Unemployment, Aggregate Demand, and the Distribution of Liquidity

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Abstract

We develop a New-Monetarist model of unemployment in which distributional considerations matter. Households who lack commitment are subject to both employment and expenditure risk. They self-insure by accumulating real balances and, possibly, claims on firms profits. The distribution of liquidity is endogenous and responds to idiosyncratic risks and monetary policy. Despite the ex-post heterogeneity our model can be solved in closed form in a variety of cases. We show the existence of an aggregate demand channel according to which the distribution of workers across employment states, and their incomes in those states, affects the distribution of liquid wealth and firms' profits. An increase in unemployment benefits or wages has a positive effect on aggregate demand and can lead to higher employment. Moreover, an increase in productivity has a multiplier effect on firms' revenue.

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1 Introduction

We develop a New-Monetarist model of unemployment in which liquidity and distributional considerations matter. Our approach is motivated by the strong empirical evidence that household heterogeneity in terms of income and wealth, together with liquidity constraints, affect aggregate consumption and unemployment (e.g., Mian and Sufi, 2010?; Carroll et al., 2015). There is a recent literature that formalizes search frictions and liquidity constraints in goods markets that reduce the ability of firms to sell their output, their incentives to hire, and ultimately the level of unemployment (e.g., Berentsen, Menzio, and Wright, 2010; Michaillat and Saez, 2015). While these models capture some demand considerations that are absent from the canonical model of unemployment, they make assumptions, or focus on equilibria, where the distribution of (liquid) wealth is degenerate. In contrast, we unharness the ex-post heterogeneity that emerges naturally in New-Monetarist environments in the presence of both idiosyncratic expenditure risk and employment risk. Our model remains tractable and classes of equilibria are solved in closed form. A key feature of the model is that the endogenous distribution of liquidity across households responds to idiosyncratic risks and monetary policy. We use our model to investigate qualitatively and quantitatively the effects of labor market and monetary policies.

Our model is a two-good version of a Bewley (1980, 1983) economy with a frictional labor market. We interpret the two goods as early and late consumption where the preferences over the goods are driven by random shocks. Because of lack of commitment, households must accumulate assets to finance their early consumption, in spirit to the New Monetarist literature, e.g., Berentsen, Menzio, and Wright (2010), BMW thereafter. The relative price between the two goods affects firms' profits and it depends on the distribution of asset holdings, which is our aggregate demand channel. There is a frictional labor market where workers and jobs are matched according to a time-consuming process and workers receive random opportunities to consume early before they receive their income. As a result, agents accumulate liquidity in order to self-insure against idiosyncratic income and expenditure risks. BMW focus on equilibria where the income of the unemployed is sufficiently large so that all workers can accumulate their desired liquidity in a single period, thereby making the distribution of money holdings degenerate. Value functions are linear in terms of workers' income, which makes the income risk due to unemployment irrelevant. In contrast, we focus on equilibria where it takes time (several periods) for workers to reach their targeted holdings of liquid assets so that the distribution of money holdings is non-degenerate. Value functions are strictly concave so that the employment risk is relevant.

The expenditure and income risks generate ex-post heterogeneity in terms of individuals' histories and, in general, a non-degenerate distribution of wealth. Despite this heterogeneity our model is tractable both analytically and numerically and it can be solved in closed form in a variety of cases. We first focus on the class on equilibria where workers spend all their liquid assets whenever they receive an opportunity to consume early. These equilibria with full depletion of real balances are highly tractable and deliver tight analytical results. The main insight is the existence of an aggregate demand channel according to which the distribution of workers across employment states, and their incomes in those states, affects the distribution of money holdings and the expected revenue of firms. An increase in unemployment benefits has a positive effect on aggregate demand and firms' expected revenue and can lead to higher employment provided that wages are not too elastic to unemployment benefits. Similarly, an increase in wages stimulates aggregate demand through the distribution of liquidity, which can raise firms' revenue. As a result, an exogenous increase in productivity has a multiplier effect on firms' revenue. Our model also predicts that the surplus from being employed decreases with workers' liquid wealth, i.e., the poorest workers have the highest incentives to find a job.

1.1 Literature

Our paper is related to the literature on unemployment and money, e.g., Shi (1998). Our model has a similar structure as in BMW that extends the quasi-linear environment of Lagos and Wright (2005) to include a frictional labor market.[1] BMW restrict their attention to equilibria with degenerate distribution of money holdings. In such equilibria agents, irrespective of their labor status, have enough income to reach their targeted holdings of liquid assets in a single period. In contrast, we consider a larger set of equilibria, including those where agents' choice of liquid assets is constrained by their income, which leads to a non-degenerate distribution of money holdings. Agents' behavior to accumulate real balances is analogous to the

[1] In Rocheteau, Rupert, and Wright (2007) only the goods market is subject to search frictions but unemployment emerges due to indivisible labor. The BMW model has been extended to incorporate credit and various forms of liquidity, e.g., Bethune, Rocheteau, and Rupert (2015) and Branch, Petrosky-Nadeau, and Rocheteau (2016).

one in Rocheteau, Weill, and Wong (2015a,b).

There are other models with search-matching frictions in both labor and goods markets, e.g., Michaillat and Saez (2015) and Petrosky-Nadeau and Wasmer (2015). Michaillat and Saez (2015, 2016) also find that an increase in unemployment benefits or wages can have a positive effect on aggregate demand and reduce unemployment. These models, however, do not have an explicit role for liquidity and they do not explain the distribution of liquidity and its relation to firms' productivity. For instance, Michaillat and Saez (2016) simply assume that a positive inflation rate is optimal. In contrast, our model is explicit about the monetary policy trade-off between enhacing the rate of return of money and providing risk sharing (Wallace, 2015).

Search-theoretic models of monetary exchange with non-degenerate distributions of money holdings include Molico (2006), Green and Zhou (1998), Chiu and Molico (2010, 2011), and Menzio, Shi, and Sun (2013). These models have random-matching shocks in the goods market but do not have income shocks and do not incorporate a frictional labor market.

Our model can be interpreted as a two-sector Bewley (1980, 1983) economy that incorporates income and preference shocks and that has fiat money as the only asset for self-insurance. For instance, Hansen and Imrohoroglu (1992) adopt a Bewley economy to study optimal unemployment insurance with exogenous employment shocks. Algan, Challe, and Ragot (2011) study temporary and permanent changes in money growth in a Bewley economy. Relative to this literature we endogenize the frequency of income shocks by adding a frictional labor market. Krusell, Mukoyama, and Sahin (2010) consider a Mortensen-Pissarides model where agents can self-insurance against income shocks by accumulating capital and claims on firms' profits. Eeckhout and Sepahsalari (2015) study a similar model with heterogeneous firms and directed search. They show that workers with low asset holdings direct their search toward low productivity jobs. In our model workers are subject to random matching shocks in both the goods and the labor markets and can self-insure by accumulating fiat money. Moreover, because retailers have some market power the distribution of liquidity across workers affects the endogenous productivity of jobs. From a methodological viewpoint, our model is analytically tractable and can be solved in closed form for a wide class of equilibria.

Debate on unemployment benefits and unemployment. See Marinescu (2016) and related literature.

2 Environment

The economy is composed of a unit measure of workers and a large measure of entrepreneurs. Time is discrete and is indexed by $t \in \mathbb{N}$. Each period of time is divided into three stages. The first stage is a frictional labor market where unemployed workers search for vacant jobs. The second and third stages have sequential competitive markets where agents can trade goods and money. We take the late-consumption good traded in the third stage as the numéraire.

Figure 1: Timing of a representative period

The lifetime utility function of a worker is

$$\mathcal{U}^W = \mathbb{E}\sum_{t=0}^{\infty} \beta^t \left[\varepsilon_t u(y_t) + (1-e_t)\ell + c_t\right], \tag{1}$$

where $\beta = (1+\rho)^{-1} \in (0,1)$ is a discount factor, $y_t \in \mathbb{R}_+$ is the worker's early (second-stage) consumption, $\varepsilon_t \in \{0,1\}$ is a binary preference shock, $c_t \in [\underline{c}_e, \bar{c}_e]$ is the worker's late (third-stage) consumption, ℓ is the utility of leisure, and $e_t \in \{0,1\}$ is the worker's supply of indivisible labor. We interpret $\underline{c}_e$ as a subsistance level and $\bar{c}_e$ as a satiation point. The utility function, $u(y_t)$, is twice continuously differentiable, strictly increasing, and concave, with $u(0) = 0$, $u'(0) = \infty$, and $u'(\infty) = 0$. Preferences shocks, $\{\varepsilon_t\}_{t=0}^{+\infty}$, are i.i.d. across agents and time with $\Pr[\varepsilon_t = 1] = \alpha$ and $\Pr[\varepsilon_t = 0] = 1 - \alpha$. Each worker is endowed with one indivisible unit of labor. Labor services are specialized and are traded through a search process described below. Specialization is such that workers do not consume the early-consumption good that they contribute to produce. The utility of an entrepreneur is

$$\mathcal{U}^E = \mathbb{E}\sum_{t=0}^{\infty} \beta^t c_t. \tag{2}$$

Entrepreneurs are risk neutral and they do not value early consumption.

A job is a technology to produce the second-stage good (early consumption), y, and the numeraire, q, with one unit of labor as the only input. This techonology is represented by the production-possibility frontier, $q = Q(y)$, that specifies the amount of numeraire a filled job can produced if it has already produced y units of the second-stage good. The production-possibility frontier satisfies $Q(0) = \bar{q} > 0$, $Q(\bar{y}) = 0$ for some

$\bar{y} > 0$, $Q'(y) < 0$, $Q''(y) < 0$, $Q'(0) = 0$, and $Q'(\bar{y}) = -\infty$. We define the opportunity cost of producing y as $\kappa(y) \equiv Q(0) - Q(y)$. Hence, $\kappa(0) = 0$, $\kappa'(y) > 0$, $\kappa''(y) > 0$, $\kappa'(0) = 0$, and $\kappa'(\bar{y}) = +\infty$. We will denote p the real price of early consumption in terms of the numeraire.

Figure 2: Production-possibility frontier of a filled job

In order to create a job in period t, entrepreneurs must open a vacant position, which costs $k > 0$ in terms of the numeraire in $t-1$. The measure of matches between vacant jobs and unemployed workers in period t is given by $m(s_t, o_t)$, where s_t is the measure of job seekers and o_t is the measure of job openings. The matching function, m, has constant returns to scale, and it is strictly increasing and strictly concave with respect to each of its arguments. Moreover, $m(0, o_t) = m(s_t, 0) = 0$ and $m(s_t, o_t) \leq \min(s_t, o_t)$. The job finding probability for a worker is $\lambda_t = m(s_t, o_t)/s_t = m(1, \theta_t)$ where $\theta_t \equiv o_t/s_t$ is referred to as labor market tightness. The vacancy filling probability for a job is $m(s_t, o_t)/o_t = m(1/\theta_t, 1) = \lambda_t/\theta_t$. The employment (measured after the matching phase at the beginning of the second stage) is denoted n_t and the economy-wide unemployment rate (measured after the matching phase) is u_t. Therefore, $u_t + n_t = 1$. An existing match is destroyed at the beginning of a period with probability δ. A worker who lost his job in period t becomes a job seeker in period $t+1$. Therefore, $u_t = s_{t+1}$. An employed worker in period t receives a wage in terms of the numéraire good, w_1, paid in the last stage. An unemployed workers enjoys w_0 interpreted as unemployment benefits.

