disaster risk are nonnormal. I extend results of Borovička and Hansen (2014), who characterize the term structure of risk for normal shocks, to nonnormal shocks. Specifically, I describe three tractable cases of shock elasticities for nonnormal shocks: (i) autoregressive gamma process for stochastic volatility or jump intensity, (ii) Poisson mixture of exponentials for one-sided jumps such as jumps in variance, and (iii) Poisson mixture of normals for two-sided jumps. These three types of nonnormal shocks model alternative channels of time-varying risk premium proposed in asset pricing literature (e.g., Bansal and Yaron, 2004, Drechsler and Yaron, 2011, and Wachter, 2013 among others). I view this extension as a separate contribution into developing diagnostic methods for structural discrete-time models with time-varying risk premium.

I illustrate my approach by examining leading asset pricing models with alternative channels for generating time-varying equity risk premium: (i) long-run risk with stochastic volatility of consumption growth (Bansal and Yaron (2004)) and (ii) time-varying consumption disasters (Wachter (2013)). I summarize salient properties of the data by estimating unrestricted VARs motivated by the structural models and quantifying patterns of risk propagation across investment horizons. I use original calibrations of Bansal and Yaron (2004) and Wachter (2013) to compute theoretical shock elasticities. The comparison between the empirical and theoretical shock elasticities reveal if the structural models have realistic term structure implications.

For empirical analysis, I use a quarterly dataset of consumption, price-dividend ratio, and equity return observations. The sample spans a time period from 1947:I to 2015:IV. NIPA tables of the Bureau of Economic Analysis and CRSP stock dataset are the main data sources. The models under consideration feature latent state variables such as stochastic volatility and disaster intensity. To filter these states and identify normal and disaster shocks, I employ Bayesian estimation methods.

My findings reveal a tension in structural models with stochastic volatility and disaster risk between generating right level and shape of the term structure of equity risk at the same time. The model with stochastic volatility captures the level of equity risk at multiple investment horizons but fails to account for a downward sloping term structure of risk. Alternatively, the model with the disaster risk produces the right shape of the term structure of equity risk but amplifies the amount of total risk across all investment horizons.

The analysis of shock elasticities suggests that the main culprits in generating wrong model implications are the long-run risk and the disaster risk, respectively. Estimation of the empirical VAR motivated by the model of Bansal and Yaron (2004) reveals that the long-run risk dies out in equity returns with the investment horizon. In contrast, the calibrated version of the model suggests that the risk intensifies as the investment horizon grows. An estimated VAR motivated by the model of Wachter (2013) identifies the disaster risk in consumption growth as a rally in equity market. While, it is apparent that both consumption growth and equity returns are negatively skewed, the sources of their skewness appear to be different.

The result that the standard calibrations of the leading asset pricing models featuring stochastic volatility and disaster risk, have different predictions for the term structure of equity returns is novel. It is simply a direct manifestation of the fact that (at least in standard calibrations) alternative risk channels generating time variation in the risk premium work differently in the term structure. This fact does not contradict the earlier findings of Binsbergen, Brandt, and Koijen (2012), who argue that the implications of the standard asset pricing models are in conflict with the evidence on the term structure of equity risk. I analyze the term structure of equity risk along a different dimension. I do not rely on the data regarding the risk of short-term equity instruments versus the entire equity index, but analyze the term structure of the cumulative returns of investing in the equity index for alternative investment horizons.

Is it necessarily a regress that the standard calibrations of the leading models in asset pricing are in conflict with the recent evidence obtained from novel datasets (e.g., Binsbergen, Brandt, and Koijen (2012), Dew-Becker, Giglio, Le, and Rodriguez, 2015)? I do not think so. The standard calibrations target one-period asset pricing moments and do not necessarily represent inherent properties of the models in the term structure. My empirical results confirm this point. The term structure of equity risk in the empirical model with stochastic volatility is downward sloping, even though it is upward sloping in the original paper of Bansal and Yaron (2004). All it says is that future research should explore alternative assumptions on the structure of economic shocks in leading asset pricing models. An interesting example of this route is pursued in Backus, Boyarchenko, and Chernov (2015).

Related literature

[TO BE COMPLETED]

2 Term structure of risk

A Motivating example

[TO BE COMPLETED]

B Shock elasticities for nonnormal sources of risk

[TO BE COMPLETED]

3 Bridging models with evidence

A linear or loglinear approximation of a macroeconomic DSGE model has the form of a vector autoregressive model of observable variables of interest and latent states.[1] The VAR parameters in this case are complicated nonlinear functions of the structural parameters. Researchers often use unrestricted vector autoregressive models, which are empirical counterparts to the structural models without cross-equation restrictions, in order to identify a set of empirical stylized facts for theoretical DSGE models to match. Unrestricted vector autoregressions exhibit a minimal set of restrictions, such as (1) affine structure that is consistent with a loglinear representation of a DSGE model and (2) shock identifying scheme.

To test a specific theory at least on a qualitative basis, researchers compute impulse response functions under the null of the DSGE model and unrestricted VAR. The impulse response functions from the VAR represent a set of stylized facts, whereas those from the DSGE model represent the model-implied properties of economic shocks. If the two are consistent with each other in terms of shapes and magnitudes, the DSGE model is consistent with the salient features of the data. In such a case, it makes sense to test the model formally by considering all cross-equation restrictions jointly.

I use this idea of conducting inference about structural models by comparing term structure properties of risk in the theoretical model and in the data (empirical unrestricted model) in the context of macro-based asset pricing models featuring time-varying risk premium. Specifically, I examine term structure of equity risk in returns and dividend growth in the model with stochastic volatility and in the model with time-varying consumption disasters.

A The model with stochastic volatility of consumption growth

The model of Bansal and Yaron (2004) has become a standard reference in modern asset pricing as a successful framework rationalizing the magnitude of and countercyclical time variation in equity risk premium. The key modeling ingredients, such as the presence of a slow moving component in the expected consumption growth, stochastic volatility in consumption growth, and recursive preferences, have proved useful in accounting for the risk-return trade-off in other asset markets as well. In this section, I show how to develop an unrestricted empirical model that nests the model of Bansal and Yaron (2004) with a subsequent purpose of testing model implications as it concerns the term structure of equity risk.

The joint data generating process (DGP) for consumption growth $\log g_{t,t+1}$ and dividend

[1]See, for example, Fernandez-Villaverde, Rubio-Ramirez, Sargent, and Watson (2007)

growth $\log d_{t,t+1}$ in the model of Bansal and Yaron (2004) is

$$\log g_{t,t+1} = \log g + x_t + \gamma_g v_t^{1/2} \varepsilon_{g,t+1}, \tag{1}$$

$$\log d_{t,t+1} = \log d + \mu_x x_t + \gamma_d v_t^{1/2} \varepsilon_{d,t+1}, \tag{2}$$

$$x_{t+1} = \varphi_x x_t + \gamma_x v_t^{1/2} \varepsilon_{x,t+1}, \tag{3}$$

$$v_{t+1} = (1-\varphi_v) + \varphi_v v_t + \sigma_v((1-\varphi_v + 2\varphi_v v_t)/2)^{1/2} \varepsilon_{vt+1}, \tag{4}$$

where stochastic variance follows the scalar autoregressive process of order one $v_{t+1} \sim \mathcal{ARG}(1)$ with the scale parameter $\sigma_v^2/2$, the degrees of freedom $2(1-\varphi_v)/\sigma_v^2$, and the serial correlation φ_v. The shocks $\varepsilon_{g,t+1} \sim \mathcal{N}(0,1)$ and $\varepsilon_{x,t+1} \sim \mathcal{N}(0,1)$ are normal, whereas the stochastic volatility shocks $\varepsilon_{v,t+1}$ are not normal but have a mean of zero and standard deviation of one. All the shocks are orthogonal to each other. While the original formulation of the model features an autoregressive process for stochastic variance, I choose to work with the autoregressive gamma process.[2] The latter process guarantees positivity of the stochastic variance regardless of the realization of the shock $\varepsilon_{v,t+1}$. This change of the process for the stochastic variance does not alter quantitative implications of the model.

A critical building block of the model that helps generating realistic magnitudes of the equity risk premium is recursive utility. Applying recursive preferences

$$U_t = [(1-\beta)c_t^\rho + \beta\mu_t(U_{t+1})^\rho]^{1/\rho}, \tag{5}$$

$$\mu_t(U_{t+1}) = [E_t(U_{t+1}^\alpha)]^{1/\alpha}, \tag{6}$$

where c_t is consumption at time t, U_t is utility from time t onwards, $1-\alpha$ is the coefficient of relative risk aversion, $1/(1-\rho)$ is the elasticity of the intertemporal substitution (EIS), and β is the subjective discount factor, to consumption growth (1), (3)-(4), I derive the representative agent pricing kernel

$$\log m_{t,t+1} = \log m + m_x x_t + m_v v_t + m_{\varepsilon g} v_t^{1/2} \varepsilon_{g,t+1} + m_{\varepsilon x} v_t^{1/2} \varepsilon_{x,t+1} + m_{\varepsilon v} v_{t+1}. \tag{7}$$

The Euler equation

$$E_t[m_{t,t+1} r_{t+1}] = 1$$

along with the specification of the dividend growth process (2)-(4) implies that the price-dividend ratio is an affine function of the state variables x_t and v_t

$$\log(p_t/d_t) = q_0 + q_x x_t + q_v v_t \tag{8}$$

and the process for the equity return is

$$r_{t+1} = \log r + r_x x_t + r_v v_t + r_{\varepsilon v} v_{t+1} + r_{\varepsilon x} v_t^{1/2} \varepsilon_{x,t+1} + r_{\varepsilon d} v_t^{1/2} \varepsilon_{d,t+1}. \tag{9}$$

[2]The autoregressive gamma process can be viewed as the discrete-time counterpart to the continuous-time square-root model. See, for example, Gourieroux and Jasiak (2006) and Le, Singleton, and Dai (2010).

Parameters of the pricing kernel process (7), price-dividend relationship (8), and return process (9) are nonlinear functions of the deep parameters of the model. Appendix B presents a detailed solution of the model.

