An Endogenous Growth Model with a Health Sector*

Jesus Fernandez-Villaverde† Dirk Krueger†
Alexander Ludwig‡ Matthias Schön§

Preliminary and Incomplete

Abstract

We develop an overlapping generations model with endogenous growth and a health sector, in order to explain three secular facts characterizing the U.S. economy: a substantial increase in life expectancy, a rise in the share of GDP devoted to health-related expenditures as well as an increase in the relative price of medical goods. We show how to interpret these observations as the equilibrium outcome of a model in which technological progress through quality improvements is endogenously directed to the sector producing medical goods.

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†University of Pennsylvania, CEPR and NBER.
‡SAFE, Goethe University Frankfurt.
§CMR, University of Cologne.

1 Introduction

We are motivated by three salient observations about health. A strongly increasing expenditure share, longer life expectancy (and thus an aging population, given birth rates) and an increasing price of health goods (as measured by the CPI on health goods, relative to the overall CPI). We proceed by developing an overlapping generations model with endogenous growth and a health sector, in order to explain these observations as the equilibrium outcome of a model in which technological progress is endogenously directed to the sector producing medical goods. An important element of our model is an explicit notion of the relative quality of health goods. Growth in the economy is generated through quality improvements. With this element we are able to decompose the relative price changes of health goods into quality adjusted and non-adjusted price indices.

[TBC]

The remainder of this paper proceeds as follows. Section 2 presents stylized facts on life-expectancy, aggregate health spending and prices of health goods in the US. Section 3 develops a two sector endogenous growth model with a health and a consumption goods sector. Endogenous growth is modeled through quality improvements. The household sector is very stylized in that households have two periods of life and consume only in the second period. The model delivers important qualitative (and, in part, analytical) insights on the relationship between health spending, trends in life expectancies, relative prices of medical goods and their quality as well as the health spending share as a fraction of aggregate income. Next, we expand in Section 4 the household sector of the theoretical model to a multi-period quantitative model. At this stage, we also model some institutional features of a (private and public) health insurance system. Our objective is to add realistic life-cycle features of health spending, in particular, the age-increasing health expenditures. These features are important when we combine and calibrate the models in Section 5. Specifically, we will calibrate the model so that the endogenous trends in life-expectancies, health spending and growth match the data. We will also add

exogenous trends in fertility, whereby the the baby-boom baby-bust cycle will be of key relevance. Combined with the endogenous trends in life-expectancies this gives rise to an overall aging pattern in that working age population ratios will be decreasing and old-age dependency will be increasing.[1] Given the age increasing health spending predicted in the household sector, these demographic developments will add additional movement to aggregate spending patterns. Section 6 will predict spending trends, growth and life-expectancies by use of the calibrated model over the next decades. Finally, Section 7 concludes the paper.

2 Stylized Facts

Panel (a) of Figure 1 displays the share of total output (GDP) in the US economy devoted to health goods and services and Panel (b) the evolution of life expectancy at birth for the U.S. over time. Both plots show a strong upward trend, with life expectancy increasing by about 1.5 years every decade, and the share of GDP produced by the health sector raising from less than 5% of GDP in 1960 to more than 17.5% in recent years (an increase of about 2 percentage points every decade).

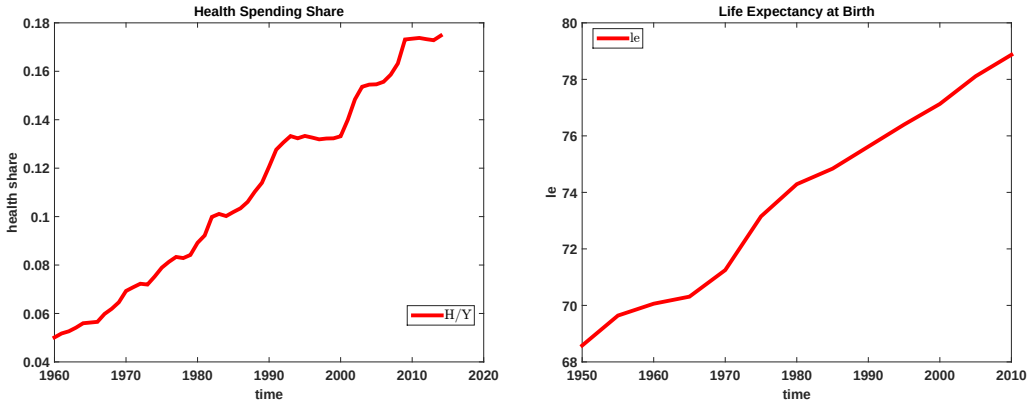
3 A Simple Model

The model is populated by overlapping generations with n_t^y identical young and n_t^o identical old households in period t. Total population is denoted by $n_t = n_t^y + n_t^o$. Households work, earn income, spend resources on health and save in the first period of their lives and consume in the second period of their lives. In addition, there are competitive firms in three sectors, one that produces consumption goods, one that produces health goods, and one that does research and development.

[1] Notice that these trends would also arise with constant fertility rates. Time varying fertility as observed in the data will strengthen the decrease of the working age population ratio and the corresponding increase of the old-age dependency ratio.

Figure 1: Health Spending Share and Life Expectancy in the US

(a) H/Y (b) LE



Notes: Panel (a) of this figure shows aggregate health spending in the US—total expenditures for health care goods and services, public health activities, government administration, the net cost of health insurance, and investment related to health care—as a fraction of GDP. Panel (b) shows US life expectancy at birth as the average for both sexes.
Source: National Health Expenditure Accounts, Centers for Medicare and Medicaid Services; National Income and Product Accounts, Bureau of Economic Analysis and World Population Prospects, United Nations.

3.1 Households

Households have preferences represented by the utility function

$$\psi\left(d_{t+1}\right) u(c_{t+1}^{o}) \tag{1}$$

and maximize it subject to the constraints

$$p_t i_t + s_t = w_t + T_t \tag{2}$$

$$c_{t+1}^{o} = r_{t+1} s_t. \tag{3}$$

where p_t is the relative price of health goods in terms of consumption goods (which we define formally below), c_{t+1}^{o} is consumption in old age, i_t is the health investment and $\psi\left(d_{t+1}\right)$ is the probability. Survival is contingent on accumulated deficits d_{t+1}. We assume that $d_{t+1} = g(i_t)$ for some mono-

tone function $g(i_t)$ with the properties $g_i < 0$ and $\lim_{i_t \to \infty} g(i_t) > 0$. On the survival probability itself we assume that $\psi(0) = 1$, $\psi_{d_{t+1}}(d_{t+1}) < 0$ and $\lim_{d_{t+1} \to \infty} \psi(d_{t+1}) > 0$. T_t are transfers from accidental bequests from previous generations which households take exogenous.

We assume that the depreciation rate on capital is 1, so that the gross return on saving s_t is r_{t+1} which will equal the marginal product on capital in equilibrium. Absent borrowing constraints the budget constraints can be consolidated to the lifetime budget constraint

$$p_t i_t + \frac{c^o_{t+1}}{r_{t+1}} = w_t + T_t = x_t. \tag{4}$$

where x_t is cash-on-hand of the household. In equilibrium, transfers to generation born in period t due to accidental bequests from generation $t-1$ are given by:

$$T_t = r_t s_{t-1}(1 - \psi(d_t))/\gamma^n_t \tag{5}$$

where $\gamma^n_t = \frac{n^y_{t-1}}{n^y_t}$ is the growth rate of the young population.

Thus transfers are positive if and only if $\psi\left(d_{t+1}\right) < 1$ and thus households die with positive probability between young and old ages.

3.2 Firms, Production and R&D

3.2.1 Final Goods Producers

Let $j \in \{f, h\}$ stand for the final and the health sector of the economy, respectively, with and p_{jt} for the price of the output of each of the two sectors. We will normalize p_{ft} to 1 and simply let p_t denote the relative price of health goods whenever it notationally more convenient and there is no room for confusion. In each sector a representative firm uses a continuum of intermediate inputs indexed by i and labor to produce sectorial output y_{jt} according to the

production function:

$$y_{jt} = \left(\int_0^1 q_{jit}^{1-\alpha} y_{jit}^{\alpha} di \right) l_{jt}^{1-\alpha} \tag{6}$$

where $0 < \alpha < 1$ and y_{jit} is the quantity of intermediate input i used to produce the output good in sector j at date t and l_{jt} is the number of workers employed in sector j. The entity q_{jit} denotes the quality of intermediate input i at date t in sector j. Growth in this model results from innovations that increase the quality q_{jit} of intermediate inputs. Since the final good producer is competitive and takes factor input prices as given, she hires labor and intermediate inputs to equate marginal productivities to these input prices, taking as given their qualities q_{jit}. Let the wage rate be given by w_t and the price of one unit of intermediate good i in sector j is p_{jit}. The first order conditions read as

$$p_{jt} (1 - \alpha) \left(\int_0^1 q_{jit}^{1-\alpha} y_{jit}^{\alpha} di \right) l_{jt}^{-\alpha} = w_t \tag{7}$$

for labor demand and

$$p_{jt} \alpha q_{jit}^{1-\alpha} y_{jit}^{\alpha-1} l_{jt}^{1-\alpha} = p_{jit} \tag{8}$$

for the demand for intermediate goods, given their quality q_{jit}.