In order to generate a role for liquid assets we assume that workers are anonymous and cannot commit to repay their debt. Hence, they cannot finance consumption or asset purchases with IOUs. Claims on jobs' profits can only be traded among entrepreneurs.[2] Fiat money, which is perfectly divisible, storable, and non-counterfeitable, can be traded among all agents. Its supply, M_t, grows as the constant rate π. For now we assume that the proceeds of money creation are given to entrepreneurs in a lump-sum fashion.

Discussion

We now discuss some key assumptions of the model. We assume that the utility in terms of c is linear. This formulation is very tractable, it admits several canonical models as special cases, and it will allow us to

[2] Rocheteau and Rodriguez (2014) study a version of the model where claims on firms' profits can serve as media of exchange.

characterize classes of equilibria in closed form. For instance, environments where α is high and $c \geq \underline{c}^e$ binds most of the time resemble one-sector Bewley models with income risks. In contrast to Pissarides (2000) and Lagos and Wright (2005), the linearity of preferences over c does not imply that agents are neutral with respect to the risk associated with their labor income. Agents have strictly concave preferences in terms of their early consumption, y, and they might be constrained by their labor income when choosing their holdings of liquid assets. As a result, we will show that value functions are strictly concave over some range and our model will generate a non-degenerate distribution of asset holdings.[3]

We are agnostic in terms of the determination of wages in the labor market. There are many mechanism for the wage formation in meetings between workers and entrepreneurs. When characterizing equilibria analytically we will either take the wage as exogenous or as a weighted average of the worker's income when unemployed and the entrepreneur's expected revenue. In all cases, the wage will satisfy individual rationality constraints.

We distinguish two classes of agents, workers and entrepreneurs, and we assume that claims on jobs' profits held by entrepreneurs are illiquid, i.e., they cannot be accumulated by workers to self-insure against idiosyncratic expenditure and employment shocks. With these assumptions money is the only liquid asset and the real interest rate used to discount firms' profits is equal to agents' rate of time preference, as in Pissarides (2000) and Berentsen, Menzio, and Wright (2010). In Section 6 we relax these assumptions and study a version of our model with a single class of agents who can hold a portfolio of money and shares. In this alternative formulation the real interest rate is endogenous.

3 Equilibrium

We characterize steady-state equilibria where aggregate real balances are constant over time and the rate of return of money is $R = (1+\pi)^{-1}$. We first describe the worker's consumption/saving problem taking the price of early consumption, p, as given. Second, we characterize the distribution of real balances through a functional equation. Third, we study the entrepreneur's problem of opening vacancies. Fourth, we determine p by market clearing.

[3]Moreover, as shown in Rocheteau, Weill, and Wong (2015), we could have strictly concave preferences over c and still go a long way in terms of characterizing equilibria analytically.

3.1 Workers

Bellman equations The state of a worker when entering the last stage is composed of his employment status, $e \in \{0,1\}$, and his real balances, $z \in \mathbb{R}_+$. His value function solves:

$$W_e(z) = \max_{c,\hat{z}} \{c + (1-e)\ell + \beta \mathbb{E}_e V_{e'}(\hat{z})\} \quad \text{s.t.} \quad \hat{z} = R(z + w_e - c) \geq 0, \quad c \in [\underline{c}_e, \bar{c}_e], \tag{3}$$

where V_e is the value function of the worker in the employment state $e \in \{0,1\}$ before the retail market opens and $\mathbb{E}_e$ is the expectation operator with respect to the worker's future employment state, e', conditional on his current state, e. According to (3) the worker chooses his current consumption, c, and next-period real balances, $\hat{z}$, in order to maximize his utility of consumption and leisure plus his discounted continuation value in the next retail goods market. The budget identity specifies that next-period real balances are equal to the sum of the current real balances and savings (income net of consumption) multiplied by the gross rate of return of money. With no loss in generality, we set $\underline{c}_e = 0$ as one can normalize w_e to be the income net of the subsistance level and we assume $c \leq \bar{c}_e$ does not bind along the equilibrium path.

In the second-stage retail goods market each worker chooses his consumption, y, taking the real price, p, as given. The total value of the purchases, py, cannot exceed the worker's holdings of liquid assets, z, because debt is not incentive-feasible due to the lack of enforcement and monitoring. Hence, the lifetime expected discounted utility of a worker at the beginning of the retail goods market is:

$$V_e(z) = \alpha \max_{py \leq z} [u(y) + W_e(z - py)] + (1-\alpha) W_e(z). \tag{4}$$

With probability α the worker receives an expenditure shock, $\varepsilon = 1$, in which case he chooses a consumption level, y, in exchange for a payment py. With probability $1-\alpha$ the worker does not wish to consume early and enters the third stage with z real balances.

Proposition 1 ***(Workers' Value Functions.)*** *There is a unique pair of value functions, (W_e, V_e), solutions to (3)-(4) in the space of continuous and bounded functions over $\mathbb{R}_+ \times \{0,1\}$. Moreover, W_e and V_e are increasing and concave and continuously differentiable where $W_e'(z)$ and $V_e'(z)$ solve:*

$$W_e'(z) = R\beta \mathbb{E}_e \left[\frac{\alpha}{p} u'[y_{e'}(\hat{z})] + (1-\alpha) W_{e'}'(\hat{z}) \right] \tag{5}$$

$$V_e'(z) = \frac{\alpha}{p} u'[y_e(z)] + (1-\alpha) W_e'(z), \tag{6}$$

where $y_e(z)$ is the policy function associated with (4). Finally, $W_e'(0^+) < +\infty$.

The value functions, $W_e(z)$ and $V_e(z)$, are fixed points to a contraction mapping and have standard properties in terms of concavity and differentiability. The derivatives are defined recursively by (5) and (6). From (6) the marginal value of real balances in the retail market is equal to the marginal utility of early consumption with probability α and the marginal utility of real balances in the third stage with probability $1-\alpha$. From (5) $W_e'(z) = R\beta V_e'(\hat{z})$.

Policy functions Substituting $c = w_e + z - \hat{z}/R$ into (3), the first-order condition for the worker's choice of real balances is

$$-(1+\xi_e) + R\beta \mathbb{E} V_{e'}'(\hat{z}) \leq 0, \quad \text{“} = \text{”} \text{ if } \hat{z} > 0, \tag{7}$$

where ξ_e denote the Lagrange multiplier associated with $c \geq 0$. The quantity $1+\xi_e$ measures the opportunity cost of accumulating real balances, and $R\beta \mathbb{E} V_{e'}'(\hat{z})$ is the expected marginal benefit of real balances in the retail goods market. If $\xi_e = 0$, then $c \geq 0$ does not bind (recall that we set the subsistance level, $\underline{c}^e$, to zero), $\hat{z}$ is independent of z, and W_e is linear in the worker's income, w_e. In that case workers are neutral relative to the labor market risk. However, if $c \geq 0$ binds, $\xi_e > 0$, then the choice of real balances is no longer independent of current wealth and W_e is strictly concave. In that case workers are averse to the labor market risk. We define a worker's targeted real balances, $z_e^\star$, as a solution to (7) when $c \geq 0$ does not bind, $\xi_e = 0$. It solves

$$R\beta \mathbb{E} V_{e'}'(z_e^\star) = 1. \tag{8}$$

If $z < z_e^\star$ workers reduce their consumption so as to bring to their real balances as close as possible to their target.

The policy function for early consumption is the solution to the following concave problem, $y_e(z) \in \arg\max_{py \leq z} \left[u(y) + W_e(z - py)\right]$. We characterize the policy functions in the following proposition.

Proposition 2 ***(Workers' Policy Functions)*** *In the third-stage competitive market the worker's policy functions are given by:*

$$\hat{z}_e(z) = \min\left\{R(w_e + z), z_e^\star\right\}, \quad c_e(z) = \max\left\{w_e + z - \frac{z_e^\star}{R}, 0\right\}. \tag{9}$$

In the second-stage retail goods market, the worker's policy function is:

$$y_e(z) = \begin{cases} z \\ \{y_e : u'(y_e) = pW'_e(z - py_e)\} \end{cases} \quad if \;\; z \begin{cases} \leq \\ > \end{cases} \bar{z}_e, \tag{10}$$

where $\bar{z}_e > 0$ *the solution to* $u'(\bar{z}_e/p) = pW'_e(0)$.

From (9) workers save their full income and keep their last-stage consumption to its minimum until they reach their targeted real balances. If $z_e^\star > R(w_e + z_e^\star)$ then the target cannot be sustained because the income of the worker is not sufficient to compensate for the inflation rate. In what follows we focus on equilibria where R is not too negative (i.e., inflation is not too high) so that $z_e^\star$ can be reached.

From (10) the optimal consumption in the retail market equalizes the worker's marginal utility from spending a unit of real balances in the retail market, $u'(y)/p$, with the marginal value of real balances in the last stage, $W'_e(z - py)$ provided that the constraint $py \leq z$ does not bind. From the concavity of W, y is a non-decreasing function of z. If $u'(z/p) \geq pW'_e(0)$, then the worker spends all his real balances. This occurs if the worker's real balances are below a threshold, $\bar{z}_e$.

3.2 Distribution of liquidity

We use the worker's policy functions from Proposition 2 to obtain a functional equation for the distribution of real balances. We denote $G_e(z)$ the measure of workers in state $e \in \{0, 1\}$ holding no more than z real balances at the start of the last stage (before late consumption). It solves:

$$\begin{aligned} G_0(z) &= (1-\lambda) \int \left[\alpha \mathbb{I}_{\{\hat{z}_0(x) - py_0[\hat{z}_0(x)] \leq z\}} + (1-\alpha)\mathbb{I}_{\{\hat{z}_0(x) \leq z\}}\right] dG_0(x) && (11) \\ &\quad + \delta \int \left[\alpha \mathbb{I}_{\{\hat{z}_1(x) - py_0[\hat{z}_1(x)] \leq z\}} + (1-\alpha)\mathbb{I}_{\{\hat{z}_1(x) \leq z\}}\right] dG_1(x) \\ G_1(z) &= (1-\delta) \int \left[\alpha \mathbb{I}_{\{\hat{z}_1(x) - py_1[\hat{z}_1(x)] \leq z\}} + (1-\alpha)\mathbb{I}_{\{\hat{z}_1(x) \leq z\}}\right] dG_1(x) && (12) \\ &\quad + \lambda \int \left[\alpha \mathbb{I}_{\{\hat{z}_0(x) - py_1[\hat{z}_0(x)] \leq z\}} + (1-\alpha)\mathbb{I}_{\{\hat{z}_0(x) \leq z\}}\right] dG_0(x) \\ G(z) &= G_0(z) + G_1(z) && (13) \end{aligned}$$

The first term on the right side of (11) is the measure of unemployed workers with x real balances in the previous period who did not find a job and exited the retail market with less than z. If those workers did not receive a spending opportunity in the retail market, their real balances are $\hat{z}_0(x)$. If they did receive a consumption opportunity in the retail market, then their real balances are $\hat{z}_0(x) - py_0[\hat{z}_0(x)]$. The second term has a similar interpretation for previously-employed workers who just lost their job.

