

Safety, Liquidity, and the Natural Rate of Interest

Marco Del Negro, Domenico Giannone, Marc P. Giannoni, Andrea Tambalotti*

Federal Reserve Bank of New York

February 13, 2017

PLEASE DO NOT DISTRIBUTE/CIRCULATE/POST

Abstract

Why are interest rates so low in the United States? We find that they are low mostly because the premium for safety and liquidity has increased since the late 1990s. We reach this conclusion using two complementary perspectives: a flexible time series model of trends in nominal rates, Treasury and corporate yields, inflation, and long term expectations, and a medium-scale DSGE model. We discuss the implications of this finding for the natural rate of interest.

JEL CLASSIFICATION: E43, E44, C32, C11, C54

KEY WORDS: Natural Rate of Interest, r^* , DSGE Models, Liquidity, Safety, Convenience Yield

*Prepared for the March 2017 *Brookings Conference on Economic Activity*. Correspondence: Marco Del Negro (marco.delnegro@ny.frb.org), Domenico Giannone (domenico.giannone@ny.frb.org), Marc P. Giannoni (marc.giannoni@ny.frb.org), Andrea Tambalotti (andrea.tambalotti@ny.frb.org): Research Department, Federal Reserve Bank of New York, 33 Liberty Street, New York NY 10045. We are grateful to Jim Stock for his guidance, to X, Y for comments, and to Todd Clark and Egon Zakrajsek for sharing their data. We also thank Abhi Gupta, Pearl Li, and Erica Moszkowski for excellent research assistance. The views expressed in this paper do not necessarily reflect those of the Federal Reserve Bank of New York or the Federal Reserve System.

1 Introduction

Interest rates have been so persistently low in the United States and many advanced economies that this state of affairs has awoken the specter of secular stagnation (e.g., Summers, 2014; Hansen, 1939), a chronic economic malaise characterized by low interest rates. Why are rates so low?

This paper approaches these questions from two complementary perspectives. First, we estimate a flexible time series model of trends in nominal rates, Treasury and corporate yields, inflation, and long term expectations. Second, we estimate a medium-scale DSGE model, which provides a structural view of the underlying forces driving interest rates. The common thread running through these two empirical approaches is that they both focus on recovering the properties of the natural rate of interest. In line with the New Keynesian tradition, we define the natural rate of interest as the real rate of return that would prevail in the absence of nominal rigidities. The appeal of this construct is that, without nominal rigidities, monetary policy would have no effect on real variables, at least according to our DSGE framework. Therefore, the natural rate of interest provides a perspective on the real underlying drivers of interest rates that takes the influence of monetary policy off the table by construction, and addresses the question: What would the level of real interest rates be in the absence of monetary policy?

We find that at least since the late 1990s the main driver of both secular trends and cyclical fluctuations in the natural rate of interest, and more broadly in the rate of return for safe and liquid assets such as government securities, are the premiums for safety and liquidity (see Krishnamurthy and Vissing-Jorgensen, 2012). Not all real rates are low. Yields on corporate bonds, for instance, have not declined as much as yields on Treasuries with comparable maturity, and this cannot be explained away by an increase in default probabilities. Even yields on very safe, but not as liquid, corporate bonds have not declined as much as yields on comparable Treasuries. Using the DSGE model, we find that changes in the premiums for liquidity and safety – or convenience yields – have important implications also for the cyclical component of interest rates, and for business cycles more broadly.¹

¹As in Krishnamurthy and Vissing-Jorgensen (2012) and the related literature discussed below, our notion of liquidity is quite broad, and encompasses all costs, frictions, and balance sheet constraints that generate the deviations from the law of one price documented by the finance literature (again, see the discussion below), and that can lead to market freezes such as those experienced in the aftermath of the Lehman crisis.

The remainder of the introduction elaborates on the relationship between the natural rate of interest and the convenience yield for safety and liquidity, and puts this paper in the context of the literature. The natural rate of interest – which we will refer to as r^* – is not directly observable.² Dynamic stochastic general equilibrium models (DSGEs) provide a mapping between macroeconomic observables (the real rate, inflation, output and consumption, *et cetera*) and r^* —as discussed in section 5. However, the current DSGE models are not designed to capture very low frequency movements in r^* . For this reason we will characterize trends in r^* using reduced form models, as described in the next section. Nonetheless, we will still use economic theory to relate the information in the data to the specific counterfactual object we are interested in.

To organize the discussion, let us start with the Euler equation for a liquid, safe, short term nominal government security, such as a 3-month US Treasury bill:

$$u'(c_t) = \beta E_t \left[\frac{R_t}{\pi_{t+1}} (1 + CY_{t+1}) u'(c_{t+1}) \right], \quad (1)$$

where c_t, π_t, R_t represent consumption, gross inflation, and the gross short term nominal return, respectively, and β is the discount rate. Expression (1) is a standard Euler equation except for the presence of the term CY_{t+1} . A growing body of recent literature (e.g., Krishnamurthy and Vissing-Jorgensen, 2012; Greenwood et al., 2015) argues that assets such as Treasuries are valued not only for their pecuniary return, but also for their liquidity/safety attributes. As Greenwood et al. (2015) put it, this literature “documents significant deviations from the predictions of standard asset pricing models – patterns that can be thought of as reflecting money-like convenience services – in the pricing of Treasury securities generally, and in the pricing of short-term T-bills more specifically.” As a consequence, these assets carry a premium (CY_{t+1}) relative to assets with the same pecuniary payoff but no such attributes, which Krishnamurthy and Vissing-Jorgensen (2012) call the *convenience yield*.³ We

²The symbol r^* is used with various meanings and in different context, as described in the literature review (section 2). Here we use it as a shortcut to refer to the natural rate of interest.

³Krishnamurthy and Vissing-Jorgensen (2012) measure the historical convenience yield on Treasuries and show that it is sizable – 73 basis points per year. From the theoretical point of view, they model the convenience yield as arising from agents deriving direct utility from holding safe/liquid assets. In Kiyotaki and Moore (2012) the liquidity-related component of the convenience yield arises from so-called liquidity (or resaleability) constraints facing actors in financial markets: liquid assets are valued as they relax such constraint. In equation (1) we introduce the convenience yield following the specification in Kiyotaki and Moore (2012).

include this term in our analysis as it plays a potentially important role in the determination of the natural rate of interest, as we discuss next.

We define the natural rate of interest r_t^* as the real return to an asset with the *same safety/liquidity* attributes as 3-month US Treasury bill in a counterfactual economy without nominal rigidities. We adopt this definition because central banks' operational targets are returns on short-term liquid/safe assets (e.g., money market rates such as the federal funds rates, or interest rates on reserves held at the central bank such as the IOER in the U.S.). If r^* is meant to be a benchmark for monetary policy, it better be associated with the return of an asset that possesses such attributes.⁴ The natural rate of interest r_t^* must therefore satisfy

$$u'(c_t^*) = \beta E_t[r_t^*(1 + CY_{t+1}^*)u'(c_{t+1}^*)], \quad (2)$$

where the $*$ denotes the variables in the counterfactual economy. Equation (2) highlights that variations in the convenience yield have a direct effect on r^* , as discussed in Del Negro et al. (forthcoming): changes in CY^* map one to one into changes in r^* (for given c_t^* and c_{t+1}^*).⁵ In the long run, this implies that trends in the convenience yield may drive trends in r^* , an hypothesis we will explore quantitatively in this paper.

The first part of the paper uses a reduced form model to extract such trends. Specifically, we estimate common trends in inflation, short term rates, and the convenience yield using a panel of yields at various maturities, and with different degrees of liquidity and safety, as well as data on inflation and long run survey expectations for inflation and the nominal rate. The reduced form model we use – which we (facetiously) call Trendy VARs – amounts to a multivariate version of an unobserved component model (e.g., Watson, 1986) where the data are driven by common trends and the residuals are modeled as a stationary vector autoregression.⁶ We first apply this model to Treasury yields and expectations only in order

⁴There is arguably confusion in the literature as to whether r^* should be measured using the return on safe and liquid assets or whether spreads should be included in the definition (see for instance Rachel and Smith, 2015). Our framework provides some guidance in thinking about these issues.

⁵Del Negro et al. (forthcoming) argue that the liquidity shocks experienced after the Lehman crisis had the *direct* effect on r^* associated with the increase in the convenience yield, and an *indirect* effect related to the fact that in the counterfactual economy consumption has to increase to compensate for the fall in investment (recall that in the RBC version of the model output is supply determined), thereby pushing r^* down further.

⁶Crump et al. (2016) and Johannsen and Mertens (2016) have recently used very similar approaches.

to conduct inference on the trend for the short term real rate. We find that this trend has been rising smoothly from the 1960s to the early 1980s, remained stable until the late 1990s, and then decreased substantially. When we use spreads between Treasuries and yields on assets that are less liquid/safe, we find that an increase in the convenience yield for Treasuries drives much of the decline in the trend for the real rate, leaving little room for many of the other factors that are often held responsible for the low interest rates, such as productivity, demographics, and the savings glut. To be more specific, some of these factors may indeed play an important role, as discussed below, but only via their effect on the convenience yield.

The second part of the paper considers a medium-scale DSGE model that includes nominal, real and financial frictions, and estimates it using numerous aggregate time series. The model has fairly sharp predictions for the natural rate of interest at various maturities. We find that the estimate of r^* fluctuates substantially in the short term and at business-cycle frequencies. For instance, after rising during the mid-2000s, the natural rate collapses during the Great Recession and has remained very low since. The DSGE model's estimate of r^* displays also longer-run fluctuations that are similar to those captured by the Trendy VAR. Looking at the sources of fluctuations in r^* , we find again that financial frictions and associated changes in the convenience yield have played a key role in lowering r^* since the late 1990s.

Our findings are very much in line with the recent literature discussing the causes, and the macroeconomic consequences, of the shortage of safe assets (e.g., Caballero and Krishnamurthy, 2009; Caballero, 2010; Caballero and Farhi, 2014; Caballero et al., 2015; Gourinchas and Rey, 2016).⁷ One implication of this shortage is that the yield of safe assets, relative to assets that are less safe, should have seen a secular decline, which is exactly what we find.⁸ Interestingly, Gourinchas and Rey (2016) reach very similar conclusions to ours using a very different approach based on the determinants of the consumption-to-wealth ratio.

⁷See Gorton (2016) for a definition of safe assets and for a broad discussion of their role in economics. Hall (2016) takes a related but slightly different perspective, as he emphasizes heterogeneity in beliefs and risk aversion, and how changes in the wealth distribution in favor of more risk averse/pessimistic investors can lead to a decline in the real rate on safe securities.

⁸Caballero and Farhi (2014) also show that the expected return on stocks is currently much higher than the yield of safe assets, consistently with their theory. However, arguably our empirical analysis is cleaner in that the safety premium is only one determinant of the stock market risk premium, while we are able to identify the convenience yield more sharply using spreads.

This literature also provides a potential explanation for this decline, which we observe since the late 1990s (see especially Caballero, 2010, for a very compelling account). In essence, over the past twenty years a sequence of foreign and domestic financial crises (the Asian crisis, the NASDAQ crash, the great recession) both increased the demand for safe and liquid assets and, in case of the recent financial crisis, severely curtailed their supply. Note that in this narrative the saving glut from foreign economies plays a key role, except that this glut did not translate into a generic demand for assets, but into a specific one for safe (and liquid) assets (Bernanke et al., 2011, provide evidence based on the flow of funds and other sources that from 2003 to 2007 foreign investors acquired substantial amounts of U.S. Treasuries, Agency debt, and Agency-sponsored mortgage-backed securities; Greenwood et al., 2016 show that foreign holdings of U.S.-produced money-like claims have risen sharply since the early 2000s).⁹

While much of the macro literature on secular stagnation emphasizes safety, we also stress the role of liquidity. Liquidity has long played a prominent role in finance (see Longstaff et al., 2004; Acharya and Pedersen, 2005; Longstaff et al., 2005; Amihud et al., 2006; Garleanu and Pedersen, 2011; Amihud et al., 2012; Fleckenstein et al., 2014, among many others) and and, starting with Kiyotaki and Moore (2012, the first draft was written in 2001), has also been incorporated in macro models.¹⁰ A number of recent papers (Kurlat, 2013; Bigio, 2015; Ajello, forthcoming; Del Negro et al., forthcoming; Cui and Radde, 2014; Guerron-Quintana and Jinnai, 2015) have studied the role of liquidity for business cycles in general and the Great Recession in particular. We show that the liquidity convenience yield plays an important role in explaining why interest rates on liquid assets are currently low, and argue

⁹To use Caballero (2010)’s words: “...it is not to say that global imbalances did not play a role. Indeed, there is a connection between the safe-assets imbalance and the more visible global imbalances: The latter were caused by the funding countries’ demand for financial assets in excess of their ability to produce them ..., but this gap is particularly acute for safe assets since emerging markets have very limited institutional capability to produce these assets.”

¹⁰For instance, Fleckenstein et al. (2014) provide ample evidence of what they call the “TIPS-Treasury bond puzzle,” that is, of differences in prices between Treasury bonds of various maturities and inflation-swapped Treasury Inflation-Protected Securities (TIPS) issues exactly replicating the cash flows of the Treasury bond of the same maturities. Specifically, they find that the price of a Treasury bond and an inflation-swapped TIPS issue exactly replicating the cash flows of the Treasury bond can differ by more than \$20 per \$100 notional – a difference that, they argue, is orders of magnitude larger than the transaction costs of executing the arbitrage strategy.

more broadly that for both secular trends and cyclical movements in interest rate liquidity plays a role that is as important as that of safety.¹¹

The remainder of the paper proceeds as follows. Section 2 reviews the literature on the causes of low interest rates in particular, and on the determinants of trends in r^* in particular. Section 3 introduces the Trendy VARs, while section 4 applies them to conduct inference on trends in interest rates. Section 5 presents the results from the DSGE model and section 6 concludes.

2 Literature Review

The extremely low levels of interest rates since the Great Recession have received a great deal of attention in the literature. Three main classes of explanations have been proposed for this phenomenon. A first set of papers connects the low rates directly to low productivity and output growth, based on the relationship between these variables captured by the intertemporal Euler equation. Laubach and Williams (2016) (LW henceforth), for instance, find a close relationship between the permanent components of the real interest rate and output growth, with the former estimated to be slightly negative since 2012. Other authors, however, are more skeptical of this connection, due to the well-known empirical deficiencies of the consumption Euler equation (e.g., Hamilton et al., 2015). Intuitively, this relationship can only go so far in accounting for the secular decline in rates over the last few decades, even if it fits the low growth and low rates environment of the last few years, since rates were high in the 1970s and 1980s, when productivity growth was low and they started declining in the 1990s, when productivity accelerated.

A second class of explanations stems from the textbook observation that the real interest rate is the price that clears the saving and investment market. Of course, equilibrium in this market also underlies the consumption Euler equation discussed above. However, papers that look at interest rate determination through this lens tend to connect rates more loosely

¹¹The discussion in the paper treats safety and liquidity as if they were independent factors. While empirically we can arguably try to distinguish safety and liquidity by looking at assets with different characteristics, from the conceptual point of view safety and liquidity are clearly interrelated. For instance, in Kurlat (2013) market freezes (illiquidity) take place precisely because agents are uncertain about the safety of the assets that are in the market.

to factors that can be expected to shift desired saving and investment¹². Among them, the most prominent is arguably the ongoing demographic transition whereby increased life expectancy interacts with slower population growth to create changes in the dependency ratio that can have potentially significant repercussions on aggregate saving behavior (e.g., Carvalho et al., 2016; Gagnon et al., 2016). Another factor that is often cited as contributing to an increase in desired saving and hence to lower interest rates is rising inequality, since richer households tend to save more out of marginal income. However, Auclert and Rognlie (2016) point out that, in general equilibrium, the fall in the interest rate tends to result in a boom in investment and output, which is clearly not a feature of the current environment. Increased uncertainty also has the potential to both increase precautionary saving and to depress investment through the channels recently emphasized by Bloom (2009). Also on the investment side, the decline in the price of capital associated with rapid investment-specific technical change, by reducing the amount of saving needed to finance each unit of capital, might create an imbalance between desired saving and investment that would put downward pressure on the interest rate (e.g., Eichengreen, 2015). Finally, in the open economy context, this framework underlies the very influential saving glut hypothesis first proposed by Bernanke (2005). According to this view, the current account imbalances that grew from the late 1990s to just before the Great Recession, and the globally low rates that accompanied them, were the result of a massive shift in desired saving in developing economies following the Asian crisis of 1997. The extent to which this increase in global desired saving remains a salient feature of the post-crisis economic environment, and an important driver of the low interest rates that are currently prevalent around the developed world is debated for instance by the growing literature on safe asset shortages (e.g., Bernanke et al., 2011).

Finally, the third class of explanations for the prevalence of low rates in the US and around the world since the financial crisis revolves around the idea of secular stagnation. The defining feature of this hypothesis, compared to all the others we have mentioned so far, is that it is built around a permanent aggregate demand deficiency, or equivalently an imbalance between desired saving and investment, which cannot be cleared by a sufficient fall in the real interest rate. Such a barrier to lower real rates can be connected most

¹²Rachel and Smith (2015) provide a comprehensive overview of this literature and some back of the envelope quantification of the impact of many of the potential shifters of desired saving and investment on the real interest rate

naturally to a binding zero lower bound, as in Eggertsson and Mehrotra (2014), where real rates are permanently pushed against this barrier by a deleveraging shock interacted with an overlapping generation structure.

The concept of the natural rate of interest, which originates in the writings of Wicksell (1898), was formalized in the context of modern dynamic macroeconomics by Woodford (2003). Its distinguishing feature is that it is defined explicitly through a counterfactual experiment, which consists of removing the effect of nominal rigidities on the economy. As such, its construction in general requires taking a stance on a fully specified general equilibrium model. As we discuss more fully below, though, its low frequency behavior can be characterized through a more flexible time series model that does not impose any of the tight cross-equation restrictions that are typical of DSGE models. Spelling out the conditions under which this procedure recovers (the low frequency component of) the same object identified in the DSGE context is one of the contributions of this paper.

This contribution is especially useful because it helps to connect some of the different concepts related to the natural rate used in the literature. In particular, it clarifies the relationship between estimates of the natural rate based on DSGE models, such as those in Justiniano and Primiceri (2010), and estimates based on statistical filters, as in Laubach and Williams (2003). As the then Vice-Chair of the FOMC lamented "Economists famously cannot agree on much. In this case, we cannot even agree on the name of the benchmark concept that I have just described. The real interest rate consistent with the eventual full utilization of resources has been called the equilibrium real federal funds rate, the natural rate of interest, and the neutral real rate. (...) Even if economists settled on a name, however, we are not likely to agree on a single model to describe a system as richly complicated as the U.S. economy. Thus, there are as many ways of estimating the equilibrium real federal funds rate as there are different economic models." (Ferguson, 2004). Among these three adjectives, we choose "natural" because it is the one most closely related to the construct proposed by Woodford (2003), even if Kahn often uses the term neutral in his translation of Wicksell (1898). Both the other names mentioned by Ferguson (2004), however, could also be applied to our concept. Equilibrium, because the market for saving and investment indeed clears in the counterfactual economy with no nominal rigidities, although it also does in the actual economy with frictions. Neutral, because monetary policy is neutral in the

natural equilibrium, as we discussed above.¹³

The most popular approach to the estimation of r^* in the literature is that proposed by LW. Their strategy blends aspects of both our VAR and DSGE estimations. Like in the trendy VAR, LW focus on the low frequency component of the natural rate, which they also model as an I(1) process. However, they impose more economic structure on the estimation problem than we do in the VAR. In particular, they assume that r_t^* is a linear function of the growth rate of trend output, which they model as a unit root process (i.e. the level of the GDP trend is an I(2) process, with an I(1) growth rate). This long-run restriction is inspired by the steady state relationship between the real rate of return and consumption (and hence output) growth that holds in the neoclassical growth model. In addition, LW posit: (i) a reduced-form dynamic IS equation linking the gap between the actual and the natural real rate of interest to the output gap —the deviation of output from trend —and (ii) a Phillips curve linking inflation to the output gap. Together, these assumptions yield an unobserved components model through which the low frequency movements in the natural rate can be inferred based on observations on the interest rate, inflation, and output using a Kalman filter.¹⁴

The reduced-form relationships described in (i) and (ii) are inspired by the intertemporal Euler equation and by the optimal pricing equation that usually hold in fully-specified DSGE models, including ours. Unlike those equations, though, they are exclusively backward-looking and they do not impose the cross-equation restrictions implied by the microfoundations underlying most DSGE models. Therefore, LW’s empirical model can be considered on the one hand as a restricted version of our trendy VAR, and on the other hand as a less tightly parametrized, but also less theoretically coherent, version of our DSGE. The main drawback of this less restricted structure compared to a fully specified DSGE model is that the latter provides a more precise notion of the counterfactual that defines the natural rate, as detailed in Section 5.

Laubach and Williams (2016) update their earlier estimates of the natural rate with data through the first half of 2015. They find a dramatic decline in r^* during the Great Recession

¹³The literature also uses the terms "efficient" and "potential" to refer to counterfactual equilibria, and hence interest rates, closely related to the natural one. See for instance Justiniano et al. (2013) for detailed definitions.

¹⁴Taylor and Wieland (2016) offer a critique of r^* estimates based on Phillip curves and IS equations. Their critique arguably does not apply to our reduced form approach.

and in the years that followed it, from about 2% to slightly negative values in the most recent period. According to their estimates, this fall continues a negative trend that started in the 1980s, when the natural rate was as high as 4%. They attribute a large fraction of this secular decline to a fall in the growth rate of trend output over the same period. Several other recent papers use unobserved component models to estimate a trend in the real interest rate and to analyze its contribution to the observed declines in actual rates over the last few decades (e.g. Kiley, 2015; Pescatori and Turunen, 2015; Johannsen and Mertens, 2016). They all tend to find that this contribution is meaningful, although generally not as pronounced as in LW, and not as concentrated around the Great Recession. These estimates are all based on economically motivated reduced-form restrictions in the spirit of LW, which focus on the connection between interest rates and macroeconomic variables such as unemployment, output and inflation. Like in LW, these restrictions help to uncover a meaningful connection between the trends in the real interest rate and in growth. Compared to this literature, our VAR analysis mostly emphasizes data on interest rates and on spreads between Treasury and corporate bonds, rather than their connection with macroeconomic variables. As a result, we uncover a prominent role for low frequency movements in the convenience yield in accounting for the observed decline in real interest rates which was previously ignored by the literature.¹⁵

3 Trendy VARs

The model is given by the measurement equation

$$y_t = \Lambda \bar{y}_t + \tilde{y}_t, \quad (3)$$

where y_t is an $n \times 1$ vector of observables, $\bar{y}_t$ is a $r \times 1$ vector of trends, with $r \leq n$, $\Lambda(\lambda)$ is a $n \times r$ matrix of loadings which is restricted and depends on the vector of free parameters

¹⁵Kiley (2015) includes a corporate spread as an exogenous variable in his IS equation, since it helps to forecast output. He finds that this modification to the LW specification reduces the estimated movements in r^* , especially around the Great Recession. This result can also be interpreted as suggesting that financial turbulence, as proxied by high credit spreads, tends to depress the natural rate, if the latter is defined to include the effects of financial shocks and frictions, as we do in our DSGE model. Consistent with this interpretation, Pescatori and Turunen (2015) find that proxies for the demand for safe assets help to explain some of the (cyclical) movements in their estimate of r^* , especially since the late 1990s.