3.2.2 Intermediate Goods Producers

Each intermediate good producer i is a monopolist that takes the demand function (8) as given and uses capital (which depreciates immediately after use) to produce the intermediate good according to:

$$y_{jit} = k_{jit}. \tag{9}$$

The rental rate of capital is given by r_t, so that each intermediate goods monopolist producer maximizes profits, taking as given the demand function of the final goods producer:

$$\pi_{jit} = \max_{y_{jit}} \left\{ \left[p_{jt} \alpha q_{jit}^{1-\alpha} y_{jit}^{\alpha-1} l_{jt}^{1-\alpha} \right] y_{jit} - r_t y_{jit} \right\}$$

with first order condition

$$y_{jit} = \left(\frac{p_{jt}\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} q_{jit} l_{jt} \tag{10}$$

and profits

$$\pi_{jit} = \frac{1-\alpha}{\alpha} r_t y_{jit} > 0 \tag{11}$$

The monopolistic price follows from using (10) in (8) as

$$p_{jit} = \frac{1}{\alpha} r_t > r_t \tag{12}$$

hence featuring the standard markup over marginal costs, r_t. It is the same across all intermediate input producers i and sectors j. Furthermore, the markup over marginal costs is also constant.

Finally, observe from (10) that $\frac{y_{jit}}{q_{jit}}$ is constant across varieties i. Likewise the ratio of profits to quality is constant, which we state for further reference using (10) in (11) to get

$$\frac{\pi_{jit}}{q_{jit}} = \left(\frac{p_{jt}\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} l_{jt}. \tag{13}$$

3.2.3 Aggregation of Production Sector

Because the ratios of variety-specific intermediate outputs to quality y_{jit}/q_{jit} and profits to output (or quality) π_{jit}/y_{jit} (π_{jit}/q_{jit}) are constant across varieties i we get immediate aggregation results for each sector.

For each production sector j we can determine aggregate capital input and production as

$$k_{jt} = \int_0^1 k_{jit} di = \int_0^1 y_{jit} di = \left(\frac{p_{jt}\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} q_{jt} l_{jt} \tag{14}$$

where

$$q_{jt} = \int_0^1 q_{jit} di \tag{15}$$

is an aggregate quality index of intermediate inputs in sector j. Furthermore, exploiting (10) and (14) in (6) yields as aggregate production function for sector j

$$y_{jt} = k_{jt}^{\alpha} \left(q_{jt} l_{jt}\right)^{1-\alpha}. \tag{16}$$

Using equations (7) and (14) delivers as factor prices for labor inputs and capital inputs:

$$w_t = (1-\alpha) \frac{p_{jt} y_{jt}}{l_{jt}} \tag{17a}$$

$$r_t = \alpha^2 \frac{p_{jt} y_{jt}}{k_{jt}}. \tag{17b}$$

Finally we can use (11) and (14) to determine aggregate profits in each sector j as

$$\pi_{jt} = \alpha (1-\alpha) p_{jt} y_{jt} \tag{18}$$

and thus in each sector j output exhausts factor input payments plus profits:

$$p_{jt} y_{jt} = \pi_{jt} + r_t k_{jt} + w_t l_{jt} \tag{19}$$

To summarize the aggregation result, in each of the two sectors output is produced with a Cobb-Douglas production function with capital and labor inputs in which the level of technology is given by q_{jt}. However, final goods producers cannot rent capital directly, but have to go through monopolistically competitive intermediaries. As a consequence owners of the capital (which will be the old households in equilibrium) command only a fraction α^2 of the value of output, with a fraction $\alpha(1-\alpha)$ accruing to the monopolist intermediaries.

3.2.4 Research and Development

An R&D developer that specializes in intermediate good i that spends resources of the final consumption good z_{jit} on R&D to achieve innovation. If successful in innovation, the quality of the intermediate good increases from q_{jit-1} to

$$q_{jit} = \lambda q_{jit-1} \tag{20}$$

where $\lambda > 1$ is a parameter. The successful innovator immediately becomes the monopolist, and for one period enjoys monopoly profits π_{jit} associated with technology level $q_{jit} = \lambda q_{jit-1}$. In a product line i in which innovation is not successful a randomly chosen entrepreneur becomes the monopolist and produces at quality $q_{jit} = q_{jit-1}$ with associated profits.

We assume that the probability of innovating is related to the quality reached when successfully innovating given by λq_{jit-1}. We also assume that the probability of innovating depends on the size of the economy. As formulated in Young (1998) and others this offsets scale effects. Specifically, we assume that the probability of innovation varies inversely with the size of the period t population to the effect that the probability of innovating is given by

$$\varphi \left(\frac{z_{jit}}{\lambda q_{jit-1}} \right)^{\gamma} \cdot n_t^{-1} \tag{21}$$

with $\gamma \in (0, 1)$ and $\varphi > 0$. The specification implies that an increase of the scale of the economy (here measured in terms of the size of the total population), dilutes the effects of research outlays, z_{jit}. Intuitively, this captures Young (1998)'s insight that as population grows, the effectiveness of research aimed at quality improvement is reduced by being spread more thinly thus dissipating the effect on the overall rate of productivity growth. Similarly, the inverse relationship between the success probability and current quality q_{jit-1} reflects the fact that it becomes increasingly harder to innovate if already a level of quality is reached for variety i.

Thus the R&D entrepreneur spends resources z_{jit} and, if successful, collects profits π_{jit}. Hence the problem reads as

$$\max_{z_{jit}} \left\{ \pi_{jit} \varphi \left(\frac{z_{jit}}{\lambda q_{jit-1}} \right)^{\gamma} n_t^{-1} - z_{jit} \right\} \tag{22}$$

with first order condition

$$\frac{\pi_{jit}}{\lambda q_{jit-1} n_t} \varphi \gamma \left(\frac{z_{jit}}{\lambda q_{jit-1}} \right)^{\gamma - 1} = 1. \tag{23}$$

which yields as solution a ratio of R&D spending to potential period t technology $\frac{z_{jit}}{\lambda q_{jit-1}}$

$$\frac{z_{jit}}{\lambda q_{jit-1}} = \left[\varphi\gamma \frac{\pi_{jit}}{\lambda q_{jit-1} n_t}\right]^{\frac{1}{1-\gamma}} \tag{24}$$

Noticing that in case of success $q_{jit} = \lambda q_{jit-1}$ we can now use equation (13) in the above to get

$$\frac{z_{jit}}{\lambda q_{jit-1}} = \left[\varphi\gamma \left(\frac{p_{jt}\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} \ell_{jt}\right]^{\frac{1}{1-\gamma}} \tag{25}$$

where $\ell_{jt} = \frac{l_{jt}}{n_t}$ is the working age population share in sector j. Using the above back in (21) we observe the share of varieties innovating is (due to the law of large numbers)

$$\mu_{jt} = \int \varphi \left(\frac{z_{jit}}{\lambda q_{jit-1}}\right)^{\gamma} di = \varphi^{\frac{1}{1-\gamma}} \left[\gamma \left(\frac{p_{jt}\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} \ell_{jt}\right]^{\frac{\gamma}{1-\gamma}} \tag{26}$$

and is thus independent of the distribution of qualities across varieties i.

For future reference, also observe that resources spend by entrepreneur i are

$$z_{jit} = \left[\varphi\gamma \left(\frac{p_{jt}\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} \ell_{jt}\right]^{\frac{1}{1-\gamma}} \lambda q_{jit-1}$$

so that total resources devoted to R&D in sector j are equal to

$$z_{jt} = \int z_{jit} di = \left[\varphi\gamma \left(\frac{p_{jt}\alpha^2}{r_t}\right)^{\frac{1}{1-\alpha}} \ell_{jt}\right]^{\frac{1}{1-\gamma}} \lambda q_{jt-1} \tag{27}$$

which are also independent of the distribution of qualities across varieties in sector j.

3.3 Equilibrium and Income and Product Accounting

3.3.1 Definition of Equilibrium

In this section we define a competitive equilibrium for our economy. We immediately proceed to defining equilibrium for the aggregate economy, thereby already exploiting the aggregation results developed in sections 3.2.3 and 3.2.4. Noticing that we can define either good as numeraire, we normalize $p_{ft} = 1$ and define all equilibrium conditions in terms of the price of health goods $p_t = \frac{p_{ht}}{p_{ft}}$.