It will also be useful to determine the value of money by market clearing to have the distribution of real balances across workers at the start of the retail market (before early consumption). The measure of workers in state e holding no more than z real balances at the start of the second stage, denoted $F_e(z)$, solves:

$$
\begin{aligned}
F_0(z) &= (1-\lambda)\int \mathbb{I}_{\{\hat{z}_0(x)\leq z\}} dG_0(x) + \delta \int \mathbb{I}_{\{\hat{z}_1(x)\leq z\}} dG_1(x) && (14)\\
F_1(z) &= (1-\delta)\int \mathbb{I}_{\{\hat{z}_1(x)\leq z\}} dG_1(x) + \lambda \int \mathbb{I}_{\{\hat{z}_0(x)\leq z\}} dG_0(x) && (15)\\
F(z) &= F_0(z) + F_1(z). && (16)
\end{aligned}
$$

A worker holds no more than z at the start of the retail market if his real balances at the end of the previous period, x, satisfy $\hat{z}_e(x) \leq z$.

Proposition 3 ***(Stationary Distributions)*** *There exists a unique pair (F, G) solutions to (11)-(13) and (14)-(16). Employment at the steady state is $n = F_1(\infty) = G_1(\infty)$, which solves*

$$n = \frac{m(1,\theta)}{\delta + m(1,\theta)}. \tag{17}$$

At the start of the retail market all the money is held by workers. (However, at the start of the last stage some money is held by entrepreneurs.) So the value of money is determined by the following money market clearing condition:

$$\phi_t M_t = \int x dF(x). \tag{18}$$

The mean of the real balances across all workers at the start of the retail market is equal to the aggregate real balances $\phi_t M_t$.

3.3 Entrepreneurs

We now turn to the decision of entrepreneurs to open jobs. Let $E(a)$ denote the lifetime expected utility of an entrepreneur at the beginning of the last stage with a units of wealth (in terms of numeraire). It solves:

$$E(a) = \max_{c,a'} \{c + \beta E(\hat{a})\} \quad \text{s.t.} \quad \hat{a} = R^f(a + T - c), \tag{19}$$

where R^f is the expected gross rate of return of entrepreneurs' wealth (which includes investment in old and new jobs) and T is a real transfer received by entrepreneurs. Note that the rate of return on entrepreneur's wealth can be greater than R because only entrepreneurs hold claims on firms/jobs' profits. From (19) an entrepreneur chooses his consumption, c, and his next period wealth, $\hat{a}$, in order to maximize the discounted sum of his consumption flows. The value of a filled job solves:

$$J = q - w_1 + (1-\delta)\frac{J}{R^f}. \tag{20}$$

It is equal to its expected revenue, q, net of the wage, w_1, plus the expected discounted profits of the job if it is not destroyed with probability $1-\delta$. The value of a vacant job is equal to

$$-k + \frac{\lambda(\theta)}{\theta}\frac{J}{R^f} \leq 0, \text{ " = " if } \theta > 0. \tag{21}$$

The first term on the right side of (21) is the cost of opening a vacancy while the second term is the expected discounted value of a job, where the vacant job is filled with probability λ/θ. The first-order condition with respect to $\hat{a}$ gives:

$$\frac{-1}{R^f} + \beta \leq 0, \text{ " = " if } \hat{a} > 0.$$

Provided that jobs are created $\beta R^f = 1$, i.e., the rate of return on jobs is equal to the discount rate. Because $R^f > R$, claims on jobs profits dominate fiat money in its rate of return.

Next, we turn to the decision of entrepreneurs to sell output to early consumers. The revenue of a job expressed in terms of the numeraire is:

$$q(p) = \max_{y\in[0,\bar{y}]}\{py + Q(y)\} = \bar{q} + \max_{y\in[0,\bar{y}]}\{py - \kappa(y)\}, \tag{22}$$

where the second equality is obtained by using that $\kappa(y) = \bar{q} - Q(y)$. The first term on the right side is the firm's total output in terms of numeraire. The second term represents the firm's profits from selling to early consumers. Using that $\kappa'(\bar{y}) = +\infty$, the solution is interior and the optimal supply of goods in the retail market is

$$y_t^s = \kappa'^{-1}(p). \tag{23}$$

The price of early consumption is equal to the firm's marginal cost from producing early.

From the free-entry condition, (21), market tightness solves:

$$-\frac{(\rho+\delta)\theta k}{\lambda(\theta)} + q - w_1 \leq 0, \text{ " = " if } \theta > 0. \tag{24}$$

For now we are agnostic about wage formation in pairwise meetings and take w_1 as exogenous. However, we impose that it satisfies the participation constraint of the worker and the entrepreneur, i.e.,

$$\begin{aligned} V_1(z) &\geq V_0(z) \text{ for all } z \\ J &\geq 0. \end{aligned}$$

3.4 Steady-state equilibrium

So far we have characterized individual problems taking the price of early-consumption as given. The price, p, is determined so as to clear the market for early consumption:

$$ny^s = \alpha y^b = \alpha \left[\int y_0(z) dF_0(z) + \int y_1(z) dF_1(z) \right]. \tag{25}$$

There is a measure n of active jobs, each of which produces y^s in the retail market. There is a measure α of households with an average demand, y^b, defined on the right side of (25). The average demand is the weighted sum of individual demands, $y_e(z)$, across employed and unemployed workers. We now have the different components to define an equilibrium.

Definition 1 *A steady-state monetary equilibrium is composed of:*

(i) Value functions, W_e and V_e, satisfying (3) and (4);

(ii) Policy functions, $\hat{z}_e(z)$, $c_e(z)$, and $y_e(z)$, satisfying (9) and (10).

(iii) Distribution, (G_e, G, F_e, F), satisfying (11)-(13) and (14)-(16).

(iv) Market tightness, θ, satisying (24).

(v) Price of early consumption, p, satisfying (25).

Given p the equilibrium has a simple recursive structure. The workers' value functions, W_e and V_e, are obtained as the unique fixed point of the Bellman equations (3) and (4). The associated policy functions, $\hat{z}_e(z)$, $c_e(z)$, and $y_e(z)$, given by (9) and (10), are used to determine the distributions G_e and G as the fixed points of (11)-(13). Given p we can solve for productivity, market tightness, and aggregate employment from (29), (24), and (17). Finally, p must be consistent with market clearing.

4 Equilibria in closed form

We first study a class of equilibria analytically. We assume $Q(y) = \bar{q} - y$, which means that each filled job produces $\bar{q}$ of output, which can be sold early or late. This specification implies a linear cost, $\kappa(y) = y$. If prices were set at the marginal cost, then $p = 1$ and $q = \bar{q}$. In that case, the distribution of liquidity does not affect firms' revenue. In the following, we assume that firms set their price at a markup over costs, $p > 1$, and we take this markup as exogenous. This pricing mechanism satisfies the notion of individually rational implementability of Hu, Kennan, and Wallace (2009).[4] Feasibility in the market for early consumption requires:

$$n\bar{q} \geq \alpha \left[\int y_0(z) dF_0(z) + \int y_1(z) dF_1(z) \right]. \tag{26}$$

The total output produced by the n firms in the market must be greater than the aggregate demand for early consumption. This condition holds provided that $\bar{q}$ is sufficiently large. Finally, we assume that the total demand from workers is divided evenly among entrepreneurs.

We focus on simple equilibria where workers deplete their real balances in full whenever they are matched in the retail market, $z_e^\star \leq \bar{z}_e$ for all $e \in \{0, 1\}$. This subset of equilibria includes the equilibrium with degenerate distribution of money holdings as in BMW. Assuming that the constraint $c \geq 0$ does not bind at the target, (8) implies:

$$u'(z^\star) = p\left(1 + \frac{i}{\alpha}\right), \tag{27}$$

where $i = (1 + \pi)(1 + \rho) - 1$. Workers target for the same real balances irrespective of their employment status. This target is such that there is a wedge between the marginal utility of early consumption and the marginal utility of late consumption, $u'(y) > 1$. This wedge has two components: the markup in the retail goods market, $p > 1$, and the cost of holding real balances, $R\beta < 1$. The condition for full depletion of real balances is:

$$p^{-1} u'\left(\frac{z^\star}{p}\right) \geq W_e'(0). \tag{28}$$

Workers find it optimal to deplete their real balances in full if their marginal utility of consumption at the target is greater than the marginal value of real balances.

[4] A difference with respect to Hu, Kennan, and Wallace (2009) is that we do not impose pairwise meetings. However, it would be straightforward to reinterpret this version of our model as one with random pairwise meetings, in which case $p > 1$ would be akin to giving some bargaining power to sellers in the retail market. Alternatively, we can interpret $p > 1$ as arising from informational frictions about the quality of goods as described in the Appendix.

The distributions of real balances have a finite support, and the money holdings in that support are of the form $z = \min\left\{\sum_{n=1}^{N} R^n w_{e_n}, z^\star\right\}$ where the sequence $\{e_n\}_1^N$ represents the employment history of a worker since his last consumption opportunity in the retail market. The expected revenue of a job is:

$$q = \frac{\alpha}{n}(p-1)\left[\int y_0(z)dF_0(z) + \int y_1(z)dF_1(z)\right] + \bar{q}. \tag{29}$$

The first term on the right side of (29) is the expected surplus from sales in the retail market. The sales, $y_e(z)$, depend on both the employment status of the worker and his real balances. The net markup on the sales is $p-1$. Given that $y_e(z) = z$ along the equilibrium path, the revenue of a filled job reduces to:

$$q = \frac{\alpha}{n}\left(\frac{p-1}{p}\right)\phi M + \bar{q}. \tag{30}$$

Aggregate demand in the retail market is proportional to aggregate real balances.