Given the mapping (8), I substitute out the latent state x_t from the DGP for consumption growth and dividend growth as a linear combination of $\log(p_t/d_t)$ and v_t

$$x_t = -\frac{q_0}{q_x} + \frac{1}{q_x}\log(p_t/d_t) - \frac{q_v}{q_x}v_t.$$

I group all variables of interest into a vector $Y_t = (\log g_{t-1,t},\ \log(p_t/d_t),\ \log r_{t-1,t},\ \log d_{t-1,t})'$ whose dynamics is described by a restricted discrete-time model with a state vector $X_t = (1,\ \log g_{t-1,t},\ \log(p_t/d_t),\ v_t)'$, shocks $\varepsilon_{t=1} = (\varepsilon_{gt+1},\ \varepsilon_{xt+1},\ \varepsilon_{dt+1})'$ and stochastic variance v_t

$$Y_{t+1} = \underset{4\times 4}{G} X_t + \underset{4\times 1}{H_v}(v_{t+1} - E_t v_{t+1}) + \underset{4\times 3}{H} v_t^{1/2}\varepsilon_{t+1}, \tag{10}$$

$$v_{t+1} = (1-\varphi_v) + \varphi_v v_t + \sigma_v((1-\varphi_v + 2\varphi_v v_t)/2)^{1/2}\varepsilon_{vt+1}. \tag{11}$$

The restrictions are embedded in matrices G, H_v, and H:

$$G = \begin{pmatrix} \log g - q_0/q_x & 0 & 1/q_x & -q_v/q_x \\ (1-\varphi_x)q_0 + q_v(1-\varphi_v) & 0 & \varphi_x & -\varphi_x q_v + q_v\varphi_v \\ \log r - r_x q_0/q_x + r_{\varepsilon v}(1-\varphi_v) & 0 & r_x/q_x + r_{\varepsilon v}\varphi_v & r_v - r_x q_v/q_x \\ \log d - \mu_x q_0/q_x & 0 & \mu_x/q_x & -\mu_x q_v/q_x \end{pmatrix}$$

$$H_v = \begin{pmatrix} 0 \\ q_v \\ r_{\varepsilon v} \\ 0 \end{pmatrix}, \quad H = \begin{pmatrix} \gamma_g & 0 & 0 \\ 0 & \gamma_x q_x & 0 \\ 0 & r_{\varepsilon x} & r_{\varepsilon d} \\ 0 & 0 & \gamma_d \end{pmatrix}.$$

I use Bayesian MCMC methods to estimate the unrestricted version of the discrete-time model (10)-(11) (keep matrices G and H free) on a sample of quarterly US macroeconomic and financial data from 1947 to 2015. Because there is a linear relationship between log returns, price-dividend ratio, and dividend growth, I exclude one (dividend growth) equation from the system. I recover dividend dynamics from the log-linear approximation of the equity return as

$$\log d_{t,t+1} = \log r_{t,t+1} - \kappa_0 - \kappa_1 \log(p_{t+1}/d_{t+1}) + \log(p_t/d_t).$$

As a part of the estimation output, I obtain the parameter estimates of $\Sigma = \tilde{H}\tilde{H}'$ and recover disturbances $w_{t+1} \sim \mathcal{MN}(0,I)$ that are unknown linear functions of structural shocks $\varepsilon_{t+1} \sim \mathcal{MN}(0,I)$:

$$\tilde{H}\varepsilon_{t+1} = \Sigma^{1/2}w_{t+1}. \tag{12}$$

The unknown part in the relationship (12) is a matrix $\tilde{H}$. I augment the empirical model with a number of identifying restrictions and use methods of structural macroeconometrics to estimate the matrix $\tilde{H}$ and recover the structural shocks ε_{t+1}. The original model of Bansal and Yaron (2004) motivates the choice of shock identifying restrictions. I consider zero restrictions and work with a globally just-identified system (Rubio-Ramirez, Waggoner, and Zha, 2010) : $\tilde{H}_{12} = \tilde{H}_{13} = \tilde{H}_{23} = 0$. The original model in Bansal and Yaron (2004) is overidentified as it features two additional restrictions $\tilde{H}_{21} = \tilde{H}_{32} = 0$ which I can test ex-post.

B The model with time varying consumption disasters

The model of Wachter (2013) proposes an alternative channel generating time variation in the equity risk premium that is time varying consumption disasters. The author posits that a representative investor fears big negative fundamental shocks which lead to dramatic drop in cash flows, hence big magnitude of the risk premium, with a time-varying rate of arrival, hence time-variation in the equity risk premium. This idea has proved useful in rationalizing the risk-return tradeoff not only in the stock market, but also in the foreign exchange market, fixed-income market, and credit market. In this section, I develop an unrestricted empirical model that nests the structural model of Wachter (2013) that can be used to test the term structure implications of the structural model.

I consider a discrete-time version of the continuous-time model of Wachter (2013). The joint data generating process (DGP) for consumption growth $\log g_{t,t+1}$ and dividend growth $\log d_{t,t+1}$ is

$$\begin{aligned}
\log g_{t,t+1} &= \log g + \gamma_g \varepsilon_{gt+1} - z_{t+1}, && (13)\\
\log d_{t,t+1} &= \log d + \varphi_d \gamma_g \varepsilon_{gt+1} + \gamma_d \varepsilon_{dt+1} - \varphi_d z_{t+1}, && (14)\\
z_{t+1} | p_{zt+1} &\sim \gamma(p_{zt+1}, \theta_g), && (15)\\
p_{zt+1} | \lambda_t &\sim \text{Poisson}(h_\lambda \lambda_t), && (16)\\
\lambda_{t+1} &= (1 - \varphi_\lambda) + \varphi_\lambda \lambda_t + \sigma_\lambda ((1 - \varphi_\lambda + \varphi_\lambda \lambda_t)/2)^{1/2} \varepsilon_{\lambda,t+1}, && (17)
\end{aligned}$$

where the jump component z_{t+1} is a Poisson mixture of gammas. Its central ingredient p_{zt} is a Poisson random variable. The mean of p_{zt+1} is a jump-arrival intensity $h_\lambda \lambda_t$. The normalized jump intensity λ_t follows the scalar autoregressive process of order one $\lambda_{t+1} \sim \mathcal{ARG}(1)$ with the scale parameters $\sigma_\lambda^2/2$, the degrees of freedom $2(1 - \varphi_\lambda)/\sigma_\lambda^2$, and the serial correlation φ_λ. The normal shocks $\varepsilon_{gt+1} \sim \mathcal{N}(0, 1)$ and $\varepsilon_{dt+1} \sim \mathcal{N}(0, 1)$ are orthogonal to each other. The nonnormal shock $\varepsilon_{\lambda t+1}$ has a mean of zero and standard deviation of one.

Similar to the case of the model of Bansal and Yaron (2004), this model features a representative agent with recursive preferences (5)-(6). I apply recursive utility to the consumption growth process (13), (15)-(17) and obtain the following process for the pricing kernel

$$\log m_{t,t+1} = \log m + m_\lambda \lambda_t + m_{\varepsilon\lambda} \lambda_{t+1} + m_{\varepsilon g} \varepsilon_{gt+1} + m_z z_{t+1}. \quad (18)$$

The Euler equation

$$E_t[m_{t,t+1} r_{t+1}] = 1$$

together with the specification of the dividend growth process (14)-(17) implies that the price-dividend ratio is an affine function of λ_t

$$\log\left(p_t/d_t\right) \quad = \quad q_0 + q_\lambda \lambda_t \tag{19}$$

and the process for the equity return is

$$\log r_{t+1} \quad = \quad \log r + r_\lambda \lambda_t + r_{\varepsilon\lambda}\lambda_{t+1} + r_{\varepsilon g}\varepsilon_{gt+1} + r_z z_{t+1} + r_{\varepsilon d}\varepsilon_{dt+1}. \tag{20}$$

Parameters of the pricing kernel process (18), price-dividend mapping into the state variable λ_t (19), and return process (20) are nonlinear functions of the deep parameters of the model. Appendix C presents a detailed solution of the model.

I group consumption growth, return, and dividend growth into a vector $Y_t = (\log g_{t-1,t},\ \log r_{t-1,t},\ \log d_{t-1,t})'$ whose dynamics is described by a restricted discrete-time model with a state vector $X_t = (1,\ \log g_t,\ \lambda_t)'$, shocks $\varepsilon_{t+1} = (\varepsilon_{gt+1},\ \varepsilon_{dt+1})$, and time-varying disasters z_{t+1} with time-varying intensity λ_t:

$$\begin{aligned} Y_{t+1} \quad &= \quad \underset{2\times 3}{G} X_t + \underset{2\times 1}{H_\lambda}(\lambda_{t+1} - E_t\lambda_{t+1}) + \underset{3\times 2}{H}\varepsilon_{t+1} + \underset{2\times 1}{\Gamma} z_{t+1}, && (21)\\ \lambda_{t+1} \quad &= \quad (1-\varphi_\lambda) + \varphi_\lambda\lambda_t + \sigma_\lambda((1-\varphi_\lambda + \varphi_\lambda\lambda_t)/2)^{1/2}\varepsilon_{\lambda t+1}. && (22) \end{aligned}$$

The restrictions are embedded in matrices G, H_λ, H, and Γ:

$$G = \begin{pmatrix} \log g & 0 \\ \log r + (1-\varphi_\lambda)r_{\varepsilon\lambda} & r_\lambda + r_{\varepsilon\lambda}\varphi_\lambda \\ \log d & 0 \end{pmatrix}, \qquad H_\lambda = \begin{pmatrix} 0 \\ r_{\varepsilon\lambda} \\ 0 \end{pmatrix},$$

$$H = \begin{pmatrix} \gamma_g & 0 \\ r_{\varepsilon g} & r_{\varepsilon d} \\ \varphi_d\gamma_g & \gamma_d \end{pmatrix}, \qquad \Gamma = \begin{pmatrix} -1 \\ r_z \\ -\varphi_d \end{pmatrix}.$$

I use Bayesian MCMC methods to estimate the unrestricted version of the discrete-time model (21)-(22) on a sample of quarterly US macroeconomic and financial data from 1947 to 2015. The MCMC methods are particularly useful in this case as they allow me to identify disaster shocks in the data. I recover both sizes of the realized disasters and times when they occur.

Because consumption disasters are, by definition, rare events, estimating the jump intensity λ_t on a sample with quarterly observations is challenging. In response to this challenge, I augment the model with one additional observation equation

$$\log\left(p_t/d_t\right) \quad = \quad q_0 + q_\lambda\lambda_t + \sigma_{pd}\varepsilon_{pd,t}, \tag{23}$$

where q_0 and q_λ are free parameters. The shock $\varepsilon_{pd,t}$ is assumed to be orthogonal to the economic shocks ε_{gt}, ε_{dt}, and $\varepsilon_{\lambda t}$, whereas the standard deviation σ_{pd} is small. One interpretation of the component $\sigma_{pd}\varepsilon_{pd,t+1}$ is that it represents a measurement error. The observation equation (23) makes the jump intensity λ_t almost observable that provides comforting ground to test quantitative implications of Wachter (2013) even on the sample without disasters.

Because there is a linear relationship between log returns, dividends and price-dividend ratio, I exclude the dividend growth equation from the unrestricted version of the system (21)-(22), similar to what I do in case of Bansal and Yaron (2004). Later on, I recover the dividend growth dynamics by using the estimated dynamics of the log return, jump intensity, the mapping between jump intensity and price-dividend ratio, and the log-linear approximation of the equity return

$$\log d_{t,t+1} \quad = \quad \log r_{t,t+1} - \kappa_0 - \kappa_1 \log\left(p_{t+1}/d_{t+1}\right) + \log\left(p_t/d_t\right).$$

As a part of the estimation output, I obtain the parameter estimates $\Sigma = \tilde{H}\tilde{H}'$ and recover disturbances $\omega_{t+1} \sim \mathcal{MN}(0, I)$ that are unknown linear functions of structural shocks $\varepsilon_{t+1} \sim \mathcal{MN}(0, I)$:

$$\tilde{H}\varepsilon_{t+1} \quad = \quad \Sigma^{1/2}\omega_{t+1}. \tag{24}$$

The unknown part in the relationship (24) is a matrix $\tilde{H}$. I augment the empirical model with an identifying zero-restriction, that is $\tilde{H}_{12} = 0$, thereby, recovering the matrix $\tilde{H}$ and structural shocks ε_{t+1}. The identifying restriction conveys the idea that the consumption growth shock contemporaneously affects both consumption growth and returns, whereas the dividend shock contemporaneously affects only returns. The original structural model of Wachter (2013) is overidentified as it has one additional restriction $\tilde{H}_{21} = 0$ which I can test ex-post.

4 Empirical results

This section analyzes empirical properties of the two leading competing macro-based models of equity risk premium. I start with a quick overview of the estimation output. I relegate the details of the estimation procedure to the online Appendix. Next, I analyze the term structure implications. I start with the term structures of equity risk premium and dividend growth under the null of the empirical models and theoretical models. I pay attention to the shapes and levels of the term structures. I proceed by inspecting mechanisms of the models. My main interest is to identify which shocks play the most prominent role in the term structure in the model and in the data and if the models agree with the data. I conclude by highlighting potential avenues which would bring models closer to the data.