λ , and $\tilde{y}_t$ is an $n \times 1$ vector of stationary components. Both $\bar{y}_t$ and $\tilde{y}_t$ are latent and evolve according to a random walk

$$\bar{y}_t = \bar{y}_{t-1} + e_t \quad (4)$$

and a VAR

$$\Phi(L)\tilde{y}_t = \varepsilon_t, \quad (5)$$

respectively, where $\Phi(L) = I - \sum_{l=1}^p \Phi_l L^l$ and the Φ_l 's are $n \times n$ matrices and the $(r+n) \times 1$ vector of shocks $(e'_t, \varepsilon'_t)'$ is independently and identically distributed according to

$$\begin{pmatrix} e_t \\ \varepsilon_t \end{pmatrix} \sim \mathcal{N} \left(\begin{pmatrix} 0_n \\ 0_r \end{pmatrix}, \begin{pmatrix} \Sigma_e & 0 \\ 0 & \Sigma_\varepsilon \end{pmatrix} \right), \quad (6)$$

with the Σ 's being conforming positive definite matrices, and where $\mathcal{N}(\cdot, \cdot)$ denotes the multivariate Gaussian distribution. Equations (4) and (5) represent the transition equations in the state space model. The initial conditions $\bar{y}_0$ and $\tilde{y}_{0:-p+1} = (\tilde{y}'_0, \dots, \tilde{y}'_{-p+1})'$ are distributed according to

$$\bar{y}_0 \sim \mathcal{N}(\underline{y}_0, \underline{V}_0), \quad \tilde{y}_{0:-p+1} \sim \mathcal{N}(0, V(\Phi, \Sigma_\varepsilon)) \quad (7)$$

where $V(\Phi, \Sigma_\varepsilon)$ is the unconditional variance of $\tilde{y}_{0:-p+1}$ implied by (5).¹⁶ Constants can be easily accommodated into this framework by introducing a deterministic trend in the vector $\bar{y}_t$. The procedure also straightforwardly accommodates missing observations.

The model above, which we dub Trendy VAR, is essentially Villani (2009)'s model, except that his deterministic trend is replaced by the stochastic trend (4).¹⁷ It is nothing but a multivariate extension of a standard unobserved component model (e.g., Watson (1986), Stock and Watson (2007), Kozicki and Tinsley (2012)). More recently, Crump et al. (2016) and Johannsen and Mertens (2016) propose models that are very similar to ours. Crump et al. (2016) however estimate the model's parameters using maximum likelihood (treating equations (3) through (5) as a state space model), which poses serious constraints on the size of the model that can be estimated. Johannsen and Mertens (2016) use Bayesian methods, like we do, but impose that the elements of the matrix Λ are known.¹⁸ On the contrary,

¹⁶We impose stationarity to the VAR (5), as discussed below, so that $V(\Phi, \Sigma_\varepsilon)$ is always well defined.

¹⁷Villani (2009) also considers a VECM version of his model.

¹⁸Johannsen and Mertens (2016)'s sophisticated model allows for stochastic volatility in the shocks distribution and for explicit treatment of the zero lower bound on nominal rates. Our model can certainly be amended to accommodate the former, along the lines of Del Negro and Primiceri (2015), and in principle also the latter, following Johannsen and Mertens (2016)'s approach.

our Gibbs sampler accommodates VARs of any size and with any estimated cointegrating relationship. As such, Trendy VARs can be broadly applied.

The priors for the VAR coefficients $\Phi = (\Phi_1, \dots, \Phi_p)'$ and the covariance matrices Σ_ε and Σ_e have standard form, namely

$$p(\varphi|\Sigma_\varepsilon) = \mathcal{N}(\text{vec}(\Phi), \Sigma_\varepsilon \otimes \underline{\Omega})\mathcal{I}(\varphi), \quad p(\Sigma_\varepsilon) = \mathcal{IW}(\kappa_\varepsilon, (\kappa_\varepsilon + n + 1)\underline{\Sigma}_\varepsilon), \\ p(\Sigma_e) = \mathcal{IW}(\kappa_e, (\kappa_e + r + 1)\underline{\Sigma}_e), \quad (8)$$

where $\varphi = \text{vec}(\Phi)$, $\mathcal{IW}(\kappa, (\kappa + n + 1)\underline{\Sigma})$ denotes the inverse Wishart distribution with mode $\underline{\Sigma}$ and κ degrees of freedom, and $\mathcal{I}(\varphi)$ is an indicator function which is equal to zero if the VAR is explosive (some of the roots of $\Phi(L)$ are less than one) and to one otherwise.¹⁹ The prior for λ is given by $p(\lambda)$, the product of independent Beta, Gamma, or Gaussian distributions for each element of the vector λ (all the details, as well as the actual values used in the prior, will be given when discussing the application). The model (3) through (7) is a linear Gaussian state space model and is therefore straightforward to estimate efficiently in spite of the large size of the state space using modern simulation smoothing techniques (Carter and Kohn, 1994, or Durbin and Koopman, 2002). Section A in the Appendix describes the Gibbs sampler.

4 Estimating Trends in r

In this section we jointly estimate long run trends in r_t and the convenience yield using both the nominal and the real yield curve, as well as data on inflation, inflation expectations, and spreads associated with liquidity and safety. We do so using the Trendy VAR model discussed in section 3 where the set of observables and the restrictions imposed on the model are broadly motivated by the theory. For any variable x_t , denote its long run trend as

$$\bar{x}_t = \lim_{h \rightarrow \infty} E_t[x_{t+h}].$$

¹⁹ The inverse-Wishart distribution with parameters κ and $(\kappa + m + 1)\underline{\Sigma}$ is given by

$$p(\Sigma; \kappa, (\kappa + m + 1)\underline{\Sigma}) = \frac{|(\kappa + m + 1)\underline{\Sigma}|^{\kappa/2}}{2^{m\kappa/2}\Gamma(\kappa/2)} |\Sigma|^{-(\kappa+m+1)/2} \exp\left(-\frac{\kappa + m + 1}{2} \text{tr}(\Sigma^{-1}\underline{\Sigma})\right),$$

where m is the size of Σ . Under this parametrization $\underline{\Sigma}$ is the mode and κ are the degrees of freedom.

To the extent that trends in the observed real rate ($\bar{r}_t$) and in r^* ($\bar{r}_t^*$) coincide – in other words, if the gap between r_t and r_t^* is stationary – we can learn about $\bar{r}_t^*$ by conducting inference on $\bar{r}_t$. We will show in section 5 that trends in $\bar{r}_t$ estimated using the Trendy VAR, and trends in the natural rate of interest estimated using the DSGE model nearly coincide, lending credence to this assumption.

The assumption that the gap between r_t and r_t^* is stationary – that is, that monetary policy cannot affect the trend growth rate of the economy – is a generally accepted assumption in macroeconomics, but is not uncontroversial. For instance, in models featuring endogenous growth with nominal rigidities (e.g. Benigno and Fornaro, 2016) this assumption would be violated. Perhaps more importantly, assumption (4) implies that long run trends evolve smoothly over time. This implies that our approach is unlikely to fully capture abrupt shifts from one long run regime to another, as envisioned in the theory of Secular Stagnation (e.g., Summers, 2014; Eggertsson and Mehrotra, 2014). This should be kept in mind when interpreting the results.

4.1 Extracting Trends in r from Trends in Short- and Long-Term Yields, Inflation, and Long-Run Expectations

Call $r_{\tau,t}^N$ the logarithm of the gross yield on a nominal Treasury of maturity τ (with τ expressed in quarters). Following the trendy VAR (3), of section 3 we decompose the term structure as the sum of a trend $\bar{r}_{\tau,t}^N$ and a stationary component $\tilde{r}_{\tau,t}^N$

$$r_{\tau,t}^N = \bar{r}_{\tau,t}^N + \tilde{r}_{\tau,t}^N. \quad (9)$$

Define r_t as the real return on an asset that is as liquid and safe as a 3-month Treasury bill, and that therefore satisfies the no arbitrage condition:

$$E_t[(e^{r_t} - e^{r_{1,t}^N - \pi_{t+1}})(1 + CY_{t+1})M_{t+1}] = 0$$

where $M_{t+1} = \beta \frac{u'(c_{t+1})}{u'(c_t)}$ is the stochastic discount factor. If we take a second order approximation of the expression above and assume that for maturity as short as one quarter the covariance term between inflation π_{t+1} and the term $(1 + CY_{t+1})M_{t+1}$ is either negligible or

at least stationary (more on this below), we obtain that the Fisher equation holds in terms of trends, that is

$$\bar{r}_{1,t}^N = \bar{r}_t + \bar{\pi}_t.$$

For a nominal 3-month bill ($\tau = 1$) we can therefore write equation (9) as

$$r_{1,t}^N = \bar{r}_t + \bar{\pi}_t + \tilde{r}_{1,t}^N. \quad (10)$$

From (10) we cannot separately disentangle movements in $\bar{r}_t$ and $\bar{\pi}_t$. We address this problem by extracting the nominal trend $\bar{\pi}_t$ from inflation π_t (measured as log changes in the GDP deflator) and—whenever available— inflation expectations obtained from surveys π_t^e using an unobserved component model a la Stock and Watson (1999):

$$\begin{aligned} \pi_t &= \bar{\pi}_t + \tilde{\pi}_t, \\ \pi_t^e &= \bar{\pi}_t + \tilde{\pi}_t^e. \end{aligned} \quad (11)$$

In principle expressions (10) and (11) are enough to conduct inference on $\bar{r}_t$. However, we do not want to use short rates information for the zero lower bound (henceforth, ZLB) period as we are concerned that these may distort our inference on the trends – which implies that we do not use data on $r_{1,t}^N$ after 2008Q3. Moreover, inference on trends can be made sharper by using two additional sources of information – the one from long maturity Treasury yields and that from forecasters expectations of long run averages of the short term rate. If the expectation hypothesis were correct, long maturity Treasuries would indeed be the ideal observable for extracting trends, being simply averages of expected short term rates. Of course, the expectation hypothesis does not hold and movements in the term premium are key drivers of yields. Several papers (most recently Johansson and Mertens, 2016) assume that the term premium is stationary. In order to avoid contaminating inference about the trend $\bar{r}_t$ with possible trends in the term structure we choose instead to model possible trends in the nominal term premium using exogenous component $\bar{tp}_t$.²⁰ We use the yield on 20-year Treasuries as a measure of long term yields and model it as

$$r_{80,t}^N = \bar{r}_t + \bar{\pi}_t + \bar{tp}_t + \tilde{r}_{80,t}^N, \quad (12)$$

²⁰We have also considered a constant term premium and found the results to be robust.

where $\tilde{r}_{80,t}^N$ captures stationary movements in long term yields.²¹ Recall that we allow for a correlation in the innovations to the trend, hence expression (10) or (12) do not necessarily imply that trends in $\bar{r}_t$, $\bar{\pi}_t$, or $\bar{t}p_t$ are independent from one another. However, since we impose a fairly strong prior that the correlation matrix is diagonal, we have also explored the possibility that trends in inflation affect the term premium by introducing a term premium component that is proportional to trends in inflation $\gamma^{tp}\bar{\pi}_t$ with $\gamma^{tp} > 0$:

$$r_{80,t}^N = \bar{r}_t + \bar{\pi}_t + \bar{t}p_t + \gamma^{tp}\bar{\pi}_t + \tilde{r}_{80,t}^N.$$

We found that results are unaffected under this alternative specification (see the discussion in section 4.4.2).

Finally, inspired by Crump et al. (2016), we also use forecasters expectations of long run averages of the short term rate, which we call $r_{1,t}^{N,e}$, and model them as

$$r_{1,t}^{N,e} = \bar{r}_t + \bar{\pi}_t + \tilde{r}_{1,t}^{N,e}. \quad (13)$$

4.1.1 Results

The system of equations (10) through (13) can be expressed as the Trendy VAR (3) where $y_t = (\pi_t, \pi_t^e, r_{1,t}^N, r_{80,t}^N, r_{1,t}^{N,e})$ and $\bar{y}_t = (\bar{r}_t, \bar{\pi}_t, \bar{t}p_t)$ evolves according to (4), the stationary components $(\tilde{\pi}_t, \tilde{\pi}_t^e, \tilde{r}_{1,t}^N, \tilde{r}_{80,t}^N, \tilde{r}_{1,t}^{N,e})$ evolves according to (5). and where the matrix of loadings Λ can be deduced from expressions (10) through (13). We estimate this Trendy VAR using as observables annualized PCE inflation, long-run (10-year averages) PCE inflation expectations, the 3-month Treasury Bill rate, the long-run (10-year averages) expectations for the 3-month Treasury Bill rate, and the 20-year Treasury constant maturity rate.²² With the exception of long run expectations, all the data are available from 1954Q1 to 2016Q4.

²¹We use the 20-year yield because that is the natural counterpart in terms of maturity for the corporate bonds we will use in the next section (see Krishnamurthy and Vissing-Jorgensen, 2012). Results obtained using the 10-year yield are very similar.

²²Annualized PCE inflation, the 3-month Treasury Bill rate and the 20-year Treasury constant maturity rate are available from FRED and their mnemonics are DPCERD3Q086SBEA, TB3MS, and GS20, respectively. The long run PCE inflation expectations are obtained from the Survey of Professional Forecasters (henceforth, SPF) from 2007 onward, while from 1970 to 2006 we use the the survey-based long-run (5- to 10-year-ahead) PCE inflation expectations series of the Federal Reserve Board of Governors FRB/US econometric model. This same dataset employed by Clark and Doh (2014), and we are grateful to Todd Clark for making the data available. The long-run expectations for the 3-month Treasury Bill rate are also

We use the period 1954Q1-1959Q4 as presample and estimate the model over the sample 1960Q1-2016Q4. Because of the zero lower bound on interest rates, we treat the short term rate as unobservable from 2008Q4 onward.

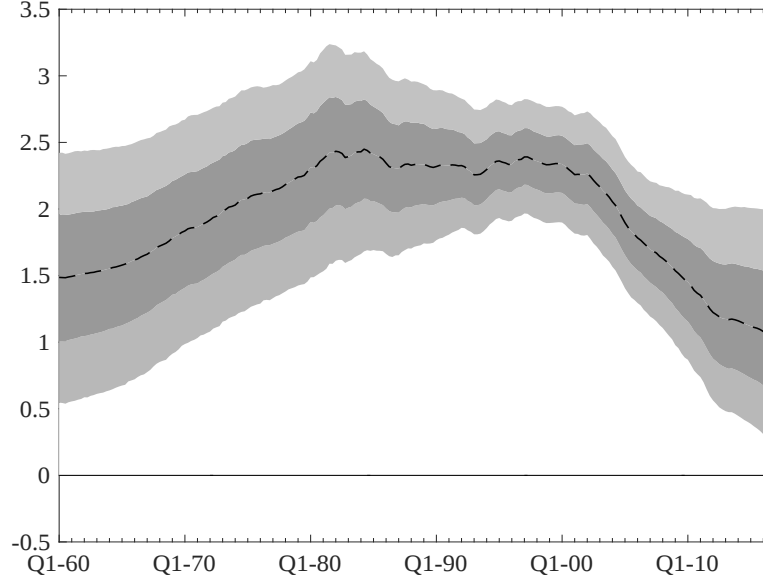
Our prior for Σ_ϵ , the variance covariance matrix of the innovations to the trends $\bar{y}_t$, is very conservative in that we do not want to find trends where there aren't any. The matrix Σ_ϵ is therefore diagonal with elements equal to $1/400$, which imply that a priori the standard deviation of the expected change in the trend over one century is only one percentage point, arguably a fairly low number. For the trend in inflation we use the slightly higher, but still very conservative, prior of $1/200$ (one percentage point in fifty years).²³ In addition, our prior is quite tight, as we set $\kappa_\epsilon = 100$. We will show below that this tight, conservative prior will not prevent us from finding trends where these are clearly present, e.g., in inflation or, as we will see in the next section, in the convenience yield. The robustness section will show that a looser prior will simply have the effect of making $\bar{y}_t$ capture some higher frequency movements, but will not substantially change our results. The prior for the VAR parameters describing the components $\tilde{y}_t$ is a standard Minnesota prior with standard hyperparameter for the overall tightness equal to .2 (see Giannone et al., 2015), except that of course it is centered at zero rather than one as we are describing stationary processes.²⁴ The initial conditions $\underline{y}_0$ for the trend components $\bar{y}_t$ are set at presample averages for inflation, the real rate, and the term spread (2, .5, and 1 for $\bar{\pi}_0$, $\bar{r}_0$, and $\bar{tp}_0$, respectively), with $\underline{V}_0$ being the identity matrix. Finally, the VAR uses five lags ($p = 5$).

Figure 1 shows the estimates of $\bar{r}_t$. Specifically, dashed black line shows the posterior median of $\bar{r}_t$ while the shaded areas show the 68 and 95 percent posterior coverage intervals

obtained from the SPF and are available once a year starting in 1992Q1. The 20-year Treasury constant maturity rate is not available from 1987Q1 to 1993Q3. For this period, following Haver Analytics we use instead an average of the 10 and 30-year Treasury constant maturity rates (GS10 and GS30, respectively). We use quarterly averages for all variables that are available at higher frequency than quarterly.

²³Results with a tighter prior of $1/400$ for the variance of the inflation trend only change in that the trend in inflation does not rise as much as long run inflation expectations in the mid-1970s, but are otherwise very similar to the ones shown here.

²⁴Our prior for the variance Σ_ϵ is a very uninformative Inverse Wishart distribution centered at a diagonal matrix with unitary elements (except for inflation, for which the diagonal element is 2, and expectations, for which the variance is .5; these numbers reflect presample variances, except for expectations which are not available) with just enough degrees of freedom ($n + 2$) to have a well-defined prior mean. We do not use the “co-persistence” or “sum-of-coefficients” priors of Sims and Zha (1998).

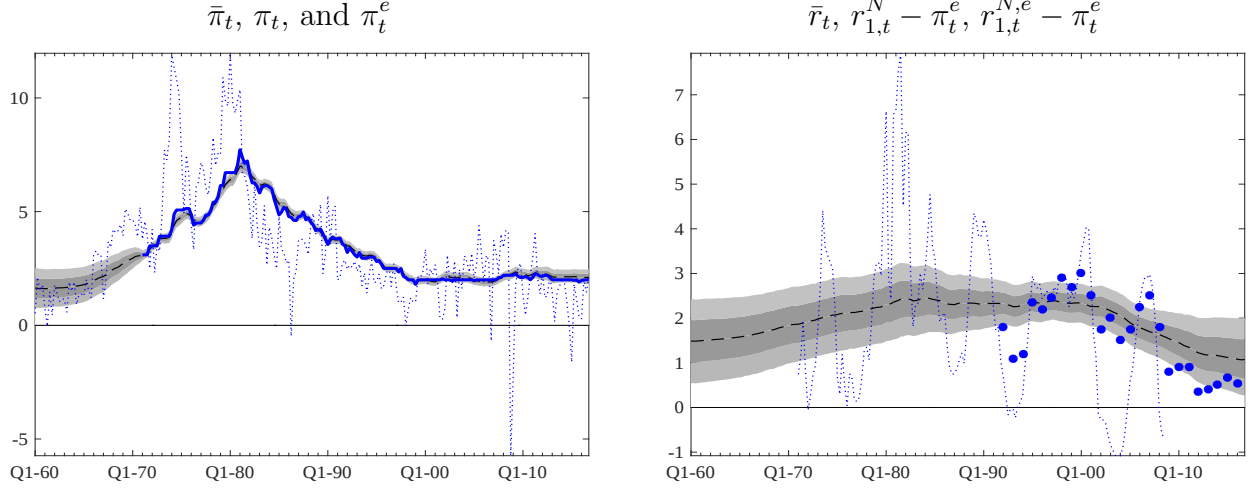
Figure 1: $\bar{r}_t$ 

Note: The dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

(this convention applies to all latent variables shown below; also, all figures showing estimates of the trends use the same scale, so that the magnitude of the changes and the inference uncertainty are easily assessed). $\bar{r}_t$ rises from the 1960s to the early 1980s, remains roughly constant until the late 1990s, and then begins to decline. The median decline in $\bar{r}_t$ from 1999Q1 to 2016Q4 is slightly above 1 percent, from 2.5 to 1.4. It is apparent from the figure that the use of long run short rate expectations helps in terms of the inference on the trend, as the bands for $\bar{r}_t$ narrow considerably when these data become available (the bands become somewhat wider again in the ZLB period as we are not using data on the short term rate during this period).

We now discuss the trends in the context of the data that inform the trend extraction. The left panel of Figure 2 shows the data, π_t (dotted blue line), and π_t^e (solid blue line), together with the trend $\bar{\pi}_t$. We find that $\bar{\pi}_t$ appears to capture well the trend in inflation and essentially coincides with long run inflation expectations, whenever these are available, even though the model only imposes that π_t , and π_t^e share a common trend. The right panel shows the short term rate $r_{1,t}^N$ and the long run expectations for the short term rate $r_{1,t}^{N,e}$, both expressed in deviations from long run inflation expectations π_t^e so that trends in the real

Figure 2: Trends and Observables, Baseline Model



Note: The left panel shows π_t (dotted blue line), and π_t^e (solid blue line), together with the trend $\bar{\pi}_t$. The right panel shows $r_{1,t}^N - \pi_t^e$ (dotted blue line), and $r_{1,t}^{N,e} - \pi_t^e$ (blue dots), together with the trend $\bar{r}_t$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

variables become more apparent, along with the estimates of the trend $\bar{r}_t$.²⁵ it is apparent that $\bar{r}_t$ declines since the late 1990s along with the decline in long term expectations for the short-term real rate $r_{1,t}^{N,e} - \pi_t^e$. Toward the end of the sample the trend remains above the data for $r_{1,t}^{N,e} - \pi_t^e$, which is arguably reasonable in light of the fact that these 10-year averages partly reflect cyclical movements – e.g., the slow renormalization of real rates in the aftermath of the crisis.²⁶

²⁵The time series for $r_{1,t}^N - \pi_t^e$ begins in 1970 simply because long run inflation expectations were not available before then. Figure A2 in the Appendix shows the entire time series for $r_{1,t}^N$ and the associated trends.

²⁶ Figure A1 in the Appendix shows the estimated trends in the term premium together with the term spread $r_{80,t}^N - r_{1,t}^N$. Figure A2 shows all the data y_t used in the estimation together with $\Lambda \bar{y}_t$ and $\tilde{y}_t$, the non-stationary and stationary components, respectively. The figure shows that the model fits the trends in the data reasonably well, including those in the 20-year yield, in that the $\tilde{y}_t$'s do indeed look stationary. In the aftermath of the Great Recession, however, all of the stationary components are persistently negative, including those for inflation and long run rates expectations. The model suggests that the Great Recession has had a persistently negative effect on the cyclical component of inflation and interest rates, possibly capturing headwinds to the recovery.

4.2 Decomposing Trends in r : The Role of the Convenience Yield

In this section we refine the approach outlined above with the goal of assessing the component of long term movements in r due to changes in the convenience yield. In order to do that we bring into the analysis assets whose safety/liquidity attributes are not the same as those of nominal Treasuries.

The Euler equation

$$E_t[e^{r_t}(1 + CY_{t+1})M_{t+1}] = 0$$

implies that trends in r_t are driven by trends in the convenience yield CY_t and in the stochastic discount factor M_t . In order to proceed we make the assumption that the covariance between CY_t and M_t is stationary and write:

$$\bar{r}_t = \bar{m}_t - \bar{cy}_t, \quad (14)$$

where $cy_t = \log(1 + CY_t)$ and $m_t = -\log M_t$. In addition, we assume that the trends $\bar{cy}_t$ and $\bar{m}_t$ evolve independently from one another according to equation (4) (although shocks to the trend are allowed to be correlated). Trends in the discount factor could be due to trends in the growth rate of productivity, demographics, or the discount rate – we will explore these factors in subsequent sections.

Using the above decomposition we can write expression (10), (12), and (13) as

$$r_{1,t}^N = \bar{m}_t - \bar{cy}_t + \bar{\pi}_t + \tilde{r}_{1,t}^N, \quad (15)$$

$$r_{80,t}^N = \bar{m}_t - \bar{cy}_t + \bar{\pi}_t + \bar{tp}_t + \tilde{r}_{80,t}^N, \text{ and} \quad (16)$$

$$r_{1,t}^{N,e} = \bar{m}_t - \bar{cy}_t + \bar{\pi}_t + \tilde{r}_{1,t}^{N,e}. \quad (17)$$

Note that since we are assuming that in the long run all Treasuries, regardless of maturity, benefit in equal measure of the same safety and liquidity attributes as 3-month bills (an assumption we discuss below), the system (16) is of no use in disentangling $\bar{cy}_t$ from $\bar{m}_t$.