4.1 Degenerate distribution of liquidity

We consider first equilibria where $Rw_0 > z^\star$. So both employed and unemployed workers can accumulate their targeted real balances in a single period. This equilibrium corresponds to the one studied in Berentsen, Menzio, and Wright (2010). The distribution of real balances across workers has a single mass point at $z^\star$ and it depends on the idiosyncratic risk in the goods market through α, but it does not depend on the idiosyncratic risk in the labor market. Firm's productivity is $q = \frac{\alpha}{n}(1-p^{-1})z^\star + \bar{q}$. The equilibrium reduces to a triple $(\theta, z^\star, n)$ solution to:

$$\frac{(\rho+\delta)\theta k}{\lambda(\theta)} = \frac{\alpha}{n}(1-p^{-1})z^\star + \bar{q} - w_1 \tag{31}$$

$$u'\left(\frac{z^\star}{p}\right) = p\left(1+\frac{i}{\alpha}\right) \tag{32}$$

$$n = \frac{\lambda(\theta)}{\delta+\lambda(\theta)} \tag{33}$$

Equilibrium is unique.[5] Real balances, $z^\star$, are determined from (32). By substituting n from (33) into (31) we determine market tightness. If w_1 is not affected by w_0 then a change in the income of the unemployed has not effect on the equilibrium. Suppose $w_1 = \gamma p + (1-\gamma)(w_0 + \ell)$. An increase in w_0 raises the wage and reduces employment. This is the standard effect of unemployment benefits in the Mortensen-Pissarides

[5] In Berentsen, Menzio, and Wright (2010) the steady-state equilibrium might not be unique because the arrival rate of spending opportunities, α, is an increasing function of n, which creates strategic complementarities between firms' entry decision and households' choice of real balances.

model. Finally, in increase of the inflation raises i, and reduces $z^\star$ which leads to lower market tightness and lower employment. This result is in accordance with Friedman's (1977) idea according to which the long-run Phillips curve might be upward sloping.

Proposition 4 ***(Degenerate distribution.)*** *Suppose $Rw_0 > z^\star$. The equilibrium features a degenerate distribution of money holdings across workers. An increase in w_0 and π reduces productivity, market tightness and employment.*

4.2 The aggregate demand effect of unemployment benefits

We now depart from the assumption in BMW according to which unemployed workers can accumulate enough liquidity in a single period to self-insure against expenditure shocks. Suppose that $Rw_1 > z^\star$ but $Rw_0 < z^\star$. While employed workers can reach their targeted real balances in a single period – they resemble their counterparts in the BMW model — unemployed workers are only able to reach their desired level of real balances after multiple periods. We start with the simplest case where F has two mass points. This class of equilibria allows us to illustrate how transfers to workers have an aggregate demand effect that can lead to higher output and lower unemployment. We will then generalize our analysis to distributions with an arbitrary number, N, of mass points.

4.2.1 Two-point distribution

For simplicity, assume $R = 1$ (i.e., money supply is constant) and $w_0 > z^\star/2$, unemployed workers can reach their targeted real balances in two periods. The distribution of real balances has two mass points, $f(w_0) = u\alpha$ and $f(z^\star) = 1 - \alpha u$. A measure αu of workers hold low real balances equal to w_0 because they received an expenditure shock while they were unemployed. As a result, their wealth is composed of their last income. All the other workers hold their targeted real balances, $z^\star$. So the distribution of real balances is now directly affected by the income of the unemployed and the unemployment rate. The condition for full depletion of real balances is

$$u'(w_0) \leq p\left[\frac{(\alpha+\rho)^2 + \rho(1-\alpha)}{\alpha^2}\right]. \tag{34}$$

Unemployed workers deplete their money holdings if w_0 is not too low, so that they can replenish their money holdings quickly.

Given the distribution of liquidity, the expected revenue of a job is:

$$q = \frac{\alpha}{n}(1 - p^{-1})\left[(1 - \alpha u) z^{\star} + \alpha u w_0\right] + \bar{q}. \tag{35}$$

As w_0 increases, unemployed workers with low real balances can raise their consumption in the event of an expenditure shock. This increase in sales translates into a higher q provided that entrepreneurs set a markup in the retail goods market, $p > 1$. The size of this aggregate demand effect is measured by

$$\frac{\partial q}{\partial w_0} = \frac{\alpha^2 u}{1 - u}(1 - p^{-1}).$$

It increases with the unemployment rate, the frequency of expenditure shocks, and the markup.

The general equilibrium effect of an increase in w_0 depends on how it is financed and how it affects wages. If w_0 is financed by lump-sum taxes on entrepreneurs, then the financing of unemployment benefits has no distortionary effect since their decision to open jobs is independent of their wealth. Similarly, a tax on employed workers has no employment effect since employed workers are not constrained in their choice of real balances and they do not make any participation decision. Financing w_0 by money creation affects $z^{\star}$ by reducing R, but provided that $z^{\star}$ is sufficiently inelastic with respect to R, an increase in w_0 can raise q.

So far we assumed that a change in w_0 does not affect w_1. A more general expression for the wage is $w_1(q, w_0, \ell)$, where the wage depends on labor productivity, unemployment benefits, and utility of leisure. An increase in w_0 can raise employment provided that w_1 is sufficiently inelastic with respect to w_0. If $w_1 = \gamma p + (1 - \gamma)(w_0 + \ell)$, then market tightness solves:

$$\frac{(\rho + \delta)\theta k}{\lambda(\theta)} = (1 - \gamma)\left\{\frac{\alpha}{n}(1 - p^{-1})\left\{\left[1 - \alpha(1 - n)\right] z^{\star} + \alpha(1 - n) w_0\right\} + \bar{q} - w_0 - \ell\right\}.$$

Entry has a positive externality on other entrepreneurs by improving the distribution of liquidity across workers. Indeed, a higher n affects F by reducing the probability that an entrepreneur meets a worker with low real balances. Note also that ℓ and w_0 do not affect market tightness symmetrically. An increase in ℓ affects θ through the wage formation only. As ℓ increases the worker's reservation wage increases, which leads to an increase in w_1. An increase in w_0 has a similar effect on w_1 but it also has an effect on aggregate demand by raising the real balances of the poorest workers. As a result, an increase in w_0 will be less detrimental to employment than an increase in ℓ.

In order to disentangle the monetary from the fiscal side of money growth, suppose that it is implemented through lump-sum transfers to entrepreneurs. Inflation affects the distribution of liquidity by raising the opportunity cost of real balances, thereby reducing $z^{\star}$, and by reducing the amount that the poorest workers can save, Rw_0. So inflation has a direct effect on aggregate demand even if $z^{\star}$ is inelastic to a change in R. If money growth is implemented by transfers to workers, this second effect can be mitigated.

4.2.2 General distribution

We now consider equilibria with any arbitrary number of mass points for the distribution of real balances across unemployed workers. Assuming $R = 1$, if the distribution has H mass points then $z^{\star} \in ((H-1)w_0, Hw_0]$, i.e., $(H-1)w_0 < u'^{-1}\left(1+\frac{\rho}{\alpha}\right) \leq Hw_0$. Along such equilibria it takes H periods for an unemployed worker with depleted real balances to rebuild his real balances and reach his target. The distribution of real balances solves:

$$f(w_0) = u\alpha \tag{36}$$

$$f(hw_0) = f\left[(h-1)w_0\right](1-\alpha)(1-\lambda), \quad h \in \{2, ..., H-1\} \tag{37}$$

$$f(z^{\star}) = 1 - \sum_{h=1}^{H-1} f(hw_0). \tag{38}$$

According to (36) in each period there is a measure α of workers who receive an expenditure shock and those workers deplete their money balances. The fraction u who are unemployed accumulate w_0 real balances. According to (37) if a worker holds $(h-1)w_0$ real balances at the start of the retail market, he will hold hw_0 in the following period provided that he did not find a job in the labor market preceeding the retail market, with probability $1-\lambda$, and he is unmatched in the retail market, with probability $1-\alpha$. The closed-form solution to (36)-(38) is:

$$f(hw_0) = u\alpha\left[(1-\alpha)(1-\lambda)\right]^{n-1}, \quad h \in \{1, ..., H-1\} \tag{39}$$

$$f(z^{\star}) = 1 - u\alpha\frac{1-\left[(1-\alpha)(1-\lambda)\right]^{H-1}}{1-(1-\alpha)(1-\lambda)} \tag{40}$$

From (39)-(40) the distribution of real balances depends on the idiosyncratic risk in both the goods and the labor market, as represented by α and λ. It also depends directly on the unemployment rate, u, and the income of the unemployed through H that determines the number of mass points of the distribution. The equilibrium features full depletion if

$$W_0'(0) = \sum_{h=1}^{H-1} \beta^h \left[(1-\alpha)(1-\lambda)\right]^{h-1} \left[\frac{\alpha u'(hw_0)}{p} + (1-\alpha)\lambda\right] + \left[(1-\alpha)(1-\lambda)\beta\right]^{H-1} \leq 1 + \frac{\rho}{\alpha}. \tag{41}$$

The condition for full depletion holds provided that w_0 is not too low, since otherwise unemployed workers would hoard liquid assets in order to build insurance toward their next expenditure shock.

Using the distribution of liquidity, (39)-(40), we compute the expected revenue of a job as

$$q = \frac{\alpha}{n}(1-p^{-1}) \left\{ u\alpha \sum_{h=1}^{H-1} \left[(1-\alpha)(1-\lambda)\right]^{h-1} hw_0 + \left[1 - u\alpha \frac{1-\left[(1-\alpha)(1-\lambda)\right]^{H-1}}{1-(1-\alpha)(1-\lambda)}\right] z^\star \right\} + \bar{q}. \tag{42}$$

The first term on the right side of (42) increases with w_0. The effect is non-linear because H is a decreasing function of w_0. The effect of w_0 on q is stronger when w_0 is small and H is large. Job productivity also depends on the aggregate state of the labor market, u, and the idiosyncratic risks in both goods and labor markets, α and λ.

Our model has implications for how the worker's surplus from finding a job depends on his holdings of liquid assets. From (4) the surplus from being employed when exiting the labor market is

$$V_1(z) - V_0(z) = \alpha\left[W_1(0) - W_e(0)\right] + (1-\alpha)\left[W_1(z) - W_0(z)\right].$$

In an equilibrium with full depletion of real balances, workers with the same liquid wealth consume the same amount irrespective of their labor status. As a result, the surplus from being employed is equal to the expected surplus from being employed in the last stage centralized market. With probability α the worker enters this last stage with depleted real balances and with probability $1-\alpha$ the worker enters the last stage with his beginning-of-period real balances. The marginal effect of real balances on this surplus is

$$\frac{\partial\left[V_1(z) - V_0(z)\right]}{\partial z} = (1-\alpha)\left[1 - W_0'(z)\right].$$

For all $z < z^\star - w_0$, $W_0'(z) > 1$ is decreasing in z. So the surplus from being employed decreases with real balances. We summarize the results of this subsection in the following Proposition.