A Estimation output

I use the data displayed in panels (a)-(c) of Figure 1 to estimate the unrestricted versions of the model with stochastic volatility (10)-(11) and the model with consumption disasters (21)-(22).

Model with stochastic volatility of consumption growth

Panel (d) of Figure 1 displays the estimated stochastic variance factor with the 95% credible interval. The estimated stochastic variance exhibits significantly higher variance of variance $\sigma_v = 0.178$ (calibrated parameter is 0.0653) and smaller parameter of the serial correlation $\varphi_v = 0.8008$ (calibrated value is 0.9615). Table 2 summarizes parameter estimates with the 95% credible intervals and reports calibrated parameter values from the original paper. The column corresponding to the dividend growth equation reports parameter estimates implied by the estimated parameters of the price-dividend and return processes and the log-linear return approximation.

The loadings of the variables of interest on the variance, that is components of the matrix H_v, have an opposite sign than those parameters in the calibrated version of the model. These loadings are not statistically significant at the 95% credible interval, yet it is an interesting fact given that it is hard to estimate parameters of the model with stochastic variance on the sample of 275 observations. Such a stark difference in parameter values may lead to different shapes of the term structure of the variance risk.

The loading of consumption growth on the price-dividend ratio is one of the key parameters in Bansal and Yaron (2004). The presence of the persistent slow moving component in the consumption growth contributes most to the high level of the equity premium in the structural model. The estimate of this parameter is approximately ten times smaller than the calibrated value. Yet volatility of the price-dividend ratio related to the shock ε_{xt+1} is about equal to the calibrated parameter, whereas the estimated persistence of the price-dividend ratio is significantly higher in the data than in the model. Taken these facts together, it seems that the data suggest a steeper term structure of risk ε_{xt+1} in the macroeconomy than that assumed in the structural model.

The structural model implies a wrong sign of return predictability not only by the stochastic variance but also by the price-dividend ratio. The estimated loading of the log return on the price-dividend ratio is negative and significant whereas it is positive in the calibration. The implied dividend process exhibits insignificant loadings on the states of the model. Qualitatively, the predictability of the dividend growth by the price-dividend ratio is in line with the model as the corresponding loading has the same sign as the calibrated parameter.

Model with consumption disaster

Panel (d) of Figure 2 displays the identified consumption disaster risk, along with the estimated jump intensity. The structural model of Wachter (2013) posits that the big negative jumps in consumption growth, that is consumption disasters, coincide with even larger negative jumps in dividend growth, and returns. This is the key mechanism of the model which generates high equity risk premium: the representative agent experiences a big loss of wealth at the worst states of the world. At first, the presence of a correlated jump in consumption growth and returns seems like a plausible feature of the data: both time series exhibit negative skewness and excess kurtosis. However, as the data suggest if anything negative jumps in consumption growth coincide with positive jumps in equity returns. This fact puts at risk the channel that facilitates matching the level of the risk premium in the structural model.

Table 3 reports parameter estimates along with their 95% credible intervals. The data agrees with the model on the sign of return predictability by the jump intensity and the lack of dividend growth predictability. The estimated variance of the jump intensity $\sigma_\lambda = 0.0636$ is significantly smaller than the calibrated parameter equal to 0.2364, whereas the estimated parameter of the serial correlation of the jump intensity $\varphi_\lambda = 0.9855$ is comparable to the assumed one (0.9802) in the original calibration. It is interesting that the risk premium's time variation channels in the models have some distinct features. Data suggest that time varying jump intensity exhibits much higher time variation than that in the stochastic volatility. This fact may translate into sizeable difference in the equilibrium prices of the two sources of risk. In other words, the jump intensity shock may contribute much more meaningfully to the level of the equity premium comparing to the contribution of the volatility shock in alternative model.

B Term structure of equity risk

In this section, I study term structure implications of the leading competing models in the equity literature. The main questions of interest are two: (i) how total risk propagates in equity cash flows and returns, that is whether risk amplifies or dies out with the investment horizon, and (ii) how alternative sources of risk contributes to the term structure of equity risk. Answers to these questions provide a qualitative assessment of the structural models with a specific focus on the model's long-term behavior.

The main instruments of my analysis are expected cumulative returns at alternative horizons scaled per quarterly basis and expected dividend growth. The term structure of dividend growth describes how macroeconomic risk propagates in cash flows over time. Because equilibrium returns are by-product of the interaction between the underlying process for cash flows, macroeconomy, and preferences, the term structure of returns provides evidence about the interaction of these three components.

The model of Wachter (2013) agrees whereas the model of Bansal and Yaron (2004) conflicts with the data regarding the shape of the term structures of equity returns and dividend growth. Alternatively, the model of Bansal and Yaron (2004) captures the level of the term structures of the expected returns and dividends, whereas the model of Wachter (2013) fall short by implying amplified levels of both term structures. Based on these facts, I conclude that standard calibrations of both leading structural models of time-varying equity risk premium exhibit tension between capturing the level and the shape of the term structures of equity returns and dividend growth. I refer to the data when I describe the properties of the empirical models.

Figures 3 and 4 display the term structure of cumulative expected equity returns and dividend growth. Data, through the lens of both empirical models, suggest that the term structure of equity returns is downward sloping. The ten-year spread in cumulative equity returns is -0.58% under the null of the empirical model with the stochastic volatility of consumption growth, whereas it is -0.48% under the null of the empirical model with time varying consumption disasters. The structural models disagree: the model of Bansal and Yaron (2004) implies a slightly upward sloping term structure of equity returns with a ten-year spread of 0.22%, whereas the model of Wachter (2013) implies a downward sloping term structure with a ten-year return spread of -0.64%.

The term structure of dividend growth is flat both under the null of the structural model in Wachter (2013) and in the data. The estimated empirical model motivated by Bansal and Yaron (2004) delivers a flat term structure of dividend growth counter to that implied by the calibrated version of the structural model. The ten-year slope in expected dividend growth is 0.89% in the structural model, whereas it is about 0.02% in the data.

While the model of Wachter (2013) meets better the evidence with respect to the shape of the term structure of returns and cash flows, the model of Bansal and Yaron (2004) is more successful with respect to the levels of expected returns and dividends across alternative investment horizons. The 95% credible intervals for the level of returns and dividends in the estimated model of Bansal and Yaron (2004) contain the term structures implied by the calibrated model. The model of Wachter (2013) implies higher returns and dividend growth than that implied by the data.

C Shock elasticities

As a next natural step of my analysis, I zoom in and inspect the mechanisms of the models. The purpose of this analysis is to identify sources of risk which are responsible for the specific shapes and magnitudes of the term structure of equity in the data and in the models. The shapes of shock elasticities are informative about shock propagation mechanisms in asset prices. If the absolute values of shock elasticities for a specific source of risk increase with the investment horizons in equity returns or cash flows, this source of risk contributes towards a positive slope of expected equity returns or cash flows, respectively. Alternatively, if shock elasticities converge to zero as investment horizon grows, the shock is gradually dying out in equity returns or dividends, and correspondingly contributes towards a negative slope.

The main focus of the analysis are the shocks which contributes most into generating the high magnitude and time-variation of the equity risk premium. In the model of Bansal and Yaron (2004), these shocks are the long-run risk shock ε_{xt+1} (magnitude of the risk premium) and stochastic variance shock ε_{vt+1} (time-variation in the risk premium). In the model of Wachter (2013), these shocks are the consumption disaster risk z_{gt+1} (magnitude of the risk premium) and stochastic intensity shock (time-variation in the risk premium).

Figures 5 and 6 display how, according to the model of Bansal and Yaron (2004) and its unrestricted version, the term structure of risk is reflected in equity returns and dividend growth.

The structural model implies that in the equity returns the shock $\varepsilon_{x,t+1}$ cumulates as the investment horizon grows, whereas the empirical model suggests that the shock gradually dies out. The difference between the sensitivities of the long-term equity returns to the shock in the model and in the data is large. In addition, the shock exhibits the steepest term structure of risk in returns, therefore, contributing most to the upward sloping term structure of equity returns in the model and downward sloping term structure of equity returns in the data.

Similarly, the model and the data do not agree on the properties of the shock ε_{xt+1} in the term structure of dividend growth. The standard calibration of the structural model implies a steep upward sloping term structure of risk, thereby, steep upward sloping term structure of dividend growth. The empirical model reveals a uniformly low sensitivity of dividend growth to the shock at any investment horizon.

The term structure properties of the variance shock is hard to pinpoint with a good degree of precision. The stochastic variance is a latent state variable estimated on a rather short sample of two hundred seventy five quarterly observations. Consequently, the estimates related to the loadings of the economic variables on the stochastic variance or its shock are associated with a high degree of statistical uncertainty. The average estimates for the sensitivities of both dividends and returns to the variance shock at long investment horizons differ from the calibrated values implied by the structural model. Interestingly, the variance shock is associated with the negative increasing term structure of risk that contributes towards an upward sloping term structure of returns and dividends, whereas the observed term structures of returns and dividends are downward sloping.

So what are the challenges in front of the model of Bansal and Yaron (2004)? And how can we improve the model? The standard calibration critically relies on the presence of the upward sloping term structure of ε_{xt+1}. As the shock has a long lasting impact on the macroeconomy and asset prices, the cash flow's risk exposure is highly compensated in the marketplace, hence a high magnitude of the risk premium. However, the data suggest a rather opposite situation – the shock ε_{xt+1} dies out in equity returns as the investment horizon grows. One potential candidate to make up for the missing role of the long-run risk shock is the variance shock. There are at least two properties of the variance shock that future calibrations should take into account: (i) the variance shock is associated with a much higher average standard deviation in the process for the stochastic variance than that assumed in the earlier structural work, (ii) all economic variables load on the stochastic variance with the opposite sign. The first property may lead to a higher price of variance risk ceteris paribus. The second property may rationalize the empirical shape of the term structures of risk in equity returns and dividend growth in the structural model.

Figures 7 and 8 display the term structure of equity risk in the structural model of Wachter (2013) and in its unrestricted version. As I have already outlined earlier, the model of Wachter (2013) agrees with the data regarding the shape of the term structure of risk in returns and cash flows. Now I explore how disaster shock z_{t+1} and stochastic intensity shock $\varepsilon_{\lambda,t+1}$ contribute to the overall term structure of total risk.

The intensity shock $\varepsilon_{\lambda t+1}$ is the main contributor generating a downward sloping term structure of equity returns in the model and in the data. The calibrated structural model and estimated empirical model exhibit strikingly similar levels and shapes of the term structure of risk. The shock elasticities for the disaster and consumption growth shocks have flat profiles both in the data and in the model. However, the levels of the empirical and theoretical term structures of risk are very different. The model posits that the consumption disaster, a rare big negative shock in consumption growth, coincides with an even larger negative shock in dividends and stock returns. The data imply that (i) consumption disasters in the post-world war II US data are not as severe as the structural model assumes and (ii) consumption disasters are stock market rallies.

The discrepancy between what the model assumes and what the data suggest for the disaster shock represents a major challenge for the rare consumption disaster hypothesis as it is formulated in Wachter (2013). If the consumption disaster, which occurs in a bad state of the world and thus has a negative price of risk, coincides with a big positive shock in the stock market return, the equity risk premium earned as the risk compensation for the cash flow exposure to the shock z_{t+1} is negative. In such a case, there is a high pressure on the stochastic intensity shock to produce a big positive risk premium. In the data, however, the stochastic intensity process is associated with much smaller average conditional volatility of intensity σ_λ, and hence, smaller price of risk ceteris paribus.