Definition 1. *Given an initial population, n_t^y, n_t^o, and initial conditions s_0, i_0, q_{f0}, q_{h0}, a competitive equilibrium is a sequence of household allocations $c_1^o, d_1, \{s_t, i_t, d_{t+1}, c_{t+1}^o\}_{t=1}^{\infty}$, a sequence capital and labor inputs of goods producers $\{k_{jt}, l_{jt}\}_{t=1}^{\infty}$, a sequence of R&D expenditures, profits and consumption of R&D developers $\{z_{jt}, \pi_{jt}, c_{jt}\}_{t=1}^{\infty}$, a sequence of aggregate capital and technology $\{k_t, q_{ft}, q_{ht}\}_{t=1}^{\infty}$, prices $\{p_t, w_t, r_t\}_{t=1}^{\infty}$ and transfers $\{T_t\}_{t=1}^{\infty}$ and a law of motion of the population n_t^y, n_t^o such that*

1. *Household maximization: for each $t \geq 1$, given prices and transfers w_t, p_t, r_{t+1}, T_t, the allocations $i_t, s_t, d_{t+1}, c_{t+1}^o$ maximize (1) subject to (4).*
2. *Transfers T_t satisfy equation (5).*
3. *Factor prices satisfy equations (17a) and (17b).*
4. *Optimal R&D spending z_{jt} in each sector is given by (27) and consumption of R&D entrepreneurs is determined as $c_{jt} = \pi_{jt} - z_{jt}$.*
5. *The equilibrium innovation intensity μ_{jt} is given by equation (26) and technology in each sector evolves according to*

$$q_{jt} = (1 - \mu_{jt}) q_{jt-1} + \mu_{jt} \lambda q_{jt-1} \tag{28}$$

6. *Markets clear: for all $t \geq 1$*

   (a) *Labor Market*

$$n_t^y = l_t = \sum_j l_{jt} \tag{29}$$

(b) Capital Market

$$\sum_j k_{jt} = k_t \tag{30}$$

(c) Asset Market

$$k_t = s_{t-1} n_{t-1}^y \tag{31}$$

(d) Final Goods Market

$$s_t n_t^y + c_t^o n_t^o + \sum_j [c_{jt} + z_{jt}] = k_{ft}^\alpha \left(q_{ft} l_{ft}\right)^{1-\alpha} \tag{32}$$

(e) Health Goods Market

$$i_t n_t^y = k_{ht}^\alpha \left(q_{ht} l_{ht}\right)^{1-\alpha} \tag{33}$$

7. The population evolves according to

$$n_t^o = \psi(d_t) n_{t-1}^y, \qquad n_t^y \text{ given.} \tag{34}$$

3.3.2 National Income and Product Accounting in this Economy

In order to map the equilibrium in our model to the data we now derive GDP, total income and total spending in this economy and verify that value added equals income and equals spending. We do this both to assure that the model is internally consistent as well as to make explicit the map between model variables and empirical counterparts.

Aggregate Income In each period t the number n_t^y of young non-entrepreneur households receive labor income and income from accidental bequests

$$w_t(l_{ft} + l_{ht}) + T_t n_t^y$$

whereas the number $n_t^o = \psi\left(d_t\right) n_{t-1}^y$ of surviving old households earn capital income $r_t s_{t-1}$. Recall that transfers are given by equation (5) so that aggregate

transfers write as

$$T_t n_t^y = [1 - \psi(d_t)] r_t s_{t-1} n_{t-1}^y.$$

R&D entrepreneurs in both sectors earn total profits given by $\pi_{ft} + \pi_{ht}$ and purchase inputs $z_{ft} + z_{ht}$. Consequently their aggregate income is

$$\pi_{ft} + \pi_{ht} - (z_{ft} + z_{ht}).$$

Thus total income earned by non-entrepreneur and entrepreneur households equals

$$w_t(l_{ft} + l_{ht}) + r_t s_{t-1} [1 - \psi(d_t)] n_{t-1}^y + r_t s_{t-1} \psi(d_t) n_{t-1}^y + \pi_{ft} + \pi_{ht} - z_{ft} - z_{ht}$$

Spending Non-entrepreneur households spend $p_t i_t n_t^y$ on health goods, $s_t n_t^y = k_{t+1}$ on investment goods and $c_t^o n_t^o$ on consumption goods (recall that the final output good is the numeraire and used for both consumption and investment). Monopolists spend $c_{ft} + c_{ht}$ on final consumption goods.

Thus the value of total spending equals

$$c_t^o \psi(d_t) n_{t-1}^y + (s_t + p_t i_t) n_t^y + (c_{ft} + c_{ht}).$$

We note that total spending equals total income since

$$\begin{aligned} c_t^o \psi(d_t) n_{t-1}^y &= r_t s_{t-1} \psi(d_t) n_{t-1}^y && (35) \\ (s_t + p_t i_t) n_t^y &= w_t(l_{ft} + l_{ht}) + [1 - \psi(d_t)] r_t s_{t-1} n_{t-1}^y && (36) \\ c_{jt} &= \pi_{jt} - z_{jt} && (37) \end{aligned}$$

from the nonentrepreneur household budget constraints and the definition of consumption c_{jt} of entrepreneurs in both sectors j.

Output (Value Added) Total value added equals the value of output in the final goods sector, the health sector (the intermediate goods sector produces capital goods whose value exactly nets out with the capital inputs of the health and final goods sector), net of the inputs in the R&D sector. Thus value added

is given by

$$y_{ft} + p_t y_{ht} - (z_{ft} + z_{ht}). \tag{38}$$

But we established that

$$p_{jt} y_{jt} = \pi_{jt} + r_t k_{jt} + w_t l_{jt}$$

for both sectors, and thus value added

$$y_{ft} + p_t y_{ht} - (z_{ft} + z_{ht}) = r_t k_t + w_t l_t + \sum_j [\pi_{jt} - z_{jt}]$$

equals income (which in turn, as shown in the previous section, equals spending). Therefore the basic accounting identities are satisfied in our model.

3.4 Analysis of the Decision Problems

3.4.1 Analysis of Household Problem

For given prices and transfers and resulting cash-on-hand, $\{p_t, x_t, r_{t+1}\}$, where $x_t = w_t + T_t$, the household problem boils down to a one-dimensional maximization problem choosing i_t and thus the survival probability $\psi\left(d_{t+1}\right) = \psi(g(i_t))$. It reads as

$$\max_{0 \leq p_t i_t \leq x_t} \psi\left(g(i_t)\right) u\left(r_{t+1}\left[x_t - p_t i_t\right]\right)$$

with first order conditions:

$$\frac{\psi_g g_i\left(i_t\right) u\left(r_{t+1}\left[x_t - p_t i_t\right]\right)}{\psi\left(g(i_t)\right) u'\left(r_{t+1}\left[x_t - p_t i_t\right]\right)} = r_{t+1} p_t. \tag{39}$$

This is one nonlinear equation in one unknown i_t which delivers the demand functions

$$\begin{aligned} i_t &= i_t(p_t, x_t, r_{t+1}) \\ s_t &= x_t - p_t i_t = s_t(p_t, x_t, r_{t+1}) \\ c^o_{t+1} &= r_{t+1}\left[x_t - p_t i_t\right] = c^o_{t+1}(p_t, x_t, r_{t+1}) \end{aligned}$$

Unfortunately equation (39) does not have a closed form solution and we cannot establish consistency with a BGP unless very specific forms of the survival function $\psi(\cdot)$, the deficit accumulation function $g(i_t)$ and the period utility function u are assumed.

Assumption 1. *Hall and Jones (2007) use a utility function of the form*

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} + b$$

where $\sigma \geq 0$ and $b > 0$ are parameters. b measures the value of life and if $\sigma \geq 1$ has to be chosen sufficiently large such that $u(c) > 0$ for all relevant levels of consumption.

Assumption 2. *The survival function is given by*

$$\psi(d_{t+1}) = \frac{1}{(1+d_{t+1})^{\varrho}}$$

where $\varrho > 0$ is a parameter.

Assumption 3. *The deficit accumulation function is given by*

$$d_{t+1} = g(i_t) = \left(1 + \delta \frac{1}{(1+i_t)^{\xi}}\right)$$

Here δ is the deficit appreciation factor. Observe that for $i_t \to \infty$ we have $d_{t+1} \to 1$ so that the limit survival rate is $\frac{1}{2^{\varrho}}$. Later, when moving to the quantitative model, we will add additional calibration parameters to both functions.

