# An Equilibrium Search Model of Fire Sales

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**Abstract**

This paper presents a novel approach to the theory of fire sales, where an asset is sold for a price below its fundamental value. Specifically, we model the "urgency to sell" motivation that pushes some unlucky sellers in the market to become anxious (desperate) about liquidating their asset because failing to do so would cause them to incur a cost. Homogenous buyers strategically choose when to enter the market and what price to offer, knowing that some sellers are more motivated, but do not know the type of the seller they meet. We characterize equilibrium prices in this economy both in the steady state and on the transition path as the economy converges to a new steady state after a permanent demand shock. We find that prices oscillate, rather than monotonically converging to the steady state. Moreover, the dynamic behavior of prices indicate overshooting, where sometimes prices fall below their long run equilibrium value and rise above it, depending on the proportion of desperate to relaxed sellers as well as the number of buyers that would clear the market.

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JEL Classification: D83

## 1 Introduction

A fire sale is the forced sale of an asset at a dislocated price (Schleifer and Vishny, 2011), meaning the sales price is well below the fundamental value of an asset. For example, a bank or a homeowner might need to liquidate assets to cover short-term borrowing costs. Campbell, *et al* (2011) report that such assets sell at a 27% discount relative to the market value. Another good example is airlines selling airplanes to

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pay back loans. Indeed, according to Pulvino (1998), the discount for those airplanes is about 10-20%. Finally, hedge funds often find themselves obligated to sell assets because they have to pay back clients withdrawal demands. What is common to all the examples one can think of, is the fact that some sellers in the market face strict deadlines to fulfill their obligations, which in turn motivates them to sell in a hurry. In other words, the reservation prices for these sellers will be below those who do not face such deadlines and therefore feel relaxed.

In this paper, we analyze equilibrium behavior in a dynamic economy with homogenous buyers and sellers who are all patient (or relaxed) initially. Some sellers then experience random shocks that make them desperate to liquidate their assets. These desperate sellers incur a cost if they fail to sell. The type of a seller (relaxed or desperate) is private information. Buyers strategically choose when to enter the market and what price to offer, knowing that there are some sellers in the market who are more motivated than others. Meetings between buyers and sellers happen randomly. Upon meeting, buyer makes it a take-it-or-leave-it offer to the seller. This offer can either be the low-price that will only be accepted by a desperate seller, or a high-price that will be accepted by both types. If the seller accepts an offer, both the buyer and the seller exit the market. If the offer is turned down, the buyer keeps searching and the seller waits to entertain other offers.

In this environment, when the economy experiences a shock that temporarily increase the share of desperate sellers in the market (as it happens in a financial crisis), we find that both the low price and the high price initially go below their steady state value. The number of buyers (who are strategic about when to enter the market) increases increase as a response to falling prices. However, as more sales happen at the low price, the fraction of desperate sellers in the market falls, slowly leading to higher prices. As prices go up, fewer buyers decide to enter, causing fewer transactions to happen, pushing prices down again. These dynamics continue until the prices, the number of buyers, and the fraction of desperate sellers in the market return back to their steady-state value. Strikingly, the possibility of some relaxed sellers to fall into desperation causes these oscillations in equilibrium prices  without this feature, we find that the transition to the steady state would be monotonic.

In a second experiment, we analyze the dynamic effects of a permanent demand shock. More specifically, we permanently reduce the value to the buyers of acquiring the asset. In this case, the number of buyers as well as the prices initially go down. As a response, the fraction of those sellers who are desperate grows. But, this causes further increases in the number of buyers who enter the market, leading to more transactions happening at the lower price, slowly eliminating the stock of the desperate. Hence, prices start going up again. These dynamics bring the economy to a new steady state, where both prices are lower, there are more buyers in the market, and the fraction of desperate sellers is lower than the initial steady state.

Our main contribution to the literature is three-fold. First, our model and its solution analyze not just the steady state behavior of endogenously generated prices under private information, but we go further and characterize the dynamic behavior fully on the transition path. Albrecht and Axell (1984), Akin and Platt (2012, 2014), Albrecht and Vroman (2005), and Albrecht, *et al* (2007) are search models that analyze equilibrium prices in the steady state. Second, and perhaps more importantly, the structure we put in the model entails a non-monotonic convergence to the steady state, in line with the empirical evidence on asset prices observed during downturns of the economy. For example, in the Great Recession, the price of housing showed a non-monotonic behavior. This latter feature is absent from the dynamic pricing literature. A related work is Guren and McQuade (2013), which examines how foreclosures exacerbate downturns in the housing market; however, equilibrium prices in their model exhibit monotonic transition. Third, our model offers a propagation mechanism the likelihood of falling into desperation pushes the sellers to accept prices that they otherwise would not, hence this random shock that separates the ex-ante homogenous sellers into two types (ex-post) generates a low price and a high price in the economy that oscillate as exogenous shocks lead to a bigger number of anxious sellers for a while and buyers strategically choosing the best time to enter the market, anticipating these changes in prices of the asset.

## 2 Baseline Model

Consider a continuous time environment, with infinitesimal buyers and sellers. There are two types of sellers: relaxed (entering the market at rate $\eta$) and desperate (entering at rate $\delta$). Either type only exits after selling their single unit of an indivisible good, and seller type is private information. A continuum of ex-ante homogeneous potential buyers can enter (or exit) the market freely. We denote the measure of relaxed sellers in the market at time $t$ as $h_r(t)$, desperate sellers as $h_d(t)$, total sellers

$h(t) \equiv h_r(t)+h_d(t)$, and buyers as $g(t)$. All endogenous variables may potentially vary over time in the equilibrium solution; for notational simplicity, we omit the function of time and use Newton's notation for time derivatives.