Given the presence of the negative skewness in consumption growth and stock returns, a model with a disaster risk appears to be a potentially successful working hypothesis to explain the joint behavior of macroeconomy and asset markets. However, as the data suggest, the sources of negative skewness in the two variables of interest appear to be different. I leave it for future research to find a more realistic configuration of the disaster hypothesis in the macroeconomy and stock market.

5 Concluding remarks

I propose a methodology (i) to describe the term structure of risk in asset prices and (ii) to test multi-period implications of discrete-time macro-based asset pricing models without relying on the availability of prices of similar claims with different maturities. Flexible empirical models of the joint dynamics of consumption growth and asset prices featuring prominent sources of time-varying risk premium are informative about shocks which are priced in financial markets. Shock elasticities, also known as asset pricing counterparts to impulse response functions, manifest how these sources of risk propagate in asset prices and cash flows across alternative investment horizons. The shapes and levels of the term structure of risk suggested by flexible models serve as a set of stylized facts which structural models of asset pricing should match.

To illustrate how my methodology works, I examine two leading hypotheses of time varying risk premium: long-run risk with stochastic volatility in consumption growth (Bansal and Yaron, 2004) and time-varying consumption disaster risk (Wachter, 2013). I document tension in both models to produce at the same time the right level and shape of the term structure of equity returns. The model of Bansal and Yaron (2004) implies that the long-run risk cumulates in equity returns as investment horizon grows contrary to the downward sloping term structure of risk in the data. In addition, the data and the model of Bansal and Yaron (2004) disagree about the sign of the marginal sensitivity of the long-horizon equity returns to the variance shock. The model of Wachter (2013) generates a realistic term structure of the disaster intensity risk in equity returns but in the data consumption disasters turn out to be stock market rallies.

On a positive side, my results suggest that alternative calibrations of the leading macro-based asset pricing models could be capable in producing asset pricing implications which are in line with the data. I leave it for future research to explore if alternative configurations of the models with stochastic volatility and/or time-varying disasters can get closer to the data.

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Table 1
Calibrations of the leading equity premium models

Quarterly calibrations of Bansal and Yaron (2004) and Wachter (2013) models. Bansal and Yaron (2004) model: I choose parameters to match means, unconditional volatility, and persistence of the variables implied by the original monthly calibration of Bansal and Yaron (2004). I posit that the stochastic variance follows the autoregressive process of order one. The calibration inputs are as follows: $g = 0.0045$, $\gamma_g = 0.0135$, $\varphi_x = 0.9383$, $\gamma_x = 0.0010$, $\varphi_v = 0.9615$, $\sigma_v = 0.0653$, $d = 0.0045$, $\mu_x = 6$, $\gamma_d = 0.0653$, $\alpha = -9$, $\rho = 1/3$, $\beta = 0.998$, $\kappa_0 = 0.0449$, $\kappa_1 = 0.9923$. Wachter (2013): I posit that consumption disasters are binomial mixtures of exponentials, whereas the stochastic disaster intensity is an autoregressive gamma process of order one. The calibration inputs are as follows: $g = 0.063$, $\gamma_g = 0.01$, $h_\lambda = 0.0075$, $\varphi_\lambda = 0.9802$, $\sigma_\lambda = 0.1743$, $\varphi_d = 2.6$, $d = 0.0163$, $\gamma_d = 0$, $\theta_g = 0.2$, $\alpha_p - 2 =$, $\rho = 0$, $\beta = 0.997$, $\kappa_0 = 0.0449$, $\kappa_1 = 0.9923$. Annualized.

	Bansal and Yaron (2004)	Wachter (2013)
Log equity premium	1.1785	1.0356
Std dev of equity return	10.1507	7.6654
Risk-free rate	0.3125	0.2754
Std dev of the risk-free rate	0.1993	0.5474
Mean consumption growth	0.4500	0.4750
Std dev of consumption growth	1.3821	2.6458
Mean dividend growth	0.4500	1.2350
Std dev of dividend growth	6.7602	6.8789

Table 2
Model of Bansal and Yaron (2004).
Parameter estimates with the 95% credible intervals and calibration from the original paper. Shock $\tilde{\varepsilon}_{vt+1}$ stands for $\sigma_v(1-\varphi_v+2\varphi_v v_t)^{1/2}\varepsilon_{vt+1}$.

Loadings	Growth eq	Price/Dividend eq	Return eq	Dividend eq
Estimated				
$\log g_t$	0.218	-0.065	0.000	0.0549
	(0.004, 0.414)	(-0.447, 0.376)	(-0.188, 0.179)	(-0.4021, 0.4456)
$\log(p_t/d_t)$	0.001	0.979	-0.023	0.0059
	(-0.005, 0.007)	(0.961, 0.994)	(-0.033, -0.014)	(-0.0067, 0.0224)
v_t	-0.002	0.001	-0.005	-0.0068
	(-0.011, 0.007)	(-0.014, 0.018)	(-0.017, 0.005)	(-0.0255, 0.0101)
$\varepsilon_{g,t+1}$	0.0061	0.0106	0.0101	-0.0004
	(0.0045, 0.0100)	(-0.0052, 0.0221)	(0.0005, 0.0203)	(-0.0084, 0.0094)
$\varepsilon_{x,t+1}$	0	0.0762	0.0689	-0.0068
		(0.0624, 0.0953)	(0.0559, 0.0882)	(-0.0107, -0.0032)
$\varepsilon_{d,t+1}$	0	0	0.0305	0.0305
			(0.0266, 0.0368)	(0.0266, 0.0368)
$\tilde{\varepsilon}_{v,t+1}$	-0.0098	-0.0783	-0.0911	-0.0134
	(-0.0529, 0.0355)	(-0.1958, 0.0375)	(-0.1844, -0.0093)	(-0.0808, 0.0502)
Calibrated				
$\log g_t$	0	0	0	0
$\log(p_t/d_t)$	0.0129	0.938	0.0086	0.0775
v_t	0.0014	-0.0025	0.0059	0.0083
$\varepsilon_{g,t+1}$	0.0135	0	0	0
$\varepsilon_{x,t+1}$	0	0.0780	0.0774	0
$\varepsilon_{d,t+1}$	0	0	0.0653	0.0653
$\tilde{\varepsilon}_{v,t+1}$	0	-0.1076	-0.1068	0

Table 3

Parameter estimates and calibration Parameter estimates with the 95% credible intervals and calibration from the original paper. Shock $\tilde{\varepsilon}_{\lambda,t+1}$ stands for $\sigma_\lambda(1-\varphi_\lambda+2\varphi_\lambda\lambda_t)^{1/2}\varepsilon_{\lambda t+1}$.

Loadings on states	Growth eq	Return eq	Dividend eq
Estimated			
$\log g_t$	0.1732	0.0972	0.0972
	(0.0003, 0.2979)	(-0.0879, 0.2914)	(-0.0879, 0.2914)
$\log \lambda_t$	-0.0007	0.0237	0.0003
	(-0.0068, 0.0032)	(0.0105, 0.0383)	(-0.0137, 0.0110)
$\varepsilon_{g,t+1}$	0.0052	0.0036	0.0036
	(0.0047, 0.0059)	(-0.0020, 0.0075)	(-0.0020, 0.0075)
$\varepsilon_{d,t+1}$	0	0.0216	0.0216
		(0.0193, 0.0250)	(0.0193, 0.0250)
$\lambda_{t+1} - E_t\lambda_{t+1}$	0.0025	-0.9612	0.0888
	(-0.0364, 0.0243)	(-1.0177, -0.9051)	(0.0416, 0.1403)
z_{t+1}	-1	7.6336	7.6336
		(3.8126, 10.8394)	(3.8126, 10.8394)
Calibrated			
$\log g_t$	0.0000	0.0000	0.0000
$\log \lambda_t$	0.0000	0.3156	0.0000
$\varepsilon_{g,t+1}$	0.0100	0.0260	0.0000
$\varepsilon_{d,t+1}$	0.0000	0.0000	0.0000
$\lambda_{t+1} - E_t\lambda_{t+1}$	0.0000	-0.3125	0.0000
z_{t+1}	-1.0000	-2.6000	-2.6000

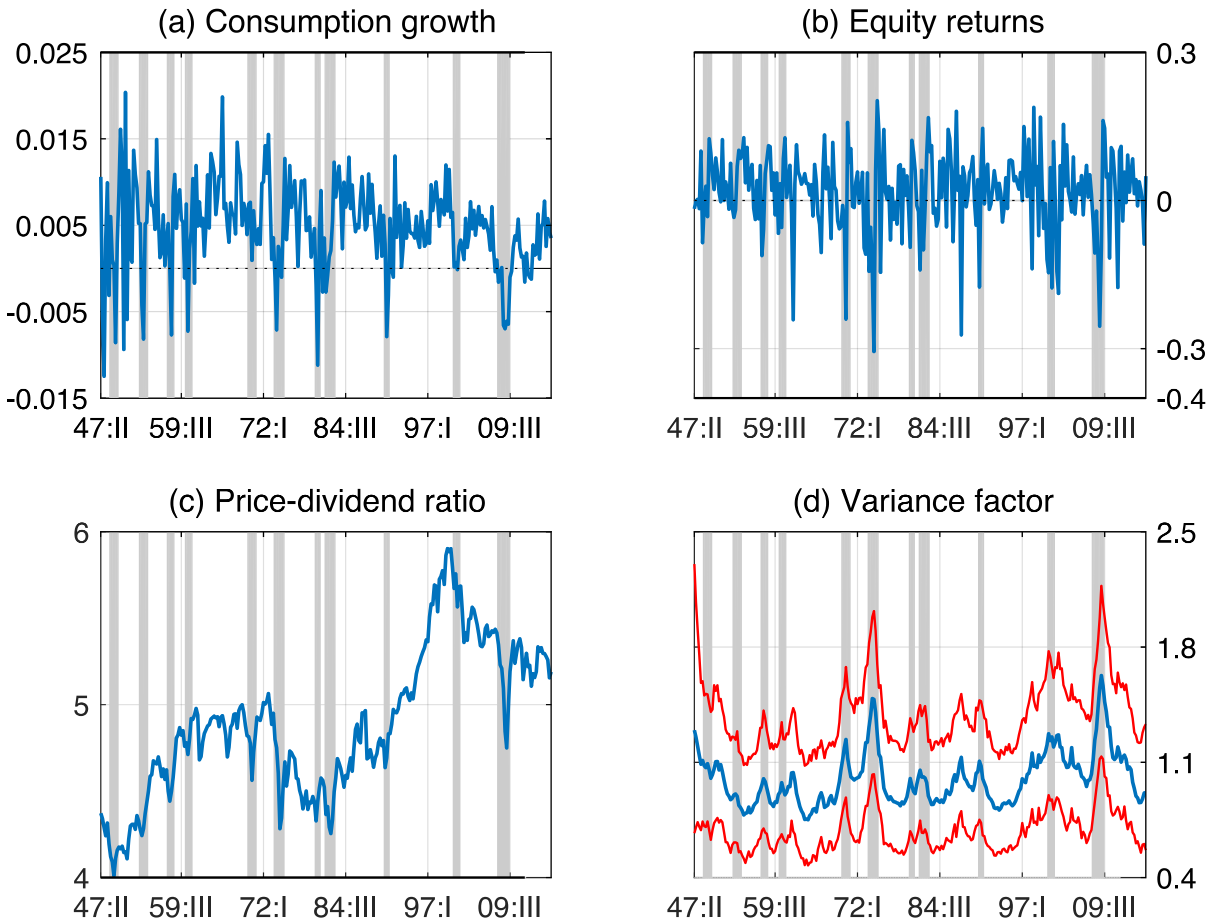

Figure 1
Macroeconomy. Bansal and Yaron (2004) Panels (a), (b), (c) display quarterly observations of consumption growth, log stock returns, and log price-dividend ratio, respectively. Panel (d) displays mean stochastic variance factor (blue line) with the 95% credible interval (red lines). Sample period: second quarter of 1947 to fourth quarter of 2015. Grey bars are the NBER recessions. Quarterly.