Proposition 5 ***(Ex-Post Heterogeneity across Unemployed Workers.)*** *Suppose there is a $H \geq 2$ such that $(H-1)w_0 < u'^{-1}\left(1+\frac{\rho}{\alpha}\right) \leq Hw_0$ and (41) holds.*

1. *The equilibrium features full depletion of real balances, a degenerate distribution of money holdings across employed workers, and a H-point distribution of money holdings across unemployed workers.*

2. *An increase in unemployment benefits financed by a lump-sum tax on entrepreneurs or employed workers can raise aggregate real balances and employment.*

3. *The worker's surplus from being employed, $V_1(z) - V_0(z)$, decreases with real balances.*

4.3 The aggregate demand effect of wages

Suppose now that both employed and unemployed workers need two periods to reach their targeted real balances, $Rw_1 < z^\star$ and $w_0R(1+R) > z^\star$. Assuming the money supply is constant, $R = 1$, the distribution of real balances has three mass points: $f(w_0) = \alpha u$, $f(w_1) = \alpha n$, and $f(z^\star) = 1 - \alpha$. There are now both employed and unemployed workers who are constrained by their income when choosing their holdings of liquid assets, and there is ex-post heterogeneity across both employed and unemployed workers. From (28) the condition for full depletion of real balances is:

$$W_0'(0) = \beta \left[\frac{\alpha}{p} u'(w_0) + 1 - \alpha \right] \leq 1 + \frac{\rho}{\alpha}. \tag{43}$$

If (43) holds, then employed workers also have incentives to deplete their real balances in full since $w_1 > w_0$.

Given the distribution of real balances, $f(z)$, the expected revenue of a job is:

$$q = \frac{\alpha}{n}(1 - p^{-1}) \left\{ \alpha u w_0 + \alpha n w_1 + (1 - \alpha) z^\star \right\} + \bar{q}. \tag{44}$$

The novelty is that q is now an increasing function of w_1. Indeed, there is a measure αn of workers whose holdings of liquidity is constrained by their wage. As their wage increases, so does their real balances and their expenditure in the retail goods market. The size of this aggregate demand effect, $\partial q / \partial w_1 = \alpha^2(1 - p^{-1})$, increases with α and with the markup.

Suppose the wage is endogenous and takes the form $w_1 = \gamma p + (1 - \gamma)(w_0 + \ell)$. Then,

$$q = \frac{\frac{\alpha}{n}(1 - p^{-1}) \left\{ \alpha \left(1 - n\gamma\right) w_0 + \alpha n (1 - \gamma) \ell + (1 - \alpha) z^\star \right\} + \bar{q}}{1 - \alpha^2 (1 - p^{-1}) \gamma}. \tag{45}$$

A change in exogenous productivity, $\bar{q}$, has a multiplier effect on the revenue of the entrepeneur:

$$\frac{\partial q}{\partial \bar{q}} = \frac{1}{1 - \alpha^2 (1 - p^{-1}) \gamma} > 1.$$

Indeed, if $\bar{q}$ increases, then w_1 increases by γ (the proxy for the worker's bargaining power). Because some workers are constrained by w_1 when choosing their real balances, the aggregate demand in the retail market increases, which raises q. This generates a second wave of increase in w_1. And so on. This multiplier effect increases with α, with the markup in the retail goods market, and with workers' bargaining power. The entrepreneur's instantaneous profits are:

$$q - w_1 = (1-\gamma)\left\{\frac{\frac{\alpha}{n}(1-p^{-1})(1-\alpha)z^\star - \left[1-\frac{\alpha^2}{n}(1-p^{-1})\right]w_0 - \left[1-\alpha^2(1-p^{-1})\right]\ell + \bar{q}}{1-\alpha^2(1-p^{-1})\gamma}\right\}.$$

Through the multiplier effect the effect of a change in $\bar{q}$ on profits is larger than the entrepreneur's bargaining power,

$$\frac{\partial(q-w_1)}{\partial\bar{q}} = \frac{1-\gamma}{1-\alpha^2(1-p^{-1})\gamma} > 1-\gamma.$$

The aggregate demand effect switches the bargaining power toward entrepreneurs. Even though workers receive a fraction γ of exogenous productivity gains, a fraction of them (αn) are constrained by their wage and will consume whatever they receive in the form of early consumption (with probability α) at a markup $(p-1)$.

We summarize these results in the following Proposition.

Proposition 6 ***(Ex-Post Heterogeneity across Employed Workers.)*** *Suppose $w_1 < z^\star$ but $2w_0 > z^\star$, and (43) holds. The equilibrium features full depletion of real balances, and a three-point distribution of money holdings across workers. An increase in w_1 raises q through an aggregate demand effect. An increase in $\bar{q}$ has a multiplier effect on q.*

5 Quantitative analysis

5.1 Calibration

The model is calibrated to a monthly frequency. We set $\beta = 1/(1+\rho) = 0.996$, which implies an annual discount rate of 5%. The model has several parameters that can be calibrated in a way that is standard in the macro-labor literature. The matching function takes the form $m(s,o) = so/(s^\gamma + o^\gamma)^{1/\gamma}$. Using data on job finding rates, unemployment, and vacancies, Schaal (2015) estimates $\gamma = 1.6$. We set the separation rate,δ, to imply a quarterly job destruction rate of 10% and set vacancy costs, k, to imply a monthly job finding rate of 45%, as in Shimer (2005). The calibration results in $k = 3.8$, which implies that vacancy costs are 5% of quarterly the quarterly wage bill. Silva and Toledo (2009) estimate recruiting costs to be about

14% of the quarterly wage bill, so our measure is slightly low. Targeting the separation and job finding rate above implies a steady-state unemployment rate of 7.2%. The flow value of leisure is calibrated to target the relative standard deviation the unemployment rate and (detrended) output per job, $q(p)$. We use the standard U3 measure of unemployment and output per job given by the BLS productivity and cost series. We follow Shimer (2005) and detrend using an HP filter with smoothing parmaeter $10e5$. This results in $sd(u) = 0.213$, $sd(q) = 0.024$, and hence $sd(u)/sd(q) = 8.79$.

The labor income process in the model is a Markov chain with two states, w_1 and w_0, and transition probabilities given by the job finding and separation rate. We normalize the level such that the average wage is one, $uw_0+(1-u)w_1 = 1$, and set the distance between wages to target the unconditional variance of transitory annual labor income shocks from Meghir and Pistaferri (2004) of 0.03.[6] Given we match the transition rates, the stationary distribution is given by the Beveride curve, $u = \delta/(\delta + f)$, and the unconditional variance of log earnings is given by $uln(w_0)^2+(1-u)ln(w_1)^2-[uln(w_0) + (1-u)ln(w_1)]^2 = 0.03$. This gives 2 equations in 2 unkonwns and implies $w_1 = 1.03$ and $w_0 = 0.53$. Interpreting w_0 as unemployment benefits implies a replacement rate of $w_0/w_1 = 0.52$. Additionally, the stationary equilibrium implies an average consumption decline upon job loss of 9.3%, in line with estimates from Hurd and Rohwedder (2010) of 11% or Browning and Crossley (2001) of 16%. We then set the technology parameter, $\bar{q}$, such that we get an aggregate labor share of 66%. In the model, aggregate output is given by $n[\bar{q}+py-\kappa(y)]$ and labor expenses are nw_1. Hence, the aggregate labor share is then given by $w_1/(\bar{q} + py - \kappa(y))$. This leads to $\bar{q} = 0.81$.

The remaining parameters to calibrate are the rate of expenditure shocks, α, household preferences for early consumption, $u(y)$, and the subsistence parameter, $\underline{c}$. These parameters help discipline the incentive for precautionary savings; α pins down the level of liquidity risk and the preference parameters determine the consumption response upon an expenditure shock and the rate of precautionary savings. We use available evidence on the distribution of liquid wealth to pin down these features of the environment. Household balance sheet data is from the 2004 Survey of Consumer Finances (SCF) and we follow Kaplan and Violante (2014) in defining liquid assets as all short-term deposit accounts, such as checking, savings, and money-market demand accounts, as well as certificates of deposit, short-term bonds, and mutual funds net of revolving debt on credit card balances.

[6]This is in-line with calibration targets from the incomplete markets literature, for instance 0.04 in Aiyagari (1994).

In the data, there are many determinants that lead to variation in households' liquid wealth that are not included in our model, such as differences in permanent income or ability, age, and access to credit markets. In our model, differences in liquid wealth arise from idiosyncratic (and transitory) expenditure and income shocks. Hence, in order to target an empirical distribution that is most in line with the one given from our model, we estimate a *residual* distribution of liquid wealth to earnings controlling for observable differences amongst households in the data, that are not included in our model.

We describe our procedure in detail in the Appendix, but as a summary, we estimate a regression of liquid wealth to annual earnings on demographics (age, sex, race, and education), industry and occupation, non-liquid wealth (real estate and vehicle equity), and a measure of permanent income. We construct permanent income from a series of questions in the SCF that ask households if their current income is "unusually high or low compared to what you would expect in a 'normal' year" and what income they would usually expect in a 'normal' year. In the sample, 72% of respondents reported having a normal income, 9% reported having unusually high income, and 19% reported having unusually low income. Table **??** reports the results of our estimation and Figure 3 illustrates the residual distribution compared to the raw data.

We set R to match an annual real rate of return of liquid assets of -1.46% from Kaplan and Violante (2014) and let $u(y) = A/(1-a)[(y+\epsilon)^{(1-a)} - \epsilon^{(1-a)}$ where $\epsilon = 1e-5$. This specification allows $a > 1$ while ensuring that $u(0) = 0$. We jointly target $(\alpha, a, A, \underline{c})$ to match the residual distribution of liquid assets. Figure 4 illustrates the fit of our model compared to the data. We match the median liquid assets to income but underestimate the dispersion; there are less households with both low and high liquid wealth than in the data. This procedure results in a rate of expenditure shocks of $\alpha = 0.49$, or an shock on average every 2 months. The preference parameters are $a = 0.40$ and $A = 1.9$, which implies an average consumption increase in the event of an expenditure shock of 13% and an average marginal propensity to consume out of liquidity, z, of 25%. Finally, $\underline{c} = 0.3$, which implies an average subsistence rate of consumption of 12%.[7]

The last parameters to calibrate determine the cost of production for firms, $\kappa(y)$. We let $\kappa(y) = By^{(1+b)}/(1+b)$. The revenue of the firm, using $p = \kappa'(y)$, can be expressed as $q = \bar{q} + \mu\kappa(y)$, where $\mu = \kappa'(y)y/\kappa(y) - 1$. We refer to μ as a markup over the firm's average cost of early production and set

[7]In the model, $\underline{c}$ is the minimum level of late stage consumption. The interpretation is that even if a household receives a liquidity shock, they have a minimum set of consumption expenditures within the month that they cannot avoid.