Meetings occur with a Cobb-Douglas matching function with equal weights: $m \equiv \Psi g^{\frac{1}{2}} h^{\frac{1}{2}}$. Thus, a seller encounters buyers at rate $\lambda \equiv \frac{m}{h}$, while buyers encounter sellers at rate $\alpha \equiv \frac{m}{g}$. On meeting, buyers make a take-it-or-leave-it offer; if rejected, both parties continue their search, with no recall of past offers. All agents discount at rate $\rho$.

## 2.1 Buyers

Buyers pay a search cost $k$ for each unit of time on the market, and after purchasing the asset from any source, enjoys instantaneous value $y$ from the good perpetually. On meeting a seller, the buyer must decide whether to offer price $p_d$, which only desperate sellers are willing to accept, or price $p_r$, which any seller will accept. Since he cannot distinguish seller types, he faces the tradeoff that the former will cost him less but is less likely to be accepted. Specifically, the fraction who would accept the lower price is denoted:

$$\phi \equiv \frac{h_d}{h_r + h_d}. \tag{1}$$

Let $B_r$ denote the present value of expected profit for a buyer that always offers $p_r$. This expected value can be recursively expressed as:

$$\rho B_r = -k + \dot{B}_r + \alpha \left( \frac{y}{\rho} - p_r - B_r \right). \tag{2}$$

Here, time is a state variable in case the rate of meeting or the offered price change over time. Since the state variable increases steadily over time, $\dot{B}_r$ captures the change in profit due to the state variable. In the last term, meetings occur at rate $\alpha$, and since the proposed price is always accepted, the buyer will stop searching $(-B_r)$ and realize profit of selling $(\frac{y}{\rho} - p_r)$.

If the buyer instead offers $p_d$, the present value of expected profit $B_d$ is expressed as:

$$\rho B_d = -k + \dot{B}_d + \alpha\phi \left( \frac{y}{\rho} - p_d - B_d \right). \tag{3}$$

Note that this recursive formulation is equivalently except in two aspects. First,

$\phi$ reflects the fact that offers may be rejected (by all relaxed sellers). Second, the realized profit is greater $(\frac{y}{\rho} - p_d)$.

We express the buyer's strategy at each point in time as the probability $\mu \in [0, 1]$ of offering $p_d$. Individual rationality requires that $\mu = 1$ if $B_d > B_r$ and $\mu = 0$ if $B_d < B_r$. When a mixed strategy is employed, this can be interpreted as randomization by the individual buyer, or as a fraction of the buyer population who always offers the desperate price.

## 2.2 Sellers

The asset to be sold still provides income to the seller until it is sold: relaxed sellers collect $x$ per unit of time, while desperate sellers collect $x-c$, where $c > 0$. Recall that the asset is homogenous from the perspective of the buyers; so any difference in the asset value is idiosyncratic to the seller's needs rather than the asset's fundamental productivity. We assume that $y \geq x$, so that it sales by relaxed sellers are efficient; indeed, if this were not the case, all relaxed sellers could be ignored in this market.

Let $V_d$ denote the present value of expected utility for a desperate seller at time $t$, which can be recursively expressed as:

$$\rho V_d = x - c + \dot{V}_d + \lambda \left[\mu \left(p_d - V_d\right) + (1 - \mu) \left(p_r - V_d\right)\right]. \tag{4}$$

Note that desperate sellers are willing to accept either offered price. Indeed, the equilibrium desperate price is chosen such that the seller is indifferent between accepting the offer or continuing her search:

$$p_d = V_d. \tag{5}$$

Similarly, let $V_r$ denote the present value of expected utility for a relaxed seller at time $t$, which gives us:

$$\rho V_r = x + \dot{V}_r + \lambda(1 - \mu) \left(p_r - V_r\right). \tag{6}$$

Here, the equilibrium price will push the relaxed seller to indifference:

$$p_r = V_r. \tag{7}$$

As a consequence, $p_r > p_d$, and thus it is always optimal to reject desperate price offers.

## 2.3 Population Dynamics

The fraction of desperate sellers in the market is particularly important in this model, since buyers react to this fraction in their frequency desperate price offers. We thus track how the stock of sellers in the market adjusts over time. First, consider the population of desperate sellers. These enter the market at rate $\delta$, but they accept any offer, and thus exit the market at rate $\lambda h_d$. Thus, the net change in the population of desperate sellers is:

$$\dot{h}_d = \delta - \lambda h_d. \tag{8}$$

Relaxed sellers, on the other hand, enter the market at rate $\eta$, but only accept relaxed price offers, exiting at rate $\lambda(1-\mu)$. Thus, the net change in the population of relaxed sellers is:

$$\dot{h}_r = \eta - \lambda(1-\mu)h_r. \tag{9}$$

## 2.4 Equilibrium Definition

For a given initial seller population $h_r(0)$ and $h_d(0)$, a search equilibrium consists of price functions $p_r$ and $p_d$, seller expected utility functions $V_r$ and $V_d$, buyer expected profit functions $B_r$ and $B_d$, population functions $h_r$, $h_d$ and $g$, and buyer strategy function $\mu$, such that:

1. Prices are optimally set for their respective targets (Eqs. 5 and 7).
2. Seller utility correctly anticipates market conditions (Eqs. 4 and 6).
3. Seller populations obey the law of motion (Eqs. 8 and 9).
4. Buyers cannot earn strictly positive expected profits: $B_r \leq 0$ and $B_d \leq 0$.
5. Buyers use optimal pricing strategies: $B_r = 0$ if $\mu < 1$, and $B_d = 0$ if $\mu > 0$.

The fourth requirement arises due to the free entry of buyers. If positive profits were expected, more buyers would enter the market, driving down the rate of meeting $\alpha$ until profits are again zero.