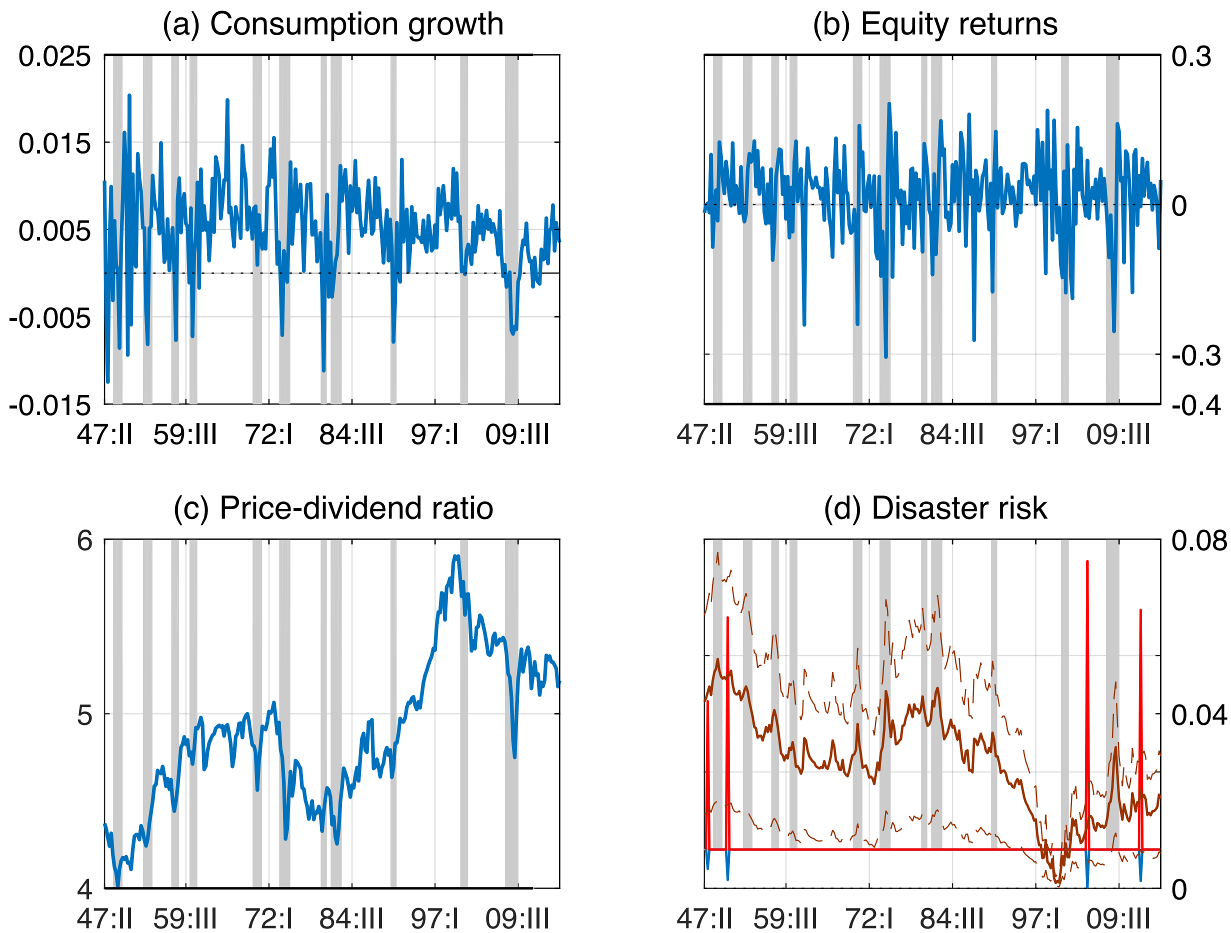

Figure 2
Macroeconomy. Wachter (2013) Panels (a), (b), (c) display quarterly observations of consumption growth, log stock returns, and log price-dividend ratio, respectively. Panel (d) displays consumption disaster risk (blue lines), jump risk in stock returns (red lines). A brown line corresponds to the estimated jump intensity $h_\lambda \lambda_t$, the dashed lines correspond to the 95% credible interval. Sample period: second quarter of 1947 to fourth quarter of 2015. Grey bars are the NBER recessions. Quarterly.

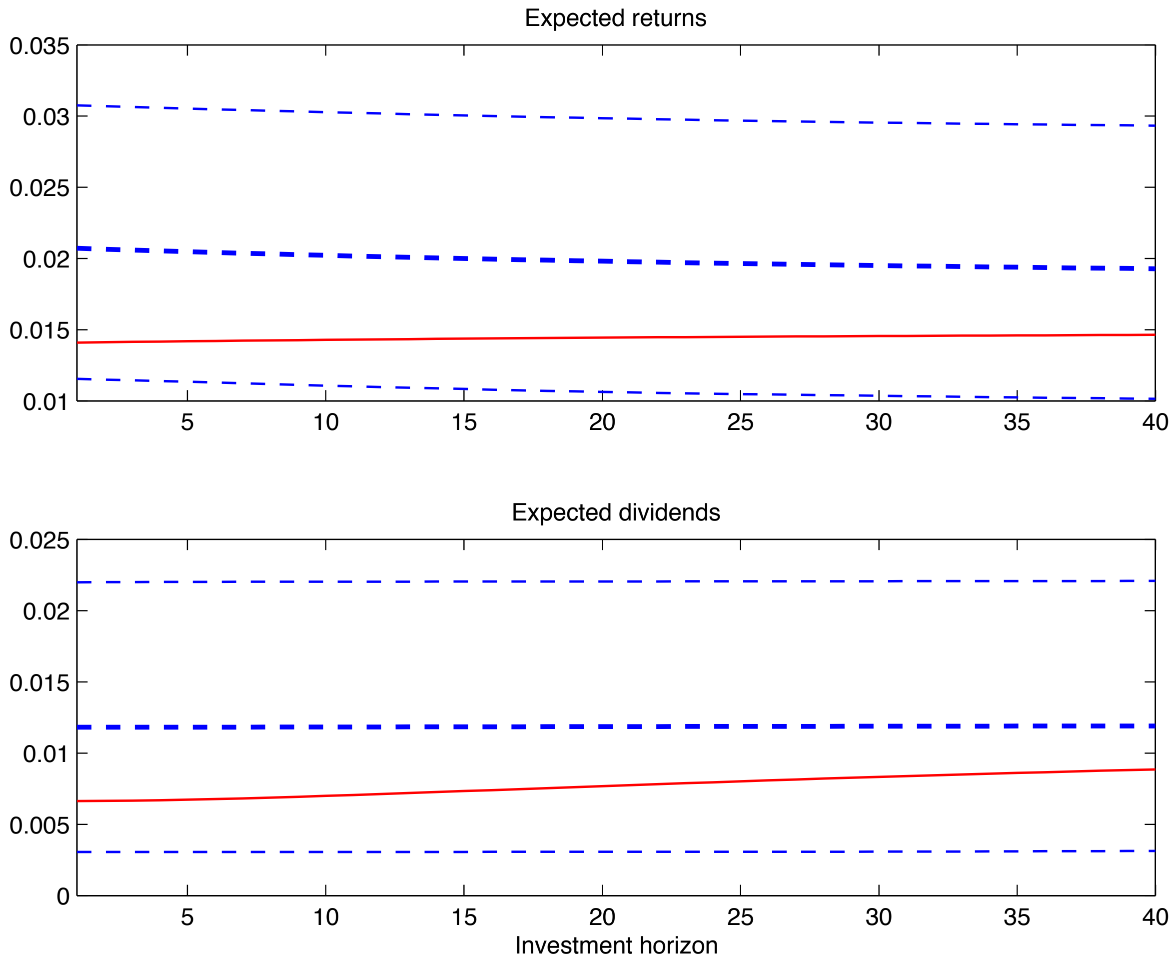

Figure 3
Term structure of equity returns and dividends. Bansal and Yaron (2004). Term structure of expected cumulative equity returns and dividend growth. The red lines correspond to the implications of the structural model in Bansal and Yaron (2004). The thick blue dashed line corresponds to the implications of the empirical unrestricted model. Quarterly.

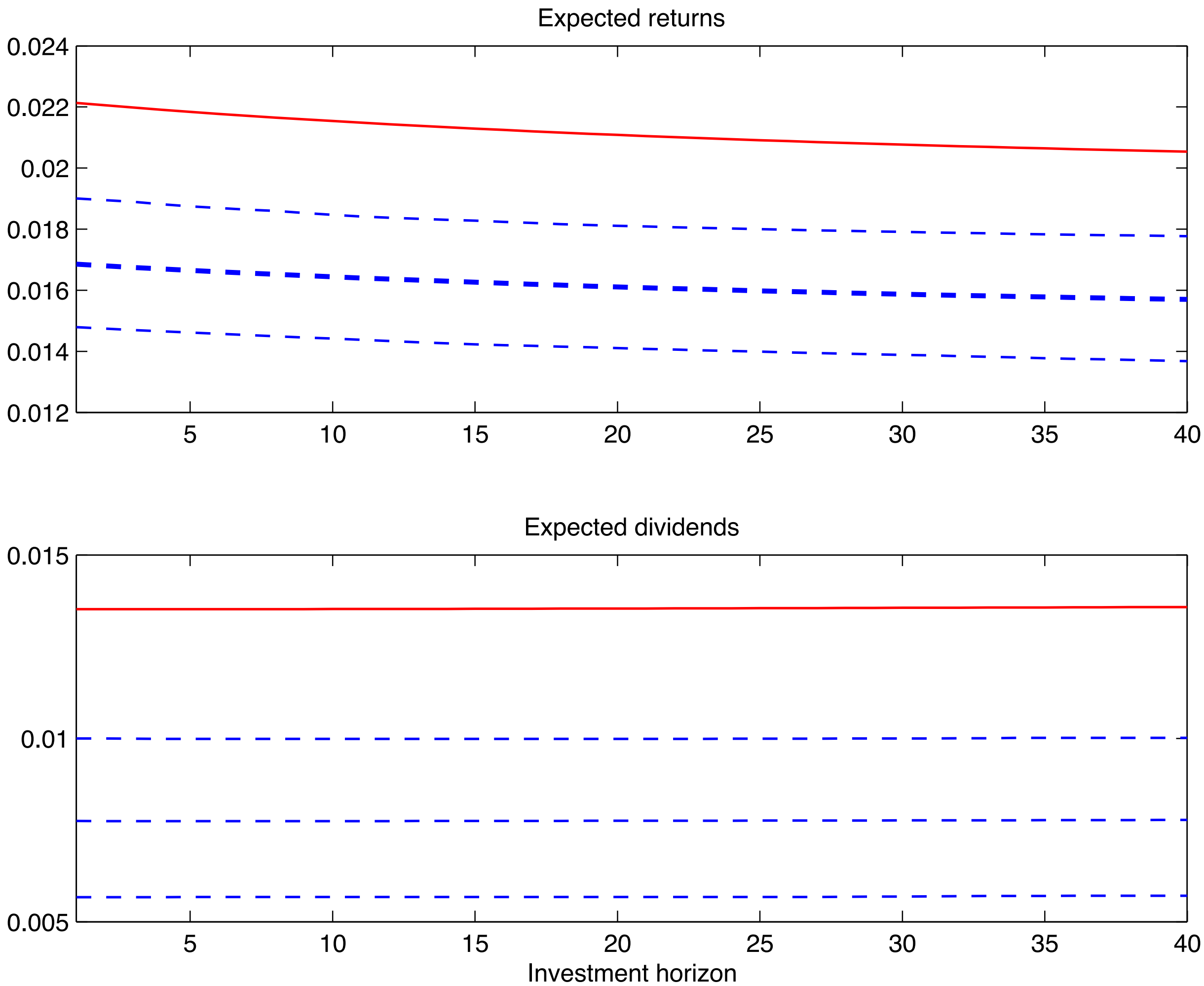

Figure 4
Term structure of equity returns and dividends. Wachter (2013).
Term structure of expected cumulative equity returns and dividend growth. The red lines correspond to the implications of the structural model in Wachter (2013). The thick blue dashed line corresponds to the implications of the empirical unrestricted model. Quarterly.