In order to do that we need to consider assets who carry less of a convenience yield than Treasuries. Krishnamurthy and Vissing-Jorgensen (2012) use the spread between Baa corporate bonds and Treasuries to identify the convenience yield. We follow their lead and augment the set of observables with the yield of Baa corporate bonds, which we model as follows:

$$r_t^{Baa} = \bar{m}_t - \lambda_{cy}^{Baa} \bar{cy}_t + \bar{\pi}_t + \bar{tp}_t + \tilde{r}_t^{Baa}, \quad (18)$$

where $0 \leq \lambda_{cy}^{Baa} < 1$, indicating that Baa corporate bonds are less liquid/safe than Treasuries. Note that we use the same term premium that we use in equivalent maturity Treasuries (following Krishnamurthy and Vissing-Jorgensen, 2012, we use 20-year Treasury yields as the reference Treasury yield), which means that we constrain the term premium to be the same, at least in the long run. Of course the spread between Baa corporate bonds and Treasuries reflects also actual probability of default, in addition to the convenience yield. The stationary component $\tilde{r}_t^{Baa}$ will therefore also reflect changes in default probabilities. However, we will show later that the data on “distance to default” used in Gilchrist and Zakrajsek (2012) show that there is no apparent trend in average default probabilities and, to the extent that such trend exists, it is downward since the late 1990s.

From equations (18) and (16) it follows that the spread between Baa corporate bonds yields and equivalent maturity Treasuries is given by

$$r_t^{Baa} - r_{80,t}^N = (1 - \lambda_{cy}^{Baa})\overline{cy}_t + \tilde{r}_t^{Baa} - \tilde{r}_{80,t}^N, \quad (19)$$

which implies that we can measure trends in the convenience yield by measuring trends in the spread. Specifically, we will assume that $\lambda_{cy}^{Baa} = 0$, that is, that Baa corporates do not have any convenience yield whatsoever. Given the measured difference in trends $\tilde{r}_t^{Baa} - \tilde{r}_{80,t}^N$ between Baa corporate bonds yields and equivalent maturity Treasuries this assumption is the most conservative in terms of extracting $\overline{cy}_t$. We should also stress that our results focus on secular *changes* in the convenience yield, as opposed to its *level*. The level of the Baa/Treasury spread may be affected by factors other than safety and liquidity premiums (e.g., default probabilities, as discussed above). The key identifying assumption we use is that secular changes in the spread primarily reflect secular changes in the convenience yield.

Equation (19) deserves additional comments. First, as explained very clearly in Krishnamurthy and Vissing-Jorgensen (2012) the spread $r_t^{Baa} - r_{80,t}^N$ captures not just the current value of the convenience yield, but rather the expected average convenience yield throughout the remaining maturity of the bond. But this is precisely what we need since we are after trends in the convenience yield. Second, we assumed that long term Treasuries benefit of the same convenience yield as short term Treasuries. In making this assumption, we are arguably underestimating the convenience yield on short term Treasuries, which is what we are after. All Treasuries are equally safe, irrespective of their maturity, hence it is reasonable to assume that the component of the convenience yield deriving from safety applies evenly

across maturities. As for the component associated with liquidity, Greenwood et al. (2015) provide some evidence that the liquidity premium is a decreasing function of maturity. They compute what they call *z-spreads*, which capture deviations in the pricing of Treasury Bills from the an extrapolation based on the rest of the yield curve, and argue that these *z-spreads*, which are sizable, “reflect a money-like premium on short-term T-bills, above and beyond the liquidity and safety premia embedded in longer term Treasury yields” (pg. 1687). In conclusion, for these reasons we think that our assumption that the convenience yields extracted from long term Treasuries applies in the same measure to Treasury Bills is conservative, it is an assumption nonetheless and one should bear that in mind in interpreting our results.

4.2.1 Results

The system of equations given by (11) and (15) through (18) can be expressed as a Trendy VAR where $y_t = (\pi_t, \pi_t^e, r_{1,t}^N, r_{80,t}^N, r_{1,t}^{N,e}, r_t^{Baa})$ and $\bar{y}_t = (\bar{r}_t, \bar{\pi}_t, \bar{c}y_t, \bar{t}p_t)$ is the vector of trends.²⁷ We use exactly the same priors as described in section 4.1.1, except that since we decompose the trend $\bar{r}_t$ into two components, $\bar{m}_t$ and $\bar{c}y_t$, we center the corresponding diagonal value of $\underline{\Sigma}_\varepsilon$ to a number that is 1/2 the value chosen for $\bar{r}_t$ (we use 1/800 as opposed to 1/400).²⁸

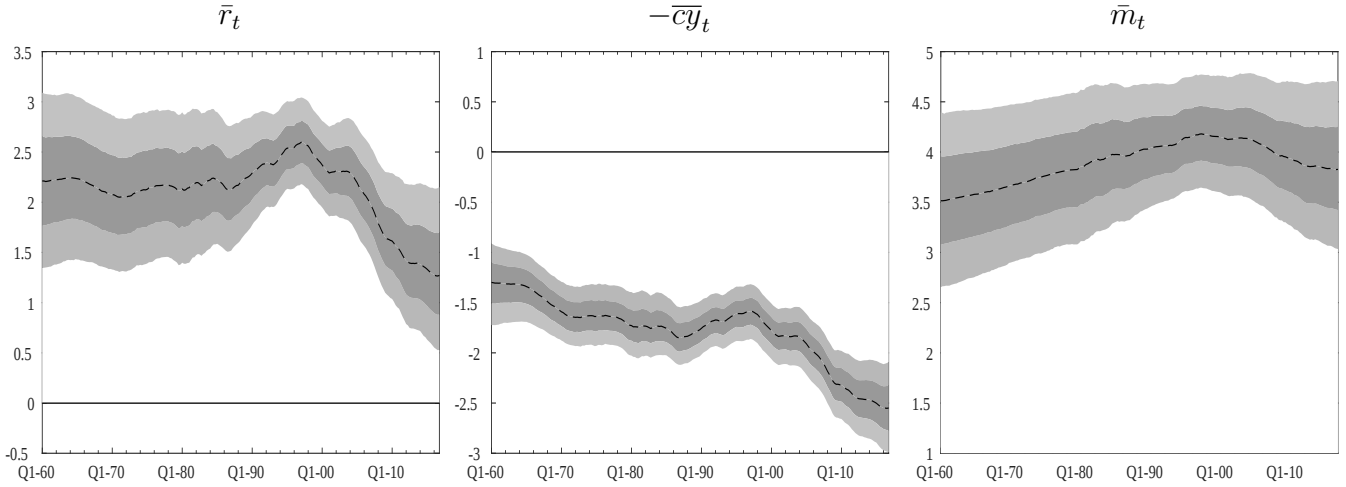
Figure 3 shows the trend $\bar{r}_t$ and its decomposition between trends in the convenience yield for safety/liquidity $\bar{c}y_t$ and the stochastic discount factor $\bar{m}_t$. The three time series are purposefully plotted using the same scale, so that the magnitude of their changes can be properly compared. The time series of $\bar{r}_t$ is similar to that shown in Figure 1, albeit not identical (recall we are now using a larger cross section of yields to pin down $\bar{r}_t$). Most importantly however in terms of the question this papers addresses, the decline in $\bar{r}_t$ from the late 90s to the present is of the same magnitude obtained before – slightly larger than 1 percent. The other two panels show that much of this decline is attributable to an increase in the convenience yield, rather than to a fall in $\bar{m}_t$. The converse of the convenience yield (

²⁷The Baa yield is available from FRED (mnemonic, Baa). As described in Krishnamurthy and Vissing-Jorgensen (2012, pg. 262) “The Moodys Baa index is constructed from a sample of long-maturity (≥ 20 years) industrial and utility bonds (industrial only from 2002 onward).” This series is available throughout the whole sample, but ends in 2016Q3.

²⁸The initial condition $\bar{c}y_0$ is set at 1 using presample averages for the Baa/Treasury spread, and correspondingly set $\bar{m}_0$ to 1.5 ($\bar{r}_0 + \bar{c}y_0$). The variance of the initial conditions is 1, as is the case for all other trends.

$-\overline{cy}_t$) falls by a bit more than one percent, and the decrease is very precisely estimated. $\bar{m}_t$ also declines a bit in the new century, but it barely budes and it is estimated with much uncertainty. We should stress once again that the reader should not focus on the level of $\bar{m}_t$ and $\overline{cy}_t$, but on their changes. Our statement is not “Were it not for the convenience yield from liquidity/safety, the secular components of real rates would be almost 4 percent” but rather “Much of the decline in rates over the past twenty years is due to the convenience yield.” This is because the level of the spread spread $r_t^{Baa} - r_{80,t}^N$ is affected by other factors – such as the probability of default, as discussed below – other than the convenience yield.

Figure 3: $\bar{r}_t$, $\overline{cy}_t$, and $\bar{m}_t$



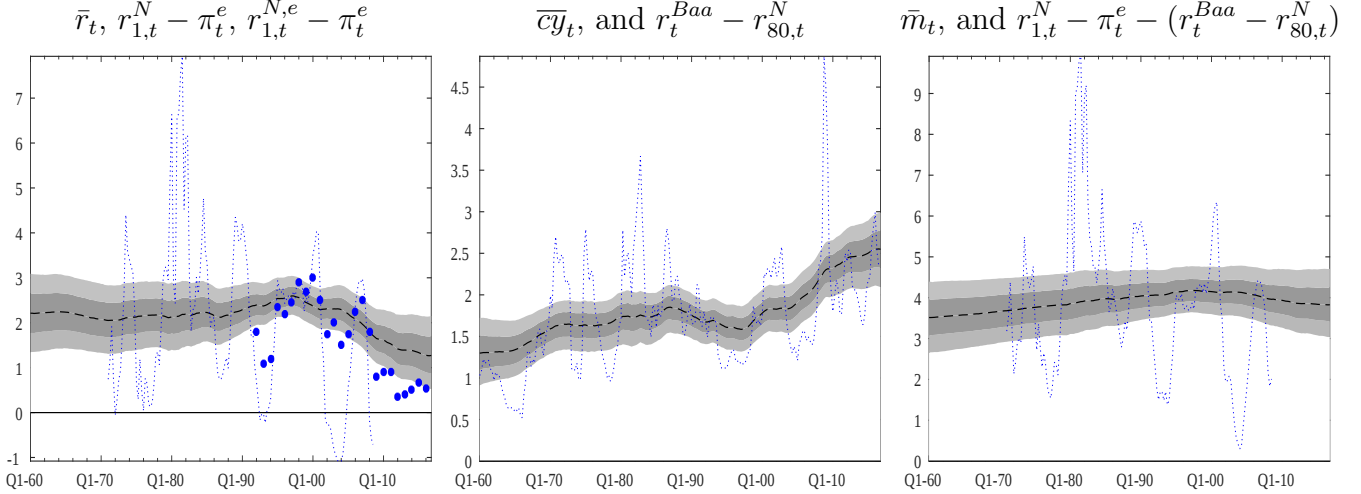
Note: The dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure 4 shows the information on which the inference on the trends is based.²⁹ The left panel shows $\bar{r}_t$ together the short term rate $r_{1,t}^N$ and the long run expectations for the short term rate $r_{1,t}^{N,e}$, both expressed in deviations from long run inflation expectations π_t^e , similarly to the right panel of Figure 2. It shows that our inference on the decline in $\bar{r}_t$ after the late 1990s is based on the fact that real rates decline, with much of the information coming from the less volatile long run expectations. The middle panel shows $\overline{cy}_t$, and the spread between Baa securities and comparable Treasuries $r_t^{Baa} - r_{80,t}^N$. This spread has a clear upward trend, especially starting right before the turn of the century, which $\overline{cy}_t$ picks up. The right panel

²⁹Figure A3 in the Appendix shows the remaining estimated trends ($\bar{\pi}_t$ and $\bar{tp}_t$) along with the relevant data. Figure A4 shows all the data y_t used in the estimation together with $\Lambda\bar{y}_t$ and $\tilde{y}_t$, the non-stationary and stationary components, respectively.

shows the “real rate” $r_{1,t}^N - \pi_t^e$ minus the spread $r_t^{Baa} - r_{80,t}^N$. It shows that once we take out the part of the decline in rates due to the increase in the convenience yield, it is not clear that there is much of a trend to explain, which is why $\bar{m}_t$ barely moves.

Figure 4: Trends and Observables, Convenience Yield Model



Note: The left panel shows $r_{1,t}^N - \pi_t^e$ (dotted blue line), and $r_{1,t}^{N,e} - \pi_t^e$ (blue dots), together with the trend $\bar{r}_t$. The middle panel shows the Baa/Treasury spread $r_t^{Baa} - r_{80,t}^N$ (dotted blue line), together with the trend $\bar{c}y_t$. The right panel shows $r_{1,t}^N - \pi_t^e - (r_t^{Baa} - r_{80,t}^N)$ (dotted blue line), together with the trend $\bar{m}_t$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

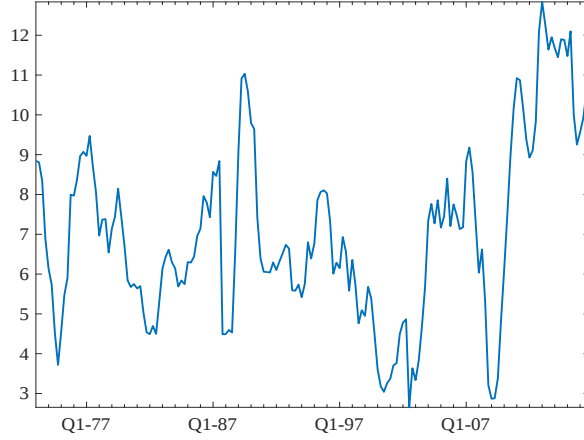
Another perspective on what we find is that the secular decline in real rates for unsafe/illiquid securities has been much less pronounced, if it has taken place at all, than that for safe/liquid securities. This lack of decline is not explained by a secular increase in the probability of default. Figure 5 shows the median distance to default (DD) in the non-financial corporate sector used in Gilchrist and Zakrajsek (2012) – the higher DD the *lower* the default probability.³⁰ This measure has a clear upward trend since the late 1990s, implying that there is, if anything, a secular *decrease* in the default probability.

As discussed in the introduction, the trend increase in the safety/liquidity convenience yield since the late 1990s is very much in line with the narrative put forth by Caballero (2010) and the “safe assets” literature more broadly. The Asian crisis first resulted in excess supply of savings which, being institutional (that is, intermediated via central banks), was naturally directed toward safe and liquid assets. The NADSAQ crash further rendered safe

³⁰These are the data shown in Figure 2 of Gilchrist and Zakrajsek (2012). We profusely thank Egon Zakrajsek for providing us with updated estimates.

assets more attractive. The housing boom and the related creation of allegedly safe securities partly met this increased demand, but this suddenly came to a halt with the housing crisis and the great recession, which resulted in an additional increased demand, and reduced supply, of safe and liquid assets.

Figure 5: Distance to Default



Note: The figure shows the median distance to default (DD) in the non-financial corporate sector used in Gilchrist and Zakrajsek (2012).

4.3 Trends in the Compensation for Safety and Liquidity

Following Krishnamurthy and Vissing-Jorgensen (2012), we distinguish the convenience yield $(1 + CY_t)$ into two components, one due to liquidity $(1 + CY_t^l)$ and one to safety $(1 + CY_t^s)$ and write the Euler equation for a safe/liquid security as

$$E_t[e^{r_t}(1 + CY_{t+1}^l)(1 + CY_{t+1}^s)M_{t+1}] = 0.$$

Under the assumption that the covariance between CY_t^l , CY_t^s , and M_t is stationary we obtain that:

$$\bar{r}_t = \bar{m}_t - \bar{cy}_t^l - \bar{cy}_t^s, \quad (20)$$

which implies that expression (10), (12), and (13) become

$$r_{1,t}^N = \bar{m}_t - \bar{cy}_t^l - \bar{cy}_t^s + \bar{\pi}_t + \tilde{r}_{1,t}^N, \quad (21)$$

$$r_{80,t}^N = \bar{m}_t - \bar{cy}_t^l - \bar{cy}_t^s + \bar{\pi}_t + \bar{tp}_t + \tilde{r}_{80,t}^N, \text{ and } \quad (22)$$

$$r_{1,t}^{N,e} = \bar{m}_t - \bar{cy}_t^l - \bar{cy}_t^s + \bar{\pi}_t + \tilde{r}_{1,t}^{N,e}. \quad (23)$$

As before, we assume that the trends $\overline{cy}_t^l$, $\overline{cy}_t^s$, and $\overline{m}_t$ evolve independently from one another according to equation (4) (although shocks to the trend are allowed to be correlated).

The distinction between liquidity and safety has two benefits. First, from an economic point of view, it allows us to disentangle the importance of the two components in explaining trends in r^* . In order to do so, of course, we have to be able to identify the two trends separately. Following once again Krishnamurthy and Vissing-Jorgensen (2012) we do so by bringing into the analysis the Aaa corporate yield, an index of securities who virtually never default, and hence carry as much of a safety discount as Treasuries, but are less liquid than Treasuries, and hence enjoy less of a liquidity premium. We therefore write:

$$r_t^{Aaa} = \overline{m}_t - \lambda_l^{Aaa} \overline{cy}_t^l - \overline{cy}_t^s + \bar{\pi}_t + \overline{tp}_t + \tilde{r}_t^{Baa}, \quad (24)$$

$$r_t^{Baa} = \overline{m}_t - \lambda_l^{Aaa} \overline{cy}_t^l - \lambda_s^{Baa} \overline{cy}_t^s + \bar{\pi}_t + \overline{tp}_t + \tilde{r}_t^{Baa}, \quad (25)$$

where $0 \leq \lambda_l^{Aaa} < 1$ and $0 \leq \lambda_s^{Baa} < 1$, indicating that both Aaa and Baa corporate bonds are less liquid than Treasuries (we assume that their degree of illiquidity is the same, hence $\lambda_{Baa}^l = \lambda_l^{Aaa}$), and that Baa corporate bonds are less safe than Treasuries. From equations (24), (25) and (22) it follows that

$$\begin{aligned} \bar{r}_t^{Aaa} - \bar{r}_{80,t}^N &= (1 - \lambda_l^{Aaa}) \overline{cy}_t^l. \\ \bar{r}_t^{Baa} - \bar{r}_t^{Aaa} &= (1 - \lambda_s^{Baa}) \overline{cy}_t^s. \end{aligned}$$

As before, we will make the conservative assumptions that Baa bonds earn no safety and liquidity premium whatsoever, and that Aaa bonds are completely illiquid. This assumptions are conservative in the sense that they minimize time variation in the trends $\overline{cy}_t^l$ and $\overline{cy}_t^s$ given the observed trends in the spreads $\bar{r}_t^{Aaa} - \bar{r}_{80,t}^N$ and $\bar{r}_t^{Baa} - \bar{r}_t^{Aaa}$.

4.3.1 Results

The system of equations given by (11) and (26) through (25) can be expressed as the Trendy VAR where $y_t = \left(\pi_t, \pi_t^e, r_{1,t}^N, r_{80,t}^N, r_{1,t}^{N,e}, r_t^{Aaa}, r_t^{Baa} \right)$ and trends $(\bar{r}_t, \bar{\pi}_t, \overline{cy}_t^s, \overline{cy}_t^l, \overline{tp}_t)$.³¹ We use exactly the same priors as described in sections 4.1.1 and 4.2.1, except that since we decompose the trend $\overline{cy}_t$ into two components, $\overline{cy}_t^s$ and $\overline{cy}_t^l$, we center the corresponding diagonal

³¹The Aaa yield is also available from FRED (mnemonic, AAA) and has similar characteristics as the Baa index in terms of maturity. This series is available throughout the whole sample, but ends in 2016Q3.

values of $\underline{\Sigma}_\varepsilon$ to a number that is 1/2 the value chosen for $\overline{cy}_t$ (we use 1/1600 as opposed to 1/800).³² This obviously makes it harder to find a trend in these convenience yields.

Figure 6 shows the estimated trends in the overall convenience yield $\overline{cy}_t$, and the convenience yields attributed to safety ($\overline{cy}_t^s$) and liquidity ($\overline{cy}_t^l$), along with the information that the model uses to extract these trends (Figure A5 in the Appendix shows the estimated trend $\bar{r}_t$ and its decomposition between $\overline{cy}_t$ and $\bar{m}_t$ for this model; these results are nearly identical to those shown above in Figure 3).³³ The left panel shows $\overline{cy}_t = \overline{cy}_t^s + \overline{cy}_t^l$, and the spread between Baa securities and Treasuries $r_t^{Baa} - r_{80,t}^N$. Again, in spite of the fact that now the trends $\overline{cy}_t^s$ and $\overline{cy}_t^l$ are now separately estimated, the inference for $\overline{cy}_t$ is the same as that shown in Figure 4. The middle panel shows $\overline{cy}_t^s$ and the spread between Baa and Aaa bonds $r_t^{Baa} - r_t^{Aaa}$. The trend in this spread, according to the model, has less of a secular increase in the overall sample than the overall convenience yield. The trend in the safety premium increases in the 1970s, reaches a pick in the early eighties, declines progressively until the NASDAQ crash, and finally increases by about 50 basis points until the end of the sample.

The right panel shows $\overline{cy}_t^l$, and the spread between Aaa securities and Treasuries $r_t^{Aaa} - r_{80,t}^N$. The trend $\overline{cy}_t^l$ has a more pronounced secular increase in the overall sample.³⁴ From the perspective of the focus of the paper – the sources of the decline in real rates since the 1990s – the right panel shows an increase in $\overline{cy}_t^l$ by about 50 basis points since the turn of the century.³⁵ Much of this increase occurred during and after the financial crisis. This is not surprising, because the liquidity shock in the aftermath of the Lehman crisis drastically curtailed the supply of liquid assets (as several asset classes became less liquid;

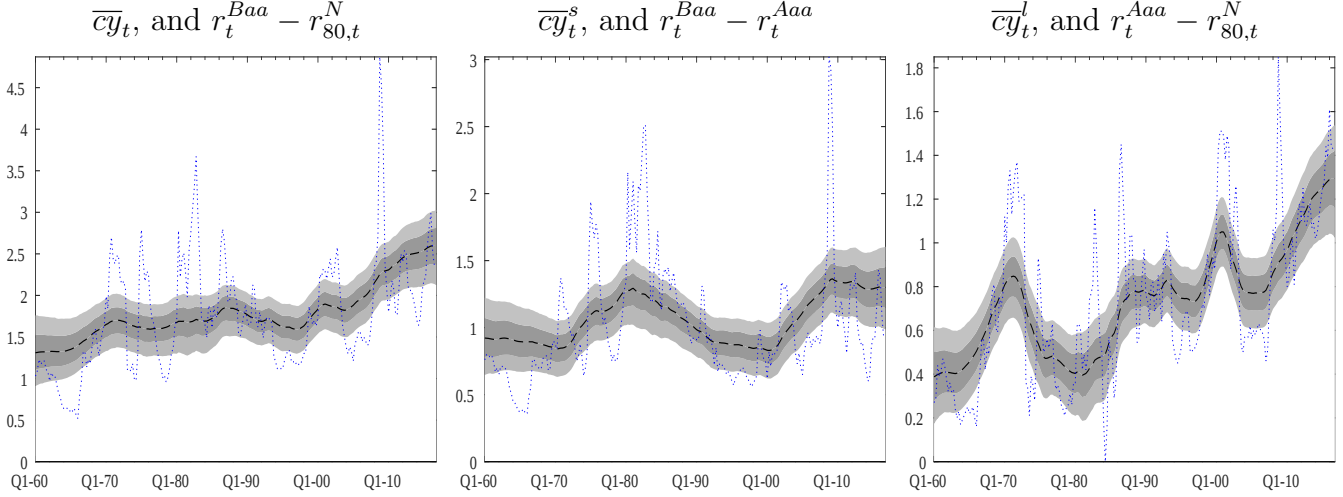
³²The initial conditions $\overline{cy}_0^s$ and $\overline{cy}_0^l$ are set at .75 and .25 using presample averages for the Baa/Aaa and the Aaa/Treasury spreads. The variance of the initial conditions is 1, as is the case for all other trends.