Indeed, the last fact implies that whenever $\mu < 1$ for some open interval of time, then $B_r = 0$ and thus $\dot{B}_r = 0$. Thus, from Eq. 2, we find that

$$\alpha = \frac{\rho k}{y - \rho p_r} \text{ if } \mu < 1. \tag{10}$$

Similarly, if $\mu > 0$, then $B_d = 0$ and $\dot{B}_d = 0$, so:

$$\alpha = \frac{\rho k}{\phi(y - \rho p_d)} \text{ if } \mu > 0. \tag{11}$$

When both apply, then we obtain:

$$p_r = \phi p_d + \frac{y(1-\phi)}{\rho} \text{ if } \mu \in (0,1). \tag{12}$$

## 2.5 Steady State Equilibrium

The preceding equilibrium definition applies for any initial condition, including the steady state equilibrium. For that, we also solve for the initial conditions $h_r(0)$ and $h_d(0)$ such that $\dot{h}_r = \dot{h}_d = 0$ for all $t$. Two possibilities exist, depending on parameter values. In one, only the relaxed sellers are targeted, because there are too few desperate sellers to be worth offering $p_d$. In the other, both prices are offered in equilibrium. The third option, where only desperate prices are offered, is not possible in steady state, because relaxed sellers would never exit, leading to $h_r$ growing without bound. The solutions are reported below in Table 1.

The degenerate equilibrium can only occur if $B_r \leq 0$ under the proposed solution. This works out to be equivalent to requiring that $c\delta k\rho^2 - \eta\psi^2(y-x)^2 \leq \eta k\rho^2(y-x)$. Note that if this holds, then the dispersed solution cannot exist, since the proposed $\mu$ would be weakly negative. Indeed, the steady state solution is unique, with either the degenerate or dispersed occurring, but not both. In particular, if the preceding condition does not hold, then the proposed dispersed $\mu$ is both strictly positive and strictly less than 1, while the proposed $h_r$ is strictly positive.

## 2.6 Dynamic Equilibrium

We can also solve for the dynamic equilibrium path for any particular deviation from steady state. In particular, we can consider two classes of shocks: temporary or

Table 1: Steady State Solution

| | Dispersed | Degnerate |
|---|---|---|
| $p_r$ | $\frac{x}{\rho}$ | |
| $p_d$ | $\frac{x-c}{\rho}+\frac{\eta(y-x)^2\psi^2}{\delta k\rho^3}$ | $\frac{x}{\rho}-\frac{\rho ck}{k\rho^2+(y-x)\psi^2}$ |
| $h_r$ | $\frac{\rho\delta ck}{(y-x)^2\psi^2}-\frac{\eta}{\rho}$ | $\frac{\rho\eta k}{(y-x)\psi^2}$ |
| $h_d$ | $\frac{\rho\delta k}{(y-x)\psi^2}$ | |
| $\mu$ | $1-\frac{\eta k(y-x)\rho^2}{c\delta k\rho^2-\eta\psi^2(y-x)^2}$ | $0$ |
| $\phi$ | $\frac{\delta k(y-x)\rho^2}{\delta k\rho^2(y-x+c)-\eta\psi^2(y-x)^2}$ | $\frac{\delta}{\delta+\eta}$ |
| $g$ | $\frac{\delta k\rho^2(y-x+c)-\eta\psi^2(y-x)^2}{k^2\rho^3}$ | $\frac{(\delta+\eta)(y-x)}{k\rho}$ |

permanent. For temporary shocks, the market receives a sudden infusion of sellers of one type or the other; that is, we start from an $h_r(0)$ or $h_d(0)$ away from steady state and follow the market behavior thereafter. This does not affect the long run steady state to which the market will eventually return, but the frictions inherent in random matching will prevent this from occurring immediately. Indeed, immediately after the one-time shock occurs, all participants will correctly anticipate the future path of prices and the stock of sellers, but they cannot arbitrage price differences over time because of the slow pace of finding trading partners — all such opportunities are already priced into the anticipated price adjustments.

For a permanent shock, we begin with both initial seller populations at their steady state levels under one set of parameters, then consider what would occur if one of the parameters were to unanticipatedly and permanently change. Again, the population will slowly adjust towards the new steady state, with all participants correctly anticipating the dynamic equilibrium transition path.

Regardless of the type of shock, the prices targeting relaxed sellers are the same. If we substitute Eq. 7 into Eq. 6, we find that at any time, the following must hold:

$$\rho p_r = x + \dot{p}_r. \tag{13}$$

Table 2: Dynamic Transition Equations

| | Desperate Degenerate | Dispersed | Relaxed Degnerate |
|---|---|---|---|
| $p_r$ | $p_r = \frac{x}{\rho}$ | | |
| $p_d$ | $\dot{p}_d = \left(1 + \frac{(1-\mu)(y-x)\psi^2}{\rho^2 k}\right)(\rho p_d - x) + c$ | | |
| $h_r$ | $\dot{h}_r = \eta - \frac{\psi^2}{\rho k}(y-x)h_r$ | | |
| $h_d$ | $\dot{h}_d = \delta - \frac{\psi^2 \phi}{\rho k}(y - \rho p_d)h_d$ | $\dot{h}_d = \delta - \frac{\psi^2}{\rho k}(y-x)h_d$ | |
| $\mu$ | $\mu = 1$ | $y - \rho p_r = \phi\,(y - \rho p_d)$ | $\mu = 0$ |
| $\phi$ | $\phi = \frac{h_d}{h_d + h_r}$ | | |

This differential equation only has one solution that converges: $p_r = \frac{x}{\rho}$ for all time. Any other solution would diverge to $+\infty$ or $-\infty$. Intuitively, the relaxed sellers have no reason to accept a lower price, since this exactly replaces their current utility; and buyers have no reason to make a larger take-it-or-leave it offer, since everyone will willing to accept that price.

The solutions for the remaining equations must be considered in cases. Initially, the shock will lead to an imbalance in the number of either relaxed and desperate sellers, which will make it profitable for buyers to strictly target the group that is more plentiful relative to steady state. We refer to this as a *desperate degenerate* or *relaxed degenerate* phase, depending on the group that is targeted. If the steady state yields a dispersed equilibrium, eventually, it will return to the *dispersed* phase as it approaches (or in some cases, reaches) steady state.