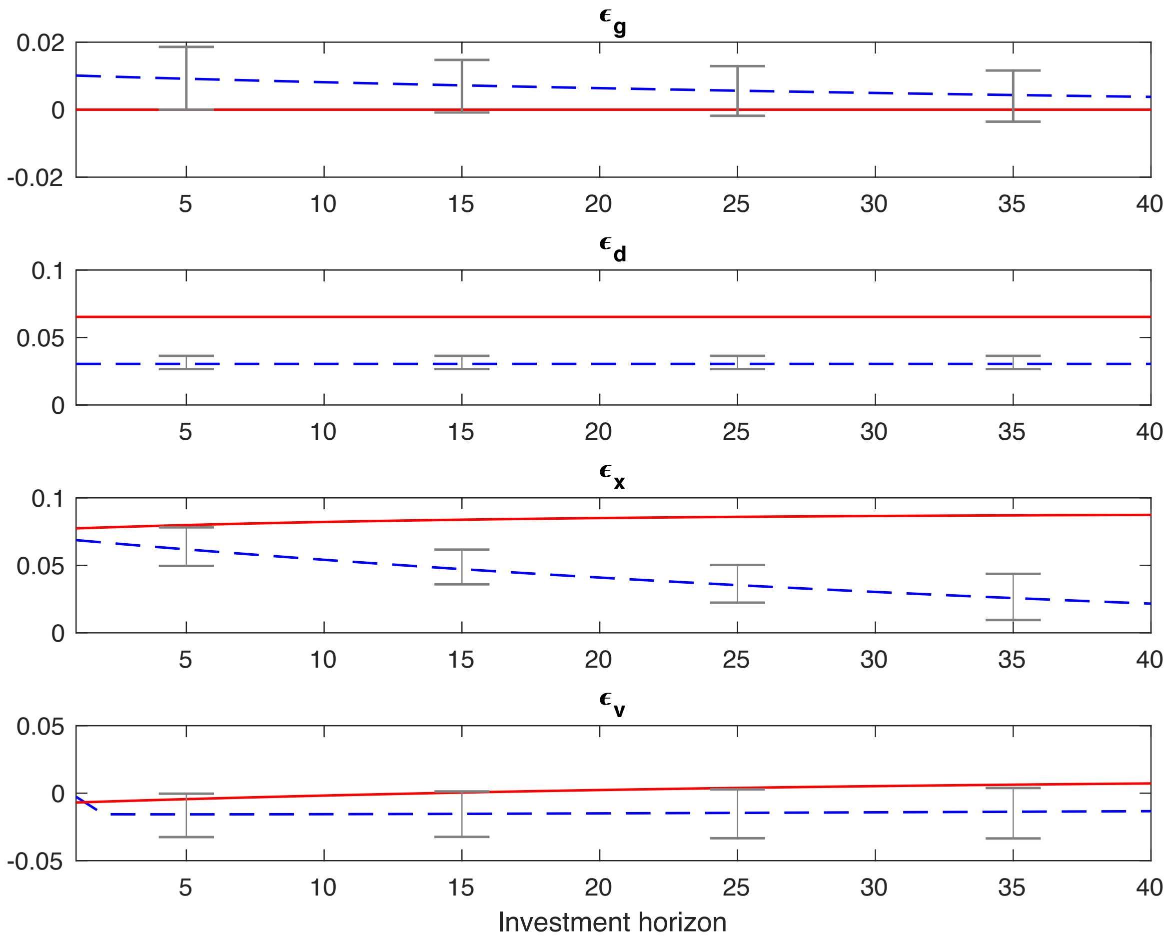

Figure 5
Term structure of risk in equity returns. Bansal and Yaron (2004).
Term structure of risk in equity returns. The red lines correspond to the implications of the structural model in Bansal and Yaron (2004). The blue dashed lines with vertical grey bars correspond to the implications of the empirical unrestricted model. Quarterly.

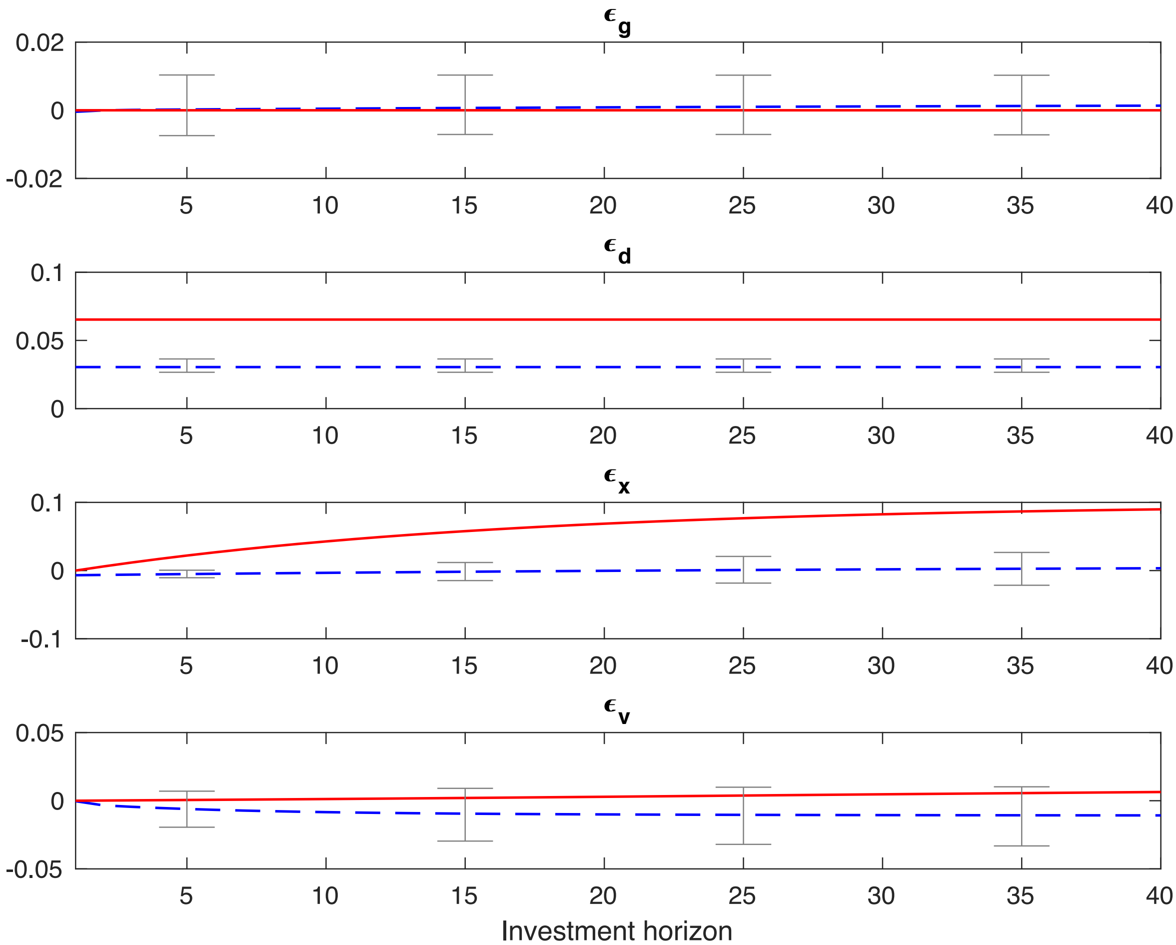

Figure 6
Term structure of risk in stock dividends. Bansal and Yaron (2004).
Term structure of risk in dividend growth. The red lines correspond to the implications of the structural model in Bansal and Yaron (2004). The blue dashed lines with vertical grey bars correspond to the implications of the empirical unrestricted model. Quarterly.

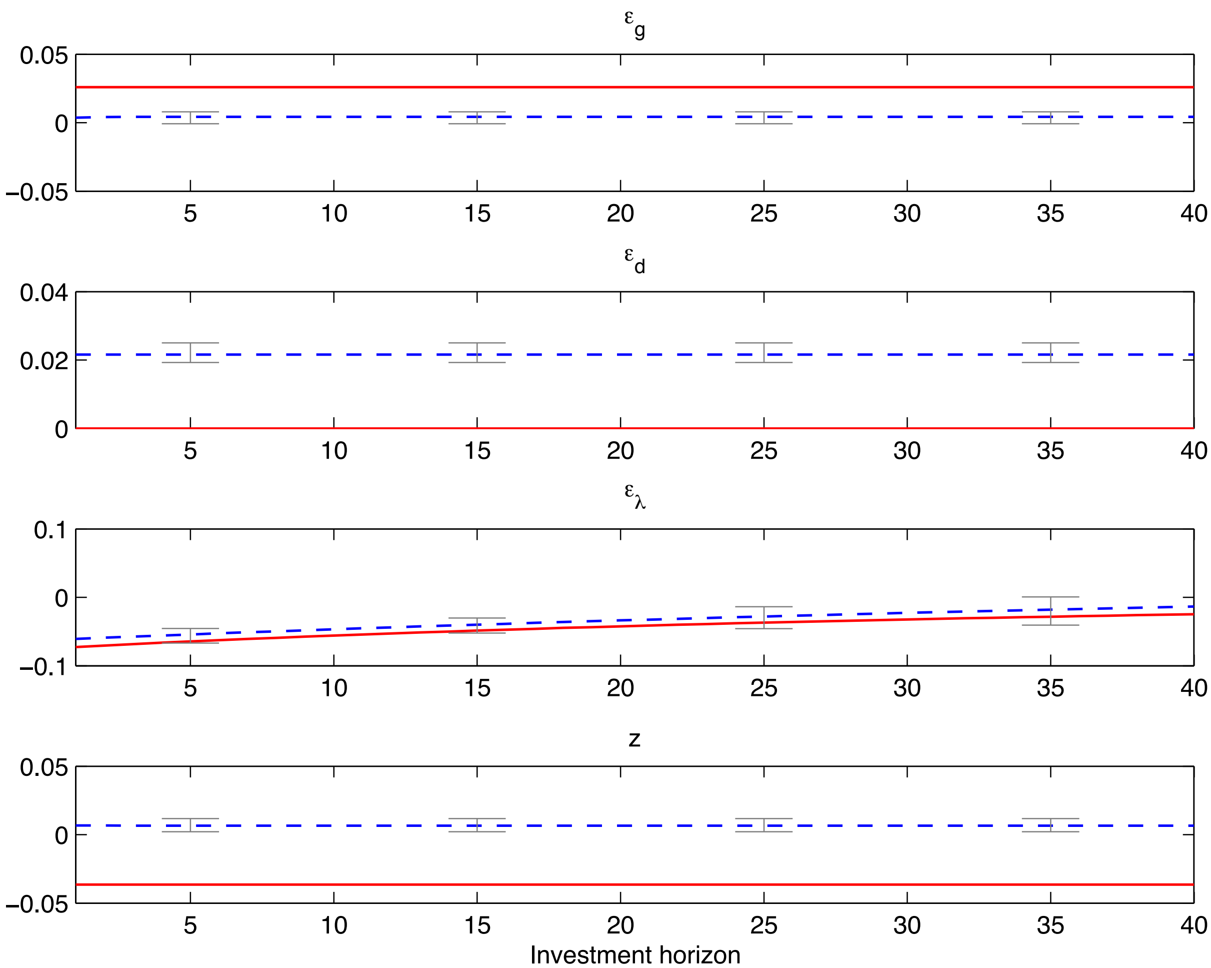

Figure 7
Term structure of risk in equity returns. Wachter (2013). Term structure of risk in equity returns. The red lines correspond to the implications of the structural model in Wachter (2013). The blue dashed lines with vertical grey bars correspond to the implications of the empirical unrestricted model. Quarterly.