³³Figure A7 in the Appendix shows the remaining estimated trends ($\bar{\pi}_t$, $\bar{r}_t$, $\bar{m}_t$, and $\bar{tp}_t$) along with the relevant data. Figure A8 shows all the data y_t used in the estimation together with $\Lambda \bar{y}_t$ and $\tilde{y}_t$, the non-stationary and stationary components, respectively.

³⁴While the transitory spikes in the convenience yield are easily explained by financial events (e.g., the stock market crash of 1987, the burst of the 1990s stock market bubble and September 11, the Lehman crisis) this secular increase is for us not straightforward to explain but we find it an interesting question for future research.

³⁵Note that the high frequency spike in illiquidity occurred during the financial crisis does not seem to play an important role in the extraction of the trend; in other words, the increase in the compensation for liquidity appears to be mostly driven by the low frequency movements in the spreads.

Figure 6: Trends in Compensation for Safety and Liquidity, and Observables



Note: The left panel shows the Baa/Treasury spread $r_t^{Baa} - r_{80,t}^N$ (dotted blue line), together with the trend $\overline{cy}_t$. The middle panel shows the Baa/Aaa spread $r_t^{Baa} - r_t^{Aaa}$ (dotted blue line), together with the trend $\overline{cy}_t^s$. The right panel shows the Aaa/Treasury spread $r_t^{Aaa} - r_{80,t}^N$ (dotted blue line), together with the trend $\overline{cy}_t^l$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

see for instance Del Negro et al., forthcoming; Gorton and Metrick, 2012) and at the same time increased its demand. In addition, the regulatory changes after the crisis (see the liquidity requirements for financial institutions under Basel III; Basel Committee on Banking Supervision, 2013) also led to an increased demand for liquid assets, as well as a decline in the supply of liquid liabilities from the financial system. In conclusion, we find that the increase in the convenience yield since the late 1990s is roughly evenly split between compensation for safety and liquidity.

4.4 Robustness

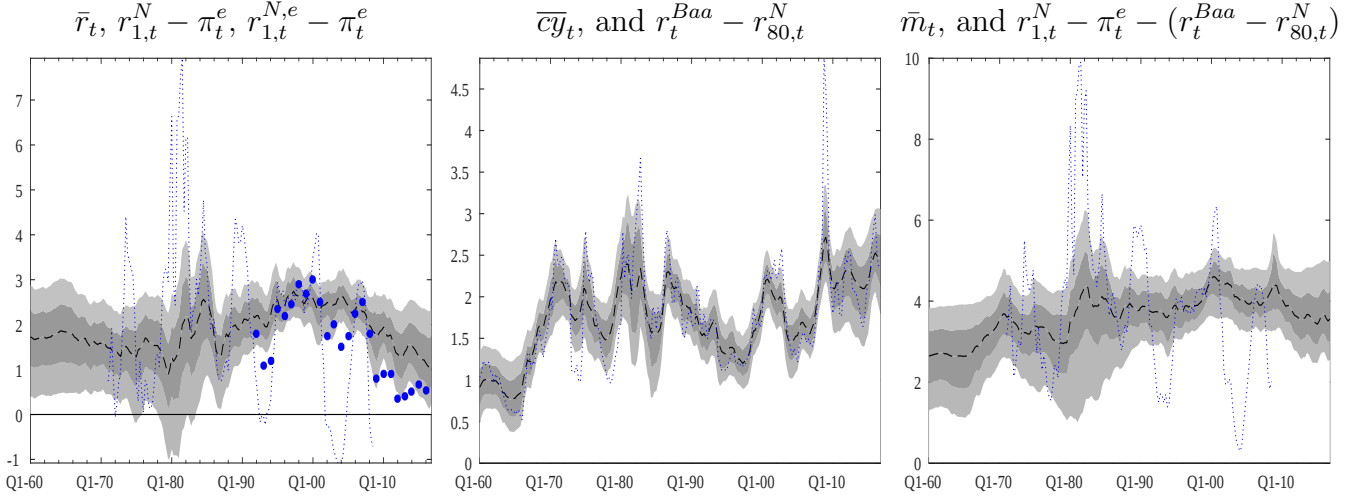
This section considers some variants to our baseline specification.

4.4.1 Loose Prior on the Trend

One of our main results is that after accounting for trends in liquidity there is not much to explain in terms of trends in the discount factor $\bar{m}_t$. One natural objection to our argument is that our prior on the standard deviation of the trends innovation is too conservative and

too tight. If we had a looser prior, we may possibly find more evidence of a trend in $\bar{r}_t$ and consequently $\bar{m}_t$.

Figure 7: Trends and Observables, Loose Prior on the Trend



Note: The left panel shows $r_{1,t}^N - \pi_t^e$ (dotted blue line), and $r_{1,t}^{N,e} - \pi_t^e$ (blue dots), together with the trend $\bar{r}_t$. The middle panel shows the Baa/Treasury spread $r_t^{Baa} - r_{80,t}^N$ (dotted blue line), together with the trend $\bar{c}y_t$. The right panel shows $r_{1,t}^N - \pi_t^e - (r_t^{Baa} - r_{80,t}^N)$ (dotted blue line), together with the trend $\bar{m}_t$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure 7 dispels this notion. This figure shows the outcome of reestimating the model of section 4.3 loosening the prior on variance-covariance matrix as much as possible (we use 8 degrees of freedom, barely enough so that the prior has a well defined mean, as opposed to the 100 used in the baseline specification). The result is that the trend is no longer a trend in the sense that it also captures high frequency fluctuations in the observables – which is why we think the original tight prior is appropriate. But the broad contours of the results is the same: $\bar{c}y_t$ trends upward while $\bar{m}_t$ does not move much.³⁶

4.4.2 Inflation Affecting the Nominal Term Premium

As anticipated, we also allow for the possibility that trends in inflation affect the nominal term premium – that is, we model the term premium as the sum of an exogenous component $\bar{t}p_t$ and a component $\gamma^{tp}\bar{\pi}_t$, where γ^{tp} is estimated (we use an exponential distribution with

³⁶We obtain similar results when we instead quadruple the variance of the trend innovations, but leave intact the number of degrees of freedom.

mean 1/10 as the prior).³⁷ We therefore replace equations (22), (24), and (25) with

$$r_{80,t}^N = \bar{m}_t - \bar{cy}_t^l - \bar{cy}_t^s + \bar{\pi}_t + \bar{tp}_t + \gamma^{tp}\bar{\pi}_t + \tilde{r}_{80,t}^N, \quad (26)$$

$$r_t^{Aaa} = \bar{m}_t - \lambda_t^{Aaa}\bar{cy}_t^l - \bar{cy}_t^s + \bar{\pi}_t + \bar{tp}_t + \gamma^{tp}\bar{\pi}_t + \tilde{r}_t^{Baa}, \text{ and} \quad (27)$$

$$r_t^{Baa} = \bar{m}_t - \lambda_t^{Aaa}\bar{cy}_t^l - \lambda_s^{Baa}\bar{cy}_t^s + \bar{\pi}_t + \bar{tp}_t + \gamma^{tp}\bar{\pi}_t + \tilde{r}_t^{Baa}. \quad (28)$$

The results under this specification are nearly identical to those shown in section 4.3.1 and therefore are relegated to the online appendix (Figure A10; Figure A9 shows the posterior distribution of γ^{tp}).

4.4.3 Callability

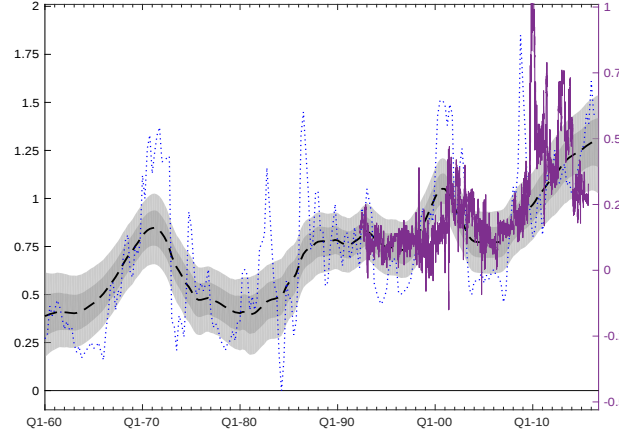
Many corporate bonds are callable, while Treasuries are not (at least since 1985). One may wonder whether secular changes in the value of the call option drive secular changes in the spread. Fortunately, there are other spreads mainly reflecting liquidity other than the Aaa/Treasuries spread. The Refcorp/Treasury spread is one of them. This spread, according to Longstaff et al. (2004), is mostly (if not entirely) due to liquidity as Refcorp bonds are effectively guaranteed by the U.S. government, are subject to the same taxation and, importantly, to the best of our knowledge they are not callable.³⁸

Figure 8 plots the estimated trend in the liquidity convenience yield $\bar{cy}_t^l$ shown in the right panel of Figure A6 together with daily data on the Refcorp/Treasury spread collected by Del Negro et al. (forthcoming) from 4/16/1991 to 9/06/2014. The figure shows that the trend in liquidity estimated using the Aaa/Treasury spread matches very well the trends in the Refcorp/Treasury spread, whenever this is available. In addition to suggesting that

³⁷The exponential distribution with parameter $\underline{\gamma}^{-1}$ is $p(\gamma; \underline{\gamma}^{-1}) = \underline{\gamma}^{-1} \exp(-\underline{\gamma}^{-1}\gamma) \mathcal{I}\{\gamma \geq 0\}$, where $\mathcal{I}\{\cdot\}$ is an indicator function.

³⁸ Refcorp bonds differ from most other agency bonds in that their principal is fully collateralized by Treasury bonds and full payment of coupons is guaranteed by the Treasury under the provisions of the Financial Institutions Reform, Recovery, and Enforcement Act of 1989. Longstaff et al. (2004) does not mention callability as a feature of these bonds. Lehman Brother's "Guide to Agency and Government-Related Securities" does not mention callability in reference to Refcorp bonds, while it discusses callability for other agency securities. As in Longstaff et al. (2004), we measure the spread by taking the differences between the constant maturity 10-year points on the Bloomberg fair value curves for Refcorp and Treasury zero-coupon bonds. The Bloomberg mnemonics are 'C091[X]Y Index' and 'C079[X]Y Index', respectively, where [X] represents the maturity.

Figure 8: Trends in the Liquidity Convenience Yield and the Refcorp Treasury Spread



Note: The figure shows the estimated trend in the liquidity convenience yield $\bar{cy}_t^l$ (the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals) and the Refcorp/Treasury spread. See footnote 38 for a description of how this spread is constructed.

callability is not the driving force behind secular movements in the Aaa/Treasury spread, Figure 8 provides important external validation to our analysis.³⁹

5 The Natural Rate of Interest in DSGE Models

Our analysis so far focused on the long-run trends in r^* and the factors that drive them. However, the natural rate of interest also fluctuates over the business cycle. For instance, the simple consumption Euler equation (2) above suggests that business cycle movements in the marginal utility of consumption and in the convenience yield will affect r^* .

This section presents estimates of r^* based on an empirical medium-scale Dynamic Stochastic General Equilibrium (DSGE) model that features nominal price and wage rigidities, as well as a host of real and financial frictions. Within this New Keynesian environment,

³⁹An alternative approach to addressing the issue of callability is the one taken by Gilchrist and Zakrajsek (2012) in the construction of the excess bond premium, who use a panel regression where they regress individual corporate spreads on individual measures of default probability as well as variables that likely capture the value of the call option. While this micro-level approach has several advantages relative to our aggregate approach – especially in terms of the exact computation of the spreads in terms of maturity and the removal of default probability – it has one important drawback from our perspective: in order to remove the call option the spreads are regressed on the level of the interest rate, among other variables, thereby removing the very trends we are interested in.

we define r^* as the real interest rate that would prevail in equilibrium in the absence of sticky prices and wages. The literature usually refers to this construct as the natural rate of interest, in homage to an idea proposed by Wicksell (1898) and brought into modern macroeconomics by Woodford (2003).⁴⁰

This particular notion of r^* is a useful tool in macroeconomic and monetary analysis for several related reasons. First, the natural rate does not depend on monetary policy. In the equilibrium without nominal rigidities, monetary policy is neutral, in the sense that it does not affect any real variable, including the real interest rate.⁴¹ Therefore, the natural rate answers the question: what would the real interest rate be, “without” monetary policy? Answering this counterfactual question requires a general equilibrium framework, such as that provided by our estimated DSGE model.

Second, the gap between actual interest rates and their natural level is a more appropriate measure of the impetus (or restraint) imparted by monetary policy to aggregate demand than the level of the policy rate itself, as further discussed in Section 5.2.2. In Wicksell’s own words, “it is not a high or low rate of interest in the absolute sense which must be regarded as influencing the demand for raw materials, labour, and land or other productive resources, and so indirectly as determining the movement of prices. The causality factor is the current rate of interest on loans as compared to [the natural rate].”

This property of the natural rate does not imply that closing the interest rate gap is optimal. This is the case only in extremely simple models that do not feature a trade-off between real and nominal stabilization. In these models, closing the interest rate gap stabilizes the output gap, and at the same time inflation. In larger, more realistic DSGE models, this “divine coincidence” (Blanchard and Galí, 2007) between price stability and full employment does not hold. Nonetheless, a monetary policy strategy in which the real policy rate tracks the natural rate generally promotes stable inflation and economic activity

⁴⁰Neiss and Nelson (2003) were the first to evaluate the properties of the natural rate in a calibrated DSGE model. Edge et al. (2008), Justiniano and Primiceri (2008), Barsky et al. (2014), and ? do so in estimated models.

⁴¹This statement holds in the model proposed below, but it might need to be qualified in other environments. Depending on the exact specification of the financial and real frictions, monetary policy might affect real variables even in the absence of nominal rigidities. However, these effects tend to be quantitatively limited in empirical models.

even in those models, providing a more explicitly normative rationale for using estimates of the natural rate as an input in monetary policy making.⁴²

The DSGE perspective on r^* described above is complementary to that explored in the first part of the paper, since it focuses on the effect of economic disturbances on r^* at business cycle (and higher) frequencies. While the VAR only provides an estimate of the low frequency component of r^* , a fully specified DSGE model gives us the entire time-path of r^* , including all frequencies. In fact, one of the key findings of our DSGE exercise is that the estimated model does allow for fluctuations in r^* at very low frequencies, as illustrated in Section 5.2.1. Having an estimate of the value of the natural rate today, and not just of its permanent component, is especially relevant in a policy context, if it is to be used to inform decisions on the appropriate level of the policy rate.

Furthermore, the structure of the DSGE model allows to determine the underlying sources of fluctuations in r^* . For instance, financial disturbances tend to depress r^* at the same time as they depress real activity, explaining why r^* reached historical lows during the Great recession, as we will see in Section 5.2.2.

Of course, the flip side of this more comprehensive view of the movements in r^* provided by the DSGE approach is that inference is conditional on the exact structure of the DSGE model. As such, it is more likely to be affected by model misspecification than the VAR estimates. However, studies based on a variety of empirical DSGE models tend to deliver a fairly consistent view of the business cycle fluctuations in r^* , even if their estimates do not coincide at any given point in time (see for instance Figure 1 in Yellen (2015)). In particular, there is widespread agreement on the fact that the natural rate plunged to its historical lows during the Great Recession, making the lower bound on nominal interest rates binding, and hence severely impairing the ability of the Federal Reserve to stabilize the economy through its conventional policy tool, as we discuss further in Section 5.2.2. This assessment of the behavior of the natural rate over the most recent cycle reinforces the VAR results, which already attributed part of the decline in rates over the past 15 or 20 years to a secular decline in the convenience yield.

⁴²For instance, Justiniano et al. (2013) find that there is a minimal trade-off between nominal and real stabilization in an estimated model similar to the one used here, approximating the divine coincidence that holds exactly in much simpler environments.

5.1 DSGE model

The DSGE model considered here is a version of the FRBNY DSGE model described in Del Negro et al. (2015). It builds on the model of Christiano et al. (2005) and Smets and Wouters (2007), and expands it with various features, most notably financial frictions similarly to Bernanke et al. (1999b) and Christiano et al. (2014). At the core of the model lies a frictionless neoclassical structure in which monetary policy has no effects. This neoclassical core is augmented with frictions such as stickiness of nominal prices and wages (i.e., nominal frictions), various real frictions (such as adjustment costs of capital) and financial frictions that interfere with the flow of funds from savers to borrowers. In addition, the model includes several structural shocks which are the ultimate causes of economic fluctuations, such as shocks to productivity, the marginal efficiency of investment (e.g., Greenwood et al. (1998), Justiniano et al. (2010)), and price and wage markup shocks. We also allow for shocks to liquidity, safety, credit premia in line with the empirical model of Section 4, as well as anticipated policy shocks as in Laseen and Svensson (2011) to account for the zero lower bound on nominal interest rates and forward guidance in monetary policy. The equilibrium conditions are approximated around the non-stochastic steady state.

The model is estimated via Bayesian methods using numerous macroeconomic data series over the 1960Q1-2016Q3 period. The model specification, the data used and the prior distributions assumed for estimation are detailed in the Appendix B. Here, we focus on the parts of the model most closely related to the natural rate of interest.

The *optimal allocation of consumption* satisfies the following Euler equation:

$$c_t = -\frac{1 - \bar{h}}{\sigma_c(1 + \bar{h})} (R_t - \mathbb{E}_t[\pi_{t+1}] + cy_t) + \frac{\bar{h}}{1 + \bar{h}} (c_{t-1} - z_t) \\ + \frac{1}{1 + \bar{h}} \mathbb{E}_t[c_{t+1} + z_{t+1}] + \frac{(\sigma_c - 1)}{\sigma_c(1 + \bar{h})} \frac{w_* L_*}{c_*} (L_t - \mathbb{E}_t[L_{t+1}]), \quad (29)$$

where c_t is consumption, L_t denotes hours worked, R_t is the nominal interest rate, π_t is inflation, and z_t is productivity growth. All variables are expressed in (log) deviations from the steady state. The parameter σ_c captures the degree of relative risk aversion while $\bar{h} \equiv h e^{-\gamma}$ depends on the degree of habit persistence in consumption, h , and steady-state growth, γ . This equation includes hours worked because utility is non-separable in consumption and leisure. It generalizes the simple Euler equation (1).

The variable cy_t refers again to the convenience yield discussed in Section 4.2. It drives a wedge between the intertemporal marginal utility of consumption and the riskless real return $R_t - \mathbb{E}_t[\pi_{t+1}]$. Del Negro et al. (forthcoming) propose a mechanism by which liquidity frictions give rise to this convenience yield, as liquid assets help relax financing constraints. Alternatively, as shown in Fisher (2015), cy_t may reflect exogenous fluctuations in households' preference for holding risk-free nominal bonds. For simplicity, we assume here that cy_t evolves exogenously. Given our emphasis on the role of convenience yields in explaining the trend behavior of interest rates, we let the shock cy_t follow a flexible process. In particular we assume, as in Section 4.3, that cy_t contains both a liquidity component cy_t^l and a safety component cy_t^s

$$cy_t = cy_t^l + cy_t^s, \quad (30)$$

and we let each premium be given by the sum of two AR(1) processes, one that captures transitory fluctuations, and one that captures the highly persistent movements discussed in Section 4.3. Using data on various spreads will allow us to identify the relevant components, as detailed below.

We include financial frictions in the model, building on the work of Bernanke et al. (1999a), Christiano et al. (2003), De Graeve (2008), and Christiano et al. (2014). We assume that banks collect deposits from households and lend to entrepreneurs who use these funds as well as their own wealth to acquire physical capital, which is rented to intermediate goods producers. Entrepreneurs are subject to idiosyncratic disturbances that affect their ability to manage capital. Their revenue may thus turn out to be too low to pay back the loans received by the banks. The banks therefore protect themselves against default risk by pooling all loans and charging a spread over the deposit rate. This spread may vary as a function of entrepreneurs' leverage and riskiness. The *excess return on capital* (the spread between the expected return on capital and the riskless rate) can be expressed as a function of the convenience yield cy_t , the entrepreneurs' leverage (i.e. the ratio of the value of capital to net worth), and "risk shocks" $\tilde{\sigma}_{\omega,t}$ capturing mean-preserving changes in the cross-sectional dispersion of ability across entrepreneurs (see Christiano et al. (2014)):

$$\mathbb{E}_t \left[\tilde{R}_{t+1}^k - R_t \right] = cy_t + \zeta_{sp,b} (q_t^k + \bar{k}_t - n_t) + \tilde{\sigma}_{\omega,t}, \quad (31)$$

where q_t^k is the value of capital, $\bar{k}_t$ is installed capital, n_t is entrepreneurs' net worth, $\zeta_{sp,b}$ is the elasticity of the credit spread to the entrepreneurs' leverage ($q_t^k + \bar{k}_t - n_t$), and $\tilde{\sigma}_{\omega,t}$ follows an AR(1) process with parameters ρ_{σ_ω} and σ_{σ_ω} .

The estimation of the model is based on data on real output growth (including both GDP and GDI measures), consumption growth, investment growth, real wage growth, hours worked, inflation (measured by both core PCE and GDP deflators), the federal funds rate, and the ten-year Treasury yield. To identify shocks to liquidity and safety premia as well as “risk” shocks, we use data on spreads between Baa and 20-year Treasury yields, and spreads between Aaa and 20-year Treasury yields. We also use survey-based long-run inflation expectations to capture information about the public’s perception of the Federal Reserve’s inflation objective, market data on expectations of future federal funds rates up to 6 quarters ahead to incorporate the effects of forward guidance on the policy rate, as well as data on total factor productivity.

The measurement equations focusing on interest rates relate the observables to the model variables as follows:

$$\begin{aligned}
\text{FFR} &= R_* + R_t \\
\text{FFR}_{t,t+j}^e &= R_* + \mathbb{E}_t [R_{t+j}], \quad j = 1, \dots, 6 \\
\text{10y Nominal Bond Yield} &= R_* + \mathbb{E}_t \left[\frac{1}{40} \sum_{j=0}^{39} R_{t+j} \right] + e_t^{10y} \\
\text{Aaa - 20-year Treasury Spread} &= cy_*^l + \mathbb{E}_t \left[\frac{1}{80} \sum_{j=0}^{79} cy_{t+j}^l \right] \\
\text{Baa - 20-year Treasury Spread} &= cy_*^l + cy_*^s + SP_* + \mathbb{E}_t \frac{1}{80} \sum_{j=0}^{79} [\tilde{R}_{t+j+1}^k - R_{t+j}].
\end{aligned}$$

All variables are measured in percent and R_* measures the net steady-state short-term nominal interest rate, expressed in percentage terms. We assume that some of the observables equal the model implied value plus an AR(1) exogenous process e_t^* that can be thought of either measurement error or some other unmodeled source of discrepancy between the model and the data as in Boivin and Giannoni (2006).⁴³

To measure spreads, we use again the Baa corporate bond yield spread over the 20-year Treasury bond yield at constant maturity and the Aaa corporate bond yield spread over

⁴³For instance, these processes capture discrepancies between the noisy measures of output (real GDP and real GDI) and the corresponding model concept, and the gaps between the measures of inflation (based on the core PCE deflator and the GDP deflator) and inflation in the model. Instead, e_t^{10y} represents fluctuations in the 10-year Treasury bond yield above and beyond those due to expectations of future short-term rates, such as movements in the term premium.

the 20-year Treasury bond yield at constant maturity. As discussed in Section 4, the Aaa - 20-year Treasury yield spread is mostly capturing a liquidity premium. We thus relate that spread to cy_t^l in our model. Furthermore, given that the maturity of this spreads is about 20 years, the data should correspond in the model to the sum of expected future values of cy_t^l (in deviations from steady state) over the next 80 quarters. In addition to the liquidity premium, we assume that the Baa - 20-year Treasury spread reflects also the safety premium and credit risk over the next 20 years. This spread provides thus information about the credit spread $\tilde{R}_{t+1}^k - R_t$ over that horizon, in the model. This implies, using (30) and (31), that the Baa spread corresponds to current and expected future values of

$$cy_t^l + cy_t^s + [\zeta_{sp,b} (q_t^k + \bar{k}_t - n_t) + \tilde{\sigma}_{\omega,t}], \quad (32)$$

where the term in square brackets captures the credit risk. The model can identify each of these terms because the Aaa spread provides information about cy_t^l , the consumption Euler equation (29) involves the sum of cy_t^l and cy_t^s , and the credit spread equation (31) involves all three terms.