Note that $\mu$ and $\phi$ can be eliminated through substitution. In a dispersed equilibrium, $\mu$ is found from the equation governing $p_d$, but this must be consistent with both price offers being equally profitable, which is the condition listed in the $\mu$ row and Dispersed column. Despite the system of three first-order differential equations (in $p_d$, $h_r$, and $h_d$), the solution technique is fairly straight forward.

We begin by assuming that we enter the dispersed equilibrium at some endogenous time $T$ with arbitrary initial conditions $h_d(T)$ and $h_r(T)$. These in turn pin down the initial condition $p_d(T)$ because of $\phi$. The differential equation for $h_d$ can then be solved independent of the other two systems of equation. We then take the first derivative of the equation governing $p_d$, and substitute for $p_d$ and $\dot{p}_d$ in $\mu$ and therefore $h_r$. This leaves us with a first-order differential equation of $h_r$. In order for the solution to $h_r$ to approach a finite number as $t \to \infty$, this will require a particular initial condition $h_r(T)$.

Since this required initial condition is not generically equal to the $h_r(0)$ associated with an exogenous shock, we then compute an initial phase for $t \in [0, T)$, which will be desperate degenerate if $h_r(0) < h_r(T)$ and relaxed degenerate if $h_r(0) > h_r(T)$. In either case, the differential equation $h_r$ can be solved independently from the exogenous initial condition $h_r(0)$, as can $p_d$ up to an arbitrary terminal price $p_d(T)$. Then $h_d$ can also be solved from initial condition $h_d(0)$

The process concludes by using the continuity of $h_r$ and $h_d$ at time $T$ to solve for $T$ and $p_d(T)$. That is, we solve for $T$ such that $h_r$ yields the same population of relaxed sellers at time $T$ whether approached from above or below. Once that is determined, we set the terminal price $p_d(T)$ so that the population of desperate sellers at time $T$ is also continuous. The equilibrium price $p_d$, on the other hand, need not be continuous.

We now illustrate the dynamic market adjustment after several shocks. The first two illustrate a temporary shock to the population in the market without altering the steady state. The latter two give an examples of a permanent shock which leads to a new steady state, with the dynamic transition between them.

### 2.6.1 Experiment 1: Additional Relaxed Sellers

In the first experiment, suppose that the market has been at steady state, but then experiences a sudden influx of additional relaxed sellers without affecting the number of desperate sellers. This could be thought of as temporary surge in the market, with more sellers willing to part with their asset, but only at the right price.

In Figure 2.6.1, we illustrate the equilibrium dynamic response by doubling the number of relaxed sellers at time $t = 0$. In the top left panel, one can see that the market immediately draws down this excess of relaxed sellers, which declines monotonically until reaching the steady state in finite time. This decline occurs because buyers exclusively target relaxed sellers during this period ($\mu = 0$, as seen in the bottom left panel), though desperate sellers also accept such offers. Indeed, the

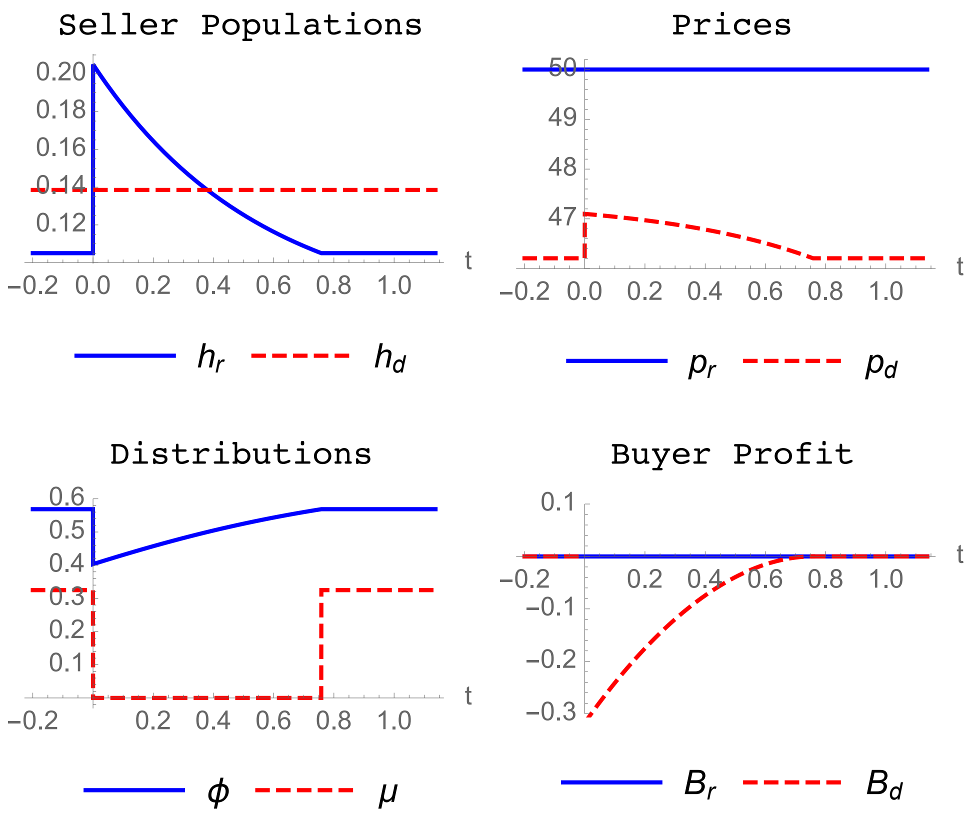


Figure 1: Dynamic response from doubling the number of relaxed sellers at time $t = 0$

shock has no effect on the population of desperate sellers throughout, as they receive and accept offers at the same rate as before.