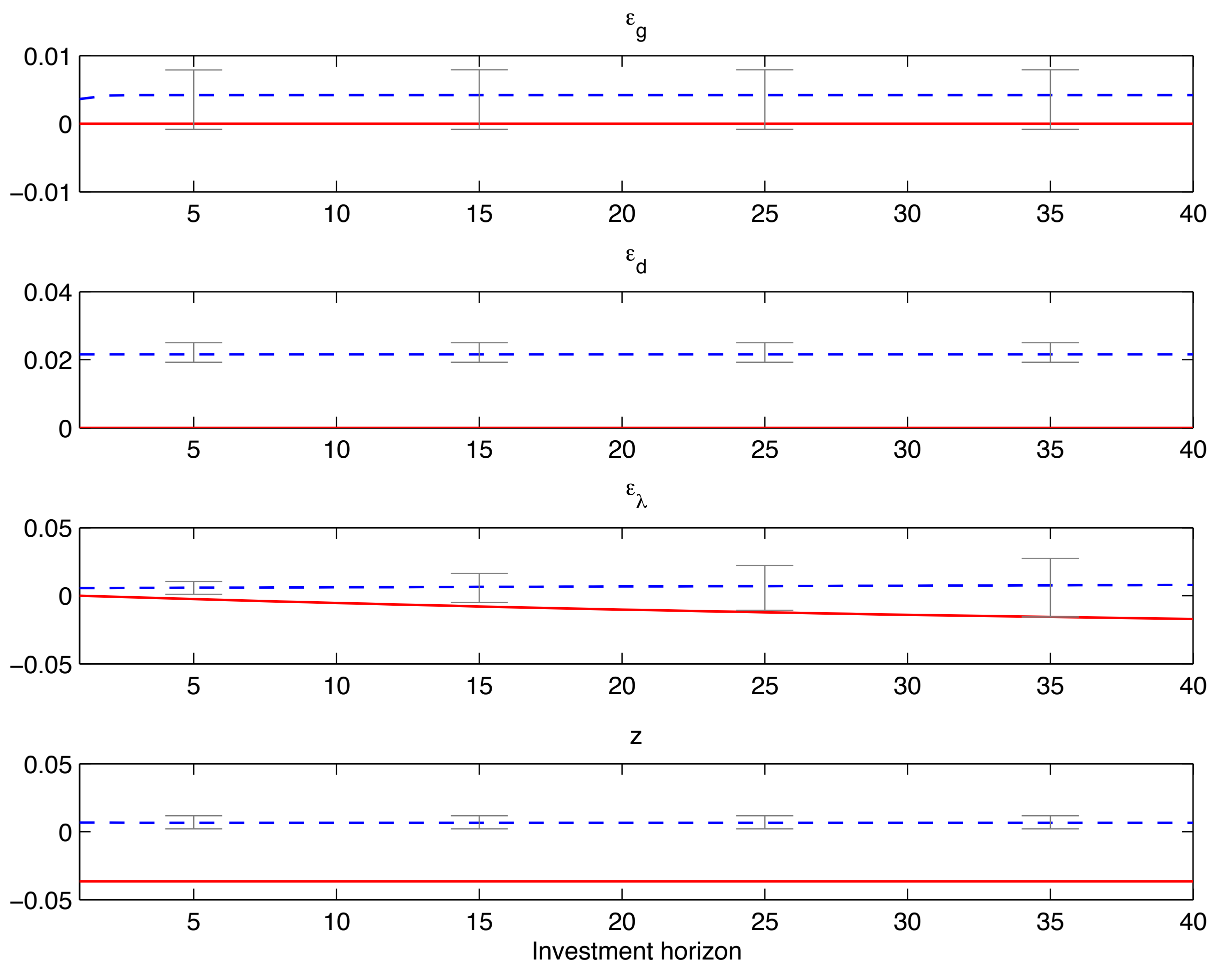

Figure 8
Term structure of risk in stock dividends. Wachter (2013). Term structure of risk in dividend growth. The red lines correspond to the implications of the structural model in Wachter (2013). The blue dashed lines with vertical grey bars correspond to the implications of the empirical unrestricted model. Quarterly.

A Appendix

A Toolkit to compute shock elasticities for nonnormal sources of risk in discrete-time models

Consider three types of shocks: (1) stochastic variance or stochastic jump intensity, (2) one-side disaster risk, and (3) two-side disaster risk.

[TO BE COMPLETED]

B Solution of the Bansal and Yaron (2004) model

The model for consumption growth with stochastic variance is

$$\begin{aligned}
\log g_{t,t+1} &= g + x_t + \gamma_g v_t^{1/2} \varepsilon_{gt+1}, \\
x_{t+1} &= \varphi_x x_t + \gamma_x v_t^{1/2} \varepsilon_{xt+1}, \\
v_{t+1} &= (1-\varphi_v) + \varphi_v v_t + \sigma_v ((1-\varphi_v + 2\varphi_v v_t)/2)^{1/2} \varepsilon_{vt+1}.
\end{aligned}$$

A representative agent has recursive preferences

$$\begin{aligned}
U_t &= [(1-\beta)c_t^\rho + \beta \mu_t(U_{t+1})^\rho]^{1/\rho}, \qquad (25) \\
\mu_t(U_{t+1}) &= [E_t(U_{t+1}^\alpha)]^{1/\alpha}.
\end{aligned}$$

Divide expression (25) by c_t, denote $u_t = U_t/c_t$ and $g_{t,t+1} = c_{t+1}/c_t$ and obtain

$$u_t = [(1-\beta) + \beta \mu_t (u_{t+1} g_{t+1})^\rho]^{1/\rho}, \qquad (26)$$

Solve a recursive problem that is a log-linear approximation of the Bellman equation (26)

$$\log u_t \approx b_0 + b_1 \log \mu_t (g_{t+1} u_{t+1}),$$

where

$$\begin{aligned}
b_1 &= \beta e^{\rho \log \mu} / (1 - \beta + \beta e^{\rho \log \mu}), \\
b_0 &= \frac{1}{\rho} \log \left((1-\beta) + \beta e^{\rho \log \mu} \right) - b_1 \log \mu.
\end{aligned}$$

Guess the value function

$$\log u_t = u + p_x x_t + p_v v_t.$$

Compute

$$\log u_{t+1} + \log g_{t+1} = u + g + (p_x\varphi_x + 1)x_t + p_v v_{t+1} + \gamma_g v_t^{1/2}\varepsilon_{gt+1} + p_x\gamma_x v_t^{1/2}\varepsilon_{xt+1}$$

Recall that the cumulant generating function for the variance represented by a gamma autoregressive process of order one is

$$\kappa(s; v_{t+1}) = \frac{\varphi_v s}{1 - s\sigma_v^2/2} v_t - \frac{(1-\varphi_v)\log(1 - s\sigma_v^2/2)}{\sigma_v^2/2}.$$

Compute

$$\begin{aligned}\log \mu_t(u_{t+1}g_{t+1}) &= u + g - \frac{(1-\varphi_v)\log(1-\alpha p_v\sigma_v^2/2)}{\alpha\sigma_v^2/2} + (p_x\varphi_x + 1)x_t \\ &+ \left(\frac{\alpha}{2}\gamma_g^2 + \frac{\alpha}{2}p_x^2\gamma_x^2 + \frac{\varphi_v p_v}{1-\alpha p_v\sigma_v^2/2}\right)v_t.\end{aligned}$$

Hence

$$\log\mu = u + g + \frac{\alpha}{2}p_x^2\gamma_x^2 + \frac{\alpha}{2}\gamma_g^2 + \frac{p_v\varphi_v}{1-\alpha p_v\sigma_v^2/2} - \frac{(1-\varphi_v)\log(1-\alpha p_v\sigma_v^2/2)}{\alpha\sigma_v^2/2}.$$

Solve the following system of three equations in three unknowns to verify the guess of the value function

$$\begin{aligned}u &= \frac{1}{1-b_1}\left(b_0 + b_1 g - b_1\frac{(1-\varphi_v)\log(1-\alpha p_v\sigma_v^2/2)}{\alpha\sigma_v^2/2}\right), \\ p_x &= \frac{b_1}{1-b_1\varphi_x}, \\ p_v &= b_1\left(\frac{\alpha}{2}\gamma_g^2 + \frac{\alpha}{2}p_x^2\gamma_x^2 + \frac{\varphi_v p_v}{1-\alpha p_v\sigma_v^2/2}\right).\end{aligned}$$

The quadratic equation for p_v has two roots. I choose root that satisfies the requirement of stochastic stability (Hansen, 2012).

Compute

$$\begin{aligned}\log(u_{t+1}g_{t+1}) - \log\mu_t(u_{t+1}g_{t+1}) &= \frac{(1-\varphi_v)\log(1-\alpha p_v\sigma_v^2/2)}{\alpha\sigma_v^2/2} - \left(\frac{\alpha}{2}\gamma_g^2 + \frac{\alpha}{2}p_x^2\gamma_x^2 + \frac{\varphi_v p_v}{1-\alpha p_v\sigma_v^2/2}\right)v_t \\ &+ p_v v_{t+1} + \gamma_g v_t^{1/2}\varepsilon_{gt+1} + p_x\gamma_x v_t^{1/2}\varepsilon_{xt+1}.\end{aligned}$$

The pricing kernel is

$$\begin{aligned}
\log m_{t,t+1} &= \log\beta + (\rho-1)\log g_{t,t+1} + (\alpha-\rho)(\log(u_{t+1}g_{t+1}) - \log\mu_t(u_{t+1}g_{t+1})) \\
&= \underbrace{\log\beta + (\rho-1)g + (\alpha-\rho)\frac{(1-\varphi_v)\log(1-\alpha p_v\sigma_v^2/2)}{\alpha\sigma_v^2/2}}_{\log m} \\
&+ \underbrace{(\rho-1)}_{m_x} x_t \underbrace{-(\alpha-\rho)\left(\frac{\alpha}{2}\gamma_g^2 + \frac{\alpha}{2}p_x^2\gamma_x^2 + \frac{\varphi_v p_v}{1-\alpha p_v\sigma_v^2/2}\right)}_{m_v} v_t \\
&+ \underbrace{(\alpha-\rho)p_v}_{m_{\varepsilon v}} v_{t+1} + \underbrace{(\alpha-1)\gamma_g}_{m_{\varepsilon g}} v_t^{1/2}\varepsilon_{gt+1} + \underbrace{(\alpha-\rho)p_x\gamma_x}_{m_{\varepsilon x}} v_t^{1/2}\varepsilon_{xt+1} \\
&= \log m + m_x x_t + m_v v_t + m_{\varepsilon g}v_t^{1/2}\varepsilon_{gt+1} + m_{\varepsilon x}v_t^{1/2}\varepsilon_{xt+1} + m_{\varepsilon v}v_{t+1}.
\end{aligned}$$