Under these functional form assumptions we can establish the following condition on the existence and the properties of the balanced growth:

Proposition 1. *Under assumptions 1, 2 and 3 a BGP with growth rate $\lambda > 1$ exists if and only if $\sigma \leq 1 + \xi$. For $\sigma < 1 + \xi$ health investments in the BGP are equal to zero, hence $c_t = x_t, i_t = 0$. For $\sigma = 1 + \xi$ health investments of young households and consumption of old households are given by*

$$\theta(p, r) \equiv \frac{pi}{x} = \frac{1}{1 + \left(\frac{2}{b\varrho\delta\xi(pr)^{\xi}}\right)^{\frac{1}{1+\xi}}} \tag{40a}$$

$$1 - \theta(p, r) \equiv \frac{c/r}{x} = \frac{1}{1 + \left(\frac{2}{b\varrho\delta\xi(pr)^{\xi}}\right)^{\frac{-1}{1+\xi}}} \tag{40b}$$

Proof. See Appendix A.1 □

Observations:

- Health spending share increases in p: more expensive health goods
- Health spending share increases in r: increasing r increases second period consumption and hence also health spending
- Increasing b, ϱ, δ all increase health spending share for obvious reasons.

We can further characterize the evolution of the health expenditure share along the transition. Now assume that $\sigma = 2$ (and thus, to permit a meaningful BGP, $\xi = 1$). Also, for simplicity, $\varrho = \delta = 1$.

Proposition 2. *Under assumptions 1, 2 and 3 and for $\sigma = 2 \Leftrightarrow \xi = 1$ and $\delta = \varrho = 1$ the health expenditure share along the transition is given by*

$$\theta_t \equiv \frac{p_t i_t}{x_t} = \frac{\left(\kappa_t + \frac{1.5p_t}{2x_t}\right)}{(\kappa_t - 1)} - \sqrt{\left(\frac{\left(\kappa_t + \frac{1.5p_t}{2x_t}\right)}{(\kappa_t - 1)}\right)^2 - \frac{\left(\kappa_t - \frac{p_t}{2x_t} - \frac{3p_t^2}{2x_t^2}\right)}{(\kappa_t - 1)}}$$

where $\kappa_t = \frac{p_t r_{t+1} b}{2}$. The health expenditure share converges to (40a) as $t \to \infty$.

Proof. See Appendix A.1 □

3.4.2 Relative Prices, Qualities and the Capital Intensity

The aggregation results give rise to the relationship between relative prices and qualities in the two sectors from (16)–(17b) as

$$\frac{p_{ht}}{p_{ft}} = \left(\frac{q_{ft}}{q_{ht}}\right)^{1-\alpha}, \tag{41}$$

cf. Appendix A.2.1 for details on the derivation. The relationship is quite intuitive. If the quality (or productivity) of sector h improves relative to sector f, then relative prices of health goods are decreasing.

The relationship also implies that along the transition the ratio of unadjusted price indices (no adjustment for quality) is

$$\frac{p_{ht}q_{ht}}{p_{ft}q_{ft}} = \left(\frac{q_{ht}}{q_{ft}}\right)^{\alpha}.$$

Therefore, if the quality (or productivity) of sector h improves relative to sector f, then relative non-quality adjusted prices of health goods are increasing.

Also, observe from the equations for the wage rate and the interest rate, equations (17a) and (17b) that the capital intensities in the two sectors are identical. Hence, in all periods t the economy wide capital intensity satisfies $\left(\frac{k_t}{l_t}\right) = \left(\frac{k_{ht}}{l_{ht}}\right) = \left(\frac{k_{ft}}{l_{ft}}\right)$.

3.4.3 Economic Growth

Economic growth is driven by quality improvements. In each sector a deterministic (by the law of large numbers) fraction μ_{jt} of intermediates experiences a quality improvement of size λ, and the remaining fraction $1-\mu_{jt}$ has constant quality. Thus the growth rate of the quality index q_{jt} in each sector is given by

$$g_{jt} = \frac{q_{jt}}{q_{jt-1}} = \frac{(1-\mu_{jt})\, q_{jt-1} + \mu_{jt}\lambda q_{jt-1}}{q_{jt-1}} = 1 + (\lambda - 1)\mu_{jt}. \tag{42}$$

Note that given the timing assumptions whereas q_{jt-1} is predetermined in period t and thus a state variable, current technology q_{jt} can be controlled today via resources z_{jt} spent on innovation. Recall that the fraction of intermediate goods μ_{jt} that experience an innovation in each sector j is endogenous and given by equation (26). Using this in (42) gives

$$g_{jt} = 1 + (\lambda - 1)\varphi^{\frac{1}{1-\gamma}} \left[\gamma \left(\frac{p_{jt}\alpha^2}{r_t} \right)^{\frac{1}{1-\alpha}} \ell_{jt} \right]^{\frac{\gamma}{1-\gamma}}.$$

Hence, the growth rate of the quality index q_{jt} in sector j is a function of the relative price p_{jt}, the interest rate r_t and the (relative) market size of sector j, as measured by the working age population share employed in this sector, $\ell_{jt} = \frac{l_{jt}}{n_t}$.

3.5 Balanced Growth Path Analysis

We assume a stationary population in the balanced growth and therefore denote the young population by n^y. To characterize the BGP equilibrium, notice that, in general, the dynamics of this model are determined by the dynamics of four variables. On the household side, young today households choose how much to spend on health, i_t and how much to save, s_t. Innovation firms decide how much to spend on R&D which determines q_{jt} for both sectors $j \in \{f, h\}$. Thus the state variables at the beginning of period $t+1$ are $(i_t, s_t, q_{ft}, q_{ht})$. In a balanced growth path these variables will grow at a constant rate (alongside with wages w_t, transfers b_t etc.), and with relative prices $(r_t, p_t = \frac{p_{ht}}{p_{ft}})$ being constant. Let g be the BGP gross growth rate. Denote by

$$\begin{aligned} \tilde{x}_t &= x_t/g^t \\ x_t &= \tilde{x}_t g^t \end{aligned}$$

the growth-deflated variables. We now want to characterize a balanced growth path of this model prior to discussing the full dynamics of the model.

We have already established the properties of the household model in the

balanced growth path in Proposition 1. We now turn to the production side.

3.5.1 Production Side

In the balanced growth path the growth rates of technology in both sectors are given by:

$$\begin{aligned} g_h &= 1+(\lambda-1)\varphi^{\frac{1}{1-\gamma}}\left[\gamma\left(\frac{p\alpha^2}{r}\right)^{\frac{1}{1-\alpha}}\ell_h\right]^{\frac{\gamma}{1-\gamma}} \\ g_f &= 1+(\lambda-1)\varphi^{\frac{1}{1-\gamma}}\left[\gamma\left(\frac{\alpha^2}{r}\right)^{\frac{1}{1-\alpha}}\ell_f\right]^{\frac{\gamma}{1-\gamma}} \end{aligned}$$

Since in the long run both sectors have to grow at the same rate we have

$$\frac{\ell_f}{\ell_h} = p^{\frac{1}{1-\alpha}}$$

and exploiting the labor market clearing condition

$$\ell_f + \ell_h = \ell$$

where

$$\ell = \frac{n^y}{n} = \frac{n^y}{n^y(1+\psi(d(r,p)))} = \frac{1}{1+\psi(d(r,p))} \equiv \ell(r,p)$$

is the economy wide working age population ratio (we show that $\psi \equiv \psi(d(r,p))$ as part of the solution of the household problem below) we have

$$\begin{aligned} \ell_h &= \frac{1}{1+p^{\frac{1}{1-\alpha}}}\ell(r,p) \\ \ell_f &= \frac{p^{\frac{1}{1-\alpha}}}{1+p^{\frac{1}{1-\alpha}}}\ell(r,p). \end{aligned}$$

This implies that the relative price of health goods p and the interest rate r uniquely pin down the labor ratios ℓ_h, ℓ_f. Using this in the expression for growth in the two sectors we find that the balanced growth path growth rate

in the two sectors is given by

$$g = 1 + (\lambda - 1)\varphi^{\frac{1}{1-\gamma}} \left[\gamma \left(\frac{\alpha^2}{r} \right)^{\frac{1}{1-\alpha}} \frac{p^{\frac{1}{1-\alpha}}}{1 + p^{\frac{1}{1-\alpha}}} \ell(r,p) \right]^{\frac{\gamma}{1-\gamma}} \equiv g(r,p). \tag{43}$$

From the firms' optimality conditions and the production function the other growth-deflated variables in the BGP are given by:

$$\tilde{q}_f = \frac{\tilde{w} r^{\frac{\alpha}{1-\alpha}}}{\alpha^{\frac{2\alpha}{1-\alpha}} (1-\alpha)} \tag{44a}$$

$$\tilde{q}_h = \frac{\tilde{w} r^{\frac{\alpha}{1-\alpha}}}{\alpha^{\frac{2\alpha}{1-\alpha}} (1-\alpha) p^{\frac{1}{1-\alpha}}} \tag{44b}$$

$$\tilde{y}_f = \frac{\tilde{w}}{(1-\alpha)} \frac{p^{\frac{1}{1-\alpha}}}{1 + p^{\frac{1}{1-\alpha}}} n^y \tag{44c}$$

$$\tilde{y}_h = \frac{\tilde{w}}{(1-\alpha)} \frac{1}{p\left(1 + p^{\frac{1}{1-\alpha}}\right)} n^y \tag{44d}$$

$$\tilde{k}_f = \frac{\alpha^2 \tilde{w}}{(1-\alpha) r} \frac{p^{\frac{1}{1-\alpha}}}{1 + p^{\frac{1}{1-\alpha}}} n^y \tag{44e}$$

$$\tilde{k}_h = \frac{\alpha^2 \tilde{w}}{(1-\alpha) r} \frac{1}{\left(1 + p^{\frac{1}{1-\alpha}}\right)} n^y \tag{44f}$$

which are all just functions of the BGP prices $r, p, \tilde{w}$.