The relaxed price is unaffected by the shock (as shown in the upper right panel), but because buyers are only offering the relaxed price, the desperate sellers have a higher reservation price during the transition. Of course, no one offers this price because it would be unprofitable, as verified in the lower right panel.

In summary, we learn that the market responds to this deviation from steady state by completely targeting the excess relaxed sellers, and returns to the prior steady state in a finite amount of time.

#### 2.6.2 Experiment 2: Additional Desperate Sellers

In the second experiment, we consider another sudden, temporary influx of sellers, but make them desperate instead. For instance, this could be thought of as a fire

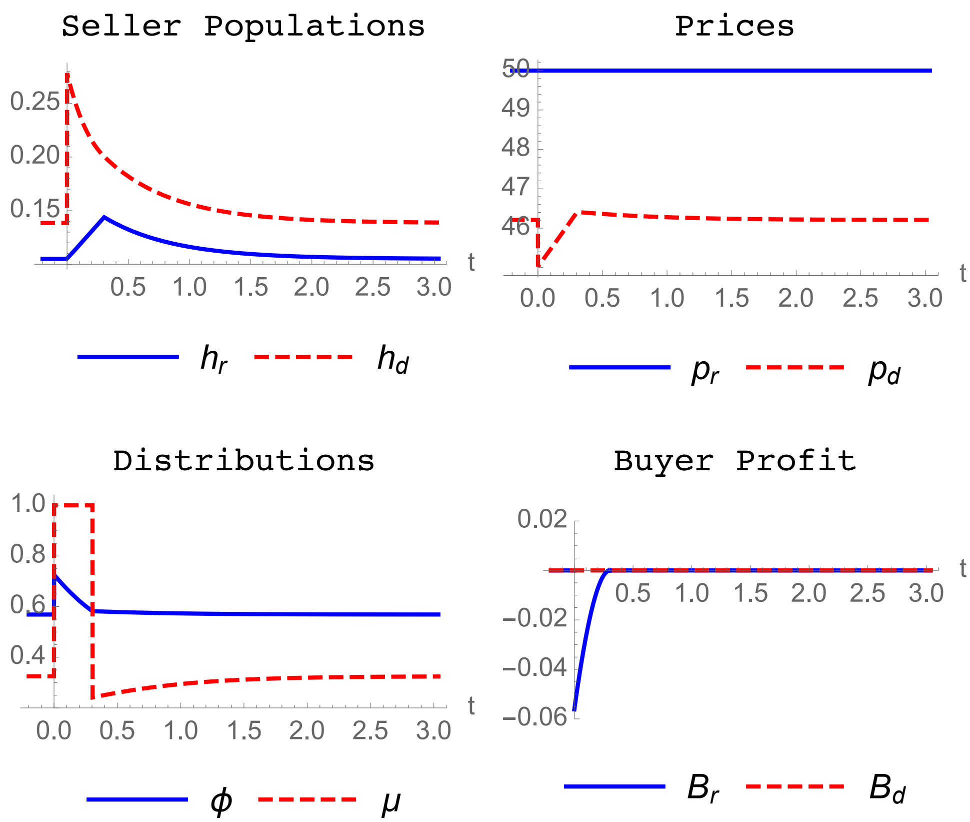


Figure 2: Dynamic response from doubling the number of desperate sellers at time $t = 0$

sale, motivated by a sudden need for liquidity that requires quick sale of an asset.

In Figure 2.6.2, we illustrate the consequence of doubling the number of desperate sellers in the market at time $t = 0$. Here, the equilibrium dynamic response differs from the previous experiment in several key ways. First, this increase in the desperate population also affect the relaxed population over time, as seen in the top left panel. This is because buyers initially target the excess desperate sellers ($\mu = 1$, as seen in the bottom left panel), meaning that no offers on the market will be acceptable to the relaxed sellers. This causes them to accumulate as new relaxed sellers continue to enter the market.

Eventually, buyers resume making some relaxed offers, and both populations begin to fall. Surprisingly, more relaxed offers occur than in the steady state. This cause the equilibrium desperate price to slightly overshoot its steady state level then gradually return. Unlike the previous experiment, the market does not reach the steady state in finite time, but only approaches it.

Bear in mind that while the shock itself was unanticipated, the dynamic transition thereafter has perfect foresight. That is, desperate sellers are perfectly aware that prices are temporarily depressed initially but will eventually recover. This foresight is built into their reservation prices, so it is still optimal to accept the depressed prices during the fire sale if offered, as the cost of waiting makes them indifferent about holding out for better market conditions.

### 2.6.3 Experiment 3: Higher Flow of Relaxed Sellers

In the third experiment, we consider a permanent increase in the flow of relaxed sellers into the market. Unlike the previous two experiments, this shifts the market to a new steady state. Indeed, if one is only interested in the eventual outcome, the comparative statics can be easily derived from the steady state solution in Table 1. For instance, as $\eta$ increases, the buyers will target more offers towards the higher flow of relaxed sellers, causing $\mu$ to rise. In fact, this re-targeting is sufficiently large that the stock of relaxed sellers $h_r$ actually falls. Also, desperate sellers benefit from the greater number of high-priced offers, and thus increase their reservation price $p_d$.

The dynamic transition reveals a richer story, as indicated in Figure 2.6.3. Indeed, in the immediate wake of the shock, buyers in the market *exclusively* target relaxed sellers, which gradually draws down the stock of relaxed buyers as they accept offers at higher than the eventual steady-state rate. This means that market conditions are even more favorable for desperate buyers during the transition — the desperate price immediately shoots up, then gradually decays to its new steady state level. As in the first experiment, the population of desperate sellers remains steady throughout the transition, because their rate of receiving offers is unchanged.