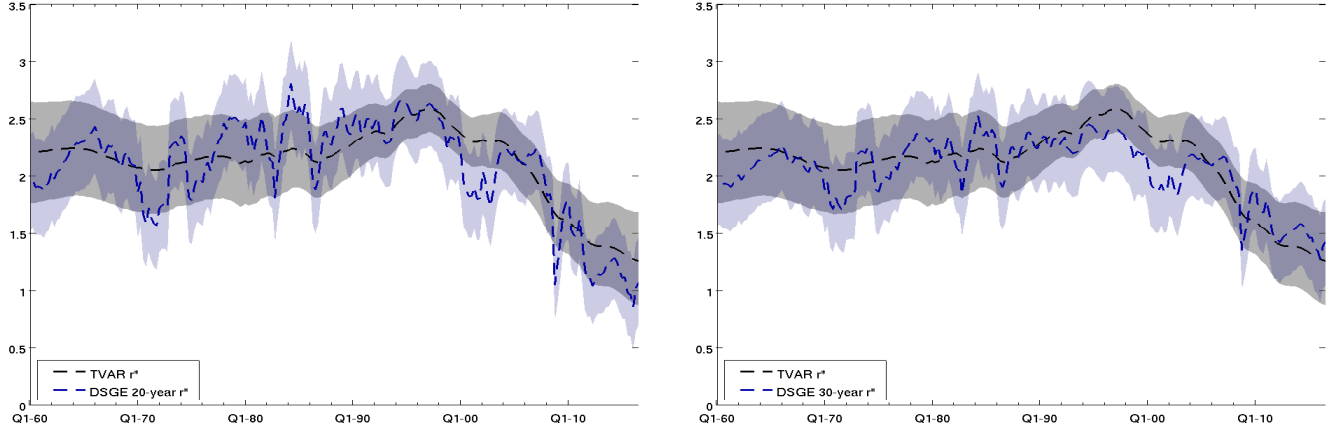
We estimate the model using Bayesian techniques. This requires the specification of a prior distribution for the model parameters. For most parameters common with Smets and Wouters (2007), we use the same marginal prior distributions. We specify priors for the additional parameters such as those related to financial frictions (cy_*^l , cy_*^s , SP_* , $\zeta_{sp,b}$, ρ_{σ_ω} , and σ_{σ_ω}) and the ones related to measurement error. Information on the priors and posterior mean is provided in Table 1 in the Appendix.

5.2 DSGE Estimates of r^*

5.2.1 Long-term forecasts of the natural rate

We start our discussion of the DSGE estimates of the natural rate by focusing on its persistent component, since this is the dimension in which the Trendy VAR and DSGE approaches are most directly comparable. Remarkably, the two models provide a very similar characterization of this component of interest rates, both in terms of their time-series behavior, as well as with regard to their fundamental drivers. This consistency between the two models is all the more notable given their very different characteristics in terms of their theoretical assumptions as well as of the data used in their estimation.

Figure 9: Forward Natural Rates $E_t r_{t+h}^*$ in the DSGE vs $\bar{r}_t$
 20 years ($h = 80$) 30 years ($h = 120$)



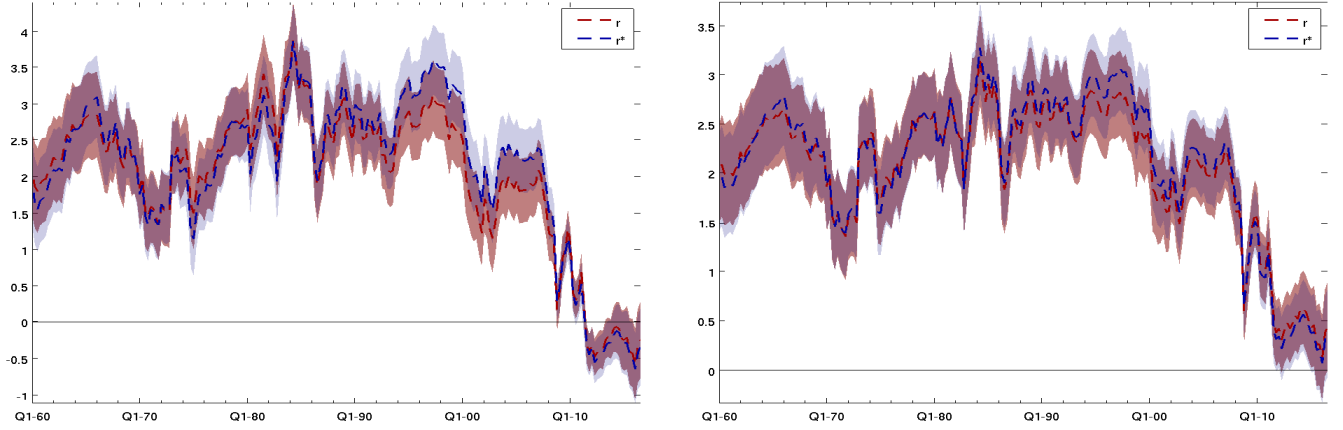
Note: The dashed black line shows the posterior median and the shaded gray areas show the 68 percent posterior coverage intervals for the Trendy VAR estimates of $\bar{r}_t$. The dashed blue lines show the posterior median and the shaded blue areas show the 68 percent posterior coverage intervals for the DSGE estimates of $E_t r_{t+h}^*$.

As a way of isolating persistent movements in real rates, Figure 9 compares forecasts of the short-term natural rate at a 20 and 30 year horizon from the DSGE model and the VAR. We refer to these forecasts as (implied) forward rates. The key result highlighted by this graph is that forecasts of the real interest rate at long horizons behave very similarly in the two models.⁴⁴ This is true whether we use forecasts of either natural or actual rates in the DSGE, since the two are essentially identical starting at horizons of around 10 years. This similarity is illustrated in Figure 10, which compares 10- and 5-year forecasts of the two rates from the DSGE estimation. Actual and natural rates are quite close at the shorter horizon, but they do diverge at times by as much as about 50 basis points. However, this distance shrinks to just a few basis points at the 10 year horizon.

The projected convergence between actual and natural rates in the DSGE model, at horizons at which both rates are still expected to display significant variation, is an important piece of evidence in favor of the key assumption needed for the VAR to be informative on the natural rate. As we discussed in Section 4, the VAR provides useful information on the persistent component of the natural rate of interest only if the gap between the actual and

⁴⁴Comin and Gertler (2006) refer to fluctuations over these horizons as medium-term cycles, in contrast to business cycles that take place at frequencies of 2 to 8 years. Given our focus on interest rates, we stick to the "long" horizon characterization, since this is the adjective most commonly used to define the yield on bonds with maturities in this range.

Figure 10: Forward Natural Rates $\mathbb{E}_t r_{t+h}^*$ and Forward Actual Rates $\mathbb{E}_t r_{t+h}$
 5 years ($h = 20$) 10 years ($h = 40$)



Note: The dashed blue lines shows the posterior median and the shaded blue areas show the 68 percent posterior coverage intervals for the DSGE estimates of $\mathbb{E}_t r_{t+h}^*$. The dashed red lines shows the posterior median and the shaded red areas show the 68 percent posterior coverage intervals for the DSGE estimates of $\mathbb{E}_t r_{t+h}$.

the natural economy is less persistent than the natural variables themselves. The evidence presented above suggests that this is indeed the case according to our DSGE estimates.

The fact that the DSGE model projects meaningful fluctuations in the interest rate at very long horizons, and even more that these fluctuations resemble those identified by the VAR, is a surprising finding. The (transformed) DSGE model is stationary around its steady state. Therefore, its infinite horizon forecasts of interest rates are constant, unlike those of the Trendy VAR that are affected by its permanent shocks.⁴⁵ Yet, the estimated model picks up enough persistence from the data to closely approximate the behavior of the VAR down to frequencies associated with forecast horizons around 30 years.⁴⁶ We conclude from this

⁴⁵Along the model's balanced growth path, the log levels of output, consumption and investment share a unit root that they inherit from productivity. As a result, their (log) ratios are stationary, and so are all the other variables, including interest rates. Given our specification, however, we could relax this assumption and only require that real interest rates *cum* convenience be stationary. Under this set-up, the natural rate of interest for riskless and safe securities and the convenience yield would share a common trend.

⁴⁶This persistence in the interest rate depends in part on the roots very close to one (0.99) that we impose on the persistent components of the liquidity and safety shocks. However, the standard deviations of the innovations to these processes are estimated, using a conservative prior regarding their size. Therefore, their importance in the posterior reflects meaningful features of the data. In fact, the far forward rates remain volatile also in models in which the distribution of the shocks to liquidity and safety does not assume the presence of a very persistent component.

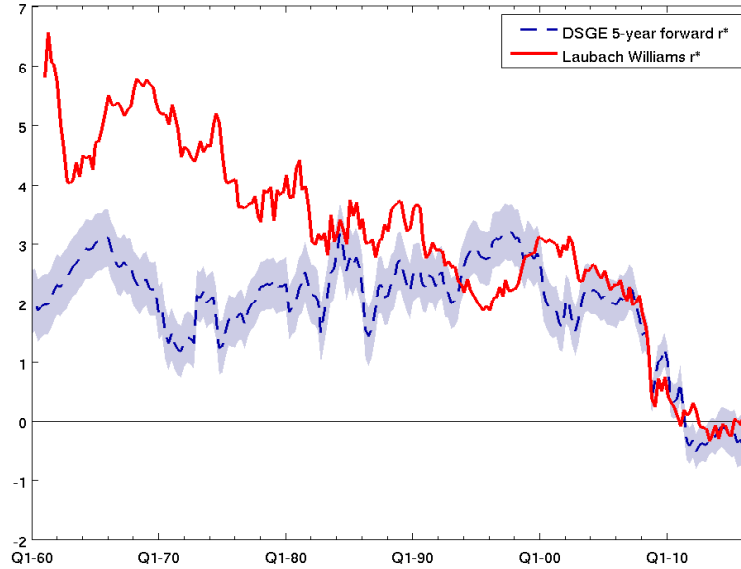
result that in practice the DSGE model describes the trend in the real interest rate as well as the VAR, even if it has no power at exactly zero frequency.⁴⁷

The flexibility of our estimated DSGE model as a tool to characterize the persistent component of real interest rates allows us to address a still open question, namely how to integrate "longer-run" estimates of the natural rate, such as those provided by Laubach and Williams (2016), and "shorter-run" estimates derived from DSGE models, such as those presented in the next section and in Barsky et al. (2014) and Curdia et al. (2015). The presumption in the literature so far has been that the two types of estimates are mostly complementary, since longer-run approaches focus exclusively on permanent movements in the natural rate, while shorter-run approaches assume that the natural rate is stationary (see for instance the discussion on this point in (Laubach and Williams, 2016, , especially Section 6.). In contrast, our results suggest that thinking of "longer-run" and "shorter-run" measures of the natural rate as estimates of two largely independent components might overstate the amount of overall variation in this variable. This conclusion does not make time-series models of the natural rate any less useful, given these models' flexibility, as also illustrated by our Trendy VAR. However, it does suggest that DSGE models can provide a more comprehensive view of the fluctuations in the natural rate across frequencies than generally assumed until now.

As a further illustration of this point, Figure 11 shows that LW's estimates of the natural rate co-move quite closely with the 5-year forward natural rate derived from our DSGE model, at least starting in the early 1980s. This similarity with a relatively short-horizon forward rate suggests that LW's model includes a fair amount of transitory variation in its estimates of the natural rate, even if it follows an $I(1)$ process. This result therefore confirms the blurry line between short and long run estimates of the natural rate. Unlike after 1980, the two models disagree strongly earlier in the sample. LW estimate natural real rates as high as 6% in the 1960s, while the DSGE model sees them fluctuating around levels very similar to those that prevail in the subsequent decades. This stationarity of the DSGE estimates in

⁴⁷The ability of a stationary DSGE model to approximate the low frequency behavior implied by the Trendy VAR is related to the approach of Stock and Watson (1998) and Stock and Watson (2007). They characterize the persistent component of what look like stationary variables, such as GDP growth and inflation, through unit root processes with a "small" variance. Our results suggest a similarly blurred line between the stationary vs unit root characterization of interest rates provided by the DSGE and the Trendy VAR.

Figure 11: DSGE Estimates of 5-Year Implied Natural Rate and Laubach-Williams Estimate



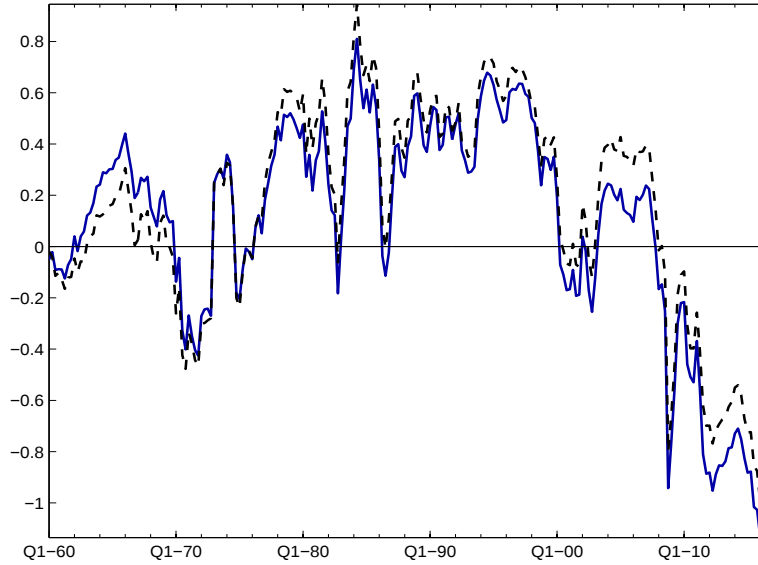
Note: The figure shows the 5-year forward real natural rate (i.e. the quarterly real natural rate expected to prevail 5 years in the future) from the DSGE model (blue dashed line), and Laubach and Williams' one-sided estimate of r^* (red line). The blue shaded area represents the 68 percent posterior coverage interval for the former.

the first twenty years of the sample is quite consistent with the trend from the VAR, which is essentially flat through the 1990s.

The consistency of the long-horizon forecasts of the real interest rate implied by the Trendy VAR and DSGE models strengthens our substantive conclusions, especially given the significant differences between the two empirical approaches. Next, we show that the two models also agree qualitatively on the main sources of persistent fluctuations in the natural rate. This result is illustrated in Figure 12, which shows the 20-year forward real natural rate in deviations from its steady state (solid blue line) and the combined contribution of liquidity and safety shocks (dashed black line) – the very shocks driving the Aaa and Baa corporate spreads. Liquidity and safety, which together represent the convenience yield, account for almost all the low frequency movements in the natural rate. This result is consistent with that obtained from the VAR as it attributes a large portion of the persistent movements in the natural rate to shifts in the convenience yield (see Figure 6).

This section established that the view of persistent fluctuations in the natural rate of interest provided by the DSGE model is surprisingly consistent with that gleaned through the lens of the VAR. With this reassuring consistency in hand, the next section levers the

Figure 12: 20-Year Forward Natural Rate and Component Attributable to Liquidity and Safety



Note: The figure shows the 20-year forward real natural rate in deviations from the steady state (i.e. the quarterly real natural rate expected to prevail 20 years into the future) from the DSGE model (blue solid line), and the component attributable to safety and liquidity shocks (dashed black line). The blue shaded area represents the 68 percent posterior coverage interval for the former.

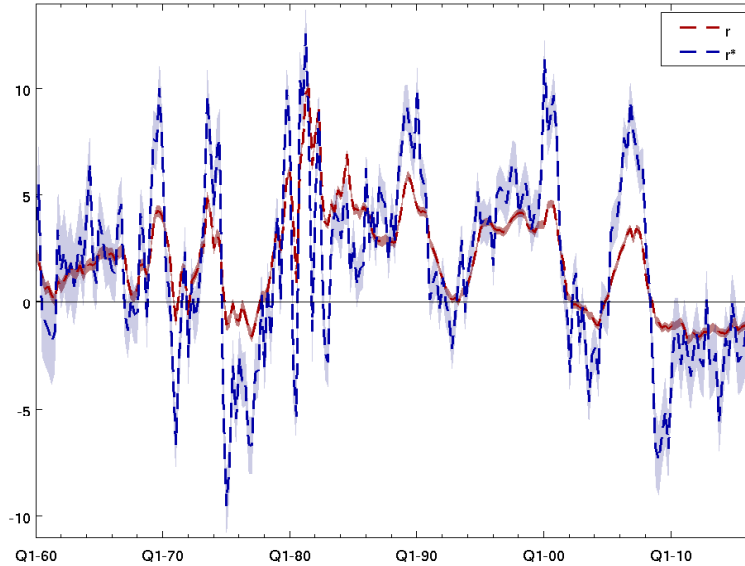
main advantage of the DSGE framework, its detailed general equilibrium structure, to derive estimates of the entire time-path of the natural rate, including all frequencies.

5.2.2 Short-term r^*

As mentioned above, the DSGE model provides estimates of the short-term natural rate. Figure 13 shows the estimate of r^* using the DSGE model described above, along with the real federal funds rate (measured as the nominal federal funds rate minus the model-based expected inflation), from 1960 through 2016. Several observations stand out. First, the estimate of r^* moves considerably over time. This is at odds with the assumptions commonly made of either a constant or slow-moving r^* . Second, r^* displays a clear cyclical pattern: it tends to be high and rising during booms, while it declines quite abruptly in recession. This decline in r^* is especially pronounced during the Great Recession, when the model sees it fall into negative territory. The estimate of r^* remains persistently low through the first phase of the recovery, but is estimated to have increased somewhat at the end of the sample. Third, the estimate of r^* displays fairly pronounced high frequency variation.

These quarter-to-quarter gyrations reflect the short-run nature of the natural rate, which moves in reaction to many of the shocks that buffet the economy.

Figure 13: DSGE estimate of r^* and actual short-term real rate



Note: The figure shows the estimated real natural rate of interest (blue dashed line) and the model-implied real interest rate (red line). The red and blue shaded areas represent the respective 68 percent posterior coverage intervals.

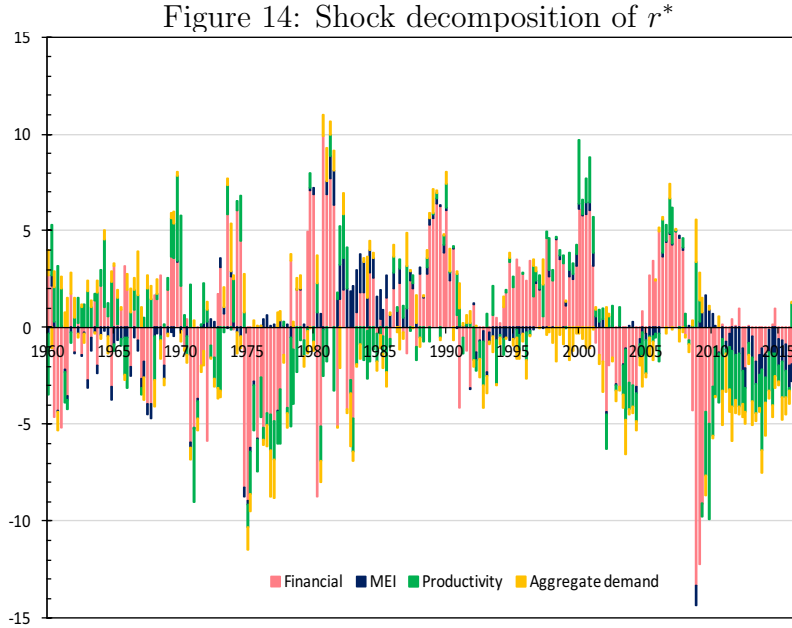
This figure makes clear that monetary policy was constrained by the zero lower bound when the nominal natural rate fell to sharply negative values in the wake of the Great Recession. From 2009 to 2014, the natural rate hovered below the real federal funds rate, leading the FOMC to engage in unconventional monetary policy (i.e., large-scale asset purchases and forward guidance) to mitigate these effects. By the end of our sample though, both rates are close to each other.

Fluctuations in r^* are driven by real and financial factors, but not by monetary factors, since monetary policy has no effect in the absence of price and wage rigidities, in the DSGE model. Therefore, the natural rate provides a useful benchmark that determines the level of the real interest rate absent monetary policy. To understand some of the drivers of the natural rate, consider the consumption Euler equation (29). The same equation holds also in the counterfactual economy in which prices and wages are fully flexible. Solving that

equation for r_t^* , we obtain

$$r_t^* = -cy_t + \frac{\sigma_c}{1-\bar{h}} \left(\mathbb{E}_t [c_{t+1}^* - c_t^* + z_{t+1}] - \bar{h} (c_t^* - c_{t-1}^* + z_t) \right) - \frac{(\sigma_c - 1) w_* L_*}{(1-\bar{h}) c_*} \mathbb{E}_t [L_{t+1}^* - L_t^*], \quad (33)$$

where c_t^* and L_t^* denote respectively the level of consumption and hours worked in the flexible price economy, i.e., at “full employment.” This reveals that r_t^* falls one-for-one with any increase in the convenience yield cy_t . The natural rate also increases with higher expected consumption growth or lower growth in labor supply in the flexible price economy.



Note: The figure shows the contribution of financial shocks (in red), investment shocks (in blue), productivity shocks (in green) and aggregate demand shocks (in orange) in causing fluctuations in the short-term real natural rate of interest. Financial shock include exogenous fluctuations in convenience yields and risk shocks $\tilde{\sigma}_{\omega,t}$.

Figure 14 decomposes the r^* estimate shown in Figure 13 in terms of the shocks that account for its movements. We group under the heading *financial shocks* (in red) exogenous fluctuations in convenience yields cy_t as well as the risk shocks $\tilde{\sigma}_{\omega,t}$. These shocks increase households desire hold safe bonds, leading to an increase in saving and a reduction in their consumption demand. The risk shocks $\tilde{\sigma}_{\omega,t}$ increase the cost of external finance for firms, reducing the demand for investment. The investment shocks, i.e., shocks to the marginal efficiency of investment (marked in blue) capture forces restraining investment over and above those that result in an increase in credit spreads, such as reductions in the firms willingness or ability to invest in physical capital. Productivity shocks (in green) that reduce total

factor productivity also depress desired investment and the natural rate. Finally, aggregate demand shocks (in orange) capture exogenous movements in government expenditures and foreign demand.

The main lessons we draw from Figure 14 is that r^* plunged during the recent financial crisis and the recession that followed is due to an unusual combination of severe financial, productivity, investment and aggregate demand shocks, all exerting negative pressure on r^* . Second, among these negative contributions, those of financial shocks (the red bars) and of productivity shocks (the green bars) have particularly pronounced effects.

Looking more in detail at the financial shocks, we observe that the convenience for liquidity has played an important role in lowering the natural rate of interest since the early 1990's, in line with the findings of our Trendy VAR, in Section 4. This was partly offset by the convenience for safety from the early 1990s to 2006. However, since the recent financial crisis the convenience for safety compounded with the one for liquidity in lowering the natural rate of interest.

6 Conclusion

We estimated the natural rate of interest and its fundamental drivers using two very different methodologies. The first one is a trendy VAR, a flexible multivariate unobserved component model that we can use to make inference on slow-moving trends in the natural rate. We estimate it using data on Treasury and corporate bond yields of various maturities, inflation, and inflation expectations. The second model is a medium scale DSGE with nominal and financial frictions, whose tighter structure allows us to recover the entire time path of the natural rate, rather than exclusively its low frequency component as in the case of the trendy VAR. We estimate this model using a fairly large set of macroeconomic information along with the same data on returns used in the trendy VAR.

The two approaches yield remarkably consistent results. First, they both isolate a slow-moving trend in the real interest rate that is fairly flat between 2 and 2.5 percent until the late 1990s, when it starts declining towards a recent trough around 1 percent. Second, they both attribute most of this decline to an increase in the convenience yield on Treasuries, which they identify as a low-frequency component in the spreads between corporate and

Treasury bonds with the same maturity, but different characteristics in terms of liquidity and safety. In addition, the DSGE model sees these factors as also playing an important role in the movements of the natural rate at business cycle frequencies, although other shocks are also relevant. Finally, the DSGE also suggests that the short-term interest rate was severely constrained by the effective lower bound on nominal interest rates since late 2008, when the natural rate plunged well into negative territory.