3.5.2 Household Side in the BGP

The demands for investment goods and savings come from the household problem and are given by

$$\tilde{i}(p, r, \tilde{w}, \tilde{T}) = \frac{\tilde{w} + \tilde{T}}{p} \theta(p, r) \tag{45a}$$

$$\tilde{s}(p, r, \tilde{w}, \tilde{T}) = \left(\tilde{w} + \tilde{T}\right) [1 - \theta(p, r)] \tag{45b}$$

cf. Proposition 1.

Next, we substitute out transfers from the above equations. From (5) we

have that in the BGP where $d = 1$ and $\gamma^n = 1$

$$Tg^{t+1} = (1 - \psi(1))rsg^t.$$

Since $\psi(1)$ is

$$\psi(1) = \frac{1}{2^\varrho},$$

we get:

$$\tilde{T} = \frac{(2^\varrho - 1)r}{2^\varrho}\tilde{s}(p, r, \tilde{w}) \tag{46}$$

We can use this in (45b) to get

$$\tilde{s}(p, r, \tilde{w}) = \frac{2^\varrho \left(1 - \theta(p, r, \tilde{w})\right)}{2^\varrho + (2^\varrho - 1)r \left(1 - \theta(p, r)\right)}\tilde{w}.$$

Using this again in (46) we get

$$\begin{aligned}\tilde{T}(r, p, \tilde{w}) &= \frac{(2^\varrho - 1)r}{2^\varrho}\frac{2^\varrho \left(1 - \theta(p, r, \tilde{w})\right)}{2^\varrho + (2^\varrho - 1)r \left(1 - \theta(p, r)\right)}\tilde{w} \\ &= \frac{(2^\varrho - 1)r \left(1 - \theta(p, r, \tilde{w})\right)}{2^\varrho + (2^\varrho - 1)r \left(1 - \theta(p, r, \tilde{w})\right)}\tilde{w}\end{aligned}$$

and, using this in (45a) we get

$$\tilde{\tilde{i}}(p, r, \tilde{w}) = \left(1 + \frac{(2^\varrho - 1)r \left(1 - \theta(p, r)\right)}{2^\varrho + (2^\varrho - 1)r \left(1 - \theta(p, r, \tilde{w})\right)}\right)\theta(p, r, \tilde{w})\frac{\tilde{w}}{p}.$$

We can therefore rewrite (45) as

$$\tilde{\tilde{i}}(p, r, \tilde{w}) = \left(1 + \frac{(2^\varrho - 1)r \left(1 - \theta(p, r)\right)}{2^\varrho + (2^\varrho - 1)r \left(1 - \theta(p, r, \tilde{w})\right)}\right)\theta(p, r)\frac{\tilde{w}}{p} \tag{47a}$$

$$\tilde{s}(p, r, \tilde{w}) = \frac{2^\varrho \left(1 - \theta(p, r)\right)}{2^\varrho + (2^\varrho - 1)r \left(1 - \theta(p, r)\right)}\tilde{w} \tag{47b}$$

3.5.3 BGP Equilibrium

Proposition 3. *In the BGP, the level of qualities in the two sectors $\tilde{q}_h$ and $\tilde{q}_f$ is not determined. The economy scales in absolute qualities.*

Proof of Proposition 3. Observe from Appendix A.2.1 that

$$w_t = (1-\alpha)\left(\frac{\alpha^2}{r_t}\right)^{\frac{\alpha}{1-\alpha}} p_{jt}^{\frac{1}{1-\alpha}} q_{jt} \tag{48}$$

hence, for given relative prices p_{jt} scaling qualities by some factor $\lambda > 0$ scales wages by factor λ. From (47) we observe that this scales the demands from the household sector by factor λ. From the equality of capital intensities in the two sectors, cf. Section 3.4.2, we observe that we can compute the capital intensity directly from the solution of the household model. We therefore have in any period t (also outside the BGP)

$$\frac{\tilde{k}_t}{l_t} = \frac{\tilde{s}_{t-1}(\cdot)/g \cdot n_t^y}{l_t} = \tilde{s}_{t-1}(\cdot)/g.$$

because $l_t = n_t^y$. By the equality of capital intensities in both sectors we therefore have

$$\frac{k_{ht}}{l_{ht}} = \tilde{s}_{t-1}(\cdot)/g.$$

Using results from Section 3.5.1 we can therefore compute the BGP capital stock in the health sector as

$$\begin{aligned}
k_h(p, r, \tilde{w}) &= \tilde{s}(p, r, \tilde{w})/g(r,p) l_h \\
&= \tilde{s}(p, r, \tilde{w})/g(r,p) \ell_h(r,p) n \\
&= \tilde{s}(p, r, \tilde{w})/g(r,p) \frac{1}{1+p^{\frac{1}{1-\alpha}}} \ell(r,p) n \\
&= \tilde{s}(p, r, \tilde{w})/g(r,p) \frac{1}{1+p^{\frac{1}{1-\alpha}}} \ell(r,p)(1+\psi(1)) n^y \\
&= \tilde{s}(p, r, \tilde{w})/g(r,p) \frac{1}{1+p^{\frac{1}{1-\alpha}}} n^y
\end{aligned} \tag{49}$$

where the last line follows because, in the BGP, $\ell(r,p) = \frac{1}{1+\psi(1)}$ and where $\tilde{s}\,(p,r,\tilde{w})$ follows from the household side, cf. Section 3.5.2. From this we observe that any scaling by the demands of households by factor λ will also scale capital in the economy by factor λ. Finally, observe from the homogeneity of aggregate production in the two sectors, cf. equation (16), that scaling both inputs k_{jt}, q_{jt} by factor λ under constant relative prices (and therefore constant labor) leads to scaling of output by factor λ. □

In consequence, we can normalize the absolute quality in the final goods sector to some arbitrary constant in the BGP, denoted $\bar{\tilde{q}}_f$. From equation (48) and recalling that we normalize $p_f = 1$ we can therefore express the level of wages in the BGP as a function of the interest rate r:

$$\tilde{w}(r) = (1-\alpha)\left(\frac{\alpha^2}{r}\right)^{\frac{\alpha}{1-\alpha}} \bar{\tilde{q}}_f \tag{50}$$

Therefore, demands in the household sector in equation (47) are also functions only of (p,r), denoted $\tilde{s}(p,r), \tilde{i}(p,r)$. Then, combining equations (49) and (44f) gives the equilibrium condition for the capital market,

$$\tilde{s}\,(p,r)\,/g(p,r) = \frac{\alpha^2 \tilde{w}(r)}{(1-\alpha)r} \tag{51}$$

and from (44d) we observe that investment demand for health goods satisfies

$$\tilde{i}\,(p,r) = \frac{\tilde{w}(r)}{(1-\alpha)} \frac{1}{p\left(1+p^{\frac{1}{1-\alpha}}\right)} n^y \tag{52}$$

where the respective demand equations $\tilde{s}\,(p,r)\,, \tilde{i}\,(p,r)$ are given in (47). These two equations uniquely determine the equilibrium (noticing that the equilibrium in the final goods market is implied by Walras' law).

We describe a (slightly modified) fixed-point solution algorithm to determine market clearing prices and interest rates in Section B.1.1 of the Appendix.

3.6 Dynamics

Characterization of the dynamics of the economy

1. On the household side we have a representative household per generation, so no question that in period t savings and health investment of the current old generation s_{t-1}, h_{t-1} fully characterizes the aggregate state of the household sector.

2. On the firm side the cross-sectional distribution over q_{jit-1} is not a state variable. The law of motion for q_{jt-1} is given by

$$q_{jt} = (1 - \mu_{jt})q_{jt-1} + \mu_{jt}\lambda q_{jt-1} \tag{53}$$

   The significance of this result is that we do not keep track of the cross sectional quality distribution in each sector, but rather only of the two aggregate indices q_{ft}, q_{ht}.