### 2.6.4 Experiment 4: Lower Buyer Utility

The fourth experiment considers permanent changes in demand for the good. Suppose that buyers suddenly get less utility from the good in question. Again, one can readily derive the comparative statics of an decrease in $y$ on the steady state from Table 1. These less-enthusiastic buyers target their offers more heavily towards desperate sellers (higher $\mu$), hoping for a low price. As a consequence, the relaxed sellers become

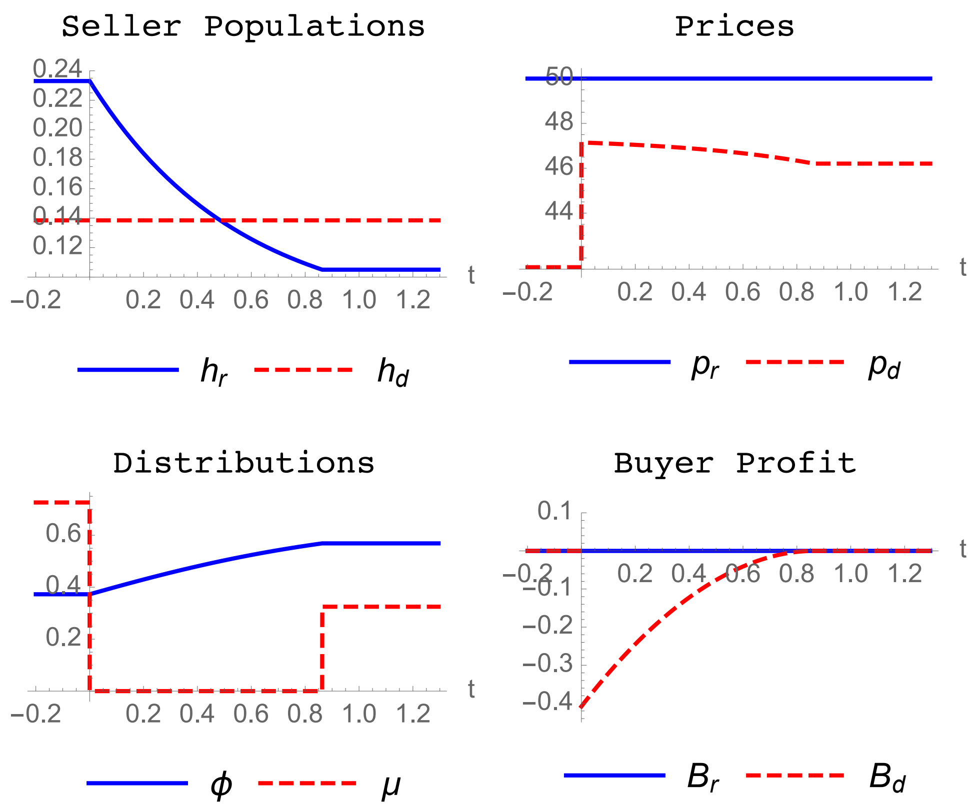


Figure 3: Dynamic response from a permanent 10% increase in the flow of relaxed sellers, starting at time $t = 0$

relatively more plentiful, and the fraction who are desperate ($\phi$) falls. This worsens the prospects for desperate sellers, causing them to reduce their reservation price.

In the dynamic equilibrium, illustrated in Figure 2.6.4, we see that immediately after the shock, buyers exclusively target the desperate sellers. In effect, buyers make "low ball" offers to everyone to compensate for their lower utility. This draws down the number of desperate sellers in the market, while the number of relaxed sellers steadily rises. After this initial strategy, buyers revert to a mixed strategy, though still targeting desperate sellers more heavily than before the shock. This causes both populations to rise towards their new steady state levels, with the fraction of desperate buyers in the market rebounding slightly.

Similarly, the offered price drops discretely at the time of the shock, then steadily recovers in the initial phase of the transition. Thereafter, it continues a very shallow climb toward the new steady state price, which is slightly lower than the pre-shock price.


Figure 4: Dynamic response from a permanent 0.2% decline in the good's value to buyers, starting at time $t = 0$

# 3 Urgency Model

Our baseline model captures heterogeneity in the market place by exogenously assuming that some sellers enter the market with greater urgency to sell. An alternative approach is to assume that all sellers enter the market in a relaxed state, but at some point they randomly become desperate. As in the previous setting, relaxed sellers are only willing to entertain the best price offers, but at some point, that same relaxed seller becomes desperate. For instance, a major life event like a wedding or job change may require liquidation of financial or real estate assets.

We refer to this as the Urgency model, since sellers (stochastically) increase in urgency over time. All sellers enter as relaxed at rate $\eta$, but at rate $\delta$ a relaxed individual becomes desperate. Many aspects of the baseline model carry over directly into the urgency model; here we note the two significant changes.

First, the steady state population dynamics now proceed as follows. Relaxed sellers enter at rate $\eta$, but can exit by accepting a relaxed offer, which are offered at rate $\lambda(1-\mu)$; or by becoming desperate, which occurs at rate $\delta$.

$$\dot{h}_r = \eta - \lambda(1-\mu)h_r - \delta h_r. \tag{14}$$

The desperate population is drawn down as the sellers get any offer (at rate $\lambda$), as before; but instead of entering at an exogenous rate $\delta$, the flow of newly desperate workers now depends on how many relaxed workers are left to increase their urgency at rate $\delta$.

$$\dot{h}_d = \delta h_r - \lambda h_d. \tag{15}$$

The other pivotal change lie in the Bellman equation for the relaxed seller; here, we must add a term that indicates the rate of increasing urgency ($\delta$) time the realized profit from doing so ($V_d - V_r$):

$$\rho V_r = x + \dot{V}_r + \lambda(1-\mu)\left(p_r - V_r\right) - \delta(V_d - V_r) \tag{16}$$

All other governing equations are simply copied from the Baseline Model, as are the equilibrium requirements.