Marrying the flexible estimation technology of our trendy VAR with the no-arbitrage restrictions typically imposed across returns of various maturities in the term-structure literature would be an interesting extension of our empirical framework, which however is beyond the scope of the present paper.

References

- Acharya, Viral V and Lasse Heje Pedersen**, “Asset pricing with liquidity risk,” *Journal of financial Economics*, 2005, 77 (2), 375–410.
- Ajello, Andrea**, “Financial Intermediation, Investment Dynamics and Business Cycle Fluctuations,” *American Economic Review*, forthcoming.
- Amihud, Yakov, Haim Mendelson, and Lasse Heje Pedersen**, *Market liquidity: asset pricing, risk, and crises*, Cambridge University Press, 2012.
- , —, — **et al.**, “Liquidity and asset prices,” *Foundations and Trends® in Finance*, 2006, 1 (4), 269–364.
- Aruoba, S. Borağan and Frank Schorfheide**, “Insights from an Estimated Search-Based Monetary Model with Nominal Rigidities,” *Working Paper*, 2008.
- Auclert, Adrien and Matthew Rognlie**, “Inequality and Aggregate Demand,” Technical Report, Stanford University 2016.
- Barsky, Robert, Alejandro Justiniano, and Leonardo Melosi**, “The Natural Rate of Interest and Its Usefulness for Monetary Policy,” *American Economic Review*, May 2014, 104 (5), 37–43.

Basel Committee on Banking Supervision, “Basel III: The Liquidity Coverage Ratio and liquidity risk monitoring tools,” *Bank for International Settlements*, 2013.

Benigno, Gianluca and Luca Fornaro, “Stagnation traps,” *Mimeo, LSE and UPF*, 2016.

Bernanke, Ben, Mark Gertler, and Simon Gilchrist, “The Financial Accelerator in a Quantitative Business Cycle Framework,” in John B. Taylor and Michael Woodford, eds., *Handbook of Macroeconomics*, Vol. 1C, North Holland, Amsterdam, 1999.

Bernanke, Ben S., “The global saving glut and the U.S. current account deficit,” 2005.

—, **Carol C. Bertaut, Laurie DeMarco, and Steven B. Kamin**, “International capital flows and the return to safe assets in the united states, 2003-2007,” *Board of Governors of the Federal Reserve System International Finance Discussion Papers 1014*, 2011.

—, **Mark Gertler, and Simon Gilchrist**, “The Financial Accelerator in a Quantitative Business Cycle Framework,” in John B. Taylor and Michael Woodford, eds., *Handbook of Macroeconomics*, Vol. 1C, Amsterdam: North-Holland, 1999, chapter 21, pp. 1341–93.

Bigio, Saki, “Endogenous Liquidity and the Business Cycle,” *American Economic Review*, 2015, 105 (6), 1883–1927.

Blanchard, Olivier and Jordi Galí, “Real Wage Rigidities and the New Keynesian Model,” *Journal of Money, Credit and Banking*, 02 2007, 39 (s1), 35–65.

Bloom, Nicholas, “The impact of uncertainty shocks,” *Econometrica*, 2009, 77 (3), 623–685.

Boivin, Jean and Marc P. Giannoni, “DSGE Models in a Data Rich Enviroment,” *NBER Working Paper*, 2006, 12772.

Caballero, Ricardo J., “The “other” imbalance and the financial crisis,” *National Bureau of Economic Research WP 15636*, 2010.

Caballero, Ricardo J and Arvind Krishnamurthy, “Global imbalances and financial fragility,” *National Bureau of Economic Research WP 14688*, 2009.

Caballero, Ricardo J. and Emmanuel Farhi, “The safety trap,” *National Bureau of Economic Research WP 19927*, 2014.

- , – , and **Pierre-Olivier Gourinchas**, “Global Imbalances and Currency Wars at the ZLB,” *National Bureau of Economic Research WP 21670*, 2015.
- Carter, C.K. and Robert Kohn**, “On Gibbs Sampling for State Space Models,” *Biometrika*, 1994, *81* (3), 541–553.
- Carvalho, Carlos, Andrea Ferrero, and Fernanda Nechio**, “Demographics and real interest rates: Inspecting the mechanism,” *European Economic Review*, 2016, pp. –.
- Christiano, Lawrence J., Martin Eichenbaum, and Charles L. Evans**, “Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy,” *Journal of Political Economy*, 2005, *113*, 1–45.
- , **Roberto Motto, and Massimo Rostagno**, “The Great Depression and the Friedman-Schwartz Hypothesis,” *Journal of Money, Credit and Banking*, 2003, *35*, 1119–1197.
- , – , and – , “Risk Shocks,” *American Economic Review*, 2014, *104* (1), 27–65.
- Clark, Todd E and Taeyoung Doh**, “Evaluating alternative models of trend inflation,” *International Journal of Forecasting*, 2014, *30* (3), 426–448.
- Comin, Diego and Mark Gertler**, “Medium-Term Business Cycles,” *American Economic Review*, June 2006, *96* (3), 523–551.
- Crump, Richard K, Stefano Eusepi, and Emanuel Moench**, “The term structure of expectations and bond yields,” *Federal Reserve Bank of New York Staff Report No. 775*, 2016.
- Cui, Wei and Soren Radde**, “Search-Based Endogenous Illiquidity and the Macroeconomy,” *Available at SSRN 2432935*, 2014.
- Curdia, Vasco, Andrea Ferrero, Ging Cee Ng, and Andrea Tambalotti**, “Has U.S. monetary policy tracked the efficient interest rate?,” *Journal of Monetary Economics*, 2015, *70* (C), 72–83.
- Del Negro, Marco and Giorgio E Primiceri**, “Time varying structural vector autoregressions and monetary policy: a corrigendum,” *The review of economic studies*, 2015, *82* (4), 1342–1345.

- **and Stefano Eusepi**, “Fitting Observed Inflation Expectations,” *Journal of Economic Dynamics and Control*, 2011, *35*, 2105–2131.
- **, Gauti Eggertsson, Andrea Ferrero, and Nobuhiro Kiyotaki**, “The Great Escape? A Quantitative Evaluation of the Feds Liquidity Facilities,” *American Economic Review*, forthcoming.
- **, Marc P Giannoni, and Frank Schorfheide**, “Inflation in the Great Recession and New Keynesian Models,” *American Economic Journal: Macroeconomics*, 2015, *7* (1), 168–196.
- Durbin, James and Siem Jan Koopman**, “A Simple and Efficient Simulation Smoother for State Space Time Series Analysis,” *Biometrika*, 2002, *89* (3), 603–616.
- Edge, Rochelle M., Michael T. Riley, and Jean-Philippe Laforte**, “Natural rate measures in an estimated DSGE model of the U.S. economy,” *Journal of Economic Dynamics and Control*, 2008, *32* (8), 2512 – 2535.
- Eggertsson, Gauti B. and Neil R. Mehrotra**, “A Model of Secular Stagnation,” NBER Working Papers 20574, National Bureau of Economic Research, Inc October 2014.
- Eichengreen, Barry**, “Secular Stagnation: The Long View,” *American Economic Review*, May 2015, *105* (5), 66–70.
- Ferguson, Roger W**, “Equilibrium real interest rate: Theory and application,” *Speech*, see *www.federalreserve.gov*, 2004.
- Fisher, Jonas**, “On the Structural Interpretation of the Smets–Wouters Risk Premium? Shock,” *Journal of Money, Credit and Banking*, 2015, *47* (2-3), 511–516.
- Fleckenstein, Matthias, Francis A Longstaff, and Hanno Lustig**, “The TIPS-Treasury Bond Puzzle,” *The Journal of Finance*, 2014, *69* (5), 2151–2197.
- Gagnon, Etienne, Benjamin K. Johannsen, and J. David Lopez-Salido**, “Understanding the New Normal : The Role of Demographics,” Finance and Economics Discussion Series 2016-080, Board of Governors of the Federal Reserve System (U.S.) September 2016.

- Garleanu, Nicolae and Lasse Heje Pedersen**, “Margin-Based Asset Pricing and Deviations from the Law of One Price,” *Review of Financial Studies*, 2011, 24 (6), 1980–2022.
- Giannone, Domenico, Michele Lenza, and Giorgio E Primiceri**, “Prior selection for vector autoregressions,” *Review of Economics and Statistics*, 2015, 97 (2), 436–451.
- Gilchrist, Simon and Egon Zakrajsek**, “Credit Spreads and Business Cycle Fluctuations,” *American Economic Review*, 2012, 102 (4), 1692–1720.
- Gorton, Gary and Andrew Metrick**, “Securitized banking and the run on repo,” *Journal of Financial economics*, 2012, 104 (3), 425–451.
- Gorton, Gary B.**, “The History and Economics of Safe Assets,” *National Bureau of Economic Research WP 22210*, 2016.
- Gourinchas, Pierre-Olivier and Hélène Rey**, “Real Interest Rates, Imbalances and the Curse of Regional Safe Asset Providers at the Zero Lower Bound,” *National Bureau of Economic Research WP 22618*, 2016.
- Graeve, Ferre De**, “The External Finance Premium and the Macroeconomy: US Post-WWII Evidence,” *Journal of Economic Dynamics and Control*, 2008, 32 (11), 3415 – 3440.
- Greenwood, Jeremy, Zvi Hercowitz, and Per Krusell**, “Long-Run Implications of Investment-Specific Technological Change,” *American Economic Review*, 1998, 87 (3), 342–36.
- Greenwood, Robin, Samuel G Hanson, and Jeremy C Stein**, “A Comparative-Advantage Approach to Government Debt Maturity,” *The Journal of Finance*, 2015, 70 (4), 1683–1722.
- , – , and – , “The Federal Reserves balance sheet as a financial-stability tool,” in ““Designing Resilient Monetary Policy Frameworks for the Future,” Jackson Hole Symposium: Federal Reserve Bank of Kansas City” 2016.
- Guerron-Quintana, Pablo and Ryo Jinnai**, “Liquidity Shocks and Asset Prices,” 2015. Working Paper, Federal Reserve Bank of Philadelphia.

Hall, Robert E., “Understanding the decline in the safe real interest rate,” *National Bureau of Economic Research WP 22196*, 2016.

Hamilton, James D., Ethan S. Harris, Jan Hatzius, and Kenneth D. West, “The Equilibrium Real Funds Rate: Past, Present and Future,” NBER Working Papers 21476, National Bureau of Economic Research, Inc August 2015.

Hansen, Alvin H., “Economic Progress and Declining Population Growth,” *The American Economic Review*, 1939, *29* (1), 1–15.

Holston, Kathryn, Thomas Laubach, and John C. Williams, “Measuring the Natural Rate of Interest: International Trends and Determinants,” *Federal Reserve Bank of San Francisco Working Paper*, 2016, *2016-11*.

Johannsen, Benjamin K. and Elmar Mertens, “A Time Series Model of Interest Rates With the Effective Lower Bound,” Finance and Economics Discussion Series 2016-033, Board of Governors of the Federal Reserve System April 2016.

Justiniano, Alejandro and Giorgio E. Primiceri, “The Time-Varying Volatility of Macroeconomic Fluctuations,” *American Economic Review*, 2008, *98* (3), 604–641.

— and —, “Measuring the equilibrium real interest rate,” *Economic Perspectives*, 2010, (Q I), 14–27.

—, **Giorgio Primiceri, and Andrea Tambalotti**, “Investment Shocks and Business Sycles,” *Journal of Monetary Economics*, 2010, *57* (2), 132–145.

—, **Giorgio E. Primiceri, and Andrea Tambalotti**, “Is There a Trade-Off between Inflation and Output Stabilization?,” *American Economic Journal: Macroeconomics*, April 2013, *5* (2), 1–31.

Kiley, Michael T., “What Can the Data Tell Us About the Equilibrium Real Interest Rate?,” Finance and Economics Discussion Series 2015-77, Board of Governors of the Federal Reserve System August 2015.

Kiyotaki, Nobuhiro and John Moore, “Liquidity, Business Cycles, and Monetary Policy,” *NBER Working Paper No. 17934*, 2012.

- Kozicki, Sharon and Peter A Tinsley**, “Effective use of survey information in estimating the evolution of expected inflation,” *Journal of Money, Credit and Banking*, 2012, 44 (1), 145–169.
- Krishnamurthy, Arvind and Annette Vissing-Jorgensen**, “The Aggregate Demand for Treasury Debt,” *Journal of Political Economy*, 2012, 120 (2), 233 – 267.
- Kurlat, Pablo**, “Lemons Markets and the Transmission of Aggregate Shocks,” *American Economic Review*, 2013, 103 (4), 1463–1489.
- Laseen, Stefan and Lars E.O. Svensson**, “Anticipated Alternative Policy-Rate Paths in Policy Simulations,” *International Journal of Central Banking*, 2011, 7 (3), 1–35.
- Laubach, Thomas and John C. Williams**, “Measuring the Natural Rate of Interest,” *The Review of Economics and Statistics*, November 2003, 85 (4), 1063–1070.
- **and John C Williams**, “Measuring the Natural Rate of Interest Redux,” *Business Economics*, 2016, 51 (2), 57–67.
- Longstaff, Francis, Sanjay Mithal, and Eric Neis**, “The Flight-to-Liquidity Premium in U.S. Treasury Bond Prices,” *Journal of Business*, 2004, 77 (3), 511–526.
- , – , **and** – , “Corporate Yield Spreads: Default Risk or Liquidity? New Evidence from the Credit Default Swap Market,” *Journal of Finance*, 2005, 60 (5), 2213–2253.
- Neiss, Katharine S. and Edward Nelson**, “The Real-Interest-Rate Gap As An Inflation Indicator,” *Macroeconomic Dynamics*, April 2003, 7 (02), 239–262.
- Pescatori, Andrea and Jarkko Turunen**, “Lower for Longer; Neutral Rates in the United States,” IMF Working Papers 15/135, International Monetary Fund June 2015.
- Primiceri, Giorgio**, “Why Inflation Rose and Fell: Policymakers’ Beliefs and US Postwar Stabilization Policy,” *Quarterly Journal of Economics*, 2006, 121, 867–901.
- Rachel, Lukasz and Thomas Smith**, “Secular Drivers of the Global Real Interest Rate,” Discussion Papers 1605, Centre for Macroeconomics (CFM) December 2015.
- Sargent, Thomas J.**, *The Conquest of American Inflation*, Princeton University Press, Princeton, 1999.

- Sims, Christopher A.**, “Comment on Cogley and Sargent’s ‘Evolving post World War II U.S. Inflation Dynamics’ ,” in Ben S. Bernanke and Kenneth Rogoff, eds., *NBER Macroeconomics Annual 2001*, Vol. 16, MIT Press, Cambridge, 2002, pp. 373–379.
- **and Tao Zha**, “Bayesian Methods for Dynamic Multivariate Models,” *International Economic Review*, 1998, *39* (4), 949–968.
- Smets, Frank and Raf Wouters**, “Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach,” *American Economic Review*, 2007, *97* (3), 586 – 606.
- Stock, James H. and Mark W. Watson**, “Median Unbiased Estimation of Coefficient Variance in a Time-Varying Parameter Model,” *Journal of the American Statistical Association*, August 1998, (441), 349–358.
- **and —** , “Forecasting Inflation,” *Journal of Monetary Economics*, 1999, *44* (2), 293–335.
- Stock, James H and Mark W Watson**, “Why has US inflation become harder to forecast?,” *Journal of Money, Credit and banking*, 2007, *39* (s1), 3–33.
- Summers, Lawrence H.**, *Secular Stagnation: Facts, Causes and Cures*, CEPR Press,
- Taylor, John B and Volker Wieland**, “Finding the equilibrium real interest rate in a fog of policy deviations,” *Business Economics*, 2016, *51* (3), 147–154.
- Villani, Mattias**, “Steady State Priors for Vector Autoregressions,” *Journal of Applied Econometrics*, 2009, *24* (4), 630–650.
- Watson, Mark W**, “Univariate detrending methods with stochastic trends,” *Journal of monetary economics*, 1986, *18* (1), 49–75.
- Wicksell, Knut**, *Interest and Prices*, english translation by r.f. kahn, 1936 ed., McMillan, London, 1898.
- Woodford, Michael**, *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, 2003.
- Yellen, Janet**, “The Economic Outlook and Monetary Policy,” December 2 2015.

Online Appendix for “Safety, Liquidity, and the Natural Rate of Interest”

Marco Del Negro, Domenico Giannone, Marc Giannoni, Andrea Tambalotti

A Gibbs Sampler for Trendy VARs

Let use the notation $x_{i:j}$ to denote the sequence $\{x_i, \dots, x_j\}$ for a generic variable x_t . The Gibbs sampler is structured according to the following blocks:

1. $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
 - (a) $\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
 - (b) $\bar{y}_{0:T}, \tilde{y}_{-p+1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$
2. $\varphi, \Sigma_\varepsilon, \Sigma_e | \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$
 - (a) $\Sigma_\varepsilon, \Sigma_e | \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$
 - (b) $\varphi | \Sigma_\varepsilon, \Sigma_e, \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$

Details of each step follow:

1. $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$

This is given by the product of the marginal posterior distribution of λ (conditional on the other parameters) times the distribution of $\bar{y}_{0:T}, \tilde{y}_{-p+1:T}$ conditional on λ (and the other parameters).

- (a) $\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$

The marginal posterior distribution of λ (conditional on the other parameters) is given by

$$p(\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}) \propto L(y_{1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e) p(\lambda),$$

where $L(y_{1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e)$ is the likelihood obtained from the Kalman filter applied to the state space system (3) through (7). $p(\lambda | \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T})$ does not have a known form so we will use a Metropolis Hastings step.

(b) $\bar{y}_{0:T}, \tilde{y}_{-p+1:T} | \lambda, \varphi, \Sigma_\varepsilon, \Sigma_e, y_{1:T}$

Given λ and the other parameters of the state space model we can use Durbin and Koopman (2002)'s simulation smoother to obtain draws for the latent states $\bar{y}_{0:T}$ and $\tilde{y}_{-p+1:T}$. Note that in addition to $\bar{y}_{1:T}$ and $\tilde{y}_{1:T}$ we also need to draw the initial conditions $\bar{y}_0$ and $\tilde{y}_{-p+1:0}$ in order to estimate the parameters of (5) and (4) in the next Gibbs sampler step.

Note that missing observations do not present any difficulty in terms of carrying out this step: if the vector y_{t_0} has some missing elements, the corresponding rows of the observation equation (3) are simply deleted for $t = t_0$.

2. $\varphi, \Sigma_\varepsilon, \Sigma_e | \bar{y}_{0:T}, \tilde{y}_{-p+1:T}, \lambda, y_{1:T}$

This step is straightforward because for given $\bar{y}_{0:T}$ and $\tilde{y}_{-p+1:T}$ equations (4) and (5) are standard VARs where in case of (4) we actually know the autoregressive matrices. The posterior distribution of Σ_e is given by

$$p(\Sigma_e | \bar{y}_{0:T}) = \mathcal{IW}(\underline{\Sigma}_e + \hat{S}_e, \kappa_e + T)$$

where $\hat{S}_e = \sum_{t=1}^T (\bar{y}_t - \bar{y}_{t-1})(\bar{y}_t - \bar{y}_{t-1})'$. The posterior distribution of φ and Σ_ε is given by

$$\begin{aligned} p(\Sigma_\varepsilon | \tilde{y}_{0:T}) &= \mathcal{IW}(\underline{\Sigma}_\varepsilon + \hat{S}_\varepsilon, \kappa_\varepsilon + T), \\ p(\varphi | \Sigma_\varepsilon, \tilde{y}_{0:T}) &= \mathcal{N} \left(\text{vec}(\hat{\Phi}), \Sigma_\varepsilon \otimes \left(\sum_{t=1}^T \tilde{x}_t \tilde{x}_t' + \underline{\Omega}^{-1} \right)^{-1} \right), \end{aligned}$$

where $\tilde{x}_t = (\tilde{y}_{t-1}', \dots, \tilde{y}_{t-p}')'$ collects the VAR regressors,

$$\hat{\Phi} = \left(\sum_{t=1}^T \tilde{x}_t \tilde{x}_t' + \underline{\Omega}^{-1} \right)^{-1} \left(\sum_{t=1}^T \tilde{x}_t \tilde{y}_t' + \underline{\Omega}^{-1} \underline{\Phi} \right), \quad \hat{S}_\varepsilon = \sum_{t=1}^T \hat{\varepsilon}_t \hat{\varepsilon}_t' + (\hat{\Phi} - \underline{\Phi})' \underline{\Omega}^{-1} (\hat{\Phi} - \underline{\Phi}),$$

and $\hat{\varepsilon}_t = \tilde{y}_t - \hat{\Phi}' \tilde{x}_t$ are the VAR residuals.

We use 40,000 draws and discard the first 20,000.

B DSGE model

This section describes the model specification, the data used, how they relate to the model concepts, and the priors distributions assumed for estimation.

The model economy is populated by eight classes of agents: 1) a continuum of households, who consume and supply differentiated labor; 2) competitive labor aggregators that combine labor supplied by individual households; 3) competitive final good-producing firms that aggregate the intermediate goods into a final product; 4) a continuum of monopolistically competitive intermediate good producing firms; 5) competitive capital producers that convert final goods into capital; 6) a continuum of entrepreneurs who purchase capital using both internal and borrowed funds and rent it to intermediate good producing firms; 7) a representative bank collecting deposits from the households and lending funds to the entrepreneurs; and finally 8) a government, composed of a monetary authority that sets short-term interest rates and a fiscal authority that sets public spending and collects taxes. We solve each agent's problem, and derive the resulting equilibrium conditions, which we approximate around the non-stochastic steady state. Since the derivation follows closely the literature (e.g., Christiano et al. (2005)), we describe here the log-linearized conditions.

Growth in the economy is driven by technological progress, $Z_t^* = e^{\frac{1}{1-\alpha}\tilde{z}_t} Z_t^p e^{\gamma t}$, which is assumed to include a deterministic trend ($e^{\gamma t}$), a stochastic trend (Z_t^p), and a stationary component ($\tilde{z}_t$), where α is the income share of capital (after paying mark-ups and fixed costs in production). Trending variables are divided by Z_t^* to express the model's equilibrium conditions in terms of the stationary variables. In what follows, all variables are expressed in log deviations from their steady state, and steady-state values are denoted by *-subscripts.

The stationary component of productivity $\tilde{z}_t$ and the growth rate of the stochastic trend $z_t^p = \log(Z_t^p/Z_{t-1}^p)$ are assumed to follow AR(1) processes:

$$\tilde{z}_t = \rho_z \tilde{z}_{t-1} + \sigma_z \varepsilon_{z,t}, \quad \varepsilon_{z,t} \sim N(0, 1). \quad (\text{A-1})$$

$$z_t^p = \rho_{z^p} z_{t-1}^p + \sigma_{z^p} \epsilon_{z^p,t}, \quad \epsilon_{z^p,t} \sim N(0, 1). \quad (\text{A-2})$$

The growth rate of technology evolves thus according to

$$z_t \equiv \log(Z_t^*/Z_{t-1}^*) - \gamma = \frac{1}{1-\alpha}(\rho_z - 1)\tilde{z}_{t-1} + \frac{1}{1-\alpha}\sigma_z \epsilon_{z,t} + z_t^p, \quad (\text{A-3})$$

where γ is the steady-state growth rate of the economy.

The *optimal allocation of consumption* satisfies the following Euler equation:

$$c_t = -\frac{1 - \bar{h}}{\sigma_c(1 + \bar{h})} (R_t - \mathbb{E}_t[\pi_{t+1}] + cy_t) + \frac{\bar{h}}{1 + \bar{h}} (c_{t-1} - z_t) + \frac{1}{1 + \bar{h}} \mathbb{E}_t [c_{t+1} + z_{t+1}] + \frac{(\sigma_c - 1)}{\sigma_c(1 + \bar{h})} \frac{w_* L_*}{c_*} (L_t - \mathbb{E}_t[L_{t+1}]), \quad (\text{A-4})$$

where c_t is consumption, L_t denotes hours worked, R_t is the nominal interest rate, and π_t is inflation. The parameter σ_c captures the degree of relative risk aversion while $\bar{h} \equiv h e^{-\gamma}$ depends on the degree of habit persistence in consumption, h , and steady-state growth. This equation includes hours worked because utility is non-separable in consumption and leisure.