Table 1: Calibration Summary - Parameters and Targets

Description	Value	Source/Target
Parameters Set Directly		
discount rate, β	0.996	95% annual discount rate
production cost curvature, b	1.300	30% markup, Retail Trade Survey
matching curvature, c	1.600	Schaal (2016)
rate of return, R	0.998	-1.46% annually, Kaplan and Violante (2014)
separation rate, δ	0.035	10% monthly rate, Shimer (2005)
employed earnings, w_1	1.050	Jointly match earnings volatility, Meghir
unemployed earnings, w_0	0.530	and Pistaferri (2004), and mean wage=1
Parameters Estimated Jointly		**Moments used in optimization**
utility curvature, a	0.40	45% monthly job finding rate
utility level, A	1.90	66% aggregate labor share
production cost level, B	1.02	10% aggregate profit share
production technology, $\bar{q}$	0.81	8.8 relative SD of unemp. to productivity
vacancy posting costs, k	3.80	Residual distribution of liquid wealth to
liquidity shock, α	0.49	earnings, see Figure 4
subsistence consumption, $\underline{c}$	0.30	
utility from leisure, ℓ	0.32	

b to target a markup of 30% using evidence from the Retail Trade Survey. Our functional form for $\kappa(y)$ implies that $\mu = b = 1.3$. Then, we calibrate B to target an aggregate profit share of 10%. In the model, capital expenses are $n\kappa(y) + n\theta k$ and total output is defined above. The aggregate profit share is given by $\bar{q} + py - w_1 - \theta k/\bar{q} + py - \kappa(y)$. Table 1 summarizes the calibrated parameters.

5.2 Baseline model performance

5.3 Comparative Statics: Return on Liquid Wealth

5.4 Comparative Statics: Unemployment Benefits

6 Endogenous interest rates

We now drop the distinction between entrepreneurs and workers and only consider households who can save by accumulating both money and claims on firms' profits where firms. The entry cost of firms is in terms of the numeraire. As in the Ayiagari (1989) model the real interest rate, r^f, is endogeneous and the gross rate of return is denoted R^f. The gross rate of return of money is $R^m = 1/(1+\pi)$. We maintain the assumption that fiat money is the only liquid asset to finance early consumption. For instance, only money can be autenticated at no cost in the retail market.

The problem of the household in the last stage with total wealth $z = z^m + z^f$, where z^f denotes real balances and z^f denotes real stock holdings, becomes:

$$W_e(z) = \max_{c,\hat{y}_0,\hat{y}_1,\hat{z}^m,\hat{z}^f} \left\{c + (1-e)\ell + \beta\mathbb{E}\left\{\alpha\left[u(\hat{y}_{e'}) + W_{e'}\left(\hat{z}^m - \mu\hat{y}_{e'} + \hat{z}^f\right)\right] + (1-\alpha)W_{e'}(\hat{z}^m + \hat{z}^f)\right\}\right\} \quad (46)$$

$$\text{s.t. } \hat{z}^f = R^f\left(z + w_e - c - \frac{\hat{z}^m}{R^m}\right) \geq 0 \text{ and } \mu\hat{y}_{e'} \leq \hat{z}^m, \quad e' \in \{0,1\}$$

The novelty is that households can now accumulate claims on firms' profits, $\hat{z}_f$ (or shares of mutual funds investing in stocks). These claims can only be traded in the competitive market in the last stage. Hence, purchases in the retail market cannot exceed holdings of liquid assets, $\mu\hat{y}_{e'} \leq \hat{z}^m$. For stocks to be held their rate of return cannot be less than the rate of return of money, $R^f \geq R^m$. As before, households have a target for their wealth, conditional on their labor status, that solves the following Euler equation:

$$1 = \beta R^f\mathbb{E}_e\left\{\alpha u'\left[\hat{y}_{e'}\left(z_e^\star\right)\right] + (1-\alpha)W'_{e'}(z_e^\star)\right\}, \quad e \in \{0,1\}.$$

The value of a job satisfies (20) where β is replaced with R^f, i.e.,

$$J = q - w_1 + R^f(1-\delta)J.$$

The distribution of wealth across households in the third stage satisfies (11)-(13). The distribution of real balances at the start of the retail market is given by:

$$F_0^m(z^m) = (1-\lambda)\int \hat{z}_0^m(x)dG_0(x) + \delta\int \hat{z}_1^m(x)dG_1(x) \quad (47)$$

$$F_1^m(z^m) = (1-\delta)\int \hat{z}_1^m(x)dG_1(x) + \lambda\int \hat{z}_0^m(x)dG_0(x). \quad (48)$$

The firm's productivity is given by (29) where F_e is replaced with F_e^m. By a similar reasoning one can obtain the distribution of stock holdings at the start of the retail market. The rate of return of stocks, R^f, is determined by the market clearing condition:

$$nJ = \int \hat{z}_0^f(x)dG_0(x) + \int \hat{z}_1^f(x)dG_1(x). \quad (49)$$

The right side of (49) represents aggregate stock holdings across households while the left side is total market capitalization.

A steady-state monetary equilibrium is composed of a value functions, W_e, satisfying (46) and associated policy functions, $\hat{z}_e^m(z)$, $\hat{z}_e^f(z)$, $c_e(z)$, and $y_e(z)$; Distributions of wealth, (G_e, G), satisfying (11)-(13) and distribution of real balances, F_e^m, solving (47)-(48); Market tightness, θ, satisying (24); Price of early consumption, p, satisfying (25).

Suppose the money supply is constant, $R^m = 1$, and $w_e > z^\star$. The distribution of real balances, F, is degenerate, and the value function W_e is linear. The first-order condition with respect to $\hat{z}^f$ gives $R^f\beta = 1$, i.e., the real interest rate on firms' claims is equal to agents' discount rate. If $w_e > z^\star$ does not hold, then the distribution of portfolios is non-degenerate.

7 Conclusion

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8 Appendix: Proofs of Propositions and Lemmas

Proof of Propositions 1 and 2

This proof adapts the Appendix from Rocheteau, Weill, and Wong (2015) to an environment with both income and expenditure shocks. Consider the pair of Bellman equations (3) and (4):

$$V_e(z) = \alpha \sup_{py \leq z} \left[u(y) + W_e\left(z - py\right)\right] + (1-\alpha)W_e(z) \tag{50}$$

$$W_e(z) = \sup_{c,\hat{z}} \left\{c + (1-e)\ell + \beta \mathbb{E}_e V_{e'}(\hat{z})\right\} \tag{51}$$

subject to $\hat{z} = R(w_e - c + z) \geq 0$ and $c \in [\underline{c}_e, \bar{c}_e]$. First, we substitute the Bellman equation for $V_e(z)$ into the Bellman equation for $W_e(z)$. The functions $W_e(z)$ and $V_e(z)$ solve the Bellman equations (50)-(51) if and only if

$$W_e(z) = \max_{c,\hat{y}_0,\hat{y}_1,\hat{z}} \left\{c + (1-e)\ell + \beta \mathbb{E}\left\{\alpha\left[u(\hat{y}_{e'}) + W_{e'}\left(\hat{z} - \mu\hat{y}_{e'}\right)\right] + (1-\alpha)W_{e'}(\hat{z})\right\}\right\}$$

$$\text{s.t.} \quad \mu\hat{y}_{e'} \leq \hat{z} = R(w_e - c + z) \geq 0, \quad e' \in \{0,1\}.$$

Next, we apply standard contraction-mapping arguments to this Bellman equation. We obtain:

Lemma 1 *The Bellman equations (50)-(51) have unique bounded solutions, $V_e(z)$ and $W_e(z)$. The functions $V_e(z)$ and $W_e(z)$ are continuous, concave, strictly increasing, and*

$$\|W\| \leq \frac{\max\{\bar{c}_0, \bar{c}_1\} + \ell + \beta\alpha\|u\|}{1-\beta} \text{ and } \|V\| \leq \frac{\max\{\bar{c}_0, \bar{c}_1\} + \ell + \alpha\|u\|}{1-\beta}.$$

Proof. Consider the space $C\left(\{0,1\} \times \mathbb{R}_+\right)$ of bounded and continuous functions from $\{0,1\} \times \mathbb{R}_+$ to $\mathbb{R}$, equipped with the sup norm. By Theorem 3.1 in Stokey, Lucas, and Edward Prescott (1989, henceforth SLP), this is a complete metric space. Now, for any $f \in C\left(\{0,1\} \times \mathbb{R}_+\right)$, consider the Bellman operator:

$$T\left[f\right]_e(z) = \max\left\{c + (1-e)\ell + \beta\mathbb{E}\left\{\alpha\left[u(\hat{y}_{e'}) + f_{e'}\left(\hat{z} - \mu\hat{y}_{e'}\right)\right] + (1-\alpha)f_{e'}(\hat{z})\right\}\right\}$$

with respect to $c \in [\underline{c}_e, \bar{c}_e]$, $\hat{y}_0 \leq \hat{z}/p$, $\hat{y}_1 \leq \hat{z}/p$, and $\hat{z} = R(w_e - c + z) \geq 0$. It is straightforward to verify that T satisfies the Blackwell sufficient condition for a contraction (Theorem 3.3 in SLP). Moreover, assuming $w_e \geq \underline{c}_e$ the constraint set is non-empty, compact valued, and continuous. Hence by the Theorem of the Maximum (Theorem 3.6 in SLP), we obtain that $T[f]$ is continuous. It is clearly bounded since all the functions on the right-hand side of the Bellman equation are bounded. Note as well that if f is concave, then

$T[f]$ is also concave since the objective is concave and the constraint correspondence has a convex graph. An application of the Contraction Mapping Theorem (Theorem 3.2 in SLP) implies that the fixed point problem $f = T[f]$ has a unique bounded solution, $W_e(z)$, and that this solution is continuous and concave.

Also, consider any z^1 and some feasible $c^1, \hat{z}^1, \hat{y}_0^1, \hat{y}_1^1$. Then, for $z^2 \geq z^1$, the following choice is feasible: $c^2 = c^1 + z^2 - z^1$, $\hat{z}_2 = \hat{z}_1$ and $\left(\hat{y}_0^2, \hat{y}_1^2\right) = \left(\hat{y}_0^1, \hat{y}_1^1\right)$. That is, a worker starting with z^2 can always consume $z^2 - z^1$ and otherwise behave as if he started with z^1. Since this yield (weakly) higher utility this implies that $T[W]_e(z) = W_e(z)$ is increasing. One also sees that it must be strictly increasing provided that $c \leq \bar{c}_e$ does not bind.

Given a fixed point $W_e(z)$ of the Bellman operator T, we can define $V_e(z)$ as in equation (50). By identical arguments as above, one sees that $V_e(z)$ is bounded, continuous, concave, and strictly increasing.