## 3.1 Steady State Equilibrium

Again, we can readily solve for the steady state equilibrium by requiring that the derivatives with respect to time of each variable remain at 0. After substitution of $p_d$, this results in a quadratic equation for $p_r$, but one of the two solutions is easily dismissed as it results in negative populations. As in the baseline model, the steady state can either be degenerate or dispersed, but here it is possible to have a degenerate equilibrium where only the desperate are targeted. Before, this was not possible because relaxed sellers would never exit, eventually accumulating to the a population that is too profitable to ignore. Here, relaxed sellers eventually convert to desperate sellers; that that point, the will accept offers and exit the market. As one might guess, this only occurs when the rate of conversion to desperation $\delta$ is rather high.

Table 3 reports the equilibrium solution for all three cases. For more compact notation in the table, we define $\theta_d = \sqrt{1 + \frac{4\psi^2(y-x+c)}{\rho\delta k}}$, while $\theta_u = \frac{\rho^2 k}{2\psi^2}\left(1 - \sqrt{1 - \frac{4\psi^2(y-x)}{\rho^2 k}}\right)$ and $\tau_u = \frac{1}{\rho}\left(1 + \frac{\delta c}{\rho(y-x)}\right)$. Finally, let $\theta_r = y - x - \frac{\tau_r - \sqrt{4c\delta k\rho\psi^2 + \tau_r^2}}{2\psi^2}$ and $\tau_r = k\rho(\delta+\rho) + \psi^2(y-x)$.

Table 3: Urgency Model: Steady State Solution

| | Desperate Degen. | Dispersed | Relaxed Degen. |
|---|---|---|---|
| $p_r$ | $\frac{x-c}{\rho} + \frac{c}{\delta+\rho}$ | $\frac{y-\theta_u}{\rho}$ | $\frac{y-\theta_r}{\rho}$ |
| $p_d$ | $\frac{x-c}{\rho}$ | $\frac{(\rho+\delta)(y-\theta_u)-\rho x}{\rho\delta}$ | $\frac{(\rho+\delta)(y-\theta_r)-\rho x}{\rho\delta}$ |
| $h_r$ | $\frac{\eta}{\delta}$ | $\frac{\eta\psi^2(\theta_u-\rho\tau_u(\theta_u+x-y))}{\rho^3(k(\rho\tau_u-1)-\tau_u^2\psi^2(y-x))}$ | $\frac{k\eta\rho}{k\delta\rho+\theta_r\psi^2}$ |
| $h_d$ | $\frac{2\eta}{\delta(\theta_d-1)}$ | $\frac{\delta\eta\left(k\rho-\theta_u\tau_u\psi^2\right)}{\rho^3(k(\rho\tau_u-1)-\tau_u^2\psi^2(y-x))}$ | $\frac{k^2\delta\eta\rho^2}{\theta_r\psi^2(k\delta\rho+\theta_r\psi^2)}$ |
| $\mu$ | 1 | $1 + \frac{\delta k\rho}{\theta\psi^2} - \frac{k\rho^4\left(k(1-\rho\tau_u)+\tau_u^2\psi^2(y-x)\right)}{\theta\psi^4(\rho\tau_u(\theta+x-y)-\theta)}$ | 0 |
| $\phi$ | $\frac{2}{\theta_d+1}$ | $\frac{\delta\left(k\rho-\theta_u\tau_u\psi^2\right)}{\delta k\rho+\psi^2(\theta_u(1-\tau_u(\delta+\rho))+\rho\tau_u(y-x))}$ | $\frac{k\delta\rho}{k\delta\rho+\theta_r\psi^2}$ |

## 3.2 Dynamic Transition

The key distinction in solving this model is that the differential equations are interrelated. Through substitution, one can reduce this to a system of three first-order differential equations (on $h_r$, $h_d$, and $p_r$). These must be numerically solved. We illustrate behavior in the following experiments. Strikingly, the transition is no longer monotonic, but oscillates above and below the eventual steady-state values.

### 3.2.1 Experiment 5: Switching Relaxed to Desperate

We first consider a scenario in which a mass of relaxed sellers are instantaneously given added urgency, becoming desperate. As depicted in Figure 3.2.1, when there

Table 4: Urgency Model: Dynamic Transition Equations

| | Desperate Degenerate | Dispersed | Relaxed Degnerate |
|---|---|---|---|
| $p_r$ | $\dot{p}_r = (\delta + \rho)p_r - x - \delta p_d$ | | |
| $p_d$ | $\dot{p}_d = \frac{(1-\mu)(y-\rho p_r)\psi^2}{\rho k}(p_d - p_r) + \rho p_d - x + c$ | | |
| $h_r$ | $\dot{h}_r = \eta - \frac{(1-\mu)(y-\rho p_r)\psi^2}{\rho k}h_r - \delta h_r$ | | |
| $h_d$ | $\dot{h}_d = \delta h_r - \frac{\phi(y-\rho p_d)\psi^2}{\rho k}h_d$ | $\dot{h}_d = \delta h_r - \frac{(y-\rho p_r)\psi^2}{\rho k}h_d$ | |
| $\mu$ | $\mu = 1$ | $y - \rho p_r = \phi\,(y - \rho p_d)$ | $\mu = 0$ |
| $\phi$ | $\phi = \frac{h_d}{h_d+h_r}$ | | |

are more desperate sellers in the market, both prices instantly drop, but slowly recover as the market works its way through the matches. Prices over-shoot their steady-state levels, though, rather than monotonically approaching steady state. This reminds us of the behavior of house prices in the Great Recession. As the troubled bank were selling foreclosed homes in bulk, the market saw an over-correction in home prices.

The shortcoming of this framework is that, the deadline penalty does not affect the equilibrium prices. This is because of the random transition – every seller faces the same risk of falling into desperation, so it is as if these sellers are exactly in the same position when it comes to setting a price under the same threat of being at the deadline state. In the next section, we are going to present a model a la Akin and Platt(2012), where there is a deterministic approach to steady state; that is, sellers differ in how long they have until they fall into the desperation state. This framework indeed gives us prices that depend on the deadline penalty.