   Conjecture 1. *There exists a competitive equilibrium in which aggregate allocations do not depend on the initial cross sectional distributions.*

   Proof. See appendix □

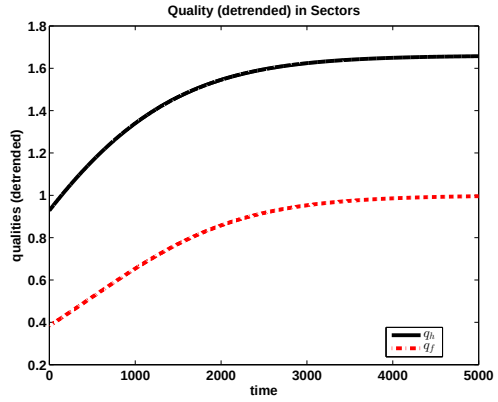
3. Under the previous result the state variables of the system are $(q_{t-1}, q_{ht-1}, s_{t-1}, h_{t-1})$.

3.7 Illustration of the Simple Model

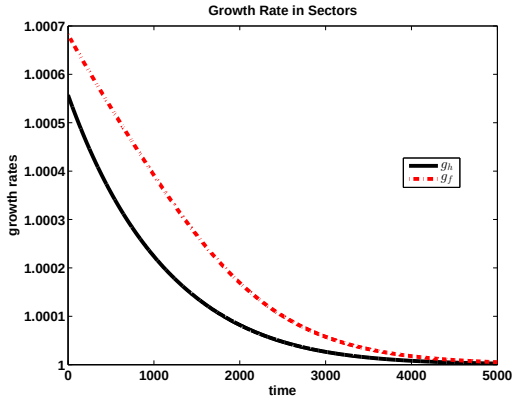
For an illustration of the simple model, we set the capital share in production to $\alpha = 0.33$, and parameterize the innovation probability in equation (21) such that the gross growth rate of quality improvements is $\lambda = 1.015^{40}$ (i.e., assuming annual growth rate of 1.5% and interpreting each model period as lasting for 40 years), and the productivity and curvature are set to $\varphi = \gamma = 0.5$. Initial deficits are set to $d_0 =$ and the value of life parameter $b = 100$. The detrended value of the BGP quality in the final goods sector is normalized to $\bar{\tilde{q}}_f = 1$. Results are presented in Figure 2.

Figure 2: Simple Model: Illustration of Detrended Variables in Transition

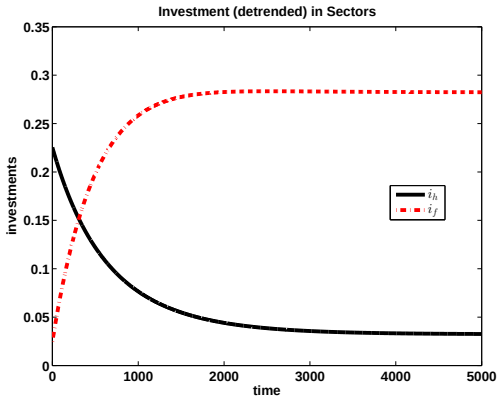
(a) Qualities



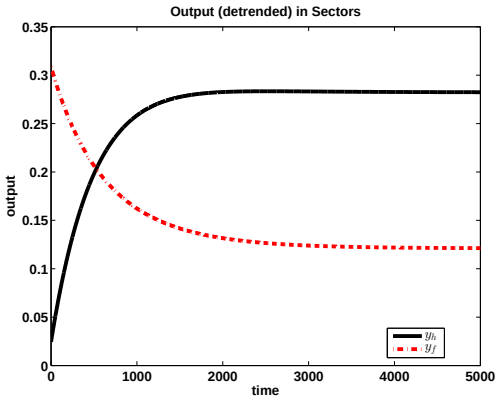
(b) Growth Rates



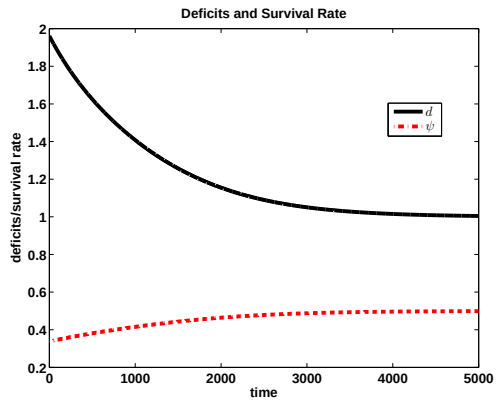
(c) Investment



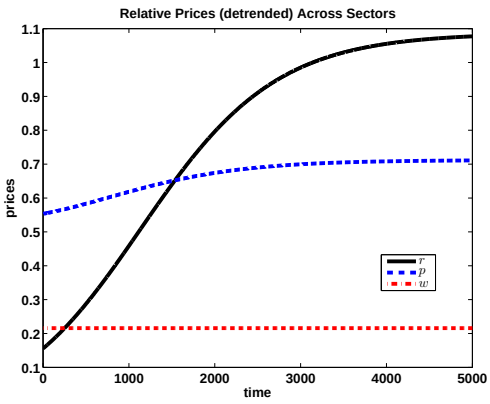
(d) Output



(e) Deficits and Survival Rates



(f) Relative Prices



Notes: Illustration of Simple Model. Detrending with BGP growth rate of $g = 1.001234$.

4 Multi-Period Household Model

We extend the household model to a multi-period structure. For sake of parsimony, we let health only influence the survival rate as well as retirement decisions by modeling stochastic retirement. We take the setup of Dalgaard and Strulik (2014). Accordingly, instead of modeling a health shock as in the standard Grossman model, we take the notion of accumulating deficits over a household's life-cycle.

4.1 Preferences and Survival Risk

A household born in period k (cohort k) values consumption over the life cycle according to

$$\sum_{t=0}^{\infty} \beta^t \left[\prod_{s=1}^{t} \psi(d_s) \right] u\left(c_{ks}\right)$$

where $\psi(d_s)$ is the survival rate depending on the accumulated deficits at date s where $d \geq d_0$ for initial deficits d_0. As in the simple model, we assume that $\psi(d) \in (0,1), \psi_d(d) < 0, \psi_{dd}(d) > 0, \lim_{d\to\infty} \psi(d) = \bar{\psi} < 1$. This latter feature means that the model collapses to a perpetual youth model with constant survival risk in the limit. Descendants in each period t are replaced by newborn cohorts of some size so that the model is consistent with data on population growth.

4.2 Budget Constraint

Households either work for some exogenous period t wages w_t or are retired and earn a lump-sum pension income b_t^p. Income is subject to health contributions at rate τ_t. Labor income is subject to contributions to a PAYG financed social security system where the contribution rate is denoted by τ_t^p.

Introducing the indicator variables $\mathbf{1_r}$ with $\mathbf{1_r} = 1$ being retirement, income writes as

$$y_t = (1-\tau_t) \cdot ((1-\mathbf{1_r}) w_t (1-\tau_t^p) + \mathbf{1_r} b_t^p) = (1-\tau_t) y_t^g$$

where $y_t^g = (1 - \mathbf{1_r})w_t(1 - \tau_t^p) + \mathbf{1_r}b_t^p$ is gross income after contributions to social security.

Transition into retirement is exogenous and depends on accumulated health deficits d. We model this through a probability $\pi(d)$ where $\pi(d)$ denotes the probability to retire with $\pi_d > 0$, and $\pi_{dd} \geq 0$ at least initially, i.e., for relatively low d (see calibration below).

Households start each period with holdings of physical assets a and a stock of accumulated deficits, d. They invest in a risk-free bond that earns interest r. Households also invest in health, ϕi, units at relative price p_t where ϕ is a copayment factor to be specified below ($1 - \phi$ of health investments i are covered by the insurance company). We assume a borrowing constraint so that assets next period, a', cannot be negative. Accordingly, the dynamic budget constraint writes as (recall that r denotes the gross interest rate)

$$a' = ar_t + y_t - (c + p_t\phi i) \geq 0.$$

4.3 Accumulation of Deficits

In contrast to the standard literature we do not model a health stock that depreciates as the agent gets older. Instead, we model a stock of health-deficits d that increases over time. This is done because on the one hand it is more realistic to loosen the relationship between the chronological age and the aging process (increase in frailty) of an agent on the other side it circumvented some unrealistic model-implications that are part of the standard model.

So at the beginning of his life an agent has a certain amount of initial deficits $d_0 > 0$. These deficits increases from one period to the next at mean rate $\delta > 0$. The process is accordingly given by

$$d' = (1 + \delta)d - f(i, d) \equiv g(i, d) \tag{54}$$

As in the simple model, we restrict the deficit production function by assuming that $f_i(i, d) > 0$, $f_{ii}(i, d) < 0$, $f(i = 0, d = 0) = 0$, $\lim_{i\to\infty} f(i, d) < \delta d$.

4.4 Health Insurance: Institutional Features and Modelling Approach

Aiming at approximating the U.S. system, there are three basic ways to obtain health insurance during working ages.