Finally, we can derive upper bounds for $W_e(z)$ and $V_e(z)$. From the Bellman equation we have:

$$\|W\| \leq \max\{\bar{c}_0, \bar{c}_1\} + \ell + \beta\alpha\left[\|u\| + \|W\|\right] + \beta(1-\alpha)\|W\| \Rightarrow \|W\| \leq \frac{\max\{\bar{c}_0, \bar{c}_1\} + \ell + \beta\alpha\|u\|}{1-\beta}.$$

We obtain the bound on $\|V\|$ following identical arguments but for $V_e(z)$, i.e.,

$$\|V\| \leq \alpha\left[\|u\| + \|W\|\right] + (1-\alpha)\|W\| \;\Rightarrow\; \|V\| \leq \frac{\max\{\bar{c}_0, \bar{c}_1\} + \ell + \alpha\|u\|}{1-\beta}.$$

■

Elementary properties of decision rules

In order to characterize decisions rules it will be convenient to use the indirect utility for real balance in the retail goods market as:

$$\Omega_e(z) = \max_{0 \leq y \leq z/p} \left\{u(y) + W_e(z - py)\right\}. \tag{52}$$

With this new notation, and by substituting the budget constraint, $c = w_e + z - \hat{z}/R$, into the objective the Bellman equation becomes:

$$W_e(z) = \max_{\hat{z}} \left\{ w_e + z - \frac{\hat{z}}{R} + (1-e)\ell + \beta\mathbb{E}\left\{\alpha\Omega_{e'}(\hat{z}) + (1-\alpha)W_{e'}(\hat{z})\right\} \right\}. \tag{53}$$

subject to $R\left(z + w_e - \bar{c}_e\right) \leq \hat{z} \leq R\left(z + w_e - \underline{c}_e\right)$.

Lemma 2 *The worker's problem in the retail market, (52), has a unique solution, $y_e(z)$. This solution is continuous, increasing, satisfies $\lim_{z\to 0} y_e(z) = 0$ and $\lim_{z\to\infty} y_e(z) = \infty$. The value of the worker in the retail market, $\Omega_e(z)$, is continuous, strictly increasing, concave, and satisfies $\Omega_e'(z^+) \geq u'[y_e(z)]/p$.*

Proof. Note first that this problem is strictly concave since we have assumed that $u(y)$ is strictly concave. Hence, it has a unique solution, which we denote by $y_e(z)$. Together with the Theorem of the Maximum (SLP Theorem 3.6), uniqueness implies that $y_e(z)$ is continuous. To show that $y_e(z)$ is increasing, consider any two $z^1 < z^2$. If $py_e(z^2) \geq z_1$, then by feasibility it immediately follows that $py_e(z^1) \leq z^1 \leq py_e(z^2)$. Otherwise, if $py_e(z^2) < z^1$, then a first-order condition for z^2 is that

$$u'\left[y_e(z^2)\right] \leq W_e'\left[z^2 - py_e(z^2)\right]^+$$

where we recall that $W_e(z)$ is concave and so it has left- and right-derivatives for all $z > 0$. By concavity we also have that $W_e'(z^+)$ is decreasing, i.e.,

$$W_e'\left[z^1 - py_e(z^2)\right]^+ \geq W_e'\left[z^2 - py_e(z^2)\right]^+ \geq u'\left[y_e(z^2)\right].$$

so $y_e(z^1) \leq y_e(z^2)$.

Because of the feasibility constraint, $0 \leq y_e \leq z$, it immediately follows that $\lim_{z\to 0} y_e(z) = 0$. Suppose that $y_e(z)$ is bounded away from infinity. Then the first-order condition $u'(y_e) \leq W_e\left[(z - py_e)^-\right]$ cannot hold for z large enough because $W_e(z)$ must satisfy Inada condition at infinity. Indeed, since $W_e(z)$ is bounded, increasing, and concave, we have $0 \leq W_e'(z^-)z \leq W_e(z) - W_e(0) \leq \|W\|$, so that $\lim_{z\to\infty} W_e'(z^-) = 0$.

The value $\Omega_e(z)$ is continuous by the Theorem of the Maximum. It is strictly increasing because $W_e(z)$ is strictly increasing. To establish the lower bound on the right derivative, we note that, $y_e(z)$ is feasible for any $z' \geq z$. This implies that, for all $z' \geq z$:

$$\Omega_e(z') \geq u\left[y_e(z) + \frac{z' - z}{p}\right] + W_e\left[z' - py_e(z)\right],$$

with equality if $z = z'$. The result follows by subtracting the equality for $z = z'$ to the above inequality, dividing by $z' - z$, and letting $z' \to z^+$. ■

To solve for the optimal choice of real balances, we define the set of optimal money holdings if the worker faces no constraint on c as:

$$Z_e^\star = \arg\max\left\{-\frac{\hat{z}}{R} + \beta\mathbb{E}\left[\alpha\left[u(y_{e'}) + W_{e'}(\hat{z} - py_{e'})\right] + \beta(1-\alpha)W_{e'}(\hat{z})\right]\right\},$$

with respect to $\hat{z} \geq 0$. The set $Z_e^\star$ represent the set of "targeted" money holding for the worker in state e.

Lemma 3 *The set $Z_e^\star$ is convex, bounded above, and bounded away from zero. Given any $z_e^\star \in Z_e^\star$, an optimal choice of real balances by a worker in state e is:*

$$\hat{z} = \max\left\{R(z + w_e - \bar{c}_e), \min\left\{R(z + w_e - \underline{c}_e), z_e^\star\right\}\right\} \tag{54}$$

Proof. The set $Z_e^\star$ is bounded above because both $u(y)$ and $W_e(z)$ are concave and bounded, implying that they satisfy Inada condition at infinity. To see that it is bounded away from zero, recall that $\Omega_e'(z) \geq u'\left[y_e(z)\right]$ and $\lim_{z\to 0} y_e(z) = 0$. Since $u(y)$ satisfies an Inada condition at zero, it follows that $\lim_{z\to 0}\Omega_e(z^+) = \infty$. This implies that, near zero, the right-derivative of the optimization program defining $Z_e^\star$ are strictly positive. Hence $0 < \min Z_e^\star$. The rest of the proposition follows because the optimization program defining $Z_e^\star$ is concave. ■

Lemma 4 *The derivative of the value function is bounded, $W_e'(0^+) < \infty$.*

Proof. Choose any $z_e^\star \in Z_e^\star$ and consider the following two cases.

If $z_e^\star \leq R(w_e - \underline{c}_e)$ then, for all $z > 0$ and close enough to zero, an optimal choice of real balances is $\hat{z} = z_e^\star$. Substituting this into the Bellman equation, we obtain:

$$W_e(z) = z + w_e + (1-e)\ell - \frac{z_e^\star}{R} + \beta\mathbb{E}\left[\alpha\Omega_{e'}(z_e^\star) + \beta(1-\alpha)W_{e'}(z_e^\star)\right],$$

which immediately implies that $W_e'(0^+) = 1$.

If $z_e^\star > R(w_e - \underline{c}_e)$ then for all $z > 0$ and close enough to zero, an optimal choice of real balances is $\hat{z} = R\left(z + w_e - \underline{c}_e\right)$. Substituting this in the Bellman equation we obtain:

$$W_e(z) = \underline{c}_e + (1-e)\ell + \beta\mathbb{E}\left\{\alpha\Omega_{e'}\left[R\left(z + w_e - \underline{c}_e\right)\right] + \beta(1-\alpha)W_{e'}\left[R\left(z + w_e - \underline{c}_e\right)\right]\right\}.$$

Since $R > 0$, $\hat{z} = R\left(z + w_e - \underline{c}_e\right)$ lies in the interior of the domain of $\Omega_e(\hat{z})$ and $W_e(\hat{z})$. Given that these functions are concave, they have right-derivative at this interior point. Hence, $W_e(z)$ has a right-derivative at zero, i.e., $W_e'(0^+) < \infty$. ■

With this result we establish:

Lemma 5 *For all $z > 0$, $y_e(z) > 0$. Moreover, $\Omega_e(z)$, is continuously differentiable over $(0, \infty)$ with $\Omega_e'(z) = u'\left[y_e(z)\right]/p$.*

Proof. The first point follows from the fact, shown in Lemma 4, that $W_e(0^+) < \infty$ but $u'(0) = \infty$. For the second point consider some $z > 0$. Since $y_e(z) > 0$, $py_e(z) - (z - z')$ is feasible for $z' < z$ and close enough to z. For such z', we have

$$\Omega_e(z') \geq u\left[y_e(z) - \frac{z - z'}{p}\right] + W_e\left[z - py_e(z)\right],$$

with an equality for $z = z'$. Subtracting the inequality for $z' < z$ to the equality for $z' = z$, and dividing through by $z - z'$, we obtain:

$$\frac{\Omega_e(z) - \Omega_e(z')}{z - z'} \leq \frac{u\left[y_e(z)\right] - u\left[y_e(z) - \frac{z-z'}{p}\right]}{z - z'}.$$

Letting $z' \to z$, we obtain $\Omega'_e(z^-) \leq u'\left[y_e(z)\right]/p$. Since we have already shown in Lemma 2 that $\Omega_e(z^+) \geq u'\left[y_e(z)\right]/p$, and since $\Omega_e(z)$ is concave, we obtain that, for all $z > 0$, $\Omega_e(z)$ is differentiable with $\Omega'_e(z) = u'\left[y_e(z)\right]/p$. ∎

Differentiability of the value function

In this section we establish the differentiability of the value function and provide an explicit formula for the derivative. We assume $\beta R < 1$. The first preliminary result is:

Lemma 6 *Optimal real balances are bounded below by* $\underline{z} = \min\left\{R(w_e - \underline{c}_e), p\left(u'\right)^{-1}\left(\frac{p}{\beta\alpha R}\right)\right\}$.

Proof. A first-order condition for an optimal choice of target ($c \geq 0$ does not bind) of a worker in state e is:

$$-\frac{1}{R} + \beta\alpha\mathbb{E}_e\left\{\frac{u'\left[y_{e'}(z)\right]}{p} + \beta(1-\alpha)W'_{e'}\left(z^+\right)\right\} \leq 0.$$

Since the value function is increasing, this implies that $\beta\alpha\mathbb{E}_e\left\{u'\left[y_{e'}(z)\right]\right\}/p \leq 1/R$. Since $z \geq py_{e'}(z)$, we obtain that $\beta\alpha u'(z/p) \leq p/R$. Since $u'(z)$ is decreasing, this implies:

$$z \geq p\left(u'\right)^{-1}\left(\frac{p}{\beta\alpha R}\right) \text{ for all } z \in Z^{\star}_{t+1}.$$

The result then follows from the policy function for optimal real balances in (54). ∎

We now can state our differentiability result:

Proposition 7 *The value function is continuously differentiable, with:*

$$W'_e(z) = \mathbb{E}_{e_t}\left\{\sum_{i=1}^{+\infty}(\beta R)^i(1-\alpha)^{i-1}\alpha\frac{u'\left[y_{e_{t+i}}(z_{t+i})\right]}{p}\right\},$$

where z_{t+i} is a stochastic sequence of optimal real balances starting starting from $z_t = z$.