The convenience yield cy_t contains both a liquidity component cy_t^l and a safety component cy_t^s

$$cy_t = cy_t^l + cy_t^s, \quad (\text{A-5})$$

where we let each premium be given by the sum of two AR(1) processes, one that captures transitory fluctuations, and one that captures the highly persistent movements discussed in Section 4.3.

The *optimal investment decision* satisfies the following relationship between the level of investment i_t , measured in terms of consumption goods, and the value of capital in terms of consumption q_t^k :

$$i_t = \frac{q_t^k}{S'' e^{2\gamma}(1 + \bar{\beta})} + \frac{1}{1 + \bar{\beta}} (i_{t-1} - z_t) + \frac{\bar{\beta}}{1 + \bar{\beta}} \mathbb{E}_t [i_{t+1} + z_{t+1}] + \mu_t. \quad (\text{A-6})$$

This relationship shows that investment is affected by investment adjustment costs (S'' is the second derivative of the adjustment cost function) and by an exogenous process μ_t , which we call “marginal efficiency of investment”, that alters the rate of transformation between consumption and installed capital (see Greenwood et al. (1998)). The shock μ_t follows an AR(1) process with parameters ρ_μ and σ_μ . The parameter $\bar{\beta} \equiv \beta e^{(1-\sigma_c)\gamma}$ depends on the intertemporal discount rate in the household utility function, β , on the degree of relative risk aversion σ_c , and on the steady-state growth rate γ .

The *capital stock*, $\bar{k}_t$, which we refer to as “installed capital”, evolves as

$$\bar{k}_t = \left(1 - \frac{i_*}{\bar{k}_*}\right) (\bar{k}_{t-1} - z_t) + \frac{i_*}{\bar{k}_*} i_t + \frac{i_*}{\bar{k}_*} S'' e^{2\gamma} (1 + \bar{\beta}) \mu_t, \quad (\text{A-7})$$

where $i_*/\bar{k}_*$ is the steady state investment to capital ratio. Capital is subject to variable capacity utilization u_t ; *effective capital* rented out to firms, k_t , is related to $\bar{k}_t$ by:

$$k_t = u_t - z_t + \bar{k}_{t-1}. \quad (\text{A-8})$$

The optimality condition determining the *rate of capital utilization* is given by

$$\frac{1-\psi}{\psi} r_t^k = u_t, \quad (\text{A-9})$$

where r_t^k is the rental rate of capital and ψ captures the utilization costs in terms of foregone consumption.

Real marginal costs for firms are given by

$$mc_t = w_t + \alpha L_t - \alpha k_t, \quad (\text{A-10})$$

where w_t is the real wage. From the optimality conditions of goods producers it follows that all firms have the same *capital-labor ratio*:

$$k_t = w_t - r_t^k + L_t. \quad (\text{A-11})$$

We include financial frictions in the model, building on the work of Bernanke et al. (1999a), Christiano et al. (2003), De Graeve (2008), and Christiano et al. (2014). We assume that banks collect deposits from households and lend to entrepreneurs who use these funds as well as their own wealth to acquire physical capital, which is rented to intermediate goods producers. Entrepreneurs are subject to idiosyncratic disturbances that affect their ability to manage capital. Their revenue may thus turn out to be too low to pay back the loans received by the banks. The banks therefore protect themselves against default risk by pooling all loans and charging a spread over the deposit rate. This spread may vary as a function of entrepreneurs' leverage and riskiness.

The *realized return on capital* is given by

$$\tilde{R}_t^k - \pi_t = \frac{r_*^k}{r_*^k + (1-\delta)} r_t^k + \frac{(1-\delta)}{r_*^k + (1-\delta)} q_t^k - q_{t-1}^k, \quad (\text{A-12})$$

where $\tilde{R}_t^k$ is the gross nominal return on capital for entrepreneurs, r_*^k is the steady state value of the rental rate of capital r_t^k , and δ is the depreciation rate.

The *excess return on capital* (the spread between the expected return on capital and the riskless rate) can be expressed as a function of the convenience yield cy_t , the entrepreneurs' leverage (i.e. the ratio of the value of capital to net worth), and “risk shocks” $\tilde{\sigma}_{\omega,t}$ capturing mean-preserving changes in the cross-sectional dispersion of ability across entrepreneurs (see Christiano et al. (2014)):

$$E_t \left[\tilde{R}_{t+1}^k - R_t \right] = cy_t + \zeta_{sp,b} (q_t^k + \bar{k}_t - n_t) + \tilde{\sigma}_{\omega,t}, \quad (\text{A-13})$$

where n_t is entrepreneurs' net worth, $\zeta_{sp,b}$ is the elasticity of the credit spread to the entrepreneurs' leverage ($q_t^k + \bar{k}_t - n_t$). $\tilde{\sigma}_{\omega,t}$ follows an AR(1) process with parameters ρ_{σ_ω} and σ_{σ_ω} . *Entrepreneurs' net worth* n_t evolves in turn according to

$$\begin{aligned} n_t = & \zeta_{n,\tilde{R}^k} \left(\tilde{R}_t^k - \pi_t \right) - \zeta_{n,R} (R_{t-1} - \pi_t + cy_{t-1}) + \zeta_{n,qK} (q_{t-1}^k + \bar{k}_{t-1}) + \zeta_{n,n} n_{t-1} \\ & - \gamma_* \frac{v_*}{n_*} z_t - \frac{\zeta_{n,\sigma_\omega}}{\zeta_{sp,\sigma_\omega}} \tilde{\sigma}_{\omega,t-1}, \end{aligned} \quad (\text{A-14})$$

where the ζ 's denote elasticities, that depend among others on the entrepreneurs' steady-state default probability $F(\bar{\omega})$, where γ_* is the fraction of entrepreneurs that survive and continue operating for another period, and where v_* is the entrepreneurs' real equity divided by Z_t^* , in steady state.

The *production function* is

$$y_t = \Phi_p (\alpha k_t + (1 - \alpha) L_t), \quad (\text{A-15})$$

where $\Phi_p = 1 + \Phi/y_*$, and Φ measures the size of fixed costs in production. The *resource constraint* is:

$$y_t = g_* g_t + \frac{c_*}{y_*} c_t + \frac{i_*}{y_*} i_t + \frac{r_*^k k_*}{y_*} u_t. \quad (\text{A-16})$$

where $g_t = \log(\frac{G_t}{Z_t^* y_* g_*})$ and $g_* = 1 - \frac{c_* + i_*}{y_*}$. *Government spending* g_t is assumed to follow the exogenous process:

$$g_t = \rho_g g_{t-1} + \sigma_g \varepsilon_{g,t} + \eta_{gz} \sigma_z \varepsilon_{z,t}.$$

Optimal decisions for price and wage setting deliver the *price and wage Phillips curves*, which are respectively:

$$\pi_t = \kappa mc_t + \frac{\iota_p}{1 + \iota_p \bar{\beta}} \pi_{t-1} + \frac{\bar{\beta}}{1 + \iota_p \bar{\beta}} \mathbb{E}_t[\pi_{t+1}] + \lambda_{f,t}, \quad (\text{A-17})$$

and

$$w_t = \frac{(1 - \zeta_w \bar{\beta})(1 - \zeta_w)}{(1 + \bar{\beta})\zeta_w((\lambda_w - 1)\epsilon_w + 1)} (w_t^h - w_t) - \frac{1 + \iota_w \bar{\beta}}{1 + \bar{\beta}} \pi_t + \frac{1}{1 + \bar{\beta}} (w_{t-1} - z_t + \iota_w \pi_{t-1}) + \frac{\bar{\beta}}{1 + \bar{\beta}} \mathbb{E}_t [w_{t+1} + z_{t+1} + \pi_{t+1}] + \lambda_{w,t}, \quad (\text{A-18})$$

where $\kappa = \frac{(1 - \zeta_p \bar{\beta})(1 - \zeta_p)}{(1 + \iota_p \bar{\beta})\zeta_p((\Phi_p - 1)\epsilon_p + 1)}$, the parameters ζ_p , ι_p , and ϵ_p are the Calvo parameter, the degree of indexation, and the curvature parameter in the Kimball aggregator for prices, and ζ_w , ι_w , and ϵ_w are the corresponding parameters for wages. w_t^h measures the household's marginal rate of substitution between consumption and labor, and is given by:

$$w_t^h = \frac{1}{1 - \bar{h}} (c_t - \bar{h}c_{t-1} + \bar{h}z_t) + \nu_l L_t, \quad (\text{A-19})$$

where ν_l characterizes the curvature of the disutility of labor (and would equal the inverse of the Frisch elasticity in the absence of wage rigidities). The mark-ups $\lambda_{f,t}$ and $\lambda_{w,t}$ follow the exogenous ARMA(1,1) processes:

$$\lambda_{f,t} = \rho_{\lambda_f} \lambda_{f,t-1} + \sigma_{\lambda_f} \varepsilon_{\lambda_f,t} - \eta_{\lambda_f} \sigma_{\lambda_f} \varepsilon_{\lambda_f,t-1},$$

and

$$\lambda_{w,t} = \rho_{\lambda_w} \lambda_{w,t-1} + \sigma_{\lambda_w} \varepsilon_{\lambda_w,t} - \eta_{\lambda_w} \sigma_{\lambda_w} \varepsilon_{\lambda_w,t-1}.$$

Finally, the monetary authority follows a *policy feedback rule*:

$$\begin{aligned} R_t = & \rho_R R_{t-1} + (1 - \rho_R) (\psi_1 (\pi_t - \pi_t^*) + \psi_2 (y_t - y_t^*)) \\ & + \psi_3 ((y_t - y_t^*) - (y_{t-1} - y_{t-1}^*)) + r_t^m. \end{aligned} \quad (\text{A-20})$$

where π_t^* is a time-varying inflation target, y_t^* is a measure of the “full-employment level of output,” and r_t^m captures exogenous departures from the policy rule.

The time-varying inflation target π_t^* is meant to capture the rise and fall of inflation and interest rates in the estimation sample.⁴⁸ As in Aruoba and Schorfheide (2008) and Del Negro and Eusepi (2011), we use data on long-run inflation expectations in the estimation of the model. This allows us to pin down the target inflation rate to the extent that long-run

⁴⁸The assumption that the inflation target moves exogenously is of course a simplification. A more realistic model would for instance relate movements in trend inflation to the evolution of the policy makers' understanding of the output-inflation trade-of, as in Sargent (1999) or Primiceri (2006).

inflation expectations contain information about the central bank's objective. The time-varying *inflation target* evolves according to

$$\pi_t^* = \rho_{\pi^*} \pi_{t-1}^* + \sigma_{\pi^*} \epsilon_{\pi^*,t}, \quad (\text{A-21})$$

where $0 < \rho_{\pi^*} < 1$ and $\epsilon_{\pi^*,t}$ is an iid shock. We model π_t^* as a stationary process, although our prior for ρ_{π^*} will force this process to be highly persistent.

The “full-employment level of output” y_t^* represents the level of output that would obtain if prices and wages were fully flexible and if there were no markup shocks. This variable along with the natural rate of interest r_t^* are obtained by solving the model without nominal rigidities and markup shocks (that is, equations (A-4) through (A-19) with $\zeta_p = \zeta_w = 0$, and $\lambda_{f,t} = \lambda_{w,t} = 0$).

The exogenous component of the policy rule r_t^m evolves according to the following process

$$r_t^m = \rho_{r^m} r_{t-1}^m + \epsilon_t^R + \sum_{k=1}^K \epsilon_{k,t-k}^R, \quad (\text{A-22})$$

where ϵ_t^R is the usual contemporaneous policy shock, and $\epsilon_{k,t-k}^R$ is a policy shock that is known to agents at time $t - k$, but affects the policy rule k periods later, that is, at time t . We assume that $\epsilon_{k,t-k}^R \sim N(0, \sigma_{k,r}^2)$, *i.i.d.* As argued in Laseen and Svensson (2011), such anticipated policy shocks allow us to capture the effects of the zero lower bound on nominal interest rates, as well as the effects of forward guidance in monetary policy.

B.1 State Space Representation and Data

We use the method in Sims (2002) to solve the system of log-linear approximate equilibrium conditions and obtain the transition equation, which summarizes the evolution of the vector of state variables s_t :

$$s_t = \mathcal{T}(\theta) s_{t-1} + \mathcal{R}(\theta) \epsilon_t. \quad (\text{A-23})$$

where θ is a vector collecting all the DSGE model parameters and ϵ_t is a vector of all structural shocks. The state-space representation of our model is composed of the transition equation (A-23), and a system of measurement equations:

$$Y_t = \mathcal{D}(\theta) + \mathcal{Z}(\theta) s_t, \quad (\text{A-24})$$

mapping the states into the observable variables Y_t , which we describe in detail next.

The estimation of the model is based on data on real output growth (including both GDP and GDI measures), consumption growth, investment growth, real wage growth, hours worked, inflation (measured by core PCE and GDP deflators), short- and long- term interest rates, 10-year inflation expectations, market expectations for the federal funds rate up to 6 quarters ahead, Aaa and Baa credit spreads, and total factor productivity. Measurement equations (A-24) relate these observables to the model variables as follows:

$$\begin{aligned}
\text{GDP growth} &= 100\gamma + (y_t - y_{t-1} + z_t) + e_t^{gdp} - e_{t-1}^{gdp} \\
\text{GDI growth} &= 100\gamma + (y_t - y_{t-1} + z_t) + e_t^{gdi} - e_{t-1}^{gdi} \\
\text{Consumption growth} &= 100\gamma + (c_t - c_{t-1} + z_t) \\
\text{Investment growth} &= 100\gamma + (i_t - i_{t-1} + z_t) \\
\text{Real Wage growth} &= 100\gamma + (w_t - w_{t-1} + z_t) \\
\text{Hours} &= \bar{L} + L_t \\
\text{Core PCE Inflation} &= \pi_* + \pi_t + e_t^{pce} \\
\text{GDP Deflator Inflation} &= \pi_* + \delta_{gdpdef} + \gamma_{gdpdef} * \pi_t + e_t^{gdpdef} \\
\text{FFR} &= R_* + R_t \\
\text{FFR}_{t,t+j}^e &= R_* + \mathbb{E}_t [R_{t+j}], \quad j = 1, \dots, 6 \\
\text{10y Nominal Bond Yield} &= R_* + \mathbb{E}_t \left[\frac{1}{40} \sum_{j=0}^{39} R_{t+j} \right] + e_t^{10y} \\
\text{10y Infl. Expectations} &= \pi_* + \mathbb{E}_t \left[\frac{1}{40} \sum_{j=0}^{39} \pi_{t+j} \right] \\
\text{Aaa - 20-year Treasury Spread} &= cy_*^l + \mathbb{E}_t \left[\frac{1}{80} \sum_{j=0}^{79} cy_{t+j}^l \right] + e_t^{Aaa} \\
\text{Baa - 20-year Treasury Spread} &= cy_*^l + cy_*^s + SP_* + \mathbb{E}_t \frac{1}{80} \sum_{j=0}^{79} \left[\tilde{R}_{t+j+1}^k - R_{t+j} \right] + e_t^{Baa} \\
\text{TFP growth, demeaned} &= z_t + \frac{\alpha}{1-\alpha} (u_t - u_{t-1}) + e_t^{tfp}.
\end{aligned} \tag{A-25}$$

All variables are measured in percent. The terms π_* and R_* measure respectively the net steady-state inflation rate and short-term nominal interest rate, expressed in percentage terms, and $\bar{L}$ captures the mean of hours (this variable is measured as an index). We assume that some of the variables are measured with “error,” that is, the observed value equals the model implied value plus an AR(1) exogenous process e_t^* that can be thought of either

measurement errors or some other unmodeled source of discrepancy between the model and the data, as in Boivin and Giannoni (2006). For instance, the terms e_t^{gdp} and e_t^{gdi} capture measurement error of total output.⁴⁹ Alternatively, or the long-term nominal interest rate, the term e_t^{10y} captures fluctuations in term premia not captured by the model.

The 10-year inflation expectations contain information about low-frequency movements of inflation and are obtained from the Blue Chip Economic Indicators survey and the Survey of Professional Forecasters.

B.2 Inference, Prior and Posterior Parameter Estimates

We estimate the model using Bayesian techniques. This requires the specification of a prior distribution for the model parameters. For most parameters common with Smets and Wouters (2007), we use the same marginal prior distributions. As an exception, we favor a looser prior than Smets and Wouters (2007) for the quarterly steady state inflation rate π_* ; it is centered at 0.75% and has a standard deviation of 0.4%. Regarding the financial frictions, we specify priors for the parameters SP_* , $\zeta_{sp,b}$, ρ_{σ_ω} , and σ_{σ_ω} , while we fix the parameters corresponding to the steady state default probability and the survival rate of entrepreneurs, respectively. In turn, these parameters imply values for the parameters of (A-14). Information on the priors and posterior mean is provided in Table [??].

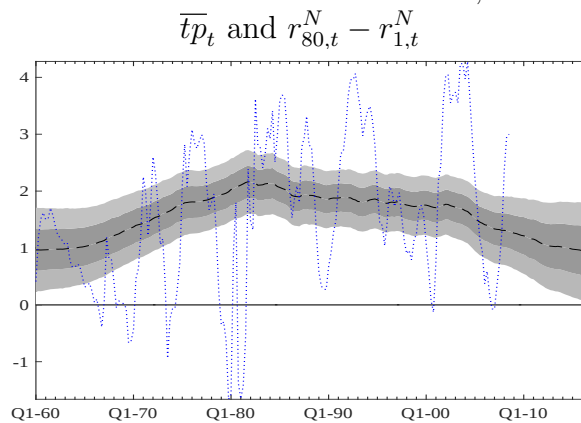
⁴⁹We introduce correlation in the measurement errors for GDP and GDI, which evolve as follows:

$$\begin{aligned} e_t^{gdp} &= \rho_{gdp} \cdot e_{t-1}^{gdp} + \sigma_{gdp} \epsilon_t^{gdp}, \quad \epsilon_t^{gdp} \sim i.i.d.N(0, 1) \\ e_t^{gdi} &= \rho_{gdi} \cdot e_{t-1}^{gdi} + \varrho_{gdp} \cdot \sigma_{gdp} \epsilon_t^{gdp} + \sigma_{gdi} \epsilon_t^{gdi}, \quad \epsilon_t^{gdi} \sim i.i.d.N(0, 1). \end{aligned}$$

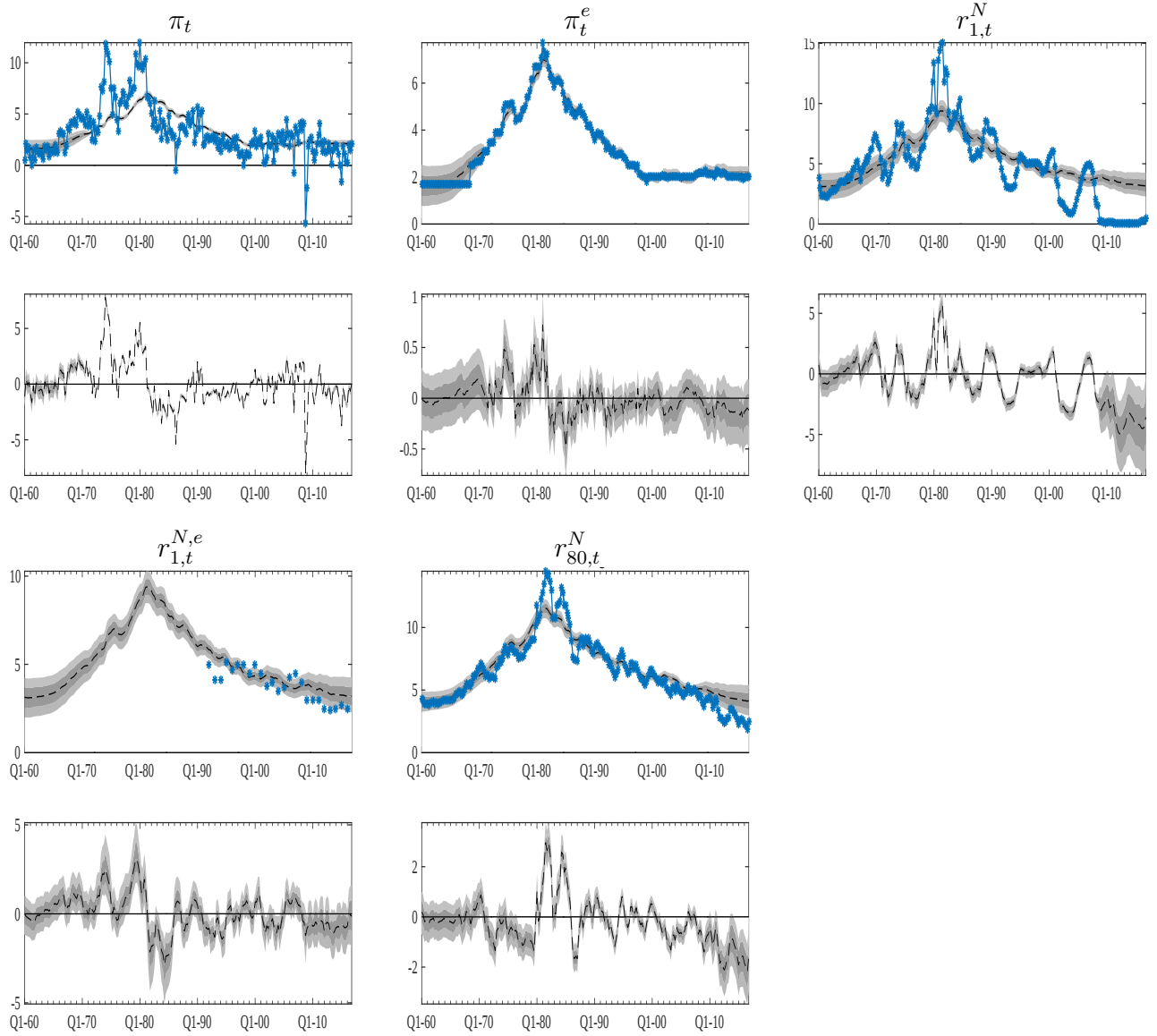
The measurement errors for GDP and GDI are thus stationary in *levels*, and enter the observation equation in first differences (e.g. $e_t^{gdp} - e_{t-1}^{gdp}$ and $e_t^{gdi} - e_{t-1}^{gdi}$). GDP and GDI are also cointegrated as they are driven by a common stochastic trend.