4.4.1 Insurance Options

First, most people have employer-provided health insurance. The key features of this system are that part of the premium is paid by the employer, and this part of the worker's compensation is not subject to income taxation. Thus employer-provided health insurance enjoys a tax advantage. The other feature is that the health insurance premium cannot depend on observables such as age k and health status (as proxied by d in our model).

The second alternative is private insurance, which has no tax advantages and a premium that depends on d.

The third option is Medicaid, which is a tax-financed, income (and asset) means-tested public health insurance program. The fourth option is to have no insurance.

4.4.2 Access

Access: whether to access Medicaid or private insurance is a household choice. For employer-provided insurance, the literature typically models this as an exogenous, possibly age (and health) dependent probability $\pi(d)$ of receiving an employer-sponsored health insurance contract. Ideally one would model this as a choice in combination with an employment choice (this is what Brueggemann and Manovskii do), but this leads us too far astray. The access to Medicaid can be modeled as a cash-at-hands means test: households with $x \leq \bar{x}_M$ have access to Medicaid.

4.4.3 Contracts

The way Jung and Tran model insurance contracts is that there is perfect competition in the insurance provision market, firms charge a premium over cost C_i that depends on the insurance option (i stands for group, individual insurance or Medicaid, where group insurance pools across types and Medicaid is free to households but tax-financed) and the health insurance pays out a fraction $(1-\phi_i)$ of the health expenditures, with ϕ_i representing the co-pay rate (the fraction the household has to bear). Nonlinear contracts with a cap could also be considered. Note that these contracts are subject to an ex-post moral hazard problem: given that the health insurance company pays for most [$(1-\phi_i)$ fraction] of the expenditures, households do not internalize the full cost of health expenditures and thus over-consume health.

4.4.4 Our Modelling Approach

Households have access to public, tax-financed health insurance with co-pay and perhaps a cap. Health insurance providers break even (perhaps subject to an administrative markup). We denote the copay ratio by ϕ. For total health investments i the private expenditures are therefore ϕpi where p is the relative price of health. $(1-\phi)pi$ is the amount paid by a tax financed health insurance.

4.5 Government Budget Constraints

As described below in the solution of the household model, policy functions of health spending are stored on the continuous state variables assets and deficits a, d as well as the discrete indicator variable $\mathbf{1_r}$. Accordingly, denote by $i(a, d, \mathbf{1_r})$ the optimal health spending policy function, and by $d'(\cdot)$ the according deficit function and by $a'(\cdot)$ the saving function. Furthermore, denote by $\Phi_t(\cdot) = \Phi_t(a, d, \mathbf{1_r})$ the cross sectional distribution of households alive in period t.[2] Accidental bequests arising because of prematurely death enter the

[2] $\Phi_t(\cdot)$ is not a distribution in the strict sense because it varies with the size of the population to the effect that $\frac{1}{n_t}\int \Phi_t(\cdot) = 1$.

health insurance budget. The tax rate then has to satisfy

$$\tau_t \int y_t^g d\Phi_t(\cdot) + \int a'(\cdot)(1-\psi(d'(\cdot)))d\Phi_t(\cdot) = (1-\phi)\int p_t i(a,d,\mathbf{1_r})d\Phi_t(\cdot)$$

and thus

$$\tau_t = \frac{(1-\phi)\int p_t i(a,d,\mathbf{1_r})d\Phi_t(\cdot) - \int a'(\cdot)(1-\psi(d'(\cdot)))d\Phi_t(\cdot)}{\int y_t^g d\Phi_t(\cdot)} \tag{55}$$

The tax rate is a simple function of the aggregate share of (gross) income devoted to health spending. The parameter ϕ nicely varies the size of the system, with $\phi = 1$ corresponding to the situation with no health insurance, and $\phi = 0$ corresponding to the situation where all health risks are fully insured. Of course the latter parameterization will have no solution as $i = \infty$ is the optimal household choice, an extreme version of the ex-post moral hazard problem identified by Jung and Tran.

The social security system also runs a balanced budget. Pension payments are lump-sum and denoted by b^p. We relate pension payment to average net wages by defining the net replacement rate ρ^p. Accordingly, pension payments are given by $b^p = \rho^p w(1-\tau^p)$. From the pension budget constraint

$$\tau_t^p w_t \int d\Phi_t(\cdot, \mathbf{1_r} = 0) = b_t^p \int d\Phi_t(\cdot, \mathbf{1_r} = 1)$$

we therefore get

$$\tau_t^p = \frac{\rho_t^p \int d\Phi_t(\cdot, \mathbf{1_r} = 1)}{\int d\Phi_t(\cdot, \mathbf{1_r} = 0) + \rho_t^p \int d\Phi_t(\cdot, \mathbf{1_r} = 1)}. \tag{56}$$

4.6 Dynamic Problem

Given the continuous endogenous state variables a, d and the discrete exogenous state variable $\mathbf{1_r}$, the Bellman equation reads as

$$V_t(a,d,\mathbf{1_r}) = \max_{c,i,a',d'} \left\{ u(c) + \beta\psi(d')\bar{V}_{t+1}(a',d',\mathbf{1_r}') \right\}$$

where

$$\bar{V}_{t+1}(x', d', \mathbf{1_r}') = \begin{cases} (1-\pi(d'))V_{t+1}(x', d', 0) + \pi(d')V_{t+1}(x', d', 1) & \text{for } \mathbf{1_r} = 0 \\ V_{t+1}(x', d', 1) & \text{for } \mathbf{1_r} = 1 \end{cases}$$

with constraints

$$\begin{aligned} a' &= ar_t + y_t - c - p_t\phi i \geq 0 \\ d' &= g(i, d) \\ i &\geq 0 \end{aligned}$$

where

$$y_t = (1 - \mathbf{1_r})w_t(1 - \tau_t - \tau_t^p) + \mathbf{1_r}b_t^p(1 - \tau_t)$$

is income in period t.

4.7 Definition of Equilibrium

Definition 2. *Let $d \in \mathcal{D}$ and $a \in \mathcal{A}$ and $\mathbf{1_r} \in \mathcal{I}$. Given an initial distribution of households, $\Phi_0(a, d, \mathbf{1_r})$, initial capital stocks and qualities in the two sectors, $\{k_{j0}, q_{j0}\}_{j\in\{f,h\}}$, a competitive equilibrium is a sequence of household policy functions $\{c_t(a, d, \mathbf{1_r}), i_t(\cdot), d_t'(\cdot), a_t'(\cdot)\}_{t=0}^{\infty}$, a sequence of capital and labor inputs of goods producers $\{\{k_{jt+1}, l_{jt}\}_{j\in\{f,h\}}\}_{t=0}^{\infty}$, a sequence of R&D expenditures, profits and consumption of R&D developers $\{\{z_{jt}, \pi_{jt}, c_{jt}\}_{j\in\{f,h\}}\}_{t=0}^{\infty}$, a sequence of aggregate capital and technology $\{k_{t+1}, \{q_{jt+1}\}_{j\in\{f,h\}}\}_{t=0}^{\infty}$, prices $\{p_t, w_t, r_t\}_{t=1}^{\infty}$, and contribution rates to social insurance $\{\tau_t, \tau_t^p\}_{t=1}^{\infty}$ such that*

1. *Household maximization: for each $t \geq 1$, given prices and contribution rates $w_t, p_t, r_{t+1}, \tau_t, \tau_t^p$, and pension payments $b_t^p = \rho_t^p w_t(1 - \tau_t^p)$ the policy functions $\{c_t(\cdot), i_t(\cdot), d_t'(\cdot), a_t'(\cdot)\}_{t=0}^{\infty}$ are the solutions to the dynamic problem stated in Section 4.6.*

2. *Contribution rates to health and pension insurance, τ_t, τ_t^p satisfy equa-*

tions (55) and (56), respectively.