The model for the dividends is

$$\log d_{t,t+1} = \log d + \mu_x x_t + \gamma_d v_t^{1/2}\varepsilon_{dt+1}.$$

Guess that the price-dividend ratio is

$$\log(p_t/d_t) = q + q_x x_t + q_v v_t.$$

The log-linearized return is

$$\begin{aligned}
\log r_{t+1} &= \kappa_0 + \kappa_1\log(p_{t+1}/d_{t+1}) + \log d_{t,t+1} - \log(p_t/d_t) \\
&= \underbrace{k_0 + (k_1-1)q + \log d}_{\log r} + \underbrace{(\mu_x + q_x(k_1\varphi_x - 1)}_{r_x} x_t \underbrace{-q_v}_{r_v} v_t \\
&+ \underbrace{k_1 q_v}_{r_{\varepsilon v}} v_{t+1} + \underbrace{k_1 q_x\gamma_x}_{r_{\varepsilon x}} v_t^{1/2}\varepsilon_{xt+1} + \underbrace{\gamma_d}_{r_{\varepsilon d}} v_t^{1/2}\varepsilon_{dt+1},
\end{aligned}$$

where

$$\begin{aligned}
\kappa_0 &= \log(1+p/d) - \frac{\log(p/d)\cdot p/d}{1+p/d}, \\
\kappa_1 &= \frac{p/d}{1+p/d}, \\
p/d &= E(p_t/d_t).
\end{aligned}$$

Or in compact form

$$\log r_{t+1} = \log r + r_x x_t + r_v v_t + r_{\varepsilon x}v_t^{1/2}\varepsilon_{xt+1} + r_{\varepsilon d}v_t^{1/2}\varepsilon_{dt+1} + r_{\varepsilon v}v_{t+1}.$$

Use the law of one price $E_t[m_{t,t+1} r_{t+1}] = 1$ to obtain

$$\log r + \log m - \frac{(1-\varphi_v)\log\left(1-(r_{\varepsilon v}+m_{\varepsilon v})\sigma_v^2/2\right)}{\sigma_v^2/2} = 0$$
$$r_x + m_x = 0,$$
$$r_v + m_v + \frac{1}{2}m_{\varepsilon g}^2 + \frac{1}{2}(r_{\varepsilon x}+m_{\varepsilon x})^2 + \frac{1}{2}r_{\varepsilon d}^2 + \frac{\varphi_v(r_{\varepsilon v}+m_{\varepsilon v})}{1-(r_{\varepsilon v}+m_{\varepsilon v})\sigma_v^2/2} = 0.$$

Solve for q, q_x, and q_v:

$$q_x = \frac{\mu_x+\rho-1}{1-k_1\varphi_x},$$
$$-q_v + \frac{\varphi_v(k_1 q_v + m_{\varepsilon v})}{1-(k_1 q_v + m_{\varepsilon v})\sigma_v^2/2} + D = 0,$$
$$q = \frac{1}{1-k_1}\left(k_0 + \log d + \log m - \frac{(1-\varphi_v)\log\left(1-(r_{\varepsilon v}+m_{\varepsilon v})\sigma_v^2/2\right)}{\sigma_v^2/2}\right),$$

where

$$D = m_v + m_{\varepsilon g}^2/2 + (r_{\varepsilon x}+m_{\varepsilon x})^2/2 + r_{\varepsilon d}^2/2.$$

The quadratic equation for q_v has two roots. I choose one that satisfies the requirement of stochastic stability (Hansen, 2012).

C Solution of the Wachter (2013) model

The model with time-varying consumption disasters is

$$\begin{aligned}
\log g_{t+1} &= \log g + \gamma_g \varepsilon_{gt+1} - z_{t+1},\\
j_{gt+1}|_{\lambda_t} &\sim \text{Poisson}(h_\lambda \lambda_t),\\
z_{t+1}|_{j_{gt+1}} &\sim \text{Gamma}(j_{gt+1}, \theta_g),\\
\lambda_{t+1} &= (1-\varphi_\lambda) + \varphi_\lambda \lambda_t + \sigma_\lambda[((1-\varphi_\lambda)+2\varphi_\lambda\lambda_t)/2]^{1/2}\varepsilon_{\lambda t+1},\\
\log(d_{t+1}/d_t) &= \log d + \varphi_d \gamma_g \varepsilon_{gt+1} + \gamma_d \varepsilon_{dt+1} - \varphi_d z_{t+1}.
\end{aligned}$$

Assume the value function

$$\log u_{t+1} = \log u + p_\lambda \lambda_{t+1}.$$

Compute

$$\begin{aligned}
\log u_{t+1} + \log g_{t+1} &= (\log u + \log g) + \gamma_g \varepsilon_{gt+1} + p_\lambda \lambda_{t+1} - z_{t+1}, \\
\log \mu_t(u_{t+1} g_{t+1}) &= (\log u + \log g) + \frac{\alpha \gamma_g^2}{2} + \frac{\varphi_\lambda \lambda_t p_\lambda}{1 - \alpha p_\lambda \sigma_\lambda^2/2} - \frac{(1-\varphi_\lambda)\log(1 - \alpha p_\lambda \sigma_\lambda^2/2)}{\alpha \sigma_\lambda^2/2} \\
&- \frac{\theta_g h_\lambda \lambda_t}{1 + \alpha \theta_g}, \\
\log u_{t+1} g_{t+1} - \log \mu_t(u_{t+1} g_{t+1}) &= \gamma_g \varepsilon_{gt+1} + p_\lambda \lambda_{t+1} - z_{t+1} - \frac{\varphi_\lambda p_\lambda \lambda_t}{1 - \alpha p_\lambda \sigma_\lambda^2/2} - \frac{\alpha \gamma_g^2}{2} \\
&+ \frac{(1-\varphi_\lambda)\log(1 - \alpha p_\lambda \sigma_\lambda^2/2)}{\alpha \sigma_\lambda^2/2} + \frac{\theta_g h_\lambda \lambda_t}{1 + \alpha \theta_g}.
\end{aligned}$$

Solve for the parameters of the value function.

There is a quadratic equation for p_λ:

$$C_2 p_\lambda^2 + C_1 p_\lambda + C_0 = 0,$$

where

$$\begin{aligned}
C_2 &= \alpha \sigma_\lambda^2/2, \\
C_1 &= b_1 \varphi_\lambda - 1 - A\alpha \sigma_\lambda^2/2, \\
C_0 &= A, \\
A &= -\frac{b_1 h_\lambda \theta_g}{1 + \alpha \theta_g}.
\end{aligned}$$

Finally

$$\log u = \frac{1}{1 - b_1}\left(b_0 + b_1 \log g + \frac{\alpha b_1 \gamma_g^2}{2} - \frac{b_1(1-\varphi_\lambda)\log(1 - \alpha p_\lambda \sigma_\lambda^2/2)}{\alpha \sigma_\lambda^2/2}\right).$$

The pricing kernel is

$$\begin{aligned}
\log m_{t+1} &= \log \beta + (\rho - 1)\log g_{t+1} + (\alpha - \rho)(\log u_{t+1} g_{t+1} - \log \mu_t(u_{t+1} g_{t+1})) \\
&= \log m + m_\lambda \lambda_t + m_{\varepsilon\lambda} \lambda_{t+1} + m_{\varepsilon g} \varepsilon_{gt+1} + m_z z_{t+1},
\end{aligned}$$

where

$$\begin{aligned}
\log m &= \log \beta + (\rho - 1)\log g + (\alpha - \rho)\frac{(1-\varphi_\lambda)\log(1 - \alpha p_\lambda \sigma_\lambda^2/2)}{\alpha \sigma_\lambda^2/2} - \frac{\alpha \gamma_g^2(\alpha - \rho)}{2}, \\
m_\lambda &= -(\alpha - \rho)\left(\frac{\varphi_\lambda p_\lambda}{1 - \alpha p_\lambda \sigma_\lambda^2/2} - \frac{\theta_g h_\lambda}{1 + \alpha \theta_g}\right), \\
m_{\varepsilon g} &= (\alpha - 1)\gamma_g, \\
m_z &= -(\alpha - 1), \\
m_{\varepsilon\lambda} &= (\alpha - \rho) p_\lambda.
\end{aligned}$$

Guess that the price-dividend ratio is

$$\log(p_{t+1}/d_{t+1}) \quad = \quad q_0 + q_\lambda \lambda_{t+1}.$$

The log return is

$$\begin{aligned}
\log r_{t+1} &= k_0 + k_1 \log(p_{t+1}/d_{t+1}) + \log(d_{t+1}/d_t) - \log(p_t/d_t) \\
&= k_0 + k_1 q_0 - q_0 + \log d \\
&- q_\lambda \lambda_t \\
&+ k_1 q_\lambda \lambda_{t+1} \\
&+ \varphi_d \gamma_g \varepsilon_{gt+1} \\
&+ \gamma_d \varepsilon_{dt+1} \\
&- \varphi_d z_{t+1} \\
&= \log r + r_\lambda \lambda_t + r_{\varepsilon\lambda} \lambda_{t+1} + r_{\varepsilon g} \varepsilon_{gt+1} + r_z z_{t+1} + r_{\varepsilon d} \varepsilon_{dt+1}.
\end{aligned}$$

where

$$\begin{aligned}
\log r &= k_0 + k_1 \log q_0 - \log q_0 + \log d, \\
r_\lambda &= -q_\lambda, \\
r_{\varepsilon\lambda} &= k_1 q_\lambda, \\
r_{\varepsilon g} &= \varphi_d \gamma_g, \\
r_{\varepsilon d} &= \gamma_d, \\
r_z &= -\varphi_d.
\end{aligned}$$

Use the law of one price

$$E_t(r_{t+1} m_{t+1}) = 1,$$

to solve for q_0 and q_λ.

The two equations are

$$\log r + \log m - \frac{(1-\varphi_\lambda)\log(1-(m_{\varepsilon\lambda}+r_{\varepsilon\lambda})\sigma_\lambda^2/2)}{\sigma_\lambda^2/2} + \frac{1}{2}(m_{\varepsilon g} + r_{\varepsilon g})^2 + \frac{1}{2} r_{\varepsilon d}^2 = 0,$$

$$m_\lambda + r_\lambda + \frac{\varphi_\lambda(m_{\varepsilon\lambda}+r_{\varepsilon\lambda})}{1-(m_{\varepsilon\lambda}+r_{\varepsilon\lambda})\sigma_\lambda^2/2} + \frac{(m_{zg}+r_{zg})h_\lambda\theta_g}{1-(m_{zg}+r_{zg})\theta_g} = 0.$$

q_λ is the solution to the quadratic equation

$$C_0 q_\lambda^2 + C_1 q_\lambda + C_2 = 0,$$

where

$$
\begin{aligned}
A &= m_\lambda + \frac{(m_{zg} + r_{zg})h_\lambda \theta_g}{1 - (m_{zg} + r_{zg})\theta_g}, \\
C_0 &= k_1 \sigma_\lambda^2/2, \\
C_1 &= k_1 \varphi_\lambda + m_{\varepsilon\lambda}\sigma_\lambda^2/2 - 1 - k_1 A \sigma_\lambda^2/2, \\
C_2 &= \varphi_\lambda m_{\varepsilon\lambda} + A - A m_{\varepsilon\lambda}\sigma_\lambda^2/2.
\end{aligned}
$$

Also

$$
q_0 = \frac{1}{1-k_1}\left(k_0 + \log d + \log m - \frac{(1-\varphi_\lambda)\log\left(1 - (m_{\varepsilon\lambda} + r_{\varepsilon\lambda})\sigma_\lambda^2/2\right)}{\sigma_\lambda^2/2} + \frac{1}{2}(m_{\varepsilon g} + r_{\varepsilon g})^2 + \frac{1}{2} r_{\varepsilon d}^2\right).
$$