C Additional Tables and Figures – Trendy VARs (Section 4)

Figure A1: Other Trends and Observables, Baseline Model

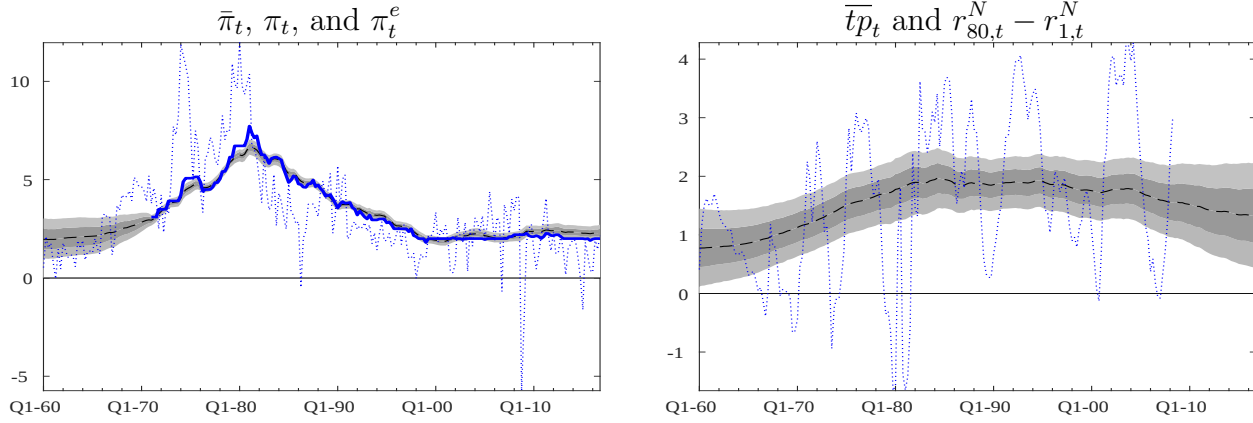


Note: The figure shows $r_{80,t}^N - r_{1,t}^N$ (dotted blue line) together with the trend $\bar{t}p_t$. For the trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

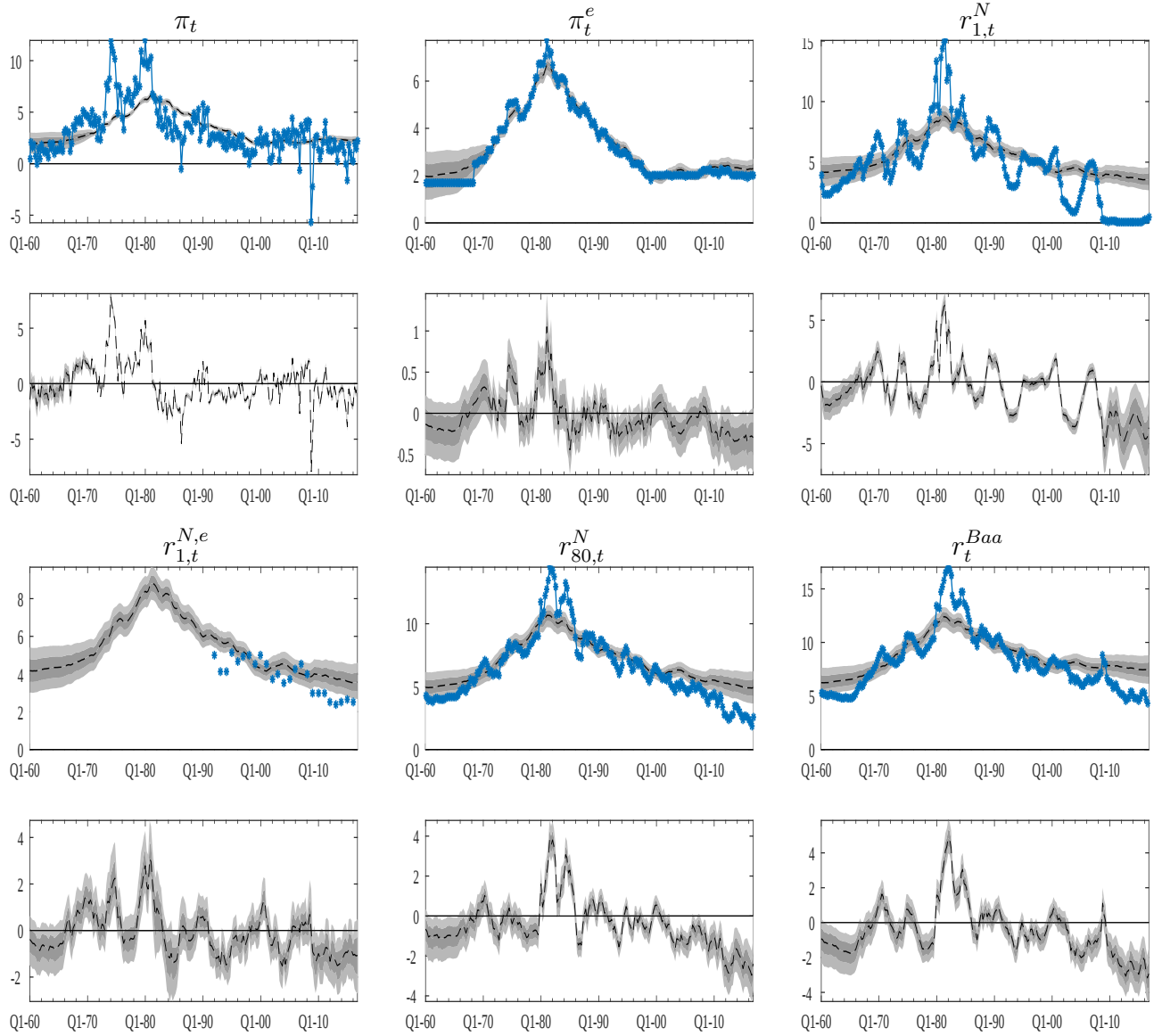
Figure A2: y_t , $\Lambda\bar{y}_t$, and $\tilde{y}_t$; Baseline Model

Note: For each variable the top panel shows the data y_t and the trend component $\Lambda\bar{y}_t$, and the bottom panel shows the stationary component $\tilde{y}_t$. For each latent variable, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

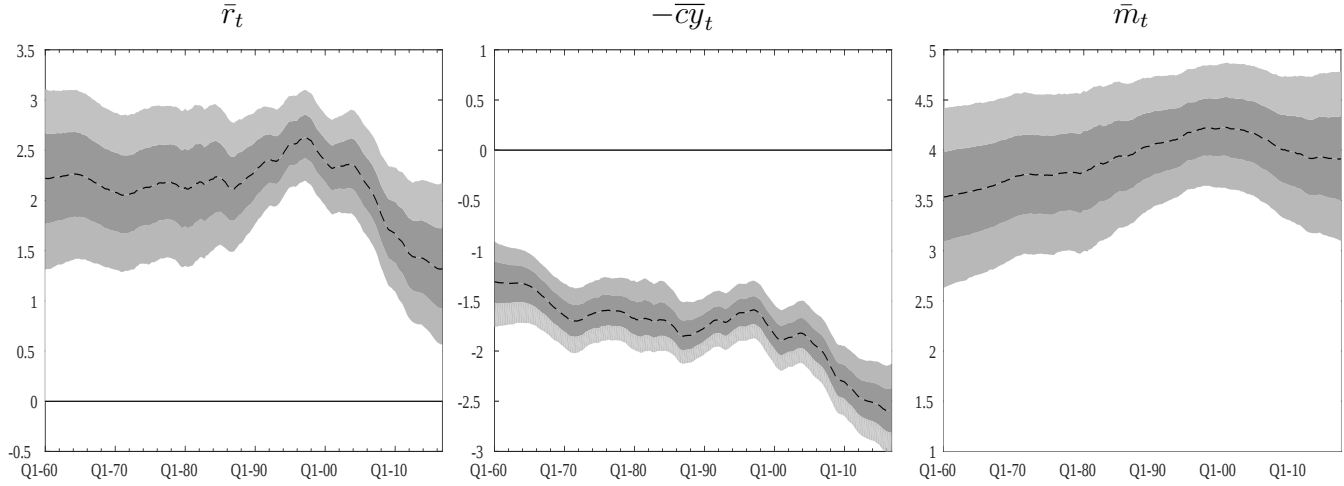
Figure A3: Other Trends and Observables, Convenience Yield Model



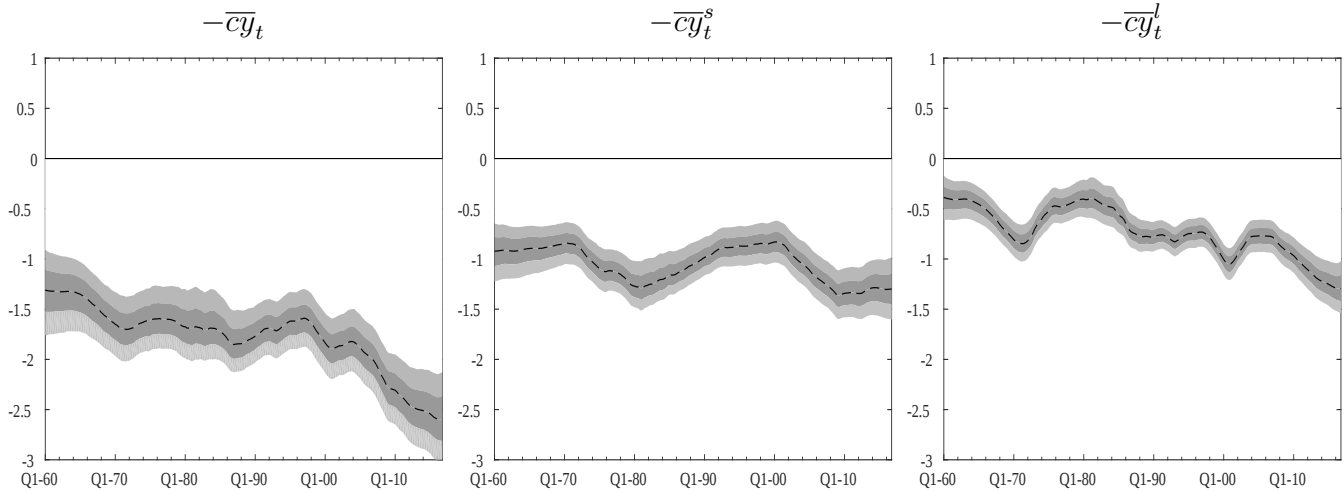
Note: The left panel shows π_t (dotted blue line), and π_t^e (solid blue line), together with the trend $\bar{\pi}_t$. The right panel shows $r_{80,t}^N - r_{1,t}^N$ (dotted blue line) together with the trend $\bar{tp}_t$. For the trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A4: y_t , $\Lambda\bar{y}_t$, and $\tilde{y}_t$; Convenience Yield Model

Note: For each variable the top panel shows the data y_t and the trend component $\Lambda\bar{y}_t$, and the bottom panel shows the stationary component $\tilde{y}_t$. For each latent variable, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

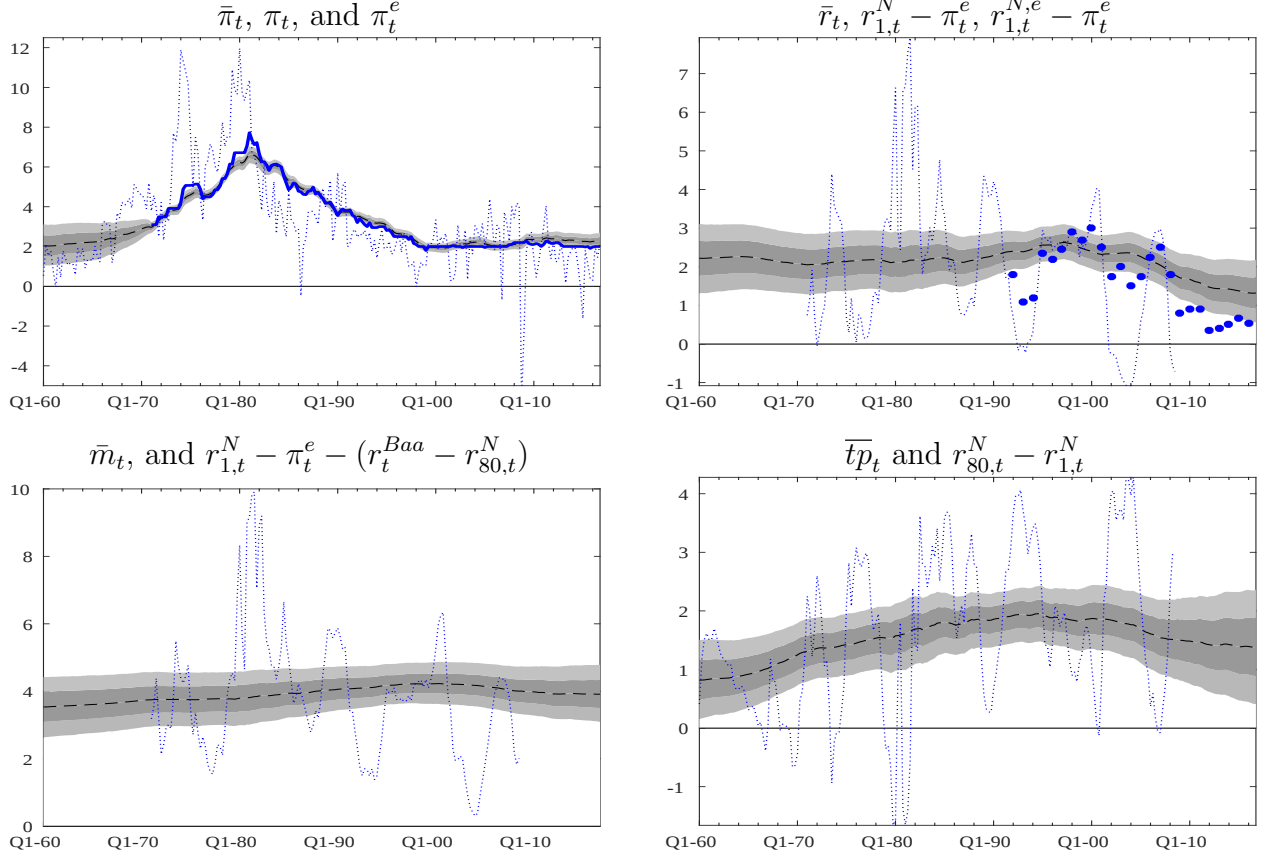
Figure A5: $\bar{r}_t$, $\bar{cy}_t$, and $\bar{m}_t$, Safety and Liquidity Model

Note: The dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

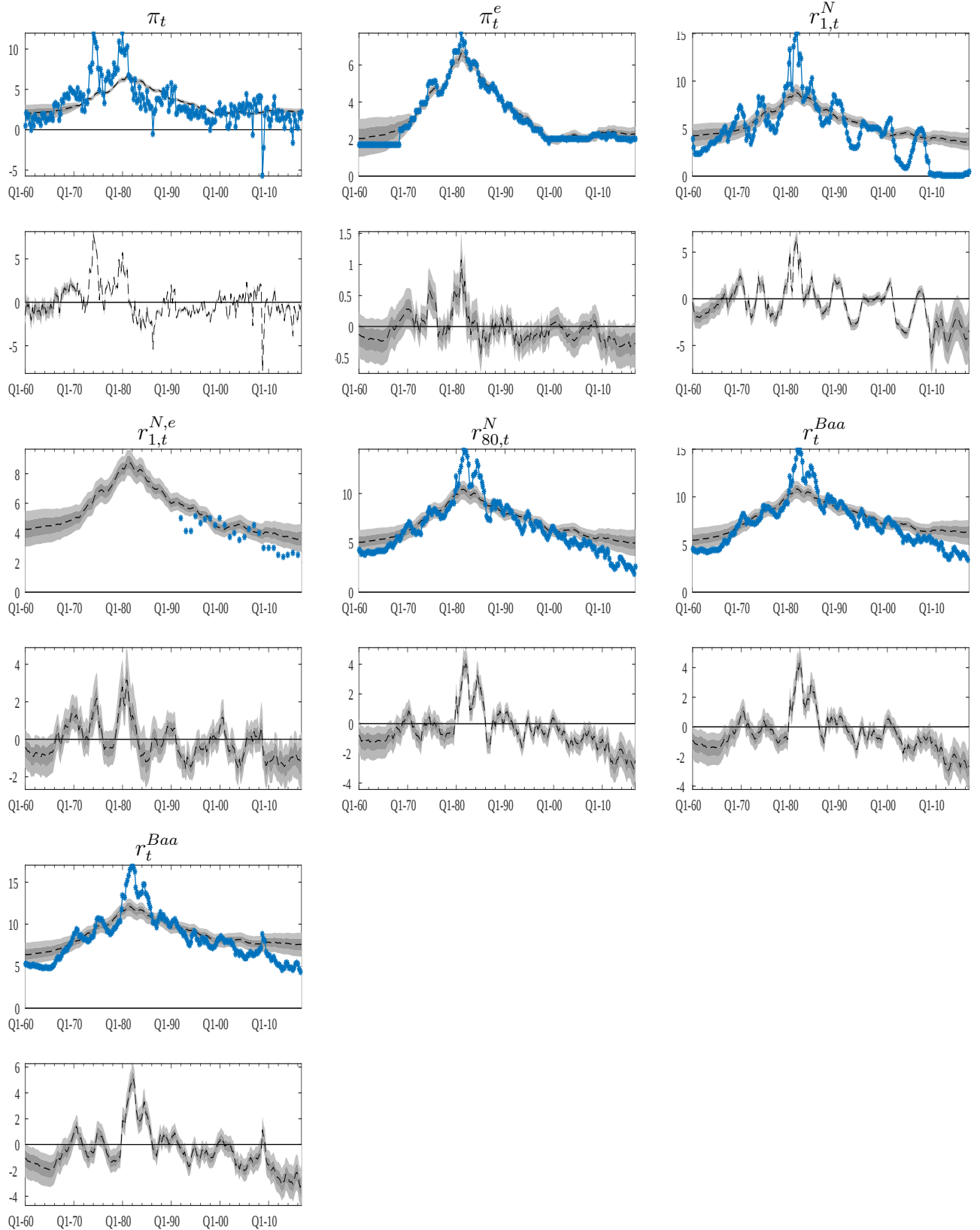
Figure A6: $\bar{cy}_t$, $\bar{cy}_t^s$, and $\bar{cy}_t^l$, Safety and Liquidity Model

Note: The dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A7: Other Trends and Observables, Safety and Liquidity Model



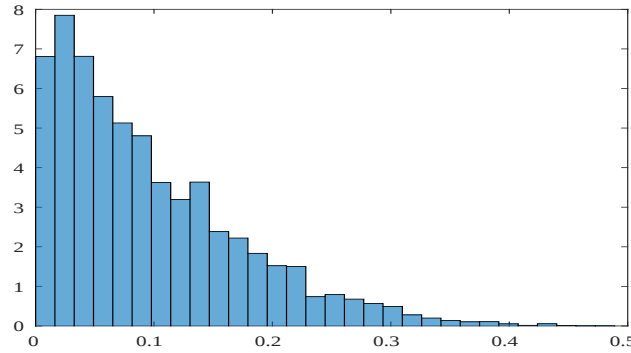
Note: The top left panel shows π_t (dotted blue line), and π_t^e (solid blue line), together with the trend $\bar{\pi}_t$. The top right panel shows $r_{1,t}^N - \pi_t^e$ (dotted blue line), and $r_{1,t}^{N,e} - \pi_t^e$ (blue dots), together with the trend $\bar{r}_t$. The bottom left panel shows $r_{1,t}^N - \pi_t^e - (r_t^{Baa} - r_{80,t}^N)$ (dotted blue line), together with the trend $\bar{m}_t$. The bottom right panel shows $r_{80,t}^N - r_{1,t}^N$ (dotted blue line) together with the trend $\bar{tp}_t$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

Figure A8: y_t , $\Lambda \bar{y}_t$, and $\tilde{y}_t$; Safety and Liquidity Model

Note: For each variable the top panel shows the data y_t and the trend component $\Lambda \bar{y}_t$, and the bottom panel shows the stationary component $\tilde{y}_t$. For each latent variable, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

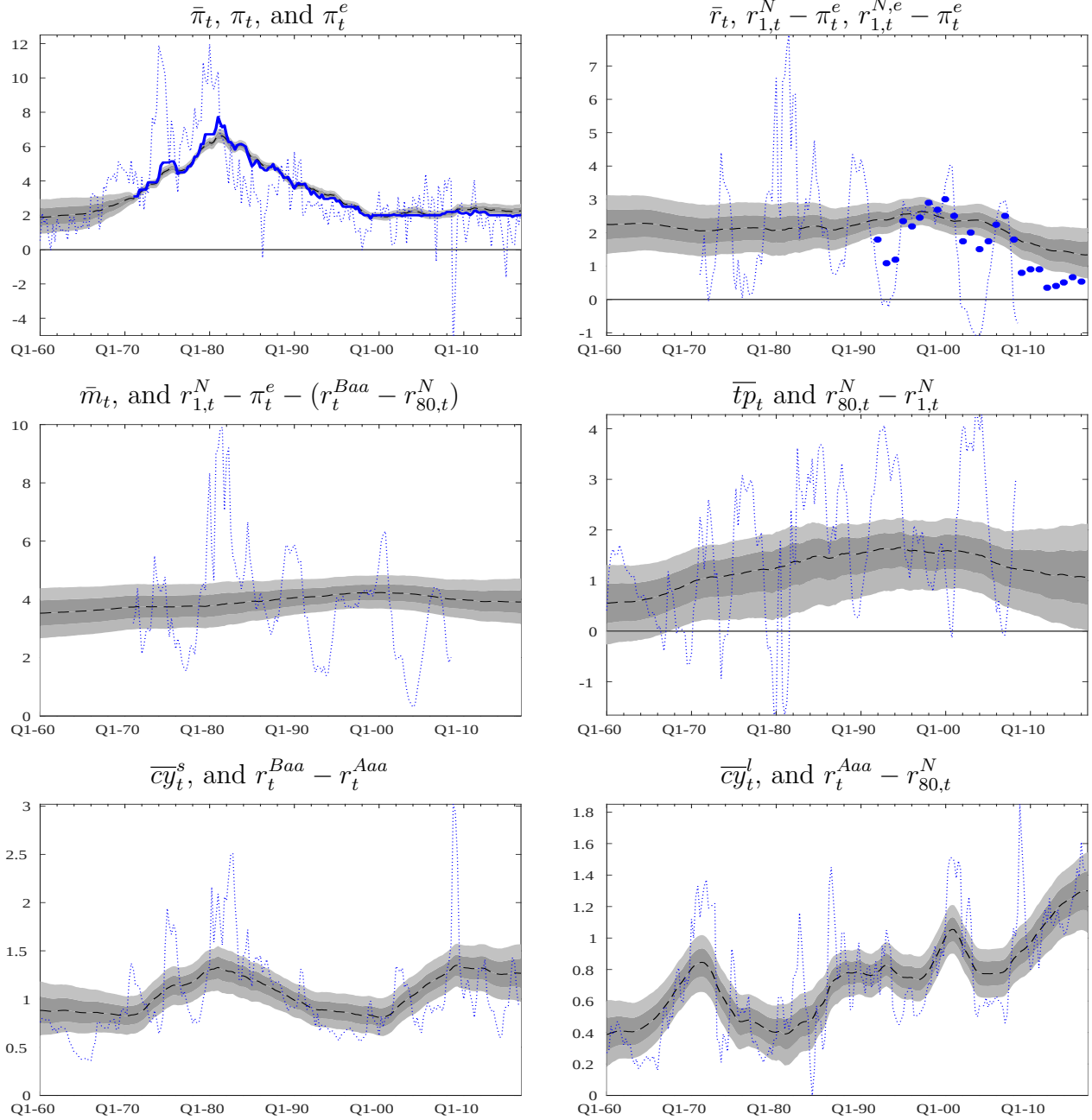
D Robustness – Trendy VARs

Figure A9: Posterior Distribution of γ^{tp} – Model with Inflation Affecting the Nominal Term Premium



Note: The figure shows the posterior distribution of γ^{tp} . The prior is an exponential with mean .10.

Figure A10: Trends and Observables, Inflation Affecting the Nominal Term Premiuml



Note: The top left panel shows π_t (dotted blue line), and π_t^e (solid blue line), together with the trend $\bar{\pi}_t$. The top right panel shows $r_{1,t}^N - \pi_t^e$ (dotted blue line), and $r_{1,t}^{N,e} - \pi_t^e$ (blue dots), together with the trend $\bar{r}_t$. The middle left panel shows $r_{1,t}^N - \pi_t^e - (r_t^{Baa} - r_{80,t}^N)$ (dotted blue line), together with the trend $\bar{m}_t$. The middle right panel shows $r_{80,t}^N - r_{1,t}^N$ (dotted blue line), together with the trend $\bar{tp}_t$. The bottom left panel shows the Baa/Aaa spread $r_t^{Baa} - r_t^{Aaa}$ (dotted blue line), together with the trend $\bar{cy}_t^s$. The bottom right panel shows the Baa/Treasury spread $r_t^{Aaa} - r_{80,t}^N$ (dotted blue line), together with the trend $\bar{cy}_t^l$. For each trend, the dashed black line shows the posterior median and the shaded areas show the 68 and 95 percent posterior coverage intervals.

E Additional Tables and Figures – DSGE (Section 5)