3. *Optimal R&D spending z_{jt} in each sector is given by (27) and consumption of R&D entrepreneurs is determined as $c_{jt} = \pi_{jt} - z_{jt}$.*

4. *Factor prices satisfy equations (17a) and (17b).*

5. *The equilibrium innovation intensity μ_{jt} is given by equation (26) and technology in each sector evolves according to*

$$q_{jt} = (1 - \mu_{jt})q_{jt-1} + \mu_{jt}\lambda q_{jt-1} \tag{57}$$

6. *Aggregate saving, health investment and household consumption are given by*

$$\begin{aligned} A'_t &= \int a'_t(a, d, \mathbf{1_r}) d\Phi(a, d, \mathbf{1_r}) \\ I_t &= \int i_t(a, d, \mathbf{1_r}) d\Phi(a, d, \mathbf{1_r}) \\ C_t &= \int c_t(a, d, \mathbf{1_r}) d\Phi(a, d, \mathbf{1_r}) \end{aligned}$$

7. *Markets clear: for all $t \geq 1$:*

   (a) *Labor Market*

$$l_t = \sum_j l_{jt} \tag{58}$$

   (b) *Capital Market*

$$k_{t+1} = \sum_j k_{jt+1} = A'_t \tag{59}$$

   (c) *Final Goods Market*

$$A'_t + C_t + \sum_j [c_{jt} + z_{jt}] = k_{ft}^{\alpha} \left(q_{ft} l_{ft}\right)^{1-\alpha} \tag{60}$$

(d) Health Goods Market

$$I_t = k_{ht}^{\alpha} \left(q_{ht} l_{ht}\right)^{1-\alpha} \tag{61}$$

8. *Cross-sectional measures evolve according to*

$$\Phi_{t+1}(\mathcal{A} \, x \, \mathcal{D} \, x \, \mathcal{I}) = \int H_t \left((a, d, \mathbf{1_r}) , \mathcal{A} \, x \, \mathcal{D} \, x \, \mathcal{I} \right) \Phi_t \left(a, d, \mathbf{1_r}\right)$$

for all sets $\mathcal{A}, \mathcal{D}, \mathcal{I}$, where the Markov transition functions H_t are induced by an exogenous fertility process and the endogenous processes for asset accumulation, health deficits and stochastic retirement, $a', d', \mathbf{1_r}'$. They are given by

$$H_t \left(\mathcal{A} \, x \, \mathcal{D} \, x \, \mathcal{I}\right) = \begin{cases} \psi(d'(\cdot)) & \text{if } a'(\cdot) \in \mathcal{A}, d' \in \mathcal{D}, \mathbf{1_r}' \in \mathcal{I} \\ 0 & \text{else.} \end{cases}$$

For period t newborns the initial distribution is given as

$$\Phi_t(\{1\} \, x \, \mathcal{D} \, x \, \mathcal{I}) = \begin{cases} n_t^0 & \text{if } 0 \in \mathcal{A} \\ 0 & \text{else.} \end{cases}$$

where n_t^0 is the initial young generation.

4.8 Functional Forms

Utility. Let

$$u(c) = \frac{c^{1-\sigma}}{1-\sigma} + b \tag{62}$$

for some $b > 0$.

Survival Rate. We assume that

$$\psi(d) = \frac{1}{(1+\chi d)^{\varrho}} \tag{63}$$

Hence:

$$\psi_d(d) = -\varrho\chi \frac{1}{(1+\chi d)^{1+\varrho}} \tag{64a}$$

$$\frac{\psi_d(d)}{\psi(d)} = -\varrho\chi \frac{1}{1+\chi d} \tag{64b}$$

Accumulation of Health Deficits. Let

$$f(i,d) = \delta d \left(1 - \frac{1}{(1+\zeta i)^{\xi}}\right)$$

so that

$$d' = g(i,d) = d\left(1 + \delta \frac{1}{(1+\zeta i)^{\xi}}\right) \tag{65}$$

is the deficit accumulation function with initial deficit stock $d_0 > 0$. $\delta > 0, \zeta > 0, \xi > 0$ are parameters.

We accordingly get

$$g_i(i,d) = -d_0\delta\xi\zeta \frac{1}{(1+\zeta i)^{1+\xi}} \tag{66a}$$

$$g_d(i,d) = \left(1 + \delta \frac{1}{(1+\zeta i)^{\xi}}\right) \tag{66b}$$

$$\frac{g_d}{g_i} = -\frac{1}{d\delta\zeta\xi}(1+\zeta i)\left(\delta + (1+\zeta i)^{\xi}\right) \tag{66c}$$

Retirement Probability. For the probability to work, $1 - \pi(d)$, we specify a Weibull survival function. Accordingly the retirement probability is given

by 1 minus the Weibull survival function,

$$\pi(d) = 1 - \exp(-\upsilon(d - d_0)^{\nu}). \tag{67}$$

Observe that $\pi(d = d_0) = 0$, $\lim_{d\to\infty} \pi(d) = 1$.

4.9 Properties of the BGP

We assume that the economy is in a BGP from some period 0 on. In the BGP where $i_t = i_0 g^t$, $c = c_0 g^t$, hence health investment will be positive ($\mu^i = 0$).

In the BGP, the survival rate is constant at

$$\psi(d) = \frac{1}{(1 + \chi d_0)^{\varrho}} < 1$$

The retirement probability is equal to zero, cf. equation (67) so that there are only workers in the economy.

Proposition 4. *As in the simple model, a BGP exists if* $\sigma = 1 + \xi$.

Proof. See Appendix A.1. □

Proposition 5. *Let* $g_c \equiv (\beta\psi(d_0)r)^{\frac{1}{\sigma}} = \frac{c_{t+s+1}}{c_{t+s}}$ *be the growth rate of cohort* t *consumption in period* $s \geq 0$ *in the steady state. If* $\psi(d_0)g_c < 1 < g < g_c < r$ *then the borrowing constraint will not be binding in the BGP. The solution of the household problem in the BGP gives period* t *consumption and investment in health of cohort* t *as*

$$c_t = \frac{1}{1 + \iota(p; d_0, \delta, \xi, \varrho, \chi, \phi, \zeta)^{-1}} \frac{r - g_c}{r - g} y_t \tag{68}$$

and

$$p\phi i_t = \frac{1}{1 + \iota(p; d_0, \delta, \xi, \varrho, \chi, \phi, \zeta)} \frac{r - g_c}{r - g} y_t \tag{69}$$

where

$$\iota(p; d_0, \delta, \xi, \varrho, \chi, \phi, \zeta) = \left(\frac{\zeta}{p\phi}\right)^{\frac{\xi}{1+\xi}} \left(\frac{(1-\beta\psi(d_0))^2}{d_0\delta\xi\varrho\chi\psi(d_0)^{\frac{1}{\varrho}}}\right)^{\frac{1}{1+\xi}} = \frac{c}{p\phi i} \tag{70}$$

is the constant consumption to health expenditure ratio (constant across all cohorts and periods of life of a cohort). From the Euler equation for consumption we get consumption of cohort t in any period $s \geq t$ as

$$c_s = \left((\beta\psi(d_0)r)^{\frac{1}{\sigma}(s-t)}\right) c_t. \tag{71}$$

The growth rate of assets $g_{a,t+s} = \frac{a_{t+s+1}}{a_{t+s}}$ converges from above to g_c. [we still have to establish the monotone convergence] These equations uniquely pin down the solution of the household problem.

5 Calibration

[TBC]

6 Results

[TBC]

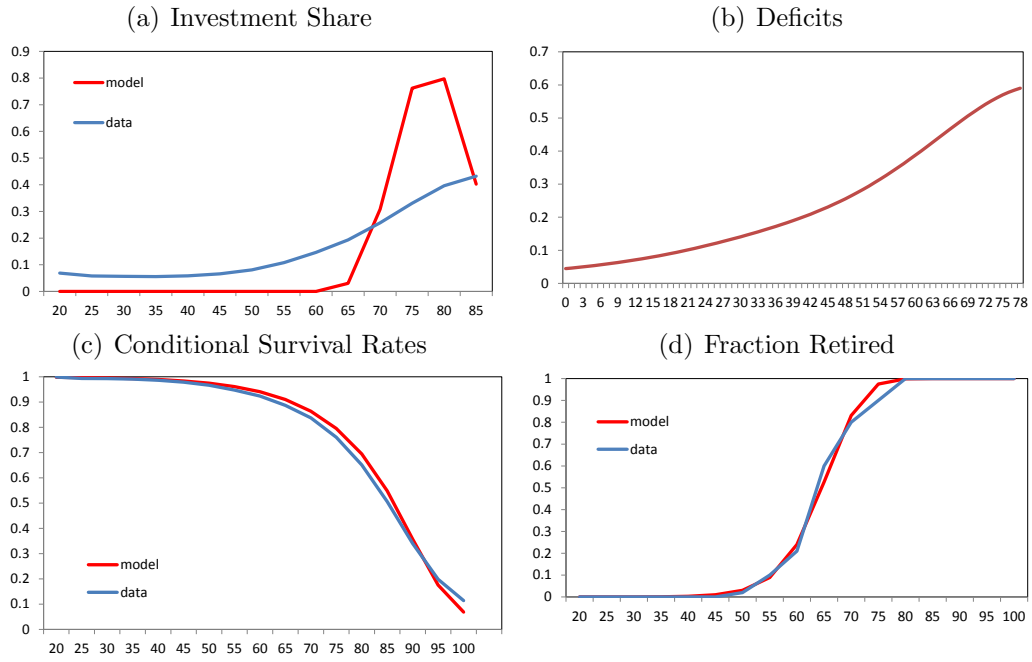
6.1 Preliminary Life-Cycle Profiles

Figure 3 shows life-cycle profiles for a very loose and preliminary calibration of the model.

7 Conclusion

[TBC]

Figure 3: Quantitative Model: Life-Cycle Profiles



Notes: Life-cycle profiles in quantitative model.