Proof. We first use the Envelope Theorem stated in Corollary 5 of Milgrom and Segal (2002), which applies to optimization problems with parameterized constraints. To see that all the conditions are satisfied, we first note that, given $R > 0$, there exists a single $\hat{z} \geq 0$ and a neighborhood of z such that, for all real balances in this neighborhood, $\hat{z}$ lies in the interior of the constraint set. Note as well that the objective function and the function defining the constraint are continuous and concave, and have partial derivatives with respect to z which are continuous in $(z, \hat{z})$. Let the Lagrangian associated with the optimization problem (53) is:

$$\begin{aligned}\mathcal{L}_e(z, \hat{z}, \lambda) \quad &= \quad z - \frac{\hat{z}}{R} + \beta \mathbb{E}\left[\alpha \Omega_{e'}(\hat{z}) + (1-\alpha) W_{e'}(\hat{z})\right] \\ &\quad\quad +\bar{\lambda}\left[R\left(z + w_e - \underline{c}_e\right) - \hat{z}\right] + \underline{\lambda}\left[\hat{z} - R\left(z + w_e - \bar{c}_e\right)\right],\end{aligned}$$

where $\lambda = (\bar{\lambda}, \underline{\lambda})$ denote the vector of Lagrange multipliers. Let $\Lambda^\star$ denote the set of Kuhn-Tucker multipliers and $\Xi^\star$ denote the set of optima associated with this optimization problem. These sets are non empty and compact under the stated conditions. Then by the above mentioned Envelope Theorem, we have:

$$W'_e(z^+) = \min_{\lambda \in \Lambda^\star} \max_{z' \in \Xi^\star} \frac{\partial \mathcal{L}_e}{\partial z}(z, \hat{z}, \lambda) = \min_{\lambda \in \Lambda^\star} \left\{1 + R\left(\bar{\lambda} - \underline{\lambda}\right)\right\}, \tag{55}$$

for all $z \geq 0$,

$$W'_e(z^-) = \max_{\lambda \in \Lambda^\star} \min_{z' \in \Xi^\star} \frac{\partial \mathcal{L}_e}{\partial z}(z, \hat{z}, \lambda) = \max_{\lambda \in \Lambda^\star} \left\{1 + R\left(\bar{\lambda} - \underline{\lambda}\right)\right\}, \tag{56}$$

for all $z > 0$. By taking derivative of the Lagrangian with respect to $\hat{z}$, we can obtain natural bounds for the Kuhn-Tucker multipliers entering the above expression for the left- and right-derivatives. Namely, fix some optimal real balances $\tilde{z} \in \Xi^\star$. Then, by Theorem 28.3 in Rockafellar (1970), we know that any Kuhn-Tucker multipliers $\lambda \in \Lambda^\star$ must satisfy:

$$\frac{\partial L}{\partial \hat{z}}(z, \tilde{z}^+, \lambda) \leq 0 \leq \frac{\partial L}{\partial \hat{z}}(z, \tilde{z}^-, \lambda).$$

Taking derivatives explicitly and rearranging the resulting first-order condition, we obtain that for any $\lambda \in \Lambda^\star$:

$$\begin{aligned}\bar{\lambda} - \underline{\lambda} \quad &\geq \quad -\frac{1}{R} + \beta \mathbb{E}_e\left\{\alpha \frac{u'\left[y_{e'}(\tilde{z})\right]}{p} + (1-\alpha) W'_{e'}(\tilde{z}^+)\right\} \\ \bar{\lambda} - \underline{\lambda} \quad &\leq \quad -\frac{1}{R} + \beta \mathbb{E}_e\left\{\alpha \frac{u'\left[y_{e'}(\tilde{z})\right]}{p} + (1-\alpha) W'_{e'}(\tilde{z}^-)\right\}.\end{aligned}$$

Plugging this back into the expression for $W_e'(z^+)$ and $W_e'(z^-)$, we obtain the inequalities:

$$\begin{aligned} W_e'(z^+) &\geq \beta R \mathbb{E}_e \left\{ \alpha \frac{u'\left[y_{e'}(\hat{z})\right]}{p} + (1-\alpha) W_{e'}'(\hat{z}^+) \right\}, \quad \forall z \geq 0, \\ W_e'(z^-) &\leq \beta R \mathbb{E}_e \left\{ \alpha \frac{u'\left[y_{e'}(\hat{z})\right]}{p} + (1-\alpha) W_{e'}'(\hat{z}^-) \right\}, \quad \forall z > 0. \end{aligned}$$

Iterating forward, we obtain:

$$W_{e_t}'(z^+) \geq \mathbb{E}_{e_t} \left\{ \sum_{i=1}^{I} (\beta R)^i (1-\alpha)^{i-1} \alpha \frac{u'\left[y_{e_{t+i}}(z_{t+i})\right]}{p} + (\beta R)^I (1-\alpha)^I W_{e_{t+I}}'(z_{t+I}^+) \right\},$$

for all $z \geq 0$, and

$$W_{e_t}'(z^-) \leq \mathbb{E}_{e_t} \left\{ \sum_{i=1}^{I} (\beta R)^i (1-\alpha)^{i-1} \alpha \frac{u'\left[y_{e_{t+i}}(z_{t+i})\right]}{p} + (\beta R)^I (1-\alpha)^I W_{e_{t+I}}'(z_{t+I}^-) \right\},$$

for all $z > 0$, where $\{z_{t+i}\}$ denote some stochastic sequence of optimal money holdings decisions generated by (54) starting starting from $z_t = z$ and function of a random employment history $\{e_{t+i}\}$. From Lemma 6, we know that optimal money holdings are bounded below by $\underline{z}$. This implies that consumption in the retail market is bounded below by $y_e(\underline{z})$ and the upper bounds $u'\left[y_e(z_{t+i})\right] \leq \|u\|/y_e(\underline{z})$ and $W_{t+I}'(z_{t+I}^{\pm}) \leq \|W\|/\xi z$. These upper bounds allow us to take limits as $I \to \infty$ in the above expressions, and we obtain:

$$W_t'(z^+) \geq \mathbb{E}_{e_t} \left\{ \sum_{i=1}^{+\infty} (\beta R)^i (1-\alpha)^{i-1} \alpha \frac{u'\left[y_{e_{t+i}}(z_{t+i})\right]}{p} \right\},$$

for all $z \geq 0$,

$$W_t'(z^-) \leq \mathbb{E}_{e_t} \left\{ \sum_{i=1}^{+\infty} (\beta R)^i (1-\alpha)^{i-1} \alpha \frac{u'\left[y_{e_{t+i}}(z_{t+i})\right]}{p} \right\},$$

for all $z > 0$. Given that $W_e'(z^+) \leq W_e'(z^-)$, this implies that $W_e(z)$ is differentiable at z, and that the derivative is as stated in the Proposition. The derivative is continuous by Theorem 24.1 in Rockafeller (1970).

■

Proof of Proposition 3

[TO CHECK] Given $(z_0^\star, z_1^\star)$ we define $Q((z,e),[0,z'] \times \{e'\})$ as the probability that a worker in state $e \in \{0,1\}$ with $z \in [0, z_e^\star]$ real balances at the start of period t ends up in state e' with less than z' at the start of period $t+1$. It satisfies:

$$
\begin{aligned}
Q((z,e'),[0,z']\times\{e'\}) &= 0 \text{ if } z' < \min\{R\left[z - y_{e'}(z) + w_{e'}\right], z^{\star}_{e'}\} \\
&= \alpha \Pr\left[e' \,|e\right] \text{ if } z' \geq \min\{R\left[z - y_{e'}(z) + w_{e'}\right], z^{\star}_{e'}\} \text{ and } z' < \min\{R\left(z + w_{e'}\right), z^{\star}_{e'}\} \\
&= \Pr\left[e' \,|e\right] \text{ if } z' \geq \min\{R\left[z - y_{e'}(z) + w_{e'}\right], z^{\star}_{e'}\}.
\end{aligned}
$$

Since both z and $z - y_e(z)$ are weakly increasing, the transition probability is monotone. Moreover, it satisfies the Feller property since z and $z - y_e(z)$ are continuous. Existence and uniqueness of a stationary distribution, $H(z,e)$, follow from Theorem 12.10 in Stokey and Lucas (1989). The distribution F satisfies $F_1(z) = (1-\delta)H(z,1) + fH(z,0)$. Similarly, $F_0(z) = (1-f)H(z,0) + \delta H(z,1)$.

Appendix: Microfoundations for entrepreneurs' rent in retail market

We assume a simple pricing mechanism in the retail goods market according to which entrepreneurs charge a constant markup over their cost. The markup or margin, $p > 1$, can be justified by the following moral hazard problem. In the second stage entrepreneurs can either sell genuine output or they can produce fake output at no cost. The consumer cannot distinguish fake from genuine output before consuming the goods. However, if the output is fake, the consumer has recourse against the producer and can recover a fraction $1/p$ of his payment. An entrepreneur who is asked to produce y in exchange for d real dollars has incentive to produce high quality if $-y+d \geq (p-1)d/p$, where the right side is the payoff from producing fake output, i.e., there is no production cost but only a fraction $1-p^{-1}$ of the payment is kept. The incentive constraint can be rewritten as $d \geq py$.

Alternatively, if $\alpha/n < 1$, we could think of agents as trading in pairwise, like in the New-Monetarist literature. Tthere are many ways to determine terms of trade in decentralized goods markets with pairwise meetings: axiomatic bargaining solutions, sequential bargaining games, posting, auctions, and so on. Here we choose a pricing mechanism that is a small departure from take-it-or-leave-it offer bargaining games (they are equivalent when $p = 1$) but give some market power to sellers ($p > 1$). The mechanism is individually rational and it satisfies the pairwise core requirement for a class of equilibria we will characterize in closed form.

Appendix: SCF data description and measuring the residual liquid wealth distribution

Appendix: Figures and Tables

Table 2: Measuring the residual distribution of liquid wealth to earnings, SCF 2004.

	(1) ln(liquid wealth/income)
Normal Income ($1,000)	0.007***
	(0.001)
Real Estate Equity ($1,000)	0.003**
	(0.002)
Vehicle Equity ($1,000)	0.002
	(0.002)
Male	0.247**
	(0.120)
Age	0.038***
	(0.004)
< High School	-1.464***
	(0.185)
High School	-0.609***
	(0.113)
Some College	-0.392***
	(0.118)
Black	-0.835***
	(0.139)
Hispanic	-0.570***
	(0.160)
Observations	4,519
Industry & Occupation FE	YES

Note: Significance levels: *** $p < 0.01$, ** $p < 0.05$, *$p < 0.1$. Observations are weighted using SCF 2004 probability weights. The sample consists of all primary economic units (PEUs) that are between the ages of 30 and 65. Dependent and independent variables have been Windsorized at the 95% level.

Figure 3: Distribution of Liquid Wealth to Labor Income, 2004 SCF

Figure 4: Distribution of Liquid Wealth to Labor Income, Model and Data