#### 3.2.2 Experiment 6: Permanent decrease in buyer utiltiy

# 4 Conclusion

In this paper, we investigate dynamic price formation in an economy where a downturn causes some sellers to become more impatient to sell their assets because they

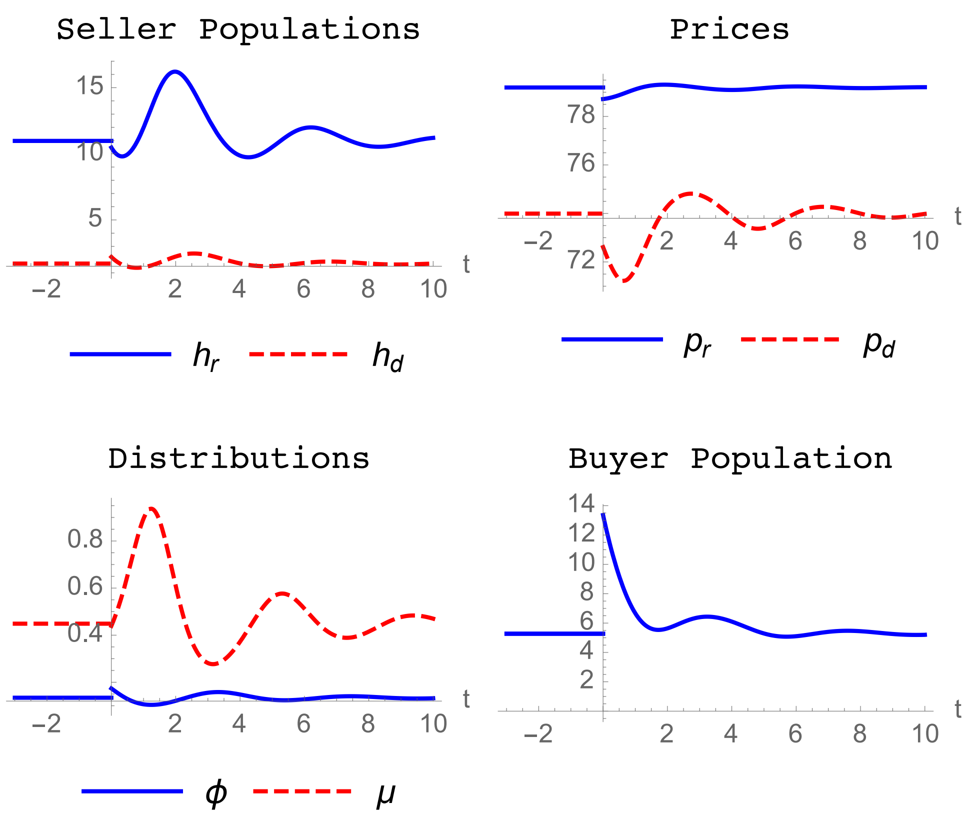


Figure 5: Dynamic response from shifting 0.5 sellers at time $t = 0$ from relaxed to desperate

need to fulfill their loan obligations. Buyers strategically choose when to enter the market, knowing that they might meet with different types of sellers, but without being informed about the type of a particular seller. Hence, they must also optimally choose what price they will offer, anticipating that a low price offer will be turned down by a relaxed seller who doesnt feel the pressure to liquidate his asset.

We have three main conclusions. First, a temporary shock such as a one-time increase in the fraction of desperate sellers causes prices to initially go down below the steady state; but prices work their way back to the steady state in a non-monotonic way, reacting to the endogenous entry of buyers as well as the fraction of desperate sellers in the economy. Second, a permanent shock such as a reduced value of the asset for the buyers (replicating a reduced value attached to home-ownership) leads to lower prices on the new steady state. These lower prices lead to more buyers in the economy. Again, prices converge to this steady state in a non-monotonic manner. Third, the probability of falling into the desperate state is crucial for these oscillations during the transition path. Without this possibility, there will be a monotonic convergence to the steady state.

# References


Akin, S. Nuray, and B. Platt. (2012) "Running Out of Time: Limited Unemployment Benefits and Reservation Wages," *Review of Economic Dynamics* **15**,149-170.

Akin, S. Nuray, and B. Platt. (2014) "A Theory of Search with Deadlines and Uncertain Recall," *Economic Theory*, **55**, 101-133

Albrecht, James, and B. Axell. (1984) "An Equilibrium Model of Search Unemployment," *Journal of Political Economy* **92**, 824-840.

Albrecht, James, and S. Vroman. (2005) "Equilibrium Search with Time-Varying Benefits," *The Economic Journal* **115**, 631-648.

Albrecht, James, A. Anderson, E. Smith, and S. Vroman. (2007) "Opportunistic Matching in the Housing Market," *International Economic Review* **48**, 641-664.

Arnold, Michael A. (1999). "Search, Bargaining, and Optimal Asking Prices, " *Real Estate Economics* **27**, 453-481.

Burdett, Kenneth, and D. Mortensen (1998) "Wage Differentials, Employer Size, and Unemployment, " *International Economic Review* **39**,257-273.

Campbell, John Y., S. Giglio, and P. Pathak. (2011) "Forced Sales and House Prices, " *American Economic Review* **101**, 2108-2131.

Guren, Adam and T. McQuade. (2013) "How do Foreclosures Exacerbate Housing Downturns?" Working Paper, Harvard University, Department of Economics.

Pulvino, Todd C. (1998) "Do Asset Fire Sales Exist? An Empirical Investigation of Commercial Aircraft Transactions, " *Journal of Finance* **53**, 939-978.

Schleifer, Andrei and R. Vishny. (2011). "Fire Sales in Finance and Macroeconomics, " *Journal of Economic Perspectives* **25**, 29-48.

van den Berg, Gerard. (1990) “Nonstationarity in Job Search Theory,” *Review of Economic Studies* **57**, 255-277.